Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.4-e-x-^m-a+b-x^n-^p-c+d-x^n-^q

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June 28, 2021 Compiled on June 28, 2021 at 11:38pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [1081]. This is test number [27].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (1081)	% 0.00 (0)
Mathematica	% 100.00 (1081)	% 0.00 (0)
Maple	% 69.29 (749)	% 30.71 (332)
Maxima	% 36.17 (391)	% 63.83 (690)
Fricas	% 61.24 (662)	% 38.76 (419)
Sympy	% 35.15 (380)	% 64.85 (701)
Giac	% 49.49 (535)	% 50.51 (546)
Mupad	% 49.12 (531)	% 50.88 (550)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

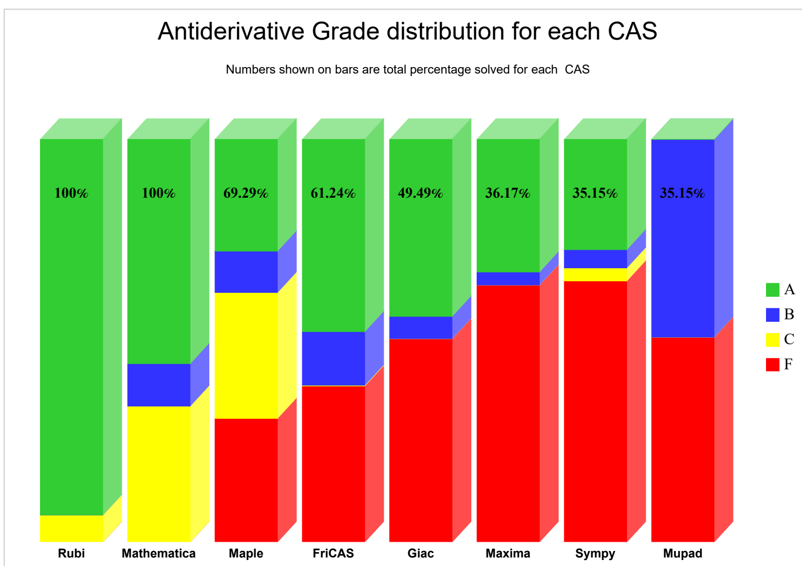
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

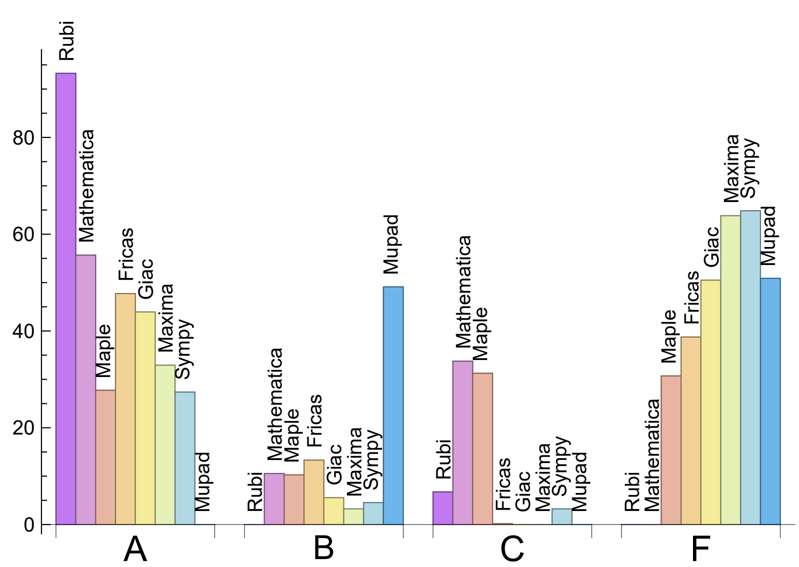
System	% A grade	% B grade	% C grade	% F grade
Rubi	93.25	0.00	6.75	0.00
Mathematica	55.69	10.55	33.77	0.00
Maple	27.75	10.27	31.27	30.71
Maxima	32.93	3.24	0.00	63.83
Fricas	47.73	13.32	0.19	38.76
Sympy	27.38	4.53	3.24	64.85
Giac	43.94	5.55	0.00	50.51
Mupad	0.00	49.12	0.00	50.88

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	332	100.00 %	0.00 %	0.00 %
Maxima	690	89.28 %	0.00 %	10.72 %
Fricas	419	58.71 %	39.86 %	1.43 %
Sympy	701	69.19 %	28.67 %	2.14 %
Giac	546	90.29 %	1.10 %	8.61 %
Mupad	550	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

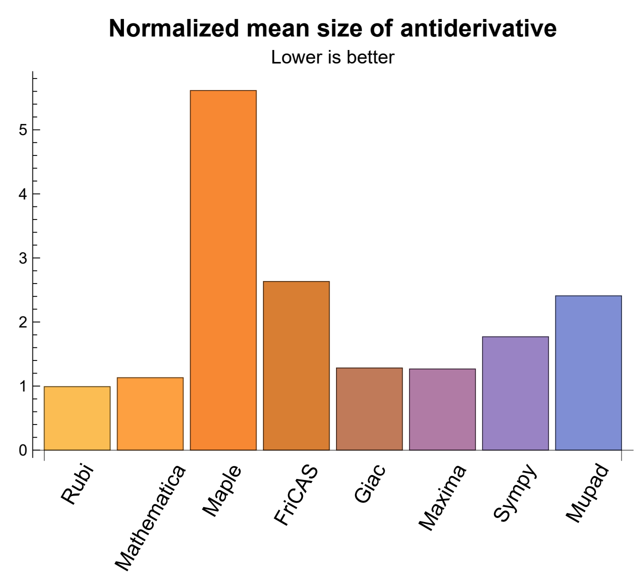
1.3 Performance

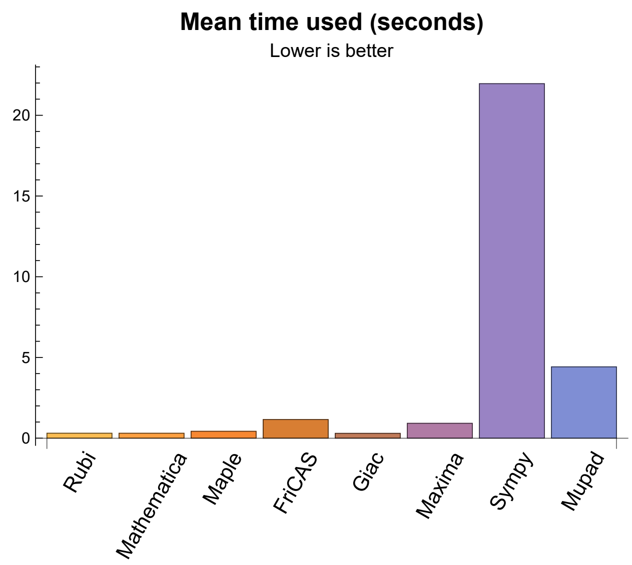
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.31	203.50	0.99	106.00	1.00
Mathematica	0.31	131.69	1.13	104.00	0.89
Maple	0.43	882.05	5.61	441.00	1.75
Maxima	0.92	119.81	1.27	97.00	1.02
Fricas	1.15	391.91	2.63	211.00	1.98
Sympy	21.96	180.66	1.77	107.00	1.04
Giac	0.30	151.79	1.28	114.00	1.04
Mupad	4.42	439.68	2.41	117.00	1.12

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {455, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 634, 635, 636, 637, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 754, 755, 756, 758, 759, 760, 795, 796, 797, 798, 799, 800, 816, 817, 818, 819, 820, 821}

Mathematica {264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 574, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 724, 725, 726, 727, 730, 731, 738, 739, 740, 741, 742, 743, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 813, 814, 815, 816, 817, 818, 819, 820, 821, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 850, 861, 862, 863, 864, 865, 866, 867, 868, 869, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1011, 1012}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

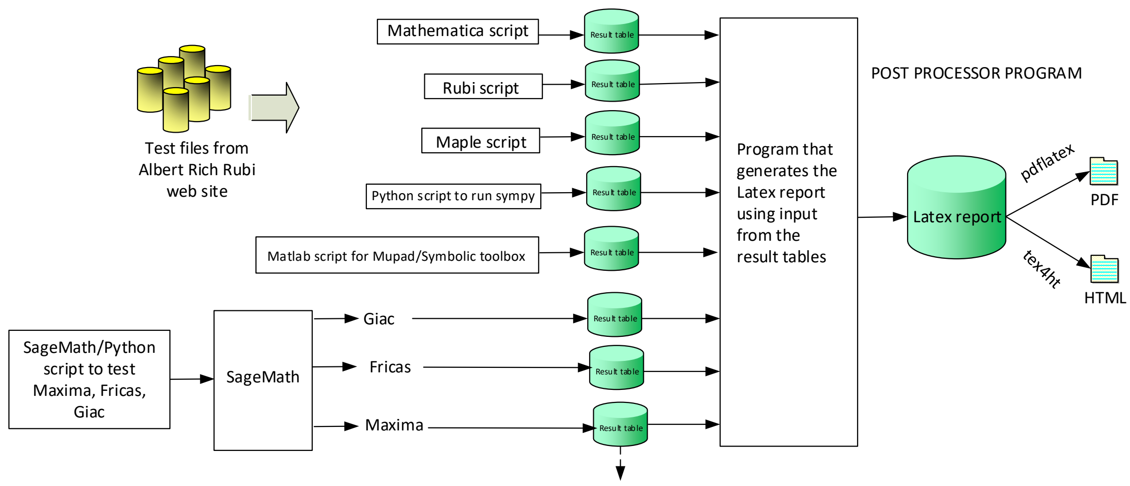
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form “{n,n,..}” which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 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673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 757, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081 }

B grade: { }

C grade: { 455, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 620, 621, 622, 634, 635, 636, 637, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 754, 755, 756, 758, 759, 760 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 244, 245, 246, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 278, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 321, 327, 358, 359, 360, 361, 362, 364, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 383, 384, 389, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 443, 460, 461, 462, 463, 464, 470, 471, 473, 474, 480, 481, 482, 483, 484, 490, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 597, 598, 599, 605, 606, 607, 608, 609, 610, 611, 614, 616, 617, 623, 624, 625, 626, 627, 628, 641, 650, 651, 658, 659, 660, 661, 662, 663, 664, 693, 696, 697, 698, 699, 700, 701, 723, 728, 729, 732, 733, 734, 735, 736, 737, 744, 760, 763, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 793, 794, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 832, 833, 841, 842, 843, 844, 846, 847, 848, 849, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 863, 864, 865, 870, 871, 872, 873, 874, 875, 876, 879, 887, 888, 889, 890, 891, 892, 893, 894, 903, 904, 908, 909, 910, 911, 912, 913, 914, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 945, 946, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 960, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1054, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078 }

B grade: { 30, 54, 267, 268, 269, 279, 280, 322, 323, 324, 338, 339, 340, 341, 363, 365, 366, 367, 373, 374, 375, 376, 377, 385, 386, 387, 393, 394, 395, 396, 397, 437, 438, 439, 440, 441, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 513, 514, 515, 672, 673, 674, 675, 676, 691, 692, 694, 695, 709, 710, 711, 712, 713, 727, 730, 731, 743, 745, 746, 761, 762, 764, 765, 766, 804, 845, 850, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 905, 906, 907, 924, 925, 926, 927, 928, 1014, 1051, 1052, 1053 }

C grade: { 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 275, 276, 277, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 325, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 388, 390, 391, 392, 405, 406, 407, 408, 409, 410, 414, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 442, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 472, 491, 492, 493, 494, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 600, 601, 602, 603, 604, 612, 613, 615, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 702, 703, 704, 705, 706, 707, 708, 714, 715, 716, 717, 718, 719, 720, 721, 722, 724, 725, 726, 738, 739, 740, 741, 742, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 792, 795, 796, 797, 798, 799, 800, 813, 815, 816, 817, 818, 819, 820, 821, 831, 834, 835, 836, 837, 838, 839, 840, 861, 862, 877, 878, 895, 896, 897, 898, 899, 900, 901, 902, 915, 916, 918, 919, 920, 921, 922, 923, 944, 947, 948, 959, 961, 984, 985, 1024, 1055, 1079, 1080, 1081 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 557, 562, 565, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 962, 963, 964, 965, 966, 967, 968, 969, 970, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1029, 1036, 1040, 1041, 1042, 1043, 1044, 1047, 1048, 1049, 1050, 1056, 1057, 1058, 1059, 1078, 1079, 1080 }

B grade: { 27, 30, 54, 124, 125, 126, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 932, 943, 944, 947, 948, 959, 960, 961, 971, 1005, 1014, 1023, 1039, 1045, 1046, 1051, 1052, 1053, 1054, 1060, 1061, 1062, 1063, 1065 }

C grade: { 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 605, 606, 607, 608, 609, 614, 615, 616, 617, 623, 624, 625, 626, 631, 632, 633, 634, 637, 638, 639, 640, 641, 642, 647, 648, 649, 650, 651, 795, 796, 797, 798, 799, 800, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 1064 }

F grade: { 127, 128, 129, 130, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 610, 611, 612, 613, 618, 619, 620, 621, 622, 627, 628, 629, 630, 635, 636, 643, 644, 645, 646, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 801, 802, 803, 804, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1024, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1055, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1081 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 228, 229, 230, 231, 244, 245, 246, 247, 248, 259, 260, 261, 270, 271, 272, 282, 283, 284, 285, 295, 296, 297, 298, 308, 309, 310, 311, 325, 326, 327, 328, 398, 399, 400, 401, 411, 412, 413, 414, 424, 425, 426, 427, 442, 443, 444, 445, 525, 567, 568, 569, 570, 586, 587, 588, 589, 605, 606, 607, 608, 609, 623, 624, 625, 626, 638, 639, 640, 641, 642, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 964, 965, 966, 967, 968, 969, 970, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1036, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1055, 1056, 1057, 1058, 1064, 1078, 1079, 1080, 1081 }

B grade: { 30, 54, 184, 217, 218, 232, 233, 929, 930, 943, 944, 945, 959, 960, 961, 962, 963, 971, 972, 973, 974, 984, 985, 1014, 1044, 1050, 1051, 1052, 1053, 1054, 1059, 1060, 1061, 1062, 1063 }

C grade: { }

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2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 161, 163, 166, 169, 171, 174, 177, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 390, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 464, 470, 471, 472, 473, 474, 481, 483, 484, 506, 509, 516, 519, 522, 525, 527, 530, 533, 535, 538, 541, 543, 546, 549, 551, 554, 557, 559, 562, 565, 567, 568, 569, 570, 572, 573, 574, 575, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 605, 606, 607, 608, 609, 611, 612, 623, 624, 625, 626, 627, 628, 629, 630, 634, 637, 638, 639, 640, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 665, 667, 682, 683, 696, 697, 698, 699, 700, 701, 702, 703, 714, 715, 719, 720, 721, 722, 751, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 787, 788, 789, 790, 791, 792, 793, 794, 805, 806, 807, 808, 809, 810, 811, 814, 822, 824, 826, 827, 828, 829, 833, 853, 854, 855, 856, 857, 858, 859, 862, 871, 873, 874, 875, 879, 887, 888, 889, 890, 891, 892, 893, 896, 909, 911, 912, 913, 917, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1029, 1039, 1040, 1041, 1042, 1043, 1045, 1046, 1047, 1048, 1049, 1056, 1057, 1058, 1059, 1064, 1066, 1067, 1068, 1069, 1071, 1072, 1073, 1074, 1075, 1078, 1079, 1080, 1081 }

B grade: { 27, 30, 54, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 124, 125, 126, 156, 157, 159, 160, 162, 164, 165, 167, 168, 170, 172, 173, 175, 176, 178, 276, 281, 317, 356, 357, 388, 389, 391, 392, 463, 480, 482, 490, 491, 492, 493, 494, 507, 508, 571, 576, 614, 615, 616, 617, 620, 631, 632, 633, 641, 645, 646, 647, 648, 649, 650, 651, 666, 677, 678, 679, 680, 681, 684, 685, 686, 716, 717, 718, 732, 733, 734, 735, 736, 737, 738, 739, 747, 748, 749, 750, 752, 753, 754, 755, 779, 780, 781, 782, 783, 784, 785, 786, 812, 813, 823, 825, 830, 831, 832, 860, 861, 870, 872, 876, 877, 878, 894, 895, 908, 910, 914, 915, 916, 949, 975, 1014, 1036, 1044, 1050, 1051, 1052, 1053, 1054, 1055, 1060, 1061, 1062, 1063, 1065, 1070, 1076, 1077 }

C grade: { 610, 613 }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 536, 537, 539, 540, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 558, 560, 561, 563, 564, 566, 577, 578, 579, 580, 581, 582, 583, 584, 585, 596, 597, 598, 599, 600, 601, 602, 603, 604, 618, 619, 621, 622, 635, 636, 652, 653, 654, 655, 656, 657, 668, 669, 670, 671, 672, 673, 674, 675, 676, 687, 688, 689, 690, 691, 692, 693, 694, 695, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 723, 724, 725, 726, 727, 728, 729, 730, 731, 740, 741, 742, 743, 744, 745, 746, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 863, 864, 865, 866, 867, 868, 869, 880, 881, 882, 883, 884, 885, 886, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 114, 115, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 179, 180, 181, 182, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 250, 251, 255, 256, 259, 260, 261, 262, 271, 272, 273, 282, 283, 284, 285, 286, 296, 297, 298, 299, 310, 311, 312, 327, 328, 329, 358, 359, 360, 361, 368, 369, 370, 371, 380, 381, 390, 391, 519, 522, 525, 543, 546, 549, 554, 787, 789, 791, 807, 808, 855, 856, 889, 890, 930, 931, 932, 933, 934, 935, 936, 941, 942, 943, 947, 948, 949, 958, 962, 963, 964, 965, 966, 967, 969, 970, 971, 972, 973, 975, 976, 977, 978, 979, 982, 985, 1023, 1029, 1039, 1040, 1041, 1047, 1056, 1057, 1058, 1059, 1078 }

B grade: { 27, 30, 110, 111, 113, 184, 201, 218, 232, 516, 530, 533, 538, 541, 769, 770, 774, 777, 929, 937, 938, 939, 940, 944, 945, 950, 951, 952, 953, 954, 955, 956, 957, 959, 960, 961, 968, 974, 980, 981, 983, 984, 1024, 1051, 1052, 1053, 1060, 1061, 1062 }

C grade: { 127, 500, 501, 502, 503, 504, 517, 518, 520, 521, 523, 524, 526, 528, 529, 531, 532, 534, 537, 539, 540, 542, 544, 545, 547, 548, 550, 555, 556, 841, 842, 843, 848, 849, 1013 }

F grade: { 50, 51, 52, 53, 54, 55, 107, 108, 109, 116, 117, 118, 119, 120, 121, 122, 123, 128, 129, 130, 155, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 248, 249, 252, 253, 254, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 274, 275, 276, 277, 278, 279, 280, 281, 287, 288, 289, 290, 291, 292, 293, 294, 295, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 362, 363, 364, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 379, 382, 383, 384, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 527, 535, 536, 551, 552, 553, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 771, 772, 773, 775, 776, 778, 779, 780, 781, 782, 783, 784, 785, 786, 788, 790, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 844, 845, 846, 847, 850, 851, 852, 853, 854, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 946, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1042, 1043, 1044, 1045, 1046, 1048, 1049, 1050, 1054, 1055, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1079, 1080, 1081 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 516, 519, 525, 543, 546, 549, 551, 554, 559, 562, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 681, 682, 683, 696, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 812, 813, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 860, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 894, 895, 908, 909, 910, 911, 912, 929, 930, 931, 937, 938, 939, 940, 942, 943, 944, 945, 946, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 976, 1002, 1003, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1023, 1024, 1051, 1056, 1057, 1058, 1060, 1061, 1062, 1064, 1078 }

B grade: { 27, 30, 54, 124, 125, 126, 508, 509, 527, 535, 679, 680, 697, 750, 751, 792, 794, 814, 828, 829, 830, 831, 832, 833, 859, 875, 877, 896, 913, 914, 915, 916, 917, 932, 933, 934, 935, 936, 941, 947, 948, 949, 950, 951, 952, 965, 966, 967, 1004, 1005, 1007, 1008, 1014, 1052, 1053, 1054, 1055, 1079, 1080, 1081 }

C grade: { }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 276, 277, 278, 279, 280, 281, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 517, 518, 520, 521, 522, 523, 524, 526, 528, 529, 530, 531, 532, 533, 534, 536, 537, 538, 539, 540, 541, 542, 544, 545, 547, 548, 550, 552, 553, 555, 556, 557, 558, 560, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 815, 816, 817, 818, 819, 820, 821, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 861, 862, 863, 864, 865, 866, 867, 868, 869, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 964, 968, 969, 970, 971, 972, 973, 974, 975, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1006, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1059, 1063, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 196, 197, 198, 199, 200, 201, 213, 214, 215, 216, 217, 218, 228, 229, 230, 231, 232, 233, 244, 245, 246, 247, 248, 259, 260, 261, 262, 263, 270, 271, 272, 273, 274, 276, 281, 282, 283, 284, 285, 286, 287, 288, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 317, 325, 326, 327, 328, 329, 330, 331, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 404, 411, 412, 413, 414, 415, 416, 417, 424, 425, 426, 427, 428, 429, 430, 442, 443, 444, 445, 446, 447, 448, 460, 461, 462, 463, 464, 470, 471, 472, 473, 474, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 506, 507, 508, 509, 557, 562, 565, 567, 568, 569, 570, 571, 572, 573, 586, 587, 588, 589, 590, 591, 592, 605, 606, 607, 608, 609, 610, 611, 623, 624, 625, 626, 627, 628, 638, 639, 640, 641, 642, 643, 644, 658, 659, 660, 661, 662, 663, 664, 677, 678, 679, 680, 681, 682, 683, 696, 697, 698, 699, 700, 701, 714, 715, 716, 717, 718, 719, 720, 732, 733, 734, 735, 736, 737, 747, 748, 749, 750, 751, 752, 753, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 789, 791, 793, 805, 806, 807, 808, 809, 822, 823, 824, 825, 826, 827, 853, 854, 855, 856, 857, 870, 871, 872, 873, 874, 887, 888, 889, 890, 891, 908, 909, 910, 911, 912, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 959, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1014, 1015, 1016, 1017, 1023, 1024, 1029, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1078, 1079, 1080, 1081 }

C grade: { }

F grade: { 127, 128, 129, 130, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 264, 265, 266, 267, 268, 269, 275, 277, 278, 279, 280, 289, 290, 291, 292, 293, 294, 302, 303, 304, 305, 306, 307, 315, 316, 318, 319, 320, 321, 322, 323, 324, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 405, 406, 407, 408, 409, 410, 418, 419, 420, 421, 422, 423, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 475, 476, 477, 478, 479, 485, 486, 487, 488, 489, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 558, 559, 560, 561, 563, 564, 566, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 629, 630, 631, 632, 633, 634, 635, 636, 637, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 738, 739, 740, 741, 742, 743, 744, 745, 746, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 788, 790, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 943, 944, 957, 958, 960, 961, 972, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1004, 1013, 1018, 1019, 1020, 1021, 1022, 1025, 1026, 1027, 1028, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	29	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85
time (sec)	N/A	0.033	0.009	0.037	0.565	0.712	0.066	0.179	0.200

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	29	29	29	28
normalized size	1	1.00	1.00	0.85	0.82	0.88	0.88	0.88	0.85
time (sec)	N/A	0.017	0.007	0.045	0.630	1.014	0.067	0.148	2.483

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.013	0.007	0.036	0.563	0.942	0.064	0.150	0.034

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	28	25	27	28	26
normalized size	1	1.00	1.00	0.97	0.97	0.86	0.93	0.97	0.90
time (sec)	N/A	0.021	0.009	0.038	0.571	0.913	0.113	0.150	0.035

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	27	29	26	29	28
normalized size	1	1.00	1.00	0.97	0.87	0.94	0.84	0.94	0.90
time (sec)	N/A	0.015	0.011	0.049	0.504	0.599	0.108	0.147	0.039

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	24	28	24	23	24
normalized size	1	1.00	1.00	0.86	0.86	1.00	0.86	0.82	0.86
time (sec)	N/A	0.016	0.009	0.044	0.478	0.914	0.115	0.159	2.337

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	28	30	26	40	25
normalized size	1	1.00	1.00	0.90	0.97	1.03	0.90	1.38	0.86
time (sec)	N/A	0.022	0.013	0.050	0.475	0.721	0.210	0.164	0.040

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	32	28	29	29	31	31	29
normalized size	1	1.00	1.03	0.90	0.94	0.94	1.00	1.00	0.94
time (sec)	N/A	0.017	0.013	0.046	0.632	0.798	0.250	0.184	0.033

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	25	27	29	29	29	28
normalized size	1	1.00	1.07	0.89	0.96	1.04	1.04	1.04	1.00
time (sec)	N/A	0.016	0.012	0.041	0.488	1.136	0.287	0.184	2.316

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	28	30	31	29	37	29
normalized size	1	1.00	1.07	0.97	1.03	1.07	1.00	1.28	1.00
time (sec)	N/A	0.021	0.018	0.044	0.509	0.944	0.542	0.164	0.052

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	52	51	53	54	53	51
normalized size	1	1.00	1.21	1.24	1.21	1.26	1.29	1.26	1.21
time (sec)	N/A	0.072	0.016	0.039	0.491	0.605	0.076	0.153	2.375

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	53	54	53	51
normalized size	1	1.00	1.00	0.95	0.93	0.96	0.98	0.96	0.93
time (sec)	N/A	0.034	0.008	0.046	0.608	0.829	0.076	0.149	0.043

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	51	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96
time (sec)	N/A	0.025	0.039	0.035	0.663	0.965	0.076	0.168	0.043

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	51	52	52	49	53	52	49
normalized size	1	1.00	1.11	1.13	1.13	1.07	1.15	1.13	1.07
time (sec)	N/A	0.033	0.038	0.042	0.538	1.001	0.145	0.149	0.038

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	51	53	49	52	50
normalized size	1	1.00	1.00	1.00	0.96	1.00	0.92	0.98	0.94
time (sec)	N/A	0.028	0.022	0.043	0.591	0.848	0.153	0.151	0.050

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	53	49	48	48
normalized size	1	1.00	1.00	0.98	0.96	1.06	0.98	0.96	0.96
time (sec)	N/A	0.027	0.016	0.045	0.487	0.957	0.152	0.169	0.046

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	51	52	54	51	69	49
normalized size	1	1.00	0.96	1.00	1.02	1.06	1.00	1.35	0.96
time (sec)	N/A	0.050	0.026	0.052	0.483	0.787	0.273	0.148	0.043

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	50	53	53	53	54	52
normalized size	1	1.00	0.96	0.94	1.00	1.00	1.00	1.02	0.98
time (sec)	N/A	0.031	0.017	0.049	0.724	0.925	0.310	0.149	0.049

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	46	51	53	53	51	50
normalized size	1	1.00	1.00	0.92	1.02	1.06	1.06	1.02	1.00
time (sec)	N/A	0.028	0.020	0.058	0.574	0.752	0.354	0.150	2.371

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	51	54	55	51	70	52
normalized size	1	1.00	1.00	1.00	1.06	1.08	1.00	1.37	1.02
time (sec)	N/A	0.038	0.022	0.067	0.666	0.999	0.756	0.207	2.359

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	48	54	53	58	56	53
normalized size	1	1.00	1.02	0.91	1.02	1.00	1.09	1.06	1.00
time (sec)	N/A	0.029	0.017	0.045	0.567	1.195	0.874	0.152	0.045

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	51	53	54	53	50
normalized size	1	1.00	1.00	0.90	1.02	1.06	1.08	1.06	1.00
time (sec)	N/A	0.029	0.031	0.046	0.493	1.000	0.986	0.148	2.342

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	125	136	125	107
normalized size	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91
time (sec)	N/A	0.098	0.022	0.038	0.460	0.815	0.096	0.160	0.050

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	107	124	119	125	136	125	107
normalized size	1	1.00	1.13	1.31	1.25	1.32	1.43	1.32	1.13
time (sec)	N/A	0.237	0.029	0.038	0.473	1.006	0.096	0.173	2.338

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	125	134	125	107
normalized size	1	1.00	1.00	1.06	1.02	1.07	1.15	1.07	0.91
time (sec)	N/A	0.075	0.020	0.042	0.475	0.956	0.096	0.153	0.041

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	125	136	125	107
normalized size	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91
time (sec)	N/A	0.068	0.020	0.033	0.588	0.951	0.096	0.152	0.041

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	107	124	119	125	138	125	107
normalized size	1	1.00	1.60	1.85	1.78	1.87	2.06	1.87	1.60
time (sec)	N/A	0.148	0.025	0.040	0.447	1.264	0.100	0.156	0.040

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	125	136	125	107
normalized size	1	1.00	1.00	1.06	1.02	1.07	1.16	1.07	0.91
time (sec)	N/A	0.070	0.021	0.042	0.644	0.958	0.098	0.227	0.041

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	124	133	124	107
normalized size	1	1.00	1.00	1.06	1.02	1.06	1.14	1.06	0.91
time (sec)	N/A	0.065	0.018	0.039	0.443	1.262	0.096	0.156	0.040

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	107	124	119	125	136	125	107
normalized size	1	1.00	2.55	2.95	2.83	2.98	3.24	2.98	2.55
time (sec)	N/A	0.069	0.030	0.042	0.443	1.168	0.095	0.173	0.041

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	124	119	124	134	124	106
normalized size	1	1.00	1.00	1.06	1.02	1.06	1.15	1.06	0.91
time (sec)	N/A	0.063	0.018	0.043	0.646	1.104	0.094	0.153	0.040

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	121	115	120	128	120	103
normalized size	1	1.00	1.00	1.11	1.06	1.10	1.17	1.10	0.94
time (sec)	N/A	0.055	0.017	0.039	0.599	0.847	0.093	0.169	0.041

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	124	120	117	134	124	105
normalized size	1	1.00	1.28	1.41	1.36	1.33	1.52	1.41	1.19
time (sec)	N/A	0.067	0.029	0.042	0.449	1.070	0.242	0.155	0.046

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	125	118	121	129	124	106
normalized size	1	1.00	1.00	1.12	1.05	1.08	1.15	1.11	0.95
time (sec)	N/A	0.061	0.035	0.049	0.539	1.052	0.248	0.151	0.042

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	112	120	116	121	128	119	104
normalized size	1	1.00	1.00	1.07	1.04	1.08	1.14	1.06	0.93
time (sec)	N/A	0.061	0.032	0.040	0.607	0.681	0.244	0.155	0.043

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	120	123	133	143	105
normalized size	1	1.00	1.02	1.09	1.06	1.09	1.18	1.27	0.93
time (sec)	N/A	0.117	0.045	0.054	0.553	1.037	0.368	0.158	2.348

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	115	123	121	121	133	127	109
normalized size	1	1.00	1.02	1.09	1.07	1.07	1.18	1.12	0.96
time (sec)	N/A	0.067	0.063	0.047	0.489	0.833	0.422	0.157	0.042

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	119	120	121	133	124	108
normalized size	1	1.00	1.00	1.05	1.06	1.07	1.18	1.10	0.96
time (sec)	N/A	0.063	0.057	0.043	0.733	1.171	0.478	0.179	0.043

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	124	122	123	131	148	113
normalized size	1	1.00	0.93	1.09	1.07	1.08	1.15	1.30	0.99
time (sec)	N/A	0.106	0.057	0.049	0.649	1.125	0.941	0.156	0.049

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	117	121	121	129	127	113
normalized size	1	1.00	1.00	1.06	1.10	1.10	1.17	1.15	1.03
time (sec)	N/A	0.067	0.038	0.047	0.513	0.812	1.118	0.173	2.341

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	113	114	120	121	133	124	111
normalized size	1	1.00	1.00	1.01	1.06	1.07	1.18	1.10	0.98
time (sec)	N/A	0.062	0.038	0.044	0.593	1.534	1.211	0.162	0.044

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	106	124	123	123	129	150	118
normalized size	1	1.00	0.93	1.09	1.08	1.08	1.13	1.32	1.04
time (sec)	N/A	0.104	0.055	0.049	0.469	0.894	2.244	0.154	0.050

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	111	122	121	131	127	118
normalized size	1	1.00	1.03	0.97	1.06	1.05	1.14	1.10	1.03
time (sec)	N/A	0.063	0.024	0.053	0.465	0.890	2.931	0.154	2.364

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	109	108	120	121	131	124	116
normalized size	1	1.00	1.00	0.99	1.10	1.11	1.20	1.14	1.06
time (sec)	N/A	0.064	0.042	0.054	0.534	0.957	3.727	0.183	0.067

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	118	124	123	123	129	149	122
normalized size	1	1.00	1.04	1.09	1.08	1.08	1.13	1.31	1.07
time (sec)	N/A	0.099	0.048	0.051	0.634	1.117	6.644	0.157	0.064

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	117	107	122	121	134	128	123
normalized size	1	1.00	1.02	0.93	1.06	1.05	1.17	1.11	1.07
time (sec)	N/A	0.062	0.051	0.058	0.558	1.099	10.765	0.159	2.375

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	102	119	121	129	123	120
normalized size	1	1.00	1.00	0.93	1.08	1.10	1.17	1.12	1.09
time (sec)	N/A	0.068	0.071	0.052	0.614	1.313	22.983	0.163	2.392

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	116	123	123	123	129	145	121
normalized size	1	1.00	1.03	1.09	1.09	1.09	1.14	1.28	1.07
time (sec)	N/A	0.091	0.068	0.049	0.586	1.223	29.301	0.155	0.079

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	104	122	121	134	128	121
normalized size	1	1.00	1.03	0.90	1.06	1.05	1.17	1.11	1.05
time (sec)	N/A	0.062	0.033	0.043	0.498	1.362	66.092	0.160	2.363

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	101	119	121	0	125	119
normalized size	1	1.00	1.00	0.92	1.08	1.10	0.00	1.14	1.08
time (sec)	N/A	0.061	0.053	0.048	0.518	1.277	0.000	0.171	0.077

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	121	124	123	123	0	136	121
normalized size	1	1.00	1.33	1.36	1.35	1.35	0.00	1.49	1.33
time (sec)	N/A	0.054	0.045	0.047	0.508	1.394	0.000	0.158	0.093

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	119	104	121	121	0	127	119
normalized size	1	1.00	1.05	0.92	1.07	1.07	0.00	1.12	1.05
time (sec)	N/A	0.064	0.032	0.043	0.460	1.253	0.000	0.153	2.369

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	121	104	121	121	0	127	121
normalized size	1	1.00	1.03	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.059	0.037	0.048	0.552	0.859	0.000	0.153	2.347

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	118	104	121	121	0	127	122
normalized size	1	1.00	2.46	2.17	2.52	2.52	0.00	2.65	2.54
time (sec)	N/A	0.032	0.036	0.046	0.612	0.822	0.000	0.183	2.370

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	104	121	121	0	127	121
normalized size	1	1.00	1.00	0.89	1.03	1.03	0.00	1.09	1.03
time (sec)	N/A	0.065	0.050	0.049	0.697	0.849	0.000	0.165	0.064

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	171	249	182	167	114	217	164
normalized size	1	1.00	0.93	1.36	0.99	0.91	0.62	1.19	0.90
time (sec)	N/A	0.149	0.135	0.044	1.431	0.833	1.372	0.237	0.268

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	47	62	50	51	46	52	52
normalized size	1	1.00	0.87	1.15	0.93	0.94	0.85	0.96	0.96
time (sec)	N/A	0.057	0.022	0.041	0.499	0.956	0.960	0.170	0.079

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	154	226	157	162	114	207	144
normalized size	1	1.00	0.92	1.35	0.94	0.97	0.68	1.24	0.86
time (sec)	N/A	0.120	0.093	0.045	1.096	0.945	0.872	0.186	2.545

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	152	221	154	145	87	186	162
normalized size	1	1.00	0.94	1.36	0.95	0.90	0.54	1.15	1.00
time (sec)	N/A	0.116	0.093	0.043	1.084	1.033	1.147	0.170	2.610

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	40	31	30	27	32	31
normalized size	1	1.00	0.89	1.14	0.89	0.86	0.77	0.91	0.89
time (sec)	N/A	0.033	0.029	0.040	0.540	0.977	0.883	0.217	0.063

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	152	198	131	382	92	161	126
normalized size	1	1.00	1.01	1.32	0.87	2.55	0.61	1.07	0.84
time (sec)	N/A	0.090	0.057	0.041	1.144	1.038	1.106	0.189	2.566

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	128	369	71	133	123
normalized size	1	1.00	0.89	1.34	0.88	2.54	0.49	0.92	0.85
time (sec)	N/A	0.077	0.089	0.045	1.157	1.094	1.016	0.176	2.541

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	37	35	32	26	34	36
normalized size	1	1.00	1.00	1.09	1.03	0.94	0.76	1.00	1.06
time (sec)	N/A	0.033	0.014	0.050	0.590	0.934	2.116	0.193	0.105

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	134	195	140	372	90	155	126
normalized size	1	1.00	0.91	1.33	0.95	2.53	0.61	1.05	0.86
time (sec)	N/A	0.086	0.094	0.057	1.157	0.981	0.940	0.223	2.545

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	135	195	140	411	73	161	126
normalized size	1	1.00	0.91	1.31	0.94	2.76	0.49	1.08	0.85
time (sec)	N/A	0.093	0.103	0.048	1.223	1.080	1.232	0.176	0.244

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	49	56	48	47	41	69	46
normalized size	1	1.00	0.98	1.12	0.96	0.94	0.82	1.38	0.92
time (sec)	N/A	0.049	0.022	0.048	0.483	0.919	2.360	0.165	2.405

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	154	216	147	158	112	197	178
normalized size	1	1.00	0.93	1.31	0.89	0.96	0.68	1.19	1.08
time (sec)	N/A	0.115	0.141	0.046	1.137	0.964	0.961	0.225	2.591

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	154	217	148	176	99	176	145
normalized size	1	1.00	0.92	1.29	0.88	1.05	0.59	1.05	0.86
time (sec)	N/A	0.119	0.134	0.048	1.281	1.032	0.932	0.176	2.564

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	81	70	73	61	99	70
normalized size	1	1.00	1.01	1.17	1.01	1.06	0.88	1.43	1.01
time (sec)	N/A	0.064	0.033	0.050	0.492	0.942	2.688	0.186	0.127

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	173	247	178	180	139	216	161
normalized size	1	1.00	0.94	1.34	0.97	0.98	0.76	1.17	0.88
time (sec)	N/A	0.134	0.144	0.057	1.289	0.927	1.135	0.183	2.581

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	203	288	218	271	156	244	209
normalized size	1	1.00	0.87	1.24	0.94	1.16	0.67	1.05	0.90
time (sec)	N/A	0.138	0.169	0.057	1.233	1.066	2.116	0.201	2.620

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	97	82	121	82	106	86
normalized size	1	1.00	0.88	1.18	1.00	1.48	1.00	1.29	1.05
time (sec)	N/A	0.091	0.101	0.053	0.492	0.594	2.143	0.183	0.083

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	266	192	257	151	236	179
normalized size	1	1.00	0.86	1.24	0.89	1.20	0.70	1.10	0.83
time (sec)	N/A	0.140	0.169	0.048	1.102	0.883	2.120	0.183	0.268

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	181	257	187	240	126	211	193
normalized size	1	1.00	0.85	1.21	0.88	1.13	0.59	0.99	0.91
time (sec)	N/A	0.130	0.150	0.086	1.012	0.980	2.536	0.180	2.621

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	74	60	81	56	91	62
normalized size	1	1.00	0.83	1.23	1.00	1.35	0.93	1.52	1.03
time (sec)	N/A	0.058	0.063	0.052	0.458	0.931	1.608	0.182	0.081

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	165	235	162	578	126	189	158
normalized size	1	1.00	0.84	1.20	0.83	2.95	0.64	0.96	0.81
time (sec)	N/A	0.110	0.140	0.080	1.171	0.775	2.093	0.208	2.578

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	160	228	157	573	102	166	150
normalized size	1	1.00	0.84	1.20	0.83	3.02	0.54	0.87	0.79
time (sec)	N/A	0.106	0.170	0.044	1.216	1.011	1.668	0.185	2.577

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	40	44	36	65	37
normalized size	1	1.00	1.00	1.15	0.98	1.07	0.88	1.59	0.90
time (sec)	N/A	0.038	0.032	0.055	0.582	0.870	1.225	0.184	2.348

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	146	223	160	548	117	186	145
normalized size	1	1.00	0.85	1.30	0.94	3.20	0.68	1.09	0.85
time (sec)	N/A	0.088	0.134	0.049	1.210	0.877	1.482	0.181	0.250

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	158	537	97	160	143
normalized size	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.086	0.139	0.057	1.102	0.909	1.434	0.177	2.550

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	53	51	70	46	61	47
normalized size	1	1.00	0.90	1.04	1.00	1.37	0.90	1.20	0.92
time (sec)	N/A	0.046	0.034	0.054	0.483	0.829	1.162	0.214	0.141

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	196	164	241	166	570	122	180	156
normalized size	1	1.01	0.84	1.24	0.85	2.92	0.63	0.92	0.80
time (sec)	N/A	0.106	0.144	0.052	1.078	0.953	1.417	0.183	2.569

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	163	237	172	618	109	188	159
normalized size	1	1.00	0.83	1.21	0.88	3.15	0.56	0.96	0.81
time (sec)	N/A	0.103	0.156	0.048	1.232	0.927	1.927	0.174	2.565

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	87	76	118	70	80	78
normalized size	1	1.00	0.84	1.14	1.00	1.55	0.92	1.05	1.03
time (sec)	N/A	0.076	0.072	0.060	0.504	0.959	1.444	0.189	2.432

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	185	257	186	259	153	231	209
normalized size	1	1.00	0.86	1.20	0.87	1.20	0.71	1.07	0.97
time (sec)	N/A	0.127	0.238	0.057	1.421	0.884	2.324	0.221	2.615

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	183	252	186	277	138	206	176
normalized size	1	1.00	0.85	1.17	0.87	1.29	0.64	0.96	0.82
time (sec)	N/A	0.127	0.180	0.053	1.319	0.924	1.843	0.197	2.574

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	85	116	106	154	100	149	100
normalized size	1	1.00	0.88	1.20	1.09	1.59	1.03	1.54	1.03
time (sec)	N/A	0.099	0.115	0.075	0.652	0.887	2.397	0.165	0.144

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	94	134	115	179	112	131	117
normalized size	1	1.00	0.88	1.25	1.07	1.67	1.05	1.22	1.09
time (sec)	N/A	0.140	0.072	0.054	0.538	0.859	4.941	0.183	0.097

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	92	110	94	142	94	93	94
normalized size	1	1.00	1.05	1.25	1.07	1.61	1.07	1.06	1.07
time (sec)	N/A	0.088	0.040	0.048	0.524	0.920	3.945	0.180	2.402

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	81	72	89	70	61	70
normalized size	1	1.00	0.97	1.23	1.09	1.35	1.06	0.92	1.06
time (sec)	N/A	0.068	0.025	0.051	0.474	0.870	3.692	0.188	2.385

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	39	42	42	42	28	44
normalized size	1	1.00	0.94	1.22	1.31	1.31	1.31	0.88	1.38
time (sec)	N/A	0.022	0.015	0.053	0.457	0.862	1.815	0.183	2.332

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	59	68	77	119	75	74	71
normalized size	1	1.00	0.87	1.00	1.13	1.75	1.10	1.09	1.04
time (sec)	N/A	0.058	0.051	0.059	0.454	0.900	1.481	0.203	0.161

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	87	117	109	197	107	136	107
normalized size	1	1.00	0.86	1.16	1.08	1.95	1.06	1.35	1.06
time (sec)	N/A	0.100	0.072	0.060	0.504	0.865	3.272	0.195	2.458

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	147	136	229	133	131	130
normalized size	1	1.00	0.89	1.20	1.11	1.88	1.09	1.07	1.07
time (sec)	N/A	0.130	0.110	0.058	0.519	0.913	3.553	0.200	0.153

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	216	308	228	364	192	259	213
normalized size	1	1.00	0.88	1.25	0.93	1.48	0.78	1.05	0.87
time (sec)	N/A	0.156	0.323	0.061	1.366	0.847	6.239	0.330	2.582

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	210	299	223	347	163	234	227
normalized size	1	1.00	0.86	1.23	0.91	1.42	0.67	0.96	0.93
time (sec)	N/A	0.165	0.244	0.056	1.237	0.878	4.009	0.238	0.320

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	194	275	196	792	162	210	187
normalized size	1	1.00	0.87	1.24	0.88	3.57	0.73	0.95	0.84
time (sec)	N/A	0.140	0.237	0.062	1.322	0.677	5.854	0.211	2.558

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	188	268	191	789	141	187	183
normalized size	1	1.00	0.85	1.22	0.87	3.59	0.64	0.85	0.83
time (sec)	N/A	0.126	0.273	0.054	1.361	0.875	3.392	0.196	2.603

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	181	241	195	756	155	206	175
normalized size	1	1.00	0.90	1.20	0.97	3.76	0.77	1.02	0.87
time (sec)	N/A	0.107	0.290	0.052	1.266	0.987	4.749	0.199	0.267

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	178	239	193	743	136	187	173
normalized size	1	1.00	0.89	1.20	0.97	3.73	0.68	0.94	0.87
time (sec)	N/A	0.113	0.244	0.049	1.253	1.014	3.253	0.192	2.559

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	178	251	195	752	153	207	175
normalized size	1	1.00	0.89	1.25	0.97	3.74	0.76	1.03	0.87
time (sec)	N/A	0.115	0.241	0.053	1.399	0.985	2.032	0.215	0.268

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	192	743	133	180	173
normalized size	1	1.00	0.89	1.26	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.103	0.185	0.056	1.370	1.006	1.513	0.195	0.256

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	193	281	199	776	162	204	185
normalized size	1	1.00	0.85	1.24	0.88	3.42	0.71	0.90	0.81
time (sec)	N/A	0.123	0.280	0.063	1.409	0.949	1.902	0.196	2.598

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	189	277	201	812	143	209	188
normalized size	1	1.00	0.83	1.22	0.89	3.58	0.63	0.92	0.83
time (sec)	N/A	0.131	0.261	0.054	1.198	1.056	1.731	0.191	2.583

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	214	299	221	366	189	254	240
normalized size	1	1.00	0.87	1.22	0.90	1.49	0.77	1.03	0.98
time (sec)	N/A	0.158	0.274	0.056	1.419	0.984	1.905	0.198	2.637

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	210	295	221	384	173	229	207
normalized size	1	1.00	0.85	1.20	0.90	1.56	0.70	0.93	0.84
time (sec)	N/A	0.142	0.317	0.062	1.465	1.012	2.176	0.193	2.584

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
normalized size	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.066	0.067	0.047	0.585	1.053	0.000	0.184	2.844

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	242	269	324	273	0	311	1751
normalized size	1	1.00	0.80	0.89	1.08	0.91	0.00	1.03	5.82
time (sec)	N/A	0.310	0.240	0.050	1.271	1.026	0.000	0.214	11.360

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	238	266	349	228	0	308	873
normalized size	1	1.00	0.80	0.90	1.18	0.77	0.00	1.04	2.95
time (sec)	N/A	0.267	0.165	0.052	1.355	1.009	0.000	0.212	1.829

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	49	42	144	51	51
normalized size	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.050	0.061	0.051	0.463	0.968	6.748	0.186	0.313

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	246	289	244	573	286	1364
normalized size	1	1.00	0.78	0.85	1.00	0.85	1.99	0.99	4.74
time (sec)	N/A	0.154	0.127	0.049	1.272	0.981	123.979	0.246	9.046

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	246	317	199	342	278	1265
normalized size	1	1.00	0.78	0.85	1.10	0.69	1.19	0.97	4.39
time (sec)	N/A	0.148	0.124	0.054	1.206	0.943	20.201	0.204	8.117

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	41	31	138	51	602
normalized size	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	13.38
time (sec)	N/A	0.031	0.020	0.046	0.483	0.853	2.474	0.183	0.257

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	265	201	515	290	982
normalized size	1	1.00	0.78	0.77	0.92	0.70	1.79	1.01	3.41
time (sec)	N/A	0.143	0.144	0.047	1.287	0.931	14.506	0.247	5.415

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	293	254	447	278	1364
normalized size	1	1.00	0.78	0.77	1.02	0.88	1.55	0.97	4.74
time (sec)	N/A	0.142	0.182	0.050	1.343	1.103	133.334	0.213	9.007

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	61	54	0	71	58
normalized size	1	1.00	0.87	0.95	0.98	0.87	0.00	1.15	0.94
time (sec)	N/A	0.060	0.050	0.049	0.543	1.677	0.000	0.184	2.840

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	244	257	300	238	0	305	716
normalized size	1	1.00	0.82	0.86	1.00	0.80	0.00	1.02	2.39
time (sec)	N/A	0.272	0.213	0.059	1.200	0.980	0.000	0.252	3.854

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	259	257	328	301	0	309	1829
normalized size	1	1.00	0.86	0.85	1.09	1.00	0.00	1.03	6.08
time (sec)	N/A	0.255	0.224	0.052	1.136	3.350	0.000	0.206	11.834

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	87	99	0	111	87
normalized size	1	1.00	1.01	1.00	1.00	1.14	0.00	1.28	1.00
time (sec)	N/A	0.092	0.106	0.060	0.485	5.069	0.000	0.193	3.223

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	282	291	341	305	0	328	1734
normalized size	1	1.00	0.89	0.92	1.07	0.96	0.00	1.03	5.45
time (sec)	N/A	0.380	0.265	0.052	1.126	3.280	0.000	0.243	11.370

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	282	293	369	356	0	336	1860
normalized size	1	1.00	0.88	0.91	1.15	1.11	0.00	1.05	5.79
time (sec)	N/A	0.456	0.221	0.054	1.231	1.024	0.000	0.206	11.573

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	119	124	117	127	0	165	118
normalized size	1	1.00	1.00	1.04	0.98	1.07	0.00	1.39	0.99
time (sec)	N/A	0.129	0.063	0.056	0.520	12.870	0.000	0.261	3.212

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	304	334	376	332	0	377	1814
normalized size	1	1.00	0.86	0.95	1.07	0.94	0.00	1.07	5.15
time (sec)	N/A	0.501	0.245	0.053	1.160	1.372	0.000	0.215	11.909

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	137	1078	205	851	5418	1331	559
normalized size	1	1.00	0.93	7.28	1.39	5.75	36.61	8.99	3.78
time (sec)	N/A	0.099	0.264	0.052	0.518	0.879	23.936	0.285	3.213

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	262	91	215	1057	332	177
normalized size	1	1.00	0.93	3.69	1.28	3.03	14.89	4.68	2.49
time (sec)	N/A	0.040	0.058	0.049	0.495	0.942	6.312	0.201	2.718

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	42	110	53	92	410	143	95
normalized size	1	1.00	0.93	2.44	1.18	2.04	9.11	3.18	2.11
time (sec)	N/A	0.021	0.047	0.043	0.613	1.071	2.891	0.166	2.655

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	190	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	2.88	0.00	-0.02
time (sec)	N/A	0.035	0.071	0.427	0.000	0.758	22.061	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.081	0.416	0.000	0.706	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	80	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.082	0.426	0.000	0.758	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	86	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.092	0.616	0.000	0.926	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.015	0.017	0.046	0.448	0.778	20.862	0.168	0.053

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.016	0.016	0.045	0.465	0.609	12.714	0.155	2.560

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	32	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.82	1.18	0.74	0.79
time (sec)	N/A	0.015	0.015	0.046	0.449	0.561	6.742	0.178	0.044

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	32	27	30	46	29	31
normalized size	1	1.00	0.85	0.82	0.69	0.77	1.18	0.74	0.79
time (sec)	N/A	0.016	0.023	0.045	0.467	0.837	3.370	0.150	0.041

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	33	32	27	29	44	29	31
normalized size	1	1.00	0.89	0.86	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.015	0.070	0.045	0.505	0.799	2.299	0.153	2.590

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	35	32	27	29	44	29	31
normalized size	1	1.00	0.95	0.86	0.73	0.78	1.19	0.78	0.84
time (sec)	N/A	0.015	0.011	0.044	0.461	0.722	2.772	0.149	2.599

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	34	32	27	28	46	29	31
normalized size	1	1.00	0.87	0.82	0.69	0.72	1.18	0.74	0.79
time (sec)	N/A	0.015	0.013	0.042	0.482	0.850	3.736	0.157	0.040

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	32	27	29	42	29	30
normalized size	1	1.00	0.97	0.86	0.73	0.78	1.14	0.78	0.81
time (sec)	N/A	0.016	0.012	0.041	0.453	0.703	4.205	0.261	0.042

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.031	0.064	0.049	0.558	0.816	47.810	0.153	2.567

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	51	56	80	53	51
normalized size	1	1.00	1.00	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.031	0.046	0.043	0.544	0.857	29.392	0.175	0.047

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	56	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.89	1.27	0.84	0.81
time (sec)	N/A	0.030	0.060	0.045	0.499	0.888	21.960	0.163	0.047

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	56	51	54	80	53	51
normalized size	1	1.00	0.84	0.89	0.81	0.86	1.27	0.84	0.81
time (sec)	N/A	0.029	0.110	0.043	0.537	0.882	5.314	0.155	0.049

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	56	51	53	78	53	51
normalized size	1	1.00	0.87	0.92	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.029	0.067	0.046	0.512	0.653	9.219	0.227	0.045

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	60	56	51	53	78	53	51
normalized size	1	1.00	0.98	0.92	0.84	0.87	1.28	0.87	0.84
time (sec)	N/A	0.030	0.070	0.049	0.539	0.574	7.796	0.154	0.050

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	57	56	51	53	80	53	51
normalized size	1	1.00	0.90	0.89	0.81	0.84	1.27	0.84	0.81
time (sec)	N/A	0.030	0.067	0.054	0.510	0.607	11.737	0.160	0.048

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	57	56	51	53	76	53	51
normalized size	1	1.00	0.93	0.92	0.84	0.87	1.25	0.87	0.84
time (sec)	N/A	0.031	0.018	0.047	0.458	0.820	18.414	0.155	0.048

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	73	78	114	77	69
normalized size	1	1.00	0.84	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.044	0.062	0.043	0.568	0.838	92.986	0.164	2.519

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.041	0.100	0.043	0.504	0.854	58.699	0.154	0.030

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	80	73	78	114	77	69
normalized size	1	1.00	1.00	0.94	0.86	0.92	1.34	0.91	0.81
time (sec)	N/A	0.042	0.083	0.049	0.481	0.943	39.599	0.170	0.032

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	71	80	73	76	114	77	69
normalized size	1	1.00	0.84	0.94	0.86	0.89	1.34	0.91	0.81
time (sec)	N/A	0.041	0.066	0.046	0.673	0.848	6.341	0.155	0.033

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	80	73	75	112	77	69
normalized size	1	1.00	1.00	0.96	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.043	0.061	0.041	0.557	1.453	23.640	0.161	0.030

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	81	80	73	75	112	77	69
normalized size	1	1.00	0.98	0.96	0.88	0.90	1.35	0.93	0.83
time (sec)	N/A	0.041	0.044	0.045	0.561	1.179	19.488	0.156	0.033

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	80	73	75	112	77	69
normalized size	1	1.00	0.91	0.94	0.86	0.88	1.32	0.91	0.81
time (sec)	N/A	0.045	0.040	0.043	0.641	1.118	28.368	0.161	0.034

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	80	73	75	110	77	69
normalized size	1	1.00	0.94	0.96	0.88	0.90	1.33	0.93	0.83
time (sec)	N/A	0.041	0.085	0.041	0.549	1.086	29.278	0.163	0.032

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	78	58	143	0	64	111
normalized size	1	1.00	0.92	1.07	0.79	1.96	0.00	0.88	1.52
time (sec)	N/A	0.048	0.124	0.047	1.316	0.705	0.000	0.171	2.607

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	54	377	295	2433	881	289	1933
normalized size	1	1.00	0.19	1.31	1.02	8.45	3.06	1.00	6.71
time (sec)	N/A	0.519	0.093	0.178	1.173	1.311	177.800	0.211	2.889

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	45	356	212	3635	857	280	1640
normalized size	1	1.00	0.17	1.32	0.79	13.46	3.17	1.04	6.07
time (sec)	N/A	0.551	0.071	0.164	1.139	1.299	64.299	1.351	2.853

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	53	39	108	537	39	93
normalized size	1	1.00	0.98	1.00	0.74	2.04	10.13	0.74	1.75
time (sec)	N/A	0.036	0.080	0.054	1.206	0.897	22.390	0.166	2.600

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	43	353	278	2424	833	280	1915
normalized size	1	1.00	0.16	1.32	1.04	9.04	3.11	1.04	7.15
time (sec)	N/A	0.473	0.029	0.151	1.424	0.771	23.158	0.213	2.885

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	46	355	212	3663	836	280	1700
normalized size	1	1.00	0.17	1.32	0.79	13.67	3.12	1.04	6.34
time (sec)	N/A	0.552	0.014	0.157	1.343	1.182	59.226	0.408	2.856

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	39	120	527	39	102
normalized size	1	1.00	1.00	1.00	0.74	2.26	9.94	0.74	1.92
time (sec)	N/A	0.037	0.050	0.056	1.400	0.943	156.584	0.166	0.101

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	47	358	278	2424	0	280	2023
normalized size	1	1.00	0.17	1.33	1.03	8.98	0.00	1.04	7.49
time (sec)	N/A	0.485	0.019	0.175	1.424	1.193	0.000	0.215	2.913

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	93	68	222	0	68	116
normalized size	1	1.00	0.81	0.98	0.72	2.34	0.00	0.72	1.22
time (sec)	N/A	0.055	0.179	0.064	1.239	0.942	0.000	0.181	2.650

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	76	405	311	2566	0	313	1884
normalized size	1	1.00	0.24	1.30	1.00	8.22	0.00	1.00	6.04
time (sec)	N/A	0.502	0.087	0.166	1.355	1.048	0.000	0.330	2.888

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	62	387	235	3787	0	302	1578
normalized size	1	1.00	0.21	1.34	0.81	13.10	0.00	1.04	5.46
time (sec)	N/A	0.549	0.098	0.164	1.470	1.220	0.000	1.377	2.869

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	74	61	190	0	63	115
normalized size	1	1.00	1.00	1.04	0.86	2.68	0.00	0.89	1.62
time (sec)	N/A	0.042	0.055	0.056	1.399	0.882	0.000	0.284	0.141

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	68	387	301	2555	0	302	1922
normalized size	1	1.00	0.24	1.34	1.04	8.84	0.00	1.04	6.65
time (sec)	N/A	0.468	0.032	0.163	1.203	1.150	0.000	0.311	2.917

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	70	401	240	3798	0	307	1757
normalized size	1	1.00	0.22	1.26	0.75	11.94	0.00	0.97	5.53
time (sec)	N/A	0.686	0.137	0.162	1.159	1.224	0.000	0.468	2.912

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	79	93	67	232	0	66	139
normalized size	1	1.01	0.82	0.97	0.70	2.42	0.00	0.69	1.45
time (sec)	N/A	0.057	0.130	0.066	1.187	0.823	0.000	0.161	0.152

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	74	395	312	2584	0	313	2080
normalized size	1	1.00	0.23	1.24	0.98	8.13	0.00	0.98	6.54
time (sec)	N/A	0.504	0.111	0.159	1.193	1.150	0.000	0.219	2.957

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	96	96	314	0	84	133
normalized size	1	1.00	0.89	0.92	0.92	3.02	0.00	0.81	1.28
time (sec)	N/A	0.057	0.194	0.066	1.430	0.976	0.000	0.206	2.761

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	92	416	341	2714	0	328	1944
normalized size	1	1.00	0.28	1.27	1.04	8.30	0.00	1.00	5.94
time (sec)	N/A	0.493	0.110	0.161	1.396	1.183	0.000	0.232	2.984

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	62	411	271	3951	0	328	1672
normalized size	1	1.00	0.19	1.26	0.83	12.08	0.00	1.00	5.11
time (sec)	N/A	0.597	0.065	0.161	1.226	1.292	0.000	0.368	2.893

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	97	96	313	0	84	136
normalized size	1	1.00	0.90	0.93	0.92	3.01	0.00	0.81	1.31
time (sec)	N/A	0.060	0.108	0.067	1.273	0.900	0.000	0.180	2.706

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	91	407	336	2674	0	322	1952
normalized size	1	1.00	0.28	1.27	1.05	8.33	0.00	1.00	6.08
time (sec)	N/A	0.522	0.048	0.169	1.215	1.179	0.000	0.235	2.950

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	113	441	273	3904	0	329	1786
normalized size	1	1.00	0.32	1.26	0.78	11.12	0.00	0.94	5.09
time (sec)	N/A	0.616	0.133	0.170	1.289	1.136	0.000	0.482	2.911

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	130	102	133	100	347	0	88	163
normalized size	1	1.01	0.79	1.03	0.78	2.69	0.00	0.68	1.26
time (sec)	N/A	0.072	0.185	0.072	1.346	0.914	0.000	0.182	2.728

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	96	435	346	2690	0	334	2109
normalized size	1	1.00	0.27	1.24	0.99	7.66	0.00	0.95	6.01
time (sec)	N/A	0.541	0.122	0.171	1.342	0.785	0.000	0.241	2.959

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	75	77	118	99	219	104	154
normalized size	1	1.00	0.73	0.75	1.15	0.96	2.13	1.01	1.50
time (sec)	N/A	0.084	0.062	0.047	0.618	0.618	4.166	0.159	2.717

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	84	75	168	73	114
normalized size	1	1.00	0.78	0.73	1.15	1.03	2.30	1.00	1.56
time (sec)	N/A	0.060	0.044	0.041	0.507	0.856	1.818	0.175	2.662

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	49	50	117	44	44
normalized size	1	1.00	0.74	0.67	1.07	1.09	2.54	0.96	0.96
time (sec)	N/A	0.040	0.031	0.041	0.591	0.958	0.738	0.153	2.598

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	60	50	67	125	76	61	80
normalized size	1	1.00	0.94	0.78	1.05	1.95	1.19	0.95	1.25
time (sec)	N/A	0.043	0.051	0.046	1.204	0.974	25.832	0.153	2.714

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	63	72	107	143	134	68	76
normalized size	1	1.00	0.75	0.86	1.27	1.70	1.60	0.81	0.90
time (sec)	N/A	0.063	0.046	0.048	1.264	0.941	43.661	0.168	2.933

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	96	158	172	160	120	93
normalized size	1	1.00	1.06	1.09	1.80	1.95	1.82	1.36	1.06
time (sec)	N/A	0.070	0.075	0.049	1.317	1.018	129.138	0.200	3.119

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	89	658	0	0	83	0	-1
normalized size	1	1.00	0.29	2.17	0.00	0.00	0.27	0.00	-0.00
time (sec)	N/A	0.151	0.166	0.053	0.000	0.939	3.470	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	75	618	0	0	82	0	-1
normalized size	1	1.00	0.28	2.31	0.00	0.00	0.31	0.00	-0.00
time (sec)	N/A	0.091	0.092	0.048	0.000	0.897	3.236	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	81	596	0	0	85	0	-1
normalized size	1	1.00	0.30	2.22	0.00	0.00	0.32	0.00	-0.00
time (sec)	N/A	0.099	0.075	0.050	0.000	0.999	3.377	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	80	616	0	0	94	0	-1
normalized size	1	1.00	0.29	2.26	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.103	0.179	0.054	0.000	0.643	3.631	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	80	660	0	0	97	0	-1
normalized size	1	1.00	0.26	2.16	0.00	0.00	0.32	0.00	-0.00
time (sec)	N/A	0.138	0.157	0.054	0.000	0.925	3.833	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	91	966	0	0	83	0	-1
normalized size	1	1.00	0.16	1.66	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.355	0.152	0.046	0.000	0.977	3.003	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	75	926	0	0	83	0	-1
normalized size	1	1.00	0.14	1.69	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.241	0.110	0.047	0.000	0.882	2.693	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	81	902	0	0	85	0	-1
normalized size	1	1.00	0.15	1.66	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.249	0.078	0.062	0.000	0.765	3.051	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	80	920	0	0	92	0	-1
normalized size	1	1.00	0.15	1.68	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.251	0.095	0.056	0.000	1.002	3.251	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	80	964	0	0	97	0	-1
normalized size	1	1.00	0.14	1.66	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.320	0.101	0.093	0.000	0.883	3.784	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	80	1006	0	0	97	0	-1
normalized size	1	1.00	0.13	1.64	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.379	0.104	0.083	0.000	0.678	5.136	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	118	124	267	104	206
normalized size	1	1.00	0.76	0.75	1.15	1.20	2.59	1.01	2.00
time (sec)	N/A	0.079	0.092	0.051	0.615	0.959	8.440	0.162	2.655

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	57	53	84	99	216	73	211
normalized size	1	1.00	0.78	0.73	1.15	1.36	2.96	1.00	2.89
time (sec)	N/A	0.058	0.055	0.047	0.610	0.948	4.755	0.182	2.715

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	31	49	73	165	44	150
normalized size	1	1.00	0.74	0.67	1.07	1.59	3.59	0.96	3.26
time (sec)	N/A	0.040	0.028	0.047	0.493	0.881	2.520	0.166	3.346

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	66	80	172	82	80	131
normalized size	1	1.00	0.99	0.81	0.99	2.12	1.01	0.99	1.62
time (sec)	N/A	0.057	0.082	0.049	1.257	0.955	66.499	0.165	2.793

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	80	101	134	169	223	103	111
normalized size	1	1.00	0.73	0.92	1.22	1.54	2.03	0.94	1.01
time (sec)	N/A	0.084	0.090	0.049	1.438	0.810	58.401	0.202	3.379

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	59	107	171	191	243	131	110
normalized size	1	1.00	0.51	0.93	1.49	1.66	2.11	1.14	0.96
time (sec)	N/A	0.087	0.036	0.052	1.306	0.956	153.107	0.185	3.472

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	93	694	0	0	172	0	-1
normalized size	1	1.00	0.28	2.07	0.00	0.00	0.51	0.00	-0.00
time (sec)	N/A	0.175	0.174	0.053	0.000	0.901	5.056	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	77	654	0	0	170	0	-1
normalized size	1	1.00	0.26	2.19	0.00	0.00	0.57	0.00	-0.00
time (sec)	N/A	0.124	0.075	0.043	0.000	0.955	4.749	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	83	629	0	0	172	0	-1
normalized size	1	1.00	0.28	2.13	0.00	0.00	0.58	0.00	-0.00
time (sec)	N/A	0.127	0.055	0.053	0.000	1.000	5.378	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	82	626	0	0	184	0	-1
normalized size	1	1.00	0.28	2.11	0.00	0.00	0.62	0.00	-0.00
time (sec)	N/A	0.121	0.079	0.051	0.000	0.953	6.178	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	82	653	0	0	196	0	-1
normalized size	1	1.00	0.27	2.16	0.00	0.00	0.65	0.00	-0.00
time (sec)	N/A	0.126	0.147	0.093	0.000	0.938	6.531	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	96	1002	0	0	172	0	-1
normalized size	1	1.00	0.16	1.63	0.00	0.00	0.28	0.00	-0.00
time (sec)	N/A	0.370	0.243	0.052	0.000	0.898	5.795	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	78	962	0	0	172	0	-1
normalized size	1	1.00	0.13	1.66	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.292	0.249	0.043	0.000	0.952	4.759	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	83	937	0	0	173	0	-1
normalized size	1	1.00	0.14	1.64	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.294	0.039	0.057	0.000	0.696	5.674	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	85	932	0	0	182	0	-1
normalized size	1	1.00	0.15	1.61	0.00	0.00	0.31	0.00	-0.00
time (sec)	N/A	0.292	0.097	0.053	0.000	0.912	6.540	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	576	576	82	957	0	0	194	0	-1
normalized size	1	1.00	0.14	1.66	0.00	0.00	0.34	0.00	-0.00
time (sec)	N/A	0.297	0.081	0.095	0.000	0.988	6.061	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	82	1002	0	0	199	0	-1
normalized size	1	1.00	0.13	1.65	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.355	0.083	0.099	0.000	1.119	6.884	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	78	77	118	76	175	101	104
normalized size	1	1.00	0.76	0.75	1.15	0.74	1.70	0.98	1.01
time (sec)	N/A	0.074	0.079	0.043	0.588	1.003	3.441	0.160	2.676

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	53	83	52	124	70	52
normalized size	1	1.00	0.77	0.73	1.14	0.71	1.70	0.96	0.71
time (sec)	N/A	0.055	0.058	0.047	0.450	0.937	1.825	0.160	2.649

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	48	29	75	38	29
normalized size	1	1.00	0.72	0.65	1.04	0.63	1.63	0.83	0.63
time (sec)	N/A	0.038	0.035	0.043	0.462	0.982	0.976	0.157	2.604

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	37	54	105	65	40	57
normalized size	1	1.00	1.00	0.77	1.12	2.19	1.35	0.83	1.19
time (sec)	N/A	0.032	0.047	0.044	1.080	1.083	11.270	0.187	2.715

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	62	109	126	80	62	67
normalized size	1	1.00	0.98	1.07	1.88	2.17	1.38	1.07	1.16
time (sec)	N/A	0.046	0.074	0.053	1.171	0.953	31.833	0.161	2.894

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	102	178	173	163	121	95
normalized size	1	1.00	0.90	1.13	1.98	1.92	1.81	1.34	1.06
time (sec)	N/A	0.070	0.286	0.075	1.199	1.091	71.181	0.194	2.995

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	89	624	0	0	80	0	-1
normalized size	1	1.00	0.33	2.31	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.101	0.128	0.050	0.000	1.071	3.172	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	74	586	0	0	78	0	-1
normalized size	1	1.00	0.31	2.45	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.063	0.046	0.053	0.000	0.856	2.119	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	78	587	0	0	82	0	-1
normalized size	1	1.00	0.32	2.42	0.00	0.00	0.34	0.00	-0.00
time (sec)	N/A	0.069	0.046	0.052	0.000	1.043	2.416	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	78	625	0	0	90	0	-1
normalized size	1	1.00	0.28	2.28	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.102	0.069	0.052	0.000	1.011	2.872	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	91	932	0	0	80	0	-1
normalized size	1	1.00	0.17	1.70	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.245	0.122	0.051	0.000	0.795	3.171	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	517	75	892	0	0	80	0	-1
normalized size	1	1.00	0.15	1.73	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.186	0.058	0.052	0.000	0.990	2.878	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	77	891	0	0	82	0	-1
normalized size	1	1.00	0.15	1.75	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.195	0.042	0.059	0.000	0.851	2.465	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	78	929	0	0	88	0	-1
normalized size	1	1.00	0.14	1.69	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.252	0.055	0.055	0.000	1.057	2.679	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	78	970	0	0	94	0	-1
normalized size	1	1.00	0.13	1.67	0.00	0.00	0.16	0.00	-0.00
time (sec)	N/A	0.308	0.038	0.083	0.000	1.144	3.139	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	77	77	116	88	175	114	152
normalized size	1	1.00	0.75	0.75	1.13	0.85	1.70	1.11	1.48
time (sec)	N/A	0.077	0.053	0.045	0.590	0.806	3.902	0.175	2.771

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	55	52	81	63	124	77	60
normalized size	1	1.00	0.75	0.71	1.11	0.86	1.70	1.05	0.82
time (sec)	N/A	0.055	0.044	0.049	0.648	0.962	1.844	0.176	2.680

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	47	41	75	38	33
normalized size	1	1.00	0.72	0.65	1.02	0.89	1.63	0.83	0.72
time (sec)	N/A	0.037	0.027	0.046	0.556	0.623	1.082	0.177	2.615

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	57	70	170	56	53	65
normalized size	1	1.00	1.00	0.98	1.21	2.93	0.97	0.91	1.12
time (sec)	N/A	0.040	0.066	0.047	1.352	0.935	20.823	0.162	2.769

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	57	100	144	233	264	99	131
normalized size	1	1.00	0.66	1.16	1.67	2.71	3.07	1.15	1.52
time (sec)	N/A	0.065	0.020	0.061	1.388	0.967	79.790	0.168	2.929

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	120	60	141	215	289	192	137	167
normalized size	1	1.02	0.51	1.19	1.82	2.45	1.63	1.16	1.42
time (sec)	N/A	0.090	0.023	0.048	1.343	0.699	179.010	0.178	3.180

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	103	666	0	0	80	0	-1
normalized size	1	1.00	0.34	2.22	0.00	0.00	0.27	0.00	-0.00
time (sec)	N/A	0.139	0.150	0.081	0.000	0.914	29.493	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	78	627	0	0	80	0	-1
normalized size	1	1.00	0.29	2.33	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.100	0.101	0.049	0.000	0.646	14.089	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	73	613	0	0	78	0	-1
normalized size	1	1.00	0.29	2.44	0.00	0.00	0.31	0.00	-0.00
time (sec)	N/A	0.063	0.042	0.056	0.000	0.775	10.050	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	86	631	0	0	82	0	-1
normalized size	1	1.00	0.32	2.32	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.101	0.063	0.055	0.000	0.930	26.009	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	72	667	0	0	90	0	-1
normalized size	1	1.00	0.24	2.19	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.136	0.059	0.048	0.000	0.839	79.845	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	547	547	79	937	0	0	80	0	-1
normalized size	1	1.00	0.14	1.71	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.255	0.080	0.055	0.000	0.718	13.659	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	524	71	921	0	0	80	0	-1
normalized size	1	1.00	0.14	1.76	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.195	0.064	0.047	0.000	0.850	9.526	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	548	72	939	0	0	82	0	-1
normalized size	1	1.00	0.13	1.71	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.244	0.037	0.046	0.000	0.721	20.348	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	72	975	0	0	88	0	-1
normalized size	1	1.00	0.12	1.68	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.304	0.039	0.051	0.000	0.814	58.880	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	611	72	1018	0	0	94	0	-1
normalized size	1	1.00	0.12	1.67	0.00	0.00	0.15	0.00	-0.00
time (sec)	N/A	0.362	0.039	0.095	0.000	0.821	136.390	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	73	76	116	98	338	104	145
normalized size	1	1.00	0.71	0.74	1.13	0.95	3.28	1.01	1.41
time (sec)	N/A	0.078	0.067	0.048	0.592	0.663	5.183	0.166	2.797

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	84	75	240	63	60
normalized size	1	1.00	0.74	0.73	1.15	1.03	3.29	0.86	0.82
time (sec)	N/A	0.056	0.047	0.050	0.488	0.603	2.424	0.199	2.761

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	30	49	52	144	32	33
normalized size	1	1.00	0.72	0.65	1.07	1.13	3.13	0.70	0.72
time (sec)	N/A	0.036	0.029	0.046	0.499	0.672	1.335	0.162	2.678

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	62	85	81	243	76	67	80
normalized size	1	1.00	0.81	1.10	1.05	3.16	0.99	0.87	1.04
time (sec)	N/A	0.050	0.047	0.086	1.169	0.688	38.427	0.168	2.782

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	57	157	170	351	0	101	198
normalized size	1	1.00	0.50	1.39	1.50	3.11	0.00	0.89	1.75
time (sec)	N/A	0.087	0.029	0.091	1.254	0.638	0.000	0.186	2.971

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	108	683	0	0	0	0	-1
normalized size	1	1.00	0.36	2.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.132	0.166	0.098	0.000	0.690	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	99	669	0	0	80	0	-1
normalized size	1	1.00	0.35	2.36	0.00	0.00	0.28	0.00	-0.00
time (sec)	N/A	0.103	0.186	0.077	0.000	0.778	115.031	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	103	674	0	0	78	0	-1
normalized size	1	1.00	0.36	2.38	0.00	0.00	0.28	0.00	-0.00
time (sec)	N/A	0.099	0.065	0.073	0.000	0.693	79.479	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	116	689	0	0	0	0	-1
normalized size	1	1.00	0.39	2.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.094	0.082	0.000	0.765	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	83	722	0	0	0	0	-1
normalized size	1	1.00	0.25	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.056	0.086	0.000	0.860	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	577	109	997	0	0	0	0	-1
normalized size	1	1.00	0.19	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.112	0.099	0.000	0.699	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	559	559	92	981	0	0	80	0	-1
normalized size	1	1.00	0.16	1.75	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.265	0.097	0.082	0.000	0.804	162.516	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	563	563	81	986	0	0	80	0	-1
normalized size	1	1.00	0.14	1.75	0.00	0.00	0.14	0.00	-0.00
time (sec)	N/A	0.248	0.076	0.077	0.000	0.559	79.718	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	86	1001	0	0	0	0	-1
normalized size	1	1.00	0.15	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.041	0.096	0.000	0.759	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	610	610	83	1034	0	0	0	0	-1
normalized size	1	1.00	0.14	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.356	0.058	0.115	0.000	0.857	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	77	506	69	156	85	82	109
normalized size	1	1.00	0.79	5.22	0.71	1.61	0.88	0.85	1.12
time (sec)	N/A	0.094	0.087	4.458	1.153	0.618	38.268	0.169	4.539

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	65	446	53	129	68	64	88
normalized size	1	1.00	0.86	5.87	0.70	1.70	0.89	0.84	1.16
time (sec)	N/A	0.062	0.041	0.170	1.299	0.686	16.958	0.192	4.278

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	425	42	110	51	44	71
normalized size	1	1.00	0.95	7.46	0.74	1.93	0.89	0.77	1.25
time (sec)	N/A	0.046	0.016	0.152	1.208	0.717	6.097	0.159	3.866

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	468	0	147	66	50	93
normalized size	1	1.00	0.91	7.20	0.00	2.26	1.02	0.77	1.43
time (sec)	N/A	0.056	0.018	0.187	0.000	0.739	13.281	0.165	4.658

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	511	0	194	0	72	113
normalized size	1	1.00	1.00	5.81	0.00	2.20	0.00	0.82	1.28
time (sec)	N/A	0.077	0.043	0.191	0.000	0.623	0.000	0.172	4.858

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	133	1309	0	0	0	0	-1
normalized size	1	1.00	0.19	1.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.477	0.084	0.257	0.000	5.590	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	659	659	63	848	0	0	0	0	-1
normalized size	1	1.00	0.10	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.167	0.035	0.227	0.000	2.302	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	136	1306	0	0	0	0	-1
normalized size	1	1.00	0.20	1.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.080	0.255	0.000	1.618	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	236	1003	0	0	0	0	-1
normalized size	1	1.00	3.58	15.20	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.363	0.260	0.000	4.773	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	696	0	0	0	0	-1
normalized size	1	1.00	2.58	10.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.196	0.145	0.000	1.088	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	244	1002	0	0	0	0	-1
normalized size	1	1.00	3.70	15.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.136	0.191	0.000	2.394	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	467	53	129	0	64	88
normalized size	1	1.00	0.83	5.99	0.68	1.65	0.00	0.82	1.13
time (sec)	N/A	0.074	0.070	0.245	1.345	0.722	0.000	0.158	5.383

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	56	425	43	112	65	49	71
normalized size	1	1.00	0.95	7.20	0.73	1.90	1.10	0.83	1.20
time (sec)	N/A	0.048	0.022	0.180	1.455	0.744	15.581	0.156	4.859

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	413	29	87	37	29	56
normalized size	1	1.00	1.00	10.32	0.72	2.18	0.92	0.72	1.40
time (sec)	N/A	0.036	0.010	0.175	1.244	0.703	10.089	0.183	5.210

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	433	0	148	63	53	94
normalized size	1	1.00	0.91	6.66	0.00	2.28	0.97	0.82	1.45
time (sec)	N/A	0.055	0.021	0.192	0.000	0.790	12.247	0.167	5.507

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	477	0	194	0	72	112
normalized size	1	1.00	1.00	5.42	0.00	2.20	0.00	0.82	1.27
time (sec)	N/A	0.077	0.035	0.227	0.000	0.664	0.000	0.158	5.719

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	667	667	67	848	0	0	0	0	-1
normalized size	1	1.00	0.10	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	0.037	0.252	0.000	4.771	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	67	416	0	2274	0	0	453
normalized size	1	1.00	0.33	2.02	0.00	11.04	0.00	0.00	2.20
time (sec)	N/A	0.032	0.026	0.178	0.000	1.831	0.000	0.000	25.804

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	697	697	136	874	0	0	0	0	-1
normalized size	1	1.00	0.20	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	0.097	0.191	0.000	2.422	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	0	0	0	-1
normalized size	1	1.00	1.02	10.55	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.061	0.239	0.000	1.237	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	165	416	0	0	0	0	-1
normalized size	1	1.00	2.58	6.50	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.069	0.151	0.000	1.736	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	243	722	0	0	0	0	-1
normalized size	1	1.00	3.68	10.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.183	0.191	0.000	3.579	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	28	164	0	1191	0	0	653
normalized size	1	1.00	0.22	1.29	0.00	9.38	0.00	0.00	5.14
time (sec)	N/A	0.019	0.029	0.796	0.000	0.843	0.000	0.000	3.419

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	81	582	96	169	99	100	118
normalized size	1	1.00	0.73	5.24	0.86	1.52	0.89	0.90	1.06
time (sec)	N/A	0.097	0.111	0.354	1.306	0.708	60.266	0.163	3.510

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	507	82	147	82	83	98
normalized size	1	1.00	0.78	5.63	0.91	1.63	0.91	0.92	1.09
time (sec)	N/A	0.085	0.064	0.161	1.209	0.710	30.233	0.171	3.404

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	58	446	66	121	65	65	78
normalized size	1	1.00	0.84	6.46	0.96	1.75	0.94	0.94	1.13
time (sec)	N/A	0.057	0.040	0.180	1.304	0.612	14.908	0.211	3.508

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	47	425	56	101	46	43	59
normalized size	1	1.00	0.94	8.50	1.12	2.02	0.92	0.86	1.18
time (sec)	N/A	0.043	0.027	0.152	1.154	0.650	5.109	0.160	3.496

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	468	0	138	60	48	125
normalized size	1	1.00	0.91	8.07	0.00	2.38	1.03	0.83	2.16
time (sec)	N/A	0.054	0.018	0.158	0.000	0.731	8.219	0.164	4.687

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	511	0	186	0	73	69
normalized size	1	1.00	1.00	6.31	0.00	2.30	0.00	0.90	0.85
time (sec)	N/A	0.072	0.029	0.176	0.000	0.719	0.000	0.170	3.748

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	96	574	0	188	0	100	83
normalized size	1	1.00	0.90	5.36	0.00	1.76	0.00	0.93	0.78
time (sec)	N/A	0.094	0.060	0.186	0.000	0.570	0.000	0.176	3.910

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	648	648	150	1788	0	0	0	0	-1
normalized size	1	1.00	0.23	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.888	0.095	0.237	0.000	44.319	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	130	1310	0	0	0	0	-1
normalized size	1	1.00	0.21	2.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.747	0.082	0.171	0.000	11.880	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	63	848	0	0	0	0	-1
normalized size	1	1.00	0.10	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	0.040	0.156	0.000	2.349	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	137	1306	0	0	0	0	-1
normalized size	1	1.00	0.22	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.717	0.111	0.168	0.000	0.830	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	153	1782	0	0	0	0	-1
normalized size	1	1.00	0.23	2.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.838	0.140	0.178	0.000	1.575	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	678	678	164	2280	0	0	0	0	-1
normalized size	1	1.00	0.24	3.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.951	0.111	0.180	0.000	3.485	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	93	634	110	191	0	117	135
normalized size	1	1.00	0.72	4.88	0.85	1.47	0.00	0.90	1.04
time (sec)	N/A	0.113	0.133	0.252	1.285	0.456	0.000	0.165	3.525

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	81	541	96	169	110	100	115
normalized size	1	1.00	0.74	4.96	0.88	1.55	1.01	0.92	1.06
time (sec)	N/A	0.102	0.087	0.157	1.335	0.440	108.786	0.163	3.495

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	70	462	82	147	90	83	95
normalized size	1	1.00	0.80	5.25	0.93	1.67	1.02	0.94	1.08
time (sec)	N/A	0.072	0.039	0.176	1.440	0.496	62.737	0.184	3.521

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	58	441	68	121	65	65	75
normalized size	1	1.00	0.87	6.58	1.01	1.81	0.97	0.97	1.12
time (sec)	N/A	0.057	0.038	0.158	1.388	0.485	29.253	0.163	3.449

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	500	0	152	73	61	89
normalized size	1	1.00	1.00	6.85	0.00	2.08	1.00	0.84	1.22
time (sec)	N/A	0.070	0.048	0.189	0.000	0.475	24.743	0.177	5.893

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	556	0	186	0	64	56
normalized size	1	1.00	1.00	7.13	0.00	2.38	0.00	0.82	0.72
time (sec)	N/A	0.072	0.046	0.262	0.000	0.474	0.000	0.166	3.524

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	96	617	0	218	0	101	87
normalized size	1	1.00	0.92	5.93	0.00	2.10	0.00	0.97	0.84
time (sec)	N/A	0.095	0.106	0.235	0.000	0.467	0.000	0.190	3.737

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	669	669	163	1840	0	0	0	0	-1
normalized size	1	1.00	0.24	2.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	0.116	0.275	0.000	50.834	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	645	645	150	1344	0	0	0	0	-1
normalized size	1	1.00	0.23	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.827	0.113	0.164	0.000	16.092	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	127	864	0	0	0	0	-1
normalized size	1	1.00	0.20	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	0.118	0.179	0.000	3.569	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	626	626	137	1339	0	0	0	0	-1
normalized size	1	1.00	0.22	2.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.102	0.209	0.000	0.000	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	651	651	154	1810	0	0	0	0	-1
normalized size	1	1.00	0.24	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.844	0.111	0.214	0.000	3.742	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	167	2306	0	0	0	0	-1
normalized size	1	1.00	0.25	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.936	0.108	0.201	0.000	5.889	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	69	528	82	146	0	82	98
normalized size	1	1.00	0.77	5.87	0.91	1.62	0.00	0.91	1.09
time (sec)	N/A	0.081	0.106	0.281	1.264	1.123	0.000	0.158	3.221

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	58	468	66	121	0	65	78
normalized size	1	1.00	0.82	6.59	0.93	1.70	0.00	0.92	1.10
time (sec)	N/A	0.069	0.069	0.158	1.312	0.985	0.000	0.187	3.393

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	425	56	103	61	48	60
normalized size	1	1.00	0.94	8.17	1.08	1.98	1.17	0.92	1.15
time (sec)	N/A	0.044	0.020	0.168	1.400	0.917	16.347	0.155	3.271

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	413	42	78	32	27	45
normalized size	1	1.00	1.00	12.52	1.27	2.36	0.97	0.82	1.36
time (sec)	N/A	0.033	0.011	0.195	1.213	0.630	11.202	0.160	3.233

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	51	433	0	139	58	54	47
normalized size	1	1.00	0.88	7.47	0.00	2.40	1.00	0.93	0.81
time (sec)	N/A	0.052	0.029	0.174	0.000	0.845	12.219	0.171	3.283

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	477	0	184	0	73	73
normalized size	1	1.00	1.00	5.89	0.00	2.27	0.00	0.90	0.90
time (sec)	N/A	0.073	0.052	0.186	0.000	0.973	0.000	0.164	3.417

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	95	540	0	217	0	101	94
normalized size	1	1.00	0.89	5.05	0.00	2.03	0.00	0.94	0.88
time (sec)	N/A	0.098	0.071	0.207	0.000	0.991	0.000	0.180	3.501

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	130	1311	0	0	0	0	-1
normalized size	1	1.00	0.21	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	0.087	0.237	0.000	31.988	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	67	848	0	0	0	0	-1
normalized size	1	1.00	0.11	1.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.581	0.034	0.170	0.000	7.626	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	67	416	0	2459	0	0	272
normalized size	1	1.00	0.48	2.95	0.00	17.44	0.00	0.00	1.93
time (sec)	N/A	0.413	0.061	0.165	0.000	4.330	0.000	0.000	40.219

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	137	874	0	0	0	0	-1
normalized size	1	1.00	0.22	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	0.091	0.167	0.000	2.567	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	654	654	152	1351	0	0	0	0	-1
normalized size	1	1.00	0.23	2.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.830	0.102	0.199	0.000	5.367	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	678	678	167	1849	0	0	0	0	-1
normalized size	1	1.00	0.25	2.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.929	0.156	0.201	0.000	13.261	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	67	696	0	0	0	0	-1
normalized size	1	1.00	1.02	10.55	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.040	0.240	0.000	1.825	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	166	416	0	0	0	0	-1
normalized size	1	1.00	2.59	6.50	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.172	0.163	0.000	1.708	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	722	0	0	0	0	-1
normalized size	1	1.00	3.67	10.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.244	0.154	0.000	4.088	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1047	0	0	0	0	-1
normalized size	1	1.00	3.95	15.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.246	0.196	0.000	11.250	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	66	560	82	189	0	82	95
normalized size	1	1.00	0.73	6.22	0.91	2.10	0.00	0.91	1.06
time (sec)	N/A	0.104	0.057	0.297	1.251	1.222	0.000	0.197	3.782

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	53	501	68	161	0	58	75
normalized size	1	1.00	0.75	7.06	0.96	2.27	0.00	0.82	1.06
time (sec)	N/A	0.073	0.055	0.170	1.122	1.084	0.000	0.168	3.712

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	456	56	149	58	47	60
normalized size	1	1.00	0.94	8.77	1.08	2.87	1.12	0.90	1.15
time (sec)	N/A	0.046	0.035	0.184	1.209	0.808	27.571	0.165	3.677

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	43	435	58	147	51	48	63
normalized size	1	1.00	0.78	7.91	1.05	2.67	0.93	0.87	1.15
time (sec)	N/A	0.049	0.012	0.214	1.228	0.711	25.526	0.180	3.630

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	63	485	0	213	78	68	68
normalized size	1	1.00	0.83	6.38	0.00	2.80	1.03	0.89	0.89
time (sec)	N/A	0.070	0.028	0.189	0.000	0.822	17.016	0.162	3.662

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	77	549	0	272	0	100	88
normalized size	1	1.00	0.77	5.49	0.00	2.72	0.00	1.00	0.88
time (sec)	N/A	0.095	0.036	0.188	0.000	0.998	0.000	0.179	3.801

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	91	636	0	303	0	118	112
normalized size	1	1.00	0.71	4.97	0.00	2.37	0.00	0.92	0.88
time (sec)	N/A	0.124	0.079	0.217	0.000	0.903	0.000	0.165	4.025

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	629	629	127	1810	0	0	0	0	-1
normalized size	1	1.00	0.20	2.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	0.124	0.281	0.000	16.291	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	126	1346	0	0	0	0	-1
normalized size	1	1.00	0.20	2.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.730	0.111	0.162	0.000	2.269	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	632	632	124	875	0	0	0	0	-1
normalized size	1	1.00	0.20	1.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.709	0.099	0.171	0.000	2.736	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	140	1361	0	0	0	0	-1
normalized size	1	1.00	0.21	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.826	0.099	0.225	0.000	3.299	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	153	1864	0	0	0	0	-1
normalized size	1	1.00	0.23	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	0.100	0.216	0.000	9.549	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	699	699	167	2389	0	0	0	0	-1
normalized size	1	1.00	0.24	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.098	0.119	0.199	0.000	21.417	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	233	1038	0	0	0	0	-1
normalized size	1	1.00	3.53	15.73	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.209	0.243	0.000	2.552	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	230	721	0	0	0	0	-1
normalized size	1	1.00	3.59	11.27	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.158	0.161	0.000	2.525	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	248	1053	0	0	0	0	-1
normalized size	1	1.00	3.76	15.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.149	0.200	0.000	6.407	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	261	1402	0	0	0	0	-1
normalized size	1	1.00	3.95	21.24	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.064	0.238	0.191	0.000	18.152	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	80	977	0	0	0	0	-1
normalized size	1	1.00	0.11	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	0.094	0.775	0.000	64.558	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	757	757	80	924	0	0	0	0	-1
normalized size	1	1.00	0.11	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.085	4.354	0.000	57.713	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	87	926	0	0	0	0	-1
normalized size	1	1.00	0.11	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.085	0.365	0.000	57.442	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	90	983	0	0	0	0	-1
normalized size	1	1.00	0.12	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.083	0.368	0.000	58.096	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	738	738	80	977	0	0	0	0	-1
normalized size	1	1.00	0.11	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.193	0.741	0.000	58.341	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	758	758	80	924	0	0	0	0	-1
normalized size	1	1.00	0.11	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.082	0.768	0.000	46.796	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	89	926	0	0	0	0	-1
normalized size	1	1.00	0.11	1.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.237	0.048	0.390	0.000	42.372	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	89	983	0	0	0	0	-1
normalized size	1	1.00	0.12	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.075	0.418	0.000	41.767	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	83	538	0	0	0	0	-1
normalized size	1	1.00	0.26	1.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.064	0.104	0.388	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	83	509	0	0	0	0	-1
normalized size	1	1.00	0.26	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.070	0.076	0.410	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	85	510	0	0	0	0	-1
normalized size	1	1.00	0.26	1.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.067	0.123	0.413	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	87	541	0	0	0	0	-1
normalized size	1	1.00	0.26	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.067	0.110	0.365	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	83	538	0	0	0	0	-1
normalized size	1	1.00	0.27	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.051	0.149	0.425	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	83	509	0	0	0	0	-1
normalized size	1	1.00	0.26	1.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.099	0.372	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	84	510	0	5060	0	0	-1
normalized size	1	1.00	0.26	1.59	0.00	15.81	0.00	0.00	-0.00
time (sec)	N/A	0.053	0.106	0.364	0.000	16.175	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	86	541	0	5060	0	0	-1
normalized size	1	1.00	0.27	1.68	0.00	15.71	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.132	0.392	0.000	15.484	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	121	514	0	280	128	139	176
normalized size	1	1.00	0.97	4.11	0.00	2.24	1.02	1.11	1.41
time (sec)	N/A	0.129	0.307	0.425	0.000	0.511	30.272	0.174	6.171

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	458	0	195	95	96	136
normalized size	1	1.00	0.95	4.92	0.00	2.10	1.02	1.03	1.46
time (sec)	N/A	0.078	0.093	0.244	0.000	0.476	14.674	0.162	6.058

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	434	0	156	68	66	82
normalized size	1	1.00	1.00	6.20	0.00	2.23	0.97	0.94	1.17
time (sec)	N/A	0.059	0.037	0.229	0.000	0.470	6.349	0.157	6.156

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	476	0	383	85	79	114
normalized size	1	1.00	0.95	5.60	0.00	4.51	1.00	0.93	1.34
time (sec)	N/A	0.074	0.052	0.262	0.000	0.477	12.248	0.172	7.938

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	518	0	513	0	107	137
normalized size	1	1.00	0.93	4.50	0.00	4.46	0.00	0.93	1.19
time (sec)	N/A	0.125	0.178	0.249	0.000	0.499	0.000	0.192	5.130

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	241	1012	0	0	0	0	-1
normalized size	1	1.00	3.77	15.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.619	0.349	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	857	0	0	0	0	-1
normalized size	1	1.00	1.02	13.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.059	0.326	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	705	0	0	0	0	-1
normalized size	1	1.00	2.73	11.95	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.187	0.236	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	139	1314	0	0	0	0	-1
normalized size	1	1.00	2.24	21.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.115	0.269	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	335	1010	0	0	0	0	-1
normalized size	1	1.00	5.23	15.78	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.253	0.333	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	145	605	0	410	153	193	330
normalized size	1	1.00	0.94	3.93	0.00	2.66	0.99	1.25	2.14
time (sec)	N/A	0.155	0.236	0.359	0.000	0.905	129.923	0.174	6.122

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	531	0	297	116	151	215
normalized size	1	1.00	0.92	4.42	0.00	2.48	0.97	1.26	1.79
time (sec)	N/A	0.104	0.121	0.324	0.000	1.760	70.599	0.189	6.134

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	85	507	0	204	90	113	143
normalized size	1	1.00	0.89	5.28	0.00	2.12	0.94	1.18	1.49
time (sec)	N/A	0.081	0.086	0.246	0.000	1.341	35.293	0.170	5.908

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	565	0	486	102	112	155
normalized size	1	1.00	1.01	5.43	0.00	4.67	0.98	1.08	1.49
time (sec)	N/A	0.112	0.097	0.261	0.000	1.418	39.290	0.177	7.879

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	108	620	0	538	0	121	167
normalized size	1	1.00	0.93	5.34	0.00	4.64	0.00	1.04	1.44
time (sec)	N/A	0.145	0.133	0.280	0.000	1.208	0.000	0.174	9.517

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	1101	0	0	0	0	-1
normalized size	1	1.00	4.31	16.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.570	0.375	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	149	930	0	0	0	0	-1
normalized size	1	1.00	2.29	14.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.208	0.366	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	351	776	0	0	0	0	-1
normalized size	1	1.00	5.85	12.93	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.361	0.259	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	148	1404	0	0	0	0	-1
normalized size	1	1.00	2.35	22.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.147	0.257	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	1096	0	0	0	0	-1
normalized size	1	1.00	5.28	16.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.326	0.305	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	488	0	289	0	106	121
normalized size	1	1.00	0.88	4.69	0.00	2.78	0.00	1.02	1.16
time (sec)	N/A	0.105	0.216	0.339	0.000	1.093	0.000	0.172	5.426

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	448	0	205	0	64	86
normalized size	1	1.00	1.00	6.05	0.00	2.77	0.00	0.86	1.16
time (sec)	N/A	0.070	0.116	0.227	0.000	1.293	0.000	0.172	5.098

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	426	0	130	39	40	70
normalized size	1	1.00	1.00	8.35	0.00	2.55	0.76	0.78	1.37
time (sec)	N/A	0.050	0.017	0.275	0.000	1.121	10.261	0.157	5.888

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	453	0	431	70	71	114
normalized size	1	1.00	0.95	5.33	0.00	5.07	0.82	0.84	1.34
time (sec)	N/A	0.076	0.100	0.245	0.000	0.836	19.391	0.166	7.313

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	151	498	0	565	0	104	142
normalized size	1	1.00	1.29	4.26	0.00	4.83	0.00	0.89	1.21
time (sec)	N/A	0.118	0.131	0.253	0.000	1.206	0.000	0.192	8.424

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	719	0	0	0	0	-1
normalized size	1	1.00	1.02	11.23	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.038	0.366	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	429	0	0	0	0	-1
normalized size	1	1.00	1.02	6.70	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.031	0.316	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	429	0	0	0	0	-1
normalized size	1	1.00	2.73	7.27	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.047	0.256	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	890	0	0	0	0	-1
normalized size	1	1.00	2.27	14.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.118	0.259	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	339	738	0	0	0	0	-1
normalized size	1	1.00	5.30	11.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.320	0.249	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	100	527	0	440	0	103	115
normalized size	1	1.00	0.93	4.93	0.00	4.11	0.00	0.96	1.07
time (sec)	N/A	0.121	0.080	0.341	0.000	1.680	0.000	0.183	6.457

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	88	487	0	326	0	78	94
normalized size	1	1.00	1.07	5.94	0.00	3.98	0.00	0.95	1.15
time (sec)	N/A	0.074	0.121	0.271	0.000	1.544	0.000	0.191	5.990

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	52	463	0	236	66	73	89
normalized size	1	1.00	0.68	6.01	0.00	3.06	0.86	0.95	1.16
time (sec)	N/A	0.066	0.026	0.229	0.000	1.243	32.782	0.189	5.848

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	89	512	0	790	104	111	139
normalized size	1	1.00	0.78	4.49	0.00	6.93	0.91	0.97	1.22
time (sec)	N/A	0.113	0.048	0.286	0.000	1.191	24.626	0.166	8.444

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	117	575	0	1120	0	173	597
normalized size	1	1.00	0.74	3.64	0.00	7.09	0.00	1.09	3.78
time (sec)	N/A	0.222	0.058	0.408	0.000	1.593	0.000	0.174	10.472

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	231	1069	0	0	0	0	-1
normalized size	1	1.00	3.45	15.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.210	0.390	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	142	907	0	0	0	0	-1
normalized size	1	1.00	2.12	13.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.123	0.365	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	338	753	0	0	0	0	-1
normalized size	1	1.00	5.45	12.15	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.327	0.263	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	1392	0	0	0	0	-1
normalized size	1	1.00	2.97	21.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.237	0.252	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	425	1084	0	0	0	0	-1
normalized size	1	1.00	6.34	16.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.726	0.233	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	101	952	107	219	0	110	127
normalized size	1	1.00	0.86	8.14	0.91	1.87	0.00	0.94	1.09
time (sec)	N/A	0.094	0.106	0.276	1.238	0.909	0.000	0.164	4.087

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	90	892	91	191	0	93	107
normalized size	1	1.00	0.88	8.75	0.89	1.87	0.00	0.91	1.05
time (sec)	N/A	0.076	0.086	0.207	1.220	0.852	0.000	0.184	4.013

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	874	79	165	0	69	87
normalized size	1	1.00	0.96	10.66	0.96	2.01	0.00	0.84	1.06
time (sec)	N/A	0.059	0.042	0.166	1.255	0.868	0.000	0.159	3.985

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	439	66	149	0	53	72
normalized size	1	1.00	0.95	6.86	1.03	2.33	0.00	0.83	1.12
time (sec)	N/A	0.049	0.062	0.179	1.392	0.868	0.000	0.159	3.926

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	102	912	0	226	0	79	76
normalized size	1	1.00	1.16	10.36	0.00	2.57	0.00	0.90	0.86
time (sec)	N/A	0.072	0.049	0.179	0.000	0.974	0.000	0.160	3.981

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	97	957	0	278	0	113	117
normalized size	1	1.00	0.78	7.72	0.00	2.24	0.00	0.91	0.94
time (sec)	N/A	0.102	0.151	0.211	0.000	0.875	0.000	0.172	4.212

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	112	1020	0	310	0	105	154
normalized size	1	1.00	0.68	6.22	0.00	1.89	0.00	0.64	0.94
time (sec)	N/A	0.131	0.211	0.188	0.000	0.927	0.000	0.181	4.453

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	663	663	176	2198	0	0	0	0	-1
normalized size	1	1.00	0.27	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.851	0.197	0.252	0.000	24.845	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1740	0	0	0	0	-1
normalized size	1	1.00	0.26	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	0.151	0.180	0.000	4.847	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	164	882	0	0	0	0	-1
normalized size	1	1.00	0.25	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	0.095	0.162	0.000	1.174	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	179	2193	0	0	0	0	-1
normalized size	1	1.00	0.27	3.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.824	0.166	0.206	0.000	1.437	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	199	2671	0	0	0	0	-1
normalized size	1	1.00	0.29	3.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.944	0.141	0.182	0.000	3.860	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	209	3169	0	0	0	0	-1
normalized size	1	1.00	0.29	4.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.071	0.165	0.187	0.000	8.645	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	111	998	119	239	0	127	147
normalized size	1	1.00	0.83	7.45	0.89	1.78	0.00	0.95	1.10
time (sec)	N/A	0.110	0.082	0.275	1.183	0.907	0.000	0.173	4.104

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	102	920	107	219	0	111	127
normalized size	1	1.00	0.86	7.73	0.90	1.84	0.00	0.93	1.07
time (sec)	N/A	0.094	0.126	0.189	1.391	0.722	0.000	0.197	4.052

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	90	902	93	192	0	93	107
normalized size	1	1.00	0.93	9.30	0.96	1.98	0.00	0.96	1.10
time (sec)	N/A	0.075	0.061	0.166	1.333	0.620	0.000	0.169	4.044

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	43	451	79	162	0	69	87
normalized size	1	1.00	0.56	5.86	1.03	2.10	0.00	0.90	1.13
time (sec)	N/A	0.060	0.015	0.177	1.311	0.792	0.000	0.169	3.987

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	100	956	0	220	0	70	101
normalized size	1	1.00	1.18	11.25	0.00	2.59	0.00	0.82	1.19
time (sec)	N/A	0.079	0.055	0.174	0.000	0.628	0.000	0.169	4.722

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	1014	0	280	0	114	110
normalized size	1	1.00	0.80	8.38	0.00	2.31	0.00	0.94	0.91
time (sec)	N/A	0.103	0.156	0.189	0.000	0.597	0.000	0.188	4.235

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	112	1075	0	310	0	129	151
normalized size	1	1.00	0.70	6.68	0.00	1.93	0.00	0.80	0.94
time (sec)	N/A	0.133	0.240	0.189	0.000	0.668	0.000	0.178	4.618

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	681	681	191	2223	0	0	0	0	-1
normalized size	1	1.00	0.28	3.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.939	0.170	0.243	0.000	50.729	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	176	1747	0	0	0	0	-1
normalized size	1	1.00	0.27	2.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.805	0.183	0.180	0.000	14.010	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	638	141	873	0	0	0	0	-1
normalized size	1	1.00	0.22	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	0.204	0.191	0.000	3.566	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	242	2217	0	0	0	0	-1
normalized size	1	1.00	0.46	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.699	0.189	0.000	0.896	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	684	684	199	2690	0	0	0	0	-1
normalized size	1	1.00	0.29	3.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.940	0.153	0.204	0.000	3.711	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	212	3186	0	0	0	0	-1
normalized size	1	1.00	0.30	4.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.076	0.153	0.184	0.000	5.738	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	91	916	93	195	0	93	107
normalized size	1	1.00	0.96	9.64	0.98	2.05	0.00	0.98	1.13
time (sec)	N/A	0.073	0.048	0.296	1.344	0.637	0.000	0.194	4.061

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	874	79	167	0	69	87
normalized size	1	1.00	0.99	10.53	0.95	2.01	0.00	0.83	1.05
time (sec)	N/A	0.064	0.053	0.199	1.561	0.635	0.000	0.174	3.997

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	861	67	155	0	58	72
normalized size	1	1.00	0.98	13.45	1.05	2.42	0.00	0.91	1.12
time (sec)	N/A	0.051	0.052	0.174	1.322	0.718	0.000	0.162	4.009

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	442	72	153	0	59	75
normalized size	1	1.00	0.96	6.60	1.07	2.28	0.00	0.88	1.12
time (sec)	N/A	0.049	0.037	0.190	1.389	0.542	0.000	0.157	3.977

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	880	0	226	0	79	80
normalized size	1	1.00	0.94	10.00	0.00	2.57	0.00	0.90	0.91
time (sec)	N/A	0.075	0.096	0.186	0.000	0.765	0.000	0.190	4.008

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	97	926	0	280	0	114	117
normalized size	1	1.00	0.78	7.47	0.00	2.26	0.00	0.92	0.94
time (sec)	N/A	0.101	0.149	0.197	0.000	0.616	0.000	0.163	4.111

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	112	989	0	310	0	128	155
normalized size	1	1.00	0.68	6.03	0.00	1.89	0.00	0.78	0.95
time (sec)	N/A	0.129	0.208	0.197	0.000	0.785	0.000	0.181	4.368

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	641	641	167	1737	0	0	0	0	-1
normalized size	1	1.00	0.26	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	0.185	0.309	0.000	9.697	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	166	1304	0	0	0	0	-1
normalized size	1	1.00	0.26	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.715	0.104	0.184	0.000	1.366	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	164	882	0	0	0	0	-1
normalized size	1	1.00	0.25	1.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.699	0.091	0.187	0.000	1.746	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	180	1761	0	0	0	0	-1
normalized size	1	1.00	0.27	2.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.828	0.173	0.217	0.000	1.637	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	196	2240	0	0	0	0	-1
normalized size	1	1.00	0.29	3.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.927	0.198	0.239	0.000	6.384	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	711	711	212	2738	0	0	0	0	-1
normalized size	1	1.00	0.30	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.044	0.232	0.191	0.000	15.149	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	239	1431	0	0	0	0	-1
normalized size	1	1.00	3.62	21.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.331	0.285	0.000	1.827	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	237	1150	0	0	0	0	-1
normalized size	1	1.00	3.59	17.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.233	0.258	0.000	1.517	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	237	728	0	0	0	0	-1
normalized size	1	1.00	3.70	11.38	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.188	0.177	0.000	2.108	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	266	1455	0	0	0	0	-1
normalized size	1	1.00	4.03	22.05	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.205	0.200	0.000	4.690	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	279	1782	0	0	0	0	-1
normalized size	1	1.00	4.23	27.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.294	0.179	0.000	9.920	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	88	970	98	233	0	88	111
normalized size	1	1.00	0.93	10.21	1.03	2.45	0.00	0.93	1.17
time (sec)	N/A	0.076	0.046	0.288	1.212	0.625	0.000	0.189	4.383

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	71	926	81	223	0	67	94
normalized size	1	1.00	0.86	11.16	0.98	2.69	0.00	0.81	1.13
time (sec)	N/A	0.065	0.105	0.178	1.412	0.617	0.000	0.175	4.300

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	68	908	83	223	0	76	96
normalized size	1	1.00	0.80	10.68	0.98	2.62	0.00	0.89	1.13
time (sec)	N/A	0.062	0.025	0.183	1.234	0.589	0.000	0.189	4.264

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	43	463	85	219	0	72	97
normalized size	1	1.00	0.49	5.26	0.97	2.49	0.00	0.82	1.10
time (sec)	N/A	0.064	0.013	0.190	1.270	0.660	0.000	0.161	4.261

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	97	953	0	316	0	93	101
normalized size	1	1.00	0.92	8.99	0.00	2.98	0.00	0.88	0.95
time (sec)	N/A	0.095	0.045	0.171	0.000	0.724	0.000	0.172	4.332

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	1019	0	368	0	129	133
normalized size	1	1.00	0.82	7.13	0.00	2.57	0.00	0.90	0.93
time (sec)	N/A	0.125	0.064	0.195	0.000	0.654	0.000	0.165	4.562

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	135	1106	0	398	0	149	171
normalized size	1	1.00	0.73	5.98	0.00	2.15	0.00	0.81	0.92
time (sec)	N/A	0.163	0.064	0.210	0.000	0.651	0.000	0.181	4.763

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	668	668	168	2255	0	0	0	0	-1
normalized size	1	1.00	0.25	3.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	0.137	0.259	0.000	1.612	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	671	671	169	1788	0	0	0	0	-1
normalized size	1	1.00	0.25	2.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.800	0.131	0.187	0.000	1.940	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	665	665	167	903	0	0	0	0	-1
normalized size	1	1.00	0.25	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.799	0.139	0.177	0.000	2.055	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	180	2269	0	0	0	0	-1
normalized size	1	1.00	0.26	3.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.923	0.144	0.197	0.000	2.536	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	708	708	198	2774	0	0	0	0	-1
normalized size	1	1.00	0.28	3.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.032	0.198	0.194	0.000	8.019	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	732	732	210	3299	0	0	0	0	-1
normalized size	1	1.00	0.29	4.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.178	0.234	0.234	0.000	15.470	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	66	189	1791	0	0	0	0	-1
normalized size	1	0.26	0.74	7.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.060	0.673	0.320	0.000	0.710	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	242	1478	0	0	0	0	-1
normalized size	1	1.00	3.67	22.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.317	0.232	0.000	3.223	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	253	747	0	0	0	0	-1
normalized size	1	1.00	3.95	11.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.332	0.174	0.000	3.407	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	259	1805	0	0	0	0	-1
normalized size	1	1.00	3.92	27.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.212	0.167	0.000	7.488	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	283	2156	0	0	0	0	-1
normalized size	1	1.00	4.29	32.67	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.284	0.185	0.000	21.302	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	147	917	0	469	0	136	202
normalized size	1	1.00	0.91	5.70	0.00	2.91	0.00	0.84	1.25
time (sec)	N/A	0.192	0.291	0.359	0.000	0.838	0.000	0.179	6.842

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	117	897	0	334	0	102	152
normalized size	1	1.00	0.86	6.60	0.00	2.46	0.00	0.75	1.12
time (sec)	N/A	0.109	0.154	0.263	0.000	0.839	0.000	0.172	6.087

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	453	0	255	0	79	125
normalized size	1	1.00	1.00	5.66	0.00	3.19	0.00	0.99	1.56
time (sec)	N/A	0.065	0.114	0.270	0.000	0.769	0.000	0.202	5.686

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	112	934	0	856	0	114	182
normalized size	1	1.00	0.93	7.72	0.00	7.07	0.00	0.94	1.50
time (sec)	N/A	0.118	0.294	0.285	0.000	0.894	0.000	0.171	8.281

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	190	978	0	870	0	183	438
normalized size	1	1.00	1.18	6.07	0.00	5.40	0.00	1.14	2.72
time (sec)	N/A	0.218	0.283	0.262	0.000	0.820	0.000	0.215	9.692

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	235	1468	0	0	0	0	-1
normalized size	1	1.00	3.67	22.94	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.272	0.355	0.000	0.000	0.000	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	153	908	0	0	0	0	-1
normalized size	1	1.00	2.39	14.19	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.130	0.361	0.000	0.000	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	753	0	0	0	0	-1
normalized size	1	1.00	3.93	12.76	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.251	0.266	0.000	0.000	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	172	2227	0	0	0	0	-1
normalized size	1	1.00	2.77	35.92	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.163	0.373	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	1768	0	0	0	0	-1
normalized size	1	1.00	5.28	27.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.359	0.264	0.000	0.000	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	162	1003	0	443	0	211	331
normalized size	1	1.00	0.86	5.31	0.00	2.34	0.00	1.12	1.75
time (sec)	N/A	0.237	0.185	0.367	0.000	1.321	0.000	0.177	7.752

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	125	983	0	314	0	173	229
normalized size	1	1.00	0.77	6.03	0.00	1.93	0.00	1.06	1.40
time (sec)	N/A	0.137	0.136	0.269	0.000	1.196	0.000	0.187	7.379

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	54	466	0	234	0	122	170
normalized size	1	1.00	0.57	4.96	0.00	2.49	0.00	1.30	1.81
time (sec)	N/A	0.081	0.024	0.266	0.000	0.983	0.000	0.169	7.354

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	122	1036	0	686	0	155	214
normalized size	1	1.00	0.93	7.91	0.00	5.24	0.00	1.18	1.63
time (sec)	N/A	0.140	0.186	0.267	0.000	1.259	0.000	0.174	9.143

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	142	1093	0	838	0	216	531
normalized size	1	1.00	0.84	6.43	0.00	4.93	0.00	1.27	3.12
time (sec)	N/A	0.258	0.333	0.248	0.000	1.167	0.000	0.217	10.816

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	338	1587	0	0	0	0	-1
normalized size	1	1.00	5.20	24.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.479	0.349	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	177	955	0	0	0	0	-1
normalized size	1	1.00	2.72	14.69	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.041	0.263	0.355	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	339	801	0	0	0	0	-1
normalized size	1	1.00	5.65	13.35	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.426	0.259	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	190	2364	0	0	0	0	-1
normalized size	1	1.00	3.02	37.52	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.206	0.265	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	370	1902	0	0	0	0	-1
normalized size	1	1.00	5.69	29.26	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.378	0.245	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	911	0	475	0	134	160
normalized size	1	1.00	0.87	7.41	0.00	3.86	0.00	1.09	1.30
time (sec)	N/A	0.144	0.276	0.424	0.000	1.388	0.000	0.226	7.290

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	98	892	0	348	0	116	111
normalized size	1	1.00	0.99	9.01	0.00	3.52	0.00	1.17	1.12
time (sec)	N/A	0.084	0.097	0.272	0.000	0.980	0.000	0.166	6.846

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	457	0	302	0	93	104
normalized size	1	1.00	0.98	5.25	0.00	3.47	0.00	1.07	1.20
time (sec)	N/A	0.073	0.104	0.256	0.000	0.901	0.000	0.164	6.373

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	915	0	862	0	139	162
normalized size	1	1.00	0.93	6.93	0.00	6.53	0.00	1.05	1.23
time (sec)	N/A	0.139	0.265	0.276	0.000	1.151	0.000	0.163	9.647

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	961	0	1236	0	257	355
normalized size	1	1.00	0.88	5.19	0.00	6.68	0.00	1.39	1.92
time (sec)	N/A	0.243	0.611	0.287	0.000	1.281	0.000	0.172	11.551

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	238	1207	0	0	0	0	-1
normalized size	1	1.00	3.72	18.86	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.209	0.370	0.000	0.000	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	923	0	0	0	0	-1
normalized size	1	1.00	2.69	14.42	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.163	0.398	0.000	0.000	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	392	769	0	0	0	0	-1
normalized size	1	1.00	6.64	13.03	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.307	0.279	0.000	0.000	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	1818	0	0	0	0	-1
normalized size	1	1.00	3.65	29.32	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.255	0.267	0.000	0.000	0.000	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	411	1512	0	0	0	0	-1
normalized size	1	1.00	6.42	23.62	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.561	0.265	0.000	0.000	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	149	134	978	0	746	0	195	367
normalized size	1	0.99	0.89	6.52	0.00	4.97	0.00	1.30	2.45
time (sec)	N/A	0.195	0.298	0.364	0.000	1.354	0.000	0.185	7.898

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	91	958	0	630	0	181	247
normalized size	1	1.00	0.68	7.15	0.00	4.70	0.00	1.35	1.84
time (sec)	N/A	0.114	0.033	0.274	0.000	1.392	0.000	0.196	7.704

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	54	485	0	450	0	153	199
normalized size	1	1.00	0.50	4.49	0.00	4.17	0.00	1.42	1.84
time (sec)	N/A	0.091	0.018	0.249	0.000	1.424	0.000	0.207	7.429

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	123	1002	0	1819	0	226	288
normalized size	1	1.00	0.72	5.83	0.00	10.58	0.00	1.31	1.67
time (sec)	N/A	0.223	0.127	0.273	0.000	1.756	0.000	0.185	12.182

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	189	1067	0	2384	0	367	18847
normalized size	1	1.00	0.78	4.43	0.00	9.89	0.00	1.52	78.20
time (sec)	N/A	0.357	0.156	0.246	0.000	2.065	0.000	0.222	19.626

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	381	1593	0	0	0	0	-1
normalized size	1	1.00	5.69	23.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.359	0.337	0.000	0.000	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	216	986	0	0	0	0	-1
normalized size	1	1.00	3.22	14.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.298	0.346	0.000	0.000	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	381	830	0	0	0	0	-1
normalized size	1	1.00	6.15	13.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.548	0.265	0.000	0.000	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	308	2383	0	0	0	0	-1
normalized size	1	1.00	4.74	36.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.433	0.297	0.000	0.000	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	515	1919	0	0	0	0	-1
normalized size	1	1.00	7.69	28.64	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.964	0.258	0.000	0.000	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	113	0	0	0	388	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	2.90	0.00	-0.01
time (sec)	N/A	0.079	0.124	0.451	0.000	1.040	44.368	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	111	0	0	0	252	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	1.91	0.00	-0.01
time (sec)	N/A	0.074	0.090	0.413	0.000	1.266	21.291	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	122	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.93	0.00	-0.01
time (sec)	N/A	0.074	0.090	0.410	0.000	1.212	5.811	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	110	0	0	0	119	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.91	0.00	-0.01
time (sec)	N/A	0.071	0.106	0.427	0.000	1.216	4.871	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	119	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.077	0.108	0.381	0.000	1.295	88.236	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	113	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	0.134	0.398	0.000	1.066	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	123	0	0	256	0	104	283
normalized size	1	1.00	1.40	0.00	0.00	2.91	0.00	1.18	3.22
time (sec)	N/A	0.093	0.266	0.177	0.000	0.874	0.000	0.211	9.251

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	85	0	0	194	0	54	49
normalized size	1	1.00	1.77	0.00	0.00	4.04	0.00	1.12	1.02
time (sec)	N/A	0.059	0.096	0.652	0.000	0.890	0.000	0.184	5.043

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	204	0	89	136
normalized size	1	1.00	1.00	0.00	0.00	4.25	0.00	1.85	2.83
time (sec)	N/A	0.048	0.019	0.585	0.000	0.912	0.000	0.190	7.572

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	91	0	0	278	0	413	481
normalized size	1	1.00	1.00	0.00	0.00	3.05	0.00	4.54	5.29
time (sec)	N/A	0.080	0.065	0.122	0.000	0.991	0.000	0.235	10.775

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.059	0.714	0.000	0.952	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.054	0.612	0.000	0.705	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.049	0.603	0.000	1.136	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	170	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.327	0.592	0.000	0.698	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	189	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.256	0.180	0.000	0.850	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	365	0	0	0	0	0	-1
normalized size	1	1.00	4.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.272	0.195	0.000	0.915	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	145	7293	0	295	292	251	-1
normalized size	1	1.00	0.90	45.30	0.00	1.83	1.81	1.56	-0.01
time (sec)	N/A	0.113	0.242	8.015	0.000	1.874	130.857	0.754	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	112	4175	0	0	97	0	-1
normalized size	1	1.00	0.35	12.89	0.00	0.00	0.30	0.00	-0.00
time (sec)	N/A	0.320	0.155	1.116	0.000	0.730	45.347	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	94	5358	0	0	97	0	-1
normalized size	1	1.00	0.16	9.22	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.621	0.113	1.057	0.000	0.976	14.975	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	6858	0	221	201	137	-1
normalized size	1	1.00	0.98	56.68	0.00	1.83	1.66	1.13	-0.01
time (sec)	N/A	0.089	0.145	1.092	0.000	1.885	9.670	0.419	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	93	3721	0	0	97	0	-1
normalized size	1	1.00	0.33	13.01	0.00	0.00	0.34	0.00	-0.00
time (sec)	N/A	0.229	0.067	1.214	0.000	0.741	4.841	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	98	5736	0	0	100	0	-1
normalized size	1	1.00	0.17	9.89	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.592	0.070	1.019	0.000	1.007	5.504	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	87	6668	0	207	160	0	-1
normalized size	1	1.00	0.74	56.51	0.00	1.75	1.36	0.00	-0.01
time (sec)	N/A	0.084	0.209	1.040	0.000	1.896	9.715	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	97	3512	0	0	100	0	-1
normalized size	1	1.00	0.34	12.41	0.00	0.00	0.35	0.00	-0.00
time (sec)	N/A	0.227	0.066	1.016	0.000	0.999	25.041	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	564	564	81	5911	0	0	97	0	-1
normalized size	1	1.00	0.14	10.48	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.530	0.105	1.729	0.000	1.019	22.005	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	87	3759	81	180	131	109	-1
normalized size	1	1.00	1.10	47.58	1.03	2.28	1.66	1.38	-0.01
time (sec)	N/A	0.046	0.181	0.999	1.217	1.370	61.862	0.246	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	80	3690	0	0	97	0	-1
normalized size	1	1.00	0.30	13.72	0.00	0.00	0.36	0.00	-0.00
time (sec)	N/A	0.199	0.090	1.100	0.000	0.982	172.406	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	167	7705	0	355	0	387	-1
normalized size	1	1.00	0.83	38.33	0.00	1.77	0.00	1.93	-0.00
time (sec)	N/A	0.139	0.335	1.142	0.000	1.961	0.000	1.455	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	116	4619	0	0	199	0	-1
normalized size	1	1.00	0.32	12.69	0.00	0.00	0.55	0.00	-0.00
time (sec)	N/A	0.309	0.174	1.316	0.000	0.968	125.430	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	96	5790	0	0	199	0	-1
normalized size	1	1.00	0.15	9.32	0.00	0.00	0.32	0.00	-0.00
time (sec)	N/A	0.661	0.134	0.958	0.000	0.881	45.758	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	143	7290	0	273	335	0	-1
normalized size	1	1.00	0.89	45.28	0.00	1.70	2.08	0.00	-0.01
time (sec)	N/A	0.113	0.182	1.064	0.000	1.904	25.464	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	96	4173	0	0	199	0	-1
normalized size	1	1.00	0.30	12.88	0.00	0.00	0.61	0.00	-0.00
time (sec)	N/A	0.256	0.077	1.171	0.000	0.903	14.830	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	614	84	6142	0	0	202	0	-1
normalized size	1	1.00	0.14	10.00	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.644	0.121	1.142	0.000	0.972	15.709	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	126	7108	0	255	289	0	-1
normalized size	1	1.00	0.83	46.76	0.00	1.68	1.90	0.00	-0.01
time (sec)	N/A	0.104	0.154	1.183	0.000	1.977	24.317	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	85	3966	0	0	202	0	-1
normalized size	1	1.00	0.27	12.63	0.00	0.00	0.64	0.00	-0.00
time (sec)	N/A	0.247	0.078	0.975	0.000	0.940	41.464	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	188	8117	0	409	0	563	-1
normalized size	1	1.00	0.78	33.68	0.00	1.70	0.00	2.34	-0.00
time (sec)	N/A	0.159	0.314	1.065	0.000	1.807	0.000	1.488	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	116	5063	0	0	0	0	-1
normalized size	1	1.00	0.29	12.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.344	0.194	1.094	0.000	1.020	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	661	661	99	6202	0	0	308	0	-1
normalized size	1	1.00	0.15	9.38	0.00	0.00	0.47	0.00	-0.00
time (sec)	N/A	0.708	0.180	1.036	0.000	0.956	104.339	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	146	7702	0	323	413	0	-1
normalized size	1	1.00	0.73	38.32	0.00	1.61	2.05	0.00	-0.00
time (sec)	N/A	0.127	0.426	1.104	0.000	2.253	56.284	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	84	4617	0	0	308	0	-1
normalized size	1	1.00	0.23	12.68	0.00	0.00	0.85	0.00	-0.00
time (sec)	N/A	0.284	0.067	1.064	0.000	1.038	40.001	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	87	6530	0	0	311	0	-1
normalized size	1	1.00	0.13	10.05	0.00	0.00	0.48	0.00	-0.00
time (sec)	N/A	0.698	0.046	1.147	0.000	0.863	39.745	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	150	7544	0	309	403	0	-1
normalized size	1	1.00	0.80	40.13	0.00	1.64	2.14	0.00	-0.01
time (sec)	N/A	0.128	0.213	1.157	0.000	2.036	63.945	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	88	4422	0	0	311	0	-1
normalized size	1	1.00	0.25	12.56	0.00	0.00	0.88	0.00	-0.00
time (sec)	N/A	0.287	0.045	1.102	0.000	0.885	85.783	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	97	6861	0	245	194	114	-1
normalized size	1	1.00	0.80	56.70	0.00	2.02	1.60	0.94	-0.01
time (sec)	N/A	0.086	0.169	1.010	0.000	2.098	95.303	0.365	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	98	3723	0	0	94	0	-1
normalized size	1	1.00	0.34	13.02	0.00	0.00	0.33	0.00	-0.00
time (sec)	N/A	0.222	0.149	1.030	0.000	1.189	32.075	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	80	4914	0	0	94	0	-1
normalized size	1	1.00	0.15	9.05	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.501	0.130	1.093	0.000	0.896	10.950	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	6424	0	184	107	72	-1
normalized size	1	1.00	0.94	77.40	0.00	2.22	1.29	0.87	-0.01
time (sec)	N/A	0.067	0.059	1.148	0.000	1.890	6.806	0.363	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	80	3275	0	0	94	0	-1
normalized size	1	1.00	0.32	13.15	0.00	0.00	0.38	0.00	-0.00
time (sec)	N/A	0.183	0.100	1.093	0.000	0.955	3.538	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	83	5385	0	0	97	0	-1
normalized size	1	1.00	0.15	9.94	0.00	0.00	0.18	0.00	-0.00
time (sec)	N/A	0.505	0.054	1.022	0.000	0.785	4.266	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	3397	0	183	60	110	-1
normalized size	1	1.00	0.87	45.29	0.00	2.44	0.80	1.47	-0.01
time (sec)	N/A	0.057	0.064	1.003	0.000	1.496	10.112	0.260	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	82	3303	0	0	97	0	-1
normalized size	1	1.00	0.33	13.43	0.00	0.00	0.39	0.00	-0.00
time (sec)	N/A	0.183	0.057	1.072	0.000	0.872	32.024	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	109	7016	0	307	0	107	-1
normalized size	1	1.00	0.91	58.47	0.00	2.56	0.00	0.89	-0.01
time (sec)	N/A	0.086	0.176	1.132	0.000	1.993	0.000	0.366	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	87	3760	0	0	0	0	-1
normalized size	1	1.00	0.30	13.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.169	1.079	0.000	0.742	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	77	5392	0	0	0	0	-1
normalized size	1	1.00	0.14	9.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.503	0.119	1.032	0.000	0.921	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	93	3654	0	234	95	75	-1
normalized size	1	1.00	1.09	42.99	0.00	2.75	1.12	0.88	-0.01
time (sec)	N/A	0.062	0.099	1.000	0.000	1.302	33.620	0.619	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	79	3565	0	0	94	0	-1
normalized size	1	1.00	0.31	13.82	0.00	0.00	0.36	0.00	-0.00
time (sec)	N/A	0.188	0.063	1.034	0.000	0.980	72.364	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	585	585	77	5563	0	0	97	0	-1
normalized size	1	1.00	0.13	9.51	0.00	0.00	0.17	0.00	-0.00
time (sec)	N/A	0.571	0.051	1.077	0.000	0.740	144.001	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	39	0	57	0	0	70
normalized size	1	1.00	0.67	0.58	0.00	0.85	0.00	0.00	1.04
time (sec)	N/A	0.029	0.023	0.047	0.000	0.911	0.000	0.000	4.747

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	95	3783	0	0	0	0	-1
normalized size	1	1.00	0.34	13.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.058	1.092	0.000	0.544	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	119	7081	0	345	0	102	-1
normalized size	1	1.00	1.04	62.11	0.00	3.03	0.00	0.89	-0.01
time (sec)	N/A	0.075	0.303	1.053	0.000	1.351	0.000	0.476	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	108	7083	0	0	0	0	-1
normalized size	1	1.00	0.36	23.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.229	0.225	0.997	0.000	0.861	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	86	10786	0	0	0	0	-1
normalized size	1	1.00	0.14	18.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.124	1.068	0.000	0.990	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	44	39	0	59	0	64	73
normalized size	1	1.00	0.56	0.49	0.00	0.75	0.00	0.81	0.92
time (sec)	N/A	0.031	0.039	0.049	0.000	0.592	0.000	0.324	4.628

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	107	7077	0	0	0	0	-1
normalized size	1	1.00	0.36	23.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.107	1.114	0.000	0.925	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	85	10961	0	0	0	0	-1
normalized size	1	1.00	0.14	17.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.639	0.049	1.141	0.000	0.878	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	65	62	0	93	0	0	115
normalized size	1	1.00	0.62	0.60	0.00	0.89	0.00	0.00	1.11
time (sec)	N/A	0.046	0.036	0.049	0.000	0.862	0.000	0.000	4.804

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	121	7299	0	0	0	0	-1
normalized size	1	1.00	0.38	22.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.258	0.088	1.056	0.000	0.901	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	230	0	183	186	0	0	240
normalized size	1	1.00	1.05	0.00	0.83	0.85	0.00	0.00	1.09
time (sec)	N/A	0.236	0.259	0.654	1.554	0.895	0.000	0.000	4.809

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	207	0	155	174	0	0	219
normalized size	1	1.00	1.19	0.00	0.89	1.00	0.00	0.00	1.26
time (sec)	N/A	0.178	0.139	0.631	1.406	0.868	0.000	0.000	4.650

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	186	0	153	157	0	0	200
normalized size	1	1.00	1.08	0.00	0.89	0.91	0.00	0.00	1.16
time (sec)	N/A	0.149	0.096	0.636	1.434	0.638	0.000	0.000	4.661

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	167	0	139	144	0	0	194
normalized size	1	1.00	1.11	0.00	0.93	0.96	0.00	0.00	1.29
time (sec)	N/A	0.119	0.053	0.636	1.271	0.932	0.000	0.000	4.638

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	233	0	0	1046	0	0	345
normalized size	1	1.00	1.09	0.00	0.00	4.89	0.00	0.00	1.61
time (sec)	N/A	0.174	0.107	0.598	0.000	0.741	0.000	0.000	5.097

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	280	0	0	321	0	0	455
normalized size	1	1.00	1.04	0.00	0.00	1.20	0.00	0.00	1.70
time (sec)	N/A	0.247	0.117	0.615	0.000	1.022	0.000	0.000	5.345

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	314	0	0	345	0	0	490
normalized size	1	1.00	1.11	0.00	0.00	1.22	0.00	0.00	1.73
time (sec)	N/A	0.264	0.138	0.652	0.000	0.941	0.000	0.000	5.442

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	268	66	177	0	0	362	0	0	-1
normalized size	1	0.25	0.66	0.00	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.060	0.210	0.649	0.000	1.068	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	66	160	0	0	338	0	0	-1
normalized size	1	0.28	0.69	0.00	0.00	1.45	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.166	0.737	0.000	1.015	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	66	63	0	0	313	0	0	-1
normalized size	1	0.33	0.31	0.00	0.00	1.56	0.00	0.00	-0.00
time (sec)	N/A	0.043	0.041	0.710	0.000	0.973	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	77	45	0	0	0	0	0	-1
normalized size	1	0.49	0.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.018	0.842	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	117	125	0	0	0	0	0	-1
normalized size	1	0.64	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.422	5.104	0.622	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	244	135	0	0	0	0	0	-1
normalized size	1	1.16	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	19.780	5.122	0.612	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	237	423	130	0	0	0	0	0	-1
normalized size	1	1.78	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	29.209	5.201	0.625	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	66	234	0	0	0	0	0	-1
normalized size	1	0.13	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.377	0.584	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	66	225	0	0	0	0	0	-1
normalized size	1	0.13	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.062	0.212	0.609	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	416	61	154	0	0	0	0	0	-1
normalized size	1	0.15	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.031	0.154	0.631	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	66	231	0	0	0	0	0	-1
normalized size	1	0.13	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.064	0.149	0.614	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	66	243	0	0	0	0	0	-1
normalized size	1	0.13	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.068	0.166	0.645	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	163	0	183	209	0	0	261
normalized size	1	1.00	0.73	0.00	0.82	0.94	0.00	0.00	1.17
time (sec)	N/A	0.250	0.276	0.576	1.252	1.283	0.000	0.000	4.846

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	153	0	155	197	0	0	206
normalized size	1	1.00	0.86	0.00	0.88	1.11	0.00	0.00	1.16
time (sec)	N/A	0.200	0.172	0.575	1.169	1.154	0.000	0.000	4.915

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	143	0	155	181	0	0	221
normalized size	1	1.00	0.82	0.00	0.89	1.03	0.00	0.00	1.26
time (sec)	N/A	0.156	0.147	0.605	1.120	1.221	0.000	0.000	4.839

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	130	0	140	167	0	0	186
normalized size	1	1.00	0.85	0.00	0.92	1.09	0.00	0.00	1.22
time (sec)	N/A	0.126	0.092	0.622	1.314	1.441	0.000	0.000	4.830

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	164	0	0	530	0	0	369
normalized size	1	1.00	0.77	0.00	0.00	2.48	0.00	0.00	1.72
time (sec)	N/A	0.182	0.080	0.682	0.000	1.575	0.000	0.000	5.855

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	213	0	0	612	0	0	490
normalized size	1	1.00	0.79	0.00	0.00	2.28	0.00	0.00	1.82
time (sec)	N/A	0.263	0.093	0.624	0.000	1.651	0.000	0.000	5.561

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	247	0	0	660	0	0	513
normalized size	1	1.00	0.87	0.00	0.00	2.32	0.00	0.00	1.81
time (sec)	N/A	0.279	0.156	0.574	0.000	0.881	0.000	0.000	5.455

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	66	244	0	0	701	0	0	-1
normalized size	1	0.25	0.92	0.00	0.00	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.305	0.595	0.000	0.770	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	66	216	0	0	653	0	0	-1
normalized size	1	0.29	0.94	0.00	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.063	0.217	0.583	0.000	1.293	0.000	0.000	0.000

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	61	156	0	0	611	0	0	-1
normalized size	1	0.30	0.78	0.00	0.00	3.06	0.00	0.00	-0.00
time (sec)	N/A	0.033	0.163	0.630	0.000	1.135	0.000	0.000	0.000

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	79	47	0	0	0	0	0	-1
normalized size	1	0.50	0.30	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.040	0.635	0.000	0.000	0.000	0.000	0.000

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	121	170	0	0	0	0	0	-1
normalized size	1	0.66	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.429	5.203	0.635	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	209	244	179	0	0	0	0	0	-1
normalized size	1	1.17	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	10.676	5.268	0.640	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	391	196	0	0	0	0	0	-1
normalized size	1	1.66	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	17.601	5.350	0.580	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	66	147	0	0	0	0	0	-1
normalized size	1	0.13	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.066	0.114	0.595	0.000	0.000	0.000	0.000	0.000

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	66	127	0	0	0	0	0	-1
normalized size	1	0.14	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.066	0.174	0.637	0.000	0.000	0.000	0.000	0.000

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	457	66	63	0	0	0	0	0	-1
normalized size	1	0.14	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.047	0.043	0.608	0.000	0.000	0.000	0.000	0.000

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	64	136	0	0	0	0	0	-1
normalized size	1	0.13	0.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	0.112	0.650	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	512	66	148	0	0	0	0	0	-1
normalized size	1	0.13	0.29	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.069	0.090	0.572	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	113	792	119	118	0	134	133
normalized size	1	1.00	0.89	6.24	0.94	0.93	0.00	1.06	1.05
time (sec)	N/A	0.093	0.104	9.325	1.241	0.754	0.000	0.173	5.034

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	128	683	119	137	0	127	133
normalized size	1	1.00	1.00	5.34	0.93	1.07	0.00	0.99	1.04
time (sec)	N/A	0.088	0.057	8.409	1.248	0.945	0.000	0.176	4.730

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	96	673	97	106	0	98	111
normalized size	1	1.00	0.99	6.94	1.00	1.09	0.00	1.01	1.14
time (sec)	N/A	0.077	0.066	6.727	1.190	1.281	0.000	0.201	4.649

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	671	97	125	0	98	111
normalized size	1	1.00	0.97	6.85	0.99	1.28	0.00	1.00	1.13
time (sec)	N/A	0.064	0.033	3.820	1.214	1.300	0.000	0.176	4.685

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	476	86	90	0	87	100
normalized size	1	1.00	0.89	5.80	1.05	1.10	0.00	1.06	1.22
time (sec)	N/A	0.059	0.025	5.594	1.300	1.282	0.000	0.181	4.894

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	133	0	0	410	0	149	256
normalized size	1	1.00	0.97	0.00	0.00	2.99	0.00	1.09	1.87
time (sec)	N/A	0.091	0.045	2.438	0.000	2.464	0.000	0.197	4.798

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	153	0	0	187	0	163	382
normalized size	1	1.00	0.97	0.00	0.00	1.19	0.00	1.04	2.43
time (sec)	N/A	0.103	0.218	2.822	0.000	0.850	0.000	0.188	4.860

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	226	144	0	0	201	0	0	-1
normalized size	1	1.47	0.94	0.00	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.158	1.916	0.000	0.791	0.000	0.000	0.000

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	207	26	0	0	452	0	0	-1
normalized size	1	1.53	0.19	0.00	0.00	3.35	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.020	1.737	0.000	2.931	0.000	0.000	0.000

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	112	931	0	253	0	0	-1
normalized size	1	1.39	1.27	10.58	0.00	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.080	4.935	0.000	5.062	0.000	0.000	0.000

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	139	82	929	0	307	0	0	-1
normalized size	1	1.32	0.78	8.85	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.597	4.490	0.000	4.990	0.000	0.000	0.000

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	140	123	955	0	283	0	0	-1
normalized size	1	1.13	0.99	7.70	0.00	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.088	5.246	2.747	0.000	4.607	0.000	0.000	0.000

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	175	133	963	0	320	0	0	-1
normalized size	1	1.24	0.94	6.83	0.00	2.27	0.00	0.00	-0.01
time (sec)	N/A	0.092	5.115	4.067	0.000	4.948	0.000	0.000	0.000

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	271	26	40	0	0	0	0	0	-1
normalized size	1	0.10	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.017	0.040	0.593	0.000	4.390	0.000	0.000	0.000

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	26	26	0	0	0	0	0	-1
normalized size	1	0.10	0.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.016	0.019	0.573	0.000	4.695	0.000	0.000	0.000

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	26	26	0	0	373	0	0	-1
normalized size	1	0.11	0.11	0.00	0.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.013	0.017	3.084	0.000	3.175	0.000	0.000	0.000

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	24	67	0	0	0	0	0	-1
normalized size	1	0.09	0.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.017	0.051	0.568	0.000	3.408	0.000	0.000	0.000

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	26	76	0	0	0	0	0	-1
normalized size	1	0.09	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.018	0.057	0.551	0.000	3.331	0.000	0.000	0.000

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	1579	119	142	0	127	135
normalized size	1	1.00	1.21	12.63	0.95	1.14	0.00	1.02	1.08
time (sec)	N/A	0.095	0.107	7.448	1.522	0.651	0.000	0.179	5.185

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	135	1584	97	114	0	98	113
normalized size	1	1.00	1.38	16.16	0.99	1.16	0.00	1.00	1.15
time (sec)	N/A	0.075	0.089	7.461	1.192	0.588	0.000	0.178	4.976

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	118	1582	97	130	0	98	113
normalized size	1	1.00	1.24	16.65	1.02	1.37	0.00	1.03	1.19
time (sec)	N/A	0.062	0.050	5.099	1.384	0.641	0.000	0.234	4.892

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	94	529	86	98	0	87	102
normalized size	1	1.00	1.13	6.37	1.04	1.18	0.00	1.05	1.23
time (sec)	N/A	0.056	0.035	6.280	1.176	0.550	0.000	0.197	5.052

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	179	0	0	182	0	149	344
normalized size	1	1.00	1.31	0.00	0.00	1.33	0.00	1.09	2.51
time (sec)	N/A	0.090	0.052	2.376	0.000	0.749	0.000	0.240	4.921

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	196	0	0	195	0	163	368
normalized size	1	1.00	1.24	0.00	0.00	1.23	0.00	1.03	2.33
time (sec)	N/A	0.103	0.100	3.079	0.000	0.604	0.000	0.215	5.073

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	228	78	0	0	232	0	0	-1
normalized size	1	1.42	0.49	0.00	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.086	2.045	0.000	0.636	0.000	0.000	0.000

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	207	26	0	0	197	0	0	-1
normalized size	1	1.49	0.19	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.051	1.759	0.000	0.681	0.000	0.000	0.000

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	122	38	938	0	283	0	0	-1
normalized size	1	1.39	0.43	10.66	0.00	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.007	4.243	0.000	3.024	0.000	0.000	0.000

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	137	81	1387	0	272	0	0	-1
normalized size	1	1.33	0.79	13.47	0.00	2.64	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.555	4.057	0.000	2.654	0.000	0.000	0.000

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	140	145	1386	0	312	0	0	-1
normalized size	1	1.13	1.17	11.18	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.084	13.899	3.514	0.000	3.020	0.000	0.000	0.000

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	291	26	115	696	0	356	0	0	-1
normalized size	1	0.09	0.40	2.39	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.019	0.163	14.511	0.000	3.160	0.000	0.000	0.000

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	26	26	0	0	0	0	0	-1
normalized size	1	0.09	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.019	0.037	1.784	0.000	3.669	0.000	0.000	0.000

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	21	111	0	0	0	0	0	-1
normalized size	1	0.07	0.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.009	0.206	1.941	0.000	3.686	0.000	0.000	0.000

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	26	120	991	0	396	0	0	-1
normalized size	1	0.09	0.41	3.37	0.00	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.019	0.100	14.426	0.000	3.633	0.000	0.000	0.000

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	53	682	128	140	0	136	148
normalized size	1	1.00	0.38	4.84	0.91	0.99	0.00	0.96	1.05
time (sec)	N/A	0.114	0.037	3.784	1.066	0.712	0.000	0.195	4.861

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	48	497	119	159	0	120	139
normalized size	1	1.00	0.37	3.82	0.92	1.22	0.00	0.92	1.07
time (sec)	N/A	0.101	0.019	3.791	1.274	0.583	0.000	0.180	5.150

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	41	672	108	130	0	109	128
normalized size	1	1.00	0.36	5.84	0.94	1.13	0.00	0.95	1.11
time (sec)	N/A	0.090	0.014	3.782	1.271	0.735	0.000	0.175	4.890

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	667	97	148	0	98	117
normalized size	1	1.00	0.95	6.67	0.97	1.48	0.00	0.98	1.17
time (sec)	N/A	0.068	0.093	3.763	1.526	0.729	0.000	0.184	4.846

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	34	667	97	125	0	98	117
normalized size	1	1.00	0.34	6.67	0.97	1.25	0.00	0.98	1.17
time (sec)	N/A	0.068	0.010	3.867	1.338	0.660	0.000	0.193	4.903

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	54	0	0	226	0	160	253
normalized size	1	1.00	0.35	0.00	0.00	1.47	0.00	1.04	1.64
time (sec)	N/A	0.111	0.023	2.719	0.000	0.777	0.000	0.187	5.399

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	64	0	0	238	0	181	399
normalized size	1	1.00	0.37	0.00	0.00	1.36	0.00	1.03	2.28
time (sec)	N/A	0.116	0.032	2.293	0.000	0.566	0.000	0.215	5.024

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	26	152	0	0	271	0	0	-1
normalized size	1	0.15	0.87	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.248	1.457	0.000	0.780	0.000	0.000	0.000

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	26	142	0	0	239	0	0	-1
normalized size	1	0.17	0.93	0.00	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.164	1.685	0.000	0.669	0.000	0.000	0.000

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	38	38	627	0	318	0	0	-1
normalized size	1	0.36	0.36	5.92	0.00	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.018	0.009	4.228	0.000	3.554	0.000	0.000	0.000

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	140	100	944	0	288	0	0	-1
normalized size	1	1.32	0.94	8.91	0.00	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.048	3.009	0.000	3.235	0.000	0.000	0.000

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	C	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	204	192	570	0	340	0	0	-1
normalized size	1	1.65	1.55	4.60	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	8.121	3.108	2.916	0.000	3.944	0.000	0.000	0.000

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	397	133	851	0	316	0	0	-1
normalized size	1	2.76	0.92	5.91	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	8.318	5.158	4.018	0.000	4.985	0.000	0.000	0.000

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	C	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	643	136	649	0	351	0	0	-1
normalized size	1	3.97	0.84	4.01	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	10.595	5.205	2.684	0.000	4.956	0.000	0.000	0.000

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	26	71	0	0	0	0	0	-1
normalized size	1	0.09	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.019	0.139	0.579	0.000	2.780	0.000	0.000	0.000

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	26	66	0	0	0	0	0	-1
normalized size	1	0.09	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.018	0.122	0.655	0.000	2.784	0.000	0.000	0.000

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	26	66	0	0	0	0	0	-1
normalized size	1	0.09	0.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.018	0.088	0.601	0.000	3.085	0.000	0.000	0.000

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	26	45	0	0	0	0	0	-1
normalized size	1	0.09	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.013	0.057	0.611	0.000	3.009	0.000	0.000	0.000

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	24	76	0	0	0	0	0	-1
normalized size	1	0.08	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.019	0.064	0.572	0.000	2.809	0.000	0.000	0.000

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	26	79	0	0	0	0	0	-1
normalized size	1	0.08	0.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.020	0.102	0.590	0.000	3.119	0.000	0.000	0.000

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	270	0	0	325	0	379	442
normalized size	1	1.00	1.02	0.00	0.00	1.23	0.00	1.44	1.67
time (sec)	N/A	0.386	0.683	0.652	0.000	0.534	0.000	0.292	4.984

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	230	0	0	282	0	320	336
normalized size	1	1.00	1.05	0.00	0.00	1.28	0.00	1.45	1.53
time (sec)	N/A	0.259	0.408	0.629	0.000	0.507	0.000	0.253	4.933

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	204	0	0	222	0	276	298
normalized size	1	1.00	1.10	0.00	0.00	1.19	0.00	1.48	1.60
time (sec)	N/A	0.202	0.466	0.620	0.000	0.615	0.000	0.246	4.617

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	205	0	0	206	0	223	249
normalized size	1	1.00	1.29	0.00	0.00	1.30	0.00	1.40	1.57
time (sec)	N/A	0.155	0.287	0.607	0.000	0.531	0.000	0.271	4.621

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	268	0	0	276	0	311	1607
normalized size	1	1.00	1.09	0.00	0.00	1.12	0.00	1.26	6.53
time (sec)	N/A	0.234	0.465	0.566	0.000	0.770	0.000	0.805	4.744

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	366	0	0	429	0	351	1917
normalized size	1	1.00	1.08	0.00	0.00	1.26	0.00	1.03	5.64
time (sec)	N/A	0.384	1.242	0.617	0.000	0.725	0.000	0.776	9.990

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	411	0	0	472	0	465	2767
normalized size	1	1.00	1.11	0.00	0.00	1.28	0.00	1.26	7.48
time (sec)	N/A	0.489	1.837	0.633	0.000	3.076	0.000	0.810	12.554

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	64	226	0	0	494	0	0	-1
normalized size	1	0.19	0.67	0.00	0.00	1.47	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.373	0.628	0.000	2.867	0.000	0.000	0.000

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	64	185	0	0	452	0	0	-1
normalized size	1	0.23	0.67	0.00	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.054	0.194	0.595	0.000	0.941	0.000	0.000	0.000

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	64	65	0	0	330	0	0	-1
normalized size	1	0.27	0.28	0.00	0.00	1.41	0.00	0.00	-0.00
time (sec)	N/A	0.038	0.035	0.579	0.000	0.618	0.000	0.000	0.000

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	87	81	0	0	0	0	0	-1
normalized size	1	0.52	0.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.043	0.602	0.000	0.000	0.000	0.000	0.000

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	145	146	0	0	0	0	0	-1
normalized size	1	0.71	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	0.776	0.621	0.000	0.000	0.000	0.000	0.000

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	451	451	0	0	0	0	0	-1
normalized size	1	1.75	1.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.940	2.753	0.602	0.000	0.000	0.000	0.000	0.000

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	905	214	0	0	0	0	0	-1
normalized size	1	2.85	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.642	5.237	0.634	0.000	0.000	0.000	0.000	0.000

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	281	0	0	0	0	0	-1
normalized size	1	1.00	4.39	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.637	0.623	0.000	0.000	0.000	0.000	0.000

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	240	0	0	0	0	0	-1
normalized size	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.372	0.628	0.000	0.000	0.000	0.000	0.000

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.044	0.039	0.000	0.000	0.000	0.000	0.000

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	327	0	0	0	0	0	-1
normalized size	1	1.00	5.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.264	0.612	0.000	0.000	0.000	0.000	0.000

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	289	0	0	0	0	0	-1
normalized size	1	1.00	4.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.463	0.617	0.000	0.000	0.000	0.000	0.000

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	148	0	0	455	0	409	490
normalized size	1	1.00	0.56	0.00	0.00	1.71	0.00	1.54	1.84
time (sec)	N/A	0.322	0.124	0.605	0.000	1.262	0.000	0.287	5.127

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	104	0	0	398	0	350	385
normalized size	1	1.00	0.47	0.00	0.00	1.78	0.00	1.57	1.73
time (sec)	N/A	0.259	0.098	0.714	0.000	1.097	0.000	0.267	5.105

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	68	0	0	353	0	306	302
normalized size	1	1.00	0.36	0.00	0.00	1.88	0.00	1.63	1.61
time (sec)	N/A	0.199	0.046	0.682	0.000	0.831	0.000	0.549	5.058

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	47	0	0	323	0	259	238
normalized size	1	1.00	0.29	0.00	0.00	1.99	0.00	1.60	1.47
time (sec)	N/A	0.170	0.026	0.578	0.000	0.987	0.000	0.271	5.049

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	115	0	0	425	0	341	1963
normalized size	1	1.00	0.47	0.00	0.00	1.73	0.00	1.39	8.01
time (sec)	N/A	0.229	0.121	0.622	0.000	0.894	0.000	0.802	4.770

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	202	0	0	1030	0	400	1908
normalized size	1	1.00	0.58	0.00	0.00	2.97	0.00	1.15	5.50
time (sec)	N/A	0.389	0.158	0.608	0.000	1.148	0.000	0.807	10.571

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	240	0	0	1151	0	493	2788
normalized size	1	1.00	0.65	0.00	0.00	3.11	0.00	1.33	7.54
time (sec)	N/A	0.496	0.377	0.669	0.000	3.607	0.000	0.782	15.187

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	64	525	0	0	1164	0	0	-1
normalized size	1	0.19	1.57	0.00	0.00	3.49	0.00	0.00	-0.00
time (sec)	N/A	0.056	0.757	0.584	0.000	4.140	0.000	0.000	0.000

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	64	286	0	0	1091	0	0	-1
normalized size	1	0.24	1.05	0.00	0.00	4.01	0.00	0.00	-0.00
time (sec)	N/A	0.059	0.594	0.620	0.000	1.315	0.000	0.000	0.000

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	59	161	0	0	469	0	0	-1
normalized size	1	0.25	0.69	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.059	0.046	0.000	1.007	0.000	0.000	0.000

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	89	83	0	0	0	0	0	-1
normalized size	1	0.53	0.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.045	0.609	0.000	0.000	0.000	0.000	0.000

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	206	148	148	0	0	0	0	0	-1
normalized size	1	0.72	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.135	0.642	0.598	0.000	0.000	0.000	0.000	0.000

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	451	451	0	0	0	0	0	-1
normalized size	1	1.75	1.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.971	2.196	0.598	0.000	0.000	0.000	0.000	0.000

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	320	819	819	0	0	0	0	0	-1
normalized size	1	2.56	2.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.626	4.995	0.608	0.000	0.000	0.000	0.000	0.000

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	-1
normalized size	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.238	0.607	0.000	0.000	0.000	0.000	0.000

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.115	0.624	0.000	0.000	0.000	0.000	0.000

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.043	0.574	0.000	0.000	0.000	0.000	0.000

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	138	0	0	0	0	0	-1
normalized size	1	1.00	2.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.060	0.142	0.687	0.000	0.000	0.000	0.000	0.000

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	181	0	0	0	0	0	-1
normalized size	1	1.00	2.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.178	0.608	0.000	0.000	0.000	0.000	0.000

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	258	0	0	369	0	394	477
normalized size	1	1.00	1.03	0.00	0.00	1.47	0.00	1.57	1.90
time (sec)	N/A	0.363	0.538	0.622	0.000	1.070	0.000	0.303	5.123

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	255	0	0	298	0	348	348
normalized size	1	1.00	1.21	0.00	0.00	1.41	0.00	1.65	1.65
time (sec)	N/A	0.245	0.337	0.745	0.000	1.099	0.000	0.291	5.057

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	232	0	0	246	0	297	304
normalized size	1	1.00	1.24	0.00	0.00	1.32	0.00	1.59	1.63
time (sec)	N/A	0.208	0.325	0.647	0.000	1.046	0.000	0.297	4.725

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	331	0	0	320	0	357	796
normalized size	1	1.00	1.27	0.00	0.00	1.23	0.00	1.37	3.05
time (sec)	N/A	0.304	0.669	0.779	0.000	1.328	0.000	0.840	6.080

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	389	0	0	383	0	394	2047
normalized size	1	1.00	0.97	0.00	0.00	0.96	0.00	0.99	5.13
time (sec)	N/A	0.484	1.548	0.651	0.000	2.099	0.000	0.859	10.881

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	440	440	429	0	0	503	0	481	2841
normalized size	1	1.00	0.98	0.00	0.00	1.14	0.00	1.09	6.46
time (sec)	N/A	0.623	1.254	0.678	0.000	5.128	0.000	0.859	13.254

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	65	225	0	0	550	0	0	-1
normalized size	1	0.19	0.67	0.00	0.00	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.059	0.312	0.590	0.000	5.078	0.000	0.000	0.000

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	A	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	65	198	0	0	396	0	0	-1
normalized size	1	0.23	0.71	0.00	0.00	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.044	0.316	0.598	0.000	1.836	0.000	0.000	0.000

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	63	161	0	0	0	0	0	-1
normalized size	1	0.25	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	0.547	0.635	0.000	0.000	0.000	0.000	0.000

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	90	84	0	0	0	0	0	-1
normalized size	1	0.45	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.064	0.040	0.626	0.000	0.000	0.000	0.000	0.000

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	169	179	0	0	0	0	0	-1
normalized size	1	0.68	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.507	0.603	0.644	0.000	0.000	0.000	0.000	0.000

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	318	260	270	0	0	0	0	0	-1
normalized size	1	0.82	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.801	2.701	0.638	0.000	0.000	0.000	0.000	0.000

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	392	1446	1446	0	0	0	0	0	-1
normalized size	1	3.69	3.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	4.391	5.442	0.621	0.000	0.000	0.000	0.000	0.000

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	343	0	0	0	0	0	-1
normalized size	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.745	0.643	0.000	0.000	0.000	0.000	0.000

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	280	0	0	0	0	0	-1
normalized size	1	1.00	4.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.625	0.645	0.000	0.000	0.000	0.000	0.000

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	-1
normalized size	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.412	0.036	0.000	0.000	0.000	0.000	0.000

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	341	0	0	0	0	0	-1
normalized size	1	1.00	5.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.057	0.379	0.642	0.000	0.000	0.000	0.000	0.000

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	286	0	0	0	0	0	-1
normalized size	1	1.00	4.40	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.439	0.615	0.000	0.000	0.000	0.000	0.000

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	157	0	0	1004	0	454	438
normalized size	1	1.00	0.54	0.00	0.00	3.46	0.00	1.57	1.51
time (sec)	N/A	0.317	0.258	0.601	0.000	0.821	0.000	0.326	5.115

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	145	0	0	873	0	371	339
normalized size	1	1.00	0.59	0.00	0.00	3.58	0.00	1.52	1.39
time (sec)	N/A	0.243	0.138	0.596	0.000	0.859	0.000	0.279	5.090

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	103	0	0	768	0	313	267
normalized size	1	1.00	0.51	0.00	0.00	3.78	0.00	1.54	1.32
time (sec)	N/A	0.214	0.095	0.592	0.000	0.588	0.000	0.261	5.113

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	69	0	0	667	0	257	219
normalized size	1	1.00	0.41	0.00	0.00	3.97	0.00	1.53	1.30
time (sec)	N/A	0.161	0.031	0.612	0.000	0.651	0.000	0.279	5.097

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	50	0	0	592	0	226	208
normalized size	1	1.00	0.34	0.00	0.00	4.08	0.00	1.56	1.43
time (sec)	N/A	0.125	0.019	0.605	0.000	0.752	0.000	0.287	4.934

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	140	0	0	628	0	326	702
normalized size	1	1.00	0.57	0.00	0.00	2.57	0.00	1.34	2.88
time (sec)	N/A	0.212	0.178	0.613	0.000	0.648	0.000	0.744	6.436

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	156	0	0	837	0	383	1929
normalized size	1	1.00	0.53	0.00	0.00	2.83	0.00	1.29	6.52
time (sec)	N/A	0.314	0.612	0.593	0.000	0.972	0.000	0.885	11.355

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	394	288	0	0	826	0	0	-1
normalized size	1	1.44	1.05	0.00	0.00	3.03	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.818	0.709	0.000	0.835	0.000	0.000	0.000

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	346	65	0	0	761	0	0	-1
normalized size	1	1.48	0.28	0.00	0.00	3.27	0.00	0.00	-0.00
time (sec)	N/A	0.291	0.041	0.668	0.000	0.737	0.000	0.000	0.000

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	207	168	0	0	0	0	0	-1
normalized size	1	1.40	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.240	0.040	0.000	0.000	0.000	0.000	0.000

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	235	124	0	0	0	0	0	-1
normalized size	1	1.34	0.70	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.066	0.621	0.000	0.000	0.000	0.000	0.000

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	271	207	0	0	0	0	0	-1
normalized size	1	1.27	0.97	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	1.025	0.586	0.000	0.000	0.000	0.000	0.000

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	262	317	821	0	0	0	0	0	-1
normalized size	1	1.21	3.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.325	2.546	0.585	0.000	0.000	0.000	0.000	0.000

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	144	0	0	0	0	0	-1
normalized size	1	1.00	2.25	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.129	0.622	0.000	0.000	0.000	0.000	0.000

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.033	0.619	0.000	0.000	0.000	0.000	0.000

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.053	0.612	0.000	0.000	0.000	0.000	0.000

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.127	0.581	0.000	0.000	0.000	0.000	0.000

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	183	0	0	0	0	0	-1
normalized size	1	1.00	2.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.225	0.586	0.000	0.000	0.000	0.000	0.000

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	251	0	0	1322	0	372	331
normalized size	1	1.00	1.04	0.00	0.00	5.49	0.00	1.54	1.37
time (sec)	N/A	0.263	0.664	0.612	0.000	0.891	0.000	0.263	5.012

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	211	0	0	1156	0	312	292
normalized size	1	1.00	1.05	0.00	0.00	5.75	0.00	1.55	1.45
time (sec)	N/A	0.212	0.425	0.610	0.000	0.969	0.000	0.297	4.688

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	202	0	0	1060	0	253	232
normalized size	1	1.00	1.22	0.00	0.00	6.42	0.00	1.53	1.41
time (sec)	N/A	0.155	0.180	0.643	0.000	0.899	0.000	0.232	4.684

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	164	0	0	927	0	221	213
normalized size	1	1.00	1.13	0.00	0.00	6.39	0.00	1.52	1.47
time (sec)	N/A	0.124	0.062	0.599	0.000	0.631	0.000	0.288	4.847

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	308	0	0	472	0	321	1413
normalized size	1	1.00	1.26	0.00	0.00	1.93	0.00	1.31	5.77
time (sec)	N/A	0.211	0.446	0.611	0.000	0.464	0.000	0.756	4.942

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	303	0	0	562	0	377	1959
normalized size	1	1.00	1.01	0.00	0.00	1.88	0.00	1.26	6.55
time (sec)	N/A	0.317	0.591	0.635	0.000	1.133	0.000	0.758	11.141

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	400	190	0	0	558	0	0	-1
normalized size	1	1.43	0.68	0.00	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.502	0.291	0.639	0.000	1.140	0.000	0.000	0.000

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	346	65	0	0	530	0	0	-1
normalized size	1	1.48	0.28	0.00	0.00	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.035	0.636	0.000	0.462	0.000	0.000	0.000

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	208	83	0	0	0	0	0	-1
normalized size	1	1.40	0.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.044	0.584	0.000	0.000	0.000	0.000	0.000

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	232	128	0	0	0	0	0	-1
normalized size	1	1.34	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.221	0.065	0.653	0.000	0.000	0.000	0.000	0.000

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	269	267	0	0	0	0	0	-1
normalized size	1	1.25	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	1.888	0.612	0.000	0.000	0.000	0.000	0.000

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	249	0	0	0	0	0	-1
normalized size	1	1.00	3.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.378	0.613	0.000	0.000	0.000	0.000	0.000

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.061	0.035	0.581	0.000	0.000	0.000	0.000	0.000

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.038	0.042	0.000	0.000	0.000	0.000	0.000

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	338	0	0	0	0	0	-1
normalized size	1	1.00	5.28	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.062	0.328	0.621	0.000	0.000	0.000	0.000	0.000

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	157	0	0	1300	0	431	564
normalized size	1	1.00	0.45	0.00	0.00	3.75	0.00	1.24	1.63
time (sec)	N/A	0.441	0.265	0.458	0.000	0.967	0.000	0.257	5.189

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	147	0	0	1141	0	372	493
normalized size	1	1.00	0.58	0.00	0.00	4.51	0.00	1.47	1.95
time (sec)	N/A	0.340	0.111	0.464	0.000	1.142	0.000	0.259	5.160

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	101	0	0	1004	0	325	449
normalized size	1	1.00	0.50	0.00	0.00	4.95	0.00	1.60	2.21
time (sec)	N/A	0.242	0.068	0.428	0.000	0.991	0.000	0.248	4.986

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	77	0	0	872	0	301	412
normalized size	1	1.00	0.44	0.00	0.00	5.01	0.00	1.73	2.37
time (sec)	N/A	0.171	0.034	0.674	0.000	1.016	0.000	0.254	4.978

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	50	0	0	262	0	285	389
normalized size	1	1.00	0.30	0.00	0.00	1.57	0.00	1.71	2.33
time (sec)	N/A	0.165	0.021	0.612	0.000	0.706	0.000	0.256	4.923

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	86	0	0	975	0	389	3804
normalized size	1	1.00	0.32	0.00	0.00	3.60	0.00	1.44	14.04
time (sec)	N/A	0.316	0.045	0.708	0.000	0.783	0.000	0.793	5.342

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	117	0	0	1386	0	486	5875
normalized size	1	1.00	0.33	0.00	0.00	3.88	0.00	1.36	16.46
time (sec)	N/A	0.394	0.054	0.477	0.000	2.644	0.000	0.788	6.390

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	67	504	0	0	1329	0	0	-1
normalized size	1	0.21	1.57	0.00	0.00	4.13	0.00	0.00	-0.00
time (sec)	N/A	0.060	1.079	0.447	0.000	2.158	0.000	0.000	0.000

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	67	309	0	0	1127	0	0	-1
normalized size	1	0.26	1.19	0.00	0.00	4.33	0.00	0.00	-0.00
time (sec)	N/A	0.059	0.446	0.656	0.000	0.925	0.000	0.000	0.000

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	172	92	86	0	0	0	0	0	-1
normalized size	1	0.53	0.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.061	0.609	0.000	0.000	0.000	0.000	0.000

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	238	256	0	0	0	0	0	-1
normalized size	1	1.33	1.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.118	0.046	0.000	0.000	0.000	0.000	0.000

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	542	542	0	0	0	0	0	-1
normalized size	1	2.37	2.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.287	0.832	0.444	0.000	0.000	0.000	0.000	0.000

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	950	950	0	0	0	0	0	-1
normalized size	1	3.31	3.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.857	2.312	0.452	0.000	0.000	0.000	0.000	0.000

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	1486	278	0	0	0	0	0	-1
normalized size	1	4.23	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.295	5.856	0.460	0.000	0.000	0.000	0.000	0.000

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	194	0	0	0	0	0	-1
normalized size	1	1.00	2.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.290	0.546	0.000	0.000	0.000	0.000	0.000

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	144	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.135	0.681	0.000	0.000	0.000	0.000	0.000

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	129	0	0	0	0	0	-1
normalized size	1	1.00	1.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.117	0.990	0.000	0.000	0.000	0.000	0.000

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	141	0	0	0	0	0	-1
normalized size	1	1.00	2.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.126	0.862	0.000	0.000	0.000	0.000	0.000

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	193	0	0	0	0	0	-1
normalized size	1	1.00	2.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.238	0.493	0.000	0.000	0.000	0.000	0.000

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	264	0	0	0	0	0	-1
normalized size	1	1.00	3.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.371	0.496	0.000	0.000	0.000	0.000	0.000

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	92	89	84	100	0	88	88
normalized size	1	1.00	1.02	0.99	0.93	1.11	0.00	0.98	0.98
time (sec)	N/A	0.096	0.066	0.059	0.467	7.126	0.000	0.174	5.880

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	65	68	72	0	70	68
normalized size	1	1.00	0.94	0.93	0.97	1.03	0.00	1.00	0.97
time (sec)	N/A	0.064	0.049	0.053	0.570	2.320	0.000	0.202	5.633

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	43	50	49	42	144	51	51
normalized size	1	1.00	0.81	0.94	0.92	0.79	2.72	0.96	0.96
time (sec)	N/A	0.050	0.026	0.052	0.621	1.336	60.853	0.188	5.096

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	42	41	31	138	51	1012
normalized size	1	1.00	0.69	0.93	0.91	0.69	3.07	1.13	22.49
time (sec)	N/A	0.031	0.020	0.051	0.519	0.611	1.914	0.337	4.987

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	59	61	54	0	73	58
normalized size	1	1.00	0.87	0.95	0.98	0.87	0.00	1.18	0.94
time (sec)	N/A	0.064	0.033	0.057	0.710	2.049	0.000	0.181	5.492

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	87	87	99	0	112	87
normalized size	1	1.00	1.01	1.00	1.00	1.14	0.00	1.29	1.00
time (sec)	N/A	0.090	0.045	0.059	0.554	8.114	0.000	0.188	6.214

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	104	105	100	576	0	112	532
normalized size	1	1.00	0.93	0.94	0.89	5.14	0.00	1.00	4.75
time (sec)	N/A	0.273	0.170	0.061	1.325	2.615	0.000	0.190	5.705

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	81	80	416	932	80	518
normalized size	1	1.00	0.89	0.88	0.87	4.52	10.13	0.87	5.63
time (sec)	N/A	0.118	0.141	0.056	1.189	0.775	29.695	0.175	5.724

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	379
normalized size	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	4.80
time (sec)	N/A	0.062	0.041	0.056	1.165	0.532	0.000	0.175	5.345

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	60	59	325	0	59	399
normalized size	1	1.00	0.84	0.76	0.75	4.11	0.00	0.75	5.05
time (sec)	N/A	0.048	0.050	0.058	1.277	0.527	0.000	0.180	5.295

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	169	81	80	432	1103	80	354
normalized size	1	1.00	1.84	0.88	0.87	4.70	11.99	0.87	3.85
time (sec)	N/A	0.122	0.230	0.056	1.202	0.872	84.889	0.207	5.348

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	193	105	101	592	0	103	535
normalized size	1	1.00	1.72	0.94	0.90	5.29	0.00	0.92	4.78
time (sec)	N/A	0.218	0.262	0.062	1.513	3.455	0.000	0.184	5.530

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	377	328	375	1378	0	469	6361
normalized size	1	1.00	0.82	0.72	0.82	3.02	0.00	1.03	13.92
time (sec)	N/A	0.443	0.245	0.059	1.216	0.735	0.000	0.223	5.633

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	363	1358	0	453	2553
normalized size	1	1.00	0.76	0.71	0.81	3.02	0.00	1.01	5.69
time (sec)	N/A	0.285	0.121	0.053	1.421	0.544	0.000	0.242	5.725

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	296	361	1238	0	437	5889
normalized size	1	1.00	0.76	0.66	0.80	2.76	0.00	0.97	13.12
time (sec)	N/A	0.274	0.113	0.059	1.393	0.486	0.000	0.236	5.859

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	296	363	1258	0	477	6633
normalized size	1	1.00	0.76	0.66	0.81	2.80	0.00	1.06	14.77
time (sec)	N/A	0.270	0.139	0.057	1.207	0.520	0.000	0.228	5.497

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	365	1356	0	437	6153
normalized size	1	1.00	0.76	0.71	0.81	3.02	0.00	0.97	13.70
time (sec)	N/A	0.259	0.196	0.056	1.384	0.614	0.000	0.215	5.853

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	385	331	384	1395	0	488	5962
normalized size	1	1.00	0.84	0.72	0.83	3.03	0.00	1.06	12.96
time (sec)	N/A	0.445	0.220	0.058	1.266	0.873	0.000	0.207	6.084

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	462	462	406	343	390	1415	0	472	7459
normalized size	1	1.00	0.88	0.74	0.84	3.06	0.00	1.02	16.15
time (sec)	N/A	0.432	0.263	0.059	1.360	5.310	0.000	0.194	6.190

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	428	365	405	1456	0	483	4547
normalized size	1	1.00	0.89	0.76	0.85	3.04	0.00	1.01	9.49
time (sec)	N/A	0.597	0.378	0.066	1.235	8.922	0.000	0.235	6.012

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	1015	0	195	90	96	87
normalized size	1	1.00	0.95	10.91	0.00	2.10	0.97	1.03	0.94
time (sec)	N/A	0.082	0.109	0.272	0.000	0.441	17.444	0.165	4.688

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	114	1066	0	714	0	0	-1
normalized size	1	1.00	0.95	8.88	0.00	5.95	0.00	0.00	-0.01
time (sec)	N/A	0.154	0.192	0.278	0.000	0.545	0.000	0.000	0.000

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	69	988	0	156	65	66	54
normalized size	1	1.00	0.99	14.11	0.00	2.23	0.93	0.94	0.77
time (sec)	N/A	0.061	0.051	0.204	0.000	0.424	8.185	0.183	4.660

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	89	1000	0	612	0	0	-1
normalized size	1	1.00	0.98	10.99	0.00	6.73	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.065	0.196	0.000	0.487	0.000	0.000	0.000

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	1037	0	383	82	79	199
normalized size	1	1.00	0.95	12.20	0.00	4.51	0.96	0.93	2.34
time (sec)	N/A	0.075	0.094	0.262	0.000	0.464	12.052	0.179	4.869

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	53	1075	0	281	0	121	-1
normalized size	1	1.00	0.70	14.14	0.00	3.70	0.00	1.59	-0.01
time (sec)	N/A	0.086	0.064	0.259	0.000	0.464	0.000	1.285	0.000

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	107	1107	0	513	0	107	269
normalized size	1	1.00	0.93	9.63	0.00	4.46	0.00	0.93	2.34
time (sec)	N/A	0.124	0.134	0.270	0.000	0.454	0.000	0.169	5.387

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	130	1116	0	329	0	225	-1
normalized size	1	1.00	1.18	10.15	0.00	2.99	0.00	2.05	-0.01
time (sec)	N/A	0.160	5.330	0.263	0.000	0.518	0.000	1.749	0.000

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	857	1067	141	421	0	0	0	0	-1
normalized size	1	1.25	0.16	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.855	0.115	0.767	0.000	0.000	0.000	0.000	0.000

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	904	241	368	0	0	0	0	-1
normalized size	1	1.29	0.34	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.248	0.476	0.352	0.000	0.000	0.000	0.000	0.000

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	786	1012	65	299	0	0	0	0	-1
normalized size	1	1.29	0.08	0.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.038	0.039	0.293	0.000	0.000	0.000	0.000	0.000

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	679	881	161	273	0	0	0	0	-1
normalized size	1	1.30	0.24	0.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.978	0.206	0.268	0.000	0.000	0.000	0.000	0.000

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	809	1031	138	421	0	0	0	0	-1
normalized size	1	1.27	0.17	0.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.506	0.132	0.289	0.000	0.000	0.000	0.000	0.000

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	703	893	333	370	0	0	0	0	-1
normalized size	1	1.27	0.47	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.190	0.303	0.298	0.000	0.000	0.000	0.000	0.000

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.052	0.607	0.000	1.873	0.000	0.000	0.000

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.036	0.649	0.000	0.000	0.000	0.000	0.000

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	68	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.050	0.637	0.000	0.000	0.000	0.000	0.000

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	143	0	0	0	0	0	-1
normalized size	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.178	0.699	0.000	1.346	0.000	0.000	0.000

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	378	0	289	0	106	102
normalized size	1	1.00	0.88	3.63	0.00	2.78	0.00	1.02	0.98
time (sec)	N/A	0.112	0.257	0.283	0.000	1.236	0.000	0.162	4.825

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	335	0	205	0	64	58
normalized size	1	1.00	0.97	4.53	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.065	0.079	0.209	0.000	1.552	0.000	0.194	4.729

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	316	0	130	37	40	40
normalized size	1	1.00	1.00	6.20	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.049	0.018	0.213	0.000	1.715	14.270	0.156	4.797

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	347	0	431	66	71	652
normalized size	1	1.00	0.95	4.08	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.073	0.086	0.287	0.000	1.362	20.207	0.170	5.028

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	151	402	0	565	0	104	396
normalized size	1	1.00	1.29	3.44	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.120	0.170	0.292	0.000	1.026	0.000	0.167	5.351

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	118	408	0	739	0	0	-1
normalized size	1	1.00	0.96	3.32	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.149	0.210	0.316	0.000	2.820	0.000	0.000	0.000

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	356	0	632	0	0	-1
normalized size	1	1.00	0.99	3.91	0.00	6.95	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.069	0.333	0.000	2.332	0.000	0.000	0.000

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	95	322	0	245	0	72	-1
normalized size	1	1.00	1.76	5.96	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.044	0.063	0.324	0.000	0.988	0.000	0.179	0.000

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	179	350	0	332	0	116	-1
normalized size	1	1.00	2.24	4.38	0.00	4.15	0.00	1.45	-0.01
time (sec)	N/A	0.088	4.830	0.297	0.000	1.057	0.000	0.219	0.000

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	137	383	0	418	0	205	-1
normalized size	1	1.00	1.19	3.33	0.00	3.63	0.00	1.78	-0.01
time (sec)	N/A	0.164	5.549	0.270	0.000	1.125	0.000	1.675	0.000

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	872	872	249	363	0	0	0	0	-1
normalized size	1	1.00	0.29	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.019	0.347	0.317	0.000	0.000	0.000	0.000	0.000

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	837	65	265	0	0	0	0	-1
normalized size	1	1.31	0.10	0.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.682	0.054	0.230	0.000	0.000	0.000	0.000	0.000

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	742	161	191	0	0	0	0	-1
normalized size	1	1.16	0.25	0.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.602	0.050	0.331	0.000	0.000	0.000	0.000	0.000

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	677	864	337	288	0	0	0	0	-1
normalized size	1	1.28	0.50	0.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.991	0.376	0.306	0.000	0.000	0.000	0.000	0.000

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	804	982	65	292	0	0	0	0	-1
normalized size	1	1.22	0.08	0.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.963	0.044	0.344	0.000	0.000	0.000	0.000	0.000

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	656	756	65	191	0	0	0	0	-1
normalized size	1	1.15	0.10	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.710	0.033	0.249	0.000	0.000	0.000	0.000	0.000

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	833	1007	141	310	0	0	0	0	-1
normalized size	1	1.21	0.17	0.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.309	0.139	0.254	0.000	0.000	0.000	0.000	0.000

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	175	923	0	622	0	180	186
normalized size	1	1.00	1.00	5.27	0.00	3.55	0.00	1.03	1.06
time (sec)	N/A	0.173	0.306	0.247	0.000	1.583	0.000	0.175	5.191

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	876	0	475	0	134	144
normalized size	1	1.00	0.87	7.12	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.146	0.355	0.190	0.000	0.996	0.000	0.168	5.117

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	98	851	0	348	0	116	95
normalized size	1	1.00	0.99	8.60	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.087	0.113	0.197	0.000	0.941	0.000	0.170	4.932

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	541	0	302	0	93	84
normalized size	1	1.00	0.98	6.22	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.074	0.102	0.206	0.000	0.681	0.000	0.164	4.852

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	880	0	862	0	139	3017
normalized size	1	1.00	0.93	6.67	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.142	0.334	0.272	0.000	0.767	0.000	0.168	5.869

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	938	0	1236	0	257	3822
normalized size	1	1.00	0.88	5.07	0.00	6.68	0.00	1.39	20.66
time (sec)	N/A	0.251	0.695	0.277	0.000	1.044	0.000	0.169	7.047

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	150	953	0	1386	0	337	-1
normalized size	1	1.00	0.79	4.99	0.00	7.26	0.00	1.76	-0.01
time (sec)	N/A	0.365	0.566	0.273	0.000	2.752	0.000	0.590	0.000

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	893	0	1077	0	298	-1
normalized size	1	1.00	0.96	6.33	0.00	7.64	0.00	2.11	-0.01
time (sec)	N/A	0.162	0.224	0.238	0.000	1.500	0.000	0.545	0.000

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	124	861	0	426	0	244	-1
normalized size	1	1.00	1.33	9.26	0.00	4.58	0.00	2.62	-0.01
time (sec)	N/A	0.092	0.583	0.240	0.000	1.030	0.000	1.630	0.000

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	407	867	0	467	0	237	-1
normalized size	1	1.00	3.91	8.34	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.094	6.013	0.194	0.000	0.919	0.000	0.376	0.000

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	155	885	0	612	0	418	-1
normalized size	1	1.00	1.04	5.94	0.00	4.11	0.00	2.81	-0.01
time (sec)	N/A	0.199	5.633	0.269	0.000	1.044	0.000	1.681	0.000

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	175	923	0	760	0	395	-1
normalized size	1	1.00	0.84	4.44	0.00	3.65	0.00	1.90	-0.00
time (sec)	N/A	0.328	5.871	0.246	0.000	1.429	0.000	2.038	0.000

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	996	996	253	604	0	0	0	0	-1
normalized size	1	1.00	0.25	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.308	0.352	0.311	0.000	0.000	0.000	0.000	0.000

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	908	908	238	530	0	0	0	0	-1
normalized size	1	1.00	0.26	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	0.193	0.255	0.000	0.000	0.000	0.000	0.000

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	983	983	392	333	0	0	0	0	-1
normalized size	1	1.00	0.40	0.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.033	0.394	0.253	0.000	0.000	0.000	0.000	0.000

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1046	1046	408	626	0	0	0	0	-1
normalized size	1	1.00	0.39	0.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.557	0.703	0.247	0.000	0.000	0.000	0.000	0.000

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1146	1146	162	556	0	0	0	0	-1
normalized size	1	1.00	0.14	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.563	0.170	0.359	0.000	0.000	0.000	0.000	0.000

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1144	1144	172	359	0	0	0	0	-1
normalized size	1	1.00	0.15	0.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.441	0.155	0.247	0.000	0.000	0.000	0.000	0.000

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1225	1225	226	674	0	0	0	0	-1
normalized size	1	1.00	0.18	0.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.898	0.331	0.230	0.000	0.000	0.000	0.000	0.000

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	194	164	0	0	0	185	0	-1
normalized size	1	0.97	0.82	0.00	0.00	0.00	0.92	0.00	-0.00
time (sec)	N/A	0.212	0.177	0.561	0.000	1.085	14.272	0.000	0.000

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	115	110	0	0	0	119	0	-1
normalized size	1	0.93	0.89	0.00	0.00	0.00	0.97	0.00	-0.01
time (sec)	N/A	0.059	0.103	0.413	0.000	0.916	4.922	0.000	0.000

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	-0.01
time (sec)	N/A	0.019	0.016	0.419	0.000	1.088	1.072	0.000	0.000

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	125	0	0	0	0	0	-1
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.135	0.584	0.000	1.119	0.000	0.000	0.000

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	179	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.230	0.548	0.000	1.149	0.000	0.000	0.000

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.113	0.648	0.000	1.012	0.000	0.000	0.000

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	167	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.159	0.552	0.000	0.855	0.000	0.000	0.000

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	113	0	0	0	119	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.90	0.00	-0.01
time (sec)	N/A	0.060	0.101	0.421	0.000	0.973	61.437	0.000	0.000

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	-0.01
time (sec)	N/A	0.023	0.017	0.398	0.000	0.902	1.492	0.000	0.000

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	169	0	0	0	0	0	-1
normalized size	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.207	0.659	0.000	0.892	0.000	0.000	0.000

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.106	0.610	0.000	1.014	0.000	0.000	0.000

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.095	0.617	0.000	0.722	0.000	0.000	0.000

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	288	0	106	103
normalized size	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.107	0.277	0.634	0.000	0.951	0.000	0.170	4.866

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	0	0	205	0	64	58
normalized size	1	1.00	0.97	0.00	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.063	0.130	0.601	0.000	0.934	0.000	0.192	4.745

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	130	37	40	40
normalized size	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.046	0.020	0.572	0.000	0.819	23.762	0.167	4.725

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	0	0	431	66	71	652
normalized size	1	1.00	0.95	0.00	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.074	0.126	0.631	0.000	0.961	26.110	0.176	5.191

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	151	0	0	565	0	104	396
normalized size	1	1.00	1.29	0.00	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.119	0.181	0.725	0.000	1.152	0.000	0.177	5.615

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	118	0	0	739	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.236	0.921	0.000	1.004	0.000	0.000	0.000

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	632	0	156	-1
normalized size	1	1.00	0.99	0.00	0.00	6.95	0.00	1.71	-0.01
time (sec)	N/A	0.087	0.091	0.610	0.000	1.084	0.000	0.234	0.000

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	95	0	0	245	0	72	-1
normalized size	1	1.00	1.76	0.00	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.049	0.090	0.605	0.000	0.973	0.000	0.247	0.000

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	179	0	0	332	0	0	-1
normalized size	1	1.00	2.24	0.00	0.00	4.15	0.00	0.00	-0.01
time (sec)	N/A	0.089	1.169	0.644	0.000	1.081	0.000	0.000	0.000

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	253	0	0	416	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	3.62	0.00	0.00	-0.01
time (sec)	N/A	0.163	4.650	0.658	0.000	1.060	0.000	0.000	0.000

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.048	0.700	0.000	0.000	0.000	0.000	0.000

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.046	0.589	0.000	0.000	0.000	0.000	0.000

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.039	0.670	0.000	0.000	0.000	0.000	0.000

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.201	0.621	0.000	0.000	0.000	0.000	0.000

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.050	0.128	0.629	0.000	0.957	0.000	0.000	0.000

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.087	0.120	0.659	0.000	0.000	0.000	0.000	0.000

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.084	0.165	0.601	0.000	0.000	0.000	0.000	0.000

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	0	0	475	0	134	144
normalized size	1	1.00	0.87	0.00	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.140	0.357	0.454	0.000	1.022	0.000	0.441	5.093

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	98	0	0	348	0	116	95
normalized size	1	1.00	0.99	0.00	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.080	0.096	0.632	0.000	1.024	0.000	0.289	4.989

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	302	0	93	84
normalized size	1	1.00	0.98	0.00	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.069	0.101	0.619	0.000	0.739	0.000	0.302	4.929

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	0	0	862	0	139	3025
normalized size	1	1.00	0.93	0.00	0.00	6.53	0.00	1.05	22.92
time (sec)	N/A	0.127	0.354	0.775	0.000	1.140	0.000	0.407	6.226

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	1236	0	257	3860
normalized size	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	20.86
time (sec)	N/A	0.228	0.683	0.559	0.000	1.360	0.000	0.410	7.290

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	0	0	1077	0	343	-1
normalized size	1	1.00	0.96	0.00	0.00	7.64	0.00	2.43	-0.01
time (sec)	N/A	0.158	0.276	0.627	0.000	2.008	0.000	0.511	0.000

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	124	0	0	426	0	0	-1
normalized size	1	1.00	1.33	0.00	0.00	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.581	0.656	0.000	0.805	0.000	0.000	0.000

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	407	0	0	467	0	237	-1
normalized size	1	1.00	3.91	0.00	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.098	0.998	0.605	0.000	0.904	0.000	0.284	0.000

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	869	0	0	612	0	0	-1
normalized size	1	1.00	5.83	0.00	0.00	4.11	0.00	0.00	-0.01
time (sec)	N/A	0.194	5.401	0.483	0.000	1.036	0.000	0.000	0.000

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	175	0	0	760	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	3.65	0.00	0.00	-0.00
time (sec)	N/A	0.308	5.858	0.448	0.000	1.369	0.000	0.000	0.000

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	169	0	0	0	0	0	-1
normalized size	1	1.00	2.64	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.051	0.218	0.627	0.000	0.688	0.000	0.000	0.000

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	168	0	0	0	0	0	-1
normalized size	1	1.00	2.62	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.073	0.244	0.641	0.000	0.000	0.000	0.000	0.000

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	172	0	0	0	0	0	-1
normalized size	1	1.00	2.69	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.192	0.600	0.000	0.000	0.000	0.000	0.000

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	329	0	0	0	0	0	-1
normalized size	1	1.00	5.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.395	0.642	0.000	0.000	0.000	0.000	0.000

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	-1
normalized size	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.289	0.458	0.000	1.606	0.000	0.000	0.000

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	-1
normalized size	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.088	0.375	0.465	0.000	0.000	0.000	0.000	0.000

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	225	0	0	0	0	0	-1
normalized size	1	1.00	3.52	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.082	0.285	0.461	0.000	0.000	0.000	0.000	0.000

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	91	0	0	288	0	106	103
normalized size	1	1.00	0.88	0.00	0.00	2.77	0.00	1.02	0.99
time (sec)	N/A	0.099	0.216	0.593	0.000	1.223	0.000	0.160	4.678

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	72	0	0	205	0	64	58
normalized size	1	1.00	0.97	0.00	0.00	2.77	0.00	0.86	0.78
time (sec)	N/A	0.061	0.078	0.642	0.000	0.811	0.000	0.157	4.730

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	130	37	40	40
normalized size	1	1.00	1.00	0.00	0.00	2.55	0.73	0.78	0.78
time (sec)	N/A	0.047	0.017	0.625	0.000	0.858	44.568	0.166	4.586

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	81	0	0	431	66	71	652
normalized size	1	1.00	0.95	0.00	0.00	5.07	0.78	0.84	7.67
time (sec)	N/A	0.079	0.099	0.683	0.000	0.750	37.858	0.159	4.813

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	151	0	0	565	0	104	396
normalized size	1	1.00	1.29	0.00	0.00	4.83	0.00	0.89	3.38
time (sec)	N/A	0.113	0.139	0.637	0.000	0.727	0.000	0.176	5.507

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	118	0	0	739	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	6.01	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.240	0.581	0.000	0.919	0.000	0.000	0.000

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	90	0	0	632	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	6.95	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.068	0.596	0.000	0.723	0.000	0.000	0.000

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	95	0	0	245	0	72	-1
normalized size	1	1.00	1.76	0.00	0.00	4.54	0.00	1.33	-0.02
time (sec)	N/A	0.055	0.068	0.616	0.000	0.880	0.000	0.203	0.000

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	179	0	0	332	0	116	-1
normalized size	1	1.00	2.24	0.00	0.00	4.15	0.00	1.45	-0.01
time (sec)	N/A	0.087	0.832	0.664	0.000	0.839	0.000	0.229	0.000

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	253	0	0	416	0	205	-1
normalized size	1	1.00	2.20	0.00	0.00	3.62	0.00	1.78	-0.01
time (sec)	N/A	0.161	2.273	0.662	0.000	0.653	0.000	1.686	0.000

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	65	0	0	0	0	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.101	0.043	0.640	0.000	0.000	0.000	0.000	0.000

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	65	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.766	0.045	0.660	0.000	0.000	0.000	0.000	0.000

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	878	878	141	0	0	0	0	0	-1
normalized size	1	1.00	0.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.222	0.163	0.642	0.000	0.000	0.000	0.000	0.000

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1005	1005	65	0	0	0	0	0	-1
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.363	0.068	0.739	0.000	0.000	0.000	0.000	0.000

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	768	768	65	0	0	0	0	0	-1
normalized size	1	1.00	0.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.935	0.043	0.732	0.000	0.000	0.000	0.000	0.000

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1032	1032	141	0	0	0	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.612	0.128	0.648	0.000	0.000	0.000	0.000	0.000

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.054	0.609	0.000	0.000	0.000	0.000	0.000

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.068	0.594	0.000	0.000	0.000	0.000	0.000

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.298	0.656	0.000	0.000	0.000	0.000	0.000

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	141	0	0	0	0	0	-1
normalized size	1	1.00	2.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.059	0.147	0.640	0.000	1.057	0.000	0.000	0.000

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	141	0	0	0	0	0	-1
normalized size	1	1.00	2.20	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.138	0.638	0.000	0.000	0.000	0.000	0.000

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	107	0	0	475	0	134	144
normalized size	1	1.00	0.87	0.00	0.00	3.86	0.00	1.09	1.17
time (sec)	N/A	0.142	0.257	0.469	0.000	0.767	0.000	0.177	5.021

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	98	0	0	348	0	116	95
normalized size	1	1.00	0.99	0.00	0.00	3.52	0.00	1.17	0.96
time (sec)	N/A	0.083	0.092	0.594	0.000	1.219	0.000	0.169	4.837

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	0	0	302	0	93	84
normalized size	1	1.00	0.98	0.00	0.00	3.47	0.00	1.07	0.97
time (sec)	N/A	0.073	0.096	0.603	0.000	1.063	0.000	0.160	4.797

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	123	0	0	862	0	139	3017
normalized size	1	1.00	0.93	0.00	0.00	6.53	0.00	1.05	22.86
time (sec)	N/A	0.137	0.355	0.799	0.000	1.222	0.000	0.209	5.820

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	163	0	0	1236	0	257	3832
normalized size	1	1.00	0.88	0.00	0.00	6.68	0.00	1.39	20.71
time (sec)	N/A	0.229	0.602	0.658	0.000	1.270	0.000	0.172	7.851

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	135	0	0	1077	0	298	-1
normalized size	1	1.00	0.96	0.00	0.00	7.64	0.00	2.11	-0.01
time (sec)	N/A	0.160	0.264	0.647	0.000	1.533	0.000	0.652	0.000

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	124	0	0	426	0	244	-1
normalized size	1	1.00	1.33	0.00	0.00	4.58	0.00	2.62	-0.01
time (sec)	N/A	0.086	0.549	0.642	0.000	1.002	0.000	1.595	0.000

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	407	0	0	467	0	237	-1
normalized size	1	1.00	3.91	0.00	0.00	4.49	0.00	2.28	-0.01
time (sec)	N/A	0.090	0.941	0.601	0.000	1.257	0.000	0.389	0.000

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	869	0	0	612	0	418	-1
normalized size	1	1.00	5.83	0.00	0.00	4.11	0.00	2.81	-0.01
time (sec)	N/A	0.185	2.034	0.439	0.000	1.270	0.000	1.764	0.000

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	175	0	0	760	0	395	-1
normalized size	1	1.00	0.84	0.00	0.00	3.65	0.00	1.90	-0.00
time (sec)	N/A	0.297	5.847	0.455	0.000	1.664	0.000	2.277	0.000

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	924	924	159	0	0	0	0	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.288	0.167	0.638	0.000	0.000	0.000	0.000	0.000

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	999	999	169	0	0	0	0	0	-1
normalized size	1	1.00	0.17	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.406	0.219	0.608	0.000	0.000	0.000	0.000	0.000

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1060	1060	225	0	0	0	0	0	-1
normalized size	1	1.00	0.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.983	0.388	0.464	0.000	0.000	0.000	0.000	0.000

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	159	0	0	0	0	0	-1
normalized size	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.936	0.165	0.644	0.000	0.000	0.000	0.000	0.000

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1162	1162	169	0	0	0	0	0	-1
normalized size	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.910	0.192	0.614	0.000	0.000	0.000	0.000	0.000

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1243	1243	226	0	0	0	0	0	-1
normalized size	1	1.00	0.18	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.384	0.338	0.474	0.000	0.000	0.000	0.000	0.000

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.204	0.591	0.000	0.000	0.000	0.000	0.000

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	170	0	0	0	0	0	-1
normalized size	1	1.00	2.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.053	0.220	0.641	0.000	0.000	0.000	0.000	0.000

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	328	0	0	0	0	0	-1
normalized size	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.363	0.651	0.000	0.000	0.000	0.000	0.000

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	226	0	0	0	0	0	-1
normalized size	1	1.00	3.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.322	0.447	0.000	1.226	0.000	0.000	0.000

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	226	0	0	0	0	0	-1
normalized size	1	1.00	3.53	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.322	0.460	0.000	0.000	0.000	0.000	0.000

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	162	243	242	226	143	134
normalized size	1	1.00	0.98	1.32	1.98	1.97	1.84	1.16	1.09
time (sec)	N/A	0.094	0.257	0.057	1.361	0.930	72.291	0.193	5.713

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	100	122	159	191	144	105	93
normalized size	1	1.00	1.11	1.36	1.77	2.12	1.60	1.17	1.03
time (sec)	N/A	0.068	0.136	0.055	1.240	0.625	59.054	0.257	5.305

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	129	108	155	107	92	68
normalized size	1	1.00	0.85	1.54	1.29	1.85	1.27	1.10	0.81
time (sec)	N/A	0.058	0.188	0.056	1.206	1.181	60.432	0.289	5.104

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	109	67	166	75	163	57
normalized size	1	1.00	1.39	1.85	1.14	2.81	1.27	2.76	0.97
time (sec)	N/A	0.043	0.155	0.055	1.226	0.898	23.441	0.488	5.206

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	48	49	60	58	250	91
normalized size	1	1.00	1.02	1.04	1.07	1.30	1.26	5.43	1.98
time (sec)	N/A	0.037	0.017	0.046	0.724	0.789	4.022	0.918	4.817

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	69	70	84	85	78	310	126
normalized size	1	1.00	0.93	0.95	1.14	1.15	1.05	4.19	1.70
time (sec)	N/A	0.059	0.025	0.046	0.599	0.646	4.564	1.382	4.924

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	79	94	118	109	112	370	168
normalized size	1	1.00	0.76	0.90	1.13	1.05	1.08	3.56	1.62
time (sec)	N/A	0.079	0.064	0.048	0.565	0.653	5.210	2.061	5.224

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	90	118	152	133	146	430	210
normalized size	1	1.00	0.67	0.88	1.13	0.99	1.09	3.21	1.57
time (sec)	N/A	0.102	0.078	0.048	0.466	0.822	5.863	2.982	5.607

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	108	113	158	131	1386	175	117
normalized size	1	1.00	0.72	0.75	1.05	0.87	9.24	1.17	0.78
time (sec)	N/A	0.073	0.084	0.054	0.643	0.613	6.441	0.220	4.591

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	86	89	124	107	910	140	97
normalized size	1	1.00	0.74	0.76	1.06	0.91	7.78	1.20	0.83
time (sec)	N/A	0.062	0.065	0.047	0.564	0.664	4.920	0.168	4.518

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	64	65	90	82	422	105	77
normalized size	1	1.00	0.76	0.77	1.07	0.98	5.02	1.25	0.92
time (sec)	N/A	0.040	0.046	0.058	0.528	0.582	3.992	0.166	4.488

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	43	55	57	119	72	54
normalized size	1	1.00	0.79	0.81	1.04	1.08	2.25	1.36	1.02
time (sec)	N/A	0.025	0.027	0.050	0.661	0.527	3.040	0.213	4.442

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	84	83	75	156	107	116	80
normalized size	1	1.00	1.27	1.26	1.14	2.36	1.62	1.76	1.21
time (sec)	N/A	0.037	0.057	0.056	1.376	0.716	3.266	0.190	4.720

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	75	135	133	164	107	76	97
normalized size	1	1.00	0.88	1.59	1.56	1.93	1.26	0.89	1.14
time (sec)	N/A	0.046	0.090	0.053	1.224	0.717	4.484	0.249	5.108

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	100	175	193	194	144	130	-1
normalized size	1	1.00	1.10	1.92	2.12	2.13	1.58	1.43	-0.01
time (sec)	N/A	0.051	0.185	0.059	1.355	0.610	6.972	0.240	0.000

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	68	220	277	244	226	153	-1
normalized size	1	1.00	0.55	1.79	2.25	1.98	1.84	1.24	-0.01
time (sec)	N/A	0.081	0.030	0.059	1.273	0.643	11.421	0.301	0.000

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	162	240	243	253	144	130
normalized size	1	1.00	1.00	1.32	1.95	1.98	2.06	1.17	1.06
time (sec)	N/A	0.090	0.142	0.060	1.298	0.673	100.225	0.233	5.783

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	89	174	171	203	0	126	105
normalized size	1	1.00	0.77	1.51	1.49	1.77	0.00	1.10	0.91
time (sec)	N/A	0.084	0.245	0.055	1.327	0.693	0.000	0.293	5.703

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	78	216	134	195	187	225	95
normalized size	1	1.00	0.71	1.96	1.22	1.77	1.70	2.05	0.86
time (sec)	N/A	0.073	0.095	0.063	1.233	0.654	56.156	0.541	5.651

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	90	153	80	213	73	254	72
normalized size	1	1.00	1.18	2.01	1.05	2.80	0.96	3.34	0.95
time (sec)	N/A	0.054	0.071	0.066	1.310	0.642	54.144	1.068	5.831

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	48	49	84	138	370	122
normalized size	1	1.00	1.07	1.04	1.07	1.83	3.00	8.04	2.65
time (sec)	N/A	0.036	0.022	0.049	0.593	0.721	14.156	1.866	5.333

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	70	84	109	194	430	164
normalized size	1	1.00	0.96	0.95	1.14	1.47	2.62	5.81	2.22
time (sec)	N/A	0.056	0.033	0.049	0.642	0.807	15.608	2.691	5.759

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	94	118	134	262	490	206
normalized size	1	1.00	0.90	0.90	1.13	1.29	2.52	4.71	1.98
time (sec)	N/A	0.072	0.040	0.053	0.580	0.803	17.961	3.285	6.315

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	115	118	152	157	326	550	248
normalized size	1	1.00	0.86	0.88	1.13	1.17	2.43	4.10	1.85
time (sec)	N/A	0.096	0.061	0.052	0.630	0.946	19.447	4.630	6.812

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	110	115	158	155	3351	175	137
normalized size	1	1.00	0.73	0.77	1.05	1.03	22.34	1.17	0.91
time (sec)	N/A	0.072	0.110	0.049	0.635	0.689	12.596	0.183	4.662

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	89	91	124	132	2304	140	118
normalized size	1	1.00	0.76	0.78	1.06	1.13	19.69	1.20	1.01
time (sec)	N/A	0.057	0.090	0.051	0.490	0.688	9.646	0.179	4.569

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	66	67	90	106	1340	105	97
normalized size	1	1.00	0.79	0.80	1.07	1.26	15.95	1.25	1.15
time (sec)	N/A	0.042	0.055	0.055	0.755	0.540	7.638	0.171	4.551

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	45	55	80	498	72	77
normalized size	1	1.00	0.83	0.85	1.04	1.51	9.40	1.36	1.45
time (sec)	N/A	0.025	0.036	0.044	0.647	0.507	5.948	0.183	4.626

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	99	91	203	184	140	-1
normalized size	1	1.00	0.94	1.15	1.06	2.36	2.14	1.63	-0.01
time (sec)	N/A	0.055	0.147	0.059	1.350	0.794	5.249	0.200	0.000

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	105	170	163	190	202	115	-1
normalized size	1	1.00	0.87	1.40	1.35	1.57	1.67	0.95	-0.01
time (sec)	N/A	0.058	0.088	0.061	1.359	0.662	7.646	0.265	0.000

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	68	213	207	216	216	145	78
normalized size	1	1.00	0.61	1.90	1.85	1.93	1.93	1.29	0.70
time (sec)	N/A	0.062	0.034	0.058	1.424	0.806	11.776	0.257	5.858

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	126	259	275	246	253	173	-1
normalized size	1	1.00	1.02	2.11	2.24	2.00	2.06	1.41	-0.01
time (sec)	N/A	0.064	0.244	0.066	1.383	0.882	18.949	0.299	0.000

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	71	302	354	298	287	214	-1
normalized size	1	1.00	0.45	1.90	2.23	1.87	1.81	1.35	-0.01
time (sec)	N/A	0.089	0.049	0.073	1.573	0.604	29.163	0.374	0.000

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	95	129	178	192	150	113	99
normalized size	1	1.00	1.06	1.43	1.98	2.13	1.67	1.26	1.10
time (sec)	N/A	0.067	0.112	0.059	1.171	0.545	71.064	0.256	5.348

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	79	90	109	146	66	88	59
normalized size	1	1.00	1.34	1.53	1.85	2.47	1.12	1.49	1.00
time (sec)	N/A	0.040	0.055	0.052	1.195	0.649	84.411	0.259	5.077

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	73	70	54	130	63	0	35
normalized size	1	1.00	1.70	1.63	1.26	3.02	1.47	0.00	0.81
time (sec)	N/A	0.033	0.049	0.054	1.105	0.628	22.339	0.000	4.851

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	39	47	48	39	138	88	35
normalized size	1	1.00	0.91	1.09	1.12	0.91	3.21	2.05	0.81
time (sec)	N/A	0.036	0.048	0.049	0.534	0.654	7.054	0.410	4.565

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	60	70	83	62	204	153	58
normalized size	1	1.00	0.83	0.97	1.15	0.86	2.83	2.12	0.81
time (sec)	N/A	0.052	0.039	0.047	0.489	0.484	13.172	0.498	4.676

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	94	118	86	269	219	102
normalized size	1	1.00	0.90	0.93	1.17	0.85	2.66	2.17	1.01
time (sec)	N/A	0.069	0.048	0.047	0.576	0.620	18.001	0.623	4.716

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	67	85	59	338	0	53
normalized size	1	1.00	0.68	0.82	1.04	0.72	4.12	0.00	0.65
time (sec)	N/A	0.033	0.075	0.052	0.491	0.775	3.553	0.000	5.314

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	44	49	36	70	0	67
normalized size	1	1.00	0.67	0.86	0.96	0.71	1.37	0.00	1.31
time (sec)	N/A	0.020	0.056	0.044	0.592	0.671	2.755	0.000	4.917

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	71	73	58	131	39	0	65
normalized size	1	1.00	1.51	1.55	1.23	2.79	0.83	0.00	1.38
time (sec)	N/A	0.029	0.041	0.051	1.276	0.737	2.856	0.000	4.998

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	80	105	121	144	66	0	94
normalized size	1	1.00	1.31	1.72	1.98	2.36	1.08	0.00	1.54
time (sec)	N/A	0.038	0.064	0.057	1.177	0.705	4.921	0.000	5.529

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	107	146	200	201	150	0	-1
normalized size	1	1.00	1.15	1.57	2.15	2.16	1.61	0.00	-0.01
time (sec)	N/A	0.050	0.179	0.063	1.234	0.801	8.461	0.000	0.000

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	120	111	140	215	304	177	0	134
normalized size	1	1.02	0.94	1.19	1.82	2.58	1.50	0.00	1.14
time (sec)	N/A	0.085	0.192	0.060	1.385	0.828	103.904	0.000	6.339

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	89	115	144	249	264	0	90
normalized size	1	1.00	1.03	1.34	1.67	2.90	3.07	0.00	1.05
time (sec)	N/A	0.058	0.217	0.058	1.204	0.727	52.419	0.000	5.609

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	75	69	200	49	0	54
normalized size	1	1.00	1.40	1.44	1.33	3.85	0.94	0.00	1.04
time (sec)	N/A	0.040	0.094	0.058	1.260	0.642	20.333	0.000	5.058

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	46	46	46	68	37	46
normalized size	1	1.00	0.86	1.10	1.10	1.10	1.62	0.88	1.10
time (sec)	N/A	0.035	0.021	0.052	0.461	0.696	3.421	0.264	4.519

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	69	81	73	61	0	66
normalized size	1	1.00	0.88	1.01	1.19	1.07	0.90	0.00	0.97
time (sec)	N/A	0.052	0.027	0.050	0.611	0.600	11.071	0.000	4.642

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	81	94	116	98	90	0	91
normalized size	1	1.00	0.81	0.94	1.16	0.98	0.90	0.00	0.91
time (sec)	N/A	0.071	0.030	0.053	0.615	0.724	13.279	0.000	4.837

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	104	118	151	121	122	0	154
normalized size	1	1.00	0.83	0.94	1.20	0.96	0.97	0.00	1.22
time (sec)	N/A	0.089	0.035	0.054	0.549	0.658	16.143	0.000	4.920

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	80	91	128	95	561	0	79
normalized size	1	1.00	0.72	0.82	1.15	0.86	5.05	0.00	0.71
time (sec)	N/A	0.049	0.076	0.055	0.558	0.779	7.831	0.000	5.768

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	66	90	70	267	0	81
normalized size	1	1.00	0.72	0.84	1.14	0.89	3.38	0.00	1.03
time (sec)	N/A	0.030	0.050	0.061	0.533	0.691	7.306	0.000	5.195

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	33	43	53	47	65	0	38
normalized size	1	1.00	0.73	0.96	1.18	1.04	1.44	0.00	0.84
time (sec)	N/A	0.030	0.026	0.044	0.681	0.666	7.513	0.000	4.898

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	79	80	195	206	0	60
normalized size	1	1.00	1.20	1.34	1.36	3.31	3.49	0.00	1.02
time (sec)	N/A	0.034	0.039	0.059	1.215	0.634	11.714	0.000	5.106

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	B	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	57	132	162	248	262	0	-1
normalized size	1	1.00	0.62	1.43	1.76	2.70	2.85	0.00	-0.01
time (sec)	N/A	0.050	0.029	0.059	1.218	0.564	18.437	0.000	0.000

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	60	157	243	314	180	0	-1
normalized size	1	1.00	0.49	1.28	1.98	2.55	1.46	0.00	-0.01
time (sec)	N/A	0.069	0.030	0.063	1.284	0.744	29.807	0.000	0.000

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	101	115	0	0	0	0	0	-1
normalized size	1	0.96	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.155	0.347	0.000	0.957	0.000	0.000	0.000

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.204	0.190	0.000	0.824	0.000	0.000	0.000

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.107	0.178	0.000	0.688	0.000	0.000	0.000

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.106	0.179	0.000	0.513	0.000	0.000	0.000

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.112	0.186	0.000	0.803	0.000	0.000	0.000

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.095	0.043	0.000	0.646	0.000	0.000	0.000

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	95	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.152	0.212	0.000	0.896	0.000	0.000	0.000

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	106	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.155	0.214	0.000	0.734	0.000	0.000	0.000

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	110	0	0	0	0	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.083	0.190	0.000	0.890	0.000	0.000	0.000

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	106	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.367	0.196	0.000	0.827	0.000	0.000	0.000

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.197	0.076	0.000	1.055	0.000	0.000	0.000

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.184	0.078	0.000	0.741	0.000	0.000	0.000

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.146	0.083	0.000	0.658	0.000	0.000	0.000

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.158	0.076	0.000	0.844	0.000	0.000	0.000

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	111	0	0	0	0	0	-1
normalized size	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.168	0.076	0.000	0.867	0.000	0.000	0.000

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	111	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.120	0.176	0.082	0.000	0.789	0.000	0.000	0.000

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	111	75	57	62	0	162	831
normalized size	1	1.00	0.82	0.56	0.42	0.46	0.00	1.20	6.16
time (sec)	N/A	0.061	0.069	0.054	0.546	0.678	0.000	0.290	52.028

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	99	65	47	57	0	127	632
normalized size	1	1.00	0.95	0.62	0.45	0.55	0.00	1.22	6.08
time (sec)	N/A	0.049	0.056	0.049	0.489	0.679	0.000	0.217	31.390

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	87	52	37	52	0	92	-1
normalized size	1	1.00	1.19	0.71	0.51	0.71	0.00	1.26	-0.01
time (sec)	N/A	0.032	0.075	0.047	0.581	0.634	0.000	0.189	0.000

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	72	72	26	46	0	57	41
normalized size	1	1.00	1.95	1.95	0.70	1.24	0.00	1.54	1.11
time (sec)	N/A	0.021	0.035	0.050	0.494	0.641	0.000	0.175	5.069

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	74	47	27	55	0	0	129
normalized size	1	1.00	1.10	0.70	0.40	0.82	0.00	0.00	1.93
time (sec)	N/A	0.031	0.079	0.054	1.280	0.657	0.000	0.000	6.249

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	10	30	0	48	31
normalized size	1	1.00	1.00	0.74	0.32	0.97	0.00	1.55	1.00
time (sec)	N/A	0.009	0.016	0.052	1.320	0.698	0.000	0.229	5.257

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	36	28	21	37	0	90	43
normalized size	1	1.00	0.57	0.44	0.33	0.59	0.00	1.43	0.68
time (sec)	N/A	0.020	0.021	0.052	1.312	0.747	0.000	0.280	5.075

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	41	33	31	44	0	111	55
normalized size	1	1.00	0.44	0.35	0.33	0.47	0.00	1.18	0.59
time (sec)	N/A	0.031	0.031	0.054	1.550	0.555	0.000	0.290	5.038

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	46	38	41	49	0	132	67
normalized size	1	1.00	0.37	0.30	0.33	0.39	0.00	1.06	0.54
time (sec)	N/A	0.042	0.033	0.061	1.217	0.682	0.000	0.426	5.023

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	67	65	47	57	0	76	632
normalized size	1	1.00	0.64	0.62	0.45	0.55	0.00	0.73	6.08
time (sec)	N/A	0.045	0.054	0.058	0.612	0.572	0.000	0.231	27.092

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	55	37	52	0	59	429
normalized size	1	1.00	0.85	0.75	0.51	0.71	0.00	0.81	5.88
time (sec)	N/A	0.032	0.036	0.063	0.460	0.618	0.000	0.227	18.765

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	55	41	24	46	83	39	-1
normalized size	1	1.00	1.57	1.17	0.69	1.31	2.37	1.11	-0.03
time (sec)	N/A	0.021	0.012	0.061	0.453	0.545	27.907	0.223	0.000

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	26	40	16	27	0	20	6
normalized size	1	1.00	3.25	5.00	2.00	3.38	0.00	2.50	0.75
time (sec)	N/A	0.011	0.007	0.049	0.647	0.670	0.000	0.195	5.287

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	20	10	25	0	25	19
normalized size	1	1.00	1.00	0.69	0.34	0.86	0.00	0.86	0.66
time (sec)	N/A	0.010	0.010	0.056	1.208	0.656	0.000	0.222	5.561

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	36	25	21	34	0	48	33
normalized size	1	1.00	0.57	0.40	0.33	0.54	0.00	0.76	0.52
time (sec)	N/A	0.020	0.014	0.058	1.200	0.597	0.000	0.222	5.504

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	41	30	31	39	0	69	43
normalized size	1	1.00	0.44	0.32	0.33	0.41	0.00	0.73	0.46
time (sec)	N/A	0.033	0.029	0.064	1.297	0.672	0.000	0.257	5.658

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.043	0.993	0.000	0.698	0.000	0.000	0.000

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.018	1.074	0.000	0.543	0.000	0.000	0.000

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.026	0.953	0.000	0.649	0.000	0.000	0.000

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.070	1.758	0.000	0.854	0.000	0.000	0.000

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.029	1.006	0.000	0.881	0.000	0.000	0.000

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	36	15	11	10	16	11
normalized size	1	1.00	1.00	2.40	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.014	0.006	0.052	0.455	0.570	0.122	0.185	0.087

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	110	0	36	40	39	38	31
normalized size	1	1.00	5.00	0.00	1.64	1.82	1.77	1.73	1.41
time (sec)	N/A	0.015	0.166	0.674	0.954	0.648	8.426	0.292	4.942

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	88	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.119	1.018	0.000	0.606	0.000	0.000	0.000

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.113	0.938	0.000	0.560	0.000	0.000	0.000

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	78	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.066	0.956	0.000	0.592	0.000	0.000	0.000

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.044	0.042	0.000	0.576	0.000	0.000	0.000

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	69	69	58	332	0	162
normalized size	1	1.00	0.89	1.10	1.10	0.92	5.27	0.00	2.57
time (sec)	N/A	0.068	0.065	0.099	0.565	0.710	4.025	0.000	5.724

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.079	1.021	0.000	0.561	0.000	0.000	0.000

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	77	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.097	1.001	0.000	0.681	0.000	0.000	0.000

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	141	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.285	1.016	0.000	0.700	0.000	0.000	0.000

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	135	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	0.236	0.970	0.000	0.675	0.000	0.000	0.000

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	134	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.182	0.233	0.970	0.000	0.665	0.000	0.000	0.000

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.189	0.050	0.000	0.680	0.000	0.000	0.000

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	97	131	151	224	0	0	-1
normalized size	1	1.00	0.96	1.30	1.50	2.22	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.329	0.105	0.665	0.667	0.000	0.000	0.000

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	133	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.270	1.024	0.000	0.738	0.000	0.000	0.000

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.231	1.289	0.000	0.561	0.000	0.000	0.000

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	115	284	231	177	320	0	-1
normalized size	1	1.00	0.88	2.18	1.78	1.36	2.46	0.00	-0.01
time (sec)	N/A	0.138	0.354	0.082	0.531	0.752	135.872	0.000	0.000

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	173	150	108	202	0	-1
normalized size	1	1.00	0.91	1.92	1.67	1.20	2.24	0.00	-0.01
time (sec)	N/A	0.089	0.138	0.077	0.698	0.577	63.508	0.000	0.000

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	50	87	83	56	105	0	-1
normalized size	1	1.00	0.83	1.45	1.38	0.93	1.75	0.00	-0.02
time (sec)	N/A	0.054	0.058	0.071	0.699	0.605	26.972	0.000	0.000

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	44	59	60	45	0	0	-1
normalized size	1	1.00	0.81	1.09	1.11	0.83	0.00	0.00	-0.02
time (sec)	N/A	0.054	0.045	0.070	0.586	0.681	0.000	0.000	0.000

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	58	109	121	120	0	0	-1
normalized size	1	1.00	0.77	1.45	1.61	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.135	0.092	0.515	0.777	0.000	0.000	0.000

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	97	203	243	267	0	0	-1
normalized size	1	1.00	0.92	1.93	2.31	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.168	0.112	0.663	0.541	0.000	0.000	0.000

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	138	342	286	230	0	0	-1
normalized size	1	1.00	0.87	2.16	1.81	1.46	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.314	0.070	0.663	0.803	0.000	0.000	0.000

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	103	236	192	146	0	0	-1
normalized size	1	1.00	0.87	2.00	1.63	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.269	0.072	0.541	0.625	0.000	0.000	0.000

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	125	112	82	139	0	-1
normalized size	1	1.00	0.91	1.45	1.30	0.95	1.62	0.00	-0.01
time (sec)	N/A	0.076	0.099	0.072	0.534	0.742	57.945	0.000	0.000

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	78	81	74	0	0	-1
normalized size	1	1.00	0.93	1.10	1.14	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.105	0.096	0.626	0.790	0.000	0.000	0.000

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	163	147	166	0	0	-1
normalized size	1	1.00	0.95	1.72	1.55	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.110	0.080	0.582	0.824	0.000	0.000	0.000

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	112	214	262	301	0	0	-1
normalized size	1	1.00	0.93	1.78	2.18	2.51	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.275	0.121	0.677	0.616	0.000	0.000	0.000

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	172	155	154	154	175	13	154
normalized size	1	1.00	12.29	11.07	11.00	11.00	12.50	0.93	11.00
time (sec)	N/A	0.002	0.006	0.040	0.566	0.428	0.115	0.148	0.156

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	156	156	182	156	156
normalized size	1	1.00	11.38	9.81	9.75	9.75	11.38	9.75	9.75
time (sec)	N/A	0.041	0.006	0.038	0.731	0.444	0.122	0.188	4.925

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	156	156	185	156	156
normalized size	1	1.00	11.62	9.81	9.75	9.75	11.56	9.75	9.75
time (sec)	N/A	0.036	0.021	0.038	0.624	0.449	0.121	0.163	4.772

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
normalized size	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.016	0.190	0.079	0.644	0.547	0.000	0.572	5.209

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	107	0	16	32	0	70	25
normalized size	1	1.00	8.23	0.00	1.23	2.46	0.00	5.38	1.92
time (sec)	N/A	0.014	0.184	0.675	0.842	0.646	0.000	0.279	4.810

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	9	9	9	9	10	8	11	8
normalized size	1	1.12	1.12	1.12	1.12	1.25	1.00	1.38	1.00
time (sec)	N/A	0.006	0.018	0.042	0.478	0.447	0.124	0.152	4.662

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	18	13
normalized size	1	1.00	1.00	0.93	1.13	0.87	0.80	1.20	0.87
time (sec)	N/A	0.015	0.009	0.053	0.488	0.665	0.178	0.151	0.060

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	12	15	13
normalized size	1	1.00	1.00	0.93	1.13	0.87	0.80	1.00	0.87
time (sec)	N/A	0.014	0.020	0.048	0.554	0.476	0.193	0.162	4.628

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	47	17	29	0	15
normalized size	1	1.00	1.00	1.13	3.13	1.13	1.93	0.00	1.00
time (sec)	N/A	0.015	0.016	0.047	0.489	0.647	0.619	0.000	4.723

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	177	81	81	87	13	12
normalized size	1	1.00	1.00	12.64	5.79	5.79	6.21	0.93	0.86
time (sec)	N/A	0.002	0.024	0.061	0.612	0.587	0.886	0.170	7.087

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.014	0.047	0.063	0.638	0.445	1.356	0.155	2.315

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
normalized size	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.013	0.054	0.062	0.614	0.542	1.908	0.169	9.981

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	0	105
normalized size	1	1.00	1.00	9.67	29.14	5.00	0.00	0.00	5.00
time (sec)	N/A	0.013	0.251	0.098	0.787	0.563	0.000	0.000	4.987

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	44	86	53	46	0	56	37
normalized size	1	1.00	0.85	1.65	1.02	0.88	0.00	1.08	0.71
time (sec)	N/A	0.028	0.043	1.147	1.252	0.738	0.000	0.155	4.817

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	142	143	0	757	0	0	-1
normalized size	1	1.00	1.53	1.54	0.00	8.14	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.682	0.102	0.000	1.012	0.000	0.000	0.000

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	223	0	0	607	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	2.41	0.00	0.00	-0.00
time (sec)	N/A	0.234	1.093	1.016	0.000	0.860	0.000	0.000	0.000

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	178	0	0	469	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	2.36	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.672	0.961	0.000	0.747	0.000	0.000	0.000

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	141	0	0	359	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	2.46	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.513	1.012	0.000	0.782	0.000	0.000	0.000

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	0	0	281	0	0	-1
normalized size	1	1.00	1.38	0.00	0.00	3.16	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.321	0.974	0.000	0.560	0.000	0.000	0.000

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	122	0	0	408	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	4.48	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.901	0.947	0.000	0.954	0.000	0.000	0.000

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	57	0	0	135	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	1.42	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.075	0.918	0.000	1.071	0.000	0.000	0.000

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	274	0	0	771	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.405	2.024	0.965	0.000	0.800	0.000	0.000	0.000

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	241	0	0	607	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.316	1.007	0.934	0.000	0.654	0.000	0.000	0.000

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	191	0	0	471	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.245	0.624	0.993	0.000	1.057	0.000	0.000	0.000

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	157	0	0	361	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	2.41	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.464	0.997	0.000	0.800	0.000	0.000	0.000

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	185	0	0	540	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	4.06	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.522	1.003	0.000	0.993	0.000	0.000	0.000

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	217	0	0	769	0	0	-1
normalized size	1	1.00	1.48	0.00	0.00	5.23	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.812	1.091	0.000	0.916	0.000	0.000	0.000

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	29	25	46	35	22
normalized size	1	1.00	1.00	1.05	1.45	1.25	2.30	1.75	1.10
time (sec)	N/A	0.004	0.018	0.046	0.824	0.661	3.098	0.164	4.795

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	0	52	47
normalized size	1	1.00	3.59	0.96	1.30	1.19	0.00	1.93	1.74
time (sec)	N/A	0.009	0.132	0.044	0.889	0.686	0.000	0.191	4.882

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	97	26	35	32	0	56	47
normalized size	1	1.00	3.59	0.96	1.30	1.19	0.00	2.07	1.74
time (sec)	N/A	0.009	0.096	0.045	0.692	0.565	0.000	0.262	4.902

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	101	0	39	35	0	66	54
normalized size	1	1.00	3.74	0.00	1.44	1.30	0.00	2.44	2.00
time (sec)	N/A	0.012	0.162	0.688	0.957	0.740	0.000	0.253	4.863

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [451] had the largest ratio of [.5200]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	18	0.111
2	A	2	1	1.00	16	0.062
3	A	2	1	1.00	15	0.067
4	A	3	2	1.00	18	0.111
5	A	2	1	1.00	18	0.056
6	A	2	1	1.00	18	0.056
7	A	3	2	1.00	18	0.111
8	A	2	1	1.00	18	0.056
9	A	2	1	1.00	18	0.056
10	A	3	2	1.00	18	0.111
11	A	3	2	1.00	20	0.100
12	A	2	1	1.00	18	0.056
13	A	2	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	4	3	1.00	20	0.150
15	A	2	1	1.00	20	0.050
16	A	2	1	1.00	20	0.050
17	A	3	2	1.00	20	0.100
18	A	2	1	1.00	20	0.050
19	A	2	1	1.00	20	0.050
20	A	3	2	1.00	20	0.100
21	A	2	1	1.00	20	0.050
22	A	2	1	1.00	20	0.050
23	A	2	1	1.00	20	0.050
24	A	3	2	1.00	20	0.100
25	A	2	1	1.00	20	0.050
26	A	2	1	1.00	20	0.050
27	A	3	2	1.00	20	0.100
28	A	2	1	1.00	20	0.050
29	A	2	1	1.00	20	0.050
30	A	3	2	1.00	20	0.100
31	A	2	1	1.00	18	0.056
32	A	2	1	1.00	17	0.059
33	A	4	3	1.00	20	0.150
34	A	2	1	1.00	20	0.050
35	A	2	1	1.00	20	0.050
36	A	3	2	1.00	20	0.100
37	A	2	1	1.00	20	0.050
38	A	2	1	1.00	20	0.050
39	A	3	2	1.00	20	0.100
40	A	2	1	1.00	20	0.050
41	A	2	1	1.00	20	0.050
42	A	3	2	1.00	20	0.100
43	A	2	1	1.00	20	0.050
44	A	2	1	1.00	20	0.050
45	A	3	2	1.00	20	0.100
46	A	2	1	1.00	20	0.050
47	A	2	1	1.00	20	0.050
48	A	3	2	1.00	20	0.100
49	A	2	1	1.00	20	0.050

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	2	1	1.00	20	0.050
51	A	4	3	1.00	20	0.150
52	A	2	1	1.00	20	0.050
53	A	2	1	1.00	20	0.050
54	A	3	3	1.00	20	0.150
55	A	2	1	1.00	20	0.050
56	A	9	8	1.00	20	0.400
57	A	3	2	1.00	20	0.100
58	A	8	8	1.00	20	0.400
59	A	8	8	1.00	20	0.400
60	A	3	2	1.00	20	0.100
61	A	7	7	1.00	18	0.389
62	A	7	7	1.00	17	0.412
63	A	3	2	1.00	20	0.100
64	A	7	7	1.00	20	0.350
65	A	7	7	1.00	20	0.350
66	A	3	2	1.00	20	0.100
67	A	8	8	1.00	20	0.400
68	A	8	8	1.00	20	0.400
69	A	3	2	1.00	20	0.100
70	A	9	8	1.00	20	0.400
71	A	9	8	1.00	20	0.400
72	A	3	2	1.00	20	0.100
73	A	9	8	1.00	20	0.400
74	A	9	8	1.00	20	0.400
75	A	3	2	1.00	20	0.100
76	A	8	8	1.00	20	0.400
77	A	8	8	1.00	20	0.400
78	A	3	2	1.00	20	0.100
79	A	7	7	1.00	18	0.389
80	A	7	7	1.00	17	0.412
81	A	3	2	1.00	20	0.100
82	A	8	8	1.01	20	0.400
83	A	8	8	1.00	20	0.400
84	A	3	2	1.00	20	0.100
85	A	9	8	1.00	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	8	1.00	20	0.400
87	A	3	2	1.00	20	0.100
88	A	3	2	1.00	20	0.100
89	A	3	2	1.00	20	0.100
90	A	3	2	1.00	20	0.100
91	A	2	2	1.00	20	0.100
92	A	3	2	1.00	20	0.100
93	A	3	2	1.00	20	0.100
94	A	3	2	1.00	20	0.100
95	A	10	9	1.00	20	0.450
96	A	10	9	1.00	20	0.450
97	A	9	9	1.00	20	0.450
98	A	9	9	1.00	20	0.450
99	A	8	8	1.00	20	0.400
100	A	8	8	1.00	20	0.400
101	A	8	8	1.00	18	0.444
102	A	8	8	1.00	17	0.471
103	A	9	9	1.00	20	0.450
104	A	9	9	1.00	20	0.450
105	A	10	9	1.00	20	0.450
106	A	10	9	1.00	20	0.450
107	A	3	2	1.00	22	0.091
108	A	15	8	1.00	22	0.364
109	A	14	8	1.00	22	0.364
110	A	3	2	1.00	22	0.091
111	A	13	7	1.00	22	0.318
112	A	13	7	1.00	22	0.318
113	A	4	3	1.00	22	0.136
114	A	13	7	1.00	20	0.350
115	A	13	7	1.00	19	0.368
116	A	3	2	1.00	22	0.091
117	A	15	8	1.00	22	0.364
118	A	14	8	1.00	22	0.364
119	A	3	2	1.00	22	0.091
120	A	16	9	1.00	22	0.409
121	A	15	9	1.00	22	0.409

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	3	2	1.00	22	0.091
123	A	17	9	1.00	22	0.409
124	A	2	1	1.00	20	0.050
125	A	2	1	1.00	20	0.050
126	A	2	1	1.00	18	0.056
127	A	2	2	1.00	20	0.100
128	A	2	2	1.00	20	0.100
129	A	2	2	1.00	20	0.100
130	A	3	2	1.00	24	0.083
131	A	2	1	1.00	20	0.050
132	A	2	1	1.00	20	0.050
133	A	2	1	1.00	20	0.050
134	A	2	1	1.00	20	0.050
135	A	2	1	1.00	20	0.050
136	A	2	1	1.00	20	0.050
137	A	2	1	1.00	20	0.050
138	A	2	1	1.00	20	0.050
139	A	2	1	1.00	22	0.045
140	A	2	1	1.00	22	0.045
141	A	2	1	1.00	22	0.045
142	A	2	1	1.00	22	0.045
143	A	2	1	1.00	22	0.045
144	A	2	1	1.00	22	0.045
145	A	2	1	1.00	22	0.045
146	A	2	1	1.00	22	0.045
147	A	2	1	1.00	22	0.045
148	A	2	1	1.00	22	0.045
149	A	2	1	1.00	22	0.045
150	A	2	1	1.00	22	0.045
151	A	2	1	1.00	22	0.045
152	A	2	1	1.00	22	0.045
153	A	2	1	1.00	22	0.045
154	A	2	1	1.00	22	0.045
155	A	5	5	1.00	22	0.227
156	A	13	9	1.00	22	0.409
157	A	12	8	1.00	22	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	4	4	1.00	22	0.182
159	A	12	8	1.00	22	0.364
160	A	12	8	1.00	22	0.364
161	A	4	4	1.00	22	0.182
162	A	12	8	1.00	22	0.364
163	A	5	5	1.00	22	0.227
164	A	13	9	1.00	22	0.409
165	A	12	8	1.00	22	0.364
166	A	4	4	1.00	22	0.182
167	A	12	8	1.00	22	0.364
168	A	13	9	1.00	22	0.409
169	A	5	5	1.01	22	0.227
170	A	13	9	1.00	22	0.409
171	A	5	5	1.00	22	0.227
172	A	13	9	1.00	22	0.409
173	A	13	9	1.00	22	0.409
174	A	5	5	1.00	22	0.227
175	A	13	9	1.00	22	0.409
176	A	14	10	1.00	22	0.454
177	A	6	6	1.01	22	0.273
178	A	14	10	1.00	22	0.454
179	A	3	2	1.00	22	0.091
180	A	3	2	1.00	22	0.091
181	A	3	2	1.00	22	0.091
182	A	5	5	1.00	22	0.227
183	A	5	5	1.00	22	0.227
184	A	5	5	1.00	22	0.227
185	A	4	4	1.00	22	0.182
186	A	3	3	1.00	19	0.158
187	A	3	3	1.00	22	0.136
188	A	3	3	1.00	22	0.136
189	A	4	4	1.00	22	0.182
190	A	6	6	1.00	22	0.273
191	A	5	5	1.00	20	0.250
192	A	5	5	1.00	22	0.227
193	A	5	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	6	6	1.00	22	0.273
195	A	7	6	1.00	22	0.273
196	A	3	2	1.00	22	0.091
197	A	3	2	1.00	22	0.091
198	A	3	2	1.00	22	0.091
199	A	6	5	1.00	22	0.227
200	A	6	5	1.00	22	0.227
201	A	6	6	1.00	22	0.273
202	A	5	4	1.00	22	0.182
203	A	4	3	1.00	19	0.158
204	A	4	3	1.00	22	0.136
205	A	4	4	1.00	22	0.182
206	A	4	3	1.00	22	0.136
207	A	7	6	1.00	22	0.273
208	A	6	5	1.00	20	0.250
209	A	6	5	1.00	22	0.227
210	A	6	6	1.00	22	0.273
211	A	6	5	1.00	22	0.227
212	A	7	6	1.00	22	0.273
213	A	3	2	1.00	22	0.091
214	A	3	2	1.00	22	0.091
215	A	3	2	1.00	22	0.091
216	A	4	4	1.00	22	0.182
217	A	4	4	1.00	22	0.182
218	A	5	5	1.00	22	0.227
219	A	3	3	1.00	22	0.136
220	A	2	2	1.00	19	0.105
221	A	2	2	1.00	22	0.091
222	A	3	3	1.00	22	0.136
223	A	5	5	1.00	22	0.227
224	A	4	4	1.00	20	0.200
225	A	4	4	1.00	22	0.182
226	A	5	5	1.00	22	0.227
227	A	6	5	1.00	22	0.227
228	A	3	2	1.00	22	0.091
229	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
230	A	3	2	1.00	22	0.091
231	A	4	4	1.00	22	0.182
232	A	5	5	1.00	22	0.227
233	A	6	5	1.02	22	0.227
234	A	4	4	1.00	22	0.182
235	A	3	3	1.00	22	0.136
236	A	2	2	1.00	19	0.105
237	A	3	3	1.00	22	0.136
238	A	4	4	1.00	22	0.182
239	A	5	5	1.00	22	0.227
240	A	4	4	1.00	20	0.200
241	A	5	5	1.00	22	0.227
242	A	6	6	1.00	22	0.273
243	A	7	6	1.00	22	0.273
244	A	3	2	1.00	22	0.091
245	A	3	2	1.00	22	0.091
246	A	3	2	1.00	22	0.091
247	A	5	5	1.00	22	0.227
248	A	6	5	1.00	22	0.227
249	A	4	3	1.00	22	0.136
250	A	3	3	1.00	22	0.136
251	A	3	3	1.00	19	0.158
252	A	4	3	1.00	22	0.136
253	A	5	4	1.00	22	0.182
254	A	6	5	1.00	22	0.227
255	A	5	5	1.00	22	0.227
256	A	5	5	1.00	20	0.250
257	A	6	5	1.00	22	0.227
258	A	7	6	1.00	22	0.273
259	A	6	5	1.00	26	0.192
260	A	5	5	1.00	26	0.192
261	A	4	4	1.00	26	0.154
262	A	6	5	1.00	26	0.192
263	A	7	6	1.00	26	0.231
264	A	7	6	1.00	26	0.231
265	A	5	5	1.00	24	0.208

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
266	A	7	6	1.00	26	0.231
267	A	2	2	1.00	26	0.077
268	A	2	2	1.00	23	0.087
269	A	2	2	1.00	26	0.077
270	A	5	4	1.00	26	0.154
271	A	4	4	1.00	26	0.154
272	A	3	3	1.00	26	0.115
273	A	6	5	1.00	26	0.192
274	A	7	6	1.00	26	0.231
275	A	5	5	1.00	26	0.192
276	A	1	1	1.00	24	0.042
277	A	7	6	1.00	26	0.231
278	A	2	2	1.00	26	0.077
279	A	2	2	1.00	23	0.087
280	A	2	2	1.00	26	0.077
281	A	1	1	1.00	22	0.045
282	A	6	5	1.00	27	0.185
283	A	6	5	1.00	27	0.185
284	A	5	5	1.00	27	0.185
285	A	4	4	1.00	27	0.148
286	A	6	5	1.00	27	0.185
287	A	7	6	1.00	27	0.222
288	A	8	7	1.00	27	0.259
289	A	15	13	1.00	27	0.482
290	A	14	12	1.00	27	0.444
291	A	12	11	1.00	25	0.440
292	A	14	12	1.00	27	0.444
293	A	15	13	1.00	27	0.482
294	A	16	13	1.00	27	0.482
295	A	7	5	1.00	27	0.185
296	A	7	5	1.00	27	0.185
297	A	6	5	1.00	27	0.185
298	A	5	4	1.00	27	0.148
299	A	7	6	1.00	27	0.222
300	A	7	6	1.00	27	0.222
301	A	8	7	1.00	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	16	13	1.00	27	0.482
303	A	15	13	1.00	27	0.482
304	A	14	12	1.00	25	0.480
305	A	14	12	1.00	27	0.444
306	A	15	13	1.00	27	0.482
307	A	16	13	1.00	27	0.482
308	A	5	4	1.00	27	0.148
309	A	5	4	1.00	27	0.148
310	A	4	4	1.00	27	0.148
311	A	3	3	1.00	27	0.111
312	A	6	5	1.00	27	0.185
313	A	7	6	1.00	27	0.222
314	A	8	7	1.00	27	0.259
315	A	14	12	1.00	27	0.444
316	A	12	11	1.00	27	0.407
317	A	8	7	1.00	25	0.280
318	A	14	12	1.00	27	0.444
319	A	15	13	1.00	27	0.482
320	A	16	13	1.00	27	0.482
321	A	2	2	1.00	27	0.074
322	A	2	2	1.00	24	0.083
323	A	2	2	1.00	27	0.074
324	A	2	2	1.00	27	0.074
325	A	7	5	1.00	27	0.185
326	A	5	4	1.00	27	0.148
327	A	4	4	1.00	27	0.148
328	A	4	4	1.00	27	0.148
329	A	7	6	1.00	27	0.222
330	A	8	7	1.00	27	0.259
331	A	9	8	1.00	27	0.296
332	A	14	12	1.00	27	0.444
333	A	14	12	1.00	27	0.444
334	A	14	12	1.00	25	0.480
335	A	15	13	1.00	27	0.482
336	A	16	13	1.00	27	0.482
337	A	17	13	1.00	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	2	2	1.00	27	0.074
339	A	2	2	1.00	24	0.083
340	A	2	2	1.00	27	0.074
341	A	2	2	1.00	27	0.074
342	A	5	5	1.00	33	0.152
343	A	5	5	1.00	35	0.143
344	A	5	5	1.00	35	0.143
345	A	5	5	1.00	37	0.135
346	A	5	5	1.00	33	0.152
347	A	5	5	1.00	35	0.143
348	A	5	5	1.00	36	0.139
349	A	5	5	1.00	36	0.139
350	A	1	1	1.00	33	0.030
351	A	1	1	1.00	35	0.029
352	A	1	1	1.00	35	0.029
353	A	1	1	1.00	37	0.027
354	A	1	1	1.00	33	0.030
355	A	1	1	1.00	35	0.029
356	A	1	1	1.00	36	0.028
357	A	1	1	1.00	36	0.028
358	A	6	5	1.00	24	0.208
359	A	5	5	1.00	24	0.208
360	A	4	4	1.00	24	0.167
361	A	6	4	1.00	24	0.167
362	A	7	5	1.00	24	0.208
363	A	2	2	1.00	24	0.083
364	A	2	2	1.00	22	0.091
365	A	2	2	1.00	21	0.095
366	A	2	2	1.00	24	0.083
367	A	2	2	1.00	24	0.083
368	A	7	5	1.00	24	0.208
369	A	6	5	1.00	24	0.208
370	A	5	4	1.00	24	0.167
371	A	7	5	1.00	24	0.208
372	A	7	5	1.00	24	0.208
373	A	2	2	1.00	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	2	2	1.00	22	0.091
375	A	2	2	1.00	21	0.095
376	A	2	2	1.00	24	0.083
377	A	2	2	1.00	24	0.083
378	A	5	4	1.00	24	0.167
379	A	4	4	1.00	24	0.167
380	A	3	3	1.00	24	0.125
381	A	6	4	1.00	24	0.167
382	A	7	5	1.00	24	0.208
383	A	2	2	1.00	24	0.083
384	A	2	2	1.00	22	0.091
385	A	2	2	1.00	21	0.095
386	A	2	2	1.00	24	0.083
387	A	2	2	1.00	24	0.083
388	A	5	4	1.00	24	0.167
389	A	4	4	1.00	24	0.167
390	A	4	4	1.00	24	0.167
391	A	7	5	1.00	24	0.208
392	A	8	6	1.00	24	0.250
393	A	2	2	1.00	24	0.083
394	A	2	2	1.00	22	0.091
395	A	2	2	1.00	21	0.095
396	A	2	2	1.00	24	0.083
397	A	2	2	1.00	24	0.083
398	A	6	6	1.00	27	0.222
399	A	6	6	1.00	27	0.222
400	A	5	5	1.00	27	0.185
401	A	4	4	1.00	27	0.148
402	A	7	6	1.00	27	0.222
403	A	8	7	1.00	27	0.259
404	A	9	7	1.00	27	0.259
405	A	15	13	1.00	27	0.482
406	A	14	12	1.00	27	0.444
407	A	14	12	1.00	25	0.480
408	A	15	13	1.00	27	0.482
409	A	16	13	1.00	27	0.482

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
410	A	17	13	1.00	27	0.482
411	A	7	7	1.00	27	0.259
412	A	7	6	1.00	27	0.222
413	A	6	5	1.00	27	0.185
414	A	5	5	1.00	27	0.185
415	A	7	6	1.00	27	0.222
416	A	8	7	1.00	27	0.259
417	A	9	7	1.00	27	0.259
418	A	16	14	1.00	27	0.518
419	A	15	13	1.00	27	0.482
420	A	14	12	1.00	25	0.480
421	A	6	6	1.00	27	0.222
422	A	16	13	1.00	27	0.482
423	A	17	13	1.00	27	0.482
424	A	5	5	1.00	27	0.185
425	A	5	5	1.00	27	0.185
426	A	4	4	1.00	27	0.148
427	A	4	4	1.00	27	0.148
428	A	7	6	1.00	27	0.222
429	A	8	7	1.00	27	0.259
430	A	9	7	1.00	27	0.259
431	A	14	12	1.00	27	0.444
432	A	14	12	1.00	27	0.444
433	A	14	12	1.00	25	0.480
434	A	15	13	1.00	27	0.482
435	A	16	13	1.00	27	0.482
436	A	17	13	1.00	27	0.482
437	A	2	2	1.00	27	0.074
438	A	2	2	1.00	27	0.074
439	A	2	2	1.00	24	0.083
440	A	2	2	1.00	27	0.074
441	A	2	2	1.00	27	0.074
442	A	5	5	1.00	27	0.185
443	A	5	5	1.00	27	0.185
444	A	5	5	1.00	27	0.185
445	A	5	4	1.00	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	8	7	1.00	27	0.259
447	A	9	8	1.00	27	0.296
448	A	10	8	1.00	27	0.296
449	A	15	13	1.00	27	0.482
450	A	15	13	1.00	27	0.482
451	A	15	13	1.00	25	0.520
452	A	16	14	1.00	27	0.518
453	A	17	14	1.00	27	0.518
454	A	18	14	1.00	27	0.518
455	C	2	2	0.26	27	0.074
456	A	2	2	1.00	27	0.074
457	A	2	2	1.00	24	0.083
458	A	2	2	1.00	27	0.074
459	A	2	2	1.00	27	0.074
460	A	6	6	1.00	24	0.250
461	A	5	5	1.00	24	0.208
462	A	4	4	1.00	24	0.167
463	A	7	5	1.00	24	0.208
464	A	8	6	1.00	24	0.250
465	A	2	2	1.00	24	0.083
466	A	2	2	1.00	22	0.091
467	A	2	2	1.00	21	0.095
468	A	2	2	1.00	24	0.083
469	A	2	2	1.00	24	0.083
470	A	7	6	1.00	24	0.250
471	A	6	5	1.00	24	0.208
472	A	5	5	1.00	24	0.208
473	A	7	5	1.00	24	0.208
474	A	8	6	1.00	24	0.250
475	A	2	2	1.00	24	0.083
476	A	2	2	1.00	22	0.091
477	A	2	2	1.00	21	0.095
478	A	2	2	1.00	24	0.083
479	A	2	2	1.00	24	0.083
480	A	5	5	1.00	24	0.208
481	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
482	A	4	4	1.00	24	0.167
483	A	7	5	1.00	24	0.208
484	A	8	6	1.00	24	0.250
485	A	2	2	1.00	24	0.083
486	A	2	2	1.00	22	0.091
487	A	2	2	1.00	21	0.095
488	A	2	2	1.00	24	0.083
489	A	2	2	1.00	24	0.083
490	A	5	5	0.99	24	0.208
491	A	5	5	1.00	24	0.208
492	A	5	4	1.00	24	0.167
493	A	8	6	1.00	24	0.250
494	A	9	7	1.00	24	0.292
495	A	2	2	1.00	24	0.083
496	A	2	2	1.00	22	0.091
497	A	2	2	1.00	21	0.095
498	A	2	2	1.00	24	0.083
499	A	2	2	1.00	24	0.083
500	A	3	3	1.00	24	0.125
501	A	3	3	1.00	24	0.125
502	A	3	3	1.00	24	0.125
503	A	3	3	1.00	24	0.125
504	A	3	3	1.00	24	0.125
505	A	3	3	1.00	24	0.125
506	A	5	5	1.00	26	0.192
507	A	4	4	1.00	26	0.154
508	A	3	3	1.00	26	0.115
509	A	4	4	1.00	26	0.154
510	A	3	2	1.00	26	0.077
511	A	3	2	1.00	26	0.077
512	A	3	2	1.00	24	0.083
513	A	3	2	1.00	23	0.087
514	A	3	2	1.00	26	0.077
515	A	3	2	1.00	26	0.077
516	A	7	7	1.00	26	0.269
517	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
518	A	6	6	1.00	26	0.231
519	A	6	6	1.00	26	0.231
520	A	4	4	1.00	26	0.154
521	A	6	6	1.00	26	0.231
522	A	6	6	1.00	26	0.231
523	A	4	4	1.00	26	0.154
524	A	6	6	1.00	24	0.250
525	A	6	6	1.00	24	0.250
526	A	4	4	1.00	24	0.167
527	A	8	7	1.00	26	0.269
528	A	6	5	1.00	26	0.192
529	A	7	6	1.00	26	0.231
530	A	7	6	1.00	26	0.231
531	A	5	4	1.00	26	0.154
532	A	7	6	1.00	26	0.231
533	A	7	6	1.00	26	0.231
534	A	5	4	1.00	26	0.154
535	A	9	7	1.00	26	0.269
536	A	7	5	1.00	26	0.192
537	A	8	6	1.00	26	0.231
538	A	8	6	1.00	26	0.231
539	A	6	4	1.00	26	0.154
540	A	8	6	1.00	26	0.231
541	A	8	6	1.00	26	0.231
542	A	6	4	1.00	26	0.154
543	A	6	6	1.00	26	0.231
544	A	4	4	1.00	26	0.154
545	A	5	5	1.00	26	0.192
546	A	5	5	1.00	26	0.192
547	A	3	3	1.00	26	0.115
548	A	5	5	1.00	26	0.192
549	A	5	5	1.00	26	0.192
550	A	3	3	1.00	26	0.115
551	A	6	6	1.00	26	0.231
552	A	4	4	1.00	26	0.154
553	A	5	5	1.00	26	0.192

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
554	A	5	5	1.00	26	0.192
555	A	3	3	1.00	26	0.115
556	A	6	6	1.00	26	0.231
557	A	2	2	1.00	26	0.077
558	A	4	4	1.00	26	0.154
559	A	6	6	1.00	26	0.231
560	A	4	4	1.00	26	0.154
561	A	6	6	1.00	26	0.231
562	A	2	2	1.00	26	0.077
563	A	4	4	1.00	26	0.154
564	A	7	6	1.00	26	0.231
565	A	3	3	1.00	26	0.115
566	A	5	4	1.00	26	0.154
567	A	8	7	1.00	28	0.250
568	A	8	7	1.00	28	0.250
569	A	7	7	1.00	28	0.250
570	A	6	6	1.00	28	0.214
571	A	10	6	1.00	28	0.214
572	A	13	8	1.00	28	0.286
573	A	12	8	1.00	28	0.286
574	C	2	2	0.25	28	0.071
575	C	2	2	0.28	28	0.071
576	C	2	2	0.33	26	0.077
577	C	2	2	0.49	28	0.071
578	C	2	2	0.64	28	0.071
579	C	2	2	1.16	28	0.071
580	C	2	2	1.78	28	0.071
581	C	2	2	0.13	28	0.071
582	C	2	2	0.13	28	0.071
583	C	2	2	0.15	25	0.080
584	C	2	2	0.13	28	0.071
585	C	2	2	0.13	28	0.071
586	A	8	7	1.00	28	0.250
587	A	8	7	1.00	28	0.250
588	A	7	7	1.00	28	0.250
589	A	6	6	1.00	28	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
590	A	10	6	1.00	28	0.214
591	A	13	8	1.00	28	0.286
592	A	12	8	1.00	28	0.286
593	C	2	2	0.25	28	0.071
594	C	2	2	0.29	28	0.071
595	C	2	2	0.30	25	0.080
596	C	2	2	0.50	28	0.071
597	C	2	2	0.66	28	0.071
598	C	2	2	1.17	28	0.071
599	C	2	2	1.66	28	0.071
600	C	2	2	0.13	28	0.071
601	C	2	2	0.14	28	0.071
602	C	2	2	0.14	26	0.077
603	C	2	2	0.13	28	0.071
604	C	2	2	0.13	28	0.071
605	A	7	6	1.00	22	0.273
606	A	7	6	1.00	22	0.273
607	A	7	6	1.00	22	0.273
608	A	6	6	1.00	22	0.273
609	A	5	5	1.00	22	0.227
610	A	10	7	1.00	22	0.318
611	A	11	8	1.00	22	0.364
612	A	15	10	1.47	22	0.454
613	A	14	9	1.53	22	0.409
614	A	7	7	1.39	19	0.368
615	A	8	8	1.32	22	0.364
616	A	9	8	1.13	22	0.364
617	A	9	8	1.24	22	0.364
618	C	1	1	0.10	22	0.045
619	C	1	1	0.10	22	0.045
620	C	1	1	0.11	20	0.050
621	C	1	1	0.09	22	0.045
622	C	1	1	0.09	22	0.045
623	A	7	6	1.00	22	0.273
624	A	7	6	1.00	22	0.273
625	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
626	A	5	5	1.00	22	0.227
627	A	10	7	1.00	22	0.318
628	A	11	8	1.00	22	0.364
629	A	14	10	1.42	22	0.454
630	A	14	9	1.49	22	0.409
631	A	7	7	1.39	20	0.350
632	A	8	8	1.33	22	0.364
633	A	9	8	1.13	22	0.364
634	C	1	1	0.09	22	0.045
635	C	1	1	0.09	22	0.045
636	C	1	1	0.07	19	0.053
637	C	1	1	0.09	22	0.045
638	A	11	9	1.00	22	0.409
639	A	11	9	1.00	22	0.409
640	A	9	8	1.00	22	0.364
641	A	6	6	1.00	22	0.273
642	A	6	6	1.00	22	0.273
643	A	11	8	1.00	22	0.364
644	A	13	9	1.00	22	0.409
645	C	1	1	0.15	22	0.045
646	C	1	1	0.17	22	0.045
647	C	1	1	0.36	22	0.045
648	A	8	8	1.32	19	0.421
649	C	1	1	1.65	22	0.045
650	C	1	1	2.76	22	0.045
651	C	1	1	3.97	22	0.045
652	C	1	1	0.09	22	0.045
653	C	1	1	0.09	22	0.045
654	C	1	1	0.09	22	0.045
655	C	1	1	0.09	20	0.050
656	C	1	1	0.08	22	0.045
657	C	1	1	0.08	22	0.045
658	A	8	7	1.00	24	0.292
659	A	8	7	1.00	24	0.292
660	A	7	7	1.00	24	0.292
661	A	6	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
662	A	10	7	1.00	24	0.292
663	A	13	9	1.00	24	0.375
664	A	12	9	1.00	24	0.375
665	C	2	2	0.19	24	0.083
666	C	2	2	0.23	24	0.083
667	C	2	2	0.27	22	0.091
668	C	2	2	0.52	24	0.083
669	C	2	2	0.71	24	0.083
670	C	2	2	1.75	24	0.083
671	C	2	2	2.85	24	0.083
672	A	2	2	1.00	24	0.083
673	A	2	2	1.00	24	0.083
674	A	2	2	1.00	21	0.095
675	A	2	2	1.00	24	0.083
676	A	2	2	1.00	24	0.083
677	A	8	7	1.00	24	0.292
678	A	8	7	1.00	24	0.292
679	A	7	7	1.00	24	0.292
680	A	6	6	1.00	24	0.250
681	A	10	7	1.00	24	0.292
682	A	13	9	1.00	24	0.375
683	A	12	9	1.00	24	0.375
684	C	2	2	0.19	24	0.083
685	C	2	2	0.24	24	0.083
686	C	2	2	0.25	21	0.095
687	C	2	2	0.53	24	0.083
688	C	2	2	0.72	24	0.083
689	C	2	2	1.75	24	0.083
690	C	2	2	2.56	24	0.083
691	A	2	2	1.00	24	0.083
692	A	2	2	1.00	24	0.083
693	A	2	2	1.00	22	0.091
694	A	2	2	1.00	24	0.083
695	A	2	2	1.00	24	0.083
696	A	9	7	1.00	24	0.292
697	A	8	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
698	A	7	6	1.00	24	0.250
699	A	11	8	1.00	24	0.333
700	A	15	9	1.00	24	0.375
701	A	14	10	1.00	24	0.417
702	C	2	2	0.19	24	0.083
703	C	2	2	0.23	22	0.091
704	C	2	2	0.25	24	0.083
705	C	2	2	0.45	24	0.083
706	C	2	2	0.68	24	0.083
707	C	2	2	0.82	24	0.083
708	C	2	2	3.69	24	0.083
709	A	2	2	1.00	24	0.083
710	A	2	2	1.00	24	0.083
711	A	2	2	1.00	21	0.095
712	A	2	2	1.00	24	0.083
713	A	2	2	1.00	24	0.083
714	A	7	6	1.00	24	0.250
715	A	7	6	1.00	24	0.250
716	A	7	6	1.00	24	0.250
717	A	6	6	1.00	24	0.250
718	A	5	5	1.00	24	0.208
719	A	10	7	1.00	24	0.292
720	A	11	8	1.00	24	0.333
721	A	15	9	1.44	24	0.375
722	A	14	8	1.48	24	0.333
723	A	7	7	1.40	21	0.333
724	A	8	8	1.34	24	0.333
725	A	9	8	1.27	24	0.333
726	A	9	8	1.21	24	0.333
727	A	2	2	1.00	24	0.083
728	A	2	2	1.00	24	0.083
729	A	2	2	1.00	22	0.091
730	A	2	2	1.00	24	0.083
731	A	2	2	1.00	24	0.083
732	A	7	6	1.00	24	0.250
733	A	7	6	1.00	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
734	A	6	6	1.00	24	0.250
735	A	5	5	1.00	24	0.208
736	A	10	7	1.00	24	0.292
737	A	11	8	1.00	24	0.333
738	A	16	9	1.43	24	0.375
739	A	14	8	1.48	24	0.333
740	A	7	7	1.40	22	0.318
741	A	8	8	1.34	24	0.333
742	A	9	8	1.25	24	0.333
743	A	2	2	1.00	24	0.083
744	A	2	2	1.00	24	0.083
745	A	2	2	1.00	21	0.095
746	A	2	2	1.00	24	0.083
747	A	11	7	1.00	24	0.292
748	A	9	7	1.00	24	0.292
749	A	7	6	1.00	24	0.250
750	A	6	6	1.00	24	0.250
751	A	6	6	1.00	24	0.250
752	A	11	8	1.00	24	0.333
753	A	13	9	1.00	24	0.375
754	C	2	2	0.21	24	0.083
755	C	2	2	0.26	24	0.083
756	C	2	2	0.53	24	0.083
757	A	8	8	1.33	21	0.381
758	C	2	2	2.37	24	0.083
759	C	2	2	3.31	24	0.083
760	C	2	2	4.23	24	0.083
761	A	2	2	1.00	24	0.083
762	A	2	2	1.00	24	0.083
763	A	2	2	1.00	24	0.083
764	A	2	2	1.00	22	0.091
765	A	2	2	1.00	24	0.083
766	A	2	2	1.00	24	0.083
767	A	3	2	1.00	22	0.091
768	A	3	2	1.00	22	0.091
769	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
770	A	4	3	1.00	22	0.136
771	A	3	2	1.00	22	0.091
772	A	3	2	1.00	22	0.091
773	A	6	5	1.00	22	0.227
774	A	5	4	1.00	22	0.182
775	A	4	3	1.00	22	0.136
776	A	4	3	1.00	20	0.150
777	A	5	4	1.00	22	0.182
778	A	6	5	1.00	22	0.227
779	A	20	8	1.00	22	0.364
780	A	19	7	1.00	22	0.318
781	A	19	7	1.00	22	0.318
782	A	19	7	1.00	22	0.318
783	A	19	7	1.00	19	0.368
784	A	21	8	1.00	22	0.364
785	A	20	8	1.00	22	0.364
786	A	22	9	1.00	22	0.409
787	A	5	5	1.00	24	0.208
788	A	7	7	1.00	24	0.292
789	A	4	4	1.00	24	0.167
790	A	6	6	1.00	22	0.273
791	A	6	4	1.00	24	0.167
792	A	5	5	1.00	24	0.208
793	A	7	5	1.00	24	0.208
794	A	6	6	1.00	24	0.250
795	A	13	8	1.25	24	0.333
796	A	10	6	1.29	24	0.250
797	A	11	7	1.29	24	0.292
798	A	9	5	1.30	21	0.238
799	A	13	8	1.27	24	0.333
800	A	10	6	1.27	24	0.250
801	A	3	3	1.00	28	0.107
802	A	3	3	1.00	28	0.107
803	A	3	3	1.00	28	0.107
804	A	3	3	1.00	28	0.107
805	A	5	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
806	A	4	4	1.00	24	0.167
807	A	3	3	1.00	24	0.125
808	A	6	4	1.00	24	0.167
809	A	7	5	1.00	24	0.208
810	A	7	7	1.00	24	0.292
811	A	6	6	1.00	24	0.250
812	A	3	3	1.00	22	0.136
813	A	5	5	1.00	24	0.208
814	A	6	6	1.00	24	0.250
815	A	10	6	1.00	24	0.250
816	A	9	5	1.31	24	0.208
817	A	7	4	1.16	21	0.190
818	A	10	6	1.28	24	0.250
819	A	11	7	1.22	24	0.292
820	A	7	4	1.15	24	0.167
821	A	13	8	1.21	24	0.333
822	A	5	5	1.00	24	0.208
823	A	5	5	1.00	24	0.208
824	A	4	4	1.00	24	0.167
825	A	4	4	1.00	24	0.167
826	A	7	5	1.00	24	0.208
827	A	8	6	1.00	24	0.250
828	A	8	8	1.00	24	0.333
829	A	7	7	1.00	24	0.292
830	A	5	5	1.00	24	0.208
831	A	4	4	1.00	22	0.182
832	A	6	6	1.00	24	0.250
833	A	7	6	1.00	24	0.250
834	A	10	6	1.00	24	0.250
835	A	10	6	1.00	24	0.250
836	A	10	6	1.00	21	0.286
837	A	11	7	1.00	24	0.292
838	A	13	8	1.00	24	0.333
839	A	13	8	1.00	24	0.333
840	A	14	9	1.00	24	0.375
841	A	4	4	0.97	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
842	A	3	3	0.93	24	0.125
843	A	2	2	1.00	17	0.118
844	A	2	2	1.00	26	0.077
845	A	2	2	1.00	26	0.077
846	A	2	2	1.00	26	0.077
847	A	4	4	1.00	26	0.154
848	A	3	3	1.00	24	0.125
849	A	2	2	1.00	17	0.118
850	A	2	2	1.00	26	0.077
851	A	2	2	1.00	26	0.077
852	A	2	2	1.00	26	0.077
853	A	5	4	1.00	24	0.167
854	A	4	4	1.00	24	0.167
855	A	3	3	1.00	24	0.125
856	A	6	4	1.00	24	0.167
857	A	7	5	1.00	24	0.208
858	A	7	7	1.00	24	0.292
859	A	6	6	1.00	24	0.250
860	A	3	3	1.00	24	0.125
861	A	5	5	1.00	24	0.208
862	A	6	6	1.00	24	0.250
863	A	2	2	1.00	24	0.083
864	A	3	3	1.00	24	0.125
865	A	3	3	1.00	22	0.136
866	A	2	2	1.00	21	0.095
867	A	2	2	1.00	24	0.083
868	A	3	3	1.00	24	0.125
869	A	3	3	1.00	24	0.125
870	A	5	5	1.00	24	0.208
871	A	4	4	1.00	24	0.167
872	A	4	4	1.00	24	0.167
873	A	7	5	1.00	24	0.208
874	A	8	6	1.00	24	0.250
875	A	7	7	1.00	24	0.292
876	A	5	5	1.00	24	0.208
877	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
878	A	6	6	1.00	24	0.250
879	A	7	6	1.00	24	0.250
880	A	2	2	1.00	24	0.083
881	A	3	3	1.00	24	0.125
882	A	3	3	1.00	22	0.136
883	A	2	2	1.00	21	0.095
884	A	2	2	1.00	24	0.083
885	A	3	3	1.00	24	0.125
886	A	3	3	1.00	24	0.125
887	A	5	4	1.00	24	0.167
888	A	4	4	1.00	24	0.167
889	A	3	3	1.00	24	0.125
890	A	6	4	1.00	24	0.167
891	A	7	5	1.00	24	0.208
892	A	7	7	1.00	24	0.292
893	A	6	6	1.00	24	0.250
894	A	3	3	1.00	24	0.125
895	A	5	5	1.00	24	0.208
896	A	6	6	1.00	24	0.250
897	A	10	6	1.00	24	0.250
898	A	8	5	1.00	22	0.227
899	A	11	7	1.00	24	0.292
900	A	12	8	1.00	24	0.333
901	A	8	5	1.00	24	0.208
902	A	14	9	1.00	24	0.375
903	A	2	2	1.00	24	0.083
904	A	2	2	1.00	24	0.083
905	A	2	2	1.00	21	0.095
906	A	2	2	1.00	24	0.083
907	A	2	2	1.00	24	0.083
908	A	5	5	1.00	24	0.208
909	A	4	4	1.00	24	0.167
910	A	4	4	1.00	24	0.167
911	A	7	5	1.00	24	0.208
912	A	8	6	1.00	24	0.250
913	A	7	7	1.00	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
914	A	5	5	1.00	24	0.208
915	A	4	4	1.00	24	0.167
916	A	6	6	1.00	24	0.250
917	A	7	6	1.00	24	0.250
918	A	11	7	1.00	24	0.292
919	A	11	7	1.00	22	0.318
920	A	12	8	1.00	24	0.333
921	A	14	9	1.00	24	0.375
922	A	14	9	1.00	24	0.375
923	A	15	10	1.00	24	0.417
924	A	2	2	1.00	24	0.083
925	A	2	2	1.00	24	0.083
926	A	2	2	1.00	21	0.095
927	A	2	2	1.00	24	0.083
928	A	2	2	1.00	24	0.083
929	A	6	6	1.00	22	0.273
930	A	5	5	1.00	22	0.227
931	A	5	5	1.00	20	0.250
932	A	5	5	1.00	22	0.227
933	A	3	2	1.00	22	0.091
934	A	3	2	1.00	22	0.091
935	A	3	2	1.00	22	0.091
936	A	3	2	1.00	22	0.091
937	A	5	3	1.00	22	0.136
938	A	4	3	1.00	22	0.136
939	A	3	3	1.00	22	0.136
940	A	2	2	1.00	22	0.091
941	A	5	5	1.00	22	0.227
942	A	5	5	1.00	19	0.263
943	A	5	5	1.00	22	0.227
944	A	6	6	1.00	22	0.273
945	A	6	5	1.00	22	0.227
946	A	6	6	1.00	22	0.273
947	A	6	5	1.00	20	0.250
948	A	6	5	1.00	22	0.227
949	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
950	A	3	2	1.00	22	0.091
951	A	3	2	1.00	22	0.091
952	A	3	2	1.00	22	0.091
953	A	5	3	1.00	22	0.136
954	A	4	3	1.00	22	0.136
955	A	3	3	1.00	22	0.136
956	A	2	2	1.00	22	0.091
957	A	6	5	1.00	22	0.227
958	A	6	6	1.00	22	0.273
959	A	6	5	1.00	19	0.263
960	A	6	5	1.00	22	0.227
961	A	7	6	1.00	22	0.273
962	A	5	5	1.00	22	0.227
963	A	4	4	1.00	20	0.200
964	A	4	4	1.00	22	0.182
965	A	3	2	1.00	22	0.091
966	A	3	2	1.00	22	0.091
967	A	3	2	1.00	22	0.091
968	A	3	3	1.00	22	0.136
969	A	2	2	1.00	22	0.091
970	A	4	4	1.00	19	0.210
971	A	4	4	1.00	22	0.182
972	A	5	5	1.00	22	0.227
973	A	6	5	1.02	22	0.227
974	A	5	5	1.00	20	0.250
975	A	4	4	1.00	22	0.182
976	A	3	2	1.00	22	0.091
977	A	3	2	1.00	22	0.091
978	A	3	2	1.00	22	0.091
979	A	3	2	1.00	22	0.091
980	A	4	4	1.00	22	0.182
981	A	3	3	1.00	22	0.136
982	A	3	3	1.00	19	0.158
983	A	4	4	1.00	22	0.182
984	A	5	5	1.00	22	0.227
985	A	6	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
986	A	4	3	0.96	24	0.125
987	A	4	3	1.00	22	0.136
988	A	3	3	1.00	22	0.136
989	A	4	3	1.00	22	0.136
990	A	3	3	1.00	20	0.150
991	A	4	3	1.00	19	0.158
992	A	3	3	1.00	22	0.136
993	A	4	3	1.00	22	0.136
994	A	3	3	1.00	22	0.136
995	A	4	3	1.00	22	0.136
996	A	4	3	1.00	26	0.115
997	A	4	3	1.00	26	0.115
998	A	4	3	1.00	26	0.115
999	A	4	3	1.00	26	0.115
1000	A	4	3	1.00	26	0.115
1001	A	4	3	1.00	26	0.115
1002	A	6	4	1.00	28	0.143
1003	A	5	4	1.00	28	0.143
1004	A	4	4	1.00	28	0.143
1005	A	3	3	1.00	28	0.107
1006	A	4	4	1.00	28	0.143
1007	A	1	1	1.00	28	0.036
1008	A	2	2	1.00	28	0.071
1009	A	3	2	1.00	28	0.071
1010	A	4	2	1.00	28	0.071
1011	A	5	3	1.00	28	0.107
1012	A	4	3	1.00	28	0.107
1013	A	3	3	1.00	28	0.107
1014	A	2	2	1.00	28	0.071
1015	A	1	1	1.00	28	0.036
1016	A	2	2	1.00	28	0.071
1017	A	3	2	1.00	28	0.071
1018	A	3	3	1.00	24	0.125
1019	A	3	3	1.00	22	0.136
1020	A	3	3	1.00	21	0.143
1021	A	4	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1022	A	3	3	1.00	24	0.125
1023	A	3	2	1.00	18	0.111
1024	A	1	1	1.00	33	0.030
1025	A	3	2	1.00	24	0.083
1026	A	3	2	1.00	22	0.091
1027	A	3	2	1.00	20	0.100
1028	A	3	2	1.00	19	0.105
1029	A	3	2	1.00	22	0.091
1030	A	3	2	1.00	22	0.091
1031	A	3	2	1.00	22	0.091
1032	A	5	3	1.00	24	0.125
1033	A	5	3	1.00	22	0.136
1034	A	5	3	1.00	20	0.150
1035	A	4	3	1.00	19	0.158
1036	A	3	2	1.00	22	0.091
1037	A	5	3	1.00	22	0.136
1038	A	5	3	1.00	22	0.136
1039	A	3	2	1.00	26	0.077
1040	A	3	2	1.00	26	0.077
1041	A	3	2	1.00	24	0.083
1042	A	3	2	1.00	26	0.077
1043	A	3	2	1.00	26	0.077
1044	A	3	2	1.00	26	0.077
1045	A	3	2	1.00	26	0.077
1046	A	3	2	1.00	26	0.077
1047	A	3	2	1.00	24	0.083
1048	A	3	2	1.00	26	0.077
1049	A	3	2	1.00	26	0.077
1050	A	3	2	1.00	26	0.077
1051	A	1	1	1.00	17	0.059
1052	A	2	2	1.00	21	0.095
1053	A	2	2	1.00	21	0.095
1054	A	2	2	1.00	25	0.080
1055	A	1	1	1.00	31	0.032
1056	A	2	1	1.12	17	0.059
1057	A	3	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1058	A	3	2	1.00	21	0.095
1059	A	3	2	1.00	21	0.095
1060	A	1	1	1.00	17	0.059
1061	A	2	2	1.00	21	0.095
1062	A	2	2	1.00	21	0.095
1063	A	2	2	1.00	25	0.080
1064	A	5	5	1.00	22	0.227
1065	A	7	7	1.00	26	0.269
1066	A	8	6	1.00	30	0.200
1067	A	7	6	1.00	30	0.200
1068	A	6	6	1.00	30	0.200
1069	A	5	5	1.00	30	0.167
1070	A	5	5	1.00	30	0.167
1071	A	3	3	1.00	30	0.100
1072	A	9	7	1.00	30	0.233
1073	A	8	7	1.00	30	0.233
1074	A	7	7	1.00	30	0.233
1075	A	6	6	1.00	30	0.200
1076	A	6	6	1.00	30	0.200
1077	A	6	6	1.00	30	0.200
1078	A	1	1	1.00	17	0.059
1079	A	1	1	1.00	27	0.037
1080	A	1	1	1.00	27	0.037
1081	A	1	1	1.00	27	0.037

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(a+bx^3\right) \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=33

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

```
[Out] 1/3*a*A*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {444, 43}

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^3\right)\left(A+Bx^3\right) dx &= \frac{1}{3}\,\text{Subst}\left(\int (a+bx)(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(aA+(Ab+aB)x+bBx^2\right) dx, x, x^3\right)\\ &= \frac{1}{3}aAx^3+\frac{1}{6}(Ab+aB)x^6+\frac{1}{9}bBx^9\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{6}x^6(aB+Ab)+\frac{1}{3}aAx^3+\frac{1}{9}bBx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)*(A + B*x^3),x]
[Out] (a*A*x^3)/3 + ((A*b + a*B)*x^6)/6 + (b*B*x^9)/9
```

fricas [A] time = 0.71, size = 29, normalized size = 0.88

$$\frac{1}{9}x^9bB+\frac{1}{6}x^6aB+\frac{1}{6}x^6bA+\frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 1/9*x^9*b*B + 1/6*x^6*a*B + 1/6*x^6*b*A + 1/3*x^3*a*A
```

giac [A] time = 0.18, size = 29, normalized size = 0.88

$$\frac{1}{9}Bbx^9+\frac{1}{6}Bax^6+\frac{1}{6}Abx^6+\frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/9*B*b*x^9 + 1/6*B*a*x^6 + 1/6*A*b*x^6 + 1/3*A*a*x^3
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bb\,x^9}{9}+\frac{(Ab+Ba)\,x^6}{6}+\frac{Aa\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)*(B*x^3+A),x)
[Out] 1/3*A*a*x^3+1/6*(A*b+B*a)*x^6+1/9*b*B*x^9
```

maxima [A] time = 0.57, size = 27, normalized size = 0.82

$$\frac{1}{9}Bbx^9+\frac{1}{6}(Ba+Ab)x^6+\frac{1}{3}Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/9*B*b*x^9 + 1/6*(B*a + A*b)*x^6 + 1/3*A*a*x^3
```

mupad [B] time = 0.20, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^9}{9}+\left(\frac{A\,b}{6}+\frac{B\,a}{6}\right)\,x^6+\frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3),x)
[Out] x^6*((A*b)/6 + (B*a)/6) + (A*a*x^3)/3 + (B*b*x^9)/9
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^3}{3} + \frac{Bbx^9}{9} + x^6\left(\frac{Ab}{6} + \frac{Ba}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x**3/3 + B*b*x**9/9 + x**6*(A*b/6 + B*a/6)
```

3.2 $\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=33

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

```
[Out] 1/2*a*A*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {448}

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aAx+(Ab+aB)x^4+bBx^7\right)\,dx\\ &= \frac{1}{2}aAx^2+\frac{1}{5}(Ab+aB)x^5+\frac{1}{8}bBx^8\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{5}x^5(aB+Ab)+\frac{1}{2}aAx^2+\frac{1}{8}bBx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^2)/2 + ((A*b + a*B)*x^5)/5 + (b*B*x^8)/8
```

fricas [A] time = 1.01, size = 29, normalized size = 0.88

$$\frac{1}{8}x^8bB+\frac{1}{5}x^5aB+\frac{1}{5}x^5bA+\frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b*B + 1/5*x^5*a*B + 1/5*x^5*b*A + 1/2*x^2*a*A
```

giac [A] time = 0.15, size = 29, normalized size = 0.88

$$\frac{1}{8}Bbx^8+\frac{1}{5}Bax^5+\frac{1}{5}Abx^5+\frac{1}{2}Aax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/8*B*b*x^8 + 1/5*B*a*x^5 + 1/5*A*b*x^5 + 1/2*A*a*x^2
```

maple [A] time = 0.04, size = 28, normalized size = 0.85

$$\frac{Bb\,x^{8}}{8}+\frac{\left(Ab+Ba\right)x^{5}}{5}+\frac{Aa\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 1/2*A*a*x^2+1/5*(A*b+B*a)*x^5+1/8*b*B*x^8
```

maxima [A] time = 0.63, size = 27, normalized size = 0.82

$$\frac{1}{8}\,Bbx^{8}+\frac{1}{5}\,(Ba+Ab)x^{5}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/8*B*b*x^8 + 1/5*(B*a + A*b)*x^5 + 1/2*A*a*x^2
```

mupad [B] time = 2.48, size = 28, normalized size = 0.85

$$\frac{B\,b\,x^{8}}{8}+\left(\frac{A\,b}{5}+\frac{B\,a}{5}\right)\,x^{5}+\frac{A\,a\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^5*((A*b)/5 + (B*a)/5) + (A*a*x^2)/2 + (B*b*x^8)/8
```

sympy [A] time = 0.07, size = 29, normalized size = 0.88

$$\frac{Aax^{2}}{2}+\frac{Bbx^{8}}{8}+x^{5}\left(\frac{Ab}{5}+\frac{Ba}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] A*a*x**2/2 + B*b*x**8/8 + x**5*(A*b/5 + B*a/5)
```

3.3 $\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

```
[Out] a*A*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA+(Ab+aB)x^3+bBx^6\right)\,dx\\ &= aAx+\frac{1}{4}(Ab+aB)x^4+\frac{1}{7}bBx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(aB+Ab)+aAx+\frac{1}{7}bBx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] a*A*x + ((A*b + a*B)*x^4)/4 + (b*B*x^7)/7
```

fricas [A] time = 0.94, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7bB+\frac{1}{4}x^4aB+\frac{1}{4}x^4bA+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b*B + 1/4*x^4*a*B + 1/4*x^4*b*A + x*a*A
```

giac [A] time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}Bbx^7+\frac{1}{4}Bax^4+\frac{1}{4}Abx^4+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="giac")
[Out] 1/7*B*b*x^7 + 1/4*B*a*x^4 + 1/4*A*b*x^4 + A*a*x
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{Bb\,x^{7}}{7}+\frac{\left(Ab+Ba\right)x^{4}}{4}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A),x)
[Out] A*a*x+1/4*(A*b+B*a)*x^4+1/7*b*B*x^7
```

maxima [A] time = 0.56, size = 24, normalized size = 0.86

$$\frac{1}{7}\,Bbx^{7}+\frac{1}{4}\,(Ba+Ab)x^{4}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
[Out] 1/7*B*b*x^7 + 1/4*(B*a + A*b)*x^4 + A*a*x
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$\frac{B\,b\,x^7}{7}+\left(\frac{A\,b}{4}+\frac{B\,a}{4}\right)\,x^4+A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3),x)
[Out] x^4*((A*b)/4 + (B*a)/4) + A*a*x + (B*b*x^7)/7
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$Aax+\frac{Bbx^{7}}{7}+x^{4}\left(\frac{Ab}{4}+\frac{Ba}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A),x)
[Out] A*a*x + B*b*x**7/7 + x**4*(A*b/4 + B*a/4)
```

3.4 $\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

```
[Out] 1/3*(A*b+B*a)*x^3+1/6*b*B*x^6+a*A*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(Ab+aB+\frac{aA}{x}+bBx\right)dx,x,x^3\right)\\ &= \frac{1}{3}(Ab+aB)x^3+\frac{1}{6}bBx^6+aA\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{1}{3}x^3(aB+Ab)+aA\log(x)+\frac{1}{6}bBx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x,x]
```

```
[Out] ((A*b + a*B)*x^3)/3 + (b*B*x^6)/6 + a*A*Log[x]
```

fricas [A] time = 0.91, size = 25, normalized size = 0.86

$$\frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba + Ab)x^3 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="fricas")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + A*a*log(x)
```

giac [A] time = 0.15, size = 28, normalized size = 0.97

$$\frac{1}{6}\,Bbx^6 + \frac{1}{3}\,Bax^3 + \frac{1}{3}\,Abx^3 + Aa\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="giac")
[Out] 1/6*B*b*x^6 + 1/3*B*a*x^3 + 1/3*A*b*x^3 + A*a*log(abs(x))
```

maple [A] time = 0.04, size = 28, normalized size = 0.97

$$\frac{Bb\,x^6}{6} + \frac{Ab\,x^3}{3} + \frac{Ba\,x^3}{3} + Aa\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x,x)
[Out] 1/6*b*B*x^6+1/3*A*x^3*b+1/3*B*a*x^3+A*a*ln(x)
```

maxima [A] time = 0.57, size = 28, normalized size = 0.97

$$\frac{1}{6}\,Bbx^6 + \frac{1}{3}\,(Ba + Ab)x^3 + \frac{1}{3}\,Aa\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x,x, algorithm="maxima")
[Out] 1/6*B*b*x^6 + 1/3*(B*a + A*b)*x^3 + 1/3*A*a*log(x^3)
```

mupad [B] time = 0.03, size = 26, normalized size = 0.90

$$x^3\,\left(\frac{A\,b}{3} + \frac{B\,a}{3}\right) + \frac{B\,b\,x^6}{6} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x,x)
[Out] x^3*((A*b)/3 + (B*a)/3) + (B*b*x^6)/6 + A*a*log(x)
```

sympy [A] time = 0.11, size = 27, normalized size = 0.93

$$Aa\log\left(x\right) + \frac{Bbx^6}{6} + x^3\left(\frac{Ab}{3} + \frac{Ba}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x,x)
[Out] A*a*log(x) + B*b*x**6/6 + x**3*(A*b/3 + B*a/3)
```

3.5 $\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx$

Optimal. Leaf size=31

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

```
[Out] -a*A/x+1/2*(A*b+B*a)*x^2+1/5*b*B*x^5
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^2} \, dx &= \int \left(\frac{aA}{x^2}+(Ab+aB)x+bBx^4\right) dx\\ &= -\frac{aA}{x}+\frac{1}{2}(Ab+aB)x^2+\frac{1}{5}bBx^5\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{1}{2}x^2(aB+Ab)-\frac{aA}{x}+\frac{1}{5}bBx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + ((A*b + a*B)*x^2)/2 + (b*B*x^5)/5
```

fricas [A] time = 0.60, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^6+5\,(Ba+Ab)x^3-10\,Aa}{10\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*B*b*x^6 + 5*(B*a + A*b)*x^3 - 10*A*a)/x
```

giac [A] time = 0.15, size = 29, normalized size = 0.94

$$\frac{1}{5}\,Bbx^5+\frac{1}{2}\,Bax^2+\frac{1}{2}\,Abx^2-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="giac")
[Out] 1/5*B*b*x^5 + 1/2*B*a*x^2 + 1/2*A*b*x^2 - A*a/x
```

maple [A] time = 0.05, size = 30, normalized size = 0.97

$$\frac{Bb\,x^5}{5}+\frac{Ab\,x^2}{2}+\frac{Ba\,x^2}{2}-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^2,x)
[Out] 1/5*b*B*x^5+1/2*A*b*x^2+1/2*B*a*x^2-A*a/x
```

maxima [A] time = 0.50, size = 27, normalized size = 0.87

$$\frac{1}{5}\,Bbx^5+\frac{1}{2}\,(Ba+Ab)x^2-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^2,x, algorithm="maxima")
[Out] 1/5*B*b*x^5 + 1/2*(B*a + A*b)*x^2 - A*a/x
```

mupad [B] time = 0.04, size = 28, normalized size = 0.90

$$x^2\left(\frac{A\,b}{2}+\frac{B\,a}{2}\right)-\frac{A\,a}{x}+\frac{B\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^2,x)
[Out] x^2*((A*b)/2 + (B*a)/2) - (A*a)/x + (B*b*x^5)/5
```

sympy [A] time = 0.11, size = 26, normalized size = 0.84

$$-\frac{Aa}{x}+\frac{Bbx^5}{5}+x^2\left(\frac{Ab}{2}+\frac{Ba}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**2,x)
[Out] -A*a/x + B*b*x**5/5 + x**2*(A*b/2 + B*a/2)
```

3.6 $\int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx$

Optimal. Leaf size=28

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

```
[Out] -1/2*a*A/x^2+(A*b+B*a)*x+1/4*b*B*x^4
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) + (A*b + a*B)*x + (b*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)(A+Bx^3)}{x^3} dx &= \int \left(Ab\left(1+\frac{aB}{Ab}\right)+\frac{aA}{x^3}+bBx^3\right) dx \\ &= -\frac{aA}{2x^2}+(Ab+aB)x+\frac{1}{4}bBx^4 \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$x(aB + Ab) - \frac{aA}{2x^2} + \frac{1}{4}bBx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 + (A*b + a*B)*x + (b*B*x^4)/4
```

fricas [A] time = 0.91, size = 28, normalized size = 1.00

$$\frac{Bbx^6 + 4(Ba + Ab)x^3 - 2Aa}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(B*b*x^6 + 4*(B*a + A*b)*x^3 - 2*A*a)/x^2
```

giac [A] time = 0.16, size = 23, normalized size = 0.82

$$\frac{1}{4}Bbx^4 + Bax + Abx - \frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="giac")
[Out] 1/4*B*b*x^4 + B*a*x + A*b*x - 1/2*A*a/x^2
```

maple [A] time = 0.04, size = 24, normalized size = 0.86

$$\frac{Bb\,x^4}{4}+Abx+Bax-\frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^3,x)
[Out] 1/4*b*B*x^4+A*b*x+B*a*x-1/2*A*a/x^2
```

maxima [A] time = 0.48, size = 24, normalized size = 0.86

$$\frac{1}{4}\,Bbx^4+(Ba+Ab)x-\frac{Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^3,x, algorithm="maxima")
[Out] 1/4*B*b*x^4 + (B*a + A*b)*x - 1/2*A*a/x^2
```

mupad [B] time = 2.34, size = 24, normalized size = 0.86

$$x\,\left(A\,b+B\,a\right)-\frac{A\,a}{2\,x^2}+\frac{B\,b\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^3,x)
[Out] x*(A*b + B*a) - (A*a)/(2*x^2) + (B*b*x^4)/4
```

sympy [A] time = 0.12, size = 24, normalized size = 0.86

$$-\frac{Aa}{2x^2}+\frac{Bbx^4}{4}+x\,(Ab+Ba)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**3,x)
[Out] -A*a/(2*x**2) + B*b*x**4/4 + x*(A*b + B*a)
```

3.7 $\int \frac{(a+bx^3)(A+Bx^3)}{x^4} dx$

Optimal. Leaf size=29

$$\log(x)(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{3}bBx^3$$

```
[Out] -1/3*a*A/x^3+1/3*b*B*x^3+(A*b+B*a)*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$\log(x)(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(bB+\frac{aA}{x^2}+\frac{Ab+aB}{x}\right)dx,x,x^3\right)\\
&= -\frac{aA}{3x^3}+\frac{1}{3}bBx^3+(Ab+aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\log(x)(aB+Ab)-\frac{aA}{3x^3}+\frac{1}{3}bBx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 + (b*B*x^3)/3 + (A*b + a*B)*Log[x]
```

fricas [A] time = 0.72, size = 30, normalized size = 1.03

$$\frac{Bbx^6 + 3\,(Ba + Ab)x^3\log(x) - Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="fricas")
[Out] 1/3*(B*b*x^6 + 3*(B*a + A*b)*x^3*log(x) - A*a)/x^3
```

giac [A] time = 0.16, size = 40, normalized size = 1.38

$$\frac{1}{3}\,Bbx^3 + (Ba + Ab)\log\left(|x|\right) - \frac{Bax^3 + Abx^3 + Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="giac")
[Out] 1/3*B*b*x^3 + (B*a + A*b)*log(abs(x)) - 1/3*(B*a*x^3 + A*b*x^3 + A*a)/x^3
```

maple [A] time = 0.05, size = 26, normalized size = 0.90

$$\frac{Bb\,x^3}{3} + Ab\ln(x) + Ba\ln(x) - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^4,x)
[Out] 1/3*B*b*x^3-1/3*A*a/x^3+A*b*ln(x)+B*a*ln(x)
```

maxima [A] time = 0.48, size = 28, normalized size = 0.97

$$\frac{1}{3}\,Bbx^3 + \frac{1}{3}\,(Ba + Ab)\log\left(x^3\right) - \frac{Aa}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^4,x, algorithm="maxima")
[Out] 1/3*B*b*x^3 + 1/3*(B*a + A*b)*log(x^3) - 1/3*A*a/x^3
```

mupad [B] time = 0.04, size = 25, normalized size = 0.86

$$\ln(x)\,\left(A\,b + B\,a\right) - \frac{A\,a}{3\,x^3} + \frac{B\,b\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^4,x)
[Out] log(x)*(A*b + B*a) - (A*a)/(3*x^3) + (B*b*x^3)/3
```

sympy [A] time = 0.21, size = 26, normalized size = 0.90

$$-\frac{Aa}{3x^3} + \frac{Bbx^3}{3} + (Ab + Ba)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**4,x)
[Out] -A*a/(3*x**3) + B*b*x**3/3 + (A*b + B*a)*log(x)
```

3.8 $\int \frac{(a+bx^3)(A+Bx^3)}{x^5} dx$

Optimal. Leaf size=31

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

```
[Out] -1/4*a*A/x^4+(-A*b-B*a)/x+1/2*b*B*x^2
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (A*b + a*B)/x + (b*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^5}\,dx &= \int \left(\frac{aA}{x^5}+\frac{Ab+aB}{x^2}+bBx\right)dx\\ &= -\frac{aA}{4x^4}-\frac{Ab+aB}{x}+\frac{1}{2}bBx^2\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{-aB-Ab}{x}-\frac{aA}{4x^4}+\frac{1}{2}bBx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a*A)/x^4 + (-(A*b) - a*B)/x + (b*B*x^2)/2
```

fricas [A] time = 0.80, size = 29, normalized size = 0.94

$$\frac{2\,Bbx^6-4\,(Ba+Ab)x^3-Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*B*b*x^6 - 4*(B*a + A*b)*x^3 - A*a)/x^4
```

giac [A] time = 0.18, size = 31, normalized size = 1.00

$$\frac{1}{2}\,Bbx^2-\frac{4\,Bax^3+4\,Abx^3+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="giac")
[Out] 1/2*B*b*x^2 - 1/4*(4*B*a*x^3 + 4*A*b*x^3 + A*a)/x^4
```

maple [A] time = 0.05, size = 28, normalized size = 0.90

$$\frac{Bb\,x^2}{2}-\frac{Ab+Ba}{x}-\frac{Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^5,x)
[Out] 1/2*b*B*x^2-1/4*a*A/x^4-(A*b+B*a)/x
```

maxima [A] time = 0.63, size = 29, normalized size = 0.94

$$\frac{1}{2}\,Bbx^2-\frac{4\,(Ba+Ab)x^3+Aa}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^5,x, algorithm="maxima")
[Out] 1/2*B*b*x^2 - 1/4*(4*(B*a + A*b)*x^3 + A*a)/x^4
```

mupad [B] time = 0.03, size = 29, normalized size = 0.94

$$\frac{B\,b\,x^2}{2}-\frac{\left(A\,b+B\,a\right)\,x^3+\frac{A\,a}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^5,x)
[Out] (B*b*x^2)/2 - ((A*a)/4 + x^3*(A*b + B*a))/x^4
```

sympy [A] time = 0.25, size = 31, normalized size = 1.00

$$\frac{Bbx^2}{2}+\frac{-Aa+x^3\,(-4Ab-4Ba)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**5,x)
[Out] B*b*x**2/2 + (-A*a + x**3*(-4*A*b - 4*B*a))/(4*x**4)
```

3.9 $\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx$

Optimal. Leaf size=28

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

```
[Out] -1/5*a*A/x^5+1/2*(-A*b-B*a)/x^2+b*B*x
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$-\frac{aB+Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (A*b + a*B)/(2*x^2) + b*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^6}\,dx &= \int \left(bB+\frac{aA}{x^6}+\frac{Ab+aB}{x^3}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{Ab+aB}{2x^2}+bBx\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.07

$$\frac{-aB-Ab}{2x^2}-\frac{aA}{5x^5}+bBx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a*A)/x^5 + (-(A*b) - a*B)/(2*x^2) + b*B*x
```

fricas [A] time = 1.14, size = 29, normalized size = 1.04

$$\frac{10\,Bbx^6-5\,(Ba+Ab)x^3-2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(10*B*b*x^6 - 5*(B*a + A*b)*x^3 - 2*A*a)/x^5
```

giac [A] time = 0.18, size = 29, normalized size = 1.04

$$Bbx-\frac{5\,Bax^3+5\,Abx^3+2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="giac")
[Out] B*b*x - 1/10*(5*B*a*x^3 + 5*A*b*x^3 + 2*A*a)/x^5
```

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$Bbx - \frac{Ab + Ba}{2x^2} - \frac{Aa}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^6,x)
[Out] B*b*x-1/5*a*A/x^5-1/2*(A*b+B*a)/x^2
```

maxima [A] time = 0.49, size = 27, normalized size = 0.96

$$Bbx - \frac{5\,(Ba + Ab)x^3 + 2\,Aa}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^6,x, algorithm="maxima")
[Out] B*b*x - 1/10*(5*(B*a + A*b)*x^3 + 2*A*a)/x^5
```

mupad [B] time = 2.32, size = 28, normalized size = 1.00

$$B\,b\,x - \frac{\left(\frac{A\,b}{2} + \frac{B\,a}{2}\right)\,x^3 + \frac{A\,a}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^6,x)
[Out] B*b*x - ((A*a)/5 + x^3*((A*b)/2 + (B*a)/2))/x^5
```

sympy [A] time = 0.29, size = 29, normalized size = 1.04

$$Bbx + \frac{-2Aa + x^3\,(-5Ab - 5Ba)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**6,x)
[Out] B*b*x + (-2*A*a + x**3*(-5*A*b - 5*B*a))/(10*x**5)
```

3.10 $\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=29

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

```
[Out] -1/6*a*A/x^6+1/3*(-A*b-B*a)/x^3+b*B*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 76}

$$-\frac{aB+Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (A*b + a*B)/(3*x^3) + b*B*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)(A+Bx)}{x^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{aA}{x^3}+\frac{Ab+aB}{x^2}+\frac{bB}{x}\right)dx,x,x^3\right)\\ &= -\frac{aA}{6x^6}-\frac{Ab+aB}{3x^3}+bB\log(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.07

$$\frac{-aB-Ab}{3x^3}-\frac{aA}{6x^6}+bB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^7,x]
```

```
[Out] -1/6*(a*A)/x^6 + (-(A*b) - a*B)/(3*x^3) + b*B*Log[x]
```

fricas [A] time = 0.94, size = 31, normalized size = 1.07

$$\frac{6\,Bbx^{6}\log(x) - 2\,(Ba + Ab)x^{3} - Aa}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="fricas")
[Out] 1/6*(6*B*b*x^6*log(x) - 2*(B*a + A*b)*x^3 - A*a)/x^6
```

giac [A] time = 0.16, size = 37, normalized size = 1.28

$$Bb\log(|x|) - \frac{3\,Bbx^{6} + 2\,Bax^{3} + 2\,Abx^{3} + Aa}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="giac")
[Out] B*b*log(abs(x)) - 1/6*(3*B*b*x^6 + 2*B*a*x^3 + 2*A*b*x^3 + A*a)/x^6
```

maple [A] time = 0.04, size = 28, normalized size = 0.97

$$Bb\ln(x) - \frac{Ab}{3x^{3}} - \frac{Ba}{3x^{3}} - \frac{Aa}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^7,x)
[Out] -1/3/x^3*A*b-1/3/x^3*B*a-1/6*a*A/x^6+B*b*ln(x)
```

maxima [A] time = 0.51, size = 30, normalized size = 1.03

$$\frac{1}{3}\,Bb\log\left(x^{3}\right) - \frac{2\,(Ba + Ab)x^{3} + Aa}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^7,x, algorithm="maxima")
[Out] 1/3*B*b*log(x^3) - 1/6*(2*(B*a + A*b)*x^3 + A*a)/x^6
```

mupad [B] time = 0.05, size = 29, normalized size = 1.00

$$B\,b\,\ln(x) - \frac{\left(\frac{A\,b}{3} + \frac{B\,a}{3}\right)\,x^{3} + \frac{A\,a}{6}}{x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^7,x)
[Out] B*b*log(x) - ((A*a)/6 + x^3*((A*b)/3 + (B*a)/3))/x^6
```

sympy [A] time = 0.54, size = 29, normalized size = 1.00

$$Bb\log(x) + \frac{-Aa + x^{3}\,(-2Ab - 2Ba)}{6x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**7,x)
[Out] B*b*log(x) + (-A*a + x**3*(-2*A*b - 2*B*a))/(6*x**6)
```

3.11 $\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

```
[Out] 1/9*(A*b-B*a)*(b*x^3+a)^3/b^2+1/12*B*(b*x^3+a)^4/b^2
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^3 (Ab-aB)}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^3)/(9*b^2) + (B*(a + b*x^3)^4)/(12*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^2(A+Bx) \, dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^2}{b}+\frac{B(a+bx)^3}{b}\right) dx,x,x^3\right)\\ &= \frac{(Ab-aB)\left(a+bx^3\right)^3}{9b^2}+\frac{B\left(a+bx^3\right)^4}{12b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.21

$$\frac{1}{36}x^3 \left(12a^2A+4bx^6(2aB+Ab)+6ax^3(aB+2Ab)+3b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (x^3*(12*a^2*A + 6*a*(2*A*b + a*B)*x^3 + 4*b*(A*b + 2*a*B)*x^6 + 3*b^2*B*x^
9))/36
```

fricas [A] time = 0.60, size = 53, normalized size = 1.26

$$\frac{1}{12}x^{12}b^2B+\frac{2}{9}x^9baB+\frac{1}{9}x^9b^2A+\frac{1}{6}x^6a^2B+\frac{1}{3}x^6baA+\frac{1}{3}x^3a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^2*B + 2/9*x^9*b*a*B + 1/9*x^9*b^2*A + 1/6*x^6*a^2*B + 1/3*x^6*b
*a*A + 1/3*x^3*a^2*A
```

giac [A] time = 0.15, size = 53, normalized size = 1.26

$$\frac{1}{12}Bb^2x^{12}+\frac{2}{9}Babx^9+\frac{1}{9}Ab^2x^9+\frac{1}{6}Ba^2x^6+\frac{1}{3}Aabx^6+\frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*B*b^2*x^12 + 2/9*B*a*b*x^9 + 1/9*A*b^2*x^9 + 1/6*B*a^2*x^6 + 1/3*A*a*b
*x^6 + 1/3*A*a^2*x^3
```

maple [A] time = 0.04, size = 52, normalized size = 1.24

$$\frac{Bb^2x^{12}}{12}+\frac{\left(b^2A+2abB\right)x^9}{9}+\frac{Aa^2x^3}{3}+\frac{\left(2abA+a^2B\right)x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/12*b^2*B*x^12+1/9*(A*b^2+2*B*a*b)*x^9+1/6*(2*A*a*b+B*a^2)*x^6+1/3*A*a^2*x
^3
```

maxima [A] time = 0.49, size = 51, normalized size = 1.21

$$\frac{1}{12}Bb^2x^{12}+\frac{1}{9}\left(2Bab+Ab^2\right)x^9+\frac{1}{6}\left(Ba^2+2Aab\right)x^6+\frac{1}{3}Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/12*B*b^2*x^12 + 1/9*(2*B*a*b + A*b^2)*x^9 + 1/6*(B*a^2 + 2*A*a*b)*x^6 + 1
/3*A*a^2*x^3
```

mupad [B] time = 2.38, size = 51, normalized size = 1.21

$$x^6\left(\frac{Ba^2}{6}+\frac{Aba}{3}\right)+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+\frac{Aa^2x^3}{3}+\frac{Bb^2x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^6*((B*a^2)/6 + (A*a*b)/3) + x^9*((A*b^2)/9 + (2*B*a*b)/9) + (A*a^2*x^3)/3
 + (B*b^2*x^12)/12
```

sympy [A] time = 0.08, size = 54, normalized size = 1.29

$$\frac{Aa^2x^3}{3}+\frac{Bb^2x^{12}}{12}+x^9\left(\frac{Ab^2}{9}+\frac{2Bab}{9}\right)+x^6\left(\frac{Aab}{3}+\frac{Ba^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**3/3 + B*b**2*x**12/12 + x**9*(A*b**2/9 + 2*B*a*b/9) + x**6*(A*a*b
/3 + B*a**2/6)
```

3.12 $\int x \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

```
[Out] 1/2*a^2*A*x^2+1/5*a*(2*A*b+B*a)*x^5+1/8*b*(A*b+2*B*a)*x^8+1/11*b^2*B*x^11
```

Rubi [A] time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^3\right)^2\left(A+Bx^3\right)dx &= \int \left(a^2Ax+a(2Ab+aB)x^4+b(Ab+2aB)x^7+b^2Bx^{10}\right)dx\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{5}a(2Ab+aB)x^5+\frac{1}{8}b(Ab+2aB)x^8+\frac{1}{11}b^2Bx^{11}\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2+\frac{1}{8}bx^8(2aB+Ab)+\frac{1}{5}ax^5(aB+2Ab)+\frac{1}{11}b^2Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^2)/2 + (a*(2*A*b + a*B)*x^5)/5 + (b*(A*b + 2*a*B)*x^8)/8 + (b^2*B*
x^11)/11
```

fricas [A] time = 0.83, size = 53, normalized size = 0.96

$$\frac{1}{11}x^{11}b^2B+\frac{1}{4}x^8baB+\frac{1}{8}x^8b^2A+\frac{1}{5}x^5a^2B+\frac{2}{5}x^5baA+\frac{1}{2}x^2a^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^2*B + 1/4*x^8*b*a*B + 1/8*x^8*b^2*A + 1/5*x^5*a^2*B + 2/5*x^5*b
*a*A + 1/2*x^2*a^2*A
```

giac [A] time = 0.15, size = 53, normalized size = 0.96

$$\frac{1}{11}\,Bb^2x^{11} + \frac{1}{4}\,Babx^8 + \frac{1}{8}\,Ab^2x^8 + \frac{1}{5}\,Ba^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{2}\,Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/11*B*b^2*x^11 + 1/4*B*a*b*x^8 + 1/8*A*b^2*x^8 + 1/5*B*a^2*x^5 + 2/5*A*a*b
*x^5 + 1/2*A*a^2*x^2
```

maple [A] time = 0.05, size = 52, normalized size = 0.95

$$\frac{B\,b^2x^{11}}{11} + \frac{\left(b^2A + 2abB\right)x^8}{8} + \frac{A\,a^2x^2}{2} + \frac{\left(2abA + a^2B\right)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/11*b^2*B*x^11+1/8*(A*b^2+2*B*a*b)*x^8+1/5*(2*A*a*b+B*a^2)*x^5+1/2*a^2*A*x
^2
```

maxima [A] time = 0.61, size = 51, normalized size = 0.93

$$\frac{1}{11}\,Bb^2x^{11} + \frac{1}{8}\left(2\,Bab + Ab^2\right)x^8 + \frac{1}{5}\left(Ba^2 + 2\,Aab\right)x^5 + \frac{1}{2}\,Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/11*B*b^2*x^11 + 1/8*(2*B*a*b + A*b^2)*x^8 + 1/5*(B*a^2 + 2*A*a*b)*x^5 + 1
/2*A*a^2*x^2
```

mupad [B] time = 0.04, size = 51, normalized size = 0.93

$$x^5\left(\frac{B\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^8\left(\frac{A\,b^2}{8} + \frac{B\,a\,b}{4}\right) + \frac{A\,a^2\,x^2}{2} + \frac{B\,b^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^5*((B*a^2)/5 + (2*A*a*b)/5) + x^8*((A*b^2)/8 + (B*a*b)/4) + (A*a^2*x^2)/2
 + (B*b^2*x^11)/11
```

sympy [A] time = 0.08, size = 54, normalized size = 0.98

$$\frac{Aa^2x^2}{2} + \frac{Bb^2x^{11}}{11} + x^8\left(\frac{Ab^2}{8} + \frac{Bab}{4}\right) + x^5\left(\frac{2Aab}{5} + \frac{Ba^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x**2/2 + B*b**2*x**11/11 + x**8*(A*b**2/8 + B*a*b/4) + x**5*(2*A*a*b
/5 + B*a**2/5)
```

3.13 $\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=50

$$a^2Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

```
[Out] a^2*A*x+1/4*a*(2*A*b+B*a)*x^4+1/7*b*(A*b+2*B*a)*x^7+1/10*b^2*B*x^10
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx &= \int \left(a^2A + a(2Ab + aB)x^3 + b(Ab + 2aB)x^6 + b^2Bx^9\right) dx \\
&= a^2Ax + \frac{1}{4}a(2Ab + aB)x^4 + \frac{1}{7}b(Ab + 2aB)x^7 + \frac{1}{10}b^2Bx^{10}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.00

$$a^2Ax + \frac{1}{7}bx^7(2aB + Ab) + \frac{1}{4}ax^4(aB + 2Ab) + \frac{1}{10}b^2Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*B)*x^4)/4 + (b*(A*b + 2*a*B)*x^7)/7 + (b^2*B*x^10)/
10
```

fricas [A] time = 0.97, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}b^2B + \frac{2}{7}x^7baB + \frac{1}{7}x^7b^2A + \frac{1}{4}x^4a^2B + \frac{1}{2}x^4baA + xa^2A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^2*B + 2/7*x^7*b*a*B + 1/7*x^7*b^2*A + 1/4*x^4*a^2*B + 1/2*x^4*b
*a*A + x*a^2*A
```

giac [A] time = 0.17, size = 50, normalized size = 1.00

$$\frac{1}{10}\,Bb^2x^{10} + \frac{2}{7}\,Babx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{2}\,Aabx^4 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/10*B*b^2*x^10 + 2/7*B*a*b*x^7 + 1/7*A*b^2*x^7 + 1/4*B*a^2*x^4 + 1/2*A*a*b
*x^4 + A*a^2*x
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{B\,b^2x^{10}}{10} + \frac{\left(b^2A + 2abB\right)x^7}{7} + A\,a^2x + \frac{\left(2abA + a^2B\right)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 1/10*b^2*B*x^10+1/7*(A*b^2+2*B*a*b)*x^7+1/4*(2*A*a*b+B*a^2)*x^4+A*a^2*x
```

maxima [A] time = 0.66, size = 48, normalized size = 0.96

$$\frac{1}{10}\,Bb^2x^{10} + \frac{1}{7}\left(2\,Bab + Ab^2\right)x^7 + \frac{1}{4}\left(Ba^2 + 2\,Aab\right)x^4 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/10*B*b^2*x^10 + 1/7*(2*B*a*b + A*b^2)*x^7 + 1/4*(B*a^2 + 2*A*a*b)*x^4 + A
*a^2*x
```

mupad [B] time = 0.04, size = 48, normalized size = 0.96

$$x^4\,\left(\frac{B\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^7\,\left(\frac{A\,b^2}{7}+\frac{2\,B\,a\,b}{7}\right)+\frac{B\,b^2\,x^{10}}{10}+A\,a^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^4*((B*a^2)/4 + (A*a*b)/2) + x^7*((A*b^2)/7 + (2*B*a*b)/7) + (B*b^2*x^10)/
10 + A*a^2*x
```

sympy [A] time = 0.08, size = 51, normalized size = 1.02

$$Aa^2x + \frac{Bb^2x^{10}}{10} + x^7\left(\frac{Ab^2}{7} + \frac{2Bab}{7}\right) + x^4\left(\frac{Aab}{2} + \frac{Ba^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] A*a**2*x + B*b**2*x**10/10 + x**7*(A*b**2/7 + 2*B*a*b/7) + x**4*(A*a*b/2 +
B*a**2/4)
```

3.14 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x} dx$

Optimal. Leaf size=46

$$a^2 A\log(x)+\frac{2}{3}aAbx^3+\frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{6}Ab^2x^6$$

```
[Out] 2/3*a*A*b*x^3+1/6*A*b^2*x^6+1/9*B*(b*x^3+a)^3/b+a^2*A*ln(x)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 80, 43}

$$a^2 A\log(x)+\frac{2}{3}aAbx^3+\frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{6}Ab^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*b*x^3)/3 + (A*b^2*x^6)/6 + (B*(a + b*x^3)^3)/(9*b) + a^2*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\text{Subst}\left(\int \frac{(a+bx)^2}{x}\,dx,x,x^3\right)\\
&= \frac{B\left(a+bx^3\right)^3}{9b}+\frac{1}{3}A\,\text{Subst}\left(\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx,x,x^3\right)\\
&= \frac{2}{3}aAbx^3+\frac{1}{6}Ab^2x^6+\frac{B\left(a+bx^3\right)^3}{9b}+a^2A\log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 1.11

$$a^2A\log(x)+\frac{1}{6}bx^6(2aB+Ab)+\frac{1}{3}ax^3(aB+2Ab)+\frac{1}{9}b^2Bx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x,x]
```

```
[Out] (a*(2*A*b + a*B)*x^3)/3 + (b*(A*b + 2*a*B)*x^6)/6 + (b^2*B*x^9)/9 + a^2*A*L
og[x]
```

fricas [A] time = 1.00, size = 49, normalized size = 1.07

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+Aa^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + A*a
^2*log(x)
```

giac [A] time = 0.15, size = 52, normalized size = 1.13

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{3}\,Babx^6+\frac{1}{6}\,Ab^2x^6+\frac{1}{3}\,Ba^2x^3+\frac{2}{3}\,Aabx^3+Aa^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*B*b^2*x^9 + 1/3*B*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*B*a^2*x^3 + 2/3*A*a*b*x
^3 + A*a^2*log(abs(x))
```

maple [A] time = 0.04, size = 52, normalized size = 1.13

$$\frac{B\,b^2x^9}{9}+\frac{A\,b^2x^6}{6}+\frac{Bab\,x^6}{3}+\frac{2Aab\,x^3}{3}+\frac{B\,a^2x^3}{3}+A\,a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x,x)
```

```
[Out] 1/9*B*b^2*x^9+1/6*A*b^2*x^6+1/3*B*x^6*a*b+2/3*a*A*b*x^3+1/3*B*a^2*x^3+A*a^2
*ln(x)
```

maxima [A] time = 0.54, size = 52, normalized size = 1.13

$$\frac{1}{9}\,Bb^2x^9+\frac{1}{6}\left(2\,Bab+Ab^2\right)x^6+\frac{1}{3}\left(Ba^2+2\,Aab\right)x^3+\frac{1}{3}\,Aa^2\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^2*x^9 + 1/6*(2*B*a*b + A*b^2)*x^6 + 1/3*(B*a^2 + 2*A*a*b)*x^3 + 1/3
*A*a^2*log(x^3)
```

mupad [B] time = 0.04, size = 49, normalized size = 1.07

$$x^3\left(\frac{B\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{B\,a\,b}{3}\right)+\frac{B\,b^2\,x^9}{9}+A\,a^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x,x)
```

```
[Out] x^3*((B*a^2)/3 + (2*A*a*b)/3) + x^6*((A*b^2)/6 + (B*a*b)/3) + (B*b^2*x^9)/9
 + A*a^2*log(x)
```

sympy [**A**] time = 0.15, size = 53, normalized size = 1.15

$$Aa^2\log(x) + \frac{Bb^2x^9}{9} + x^6\left(\frac{Ab^2}{6} + \frac{Bab}{3}\right) + x^3\left(\frac{2Aab}{3} + \frac{Ba^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*b**2*x**9/9 + x**6*(A*b**2/6 + B*a*b/3) + x**3*(2*A*a*b/3
 + B*a**2/3)
```

3.15 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^2} dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

```
[Out] -a^2*A/x+1/2*a*(2*A*b+B*a)*x^2+1/5*b*(A*b+2*B*a)*x^5+1/8*b^2*B*x^8
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2 (A + Bx^3)}{x^2} dx &= \int \left(\frac{a^2A}{x^2} + a(2Ab + aB)x + b(Ab + 2aB)x^4 + b^2Bx^7\right) dx \\ &= -\frac{a^2A}{x} + \frac{1}{2}a(2Ab + aB)x^2 + \frac{1}{5}b(Ab + 2aB)x^5 + \frac{1}{8}b^2Bx^8 \end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.00

$$-\frac{a^2A}{x} + \frac{1}{5}bx^5(2aB + Ab) + \frac{1}{2}ax^2(aB + 2Ab) + \frac{1}{8}b^2Bx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + (a*(2*A*b + a*B)*x^2)/2 + (b*(A*b + 2*a*B)*x^5)/5 + (b^2*B*x
^8)/8
```

fricas [A] time = 0.85, size = 53, normalized size = 1.00

$$\frac{5\,Bb^2x^9 + 8\left(2\,Bab + Ab^2\right)x^6 + 20\left(Ba^2 + 2\,Aab\right)x^3 - 40\,Aa^2}{40\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/40*(5*B*b^2*x^9 + 8*(2*B*a*b + A*b^2)*x^6 + 20*(B*a^2 + 2*A*a*b)*x^3 - 40
*A*a^2)/x
```

giac [A] time = 0.15, size = 52, normalized size = 0.98

$$\frac{1}{8}Bb^2x^8 + \frac{2}{5}Babx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{2}Ba^2x^2 + Aabx^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*B*b^2*x^8 + 2/5*B*a*b*x^5 + 1/5*A*b^2*x^5 + 1/2*B*a^2*x^2 + A*a*b*x^2 -
 A*a^2/x
```

maple [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{B\,b^2x^8}{8} + \frac{A\,b^2x^5}{5} + \frac{2Bab\,x^5}{5} + Aab\,x^2 + \frac{B\,a^2x^2}{2} - \frac{A\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^2,x)
```

```
[Out] 1/8*b^2*B*x^8+1/5*A*x^5*b^2+2/5*B*x^5*a*b+A*a*b*x^2+1/2*B*a^2*x^2-A*a^2/x
```

maxima [A] time = 0.59, size = 51, normalized size = 0.96

$$\frac{1}{8}Bb^2x^8 + \frac{1}{5}\left(2\,Bab + Ab^2\right)x^5 + \frac{1}{2}\left(Ba^2 + 2\,Aab\right)x^2 - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^2*x^8 + 1/5*(2*B*a*b + A*b^2)*x^5 + 1/2*(B*a^2 + 2*A*a*b)*x^2 - A*a
^2/x
```

mupad [B] time = 0.05, size = 50, normalized size = 0.94

$$x^2\left(\frac{B\,a^2}{2} + A\,b\,a\right) + x^5\left(\frac{A\,b^2}{5} + \frac{2\,B\,a\,b}{5}\right) - \frac{A\,a^2}{x} + \frac{B\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^2,x)
```

```
[Out] x^2*((B*a^2)/2 + A*a*b) + x^5*((A*b^2)/5 + (2*B*a*b)/5) - (A*a^2)/x + (B*b^
2*x^8)/8
```

sympy [A] time = 0.15, size = 49, normalized size = 0.92

$$-\frac{Aa^2}{x} + \frac{Bb^2x^8}{8} + x^5\left(\frac{Ab^2}{5} + \frac{2Bab}{5}\right) + x^2\left(Aab + \frac{Ba^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*b**2*x**8/8 + x**5*(A*b**2/5 + 2*B*a*b/5) + x**2*(A*a*b + B*a
**2/2)
```

3.16 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^3}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

```
[Out] -1/2*a^2*A/x^2+a*(2*A*b+B*a)*x+1/4*b*(A*b+2*B*a)*x^4+1/7*b^2*B*x^7
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^3}\,dx &= \int \left(a(2Ab+aB)+\frac{a^2A}{x^3}+b(Ab+2aB)x^3+b^2Bx^6\right)dx\\ &= -\frac{a^2A}{2x^2}+a(2Ab+aB)x+\frac{1}{4}b(Ab+2aB)x^4+\frac{1}{7}b^2Bx^7\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2A}{2x^2}+\frac{1}{4}bx^4(2aB+Ab)+ax(aB+2Ab)+\frac{1}{7}b^2Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 + a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*x^4)/4 + (b^2*B*x^7
)/7
```

fricas [A] time = 0.96, size = 53, normalized size = 1.06

$$\frac{4\,Bb^2x^9+7\left(2\,Bab+Ab^2\right)x^6+28\left(Ba^2+2\,Aab\right)x^3-14\,Aa^2}{28\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/28*(4*B*b^2*x^9 + 7*(2*B*a*b + A*b^2)*x^6 + 28*(B*a^2 + 2*A*a*b)*x^3 - 14
*A*a^2)/x^2
```

giac [A] time = 0.17, size = 48, normalized size = 0.96

$$\frac{1}{7}Bb^2x^7 + \frac{1}{2}Babx^4 + \frac{1}{4}Ab^2x^4 + Ba^2x + 2\,Aabx - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*B*b^2*x^7 + 1/2*B*a*b*x^4 + 1/4*A*b^2*x^4 + B*a^2*x + 2*A*a*b*x - 1/2*A
*a^2/x^2
```

maple [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{B\,b^2x^7}{7} + \frac{A\,b^2x^4}{4} + \frac{Bab\,x^4}{2} + 2Aabx + B\,a^2x - \frac{A\,a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^3,x)
```

```
[Out] 1/7*b^2*B*x^7+1/4*A*b^2*x^4+1/2*B*x^4*a*b+2*a*b*A*x+B*a^2*x-1/2*A*a^2/x^2
```

maxima [A] time = 0.49, size = 48, normalized size = 0.96

$$\frac{1}{7}Bb^2x^7 + \frac{1}{4}\left(2\,Bab + Ab^2\right)x^4 + \left(Ba^2 + 2\,Aab\right)x - \frac{Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^2*x^7 + 1/4*(2*B*a*b + A*b^2)*x^4 + (B*a^2 + 2*A*a*b)*x - 1/2*A*a^2
/x^2
```

mupad [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4\left(\frac{A\,b^2}{4} + \frac{B\,a\,b}{2}\right) + x\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{2\,x^2} + \frac{B\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^3,x)
```

```
[Out] x^4*((A*b^2)/4 + (B*a*b)/2) + x*(B*a^2 + 2*A*a*b) - (A*a^2)/(2*x^2) + (B*b^
2*x^7)/7
```

sympy [A] time = 0.15, size = 49, normalized size = 0.98

$$-\frac{Aa^2}{2x^2} + \frac{Bb^2x^7}{7} + x^4\left(\frac{Ab^2}{4} + \frac{Bab}{2}\right) + x\left(2Aab + Ba^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**2/(2*x**2) + B*b**2*x**7/7 + x**4*(A*b**2/4 + B*a*b/2) + x*(2*A*a*b +
 B*a**2)
```

3.17 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

```
[Out] -1/3*a^2*A/x^3+1/3*b*(A*b+2*B*a)*x^3+1/6*b^2*B*x^6+a*(2*A*b+B*a)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{3x^3}+\frac{1}{3}bx^3(2aB+Ab)+a\log(x)(aB+2Ab)+\frac{1}{6}b^2Bx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) + (b*(A*b + 2*a*B)*x^3)/3 + (b^2*B*x^6)/6 + a*(2*A*b + a*B
)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b(Ab+2aB)+\frac{a^2A}{x^2}+\frac{a(2Ab+aB)}{x}+b^2Bx\right)dx,x,x^3\right)\\ &= -\frac{a^2A}{3x^3}+\frac{1}{3}b(Ab+2aB)x^3+\frac{1}{6}b^2Bx^6+a(2Ab+aB)\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.96

$$\frac{1}{6}\left(-\frac{2a^2A}{x^3}+2bx^3(2aB+Ab)+6a\log(x)(aB+2Ab)+b^2Bx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^4,x]
```

```
[Out] ((-2*a^2*A)/x^3 + 2*b*(A*b + 2*a*B)*x^3 + b^2*B*x^6 + 6*a*(2*A*b + a*B)*Log
[x])/6
```

fricas [A] time = 0.79, size = 54, normalized size = 1.06

$$\frac{Bb^2x^9 + 2\left(2\,Bab + Ab^2\right)x^6 + 6\left(Ba^2 + 2\,Aab\right)x^3\log\left(x\right) - 2\,Aa^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^9 + 2*(2*B*a*b + A*b^2)*x^6 + 6*(B*a^2 + 2*A*a*b)*x^3*log(x) -
 2*A*a^2)/x^3
```

giac [A] time = 0.15, size = 69, normalized size = 1.35

$$\frac{1}{6}\,Bb^2x^6 + \frac{2}{3}\,Babx^3 + \frac{1}{3}\,Ab^2x^3 + \left(Ba^2 + 2\,Aab\right)\log\left(|x|\right) - \frac{Ba^2x^3 + 2\,Aabx^3 + Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*B*b^2*x^6 + 2/3*B*a*b*x^3 + 1/3*A*b^2*x^3 + (B*a^2 + 2*A*a*b)*log(abs(x
)) - 1/3*(B*a^2*x^3 + 2*A*a*b*x^3 + A*a^2)/x^3
```

maple [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{B\,b^2x^6}{6} + \frac{A\,b^2x^3}{3} + \frac{2Bab\,x^3}{3} + 2Aab\ln(x) + B\,a^2\ln(x) - \frac{A\,a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^4,x)
```

```
[Out] 1/6*b^2*B*x^6+1/3*A*x^3*b^2+2/3*B*x^3*a*b-1/3*A*a^2/x^3+2*A*ln(x)*a*b+B*a^2
*ln(x)
```

maxima [A] time = 0.48, size = 52, normalized size = 1.02

$$\frac{1}{6}\,Bb^2x^6 + \frac{1}{3}\left(2\,Bab + Ab^2\right)x^3 + \frac{1}{3}\left(Ba^2 + 2\,Aab\right)\log\left(x^3\right) - \frac{Aa^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^2*x^6 + 1/3*(2*B*a*b + A*b^2)*x^3 + 1/3*(B*a^2 + 2*A*a*b)*log(x^3)
- 1/3*A*a^2/x^3
```

mupad [B] time = 0.04, size = 49, normalized size = 0.96

$$x^3\left(\frac{A\,b^2}{3} + \frac{2\,B\,a\,b}{3}\right) + \ln(x)\,\left(B\,a^2 + 2\,A\,b\,a\right) - \frac{A\,a^2}{3\,x^3} + \frac{B\,b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^4,x)
```

```
[Out] x^3*((A*b^2)/3 + (2*B*a*b)/3) + log(x)*(B*a^2 + 2*A*a*b) - (A*a^2)/(3*x^3)
+ (B*b^2*x^6)/6
```

sympy [A] time = 0.27, size = 51, normalized size = 1.00

$$-\frac{Aa^2}{3x^3}+\frac{Bb^2x^6}{6}+a\left(2Ab+Ba\right)\log\left(x\right)+x^3\left(\frac{Ab^2}{3}+\frac{2Bab}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**2/(3*x**3) + B*b**2*x**6/6 + a*(2*A*b + B*a)*log(x) + x**3*(A*b**2/3
+ 2*B*a*b/3)
```

3.18 $\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

```
[Out] -1/4*a^2*A/x^4-a*(2*A*b+B*a)/x+1/2*b*(A*b+2*B*a)*x^2+1/5*b^2*B*x^5
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{4x^4}+\frac{1}{2}bx^2(2aB+Ab)-\frac{a(aB+2Ab)}{x}+\frac{1}{5}b^2Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a*(2*A*b + a*B))/x + (b*(A*b + 2*a*B)*x^2)/2 + (b^2*B*x
^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^5}\,dx &= \int \left(\frac{a^2A}{x^5}+\frac{a(2Ab+aB)}{x^2}+b(Ab+2aB)x+b^2Bx^4\right)dx\\ &= -\frac{a^2A}{4x^4}-\frac{a(2Ab+aB)}{x}+\frac{1}{2}b(Ab+2aB)x^2+\frac{1}{5}b^2Bx^5\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$\frac{-5a^2A+10bx^6(2aB+Ab)-20ax^3(aB+2Ab)+4b^2Bx^9}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^5,x]
```

```
[Out] (-5*a^2*A - 20*a*(2*A*b + a*B)*x^3 + 10*b*(A*b + 2*a*B)*x^6 + 4*b^2*B*x^9)/
(20*x^4)
```

fricas [A] time = 0.93, size = 53, normalized size = 1.00

$$\frac{4\,Bb^2x^9+10\left(2\,Bab+Ab^2\right)x^6-20\left(Ba^2+2\,Aab\right)x^3-5\,Aa^2}{20\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/20*(4*B*b^2*x^9 + 10*(2*B*a*b + A*b^2)*x^6 - 20*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^4
```

giac [A] time = 0.15, size = 54, normalized size = 1.02

$$\frac{1}{5}\,Bb^2x^5 + Babx^2 + \frac{1}{2}\,Ab^2x^2 - \frac{4\,Ba^2x^3 + 8\,Aabx^3 + Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/5*B*b^2*x^5 + B*a*b*x^2 + 1/2*A*b^2*x^2 - 1/4*(4*B*a^2*x^3 + 8*A*a*b*x^3
+ A*a^2)/x^4
```

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{B\,b^2x^5}{5} + \frac{A\,b^2x^2}{2} + Bab\,x^2 - \frac{(2Ab + Ba)\,a}{x} - \frac{A\,a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^5,x)
```

```
[Out] 1/5*b^2*B*x^5+1/2*A*x^2*b^2+B*a*b*x^2-1/4*a^2*A/x^4-a*(2*A*b+B*a)/x
```

maxima [A] time = 0.72, size = 53, normalized size = 1.00

$$\frac{1}{5}\,Bb^2x^5 + \frac{1}{2}\left(2\,Bab + Ab^2\right)x^2 - \frac{4\left(Ba^2 + 2\,Aab\right)x^3 + Aa^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^2*x^5 + 1/2*(2*B*a*b + A*b^2)*x^2 - 1/4*(4*(B*a^2 + 2*A*a*b)*x^3 +
A*a^2)/x^4
```

mupad [B] time = 0.05, size = 52, normalized size = 0.98

$$x^2\left(\frac{A\,b^2}{2} + B\,a\,b\right) - \frac{x^3\,\left(B\,a^2 + 2\,A\,b\,a\right) + \frac{A\,a^2}{4}}{x^4} + \frac{B\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^5,x)
```

```
[Out] x^2*((A*b^2)/2 + B*a*b) - (x^3*(B*a^2 + 2*A*a*b) + (A*a^2)/4)/x^4 + (B*b^2*
x^5)/5
```

sympy [A] time = 0.31, size = 53, normalized size = 1.00

$$\frac{Bb^2x^5}{5} + x^2\left(\frac{Ab^2}{2} + Bab\right) + \frac{-Aa^2 + x^3\left(-8Aab - 4Ba^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**2*x**5/5 + x**2*(A*b**2/2 + B*a*b) + (-A*a**2 + x**3*(-8*A*a*b - 4*B*a
**2))/(4*x**4)
```

3.19 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx$

Optimal. Leaf size=50

$$-\frac{a^2 A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

```
[Out] -1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2+b*(A*b+2*B*a)*x+1/4*b^2*B*x^4
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$-\frac{a^2 A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^6} \, dx &= \int \left(b(Ab+2aB) + \frac{a^2 A}{x^6} + \frac{a(2Ab+aB)}{x^3} + b^2Bx^3\right) dx \\
&= -\frac{a^2 A}{5x^5} - \frac{a(2Ab+aB)}{2x^2} + b(Ab+2aB)x + \frac{1}{4}b^2Bx^4
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.00

$$-\frac{a^2 A}{5x^5} - \frac{a(aB+2Ab)}{2x^2} + bx(2aB+Ab) + \frac{1}{4}b^2Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a*(2*A*b + a*B))/(2*x^2) + b*(A*b + 2*a*B)*x + (b^2*B*x
^4)/4
```

fricas [A] time = 0.75, size = 53, normalized size = 1.06

$$\frac{5\,Bb^2x^9 + 20\left(2\,Bab + Ab^2\right)x^6 - 10\left(Ba^2 + 2\,Aab\right)x^3 - 4\,Aa^2}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(5*B*b^2*x^9 + 20*(2*B*a*b + A*b^2)*x^6 - 10*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^5
```

giac [A] time = 0.15, size = 51, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4 + 2\,Babx + Ab^2x - \frac{5\,Ba^2x^3 + 10\,Aabx^3 + 2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*B*b^2*x^4 + 2*B*a*b*x + A*b^2*x - 1/10*(5*B*a^2*x^3 + 10*A*a*b*x^3 + 2*
A*a^2)/x^5
```

maple [A] time = 0.06, size = 46, normalized size = 0.92

$$\frac{B\,b^2x^4}{4} + A\,b^2x + 2Babx - \frac{(2Ab + Ba)\,a}{2x^2} - \frac{A\,a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^6,x)
```

```
[Out] 1/4*B*b^2*x^4+b^2*A*x+2*a*b*B*x-1/5*a^2*A/x^5-1/2*a*(2*A*b+B*a)/x^2
```

maxima [A] time = 0.57, size = 51, normalized size = 1.02

$$\frac{1}{4}\,Bb^2x^4 + \left(2\,Bab + Ab^2\right)x - \frac{5\left(Ba^2 + 2\,Aab\right)x^3 + 2\,Aa^2}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^2*x^4 + (2*B*a*b + A*b^2)*x - 1/10*(5*(B*a^2 + 2*A*a*b)*x^3 + 2*A*a
^2)/x^5
```

mupad [B] time = 2.37, size = 50, normalized size = 1.00

$$x\,\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^3\,\left(\frac{B\,a^2}{2} + A\,b\,a\right) + \frac{A\,a^2}{5}}{x^5} + \frac{B\,b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^6,x)
```

```
[Out] x*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/2 + A*a*b) + (A*a^2)/5)/x^5 + (B*b^2*x^
4)/4
```

sympy [A] time = 0.35, size = 53, normalized size = 1.06

$$\frac{Bb^2x^4}{4} + x\left(Ab^2 + 2Bab\right) + \frac{-2Aa^2 + x^3\left(-10Aab - 5Ba^2\right)}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**2*x**4/4 + x*(A*b**2 + 2*B*a*b) + (-2*A*a**2 + x**3*(-10*A*a*b - 5*B*a
**2))/(10*x**5)
```

3.20 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=51

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

```
[Out] -1/6*a^2*A/x^6-1/3*a*(2*A*b+B*a)/x^3+1/3*b^2*B*x^3+b*(A*b+2*B*a)*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{a^2A}{6x^6}-\frac{a(aB+2Ab)}{3x^3}+b\log(x)(2aB+Ab)+\frac{1}{3}b^2Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a*(2*A*b + a*B))/(3*x^3) + (b^2*B*x^3)/3 + b*(A*b + 2*a
*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^2\left(A+Bx^3\right)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^2(A+Bx)}{x^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(b^2B+\frac{a^2A}{x^3}+\frac{a(2Ab+aB)}{x^2}+\frac{b(Ab+2aB)}{x}\right)dx,x,x^3\right)\\
&= -\frac{a^2A}{6x^6}-\frac{a(2Ab+aB)}{3x^3}+\frac{1}{3}b^2Bx^3+b(Ab+2aB)\log(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$\frac{1}{6}\left(-\frac{a^2\left(A+2Bx^3\right)}{x^6}+6b\log(x)(2aB+Ab)-\frac{4aAb}{x^3}+2b^2Bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^7,x]
```

```
[Out] ((-4*a*A*b)/x^3 + 2*b^2*B*x^3 - (a^2*(A + 2*B*x^3))/x^6 + 6*b*(A*b + 2*a*B)
*Log[x])/6
```

fricas [A] time = 1.00, size = 55, normalized size = 1.08

$$\frac{2\,Bb^2x^9 + 6\left(2\,Bab + Ab^2\right)x^6\log(x) - 2\left(Ba^2 + 2\,Aab\right)x^3 - Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^2*x^9 + 6*(2*B*a*b + A*b^2)*x^6*log(x) - 2*(B*a^2 + 2*A*a*b)*x^3
 - A*a^2)/x^6
```

giac [A] time = 0.21, size = 70, normalized size = 1.37

$$\frac{1}{3}\,Bb^2x^3 + \left(2\,Bab + Ab^2\right)\log\left(|x|\right) - \frac{6\,Babx^6 + 3\,Ab^2x^6 + 2\,Ba^2x^3 + 4\,Aabx^3 + Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/3*B*b^2*x^3 + (2*B*a*b + A*b^2)*log(abs(x)) - 1/6*(6*B*a*b*x^6 + 3*A*b^2*
x^6 + 2*B*a^2*x^3 + 4*A*a*b*x^3 + A*a^2)/x^6
```

maple [A] time = 0.07, size = 51, normalized size = 1.00

$$\frac{B\,b^2x^3}{3} + A\,b^2\ln(x) + 2Bab\ln(x) - \frac{2Aab}{3x^3} - \frac{B\,a^2}{3x^3} - \frac{A\,a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^7,x)
```

```
[Out] 1/3*b^2*B*x^3-2/3*a/x^3*A*b-1/3*a^2/x^3*B-1/6*a^2*A/x^6+A*ln(x)*b^2+2*B*ln(
x)*a*b
```

maxima [A] time = 0.67, size = 54, normalized size = 1.06

$$\frac{1}{3}\,Bb^2x^3 + \frac{1}{3}\left(2\,Bab + Ab^2\right)\log\left(x^3\right) - \frac{2\left(Ba^2 + 2\,Aab\right)x^3 + Aa^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3 + 1/3*(2*B*a*b + A*b^2)*log(x^3) - 1/6*(2*(B*a^2 + 2*A*a*b)*x
^3 + A*a^2)/x^6
```

mupad [B] time = 2.36, size = 52, normalized size = 1.02

$$\ln(x)\,\left(A\,b^2 + 2\,B\,a\,b\right) - \frac{x^3\left(\frac{B\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + \frac{A\,a^2}{6}}{x^6} + \frac{B\,b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^7,x)
```

```
[Out] log(x)*(A*b^2 + 2*B*a*b) - (x^3*((B*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/6)/x^6
+ (B*b^2*x^3)/3
```

sympy [A] time = 0.76, size = 51, normalized size = 1.00

$$\frac{Bb^2x^3}{3} + b\left(Ab + 2Ba\right)\log\left(x\right) + \frac{-Aa^2 + x^3\left(-4Aab - 2Ba^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**2*x**3/3 + b*(A*b + 2*B*a)*log(x) + (-A*a**2 + x**3*(-4*A*a*b - 2*B*a*
*2))/(6*x**6)
```

3.21 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2Bx^2$$

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{7x^7}-\frac{a(aB+2Ab)}{4x^4}-\frac{b(2aB+Ab)}{x}+\frac{1}{2}b^2Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^2*A)/(7*x^7) - (a*(2*A*b + a*B))/(4*x^4) - (b*(A*b + 2*a*B))/x + (b^2*B
*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^8}\,dx &= \int \left(\frac{a^2A}{x^8}+\frac{a(2Ab+aB)}{x^5}+\frac{b(Ab+2aB)}{x^2}+b^2Bx\right)dx\\ &= -\frac{a^2A}{7x^7}-\frac{a(2Ab+aB)}{4x^4}-\frac{b(Ab+2aB)}{x}+\frac{1}{2}b^2Bx^2\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.02

$$-\frac{a^2(4A+7Bx^3)+14abx^3(A+4Bx^3)-14b^2x^6(Bx^3-2A)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^8,x]
```

```
[Out] -1/28*(-14*b^2*x^6*(-2*A + B*x^3) + 14*a*b*x^3*(A + 4*B*x^3) + a^2*(4*A + 7
*B*x^3))/x^7
```

fricas [A] time = 1.20, size = 53, normalized size = 1.00

$$\frac{14\,Bb^2x^9-28\left(2\,Bab+Ab^2\right)x^6-7\left(Ba^2+2\,Aab\right)x^3-4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/28*(14*B*b^2*x^9 - 28*(2*B*a*b + A*b^2)*x^6 - 7*(B*a^2 + 2*A*a*b)*x^3 - 4
*A*a^2)/x^7
```

giac [A] time = 0.15, size = 56, normalized size = 1.06

$$\frac{1}{2}Bb^2x^2 - \frac{56\,Babx^6 + 28\,Ab^2x^6 + 7\,Ba^2x^3 + 14\,Aabx^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(56*B*a*b*x^6 + 28*A*b^2*x^6 + 7*B*a^2*x^3 + 14*A*a*b*
x^3 + 4*A*a^2)/x^7
```

maple [A] time = 0.04, size = 48, normalized size = 0.91

$$\frac{B\,b^2x^2}{2} - \frac{(Ab + 2Ba)\,b}{x} - \frac{(2Ab + Ba)\,a}{4x^4} - \frac{A\,a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^8,x)
```

```
[Out] -1/7*a^2*A/x^7-1/4*a*(2*A*b+B*a)/x^4-b*(A*b+2*B*a)/x+1/2*b^2*B*x^2
```

maxima [A] time = 0.57, size = 54, normalized size = 1.02

$$\frac{1}{2}Bb^2x^2 - \frac{28\left(2\,Bab + Ab^2\right)x^6 + 7\left(Ba^2 + 2\,Aab\right)x^3 + 4\,Aa^2}{28\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^2*x^2 - 1/28*(28*(2*B*a*b + A*b^2)*x^6 + 7*(B*a^2 + 2*A*a*b)*x^3 +
4*A*a^2)/x^7
```

mupad [B] time = 0.05, size = 53, normalized size = 1.00

$$\frac{B\,b^2\,x^2}{2} - \frac{x^3\,\left(\frac{B\,a^2}{4} + \frac{A\,b\,a}{2}\right) + x^6\,\left(A\,b^2 + 2\,B\,a\,b\right) + \frac{A\,a^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^8,x)
```

```
[Out] (B*b^2*x^2)/2 - (x^3*((B*a^2)/4 + (A*a*b)/2) + x^6*(A*b^2 + 2*B*a*b) + (A*a
^2)/7)/x^7
```

sympy [A] time = 0.87, size = 58, normalized size = 1.09

$$\frac{Bb^2x^2}{2} + \frac{-4Aa^2 + x^6\left(-28Ab^2 - 56Bab\right) + x^3\left(-14Aab - 7Ba^2\right)}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**2*x**2/2 + (-4*A*a**2 + x**6*(-28*A*b**2 - 56*B*a*b) + x**3*(-14*A*a*b
 - 7*B*a**2))/(28*x**7)
```

3.22 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^2*A)/(8*x^8) - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^9}\,dx &= \int \left(b^2B+\frac{a^2A}{x^9}+\frac{a(2Ab+aB)}{x^6}+\frac{b(Ab+2aB)}{x^3}\right)dx\\ &= -\frac{a^2A}{8x^8}-\frac{a(2Ab+aB)}{5x^5}-\frac{b(Ab+2aB)}{2x^2}+b^2Bx\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.00

$$-\frac{a^2A}{8x^8}-\frac{a(aB+2Ab)}{5x^5}-\frac{b(2aB+Ab)}{2x^2}+b^2Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^2*A)/x^8 - (a*(2*A*b + a*B))/(5*x^5) - (b*(A*b + 2*a*B))/(2*x^2) +
b^2*B*x
```

fricas [A] time = 1.00, size = 53, normalized size = 1.06

$$\frac{40\,Bb^2x^9-20\left(2\,Bab+Ab^2\right)x^6-8\left(Ba^2+2\,Aab\right)x^3-5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/40*(40*B*b^2*x^9 - 20*(2*B*a*b + A*b^2)*x^6 - 8*(B*a^2 + 2*A*a*b)*x^3 - 5
*A*a^2)/x^8
```

giac [A] time = 0.15, size = 53, normalized size = 1.06

$$Bb^2x - \frac{40\,Babx^6 + 20\,Ab^2x^6 + 8\,Ba^2x^3 + 16\,Aabx^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] B*b^2*x - 1/40*(40*B*a*b*x^6 + 20*A*b^2*x^6 + 8*B*a^2*x^3 + 16*A*a*b*x^3 +
5*A*a^2)/x^8
```

maple [A] time = 0.05, size = 45, normalized size = 0.90

$$B\,b^2x - \frac{(Ab + 2Ba)\,b}{2x^2} - \frac{(2Ab + Ba)\,a}{5x^5} - \frac{A\,a^2}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^9,x)
```

```
[Out] -1/8*a^2*A/x^8-1/5*a*(2*A*b+B*a)/x^5-1/2*b*(A*b+2*B*a)/x^2+b^2*B*x
```

maxima [A] time = 0.49, size = 51, normalized size = 1.02

$$Bb^2x - \frac{20\left(2\,Bab + Ab^2\right)x^6 + 8\left(Ba^2 + 2\,Aab\right)x^3 + 5\,Aa^2}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] B*b^2*x - 1/40*(20*(2*B*a*b + A*b^2)*x^6 + 8*(B*a^2 + 2*A*a*b)*x^3 + 5*A*a^
2)/x^8
```

mupad [B] time = 2.34, size = 50, normalized size = 1.00

$$B\,b^2\,x - \frac{x^3\,\left(\frac{B\,a^2}{5} + \frac{2\,A\,b\,a}{5}\right) + x^6\,\left(\frac{A\,b^2}{2} + B\,a\,b\right) + \frac{A\,a^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^9,x)
```

```
[Out] B*b^2*x - (x^3*((B*a^2)/5 + (2*A*a*b)/5) + x^6*((A*b^2)/2 + B*a*b) + (A*a^2
)/8)/x^8
```

sympy [A] time = 0.99, size = 54, normalized size = 1.08

$$Bb^2x + \frac{-5Aa^2 + x^6\left(-20Ab^2 - 40Bab\right) + x^3\left(-16Aab - 8Ba^2\right)}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**2*x + (-5*A*a**2 + x**6*(-20*A*b**2 - 40*B*a*b) + x**3*(-16*A*a*b - 8*
B*a**2))/(40*x**8)
```

3.23 $\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+A$$

```
[Out] 1/10*a^5*A*x^10+1/13*a^4*(5*A*b+B*a)*x^13+5/16*a^3*b*(2*A*b+B*a)*x^16+10/19
*a^2*b^2*(A*b+B*a)*x^19+5/22*a*b^3*(A*b+2*B*a)*x^22+1/25*b^4*(A*b+5*B*a)*x^
25+1/28*b^5*B*x^28
```

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{1}{10}a^5Ax^{10}+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+A$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^9 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^9+a^4(5Ab+aB)x^{12}+5a^3b(2Ab+aB)x^{15}+10a^2b^2(Ab+aB)x^{18}+5a\right.\\ &= \frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4(5Ab+aB)x^{13}+\frac{5}{16}a^3b(2Ab+aB)x^{16}+\frac{10}{19}a^2b^2(Ab+aB)x^{19}\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{10}a^5Ax^{10}+\frac{1}{13}a^4x^{13}(aB+5Ab)+\frac{5}{16}a^3bx^{16}(aB+2Ab)+\frac{10}{19}a^2b^2x^{19}(aB+Ab)+\frac{1}{25}b^4x^{25}(5aB+Ab)+\frac{5}{22}ab^3x^{22}(2aB+A$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^10)/10 + (a^4*(5*A*b + a*B)*x^13)/13 + (5*a^3*b*(2*A*b + a*B)*x^16
)/16 + (10*a^2*b^2*(A*b + a*B)*x^19)/19 + (5*a*b^3*(A*b + 2*a*B)*x^22)/22 +
 (b^4*(A*b + 5*a*B)*x^25)/25 + (b^5*B*x^28)/28
```

fricas [A] time = 0.82, size = 125, normalized size = 1.07

$$\frac{1}{28}x^{28}b^5B+\frac{1}{5}x^{25}b^4aB+\frac{1}{25}x^{25}b^5A+\frac{5}{11}x^{22}b^3a^2B+\frac{5}{22}x^{22}b^4aA+\frac{10}{19}x^{19}b^2a^3B+\frac{10}{19}x^{19}b^3a^2A+\frac{5}{16}x^{16}ba^4B+\frac{5}{8}x^{16}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/28*x^28*b^5*B + 1/5*x^25*b^4*a*B + 1/25*x^25*b^5*A + 5/11*x^22*b^3*a^2*B
+ 5/22*x^22*b^4*a*A + 10/19*x^19*b^2*a^3*B + 10/19*x^19*b^3*a^2*A + 5/16*x^
16*b*a^4*B + 5/8*x^16*b^2*a^3*A + 1/13*x^13*a^5*B + 5/13*x^13*b*a^4*A + 1/1
0*x^10*a^5*A
```

giac [A] time = 0.16, size = 125, normalized size = 1.07

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{5}Bab^4x^{25}+\frac{1}{25}Ab^5x^{25}+\frac{5}{11}Ba^2b^3x^{22}+\frac{5}{22}Aab^4x^{22}+\frac{10}{19}Ba^3b^2x^{19}+\frac{10}{19}Aa^2b^3x^{19}+\frac{5}{16}Ba^4bx^{16}+\frac{5}{8}Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/28*B*b^5*x^28 + 1/5*B*a*b^4*x^25 + 1/25*A*b^5*x^25 + 5/11*B*a^2*b^3*x^22
+ 5/22*A*a*b^4*x^22 + 10/19*B*a^3*b^2*x^19 + 10/19*A*a^2*b^3*x^19 + 5/16*B*
a^4*b*x^16 + 5/8*A*a^3*b^2*x^16 + 1/13*B*a^5*x^13 + 5/13*A*a^4*b*x^13 + 1/1
0*A*a^5*x^10
```

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{28}}{28}+\frac{\left(b^5A+5a\,b^4B\right)x^{25}}{25}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{22}}{22}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{19}}{19}+\frac{A\,a^5x^{10}}{10}+\frac{\left(10a^3b^2A+5a^4\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/28*b^5*B*x^28+1/25*(A*b^5+5*B*a*b^4)*x^25+1/22*(5*A*a*b^4+10*B*a^2*b^3)*x
^22+1/19*(10*A*a^2*b^3+10*B*a^3*b^2)*x^19+1/16*(10*A*a^3*b^2+5*B*a^4*b)*x^1
6+1/13*(5*A*a^4*b+B*a^5)*x^13+1/10*a^5*A*x^10
```

maxima [A] time = 0.46, size = 119, normalized size = 1.02

$$\frac{1}{28}Bb^5x^{28}+\frac{1}{25}\left(5\,Bab^4+Ab^5\right)x^{25}+\frac{5}{22}\left(2\,Ba^2b^3+Aab^4\right)x^{22}+\frac{10}{19}\left(Ba^3b^2+Aa^2b^3\right)x^{19}+\frac{5}{16}\left(Ba^4b+2\,Aa^3b^2\right)x^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/28*B*b^5*x^28 + 1/25*(5*B*a*b^4 + A*b^5)*x^25 + 5/22*(2*B*a^2*b^3 + A*a*b
^4)*x^22 + 10/19*(B*a^3*b^2 + A*a^2*b^3)*x^19 + 5/16*(B*a^4*b + 2*A*a^3*b^2
)*x^16 + 1/10*A*a^5*x^10 + 1/13*(B*a^5 + 5*A*a^4*b)*x^13
```

mupad [B] time = 0.05, size = 107, normalized size = 0.91

$$x^{13}\left(\frac{B\,a^5}{13}+\frac{5\,A\,b\,a^4}{13}\right)+x^{25}\left(\frac{A\,b^5}{25}+\frac{B\,a\,b^4}{5}\right)+\frac{A\,a^5\,x^{10}}{10}+\frac{B\,b^5\,x^{28}}{28}+\frac{10\,a^2\,b^2\,x^{19}\,\left(A\,b+B\,a\right)}{19}+\frac{5\,a^3\,b\,x^{16}\,\left(2\,A\,b+\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^13*((B*a^5)/13 + (5*A*a^4*b)/13) + x^25*((A*b^5)/25 + (B*a*b^4)/5) + (A*a
^5*x^10)/10 + (B*b^5*x^28)/28 + (10*a^2*b^2*x^19*(A*b + B*a))/19 + (5*a^3*b
*x^16*(2*A*b + B*a))/16 + (5*a*b^3*x^22*(A*b + 2*B*a))/22
```

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^{10}}{10}+\frac{Bb^5x^{28}}{28}+x^{25}\left(\frac{Ab^5}{25}+\frac{Bab^4}{5}\right)+x^{22}\left(\frac{5Aab^4}{22}+\frac{5Ba^2b^3}{11}\right)+x^{19}\left(\frac{10Aa^2b^3}{19}+\frac{10Ba^3b^2}{19}\right)+x^{16}\left(\frac{5Aa^3b^2}{8}+\frac{5B}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**10/10 + B*b**5*x**28/28 + x**25*(A*b**5/25 + B*a*b**4/5) + x**22*
(5*A*a*b**4/22 + 5*B*a**2*b**3/11) + x**19*(10*A*a**2*b**3/19 + 10*B*a**3*b
**2/19) + x**16*(5*A*a**3*b**2/8 + 5*B*a**4*b/16) + x**13*(5*A*a**4*b/13 +
B*a**5/13)
```

3.24 $\int x^8 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=95

$$\frac{a^2\left(a+bx^3\right)^6(Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8(Ab-3aB)}{24b^4}-\frac{a\left(a+bx^3\right)^7(2Ab-3aB)}{21b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}$$

```
[Out] 1/18*a^2*(A*b-B*a)*(b*x^3+a)^6/b^4-1/21*a*(2*A*b-3*B*a)*(b*x^3+a)^7/b^4+1/2
4*(A*b-3*B*a)*(b*x^3+a)^8/b^4+1/27*B*(b*x^3+a)^9/b^4
```

Rubi [A] time = 0.24, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{a^2\left(a+bx^3\right)^6(Ab-aB)}{18b^4}+\frac{\left(a+bx^3\right)^8(Ab-3aB)}{24b^4}-\frac{a\left(a+bx^3\right)^7(2Ab-3aB)}{21b^4}+\frac{B\left(a+bx^3\right)^9}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^2*(A*b - a*B)*(a + b*x^3)^6)/(18*b^4) - (a*(2*A*b - 3*a*B)*(a + b*x^3)^7
)/(21*b^4) + ((A*b - 3*a*B)*(a + b*x^3)^8)/(24*b^4) + (B*(a + b*x^3)^9)/(27
*b^4)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^5(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^5}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^6}{b^3}+\frac{(Ab-3\right.\right.\\ &= \frac{a^2(Ab-aB)\left(a+bx^3\right)^6}{18b^4}-\frac{a(2Ab-3aB)\left(a+bx^3\right)^7}{21b^4}+\frac{(Ab-3aB)\left(a+bx^3\right)^8}{24b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 107, normalized size = 1.13

$$\frac{x^9\left(168a^5A+126a^4x^3(aB+5Ab)+504a^3bx^6(aB+2Ab)+840a^2b^2x^9(aB+Ab)+63b^4x^{15}(5aB+Ab)+360a\right.}{1512}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^9*(168*a^5*A + 126*a^4*(5*A*b + a*B)*x^3 + 504*a^3*b*(2*A*b + a*B)*x^6 +
 840*a^2*b^2*(A*b + a*B)*x^9 + 360*a*b^3*(A*b + 2*a*B)*x^12 + 63*b^4*(A*b +
 5*a*B)*x^15 + 56*b^5*B*x^18))/1512
```

fricas [A] time = 1.01, size = 125, normalized size = 1.32

$$\frac{1}{27}x^{27}b^5B+\frac{5}{24}x^{24}b^4aB+\frac{1}{24}x^{24}b^5A+\frac{10}{21}x^{21}b^3a^2B+\frac{5}{21}x^{21}b^4aA+\frac{5}{9}x^{18}b^2a^3B+\frac{5}{9}x^{18}b^3a^2A+\frac{1}{3}x^{15}ba^4B+\frac{2}{3}x^{15}b^2a^3A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/27*x^27*b^5*B + 5/24*x^24*b^4*a*B + 1/24*x^24*b^5*A + 10/21*x^21*b^3*a^2*
B + 5/21*x^21*b^4*a*A + 5/9*x^18*b^2*a^3*B + 5/9*x^18*b^3*a^2*A + 1/3*x^15*
b*a^4*B + 2/3*x^15*b^2*a^3*A + 1/12*x^12*a^5*B + 5/12*x^12*b*a^4*A + 1/9*x^
9*a^5*A
```

giac [A] time = 0.17, size = 125, normalized size = 1.32

$$\frac{1}{27}Bb^5x^{27}+\frac{5}{24}Bab^4x^{24}+\frac{1}{24}Ab^5x^{24}+\frac{10}{21}Ba^2b^3x^{21}+\frac{5}{21}Aab^4x^{21}+\frac{5}{9}Ba^3b^2x^{18}+\frac{5}{9}Aa^2b^3x^{18}+\frac{1}{3}Ba^4bx^{15}+\frac{2}{3}Aa^3b^2x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/27*B*b^5*x^27 + 5/24*B*a*b^4*x^24 + 1/24*A*b^5*x^24 + 10/21*B*a^2*b^3*x^2
1 + 5/21*A*a*b^4*x^21 + 5/9*B*a^3*b^2*x^18 + 5/9*A*a^2*b^3*x^18 + 1/3*B*a^4
*b*x^15 + 2/3*A*a^3*b^2*x^15 + 1/12*B*a^5*x^12 + 5/12*A*a^4*b*x^12 + 1/9*A*
a^5*x^9
```

maple [A] time = 0.04, size = 124, normalized size = 1.31

$$\frac{B\,b^5x^{27}}{27}+\frac{\left(b^5A+5a\,b^4B\right)x^{24}}{24}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{21}}{21}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{18}}{18}+\frac{A\,a^5x^9}{9}+\frac{\left(10a^3b^2A+5a^4bB\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/27*b^5*B*x^27+1/24*(A*b^5+5*B*a*b^4)*x^24+1/21*(5*A*a*b^4+10*B*a^2*b^3)*x
^21+1/18*(10*A*a^2*b^3+10*B*a^3*b^2)*x^18+1/15*(10*A*a^3*b^2+5*B*a^4*b)*x^1
5+1/12*(5*A*a^4*b+B*a^5)*x^12+1/9*a^5*A*x^9
```

maxima [A] time = 0.47, size = 119, normalized size = 1.25

$$\frac{1}{27}Bb^5x^{27}+\frac{1}{24}\left(5\,Bab^4+Ab^5\right)x^{24}+\frac{5}{21}\left(2\,Ba^2b^3+Aab^4\right)x^{21}+\frac{5}{9}\left(Ba^3b^2+Aa^2b^3\right)x^{18}+\frac{1}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^{15}+\frac{1}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/27*B*b^5*x^27 + 1/24*(5*B*a*b^4 + A*b^5)*x^24 + 5/21*(2*B*a^2*b^3 + A*a*b
^4)*x^21 + 5/9*(B*a^3*b^2 + A*a^2*b^3)*x^18 + 1/3*(B*a^4*b + 2*A*a^3*b^2)*x
^15 + 1/9*A*a^5*x^9 + 1/12*(B*a^5 + 5*A*a^4*b)*x^12
```

mupad [B] time = 2.34, size = 107, normalized size = 1.13

$$x^{12}\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^{24}\left(\frac{A\,b^5}{24}+\frac{5\,B\,a\,b^4}{24}\right)+\frac{A\,a^5\,x^9}{9}+\frac{B\,b^5\,x^{27}}{27}+\frac{5\,a^2\,b^2\,x^{18}\,\left(A\,b+B\,a\right)}{9}+\frac{a^3\,b\,x^{15}\,\left(2\,A\,b+B\,a\right)}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^12*((B*a^5)/12 + (5*A*a^4*b)/12) + x^24*((A*b^5)/24 + (5*B*a*b^4)/24) + (
A*a^5*x^9)/9 + (B*b^5*x^27)/27 + (5*a^2*b^2*x^18*(A*b + B*a))/9 + (a^3*b*x^
15*(2*A*b + B*a))/3 + (5*a*b^3*x^21*(A*b + 2*B*a))/21
```

sympy [A] time = 0.10, size = 136, normalized size = 1.43

$$\frac{Aa^5x^9}{9}+\frac{Bb^5x^{27}}{27}+x^{24}\left(\frac{Ab^5}{24}+\frac{5Bab^4}{24}\right)+x^{21}\left(\frac{5Aab^4}{21}+\frac{10Ba^2b^3}{21}\right)+x^{18}\left(\frac{5Aa^2b^3}{9}+\frac{5Ba^3b^2}{9}\right)+x^{15}\left(\frac{2Aa^3b^2}{3}+\frac{Ba^4}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**9/9 + B*b**5*x**27/27 + x**24*(A*b**5/24 + 5*B*a*b**4/24) + x**21
*(5*A*a*b**4/21 + 10*B*a**2*b**3/21) + x**18*(5*A*a**2*b**3/9 + 5*B*a**3*b*
*2/9) + x**15*(2*A*a**3*b**2/3 + B*a**4*b/3) + x**12*(5*A*a**4*b/12 + B*a**
5/12)
```

3.25 $\int x^7 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+$$

```
[Out] 1/8*a^5*A*x^8+1/11*a^4*(5*A*b+B*a)*x^11+5/14*a^3*b*(2*A*b+B*a)*x^14+10/17*a
^2*b^2*(A*b+B*a)*x^17+1/4*a*b^3*(A*b+2*B*a)*x^20+1/23*b^4*(A*b+5*B*a)*x^23+
1/26*b^5*B*x^26
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{1}{8}a^5Ax^8+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^7\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \int \left(a^5Ax^7+a^4(5Ab+aB)x^{10}+5a^3b(2Ab+aB)x^{13}+10a^2b^2(Ab+aB)x^{16}+5a\right.\\ &= \frac{1}{8}a^5Ax^8+\frac{1}{11}a^4(5Ab+aB)x^{11}+\frac{5}{14}a^3b(2Ab+aB)x^{14}+\frac{10}{17}a^2b^2(Ab+aB)x^{17}+\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{8}a^5Ax^8+\frac{1}{11}a^4x^{11}(aB+5Ab)+\frac{5}{14}a^3bx^{14}(aB+2Ab)+\frac{10}{17}a^2b^2x^{17}(aB+Ab)+\frac{1}{23}b^4x^{23}(5aB+Ab)+\frac{1}{4}ab^3x^{20}(2aB+Ab)+$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^8)/8 + (a^4*(5*A*b + a*B)*x^11)/11 + (5*a^3*b*(2*A*b + a*B)*x^14)/
14 + (10*a^2*b^2*(A*b + a*B)*x^17)/17 + (a*b^3*(A*b + 2*a*B)*x^20)/4 + (b^4
*(A*b + 5*a*B)*x^23)/23 + (b^5*B*x^26)/26
```

fricas [A] time = 0.96, size = 125, normalized size = 1.07

$$\frac{1}{26}x^{26}b^5B+\frac{5}{23}x^{23}b^4aB+\frac{1}{23}x^{23}b^5A+\frac{1}{2}x^{20}b^3a^2B+\frac{1}{4}x^{20}b^4aA+\frac{10}{17}x^{17}b^2a^3B+\frac{10}{17}x^{17}b^3a^2A+\frac{5}{14}x^{14}ba^4B+\frac{5}{7}x^{14}b^2a^3A+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/26*x^26*b^5*B + 5/23*x^23*b^4*a*B + 1/23*x^23*b^5*A + 1/2*x^20*b^3*a^2*B
+ 1/4*x^20*b^4*a*A + 10/17*x^17*b^2*a^3*B + 10/17*x^17*b^3*a^2*A + 5/14*x^1
4*b*a^4*B + 5/7*x^14*b^2*a^3*A + 1/11*x^11*a^5*B + 5/11*x^11*b*a^4*A + 1/8*
x^8*a^5*A
```

giac [A] time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{26}\,Bb^5x^{26}+\frac{5}{23}\,Bab^4x^{23}+\frac{1}{23}\,Ab^5x^{23}+\frac{1}{2}\,Ba^2b^3x^{20}+\frac{1}{4}\,Aab^4x^{20}+\frac{10}{17}\,Ba^3b^2x^{17}+\frac{10}{17}\,Aa^2b^3x^{17}+\frac{5}{14}\,Ba^4bx^{14}+\frac{5}{7}\,Aa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/26*B*b^5*x^26 + 5/23*B*a*b^4*x^23 + 1/23*A*b^5*x^23 + 1/2*B*a^2*b^3*x^20
+ 1/4*A*a*b^4*x^20 + 10/17*B*a^3*b^2*x^17 + 10/17*A*a^2*b^3*x^17 + 5/14*B*a
^4*b*x^14 + 5/7*A*a^3*b^2*x^14 + 1/11*B*a^5*x^11 + 5/11*A*a^4*b*x^11 + 1/8*
A*a^5*x^8
```

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{26}}{26}+\frac{\left(b^5A+5a\,b^4B\right)x^{23}}{23}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{20}}{20}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{17}}{17}+\frac{A\,a^5x^8}{8}+\frac{\left(10a^3b^2A+5a^4b\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/26*b^5*B*x^26+1/23*(A*b^5+5*B*a*b^4)*x^23+1/20*(5*A*a*b^4+10*B*a^2*b^3)*x
^20+1/17*(10*A*a^2*b^3+10*B*a^3*b^2)*x^17+1/14*(10*A*a^3*b^2+5*B*a^4*b)*x^1
4+1/11*(5*A*a^4*b+B*a^5)*x^11+1/8*a^5*A*x^8
```

maxima [A] time = 0.47, size = 119, normalized size = 1.02

$$\frac{1}{26}\,Bb^5x^{26}+\frac{1}{23}\left(5\,Bab^4+Ab^5\right)x^{23}+\frac{1}{4}\left(2\,Ba^2b^3+Aab^4\right)x^{20}+\frac{10}{17}\left(Ba^3b^2+Aa^2b^3\right)x^{17}+\frac{5}{14}\left(Ba^4b+2\,Aa^3b^2\right)x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/26*B*b^5*x^26 + 1/23*(5*B*a*b^4 + A*b^5)*x^23 + 1/4*(2*B*a^2*b^3 + A*a*b^
4)*x^20 + 10/17*(B*a^3*b^2 + A*a^2*b^3)*x^17 + 5/14*(B*a^4*b + 2*A*a^3*b^2)
*x^14 + 1/8*A*a^5*x^8 + 1/11*(B*a^5 + 5*A*a^4*b)*x^11
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{11}\left(\frac{B\,a^5}{11}+\frac{5\,A\,b\,a^4}{11}\right)+x^{23}\left(\frac{A\,b^5}{23}+\frac{5\,B\,a\,b^4}{23}\right)+\frac{A\,a^5\,x^8}{8}+\frac{B\,b^5\,x^{26}}{26}+\frac{10\,a^2\,b^2\,x^{17}\,\left(A\,b+B\,a\right)}{17}+\frac{5\,a^3\,b\,x^{14}\,\left(2\,A\,b+\right.}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^11*((B*a^5)/11 + (5*A*a^4*b)/11) + x^23*((A*b^5)/23 + (5*B*a*b^4)/23) + (
A*a^5*x^8)/8 + (B*b^5*x^26)/26 + (10*a^2*b^2*x^17*(A*b + B*a))/17 + (5*a^3*
b*x^14*(2*A*b + B*a))/14 + (a*b^3*x^20*(A*b + 2*B*a))/4
```

sympy [A] time = 0.10, size = 134, normalized size = 1.15

$$\frac{Aa^5x^8}{8}+\frac{Bb^5x^{26}}{26}+x^{23}\left(\frac{Ab^5}{23}+\frac{5Bab^4}{23}\right)+x^{20}\left(\frac{Aab^4}{4}+\frac{Ba^2b^3}{2}\right)+x^{17}\left(\frac{10Aa^2b^3}{17}+\frac{10Ba^3b^2}{17}\right)+x^{14}\left(\frac{5Aa^3b^2}{7}+\frac{5Ba^4}{14}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**8/8 + B*b**5*x**26/26 + x**23*(A*b**5/23 + 5*B*a*b**4/23) + x**20
*(A*a*b**4/4 + B*a**2*b**3/2) + x**17*(10*A*a**2*b**3/17 + 10*B*a**3*b**2/1
7) + x**14*(5*A*a**3*b**2/7 + 5*B*a**4*b/14) + x**11*(5*A*a**4*b/11 + B*a**
5/11)
```

3.26 $\int x^6 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab$$

```
[Out] 1/7*a^5*A*x^7+1/10*a^4*(5*A*b+B*a)*x^10+5/13*a^3*b*(2*A*b+B*a)*x^13+5/8*a^2
*b^2*(A*b+B*a)*x^16+5/19*a*b^3*(A*b+2*B*a)*x^19+1/22*b^4*(A*b+5*B*a)*x^22+1
/25*b^5*B*x^25
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{1}{7}a^5Ax^7+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^6\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \int \left(a^5Ax^6+a^4(5Ab+aB)x^9+5a^3b(2Ab+aB)x^{12}+10a^2b^2(Ab+aB)x^{15}+\right.\\ &= \frac{1}{7}a^5Ax^7+\frac{1}{10}a^4(5Ab+aB)x^{10}+\frac{5}{13}a^3b(2Ab+aB)x^{13}+\frac{5}{8}a^2b^2(Ab+aB)x^{16}\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{7}a^5Ax^7+\frac{1}{10}a^4x^{10}(aB+5Ab)+\frac{5}{13}a^3bx^{13}(aB+2Ab)+\frac{5}{8}a^2b^2x^{16}(aB+Ab)+\frac{1}{22}b^4x^{22}(5aB+Ab)+\frac{5}{19}ab^3x^{19}(2aB+Ab$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^7)/7 + (a^4*(5*A*b + a*B)*x^10)/10 + (5*a^3*b*(2*A*b + a*B)*x^13)/
13 + (5*a^2*b^2*(A*b + a*B)*x^16)/8 + (5*a*b^3*(A*b + 2*a*B)*x^19)/19 + (b^
4*(A*b + 5*a*B)*x^22)/22 + (b^5*B*x^25)/25
```

fricas [A] time = 0.95, size = 125, normalized size = 1.07

$$\frac{1}{25}x^{25}b^5B+\frac{5}{22}x^{22}b^4aB+\frac{1}{22}x^{22}b^5A+\frac{10}{19}x^{19}b^3a^2B+\frac{5}{19}x^{19}b^4aA+\frac{5}{8}x^{16}b^2a^3B+\frac{5}{8}x^{16}b^3a^2A+\frac{5}{13}x^{13}ba^4B+\frac{10}{13}x^{13}b^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/25*x^25*b^5*B + 5/22*x^22*b^4*a*B + 1/22*x^22*b^5*A + 10/19*x^19*b^3*a^2*
B + 5/19*x^19*b^4*a*A + 5/8*x^16*b^2*a^3*B + 5/8*x^16*b^3*a^2*A + 5/13*x^13
*b*a^4*B + 10/13*x^13*b^2*a^3*A + 1/10*x^10*a^5*B + 1/2*x^10*b*a^4*A + 1/7*
x^7*a^5*A
```

giac [A] time = 0.15, size = 125, normalized size = 1.07

$$\frac{1}{25}Bb^5x^{25}+\frac{5}{22}Bab^4x^{22}+\frac{1}{22}Ab^5x^{22}+\frac{10}{19}Ba^2b^3x^{19}+\frac{5}{19}Aab^4x^{19}+\frac{5}{8}Ba^3b^2x^{16}+\frac{5}{8}Aa^2b^3x^{16}+\frac{5}{13}Ba^4bx^{13}+\frac{10}{13}Aa^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/25*B*b^5*x^25 + 5/22*B*a*b^4*x^22 + 1/22*A*b^5*x^22 + 10/19*B*a^2*b^3*x^1
9 + 5/19*A*a*b^4*x^19 + 5/8*B*a^3*b^2*x^16 + 5/8*A*a^2*b^3*x^16 + 5/13*B*a^
4*b*x^13 + 10/13*A*a^3*b^2*x^13 + 1/10*B*a^5*x^10 + 1/2*A*a^4*b*x^10 + 1/7*
A*a^5*x^7
```

maple [A] time = 0.03, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{25}}{25}+\frac{\left(b^5A+5a\,b^4B\right)x^{22}}{22}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{19}}{19}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{16}}{16}+\frac{A\,a^5x^7}{7}+\frac{\left(10a^3b^2A+5a^4bB\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/25*b^5*B*x^25+1/22*(A*b^5+5*B*a*b^4)*x^22+1/19*(5*A*a*b^4+10*B*a^2*b^3)*x
^19+1/16*(10*A*a^2*b^3+10*B*a^3*b^2)*x^16+1/13*(10*A*a^3*b^2+5*B*a^4*b)*x^1
3+1/10*(5*A*a^4*b+B*a^5)*x^10+1/7*a^5*A*x^7
```

maxima [A] time = 0.59, size = 119, normalized size = 1.02

$$\frac{1}{25}Bb^5x^{25}+\frac{1}{22}\left(5\,Bab^4+Ab^5\right)x^{22}+\frac{5}{19}\left(2\,Ba^2b^3+Aab^4\right)x^{19}+\frac{5}{8}\left(Ba^3b^2+Aa^2b^3\right)x^{16}+\frac{5}{13}\left(Ba^4b+2\,Aa^3b^2\right)x^{13}+\frac{1}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/25*B*b^5*x^25 + 1/22*(5*B*a*b^4 + A*b^5)*x^22 + 5/19*(2*B*a^2*b^3 + A*a*b
^4)*x^19 + 5/8*(B*a^3*b^2 + A*a^2*b^3)*x^16 + 5/13*(B*a^4*b + 2*A*a^3*b^2)*
x^13 + 1/7*A*a^5*x^7 + 1/10*(B*a^5 + 5*A*a^4*b)*x^10
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^{10}\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^{22}\left(\frac{A\,b^5}{22}+\frac{5\,B\,a\,b^4}{22}\right)+\frac{A\,a^5\,x^7}{7}+\frac{B\,b^5\,x^{25}}{25}+\frac{5\,a^2\,b^2\,x^{16}\,\left(A\,b+B\,a\right)}{8}+\frac{5\,a^3\,b\,x^{13}\,\left(2\,A\,b+B\,a\right)}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^10*((B*a^5)/10 + (A*a^4*b)/2) + x^22*((A*b^5)/22 + (5*B*a*b^4)/22) + (A*a
^5*x^7)/7 + (B*b^5*x^25)/25 + (5*a^2*b^2*x^16*(A*b + B*a))/8 + (5*a^3*b*x^1
3*(2*A*b + B*a))/13 + (5*a*b^3*x^19*(A*b + 2*B*a))/19
```

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^7}{7}+\frac{Bb^5x^{25}}{25}+x^{22}\left(\frac{Ab^5}{22}+\frac{5Bab^4}{22}\right)+x^{19}\left(\frac{5Aab^4}{19}+\frac{10Ba^2b^3}{19}\right)+x^{16}\left(\frac{5Aa^2b^3}{8}+\frac{5Ba^3b^2}{8}\right)+x^{13}\left(\frac{10Aa^3b^2}{13}+\frac{5Ba^4}{13}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**7/7 + B*b**5*x**25/25 + x**22*(A*b**5/22 + 5*B*a*b**4/22) + x**19
*(5*A*a*b**4/19 + 10*B*a**2*b**3/19) + x**16*(5*A*a**2*b**3/8 + 5*B*a**3*b*
*2/8) + x**13*(10*A*a**3*b**2/13 + 5*B*a**4*b/13) + x**10*(A*a**4*b/2 + B*a
**5/10)
```

3.27 $\int x^5 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=67

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

```
[Out] -1/18*a*(A*b-B*a)*(b*x^3+a)^6/b^3+1/21*(A*b-2*B*a)*(b*x^3+a)^7/b^3+1/24*B*(
b*x^3+a)^8/b^3
```

Rubi [A] time = 0.15, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{\left(a+bx^3\right)^7 (Ab-2aB)}{21b^3}-\frac{a\left(a+bx^3\right)^6 (Ab-aB)}{18b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] -(a*(A*b - a*B)*(a + b*x^3)^6)/(18*b^3) + ((A*b - 2*a*B)*(a + b*x^3)^7)/(21
*b^3) + (B*(a + b*x^3)^8)/(24*b^3)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^3\right)^5\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x(a+bx)^5(A+Bx)\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)(a+bx)^5}{b^2}+\frac{(Ab-2aB)(a+bx)^6}{b^2}+\frac{B(a+bx)^7}{b^2}\right)dx,\right.\\ &= -\frac{a(Ab-aB)\left(a+bx^3\right)^6}{18b^3}+\frac{(Ab-2aB)\left(a+bx^3\right)^7}{21b^3}+\frac{B\left(a+bx^3\right)^8}{24b^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 107, normalized size = 1.60

$$\frac{1}{504}x^6\left(84a^5A+56a^4x^3(aB+5Ab)+210a^3bx^6(aB+2Ab)+336a^2b^2x^9(aB+Ab)+24b^4x^{15}(5aB+Ab)+140ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^6*(84*a^5*A + 56*a^4*(5*A*b + a*B)*x^3 + 210*a^3*b*(2*A*b + a*B)*x^6 + 3
36*a^2*b^2*(A*b + a*B)*x^9 + 140*a*b^3*(A*b + 2*a*B)*x^12 + 24*b^4*(A*b + 5
*a*B)*x^15 + 21*b^5*B*x^18))/504
```

fricas [B] time = 1.26, size = 125, normalized size = 1.87

$$\frac{1}{24}x^{24}b^5B+\frac{5}{21}x^{21}b^4aB+\frac{1}{21}x^{21}b^5A+\frac{5}{9}x^{18}b^3a^2B+\frac{5}{18}x^{18}b^4aA+\frac{2}{3}x^{15}b^2a^3B+\frac{2}{3}x^{15}b^3a^2A+\frac{5}{12}x^{12}ba^4B+\frac{5}{6}x^{12}b^2a^3A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/24*x^24*b^5*B + 5/21*x^21*b^4*a*B + 1/21*x^21*b^5*A + 5/9*x^18*b^3*a^2*B
+ 5/18*x^18*b^4*a*A + 2/3*x^15*b^2*a^3*B + 2/3*x^15*b^3*a^2*A + 5/12*x^12*b
*a^4*B + 5/6*x^12*b^2*a^3*A + 1/9*x^9*a^5*B + 5/9*x^9*b*a^4*A + 1/6*x^6*a^5
*A
```

giac [B] time = 0.16, size = 125, normalized size = 1.87

$$\frac{1}{24}\,Bb^5x^{24}+\frac{5}{21}\,Bab^4x^{21}+\frac{1}{21}\,Ab^5x^{21}+\frac{5}{9}\,Ba^2b^3x^{18}+\frac{5}{18}\,Aab^4x^{18}+\frac{2}{3}\,Ba^3b^2x^{15}+\frac{2}{3}\,Aa^2b^3x^{15}+\frac{5}{12}\,Ba^4bx^{12}+\frac{5}{6}\,Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/24*B*b^5*x^24 + 5/21*B*a*b^4*x^21 + 1/21*A*b^5*x^21 + 5/9*B*a^2*b^3*x^18
+ 5/18*A*a*b^4*x^18 + 2/3*B*a^3*b^2*x^15 + 2/3*A*a^2*b^3*x^15 + 5/12*B*a^4*
b*x^12 + 5/6*A*a^3*b^2*x^12 + 1/9*B*a^5*x^9 + 5/9*A*a^4*b*x^9 + 1/6*A*a^5*x
^6
```

maple [B] time = 0.04, size = 124, normalized size = 1.85

$$\frac{B\,b^5x^{24}}{24}+\frac{\left(b^5A+5a\,b^4B\right)x^{21}}{21}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{18}}{18}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{15}}{15}+\frac{A\,a^5x^6}{6}+\frac{\left(10a^3b^2A+5a^4b\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/24*b^5*B*x^24+1/21*(A*b^5+5*B*a*b^4)*x^21+1/18*(5*A*a*b^4+10*B*a^2*b^3)*x
^18+1/15*(10*A*a^2*b^3+10*B*a^3*b^2)*x^15+1/12*(10*A*a^3*b^2+5*B*a^4*b)*x^1
2+1/9*(5*A*a^4*b+B*a^5)*x^9+1/6*a^5*A*x^6
```

maxima [A] time = 0.45, size = 119, normalized size = 1.78

$$\frac{1}{24}\,Bb^5x^{24}+\frac{1}{21}\left(5\,Bab^4+Ab^5\right)x^{21}+\frac{5}{18}\left(2\,Ba^2b^3+Aab^4\right)x^{18}+\frac{2}{3}\left(Ba^3b^2+Aa^2b^3\right)x^{15}+\frac{5}{12}\left(Ba^4b+2\,Aa^3b^2\right)x^{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/24*B*b^5*x^24 + 1/21*(5*B*a*b^4 + A*b^5)*x^21 + 5/18*(2*B*a^2*b^3 + A*a*b
^4)*x^18 + 2/3*(B*a^3*b^2 + A*a^2*b^3)*x^15 + 5/12*(B*a^4*b + 2*A*a^3*b^2)*
x^12 + 1/6*A*a^5*x^6 + 1/9*(B*a^5 + 5*A*a^4*b)*x^9
```

mupad [B] time = 0.04, size = 107, normalized size = 1.60

$$x^9\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^{21}\left(\frac{A\,b^5}{21}+\frac{5\,B\,a\,b^4}{21}\right)+\frac{A\,a^5\,x^6}{6}+\frac{B\,b^5\,x^{24}}{24}+\frac{2\,a^2\,b^2\,x^{15}\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^{12}\,\left(2\,A\,b+B\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^9*((B*a^5)/9 + (5*A*a^4*b)/9) + x^21*((A*b^5)/21 + (5*B*a*b^4)/21) + (A*a
^5*x^6)/6 + (B*b^5*x^24)/24 + (2*a^2*b^2*x^15*(A*b + B*a))/3 + (5*a^3*b*x^1
2*(2*A*b + B*a))/12 + (5*a*b^3*x^18*(A*b + 2*B*a))/18
```

sympy [B] time = 0.10, size = 138, normalized size = 2.06

$$\frac{Aa^5x^6}{6}+\frac{Bb^5x^{24}}{24}+x^{21}\left(\frac{Ab^5}{21}+\frac{5Bab^4}{21}\right)+x^{18}\left(\frac{5Aab^4}{18}+\frac{5Ba^2b^3}{9}\right)+x^{15}\left(\frac{2Aa^2b^3}{3}+\frac{2Ba^3b^2}{3}\right)+x^{12}\left(\frac{5Aa^3b^2}{6}+\frac{5Ba^4b}{12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**6/6 + B*b**5*x**24/24 + x**21*(A*b**5/21 + 5*B*a*b**4/21) + x**18
*(5*A*a*b**4/18 + 5*B*a**2*b**3/9) + x**15*(2*A*a**2*b**3/3 + 2*B*a**3*b**2
/3) + x**12*(5*A*a**3*b**2/6 + 5*B*a**4*b/12) + x**9*(5*A*a**4*b/9 + B*a**5
/9)
```

3.28 $\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

```
[Out] 1/5*a^5*A*x^5+1/8*a^4*(5*A*b+B*a)*x^8+5/11*a^3*b*(2*A*b+B*a)*x^11+5/7*a^2*b
^2*(A*b+B*a)*x^14+5/17*a*b^3*(A*b+2*B*a)*x^17+1/20*b^4*(A*b+5*B*a)*x^20+1/2
3*b^5*B*x^23
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{1}{5}a^5Ax^5+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^4+a^4(5Ab+aB)x^7+5a^3b(2Ab+aB)x^{10}+10a^2b^2(Ab+aB)x^{13}+\right.\\ &= \frac{1}{5}a^5Ax^5+\frac{1}{8}a^4(5Ab+aB)x^8+\frac{5}{11}a^3b(2Ab+aB)x^{11}+\frac{5}{7}a^2b^2(Ab+aB)x^{14}+\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{5}a^5Ax^5+\frac{1}{8}a^4x^8(aB+5Ab)+\frac{5}{11}a^3bx^{11}(aB+2Ab)+\frac{5}{7}a^2b^2x^{14}(aB+Ab)+\frac{1}{20}b^4x^{20}(5aB+Ab)+\frac{5}{17}ab^3x^{17}(2aB+Ab)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^5)/5 + (a^4*(5*A*b + a*B)*x^8)/8 + (5*a^3*b*(2*A*b + a*B)*x^11)/11
 + (5*a^2*b^2*(A*b + a*B)*x^14)/7 + (5*a*b^3*(A*b + 2*a*B)*x^17)/17 + (b^4*
(A*b + 5*a*B)*x^20)/20 + (b^5*B*x^23)/23
```

fricas [A] time = 0.96, size = 125, normalized size = 1.07

$$\frac{1}{23}x^{23}b^5B+\frac{1}{4}x^{20}b^4aB+\frac{1}{20}x^{20}b^5A+\frac{10}{17}x^{17}b^3a^2B+\frac{5}{17}x^{17}b^4aA+\frac{5}{7}x^{14}b^2a^3B+\frac{5}{7}x^{14}b^3a^2A+\frac{5}{11}x^{11}ba^4B+\frac{10}{11}x^{11}b^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/23*x^23*b^5*B + 1/4*x^20*b^4*a*B + 1/20*x^20*b^5*A + 10/17*x^17*b^3*a^2*B
 + 5/17*x^17*b^4*a*A + 5/7*x^14*b^2*a^3*B + 5/7*x^14*b^3*a^2*A + 5/11*x^11*
b*a^4*B + 10/11*x^11*b^2*a^3*A + 1/8*x^8*a^5*B + 5/8*x^8*b*a^4*A + 1/5*x^5*
a^5*A
```

giac [A] time = 0.23, size = 125, normalized size = 1.07

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{4}Bab^4x^{20}+\frac{1}{20}Ab^5x^{20}+\frac{10}{17}Ba^2b^3x^{17}+\frac{5}{17}Aab^4x^{17}+\frac{5}{7}Ba^3b^2x^{14}+\frac{5}{7}Aa^2b^3x^{14}+\frac{5}{11}Ba^4bx^{11}+\frac{10}{11}Aa^3b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/23*B*b^5*x^23 + 1/4*B*a*b^4*x^20 + 1/20*A*b^5*x^20 + 10/17*B*a^2*b^3*x^17
 + 5/17*A*a*b^4*x^17 + 5/7*B*a^3*b^2*x^14 + 5/7*A*a^2*b^3*x^14 + 5/11*B*a^4
*b*x^11 + 10/11*A*a^3*b^2*x^11 + 1/8*B*a^5*x^8 + 5/8*A*a^4*b*x^8 + 1/5*A*a^
5*x^5
```

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{23}}{23}+\frac{\left(b^5A+5a\,b^4B\right)x^{20}}{20}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{17}}{17}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{14}}{14}+\frac{A\,a^5x^5}{5}+\frac{\left(10a^3b^2A+5a^4bB\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/23*b^5*B*x^23+1/20*(A*b^5+5*B*a*b^4)*x^20+1/17*(5*A*a*b^4+10*B*a^2*b^3)*x
^17+1/14*(10*A*a^2*b^3+10*B*a^3*b^2)*x^14+1/11*(10*A*a^3*b^2+5*B*a^4*b)*x^1
1+1/8*(5*A*a^4*b+B*a^5)*x^8+1/5*a^5*A*x^5
```

maxima [A] time = 0.64, size = 119, normalized size = 1.02

$$\frac{1}{23}Bb^5x^{23}+\frac{1}{20}\left(5\,Bab^4+Ab^5\right)x^{20}+\frac{5}{17}\left(2\,Ba^2b^3+Aab^4\right)x^{17}+\frac{5}{7}\left(Ba^3b^2+Aa^2b^3\right)x^{14}+\frac{5}{11}\left(Ba^4b+2\,Aa^3b^2\right)x^{11}+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/23*B*b^5*x^23 + 1/20*(5*B*a*b^4 + A*b^5)*x^20 + 5/17*(2*B*a^2*b^3 + A*a*b
^4)*x^17 + 5/7*(B*a^3*b^2 + A*a^2*b^3)*x^14 + 5/11*(B*a^4*b + 2*A*a^3*b^2)*
x^11 + 1/5*A*a^5*x^5 + 1/8*(B*a^5 + 5*A*a^4*b)*x^8
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^8\left(\frac{B\,a^5}{8}+\frac{5\,A\,b\,a^4}{8}\right)+x^{20}\left(\frac{A\,b^5}{20}+\frac{B\,a\,b^4}{4}\right)+\frac{A\,a^5\,x^5}{5}+\frac{B\,b^5\,x^{23}}{23}+\frac{5\,a^2\,b^2\,x^{14}\,\left(A\,b+B\,a\right)}{7}+\frac{5\,a^3\,b\,x^{11}\,\left(2\,A\,b+B\,a\right)}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^8*((B*a^5)/8 + (5*A*a^4*b)/8) + x^20*((A*b^5)/20 + (B*a*b^4)/4) + (A*a^5*
x^5)/5 + (B*b^5*x^23)/23 + (5*a^2*b^2*x^14*(A*b + B*a))/7 + (5*a^3*b*x^11*(
2*A*b + B*a))/11 + (5*a*b^3*x^17*(A*b + 2*B*a))/17
```

sympy [A] time = 0.10, size = 136, normalized size = 1.16

$$\frac{Aa^5x^5}{5}+\frac{Bb^5x^{23}}{23}+x^{20}\left(\frac{Ab^5}{20}+\frac{Bab^4}{4}\right)+x^{17}\left(\frac{5Aab^4}{17}+\frac{10Ba^2b^3}{17}\right)+x^{14}\left(\frac{5Aa^2b^3}{7}+\frac{5Ba^3b^2}{7}\right)+x^{11}\left(\frac{10Aa^3b^2}{11}+\frac{5Ba^4b}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**5/5 + B*b**5*x**23/23 + x**20*(A*b**5/20 + B*a*b**4/4) + x**17*(5
*A*a*b**4/17 + 10*B*a**2*b**3/17) + x**14*(5*A*a**2*b**3/7 + 5*B*a**3*b**2/
7) + x**11*(10*A*a**3*b**2/11 + 5*B*a**4*b/11) + x**8*(5*A*a**4*b/8 + B*a**
5/8)
```

3.29 $\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}\ldots$$

```
[Out] 1/4*a^5*A*x^4+1/7*a^4*(5*A*b+B*a)*x^7+1/2*a^3*b*(2*A*b+B*a)*x^10+10/13*a^2*
b^2*(A*b+B*a)*x^13+5/16*a*b^3*(A*b+2*B*a)*x^16+1/19*b^4*(A*b+5*B*a)*x^19+1/
22*b^5*B*x^22
```

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{4}a^5Ax^4+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^3+a^4(5Ab+aB)x^6+5a^3b(2Ab+aB)x^9+10a^2b^2(Ab+aB)x^{12}+5ab^3\ldots\right.\\ &= \frac{1}{4}a^5Ax^4+\frac{1}{7}a^4(5Ab+aB)x^7+\frac{1}{2}a^3b(2Ab+aB)x^{10}+\frac{10}{13}a^2b^2(Ab+aB)x^{13}+\frac{5}{16}\ldots\end{aligned}$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{4}a^5Ax^4+\frac{1}{7}a^4x^7(aB+5Ab)+\frac{1}{2}a^3bx^{10}(aB+2Ab)+\frac{10}{13}a^2b^2x^{13}(aB+Ab)+\frac{1}{19}b^4x^{19}(5aB+Ab)+\frac{5}{16}ab^3x^{16}(2aB+Ab)+\frac{1}{22}\ldots$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^4)/4 + (a^4*(5*A*b + a*B)*x^7)/7 + (a^3*b*(2*A*b + a*B)*x^10)/2 +
(10*a^2*b^2*(A*b + a*B)*x^13)/13 + (5*a*b^3*(A*b + 2*a*B)*x^16)/16 + (b^4*(
A*b + 5*a*B)*x^19)/19 + (b^5*B*x^22)/22
```

fricas [A] time = 1.26, size = 124, normalized size = 1.06

$$\frac{1}{22}x^{22}b^5B+\frac{5}{19}x^{19}b^4aB+\frac{1}{19}x^{19}b^5A+\frac{5}{8}x^{16}b^3a^2B+\frac{5}{16}x^{16}b^4aA+\frac{10}{13}x^{13}b^2a^3B+\frac{10}{13}x^{13}b^3a^2A+\frac{1}{2}x^{10}ba^4B+x^{10}b^2a^3A+\frac{1}{7}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^5*B + 5/19*x^19*b^4*a*B + 1/19*x^19*b^5*A + 5/8*x^16*b^3*a^2*B
+ 5/16*x^16*b^4*a*A + 10/13*x^13*b^2*a^3*B + 10/13*x^13*b^3*a^2*A + 1/2*x^1
0*b*a^4*B + x^10*b^2*a^3*A + 1/7*x^7*a^5*B + 5/7*x^7*b*a^4*A + 1/4*x^4*a^5*
A
```

giac [A] time = 0.16, size = 124, normalized size = 1.06

$$\frac{1}{22}Bb^5x^{22}+\frac{5}{19}Bab^4x^{19}+\frac{1}{19}Ab^5x^{19}+\frac{5}{8}Ba^2b^3x^{16}+\frac{5}{16}Aab^4x^{16}+\frac{10}{13}Ba^3b^2x^{13}+\frac{10}{13}Aa^2b^3x^{13}+\frac{1}{2}Ba^4bx^{10}+Aa^3b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/22*B*b^5*x^22 + 5/19*B*a*b^4*x^19 + 1/19*A*b^5*x^19 + 5/8*B*a^2*b^3*x^16
+ 5/16*A*a*b^4*x^16 + 10/13*B*a^3*b^2*x^13 + 10/13*A*a^2*b^3*x^13 + 1/2*B*a
^4*b*x^10 + A*a^3*b^2*x^10 + 1/7*B*a^5*x^7 + 5/7*A*a^4*b*x^7 + 1/4*A*a^5*x^
4
```

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{B\,b^5x^{22}}{22}+\frac{\left(b^5A+5a\,b^4B\right)x^{19}}{19}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{16}}{16}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{13}}{13}+\frac{A\,a^5x^4}{4}+\frac{\left(10a^3b^2A+5a^4b\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/22*b^5*B*x^22+1/19*(A*b^5+5*B*a*b^4)*x^19+1/16*(5*A*a*b^4+10*B*a^2*b^3)*x
^16+1/13*(10*A*a^2*b^3+10*B*a^3*b^2)*x^13+1/10*(10*A*a^3*b^2+5*B*a^4*b)*x^1
0+1/7*(5*A*a^4*b+B*a^5)*x^7+1/4*a^5*A*x^4
```

maxima [A] time = 0.44, size = 119, normalized size = 1.02

$$\frac{1}{22}Bb^5x^{22}+\frac{1}{19}\left(5\,Bab^4+Ab^5\right)x^{19}+\frac{5}{16}\left(2\,Ba^2b^3+Aab^4\right)x^{16}+\frac{10}{13}\left(Ba^3b^2+Aa^2b^3\right)x^{13}+\frac{1}{2}\left(Ba^4b+2\,Aa^3b^2\right)x^{10}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/22*B*b^5*x^22 + 1/19*(5*B*a*b^4 + A*b^5)*x^19 + 5/16*(2*B*a^2*b^3 + A*a*b
^4)*x^16 + 10/13*(B*a^3*b^2 + A*a^2*b^3)*x^13 + 1/2*(B*a^4*b + 2*A*a^3*b^2)
*x^10 + 1/4*A*a^5*x^4 + 1/7*(B*a^5 + 5*A*a^4*b)*x^7
```

mupad [B] time = 0.04, size = 107, normalized size = 0.91

$$x^7\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^{19}\left(\frac{A\,b^5}{19}+\frac{5\,B\,a\,b^4}{19}\right)+\frac{A\,a^5\,x^4}{4}+\frac{B\,b^5\,x^{22}}{22}+\frac{10\,a^2\,b^2\,x^{13}\,\left(A\,b+B\,a\right)}{13}+\frac{a^3\,b\,x^{10}\,\left(2\,A\,b+B\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^7*((B*a^5)/7 + (5*A*a^4*b)/7) + x^19*((A*b^5)/19 + (5*B*a*b^4)/19) + (A*a
^5*x^4)/4 + (B*b^5*x^22)/22 + (10*a^2*b^2*x^13*(A*b + B*a))/13 + (a^3*b*x^1
0*(2*A*b + B*a))/2 + (5*a*b^3*x^16*(A*b + 2*B*a))/16
```

sympy [A] time = 0.10, size = 133, normalized size = 1.14

$$\frac{Aa^5x^4}{4}+\frac{Bb^5x^{22}}{22}+x^{19}\left(\frac{Ab^5}{19}+\frac{5Bab^4}{19}\right)+x^{16}\left(\frac{5Aab^4}{16}+\frac{5Ba^2b^3}{8}\right)+x^{13}\left(\frac{10Aa^2b^3}{13}+\frac{10Ba^3b^2}{13}\right)+x^{10}\left(Aa^3b^2+\frac{Ba^4}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**4/4 + B*b**5*x**22/22 + x**19*(A*b**5/19 + 5*B*a*b**4/19) + x**16
*(5*A*a*b**4/16 + 5*B*a**2*b**3/8) + x**13*(10*A*a**2*b**3/13 + 10*B*a**3*b
**2/13) + x**10*(A*a**3*b**2 + B*a**4*b/2) + x**7*(5*A*a**4*b/7 + B*a**5/7)
```

3.30 $\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=42

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

```
[Out] 1/18*(A*b-B*a)*(b*x^3+a)^6/b^2+1/21*B*(b*x^3+a)^7/b^2
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {444, 43}

$$\frac{\left(a+bx^3\right)^6 (Ab-aB)}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] ((A*b - a*B)*(a + b*x^3)^6)/(18*b^2) + (B*(a + b*x^3)^7)/(21*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2 \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^5(A+Bx)\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(Ab-aB)(a+bx)^5}{b}+\frac{B(a+bx)^6}{b}\right)dx,x,x^3\right)\\
&= \frac{(Ab-aB)\left(a+bx^3\right)^6}{18b^2}+\frac{B\left(a+bx^3\right)^7}{21b^2}
\end{aligned}$$

Mathematica [B] time = 0.03, size = 107, normalized size = 2.55

$$\frac{1}{126}x^3\left(42a^5A+21a^4x^3(aB+5Ab)+70a^3bx^6(aB+2Ab)+105a^2b^2x^9(aB+Ab)+7b^4x^{15}(5aB+Ab)+42ab^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (x^3*(42*a^5*A + 21*a^4*(5*A*b + a*B)*x^3 + 70*a^3*b*(2*A*b + a*B)*x^6 + 10
5*a^2*b^2*(A*b + a*B)*x^9 + 42*a*b^3*(A*b + 2*a*B)*x^12 + 7*b^4*(A*b + 5*a*
B)*x^15 + 6*b^5*B*x^18))/126
```

fricas [B] time = 1.17, size = 125, normalized size = 2.98

$$\frac{1}{21}x^{21}b^5B+\frac{5}{18}x^{18}b^4aB+\frac{1}{18}x^{18}b^5A+\frac{2}{3}x^{15}b^3a^2B+\frac{1}{3}x^{15}b^4aA+\frac{5}{6}x^{12}b^2a^3B+\frac{5}{6}x^{12}b^3a^2A+\frac{5}{9}x^9ba^4B+\frac{10}{9}x^9b^2a^3A+\frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^5*B + 5/18*x^18*b^4*a*B + 1/18*x^18*b^5*A + 2/3*x^15*b^3*a^2*B
+ 1/3*x^15*b^4*a*A + 5/6*x^12*b^2*a^3*B + 5/6*x^12*b^3*a^2*A + 5/9*x^9*b*a^
4*B + 10/9*x^9*b^2*a^3*A + 1/6*x^6*a^5*B + 5/6*x^6*b*a^4*A + 1/3*x^3*a^5*A
```

giac [B] time = 0.17, size = 125, normalized size = 2.98

$$\frac{1}{21}Bb^5x^{21}+\frac{5}{18}Bab^4x^{18}+\frac{1}{18}Ab^5x^{18}+\frac{2}{3}Ba^2b^3x^{15}+\frac{1}{3}Aab^4x^{15}+\frac{5}{6}Ba^3b^2x^{12}+\frac{5}{6}Aa^2b^3x^{12}+\frac{5}{9}Ba^4bx^9+\frac{10}{9}Aa^3b^2x^9+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/21*B*b^5*x^21 + 5/18*B*a*b^4*x^18 + 1/18*A*b^5*x^18 + 2/3*B*a^2*b^3*x^15
+ 1/3*A*a*b^4*x^15 + 5/6*B*a^3*b^2*x^12 + 5/6*A*a^2*b^3*x^12 + 5/9*B*a^4*b*
x^9 + 10/9*A*a^3*b^2*x^9 + 1/6*B*a^5*x^6 + 5/6*A*a^4*b*x^6 + 1/3*A*a^5*x^3
```

maple [B] time = 0.04, size = 124, normalized size = 2.95

$$\frac{B\,b^5x^{21}}{21}+\frac{\left(b^5A+5a\,b^4B\right)x^{18}}{18}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{15}}{15}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{12}}{12}+\frac{A\,a^5x^3}{3}+\frac{\left(10a^3b^2A+5a^4bB\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/21*b^5*B*x^21+1/18*(A*b^5+5*B*a*b^4)*x^18+1/15*(5*A*a*b^4+10*B*a^2*b^3)*x
^15+1/12*(10*A*a^2*b^3+10*B*a^3*b^2)*x^12+1/9*(10*A*a^3*b^2+5*B*a^4*b)*x^9+
1/6*(5*A*a^4*b+B*a^5)*x^6+1/3*a^5*A*x^3
```

maxima [B] time = 0.44, size = 119, normalized size = 2.83

$$\frac{1}{21}Bb^5x^{21}+\frac{1}{18}\left(5\,Bab^4+Ab^5\right)x^{18}+\frac{1}{3}\left(2\,Ba^2b^3+Aab^4\right)x^{15}+\frac{5}{6}\left(Ba^3b^2+Aa^2b^3\right)x^{12}+\frac{5}{9}\left(Ba^4b+2\,Aa^3b^2\right)x^9+\frac{1}{3}Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/21*B*b^5*x^21 + 1/18*(5*B*a*b^4 + A*b^5)*x^18 + 1/3*(2*B*a^2*b^3 + A*a*b^
4)*x^15 + 5/6*(B*a^3*b^2 + A*a^2*b^3)*x^12 + 5/9*(B*a^4*b + 2*A*a^3*b^2)*x^
9 + 1/3*A*a^5*x^3 + 1/6*(B*a^5 + 5*A*a^4*b)*x^6
```

mupad [B] time = 0.04, size = 107, normalized size = 2.55

$$x^6\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)+x^{18}\left(\frac{A\,b^5}{18}+\frac{5\,B\,a\,b^4}{18}\right)+\frac{A\,a^5\,x^3}{3}+\frac{B\,b^5\,x^{21}}{21}+\frac{5\,a^2\,b^2\,x^{12}\,\left(A\,b+B\,a\right)}{6}+\frac{5\,a^3\,b\,x^9\,\left(2\,A\,b+B\,a\right)}{9}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^6*((B*a^5)/6 + (5*A*a^4*b)/6) + x^18*((A*b^5)/18 + (5*B*a*b^4)/18) + (A*a
^5*x^3)/3 + (B*b^5*x^21)/21 + (5*a^2*b^2*x^12*(A*b + B*a))/6 + (5*a^3*b*x^9
*(2*A*b + B*a))/9 + (a*b^3*x^15*(A*b + 2*B*a))/3
```

sympy [B] time = 0.10, size = 136, normalized size = 3.24

$$\frac{Aa^5x^3}{3}+\frac{Bb^5x^{21}}{21}+x^{18}\left(\frac{Ab^5}{18}+\frac{5Bab^4}{18}\right)+x^{15}\left(\frac{Aab^4}{3}+\frac{2Ba^2b^3}{3}\right)+x^{12}\left(\frac{5Aa^2b^3}{6}+\frac{5Ba^3b^2}{6}\right)+x^9\left(\frac{10Aa^3b^2}{9}+\frac{5Ba^4}{9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**3/3 + B*b**5*x**21/21 + x**18*(A*b**5/18 + 5*B*a*b**4/18) + x**15
*(A*a*b**4/3 + 2*B*a**2*b**3/3) + x**12*(5*A*a**2*b**3/6 + 5*B*a**3*b**2/6)
 + x**9*(10*A*a**3*b**2/9 + 5*B*a**4*b/9) + x**6*(5*A*a**4*b/6 + B*a**5/6)
```

3.31 $\int x \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=117

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}$$

```
[Out] 1/2*a^5*A*x^2+1/5*a^4*(5*A*b+B*a)*x^5+5/8*a^3*b*(2*A*b+B*a)*x^8+10/11*a^2*b
^2*(A*b+B*a)*x^11+5/14*a*b^3*(A*b+2*B*a)*x^14+1/17*b^4*(A*b+5*B*a)*x^17+1/2
0*b^5*B*x^20
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{1}{2}a^5Ax^2+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^5\left(A+Bx^3\right)dx = \int \left(a^5Ax + a^4(5Ab+aB)x^4 + 5a^3b(2Ab+aB)x^7 + 10a^2b^2(Ab+aB)x^{10} + 5ab^3(\right.$$
$$= \frac{1}{2}a^5Ax^2 + \frac{1}{5}a^4(5Ab+aB)x^5 + \frac{5}{8}a^3b(2Ab+aB)x^8 + \frac{10}{11}a^2b^2(Ab+aB)x^{11} + \frac{5}{14}al$$

Mathematica [A] time = 0.02, size = 117, normalized size = 1.00

$$\frac{1}{2}a^5Ax^2+\frac{1}{5}a^4x^5(aB+5Ab)+\frac{5}{8}a^3bx^8(aB+2Ab)+\frac{10}{11}a^2b^2x^{11}(aB+Ab)+\frac{1}{17}b^4x^{17}(5aB+Ab)+\frac{5}{14}ab^3x^{14}(2aB+Ab)+\frac{1}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^2)/2 + (a^4*(5*A*b + a*B)*x^5)/5 + (5*a^3*b*(2*A*b + a*B)*x^8)/8 +
 (10*a^2*b^2*(A*b + a*B)*x^11)/11 + (5*a*b^3*(A*b + 2*a*B)*x^14)/14 + (b^4*
(A*b + 5*a*B)*x^17)/17 + (b^5*B*x^20)/20
```

fricas [A] time = 1.10, size = 124, normalized size = 1.06

$$\frac{1}{20}x^{20}b^5B+\frac{5}{17}x^{17}b^4aB+\frac{1}{17}x^{17}b^5A+\frac{5}{7}x^{14}b^3a^2B+\frac{5}{14}x^{14}b^4aA+\frac{10}{11}x^{11}b^2a^3B+\frac{10}{11}x^{11}b^3a^2A+\frac{5}{8}x^8ba^4B+\frac{5}{4}x^8b^2a^3A+\frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^5*B + 5/17*x^17*b^4*a*B + 1/17*x^17*b^5*A + 5/7*x^14*b^3*a^2*B
+ 5/14*x^14*b^4*a*A + 10/11*x^11*b^2*a^3*B + 10/11*x^11*b^3*a^2*A + 5/8*x^8
*b*a^4*B + 5/4*x^8*b^2*a^3*A + 1/5*x^5*a^5*B + x^5*b*a^4*A + 1/2*x^2*a^5*A
```

giac [A] time = 0.15, size = 124, normalized size = 1.06

$$\frac{1}{20}Bb^5x^{20}+\frac{5}{17}Bab^4x^{17}+\frac{1}{17}Ab^5x^{17}+\frac{5}{7}Ba^2b^3x^{14}+\frac{5}{14}Aab^4x^{14}+\frac{10}{11}Ba^3b^2x^{11}+\frac{10}{11}Aa^2b^3x^{11}+\frac{5}{8}Ba^4bx^8+\frac{5}{4}Aa^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/20*B*b^5*x^20 + 5/17*B*a*b^4*x^17 + 1/17*A*b^5*x^17 + 5/7*B*a^2*b^3*x^14
+ 5/14*A*a*b^4*x^14 + 10/11*B*a^3*b^2*x^11 + 10/11*A*a^2*b^3*x^11 + 5/8*B*a
^4*b*x^8 + 5/4*A*a^3*b^2*x^8 + 1/5*B*a^5*x^5 + A*a^4*b*x^5 + 1/2*A*a^5*x^2
```

maple [A] time = 0.04, size = 124, normalized size = 1.06

$$\frac{Bb^5x^{20}}{20}+\frac{\left(b^5A+5ab^4B\right)x^{17}}{17}+\frac{\left(5ab^4A+10a^2b^3B\right)x^{14}}{14}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{11}}{11}+\frac{Aa^5x^2}{2}+\frac{\left(10a^3b^2A+5a^4b\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/20*b^5*B*x^20+1/17*(A*b^5+5*B*a*b^4)*x^17+1/14*(5*A*a*b^4+10*B*a^2*b^3)*x
^14+1/11*(10*A*a^2*b^3+10*B*a^3*b^2)*x^11+1/8*(10*A*a^3*b^2+5*B*a^4*b)*x^8+
1/5*(5*A*a^4*b+B*a^5)*x^5+1/2*a^5*A*x^2
```

maxima [A] time = 0.65, size = 119, normalized size = 1.02

$$\frac{1}{20}Bb^5x^{20}+\frac{1}{17}\left(5Bab^4+Ab^5\right)x^{17}+\frac{5}{14}\left(2Ba^2b^3+Aab^4\right)x^{14}+\frac{10}{11}\left(Ba^3b^2+Aa^2b^3\right)x^{11}+\frac{5}{8}\left(Ba^4b+2Aa^3b^2\right)x^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/20*B*b^5*x^20 + 1/17*(5*B*a*b^4 + A*b^5)*x^17 + 5/14*(2*B*a^2*b^3 + A*a*b
^4)*x^14 + 10/11*(B*a^3*b^2 + A*a^2*b^3)*x^11 + 5/8*(B*a^4*b + 2*A*a^3*b^2)
*x^8 + 1/2*A*a^5*x^2 + 1/5*(B*a^5 + 5*A*a^4*b)*x^5
```

mupad [B] time = 0.04, size = 106, normalized size = 0.91

$$x^5\left(\frac{Ba^5}{5}+Aba^4\right)+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+\frac{10a^2b^2x^{11}\left(Ab+Ba\right)}{11}+\frac{5a^3bx^8\left(2Ab+Ba\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^5*((B*a^5)/5 + A*a^4*b) + x^17*((A*b^5)/17 + (5*B*a*b^4)/17) + (A*a^5*x^2
)/2 + (B*b^5*x^20)/20 + (10*a^2*b^2*x^11*(A*b + B*a))/11 + (5*a^3*b*x^8*(2*
A*b + B*a))/8 + (5*a*b^3*x^14*(A*b + 2*B*a))/14
```

sympy [A] time = 0.09, size = 134, normalized size = 1.15

$$\frac{Aa^5x^2}{2}+\frac{Bb^5x^{20}}{20}+x^{17}\left(\frac{Ab^5}{17}+\frac{5Bab^4}{17}\right)+x^{14}\left(\frac{5Aab^4}{14}+\frac{5Ba^2b^3}{7}\right)+x^{11}\left(\frac{10Aa^2b^3}{11}+\frac{10Ba^3b^2}{11}\right)+x^8\left(\frac{5Aa^3b^2}{4}+\frac{5B}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x**2/2 + B*b**5*x**20/20 + x**17*(A*b**5/17 + 5*B*a*b**4/17) + x**14
*(5*A*a*b**4/14 + 5*B*a**2*b**3/7) + x**11*(10*A*a**2*b**3/11 + 10*B*a**3*b
**2/11) + x**8*(5*A*a**3*b**2/4 + 5*B*a**4*b/8) + x**5*(A*a**4*b + B*a**5/5
)
```

3.32 $\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=109

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5$$

```
[Out] a^5*A*x+1/4*a^4*(5*A*b+B*a)*x^4+5/7*a^3*b*(2*A*b+B*a)*x^7+a^2*b^2*(A*b+B*a)
*x^10+5/13*a*b^3*(A*b+2*B*a)*x^13+1/16*b^4*(A*b+5*B*a)*x^16+1/19*b^5*B*x^19
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2b^2x^{10}(aB+Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+\frac{1}{4}a^4x^4(aB+5Ab)+a^5Ax+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^5 \left(A+Bx^3\right) \, dx &= \int \left(a^5A+a^4(5Ab+aB)x^3+5a^3b(2Ab+aB)x^6+10a^2b^2(Ab+aB)x^9+5ab^3(A\right.\\ &= a^5Ax+\frac{1}{4}a^4(5Ab+aB)x^4+\frac{5}{7}a^3b(2Ab+aB)x^7+a^2b^2(Ab+aB)x^{10}+\frac{5}{13}ab^3(A\end{aligned}$$

Mathematica [A] time = 0.02, size = 109, normalized size = 1.00

$$a^5Ax+\frac{1}{4}a^4x^4(aB+5Ab)+\frac{5}{7}a^3bx^7(aB+2Ab)+a^2b^2x^{10}(aB+Ab)+\frac{1}{16}b^4x^{16}(5aB+Ab)+\frac{5}{13}ab^3x^{13}(2aB+Ab)+\frac{1}{19}b^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] a^5*A*x + (a^4*(5*A*b + a*B)*x^4)/4 + (5*a^3*b*(2*A*b + a*B)*x^7)/7 + a^2*b
^2*(A*b + a*B)*x^10 + (5*a*b^3*(A*b + 2*a*B)*x^13)/13 + (b^4*(A*b + 5*a*B)*
x^16)/16 + (b^5*B*x^19)/19
```

fricas [A] time = 0.85, size = 120, normalized size = 1.10

$$\frac{1}{19}x^{19}b^5B+\frac{5}{16}x^{16}b^4aB+\frac{1}{16}x^{16}b^5A+\frac{10}{13}x^{13}b^3a^2B+\frac{5}{13}x^{13}b^4aA+x^{10}b^2a^3B+x^{10}b^3a^2A+\frac{5}{7}x^7ba^4B+\frac{10}{7}x^7b^2a^3A+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^5*B + 5/16*x^16*b^4*a*B + 1/16*x^16*b^5*A + 10/13*x^13*b^3*a^2*
B + 5/13*x^13*b^4*a*A + x^10*b^2*a^3*B + x^10*b^3*a^2*A + 5/7*x^7*b*a^4*B +
 10/7*x^7*b^2*a^3*A + 1/4*x^4*a^5*B + 5/4*x^4*b*a^4*A + x*a^5*A
```

giac [A] time = 0.17, size = 120, normalized size = 1.10

$$\frac{1}{19}Bb^5x^{19}+\frac{5}{16}Bab^4x^{16}+\frac{1}{16}Ab^5x^{16}+\frac{10}{13}Ba^2b^3x^{13}+\frac{5}{13}Aab^4x^{13}+Ba^3b^2x^{10}+Aa^2b^3x^{10}+\frac{5}{7}Ba^4bx^7+\frac{10}{7}Aa^3b^2x^7+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/19*B*b^5*x^19 + 5/16*B*a*b^4*x^16 + 1/16*A*b^5*x^16 + 10/13*B*a^2*b^3*x^1
3 + 5/13*A*a*b^4*x^13 + B*a^3*b^2*x^10 + A*a^2*b^3*x^10 + 5/7*B*a^4*b*x^7 +
 10/7*A*a^3*b^2*x^7 + 1/4*B*a^5*x^4 + 5/4*A*a^4*b*x^4 + A*a^5*x
```

maple [A] time = 0.04, size = 121, normalized size = 1.11

$$\frac{B\,b^5x^{19}}{19}+\frac{\left(b^5A+5a\,b^4B\right)x^{16}}{16}+\frac{\left(5a\,b^4A+10a^2b^3B\right)x^{13}}{13}+\frac{\left(10a^2b^3A+10a^3b^2B\right)x^{10}}{10}+A\,a^5x+\frac{\left(10a^3b^2A+5a^4bB\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] 1/19*b^5*B*x^19+1/16*(A*b^5+5*B*a*b^4)*x^16+1/13*(5*A*a*b^4+10*B*a^2*b^3)*x
^13+1/10*(10*A*a^2*b^3+10*B*a^3*b^2)*x^10+1/7*(10*A*a^3*b^2+5*B*a^4*b)*x^7+
1/4*(5*A*a^4*b+B*a^5)*x^4+a^5*A*x
```

maxima [A] time = 0.60, size = 115, normalized size = 1.06

$$\frac{1}{19}Bb^5x^{19}+\frac{1}{16}\left(5\,Bab^4+Ab^5\right)x^{16}+\frac{5}{13}\left(2\,Ba^2b^3+Aab^4\right)x^{13}+\left(Ba^3b^2+Aa^2b^3\right)x^{10}+\frac{5}{7}\left(Ba^4b+2\,Aa^3b^2\right)x^7+Aa^5x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 1/19*B*b^5*x^19 + 1/16*(5*B*a*b^4 + A*b^5)*x^16 + 5/13*(2*B*a^2*b^3 + A*a*b
^4)*x^13 + (B*a^3*b^2 + A*a^2*b^3)*x^10 + 5/7*(B*a^4*b + 2*A*a^3*b^2)*x^7 +
 A*a^5*x + 1/4*(B*a^5 + 5*A*a^4*b)*x^4
```

mupad [B] time = 0.04, size = 103, normalized size = 0.94

$$x^4\left(\frac{B\,a^5}{4}+\frac{5\,A\,b\,a^4}{4}\right)+x^{16}\left(\frac{A\,b^5}{16}+\frac{5\,B\,a\,b^4}{16}\right)+\frac{B\,b^5\,x^{19}}{19}+A\,a^5\,x+a^2\,b^2\,x^{10}\,\left(A\,b+B\,a\right)+\frac{5\,a^3\,b\,x^7\,\left(2\,A\,b+B\,a\right)}{7}+\frac{5\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] x^4*((B*a^5)/4 + (5*A*a^4*b)/4) + x^16*((A*b^5)/16 + (5*B*a*b^4)/16) + (B*b
^5*x^19)/19 + A*a^5*x + a^2*b^2*x^10*(A*b + B*a) + (5*a^3*b*x^7*(2*A*b + B*
a))/7 + (5*a*b^3*x^13*(A*b + 2*B*a))/13
```

sympy [A] time = 0.09, size = 128, normalized size = 1.17

$$Aa^5x+\frac{Bb^5x^{19}}{19}+x^{16}\left(\frac{Ab^5}{16}+\frac{5Bab^4}{16}\right)+x^{13}\left(\frac{5Aab^4}{13}+\frac{10Ba^2b^3}{13}\right)+x^{10}\left(Aa^2b^3+Ba^3b^2\right)+x^7\left(\frac{10Aa^3b^2}{7}+\frac{5Ba^4b}{7}\right)+x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] A*a**5*x + B*b**5*x**19/19 + x**16*(A*b**5/16 + 5*B*a*b**4/16) + x**13*(5*A
*a*b**4/13 + 10*B*a**2*b**3/13) + x**10*(A*a**2*b**3 + B*a**3*b**2) + x**7*
(10*A*a**3*b**2/7 + 5*B*a**4*b/7) + x**4*(5*A*a**4*b/4 + B*a**5/4)
```

3.33 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx$

Optimal. Leaf size=88

$$a^5 A \log(x) + \frac{5}{3}a^4 Abx^3 + \frac{5}{3}a^3 Ab^2x^6 + \frac{10}{9}a^2 Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

```
[Out] 5/3*a^4*A*b*x^3+5/3*a^3*A*b^2*x^6+10/9*a^2*A*b^3*x^9+5/12*a*A*b^4*x^12+1/15
*A*b^5*x^15+1/18*B*(b*x^3+a)^6/b+a^5*A*ln(x)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{446, 80, 43\}$

$$\frac{10}{9}a^2 Ab^3x^9 + \frac{5}{3}a^3 Ab^2x^6 + \frac{5}{3}a^4 Abx^3 + a^5 A \log(x) + \frac{5}{12}aAb^4x^{12} + \frac{B(a+bx^3)^6}{18b} + \frac{1}{15}Ab^5x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (5*a^4*A*b*x^3)/3 + (5*a^3*A*b^2*x^6)/3 + (10*a^2*A*b^3*x^9)/9 + (5*a*A*b^4
*x^12)/12 + (A*b^5*x^15)/15 + (B*(a + b*x^3)^6)/(18*b) + a^5*A*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x} dx = \frac{1}{3} \text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x} dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3} A \text{Subst}\left(\int \frac{(a+bx)^5}{x} dx, x, x^3\right)$$
$$= \frac{B(a+bx^3)^6}{18b} + \frac{1}{3} A \text{Subst}\left(\int \left(5a^4b + \frac{a^5}{x} + 10a^3b^2x + 10a^2b^3x^2 + 5ab^4x^3 + b^5x^4\right)\right.$$
$$= \frac{5}{3}a^4Abx^3 + \frac{5}{3}a^3Ab^2x^6 + \frac{10}{9}a^2Ab^3x^9 + \frac{5}{12}aAb^4x^{12} + \frac{1}{15}Ab^5x^{15} + \frac{B(a+bx^3)^6}{18b} + a$$

Mathematica [A] time = 0.03, size = 113, normalized size = 1.28

$$a^5 A \log(x)+\frac{1}{3}a^4x^3(aB+5Ab)+\frac{5}{6}a^3bx^6(aB+2Ab)+\frac{10}{9}a^2b^2x^9(aB+Ab)+\frac{1}{15}b^4x^{15}(5aB+Ab)+\frac{5}{12}ab^3x^{12}(2aB+Ab)+\frac{1}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x,x]
```

```
[Out] (a^4*(5*A*b + a*B)*x^3)/3 + (5*a^3*b*(2*A*b + a*B)*x^6)/6 + (10*a^2*b^2*(A*
b + a*B)*x^9)/9 + (5*a*b^3*(A*b + 2*a*B)*x^12)/12 + (b^4*(A*b + 5*a*B)*x^15
)/15 + (b^5*B*x^18)/18 + a^5*A*Log[x]
```

fricas [A] time = 1.07, size = 117, normalized size = 1.33

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2Aa^3b^2\right)x^6+Aa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + A*a^5*log(x) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

giac [A] time = 0.16, size = 124, normalized size = 1.41

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{3}Bab^4x^{15}+\frac{1}{15}Ab^5x^{15}+\frac{5}{6}Ba^2b^3x^{12}+\frac{5}{12}Aab^4x^{12}+\frac{10}{9}Ba^3b^2x^9+\frac{10}{9}Aa^2b^3x^9+\frac{5}{6}Ba^4bx^6+\frac{5}{3}Aa^3b^2x^6+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 1/18*B*b^5*x^18 + 1/3*B*a*b^4*x^15 + 1/15*A*b^5*x^15 + 5/6*B*a^2*b^3*x^12 +
 5/12*A*a*b^4*x^12 + 10/9*B*a^3*b^2*x^9 + 10/9*A*a^2*b^3*x^9 + 5/6*B*a^4*b*
x^6 + 5/3*A*a^3*b^2*x^6 + 1/3*B*a^5*x^3 + 5/3*A*a^4*b*x^3 + A*a^5*log(abs(x
))
```

maple [A] time = 0.04, size = 124, normalized size = 1.41

$$\frac{B\,b^5x^{18}}{18}+\frac{A\,b^5x^{15}}{15}+\frac{Ba\,b^4x^{15}}{3}+\frac{5Aa\,b^4x^{12}}{12}+\frac{5B\,a^2b^3x^{12}}{6}+\frac{10A\,a^2b^3x^9}{9}+\frac{10B\,a^3b^2x^9}{9}+\frac{5A\,a^3b^2x^6}{3}+\frac{5B\,a^4b\,x^6}{6}+\frac{5A\,a^4b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x,x)
```

```
[Out] 1/18*B*b^5*x^18+1/15*A*b^5*x^15+1/3*B*x^15*a*b^4+5/12*a*A*b^4*x^12+5/6*B*x^
12*a^2*b^3+10/9*a^2*A*b^3*x^9+10/9*B*x^9*a^3*b^2+5/3*a^3*A*b^2*x^6+5/6*B*x^
6*a^4*b+5/3*a^4*A*b*x^3+1/3*B*x^3*a^5+a^5*A*ln(x)
```

maxima [A] time = 0.45, size = 120, normalized size = 1.36

$$\frac{1}{18}Bb^5x^{18}+\frac{1}{15}\left(5Bab^4+Ab^5\right)x^{15}+\frac{5}{12}\left(2Ba^2b^3+Aab^4\right)x^{12}+\frac{10}{9}\left(Ba^3b^2+Aa^2b^3\right)x^9+\frac{5}{6}\left(Ba^4b+2Aa^3b^2\right)x^6+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 1/18*B*b^5*x^18 + 1/15*(5*B*a*b^4 + A*b^5)*x^15 + 5/12*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 10/9*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5/6*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 1/3*A*a^5*log(x^3) + 1/3*(B*a^5 + 5*A*a^4*b)*x^3
```

mupad [B] time = 0.05, size = 105, normalized size = 1.19

$$x^3\left(\frac{Ba^5}{3}+\frac{5Aba^4}{3}\right)+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+\frac{Bb^5x^{18}}{18}+Aa^5\ln(x)+\frac{10a^2b^2x^9\left(Ab+Ba\right)}{9}+\frac{5a^3bx^6\left(2Ab+B\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x,x)
```

```
[Out] x^3*((B*a^5)/3 + (5*A*a^4*b)/3) + x^15*((A*b^5)/15 + (B*a*b^4)/3) + (B*b^5*
x^18)/18 + A*a^5*log(x) + (10*a^2*b^2*x^9*(A*b + B*a))/9 + (5*a^3*b*x^6*(2*
A*b + B*a))/6 + (5*a*b^3*x^12*(A*b + 2*B*a))/12
```

sympy [A] time = 0.24, size = 134, normalized size = 1.52

$$Aa^5\log(x)+\frac{Bb^5x^{18}}{18}+x^{15}\left(\frac{Ab^5}{15}+\frac{Bab^4}{3}\right)+x^{12}\left(\frac{5Aab^4}{12}+\frac{5Ba^2b^3}{6}\right)+x^9\left(\frac{10Aa^2b^3}{9}+\frac{10Ba^3b^2}{9}\right)+x^6\left(\frac{5Aa^3b^2}{3}+\frac{5}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x,x)
```

```
[Out] A*a**5*log(x) + B*b**5*x**18/18 + x**15*(A*b**5/15 + B*a*b**4/3) + x**12*(5
*A*a*b**4/12 + 5*B*a**2*b**3/6) + x**9*(10*A*a**2*b**3/9 + 10*B*a**3*b**2/9
) + x**6*(5*A*a**3*b**2/3 + 5*B*a**4*b/6) + x**3*(5*A*a**4*b/3 + B*a**5/3)
```

3.34 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx$

Optimal. Leaf size=112

$$-\frac{a^5 A}{x}+\frac{1}{2}a^4x^2(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^1$$

```
[Out] -a^5*A/x+1/2*a^4*(5*A*b+B*a)*x^2+a^3*b*(2*A*b+B*a)*x^5+5/4*a^2*b^2*(A*b+B*a
)*x^8+5/11*a*b^3*(A*b+2*B*a)*x^11+1/14*b^4*(A*b+5*B*a)*x^14+1/17*b^5*B*x^17
```

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{4}a^2b^2x^8(aB+Ab)+a^3bx^5(aB+2Ab)+\frac{1}{2}a^4x^2(aB+5Ab)-\frac{a^5 A}{x}+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^{17}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^2} \, dx &= \int \left(\frac{a^5 A}{x^2} + a^4(5Ab+aB)x + 5a^3b(2Ab+aB)x^4 + 10a^2b^2(Ab+aB)x^7 + 5ab^3(Ab+\right. \\ &= -\frac{a^5 A}{x} + \frac{1}{2}a^4(5Ab+aB)x^2 + a^3b(2Ab+aB)x^5 + \frac{5}{4}a^2b^2(Ab+aB)x^8 + \frac{5}{11}ab^3(Ab+\end{aligned}$$

Mathematica [A] time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5 A}{x}+\frac{1}{2}a^4x^2(aB+5Ab)+a^3bx^5(aB+2Ab)+\frac{5}{4}a^2b^2x^8(aB+Ab)+\frac{1}{14}b^4x^{14}(5aB+Ab)+\frac{5}{11}ab^3x^{11}(2aB+Ab)+\frac{1}{17}b^5Bx^1$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^2,x]
```

```
[Out] -((a^5*A)/x) + (a^4*(5*A*b + a*B)*x^2)/2 + a^3*b*(2*A*b + a*B)*x^5 + (5*a^2
*b^2*(A*b + a*B)*x^8)/4 + (5*a*b^3*(A*b + 2*a*B)*x^11)/11 + (b^4*(A*b + 5*a
*B)*x^14)/14 + (b^5*B*x^17)/17
```

fricas [A] time = 1.05, size = 121, normalized size = 1.08

$$\frac{308\,Bb^5x^{18} + 374\left(5\,Bab^4 + Ab^5\right)x^{15} + 2380\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 6545\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 5236\left(Ba^4b + 2\,A\right.}{5236\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5236*(308*B*b^5*x^18 + 374*(5*B*a*b^4 + A*b^5)*x^15 + 2380*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 5236*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 5236*A*a^5 + 2618*(B*a^5 + 5*A*a^4*b)*x^3)/x
```

giac [A] time = 0.15, size = 124, normalized size = 1.11

$$\frac{1}{17}Bb^5x^{17}+\frac{5}{14}Bab^4x^{14}+\frac{1}{14}Ab^5x^{14}+\frac{10}{11}Ba^2b^3x^{11}+\frac{5}{11}Aab^4x^{11}+\frac{5}{4}Ba^3b^2x^8+\frac{5}{4}Aa^2b^3x^8+Ba^4bx^5+2Aa^3b^2x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/17*B*b^5*x^17 + 5/14*B*a*b^4*x^14 + 1/14*A*b^5*x^14 + 10/11*B*a^2*b^3*x^1
1 + 5/11*A*a*b^4*x^11 + 5/4*B*a^3*b^2*x^8 + 5/4*A*a^2*b^3*x^8 + B*a^4*b*x^5
 + 2*A*a^3*b^2*x^5 + 1/2*B*a^5*x^2 + 5/2*A*a^4*b*x^2 - A*a^5/x
```

maple [A] time = 0.05, size = 125, normalized size = 1.12

$$\frac{Bb^5x^{17}}{17}+\frac{Ab^5x^{14}}{14}+\frac{5Bab^4x^{14}}{14}+\frac{5Aab^4x^{11}}{11}+\frac{10Ba^2b^3x^{11}}{11}+\frac{5Aa^2b^3x^8}{4}+\frac{5Ba^3b^2x^8}{4}+2Aa^3b^2x^5+Ba^4bx^5+\frac{5Aa^4b}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^2,x)
```

```
[Out] 1/17*b^5*B*x^17+1/14*A*x^14*b^5+5/14*B*x^14*a*b^4+5/11*A*x^11*a*b^4+10/11*B
*x^11*a^2*b^3+5/4*A*x^8*a^2*b^3+5/4*B*x^8*a^3*b^2+2*A*x^5*a^3*b^2+B*x^5*a^4
*b+5/2*A*x^2*a^4*b+1/2*B*x^2*a^5-a^5*A/x
```

maxima [A] time = 0.54, size = 118, normalized size = 1.05

$$\frac{1}{17}Bb^5x^{17}+\frac{1}{14}\left(5Bab^4+Ab^5\right)x^{14}+\frac{5}{11}\left(2Ba^2b^3+Aab^4\right)x^{11}+\frac{5}{4}\left(Ba^3b^2+Aa^2b^3\right)x^8+\left(Ba^4b+2Aa^3b^2\right)x^5-\frac{Aa^5}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/17*B*b^5*x^17 + 1/14*(5*B*a*b^4 + A*b^5)*x^14 + 5/11*(2*B*a^2*b^3 + A*a*b
^4)*x^11 + 5/4*(B*a^3*b^2 + A*a^2*b^3)*x^8 + (B*a^4*b + 2*A*a^3*b^2)*x^5 -
A*a^5/x + 1/2*(B*a^5 + 5*A*a^4*b)*x^2
```

mupad [B] time = 0.04, size = 106, normalized size = 0.95

$$x^2\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)+x^{14}\left(\frac{A\,b^5}{14}+\frac{5\,B\,a\,b^4}{14}\right)-\frac{A\,a^5}{x}+\frac{B\,b^5\,x^{17}}{17}+\frac{5\,a^2\,b^2\,x^8\,\left(A\,b+B\,a\right)}{4}+a^3\,b\,x^5\,\left(2\,A\,b+B\,a\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^2,x)
```

```
[Out] x^2*((B*a^5)/2 + (5*A*a^4*b)/2) + x^14*((A*b^5)/14 + (5*B*a*b^4)/14) - (A*a
^5)/x + (B*b^5*x^17)/17 + (5*a^2*b^2*x^8*(A*b + B*a))/4 + a^3*b*x^5*(2*A*b
+ B*a) + (5*a*b^3*x^11*(A*b + 2*B*a))/11
```

sympy [A] time = 0.25, size = 129, normalized size = 1.15

$$-\frac{Aa^5}{x}+\frac{Bb^5x^{17}}{17}+x^{14}\left(\frac{Ab^5}{14}+\frac{5Bab^4}{14}\right)+x^{11}\left(\frac{5Aab^4}{11}+\frac{10Ba^2b^3}{11}\right)+x^8\left(\frac{5Aa^2b^3}{4}+\frac{5Ba^3b^2}{4}\right)+x^5\left(2Aa^3b^2+Ba^4b\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**2,x)
```

```
[Out] -A*a**5/x + B*b**5*x**17/17 + x**14*(A*b**5/14 + 5*B*a*b**4/14) + x**11*(5*
A*a*b**4/11 + 10*B*a**2*b**3/11) + x**8*(5*A*a**2*b**3/4 + 5*B*a**3*b**2/4)
 + x**5*(2*A*a**3*b**2 + B*a**4*b) + x**2*(5*A*a**4*b/2 + B*a**5/2)
```

3.35 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^3} dx$

Optimal. Leaf size=112

$$-\frac{a^5 A}{2x^2}+a^4x(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5B$$

```
[Out] -1/2*a^5*A/x^2+a^4*(5*A*b+B*a)*x+5/4*a^3*b*(2*A*b+B*a)*x^4+10/7*a^2*b^2*(A*
b+B*a)*x^7+1/2*a*b^3*(A*b+2*B*a)*x^10+1/13*b^4*(A*b+5*B*a)*x^13+1/16*b^5*B*
x^16
```

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+a^4x(aB+5Ab)-\frac{a^5 A}{2x^2}+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -(a^5*A)/(2*x^2) + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^3} dx = \int \left(a^4(5Ab+aB)+\frac{a^5 A}{x^3}+5a^3b(2Ab+aB)x^3+10a^2b^2(Ab+aB)x^6+5ab^3(Ab\right.$$
$$= -\frac{a^5 A}{2x^2}+a^4(5Ab+aB)x+\frac{5}{4}a^3b(2Ab+aB)x^4+\frac{10}{7}a^2b^2(Ab+aB)x^7+\frac{1}{2}ab^3(Ab$$

Mathematica [A] time = 0.03, size = 112, normalized size = 1.00

$$-\frac{a^5 A}{2x^2}+a^4x(aB+5Ab)+\frac{5}{4}a^3bx^4(aB+2Ab)+\frac{10}{7}a^2b^2x^7(aB+Ab)+\frac{1}{13}b^4x^{13}(5aB+Ab)+\frac{1}{2}ab^3x^{10}(2aB+Ab)+\frac{1}{16}b^5B$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^3,x]
```

```
[Out] -1/2*(a^5*A)/x^2 + a^4*(5*A*b + a*B)*x + (5*a^3*b*(2*A*b + a*B)*x^4)/4 + (1
0*a^2*b^2*(A*b + a*B)*x^7)/7 + (a*b^3*(A*b + 2*a*B)*x^10)/2 + (b^4*(A*b + 5
*a*B)*x^13)/13 + (b^5*B*x^16)/16
```

fricas [A] time = 0.68, size = 121, normalized size = 1.08

$$\frac{91\,Bb^5x^{18}+112\left(5\,Bab^4+Ab^5\right)x^{15}+728\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2080\left(Ba^3b^2+Aa^2b^3\right)x^9+1820\left(Ba^4b+2\,A\right.}{1456\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/1456*(91*B*b^5*x^18 + 112*(5*B*a*b^4 + A*b^5)*x^15 + 728*(2*B*a^2*b^3 + A
*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1820*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 728*A*a^5 + 1456*(B*a^5 + 5*A*a^4*b)*x^3)/x^2
```

giac [A] time = 0.16, size = 119, normalized size = 1.06

$$\frac{1}{16}Bb^5x^{16}+\frac{5}{13}Bab^4x^{13}+\frac{1}{13}Ab^5x^{13}+Ba^2b^3x^{10}+\frac{1}{2}Aab^4x^{10}+\frac{10}{7}Ba^3b^2x^7+\frac{10}{7}Aa^2b^3x^7+\frac{5}{4}Ba^4bx^4+\frac{5}{2}Aa^3b^2x^4+Ba^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*B*b^5*x^16 + 5/13*B*a*b^4*x^13 + 1/13*A*b^5*x^13 + B*a^2*b^3*x^10 + 1/
2*A*a*b^4*x^10 + 10/7*B*a^3*b^2*x^7 + 10/7*A*a^2*b^3*x^7 + 5/4*B*a^4*b*x^4
+ 5/2*A*a^3*b^2*x^4 + B*a^5*x + 5*A*a^4*b*x - 1/2*A*a^5/x^2
```

maple [A] time = 0.04, size = 120, normalized size = 1.07

$$\frac{Bb^5x^{16}}{16}+\frac{Ab^5x^{13}}{13}+\frac{5Bab^4x^{13}}{13}+\frac{Aab^4x^{10}}{2}+Ba^2b^3x^{10}+\frac{10Aa^2b^3x^7}{7}+\frac{10Ba^3b^2x^7}{7}+\frac{5Aa^3b^2x^4}{2}+\frac{5Ba^4bx^4}{4}+5Aa^4bx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^3,x)
```

```
[Out] 1/16*b^5*B*x^16+1/13*A*x^13*b^5+5/13*B*x^13*a*b^4+1/2*A*x^10*a*b^4+B*x^10*a
^2*b^3+10/7*A*x^7*a^2*b^3+10/7*B*x^7*a^3*b^2+5/2*A*x^4*a^3*b^2+5/4*B*x^4*a^
4*b+5*a^4*b*A*x+B*a^5*x-1/2*a^5*A/x^2
```

maxima [A] time = 0.61, size = 116, normalized size = 1.04

$$\frac{1}{16}Bb^5x^{16}+\frac{1}{13}\left(5Bab^4+Ab^5\right)x^{13}+\frac{1}{2}\left(2Ba^2b^3+Aab^4\right)x^{10}+\frac{10}{7}\left(Ba^3b^2+Aa^2b^3\right)x^7+\frac{5}{4}\left(Ba^4b+2Aa^3b^2\right)x^4-\frac{Aa^5}{2x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*b^5*x^16 + 1/13*(5*B*a*b^4 + A*b^5)*x^13 + 1/2*(2*B*a^2*b^3 + A*a*b^
4)*x^10 + 10/7*(B*a^3*b^2 + A*a^2*b^3)*x^7 + 5/4*(B*a^4*b + 2*A*a^3*b^2)*x^
4 - 1/2*A*a^5/x^2 + (B*a^5 + 5*A*a^4*b)*x
```

mupad [B] time = 0.04, size = 104, normalized size = 0.93

$$x\left(Ba^5+5Aba^4\right)+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+\frac{10a^2b^2x^7(Ab+Ba)}{7}+\frac{5a^3bx^4(2Ab+Ba)}{4}+\frac{ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^3,x)
```

```
[Out] x*(B*a^5 + 5*A*a^4*b) + x^13*((A*b^5)/13 + (5*B*a*b^4)/13) - (A*a^5)/(2*x^2
) + (B*b^5*x^16)/16 + (10*a^2*b^2*x^7*(A*b + B*a))/7 + (5*a^3*b*x^4*(2*A*b
+ B*a))/4 + (a*b^3*x^10*(A*b + 2*B*a))/2
```

sympy [A] time = 0.24, size = 128, normalized size = 1.14

$$-\frac{Aa^5}{2x^2}+\frac{Bb^5x^{16}}{16}+x^{13}\left(\frac{Ab^5}{13}+\frac{5Bab^4}{13}\right)+x^{10}\left(\frac{Aab^4}{2}+Ba^2b^3\right)+x^7\left(\frac{10Aa^2b^3}{7}+\frac{10Ba^3b^2}{7}\right)+x^4\left(\frac{5Aa^3b^2}{2}+\frac{5Ba^4b}{4}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**3,x)
```

```
[Out] -A*a**5/(2*x**2) + B*b**5*x**16/16 + x**13*(A*b**5/13 + 5*B*a*b**4/13) + x*
*10*(A*a*b**4/2 + B*a**2*b**3) + x**7*(10*A*a**2*b**3/7 + 10*B*a**3*b**2/7)
 + x**4*(5*A*a**3*b**2/2 + 5*B*a**4*b/4) + x*(5*A*a**4*b + B*a**5)
```

3.36 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{3x^3}+a^4\log(x)(aB+5Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5B$$

```
[Out] -1/3*a^5*A/x^3+5/3*a^3*b*(2*A*b+B*a)*x^3+5/3*a^2*b^2*(A*b+B*a)*x^6+5/9*a*b^
3*(A*b+2*B*a)*x^9+1/12*b^4*(A*b+5*B*a)*x^12+1/15*b^5*B*x^15+a^4*(5*A*b+B*a)
*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{5}{3}a^2b^2x^6(aB+Ab)+\frac{5}{3}a^3bx^3(aB+2Ab)+a^4\log(x)(aB+5Ab)-\frac{a^5A}{3x^3}+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -(a^5*A)/(3*x^3) + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + a^4*(5*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^4} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^2} dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5a^3b(2Ab+aB)+\frac{a^5A}{x^2}+\frac{a^4(5Ab+aB)}{x}+10a^2b^2(Ab+aB)x+5ab^3\right.\right.\\ &= -\frac{a^5A}{3x^3}+\frac{5}{3}a^3b(2Ab+aB)x^3+\frac{5}{3}a^2b^2(Ab+aB)x^6+\frac{5}{9}ab^3(Ab+2aB)x^9+\frac{1}{12}b^4(Ab\end{aligned}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 1.02

$$-\frac{a^5A}{3x^3}+\frac{5}{3}a^3bx^3(aB+2Ab)+\frac{5}{3}a^2b^2x^6(aB+Ab)+\log(x)\left(a^5B+5a^4Ab\right)+\frac{1}{12}b^4x^{12}(5aB+Ab)+\frac{5}{9}ab^3x^9(2aB+Ab)+\frac{1}{15}b$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^4,x]
```

```
[Out] -1/3*(a^5*A)/x^3 + (5*a^3*b*(2*A*b + a*B)*x^3)/3 + (5*a^2*b^2*(A*b + a*B)*x
^6)/3 + (5*a*b^3*(A*b + 2*a*B)*x^9)/9 + (b^4*(A*b + 5*a*B)*x^12)/12 + (b^5*
B*x^15)/15 + (5*a^4*A*b + a^5*B)*Log[x]
```

fricas [A] time = 1.04, size = 123, normalized size = 1.09

$$\frac{12\,Bb^5x^{18}+15\left(5\,Bab^4+Ab^5\right)x^{15}+100\left(2\,Ba^2b^3+Aab^4\right)x^{12}+300\left(Ba^3b^2+Aa^2b^3\right)x^9+300\left(Ba^4b+2\,Aa^3b\right.}{180\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/180*(12*B*b^5*x^18 + 15*(5*B*a*b^4 + A*b^5)*x^15 + 100*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 300*(B*a^4*b + 2*A*a^3*b^2)*
x^6 - 60*A*a^5 + 180*(B*a^5 + 5*A*a^4*b)*x^3*log(x))/x^3
```

giac [A] time = 0.16, size = 143, normalized size = 1.27

$$\frac{1}{15}\,Bb^5x^{15}+\frac{5}{12}\,Bab^4x^{12}+\frac{1}{12}\,Ab^5x^{12}+\frac{10}{9}\,Ba^2b^3x^9+\frac{5}{9}\,Aab^4x^9+\frac{5}{3}\,Ba^3b^2x^6+\frac{5}{3}\,Aa^2b^3x^6+\frac{5}{3}\,Ba^4bx^3+\frac{10}{3}\,Aa^3b^2x^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/15*B*b^5*x^15 + 5/12*B*a*b^4*x^12 + 1/12*A*b^5*x^12 + 10/9*B*a^2*b^3*x^9
+ 5/9*A*a*b^4*x^9 + 5/3*B*a^3*b^2*x^6 + 5/3*A*a^2*b^3*x^6 + 5/3*B*a^4*b*x^3
 + 10/3*A*a^3*b^2*x^3 + (B*a^5 + 5*A*a^4*b)*log(abs(x)) - 1/3*(B*a^5*x^3 +
5*A*a^4*b*x^3 + A*a^5)/x^3
```

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^{15}}{15}+\frac{A\,b^5x^{12}}{12}+\frac{5Ba\,b^4x^{12}}{12}+\frac{5Aa\,b^4x^9}{9}+\frac{10B\,a^2b^3x^9}{9}+\frac{5A\,a^2b^3x^6}{3}+\frac{5B\,a^3b^2x^6}{3}+\frac{10A\,a^3b^2x^3}{3}+\frac{5B\,a^4b\,x^3}{3}+5A\,a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^4,x)
```

```
[Out] 1/15*b^5*B*x^15+1/12*A*x^12*b^5+5/12*B*x^12*a*b^4+5/9*A*x^9*a*b^4+10/9*B*x^
9*a^2*b^3+5/3*A*x^6*a^2*b^3+5/3*B*x^6*a^3*b^2+10/3*A*x^3*a^3*b^2+5/3*B*x^3*
a^4*b-1/3*a^5*A/x^3+5*A*ln(x)*a^4*b+B*ln(x)*a^5
```

maxima [A] time = 0.55, size = 120, normalized size = 1.06

$$\frac{1}{15}\,Bb^5x^{15}+\frac{1}{12}\left(5\,Bab^4+Ab^5\right)x^{12}+\frac{5}{9}\left(2\,Ba^2b^3+Aab^4\right)x^9+\frac{5}{3}\left(Ba^3b^2+Aa^2b^3\right)x^6+\frac{5}{3}\left(Ba^4b+2\,Aa^3b^2\right)x^3-\frac{Aa^5}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/15*B*b^5*x^15 + 1/12*(5*B*a*b^4 + A*b^5)*x^12 + 5/9*(2*B*a^2*b^3 + A*a*b^
4)*x^9 + 5/3*(B*a^3*b^2 + A*a^2*b^3)*x^6 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*x^3
- 1/3*A*a^5/x^3 + 1/3*(B*a^5 + 5*A*a^4*b)*log(x^3)
```

mupad [B] time = 2.35, size = 105, normalized size = 0.93

$$x^{12}\left(\frac{A\,b^5}{12}+\frac{5\,B\,a\,b^4}{12}\right)+\ln(x)\,\left(B\,a^5+5\,A\,b\,a^4\right)-\frac{A\,a^5}{3\,x^3}+\frac{B\,b^5\,x^{15}}{15}+\frac{5\,a^2\,b^2\,x^6\,\left(A\,b+B\,a\right)}{3}+\frac{5\,a^3\,b\,x^3\,\left(2\,A\,b+B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^4,x)
```

```
[Out] x^12*((A*b^5)/12 + (5*B*a*b^4)/12) + log(x)*(B*a^5 + 5*A*a^4*b) - (A*a^5)/(
3*x^3) + (B*b^5*x^15)/15 + (5*a^2*b^2*x^6*(A*b + B*a))/3 + (5*a^3*b*x^3*(2*
A*b + B*a))/3 + (5*a*b^3*x^9*(A*b + 2*B*a))/9
```

sympy [A] time = 0.37, size = 133, normalized size = 1.18

$$-\frac{Aa^5}{3x^3}+\frac{Bb^5x^{15}}{15}+a^4\left(5Ab+Ba\right)\log\left(x\right)+x^{12}\left(\frac{Ab^5}{12}+\frac{5Bab^4}{12}\right)+x^9\left(\frac{5Aab^4}{9}+\frac{10Ba^2b^3}{9}\right)+x^6\left(\frac{5Aa^2b^3}{3}+\frac{5Ba^3b^2}{3}\right)+x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**4,x)
```

```
[Out] -A*a**5/(3*x**3) + B*b**5*x**15/15 + a**4*(5*A*b + B*a)*log(x) + x**12*(A*b
**5/12 + 5*B*a*b**4/12) + x**9*(5*A*a*b**4/9 + 10*B*a**2*b**3/9) + x**6*(5*
A*a**2*b**3/3 + 5*B*a**3*b**2/3) + x**3*(10*A*a**3*b**2/3 + 5*B*a**4*b/3)
```

3.37 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} \, dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{4x^4}-\frac{a^4(aB+5Ab)}{x}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

```
[Out] -1/4*a^5*A/x^4-a^4*(5*A*b+B*a)/x+5/2*a^3*b*(2*A*b+B*a)*x^2+2*a^2*b^2*(A*b+B
*a)*x^5+5/8*a*b^3*(A*b+2*B*a)*x^8+1/11*b^4*(A*b+5*B*a)*x^11+1/14*b^5*B*x^14
```

Rubi [A] time = 0.07, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2a^2b^2x^5(aB+Ab)+\frac{5}{2}a^3bx^2(aB+2Ab)-\frac{a^4(aB+5Ab)}{x}-\frac{a^5A}{4x^4}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5Bx^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -(a^5*A)/(4*x^4) - (a^4*(5*A*b + a*B))/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*a
*B)*x^11)/11 + (b^5*B*x^14)/14
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^5} \, dx &= \int \left(\frac{a^5A}{x^5}+\frac{a^4(5Ab+aB)}{x^2}+5a^3b(2Ab+aB)x+10a^2b^2(Ab+aB)x^4+5ab^3(Ab\right.\\ &= -\frac{a^5A}{4x^4}-\frac{a^4(5Ab+aB)}{x}+\frac{5}{2}a^3b(2Ab+aB)x^2+2a^2b^2(Ab+aB)x^5+\frac{5}{8}ab^3(Ab+\end{aligned}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 1.02

$$-\frac{a^5A}{4x^4}+\frac{5}{2}a^3bx^2(aB+2Ab)+2a^2b^2x^5(aB+Ab)+\frac{a^5(-B)-5a^4Ab}{x}+\frac{1}{11}b^4x^{11}(5aB+Ab)+\frac{5}{8}ab^3x^8(2aB+Ab)+\frac{1}{14}b^5B$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(a^5*A)/x^4 + (-5*a^4*A*b - a^5*B)/x + (5*a^3*b*(2*A*b + a*B)*x^2)/2 +
 2*a^2*b^2*(A*b + a*B)*x^5 + (5*a*b^3*(A*b + 2*a*B)*x^8)/8 + (b^4*(A*b + 5*
a*B)*x^11)/11 + (b^5*B*x^14)/14
```

fricas [A] time = 0.83, size = 121, normalized size = 1.07

$$\frac{44\,Bb^5x^{18}+56\left(5\,Bab^4+Ab^5\right)x^{15}+385\left(2\,Ba^2b^3+Aab^4\right)x^{12}+1232\left(Ba^3b^2+Aa^2b^3\right)x^9+1540\left(Ba^4b+2\,A\right.}{616\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] 1/616*(44*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 385*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 1540*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 154*A*a^5 - 616*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

giac [A] time = 0.16, size = 127, normalized size = 1.12

$$\frac{1}{14}Bb^5x^{14}+\frac{5}{11}Bab^4x^{11}+\frac{1}{11}Ab^5x^{11}+\frac{5}{4}Ba^2b^3x^8+\frac{5}{8}Aab^4x^8+2Ba^3b^2x^5+2Aa^2b^3x^5+\frac{5}{2}Ba^4bx^2+5Aa^3b^2x^2-\frac{4Ba^5x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] 1/14*B*b^5*x^14 + 5/11*B*a*b^4*x^11 + 1/11*A*b^5*x^11 + 5/4*B*a^2*b^3*x^8 +
 5/8*A*a*b^4*x^8 + 2*B*a^3*b^2*x^5 + 2*A*a^2*b^3*x^5 + 5/2*B*a^4*b*x^2 + 5*
A*a^3*b^2*x^2 - 1/4*(4*B*a^5*x^3 + 20*A*a^4*b*x^3 + A*a^5)/x^4
```

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{Bb^5x^{14}}{14}+\frac{Ab^5x^{11}}{11}+\frac{5Bab^4x^{11}}{11}+\frac{5Aab^4x^8}{8}+\frac{5Ba^2b^3x^8}{4}+2Aa^2b^3x^5+2Ba^3b^2x^5+5Aa^3b^2x^2+\frac{5Ba^4bx^2}{2}-\frac{(5Ab+Ba)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^5,x)
```

```
[Out] 1/14*b^5*B*x^14+1/11*A*x^11*b^5+5/11*B*x^11*a*b^4+5/8*A*x^8*a*b^4+5/4*B*x^8
*a^2*b^3+2*A*x^5*a^2*b^3+2*B*x^5*a^3*b^2+5*A*x^2*a^3*b^2+5/2*B*x^2*a^4*b-1/
4*a^5*A/x^4-a^4*(5*A*b+B*a)/x
```

maxima [A] time = 0.49, size = 121, normalized size = 1.07

$$\frac{1}{14}Bb^5x^{14}+\frac{1}{11}\left(5Bab^4+Ab^5\right)x^{11}+\frac{5}{8}\left(2Ba^2b^3+Aab^4\right)x^8+2\left(Ba^3b^2+Aa^2b^3\right)x^5+\frac{5}{2}\left(Ba^4b+2Aa^3b^2\right)x^2-\frac{Aa^5+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] 1/14*B*b^5*x^14 + 1/11*(5*B*a*b^4 + A*b^5)*x^11 + 5/8*(2*B*a^2*b^3 + A*a*b^
4)*x^8 + 2*(B*a^3*b^2 + A*a^2*b^3)*x^5 + 5/2*(B*a^4*b + 2*A*a^3*b^2)*x^2 -
1/4*(A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^4
```

mupad [B] time = 0.04, size = 109, normalized size = 0.96

$$x^{11}\left(\frac{A\,b^5}{11}+\frac{5\,B\,a\,b^4}{11}\right)-\frac{\frac{A\,a^5}{4}+x^3\,\left(B\,a^5+5\,A\,b\,a^4\right)}{x^4}+\frac{B\,b^5\,x^{14}}{14}+2\,a^2\,b^2\,x^5\,\left(A\,b+B\,a\right)+\frac{5\,a^3\,b\,x^2\,\left(2\,A\,b+B\,a\right)}{2}+\frac{5\,a\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^5,x)
```

```
[Out] x^11*((A*b^5)/11 + (5*B*a*b^4)/11) - ((A*a^5)/4 + x^3*(B*a^5 + 5*A*a^4*b))/
x^4 + (B*b^5*x^14)/14 + 2*a^2*b^2*x^5*(A*b + B*a) + (5*a^3*b*x^2*(2*A*b + B
*a))/2 + (5*a*b^3*x^8*(A*b + 2*B*a))/8
```

sympy [A] time = 0.42, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{14}}{14}+x^{11}\left(\frac{Ab^5}{11}+\frac{5Bab^4}{11}\right)+x^8\left(\frac{5Aab^4}{8}+\frac{5Ba^2b^3}{4}\right)+x^5\left(2Aa^2b^3+2Ba^3b^2\right)+x^2\left(5Aa^3b^2+\frac{5Ba^4b}{2}\right)+\frac{-Aa^5+x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**5,x)
```

```
[Out] B*b**5*x**14/14 + x**11*(A*b**5/11 + 5*B*a*b**4/11) + x**8*(5*A*a*b**4/8 +
5*B*a**2*b**3/4) + x**5*(2*A*a**2*b**3 + 2*B*a**3*b**2) + x**2*(5*A*a**3*b*
*2 + 5*B*a**4*b/2) + (-A*a**5 + x**3*(-20*A*a**4*b - 4*B*a**5))/(4*x**4)
```

3.38 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx$

Optimal. Leaf size=113

$$-\frac{a^5 A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

```
[Out] -1/5*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2+5*a^3*b*(2*A*b+B*a)*x+5/2*a^2*b^2*(A
*b+B*a)*x^4+5/7*a*b^3*(A*b+2*B*a)*x^7+1/10*b^4*(A*b+5*B*a)*x^10+1/13*b^5*B*
x^13
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{5}{2}a^2b^2x^4(aB+Ab)-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)-\frac{a^5 A}{5x^5}+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -(a^5*A)/(5*x^5) - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^6} dx = \int \left(5a^3b(2Ab+aB)+\frac{a^5 A}{x^6}+\frac{a^4(5Ab+aB)}{x^3}+10a^2b^2(Ab+aB)x^3+5ab^3(Ab+2a\right.$$

$$= -\frac{a^5 A}{5x^5}-\frac{a^4(5Ab+aB)}{2x^2}+5a^3b(2Ab+aB)x+\frac{5}{2}a^2b^2(Ab+aB)x^4+\frac{5}{7}ab^3(Ab+2aB$$

Mathematica [A] time = 0.06, size = 113, normalized size = 1.00

$$-\frac{a^5 A}{5x^5}-\frac{a^4(aB+5Ab)}{2x^2}+5a^3bx(aB+2Ab)+\frac{5}{2}a^2b^2x^4(aB+Ab)+\frac{1}{10}b^4x^{10}(5aB+Ab)+\frac{5}{7}ab^3x^7(2aB+Ab)+\frac{1}{13}b^5Bx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^6,x]
```

```
[Out] -1/5*(a^5*A)/x^5 - (a^4*(5*A*b + a*B))/(2*x^2) + 5*a^3*b*(2*A*b + a*B)*x +
(5*a^2*b^2*(A*b + a*B)*x^4)/2 + (5*a*b^3*(A*b + 2*a*B)*x^7)/7 + (b^4*(A*b +
 5*a*B)*x^10)/10 + (b^5*B*x^13)/13
```

fricas [A] time = 1.17, size = 121, normalized size = 1.07

$$\frac{70\,Bb^5x^{18}+91\left(5\,Bab^4+Ab^5\right)x^{15}+650\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2275\left(Ba^3b^2+Aa^2b^3\right)x^9+4550\left(Ba^4b+2\,Aa^3b\right.}{910\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] 1/910*(70*B*b^5*x^18 + 91*(5*B*a*b^4 + A*b^5)*x^15 + 650*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 4550*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 182*A*a^5 - 455*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

giac [A] time = 0.18, size = 124, normalized size = 1.10

$$\frac{1}{13}Bb^5x^{13}+\frac{1}{2}Bab^4x^{10}+\frac{1}{10}Ab^5x^{10}+\frac{10}{7}Ba^2b^3x^7+\frac{5}{7}Aab^4x^7+\frac{5}{2}Ba^3b^2x^4+\frac{5}{2}Aa^2b^3x^4+5Ba^4bx+10Aa^3b^2x-\frac{5Ba}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] 1/13*B*b^5*x^13 + 1/2*B*a*b^4*x^10 + 1/10*A*b^5*x^10 + 10/7*B*a^2*b^3*x^7 +
 5/7*A*a*b^4*x^7 + 5/2*B*a^3*b^2*x^4 + 5/2*A*a^2*b^3*x^4 + 5*B*a^4*b*x + 10
*A*a^3*b^2*x - 1/10*(5*B*a^5*x^3 + 25*A*a^4*b*x^3 + 2*A*a^5)/x^5
```

maple [A] time = 0.04, size = 119, normalized size = 1.05

$$\frac{B\,b^5x^{13}}{13}+\frac{A\,b^5x^{10}}{10}+\frac{Ba\,b^4x^{10}}{2}+\frac{5Aa\,b^4x^7}{7}+\frac{10B\,a^2b^3x^7}{7}+\frac{5A\,a^2b^3x^4}{2}+\frac{5B\,a^3b^2x^4}{2}+10A\,a^3b^2x+5B\,a^4bx-\frac{(5Ab+Ba}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^6,x)
```

```
[Out] 1/13*b^5*B*x^13+1/10*A*x^10*b^5+1/2*B*x^10*a*b^4+5/7*A*x^7*a*b^4+10/7*B*x^7
*a^2*b^3+5/2*A*x^4*a^2*b^3+5/2*B*x^4*a^3*b^2+10*A*a^3*b^2*x+5*B*a^4*b*x-1/5
*a^5*A/x^5-1/2*a^4*(5*A*b+B*a)/x^2
```

maxima [A] time = 0.73, size = 120, normalized size = 1.06

$$\frac{1}{13}Bb^5x^{13}+\frac{1}{10}\left(5\,Bab^4+Ab^5\right)x^{10}+\frac{5}{7}\left(2\,Ba^2b^3+Aab^4\right)x^7+\frac{5}{2}\left(Ba^3b^2+Aa^2b^3\right)x^4+5\left(Ba^4b+2\,Aa^3b^2\right)x-\frac{2\,Aa^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] 1/13*B*b^5*x^13 + 1/10*(5*B*a*b^4 + A*b^5)*x^10 + 5/7*(2*B*a^2*b^3 + A*a*b^
4)*x^7 + 5/2*(B*a^3*b^2 + A*a^2*b^3)*x^4 + 5*(B*a^4*b + 2*A*a^3*b^2)*x - 1/
10*(2*A*a^5 + 5*(B*a^5 + 5*A*a^4*b)*x^3)/x^5
```

mupad [B] time = 0.04, size = 108, normalized size = 0.96

$$x^{10}\left(\frac{A\,b^5}{10}+\frac{B\,a\,b^4}{2}\right)-\frac{\frac{A\,a^5}{5}+x^3\,\left(\frac{B\,a^5}{2}+\frac{5\,A\,b\,a^4}{2}\right)}{x^5}+\frac{B\,b^5\,x^{13}}{13}+\frac{5\,a^2\,b^2\,x^4\,\left(A\,b+B\,a\right)}{2}+5\,a^3\,b\,x\,\left(2\,A\,b+B\,a\right)+\frac{5\,a\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^6,x)
```

```
[Out] x^10*((A*b^5)/10 + (B*a*b^4)/2) - ((A*a^5)/5 + x^3*((B*a^5)/2 + (5*A*a^4*b)
/2))/x^5 + (B*b^5*x^13)/13 + (5*a^2*b^2*x^4*(A*b + B*a))/2 + 5*a^3*b*x*(2*A
*b + B*a) + (5*a*b^3*x^7*(A*b + 2*B*a))/7
```

sympy [A] time = 0.48, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{13}}{13}+x^{10}\left(\frac{Ab^5}{10}+\frac{Bab^4}{2}\right)+x^7\left(\frac{5Aab^4}{7}+\frac{10Ba^2b^3}{7}\right)+x^4\left(\frac{5Aa^2b^3}{2}+\frac{5Ba^3b^2}{2}\right)+x\left(10Aa^3b^2+5Ba^4b\right)+\frac{-2Aa^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**6,x)
```

```
[Out] B*b**5*x**13/13 + x**10*(A*b**5/10 + B*a*b**4/2) + x**7*(5*A*a*b**4/7 + 10*
B*a**2*b**3/7) + x**4*(5*A*a**2*b**3/2 + 5*B*a**3*b**2/2) + x*(10*A*a**3*b*
*2 + 5*B*a**4*b) + (-2*A*a**5 + x**3*(-25*A*a**4*b - 5*B*a**5))/(10*x**5)
```

3.39 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=114

$$-\frac{a^5A}{6x^6}-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)+\frac{10}{3}a^2b^2x^3(aB+Ab)+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5$$

```
[Out] -1/6*a^5*A/x^6-1/3*a^4*(5*A*b+B*a)/x^3+10/3*a^2*b^2*(A*b+B*a)*x^3+5/6*a*b^3
*(A*b+2*B*a)*x^6+1/9*b^4*(A*b+5*B*a)*x^9+1/12*b^5*B*x^12+5*a^3*b*(2*A*b+B*a
)*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$\frac{10}{3}a^2b^2x^3(aB+Ab)-\frac{a^4(aB+5Ab)}{3x^3}+5a^3b\log(x)(aB+2Ab)-\frac{a^5A}{6x^6}+\frac{1}{9}b^4x^9(5aB+Ab)+\frac{5}{6}ab^3x^6(2aB+Ab)+\frac{1}{12}b^5B$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] -(a^5*A)/(6*x^6) - (a^4*(5*A*b + a*B))/(3*x^3) + (10*a^2*b^2*(A*b + a*B)*x^
3)/3 + (5*a*b^3*(A*b + 2*a*B)*x^6)/6 + (b^4*(A*b + 5*a*B)*x^9)/9 + (b^5*B*x
^12)/12 + 5*a^3*b*(2*A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^7}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(10a^2b^2(Ab+aB)+\frac{a^5A}{x^3}+\frac{a^4(5Ab+aB)}{x^2}+\frac{5a^3b(2Ab+aB)}{x}+5a\right.\right.\\ &= -\frac{a^5A}{6x^6}-\frac{a^4(5Ab+aB)}{3x^3}+\frac{10}{3}a^2b^2(Ab+aB)x^3+\frac{5}{6}ab^3(Ab+2aB)x^6+\frac{1}{9}b^4(Ab+\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{36}\left(-\frac{6a^5A}{x^6}-\frac{12a^4(aB+5Ab)}{x^3}+180a^3b\log(x)(aB+2Ab)+120a^2b^2x^3(aB+Ab)+4b^4x^9(5aB+Ab)+30ab\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^7,x]
```

```
[Out] ((-6*a^5*A)/x^6 - (12*a^4*(5*A*b + a*B))/x^3 + 120*a^2*b^2*(A*b + a*B)*x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^6 + 4*b^4*(A*b + 5*a*B)*x^9 + 3*b^5*B*x^12 + 180
*a^3*b*(2*A*b + a*B)*Log[x])/36
```

fricas [A] time = 1.13, size = 123, normalized size = 1.08

$$\frac{3\,Bb^5x^{18} + 4\left(5\,Bab^4 + Ab^5\right)x^{15} + 30\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 120\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 180\left(Ba^4b + 2\,Aa^3b^2\right)x^6 \log}{36\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] 1/36*(3*B*b^5*x^18 + 4*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 180*(B*a^4*b + 2*A*a^3*b^2)*x^6*
log(x) - 6*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

giac [A] time = 0.16, size = 148, normalized size = 1.30

$$\frac{1}{12}\,Bb^5x^{12}+\frac{5}{9}\,Bab^4x^9+\frac{1}{9}\,Ab^5x^9+\frac{5}{3}\,Ba^2b^3x^6+\frac{5}{6}\,Aab^4x^6+\frac{10}{3}\,Ba^3b^2x^3+\frac{10}{3}\,Aa^2b^3x^3+5\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(|x|\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*B*b^5*x^12 + 5/9*B*a*b^4*x^9 + 1/9*A*b^5*x^9 + 5/3*B*a^2*b^3*x^6 + 5/6
*A*a*b^4*x^6 + 10/3*B*a^3*b^2*x^3 + 10/3*A*a^2*b^3*x^3 + 5*(B*a^4*b + 2*A*a
^3*b^2)*log(abs(x)) - 1/6*(15*B*a^4*b*x^6 + 30*A*a^3*b^2*x^6 + 2*B*a^5*x^3
+ 10*A*a^4*b*x^3 + A*a^5)/x^6
```

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^{12}}{12}+\frac{A\,b^5x^9}{9}+\frac{5Ba\,b^4x^9}{9}+\frac{5Aa\,b^4x^6}{6}+\frac{5B\,a^2b^3x^6}{3}+\frac{10A\,a^2b^3x^3}{3}+\frac{10B\,a^3b^2x^3}{3}+10A\,a^3b^2\ln(x)+5B\,a^4b\ln(x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^7,x)
```

```
[Out] 1/12*b^5*B*x^12+1/9*A*x^9*b^5+5/9*B*x^9*a*b^4+5/6*A*x^6*a*b^4+5/3*B*x^6*a^2
*b^3+10/3*A*a^2*b^3*x^3+10/3*B*a^3*b^2*x^3-5/3*a^4/x^3*A*b-1/3*a^5/x^3*B-1/
6*a^5*A/x^6+10*A*ln(x)*a^3*b^2+5*B*ln(x)*a^4*b
```

maxima [A] time = 0.65, size = 122, normalized size = 1.07

$$\frac{1}{12}\,Bb^5x^{12}+\frac{1}{9}\left(5\,Bab^4 + Ab^5\right)x^9+\frac{5}{6}\left(2\,Ba^2b^3 + Aab^4\right)x^6+\frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)x^3+\frac{5}{3}\left(Ba^4b + 2\,Aa^3b^2\right)\log\left(x^3\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*B*b^5*x^12 + 1/9*(5*B*a*b^4 + A*b^5)*x^9 + 5/6*(2*B*a^2*b^3 + A*a*b^4)
*x^6 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*x^3 + 5/3*(B*a^4*b + 2*A*a^3*b^2)*log(x
^3) - 1/6*(A*a^5 + 2*(B*a^5 + 5*A*a^4*b)*x^3)/x^6
```

mupad [B] time = 0.05, size = 113, normalized size = 0.99

$$\ln(x)\,\left(5\,B\,a^4\,b+10\,A\,a^3\,b^2\right)-\frac{\frac{A\,a^5}{6}+x^3\,\left(\frac{B\,a^5}{3}+\frac{5\,A\,b\,a^4}{3}\right)}{x^6}+x^9\,\left(\frac{A\,b^5}{9}+\frac{5\,B\,a\,b^4}{9}\right)+\frac{B\,b^5\,x^{12}}{12}+\frac{10\,a^2\,b^2\,x^3\,\left(A\,b+B\,a\right)}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^7,x)
```

```
[Out] log(x)*(10*A*a^3*b^2 + 5*B*a^4*b) - ((A*a^5)/6 + x^3*((B*a^5)/3 + (5*A*a^4*
b)/3))/x^6 + x^9*((A*b^5)/9 + (5*B*a*b^4)/9) + (B*b^5*x^12)/12 + (10*a^2*b^
2*x^3*(A*b + B*a))/3 + (5*a*b^3*x^6*(A*b + 2*B*a))/6
```

sympy [A] time = 0.94, size = 131, normalized size = 1.15

$$\frac{Bb^5x^{12}}{12}+5a^3b\left(2Ab+Ba\right)\log\left(x\right)+x^9\left(\frac{Ab^5}{9}+\frac{5Bab^4}{9}\right)+x^6\left(\frac{5Aab^4}{6}+\frac{5Ba^2b^3}{3}\right)+x^3\left(\frac{10Aa^2b^3}{3}+\frac{10Ba^3b^2}{3}\right)+\frac{-A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**7,x)
```

```
[Out] B*b**5*x**12/12 + 5*a**3*b*(2*A*b + B*a)*log(x) + x**9*(A*b**5/9 + 5*B*a*b*
*4/9) + x**6*(5*A*a*b**4/6 + 5*B*a**2*b**3/3) + x**3*(10*A*a**2*b**3/3 + 10
*B*a**3*b**2/3) + (-A*a**5 + x**3*(-10*A*a**4*b - 2*B*a**5))/(6*x**6)
```

3.40 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx$

Optimal. Leaf size=110

$$-\frac{a^5 A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

```
[Out] -1/7*a^5*A/x^7-1/4*a^4*(5*A*b+B*a)/x^4-5*a^3*b*(2*A*b+B*a)/x+5*a^2*b^2*(A*b
+B*a)*x^2+a*b^3*(A*b+2*B*a)*x^5+1/8*b^4*(A*b+5*B*a)*x^8+1/11*b^5*B*x^11
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$5a^2b^2x^2(aB+Ab)-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}-\frac{a^5 A}{7x^7}+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -(a^5*A)/(7*x^7) - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^8} \, dx &= \int \left(\frac{a^5 A}{x^8}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^2}+10a^2b^2(Ab+aB)x+5ab^3(Ab+2a\right.\\ &= -\frac{a^5 A}{7x^7}-\frac{a^4(5Ab+aB)}{4x^4}-\frac{5a^3b(2Ab+aB)}{x}+5a^2b^2(Ab+aB)x^2+ab^3(Ab+2aB)x^5\end{aligned}$$

Mathematica [A] time = 0.04, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{7x^7}-\frac{a^4(aB+5Ab)}{4x^4}-\frac{5a^3b(aB+2Ab)}{x}+5a^2b^2x^2(aB+Ab)+\frac{1}{8}b^4x^8(5aB+Ab)+ab^3x^5(2aB+Ab)+\frac{1}{11}b^5Bx^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^8,x]
```

```
[Out] -1/7*(a^5*A)/x^7 - (a^4*(5*A*b + a*B))/(4*x^4) - (5*a^3*b*(2*A*b + a*B))/x
+ 5*a^2*b^2*(A*b + a*B)*x^2 + a*b^3*(A*b + 2*a*B)*x^5 + (b^4*(A*b + 5*a*B)*
x^8)/8 + (b^5*B*x^11)/11
```

fricas [A] time = 0.81, size = 121, normalized size = 1.10

$$\frac{56\,Bb^5x^{18}+77\left(5\,Bab^4+Ab^5\right)x^{15}+616\left(2\,Ba^2b^3+Aab^4\right)x^{12}+3080\left(Ba^3b^2+Aa^2b^3\right)x^9-3080\left(Ba^4b+2\,Aa^3b\right.}{616\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] 1/616*(56*B*b^5*x^18 + 77*(5*B*a*b^4 + A*b^5)*x^15 + 616*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 3080*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 88*A*a^5 - 154*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

giac [A] time = 0.17, size = 127, normalized size = 1.15

$$\frac{1}{11}Bb^5x^{11}+\frac{5}{8}Bab^4x^8+\frac{1}{8}Ab^5x^8+2Ba^2b^3x^5+Aab^4x^5+5Ba^3b^2x^2+5Aa^2b^3x^2-\frac{140Ba^4bx^6+280Aa^3b^2x^6+7B}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] 1/11*B*b^5*x^11 + 5/8*B*a*b^4*x^8 + 1/8*A*b^5*x^8 + 2*B*a^2*b^3*x^5 + A*a*b
^4*x^5 + 5*B*a^3*b^2*x^2 + 5*A*a^2*b^3*x^2 - 1/28*(140*B*a^4*b*x^6 + 280*A*
a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 4*A*a^5)/x^7
```

maple [A] time = 0.05, size = 117, normalized size = 1.06

$$\frac{Bb^5x^{11}}{11}+\frac{Ab^5x^8}{8}+\frac{5Bab^4x^8}{8}+Aab^4x^5+2Ba^2b^3x^5+5Aa^2b^3x^2+5Ba^3b^2x^2-\frac{5(2Ab+Ba)a^3b}{x}-\frac{(5Ab+Ba)a^4}{4x^4}-\frac{A}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^8,x)
```

```
[Out] 1/11*b^5*B*x^11+1/8*A*x^8*b^5+5/8*B*x^8*a*b^4+A*a*b^4*x^5+2*B*a^2*b^3*x^5+5
*A*x^2*a^2*b^3+5*B*x^2*a^3*b^2-1/4*a^4*(5*A*b+B*a)/x^4-1/7*a^5*A/x^7-5*a^3*
b*(2*A*b+B*a)/x
```

maxima [A] time = 0.51, size = 121, normalized size = 1.10

$$\frac{1}{11}Bb^5x^{11}+\frac{1}{8}\left(5Bab^4+Ab^5\right)x^8+\left(2Ba^2b^3+Aab^4\right)x^5+5\left(Ba^3b^2+Aa^2b^3\right)x^2-\frac{140\left(Ba^4b+2Aa^3b^2\right)x^6+4Aa^5}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] 1/11*B*b^5*x^11 + 1/8*(5*B*a*b^4 + A*b^5)*x^8 + (2*B*a^2*b^3 + A*a*b^4)*x^5
 + 5*(B*a^3*b^2 + A*a^2*b^3)*x^2 - 1/28*(140*(B*a^4*b + 2*A*a^3*b^2)*x^6 +
4*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^7
```

mupad [B] time = 2.34, size = 113, normalized size = 1.03

$$x^8\left(\frac{Ab^5}{8}+\frac{5Bab^4}{8}\right)-\frac{\frac{Aa^5}{7}+x^6\left(5Ba^4b+10Aa^3b^2\right)+x^3\left(\frac{Ba^5}{4}+\frac{5Aba^4}{4}\right)}{x^7}+\frac{Bb^5x^{11}}{11}+5a^2b^2x^2(Ab+Ba)+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^8,x)
```

```
[Out] x^8*((A*b^5)/8 + (5*B*a*b^4)/8) - ((A*a^5)/7 + x^6*(10*A*a^3*b^2 + 5*B*a^4*
b) + x^3*((B*a^5)/4 + (5*A*a^4*b)/4))/x^7 + (B*b^5*x^11)/11 + 5*a^2*b^2*x^2
*(A*b + B*a) + a*b^3*x^5*(A*b + 2*B*a)
```

sympy [A] time = 1.12, size = 129, normalized size = 1.17

$$\frac{Bb^5x^{11}}{11}+x^8\left(\frac{Ab^5}{8}+\frac{5Bab^4}{8}\right)+x^5\left(Aab^4+2Ba^2b^3\right)+x^2\left(5Aa^2b^3+5Ba^3b^2\right)+\frac{-4Aa^5+x^6\left(-280Aa^3b^2-140Ba^4\right.}{28x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**8,x)
```

```
[Out] B*b**5*x**11/11 + x**8*(A*b**5/8 + 5*B*a*b**4/8) + x**5*(A*a*b**4 + 2*B*a**
2*b**3) + x**2*(5*A*a**2*b**3 + 5*B*a**3*b**2) + (-4*A*a**5 + x**6*(-280*A*
a**3*b**2 - 140*B*a**4*b) + x**3*(-35*A*a**4*b - 7*B*a**5))/(28*x**7)
```

3.41 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx$

Optimal. Leaf size=113

$$-\frac{a^5 A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

```
[Out] -1/8*a^5*A/x^8-1/5*a^4*(5*A*b+B*a)/x^5-5/2*a^3*b*(2*A*b+B*a)/x^2+10*a^2*b^2
*(A*b+B*a)*x+5/4*a*b^3*(A*b+2*B*a)*x^4+1/7*b^4*(A*b+5*B*a)*x^7+1/10*b^5*B*x
^10
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {448}

$$10a^2b^2x(aB+Ab)-\frac{5a^3b(aB+2Ab)}{2x^2}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{a^5 A}{8x^8}+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -(a^5*A)/(8*x^8) - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^9} dx = \int \left(10a^2b^2(Ab+aB)+\frac{a^5 A}{x^9}+\frac{a^4(5Ab+aB)}{x^6}+\frac{5a^3b(2Ab+aB)}{x^3}+5ab^3(Ab+2\right.$$
$$= -\frac{a^5 A}{8x^8}-\frac{a^4(5Ab+aB)}{5x^5}-\frac{5a^3b(2Ab+aB)}{2x^2}+10a^2b^2(Ab+aB)x+\frac{5}{4}ab^3(Ab+2a$$

Mathematica [A] time = 0.04, size = 113, normalized size = 1.00

$$-\frac{a^5 A}{8x^8}-\frac{a^4(aB+5Ab)}{5x^5}-\frac{5a^3b(aB+2Ab)}{2x^2}+10a^2b^2x(aB+Ab)+\frac{1}{7}b^4x^7(5aB+Ab)+\frac{5}{4}ab^3x^4(2aB+Ab)+\frac{1}{10}b^5Bx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^9,x]
```

```
[Out] -1/8*(a^5*A)/x^8 - (a^4*(5*A*b + a*B))/(5*x^5) - (5*a^3*b*(2*A*b + a*B))/(2
*x^2) + 10*a^2*b^2*(A*b + a*B)*x + (5*a*b^3*(A*b + 2*a*B)*x^4)/4 + (b^4*(A*
b + 5*a*B)*x^7)/7 + (b^5*B*x^10)/10
```

fricas [A] time = 1.53, size = 121, normalized size = 1.07

$$\frac{28\,Bb^5x^{18}+40\left(5\,Bab^4+Ab^5\right)x^{15}+350\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2800\left(Ba^3b^2+Aa^2b^3\right)x^9-700\left(Ba^4b+2\,Aa^3\right.}{280\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="fricas")
```

```
[Out] 1/280*(28*B*b^5*x^18 + 40*(5*B*a*b^4 + A*b^5)*x^15 + 350*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 700*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 35*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

giac [A] time = 0.16, size = 124, normalized size = 1.10

$$\frac{1}{10}Bb^5x^{10}+\frac{5}{7}Bab^4x^7+\frac{1}{7}Ab^5x^7+\frac{5}{2}Ba^2b^3x^4+\frac{5}{4}Aab^4x^4+10\,Ba^3b^2x+10\,Aa^2b^3x-\frac{100\,Ba^4bx^6+200\,Aa^3b^2x^6+8\,Ba}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] 1/10*B*b^5*x^10 + 5/7*B*a*b^4*x^7 + 1/7*A*b^5*x^7 + 5/2*B*a^2*b^3*x^4 + 5/4
*A*a*b^4*x^4 + 10*B*a^3*b^2*x + 10*A*a^2*b^3*x - 1/40*(100*B*a^4*b*x^6 + 20
0*A*a^3*b^2*x^6 + 8*B*a^5*x^3 + 40*A*a^4*b*x^3 + 5*A*a^5)/x^8
```

maple [A] time = 0.04, size = 114, normalized size = 1.01

$$\frac{B\,b^5x^{10}}{10}+\frac{A\,b^5x^7}{7}+\frac{5Ba\,b^4x^7}{7}+\frac{5Aa\,b^4x^4}{4}+\frac{5B\,a^2b^3x^4}{2}+10A\,a^2b^3x+10B\,a^3b^2x-\frac{5\,(2Ab+Ba)\,a^3b}{2x^2}-\frac{(5Ab+Ba)\,a^4}{5x^5}-\frac{A}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^9,x)
```

```
[Out] 1/10*b^5*B*x^10+1/7*A*b^5*x^7+5/7*B*a*b^4*x^7+5/4*A*x^4*a*b^4+5/2*B*x^4*a^2
*b^3+10*a^2*b^3*A*x+10*a^3*b^2*B*x-1/5*a^4*(5*A*b+B*a)/x^5-1/8*a^5*A/x^8-5/
2*a^3*b*(2*A*b+B*a)/x^2
```

maxima [A] time = 0.59, size = 120, normalized size = 1.06

$$\frac{1}{10}Bb^5x^{10}+\frac{1}{7}\left(5\,Bab^4+Ab^5\right)x^7+\frac{5}{4}\left(2\,Ba^2b^3+Aab^4\right)x^4+10\left(Ba^3b^2+Aa^2b^3\right)x-\frac{100\left(Ba^4b+2\,Aa^3b^2\right)x^6+5\,Aa^5}{40\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] 1/10*B*b^5*x^10 + 1/7*(5*B*a*b^4 + A*b^5)*x^7 + 5/4*(2*B*a^2*b^3 + A*a*b^4)
*x^4 + 10*(B*a^3*b^2 + A*a^2*b^3)*x - 1/40*(100*(B*a^4*b + 2*A*a^3*b^2)*x^6
 + 5*A*a^5 + 8*(B*a^5 + 5*A*a^4*b)*x^3)/x^8
```

mupad [B] time = 0.04, size = 111, normalized size = 0.98

$$x^7\left(\frac{A\,b^5}{7}+\frac{5\,B\,a\,b^4}{7}\right)-\frac{\frac{A\,a^5}{8}+x^6\left(\frac{5\,B\,a^4\,b}{2}+5\,A\,a^3\,b^2\right)+x^3\left(\frac{B\,a^5}{5}+A\,b\,a^4\right)}{x^8}+\frac{B\,b^5\,x^{10}}{10}+10\,a^2\,b^2\,x\,\left(A\,b+B\,a\right)+\frac{5\,a\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^9,x)
```

```
[Out] x^7*((A*b^5)/7 + (5*B*a*b^4)/7) - ((A*a^5)/8 + x^6*(5*A*a^3*b^2 + (5*B*a^4*
b)/2) + x^3*((B*a^5)/5 + A*a^4*b))/x^8 + (B*b^5*x^10)/10 + 10*a^2*b^2*x*(A*
b + B*a) + (5*a*b^3*x^4*(A*b + 2*B*a))/4
```

sympy [A] time = 1.21, size = 133, normalized size = 1.18

$$\frac{Bb^5x^{10}}{10}+x^7\left(\frac{Ab^5}{7}+\frac{5Bab^4}{7}\right)+x^4\left(\frac{5Aab^4}{4}+\frac{5Ba^2b^3}{2}\right)+x\left(10Aa^2b^3+10Ba^3b^2\right)+\frac{-5Aa^5+x^6\left(-200Aa^3b^2-100Ba^4\right.}{40x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**9,x)
```

```
[Out] B*b**5*x**10/10 + x**7*(A*b**5/7 + 5*B*a*b**4/7) + x**4*(5*A*a*b**4/4 + 5*B
*a**2*b**3/2) + x*(10*A*a**2*b**3 + 10*B*a**3*b**2) + (-5*A*a**5 + x**6*(-2
00*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-40*A*a**4*b - 8*B*a**5))/(40*x**8)
```

3.42 $\int \frac{\left(a+bx^3\right)^5 \left(A+Bx^3\right)}{x^{10}} \, dx$

Optimal. Leaf size=114

$$-\frac{a^5 A}{9x^9}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{5a^3b(aB+2Ab)}{3x^3}+10a^2b^2\log(x)(aB+Ab)+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

```
[Out] -1/9*a^5*A/x^9-1/6*a^4*(5*A*b+B*a)/x^6-5/3*a^3*b*(2*A*b+B*a)/x^3+5/3*a*b^3*
(A*b+2*B*a)*x^3+1/6*b^4*(A*b+5*B*a)*x^6+1/9*b^5*B*x^9+10*a^2*b^2*(A*b+B*a)*
ln(x)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$10a^2b^2\log(x)(aB+Ab)-\frac{5a^3b(aB+2Ab)}{3x^3}-\frac{a^4(aB+5Ab)}{6x^6}-\frac{a^5A}{9x^9}+\frac{1}{6}b^4x^6(5aB+Ab)+\frac{5}{3}ab^3x^3(2aB+Ab)+\frac{1}{9}b^5Bx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] -(a^5*A)/(9*x^9) - (a^4*(5*A*b + a*B))/(6*x^6) - (5*a^3*b*(2*A*b + a*B))/(3
*x^3) + (5*a*b^3*(A*b + 2*a*B)*x^3)/3 + (b^4*(A*b + 5*a*B)*x^6)/6 + (b^5*B*
x^9)/9 + 10*a^2*b^2*(A*b + a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{10}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^4}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^4}+\frac{a^4(5Ab+aB)}{x^3}+\frac{5a^3b(2Ab+aB)}{x^2}+\frac{10a^2b\right.\right.\\ &= -\frac{a^5A}{9x^9}-\frac{a^4(5Ab+aB)}{6x^6}-\frac{5a^3b(2Ab+aB)}{3x^3}+\frac{5}{3}ab^3(Ab+2aB)x^3+\frac{1}{6}b^4(Ab+5aB)\end{aligned}$$

Mathematica [A] time = 0.06, size = 106, normalized size = 0.93

$$\frac{1}{18}\left(-\frac{2a^5A}{x^9}-\frac{3a^4(aB+5Ab)}{x^6}-\frac{30a^3b(aB+2Ab)}{x^3}+180a^2b^2\log(x)(aB+Ab)+3b^4x^6(5aB+Ab)+30ab^3x^3(2aE\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^10,x]
```

```
[Out] ((-2*a^5*A)/x^9 - (3*a^4*(5*A*b + a*B))/x^6 - (30*a^3*b*(2*A*b + a*B))/x^3
+ 30*a*b^3*(A*b + 2*a*B)*x^3 + 3*b^4*(A*b + 5*a*B)*x^6 + 2*b^5*B*x^9 + 180*
a^2*b^2*(A*b + a*B)*Log[x])/18
```

fricas [A] time = 0.89, size = 123, normalized size = 1.08

$$\frac{2\,Bb^5x^{18} + 3\left(5\,Bab^4 + Ab^5\right)x^{15} + 30\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 180\left(Ba^3b^2 + Aa^2b^3\right)x^9\log(x) - 30\left(Ba^4b + 2\,Aa\right.}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(2*B*b^5*x^18 + 3*(5*B*a*b^4 + A*b^5)*x^15 + 30*(2*B*a^2*b^3 + A*a*b^4
)*x^12 + 180*(B*a^3*b^2 + A*a^2*b^3)*x^9*log(x) - 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 - 2*A*a^5 - 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

giac [A] time = 0.15, size = 150, normalized size = 1.32

$$\frac{1}{9}\,Bb^5x^9+\frac{5}{6}\,Bab^4x^6+\frac{1}{6}\,Ab^5x^6+\frac{10}{3}\,Ba^2b^3x^3+\frac{5}{3}\,Aab^4x^3+10\left(Ba^3b^2 + Aa^2b^3\right)\log\left(|x|\right)-\frac{110\,Ba^3b^2x^9 + 110\,Aa^2b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="giac")
```

```
[Out] 1/9*B*b^5*x^9 + 5/6*B*a*b^4*x^6 + 1/6*A*b^5*x^6 + 10/3*B*a^2*b^3*x^3 + 5/3*
A*a*b^4*x^3 + 10*(B*a^3*b^2 + A*a^2*b^3)*log(abs(x)) - 1/18*(110*B*a^3*b^2*
x^9 + 110*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 3*B*a^5*x^3 +
 15*A*a^4*b*x^3 + 2*A*a^5)/x^9
```

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^9}{9}+\frac{A\,b^5x^6}{6}+\frac{5Ba\,b^4x^6}{6}+\frac{5Aa\,b^4x^3}{3}+\frac{10B\,a^2b^3x^3}{3}+10A\,a^2b^3\ln(x)+10B\,a^3b^2\ln(x)-\frac{10A\,a^3b^2}{3x^3}-\frac{5B\,a^4b}{3x^3}-\frac{5A\,a^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^10,x)
```

```
[Out] 1/9*B*b^5*x^9+1/6*A*x^6*b^5+5/6*B*x^6*a*b^4+5/3*A*x^3*a*b^4+10/3*B*x^3*a^2*
b^3-10/3*a^3*b^2/x^3*A-5/3*a^4*b/x^3*B-1/9*a^5*A/x^9-5/6*a^4/x^6*A*b-1/6*a^
5/x^6*B+10*A*ln(x)*a^2*b^3+10*B*ln(x)*a^3*b^2
```

maxima [A] time = 0.47, size = 123, normalized size = 1.08

$$\frac{1}{9}\,Bb^5x^9+\frac{1}{6}\left(5\,Bab^4 + Ab^5\right)x^6+\frac{5}{3}\left(2\,Ba^2b^3 + Aab^4\right)x^3+\frac{10}{3}\left(Ba^3b^2 + Aa^2b^3\right)\log\left(x^3\right)-\frac{30\left(Ba^4b + 2\,Aa^3b^2\right)x^6 +}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*B*b^5*x^9 + 1/6*(5*B*a*b^4 + A*b^5)*x^6 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*x
^3 + 10/3*(B*a^3*b^2 + A*a^2*b^3)*log(x^3) - 1/18*(30*(B*a^4*b + 2*A*a^3*b^
2)*x^6 + 2*A*a^5 + 3*(B*a^5 + 5*A*a^4*b)*x^3)/x^9
```

mupad [B] time = 0.05, size = 118, normalized size = 1.04

$$x^6\left(\frac{A\,b^5}{6}+\frac{5\,B\,a\,b^4}{6}\right)-\frac{\frac{A\,a^5}{9}+x^6\left(\frac{5\,B\,a^4\,b}{3}+\frac{10\,A\,a^3\,b^2}{3}\right)+x^3\left(\frac{B\,a^5}{6}+\frac{5\,A\,b\,a^4}{6}\right)}{x^9}+\ln(x)\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)+\frac{B\,b^5}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^10,x)
```

```
[Out] x^6*((A*b^5)/6 + (5*B*a*b^4)/6) - ((A*a^5)/9 + x^6*((10*A*a^3*b^2)/3 + (5*B
*a^4*b)/3) + x^3*((B*a^5)/6 + (5*A*a^4*b)/6))/x^9 + log(x)*(10*A*a^2*b^3 +
10*B*a^3*b^2) + (B*b^5*x^9)/9 + (5*a*b^3*x^3*(A*b + 2*B*a))/3
```

sympy [A] time = 2.24, size = 129, normalized size = 1.13

$$\frac{Bb^5x^9}{9}+10a^2b^2\left(Ab+Ba\right)\log\left(x\right)+x^6\left(\frac{Ab^5}{6}+\frac{5Bab^4}{6}\right)+x^3\left(\frac{5Aab^4}{3}+\frac{10Ba^2b^3}{3}\right)+\frac{-2Aa^5+x^6\left(-60Aa^3b^2-30Ba^4b\right.}{18x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**10,x)
```

```
[Out] B*b**5*x**9/9 + 10*a**2*b**2*(A*b + B*a)*log(x) + x**6*(A*b**5/6 + 5*B*a*b*
*4/6) + x**3*(5*A*a*b**4/3 + 10*B*a**2*b**3/3) + (-2*A*a**5 + x**6*(-60*A*a
**3*b**2 - 30*B*a**4*b) + x**3*(-15*A*a**4*b - 3*B*a**5))/(18*x**9)
```

3.43 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=115

$$-\frac{a^5A}{10x^{10}}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{10a^2b^2(aB+Ab)}{x}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

```
[Out] -1/10*a^5*A/x^10-1/7*a^4*(5*A*b+B*a)/x^7-5/4*a^3*b*(2*A*b+B*a)/x^4-10*a^2*b
^2*(A*b+B*a)/x+5/2*a*b^3*(A*b+2*B*a)*x^2+1/5*b^4*(A*b+5*B*a)*x^5+1/8*b^5*B*
x^8
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{x}-\frac{5a^3b(aB+2Ab)}{4x^4}-\frac{a^4(aB+5Ab)}{7x^7}-\frac{a^5A}{10x^{10}}+\frac{1}{5}b^4x^5(5aB+Ab)+\frac{5}{2}ab^3x^2(2aB+Ab)+\frac{1}{8}b^5Bx^8$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] -(a^5*A)/(10*x^10) - (a^4*(5*A*b + a*B))/(7*x^7) - (5*a^3*b*(2*A*b + a*B))/
(4*x^4) - (10*a^2*b^2*(A*b + a*B))/x + (5*a*b^3*(A*b + 2*a*B)*x^2)/2 + (b^4
*(A*b + 5*a*B)*x^5)/5 + (b^5*B*x^8)/8
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{11}}\,dx &= \int \left(\frac{a^5A}{x^{11}}+\frac{a^4(5Ab+aB)}{x^8}+\frac{5a^3b(2Ab+aB)}{x^5}+\frac{10a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+\right.\\ &= -\frac{a^5A}{10x^{10}}-\frac{a^4(5Ab+aB)}{7x^7}-\frac{5a^3b(2Ab+aB)}{4x^4}-\frac{10a^2b^2(Ab+aB)}{x}+\frac{5}{2}ab^3(Ab+2\end{aligned}$$

Mathematica [A] time = 0.02, size = 118, normalized size = 1.03

$$\frac{-4a^5\left(7A+10Bx^3\right)-50a^4bx^3\left(4A+7Bx^3\right)-700a^3b^2x^6\left(A+4Bx^3\right)+1400a^2b^3x^9\left(Bx^3-2A\right)+140ab^4x^{12}\left(5\right.}{280x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^11,x]
```

```
[Out] (1400*a^2*b^3*x^9*(-2*A + B*x^3) + 140*a*b^4*x^12*(5*A + 2*B*x^3) - 700*a^3
*b^2*x^6*(A + 4*B*x^3) + 7*b^5*x^15*(8*A + 5*B*x^3) - 50*a^4*b*x^3*(4*A + 7
*B*x^3) - 4*a^5*(7*A + 10*B*x^3))/(280*x^10)
```

fricas [A] time = 0.89, size = 121, normalized size = 1.05

$$\frac{35\,Bb^5x^{18}+56\left(5\,Bab^4+Ab^5\right)x^{15}+700\left(2\,Ba^2b^3+Aab^4\right)x^{12}-2800\left(Ba^3b^2+Aa^2b^3\right)x^9-350\left(Ba^4b+2\,Aa^3\right.}{280\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] 1/280*(35*B*b^5*x^18 + 56*(5*B*a*b^4 + A*b^5)*x^15 + 700*(2*B*a^2*b^3 + A*a
*b^4)*x^12 - 2800*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 350*(B*a^4*b + 2*A*a^3*b^2)
*x^6 - 28*A*a^5 - 40*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

giac [A] time = 0.15, size = 127, normalized size = 1.10

$$\frac{1}{8}\,Bb^5x^8+Bab^4x^5+\frac{1}{5}\,Ab^5x^5+5\,Ba^2b^3x^2+\frac{5}{2}\,Aab^4x^2-\frac{1400\,Ba^3b^2x^9+1400\,Aa^2b^3x^9+175\,Ba^4bx^6+350\,Aa^3b^2x^6+}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] 1/8*B*b^5*x^8 + B*a*b^4*x^5 + 1/5*A*b^5*x^5 + 5*B*a^2*b^3*x^2 + 5/2*A*a*b^4
*x^2 - 1/140*(1400*B*a^3*b^2*x^9 + 1400*A*a^2*b^3*x^9 + 175*B*a^4*b*x^6 + 3
50*A*a^3*b^2*x^6 + 20*B*a^5*x^3 + 100*A*a^4*b*x^3 + 14*A*a^5)/x^10
```

maple [A] time = 0.05, size = 111, normalized size = 0.97

$$\frac{B\,b^5x^8}{8}+\frac{A\,b^5x^5}{5}+Ba\,b^4x^5+\frac{5Aa\,b^4x^2}{2}+5B\,a^2b^3x^2-\frac{10\,(Ab+Ba)\,a^2b^2}{x}-\frac{5\,(2Ab+Ba)\,a^3b}{4x^4}-\frac{(5Ab+Ba)\,a^4}{7x^7}-\frac{A\,a^5}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^11,x)
```

```
[Out] 1/8*b^5*B*x^8+1/5*A*x^5*b^5+B*x^5*a*b^4+5/2*A*x^2*a*b^4+5*B*x^2*a^2*b^3-5/4
*a^3*b*(2*A*b+B*a)/x^4-1/7*a^4*(5*A*b+B*a)/x^7-10*a^2*b^2*(A*b+B*a)/x-1/10*
a^5*A/x^10
```

maxima [A] time = 0.47, size = 122, normalized size = 1.06

$$\frac{1}{8}\,Bb^5x^8+\frac{1}{5}\left(5\,Bab^4+Ab^5\right)x^5+\frac{5}{2}\left(2\,Ba^2b^3+Aab^4\right)x^2-\frac{1400\left(Ba^3b^2+Aa^2b^3\right)x^9+175\left(Ba^4b+2\,Aa^3b^2\right)x^6+14\,A}{140\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] 1/8*B*b^5*x^8 + 1/5*(5*B*a*b^4 + A*b^5)*x^5 + 5/2*(2*B*a^2*b^3 + A*a*b^4)*x
^2 - 1/140*(1400*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 175*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 14*A*a^5 + 20*(B*a^5 + 5*A*a^4*b)*x^3)/x^10
```

mupad [B] time = 2.36, size = 118, normalized size = 1.03

$$x^5\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)-\frac{\frac{A\,a^5}{10}+x^6\left(\frac{5\,B\,a^4\,b}{4}+\frac{5\,A\,a^3\,b^2}{2}\right)+x^3\left(\frac{B\,a^5}{7}+\frac{5\,A\,b\,a^4}{7}\right)+x^9\left(10\,B\,a^3\,b^2+10\,A\,a^2\,b^3\right)}{x^{10}}+\frac{B\,b^5\,x^8}{8}+\frac{5\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^11,x)
```

```
[Out] x^5*((A*b^5)/5 + B*a*b^4) - ((A*a^5)/10 + x^6*((5*A*a^3*b^2)/2 + (5*B*a^4*b
)/4) + x^3*((B*a^5)/7 + (5*A*a^4*b)/7) + x^9*(10*A*a^2*b^3 + 10*B*a^3*b^2))
/x^10 + (B*b^5*x^8)/8 + (5*a*b^3*x^2*(A*b + 2*B*a))/2
```

sympy [A] time = 2.93, size = 131, normalized size = 1.14

$$\frac{Bb^5x^8}{8}+x^5\left(\frac{Ab^5}{5}+Bab^4\right)+x^2\left(\frac{5Aab^4}{2}+5Ba^2b^3\right)+\frac{-14Aa^5+x^9\left(-1400Aa^2b^3-1400Ba^3b^2\right)+x^6\left(-350Aa^3b^2-\right.}{140x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**11,x)
```

```
[Out] B*b**5*x**8/8 + x**5*(A*b**5/5 + B*a*b**4) + x**2*(5*A*a*b**4/2 + 5*B*a**2*
b**3) + (-14*A*a**5 + x**9*(-1400*A*a**2*b**3 - 1400*B*a**3*b**2) + x**6*(-
350*A*a**3*b**2 - 175*B*a**4*b) + x**3*(-100*A*a**4*b - 20*B*a**5))/(140*x*
*10)
```

3.44 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx$

Optimal. Leaf size=109

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

```
[Out] -1/11*a^5*A/x^11-1/8*a^4*(5*A*b+B*a)/x^8-a^3*b*(2*A*b+B*a)/x^5-5*a^2*b^2*(A
*b+B*a)/x^2+5*a*b^3*(A*b+2*B*a)*x+1/4*b^4*(A*b+5*B*a)*x^4+1/7*b^5*B*x^7
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{x^2}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^5A}{11x^{11}}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -(a^5*A)/(11*x^11) - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{12}}\,dx = \int \left(5ab^3(Ab+2aB)+\frac{a^5A}{x^{12}}+\frac{a^4(5Ab+aB)}{x^9}+\frac{5a^3b(2Ab+aB)}{x^6}+\frac{10a^2b^2(Ab+aB)}{x^3}\right.$$
$$=-\frac{a^5A}{11x^{11}}-\frac{a^4(5Ab+aB)}{8x^8}-\frac{a^3b(2Ab+aB)}{x^5}-\frac{5a^2b^2(Ab+aB)}{x^2}+5ab^3(Ab+2aB)x$$

Mathematica [A] time = 0.04, size = 109, normalized size = 1.00

$$-\frac{a^5A}{11x^{11}}-\frac{a^4(aB+5Ab)}{8x^8}-\frac{a^3b(aB+2Ab)}{x^5}-\frac{5a^2b^2(aB+Ab)}{x^2}+\frac{1}{4}b^4x^4(5aB+Ab)+5ab^3x(2aB+Ab)+\frac{1}{7}b^5Bx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^12,x]
```

```
[Out] -1/11*(a^5*A)/x^11 - (a^4*(5*A*b + a*B))/(8*x^8) - (a^3*b*(2*A*b + a*B))/x^
5 - (5*a^2*b^2*(A*b + a*B))/x^2 + 5*a*b^3*(A*b + 2*a*B)*x + (b^4*(A*b + 5*a
*B)*x^4)/4 + (b^5*B*x^7)/7
```

fricas [A] time = 0.96, size = 121, normalized size = 1.11

$$\frac{88\,Bb^5x^{18}+154\,(5\,Bab^4+Ab^5)x^{15}+3080\,(2\,Ba^2b^3+Aab^4)x^{12}-3080\,(Ba^3b^2+Aa^2b^3)x^9-616\,(Ba^4b+2\,Aa^3}{616\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="fricas")
```

```
[Out] 1/616*(88*B*b^5*x^18 + 154*(5*B*a*b^4 + A*b^5)*x^15 + 3080*(2*B*a^2*b^3 + A
*a*b^4)*x^12 - 3080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 616*(B*a^4*b + 2*A*a^3*b^
2)*x^6 - 56*A*a^5 - 77*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

giac [A] time = 0.18, size = 124, normalized size = 1.14

$$\frac{1}{7}Bb^5x^7+\frac{5}{4}Bab^4x^4+\frac{1}{4}Ab^5x^4+10Ba^2b^3x+5Aab^4x-\frac{440Ba^3b^2x^9+440Aa^2b^3x^9+88Ba^4bx^6+176Aa^3b^2x^6+}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="giac")
```

```
[Out] 1/7*B*b^5*x^7 + 5/4*B*a*b^4*x^4 + 1/4*A*b^5*x^4 + 10*B*a^2*b^3*x + 5*A*a*b^
4*x - 1/88*(440*B*a^3*b^2*x^9 + 440*A*a^2*b^3*x^9 + 88*B*a^4*b*x^6 + 176*A*
a^3*b^2*x^6 + 11*B*a^5*x^3 + 55*A*a^4*b*x^3 + 8*A*a^5)/x^11
```

maple [A] time = 0.05, size = 108, normalized size = 0.99

$$\frac{Bb^5x^7}{7}+\frac{Ab^5x^4}{4}+\frac{5Bab^4x^4}{4}+5Aab^4x+10Ba^2b^3x-\frac{5(Ab+Ba)a^2b^2}{x^2}-\frac{(2Ab+Ba)a^3b}{x^5}-\frac{(5Ab+Ba)a^4}{8x^8}-\frac{Aa^5}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^12,x)
```

```
[Out] 1/7*b^5*B*x^7+1/4*A*x^4*b^5+5/4*B*x^4*a*b^4+5*a*b^4*A*x+10*a^2*b^3*B*x-a^3*
b*(2*A*b+B*a)/x^5-1/8*a^4*(5*A*b+B*a)/x^8-5*a^2*b^2*(A*b+B*a)/x^2-1/11*a^5*
A/x^11
```

maxima [A] time = 0.53, size = 120, normalized size = 1.10

$$\frac{1}{7}Bb^5x^7+\frac{1}{4}\left(5Bab^4+Ab^5\right)x^4+5\left(2Ba^2b^3+Aab^4\right)x-\frac{440\left(Ba^3b^2+Aa^2b^3\right)x^9+88\left(Ba^4b+2Aa^3b^2\right)x^6+8Aa}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^12,x, algorithm="maxima")
```

```
[Out] 1/7*B*b^5*x^7 + 1/4*(5*B*a*b^4 + A*b^5)*x^4 + 5*(2*B*a^2*b^3 + A*a*b^4)*x -
 1/88*(440*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 88*(B*a^4*b + 2*A*a^3*b^2)*x^6 + 8
*A*a^5 + 11*(B*a^5 + 5*A*a^4*b)*x^3)/x^11
```

mupad [B] time = 0.07, size = 116, normalized size = 1.06

$$x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)-\frac{\frac{Aa^5}{11}+x^6\left(Ba^4b+2Aa^3b^2\right)+x^3\left(\frac{Ba^5}{8}+\frac{5Aba^4}{8}\right)+x^9\left(5Ba^3b^2+5Aa^2b^3\right)}{x^{11}}+\frac{Bb^5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^12,x)
```

```
[Out] x^4*((A*b^5)/4 + (5*B*a*b^4)/4) - ((A*a^5)/11 + x^6*(2*A*a^3*b^2 + B*a^4*b)
 + x^3*((B*a^5)/8 + (5*A*a^4*b)/8) + x^9*(5*A*a^2*b^3 + 5*B*a^3*b^2))/x^11
+ (B*b^5*x^7)/7 + 5*a*b^3*x*(A*b + 2*B*a)
```

sympy [A] time = 3.73, size = 131, normalized size = 1.20

$$\frac{Bb^5x^7}{7}+x^4\left(\frac{Ab^5}{4}+\frac{5Bab^4}{4}\right)+x\left(5Aab^4+10Ba^2b^3\right)+\frac{-8Aa^5+x^9\left(-440Aa^2b^3-440Ba^3b^2\right)+x^6\left(-176Aa^3b^2-\right.}{88x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**12,x)
```

```
[Out] B*b**5*x**7/7 + x**4*(A*b**5/4 + 5*B*a*b**4/4) + x*(5*A*a*b**4 + 10*B*a**2*
b**3) + (-8*A*a**5 + x**9*(-440*A*a**2*b**3 - 440*B*a**3*b**2) + x**6*(-176
*A*a**3*b**2 - 88*B*a**4*b) + x**3*(-55*A*a**4*b - 11*B*a**5))/(88*x**11)
```

3.45 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} \, dx$

Optimal. Leaf size=114

$$-\frac{a^5 A}{12x^{12}}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{10a^2b^2(aB+Ab)}{3x^3}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx$$

```
[Out] -1/12*a^5*A/x^12-1/9*a^4*(5*A*b+B*a)/x^9-5/6*a^3*b*(2*A*b+B*a)/x^6-10/3*a^2
*b^2*(A*b+B*a)/x^3+1/3*b^4*(A*b+5*B*a)*x^3+1/6*b^5*B*x^6+5*a*b^3*(A*b+2*B*a
)*ln(x)
```

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{10a^2b^2(aB+Ab)}{3x^3}-\frac{5a^3b(aB+2Ab)}{6x^6}-\frac{a^4(aB+5Ab)}{9x^9}-\frac{a^5 A}{12x^{12}}+\frac{1}{3}b^4x^3(5aB+Ab)+5ab^3\log(x)(2aB+Ab)+\frac{1}{6}b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -(a^5*A)/(12*x^12) - (a^4*(5*A*b + a*B))/(9*x^9) - (5*a^3*b*(2*A*b + a*B))/
(6*x^6) - (10*a^2*b^2*(A*b + a*B))/(3*x^3) + (b^4*(A*b + 5*a*B)*x^3)/3 + (b
^5*B*x^6)/6 + 5*a*b^3*(A*b + 2*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{13}} \, dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^5} \, dx, x, x^3\right) \\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(b^4(Ab+5aB)+\frac{a^5 A}{x^5}+\frac{a^4(5Ab+aB)}{x^4}+\frac{5a^3b(2Ab+aB)}{x^3}+\frac{10a^2b}{}\right.\right. \\
&= -\frac{a^5 A}{12x^{12}}-\frac{a^4(5Ab+aB)}{9x^9}-\frac{5a^3b(2Ab+aB)}{6x^6}-\frac{10a^2b^2(Ab+aB)}{3x^3}+\frac{1}{3}b^4(Ab+5a
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 1.04

$$-\frac{a^5\left(3A+4Bx^3\right)+10a^4bx^3\left(2A+3Bx^3\right)+60a^3b^2x^6\left(A+2Bx^3\right)+120a^2Ab^3x^9-180ab^3x^{12}\log(x)(2aB+Ab}{36x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^13,x]
```

```
[Out] -1/36*(120*a^2*A*b^3*x^9 - 60*a*b^4*B*x^15 - 6*b^5*x^15*(2*A + B*x^3) + 60*
a^3*b^2*x^6*(A + 2*B*x^3) + 10*a^4*b*x^3*(2*A + 3*B*x^3) + a^5*(3*A + 4*B*x
^3) - 180*a*b^3*(A*b + 2*a*B)*x^12*Log[x])/x^12
```

fricas [A] time = 1.12, size = 123, normalized size = 1.08

$$\frac{6\,Bb^5x^{18}+12\left(5\,Bab^4+Ab^5\right)x^{15}+180\left(2\,Ba^2b^3+Aab^4\right)x^{12}\log(x)-120\left(Ba^3b^2+Aa^2b^3\right)x^9-30\left(Ba^4b+2\,Aa^3\right.}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="fricas")
```

```
[Out] 1/36*(6*B*b^5*x^18 + 12*(5*B*a*b^4 + A*b^5)*x^15 + 180*(2*B*a^2*b^3 + A*a*b
^4)*x^12*log(x) - 120*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 30*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 3*A*a^5 - 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

giac [A] time = 0.16, size = 149, normalized size = 1.31

$$\frac{1}{6}\,Bb^5x^6+\frac{5}{3}\,Bab^4x^3+\frac{1}{3}\,Ab^5x^3+5\left(2\,Ba^2b^3+Aab^4\right)\log\left(|x|\right)-\frac{250\,Ba^2b^3x^{12}+125\,Aab^4x^{12}+120\,Ba^3b^2x^9+120\,Aa}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="giac")
```

```
[Out] 1/6*B*b^5*x^6 + 5/3*B*a*b^4*x^3 + 1/3*A*b^5*x^3 + 5*(2*B*a^2*b^3 + A*a*b^4)
*log(abs(x)) - 1/36*(250*B*a^2*b^3*x^12 + 125*A*a*b^4*x^12 + 120*B*a^3*b^2*
x^9 + 120*A*a^2*b^3*x^9 + 30*B*a^4*b*x^6 + 60*A*a^3*b^2*x^6 + 4*B*a^5*x^3 +
 20*A*a^4*b*x^3 + 3*A*a^5)/x^12
```

maple [A] time = 0.05, size = 124, normalized size = 1.09

$$\frac{B\,b^5x^6}{6}+\frac{A\,b^5x^3}{3}+\frac{5Ba\,b^4x^3}{3}+5Aa\,b^4\ln(x)+10B\,a^2b^3\ln(x)-\frac{10A\,a^2b^3}{3x^3}-\frac{10B\,a^3b^2}{3x^3}-\frac{5A\,a^3b^2}{3x^6}-\frac{5B\,a^4b}{6x^6}-\frac{5A\,a^4b}{9x^9}-\frac{B\,a^5}{9x^9}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^13,x)
```

```
[Out] 1/6*b^5*B*x^6+1/3*A*x^3*b^5+5/3*B*x^3*a*b^4-1/12*a^5*A/x^12-10/3*b^3*a^2/x^
3*A-10/3*b^2*a^3/x^3*B-5/9*a^4/x^9*A*b-1/9*a^5/x^9*B-5/3*a^3*b^2/x^6*A-5/6*
a^4*b/x^6*B+5*A*ln(x)*a*b^4+10*B*ln(x)*a^2*b^3
```

maxima [A] time = 0.63, size = 123, normalized size = 1.08

$$\frac{1}{6}\,Bb^5x^6+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)x^3+\frac{5}{3}\left(2\,Ba^2b^3+Aab^4\right)\log\left(x^3\right)-\frac{120\left(Ba^3b^2+Aa^2b^3\right)x^9+30\left(Ba^4b+2\,Aa^3b^2\right)x^6+}{36\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^13,x, algorithm="maxima")
```

```
[Out] 1/6*B*b^5*x^6 + 1/3*(5*B*a*b^4 + A*b^5)*x^3 + 5/3*(2*B*a^2*b^3 + A*a*b^4)*l
og(x^3) - 1/36*(120*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 30*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 3*A*a^5 + 4*(B*a^5 + 5*A*a^4*b)*x^3)/x^12
```

mupad [B] time = 0.06, size = 122, normalized size = 1.07

$$\ln(x)\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)-\frac{\frac{A\,a^5}{12}+x^6\left(\frac{5\,B\,a^4\,b}{6}+\frac{5\,A\,a^3\,b^2}{3}\right)+x^3\left(\frac{B\,a^5}{9}+\frac{5\,A\,b\,a^4}{9}\right)+x^9\left(\frac{10\,B\,a^3\,b^2}{3}+\frac{10\,A\,a^2\,b^3}{3}\right)}{x^{12}}+x^3\left(\frac{A}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^13,x)
```

```
[Out] log(x)*(10*B*a^2*b^3 + 5*A*a*b^4) - ((A*a^5)/12 + x^6*((5*A*a^3*b^2)/3 + (5
*B*a^4*b)/6) + x^3*((B*a^5)/9 + (5*A*a^4*b)/9) + x^9*((10*A*a^2*b^3)/3 + (1
0*B*a^3*b^2)/3))/x^12 + x^3*((A*b^5)/3 + (5*B*a*b^4)/3) + (B*b^5*x^6)/6
```

sympy [A] time = 6.64, size = 129, normalized size = 1.13

$$\frac{Bb^5x^6}{6}+5ab^3\left(Ab+2Ba\right)\log\left(x\right)+x^3\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+\frac{-3Aa^5+x^9\left(-120Aa^2b^3-120Ba^3b^2\right)+x^6\left(-60Aa^3b^2-\right.}{36x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**13,x)
```

```
[Out] B*b**5*x**6/6 + 5*a*b**3*(A*b + 2*B*a)*log(x) + x**3*(A*b**5/3 + 5*B*a*b**4
/3) + (-3*A*a**5 + x**9*(-120*A*a**2*b**3 - 120*B*a**3*b**2) + x**6*(-60*A*
a**3*b**2 - 30*B*a**4*b) + x**3*(-20*A*a**4*b - 4*B*a**5))/(36*x**12)
```

3.46 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx$

Optimal. Leaf size=115

$$-\frac{a^5 A}{13x^{13}}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{5a^2b^2(aB+Ab)}{2x^4}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

```
[Out] -1/13*a^5*A/x^13-1/10*a^4*(5*A*b+B*a)/x^10-5/7*a^3*b*(2*A*b+B*a)/x^7-5/2*a^
2*b^2*(A*b+B*a)/x^4-5*a*b^3*(A*b+2*B*a)/x+1/2*b^4*(A*b+5*B*a)*x^2+1/5*b^5*B
*x^5
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{2x^4}-\frac{a^4(aB+5Ab)}{10x^{10}}-\frac{5a^3b(aB+2Ab)}{7x^7}-\frac{a^5 A}{13x^{13}}+\frac{1}{2}b^4x^2(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{x}+\frac{1}{5}b^5Bx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -(a^5*A)/(13*x^13) - (a^4*(5*A*b + a*B))/(10*x^10) - (5*a^3*b*(2*A*b + a*B)
)/(7*x^7) - (5*a^2*b^2*(A*b + a*B))/(2*x^4) - (5*a*b^3*(A*b + 2*a*B))/x + (
b^4*(A*b + 5*a*B)*x^2)/2 + (b^5*B*x^5)/5
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{14}} \, dx &= \int \left(\frac{a^5 A}{x^{14}}+\frac{a^4(5Ab+aB)}{x^{11}}+\frac{5a^3b(2Ab+aB)}{x^8}+\frac{10a^2b^2(Ab+aB)}{x^5}+\frac{5ab^3(Ab+2aB)}{x^2}\right.\\ &= -\frac{a^5 A}{13x^{13}}-\frac{a^4(5Ab+aB)}{10x^{10}}-\frac{5a^3b(2Ab+aB)}{7x^7}-\frac{5a^2b^2(Ab+aB)}{2x^4}-\frac{5ab^3(Ab+2aB)}{x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 117, normalized size = 1.02

$$-\frac{a^5\left(70A+91Bx^3\right)+65a^4bx^3\left(7A+10Bx^3\right)+325a^3b^2x^6\left(4A+7Bx^3\right)+2275a^2b^3x^9\left(A+4Bx^3\right)-2275ab^4x^{12}}{910x^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^14,x]
```

```
[Out] -1/910*(-2275*a*b^4*x^12*(-2*A + B*x^3) - 91*b^5*x^15*(5*A + 2*B*x^3) + 227
5*a^2*b^3*x^9*(A + 4*B*x^3) + 325*a^3*b^2*x^6*(4*A + 7*B*x^3) + 65*a^4*b*x^
3*(7*A + 10*B*x^3) + a^5*(70*A + 91*B*x^3))/x^13
```

fricas [A] time = 1.10, size = 121, normalized size = 1.05

$$\frac{182\,Bb^5x^{18}+455\left(5\,Bab^4+Ab^5\right)x^{15}-4550\left(2\,Ba^2b^3+Aab^4\right)x^{12}-2275\left(Ba^3b^2+Aa^2b^3\right)x^9-650\left(Ba^4b+2\,Aa\right.}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="fricas")
```

```
[Out] 1/910*(182*B*b^5*x^18 + 455*(5*B*a*b^4 + A*b^5)*x^15 - 4550*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 650*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 70*A*a^5 - 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

giac [A] time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{5}\,Bb^5x^5+\frac{5}{2}\,Bab^4x^2+\frac{1}{2}\,Ab^5x^2-\frac{9100\,Ba^2b^3x^{12}+4550\,Aab^4x^{12}+2275\,Ba^3b^2x^9+2275\,Aa^2b^3x^9+650\,Ba^4bx^6}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="giac")
```

```
[Out] 1/5*B*b^5*x^5 + 5/2*B*a*b^4*x^2 + 1/2*A*b^5*x^2 - 1/910*(9100*B*a^2*b^3*x^1
2 + 4550*A*a*b^4*x^12 + 2275*B*a^3*b^2*x^9 + 2275*A*a^2*b^3*x^9 + 650*B*a^4
*b*x^6 + 1300*A*a^3*b^2*x^6 + 91*B*a^5*x^3 + 455*A*a^4*b*x^3 + 70*A*a^5)/x^
13
```

maple [A] time = 0.06, size = 107, normalized size = 0.93

$$\frac{B\,b^5x^5}{5}+\frac{A\,b^5x^2}{2}+\frac{5Ba\,b^4x^2}{2}-\frac{5\,(Ab+2Ba)\,a\,b^3}{x}-\frac{5\,(Ab+Ba)\,a^2b^2}{2x^4}-\frac{5\,(2Ab+Ba)\,a^3b}{7x^7}-\frac{(5Ab+Ba)\,a^4}{10x^{10}}-\frac{A\,a^5}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^14,x)
```

```
[Out] 1/5*b^5*B*x^5+1/2*A*x^2*b^5+5/2*B*x^2*a*b^4-5/2*a^2*b^2*(A*b+B*a)/x^4-1/13*
a^5*A/x^13-5/7*a^3*b*(2*A*b+B*a)/x^7-5*a*b^3*(A*b+2*B*a)/x-1/10*a^4*(5*A*b+
B*a)/x^10
```

maxima [A] time = 0.56, size = 122, normalized size = 1.06

$$\frac{1}{5}\,Bb^5x^5+\frac{1}{2}\left(5\,Bab^4+Ab^5\right)x^2-\frac{4550\left(2\,Ba^2b^3+Aab^4\right)x^{12}+2275\left(Ba^3b^2+Aa^2b^3\right)x^9+650\left(Ba^4b+2\,Aa^3b^2\right)}{910\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^14,x, algorithm="maxima")
```

```
[Out] 1/5*B*b^5*x^5 + 1/2*(5*B*a*b^4 + A*b^5)*x^2 - 1/910*(4550*(2*B*a^2*b^3 + A*
a*b^4)*x^12 + 2275*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 650*(B*a^4*b + 2*A*a^3*b^2
)*x^6 + 70*A*a^5 + 91*(B*a^5 + 5*A*a^4*b)*x^3)/x^13
```

mupad [B] time = 2.37, size = 123, normalized size = 1.07

$$x^2\left(\frac{A\,b^5}{2}+\frac{5\,B\,a\,b^4}{2}\right)-\frac{\frac{A\,a^5}{13}+x^{12}\left(10\,B\,a^2\,b^3+5\,A\,a\,b^4\right)+x^6\left(\frac{5\,B\,a^4\,b}{7}+\frac{10\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{10}+\frac{A\,b\,a^4}{2}\right)+x^9\left(\frac{5\,B\,a}{2}\right.}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^14,x)
```

```
[Out] x^2*((A*b^5)/2 + (5*B*a*b^4)/2) - ((A*a^5)/13 + x^12*(10*B*a^2*b^3 + 5*A*a*
b^4) + x^6*((10*A*a^3*b^2)/7 + (5*B*a^4*b)/7) + x^3*((B*a^5)/10 + (A*a^4*b)
/2) + x^9*((5*A*a^2*b^3)/2 + (5*B*a^3*b^2)/2))/x^13 + (B*b^5*x^5)/5
```

sympy [A] time = 10.76, size = 134, normalized size = 1.17

$$\frac{Bb^5x^5}{5}+x^2\left(\frac{Ab^5}{2}+\frac{5Bab^4}{2}\right)+\frac{-70Aa^5+x^{12}\left(-4550Aab^4-9100Ba^2b^3\right)+x^9\left(-2275Aa^2b^3-2275Ba^3b^2\right)+x^6\left(-1\right.}{910x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**14,x)
```

```
[Out] B*b**5*x**5/5 + x**2*(A*b**5/2 + 5*B*a*b**4/2) + (-70*A*a**5 + x**12*(-4550
*A*a*b**4 - 9100*B*a**2*b**3) + x**9*(-2275*A*a**2*b**3 - 2275*B*a**3*b**2)
 + x**6*(-1300*A*a**3*b**2 - 650*B*a**4*b) + x**3*(-455*A*a**4*b - 91*B*a**
5))/(910*x**13)
```

3.47 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} dx$

Optimal. Leaf size=110

$$-\frac{a^5 A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

```
[Out] -1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^11-5/8*a^3*b*(2*A*b+B*a)/x^8-2*a^2*
b^2*(A*b+B*a)/x^5-5/2*a*b^3*(A*b+2*B*a)/x^2+b^4*(A*b+5*B*a)*x+1/4*b^5*B*x^4
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{2a^2b^2(aB+Ab)}{x^5}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{a^5 A}{14x^{14}}-\frac{5ab^3(2aB+Ab)}{2x^2}+b^4x(5aB+Ab)+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -(a^5*A)/(14*x^14) - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{15}} dx = \int \left(b^4(Ab+5aB)+\frac{a^5 A}{x^{15}}+\frac{a^4(5Ab+aB)}{x^{12}}+\frac{5a^3b(2Ab+aB)}{x^9}+\frac{10a^2b^2(Ab+a}{x^6}\right.$$
$$=-\frac{a^5 A}{14x^{14}}-\frac{a^4(5Ab+aB)}{11x^{11}}-\frac{5a^3b(2Ab+aB)}{8x^8}-\frac{2a^2b^2(Ab+aB)}{x^5}-\frac{5ab^3(Ab+2a}{2x^2}$$

Mathematica [A] time = 0.07, size = 110, normalized size = 1.00

$$-\frac{a^5 A}{14x^{14}}-\frac{a^4(aB+5Ab)}{11x^{11}}-\frac{5a^3b(aB+2Ab)}{8x^8}-\frac{2a^2b^2(aB+Ab)}{x^5}+b^4x(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{2x^2}+\frac{1}{4}b^5Bx^4$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^15,x]
```

```
[Out] -1/14*(a^5*A)/x^14 - (a^4*(5*A*b + a*B))/(11*x^11) - (5*a^3*b*(2*A*b + a*B)
)/(8*x^8) - (2*a^2*b^2*(A*b + a*B))/x^5 - (5*a*b^3*(A*b + 2*a*B))/(2*x^2) +
 b^4*(A*b + 5*a*B)*x + (b^5*B*x^4)/4
```

fricas [A] time = 1.31, size = 121, normalized size = 1.10

$$\frac{154\,Bb^5x^{18}+616\left(5\,Bab^4+Ab^5\right)x^{15}-1540\left(2\,Ba^2b^3+Aab^4\right)x^{12}-1232\left(Ba^3b^2+Aa^2b^3\right)x^9-385\left(Ba^4b+2\right.}{616\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="fricas")
```

```
[Out] 1/616*(154*B*b^5*x^18 + 616*(5*B*a*b^4 + A*b^5)*x^15 - 1540*(2*B*a^2*b^3 +
A*a*b^4)*x^12 - 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 385*(B*a^4*b + 2*A*a^3*b
^2)*x^6 - 44*A*a^5 - 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

giac [A] time = 0.16, size = 123, normalized size = 1.12

$$\frac{1}{4}Bb^5x^4+5Bab^4x+Ab^5x-\frac{3080Ba^2b^3x^{12}+1540Aab^4x^{12}+1232Ba^3b^2x^9+1232Aa^2b^3x^9+385Ba^4bx^6+770A}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="giac")
```

```
[Out] 1/4*B*b^5*x^4 + 5*B*a*b^4*x + A*b^5*x - 1/616*(3080*B*a^2*b^3*x^12 + 1540*A
*a*b^4*x^12 + 1232*B*a^3*b^2*x^9 + 1232*A*a^2*b^3*x^9 + 385*B*a^4*b*x^6 + 7
70*A*a^3*b^2*x^6 + 56*B*a^5*x^3 + 280*A*a^4*b*x^3 + 44*A*a^5)/x^14
```

maple [A] time = 0.05, size = 102, normalized size = 0.93

$$\frac{Bb^5x^4}{4}+Ab^5x+5Bab^4x-\frac{5(Ab+2Ba)ab^3}{2x^2}-\frac{2(Ab+Ba)a^2b^2}{x^5}-\frac{5(2Ab+Ba)a^3b}{8x^8}-\frac{(5Ab+Ba)a^4}{11x^{11}}-\frac{Aa^5}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^15,x)
```

```
[Out] 1/4*b^5*B*x^4+b^5*A*x+5*a*b^4*B*x-2*a^2*b^2*(A*b+B*a)/x^5-5/8*a^3*b*(2*A*b+
B*a)/x^8-5/2*a*b^3*(A*b+2*B*a)/x^2-1/14*a^5*A/x^14-1/11*a^4*(5*A*b+B*a)/x^1
1
```

maxima [A] time = 0.61, size = 119, normalized size = 1.08

$$\frac{1}{4}Bb^5x^4+\left(5Bab^4+Ab^5\right)x-\frac{1540\left(2Ba^2b^3+Aab^4\right)x^{12}+1232\left(Ba^3b^2+Aa^2b^3\right)x^9+385\left(Ba^4b+2Aa^3b^2\right)x^6+44}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^15,x, algorithm="maxima")
```

```
[Out] 1/4*B*b^5*x^4 + (5*B*a*b^4 + A*b^5)*x - 1/616*(1540*(2*B*a^2*b^3 + A*a*b^4)
*x^12 + 1232*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 385*(B*a^4*b + 2*A*a^3*b^2)*x^6
+ 44*A*a^5 + 56*(B*a^5 + 5*A*a^4*b)*x^3)/x^14
```

mupad [B] time = 2.39, size = 120, normalized size = 1.09

$$x\left(Ab^5+5Bab^4\right)-\frac{\frac{Aa^5}{14}+x^{12}\left(5Ba^2b^3+\frac{5Aab^4}{2}\right)+x^6\left(\frac{5Ba^4b}{8}+\frac{5Aa^3b^2}{4}\right)+x^3\left(\frac{Ba^5}{11}+\frac{5Aba^4}{11}\right)+x^9\left(2Ba^3b^2+\right.}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^15,x)
```

```
[Out] x*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/14 + x^12*(5*B*a^2*b^3 + (5*A*a*b^4)/2) +
x^6*((5*A*a^3*b^2)/4 + (5*B*a^4*b)/8) + x^3*((B*a^5)/11 + (5*A*a^4*b)/11) +
 x^9*(2*A*a^2*b^3 + 2*B*a^3*b^2))/x^14 + (B*b^5*x^4)/4
```

sympy [A] time = 22.98, size = 129, normalized size = 1.17

$$\frac{Bb^5x^4}{4}+x\left(Ab^5+5Bab^4\right)+\frac{-44Aa^5+x^{12}\left(-1540Aab^4-3080Ba^2b^3\right)+x^9\left(-1232Aa^2b^3-1232Ba^3b^2\right)+x^6\left(-770\right.}{616x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**15,x)
```

```
[Out] B*b**5*x**4/4 + x*(A*b**5 + 5*B*a*b**4) + (-44*A*a**5 + x**12*(-1540*A*a*b*
*4 - 3080*B*a**2*b**3) + x**9*(-1232*A*a**2*b**3 - 1232*B*a**3*b**2) + x**6
*(-770*A*a**3*b**2 - 385*B*a**4*b) + x**3*(-280*A*a**4*b - 56*B*a**5))/(616
*x**14)
```

3.48 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx$

Optimal. Leaf size=113

$$-\frac{a^5 A}{15x^{15}}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{5a^2b^2(aB+Ab)}{3x^6}+b^4\log(x)(5aB+Ab)-\frac{5ab^3(2aB+Ab)}{3x^3}+\frac{1}{3}b^5Bx^3$$

```
[Out] -1/15*a^5*A/x^15-1/12*a^4*(5*A*b+B*a)/x^12-5/9*a^3*b*(2*A*b+B*a)/x^9-5/3*a^
2*b^2*(A*b+B*a)/x^6-5/3*a*b^3*(A*b+2*B*a)/x^3+1/3*b^5*B*x^3+b^4*(A*b+5*B*a)
*ln(x)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 76}

$$-\frac{5a^2b^2(aB+Ab)}{3x^6}-\frac{a^4(aB+5Ab)}{12x^{12}}-\frac{5a^3b(aB+2Ab)}{9x^9}-\frac{a^5 A}{15x^{15}}-\frac{5ab^3(2aB+Ab)}{3x^3}+b^4\log(x)(5aB+Ab)+\frac{1}{3}b^5Bx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -(a^5*A)/(15*x^15) - (a^4*(5*A*b + a*B))/(12*x^12) - (5*a^3*b*(2*A*b + a*B)
)/(9*x^9) - (5*a^2*b^2*(A*b + a*B))/(3*x^6) - (5*a*b^3*(A*b + 2*a*B))/(3*x^
3) + (b^5*B*x^3)/3 + b^4*(A*b + 5*a*B)*Log[x]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{16}} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^6} dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(b^5B+\frac{a^5A}{x^6}+\frac{a^4(5Ab+aB)}{x^5}+\frac{5a^3b(2Ab+aB)}{x^4}+\frac{10a^2b^2(Ab+aB)}{x^3}\right.\right.\\ &= -\frac{a^5A}{15x^{15}}-\frac{a^4(5Ab+aB)}{12x^{12}}-\frac{5a^3b(2Ab+aB)}{9x^9}-\frac{5a^2b^2(Ab+aB)}{3x^6}-\frac{5ab^3(Ab+2aB)}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 116, normalized size = 1.03

$$b^4\log(x)(5aB+Ab)-\frac{3a^5(4A+5Bx^3)+25a^4bx^3(3A+4Bx^3)+100a^3b^2x^6(2A+3Bx^3)+300a^2b^3x^9(A+2Bx^3}{180x^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^16,x]
```

```
[Out] -1/180*(300*a*A*b^4*x^12 - 60*b^5*B*x^18 + 300*a^2*b^3*x^9*(A + 2*B*x^3) +
100*a^3*b^2*x^6*(2*A + 3*B*x^3) + 25*a^4*b*x^3*(3*A + 4*B*x^3) + 3*a^5*(4*A
 + 5*B*x^3))/x^15 + b^4*(A*b + 5*a*B)*Log[x]
```

fricas [A] time = 1.22, size = 123, normalized size = 1.09

$$\frac{60\,Bb^5x^{18}+180\left(5\,Bab^4+Ab^5\right)x^{15}\log(x)-300\left(2\,Ba^2b^3+Aab^4\right)x^{12}-300\left(Ba^3b^2+Aa^2b^3\right)x^9-100\left(Ba^4b+\right.}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="fricas")
```

```
[Out] 1/180*(60*B*b^5*x^18 + 180*(5*B*a*b^4 + A*b^5)*x^15*log(x) - 300*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 100*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 12*A*a^5 - 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

giac [A] time = 0.15, size = 145, normalized size = 1.28

$$\frac{1}{3}\,Bb^5x^3+\left(5\,Bab^4+Ab^5\right)\log\left(|x|\right)-\frac{685\,Bab^4x^{15}+137\,Ab^5x^{15}+600\,Ba^2b^3x^{12}+300\,Aab^4x^{12}+300\,Ba^3b^2x^9+}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="giac")
```

```
[Out] 1/3*B*b^5*x^3 + (5*B*a*b^4 + A*b^5)*log(abs(x)) - 1/180*(685*B*a*b^4*x^15 +
 137*A*b^5*x^15 + 600*B*a^2*b^3*x^12 + 300*A*a*b^4*x^12 + 300*B*a^3*b^2*x^9
 + 300*A*a^2*b^3*x^9 + 100*B*a^4*b*x^6 + 200*A*a^3*b^2*x^6 + 15*B*a^5*x^3 +
 75*A*a^4*b*x^3 + 12*A*a^5)/x^15
```

maple [A] time = 0.05, size = 123, normalized size = 1.09

$$\frac{B\,b^5x^3}{3}+A\,b^5\ln(x)+5Ba\,b^4\ln(x)-\frac{5Aa\,b^4}{3x^3}-\frac{10B\,a^2b^3}{3x^3}-\frac{5A\,a^2b^3}{3x^6}-\frac{5B\,a^3b^2}{3x^6}-\frac{10A\,a^3b^2}{9x^9}-\frac{5B\,a^4b}{9x^9}-\frac{5A\,a^4b}{12x^{12}}-\frac{B\,a^5}{12x^{12}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^16,x)
```

```
[Out] 1/3*b^5*B*x^3-5/12*a^4/x^12*A*b-1/12*a^5/x^12*B-5/3*a*b^4/x^3*A-10/3*a^2*b^
3/x^3*B-1/15*a^5*A/x^15-10/9*a^3*b^2/x^9*A-5/9*a^4*b/x^9*B-5/3*b^3*a^2/x^6*
A-5/3*b^2*a^3/x^6*B+A*ln(x)*b^5+5*B*ln(x)*a*b^4
```

maxima [A] time = 0.59, size = 123, normalized size = 1.09

$$\frac{1}{3}\,Bb^5x^3+\frac{1}{3}\left(5\,Bab^4+Ab^5\right)\log\left(x^3\right)-\frac{300\left(2\,Ba^2b^3+Aab^4\right)x^{12}+300\left(Ba^3b^2+Aa^2b^3\right)x^9+100\left(Ba^4b+2\,Aa^3\right.}{180\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^16,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*x^3 + 1/3*(5*B*a*b^4 + A*b^5)*log(x^3) - 1/180*(300*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 300*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 100*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 12*A*a^5 + 15*(B*a^5 + 5*A*a^4*b)*x^3)/x^15
```

mupad [B] time = 0.08, size = 121, normalized size = 1.07

$$\ln(x)\,\left(A\,b^5+5\,B\,a\,b^4\right)-\frac{\frac{A\,a^5}{15}+x^{12}\,\left(\frac{10\,B\,a^2\,b^3}{3}+\frac{5\,A\,a\,b^4}{3}\right)+x^6\,\left(\frac{5\,B\,a^4\,b}{9}+\frac{10\,A\,a^3\,b^2}{9}\right)+x^3\,\left(\frac{B\,a^5}{12}+\frac{5\,A\,b\,a^4}{12}\right)+x^9\,\left(\frac{5\,B\,a}{3}\right.}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^16,x)
```

```
[Out] log(x)*(A*b^5 + 5*B*a*b^4) - ((A*a^5)/15 + x^12*((10*B*a^2*b^3)/3 + (5*A*a*
b^4)/3) + x^6*((10*A*a^3*b^2)/9 + (5*B*a^4*b)/9) + x^3*((B*a^5)/12 + (5*A*a
^4*b)/12) + x^9*((5*A*a^2*b^3)/3 + (5*B*a^3*b^2)/3))/x^15 + (B*b^5*x^3)/3
```

sympy [A] time = 29.30, size = 129, normalized size = 1.14

$$\frac{Bb^5x^3}{3}+b^4\left(Ab+5Ba\right)\log\left(x\right)+\frac{-12Aa^5+x^{12}\left(-300Aab^4-600Ba^2b^3\right)+x^9\left(-300Aa^2b^3-300Ba^3b^2\right)+x^6\left(-200\right.}{180x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**16,x)
```

```
[Out] B*b**5*x**3/3 + b**4*(A*b + 5*B*a)*log(x) + (-12*A*a**5 + x**12*(-300*A*a*b
**4 - 600*B*a**2*b**3) + x**9*(-300*A*a**2*b**3 - 300*B*a**3*b**2) + x**6*(
-200*A*a**3*b**2 - 100*B*a**4*b) + x**3*(-75*A*a**4*b - 15*B*a**5))/(180*x*
*15)
```

3.49 $\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx$

Optimal. Leaf size=115

$$-\frac{a^5A}{16x^{16}}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{x}-\frac{5ab^3(2aB+Ab)}{4x^4}+\frac{1}{2}b^5Bx^2$$

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{7x^7}-\frac{a^4(aB+5Ab)}{13x^{13}}-\frac{a^3b(aB+2Ab)}{2x^{10}}-\frac{a^5A}{16x^{16}}-\frac{5ab^3(2aB+Ab)}{4x^4}-\frac{b^4(5aB+Ab)}{x}+\frac{1}{2}b^5Bx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -(a^5*A)/(16*x^16) - (a^4*(5*A*b + a*B))/(13*x^13) - (a^3*b*(2*A*b + a*B))/
(2*x^10) - (10*a^2*b^2*(A*b + a*B))/(7*x^7) - (5*a*b^3*(A*b + 2*a*B))/(4*x^
4) - (b^4*(A*b + 5*a*B))/x + (b^5*B*x^2)/2
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5(A+Bx^3)}{x^{17}}\,dx = \int \left(\frac{a^5A}{x^{17}}+\frac{a^4(5Ab+aB)}{x^{14}}+\frac{5a^3b(2Ab+aB)}{x^{11}}+\frac{10a^2b^2(Ab+aB)}{x^8}+\frac{5ab^3(Ab+$$
$$= -\frac{a^5A}{16x^{16}}-\frac{a^4(5Ab+aB)}{13x^{13}}-\frac{a^3b(2Ab+aB)}{2x^{10}}-\frac{10a^2b^2(Ab+aB)}{7x^7}-\frac{5ab^3(Ab+2a}{4x^4}$$

Mathematica [A] time = 0.03, size = 118, normalized size = 1.03

$$-\frac{7a^5(13A+16Bx^3)+56a^4bx^3(10A+13Bx^3)+208a^3b^2x^6(7A+10Bx^3)+520a^2b^3x^9(4A+7Bx^3)+1820a}{1456x^{16}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^17,x]
```

```
[Out] -1/1456*(-728*b^5*x^15*(-2*A + B*x^3) + 1820*a*b^4*x^12*(A + 4*B*x^3) + 520
*a^2*b^3*x^9*(4*A + 7*B*x^3) + 208*a^3*b^2*x^6*(7*A + 10*B*x^3) + 56*a^4*b*
x^3*(10*A + 13*B*x^3) + 7*a^5*(13*A + 16*B*x^3))/x^16
```

fricas [A] time = 1.36, size = 121, normalized size = 1.05

$$\frac{728\,Bb^5x^{18}-1456\left(5\,Bab^4+Ab^5\right)x^{15}-1820\left(2\,Ba^2b^3+Aab^4\right)x^{12}-2080\left(Ba^3b^2+Aa^2b^3\right)x^9-728\left(Ba^4b+2\right.}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="fricas")
```

```
[Out] 1/1456*(728*B*b^5*x^18 - 1456*(5*B*a*b^4 + A*b^5)*x^15 - 1820*(2*B*a^2*b^3
+ A*a*b^4)*x^12 - 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 728*(B*a^4*b + 2*A*a^3
*b^2)*x^6 - 91*A*a^5 - 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

giac [A] time = 0.16, size = 128, normalized size = 1.11

$$\frac{1}{2}Bb^5x^2 - \frac{7280\,Bab^4x^{15} + 1456\,Ab^5x^{15} + 3640\,Ba^2b^3x^{12} + 1820\,Aab^4x^{12} + 2080\,Ba^3b^2x^9 + 2080\,Aa^2b^3x^9 + 728\,B}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="giac")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(7280*B*a*b^4*x^15 + 1456*A*b^5*x^15 + 3640*B*a^2*b^
3*x^12 + 1820*A*a*b^4*x^12 + 2080*B*a^3*b^2*x^9 + 2080*A*a^2*b^3*x^9 + 728*
B*a^4*b*x^6 + 1456*A*a^3*b^2*x^6 + 112*B*a^5*x^3 + 560*A*a^4*b*x^3 + 91*A*a
^5)/x^16
```

maple [A] time = 0.04, size = 104, normalized size = 0.90

$$\frac{B\,b^5x^2}{2} - \frac{(Ab+5Ba)\,b^4}{x} - \frac{5\,(Ab+2Ba)\,a\,b^3}{4x^4} - \frac{10\,(Ab+Ba)\,a^2b^2}{7x^7} - \frac{(2Ab+Ba)\,a^3b}{2x^{10}} - \frac{(5Ab+Ba)\,a^4}{13x^{13}} - \frac{A\,a^5}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^17,x)
```

```
[Out] -1/16*a^5*A/x^16-1/13*a^4*(5*A*b+B*a)/x^13-1/2*a^3*b*(2*A*b+B*a)/x^10-10/7*
a^2*b^2*(A*b+B*a)/x^7-5/4*a*b^3*(A*b+2*B*a)/x^4-b^4*(A*b+5*B*a)/x+1/2*b^5*B
*x^2
```

maxima [A] time = 0.50, size = 122, normalized size = 1.06

$$\frac{1}{2}Bb^5x^2 - \frac{1456\left(5\,Bab^4 + Ab^5\right)x^{15} + 1820\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 2080\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 728\left(Ba^4b + 2\,Aa^3b^2\right.}{1456\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^17,x, algorithm="maxima")
```

```
[Out] 1/2*B*b^5*x^2 - 1/1456*(1456*(5*B*a*b^4 + A*b^5)*x^15 + 1820*(2*B*a^2*b^3 +
 A*a*b^4)*x^12 + 2080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 728*(B*a^4*b + 2*A*a^3*
b^2)*x^6 + 91*A*a^5 + 112*(B*a^5 + 5*A*a^4*b)*x^3)/x^16
```

mupad [B] time = 2.36, size = 121, normalized size = 1.05

$$\frac{B\,b^5\,x^2}{2} - \frac{\frac{A\,a^5}{16} + x^6\left(\frac{B\,a^4\,b}{2} + A\,a^3\,b^2\right) + x^{12}\left(\frac{5\,B\,a^2\,b^3}{2} + \frac{5\,A\,a\,b^4}{4}\right) + x^3\left(\frac{B\,a^5}{13} + \frac{5\,A\,b\,a^4}{13}\right) + x^{15}\left(A\,b^5 + 5\,B\,a\,b^4\right) + x^9\left(\frac{10}{}\right.}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^17,x)
```

```
[Out] (B*b^5*x^2)/2 - ((A*a^5)/16 + x^6*(A*a^3*b^2 + (B*a^4*b)/2) + x^12*((5*B*a^
2*b^3)/2 + (5*A*a*b^4)/4) + x^3*((B*a^5)/13 + (5*A*a^4*b)/13) + x^15*(A*b^5
 + 5*B*a*b^4) + x^9*((10*A*a^2*b^3)/7 + (10*B*a^3*b^2)/7))/x^16
```

sympy [A] time = 66.09, size = 134, normalized size = 1.17

$$\frac{Bb^5x^2}{2} + \frac{-91Aa^5 + x^{15}\left(-1456Ab^5 - 7280Bab^4\right) + x^{12}\left(-1820Aab^4 - 3640Ba^2b^3\right) + x^9\left(-2080Aa^2b^3 - 2080Ba^3b\right.}{1456x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**17,x)
```

```
[Out] B*b**5*x**2/2 + (-91*A*a**5 + x**15*(-1456*A*b**5 - 7280*B*a*b**4) + x**12*
(-1820*A*a*b**4 - 3640*B*a**2*b**3) + x**9*(-2080*A*a**2*b**3 - 2080*B*a**3
*b**2) + x**6*(-1456*A*a**3*b**2 - 728*B*a**4*b) + x**3*(-560*A*a**4*b - 11
2*B*a**5))/(1456*x**16)
```

3.50 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx$

Optimal. Leaf size=110

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

Rubi [A] time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{a^5A}{17x^{17}}-\frac{ab^3(2aB+Ab)}{x^5}-\frac{b^4(5aB+Ab)}{2x^2}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -(a^5*A)/(17*x^17) - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{18}} \, dx &= \int \left(b^5B+\frac{a^5A}{x^{18}}+\frac{a^4(5Ab+aB)}{x^{15}}+\frac{5a^3b(2Ab+aB)}{x^{12}}+\frac{10a^2b^2(Ab+aB)}{x^9}+\frac{5ab^3(Al}{x}\right.\\ &= -\frac{a^5A}{17x^{17}}-\frac{a^4(5Ab+aB)}{14x^{14}}-\frac{5a^3b(2Ab+aB)}{11x^{11}}-\frac{5a^2b^2(Ab+aB)}{4x^8}-\frac{ab^3(Ab+2aB)}{x^5}-\end{aligned}$$

Mathematica [A] time = 0.05, size = 110, normalized size = 1.00

$$-\frac{a^5A}{17x^{17}}-\frac{a^4(aB+5Ab)}{14x^{14}}-\frac{5a^3b(aB+2Ab)}{11x^{11}}-\frac{5a^2b^2(aB+Ab)}{4x^8}-\frac{b^4(5aB+Ab)}{2x^2}-\frac{ab^3(2aB+Ab)}{x^5}+b^5Bx$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^18,x]
```

```
[Out] -1/17*(a^5*A)/x^17 - (a^4*(5*A*b + a*B))/(14*x^14) - (5*a^3*b*(2*A*b + a*B)
)/(11*x^11) - (5*a^2*b^2*(A*b + a*B))/(4*x^8) - (a*b^3*(A*b + 2*a*B))/x^5 -
 (b^4*(A*b + 5*a*B))/(2*x^2) + b^5*B*x
```

fricas [A] time = 1.28, size = 121, normalized size = 1.10

$$\frac{5236\,Bb^5x^{18}-2618\left(5\,Bab^4+Ab^5\right)x^{15}-5236\left(2\,Ba^2b^3+Aab^4\right)x^{12}-6545\left(Ba^3b^2+Aa^2b^3\right)x^9-2380\left(Ba^4b+2\right.}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="fricas")
```

```
[Out] 1/5236*(5236*B*b^5*x^18 - 2618*(5*B*a*b^4 + A*b^5)*x^15 - 5236*(2*B*a^2*b^3
 + A*a*b^4)*x^12 - 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 2380*(B*a^4*b + 2*A*a
^3*b^2)*x^6 - 308*A*a^5 - 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

giac [A] time = 0.17, size = 125, normalized size = 1.14

$$Bb^5x - \frac{13090\,Bab^4x^{15} + 2618\,Ab^5x^{15} + 10472\,Ba^2b^3x^{12} + 5236\,Aab^4x^{12} + 6545\,Ba^3b^2x^9 + 6545\,Aa^2b^3x^9 + 23}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="giac")
```

```
[Out] B*b^5*x - 1/5236*(13090*B*a*b^4*x^15 + 2618*A*b^5*x^15 + 10472*B*a^2*b^3*x^
12 + 5236*A*a*b^4*x^12 + 6545*B*a^3*b^2*x^9 + 6545*A*a^2*b^3*x^9 + 2380*B*a
^4*b*x^6 + 4760*A*a^3*b^2*x^6 + 374*B*a^5*x^3 + 1870*A*a^4*b*x^3 + 308*A*a^
5)/x^17
```

maple [A] time = 0.05, size = 101, normalized size = 0.92

$$B\,b^5x - \frac{(Ab+5Ba)\,b^4}{2x^2} - \frac{(Ab+2Ba)\,a\,b^3}{x^5} - \frac{5\,(Ab+Ba)\,a^2b^2}{4x^8} - \frac{5\,(2Ab+Ba)\,a^3b}{11x^{11}} - \frac{(5Ab+Ba)\,a^4}{14x^{14}} - \frac{A\,a^5}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^18,x)
```

```
[Out] -1/17*a^5*A/x^17-1/14*a^4*(5*A*b+B*a)/x^14-5/11*a^3*b*(2*A*b+B*a)/x^11-5/4*
a^2*b^2*(A*b+B*a)/x^8-a*b^3*(A*b+2*B*a)/x^5-1/2*b^4*(A*b+5*B*a)/x^2+b^5*B*x
```

maxima [A] time = 0.52, size = 119, normalized size = 1.08

$$Bb^5x - \frac{2618\left(5\,Bab^4 + Ab^5\right)x^{15} + 5236\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 6545\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 2380\left(Ba^4b + 2\,Aa^3b\right.}{5236\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^18,x, algorithm="maxima")
```

```
[Out] B*b^5*x - 1/5236*(2618*(5*B*a*b^4 + A*b^5)*x^15 + 5236*(2*B*a^2*b^3 + A*a*b
^4)*x^12 + 6545*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 2380*(B*a^4*b + 2*A*a^3*b^2)*
x^6 + 308*A*a^5 + 374*(B*a^5 + 5*A*a^4*b)*x^3)/x^17
```

mupad [B] time = 0.08, size = 119, normalized size = 1.08

$$B\,b^5\,x - \frac{\frac{A\,a^5}{17} + x^{12}\,\left(2\,B\,a^2\,b^3 + A\,a\,b^4\right) + x^6\,\left(\frac{5\,B\,a^4\,b}{11} + \frac{10\,A\,a^3\,b^2}{11}\right) + x^3\,\left(\frac{B\,a^5}{14} + \frac{5\,A\,b\,a^4}{14}\right) + x^{15}\,\left(\frac{A\,b^5}{2} + \frac{5\,B\,a\,b^4}{2}\right) + x^9}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^18,x)
```

```
[Out] B*b^5*x - ((A*a^5)/17 + x^12*(2*B*a^2*b^3 + A*a*b^4) + x^6*((10*A*a^3*b^2)/
11 + (5*B*a^4*b)/11) + x^3*((B*a^5)/14 + (5*A*a^4*b)/14) + x^15*((A*b^5)/2
+ (5*B*a*b^4)/2) + x^9*((5*A*a^2*b^3)/4 + (5*B*a^3*b^2)/4))/x^17
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**18,x)
```

```
[Out] Timed out
```

3.51 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} \, dx$

Optimal. Leaf size=91

$$-\frac{a^5B}{15x^{15}}-\frac{5a^4bB}{12x^{12}}-\frac{10a^3b^2B}{9x^9}-\frac{5a^2b^3B}{3x^6}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

```
[Out] -1/15*a^5*B/x^15-5/12*a^4*b*B/x^12-10/9*a^3*b^2*B/x^9-5/3*a^2*b^3*B/x^6-5/3
*a*b^4*B/x^3-1/18*A*(b*x^3+a)^6/a/x^18+b^5*B*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 43}

$$-\frac{5a^2b^3B}{3x^6}-\frac{10a^3b^2B}{9x^9}-\frac{5a^4bB}{12x^{12}}-\frac{a^5B}{15x^{15}}-\frac{A\left(a+bx^3\right)^6}{18ax^{18}}-\frac{5ab^4B}{3x^3}+b^5B\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -(a^5*B)/(15*x^15) - (5*a^4*b*B)/(12*x^12) - (10*a^3*b^2*B)/(9*x^9) - (5*a^
2*b^3*B)/(3*x^6) - (5*a*b^4*B)/(3*x^3) - (A*(a + b*x^3)^6)/(18*a*x^18) + b^
5*B*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{19}} dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^7} dx, x, x^3\right)$$
$$= -\frac{A(a+bx^3)^6}{18ax^{18}} + \frac{1}{3}B\,\text{Subst}\left(\int \frac{(a+bx)^5}{x^6} dx, x, x^3\right)$$
$$= -\frac{A(a+bx^3)^6}{18ax^{18}} + \frac{1}{3}B\,\text{Subst}\left(\int \left(\frac{a^5}{x^6} + \frac{5a^4b}{x^5} + \frac{10a^3b^2}{x^4} + \frac{10a^2b^3}{x^3} + \frac{5ab^4}{x^2} + \frac{b^5}{x}\right) dx,\right.$$
$$= -\frac{a^5B}{15x^{15}} - \frac{5a^4bB}{12x^{12}} - \frac{10a^3b^2B}{9x^9} - \frac{5a^2b^3B}{3x^6} - \frac{5ab^4B}{3x^3} - \frac{A(a+bx^3)^6}{18ax^{18}} + b^5B\log(x)$$

Mathematica [A] time = 0.05, size = 121, normalized size = 1.33

$$-\frac{2a^5(5A+6Bx^3) + 15a^4bx^3(4A+5Bx^3) + 50a^3b^2x^6(3A+4Bx^3) + 100a^2b^3x^9(2A+3Bx^3) + 150ab^4x^{12}(A+}{180x^{18}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^19,x]
```

```
[Out] -1/180*(60*A*b^5*x^15 + 150*a*b^4*x^12*(A + 2*B*x^3) + 100*a^2*b^3*x^9*(2*A
 + 3*B*x^3) + 50*a^3*b^2*x^6*(3*A + 4*B*x^3) + 15*a^4*b*x^3*(4*A + 5*B*x^3)
 + 2*a^5*(5*A + 6*B*x^3) - 180*b^5*B*x^18*Log[x])/x^18
```

fricas [A] time = 1.39, size = 123, normalized size = 1.35

$$\frac{180\,Bb^5x^{18}\log(x) - 60\left(5\,Bab^4 + Ab^5\right)x^{15} - 150\left(2\,Ba^2b^3 + Aab^4\right)x^{12} - 200\left(Ba^3b^2 + Aa^2b^3\right)x^9 - 75\left(Ba^4b + 2\,A\right.}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="fricas")
```

```
[Out] 1/180*(180*B*b^5*x^18*log(x) - 60*(5*B*a*b^4 + A*b^5)*x^15 - 150*(2*B*a^2*b
^3 + A*a*b^4)*x^12 - 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 - 75*(B*a^4*b + 2*A*a^
3*b^2)*x^6 - 10*A*a^5 - 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

giac [A] time = 0.16, size = 136, normalized size = 1.49

$$Bb^5\log(|x|) - \frac{147\,Bb^5x^{18} + 300\,Bab^4x^{15} + 60\,Ab^5x^{15} + 300\,Ba^2b^3x^{12} + 150\,Aab^4x^{12} + 200\,Ba^3b^2x^9 + 200\,Aa^2b^3x}{180\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="giac")
```

```
[Out] B*b^5*log(abs(x)) - 1/180*(147*B*b^5*x^18 + 300*B*a*b^4*x^15 + 60*A*b^5*x^1
5 + 300*B*a^2*b^3*x^12 + 150*A*a*b^4*x^12 + 200*B*a^3*b^2*x^9 + 200*A*a^2*b
^3*x^9 + 75*B*a^4*b*x^6 + 150*A*a^3*b^2*x^6 + 12*B*a^5*x^3 + 60*A*a^4*b*x^3
 + 10*A*a^5)/x^18
```

maple [A] time = 0.05, size = 124, normalized size = 1.36

$$B\,b^5\ln(x) - \frac{A\,b^5}{3x^3} - \frac{5Ba\,b^4}{3x^3} - \frac{5Aa\,b^4}{6x^6} - \frac{5B\,a^2b^3}{3x^6} - \frac{10A\,a^2b^3}{9x^9} - \frac{10B\,a^3b^2}{9x^9} - \frac{5A\,a^3b^2}{6x^{12}} - \frac{5B\,a^4b}{12x^{12}} - \frac{A\,a^4b}{3x^{15}} - \frac{B\,a^5}{15x^{15}} - \frac{A\,a^5}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^19,x)
```

```
[Out] -5/6*a^3*b^2/x^12*A-5/12*a^4*b*B/x^12-1/3*b^5/x^3*A-5/3*a*b^4*B/x^3-1/18*A*
a^5/x^18-1/3*a^4/x^15*A*b-1/15*a^5*B/x^15-10/9*b^3*a^2/x^9*A-10/9*a^3*b^2*B
/x^9-5/6*a*b^4/x^6*A-5/3*a^2*b^3*B/x^6+b^5*B*ln(x)
```

maxima [A] time = 0.51, size = 123, normalized size = 1.35

$$\frac{1}{3}Bb^5\log\left(x^3\right)-\frac{60\left(5Bab^4+Ab^5\right)x^{15}+150\left(2Ba^2b^3+Aab^4\right)x^{12}+200\left(Ba^3b^2+Aa^2b^3\right)x^9+75\left(Ba^4b+2Aa\right.}{180x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^19,x, algorithm="maxima")
```

```
[Out] 1/3*B*b^5*log(x^3) - 1/180*(60*(5*B*a*b^4 + A*b^5)*x^15 + 150*(2*B*a^2*b^3
+ A*a*b^4)*x^12 + 200*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 75*(B*a^4*b + 2*A*a^3*b
^2)*x^6 + 10*A*a^5 + 12*(B*a^5 + 5*A*a^4*b)*x^3)/x^18
```

mupad [B] time = 0.09, size = 121, normalized size = 1.33

$$Bb^5\ln(x)-\frac{\frac{Aa^5}{18}+x^{12}\left(\frac{5Ba^2b^3}{3}+\frac{5Aab^4}{6}\right)+x^6\left(\frac{5Ba^4b}{12}+\frac{5Aa^3b^2}{6}\right)+x^3\left(\frac{Ba^5}{15}+\frac{Aba^4}{3}\right)+x^{15}\left(\frac{Ab^5}{3}+\frac{5Bab^4}{3}\right)+x^9}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^19,x)
```

```
[Out] B*b^5*log(x) - ((A*a^5)/18 + x^12*((5*B*a^2*b^3)/3 + (5*A*a*b^4)/6) + x^6*(
(5*A*a^3*b^2)/6 + (5*B*a^4*b)/12) + x^3*((B*a^5)/15 + (A*a^4*b)/3) + x^15*(
(A*b^5)/3 + (5*B*a*b^4)/3) + x^9*((10*A*a^2*b^3)/9 + (10*B*a^3*b^2)/9))/x^1
8
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**19,x)
```

```
[Out] Timed out
```

3.52 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx$

Optimal. Leaf size=113

$$-\frac{a^5A}{19x^{19}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^5B}{x}$$

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{a^2b^2(aB+Ab)}{x^{10}}-\frac{a^4(aB+5Ab)}{16x^{16}}-\frac{5a^3b(aB+2Ab)}{13x^{13}}-\frac{a^5A}{19x^{19}}-\frac{5ab^3(2aB+Ab)}{7x^7}-\frac{b^4(5aB+Ab)}{4x^4}-\frac{b^5B}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -(a^5*A)/(19*x^19) - (a^4*(5*A*b + a*B))/(16*x^16) - (5*a^3*b*(2*A*b + a*B)
)/(13*x^13) - (a^2*b^2*(A*b + a*B))/x^10 - (5*a*b^3*(A*b + 2*a*B))/(7*x^7)
- (b^4*(A*b + 5*a*B))/(4*x^4) - (b^5*B)/x
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{20}} \, dx = \int \left(\frac{a^5A}{x^{20}}+\frac{a^4(5Ab+aB)}{x^{17}}+\frac{5a^3b(2Ab+aB)}{x^{14}}+\frac{10a^2b^2(Ab+aB)}{x^{11}}+\frac{5ab^3(Ab+2a}{x^8}\right.$$
$$=-\frac{a^5A}{19x^{19}}-\frac{a^4(5Ab+aB)}{16x^{16}}-\frac{5a^3b(2Ab+aB)}{13x^{13}}-\frac{a^2b^2(Ab+aB)}{x^{10}}-\frac{5ab^3(Ab+2aB)}{7x^7}-$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.05

$$-\frac{91a^5(16A+19Bx^3)+665a^4bx^3(13A+16Bx^3)+2128a^3b^2x^6(10A+13Bx^3)+3952a^2b^3x^9(7A+10Bx^3)+49}{27664x^{19}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^20,x]
```

```
[Out] -1/27664*(6916*b^5*x^15*(A + 4*B*x^3) + 4940*a*b^4*x^12*(4*A + 7*B*x^3) + 3
952*a^2*b^3*x^9*(7*A + 10*B*x^3) + 2128*a^3*b^2*x^6*(10*A + 13*B*x^3) + 665
*a^4*b*x^3*(13*A + 16*B*x^3) + 91*a^5*(16*A + 19*B*x^3))/x^19
```

fricas [A] time = 1.25, size = 121, normalized size = 1.07

$$-\frac{27664\,Bb^5x^{18}+6916\,(5\,Bab^4+Ab^5)x^{15}+19760\,(2\,Ba^2b^3+Aab^4)x^{12}+27664\,(Ba^3b^2+Aa^2b^3)x^9+10640\,(Ba}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="fricas")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

giac [A] time = 0.15, size = 127, normalized size = 1.12

$$-\frac{27664\,Bb^5x^{18} + 34580\,Bab^4x^{15} + 6916\,Ab^5x^{15} + 39520\,Ba^2b^3x^{12} + 19760\,Aab^4x^{12} + 27664\,Ba^3b^2x^9 + 2766\cdots}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="giac")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 34580*B*a*b^4*x^15 + 6916*A*b^5*x^15 + 39520*B
*a^2*b^3*x^12 + 19760*A*a*b^4*x^12 + 27664*B*a^3*b^2*x^9 + 27664*A*a^2*b^3*
x^9 + 10640*B*a^4*b*x^6 + 21280*A*a^3*b^2*x^6 + 1729*B*a^5*x^3 + 8645*A*a^4
*b*x^3 + 1456*A*a^5)/x^19
```

maple [A] time = 0.04, size = 104, normalized size = 0.92

$$-\frac{B\,b^5}{x}-\frac{(Ab+5Ba)\,b^4}{4x^4}-\frac{5\,(Ab+2Ba)\,a\,b^3}{7x^7}-\frac{(Ab+Ba)\,a^2b^2}{x^{10}}-\frac{5\,(2Ab+Ba)\,a^3b}{13x^{13}}-\frac{(5Ab+Ba)\,a^4}{16x^{16}}-\frac{A\,a^5}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^20,x)
```

```
[Out] -1/19*a^5*A/x^19-1/16*a^4*(5*A*b+B*a)/x^16-5/13*a^3*b*(2*A*b+B*a)/x^13-a^2*
b^2*(A*b+B*a)/x^10-5/7*a*b^3*(A*b+2*B*a)/x^7-1/4*b^4*(A*b+5*B*a)/x^4-b^5*B/
x
```

maxima [A] time = 0.46, size = 121, normalized size = 1.07

$$-\frac{27664\,Bb^5x^{18} + 6916\left(5\,Bab^4 + Ab^5\right)x^{15} + 19760\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 27664\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 10640\cdots}{27664\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^20,x, algorithm="maxima")
```

```
[Out] -1/27664*(27664*B*b^5*x^18 + 6916*(5*B*a*b^4 + A*b^5)*x^15 + 19760*(2*B*a^2
*b^3 + A*a*b^4)*x^12 + 27664*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 10640*(B*a^4*b +
 2*A*a^3*b^2)*x^6 + 1456*A*a^5 + 1729*(B*a^5 + 5*A*a^4*b)*x^3)/x^19
```

mupad [B] time = 2.37, size = 119, normalized size = 1.05

$$-\frac{\frac{A\,a^5}{19}+x^{12}\left(\frac{10\,B\,a^2\,b^3}{7}+\frac{5\,A\,a\,b^4}{7}\right)+x^6\left(\frac{5\,B\,a^4\,b}{13}+\frac{10\,A\,a^3\,b^2}{13}\right)+x^3\left(\frac{B\,a^5}{16}+\frac{5\,A\,b\,a^4}{16}\right)+x^{15}\left(\frac{A\,b^5}{4}+\frac{5\,B\,a\,b^4}{4}\right)+x^9\,\left(B\,a^3\cdots\right.}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^20,x)
```

```
[Out] -((A*a^5)/19 + x^12*((10*B*a^2*b^3)/7 + (5*A*a*b^4)/7) + x^6*((10*A*a^3*b^2
)/13 + (5*B*a^4*b)/13) + x^3*((B*a^5)/16 + (5*A*a^4*b)/16) + x^15*((A*b^5)/
4 + (5*B*a*b^4)/4) + x^9*(A*a^2*b^3 + B*a^3*b^2) + B*b^5*x^18)/x^19
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**20,x)
```

```
[Out] Timed out
```

3.53 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx$

Optimal. Leaf size=117

$$-\frac{a^5 A}{20x^{20}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^5 B}{2x^2}$$

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{11x^{11}}-\frac{a^4(aB+5Ab)}{17x^{17}}-\frac{5a^3b(aB+2Ab)}{14x^{14}}-\frac{a^5 A}{20x^{20}}-\frac{5ab^3(2aB+Ab)}{8x^8}-\frac{b^4(5aB+Ab)}{5x^5}-\frac{b^5 B}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -(a^5*A)/(20*x^20) - (a^4*(5*A*b + a*B))/(17*x^17) - (5*a^3*b*(2*A*b + a*B)
)/(14*x^14) - (10*a^2*b^2*(A*b + a*B))/(11*x^11) - (5*a*b^3*(A*b + 2*a*B))/
(8*x^8) - (b^4*(A*b + 5*a*B))/(5*x^5) - (b^5*B)/(2*x^2)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{21}} \, dx &= \int \left(\frac{a^5 A}{x^{21}}+\frac{a^4(5Ab+aB)}{x^{18}}+\frac{5a^3b(2Ab+aB)}{x^{15}}+\frac{10a^2b^2(Ab+aB)}{x^{12}}+\frac{5ab^3(Ab+\right.\\ &= -\frac{a^5 A}{20x^{20}}-\frac{a^4(5Ab+aB)}{17x^{17}}-\frac{5a^3b(2Ab+aB)}{14x^{14}}-\frac{10a^2b^2(Ab+aB)}{11x^{11}}-\frac{5ab^3(Ab+2a}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.04, size = 121, normalized size = 1.03

$$-\frac{154a^5(17A+20Bx^3)+1100a^4bx^3(14A+17Bx^3)+3400a^3b^2x^6(11A+14Bx^3)+5950a^2b^3x^9(8A+11Bx^3)}{52360x^{20}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^21,x]
```

```
[Out] -1/52360*(5236*b^5*x^15*(2*A + 5*B*x^3) + 6545*a*b^4*x^12*(5*A + 8*B*x^3) +
 5950*a^2*b^3*x^9*(8*A + 11*B*x^3) + 3400*a^3*b^2*x^6*(11*A + 14*B*x^3) + 1
100*a^4*b*x^3*(14*A + 17*B*x^3) + 154*a^5*(17*A + 20*B*x^3))/x^20
```

fricas [A] time = 0.86, size = 121, normalized size = 1.03

$$-\frac{26180\,Bb^5x^{18}+10472\left(5\,Bab^4+Ab^5\right)x^{15}+32725\left(2\,Ba^2b^3+Aab^4\right)x^{12}+47600\left(Ba^3b^2+Aa^2b^3\right)x^9+1870}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="fricas")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

giac [A] time = 0.15, size = 127, normalized size = 1.09

$$-\frac{26180\,Bb^5x^{18}+52360\,Bab^4x^{15}+10472\,Ab^5x^{15}+65450\,Ba^2b^3x^{12}+32725\,Aab^4x^{12}+47600\,Ba^3b^2x^9+47600\,\ldots}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="giac")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 52360*B*a*b^4*x^15 + 10472*A*b^5*x^15 + 65450*
B*a^2*b^3*x^12 + 32725*A*a*b^4*x^12 + 47600*B*a^3*b^2*x^9 + 47600*A*a^2*b^3
*x^9 + 18700*B*a^4*b*x^6 + 37400*A*a^3*b^2*x^6 + 3080*B*a^5*x^3 + 15400*A*a
^4*b*x^3 + 2618*A*a^5)/x^20
```

maple [A] time = 0.05, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{2x^2}-\frac{(Ab+5Ba)\,b^4}{5x^5}-\frac{5\,(Ab+2Ba)\,a\,b^3}{8x^8}-\frac{10\,(Ab+Ba)\,a^2b^2}{11x^{11}}-\frac{5\,(2Ab+Ba)\,a^3b}{14x^{14}}-\frac{(5Ab+Ba)\,a^4}{17x^{17}}-\frac{A\,a^5}{20x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^21,x)
```

```
[Out] -1/20*a^5*A/x^20-1/17*a^4*(5*A*b+B*a)/x^17-5/14*a^3*b*(2*A*b+B*a)/x^14-10/1
1*a^2*b^2*(A*b+B*a)/x^11-5/8*a*b^3*(A*b+2*B*a)/x^8-1/5*b^4*(A*b+5*B*a)/x^5-
1/2*b^5*B/x^2
```

maxima [A] time = 0.55, size = 121, normalized size = 1.03

$$-\frac{26180\,Bb^5x^{18}+10472\left(5\,Bab^4+Ab^5\right)x^{15}+32725\left(2\,Ba^2b^3+Aab^4\right)x^{12}+47600\left(Ba^3b^2+Aa^2b^3\right)x^9+18700\left(\ldots\right.}{52360\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^21,x, algorithm="maxima")
```

```
[Out] -1/52360*(26180*B*b^5*x^18 + 10472*(5*B*a*b^4 + A*b^5)*x^15 + 32725*(2*B*a^
2*b^3 + A*a*b^4)*x^12 + 47600*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 18700*(B*a^4*b
+ 2*A*a^3*b^2)*x^6 + 2618*A*a^5 + 3080*(B*a^5 + 5*A*a^4*b)*x^3)/x^20
```

mupad [B] time = 2.35, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{20}+x^{12}\left(\frac{5\,B\,a^2\,b^3}{4}+\frac{5\,A\,a\,b^4}{8}\right)+x^6\left(\frac{5\,B\,a^4\,b}{14}+\frac{5\,A\,a^3\,b^2}{7}\right)+x^3\left(\frac{B\,a^5}{17}+\frac{5\,A\,b\,a^4}{17}\right)+x^{15}\left(\frac{A\,b^5}{5}+B\,a\,b^4\right)+x^9\left(\frac{10\,B\,a^3\,b^2}{11}+\ldots\right.}{x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^21,x)
```

```
[Out] -((A*a^5)/20 + x^12*((5*B*a^2*b^3)/4 + (5*A*a*b^4)/8) + x^6*((5*A*a^3*b^2)/
7 + (5*B*a^4*b)/14) + x^3*((B*a^5)/17 + (5*A*a^4*b)/17) + x^15*((A*b^5)/5 +
 B*a*b^4) + x^9*((10*A*a^2*b^3)/11 + (10*B*a^3*b^2)/11) + (B*b^5*x^18)/2)/x
^20
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**21,x)
```

```
[Out] Timed out
```

3.54 $\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx$

Optimal. Leaf size=48

$$\frac{\left(a+bx^3\right)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A\left(a+bx^3\right)^6}{21ax^{21}}$$

```
[Out] -1/21*A*(b*x^3+a)^6/a/x^21+1/126*(A*b-7*B*a)*(b*x^3+a)^6/a^2/x^18
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 78, 37}

$$\frac{\left(a+bx^3\right)^6(Ab-7aB)}{126a^2x^{18}}-\frac{A\left(a+bx^3\right)^6}{21ax^{21}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -(A*(a + b*x^3)^6)/(21*a*x^21) + ((A*b - 7*a*B)*(a + b*x^3)^6)/(126*a^2*x^1
8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^5\left(A+Bx^3\right)}{x^{22}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^5(A+Bx)}{x^8}\,dx,x,x^3\right)\\ &= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}}+\frac{(-Ab+7aB)\operatorname{Subst}\left(\int \frac{(a+bx)^5}{x^7}\,dx,x,x^3\right)}{21a}\\ &= -\frac{A\left(a+bx^3\right)^6}{21ax^{21}}+\frac{(Ab-7aB)\left(a+bx^3\right)^6}{126a^2x^{18}}\end{aligned}$$

Mathematica [B] time = 0.04, size = 118, normalized size = 2.46

$$-\frac{a^5\left(6A+7Bx^3\right)+7a^4bx^3\left(5A+6Bx^3\right)+21a^3b^2x^6\left(4A+5Bx^3\right)+35a^2b^3x^9\left(3A+4Bx^3\right)+35ab^4x^{12}\left(2A+\right.}{126x^{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^22,x]
```

```
[Out] -1/126*(21*b^5*x^15*(A + 2*B*x^3) + 35*a*b^4*x^12*(2*A + 3*B*x^3) + 35*a^2*
b^3*x^9*(3*A + 4*B*x^3) + 21*a^3*b^2*x^6*(4*A + 5*B*x^3) + 7*a^4*b*x^3*(5*A
 + 6*B*x^3) + a^5*(6*A + 7*B*x^3))/x^21
```

fricas [B] time = 0.82, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right.}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="fricas")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

giac [B] time = 0.18, size = 127, normalized size = 2.65

$$-\frac{42\,Bb^5x^{18}+105\,Bab^4x^{15}+21\,Ab^5x^{15}+140\,Ba^2b^3x^{12}+70\,Aab^4x^{12}+105\,Ba^3b^2x^9+105\,Aa^2b^3x^9+42\,Ba^4b}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="giac")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 105*B*a*b^4*x^15 + 21*A*b^5*x^15 + 140*B*a^2*b^3*x^
12 + 70*A*a*b^4*x^12 + 105*B*a^3*b^2*x^9 + 105*A*a^2*b^3*x^9 + 42*B*a^4*b*x
^6 + 84*A*a^3*b^2*x^6 + 7*B*a^5*x^3 + 35*A*a^4*b*x^3 + 6*A*a^5)/x^21
```

maple [B] time = 0.05, size = 104, normalized size = 2.17

$$-\frac{B\,b^5}{3x^3}-\frac{(Ab+5Ba)\,b^4}{6x^6}-\frac{5\,(Ab+2Ba)\,a\,b^3}{9x^9}-\frac{5\,(Ab+Ba)\,a^2b^2}{6x^{12}}-\frac{(2Ab+Ba)\,a^3b}{3x^{15}}-\frac{(5Ab+Ba)\,a^4}{18x^{18}}-\frac{A\,a^5}{21x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^22,x)
```

```
[Out] -5/6*b^2*a^2*(A*b+B*a)/x^12-1/3*B*b^5/x^3-1/18*a^4*(5*A*b+B*a)/x^18-1/3*a^3
*b*(2*A*b+B*a)/x^15-5/9*a*b^3*(A*b+2*B*a)/x^9-1/6*b^4*(A*b+5*B*a)/x^6-1/21*
A*a^5/x^21
```

maxima [B] time = 0.61, size = 121, normalized size = 2.52

$$-\frac{42\,Bb^5x^{18}+21\left(5\,Bab^4+Ab^5\right)x^{15}+70\left(2\,Ba^2b^3+Aab^4\right)x^{12}+105\left(Ba^3b^2+Aa^2b^3\right)x^9+42\left(Ba^4b+2\,Aa^3b^2\right.}{126\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^22,x, algorithm="maxima")
```

```
[Out] -1/126*(42*B*b^5*x^18 + 21*(5*B*a*b^4 + A*b^5)*x^15 + 70*(2*B*a^2*b^3 + A*a
*b^4)*x^12 + 105*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 42*(B*a^4*b + 2*A*a^3*b^2)*x
^6 + 6*A*a^5 + 7*(B*a^5 + 5*A*a^4*b)*x^3)/x^21
```

mupad [B] time = 2.37, size = 122, normalized size = 2.54

$$-\frac{\frac{A a^5}{21} + x^6 \left(\frac{B a^4 b}{3} + \frac{2 A a^3 b^2}{3}\right) + x^{12} \left(\frac{10 B a^2 b^3}{9} + \frac{5 A a b^4}{9}\right) + x^3 \left(\frac{B a^5}{18} + \frac{5 A b a^4}{18}\right) + x^{15} \left(\frac{A b^5}{6} + \frac{5 B a b^4}{6}\right) + x^9 \left(\frac{5 B a^3 b^2}{6} + \cdots\right.}{x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^22,x)
```

```
[Out] -((A*a^5)/21 + x^6*((2*A*a^3*b^2)/3 + (B*a^4*b)/3) + x^12*((10*B*a^2*b^3)/9
 + (5*A*a*b^4)/9) + x^3*((B*a^5)/18 + (5*A*a^4*b)/18) + x^15*((A*b^5)/6 + (
5*B*a*b^4)/6) + x^9*((5*A*a^2*b^3)/6 + (5*B*a^3*b^2)/6) + (B*b^5*x^18)/3)/x
^21
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**22,x)
```

```
[Out] Timed out
```

3.55 $\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx$

Optimal. Leaf size=117

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{a^5A}{22x^{22}}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -(a^5*A)/(22*x^22) - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^5 (A+Bx^3)}{x^{23}} dx &= \int \left(\frac{a^5A}{x^{23}}+\frac{a^4(5Ab+aB)}{x^{20}}+\frac{5a^3b(2Ab+aB)}{x^{17}}+\frac{10a^2b^2(Ab+aB)}{x^{14}}+\frac{5ab^3(Ab+\right.\\ &= -\frac{a^5A}{22x^{22}}-\frac{a^4(5Ab+aB)}{19x^{19}}-\frac{5a^3b(2Ab+aB)}{16x^{16}}-\frac{10a^2b^2(Ab+aB)}{13x^{13}}-\frac{ab^3(Ab+2a}{2x^{10}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 117, normalized size = 1.00

$$-\frac{a^5A}{22x^{22}}-\frac{a^4(aB+5Ab)}{19x^{19}}-\frac{5a^3b(aB+2Ab)}{16x^{16}}-\frac{10a^2b^2(aB+Ab)}{13x^{13}}-\frac{b^4(5aB+Ab)}{7x^7}-\frac{ab^3(2aB+Ab)}{2x^{10}}-\frac{b^5B}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^5*(A + B*x^3))/x^23,x]
```

```
[Out] -1/22*(a^5*A)/x^22 - (a^4*(5*A*b + a*B))/(19*x^19) - (5*a^3*b*(2*A*b + a*B)
)/(16*x^16) - (10*a^2*b^2*(A*b + a*B))/(13*x^13) - (a*b^3*(A*b + 2*a*B))/(2
*x^10) - (b^4*(A*b + 5*a*B))/(7*x^7) - (b^5*B)/(4*x^4)
```

fricas [A] time = 0.85, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18}+43472\left(5\,Bab^4+Ab^5\right)x^{15}+152152\left(2\,Ba^2b^3+Aab^4\right)x^{12}+234080\left(Ba^3b^2+Aa^2b^3\right)x^9+950}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="fricas")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

giac [A] time = 0.16, size = 127, normalized size = 1.09

$$-\frac{76076\,Bb^5x^{18} + 217360\,Bab^4x^{15} + 43472\,Ab^5x^{15} + 304304\,Ba^2b^3x^{12} + 152152\,Aab^4x^{12} + 234080\,Ba^3b^2x^9 + 23}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="giac")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 217360*B*a*b^4*x^15 + 43472*A*b^5*x^15 + 3043
04*B*a^2*b^3*x^12 + 152152*A*a*b^4*x^12 + 234080*B*a^3*b^2*x^9 + 234080*A*a
^2*b^3*x^9 + 95095*B*a^4*b*x^6 + 190190*A*a^3*b^2*x^6 + 16016*B*a^5*x^3 + 8
0080*A*a^4*b*x^3 + 13832*A*a^5)/x^22
```

maple [A] time = 0.05, size = 104, normalized size = 0.89

$$-\frac{B\,b^5}{4x^4}-\frac{(Ab+5Ba)\,b^4}{7x^7}-\frac{(Ab+2Ba)\,a\,b^3}{2x^{10}}-\frac{10\,(Ab+Ba)\,a^2b^2}{13x^{13}}-\frac{5\,(2Ab+Ba)\,a^3b}{16x^{16}}-\frac{(5Ab+Ba)\,a^4}{19x^{19}}-\frac{A\,a^5}{22x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^5*(B*x^3+A)/x^23,x)
```

```
[Out] -1/22*a^5*A/x^22-1/19*a^4*(5*A*b+B*a)/x^19-5/16*a^3*b*(2*A*b+B*a)/x^16-10/1
3*a^2*b^2*(A*b+B*a)/x^13-1/2*a*b^3*(A*b+2*B*a)/x^10-1/7*b^4*(A*b+5*B*a)/x^7
-1/4*b^5*B/x^4
```

maxima [A] time = 0.70, size = 121, normalized size = 1.03

$$-\frac{76076\,Bb^5x^{18} + 43472\left(5\,Bab^4 + Ab^5\right)x^{15} + 152152\left(2\,Ba^2b^3 + Aab^4\right)x^{12} + 234080\left(Ba^3b^2 + Aa^2b^3\right)x^9 + 95095}{304304\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^5*(B*x^3+A)/x^23,x, algorithm="maxima")
```

```
[Out] -1/304304*(76076*B*b^5*x^18 + 43472*(5*B*a*b^4 + A*b^5)*x^15 + 152152*(2*B*
a^2*b^3 + A*a*b^4)*x^12 + 234080*(B*a^3*b^2 + A*a^2*b^3)*x^9 + 95095*(B*a^4
*b + 2*A*a^3*b^2)*x^6 + 13832*A*a^5 + 16016*(B*a^5 + 5*A*a^4*b)*x^3)/x^22
```

mupad [B] time = 0.06, size = 121, normalized size = 1.03

$$-\frac{\frac{A\,a^5}{22} + x^{12}\left(B\,a^2\,b^3 + \frac{A\,a\,b^4}{2}\right) + x^6\left(\frac{5\,B\,a^4\,b}{16} + \frac{5\,A\,a^3\,b^2}{8}\right) + x^3\left(\frac{B\,a^5}{19} + \frac{5\,A\,b\,a^4}{19}\right) + x^{15}\left(\frac{A\,b^5}{7} + \frac{5\,B\,a\,b^4}{7}\right) + x^9\left(\frac{10\,B\,a^3\,b^2}{13} + \right.}{x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^5)/x^23,x)
```

```
[Out] -((A*a^5)/22 + x^12*(B*a^2*b^3 + (A*a*b^4)/2) + x^6*((5*A*a^3*b^2)/8 + (5*B
*a^4*b)/16) + x^3*((B*a^5)/19 + (5*A*a^4*b)/19) + x^15*((A*b^5)/7 + (5*B*a*
b^4)/7) + x^9*((10*A*a^2*b^3)/13 + (10*B*a^3*b^2)/13) + (B*b^5*x^18)/4)/x^2
2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**5*(B*x**3+A)/x**23,x)
```

```
[Out] Timed out
```

3.56 $\int \frac{x^6(A+Bx^3)}{a+bx^3} dx$

Optimal. Leaf size=183

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}-$$

```
[Out] -a*(A*b-B*a)*x/b^3+1/4*(A*b-B*a)*x^4/b^2+1/7*B*x^7/b+1/3*a^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/b^(10/3)-1/6*a^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/b^(10/3)-1/3*a^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(10/3)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{10/3}}+\frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{10/3}}-\frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{10/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] -((a*(A*b - a*B)*x)/b^3) + ((A*b - a*B)*x^4)/(4*b^2) + (B*x^7)/(7*b) - (a^(
4/3)*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*b^(10/3)) + (a^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(10/3)) -
 (a^(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^
(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{7b}$$

$$= \frac{Bx^7}{7b} - \frac{(-7Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{7b}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^2(Ab-aB)\right)\int \frac{1}{a+bx^3}\,dx}{b^3}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{\left(a^{4/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^3} + \frac{\left(a^{4/3}(Ab-aB)\right.}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{\left(a^{4/3}(Ab-a\right.}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} + \frac{a^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{10/3}} - \frac{a^{4/3}(Ab-aB}{}$$

$$= -\frac{a(Ab-aB)x}{b^3} + \frac{(Ab-aB)x^4}{4b^2} + \frac{Bx^7}{7b} - \frac{a^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{10/3}} + \frac{a^{4/3}(Ab-aB}{}$$

Mathematica [A] time = 0.13, size = 171, normalized size = 0.93

$$\frac{14a^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-28a^{4/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)+28\sqrt{3}\,a^{4/3}(aB-Ab)\tan^{-}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (84*a*b^(1/3)*(-(A*b) + a*B)*x + 21*b^(4/3)*(A*b - a*B)*x^4 + 12*b^(7/3)*B*
x^7 + 28*Sqrt[3]*a^(4/3)*(-(A*b) + a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]] - 28*a^(4/3)*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*
(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*b^(10/3)
)
```

fricas [A] time = 0.83, size = 167, normalized size = 0.91

$$\frac{12\,Bb^2x^7-21\left(Bab-Ab^2\right)x^4-28\,\sqrt{3}\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+14\left(Ba^2-Aab\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-\cdots\right.}{84\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(12*B*b^2*x^7 - 21*(B*a*b - A*b^2)*x^4 - 28*sqrt(3)*(B*a^2 - A*a*b)*(a
/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(B*a^2
 - A*a*b)*(a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(B*a^2 -
A*a*b)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(B*a^2 - A*a*b)*x)/b^3
```

giac [A] time = 0.24, size = 217, normalized size = 1.19

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba^2-\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(\cdots\right.\right.}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a^2 - (-a*b^2)^(1/3)*A*a*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/6*((-a*b^2)^(1/3)*B*a^2 - (-a
*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 1/3*(B*a^
3*b^4 - A*a^2*b^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^7) + 1/28*(
4*B*b^6*x^7 - 7*B*a*b^5*x^4 + 7*A*b^6*x^4 + 28*B*a^2*b^4*x - 28*A*a*b^5*x)/
b^7
```

maple [A] time = 0.04, size = 249, normalized size = 1.36

$$\frac{Bx^7}{7b}+\frac{Ax^4}{4b}-\frac{Bax^4}{4b^2}+\frac{\sqrt{3}\;Aa^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{Aa^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{Aa^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{Aax}{b^2}-\cdots\sqrt{3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/7*B*x^7/b+1/4/b*A*x^4-1/4/b^2*B*x^4*a-1/b^2*a*A*x+1/b^3*a^2*B*x+1/3*a^2/b
^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3*a^3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)*B-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^3/b^4
/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3*a^2/b^3/(a/b)^(2/3)*3^
```

```
(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3*a^3/b^4/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.43, size = 182, normalized size = 0.99

$$\frac{4\,Bb^{2}x^{7}-7\left(Bab-Ab^{2}\right)x^{4}+28\left(Ba^{2}-Aab\right)x}{28\,b^{3}}-\frac{\sqrt{3}\left(Ba^{3}-Aa^{2}b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Ba^{3}-Aa^{2}b\right)\log\left(x\right.}{6\,b^{4}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*B*b^2*x^7 - 7*(B*a*b - A*b^2)*x^4 + 28*(B*a^2 - A*a*b)*x)/b^3 - 1/3
*sqrt(3)*(B*a^3 - A*a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/
3))/(b^4*(a/b)^(2/3)) + 1/6*(B*a^3 - A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/
b)^(2/3))/(b^4*(a/b)^(2/3)) - 1/3*(B*a^3 - A*a^2*b)*log(x + (a/b)^(1/3))/(b
^4*(a/b)^(2/3))
```

mupad [B] time = 0.27, size = 164, normalized size = 0.90

$$x^{4}\left(\frac{A}{4\,b}-\frac{B\,a}{4\,b^{2}}\right)+\frac{B\,x^{7}}{7\,b}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{10/3}}-\frac{a\,x\,\left(\frac{A}{b}-\frac{B\,a}{b^{2}}\right)}{b}-\frac{a^{4/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\right.}{3\,b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^4*(A/(4*b) - (B*a)/(4*b^2)) + (B*x^7)/(7*b) + (a^(4/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(10/3)) - (a*x*(A/b - (B*a)/b^2))/b - (a^(4/3)*log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*b^(10/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3
))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(10/3))
```

sympy [A] time = 1.37, size = 114, normalized size = 0.62

$$\frac{Bx^{7}}{7b}+x^{4}\left(\frac{A}{4b}-\frac{Ba}{4b^{2}}\right)+x\left(-\frac{Aa}{b^{2}}+\frac{Ba^{2}}{b^{3}}\right)+\operatorname{RootSum}\left(27t^{3}b^{10}-A^{3}a^{4}b^{3}+3A^{2}Ba^{5}b^{2}-3AB^{2}a^{6}b+B^{3}a^{7},\left(t\mapsto t\,\mathrm{lo}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**7/(7*b) + x**4*(A/(4*b) - B*a/(4*b**2)) + x*(-A*a/b**2 + B*a**2/b**3)
+ RootSum(27*_t**3*b**10 - A**3*a**4*b**3 + 3*A**2*B*a**5*b**2 - 3*A*B**2*a
**6*b + B**3*a**7, Lambda(_t, _t*log(-3*_t*b**3/(-A*a*b + B*a**2) + x)))
```

3.57 $\int \frac{x^5(A+Bx^3)}{a+bx^3} dx$

Optimal. Leaf size=54

$$-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}+\frac{x^3(Ab-aB)}{3b^2}+\frac{Bx^6}{6b}$$

```
[Out] 1/3*(A*b-B*a)*x^3/b^2+1/6*B*x^6/b-1/3*a*(A*b-B*a)*ln(b*x^3+a)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{x^3(Ab-aB)}{3b^2}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}+\frac{Bx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^3)/(3*b^2) + (B*x^6)/(6*b) - (a*(A*b - a*B)*Log[a + b*x^3])/
(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^3\right)}{a+bx^3}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b^2}+\frac{Bx}{b}+\frac{a(-Ab+aB)}{b^2(a+bx)}\right)dx,x,x^3\right)\\&=\frac{(Ab-aB)x^3}{3b^2}+\frac{Bx^6}{6b}-\frac{a(Ab-aB)\log\left(a+bx^3\right)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.87

$$\frac{bx^3\left(-2aB+2Ab+bBx^3\right)+2a(aB-Ab)\log\left(a+bx^3\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (b*x^3*(2*A*b - 2*a*B + b*B*x^3) + 2*a*(-(A*b) + a*B)*Log[a + b*x^3])/(6*b^
3)
```

fricas [A] time = 0.96, size = 51, normalized size = 0.94

$$\frac{Bb^2x^6 - 2\left(Bab - Ab^2\right)x^3 + 2\left(Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^2*x^6 - 2*(B*a*b - A*b^2)*x^3 + 2*(B*a^2 - A*a*b)*log(b*x^3 + a))/
b^3
```

giac [A] time = 0.17, size = 52, normalized size = 0.96

$$\frac{Bbx^6 - 2\,Bax^3 + 2\,Abx^3}{6\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(B*b*x^6 - 2*B*a*x^3 + 2*A*b*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(abs(b*x
^3 + a))/b^3
```

maple [A] time = 0.04, size = 62, normalized size = 1.15

$$\frac{B\,x^6}{6b} + \frac{A\,x^3}{3b} - \frac{Ba\,x^3}{3b^2} - \frac{Aa\ln\left(b\,x^3 + a\right)}{3b^2} + \frac{B\,a^2\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/6*B*x^6/b+1/3/b*A*x^3-1/3/b^2*B*a*x^3-1/3*a/b^2*ln(b*x^3+a)*A+1/3*a^2/b^3
*ln(b*x^3+a)*B
```

maxima [A] time = 0.50, size = 50, normalized size = 0.93

$$\frac{Bbx^6 - 2\,(Ba - Ab)x^3}{6\,b^2} + \frac{\left(Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(B*b*x^6 - 2*(B*a - A*b)*x^3)/b^2 + 1/3*(B*a^2 - A*a*b)*log(b*x^3 + a)/
b^3
```

mupad [B] time = 0.08, size = 52, normalized size = 0.96

$$x^3\left(\frac{A}{3\,b} - \frac{B\,a}{3\,b^2}\right) + \frac{\ln\left(b\,x^3 + a\right)\left(B\,a^2 - A\,a\,b\right)}{3\,b^3} + \frac{B\,x^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^3*(A/(3*b) - (B*a)/(3*b^2)) + (log(a + b*x^3)*(B*a^2 - A*a*b))/(3*b^3) +
(B*x^6)/(6*b)
```

<u>**sympy**</u> **[A]** time = 0.96, size = 46, normalized size = 0.85

$$\frac{Bx^6}{6b} + \frac{a(-Ab+Ba)\log\left(a+bx^3\right)}{3b^3} + x^3\left(\frac{A}{3b} - \frac{Ba}{3b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**6/(6*b) + a*(-A*b + B*a)*log(a + b*x**3)/(3*b**3) + x**3*(A/(3*b) - B*
a/(3*b**2))
```

3.58 $\int \frac{x^4(A+Bx^3)}{a+bx^3} \, dx$

Optimal. Leaf size=167

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}$$

```
[Out] 1/2*(A*b-B*a)*x^2/b^2+1/5*B*x^5/b+1/3*a^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*
x)/b^(8/3)-1/6*a^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
b^(8/3)+1/3*a^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1
/2))/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 321, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{8/3}}+\frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}}+\frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x^2)/(2*b^2) + (B*x^5)/(5*b) + (a^(2/3)*(A*b - a*B)*ArcTan[(a^
(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (a^(2/3)*(A*b
- a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(8/3)) - (a^(2/3)*(A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^5}{5b} - \frac{(-5Ab+5aB)\int \frac{x^4}{a+bx^3}\,dx}{5b}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} - \frac{(a(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{b^2}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{7/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{\left(a^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{6b^{8/3}}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} - \frac{a^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6b^{8/3}}$$

$$= \frac{(Ab-aB)x^2}{2b^2} + \frac{Bx^5}{5b} + \frac{a^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{a^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{8/3}} -$$

Mathematica [A] time = 0.09, size = 154, normalized size = 0.92

$$\frac{5a^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10a^{2/3}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}\,a^{2/3}(aB-Ab)\tan^{-1}\left(\frac{1-}{}\right.}{30b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (15*b^(2/3)*(A*b - a*B)*x^2 + 6*b^(5/3)*B*x^5 - 10*Sqrt[3]*a^(2/3)*(-(A*b)
+ a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-(A*b) + a
*B)*Log[a^(1/3) + b^(1/3)*x] + 5*a^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(30*b^(8/3))
```

fricas [A] time = 0.94, size = 162, normalized size = 0.97

$$\frac{6\,Bbx^5 - 15\,(Ba - Ab)x^2 + 10\,\sqrt{3}\,(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3\,a}\right) + 5\,(Ba - Ab)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2 - bx \cdots\right.}{30\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*B*b*x^5 - 15*(B*a - A*b)*x^2 + 10*sqrt(3)*(B*a - A*b)*(a^2/b^2)^(1/
3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(B*a - A*b
)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) - 10
*(B*a - A*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/b^2
```

giac [A] time = 0.19, size = 207, normalized size = 1.24

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^4} + \frac{\left(\left(-ab^2\right)^{\frac{2}{3}}Ba - \left(-ab^2\right)^{\frac{2}{3}}Ab\right)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(- \cdots\right.\right.}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/6*((-a*b^2)^(2/3)*B*a - (-a*b^2)^
(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2*b^3*(-
a/b)^(1/3) - A*a*b^4*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/
(a*b^5) + 1/10*(2*B*b^4*x^5 - 5*B*a*b^3*x^2 + 5*A*b^4*x^2)/b^5
```

maple [A] time = 0.04, size = 226, normalized size = 1.35

$$\frac{B\,x^5}{5b} + \frac{A\,x^2}{2b} - \frac{Ba\,x^2}{2b^2} - \frac{\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}} - 1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{Aa\ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} - \frac{Aa\ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2} + \frac{\sqrt{3}\,B\,a^2\,\mathrm{ar}\cdots}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/5*B*x^5/b+1/2/b*A*x^2-1/2/b^2*B*x^2*a+1/3*a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1
/3))*A-1/3*a^2/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B-1/6*a/b^2/(a/b)^(1/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6*a^2/b^3/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*B-1/3*a/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))*A+1/3*a^2/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))*B
```

maxima [A] time = 1.10, size = 157, normalized size = 0.94

$$\frac{\sqrt{3}\left(Ba^2 - Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5 - 5\,(Ba - Ab)x^2}{10\,b^2}+\frac{\left(Ba^2 - Aab\right)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(Ba^2 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(b^3*(a/b)^(1/3)) + 1/10*(2*B*b*x^5 - 5*(B*a - A*b)*x^2)/b^2 + 1/6*(B*
a^2 - A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) - 1/3
*(B*a^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

mupad [B] time = 2.55, size = 144, normalized size = 0.86

$$x^2\left(\frac{A}{2\,b}-\frac{B\,a}{2\,b^2}\right)+\frac{B\,x^5}{5\,b}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,b^{8/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b\right.}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^2*(A/(2*b) - (B*a)/(2*b^2)) + (B*x^5)/(5*b) + (a^(2/3)*log(b^(1/3)*x + a^
(1/3))*(A*b - B*a))/(3*b^(8/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/
3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(8/3)) - (a^(2/3)*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b
 - B*a))/(3*b^(8/3))
```

sympy [A] time = 0.87, size = 114, normalized size = 0.68

$$\frac{Bx^5}{5b}+x^2\left(\frac{A}{2b}-\frac{Ba}{2b^2}\right)+\operatorname{RootSum}\left(27t^3b^8 - A^3a^2b^3 + 3A^2Ba^3b^2 - 3AB^2a^4b + B^3a^5,\left(t\mapsto t\log\left(\frac{9t^2b^5}{A^2ab^2 - 2ABa^2b}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**5/(5*b) + x**2*(A/(2*b) - B*a/(2*b**2)) + RootSum(27*_t**3*b**8 - A**3
*a**2*b**3 + 3*A**2*B*a**3*b**2 - 3*A*B**2*a**4*b + B**3*a**5, Lambda(_t, _
t*log(9*_t**2*b**5/(A**2*a*b**2 - 2*A*B*a**2*b + B**2*a**3) + x)))
```

3.59 $\int \frac{x^3(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}$$

```
[Out] (A*b-B*a)*x/b^2+1/4*B*x^4/b-1/3*a^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(
7/3)+1/6*a^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(7/3
)+1/3*a^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b
^(7/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {459, 321, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{7/3}}+\frac{x(Ab-aB)}{b^2}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}}{\sqrt{3}\,b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] ((A*b - a*B)*x)/b^2 + (B*x^4)/(4*b) + (a^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (a^(1/3)*(A*b - a*B)
*Log[a^(1/3) + b^(1/3)*x])/(3*b^(7/3)) + (a^(1/3)*(A*b - a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{Bx^4}{4b}-\frac{(-4Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{4b}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{(a(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{b^2}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3b^2}-\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,d
\end{aligned}$$

$$\begin{aligned}
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\left(\sqrt[3]{a}\,(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}}{6b^{7/3}}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\right.}{6b^{7/3}}\\
&= \frac{(Ab-aB)x}{b^2}+\frac{Bx^4}{4b}+\frac{\sqrt[3]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{\sqrt[3]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{7/3}}+
\end{aligned}$$

Mathematica [A] time = 0.09, size = 152, normalized size = 0.94

$$\frac{-2\sqrt[3]{a}\,(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+12\sqrt[3]{b}\,x(Ab-aB)+4\sqrt[3]{a}\,(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-4\sqrt{3}\,\sqrt[3]{a}\,(}{12b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3),x]
```

[Out] (12*b^(1/3)*(A*b - a*B)*x + 3*b^(4/3)*B*x^4 - 4*Sqrt[3]*a^(1/3)*(-(A*b) + a *B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-(A*b) + a*B)* Log[a^(1/3) + b^(1/3)*x] - 2*a^(1/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b ^(1/3)*x + b^(2/3)*x^2])/(12*b^(7/3))

fricas [A] time = 1.03, size = 145, normalized size = 0.90

$$\frac{3\,Bbx^4-4\,\sqrt{3}\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)-\cdots}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] 1/12*(3*B*b*x^4 - 4*sqrt(3)*(B*a - A*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)* b*x*(-a/b)^(2/3) - sqrt(3)*a)/a) + 2*(B*a - A*b)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*(B*a - A*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/ 3)) - 12*(B*a - A*b)*x)/b^2

giac [A] time = 0.17, size = 186, normalized size = 1.15

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)\cdots\right.}{6\,b^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2 *x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 - 1/3*(B*a^2*b^2 - A *a*b^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 1/4*(B*b^3*x^4 - 4*B*a*b^2*x + 4*A*b^3*x)/b^4

maple [A] time = 0.04, size = 221, normalized size = 1.36

$$\frac{B\,x^4}{4b}-\frac{\sqrt{3}\;Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Aa\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{Ax}{b}+\frac{\sqrt{3}\;B\,a^2\arctan\left(\cdots\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(B*x^3+A)/(b*x^3+a),x)

[Out] 1/4*B*x^4/b+1/b*A*x-1/b^2*B*a*x-1/3*a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1 /3*a^2/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B+1/6*a/b^2/(a/b)^(2/3)*ln(x^2-(a/ b)^(1/3)*x+(a/b)^(2/3))*A-1/6*a^2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3*a/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)* x-1))*A+1/3*a^2/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x -1))*B

maxima [A] time = 1.08, size = 154, normalized size = 0.95

$$\frac{Bbx^4-4\,(Ba-Ab)x}{4\,b^2}+\frac{\sqrt{3}\left(Ba^2-Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(Ba^2-Aab\right)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(Ba^2-Aa\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/4*(B*b*x^4 - 4*(B*a - A*b)*x)/b^2 + 1/3*sqrt(3)*(B*a^2 - A*a*b)*arctan(1/
3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) - 1/6*(B*a^2 -
 A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) + 1/3*(B*a
^2 - A*a*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 2.61, size = 162, normalized size = 1.00

$$x\left(\frac{A}{b}-\frac{B\,a}{b^2}\right)+\frac{B\,x^4}{4\,b}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,\left(A\,b-B\,a\right)}{3\,b^{7/3}}-\frac{(-a)^{1/3}\,\ln\left(2\,a\,b^{1/3}\,x-(-a)^{4/3}-\sqrt{3}\,(-a)^{4/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}\right.}{3\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x*(A/b - (B*a)/b^2) + (B*x^4)/(4*b) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3
)*x)*(A*b - B*a))/(3*b^(7/3)) - ((-a)^(1/3)*log(2*a*b^(1/3)*x - 3^(1/2)*(-a
)^(4/3)*1i - (-a)^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*b^(7/3)) +
((-a)^(1/3)*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a))/(3*b^(7/3))
```

sympy [A] time = 1.15, size = 87, normalized size = 0.54

$$\frac{Bx^4}{4b}+x\left(\frac{A}{b}-\frac{Ba}{b^2}\right)+\operatorname{RootSum}\left(27t^3b^7+A^3ab^3-3A^2Ba^2b^2+3AB^2a^3b-B^3a^4,\left(t\mapsto t\log\left(\frac{3tb^2}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**4/(4*b) + x*(A/b - B*a/b**2) + RootSum(27*_t**3*b**7 + A**3*a*b**3 - 3
*A**2*B*a**2*b**2 + 3*A*B**2*a**3*b - B**3*a**4, Lambda(_t, _t*log(3*_t*b**
2/(-A*b + B*a) + x)))
```

3.60 $\int \frac{x^2(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=35

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

```
[Out] 1/3*B*x^3/b+1/3*(A*b-B*a)*ln(b*x^3+a)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}+\frac{Bx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3),x]
[Out] (B*x^3)/(3*b) + ((A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{a+bx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b}+\frac{Ab-aB}{b(a+bx)}\right)dx,x,x^3\right)\\
&= \frac{Bx^3}{3b}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.89

$$\frac{(Ab-aB)\log\left(a+bx^3\right)+bBx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3),x]
[Out] (b*B*x^3 + (A*b - a*B)*Log[a + b*x^3])/(3*b^2)
```

fricas [A] time = 0.98, size = 30, normalized size = 0.86

$$\frac{Bbx^3 - (Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(B*b*x^3 - (B*a - A*b)*log(b*x^3 + a))/b^2
```

giac [A] time = 0.22, size = 32, normalized size = 0.91

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/b^2
```

maple [A] time = 0.04, size = 40, normalized size = 1.14

$$\frac{B\,x^3}{3b} + \frac{A\ln\left(b\,x^3 + a\right)}{3b} - \frac{Ba\ln\left(b\,x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a),x)
[Out] 1/3*B*x^3/b+1/3/b*ln(b*x^3+a)*A-1/3/b^2*ln(b*x^3+a)*B*a
```

maxima [A] time = 0.54, size = 31, normalized size = 0.89

$$\frac{Bx^3}{3\,b} - \frac{(Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*B*x^3/b - 1/3*(B*a - A*b)*log(b*x^3 + a)/b^2
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{B\,x^3}{3\,b} + \frac{\ln\left(b\,x^3 + a\right)\,(A\,b - B\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3),x)
[Out] (B*x^3)/(3*b) + (log(a + b*x^3)*(A*b - B*a))/(3*b^2)
```

sympy [A] time = 0.88, size = 27, normalized size = 0.77

$$\frac{Bx^3}{3b} - \frac{(-Ab + Ba)\log\left(a + bx^3\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a),x)
[Out] B*x**3/(3*b) - (-A*b + B*a)*log(a + b*x**3)/(3*b**2)
```

3.61 $\int \frac{x\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=150

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

```
[Out] 1/2*B*x^2/b-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(5/3)+1/6*(A*b-B*
a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(5/3)-1/3*(A*b-B*a)*
arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {459, 292, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}b^{5/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}b^{5/3}}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3
))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*b^(5/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{Bx^2}{2b} - \frac{(-2Ab+2aB)\int \frac{x}{a+bx^3}\,dx}{2b}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}\,b^{4/3}} + \frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}\,b^{4/3}}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\int \frac{\ \ }{a^{2/3}-\sqrt[3]{\ }}}{2b^{4/3}}$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)}{\ }$$

$$= \frac{Bx^2}{2b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{5/3}} + \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{\ }\right.}{6\sqrt[3]{a}\,b^{5/}}$$

Mathematica [A] time = 0.06, size = 152, normalized size = 1.01

$$-\frac{(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}\,b^{5/3}}+\frac{(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{(aB-Ab)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}+\frac{Bx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^2)/(2*b) - ((-(A*b) + a*B)*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) + ((-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(1/3)*b^(5/3)) - ((-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(1/3)*b^(5/3))
```

fricas [A] time = 1.04, size = 382, normalized size = 2.55

$$\left[\frac{3\,Bab^2x^2-3\,\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2+\left(-ab^2\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)-\left(-a\right.}{6\,ab^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(3*B*a*b^2*x^2 - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^
2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - (-a
*b^2)^(2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3))
+ 2*(-a*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3), 1/6*(3*B
*a*b^2*x^2 - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(-a*b^2)^(1/3)/a)*arctan
(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(
2/3)*(B*a - A*b)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*(-a
*b^2)^(2/3)*(B*a - A*b)*log(b*x - (-a*b^2)^(1/3)))/(a*b^3)]
```

giac [A] time = 0.19, size = 161, normalized size = 1.07

$$\frac{Bx^2}{2\,b}-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{1}{3}}\,b}+\frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{1}{3}}\,b}+\frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3
))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b) + 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-
a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
```

maple [A] time = 0.04, size = 198, normalized size = 1.32

$$\frac{B\,x^2}{2b}+\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 1/2*B*x^2/b-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/b^2/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))*B*a+1/6/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/
```

```
b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2*3^(1/2)/(a/b)^(1/3)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

maxima [A] time = 1.14, size = 131, normalized size = 0.87

$$\frac{Bx^2}{2b} - \frac{\sqrt{3}\,(Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{(Ba - Ab)\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{(Ba - Ab)\log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^2/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3)
)/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3)
+ (a/b)^(2/3))/(b^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^
2*(a/b)^(1/3))
```

mupad [B] time = 2.57, size = 126, normalized size = 0.84

$$\frac{B\,x^2}{2\,b} - \frac{\ln\left(b^{1/3}\,x + a^{1/3}\right)\,(A\,b - B\,a)}{3\,a^{1/3}\,b^{5/3}} - \frac{\ln\left(a^{1/3} - 2\,b^{1/3}\,x + \sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b - B\,a)}{3\,a^{1/3}\,b^{5/3}} + \frac{\ln\left(2\,b^{1/3}\,x - a^{1/3} + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (B*x^2)/(2*b) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
- (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(
A*b - B*a))/(3*a^(1/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a
^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(1/3)*b^(5/3))
```

sympy [A] time = 1.11, size = 92, normalized size = 0.61

$$\frac{Bx^2}{2b} + \operatorname{RootSum}\left(27t^3ab^5 + A^3b^3 - 3A^2Bab^2 + 3AB^2a^2b - B^3a^3, \left(t \mapsto t\log\left(\frac{9t^2ab^3}{A^2b^2 - 2ABab + B^2a^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x**2/(2*b) + RootSum(27*_t**3*a*b**5 + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A*
B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(9*_t**2*a*b**3/(A**2*b**2 - 2*A*
B*a*b + B**2*a**2) + x)))
```

3.62 $\int \frac{A+Bx^3}{a+bx^3}\,dx$

Optimal. Leaf size=145

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

```
[Out] B*x/b+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(4/3)-1/6*(A*b-B*a)*ln(
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(4/3)-1/3*(A*b-B*a)*arctan
(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}+\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (B*x)/b - ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{a+bx^3}\,dx &= \frac{Bx}{b} - \frac{(-Ab+aB)\int \frac{1}{a+bx^3}\,dx}{b}\\ &= \frac{Bx}{b} + \frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b} + \frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b}\\ &= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{2\sqrt[3]{a}\,b}\\ &= \frac{Bx}{b} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(Ab-aB)\,\mathrm{Subs}}{}\\ &= \frac{Bx}{b} - \frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{}\right.}{6a^{2/3}b^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 129, normalized size = 0.89

$$\frac{-(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}Bx+2(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(Ab-aB)\tan^{-1}\left(1-\right.}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*B*x - 2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

fricas [A] time = 1.09, size = 369, normalized size = 2.54

$$\left[\frac{6\,Ba^2bx-3\,\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\,\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(a^2b\right)^{\frac{2}{3}}}{6\,a^2b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="fricas")

[Out] [1/6*(6*B*a^2*b*x - 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + (a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2), 1/6*(6*B*a^2*b*x - 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*(B*a - A*b)*log(a*b*x + (a^2*b)^(2/3)))/(a^2*b^2)]

giac [A] time = 0.18, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,\left(-ab^2\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,\left(-ab^2\right)^{\frac{2}{3}}}+\frac{Bx}{b}+\frac{(Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\right.\right.}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="giac")

[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a*b^2)^(2/3) + 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a*b^2)^(2/3) + B*x/b + 1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b)

maple [A] time = 0.04, size = 195, normalized size = 1.34

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\sqrt{3}\,Ba\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}-\frac{Ba\ln}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(b*x^3+a),x)

[Out] B*x/b+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B*a-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b^2/(a

```
/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B*a+1/3/b/(a/b)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B*a
```

maxima [A] time = 1.16, size = 128, normalized size = 0.88

$$\frac{Bx}{b}-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] B*x/b - 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b
)^(1/3))/(b^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(b^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(b^2*(a/b
)^(2/3))
```

mupad [B] time = 2.54, size = 123, normalized size = 0.85

$$\frac{B\,x}{b}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{2/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3),x)
```

```
[Out] (B*x)/b + (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(2/3)*b^(4/3)) - (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a))/(3*a^(2/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(2/3)*b^(4/3))
```

sympy [A] time = 1.02, size = 71, normalized size = 0.49

$$\frac{Bx}{b}+\operatorname{RootSum}\left(27t^3a^2b^4-A^3b^3+3A^2Bab^2-3AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(-\frac{3tab}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a),x)
```

```
[Out] B*x/b + RootSum(27*_t**3*a**2*b**4 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B**2
*a**2*b + B**3*a**3, Lambda(_t, _t*log(-3*_t*a*b/(-A*b + B*a) + x)))
```

3.63 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)} dx$

Optimal. Leaf size=34

$$\frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

```
[Out] A*ln(x)/a-1/3*(A*b-B*a)*ln(b*x^3+a)/a/b
```

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 72}

$$\frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a - ((A*b - a*B)*Log[a + b*x^3])/(3*a*b)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)} dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax} + \frac{-Ab+aB}{a(a+bx)}\right) dx, x, x^3\right) \\
&= \frac{A\log(x)}{a} - \frac{(Ab-aB)\log\left(a+bx^3\right)}{3ab}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{(aB-Ab)\log\left(a+bx^3\right)}{3ab} + \frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)),x]
```

```
[Out] (A*Log[x])/a + ((-(A*b) + a*B)*Log[a + b*x^3])/(3*a*b)
```

fricas [A] time = 0.93, size = 32, normalized size = 0.94

$$\frac{3\,Ab\log(x) + (Ba - Ab)\log\left(bx^3 + a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="fricas")
[Out] 1/3*(3*A*b*log(x) + (B*a - A*b)*log(b*x^3 + a))/(a*b)
```

giac [A] time = 0.19, size = 34, normalized size = 1.00

$$\frac{A\log\left(|x|\right)}{a} + \frac{(Ba - Ab)\log\left(\left|bx^3 + a\right|\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="giac")
[Out] A*log(abs(x))/a + 1/3*(B*a - A*b)*log(abs(b*x^3 + a))/(a*b)
```

maple [A] time = 0.05, size = 37, normalized size = 1.09

$$\frac{A\ln(x)}{a} - \frac{A\ln\left(b\,x^3 + a\right)}{3a} + \frac{B\ln\left(b\,x^3 + a\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a),x)
[Out] -1/3/a*ln(b*x^3+a)*A+1/3/b*ln(b*x^3+a)*B+A/a*ln(x)
```

maxima [A] time = 0.59, size = 35, normalized size = 1.03

$$\frac{A\log\left(x^3\right)}{3\,a} + \frac{(Ba - Ab)\log\left(bx^3 + a\right)}{3\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a),x, algorithm="maxima")
[Out] 1/3*A*log(x^3)/a + 1/3*(B*a - A*b)*log(b*x^3 + a)/(a*b)
```

mupad [B] time = 0.10, size = 36, normalized size = 1.06

$$\frac{B\,\ln\left(b\,x^3 + a\right)}{3\,b} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a} + \frac{A\,\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)),x)
[Out] (B*log(a + b*x^3))/(3*b) - (A*log(a + b*x^3))/(3*a) + (A*log(x))/a
```

sympy [A] time = 2.12, size = 26, normalized size = 0.76

$$\frac{A\log\left(x\right)}{a} + \frac{(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a),x)
[Out] A*log(x)/a + (-A*b + B*a)*log(a/b + x**3)/(3*a*b)
```

3.64 $\int \frac{A+Bx^3}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=147

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}-\frac{A}{ax}$$

```
[Out] -A/a/x+1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(2/3)-1/6*(A*b-B*a)*ln
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(2/3)+1/3*(A*b-B*a)*arcta
n(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {453, 292, 31, 634, 617, 204, 628}

$$-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] -(A/(a*x)) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(4/3)*b^(2/3)) + ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(
4/3)*b^(2/3)) - ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)}\,dx = -\frac{A}{ax}-\frac{(Ab-aB)\int \frac{x}{a+bx^3}\,dx}{a}$$

$$= -\frac{A}{ax}+\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}\sqrt[3]{b}}-\frac{(Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}\sqrt[3]{b}}$$

$$= -\frac{A}{ax}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{4/3}b^{2/3}}-\frac{(Ab-aB)\int \frac{}{a^{2/3}-\sqrt[3]{}}}{2a\sqrt[3]{}}$$

$$= -\frac{A}{ax}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}b^{2/3}}-\frac{(Ab-aB)}{}$$

$$= -\frac{A}{ax}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}b^{2/3}}-\frac{(Ab-aB)\log\left(a^{2/3}-\right.}{6a^{4/3}b^{2}}$$

Mathematica [A] time = 0.09, size = 134, normalized size = 0.91

$$\frac{-x(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-6\sqrt[3]{a}\,Ab^{2/3}+2x(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+2\sqrt{3}\,x(Ab-aB)\tan^{-1}}{6a^{4/3}b^{2/3}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-6*a^(1/3)*A*b^(2/3) + 2*Sqrt[3]*(A*b - a*B)*x*ArcTan[(1 - (2*b^(1/3)*x)/a
^(1/3))/Sqrt[3]] + 2*(A*b - a*B)*x*Log[a^(1/3) + b^(1/3)*x] - (A*b - a*B)*x
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(4/3)*b^(2/3)*x)
```

fricas [A] time = 0.98, size = 372, normalized size = 2.53

$$\left[-\frac{6\,Aab^2+3\,\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)x\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\,\log\left(\frac{2\,b^2x^3-ab-3\,\sqrt{\frac{1}{3}}\left(abx+2\left(ab^2\right)^{\frac{2}{3}}x^2-\left(ab^2\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}-3\left(ab^2\right)^{\frac{2}{3}}x}{bx^3+a}\right)-\left(ab^2\right)}{6\,a^2b^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*A*a*b^2 + 3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt(-(a*b^2)^(1/3)/a)
*log((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^
(1/3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - (a*b^2)
^(2/3)*(B*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(
a*b^2)^(2/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x), -1/6*(6*A
*a*b^2 + 6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x*sqrt((a*b^2)^(1/3)/a)*arctan(-sq
rt(1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - (a*b^2)^(2/3)*(B
*a - A*b)*x*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) + 2*(a*b^2)^(2
/3)*(B*a - A*b)*x*log(b*x + (a*b^2)^(1/3)))/(a^2*b^2*x)]
```

giac [A] time = 0.22, size = 155, normalized size = 1.05

$$\frac{\sqrt{3}\left(Ba-Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(-ab^2\right)^{\frac{1}{3}}a}-\frac{\left(Ba-Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(-ab^2\right)^{\frac{1}{3}}a}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/((-a*b^2)^(1/3)*a) - 1/6*(B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(
2/3))/((-a*b^2)^(1/3)*a) - 1/3*(B*a*(-a/b)^(1/3) - A*b*(-a/b)^(1/3))*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 - A/(a*x)
```

maple [A] time = 0.06, size = 195, normalized size = 1.33

$$-\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{B\ln}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a),x)
```

```
[Out] 1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B
-1/6/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/b/(a/b)^(1/3)*ln
```

```
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1
/2)*(2/(a/b)^(1/3)*x-1))*A+1/3*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))*B-A/a/x
```

maxima [A] time = 1.16, size = 140, normalized size = 0.95

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(1/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(1/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(1/3))
 - A/(a*x)
```

mupad [B] time = 2.54, size = 126, normalized size = 0.86

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{4/3}\,b^{2/3}}-\frac{A}{a\,x}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a)}{3\,a^{4/3}\,b^{2/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(4/3)*b^(2/3)) - A/(a*x) + (log
(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b -
B*a))/(3*a^(4/3)*b^(2/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3)
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(4/3)*b^(2/3))
```

sympy [A] time = 0.94, size = 90, normalized size = 0.61

$$-\frac{A}{ax}+\operatorname{RootSum}\left(27t^3a^4b^2-A^3b^3+3A^2Bab^2-3AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{9t^2a^3b}{A^2b^2-2ABab+B^2a^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a),x)
```

```
[Out] -A/(a*x) + RootSum(27*_t**3*a**4*b**2 - A**3*b**3 + 3*A**2*B*a*b**2 - 3*A*B
**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(9*_t**2*a**3*b/(A**2*b**2 - 2*A*B
*a*b + B**2*a**2) + x)))
```

3.65 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=149

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

```
[Out] -1/2*A/a/x^2-1/3*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(1/3)+1/6*(A*b-B
*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(1/3)+1/3*(A*b-B*a)
*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {453, 200, 31, 634, 617, 204, 628}

$$\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] -A/(2*a*x^2) + ((A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(5/3)*b^(1/3)) - ((A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a
^(5/3)*b^(1/3)) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^
2])/(6*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)}\,dx = -\frac{A}{2ax^2}-\frac{(2Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{2a}$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{5/3}}-\frac{(Ab-aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{5/3}}$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2a^{4/3}}+\frac{(Ab-aB)\int \frac{-\ldots}{a^{2/3}\ldots}}{6a^{5/\ldots}}$$

$$= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}\sqrt[3]{b}}-\frac{(Ab-a\ldots}{\ldots}$$

$$= -\frac{A}{2ax^2}+\frac{(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{(Ab-aB)\log\left(a^{2/3}\ldots\right.}{6a^{5/3}\ldots}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 0.91

$$\frac{\frac{(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{3a^{2/3}A}{x^2}+\frac{2(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(2/3)*A)/x^2 + (2*Sqrt[3]*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(1/3) + (2*(-(A*b) + a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3
) + ((A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(
6*a^(5/3))
```

fricas [A] time = 1.08, size = 411, normalized size = 2.76

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(Ba^2b-Aab^2\right)x^2\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3\left(-a^2b\right)^{\frac{1}{3}}ax-a^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(-a^2b\right)^{\frac{2}{3}}x+\left(-a^2b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3+a}\right)+\left(-a^2b\right)^{\frac{2}{3}}(\ldots}{6\,a^3b\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt((-a^2*b)^(1/3)/b)*log((2*a*
b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3
)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) + (-a^2*b)^(2/
3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*(
-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^(2/3)) + 3*A*a^2*b)/(a^3
*b*x^2), 1/6*(6*sqrt(1/3)*(B*a^2*b - A*a*b^2)*x^2*sqrt(-(-a^2*b)^(1/3)/b)*a
rctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3
)/b)/a^2) - (-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
 (-a^2*b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(B*a - A*b)*x^2*log(a*b*x + (-a^2*b)^
(2/3)) - 3*A*a^2*b)/(a^3*b*x^2)]
```

giac [A] time = 0.18, size = 161, normalized size = 1.08

$$-\frac{\left(Ba-Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*(B*a - A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^2 + 1/3*sqrt(3)*
((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^
(1/3))/(-a/b)^(1/3))/(a^2*b) + 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b) - 1/2*A/(a*x^2)
```

maple [A] time = 0.05, size = 195, normalized size = 1.31

$$-\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{B\ln\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a),x)
```

```
[Out] -1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*
B+1/6/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/b/(a/b)^(2/3)*l
```

```
n(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))*B-1/2*A/a/x^2
```

maxima [A] time = 1.22, size = 140, normalized size = 0.94

$$\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))
/(a*b*(a/b)^(2/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))
/(a*b*(a/b)^(2/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a*b*(a/b)^(2/3))
 - 1/2*A/(a*x^2)
```

mupad [B] time = 0.24, size = 126, normalized size = 0.85

$$-\frac{A}{2\,a\,x^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-B\,a\right)}{3\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b - B*a))/(3*a^(5/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a
^(5/3)*b^(1/3)) - A/(2*a*x^2) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(
1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/(3*a^(5/3)*b^(1/3))
```

sympy [A] time = 1.23, size = 73, normalized size = 0.49

$$-\frac{A}{2ax^2}+\operatorname{RootSum}\left(27t^3a^5b+A^3b^3-3A^2Bab^2+3AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{3ta^2}{-Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a),x)
```

```
[Out] -A/(2*a*x**2) + RootSum(27*_t**3*a**5*b + A**3*b**3 - 3*A**2*B*a*b**2 + 3*A
*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(3*_t*a**2/(-A*b + B*a) + x)))
```

3.66 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=50

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

```
[Out] -1/3*A/a/x^3-(A*b-B*a)*ln(x)/a^2+1/3*(A*b-B*a)*ln(b*x^3+a)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -A/(3*a*x^3) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])/(3*a
^2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}-\frac{b(-Ab+aB)}{a^2(a+bx)}\right)\,dx,x,x^3\right)\\ &= -\frac{A}{3ax^3}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.98

$$\frac{(Ab-aB)\log\left(a+bx^3\right)}{3a^2}+\frac{\log(x)(aB-Ab)}{a^2}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)),x]
```

```
[Out] -1/3*A/(a*x^3) + ((-(A*b) + a*B)*Log[x])/a^2 + ((A*b - a*B)*Log[a + b*x^3])
/(3*a^2)
```

fricas [A] time = 0.92, size = 47, normalized size = 0.94

$$-\frac{(Ba - Ab)x^3 \log\left(bx^3 + a\right) - 3\,(Ba - Ab)x^3 \log(x) + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/3*((B*a - A*b)*x^3*log(b*x^3 + a) - 3*(B*a - A*b)*x^3*log(x) + A*a)/(a^2
*x^3)
```

giac [A] time = 0.17, size = 69, normalized size = 1.38

$$\frac{(Ba - Ab)\log(|x|)}{a^2} - \frac{\left(Bab - Ab^2\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^2b} - \frac{Bax^3 - Abx^3 + Aa}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] (B*a - A*b)*log(abs(x))/a^2 - 1/3*(B*a*b - A*b^2)*log(abs(b*x^3 + a))/(a^2*
b) - 1/3*(B*a*x^3 - A*b*x^3 + A*a)/(a^2*x^3)
```

maple [A] time = 0.05, size = 56, normalized size = 1.12

$$-\frac{Ab\ln(x)}{a^2} + \frac{Ab\ln\left(b\,x^3 + a\right)}{3a^2} + \frac{B\ln(x)}{a} - \frac{B\ln\left(b\,x^3 + a\right)}{3a} - \frac{A}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*ln(b*x^3+a)*A*b-1/3/a*ln(b*x^3+a)*B-1/3*A/a/x^3-1/a^2*ln(x)*A*b+B/a
*ln(x)
```

maxima [A] time = 0.48, size = 48, normalized size = 0.96

$$-\frac{(Ba - Ab)\log\left(bx^3 + a\right)}{3\,a^2} + \frac{(Ba - Ab)\log\left(x^3\right)}{3\,a^2} - \frac{A}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*log(b*x^3 + a)/a^2 + 1/3*(B*a - A*b)*log(x^3)/a^2 - 1/3*A/
(a*x^3)
```

mupad [B] time = 2.40, size = 46, normalized size = 0.92

$$\frac{\ln\left(b\,x^3 + a\right)\,(A\,b - B\,a)}{3\,a^2} - \frac{A}{3\,a\,x^3} - \frac{\ln(x)\,(A\,b - B\,a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log(a + b*x^3)*(A*b - B*a))/(3*a^2) - A/(3*a*x^3) - (log(x)*(A*b - B*a))/a
^2
```

sympy [**A**] time = 2.36, size = 41, normalized size = 0.82

$$-\frac{A}{3ax^3}+\frac{(-Ab+Ba)\log(x)}{a^2}-\frac{(-Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a),x)
```

```
[Out] -A/(3*a*x**3) + (-A*b + B*a)*log(x)/a**2 - (-A*b + B*a)*log(a/b + x**3)/(3*
a**2)
```

3.67 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=165

$$\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+$$

```
[Out] -1/4*A/a/x^4+(A*b-B*a)/a^2/x-1/3*b^(1/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^
(7/3)+1/6*b^(1/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/
3)-1/3*b^(1/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(7/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}}+\frac{Ab-aB}{a^2x}-\frac{\sqrt[3]{b}\,(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}}-\frac{\sqrt[3]{b}\,(Ab-aB)\tan^{-1}\left(\cdots\right)}{\sqrt{3}\,a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] -A/(4*a*x^4) + (A*b - a*B)/(a^2*x) - (b^(1/3)*(A*b - a*B)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (b^(1/3)*(A*b - a*B)*
Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)) + (b^(1/3)*(A*b - a*B)*Log[a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)}\,dx = -\frac{A}{4ax^4} - \frac{(4Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{4a}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} + \frac{(b(Ab-aB))\int \frac{x}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{7/3}} + \frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\left(\sqrt[3]{b}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} + \frac{\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\right.}{6a^{7/3}}$$

$$= -\frac{A}{4ax^4} + \frac{Ab-aB}{a^2x} - \frac{\sqrt[3]{b}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}} - \frac{\sqrt[3]{b}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{7/3}} +$$

Mathematica [A] time = 0.14, size = 154, normalized size = 0.93

$$\frac{2\sqrt[3]{b}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{3a^{4/3}A}{x^4}+\frac{12\sqrt[3]{a}(Ab-aB)}{x}+4\sqrt[3]{b}(aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-4\sqrt{3}}{12a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)),x]
```

```
[Out] ((-3*a^(4/3)*A)/x^4 + (12*a^(1/3)*(A*b - a*B))/x - 4*Sqrt[3]*b^(1/3)*(A*b -
 a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-(A*b) + a*B
)*Log[a^(1/3) + b^(1/3)*x] + 2*b^(1/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(12*a^(7/3))
```

fricas [A] time = 0.96, size = 158, normalized size = 0.96

$$-\frac{4\sqrt{3}\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\,(Ba-Ab)x^4\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(-\frac{b}{a}\right)^{\frac{2}{3}}-a\left(-\frac{b}{a}\right)^{\frac{1}{3}}\right)}{12\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*(B*a - A*b)*x^4*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(
1/3) + 1/3*sqrt(3)) - 2*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)
^(2/3) - a*(-b/a)^(1/3)) + 4*(B*a - A*b)*x^4*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 12*(B*a - A*b)*x^3 + 3*A*a)/(a^2*x^4)
```

giac [A] time = 0.22, size = 197, normalized size = 1.19

$$\frac{\left(Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^3}+\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^3b}-\frac{\left(\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*(B*a*b*(-a/b)^(1/3) - A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/
b)^(1/3)))/a^3 + 1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) - 1/6*((-a*b^2)^(
2/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^
3*b) - 1/4*(4*B*a*x^3 - 4*A*b*x^3 + A*a)/(a^2*x^4)
```

maple [A] time = 0.05, size = 216, normalized size = 1.31

$$\frac{\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{B\ln\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a),x)
```

```
[Out] -1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A+1/3/a/(a/b)^(1/3)*ln(x+(a/b)^(1/
3))*B+1/6/a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/6/a/(a/b)
^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3/a^2*b*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a*3^(1/2)/(a/b)^(1/3)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/4*A/a/x^4+1/a^2/x*A*b-B/a/x
```

maxima [A] time = 1.14, size = 147, normalized size = 0.89

$$-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{4\,(Ba\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(1/3)) - 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(1/3)) + 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(1/3)
) - 1/4*(4*(B*a - A*b)*x^3 + A*a)/(a^2*x^4)
```

mupad [B] time = 2.59, size = 178, normalized size = 1.08

$$\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^3\,x\right)\,(A\,b-B\,a)}{3\,a^{7/3}}-\frac{\frac{A}{4\,a}-\frac{x^3\,(A\,b-B\,a)}{a^2}}{x^4}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b^3\,x+\sqrt{3}\,a^{1/3}\,(-b)^{8/3}\,1\cdots\right.}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)),x)
```

```
[Out] ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(A*b - B*a))/(3*a^(7/3)) - (A/(
4*a) - (x^3*(A*b - B*a))/a^2)/x^4 + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*
b^3*x + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(7/3)) - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3
)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(7/3))
```

sympy [A] time = 0.96, size = 112, normalized size = 0.68

$$\operatorname{RootSum}\left(27t^3a^7+A^3b^4-3A^2Bab^3+3AB^2a^2b^2-B^3a^3b,\left(t\mapsto t\log\left(\frac{9t^2a^5}{A^2b^3-2ABab^2+B^2a^2b}+x\right)\right)\right)+\frac{-A\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**7 + A**3*b**4 - 3*A**2*B*a*b**3 + 3*A*B**2*a**2*b**2 -
B**3*a**3*b, Lambda(_t, _t*log(9*_t**2*a**5/(A**2*b**3 - 2*A*B*a*b**2 + B**
2*a**2*b) + x))) + (-A*a + x**3*(4*A*b - 4*B*a))/(4*a**2*x**4)
```

3.68 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=168

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+$$

```
[Out] -1/5*A/a/x^5+1/2*(A*b-B*a)/a^2/x^2+1/3*b^(2/3)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)
*x)/a^(8/3)-1/6*b^(2/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)
/a^(8/3)-1/3*b^(2/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(
1/2))/a^(8/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] -A/(5*a*x^5) + (A*b - a*B)/(2*a^2*x^2) - (b^(2/3)*(A*b - a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) + (b^(2/3)*(A*b - a
*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)) - (b^(2/3)*(A*b - a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)}\,dx = -\frac{A}{5ax^5}-\frac{(5Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{5a}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{(b(Ab-aB))\int \frac{1}{a+bx^3}\,dx}{a^2}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{(b(Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{8/3}}+\frac{(b(Ab-aB))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}-\frac{\left(b^{2/3}(Ab-aB)\right)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}}}{6a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}-\frac{b^{2/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{6a^{8/3}}$$

$$= -\frac{A}{5ax^5}+\frac{Ab-aB}{2a^2x^2}-\frac{b^{2/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}+\frac{b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}}$$

Mathematica [A] time = 0.13, size = 154, normalized size = 0.92

$$\frac{5b^{2/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{15a^{2/3}(Ab-aB)}{x^2}-\frac{6a^{5/3}A}{x^5}+10b^{2/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10}{30a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)),x]
```

```
[Out] ((-6*a^(5/3)*A)/x^5 + (15*a^(2/3)*(A*b - a*B))/x^2 - 10*Sqrt[3]*b^(2/3)*(A*
b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x] + 5*b^(2/3)*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(30*a^(8/3))
```

fricas [A] time = 1.03, size = 176, normalized size = 1.05

$$-\frac{10\sqrt{3}(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-5(Ba-Ab)x^5\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2-abx\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+1}{30a^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x
*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(B*a - A*b)*x^5*(b^2/a^2)^(1/3)*log(b^
2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*(B*a - A*b)*x^5*(
b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 15*(B*a - A*b)*x^3 + 6*A*a)/(
a^2*x^5)
```

giac [A] time = 0.18, size = 176, normalized size = 1.05

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^3}+\frac{\left(Bab-Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3a^3}-\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 + 1/3*(B*a*b - A*b^2)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/a^3 - 1/6*((-a*b^2)^(1/3)*B*a - (-a*b^2)^(1/3)*A*b
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^3 - 1/10*(5*B*a*x^3 - 5*A*b*x^
3 + 2*A*a)/(a^2*x^5)
```

maple [A] time = 0.05, size = 217, normalized size = 1.29

$$\frac{\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{B\ln\left(\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a),x)
```

```
[Out] 1/3/a^2*b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A-1/3/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3
))*B-1/6/a^2*b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/a/(a/b)^
(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/3/a^2*b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A-1/3/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/5*A/a/x^5+1/2/a^2/x^2*A*b-1/2/a/x^2*B
```

maxima [A] time = 1.28, size = 148, normalized size = 0.88

$$-\frac{\sqrt{3}\,(Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba-Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba-Ab)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,(Ba\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*(B*a - A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(a^2*(a/b)^(2/3)) + 1/6*(B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(a^2*(a/b)^(2/3)) - 1/3*(B*a - A*b)*log(x + (a/b)^(1/3))/(a^2*(a/b)^(2/3)
) - 1/10*(5*(B*a - A*b)*x^3 + 2*A*a)/(a^2*x^5)
```

mupad [B] time = 2.56, size = 145, normalized size = 0.86

$$\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b-B\,a)}{3\,a^{8/3}}-\frac{\frac{A}{5\,a}-\frac{x^3\,(A\,b-B\,a)}{2\,a^2}}{x^5}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b-B\,a\cdots}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)),x)
```

```
[Out] (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(8/3)) - (A/(5*a) - (x^
3*(A*b - B*a))/(2*a^2))/x^5 - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x
 + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(8/3)) + (b^(2/3)*log(
3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B
*a))/(3*a^(8/3))
```

sympy [A] time = 0.93, size = 99, normalized size = 0.59

$$\operatorname{RootSum}\left(27t^3a^8-A^3b^5+3A^2Bab^4-3AB^2a^2b^3+B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{3ta^3}{-Ab^2+Bab}+x\right)\right)\right)+\frac{-2Aa+x^3\,(5\cdots}{10a^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**8 - A**3*b**5 + 3*A**2*B*a*b**4 - 3*A*B**2*a**2*b**3 +
B**3*a**3*b**2, Lambda(_t, _t*log(-3*_t*a**3/(-A*b**2 + B*a*b) + x))) + (-2
*A*a + x**3*(5*A*b - 5*B*a))/(10*a**2*x**5)
```

3.69 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=69

$$-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}+\frac{Ab-aB}{3a^2x^3}-\frac{A}{6ax^6}$$

```
[Out] -1/6*A/a/x^6+1/3*(A*b-B*a)/a^2/x^3+b*(A*b-B*a)*ln(x)/a^3-1/3*b*(A*b-B*a)*ln
(b*x^3+a)/a^3
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{Ab-aB}{3a^2x^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}+\frac{b\log(x)(Ab-aB)}{a^3}-\frac{A}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] -A/(6*a*x^6) + (A*b - a*B)/(3*a^2*x^3) + (b*(A*b - a*B)*Log[x])/a^3 - (b*(A
*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{ax^3}+\frac{-Ab+aB}{a^2x^2}-\frac{b(-Ab+aB)}{a^3x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)}\right)dx,x,x^3\right)\\
&= -\frac{A}{6ax^6}+\frac{Ab-aB}{3a^2x^3}+\frac{b(Ab-aB)\log(x)}{a^3}-\frac{b(Ab-aB)\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 1.01

$$\frac{6bx^6\log(x)(Ab-aB)-a\left(aA+2aBx^3-2Abx^3\right)+2bx^6(aB-Ab)\log\left(a+bx^3\right)}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)),x]
```

```
[Out] (-(a*(a*A - 2*A*b*x^3 + 2*a*B*x^3)) + 6*b*(A*b - a*B)*x^6*Log[x] + 2*b*(-(A
*b) + a*B)*x^6*Log[a + b*x^3])/(6*a^3*x^6)
```

fricas [A] time = 0.94, size = 73, normalized size = 1.06

$$\frac{2\left(Bab - Ab^2\right)x^6 \log\left(bx^3 + a\right) - 6\left(Bab - Ab^2\right)x^6 \log(x) - 2\left(Ba^2 - Aab\right)x^3 - Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a*b - A*b^2)*x^6*log(b*x^3 + a) - 6*(B*a*b - A*b^2)*x^6*log(x) -
2*(B*a^2 - A*a*b)*x^3 - A*a^2)/(a^3*x^6)
```

giac [A] time = 0.19, size = 99, normalized size = 1.43

$$-\frac{\left(Bab - Ab^2\right)\log\left(|x|\right)}{a^3} + \frac{\left(Bab^2 - Ab^3\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b} + \frac{3\,Babx^6 - 3\,Ab^2x^6 - 2\,Ba^2x^3 + 2\,Aabx^3 - Aa^2}{6\,a^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -(B*a*b - A*b^2)*log(abs(x))/a^3 + 1/3*(B*a*b^2 - A*b^3)*log(abs(b*x^3 + a)
)/(a^3*b) + 1/6*(3*B*a*b*x^6 - 3*A*b^2*x^6 - 2*B*a^2*x^3 + 2*A*a*b*x^3 - A*
a^2)/(a^3*x^6)
```

maple [A] time = 0.05, size = 81, normalized size = 1.17

$$\frac{A\,b^2\ln(x)}{a^3} - \frac{A\,b^2\ln\left(b\,x^3 + a\right)}{3a^3} - \frac{Bb\ln(x)}{a^2} + \frac{Bb\ln\left(b\,x^3 + a\right)}{3a^2} + \frac{Ab}{3a^2x^3} - \frac{B}{3a\,x^3} - \frac{A}{6a\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a),x)
```

```
[Out] -1/3*b^2/a^3*ln(b*x^3+a)*A+1/3*b/a^2*ln(b*x^3+a)*B-1/6*A/a/x^6+1/3/a^2/x^3*
A*b-1/3/a/x^3*B+b^2/a^3*ln(x)*A-b/a^2*ln(x)*B
```

maxima [A] time = 0.49, size = 70, normalized size = 1.01

$$\frac{\left(Bab - Ab^2\right)\log\left(bx^3 + a\right)}{3\,a^3} - \frac{\left(Bab - Ab^2\right)\log\left(x^3\right)}{3\,a^3} - \frac{2\,(Ba - Ab)x^3 + Aa}{6\,a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*a*b - A*b^2)*log(b*x^3 + a)/a^3 - 1/3*(B*a*b - A*b^2)*log(x^3)/a^3 -
 1/6*(2*(B*a - A*b)*x^3 + A*a)/(a^2*x^6)
```

mupad [B] time = 0.13, size = 70, normalized size = 1.01

$$\frac{\ln(x)\,\left(A\,b^2 - B\,a\,b\right)}{a^3} - \frac{\ln\left(b\,x^3 + a\right)\,\left(A\,b^2 - B\,a\,b\right)}{3\,a^3} - \frac{\frac{A}{6\,a} - \frac{x^3\,(A\,b - B\,a)}{3\,a^2}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)),x)
```

```
[Out] (log(x)*(A*b^2 - B*a*b))/a^3 - (log(a + b*x^3)*(A*b^2 - B*a*b))/(3*a^3) - (
A/(6*a) - (x^3*(A*b - B*a))/(3*a^2))/x^6
```

sympy [A] time = 2.69, size = 61, normalized size = 0.88

$$\frac{-Aa + x^3(2Ab - 2Ba)}{6a^2x^6} - \frac{b(-Ab + Ba)\log(x)}{a^3} + \frac{b(-Ab + Ba)\log\left(\frac{a}{b} + x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a),x)
```

```
[Out] (-A*a + x**3*(2*A*b - 2*B*a))/(6*a**2*x**6) - b*(-A*b + B*a)*log(x)/a**3 +
b*(-A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

3.70 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=184

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}a^{10/3}}$$

```
[Out] -1/7*A/a/x^7+1/4*(A*b-B*a)/a^2/x^4-b*(A*b-B*a)/a^3/x+1/3*b^(4/3)*(A*b-B*a)*
ln(a^(1/3)+b^(1/3)*x)/a^(10/3)-1/6*b^(4/3)*(A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(10/3)+1/3*b^(4/3)*(A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {453, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] -A/(7*a*x^7) + (A*b - a*B)/(4*a^2*x^4) - (b*(A*b - a*B))/(a^3*x) + (b^(4/3)
*(A*b - a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^
(10/3)) + (b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(10/3)) - (b^
(4/3)*(A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(10/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)}\,dx &= -\frac{A}{7ax^7}-\frac{(7Ab-7aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{7a}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}+\frac{(b(Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{a^2}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}-\frac{\left(b^2(Ab-aB)\right)\int \frac{x}{a+bx^3}\,dx}{a^3}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}}-\frac{\left(b^{5/3}(Ab-aB)\right)\int \frac{}{a^{2/3}-\sqrt[3]{}}}{3a^{10/3}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{\left(b^{4/3}(Ab-aB)\right)\int \frac{-}{a^{2/3}}}{6a^{10/3}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}}-\frac{b^{4/3}(Ab-aB)\log\left(a^{2/}\right.}{6a^{}}\\
&= -\frac{A}{7ax^7}+\frac{Ab-aB}{4a^2x^4}-\frac{b(Ab-aB)}{a^3x}+\frac{b^{4/3}(Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}}+\frac{b^{4/3}(Ab-aB)\log\left(\sqrt[3]{\right.}}{3a^{10/3}}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 173, normalized size = 0.94

$$
\frac{14b^{4/3}(aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{21a^{4/3}(Ab-aB)}{x^4}-\frac{12a^{7/3}A}{x^7}+28b^{4/3}(Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{}}{84a^{10/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)),x]
```

```
[Out] ((-12*a^(7/3)*A)/x^7 + (21*a^(4/3)*(A*b - a*B))/x^4 + (84*a^(1/3)*b*(-(A*b)
 + a*B))/x + 28*Sqrt[3]*b^(4/3)*(A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]] + 28*b^(4/3)*(A*b - a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(4/3)
*(-(A*b) + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(84*a^(10/3
))
```

fricas [A] time = 0.93, size = 180, normalized size = 0.98

$$\frac{28\sqrt{3}\left(Bab-Ab^2\right)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(Bab-Ab^2\right)x^7\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\frac{b}{a}\right)^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right.}{84\,a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)
^(1/3) - 1/3*sqrt(3)) + 14*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x^2 - a*x*
(b/a)^(2/3) + a*(b/a)^(1/3)) - 28*(B*a*b - A*b^2)*x^7*(b/a)^(1/3)*log(b*x +
 a*(b/a)^(2/3)) + 84*(B*a*b - A*b^2)*x^6 - 21*(B*a^2 - A*a*b)*x^3 - 12*A*a^
2)/(a^3*x^7)
```

giac [A] time = 0.18, size = 216, normalized size = 1.17

$$-\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{2}{3}}Ba-\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^4}-\frac{\left(Bab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-Ab^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*((-a*b^2)^(2/3)*B*a - (-a*b^2)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(
2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 - 1/3*(B*a*b^2*(-a/b)^(1/3) - A*b^3*(
-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 1/6*((-a*b^2)^(2
/3)*B*a - (-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4
+ 1/28*(28*B*a*b*x^6 - 28*A*b^2*x^6 - 7*B*a^2*x^3 + 7*A*a*b*x^3 - 4*A*a^2)/
(a^3*x^7)
```

maple [A] time = 0.06, size = 247, normalized size = 1.34

$$-\frac{\sqrt{3}\,A\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{A\,b^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{A\,b^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{\sqrt{3}\,Bb\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a),x)
```

```
[Out] 1/3/a^3*b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/3/a^2*b/(a/b)^(1/3)*ln(x+(a/b
)^(1/3))*B-1/6/a^3*b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/6/
```

```
a^2*b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B-1/3/a^3*b^2*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/3/a^2*b*3^(1/2)/(a/b
)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B-1/7*A/a/x^7+1/4/a^2/x^4*A
*b-1/4/a/x^4*B-b^2/a^3/x*A+b/a^2/x*B
```

maxima [A] time = 1.29, size = 178, normalized size = 0.97

$$\frac{\sqrt{3}\left(Bab-Ab^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\left(Bab-Ab^{2}\right)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\left(Bab-Ab^{2}\right)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{28}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(B*a*b - A*b^2)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1
/3))/(a^3*(a/b)^(1/3)) + 1/6*(B*a*b - A*b^2)*log(x^2 - x*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) - 1/3*(B*a*b - A*b^2)*log(x + (a/b)^(1/3))/(a^3*
(a/b)^(1/3)) + 1/28*(28*(B*a*b - A*b^2)*x^6 - 7*(B*a^2 - A*a*b)*x^3 - 4*A*a
^2)/(a^3*x^7)
```

mupad [B] time = 2.58, size = 161, normalized size = 0.88

$$\frac{b^{4/3}\ln\left(b^{1/3}x+a^{1/3}\right)(A\,b-B\,a)}{3\,a^{10/3}}-\frac{\frac{A}{7\,a}-\frac{x^{3}(A\,b-B\,a)}{4\,a^{2}}+\frac{b\,x^{6}(A\,b-B\,a)}{a^{3}}}{x^{7}}+\frac{b^{4/3}\ln\left(a^{1/3}-2\,b^{1/3}x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right.}{3\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)),x)
```

```
[Out] (b^(4/3)*log(b^(1/3)*x + a^(1/3))*(A*b - B*a))/(3*a^(10/3)) - (A/(7*a) - (x
^3*(A*b - B*a))/(4*a^2) + (b*x^6*(A*b - B*a))/a^3)/x^7 + (b^(4/3)*log(3^(1/
2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a))/
(3*a^(10/3)) - (b^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a))/(3*a^(10/3))
```

sympy [A] time = 1.13, size = 139, normalized size = 0.76

$$\operatorname{RootSum}\left(27t^{3}a^{10}-A^{3}b^{7}+3A^{2}Bab^{6}-3AB^{2}a^{2}b^{5}+B^{3}a^{3}b^{4},\left(t\mapsto t\log\left(\frac{9t^{2}a^{7}}{A^{2}b^{5}-2ABab^{4}+B^{2}a^{2}b^{3}}+x\right)\right)\right)+\frac{-4A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**10 - A**3*b**7 + 3*A**2*B*a*b**6 - 3*A*B**2*a**2*b**5 +
 B**3*a**3*b**4, Lambda(_t, _t*log(9*_t**2*a**7/(A**2*b**5 - 2*A*B*a*b**4 +
 B**2*a**2*b**3) + x))) + (-4*A*a**2 + x**6*(-28*A*b**2 + 28*B*a*b) + x**3*
(7*A*a*b - 7*B*a**2))/(28*a**3*x**7)
```

3.71 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-}}{3\sqrt{3}\,b^{13/3}}$$

```
[Out] -1/3*a*(7*A*b-10*B*a)*x/b^4+1/12*(7*A*b-10*B*a)*x^4/b^3-1/21*(7*A*b-10*B*a)
*x^7/a/b^2+1/3*(A*b-B*a)*x^10/a/b/(b*x^3+a)+1/9*a^(4/3)*(7*A*b-10*B*a)*ln(a
^(1/3)+b^(1/3)*x)/b^(13/3)-1/18*a^(4/3)*(7*A*b-10*B*a)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(13/3)-1/9*a^(4/3)*(7*A*b-10*B*a)*arctan(1/3*(a^(1/
3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}(7Ab-10aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{13/3}}+\frac{a^{4/3}(7Ab-10aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{13/3}}-\frac{a^{4/3}(7Ab-10aB)\tan^{-}}{3\sqrt{3}\,b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(7*A*b - 10*a*B)*x)/(3*b^4) + ((7*A*b - 10*a*B)*x^4)/(12*b^3) - ((7*A*b
 - 10*a*B)*x^7)/(21*a*b^2) + ((A*b - a*B)*x^10)/(3*a*b*(a + b*x^3)) - (a^(4
/3)*(7*A*b - 10*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*
Sqrt[3]*b^(13/3)) + (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*
b^(13/3)) - (a^(4/3)*(7*A*b - 10*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/(18*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^9\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{(-7Ab+10aB)\int \frac{x^9}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{(-7Ab+10aB)\int \left(\frac{a^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^6}{b}-\frac{a^3}{b^3\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{\left(a^2(7Ab-10a\right.}{3b}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{\left(a^{4/3}(7Ab-10\right.}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{a^{4/3}(7Ab-10a}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} + \frac{a^{4/3}(7Ab-10a}{}$$

$$= -\frac{a(7Ab-10aB)x}{3b^4} + \frac{(7Ab-10aB)x^4}{12b^3} - \frac{(7Ab-10aB)x^7}{21ab^2} + \frac{(Ab-aB)x^{10}}{3ab\left(a+bx^3\right)} - \frac{a^{4/3}(7Ab-10a}{3}$$

Mathematica [A] time = 0.17, size = 203, normalized size = 0.87

$$\frac{14a^{4/3}(10aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-28a^{4/3}(10aB-7Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+28\sqrt{3}a^{4/3}(10aB-}{252b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (252*a*b^(1/3)*(-2*A*b + 3*a*B)*x + 63*b^(4/3)*(A*b - 2*a*B)*x^4 + 36*b^(7/
3)*B*x^7 + (84*a^2*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + 28*Sqrt[3]*a^(4/
3)*(-7*A*b + 10*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 28*a^(4/
3)*(-7*A*b + 10*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*a^(4/3)*(-7*A*b + 10*a*B
)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(252*b^(13/3))
```

fricas [A] time = 1.07, size = 271, normalized size = 1.16

$$\frac{36\,Bb^3x^{10}-9\left(10\,Bab^2-7\,Ab^3\right)x^7+63\left(10\,Ba^2b-7\,Aab^2\right)x^4-28\sqrt{3}\left(10\,Ba^3-7\,Aa^2b+\left(10\,Ba^2b-7\,Aab^2\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(36*B*b^3*x^10 - 9*(10*B*a*b^2 - 7*A*b^3)*x^7 + 63*(10*B*a^2*b - 7*A*
a*b^2)*x^4 - 28*sqrt(3)*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^
3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)*a)/a) + 14*(
10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A*a*b^2)*x^3)*(a/b)^(1/3)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3)) - 28*(10*B*a^3 - 7*A*a^2*b + (10*B*a^2*b - 7*A
*a*b^2)*x^3)*(a/b)^(1/3)*log(x + (a/b)^(1/3)) + 84*(10*B*a^3 - 7*A*a^2*b)*x
)/(b^5*x^3 + a*b^4)
```

giac [A] time = 0.20, size = 244, normalized size = 1.05

$$-\frac{\sqrt{3}\left(10\left(-ab^2\right)^{\frac{1}{3}}Ba^2-7\left(-ab^2\right)^{\frac{1}{3}}Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5}+\frac{\left(10\,Ba^3-7\,Aa^2b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(10*(-a*b^2)^(1/3)*B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*arctan(1/3*
sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 + 1/9*(10*B*a^3 - 7*A*a^2*b)
*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) - 1/18*(10*(-a*b^2)^(1/3)*
B*a^2 - 7*(-a*b^2)^(1/3)*A*a*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
5 + 1/3*(B*a^3*x - A*a^2*b*x)/((b*x^3 + a)*b^4) + 1/28*(4*B*b^12*x^7 - 14*B
*a*b^11*x^4 + 7*A*b^12*x^4 + 84*B*a^2*b^10*x - 56*A*a*b^11*x)/b^14
```

maple [A] time = 0.06, size = 288, normalized size = 1.24

$$\frac{B x^7}{7b^2}+\frac{A x^4}{4b^2}-\frac{B a x^4}{2b^3}-\frac{A a^2 x}{3\left(b x^3+a\right) b^3}+\frac{B a^3 x}{3\left(b x^3+a\right) b^4}+\frac{7\sqrt{3}\, A a^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4}+\frac{7A a^2 \ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4}-\frac{7A a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/7/b^2*B*x^7+1/4/b^2*A*x^4-1/2/b^3*B*x^4*a-2/b^3*a*A*x+3/b^4*a^2*B*x-1/3*a
^2/b^3*x/(b*x^3+a)*A+1/3*a^3/b^4*x/(b*x^3+a)*B+7/9*a^2/b^4*A/(a/b)^(2/3)*ln
(x+(a/b)^(1/3))-7/18*a^2/b^4*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)+7/9*a^2/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
-10/9*a^3/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/9*a^3/b^5*B/(a/b)^(2/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-10/9*a^3/b^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.23, size = 218, normalized size = 0.94

$$\frac{\left(Ba^3-Aa^2b\right)x}{3\left(b^5x^3+ab^4\right)}+\frac{4\,Bb^2x^7-7\left(2\,Bab-Ab^2\right)x^4+28\left(3\,Ba^2-2\,Aab\right)x}{28\,b^4}-\frac{\sqrt{3}\left(10\,Ba^3-7\,Aa^2b\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)*x/(b^5*x^3 + a*b^4) + 1/28*(4*B*b^2*x^7 - 7*(2*B*a*b
- A*b^2)*x^4 + 28*(3*B*a^2 - 2*A*a*b)*x)/b^4 - 1/9*sqrt(3)*(10*B*a^3 - 7*A*
a^2*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(2/3)
) + 1/18*(10*B*a^3 - 7*A*a^2*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^5
*(a/b)^(2/3)) - 1/9*(10*B*a^3 - 7*A*a^2*b)*log(x + (a/b)^(1/3))/(b^5*(a/b)^
(2/3))
```

mupad [B] time = 2.62, size = 209, normalized size = 0.90

$$x^4\left(\frac{A}{4\,b^2}-\frac{B\,a}{2\,b^3}\right)-x\left(\frac{2\,a\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)}{b}+\frac{B\,a^2}{b^4}\right)+\frac{B\,x^7}{7\,b^2}+\frac{x\left(\frac{B\,a^3}{3}-\frac{A\,a^2\,b}{3}\right)}{b^5\,x^3+a\,b^4}+\frac{a^{4/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-10\,B\,a\right)}{9\,b^{13/3}}-\frac{a^{4/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^4*(A/(4*b^2) - (B*a)/(2*b^3)) - x*((2*a*(A/b^2 - (2*B*a)/b^3))/b + (B*a^2
)/b^4) + (B*x^7)/(7*b^2) + (x*((B*a^3)/3 - (A*a^2*b)/3))/(a*b^4 + b^5*x^3)
+ (a^(4/3)*log(b^(1/3)*x + a^(1/3))*(7*A*b - 10*B*a))/(9*b^(13/3)) - (a^(4/
3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - 10*B*a))/(9*b^(13/3)) + (a^(4/3)*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 10*B*a))/(9*b^(13/3))
```

sympy [A] time = 2.12, size = 156, normalized size = 0.67

$$\frac{Bx^7}{7b^2}+x^4\left(\frac{A}{4b^2}-\frac{Ba}{2b^3}\right)+x\left(-\frac{2Aa}{b^3}+\frac{3Ba^2}{b^4}\right)+\frac{x\left(-Aa^2b+Ba^3\right)}{3ab^4+3b^5x^3}+\mathrm{RootSum}\left(729t^3b^{13}-343A^3a^4b^3+1470A^2Ba^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**7/(7*b**2) + x**4*(A/(4*b**2) - B*a/(2*b**3)) + x*(-2*A*a/b**3 + 3*B*a
**2/b**4) + x*(-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3) + RootSum(729*_
t**3*b**13 - 343*A**3*a**4*b**3 + 1470*A**2*B*a**5*b**2 - 2100*A*B**2*a**6*
b + 1000*B**3*a**7, Lambda(_t, _t*log(-9*_t*b**4/(-7*A*a*b + 10*B*a**2) + x
)))
```

3.72 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=82

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{x^3(Ab-2aB)}{3b^3}+\frac{Bx^6}{6b^2}$$

```
[Out] 1/3*(A*b-2*B*a)*x^3/b^3+1/6*B*x^6/b^2-1/3*a^2*(A*b-B*a)/b^4/(b*x^3+a)-1/3*a
*(2*A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}+\frac{x^3(Ab-2aB)}{3b^3}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}+\frac{Bx^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - 2*a*B)*x^3)/(3*b^3) + (B*x^6)/(6*b^2) - (a^2*(A*b - a*B))/(3*b^4*(a
 + b*x^3)) - (a*(2*A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{Ab-2aB}{b^3}+\frac{Bx}{b^2}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^2}+\frac{a(-2Ab+3aB)}{b^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{(Ab-2aB)x^3}{3b^3}+\frac{Bx^6}{6b^2}-\frac{a^2(Ab-aB)}{3b^4\left(a+bx^3\right)}-\frac{a(2Ab-3aB)\log\left(a+bx^3\right)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.88

$$\frac{\frac{2a^2(aB-Ab)}{a+bx^3}+2bx^3(Ab-2aB)+2a(3aB-2Ab)\log\left(a+bx^3\right)+b^2Bx^6}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*b*(A*b - 2*a*B)*x^3 + b^2*B*x^6 + (2*a^2*(-(A*b) + a*B))/(a + b*x^3) + 2
*a*(-2*A*b + 3*a*B)*Log[a + b*x^3])/(6*b^4)
```

fricas [A] time = 0.59, size = 121, normalized size = 1.48

$$\frac{Bb^3x^9 - \left(3\,Bab^2 - 2\,Ab^3\right)x^6 + 2\,Ba^3 - 2\,Aa^2b - 2\left(2\,Ba^2b - Aab^2\right)x^3 + 2\left(3\,Ba^3 - 2\,Aa^2b + \left(3\,Ba^2b - 2\,Aab^2\right)\right.}{6\left(b^5x^3 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^3*x^9 - (3*B*a*b^2 - 2*A*b^3)*x^6 + 2*B*a^3 - 2*A*a^2*b - 2*(2*B*a
^2*b - A*a*b^2)*x^3 + 2*(3*B*a^3 - 2*A*a^2*b + (3*B*a^2*b - 2*A*a*b^2)*x^3)
*log(b*x^3 + a))/(b^5*x^3 + a*b^4)
```

giac [A] time = 0.18, size = 106, normalized size = 1.29

$$\frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(\left|bx^3 + a\right|\right)}{3\,b^4} + \frac{Bb^2x^6 - 4\,Babx^3 + 2\,Ab^2x^3}{6\,b^4} - \frac{3\,Ba^2bx^3 - 2\,Aab^2x^3 + 2\,Ba^3 - Aa^2b}{3\left(bx^3 + a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(3*B*a^2 - 2*A*a*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(B*b^2*x^6 - 4*B*a*b*
x^3 + 2*A*b^2*x^3)/b^4 - 1/3*(3*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 2*B*a^3 - A*a
^2*b)/((b*x^3 + a)*b^4)
```

maple [A] time = 0.05, size = 97, normalized size = 1.18

$$\frac{B\,x^6}{6b^2} + \frac{A\,x^3}{3b^2} - \frac{2Ba\,x^3}{3b^3} - \frac{A\,a^2}{3\left(b\,x^3 + a\right)b^3} - \frac{2Aa\ln\left(b\,x^3 + a\right)}{3b^3} + \frac{B\,a^3}{3\left(b\,x^3 + a\right)b^4} + \frac{B\,a^2\ln\left(b\,x^3 + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/6*B*x^6/b^2+1/3/b^2*A*x^3-2/3/b^3*B*a*x^3-2/3*a/b^3*ln(b*x^3+a)*A+a^2/b^4
*ln(b*x^3+a)*B-1/3*a^2/b^3/(b*x^3+a)*A+1/3*a^3/b^4/(b*x^3+a)*B
```

maxima [A] time = 0.49, size = 82, normalized size = 1.00

$$\frac{Ba^3 - Aa^2b}{3\left(b^5x^3 + ab^4\right)} + \frac{Bbx^6 - 2\,(2\,Ba - Ab)x^3}{6\,b^3} + \frac{\left(3\,Ba^2 - 2\,Aab\right)\log\left(bx^3 + a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a^3 - A*a^2*b)/(b^5*x^3 + a*b^4) + 1/6*(B*b*x^6 - 2*(2*B*a - A*b)*x^
3)/b^3 + 1/3*(3*B*a^2 - 2*A*a*b)*log(b*x^3 + a)/b^4
```

mupad [B] time = 0.08, size = 86, normalized size = 1.05

$$x^3\left(\frac{A}{3\,b^2} - \frac{2\,B\,a}{3\,b^3}\right) + \frac{\ln\left(b\,x^3 + a\right)\,\left(3\,B\,a^2 - 2\,A\,a\,b\right)}{3\,b^4} + \frac{B\,x^6}{6\,b^2} + \frac{B\,a^3 - A\,a^2\,b}{3\,b\,\left(b^4\,x^3 + a\,b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^3*(A/(3*b^2) - (2*B*a)/(3*b^3)) + (log(a + b*x^3)*(3*B*a^2 - 2*A*a*b))/(3
*b^4) + (B*x^6)/(6*b^2) + (B*a^3 - A*a^2*b)/(3*b*(a*b^3 + b^4*x^3))
```

sympy [A] time = 2.14, size = 82, normalized size = 1.00

$$\frac{Bx^6}{6b^2} + \frac{a\left(-2Ab + 3Ba\right)\log\left(a + bx^3\right)}{3b^4} + x^3\left(\frac{A}{3b^2} - \frac{2Ba}{3b^3}\right) + \frac{-Aa^2b + Ba^3}{3ab^4 + 3b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**6/(6*b**2) + a*(-2*A*b + 3*B*a)*log(a + b*x**3)/(3*b**4) + x**3*(A/(3*
b**2) - 2*B*a/(3*b**3)) + (-A*a**2*b + B*a**3)/(3*a*b**4 + 3*b**5*x**3)
```

3.73 $\int \frac{x^7(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=215

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\ \right.}{3\sqrt{3}\,b^{11/3}}$$

```
[Out] 1/6*(5*A*b-8*B*a)*x^2/b^3-1/15*(5*A*b-8*B*a)*x^5/a/b^2+1/3*(A*b-B*a)*x^8/a/
b/(b*x^3+a)+1/9*a^(2/3)*(5*A*b-8*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(11/3)-1/18*a
^(2/3)*(5*A*b-8*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(11/3)+1/9
*a^(2/3)*(5*A*b-8*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^
(11/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}(5Ab-8aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9b^{11/3}}+\frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\ \right.}{3\sqrt{3}\,b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((5*A*b - 8*a*B)*x^2)/(6*b^3) - ((5*A*b - 8*a*B)*x^5)/(15*a*b^2) + ((A*b -
a*B)*x^8)/(3*a*b*(a + b*x^3)) + (a^(2/3)*(5*A*b - 8*a*B)*ArcTan[(a^(1/3) -
2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(11/3)) + (a^(2/3)*(5*A*b - 8
*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(11/3)) - (a^(2/3)*(5*A*b - 8*a*B)*Log
[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{(-5Ab+8aB)\int \frac{x^7}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{(-5Ab+8aB)\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} - \frac{(a(5Ab-8aB))\int \frac{x}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{\left(a^{2/3}(5Ab-8aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9b^{10/3}} - \frac{\left(a^{2}\right.}{}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{11/3}} - \frac{(}{}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{a^{2/3}(5Ab-8aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{11/3}} - \frac{a}{}$$

$$= \frac{(5Ab-8aB)x^2}{6b^3} - \frac{(5Ab-8aB)x^5}{15ab^2} + \frac{(Ab-aB)x^8}{3ab\left(a+bx^3\right)} + \frac{a^{2/3}(5Ab-8aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{11/3}} + \frac{a}{}$$

Mathematica [A] time = 0.17, size = 185, normalized size = 0.86

$$\frac{5a^{2/3}(8aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-10a^{2/3}(8aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-10\sqrt{3}a^{2/3}(8aB-5Ab}{90b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (45*b^(2/3)*(A*b - 2*a*B)*x^2 + 18*b^(5/3)*B*x^5 + (30*a*b^(2/3)*(A*b - a*B
)*x^2)/(a + b*x^3) - 10*Sqrt[3]*a^(2/3)*(-5*A*b + 8*a*B)*ArcTan[(1 - (2*b^(
1/3)*x)/a^(1/3))/Sqrt[3]] - 10*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(1/3) + b^(1/
3)*x] + 5*a^(2/3)*(-5*A*b + 8*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3
)*x^2])/(90*b^(11/3))
```

fricas [A] time = 0.88, size = 257, normalized size = 1.20

$$\frac{18\,Bb^2x^8-9\left(8\,Bab-5\,Ab^2\right)x^5-15\left(8\,Ba^2-5\,Aab\right)x^2+10\,\sqrt{3}\left(\left(8\,Bab-5\,Ab^2\right)x^3+8\,Ba^2-5\,Aab\right)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\text{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/90*(18*B*b^2*x^8 - 9*(8*B*a*b - 5*A*b^2)*x^5 - 15*(8*B*a^2 - 5*A*a*b)*x^2
 + 10*sqrt(3)*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)
*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*((8*B*a*b -
5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a*b)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)
^(2/3) + a*(a^2/b^2)^(1/3)) - 10*((8*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 5*A*a
*b)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^4*x^3 + a*b^3)
```

giac [A] time = 0.18, size = 236, normalized size = 1.10

$$-\frac{\left(8\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3}-\frac{\sqrt{3}\left(8\left(-ab^2\right)^{\frac{2}{3}}Ba-5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+(\right.}{3\left(-\frac{a}{b}\right.}\right.}{9\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(8*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a*b^3) - 1/9*sqrt(3)*(8*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2
/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 1/3*(B
*a^2*x^2 - A*a*b*x^2)/((b*x^3 + a)*b^3) + 1/18*(8*(-a*b^2)^(2/3)*B*a - 5*(-
a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 + 1/10*(2*B*
b^8*x^5 - 10*B*a*b^7*x^2 + 5*A*b^8*x^2)/b^10
```

maple [A] time = 0.05, size = 266, normalized size = 1.24

$$\frac{Bx^5}{5b^2}+\frac{Aax^2}{3\left(bx^3+a\right)b^2}-\frac{Ba^2x^2}{3\left(bx^3+a\right)b^3}+\frac{Ax^2}{2b^2}-\frac{Bax^2}{b^3}-\frac{5\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5Aa\ln\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/5/b^2*B*x^5+1/2/b^2*A*x^2-1/b^3*B*x^2*a+1/3*a/b^2*x^2/(b*x^3+a)*A-1/3*a^2
/b^3*x^2/(b*x^3+a)*B+5/9*a/b^3*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18*a/b^3*A
/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9*a/b^3*A*3^(1/2)/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-8/9*a^2/b^4*B/(a/b)^(1/3)*ln(x+
(a/b)^(1/3))+4/9*a^2/b^4*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+8/
9*a^2/b^4*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.10, size = 192, normalized size = 0.89

$$-\frac{\left(Ba^2-Aab\right)x^2}{3\left(b^4x^3+ab^3\right)}+\frac{\sqrt{3}\left(8\,Ba^2-5\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5-5\,(2\,Ba-Ab)x^2}{10\,b^3}+\frac{\left(8\,Ba^2-5\,Aab\right)\log\left(x^2\right.}{18\,b^4\left(\frac{a}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x^2/(b^4*x^3 + a*b^3) + 1/9*sqrt(3)*(8*B*a^2 - 5*A*a*b
)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) + 1
/10*(2*B*b*x^5 - 5*(2*B*a - A*b)*x^2)/b^3 + 1/18*(8*B*a^2 - 5*A*a*b)*log(x^
2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) - 1/9*(8*B*a^2 - 5*A*a*b
)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

mupad [B] time = 0.27, size = 179, normalized size = 0.83

$$x^2\left(\frac{A}{2\,b^2}-\frac{B\,a}{b^3}\right)+\frac{B\,x^5}{5\,b^2}-\frac{x^2\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\left(5\,A\,b-8\,B\,a\right)}{9\,b^{11/3}}+\frac{a^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\right.}{9\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x^2*(A/(2*b^2) - (B*a)/b^3) + (B*x^5)/(5*b^2) - (x^2*((B*a^2)/3 - (A*a*b)/3
))/(a*b^3 + b^4*x^3) + (a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 8*B*a))/(
9*b^(11/3)) + (a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(5*A*b - 8*B*a))/(9*b^(11/3)) - (a^(2/3)*log(3^(1/2)*a^(
1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 8*B*a))/(9
*b^(11/3))
```

sympy [A] time = 2.12, size = 151, normalized size = 0.70

$$\frac{Bx^5}{5b^2}+x^2\left(\frac{A}{2b^2}-\frac{Ba}{b^3}\right)+\frac{x^2\left(Aab-Ba^2\right)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{11}-125A^3a^2b^3+600A^2Ba^3b^2-960AB^2a^4b+512B^3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**5/(5*b**2) + x**2*(A/(2*b**2) - B*a/b**3) + x**2*(A*a*b - B*a**2)/(3*a
*b**3 + 3*b**4*x**3) + RootSum(729*_t**3*b**11 - 125*A**3*a**2*b**3 + 600*A
**2*B*a**3*b**2 - 960*A*B**2*a**4*b + 512*B**3*a**5, Lambda(_t, _t*log(81*_
t**2*b**7/(25*A**2*a*b**2 - 80*A*B*a**2*b + 64*B**2*a**3) + x)))
```

3.74 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}}+\frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\ldots}{\sqrt{3}\ldots}\right.}{3\sqrt{3}\,b^{10/3}}$$

```
[Out] 1/3*(4*A*b-7*B*a)*x/b^3-1/12*(4*A*b-7*B*a)*x^4/a/b^2+1/3*(A*b-B*a)*x^7/a/b/
(b*x^3+a)-1/9*a^(1/3)*(4*A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(10/3)+1/18*a^(
1/3)*(4*A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(10/3)+1/9*a
^(1/3)*(4*A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(1
0/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 302, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18b^{10/3}}-\frac{x^4(4Ab-7aB)}{12ab^2}+\frac{x(4Ab-7aB)}{3b^3}-\frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\ldots\right.}{9b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((4*A*b - 7*a*B)*x)/(3*b^3) - ((4*A*b - 7*a*B)*x^4)/(12*a*b^2) + ((A*b - a*
B)*x^7)/(3*a*b*(a + b*x^3)) + (a^(1/3)*(4*A*b - 7*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*b^(10/3)) - (a^(1/3)*(4*A*b - 7*a
*B)*Log[a^(1/3) + b^(1/3)*x])/(9*b^(10/3)) + (a^(1/3)*(4*A*b - 7*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \frac{x^6}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{(-4Ab+7aB)\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2\left(a+bx^3\right)}\right)dx}{3ab}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{(a(4Ab-7aB))\int \frac{1}{a+bx^3}\,dx}{3b^3}$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\left(\sqrt[3]{a}\,(4Ab-7aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9b^3} - \left(\cdots\right.$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}} + \cdots$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} - \frac{\sqrt[3]{a}\,(4Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9b^{10/3}} + \cdots$$

$$= \frac{(4Ab-7aB)x}{3b^3} - \frac{(4Ab-7aB)x^4}{12ab^2} + \frac{(Ab-aB)x^7}{3ab\left(a+bx^3\right)} + \frac{\sqrt[3]{a}\,(4Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,b^{10/3}} - \cdots$$

Mathematica [A] time = 0.15, size = 181, normalized size = 0.85

$$\frac{-2\sqrt[3]{a}\,(7aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)+\frac{12a\sqrt[3]{b}\,x(Ab-aB)}{a+bx^3}+36\sqrt[3]{b}\,x(Ab-2aB)+4\sqrt[3]{a}\,(7aB-4Ab)\log\left(\sqrt[3]{\right.}}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*(A*b - 2*a*B)*x + 9*b^(4/3)*B*x^4 + (12*a*b^(1/3)*(A*b - a*B)*x
)/(a + b*x^3) - 4*Sqrt[3]*a^(1/3)*(-4*A*b + 7*a*B)*ArcTan[(1 - (2*b^(1/3)*x
)/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(1/3) + b^(1/3)*x] -
 2*a^(1/3)*(-4*A*b + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])
/(36*b^(10/3))
```

fricas [A] time = 0.98, size = 240, normalized size = 1.13

$$9\,Bb^2x^7-9\left(7\,Bab-4\,Ab^2\right)x^4-4\,\sqrt{3}\left(\left(7\,Bab-4\,Ab^2\right)x^3+7\,Ba^2-4\,Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}\,bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3\,a}\right)+2\,\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(9*B*b^2*x^7 - 9*(7*B*a*b - 4*A*b^2)*x^4 - 4*sqrt(3)*((7*B*a*b - 4*A*b
^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^
(2/3) - sqrt(3)*a)/a) + 2*((7*B*a*b - 4*A*b^2)*x^3 + 7*B*a^2 - 4*A*a*b)*(-a
/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3)) - 4*((7*B*a*b - 4*A*b^2)
*x^3 + 7*B*a^2 - 4*A*a*b)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 12*(7*B*a^2
- 4*A*a*b)*x)/(b^4*x^3 + a*b^3)
```

giac [A] time = 0.18, size = 211, normalized size = 0.99

$$\frac{\sqrt{3}\left(7\left(-ab^2\right)^{\frac{1}{3}}Ba-4\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4}-\frac{\left(7\,Ba^2-4\,Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^3}+\frac{\left(7\left(-ab^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(7*(-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3
)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 1/9*(7*B*a^2 - 4*A*a*b)*(-a/b)^(
1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(7*(-a*b^2)^(1/3)*B*a - 4*(-
a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 - 1/3*(B*a^2
*x - A*a*b*x)/((b*x^3 + a)*b^3) + 1/4*(B*b^6*x^4 - 8*B*a*b^5*x + 4*A*b^6*x)
/b^8
```

maple [A] time = 0.09, size = 257, normalized size = 1.21

$$\frac{B\,x^4}{4b^2}+\frac{Aax}{3\left(b\,x^3+a\right)b^2}-\frac{B\,a^2x}{3\left(b\,x^3+a\right)b^3}-\frac{4\sqrt{3}\;Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4Aa\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2Aa\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/4/b^2*B*x^4+1/b^2*A*x-2/b^3*B*a*x+1/3*a/b^2*x/(b*x^3+a)*A-1/3*a^2/b^3*x/(
b*x^3+a)*B-4/9*a/b^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+2/9*a/b^3*A/(a/b)^(2/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9*a/b^3*A/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+7/9*a^2/b^4*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-7/18*a^2/b^4*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+7/9*a^2/b^4*
B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.01, size = 187, normalized size = 0.88

$$-\frac{\left(Ba^2-Aab\right)x}{3\left(b^4x^3+ab^3\right)}+\frac{Bbx^4-4\left(2\,Ba-Ab\right)x}{4\,b^3}+\frac{\sqrt{3}\left(7\,Ba^2-4\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(7\,Ba^2-4\,Aab\right)\log\left(x^2\right.}{18\,b^4\left(\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a^2 - A*a*b)*x/(b^4*x^3 + a*b^3) + 1/4*(B*b*x^4 - 4*(2*B*a - A*b)*x
)/b^3 + 1/9*sqrt(3)*(7*B*a^2 - 4*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/
3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/18*(7*B*a^2 - 4*A*a*b)*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/9*(7*B*a^2 - 4*A*a*b)*log(x
 + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 2.62, size = 193, normalized size = 0.91

$$x\left(\frac{A}{b^2}-\frac{2\,B\,a}{b^3}\right)-\frac{x\left(\frac{B\,a^2}{3}-\frac{A\,a\,b}{3}\right)}{b^4\,x^3+a\,b^3}+\frac{B\,x^4}{4\,b^2}+\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}+a\,b^{1/3}\,x\right)\,\left(4\,A\,b-7\,B\,a\right)}{9\,b^{10/3}}-\frac{(-a)^{1/3}\,\ln\left((-a)^{4/3}-2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] x*(A/b^2 - (2*B*a)/b^3) - (x*((B*a^2)/3 - (A*a*b)/3))/(a*b^3 + b^4*x^3) + (
B*x^4)/(4*b^2) + ((-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(4*A*b - 7*B*a))
/(9*b^(10/3)) - ((-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^
(1/3)*x)*((3^(1/2)*1i)/2 + 1/2)*(4*A*b - 7*B*a))/(9*b^(10/3)) + ((-a)^(1/3)
*log(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 -
1/2)*(4*A*b - 7*B*a))/(9*b^(10/3))
```

sympy [A] time = 2.54, size = 126, normalized size = 0.59

$$\frac{Bx^4}{4b^2}+x\left(\frac{A}{b^2}-\frac{2Ba}{b^3}\right)+\frac{x\left(Aab-Ba^2\right)}{3ab^3+3b^4x^3}+\operatorname{RootSum}\left(729t^3b^{10}+64A^3ab^3-336A^2Ba^2b^2+588AB^2a^3b-343B^3a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**4/(4*b**2) + x*(A/b**2 - 2*B*a/b**3) + x*(A*a*b - B*a**2)/(3*a*b**3 +
3*b**4*x**3) + RootSum(729*_t**3*b**10 + 64*A**3*a*b**3 - 336*A**2*B*a**2*b
**2 + 588*A*B**2*a**3*b - 343*B**3*a**4, Lambda(_t, _t*log(9*_t*b**3/(-4*A*
b + 7*B*a) + x)))
```

3.75 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=60

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

```
[Out] 1/3*B*x^3/b^2+1/3*a*(A*b-B*a)/b^3/(b*x^3+a)+1/3*(A*b-2*B*a)*ln(b*x^3+a)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}+\frac{Bx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (B*x^3)/(3*b^2) + (a*(A*b - a*B))/(3*b^3*(a + b*x^3)) + ((A*b - 2*a*B)*Log[
a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{B}{b^2}+\frac{a(-Ab+aB)}{b^2(a+bx)^2}+\frac{Ab-2aB}{b^2(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Bx^3}{3b^2}+\frac{a(Ab-aB)}{3b^3(a+bx^3)}+\frac{(Ab-2aB)\log(a+bx^3)}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.83

$$\frac{\frac{a(Ab-aB)}{a+bx^3}+(Ab-2aB)\log(a+bx^3)+bBx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (b*B*x^3 + (a*(A*b - a*B))/(a + b*x^3) + (A*b - 2*a*B)*Log[a + b*x^3])/(3*b
^3)
```

fricas [A] time = 0.93, size = 81, normalized size = 1.35

$$\frac{Bb^2x^6 + Babx^3 - Ba^2 + Aab - \left(\left(2\,Bab - Ab^2\right)x^3 + 2\,Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{3\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(B*b^2*x^6 + B*a*b*x^3 - B*a^2 + A*a*b - ((2*B*a*b - A*b^2)*x^3 + 2*B*a
^2 - A*a*b)*log(b*x^3 + a))/(b^4*x^3 + a*b^3)
```

giac [A] time = 0.18, size = 91, normalized size = 1.52

$$\frac{\frac{\left(bx^3+a\right)B}{b^2} + \frac{(2\,Ba-Ab)\log\left(\frac{\left|bx^3+a\right|}{\left(bx^3+a\right)^2|b|}\right)}{b^2} - \frac{\frac{Ba^2b}{bx^3+a}-\frac{Aab^2}{bx^3+a}}{b^3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*((b*x^3 + a)*B/b^2 + (2*B*a - A*b)*log(abs(b*x^3 + a)/((b*x^3 + a)^2*ab
s(b)))/b^2 - (B*a^2*b/(b*x^3 + a) - A*a*b^2/(b*x^3 + a))/b^3)/b
```

maple [A] time = 0.05, size = 74, normalized size = 1.23

$$\frac{B\,x^3}{3b^2} + \frac{Aa}{3\left(b\,x^3 + a\right)b^2} + \frac{A\ln\left(b\,x^3 + a\right)}{3b^2} - \frac{B\,a^2}{3\left(b\,x^3 + a\right)b^3} - \frac{2Ba\ln\left(b\,x^3 + a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*B*x^3/b^2+1/3/b^2*ln(b*x^3+a)*A-2/3/b^3*ln(b*x^3+a)*B*a+1/3/b^2*a/(b*x^
3+a)*A-1/3/b^3*a^2/(b*x^3+a)*B
```

maxima [A] time = 0.46, size = 60, normalized size = 1.00

$$\frac{Bx^3}{3\,b^2} - \frac{Ba^2 - Aab}{3\left(b^4x^3 + ab^3\right)} - \frac{(2\,Ba - Ab)\log\left(bx^3 + a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^2 - 1/3*(B*a^2 - A*a*b)/(b^4*x^3 + a*b^3) - 1/3*(2*B*a - A*b)*l
og(b*x^3 + a)/b^3
```

mupad [B] time = 0.08, size = 62, normalized size = 1.03

$$\frac{B\,x^3}{3\,b^2} + \frac{\ln\left(b\,x^3 + a\right)\,\left(A\,b - 2\,B\,a\right)}{3\,b^3} - \frac{B\,a^2 - A\,a\,b}{3\,b\,\left(b^3\,x^3 + a\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^3)/(3*b^2) + (log(a + b*x^3)*(A*b - 2*B*a))/(3*b^3) - (B*a^2 - A*a*b)/
(3*b*(a*b^2 + b^3*x^3))
```

sympy [A] time = 1.61, size = 56, normalized size = 0.93

$$\frac{Bx^3}{3b^2} + \frac{Aab - Ba^2}{3ab^3 + 3b^4x^3} - \frac{(-Ab + 2Ba)\log\left(a + bx^3\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**3/(3*b**2) + (A*a*b - B*a**2)/(3*a*b**3 + 3*b**4*x**3) - (-A*b + 2*B*a
)*log(a + b*x**3)/(3*b**3)
```

3.76 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=196

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}\,b^{8/3}}-x$$

```
[Out] -1/6*(2*A*b-5*B*a)*x^2/a/b^2+1/3*(A*b-B*a)*x^5/a/b/(b*x^3+a)-1/9*(2*A*b-5*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(8/3)+1/18*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(8/3)-1/9*(2*A*b-5*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(1/3)/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 321, 292, 31, 634, 617, 204, 628}

$$\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18\sqrt[3]{a}\,b^{8/3}}-\frac{x^2(2Ab-5aB)}{6ab^2}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\tan^{-1}}{3\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((2*A*b - 5*a*B)*x^2)/(6*a*b^2) + ((A*b - a*B)*x^5)/(3*a*b*(a + b*x^3)) -
((2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt
[3]*a^(1/3)*b^(8/3)) - ((2*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(1/3
)*b^(8/3)) + ((2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(-2Ab+5aB)\int\frac{x^4}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}+\frac{(2Ab-5aB)\int\frac{x}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9\sqrt[3]{a}\,b^{7/3}}+\frac{(2Ab-5aB)\int\frac{\sqrt[3]{a}+\cdots}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\cdots}}{9\sqrt[3]{a}\,b^{7/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}+\frac{(2Ab-5aB)\int\frac{-\sqrt[3]{a}\cdots}{a^{2/3}-\cdots}}{18\sqrt[3]{a}\,b^{8/3}}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9\sqrt[3]{a}\,b^{8/3}}+\frac{(2Ab-5aB)\log\left(a^{2/3}\cdots\right.}{18\sqrt[3]{a}\cdots}$$

$$= -\frac{(2Ab-5aB)x^2}{6ab^2}+\frac{(Ab-aB)x^5}{3ab\left(a+bx^3\right)}-\frac{(2Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}}-\frac{(2Ab-5aB)\log\left(\sqrt[3]{a}\cdots\right.}{9\sqrt[3]{a}\,b^{8/3}}$$

Mathematica [A] time = 0.14, size = 165, normalized size = 0.84

$$\frac{\frac{(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}}-\frac{6b^{2/3}x^2(Ab-aB)}{a+bx^3}+\frac{2(5aB-2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}+\frac{2\sqrt{3}(5aB-2Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}+9b^{2/3}Bx^2}{18b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(2/3)*B*x^2 - (6*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3]*(-2
*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (2*(-2
*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) + ((2*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(1/3))/(18*b^(8/3))
```

fricas [A] time = 0.77, size = 578, normalized size = 2.95

$$\left[\frac{9\,Bab^3x^5+3\left(5\,Ba^2b^2-2\,Aab^3\right)x^2-3\sqrt{\frac{1}{3}}\left(5\,Ba^3b-2\,Aa^2b^2+\left(5\,Ba^2b^2-2\,Aab^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 3*sqrt(1/3)*(5*B*a
^3*b - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*
log((2*b^2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)
^(1/3)*a)*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B
*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b
^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*
A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4), 1/1
8*(9*B*a*b^3*x^5 + 3*(5*B*a^2*b^2 - 2*A*a*b^3)*x^2 - 6*sqrt(1/3)*(5*B*a^3*b
 - 2*A*a^2*b^2 + (5*B*a^2*b^2 - 2*A*a*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arc
tan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - ((5*B*a
*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2
)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b - 2*A*b^2)*x^3 + 5*B*a^2 - 2*A*
a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a*b^5*x^3 + a^2*b^4)]
```

giac [A] time = 0.21, size = 189, normalized size = 0.96

$$\frac{Bx^2}{2\,b^2}-\frac{\sqrt{3}\,(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}b^2}+\frac{(5\,Ba-2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}b^2}+\frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a - 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b
)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^2) + 1/18*(5*B*a - 2*A*b)*log(x^2
```

```
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^2) + 1/9*(5*B*a*(-a/b)^(
1/3) - 2*A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2)
+ 1/3*(B*a*x^2 - A*b*x^2)/((b*x^3 + a)*b^2)
```

maple [A] time = 0.08, size = 235, normalized size = 1.20

$$-\frac{A\,x^{2}}{3\left(b\,x^{3}+a\right)b}+\frac{Ba\,x^{2}}{3\left(b\,x^{3}+a\right)b^{2}}+\frac{B\,x^{2}}{2b^{2}}+\frac{2\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/2*B*x^2/b^2-1/3/b*x^2/(b*x^3+a)*A+1/3/b^2*x^2/(b*x^3+a)*B*a+5/9/b^3*B*a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))-5/18/b^3*B*a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+
(a/b)^(2/3))-5/9/b^3*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-2/9/b^2*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/9/b^2*A/(a/b)^(1/3)*ln(
x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/9/b^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(
1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.17, size = 162, normalized size = 0.83

$$\frac{(Ba-Ab)x^{2}}{3\left(b^{3}x^{3}+ab^{2}\right)}+\frac{Bx^{2}}{2\,b^{2}}-\frac{\sqrt{3}\,(5\,Ba-2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(5\,Ba-2\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba-2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^2/(b^3*x^3 + a*b^2) + 1/2*B*x^2/b^2 - 1/9*sqrt(3)*(5*B*a
- 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/
3)) - 1/18*(5*B*a - 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b
)^(1/3)) + 1/9*(5*B*a - 2*A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

mupad [B] time = 2.58, size = 158, normalized size = 0.81

$$\frac{B\,x^2}{2\,b^2}-\frac{x^2\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^3\,x^3+a\,b^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(2\,A\,b-5\,B\,a\right)}{9\,a^{1/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,A\,b-5\,B\,a\right.}{9\,a^{1/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x^2)/(2*b^2) - (x^2*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (log(b^(1/3
)*x + a^(1/3))*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*
1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*A*b - 5*B*a))/(9*a^(1
/3)*b^(8/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1
i)/2 + 1/2)*(2*A*b - 5*B*a))/(9*a^(1/3)*b^(8/3))
```

sympy [A] time = 2.09, size = 126, normalized size = 0.64

$$\frac{Bx^{2}}{2b^{2}}+\frac{x^{2}\left(-Ab+Ba\right)}{3ab^{2}+3b^{3}x^{3}}+\operatorname{RootSum}\left(729t^{3}ab^{8}+8A^{3}b^{3}-60A^{2}Bab^{2}+150AB^{2}a^{2}b-125B^{3}a^{3},\left(t\mapsto t\log\left(\frac{}{4A^{2}b^{2}-}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x**2/(2*b**2) + x**2*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*
_t**3*a*b**8 + 8*A**3*b**3 - 60*A**2*B*a*b**2 + 150*A*B**2*a**2*b - 125*B**
3*a**3, Lambda(_t, _t*log(81*_t**2*a*b**5/(4*A**2*b**2 - 20*A*B*a*b + 25*B*
*2*a**2) + x)))
```

3.77 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal. Leaf size=190

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(Ab-}{3a}$$

```
[Out] -1/3*(A*b-4*B*a)*x/a/b^2+1/3*(A*b-B*a)*x^4/a/b/(b*x^3+a)+1/9*(A*b-4*B*a)*ln
(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(7/3)-1/18*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(7/3)-1/9*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{2/3}b^{7/3}}+\frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{2/3}b^{7/3}}-\frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}b^{7/3}}-\frac{x(Ab-}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 4*a*B)*x)/(3*a*b^2) + ((A*b - a*B)*x^4)/(3*a*b*(a + b*x^3)) - ((A*
b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^
(2/3)*b^(7/3)) + ((A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(2/3)*b^(7/3
)) - ((A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(
2/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} + \frac{(-Ab+4aB)\int \frac{x^3}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} + \frac{(Ab-4aB)\int \frac{1}{a+bx^3}\,dx}{3b^2}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} + \frac{(Ab-4aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{2/3}b^2} + \frac{(Ab-4aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\cdots}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x\cdots}}{9a^{2/3}b^2}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{\cdots}}{a^{2/3}-\sqrt[3]{a}\cdots}}{18a^{2/3}b^{7/3}}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{2/3}b^{7/3}} - \frac{(Ab-4aB)\log\left(a^{2/3}\cdots\right)}{18a^{2/3}\cdots}$$

$$= -\frac{(Ab-4aB)x}{3ab^2} + \frac{(Ab-aB)x^4}{3ab\left(a+bx^3\right)} - \frac{(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}} + \frac{(Ab-4aB)\log\left(\sqrt[3]{a}\cdots\right)}{9a^{2/3}b^{7/3}}$$

Mathematica [A] time = 0.17, size = 160, normalized size = 0.84

$$\frac{\frac{(4aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{2(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{2\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}-\frac{6\sqrt[3]{b}x(Ab-aB)}{a+bx^3}+18\sqrt[3]{b}Bx}{18b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (18*b^(1/3)*B*x - (6*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3) + (2*Sqrt[3]*(-(A*b
) + 4*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (2*(A*b -
 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + ((-(A*b) + 4*a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(2/3))/(18*b^(7/3))
```

fricas [A] time = 1.01, size = 573, normalized size = 3.02

$$\left[\frac{18\,Ba^2b^2x^4-3\sqrt{\frac{1}{3}}\left(4\,Ba^3b-Aa^2b^2+\left(4\,Ba^2b^2-Aab^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\cdots\right.}{bx^3+a}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*B*a^2*b^2*x^4 - 3*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2
 - A*a*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*
x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
-(a^2*b)^(1/3)/b))/(b*x^3 + a)) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)
*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a
*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 6*(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3), 1/18*(18*B*a^2*b^2
*x^4 - 6*sqrt(1/3)*(4*B*a^3*b - A*a^2*b^2 + (4*B*a^2*b^2 - A*a*b^3)*x^3)*sq
rt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt((a^2*b)^(1/3)/b)/a^2) + ((4*B*a*b - A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2
*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((4*B*a*b -
A*b^2)*x^3 + 4*B*a^2 - A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*
(4*B*a^3*b - A*a^2*b^2)*x)/(a^2*b^4*x^3 + a^3*b^3)]
```

giac [A] time = 0.19, size = 166, normalized size = 0.87

$$\frac{\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}b}+\frac{(4\,Ba-Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}b}+\frac{Bx}{b^2}+\frac{(4\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|\cdots\right.\right.}{9\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(2/3)*b) + 1/18*(4*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a
```

```
/b)^(2/3))/((-a*b^2)^(2/3)*b) + B*x/b^2 + 1/9*(4*B*a - A*b)*(-a/b)^(1/3)*lo
g(abs(x - (-a/b)^(1/3)))/(a*b^2) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*b^2)
```

maple [A] time = 0.04, size = 228, normalized size = 1.20

$$-\frac{Ax}{3\left(b\,x^{3}+a\right)b}+\frac{Bax}{3\left(b\,x^{3}+a\right)b^{2}}+\frac{\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] B/b^2*x-1/3/b*x/(b*x^3+a)*A+1/3/b^2*x/(b*x^3+a)*B*a-4/9/b^3*B*a/(a/b)^(2/3)
*ln(x+(a/b)^(1/3))+2/9/b^3*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3)
)-4/9/b^3*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1
/9/b^2*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/18/b^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^
(1/3)*x+(a/b)^(2/3))+1/9/b^2*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x-1))
```

maxima [A] time = 1.22, size = 157, normalized size = 0.83

$$\frac{(Ba-Ab)x}{3\left(b^{3}x^{3}+ab^{2}\right)}+\frac{Bx}{b^{2}}-\frac{\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(4\,Ba-Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(4\,Ba-Ab}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x/(b^3*x^3 + a*b^2) + B*x/b^2 - 1/9*sqrt(3)*(4*B*a - A*b)*a
rctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 1/18
*(4*B*a - A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 1
/9*(4*B*a - A*b)*log(x + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 2.58, size = 150, normalized size = 0.79

$$\frac{B\,x}{b^{2}}-\frac{x\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^{3}\,x^{3}+a\,b^{2}}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b-4\,B\,a\right)}{9\,a^{2/3}\,b^{7/3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*x)/b^2 - (x*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (log(b^(1/3)*x + a^
(1/3))*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
 + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*
(A*b - 4*B*a))/(9*a^(2/3)*b^(7/3))
```

sympy [A] time = 1.67, size = 102, normalized size = 0.54

$$\frac{Bx}{b^{2}}+\frac{x\,(-Ab+Ba)}{3ab^{2}+3b^{3}x^{3}}+\operatorname{RootSum}\left(729t^{3}a^{2}b^{7}-A^{3}b^{3}+12A^{2}Bab^{2}-48AB^{2}a^{2}b+64B^{3}a^{3},\left(t\mapsto t\log\left(-\frac{9tab^{2}}{-Ab+4B}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*x/b**2 + x*(-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3) + RootSum(729*_t**3*a**2
*b**7 - A**3*b**3 + 12*A**2*B*a*b**2 - 48*A*B**2*a**2*b + 64*B**3*a**3, Lam
bda(_t, _t*log(-9*_t*a*b**2/(-A*b + 4*B*a) + x)))
```

3.78 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=41

$$\frac{aB-Ab}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

```
[Out] 1/3*(-A*b+B*a)/b^2/(b*x^3+a)+1/3*B*ln(b*x^3+a)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 43}

$$\frac{B\log(a+bx^3)}{3b^2}-\frac{Ab-aB}{3b^2(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -(A*b - a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^2}+\frac{B}{b(a+bx)}\right)dx, x, x^3\right)\\ &= -\frac{Ab-aB}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{aB-Ab}{3b^2(a+bx^3)}+\frac{B\log(a+bx^3)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^2,x]
```

[Out] (-(A*b) + a*B)/(3*b^2*(a + b*x^3)) + (B*Log[a + b*x^3])/(3*b^2)

fricas [A] time = 0.87, size = 44, normalized size = 1.07

$$\frac{Ba - Ab + \left(Bbx^3 + Ba\right)\log\left(bx^3 + a\right)}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")

[Out] 1/3*(B*a - A*b + (B*b*x^3 + B*a)*log(b*x^3 + a))/(b^3*x^3 + a*b^2)

giac [A] time = 0.18, size = 65, normalized size = 1.59

$$-\frac{B\left(\frac{\log\left(\frac{|bx^3+a|}{\left(bx^3+a\right)^2|b|}\right)}{b} - \frac{a}{\left(bx^3+a\right)b}\right)}{3\,b} - \frac{A}{3\left(bx^3 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")

[Out] -1/3*B*(log(abs(b*x^3 + a)/((b*x^3 + a)^2*abs(b)))/b - a/((b*x^3 + a)*b))/b - 1/3*A/((b*x^3 + a)*b)

maple [A] time = 0.06, size = 47, normalized size = 1.15

$$-\frac{A}{3\left(b\,x^3 + a\right)b} + \frac{Ba}{3\left(b\,x^3 + a\right)b^2} + \frac{B\ln\left(b\,x^3 + a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^2,x)

[Out] 1/3*B*ln(b*x^3+a)/b^2-1/3/b/(b*x^3+a)*A+1/3/b^2/(b*x^3+a)*B*a

maxima [A] time = 0.58, size = 40, normalized size = 0.98

$$\frac{Ba - Ab}{3\left(b^3x^3 + ab^2\right)} + \frac{B\log\left(bx^3 + a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")

[Out] 1/3*(B*a - A*b)/(b^3*x^3 + a*b^2) + 1/3*B*log(b*x^3 + a)/b^2

mupad [B] time = 2.35, size = 37, normalized size = 0.90

$$\frac{B\,\ln\left(b\,x^3 + a\right)}{3\,b^2} - \frac{A\,b - B\,a}{3\,b^2\,\left(b\,x^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^2,x)

[Out] (B*log(a + b*x^3))/(3*b^2) - (A*b - B*a)/(3*b^2*(a + b*x^3))

sympy [A] time = 1.23, size = 36, normalized size = 0.88

$$\frac{B\log\left(a+bx^3\right)}{3b^2}+\frac{-Ab+Ba}{3ab^2+3b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**2) + (-A*b + B*a)/(3*a*b**2 + 3*b**3*x**3)
```

3.79 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=171

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2(Ab}{3ab\left(a\right.}$$

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9*(A*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/
3)/b^(5/3)+1/18*(A*b+2*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/
3)/b^(5/3)-1/9*(A*b+2*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2)
)/a^(4/3)/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {457, 292, 31, 634, 617, 204, 628}

$$\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^2(Ab}{3ab\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^2)/(3*a*b*(a + b*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*
b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - ((A*b + 2*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(4/3)*b^(5/3)) + ((A*b + 2*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(4/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}+\frac{(Ab+2aB)\int\frac{x}{a+bx^3}\,dx}{3ab}\\
&= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{4/3}b^{4/3}}+\frac{(Ab+2aB)\int\frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{9a^{4/3}b^{4/3}}\\
&= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2aB)\int\frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{18a^{4/3}b^{5/3}}+\frac{(A
\\
&= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right.}{18a^{4/3}b^{5/3}}\\
&= \frac{(Ab-aB)x^2}{3ab\left(a+bx^3\right)}-\frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}-\frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{4/3}b^{5/3}}+\frac{(Ab+2
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 146, normalized size = 0.85

$$
\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)-\frac{6\sqrt[3]{a}\,b^{2/3}x^2(aB-Ab)}{a+bx^3}-2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-2\sqrt{3}\,(2aB+Ab}{18a^{4/3}b^{5/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(1/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(A*b + 2*a
*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(A*b + 2*a*B)*Log[a^(1/
3) + b^(1/3)*x] + (A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(18*a^(4/3)*b^(5/3))
```

fricas [A] time = 0.88, size = 548, normalized size = 3.20

$$\left[-\frac{6\left(Ba^2b^2-Aab^3\right)x^2-3\sqrt{\frac{1}{3}}\left(2\,Ba^3b+Aa^2b^2+\left(2\,Ba^2b^2+Aab^3\right)x^3\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab+3\,\sqrt{\frac{1}{3}}\left(abx+2\left(-ab^2\right)^{\frac{2}{3}}x^2\cdots\right.}{bx\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(6*(B*a^2*b^2 - A*a*b^3)*x^2 - 3*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 +
(2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b +
3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)
^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((2*B*a*b + A*b^2)*x^3 + 2*B
*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) + 2*((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x -
 (-a*b^2)^(1/3)))/(a^2*b^4*x^3 + a^3*b^3), -1/18*(6*(B*a^2*b^2 - A*a*b^3)*x
^2 - 6*sqrt(1/3)*(2*B*a^3*b + A*a^2*b^2 + (2*B*a^2*b^2 + A*a*b^3)*x^3)*sqrt
(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2
)^(1/3)/a)/b) - ((2*B*a*b + A*b^2)*x^3 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*lo
g(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((2*B*a*b + A*b^2)*x^3
 + 2*B*a^2 + A*a*b)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^4*x^3
+ a^3*b^3)]
```

giac [A] time = 0.18, size = 186, normalized size = 1.09

$$\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,\left(-ab^2\right)^{\frac{1}{3}}ab}-\frac{(2\,Ba+Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,\left(-ab^2\right)^{\frac{1}{3}}ab}-\frac{\left(2\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\cdots\right.}{9\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a*b) - 1/18*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) + (
-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b) - 1/9*(2*B*a*(-a/b)^(1/3) + A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x^2 - A*b*x
^2)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.05, size = 223, normalized size = 1.30

$$\frac{\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{2\sqrt{3}\;B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^2}-\frac{2B\ln\left(x\cdots\right.}{9\left(\frac{a}{b}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)-1/9/b/a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-2/9
/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/18/b/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3
)*x+(a/b)^(2/3))*A+1/9/b^2/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+
1/9/b/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/9/b
^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.21, size = 160, normalized size = 0.94

$$-\frac{(Ba-Ab)x^2}{3\left(ab^2x^3+a^2b\right)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(2\,Ba+Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(2\,Ba+Ab)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^2/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(2*B*a + A*b)*arctan
(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(1/3)) + 1/18*(2
*B*a + A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(1/3)) - 1/
9*(2*B*a + A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

mupad [B] time = 0.25, size = 145, normalized size = 0.85

$$\frac{x^2\,\left(A\,b-B\,a\right)}{3\,a\,b\,\left(b\,x^3+a\right)}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right)}{9\,a^{4/3}\,b^{5/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{}{}\right.}{9\,a^{4/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a
^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) - (log(b^
(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(9*a^(4/3)*b^(5/3)) + (x^2*(A*b - B*a))/(
3*a*b*(a + b*x^3))
```

sympy [A] time = 1.48, size = 117, normalized size = 0.68

$$\frac{x^2\,(Ab-Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^4b^5+A^3b^3+6A^2Bab^2+12AB^2a^2b+8B^3a^3,\left(t\mapsto t\log\left(\frac{81t^2a^3b^3}{A^2b^2+4ABab+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x**2*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**4*b**5 +
 A**3*b**3 + 6*A**2*B*a*b**2 + 12*A*B**2*a**2*b + 8*B**3*a**3, Lambda(_t, _
t*log(81*_t**2*a**3*b**3/(A**2*b**2 + 4*A*B*a*b + 4*B**2*a**2) + x)))
```

3.80 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=169

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab}{3ab(a}$$

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+1/9*(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)
/b^(4/3)-1/18*(2*A*b+B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)
/b^(4/3)-1/9*(2*A*b+B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/
a^(5/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(Ab}{3ab(a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x)/(3*a*b*(a + b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*A*b + a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*A*b + a*B)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b} + \frac{(2Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2A\ldots}{}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{3ab\left(a+bx^3\right)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2Ab+a\ldots}{}$$

Mathematica [A] time = 0.14, size = 145, normalized size = 0.86

$$\frac{-(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{6a^{2/3}\sqrt[3]{b}x(aB-Ab)}{a+bx^3}+2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(aB+2A\ldots}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x] - (2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(18*a^(5/3)*b^(4/3))
```

fricas [A] time = 0.91, size = 537, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(Ba^3b+2\,Aa^2b^2+\left(Ba^2b^2+2\,Aab^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\log\left(\frac{2\,abx^3-3\left(a^2b\right)^{\frac{1}{3}}ax-a^2+3\sqrt{\frac{1}{3}}\left(2\,abx^2+\left(a^2b\right)^{\frac{2}{3}}x-\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)}{b}}}{bx^3+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B
*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a
^2*b^2)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(B*a^3*b + 2*A*a^2*b^
2 + (B*a^2*b^2 + 2*A*a*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b + 2
*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((B*a*b + 2*A*b^2)*x^3 + B*a^2 + 2*A*a*b)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^3*b - A*a^2*b^2)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

giac [A] time = 0.18, size = 160, normalized size = 0.95

$$-\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}a}-\frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}a}-\frac{(Ba+2\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\right.\right.}{9\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*a) - 1/18*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/((-a*b^2)^(2/3)*a) - 1/9*(B*a + 2*A*b)*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/(a^2*b) - 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a*b)
```

maple [A] time = 0.06, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2}+\frac{B\ln\left(x\right.}{9\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x/a/b/(b*x^3+a)+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/9/b
^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
+(a/b)^(2/3))*A-1/18/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/
9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/9/b^2
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.10, size = 158, normalized size = 0.93

$$-\frac{(Ba-Ab)x}{3\left(ab^2x^3+a^2b\right)}+\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+2\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(Ba+2\,Ab)}{9\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x/(a*b^2*x^3 + a^2*b) + 1/9*sqrt(3)*(B*a + 2*A*b)*arctan(1
/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - 1/18*(B*a
 + 2*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3)) + 1/9*
(B*a + 2*A*b)*log(x + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 2.55, size = 143, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b+B\,a)}{9\,a^{5/3}\,b^{4/3}}+\frac{\ln\left(2\,b^{1/3}\,x-a^{1/3}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^2,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) - (log(3^(1/2)
*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/
(9*a^(5/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^
(1/2)*1i)/2 - 1/2)*(2*A*b + B*a))/(9*a^(5/3)*b^(4/3)) + (x*(A*b - B*a))/(3*
a*b*(a + b*x^3))
```

sympy [A] time = 1.43, size = 97, normalized size = 0.57

$$\frac{x\,(Ab-Ba)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-8A^3b^3-12A^2Bab^2-6AB^2a^2b-B^3a^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{2Ab+Ba}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] x*(A*b - B*a)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - 8*
A**3*b**3 - 12*A**2*B*a*b**2 - 6*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*l
og(9*_t*a**2*b/(2*A*b + B*a) + x)))
```

3.81 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=51

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

```
[Out] 1/3*(A*b-B*a)/a/b/(b*x^3+a)+A*ln(x)/a^2-1/3*A*ln(b*x^3+a)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{A\log\left(a+bx^3\right)}{3a^2}+\frac{A\log(x)}{a^2}+\frac{Ab-aB}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] (A*b - a*B)/(3*a*b*(a + b*x^3)) + (A*Log[x])/a^2 - (A*Log[a + b*x^3])/(3*a^
2)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^2}\,dx&=\frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^2}\,dx,x,x^3\right)\\&=\frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{A}{a^2x}+\frac{-Ab+aB}{a(a+bx)^2}-\frac{Ab}{a^2(a+bx)}\right)dx,x,x^3\right)\\&=\frac{Ab-aB}{3ab\left(a+bx^3\right)}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^3\right)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.90

$$\frac{\frac{a(Ab-aB)}{b\left(a+bx^3\right)}-A\log\left(a+bx^3\right)+3A\log(x)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(A*b - a*B))/(b*(a + b*x^3)) + 3*A*Log[x] - A*Log[a + b*x^3])/(3*a^2)
```

fricas [A] time = 0.83, size = 70, normalized size = 1.37

$$-\frac{Ba^2 - Aab + \left(Ab^2x^3 + Aab\right)\log\left(bx^3 + a\right) - 3\left(Ab^2x^3 + Aab\right)\log(x)}{3\left(a^2b^2x^3 + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(B*a^2 - A*a*b + (A*b^2*x^3 + A*a*b)*log(b*x^3 + a) - 3*(A*b^2*x^3 + A
*a*b)*log(x))/(a^2*b^2*x^3 + a^3*b)
```

giac [A] time = 0.21, size = 61, normalized size = 1.20

$$-\frac{A\log\left(\left|bx^3 + a\right|\right)}{3\,a^2} + \frac{A\log\left(|x|\right)}{a^2} + \frac{Ab^2x^3 - Ba^2 + 2\,Aab}{3\left(bx^3 + a\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^2 + A*log(abs(x))/a^2 + 1/3*(A*b^2*x^3 - B*a^2
 + 2*A*a*b)/((b*x^3 + a)*a^2*b)
```

maple [A] time = 0.05, size = 53, normalized size = 1.04

$$\frac{A}{3\left(b\,x^3 + a\right)a} + \frac{A\ln(x)}{a^2} - \frac{A\ln\left(b\,x^3 + a\right)}{3a^2} - \frac{B}{3\left(b\,x^3 + a\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/3*A*ln(b*x^3+a)/a^2+1/3/a/(b*x^3+a)*A-1/3/b/(b*x^3+a)*B+A/a^2*ln(x)
```

maxima [A] time = 0.48, size = 51, normalized size = 1.00

$$-\frac{Ba - Ab}{3\left(ab^2x^3 + a^2b\right)} - \frac{A\log\left(bx^3 + a\right)}{3\,a^2} + \frac{A\log\left(x^3\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)/(a*b^2*x^3 + a^2*b) - 1/3*A*log(b*x^3 + a)/a^2 + 1/3*A*log
(x^3)/a^2
```

mupad [B] time = 0.14, size = 47, normalized size = 0.92

$$\frac{A\,\ln(x)}{a^2} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a^2} + \frac{A\,b - B\,a}{3\,a\,b\,\left(b\,x^3 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^2),x)
```

```
[Out] (A*log(x))/a^2 - (A*log(a + b*x^3))/(3*a^2) + (A*b - B*a)/(3*a*b*(a + b*x^3
))
```

sympy [A] time = 1.16, size = 46, normalized size = 0.90

$$\frac{A\log(x)}{a^2} - \frac{A\log\left(\frac{a}{b} + x^3\right)}{3a^2} + \frac{Ab - Ba}{3a^2b + 3ab^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**2,x)
```

```
[Out] A*log(x)/a**2 - A*log(a/b + x**3)/(3*a**2) + (A*b - B*a)/(3*a**2*b + 3*a*b*
*2*x**3)
```

3.82 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=195

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{aB}{3}$$

```
[Out] 1/3*(-4*A*b+B*a)/a^2/b/x+1/3*(A*b-B*a)/a/b/x/(b*x^3+a)+1/9*(4*A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x)/a^(7/3)/b^(2/3)-1/18*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/a^(7/3)/b^(2/3)+1/9*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 196, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}-\frac{4A}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -(4*A*b - a*B)/(3*a^2*b*x) + (A*b - a*B)/(3*a*b*x*(a + b*x^3)) + ((4*A*b -
a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*
b^(2/3)) + ((4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(7/3)*b^(2/3)) - (
(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(7/3)*b
^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}-\frac{(4Ab-aB)\int \frac{x}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{7/3}\sqrt[3]{b}}-\frac{(4Ab-aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2 \ldots}}{9a^{7/3}\sqrt[3]{b}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\int \frac{-\sqrt[3]{a}\,\sqrt[3]{b}+2b^{2/3} \ldots}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+\ldots}}{18a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{7/3}b^{2/3}}-\frac{(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\ldots\right.}{18a^{7/3}b^{2/3}}$$

$$= -\frac{4Ab-aB}{3a^2bx}+\frac{Ab-aB}{3abx\left(a+bx^3\right)}+\frac{(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\ldots\right.}{9a^{7/3}b^{2/3}}$$

Mathematica [A] time = 0.14, size = 164, normalized size = 0.84

$$\frac{\frac{(aB-4Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{2(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{2\sqrt{3}(4Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6\sqrt[3]{a}x^2(aB-Ab)}{a+bx^3}-\frac{18\sqrt[3]{a}A}{x}}{18a^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(1/3)*A)/x + (6*a^(1/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3) + (2*Sqrt[3
]*(4*A*b - a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (2*(
4*A*b - a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) + ((-4*A*b + a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(2/3))/(18*a^(7/3))
```

fricas [A] time = 0.95, size = 570, normalized size = 2.92

$$\left[-\frac{18\,Aa^2b^2-6\left(Ba^2b^2-4\,Aab^3\right)x^3+3\sqrt{\frac{1}{3}}\left(\left(Ba^2b^2-4\,Aab^3\right)x^4+\left(Ba^3b-4\,Aa^2b^2\right)x\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,b^2x^3-ab\cdots}{\cdots}\right)}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(18*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 3*sqrt(1/3)*((B*a^2*
b^2 - 4*A*a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*lo
g((2*b^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/
3)*a)*sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((B*a*b -
4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/
3)*b*x + (a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(
a*b^2)^(2/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x), -1/18*(18
*A*a^2*b^2 - 6*(B*a^2*b^2 - 4*A*a*b^3)*x^3 + 6*sqrt(1/3)*((B*a^2*b^2 - 4*A*
a*b^3)*x^4 + (B*a^3*b - 4*A*a^2*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) - ((B*a*b - 4*A*b^2)*
x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x +
(a*b^2)^(2/3)) + 2*((B*a*b - 4*A*b^2)*x^4 + (B*a^2 - 4*A*a*b)*x)*(a*b^2)^(2
/3)*log(b*x + (a*b^2)^(1/3)))/(a^3*b^3*x^4 + a^4*b^2*x)]
```

giac [A] time = 0.18, size = 180, normalized size = 0.92

$$\frac{\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{1}{3}}a^2}-\frac{(Ba-4\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(-ab^2\right)^{\frac{1}{3}}a^2}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\cdots\right.}{9\,a^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1
/3))/((-a*b^2)^(1/3)*a^2) - 1/18*(B*a - 4*A*b)*log(x^2 + x*(-a/b)^(1/3) + (
```

```
-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2) - 1/9*(B*a*(-a/b)^(1/3) - 4*A*b*(-a/b)^(1
/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/3*(B*a*x^3 - 4*A*b*x^3
 - 3*A*a)/((b*x^4 + a*x)*a^2)
```

maple [A] time = 0.05, size = 241, normalized size = 1.24

$$-\frac{Ab\,x^2}{3\left(b\,x^3+a\right)a^2}+\frac{B\,x^2}{3\left(b\,x^3+a\right)a}-\frac{4\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{4A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{2A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^2/(b*x^3+a)*A*b+1/3/a*x^2/(b*x^3+a)*B+4/9/a^2*A/(a/b)^(1/3)*ln(x
+(a/b)^(1/3))-2/9/a^2*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-4/9/a
^2*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/9/a*B/b/
(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/18/a*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
a/b)^(2/3))+1/9/a*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-A/a^2/x
```

maxima [A] time = 1.08, size = 166, normalized size = 0.85

$$\frac{(Ba-4\,Ab)x^3-3\,Aa}{3\left(a^2bx^4+a^3x\right)}+\frac{\sqrt{3}\,(Ba-4\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-4\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(Ba-4\,Ab}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 4*A*b)*x^3 - 3*A*a)/(a^2*b*x^4 + a^3*x) + 1/9*sqrt(3)*(B*a - 4*
A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(1/3)
) + 1/18*(B*a - 4*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(1/3)) - 1/9*(B*a - 4*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

mupad [B] time = 2.57, size = 156, normalized size = 0.80

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(4\,A\,b-B\,a)}{9\,a^{7/3}\,b^{2/3}}-\frac{\frac{A}{a}+\frac{x^3\,(4\,A\,b-B\,a)}{3\,a^2}}{b\,x^4+a\,x}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(4\,A\,b-B\,a)}{9\,a^{7/3}\,b^{2/3}}-\ln($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (A/a + (x^3*
(4*A*b - B*a))/(3*a^2))/(a*x + b*x^4) + (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)
*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B*a))/(9*a^(7/3)*b^(2/3)) - (
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(4*A
*b - B*a))/(9*a^(7/3)*b^(2/3))
```

sympy [A] time = 1.42, size = 122, normalized size = 0.63

$$\frac{-3Aa+x^3\,(-4Ab+Ba)}{3a^3x+3a^2bx^4}+\mathrm{RootSum}\left(729t^3a^7b^2-64A^3b^3+48A^2Bab^2-12AB^2a^2b+B^3a^3,\left(t\mapsto t\log\left(\frac{}{16A^2b^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-4*A*b + B*a))/(3*a**3*x + 3*a**2*b*x**4) + RootSum(729*_t*
*3*a**7*b**2 - 64*A**3*b**3 + 48*A**2*B*a*b**2 - 12*A*B**2*a**2*b + B**3*a*
*3, Lambda(_t, _t*log(81*_t**2*a**5*b/(16*A**2*b**2 - 8*A*B*a*b + B**2*a**2
) + x)))
```

3.83 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=196

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}+\frac{2aB}{6a}$$

```
[Out] 1/6*(-5*A*b+2*B*a)/a^2/b/x^2+1/3*(A*b-B*a)/a/b/x^2/(b*x^3+a)-1/9*(5*A*b-2*B
*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(1/3)+1/18*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(1/3)+1/9*(5*A*b-2*B*a)*arctan(1/3*(a
^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{8/3}\sqrt[3]{b}}-\frac{5Ab-2aB}{6a^2bx^2}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(5*A*b - 2*a*B)/(6*a^2*b*x^2) + (A*b - a*B)/(3*a*b*x^2*(a + b*x^3)) + ((5*
A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*
a^(8/3)*b^(1/3)) - ((5*A*b - 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(8/3)*b^
(1/3)) + ((5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{a+bx^3}\,dx}{3a^2}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{8/3}}-\frac{(5Ab-2aB)\int \frac{2\sqrt[3]{a}-}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{}}}{9a^{8/3}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\int \frac{}{a^{2/3}-}}{6a^{7/3}}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{8/3}\sqrt[3]{b}}+\frac{(5Ab-2aB)\log\left(a^{2}\right.}{18a}$$

$$= -\frac{5Ab-2aB}{6a^2bx^2}+\frac{Ab-aB}{3abx^2\left(a+bx^3\right)}+\frac{(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{8/3}\sqrt[3]{b}}-\frac{(5Ab-2aB)\log\left(\sqrt[3]{}\right.}{9a^{8/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.16, size = 163, normalized size = 0.83

$$\frac{\frac{(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}+\frac{6a^{2/3}x(aB-Ab)}{a+bx^3}-\frac{9a^{2/3}A}{x^2}+\frac{2(2aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}(5Ab-2aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{18a^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(2/3)*A)/x^2 + (6*a^(2/3)*(-(A*b) + a*B)*x)/(a + b*x^3) + (2*Sqrt[3]
*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (2*
(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + ((5*A*b - 2*a*B)*Log[a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/b^(1/3))/(18*a^(8/3))
```

fricas [A] time = 0.93, size = 618, normalized size = 3.15

$$\left[-\frac{9\,Aa^3b-3\left(2\,Ba^3b-5\,Aa^2b^2\right)x^3+3\sqrt{\frac{1}{3}}\left(\left(2\,Ba^2b^2-5\,Aab^3\right)x^5+\left(2\,Ba^3b-5\,Aa^2b^2\right)x^2\right)\sqrt{\frac{\left(-a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^3+3 \cdots}{\cdots}\right) \cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 + 3*sqrt(1/3)*((2*B*a^2
*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/
b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (
-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) +
((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*
x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (
2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^
2*x^5 + a^5*b*x^2), -1/18*(9*A*a^3*b - 3*(2*B*a^3*b - 5*A*a^2*b^2)*x^3 - 6*
sqrt(1/3)*((2*B*a^2*b^2 - 5*A*a*b^3)*x^5 + (2*B*a^3*b - 5*A*a^2*b^2)*x^2)*s
qrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3
)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5
*A*a*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)
*a) - 2*((2*B*a*b - 5*A*b^2)*x^5 + (2*B*a^2 - 5*A*a*b)*x^2)*(-a^2*b)^(2/3)*
log(a*b*x + (-a^2*b)^(2/3)))/(a^4*b^2*x^5 + a^5*b*x^2)]
```

giac [A] time = 0.17, size = 188, normalized size = 0.96

$$-\frac{\left(2\,Ba-5\,Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}+\frac{\sqrt{3}\left(2\left(-ab^2\right)^{\frac{1}{3}}Ba-5\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b}+\frac{Bax-A \cdots}{3\left(bx^3+a \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*B*a - 5*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^3 + 1/9*sqrt
(3)*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b) + 1/3*(B*a*x - A*b*x)/((b*x^3 + a)*a^2
) + 1/18*(2*(-a*b^2)^(1/3)*B*a - 5*(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^3*b) - 1/2*A/(a^2*x^2)
```

maple [A] time = 0.05, size = 237, normalized size = 1.21

$$-\frac{Abx}{3\left(b x^3+a\right)a^2}+\frac{Bx}{3\left(b x^3+a\right)a}-\frac{5\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{5A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{5A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x/(b*x^3+a)*A*b+1/3/a*x/(b*x^3+a)*B-5/9/a^2*A/(a/b)^(2/3)*ln(x+(a/
b)^(1/3))+5/18/a^2*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-5/9/a^2*
A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+2/9/a*B/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))-1/9/a*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)
^(2/3))+2/9/a*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1
))-1/2*A/a^2/x^2
```

maxima [A] time = 1.23, size = 172, normalized size = 0.88

$$\frac{(2\,Ba-5\,Ab)x^3-3\,Aa}{6\left(a^2bx^5+a^3x^2\right)}+\frac{\sqrt{3}\,(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba-5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(2\,B}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*((2*B*a - 5*A*b)*x^3 - 3*A*a)/(a^2*b*x^5 + a^3*x^2) + 1/9*sqrt(3)*(2*B*
a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)
^(2/3)) - 1/18*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*
b*(a/b)^(2/3)) + 1/9*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3
))
```

mupad [B] time = 2.57, size = 159, normalized size = 0.81

$$-\frac{\frac{A}{2\,a}+\frac{x^3\,(5\,A\,b-2\,B\,a)}{6\,a^2}}{b\,x^5+a\,x^2}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-2\,B\,a)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b-2\,B}{9\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*
A*b - 2*B*a))/(9*a^(8/3)*b^(1/3)) - (log(b^(1/3)*x + a^(1/3))*(5*A*b - 2*B*
a))/(9*a^(8/3)*b^(1/3)) - (A/(2*a) + (x^3*(5*A*b - 2*B*a))/(6*a^2))/(a*x^2
+ b*x^5) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2
 - 1/2)*(5*A*b - 2*B*a))/(9*a^(8/3)*b^(1/3))
```

sympy [A] time = 1.93, size = 109, normalized size = 0.56

$$\frac{-3Aa+x^3(-5Ab+2Ba)}{6a^3x^2+6a^2bx^5}+\mathrm{RootSum}\left(729t^3a^8b+125A^3b^3-150A^2Bab^2+60AB^2a^2b-8B^3a^3,\left(t\mapsto t\log\left(\frac{}{-5A\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**2,x)
```

```
[Out] (-3*A*a + x**3*(-5*A*b + 2*B*a))/(6*a**3*x**2 + 6*a**2*b*x**5) + RootSum(72
9*_t**3*a**8*b + 125*A**3*b**3 - 150*A**2*B*a*b**2 + 60*A*B**2*a**2*b - 8*B
**3*a**3, Lambda(_t, _t*log(9*_t*a**3/(-5*A*b + 2*B*a) + x)))
```

3.84 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=76

$$\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{A}{3a^2x^3}$$

```
[Out] -1/3*A/a^2/x^3+1/3*(-A*b+B*a)/a^2/(b*x^3+a)-(2*A*b-B*a)*ln(x)/a^3+1/3*(2*A*
b-B*a)*ln(b*x^3+a)/a^3
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{A}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -A/(3*a^2*x^3) - (A*b - a*B)/(3*a^2*(a + b*x^3)) - ((2*A*b - a*B)*Log[x])/a
^3 + ((2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^2} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^2} \, dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^2}+\frac{-2Ab+aB}{a^3x}-\frac{b(-Ab+aB)}{a^2(a+bx)^2}-\frac{b(-2Ab+aB)}{a^3(a+bx)}\right) \, dx,x,x^3\right)\\
&= -\frac{A}{3a^2x^3}-\frac{Ab-aB}{3a^2\left(a+bx^3\right)}-\frac{(2Ab-aB)\log(x)}{a^3}+\frac{(2Ab-aB)\log\left(a+bx^3\right)}{3a^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 0.84

$$\frac{\frac{a(aB-Ab)}{a+bx^3}+(2Ab-aB)\log\left(a+bx^3\right)+3\log(x)(aB-2Ab)-\frac{aA}{x^3}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((a*A)/x^3) + (a*(-(A*b) + a*B))/(a + b*x^3) + 3*(-2*A*b + a*B)*Log[x] +
(2*A*b - a*B)*Log[a + b*x^3])/(3*a^3)
```

fricas [A] time = 0.96, size = 118, normalized size = 1.55

$$\frac{\left(Ba^2 - 2\,Aab\right)x^3 - Aa^2 - \left(\left(Bab - 2\,Ab^2\right)x^6 + \left(Ba^2 - 2\,Aab\right)x^3\right)\log\left(bx^3 + a\right) + 3\left(\left(Bab - 2\,Ab^2\right)x^6 + \left(Ba^2 - 2\,A\right.\right.}{3\left(a^3bx^6 + a^4x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((B*a^2 - 2*A*a*b)*x^3 - A*a^2 - ((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*
a*b)*x^3)*log(b*x^3 + a) + 3*((B*a*b - 2*A*b^2)*x^6 + (B*a^2 - 2*A*a*b)*x^3
)*log(x))/(a^3*b*x^6 + a^4*x^3)
```

giac [A] time = 0.19, size = 80, normalized size = 1.05

$$\frac{(Ba - 2\,Ab)\log\left(|x|\right)}{a^3} + \frac{Bax^3 - 2\,Abx^3 - Aa}{3\left(bx^6 + ax^3\right)a^2} - \frac{\left(Bab - 2\,Ab^2\right)\log\left(\left|bx^3 + a\right|\right)}{3\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] (B*a - 2*A*b)*log(abs(x))/a^3 + 1/3*(B*a*x^3 - 2*A*b*x^3 - A*a)/((b*x^6 + a
*x^3)*a^2) - 1/3*(B*a*b - 2*A*b^2)*log(abs(b*x^3 + a))/(a^3*b)
```

maple [A] time = 0.06, size = 87, normalized size = 1.14

$$-\frac{Ab}{3\left(b\,x^3 + a\right)a^2} - \frac{2Ab\ln(x)}{a^3} + \frac{2Ab\ln\left(b\,x^3 + a\right)}{3a^3} + \frac{B}{3\left(b\,x^3 + a\right)a} + \frac{B\ln(x)}{a^2} - \frac{B\ln\left(b\,x^3 + a\right)}{3a^2} - \frac{A}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 2/3*b/a^3*ln(b*x^3+a)*A-1/3/a^2*ln(b*x^3+a)*B-1/3*b/a^2/(b*x^3+a)*A+1/3/a/(
b*x^3+a)*B-1/3*A/a^2/x^3-2/a^3*ln(x)*A*b+B/a^2*ln(x)
```

maxima [A] time = 0.50, size = 76, normalized size = 1.00

$$\frac{(Ba - 2\,Ab)x^3 - Aa}{3\left(a^2bx^6 + a^3x^3\right)} - \frac{(Ba - 2\,Ab)\log\left(bx^3 + a\right)}{3\,a^3} + \frac{(Ba - 2\,Ab)\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 2*A*b)*x^3 - A*a)/(a^2*b*x^6 + a^3*x^3) - 1/3*(B*a - 2*A*b)*log
(b*x^3 + a)/a^3 + 1/3*(B*a - 2*A*b)*log(x^3)/a^3
```

mupad [B] time = 2.43, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^3 + a\right)\,\left(2\,A\,b - B\,a\right)}{3\,a^3} - \frac{\frac{A}{3\,a} + \frac{x^3\,\left(2\,A\,b - B\,a\right)}{3\,a^2}}{b\,x^6 + a\,x^3} - \frac{\ln(x)\,\left(2\,A\,b - B\,a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (log(a + b*x^3)*(2*A*b - B*a))/(3*a^3) - (A/(3*a) + (x^3*(2*A*b - B*a))/(3*
a^2))/(a*x^3 + b*x^6) - (log(x)*(2*A*b - B*a))/a^3
```

sympy [A] time = 1.44, size = 70, normalized size = 0.92

$$\frac{-Aa+x^{3}\left(-2Ab+Ba\right)}{3a^{3}x^{3}+3a^{2}bx^{6}}+\frac{\left(-2Ab+Ba\right)\log\left(x\right)}{a^{3}}-\frac{\left(-2Ab+Ba\right)\log\left(\frac{a}{b}+x^{3}\right)}{3a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**2,x)
```

```
[Out] (-A*a + x**3*(-2*A*b + B*a))/(3*a**3*x**3 + 3*a**2*b*x**6) + (-2*A*b + B*a)
*log(x)/a**3 - (-2*A*b + B*a)*log(a/b + x**3)/(3*a**3)
```

3.85 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{}}{\sqrt{3}\,\sqrt[3]{}}\right.}{3\sqrt{3}\,a^{10/3}}$$

```
[Out] 1/12*(-7*A*b+4*B*a)/a^2/b/x^4+1/3*(7*A*b-4*B*a)/a^3/x+1/3*(A*b-B*a)/a/b/x^4
/(b*x^3+a)-1/9*b^(1/3)*(7*A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)+1/18*b^
(1/3)*(7*A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)-1/9*
b^(1/3)*(7*A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
10/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 325, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{10/3}}-\frac{7Ab-4aB}{12a^2bx^4}+\frac{7Ab-4aB}{3a^3x}-\frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}}-\frac{\sqrt[3]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] -(7*A*b - 4*a*B)/(12*a^2*b*x^4) + (7*A*b - 4*a*B)/(3*a^3*x) + (A*b - a*B)/(
3*a*b*x^4*(a + b*x^3)) - (b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(a^(1/3) - 2*b^(1/
3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (b^(1/3)*(7*A*b - 4*a*B)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(10/3)) + (b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} + \frac{(7Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{(7Ab-4aB)\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} + \frac{(b(7Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\left(b^{2/3}(7Ab-4aB)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{10/3}} + \frac{\left(b^{2/\cdots\right.}{\cdots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}} + \frac{(\cdots}{\cdots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}\,(7Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{10/3}} + \frac{\sqrt[3]{\cdots}}{\cdots}$$

$$= -\frac{7Ab-4aB}{12a^2bx^4} + \frac{7Ab-4aB}{3a^3x} + \frac{Ab-aB}{3abx^4\left(a+bx^3\right)} - \frac{\sqrt[3]{b}\,(7Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{\sqrt[3]{\cdots}}{\cdots}$$

Mathematica [A] time = 0.24, size = 185, normalized size = 0.86

$$\frac{2\sqrt[3]{b}(7Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{4/3}A}{x^4}-\frac{12\sqrt[3]{a}bx^2(aB-Ab)}{a+bx^3}-\frac{36\sqrt[3]{a}(aB-2Ab)}{x}+4\sqrt[3]{b}(4aB-7Ab)\log\left(\right.}{36a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^2),x]
```

```
[Out] ((-9*a^(4/3)*A)/x^4 - (36*a^(1/3)*(-2*A*b + a*B))/x - (12*a^(1/3)*b*(-(A*b)
 + a*B)*x^2)/(a + b*x^3) - 4*Sqrt[3]*b^(1/3)*(7*A*b - 4*a*B)*ArcTan[(1 - (2
*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 4*b^(1/3)*(-7*A*b + 4*a*B)*Log[a^(1/3) + b^
(1/3)*x] + 2*b^(1/3)*(7*A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(36*a^(10/3))
```

fricas [A] time = 0.88, size = 259, normalized size = 1.20

$$-\frac{12\left(4Bab-7Ab^2\right)x^6+9\left(4Ba^2-7Aab\right)x^3+9Aa^2+4\sqrt{3}\left(\left(4Bab-7Ab^2\right)x^7+\left(4Ba^2-7Aab\right)x^4\right)\left(-\frac{b}{a}\right)^{\frac{1}{3}}\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(4*B*a*b - 7*A*b^2)*x^6 + 9*(4*B*a^2 - 7*A*a*b)*x^3 + 9*A*a^2 + 4
*sqrt(3)*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*a
rctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 2*((4*B*a*b - 7*A*b^2)*x^
7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*
(-b/a)^(1/3)) + 4*((4*B*a*b - 7*A*b^2)*x^7 + (4*B*a^2 - 7*A*a*b)*x^4)*(-b/a
)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^3*b*x^7 + a^4*x^4)
```

giac [A] time = 0.22, size = 231, normalized size = 1.07

$$\frac{\left(4Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7Ab^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^4}+\frac{\sqrt{3}\left(4\left(-ab^2\right)^{\frac{2}{3}}Ba-7\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{9a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(4*B*a*b*(-a/b)^(1/3) - 7*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
(-a/b)^(1/3)))/a^4 + 1/9*sqrt(3)*(4*(-a*b^2)^(2/3)*B*a - 7*(-a*b^2)^(2/3)*A
*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) - 1/3*(B*
a*b*x^2 - A*b^2*x^2)/((b*x^3 + a)*a^3) - 1/18*(4*(-a*b^2)^(2/3)*B*a - 7*(-a
*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) - 1/4*(4*
B*a*x^3 - 8*A*b*x^3 + A*a)/(a^3*x^4)
```

maple [A] time = 0.06, size = 257, normalized size = 1.20

$$\frac{Ab^2x^2}{3\left(bx^3+a\right)a^3}-\frac{Bbx^2}{3\left(bx^3+a\right)a^2}+\frac{7\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{7Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{7Ab\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x^2/(b*x^3+a)*A-1/3*b/a^2*x^2/(b*x^3+a)*B-7/9*b/a^3*A/(a/b)^(1/
3)*ln(x+(a/b)^(1/3))+7/18*b/a^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2
/3))+7/9*b/a^3*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1)
)+4/9/a^2*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-2/9/a^2*B/(a/b)^(1/3)*ln(x^2-(a/b
)^(1/3)*x+(a/b)^(2/3))-4/9/a^2*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*x-1))-1/4/a^2*A/x^4+2*A/a^3*b/x-B/a^2/x
```

maxima [A] time = 1.42, size = 186, normalized size = 0.87

$$-\frac{4\left(4\,Bab-7\,Ab^{2}\right)x^{6}+3\left(4\,Ba^{2}-7\,Aab\right)x^{3}+3\,Aa^{2}}{12\left(a^{3}bx^{7}+a^{4}x^{4}\right)}-\frac{\sqrt{3}\,(4\,Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{(4\,Ba-7\,Ab)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(4*(4*B*a*b - 7*A*b^2)*x^6 + 3*(4*B*a^2 - 7*A*a*b)*x^3 + 3*A*a^2)/(a^
3*b*x^7 + a^4*x^4) - 1/9*sqrt(3)*(4*B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(1/3)) - 1/18*(4*B*a - 7*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(1/3)) + 1/9*(4*B*a - 7*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(1/3))
```

mupad [B] time = 2.62, size = 209, normalized size = 0.97

$$\frac{\frac{x^{3}\,(7\,A\,b-4\,B\,a)}{4\,a^{2}}-\frac{A}{4\,a}+\frac{b\,x^{6}\,(7\,A\,b-4\,B\,a)}{3\,a^{3}}}{b\,x^{7}+a\,x^{4}}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}+b^{3}\,x\right)\,(7\,A\,b-4\,B\,a)}{9\,a^{10/3}}+\frac{(-b)^{1/3}\,\ln\left(a^{1/3}\,(-b)^{8/3}-2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(7*A*b - 4*B*a))/(4*a^2) - A/(4*a) + (b*x^6*(7*A*b - 4*B*a))/(3*a^3))
/(a*x^4 + b*x^7) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(7*A*b - 4*B
*a))/(9*a^(10/3)) + ((-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*
a^(1/3)*(-b)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
 - ((-b)^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3
)*1i)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - 4*B*a))/(9*a^(10/3))
```

sympy [A] time = 2.32, size = 153, normalized size = 0.71

$$\operatorname{RootSum}\left(729t^{3}a^{10}+343A^{3}b^{4}-588A^{2}Bab^{3}+336AB^{2}a^{2}b^{2}-64B^{3}a^{3}b,\left(t\mapsto t\log\left(\frac{81t^{2}a^{7}}{49A^{2}b^{3}-56ABab^{2}+16\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**10 + 343*A**3*b**4 - 588*A**2*B*a*b**3 + 336*A*B**2*a*
*2*b**2 - 64*B**3*a**3*b, Lambda(_t, _t*log(81*_t**2*a**7/(49*A**2*b**3 - 5
6*A*B*a*b**2 + 16*B**2*a**2*b) + x))) + (-3*A*a**2 + x**6*(28*A*b**2 - 16*B
*a*b) + x**3*(21*A*a*b - 12*B*a**2))/(12*a**4*x**4 + 12*a**3*b*x**7)
```

3.86 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=215

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{11/3}}$$

```
[Out] 1/15*(-8*A*b+5*B*a)/a^2/b/x^5+1/6*(8*A*b-5*B*a)/a^3/x^2+1/3*(A*b-B*a)/a/b/x
^5/(b*x^3+a)+1/9*b^(2/3)*(8*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)-1/18*
b^(2/3)*(8*A*b-5*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)-1/
9*b^(2/3)*(8*A*b-5*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a
^(11/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {457, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(8Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{18a^{11/3}}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-}{\sqrt{3}}\right.}{3\sqrt{3}\,a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] -(8*A*b - 5*a*B)/(15*a^2*b*x^5) + (8*A*b - 5*a*B)/(6*a^3*x^2) + (A*b - a*B)
/(3*a*b*x^5*(a + b*x^3)) - (b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11/3)) + (b^(2/3)*(8*A*b - 5*a*B)
*Log[a^(1/3) + b^(1/3)*x])/(9*a^(11/3)) - (b^(2/3)*(8*A*b - 5*a*B)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(8Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}-\frac{(8Ab-5aB)\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{3a^2}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(b(8Ab-5aB))\int \frac{1}{a+bx^3}\,dx}{3a^3}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{(b(8Ab-5aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{9a^{11/3}}+\frac{(b(8A\ldots}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{(b\ldots}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}+\frac{b^{2/3}(8Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{9a^{11/3}}-\frac{b^{2\ldots}}{\ldots}$$

$$= -\frac{8Ab-5aB}{15a^2bx^5}+\frac{8Ab-5aB}{6a^3x^2}+\frac{Ab-aB}{3abx^5\left(a+bx^3\right)}-\frac{b^{2/3}(8Ab-5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{b^{2\ldots}}{\ldots}$$

Mathematica [A] time = 0.18, size = 183, normalized size = 0.85

$$\frac{5b^{2/3}(5aB-8Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{30a^{2/3}bx(aB-Ab)}{a+bx^3}-\frac{45a^{2/3}(aB-2Ab)}{x^2}-\frac{18a^{5/3}A}{x^5}+10b^{2/3}(8Ab-5aB)\log}{90a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^2),x]
```

```
[Out] ((-18*a^(5/3)*A)/x^5 - (45*a^(2/3)*(-2*A*b + a*B))/x^2 - (30*a^(2/3)*b*(-(A
*b) + a*B)*x)/(a + b*x^3) - 10*Sqrt[3]*b^(2/3)*(8*A*b - 5*a*B)*ArcTan[(1 -
(2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 10*b^(2/3)*(8*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x] + 5*b^(2/3)*(-8*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(90*a^(11/3))
```

fricas [A] time = 0.92, size = 277, normalized size = 1.29

$$-\frac{15\left(5\,Bab-8\,Ab^2\right)x^6+9\left(5\,Ba^2-8\,Aab\right)x^3+18\,Aa^2+10\,\sqrt{3}\left(\left(5\,Bab-8\,Ab^2\right)x^8+\left(5\,Ba^2-8\,Aab\right)x^5\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/90*(15*(5*B*a*b - 8*A*b^2)*x^6 + 9*(5*B*a^2 - 8*A*a*b)*x^3 + 18*A*a^2 +
10*sqrt(3)*((5*B*a*b - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1
/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*((5*B*a*b
 - 8*A*b^2)*x^8 + (5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*
b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + 10*((5*B*a*b - 8*A*b^2)*x^8 +
(5*B*a^2 - 8*A*a*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)))/(a^3
*b*x^8 + a^4*x^5)
```

giac [A] time = 0.20, size = 206, normalized size = 0.96

$$-\frac{\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-8\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}+\frac{\left(5\,Bab-8\,Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}-\frac{\left(5\left(-ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4 + 1/9*(5*B*a*b - 8*A*b^2)*(-a/b)^
(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - 1/18*(5*(-a*b^2)^(1/3)*B*a - 8*(-a*b
^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^4 - 1/3*(B*a*b*x
- A*b^2*x)/((b*x^3 + a)*a^3) - 1/10*(5*B*a*x^3 - 10*A*b*x^3 + 2*A*a)/(a^3*x
^5)
```

maple [A] time = 0.05, size = 252, normalized size = 1.17

$$\frac{A\,b^{2}x}{3\left(b\,x^{3}+a\right)a^{3}}-\frac{Bbx}{3\left(b\,x^{3}+a\right)a^{2}}+\frac{8\sqrt{3}\;Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}+\frac{8Ab\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}-\frac{4Ab\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right.\right.}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^2,x)
```

```
[Out] 1/3*b^2/a^3*x/(b*x^3+a)*A-1/3*b/a^2*x/(b*x^3+a)*B+8/9*b/a^3*A/(a/b)^(2/3)*l
n(x+(a/b)^(1/3))-4/9*b/a^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
8/9*b/a^3*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-5/9
/a^2*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+5/18/a^2*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1
/3)*x+(a/b)^(2/3))-5/9/a^2*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*x-1))-1/5/a^2*A/x^5+1/a^3/x^2*A*b-1/2/a^2/x^2*B
```

maxima [A] time = 1.32, size = 186, normalized size = 0.87

$$-\frac{5\left(5\,Bab-8\,Ab^{2}\right)x^{6}+3\left(5\,Ba^{2}-8\,Aab\right)x^{3}+6\,Aa^{2}}{30\left(a^{3}bx^{8}+a^{4}x^{5}\right)}-\frac{\sqrt{3}\,(5\,Ba-8\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(5\,Ba-8\,Ab)\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(5*(5*B*a*b - 8*A*b^2)*x^6 + 3*(5*B*a^2 - 8*A*a*b)*x^3 + 6*A*a^2)/(a^
3*b*x^8 + a^4*x^5) - 1/9*sqrt(3)*(5*B*a - 8*A*b)*arctan(1/3*sqrt(3)*(2*x -
(a/b)^(1/3))/(a/b)^(1/3))/(a^3*(a/b)^(2/3)) + 1/18*(5*B*a - 8*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*(a/b)^(2/3)) - 1/9*(5*B*a - 8*A*b)*log(
x + (a/b)^(1/3))/(a^3*(a/b)^(2/3))
```

mupad [B] time = 2.57, size = 176, normalized size = 0.82

$$\frac{\frac{x^{3}\,(8\,A\,b-5\,B\,a)}{10\,a^{2}}-\frac{A}{5\,a}+\frac{b\,x^{6}\,(8\,A\,b-5\,B\,a)}{6\,a^{3}}}{b\,x^{8}+a\,x^{5}}+\frac{b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(8\,A\,b-5\,B\,a)}{9\,a^{11/3}}-\frac{b^{2/3}\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\right.}{9\,a^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(8*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (b*x^6*(8*A*b - 5*B*a))/(6*a^3)
)/(a*x^5 + b*x^8) + (b^(2/3)*log(b^(1/3)*x + a^(1/3))*(8*A*b - 5*B*a))/(9*a
^(11/3)) - (b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/
2)*1i)/2 + 1/2)*(8*A*b - 5*B*a))/(9*a^(11/3)) + (b^(2/3)*log(3^(1/2)*a^(1/3
)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(8*A*b - 5*B*a))/(9*a^
(11/3))
```

sympy [A] time = 1.84, size = 138, normalized size = 0.64

$$\operatorname{RootSum}\left(729t^{3}a^{11}-512A^{3}b^{5}+960A^{2}Bab^{4}-600AB^{2}a^{2}b^{3}+125B^{3}a^{3}b^{2},\left(t\mapsto t\log\left(-\frac{9ta^{4}}{-8Ab^{2}+5Bab}+x\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**2,x)
```

```
[Out] RootSum(729*_t**3*a**11 - 512*A**3*b**5 + 960*A**2*B*a*b**4 - 600*A*B**2*a*
*2*b**3 + 125*B**3*a**3*b**2, Lambda(_t, _t*log(-9*_t*a**4/(-8*A*b**2 + 5*B
*a*b) + x))) + (-6*A*a**2 + x**6*(40*A*b**2 - 25*B*a*b) + x**3*(24*A*a*b -
15*B*a**2))/(30*a**4*x**5 + 30*a**3*b*x**8)
```

3.87 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=97

$$-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{A}{6a^2x^6}$$

```
[Out] -1/6*A/a^2/x^6+1/3*(2*A*b-B*a)/a^3/x^3+1/3*b*(A*b-B*a)/a^3/(b*x^3+a)+b*(3*A
*b-2*B*a)*ln(x)/a^4-1/3*b*(3*A*b-2*B*a)*ln(b*x^3+a)/a^4
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{2Ab-aB}{3a^3x^3}-\frac{b(3Ab-2aB)\log\left(a+bx^3\right)}{3a^4}+\frac{b\log(x)(3Ab-2aB)}{a^4}-\frac{A}{6a^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -A/(6*a^2*x^6) + (2*A*b - a*B)/(3*a^3*x^3) + (b*(A*b - a*B))/(3*a^3*(a + b*
x^3)) + (b*(3*A*b - 2*a*B)*Log[x])/a^4 - (b*(3*A*b - 2*a*B)*Log[a + b*x^3])
/(3*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^2}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^2}\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^2x^3}+\frac{-2Ab+aB}{a^3x^2}-\frac{b(-3Ab+2aB)}{a^4x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^2}+\frac{b^2(-3Ab+2aB}{a^4(a+bx)}\right.\right.\\&=-\frac{A}{6a^2x^6}+\frac{2Ab-aB}{3a^3x^3}+\frac{b(Ab-aB)}{3a^3\left(a+bx^3\right)}+\frac{b(3Ab-2aB)\log(x)}{a^4}-\frac{b(3Ab-2aB)\log\left(a+b\right.}{3a^4}\end{aligned}$$

Mathematica [A] time = 0.11, size = 85, normalized size = 0.88

$$-\frac{\frac{a^2A}{x^6}+\frac{2ab(aB-Ab)}{a+bx^3}+\frac{2a(aB-2Ab)}{x^3}+2b(3Ab-2aB)\log\left(a+bx^3\right)-6b\log(x)(3Ab-2aB)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^2),x]
```

```
[Out] -1/6*((a^2*A)/x^6 + (2*a*(-2*A*b + a*B))/x^3 + (2*a*b*(-(A*b) + a*B))/(a +
b*x^3) - 6*b*(3*A*b - 2*a*B)*Log[x] + 2*b*(3*A*b - 2*a*B)*Log[a + b*x^3])/a
^4
```

fricas [A] time = 0.89, size = 154, normalized size = 1.59

$$-\frac{2\left(2\,Ba^{2}b-3\,Aab^{2}\right)x^{6}+Aa^{3}+\left(2\,Ba^{3}-3\,Aa^{2}b\right)x^{3}-2\left(\left(2\,Bab^{2}-3\,Ab^{3}\right)x^{9}+\left(2\,Ba^{2}b-3\,Aab^{2}\right)x^{6}\right)\log\left(bx^{3}+\right.}{6\left(a^{4}bx^{9}+a^{5}x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*B*a^2*b - 3*A*a*b^2)*x^6 + A*a^3 + (2*B*a^3 - 3*A*a^2*b)*x^3 - 2
*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(b*x^3 + a) +
 6*((2*B*a*b^2 - 3*A*b^3)*x^9 + (2*B*a^2*b - 3*A*a*b^2)*x^6)*log(x))/(a^4*b
*x^9 + a^5*x^6)
```

giac [A] time = 0.17, size = 149, normalized size = 1.54

$$-\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(|x|\right)}{a^{4}}+\frac{\left(2\,Bab^{2}-3\,Ab^{3}\right)\log\left(\left|bx^{3}+a\right|\right)}{3\,a^{4}b}-\frac{2\,Bab^{2}x^{3}-3\,Ab^{3}x^{3}+3\,Ba^{2}b-4\,Aab^{2}}{3\left(bx^{3}+a\right)a^{4}}+\frac{6\,Babx^{6}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a*b - 3*A*b^2)*log(abs(x))/a^4 + 1/3*(2*B*a*b^2 - 3*A*b^3)*log(abs(b*
x^3 + a))/(a^4*b) - 1/3*(2*B*a*b^2*x^3 - 3*A*b^3*x^3 + 3*B*a^2*b - 4*A*a*b^
2)/((b*x^3 + a)*a^4) + 1/6*(6*B*a*b*x^6 - 9*A*b^2*x^6 - 2*B*a^2*x^3 + 4*A*a
*b*x^3 - A*a^2)/(a^4*x^6)
```

maple [A] time = 0.08, size = 116, normalized size = 1.20

$$\frac{A\,b^{2}}{3\left(b\,x^{3}+a\right)a^{3}}+\frac{3A\,b^{2}\ln\left(x\right)}{a^{4}}-\frac{A\,b^{2}\ln\left(b\,x^{3}+a\right)}{a^{4}}-\frac{Bb}{3\left(b\,x^{3}+a\right)a^{2}}-\frac{2Bb\ln\left(x\right)}{a^{3}}+\frac{2Bb\ln\left(b\,x^{3}+a\right)}{3a^{3}}+\frac{2Ab}{3a^{3}x^{3}}-\frac{B}{3a^{2}x^{3}}-\frac{}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^2,x)
```

```
[Out] -1/a^4*b^2*ln(b*x^3+a)*A+2/3/a^3*b*ln(b*x^3+a)*B+1/3/a^3*b^2/(b*x^3+a)*A-1/
3/a^2*b/(b*x^3+a)*B-1/6*A/a^2/x^6+2/3/a^3/x^3*A*b-1/3/a^2/x^3*B+3*b^2/a^4*l
n(x)*A-2*b/a^3*ln(x)*B
```

maxima [A] time = 0.65, size = 106, normalized size = 1.09

$$-\frac{2\left(2\,Bab-3\,Ab^{2}\right)x^{6}+\left(2\,Ba^{2}-3\,Aab\right)x^{3}+Aa^{2}}{6\left(a^{3}bx^{9}+a^{4}x^{6}\right)}+\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(bx^{3}+a\right)}{3\,a^{4}}-\frac{\left(2\,Bab-3\,Ab^{2}\right)\log\left(x^{3}\right)}{3\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3 + A*a^2)/(a^3*b*x
^9 + a^4*x^6) + 1/3*(2*B*a*b - 3*A*b^2)*log(b*x^3 + a)/a^4 - 1/3*(2*B*a*b -
 3*A*b^2)*log(x^3)/a^4
```

mupad [B] time = 0.14, size = 100, normalized size = 1.03

$$\frac{\frac{x^3\,(3\,A\,b-2\,B\,a)}{6\,a^2}-\frac{A}{6\,a}+\frac{b\,x^6\,(3\,A\,b-2\,B\,a)}{3\,a^3}}{b\,x^9+a\,x^6}-\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{3\,a^4}+\frac{\ln(x)\,\left(3\,A\,b^2-2\,B\,a\,b\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^2),x)
```

```
[Out] ((x^3*(3*A*b - 2*B*a))/(6*a^2) - A/(6*a) + (b*x^6*(3*A*b - 2*B*a))/(3*a^3))
/(a*x^6 + b*x^9) - (log(a + b*x^3)*(3*A*b^2 - 2*B*a*b))/(3*a^4) + (log(x)*(
3*A*b^2 - 2*B*a*b))/a^4
```

sympy [A] time = 2.40, size = 100, normalized size = 1.03

$$\frac{-Aa^2+x^6\left(6Ab^2-4Bab\right)+x^3\left(3Aab-2Ba^2\right)}{6a^4x^6+6a^3bx^9}-\frac{b\,(-3Ab+2Ba)\log{(x)}}{a^4}+\frac{b\,(-3Ab+2Ba)\log{\left(\frac{a}{b}+x^3\right)}}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**2,x)
```

```
[Out] (-A*a**2 + x**6*(6*A*b**2 - 4*B*a*b) + x**3*(3*A*a*b - 2*B*a**2))/(6*a**4*x
**6 + 6*a**3*b*x**9) - b*(-3*A*b + 2*B*a)*log(x)/a**4 + b*(-3*A*b + 2*B*a)*
log(a/b + x**3)/(3*a**4)
```

3.88 $\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=107

$$\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}+\frac{x^3(Ab-3aB)}{3b^4}+\frac{Bx^6}{6b^3}$$

```
[Out] 1/3*(A*b-3*B*a)*x^3/b^4+1/6*B*x^6/b^3+1/6*a^3*(A*b-B*a)/b^5/(b*x^3+a)^2-1/3
*a^2*(3*A*b-4*B*a)/b^5/(b*x^3+a)-a*(A*b-2*B*a)*ln(b*x^3+a)/b^5
```

Rubi [A] time = 0.14, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}+\frac{x^3(Ab-3aB)}{3b^4}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}+\frac{Bx^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - 3*a*B)*x^3)/(3*b^4) + (B*x^6)/(6*b^3) + (a^3*(A*b - a*B))/(6*b^5*(a
 + b*x^3)^2) - (a^2*(3*A*b - 4*a*B))/(3*b^5*(a + b*x^3)) - (a*(A*b - 2*a*B)
*Log[a + b*x^3])/b^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-3aB}{b^4}+\frac{Bx}{b^3}+\frac{a^3(-Ab+aB)}{b^4(a+bx)^3}-\frac{a^2(-3Ab+4aB)}{b^4(a+bx)^2}+\frac{3a(-Ab+2aB)}{b^4(a+bx)}\right)dx\\
&= \frac{(Ab-3aB)x^3}{3b^4}+\frac{Bx^6}{6b^3}+\frac{a^3(Ab-aB)}{6b^5\left(a+bx^3\right)^2}-\frac{a^2(3Ab-4aB)}{3b^5\left(a+bx^3\right)}-\frac{a(Ab-2aB)\log\left(a+bx^3\right)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.88

$$\frac{\frac{a^3(Ab-aB)}{\left(a+bx^3\right)^2}+\frac{2a^2(4aB-3Ab)}{a+bx^3}+2bx^3(Ab-3aB)+6a(2aB-Ab)\log\left(a+bx^3\right)+b^2Bx^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*b*(A*b - 3*a*B)*x^3 + b^2*B*x^6 + (a^3*(A*b - a*B))/(a + b*x^3)^2 + (2*a
^2*(-3*A*b + 4*a*B))/(a + b*x^3) + 6*a*(-(A*b) + 2*a*B)*Log[a + b*x^3])/(6*
b^5)
```

fricas [A] time = 0.86, size = 179, normalized size = 1.67

$$\frac{Bb^4x^{12} - 2\left(2\,Bab^3 - Ab^4\right)x^9 - \left(11\,Ba^2b^2 - 4\,Aab^3\right)x^6 + 7\,Ba^4 - 5\,Aa^3b + 2\left(Ba^3b - 2\,Aa^2b^2\right)x^3 + 6\left(\left(2\,Ba^2b^2\right.\right.}{6\left(b^7x^6 + 2\,ab^6x^3 + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(B*b^4*x^12 - 2*(2*B*a*b^3 - A*b^4)*x^9 - (11*B*a^2*b^2 - 4*A*a*b^3)*x^
6 + 7*B*a^4 - 5*A*a^3*b + 2*(B*a^3*b - 2*A*a^2*b^2)*x^3 + 6*((2*B*a^2*b^2 -
 A*a*b^3)*x^6 + 2*B*a^4 - A*a^3*b + 2*(2*B*a^3*b - A*a^2*b^2)*x^3)*log(b*x^
3 + a))/(b^7*x^6 + 2*a*b^6*x^3 + a^2*b^5)
```

giac [A] time = 0.18, size = 131, normalized size = 1.22

$$\frac{\left(2\,Ba^2 - Aab\right)\log\left(\left|bx^3 + a\right|\right)}{b^5} + \frac{Bb^3x^6 - 6\,Bab^2x^3 + 2\,Ab^3x^3}{6\,b^6} - \frac{18\,Ba^2b^2x^6 - 9\,Aab^3x^6 + 28\,Ba^3bx^3 - 12\,Aa^2b^2x}{6\left(bx^3 + a\right)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (2*B*a^2 - A*a*b)*log(abs(b*x^3 + a))/b^5 + 1/6*(B*b^3*x^6 - 6*B*a*b^2*x^3
+ 2*A*b^3*x^3)/b^6 - 1/6*(18*B*a^2*b^2*x^6 - 9*A*a*b^3*x^6 + 28*B*a^3*b*x^3
 - 12*A*a^2*b^2*x^3 + 11*B*a^4 - 4*A*a^3*b)/((b*x^3 + a)^2*b^5)
```

maple [A] time = 0.05, size = 134, normalized size = 1.25

$$\frac{B\,x^6}{6b^3} + \frac{A\,x^3}{3b^3} - \frac{Ba\,x^3}{b^4} + \frac{A\,a^3}{6\left(b\,x^3 + a\right)^2 b^4} - \frac{B\,a^4}{6\left(b\,x^3 + a\right)^2 b^5} - \frac{A\,a^2}{\left(b\,x^3 + a\right)b^4} - \frac{Aa\ln\left(b\,x^3 + a\right)}{b^4} + \frac{4B\,a^3}{3\left(b\,x^3 + a\right)b^5} + \frac{2B\,a^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*B*x^6/b^3+1/3/b^3*A*x^3-1/b^4*B*a*x^3+1/6*a^3/b^4/(b*x^3+a)^2*A-1/6*a^4
/b^5/(b*x^3+a)^2*B-a/b^4*ln(b*x^3+a)*A+2*a^2/b^5*ln(b*x^3+a)*B-a^2/b^4/(b*x
^3+a)*A+4/3*a^3/b^5/(b*x^3+a)*B
```

maxima [A] time = 0.54, size = 115, normalized size = 1.07

$$\frac{7\,Ba^4 - 5\,Aa^3b + 2\left(4\,Ba^3b - 3\,Aa^2b^2\right)x^3}{6\left(b^7x^6 + 2\,ab^6x^3 + a^2b^5\right)} + \frac{Bbx^6 - 2\,(3\,Ba - Ab)x^3}{6\,b^4} + \frac{\left(2\,Ba^2 - Aab\right)\log\left(bx^3 + a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(7*B*a^4 - 5*A*a^3*b + 2*(4*B*a^3*b - 3*A*a^2*b^2)*x^3)/(b^7*x^6 + 2*a*
b^6*x^3 + a^2*b^5) + 1/6*(B*b*x^6 - 2*(3*B*a - A*b)*x^3)/b^4 + (2*B*a^2 - A
*a*b)*log(b*x^3 + a)/b^5
```

mupad [B] time = 0.10, size = 117, normalized size = 1.09

$$\frac{\frac{7\,B\,a^4-5\,A\,a^3\,b}{6\,b}+x^3\,\left(\frac{4\,B\,a^3}{3}-A\,a^2\,b\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^3\,\left(\frac{A}{3\,b^3}-\frac{B\,a}{b^4}\right)+\frac{\ln\left(b\,x^3+a\right)\,\left(2\,B\,a^2-A\,a\,b\right)}{b^5}+\frac{B\,x^6}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((7*B*a^4 - 5*A*a^3*b)/(6*b) + x^3*((4*B*a^3)/3 - A*a^2*b))/(a^2*b^4 + b^6*
x^6 + 2*a*b^5*x^3) + x^3*(A/(3*b^3) - (B*a)/b^4) + (log(a + b*x^3)*(2*B*a^2
 - A*a*b))/b^5 + (B*x^6)/(6*b^3)
```

sympy [A] time = 4.94, size = 112, normalized size = 1.05

$$\frac{Bx^6}{6b^3}+\frac{a\left(-Ab+2Ba\right)\log\left(a+bx^3\right)}{b^5}+x^3\left(\frac{A}{3b^3}-\frac{Ba}{b^4}\right)+\frac{-5Aa^3b+7Ba^4+x^3\left(-6Aa^2b^2+8Ba^3b\right)}{6a^2b^5+12ab^6x^3+6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**6/(6*b**3) + a*(-A*b + 2*B*a)*log(a + b*x**3)/b**5 + x**3*(A/(3*b**3)
- B*a/b**4) + (-5*A*a**3*b + 7*B*a**4 + x**3*(-6*A*a**2*b**2 + 8*B*a**3*b))
/(6*a**2*b**5 + 12*a*b**6*x**3 + 6*b**7*x**6)
```

3.89 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=88

$$-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^3}{3b^3}$$

```
[Out] 1/3*B*x^3/b^3-1/6*a^2*(A*b-B*a)/b^4/(b*x^3+a)^2+1/3*a*(2*A*b-3*B*a)/b^4/(b*
x^3+a)+1/3*(A*b-3*B*a)*ln(b*x^3+a)/b^4
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) - (a^2*(A*b - a*B))/(6*b^4*(a + b*x^3)^2) + (a*(2*A*b - 3*a
*B))/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8(A+Bx^3)}{(a+bx^3)^3} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^3} dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{B}{b^3}-\frac{a^2(-Ab+aB)}{b^3(a+bx)^3}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^2}+\frac{Ab-3aB}{b^3(a+bx)}\right) dx, x, x^3\right)\\ &= \frac{Bx^3}{3b^3}-\frac{a^2(Ab-aB)}{6b^4(a+bx^3)^2}+\frac{a(2Ab-3aB)}{3b^4(a+bx^3)}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 92, normalized size = 1.05

$$\frac{2aAb-3a^2B}{3b^4(a+bx^3)}+\frac{a^3B-a^2Ab}{6b^4(a+bx^3)^2}+\frac{(Ab-3aB)\log(a+bx^3)}{3b^4}+\frac{Bx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (B*x^3)/(3*b^3) + (-(a^2*A*b) + a^3*B)/(6*b^4*(a + b*x^3)^2) + (2*a*A*b - 3
*a^2*B)/(3*b^4*(a + b*x^3)) + ((A*b - 3*a*B)*Log[a + b*x^3])/(3*b^4)
```

fricas [A] time = 0.92, size = 142, normalized size = 1.61

$$\frac{2\,Bb^3x^9+4\,Bab^2x^6-5\,Ba^3+3\,Aa^2b-4\left(Ba^2b-Aab^2\right)x^3-2\left(\left(3\,Bab^2-Ab^3\right)x^6+3\,Ba^3-Aa^2b+2\left(3\,Ba^2b-A\right.\right.}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*B*b^3*x^9 + 4*B*a*b^2*x^6 - 5*B*a^3 + 3*A*a^2*b - 4*(B*a^2*b - A*a*b
^2)*x^3 - 2*((3*B*a*b^2 - A*b^3)*x^6 + 3*B*a^3 - A*a^2*b + 2*(3*B*a^2*b - A
*a*b^2)*x^3)*log(b*x^3 + a))/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

giac [A] time = 0.18, size = 93, normalized size = 1.06

$$\frac{Bx^3}{3\,b^3}-\frac{(3\,Ba-Ab)\log\left(\left|bx^3+a\right|\right)}{3\,b^4}+\frac{9\,Bab^2x^6-3\,Ab^3x^6+12\,Ba^2bx^3-2\,Aab^2x^3+4\,Ba^3}{6\left(bx^3+a\right)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*x^3/b^3 - 1/3*(3*B*a - A*b)*log(abs(b*x^3 + a))/b^4 + 1/6*(9*B*a*b^2*
x^6 - 3*A*b^3*x^6 + 12*B*a^2*b*x^3 - 2*A*a*b^2*x^3 + 4*B*a^3)/((b*x^3 + a)^
2*b^4)
```

maple [A] time = 0.05, size = 110, normalized size = 1.25

$$\frac{B\,x^3}{3b^3}-\frac{A\,a^2}{6\left(b\,x^3+a\right)^2b^3}+\frac{B\,a^3}{6\left(b\,x^3+a\right)^2b^4}+\frac{2Aa}{3\left(b\,x^3+a\right)b^3}+\frac{A\ln\left(b\,x^3+a\right)}{3b^3}-\frac{B\,a^2}{\left(b\,x^3+a\right)b^4}-\frac{Ba\ln\left(b\,x^3+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/3*B*x^3/b^3-1/6/b^3*a^2/(b*x^3+a)^2*A+1/6/b^4*a^3/(b*x^3+a)^2*B+1/3/b^3*l
n(b*x^3+a)*A-1/b^4*ln(b*x^3+a)*B*a+2/3/b^3*a/(b*x^3+a)*A-1/b^4*a^2/(b*x^3+a
)*B
```

maxima [A] time = 0.52, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3\,b^3}-\frac{5\,Ba^3-3\,Aa^2b+2\left(3\,Ba^2b-2\,Aab^2\right)x^3}{6\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}-\frac{(3\,Ba-Ab)\log\left(bx^3+a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*B*x^3/b^3 - 1/6*(5*B*a^3 - 3*A*a^2*b + 2*(3*B*a^2*b - 2*A*a*b^2)*x^3)/(
b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4) - 1/3*(3*B*a - A*b)*log(b*x^3 + a)/b^4
```

mupad [B] time = 2.40, size = 94, normalized size = 1.07

$$\frac{B\,x^3}{3\,b^3}-\frac{x^3\,\left(B\,a^2-\frac{2\,A\,a\,b}{3}\right)+\frac{5\,B\,a^3-3\,A\,a^2\,b}{6\,b}}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{\ln\left(b\,x^3+a\right)\,\left(A\,b-3\,B\,a\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x^3)/(3*b^3) - (x^3*(B*a^2 - (2*A*a*b)/3) + (5*B*a^3 - 3*A*a^2*b)/(6*b))
/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (log(a + b*x^3)*(A*b - 3*B*a))/(3*b^4)
```

sympy [A] time = 3.94, size = 94, normalized size = 1.07

$$\frac{Bx^3}{3b^3} + \frac{3Aa^2b - 5Ba^3 + x^3\left(4Aab^2 - 6Ba^2b\right)}{6a^2b^4 + 12ab^5x^3 + 6b^6x^6} - \frac{(-Ab + 3Ba)\log\left(a + bx^3\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**3/(3*b**3) + (3*A*a**2*b - 5*B*a**3 + x**3*(4*A*a*b**2 - 6*B*a**2*b))/
(6*a**2*b**4 + 12*a*b**5*x**3 + 6*b**6*x**6) - (-A*b + 3*B*a)*log(a + b*x**
3)/(3*b**4)
```

3.90 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=66

$$-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}+\frac{B\log\left(a+bx^3\right)}{3b^3}$$

```
[Out] 1/6*a*(A*b-B*a)/b^3/(b*x^3+a)^2+1/3*(-A*b+2*B*a)/b^3/(b*x^3+a)+1/3*B*ln(b*x
^3+a)/b^3
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}+\frac{B\log\left(a+bx^3\right)}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (a*(A*b - a*B))/(6*b^3*(a + b*x^3)^2) - (A*b - 2*a*B)/(3*b^3*(a + b*x^3)) +
 (B*Log[a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^3}+\frac{Ab-2aB}{b^2(a+bx)^2}+\frac{B}{b^2(a+bx)}\right)dx,x,x^3\right)\\
&= \frac{a(Ab-aB)}{6b^3\left(a+bx^3\right)^2}-\frac{Ab-2aB}{3b^3\left(a+bx^3\right)}+\frac{B\log\left(a+bx^3\right)}{3b^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.97

$$\frac{3a^2B-ab\left(A-4Bx^3\right)+2B\left(a+bx^3\right)^2\log\left(a+bx^3\right)-2Ab^2x^3}{6b^3\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (3*a^2*B - 2*A*b^2*x^3 - a*b*(A - 4*B*x^3) + 2*B*(a + b*x^3)^2*Log[a + b*x^
3])/(6*b^3*(a + b*x^3)^2)
```

fricas [A] time = 0.87, size = 89, normalized size = 1.35

$$\frac{2\left(2\,Bab-Ab^2\right)x^3+3\,Ba^2-Aab+2\left(Bb^2x^6+2\,Babx^3+Ba^2\right)\log\left(bx^3+a\right)}{6\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b + 2*(B*b^2*x^6 + 2*B*a*b*x^3
 + B*a^2)*log(b*x^3 + a))/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

giac [A] time = 0.19, size = 61, normalized size = 0.92

$$\frac{B\log\left(\left|bx^3+a\right|\right)}{3\,b^3}+\frac{2\,(2\,Ba-Ab)x^3+\frac{3\,Ba^2-Aab}{b}}{6\left(bx^3+a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*B*log(abs(b*x^3 + a))/b^3 + 1/6*(2*(2*B*a - A*b)*x^3 + (3*B*a^2 - A*a*b
)/b)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.05, size = 81, normalized size = 1.23

$$\frac{Aa}{6\left(b\,x^3+a\right)^2b^2}-\frac{B\,a^2}{6\left(b\,x^3+a\right)^2b^3}-\frac{A}{3\left(b\,x^3+a\right)b^2}+\frac{2Ba}{3\left(b\,x^3+a\right)b^3}+\frac{B\ln\left(b\,x^3+a\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2*A-1/6*a^2/b^3/(b*x^3+a)^2*B+1/3*B*ln(b*x^3+a)/b^3-1/3
/b^2/(b*x^3+a)*A+2/3/b^3/(b*x^3+a)*B*a
```

maxima [A] time = 0.47, size = 72, normalized size = 1.09

$$\frac{2\left(2\,Bab-Ab^2\right)x^3+3\,Ba^2-Aab}{6\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}+\frac{B\log\left(bx^3+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(2*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)/(b^5*x^6 + 2*a*b^4*x^3 + a^
2*b^3) + 1/3*B*log(b*x^3 + a)/b^3
```

mupad [B] time = 2.38, size = 70, normalized size = 1.06

$$\frac{\frac{3\,B\,a^2-A\,a\,b}{6\,b^3}-\frac{x^3\,(A\,b-2\,B\,a)}{3\,b^2}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{B\,\ln\left(b\,x^3+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((3*B*a^2 - A*a*b)/(6*b^3) - (x^3*(A*b - 2*B*a))/(3*b^2))/(a^2 + b^2*x^6 +
2*a*b*x^3) + (B*log(a + b*x^3))/(3*b^3)
```

sympy [A] time = 3.69, size = 70, normalized size = 1.06

$$\frac{B\log\left(a+bx^3\right)}{3b^3}+\frac{-Aab+3Ba^2+x^3\left(-2Ab^2+4Bab\right)}{6a^2b^3+12ab^4x^3+6b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*log(a + b*x**3)/(3*b**3) + (-A*a*b + 3*B*a**2 + x**3*(-2*A*b**2 + 4*B*a*b
))/(6*a**2*b**3 + 12*a*b**4*x**3 + 6*b**5*x**6)
```

3.91 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

```
[Out] -1/6*(B*x^3+A)^2/(A*b-B*a)/(b*x^3+a)^2
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {444, 37}

$$-\frac{(A+Bx^3)^2}{6(a+bx^3)^2(Ab-aB)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -(A + B*x^3)^2/(6*(A*b - a*B)*(a + b*x^3)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx^3)}{(a+bx^3)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^3}\,dx, x, x^3\right)\\ &= -\frac{(A+Bx^3)^2}{6(Ab-aB)(a+bx^3)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.94

$$-\frac{B(a+2bx^3)+Ab}{6b^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] -1/6*(A*b + B*(a + 2*b*x^3))/(b^2*(a + b*x^3)^2)
```

fricas [A] time = 0.86, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

giac [A] time = 0.18, size = 28, normalized size = 0.88

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(bx^3 + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.05, size = 39, normalized size = 1.22

$$-\frac{B}{3\left(b\,x^3 + a\right)b^2} - \frac{Ab - Ba}{6\left(b\,x^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^3,x)
[Out] -1/6*(A*b-B*a)/b^2/(b*x^3+a)^2-1/3*B/b^2/(b*x^3+a)
```

maxima [A] time = 0.46, size = 42, normalized size = 1.31

$$-\frac{2\,Bbx^3 + Ba + Ab}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
[Out] -1/6*(2*B*b*x^3 + B*a + A*b)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

mupad [B] time = 2.33, size = 44, normalized size = 1.38

$$-\frac{\frac{A\,b+B\,a}{6\,b^2} + \frac{B\,x^3}{3\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^3,x)
[Out] -((A*b + B*a)/(6*b^2) + (B*x^3)/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

sympy [A] time = 1.82, size = 42, normalized size = 1.31

$$\frac{-Ab - Ba - 2Bbx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**3,x)
[Out] (-A*b - B*a - 2*B*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.92 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=68

$$-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

```
[Out] 1/6*(A*b-B*a)/a/b/(b*x^3+a)^2+1/3*A/a^2/(b*x^3+a)+A*ln(x)/a^3-1/3*A*ln(b*x^
3+a)/a^3
```

Rubi [A] time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$\frac{A}{3a^2\left(a+bx^3\right)}-\frac{A\log\left(a+bx^3\right)}{3a^3}+\frac{A\log(x)}{a^3}+\frac{Ab-aB}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] (A*b - a*B)/(6*a*b*(a + b*x^3)^2) + A/(3*a^2*(a + b*x^3)) + (A*Log[x])/a^3
- (A*Log[a + b*x^3])/(3*a^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x}+\frac{-Ab+aB}{a(a+bx)^3}-\frac{Ab}{a^2(a+bx)^2}-\frac{Ab}{a^3(a+bx)}\right)dx,x,x^3\right)\\ &= \frac{Ab-aB}{6ab\left(a+bx^3\right)^2}+\frac{A}{3a^2\left(a+bx^3\right)}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.87

$$\frac{\frac{a\left(-a^2B+3aAb+2Ab^2x^3\right)}{b\left(a+bx^3\right)^2}-2A\log\left(a+bx^3\right)+6A\log(x)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^3),x]
```

```
[Out] ((a*(3*a*A*b - a^2*B + 2*A*b^2*x^3))/(b*(a + b*x^3)^2) + 6*A*Log[x] - 2*A*L
og[a + b*x^3])/(6*a^3)
```

fricas [A] time = 0.90, size = 119, normalized size = 1.75

$$\frac{2\,Aab^2x^3 - Ba^3 + 3\,Aa^2b - 2\left(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b\right)\log\left(bx^3 + a\right) + 6\left(Ab^3x^6 + 2\,Aab^2x^3 + Aa^2b\right)\log(x)}{6\left(a^3b^3x^6 + 2\,a^4b^2x^3 + a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*a*b^2*x^3 - B*a^3 + 3*A*a^2*b - 2*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a
^2*b)*log(b*x^3 + a) + 6*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*log(x))/(a^3
*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b)
```

giac [A] time = 0.20, size = 74, normalized size = 1.09

$$-\frac{A\log\left(\left|bx^3 + a\right|\right)}{3\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{3\,Ab^3x^6 + 8\,Aab^2x^3 - Ba^3 + 6\,Aa^2b}{6\left(bx^3 + a\right)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*A*log(abs(b*x^3 + a))/a^3 + A*log(abs(x))/a^3 + 1/6*(3*A*b^3*x^6 + 8*A
*a*b^2*x^3 - B*a^3 + 6*A*a^2*b)/((b*x^3 + a)^2*a^3*b)
```

maple [A] time = 0.06, size = 68, normalized size = 1.00

$$\frac{A}{6\left(b\,x^3 + a\right)^2 a} - \frac{B}{6\left(b\,x^3 + a\right)^2 b} + \frac{A}{3\left(b\,x^3 + a\right)a^2} + \frac{A\ln(x)}{a^3} - \frac{A\ln\left(b\,x^3 + a\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a/(b*x^3+a)^2*A-1/6/b/(b*x^3+a)^2*B-1/3*A*ln(b*x^3+a)/a^3+1/3*A/a^2/(b*
x^3+a)+A/a^3*ln(x)
```

maxima [A] time = 0.45, size = 77, normalized size = 1.13

$$\frac{2\,Ab^2x^3 - Ba^2 + 3\,Aab}{6\left(a^2b^3x^6 + 2\,a^3b^2x^3 + a^4b\right)} - \frac{A\log\left(bx^3 + a\right)}{3\,a^3} + \frac{A\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*A*b^2*x^3 - B*a^2 + 3*A*a*b)/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b) -
 1/3*A*log(b*x^3 + a)/a^3 + 1/3*A*log(x^3)/a^3
```

mupad [B] time = 0.16, size = 71, normalized size = 1.04

$$\frac{\frac{3\,A\,b-B\,a}{6\,a\,b} + \frac{A\,b\,x^3}{3\,a^2}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6} - \frac{A\,\ln\left(b\,x^3 + a\right)}{3\,a^3} + \frac{A\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^3),x)
```

```
[Out] ((3*A*b - B*a)/(6*a*b) + (A*b*x^3)/(3*a^2))/(a^2 + b^2*x^6 + 2*a*b*x^3) - (
A*log(a + b*x^3))/(3*a^3) + (A*log(x))/a^3
```

sympy [A] time = 1.48, size = 75, normalized size = 1.10

$$\frac{A\log{(x)}}{a^3} - \frac{A\log{\left(\frac{a}{b}+x^3\right)}}{3a^3} + \frac{3Aab+2Ab^2x^3-Ba^2}{6a^4b+12a^3b^2x^3+6a^2b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**3,x)
```

```
[Out] A*log(x)/a**3 - A*log(a/b + x**3)/(3*a**3) + (3*A*a*b + 2*A*b**2*x**3 - B*a
**2)/(6*a**4*b + 12*a**3*b**2*x**3 + 6*a**2*b**3*x**6)
```

3.93 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3} \, dx$

Optimal. Leaf size=101

$$\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}$$

```
[Out] -1/3*A/a^3/x^3+1/6*(-A*b+B*a)/a^2/(b*x^3+a)^2+1/3*(-2*A*b+B*a)/a^3/(b*x^3+a
)-(3*A*b-B*a)*ln(x)/a^4+1/3*(3*A*b-B*a)*ln(b*x^3+a)/a^4
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 77}

$$-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{A}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] -A/(3*a^3*x^3) - (A*b - a*B)/(6*a^2*(a + b*x^3)^2) - (2*A*b - a*B)/(3*a^3*(
a + b*x^3)) - ((3*A*b - a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^3])/(
3*a^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^3}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^2}+\frac{-3Ab+aB}{a^4x}-\frac{b(-Ab+aB)}{a^2(a+bx)^3}-\frac{b(-2Ab+aB)}{a^3(a+bx)^2}-\frac{b(-3Ab+aB)}{a^4(a+bx)}\right)dx,x\right.\\
&= -\frac{A}{3a^3x^3}-\frac{Ab-aB}{6a^2\left(a+bx^3\right)^2}-\frac{2Ab-aB}{3a^3\left(a+bx^3\right)}-\frac{(3Ab-aB)\log(x)}{a^4}+\frac{(3Ab-aB)\log\left(a+bx^3\right)}{3a^4}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 87, normalized size = 0.86

$$\frac{\frac{a^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{2a(aB-2Ab)}{a+bx^3}+2(3Ab-aB)\log\left(a+bx^3\right)+6\log(x)(aB-3Ab)-\frac{2aA}{x^3}}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^3),x]
```

```
[Out] ((-2*a*A)/x^3 + (a^2*(-(A*b) + a*B))/(a + b*x^3)^2 + (2*a*(-2*A*b + a*B))/(
a + b*x^3) + 6*(-3*A*b + a*B)*Log[x] + 2*(3*A*b - a*B)*Log[a + b*x^3])/(6*a
^4)
```

fricas [B] time = 0.86, size = 197, normalized size = 1.95

$$\frac{2\left(Ba^2b-3\,Aab^2\right)x^6-2\,Aa^3+3\left(Ba^3-3\,Aa^2b\right)x^3-2\left(\left(Bab^2-3\,Ab^3\right)x^9+2\left(Ba^2b-3\,Aab^2\right)x^6+\left(Ba^3-3\,A\right.\right.}{6\left(a^4b^2x^9+2\,a^5bx^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(B*a^2*b - 3*A*a*b^2)*x^6 - 2*A*a^3 + 3*(B*a^3 - 3*A*a^2*b)*x^3 - 2*
((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2)*x^6 + (B*a^3 - 3*A*a^2*b
)*x^3)*log(b*x^3 + a) + 6*((B*a*b^2 - 3*A*b^3)*x^9 + 2*(B*a^2*b - 3*A*a*b^2
)*x^6 + (B*a^3 - 3*A*a^2*b)*x^3)*log(x))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x
^3)
```

giac [A] time = 0.20, size = 136, normalized size = 1.35

$$\frac{(Ba-3\,Ab)\log\left(|x|\right)}{a^4}-\frac{\left(Bab-3\,Ab^2\right)\log\left(\left|bx^3+a\right|\right)}{3\,a^4b}+\frac{3\,Bab^2x^6-9\,Ab^3x^6+8\,Ba^2bx^3-22\,Aab^2x^3+6\,Ba^3-14}{6\left(bx^3+a\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] (B*a - 3*A*b)*log(abs(x))/a^4 - 1/3*(B*a*b - 3*A*b^2)*log(abs(b*x^3 + a))/(
a^4*b) + 1/6*(3*B*a*b^2*x^6 - 9*A*b^3*x^6 + 8*B*a^2*b*x^3 - 22*A*a*b^2*x^3
+ 6*B*a^3 - 14*A*a^2*b)/((b*x^3 + a)^2*a^4) - 1/3*(B*a*x^3 - 3*A*b*x^3 + A*
a)/(a^4*x^3)
```

maple [A] time = 0.06, size = 117, normalized size = 1.16

$$-\frac{Ab}{6\left(b\,x^3+a\right)^2a^2}+\frac{B}{6\left(b\,x^3+a\right)^2a}-\frac{2Ab}{3\left(b\,x^3+a\right)a^3}-\frac{3Ab\ln(x)}{a^4}+\frac{Ab\ln\left(b\,x^3+a\right)}{a^4}+\frac{B}{3\left(b\,x^3+a\right)a^2}+\frac{B\ln(x)}{a^3}-\frac{B\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^3,x)
```

```
[Out] -1/6/a^2*b/(b*x^3+a)^2*A+1/6/a/(b*x^3+a)^2*B+1/a^4*b*ln(b*x^3+a)*A-1/3/a^3*
ln(b*x^3+a)*B-2/3/a^3*b/(b*x^3+a)*A+1/3/a^2/(b*x^3+a)*B-1/3*A/a^3/x^3-3*A/a
^4*b*ln(x)+B/a^3*ln(x)
```

maxima [A] time = 0.50, size = 109, normalized size = 1.08

$$\frac{2\left(Bab-3\,Ab^2\right)x^6+3\left(Ba^2-3\,Aab\right)x^3-2\,Aa^2}{6\left(a^3b^2x^9+2\,a^4bx^6+a^5x^3\right)}-\frac{(Ba-3\,Ab)\log\left(bx^3+a\right)}{3\,a^4}+\frac{(Ba-3\,Ab)\log\left(x^3\right)}{3\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(B*a*b - 3*A*b^2)*x^6 + 3*(B*a^2 - 3*A*a*b)*x^3 - 2*A*a^2)/(a^3*b^2*
x^9 + 2*a^4*b*x^6 + a^5*x^3) - 1/3*(B*a - 3*A*b)*log(b*x^3 + a)/a^4 + 1/3*(
B*a - 3*A*b)*log(x^3)/a^4
```

mupad [B] time = 2.46, size = 107, normalized size = 1.06

$$\frac{\ln\left(b\,x^3+a\right)\,\left(3\,A\,b-B\,a\right)}{3\,a^4}-\frac{\frac{A}{3\,a}+\frac{x^3\,\left(3\,A\,b-B\,a\right)}{2\,a^2}+\frac{b\,x^6\,\left(3\,A\,b-B\,a\right)}{3\,a^3}}{a^2\,x^3+2\,a\,b\,x^6+b^2\,x^9}-\frac{\ln(x)\,\left(3\,A\,b-B\,a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^3),x)
```

```
[Out] (log(a + b*x^3)*(3*A*b - B*a))/(3*a^4) - (A/(3*a) + (x^3*(3*A*b - B*a))/(2*
a^2) + (b*x^6*(3*A*b - B*a))/(3*a^3))/(a^2*x^3 + b^2*x^9 + 2*a*b*x^6) - (lo
g(x)*(3*A*b - B*a))/a^4
```

sympy [A] time = 3.27, size = 107, normalized size = 1.06

$$\frac{-2Aa^2+x^6\left(-6Ab^2+2Bab\right)+x^3\left(-9Aab+3Ba^2\right)}{6a^5x^3+12a^4bx^6+6a^3b^2x^9}+\frac{(-3Ab+Ba)\log(x)}{a^4}-\frac{(-3Ab+Ba)\log\left(\frac{a}{b}+x^3\right)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**3,x)
```

```
[Out] (-2*A*a**2 + x**6*(-6*A*b**2 + 2*B*a*b) + x**3*(-9*A*a*b + 3*B*a**2))/(6*a*
*5*x**3 + 12*a**4*b*x**6 + 6*a**3*b**2*x**9) + (-3*A*b + B*a)*log(x)/a**4 -
 (-3*A*b + B*a)*log(a/b + x**3)/(3*a**4)
```

3.94 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=122

$$-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}-\frac{A}{6a^3x^6}$$

```
[Out] -1/6*A/a^3/x^6+1/3*(3*A*b-B*a)/a^4/x^3+1/6*b*(A*b-B*a)/a^3/(b*x^3+a)^2+1/3*
b*(3*A*b-2*B*a)/a^4/(b*x^3+a)+3*b*(2*A*b-B*a)*ln(x)/a^5-b*(2*A*b-B*a)*ln(b*
x^3+a)/a^5
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{446, 77\}$

$$\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{3Ab-aB}{3a^4x^3}-\frac{b(2Ab-aB)\log\left(a+bx^3\right)}{a^5}+\frac{3b\log(x)(2Ab-aB)}{a^5}-\frac{A}{6a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] -A/(6*a^3*x^6) + (3*A*b - a*B)/(3*a^4*x^3) + (b*(A*b - a*B))/(6*a^3*(a + b*
x^3)^2) + (b*(3*A*b - 2*a*B))/(3*a^4*(a + b*x^3)) + (3*b*(2*A*b - a*B)*Log[
x])/a^5 - (b*(2*A*b - a*B)*Log[a + b*x^3])/a^5
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^3}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{A}{a^3x^3}+\frac{-3Ab+aB}{a^4x^2}-\frac{3b(-2Ab+aB)}{a^5x}+\frac{b^2(-Ab+aB)}{a^3(a+bx)^3}+\frac{b^2(-3Ab+2aB}{a^4(a+bx)^2}\right.\right.\\ &= -\frac{A}{6a^3x^6}+\frac{3Ab-aB}{3a^4x^3}+\frac{b(Ab-aB)}{6a^3\left(a+bx^3\right)^2}+\frac{b(3Ab-2aB)}{3a^4\left(a+bx^3\right)}+\frac{3b(2Ab-aB)\log(x)}{a^5}-\frac{b(2Al}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 108, normalized size = 0.89

$$\frac{\frac{a^2b(Ab-aB)}{(a+bx^3)^2}-\frac{a^2A}{x^6}+\frac{2ab(3Ab-2aB)}{a+bx^3}-\frac{2a(aB-3Ab)}{x^3}+6b(aB-2Ab)\log\left(a+bx^3\right)+18b\log(x)(2Ab-aB)}{6a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^3),x]
```

```
[Out] (-((a^2*A)/x^6) - (2*a*(-3*A*b + a*B))/x^3 + (a^2*b*(A*b - a*B))/(a + b*x^3
)^2 + (2*a*b*(3*A*b - 2*a*B))/(a + b*x^3) + 18*b*(2*A*b - a*B)*Log[x] + 6*b
*(-2*A*b + a*B)*Log[a + b*x^3])/(6*a^5)
```

fricas [B] time = 0.91, size = 229, normalized size = 1.88

$$-\frac{6\left(Ba^2b^2-2\,Aab^3\right)x^9+9\left(Ba^3b-2\,Aa^2b^2\right)x^6+Aa^4+2\left(Ba^4-2\,Aa^3b\right)x^3-6\left(\left(Bab^3-2\,Ab^4\right)x^{12}+2\left(Ba^2b^2-\right.\right.}{6\left(a^5b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(6*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + 9*(B*a^3*b - 2*A*a^2*b^2)*x^6 + A*a^4
 + 2*(B*a^4 - 2*A*a^3*b)*x^3 - 6*((B*a*b^3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 -
 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)*x^6)*log(b*x^3 + a) + 18*((B*a*b^
3 - 2*A*b^4)*x^12 + 2*(B*a^2*b^2 - 2*A*a*b^3)*x^9 + (B*a^3*b - 2*A*a^2*b^2)
*x^6)*log(x))/(a^5*b^2*x^12 + 2*a^6*b*x^9 + a^7*x^6)
```

giac [A] time = 0.20, size = 131, normalized size = 1.07

$$-\frac{3\left(Bab-2\,Ab^2\right)\log\left(|x|\right)}{a^5}+\frac{\left(Bab^2-2\,Ab^3\right)\log\left(\left|bx^3+a\right|\right)}{a^5b}-\frac{6\,Bab^2x^9-12\,Ab^3x^9+9\,Ba^2bx^6-18\,Aab^2x^6+2\,Ba^3}{6\left(bx^6+ax^3\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*a*b - 2*A*b^2)*log(abs(x))/a^5 + (B*a*b^2 - 2*A*b^3)*log(abs(b*x^3 +
a))/(a^5*b) - 1/6*(6*B*a*b^2*x^9 - 12*A*b^3*x^9 + 9*B*a^2*b*x^6 - 18*A*a*b^
2*x^6 + 2*B*a^3*x^3 - 4*A*a^2*b*x^3 + A*a^3)/((b*x^6 + a*x^3)^2*a^4)
```

maple [A] time = 0.06, size = 147, normalized size = 1.20

$$\frac{A\,b^2}{6\left(b\,x^3+a\right)^2a^3}-\frac{Bb}{6\left(b\,x^3+a\right)^2a^2}+\frac{A\,b^2}{\left(b\,x^3+a\right)a^4}+\frac{6A\,b^2\ln(x)}{a^5}-\frac{2A\,b^2\ln\left(b\,x^3+a\right)}{a^5}-\frac{2Bb}{3\left(b\,x^3+a\right)a^3}-\frac{3Bb\ln(x)}{a^4}+\frac{Bb}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^3,x)
```

```
[Out] 1/6/a^3*b^2/(b*x^3+a)^2*A-1/6/a^2*b/(b*x^3+a)^2*B-2/a^5*b^2*ln(b*x^3+a)*A+1
/a^4*b*ln(b*x^3+a)*B+1/a^4*b^2/(b*x^3+a)*A-2/3/a^3*b/(b*x^3+a)*B-1/6*A/a^3/
x^6+1/a^4/x^3*A*b-1/3/a^3/x^3*B+6*b^2/a^5*ln(x)*A-3*b/a^4*ln(x)*B
```

maxima [A] time = 0.52, size = 136, normalized size = 1.11

$$-\frac{6\left(Bab^2-2\,Ab^3\right)x^9+9\left(Ba^2b-2\,Aab^2\right)x^6+Aa^3+2\left(Ba^3-2\,Aa^2b\right)x^3}{6\left(a^4b^2x^{12}+2\,a^5bx^9+a^6x^6\right)}+\frac{\left(Bab-2\,Ab^2\right)\log\left(bx^3+a\right)}{a^5}-\frac{\left(Bab-2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(6*(B*a*b^2 - 2*A*b^3)*x^9 + 9*(B*a^2*b - 2*A*a*b^2)*x^6 + A*a^3 + 2*(
B*a^3 - 2*A*a^2*b)*x^3)/(a^4*b^2*x^12 + 2*a^5*b*x^9 + a^6*x^6) + (B*a*b - 2
*A*b^2)*log(b*x^3 + a)/a^5 - (B*a*b - 2*A*b^2)*log(x^3)/a^5
```

mupad [B] time = 0.15, size = 130, normalized size = 1.07

$$\frac{\frac{x^3 (2 A b-B a)}{3 a^2}-\frac{A}{6 a}+\frac{b^2 x^9 (2 A b-B a)}{a^4}+\frac{3 b x^6 (2 A b-B a)}{2 a^3}}{a^2 x^6+2 a b x^9+b^2 x^{12}}-\frac{\ln\left(b x^3+a\right) \left(2 A b^2-B a b\right)}{a^5}+\frac{\ln(x) \left(6 A b^2-3 B a b\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(2*A*b - B*a))/(3*a^2) - A/(6*a) + (b^2*x^9*(2*A*b - B*a))/a^4 + (3*b
*x^6*(2*A*b - B*a))/(2*a^3))/(a^2*x^6 + b^2*x^12 + 2*a*b*x^9) - (log(a + b*
x^3)*(2*A*b^2 - B*a*b))/a^5 + (log(x)*(6*A*b^2 - 3*B*a*b))/a^5
```

sympy [A] time = 3.55, size = 133, normalized size = 1.09

$$\frac{-Aa^3+x^9\left(12Ab^3-6Bab^2\right)+x^6\left(18Aab^2-9Ba^2b\right)+x^3\left(4Aa^2b-2Ba^3\right)}{6a^6x^6+12a^5bx^9+6a^4b^2x^{12}}-\frac{3b\left(-2Ab+Ba\right)\log\left(x\right)}{a^5}+\frac{b\left(-2Ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**3,x)
```

```
[Out] (-A*a**3 + x**9*(12*A*b**3 - 6*B*a*b**2) + x**6*(18*A*a*b**2 - 9*B*a**2*b)
+ x**3*(4*A*a**2*b - 2*B*a**3))/(6*a**6*x**6 + 12*a**5*b*x**9 + 6*a**4*b**2
*x**12) - 3*b*(-2*A*b + B*a)*log(x)/a**5 + b*(-2*A*b + B*a)*log(a/b + x**3)
/a**5
```

3.95 $\int \frac{x^{10}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=246

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-}}{9\sqrt{3}b^{14/3}}$$

```
[Out] 2/9*(5*A*b-11*B*a)*x^2/b^4-4/45*(5*A*b-11*B*a)*x^5/a/b^3+1/6*(A*b-B*a)*x^11
/a/b/(b*x^3+a)^2+1/18*(5*A*b-11*B*a)*x^8/a/b^2/(b*x^3+a)+4/27*a^(2/3)*(5*A*
b-11*B*a)*ln(a^(1/3)+b^(1/3)*x)/b^(14/3)-2/27*a^(2/3)*(5*A*b-11*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(14/3)+4/27*a^(2/3)*(5*A*b-11*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(14/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 288, 302, 292, 31, 634, 617, 204, 628}

$$-\frac{2a^{2/3}(5Ab-11aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27b^{14/3}}+\frac{4a^{2/3}(5Ab-11aB)\tan^{-}}{9\sqrt{3}b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*(5*A*b - 11*a*B)*x^2)/(9*b^4) - (4*(5*A*b - 11*a*B)*x^5)/(45*a*b^3) + ((
A*b - a*B)*x^11)/(6*a*b*(a + b*x^3)^2) + ((5*A*b - 11*a*B)*x^8)/(18*a*b^2*(
a + b*x^3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(S
qrt[3]*a^(1/3))])/(9*Sqrt[3]*b^(14/3)) + (4*a^(2/3)*(5*A*b - 11*a*B)*Log[a^
(1/3) + b^(1/3)*x])/(27*b^(14/3)) - (2*a^(2/3)*(5*A*b - 11*a*B)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*b^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{10}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx = \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(-5Ab+11aB)\int \frac{x^{10}}{\left(a+bx^3\right)^2}dx}{6ab}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \frac{x^7}{a+bx^3}dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4(5Ab-11aB))\int \left(-\frac{ax}{b^2}+\frac{x^4}{b}+\frac{a^2x}{b^2\left(a+bx^3\right)}\right)dx}{9ab^2}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} - \frac{(4a(5Ab-1}{}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{\left(4a^{2/3}(5Ab-\right.}{}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-}{}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-}{}$$

$$= \frac{2(5Ab-11aB)x^2}{9b^4} - \frac{4(5Ab-11aB)x^5}{45ab^3} + \frac{(Ab-aB)x^{11}}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab-11aB)x^8}{18ab^2\left(a+bx^3\right)} + \frac{4a^{2/3}(5Ab-}{}$$

Mathematica [**A**] time = 0.32, size = 216, normalized size = 0.88

$$\frac{20a^{2/3}(11aB-5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)-40a^{2/3}(11aB-5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)-40\sqrt{3}\,a^{2/3}(11aB-5A}{270b^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (135*b^(2/3)*(A*b - 3*a*B)*x^2 + 54*b^(5/3)*B*x^5 + (45*a^2*b^(2/3)*(-(A*b)
 + a*B)*x^2)/(a + b*x^3)^2 + (30*a*b^(2/3)*(7*A*b - 10*a*B)*x^2)/(a + b*x^3
) - 40*Sqrt[3]*a^(2/3)*(-5*A*b + 11*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] - 40*a^(2/3)*(-5*A*b + 11*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*a^(2
/3)*(-5*A*b + 11*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
b^(14/3))
```

fricas [**A**] time = 0.85, size = 364, normalized size = 1.48

$$\frac{54\,Bb^3x^{11}-27\left(11\,Bab^2-5\,Ab^3\right)x^8-96\left(11\,Ba^2b-5\,Aab^2\right)x^5-60\left(11\,Ba^3-5\,Aa^2b\right)x^2+40\sqrt{3}\left(\left(11\,Bab^2-5\,A\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/270*(54*B*b^3*x^11 - 27*(11*B*a*b^2 - 5*A*b^3)*x^8 - 96*(11*B*a^2*b - 5*A
*a*b^2)*x^5 - 60*(11*B*a^3 - 5*A*a^2*b)*x^2 + 40*sqrt(3)*((11*B*a*b^2 - 5*A
*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)*x^3)*(a^2/b^2
)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 20*((11
*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5*A*a*b^2)
*x^3)*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3))
- 40*((11*B*a*b^2 - 5*A*b^3)*x^6 + 11*B*a^3 - 5*A*a^2*b + 2*(11*B*a^2*b - 5
*A*a*b^2)*x^3)*(a^2/b^2)^(1/3)*log(a*x + b*(a^2/b^2)^(2/3)))/(b^6*x^6 + 2*a
*b^5*x^3 + a^2*b^4)
```

giac [A] time = 0.33, size = 259, normalized size = 1.05

$$-\frac{4\left(11\,Ba^2\left(-\frac{a}{b}\right)^{\frac{1}{3}} - 5\,Aab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4} - \frac{4\sqrt{3}\left(11\left(-ab^2\right)^{\frac{2}{3}}Ba - 5\left(-ab^2\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\ldots}{\ldots}\right.}{27\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*(11*B*a^2*(-a/b)^(1/3) - 5*A*a*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x
 - (-a/b)^(1/3)))/(a*b^4) - 4/27*sqrt(3)*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2
)^(2/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^6 + 2/
27*(11*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/b^6 - 1/18*(20*B*a^2*b*x^5 - 14*A*a*b^2*x^5 + 17*B*a^3*x^2
- 11*A*a^2*b*x^2)/((b*x^3 + a)^2*b^4) + 1/10*(2*B*b^12*x^5 - 15*B*a*b^11*x^
2 + 5*A*b^12*x^2)/b^15
```

maple [A] time = 0.06, size = 308, normalized size = 1.25

$$\frac{7Aa\,x^5}{9\left(b\,x^3+a\right)^2b^2} - \frac{10B\,a^2x^5}{9\left(b\,x^3+a\right)^2b^3} + \frac{B\,x^5}{5b^3} + \frac{11A\,a^2x^2}{18\left(b\,x^3+a\right)^2b^3} - \frac{17B\,a^3x^2}{18\left(b\,x^3+a\right)^2b^4} + \frac{A\,x^2}{2b^3} - \frac{3Ba\,x^2}{2b^4} - \frac{20\sqrt{3}\,Aa\arctan\ldots}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/5/b^3*B*x^5+1/2/b^3*A*x^2-3/2/b^4*B*x^2*a+7/9*a/b^2/(b*x^3+a)^2*A*x^5-10/
9*a^2/b^3/(b*x^3+a)^2*B*x^5+11/18*a^2/b^3/(b*x^3+a)^2*A*x^2-17/18*a^3/b^4/(
b*x^3+a)^2*B*x^2+20/27*a/b^4*A/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27*a/b^4*A/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27*a/b^4*A*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-44/27*a^2/b^5*B/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+22/27*a^2/b^5*B/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3
))+44/27*a^2/b^5*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-
1))
```

maxima [A] time = 1.37, size = 228, normalized size = 0.93

$$-\frac{2\left(10\,Ba^2b-7\,Aab^2\right)x^5+\left(17\,Ba^3-11\,Aa^2b\right)x^2}{18\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}+\frac{4\sqrt{3}\left(11\,Ba^2-5\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{2\,Bbx^5-5\,(3\,Ba}{10\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(10*B*a^2*b - 7*A*a*b^2)*x^5 + (17*B*a^3 - 11*A*a^2*b)*x^2)/(b^6*x
^6 + 2*a*b^5*x^3 + a^2*b^4) + 4/27*sqrt(3)*(11*B*a^2 - 5*A*a*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^5*(a/b)^(1/3)) + 1/10*(2*B*b*x^
5 - 5*(3*B*a - A*b)*x^2)/b^4 + 2/27*(11*B*a^2 - 5*A*a*b)*log(x^2 - x*(a/b)^
(1/3) + (a/b)^(2/3))/(b^5*(a/b)^(1/3)) - 4/27*(11*B*a^2 - 5*A*a*b)*log(x +
(a/b)^(1/3))/(b^5*(a/b)^(1/3))
```

mupad [B] time = 2.58, size = 213, normalized size = 0.87

$$\frac{x^5\left(\frac{7\,A\,a\,b^2}{9}-\frac{10\,B\,a^2\,b}{9}\right)-x^2\left(\frac{17\,B\,a^3}{18}-\frac{11\,A\,a^2\,b}{18}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x^2\left(\frac{A}{2\,b^3}-\frac{3\,B\,a}{2\,b^4}\right)+\frac{B\,x^5}{5\,b^3}+\frac{4\,a^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b-11\,B\,a)}{27\,b^{14/3}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^5*((7*A*a*b^2)/9 - (10*B*a^2*b)/9) - x^2*((17*B*a^3)/18 - (11*A*a^2*b)/1
8))/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x^2*(A/(2*b^3) - (3*B*a)/(2*b^4)) +
 (B*x^5)/(5*b^3) + (4*a^(2/3)*log(b^(1/3)*x + a^(1/3))*(5*A*b - 11*B*a))/(2
7*b^(14/3)) + (4*a^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(5*A*b - 11*B*a))/(27*b^(14/3)) - (4*a^(2/3)*log(3^(1/
2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b - 11*B
*a))/(27*b^(14/3))
```

sympy [A] time = 6.24, size = 192, normalized size = 0.78

$$\frac{Bx^5}{5b^3}+x^2\left(\frac{A}{2b^3}-\frac{3Ba}{2b^4}\right)+\frac{x^5\left(14Aab^2-20Ba^2b\right)+x^2\left(11Aa^2b-17Ba^3\right)}{18a^2b^4+36ab^5x^3+18b^6x^6}+\operatorname{RootSum}\left(19683t^3b^{14}-8000A^3a^2b^3+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**5/(5*b**3) + x**2*(A/(2*b**3) - 3*B*a/(2*b**4)) + (x**5*(14*A*a*b**2 -
 20*B*a**2*b) + x**2*(11*A*a**2*b - 17*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x
**3 + 18*b**6*x**6) + RootSum(19683*_t**3*b**14 - 8000*A**3*a**2*b**3 + 528
00*A**2*B*a**3*b**2 - 116160*A*B**2*a**4*b + 85184*B**3*a**5, Lambda(_t, _t
*log(729*_t**2*b**9/(400*A**2*a*b**2 - 1760*A*B*a**2*b + 1936*B**2*a**3) +
x)))
```

3.96 $\int \frac{x^9(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=244

$$\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{13/3}}-\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27b^{13/3}}+\frac{7\sqrt[3]{a}\,(2Ab-5aB)\tan^{-1}}{9\sqrt{3}\,b^{13/3}}$$

```
[Out] 7/9*(2*A*b-5*B*a)*x/b^4-7/36*(2*A*b-5*B*a)*x^4/a/b^3+1/6*(A*b-B*a)*x^10/a/b
/(b*x^3+a)^2+1/9*(2*A*b-5*B*a)*x^7/a/b^2/(b*x^3+a)-7/27*a^(1/3)*(2*A*b-5*B*
a)*ln(a^(1/3)+b^(1/3)*x)/b^(13/3)+7/54*a^(1/3)*(2*A*b-5*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/b^(13/3)+7/27*a^(1/3)*(2*A*b-5*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/b^(13/3)*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 302, 200, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{a}\,(2Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54b^{13/3}}+\frac{x^7(2Ab-5aB)}{9ab^2\left(a+bx^3\right)}-\frac{7x^4(2Ab-5aB)}{36ab^3}+\frac{7x(2Ab-5aB)}{9b^4}-\frac{7\sqrt[3]{a}\,(2A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (7*(2*A*b - 5*a*B)*x)/(9*b^4) - (7*(2*A*b - 5*a*B)*x^4)/(36*a*b^3) + ((A*b
- a*B)*x^10)/(6*a*b*(a + b*x^3)^2) + ((2*A*b - 5*a*B)*x^7)/(9*a*b^2*(a + b*
x^3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*
a^(1/3))])/(9*Sqrt[3]*b^(13/3)) - (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(1/3) +
b^(1/3)*x])/(27*b^(13/3)) + (7*a^(1/3)*(2*A*b - 5*a*B)*Log[a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2])/(54*b^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[(((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^9 (A+Bx^3)}{(a+bx^3)^3}\,dx = \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(-4Ab+10aB)\int \frac{x^9}{(a+bx^3)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{(7(2Ab-5aB))\int \frac{x^6}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{(7(2Ab-5aB))\int \left(-\frac{a}{b^2}+\frac{x^3}{b}+\frac{a^2}{b^2(a+bx^3)}\right)dx}{9ab^2}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{(7a(2Ab-5a}{9b}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{(7\sqrt[3]{a}(2Ab-5}{}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{7\sqrt[3]{a}(2Ab-5a}{2}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} - \frac{7\sqrt[3]{a}(2Ab-5a}{2}$$

$$= \frac{7(2Ab-5aB)x}{9b^4} - \frac{7(2Ab-5aB)x^4}{36ab^3} + \frac{(Ab-aB)x^{10}}{6ab(a+bx^3)^2} + \frac{(2Ab-5aB)x^7}{9ab^2(a+bx^3)} + \frac{7\sqrt[3]{a}(2Ab-5}{9}$$

Mathematica [A] time = 0.24, size = 210, normalized size = 0.86

$$\frac{-14\sqrt[3]{a}(5aB-2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+\frac{18a^2\sqrt[3]{b}x(aB-Ab)}{(a+bx^3)^2}+\frac{6a\sqrt[3]{b}x(13Ab-19aB)}{a+bx^3}+108\sqrt[3]{b}x(Ab-3aB)+}{108b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (108*b^(1/3)*(A*b - 3*a*B)*x + 27*b^(4/3)*B*x^4 + (18*a^2*b^(1/3)*(-(A*b) +
 a*B)*x)/(a + b*x^3)^2 + (6*a*b^(1/3)*(13*A*b - 19*a*B)*x)/(a + b*x^3) - 28
*Sqrt[3]*a^(1/3)*(-2*A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3
]] + 28*a^(1/3)*(-2*A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x] - 14*a^(1/3)*(-2*
A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*b^(13/3))
```

fricas [A] time = 0.88, size = 347, normalized size = 1.42

$$\frac{27\,Bb^3x^{10}-54\left(5\,Bab^2-2\,Ab^3\right)x^7-147\left(5\,Ba^2b-2\,Aab^2\right)x^4-28\sqrt{3}\left(\left(5\,Bab^2-2\,Ab^3\right)x^6+5\,Ba^3-2\,Aa^2b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/108*(27*B*b^3*x^10 - 54*(5*B*a*b^2 - 2*A*b^3)*x^7 - 147*(5*B*a^2*b - 2*A*
a*b^2)*x^4 - 28*sqrt(3)*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b +
2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b
)^(2/3) - sqrt(3)*a)/a) + 14*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2
*b + 2*(5*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3)) - 28*((5*B*a*b^2 - 2*A*b^3)*x^6 + 5*B*a^3 - 2*A*a^2*b + 2*(5
*B*a^2*b - 2*A*a*b^2)*x^3)*(-a/b)^(1/3)*log(x - (-a/b)^(1/3)) - 84*(5*B*a^3
 - 2*A*a^2*b)*x)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

giac [A] time = 0.24, size = 234, normalized size = 0.96

$$\frac{7\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-2\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5}-\frac{7\left(5\,Ba^2-2\,Aab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,ab^4}+\frac{7\left(5\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 2*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/b^5 - 7/27*(5*B*a^2 - 2*A*a*b)*(-a/b)
^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4) + 7/54*(5*(-a*b^2)^(1/3)*B*a - 2*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/b^5 - 1/18*(19
*B*a^2*b*x^4 - 13*A*a*b^2*x^4 + 16*B*a^3*x - 10*A*a^2*b*x)/((b*x^3 + a)^2*b
^4) + 1/4*(B*b^9*x^4 - 12*B*a*b^8*x + 4*A*b^9*x)/b^12
```

maple [A] time = 0.06, size = 299, normalized size = 1.23

$$\frac{13Aa\,x^4}{18\left(b\,x^3+a\right)^2b^2}-\frac{19B\,a^2x^4}{18\left(b\,x^3+a\right)^2b^3}+\frac{B\,x^4}{4b^3}+\frac{5A\,a^2x}{9\left(b\,x^3+a\right)^2b^3}-\frac{8B\,a^3x}{9\left(b\,x^3+a\right)^2b^4}-\frac{14\sqrt{3}\,Aa\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^4}-\frac{14A\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/4/b^3*B*x^4+1/b^3*A*x-3/b^4*B*a*x+13/18*a/b^2/(b*x^3+a)^2*A*x^4-19/18*a^2
/b^3/(b*x^3+a)^2*B*x^4+5/9*a^2/b^3/(b*x^3+a)^2*A*x-8/9*a^3/b^4/(b*x^3+a)^2*
B*x-14/27*a/b^4*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27*a/b^4*A/(a/b)^(2/3)*ln
(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27*a/b^4*A/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+35/27*a^2/b^5*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)-35/54*a^2/b^5*B/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+35/27*a^2/b
^5*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.24, size = 223, normalized size = 0.91

$$-\frac{\left(19\,Ba^2b-13\,Aab^2\right)x^4+2\left(8\,Ba^3-5\,Aa^2b\right)x}{18\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}+\frac{Bbx^4-4\left(3\,Ba-Ab\right)x}{4\,b^4}+\frac{7\sqrt{3}\left(5\,Ba^2-2\,Aab\right)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)\cdots\right.}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^5\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((19*B*a^2*b - 13*A*a*b^2)*x^4 + 2*(8*B*a^3 - 5*A*a^2*b)*x)/(b^6*x^6
+ 2*a*b^5*x^3 + a^2*b^4) + 1/4*(B*b*x^4 - 4*(3*B*a - A*b)*x)/b^4 + 7/27*sqr
t(3)*(5*B*a^2 - 2*A*a*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3)
)/(b^5*(a/b)^(2/3)) - 7/54*(5*B*a^2 - 2*A*a*b)*log(x^2 - x*(a/b)^(1/3) + (a
/b)^(2/3))/(b^5*(a/b)^(2/3)) + 7/27*(5*B*a^2 - 2*A*a*b)*log(x + (a/b)^(1/3)
)/(b^5*(a/b)^(2/3))
```

mupad [B] time = 0.32, size = 227, normalized size = 0.93

$$\frac{x^4\left(\frac{13\,A\,a\,b^2}{18}-\frac{19\,B\,a^2\,b}{18}\right)-x\left(\frac{8\,B\,a^3}{9}-\frac{5\,A\,a^2\,b}{9}\right)}{a^2\,b^4+2\,a\,b^5\,x^3+b^6\,x^6}+x\left(\frac{A}{b^3}-\frac{3\,B\,a}{b^4}\right)+\frac{B\,x^4}{4\,b^3}+\frac{7\,{(-a)}^{1/3}\,\ln\left({(-a)}^{4/3}+a\,b^{1/3}\,x\right)\,(2\,A\,b-5\,B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^4*((13*A*a*b^2)/18 - (19*B*a^2*b)/18) - x*((8*B*a^3)/9 - (5*A*a^2*b)/9))
/(a^2*b^4 + b^6*x^6 + 2*a*b^5*x^3) + x*(A/b^3 - (3*B*a)/b^4) + (B*x^4)/(4*b
^3) + (7*(-a)^(1/3)*log((-a)^(4/3) + a*b^(1/3)*x)*(2*A*b - 5*B*a))/(27*b^(1
3/3)) - (7*(-a)^(1/3)*log((-a)^(4/3) + 3^(1/2)*(-a)^(4/3)*1i - 2*a*b^(1/3)*
x)*((3^(1/2)*1i)/2 + 1/2)*(2*A*b - 5*B*a))/(27*b^(13/3)) + (7*(-a)^(1/3)*lo
g(3^(1/2)*(-a)^(4/3)*1i - (-a)^(4/3) + 2*a*b^(1/3)*x)*((3^(1/2)*1i)/2 - 1/2
)*(2*A*b - 5*B*a))/(27*b^(13/3))
```

sympy [A] time = 4.01, size = 163, normalized size = 0.67

$$\frac{Bx^4}{4b^3}+x\left(\frac{A}{b^3}-\frac{3Ba}{b^4}\right)+\frac{x^4\left(13Aab^2-19Ba^2b\right)+x\left(10Aa^2b-16Ba^3\right)}{18a^2b^4+36ab^5x^3+18b^6x^6}+\mathrm{RootSum}\left(19683t^3b^{13}+2744A^3ab^3-20$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**4/(4*b**3) + x*(A/b**3 - 3*B*a/b**4) + (x**4*(13*A*a*b**2 - 19*B*a**2*
b) + x*(10*A*a**2*b - 16*B*a**3))/(18*a**2*b**4 + 36*a*b**5*x**3 + 18*b**6*
x**6) + RootSum(19683*_t**3*b**13 + 2744*A**3*a*b**3 - 20580*A**2*B*a**2*b*
*2 + 51450*A*B**2*a**3*b - 42875*B**3*a**4, Lambda(_t, _t*log(27*_t*b**4/(-
14*A*b + 35*B*a) + x)))
```

3.97 $\int \frac{x^7 (A+Bx^3)}{(a+bx^3)^3} \, dx$

Optimal. Leaf size=222

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}}-\frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{11/3}}-\frac{5x^2(}{}$$

```
[Out] -5/18*(A*b-4*B*a)*x^2/a/b^3+1/6*(A*b-B*a)*x^8/a/b/(b*x^3+a)^2+1/9*(A*b-4*B*
a)*x^5/a/b^2/(b*x^3+a)-5/27*(A*b-4*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/b^(11
/3)+5/54*(A*b-4*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(1/3)/b^(1
1/3)-5/27*(A*b-4*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
1/3)/b^(11/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 321, 292, 31, 634, 617, 204, 628}

$$\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54\sqrt[3]{a}\,b^{11/3}}+\frac{x^5(Ab-4aB)}{9ab^2\left(a+bx^3\right)}-\frac{5x^2(Ab-4aB)}{18ab^3}-\frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}}-\frac{5(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-5*(A*b - 4*a*B)*x^2)/(18*a*b^3) + ((A*b - a*B)*x^8)/(6*a*b*(a + b*x^3)^2)
 + ((A*b - 4*a*B)*x^5)/(9*a*b^2*(a + b*x^3)) - (5*(A*b - 4*a*B)*ArcTan[(a^(
1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(1/3)*b^(11/3)) - (5*(
A*b - 4*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(1/3)*b^(11/3)) + (5*(A*b - 4*
a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(1/3)*b^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(-2Ab+8aB)\int \frac{x^7}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{x^4}{a+bx^3}\,dx}{9ab^2}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} + \frac{(5(Ab-4aB))\int \frac{x}{a+bx^3}\,dx}{9b^3}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{(5(Ab-4aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27\sqrt[3]{a}\,b^{10/3}} + \frac{(5(A}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}} + \frac{(5}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27\sqrt[3]{a}\,b^{11/3}} + \frac{5(}{}$$

$$= -\frac{5(Ab-4aB)x^2}{18ab^3} + \frac{(Ab-aB)x^8}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-4aB)x^5}{9ab^2\left(a+bx^3\right)} - \frac{5(Ab-4aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\sqrt[3]{a}\,b^{11/3}} - \frac{5(}{}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.87

$$\frac{\frac{5(Ab-4aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} - \frac{6b^{2/3}x^2(4Ab-7aB)}{a+bx^3} + \frac{9ab^{2/3}x^2(Ab-aB)}{\left(a+bx^3\right)^2} + \frac{10(4aB-Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{10\sqrt{3}(4aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}}{54b^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (27*b^(2/3)*B*x^2 + (9*a*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 - (6*b^(2/3
)*(4*A*b - 7*a*B)*x^2)/(a + b*x^3) + (10*Sqrt[3]*(-(A*b) + 4*a*B)*ArcTan[(1
 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) + (10*(-(A*b) + 4*a*B)*Log[a^(1
/3) + b^(1/3)*x])/a^(1/3) + (5*(A*b - 4*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/a^(1/3))/(54*b^(11/3))
```

fricas [B] time = 0.68, size = 792, normalized size = 3.57

$$\left[\frac{27\,Bab^4x^8 + 24\left(4\,Ba^2b^3 - Aab^4\right)x^5 + 15\left(4\,Ba^3b^2 - Aa^2b^3\right)x^2 - 15\sqrt{\frac{1}{3}}\left(\left(4\,Ba^2b^3 - Aab^4\right)x^6 + 4\,Ba^4b - Aa^3b^2 +\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(27*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 -
A*a^2*b^3)*x^2 - 15*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*
a^3*b^2 + 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^
2*x^3 - a*b + 3*sqrt(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)
*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - 5*((4*B*a*b^2
- A*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2
/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A
*b^3)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)
*log(b*x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5), 1/54*(27
*B*a*b^4*x^8 + 24*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 15*(4*B*a^3*b^2 - A*a^2*b^3
)*x^2 - 30*sqrt(1/3)*((4*B*a^2*b^3 - A*a*b^4)*x^6 + 4*B*a^4*b - A*a^3*b^2 +
 2*(4*B*a^3*b^2 - A*a^2*b^3)*x^3)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*
(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - 5*((4*B*a*b^2 - A*b^3
)*x^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log
(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 10*((4*B*a*b^2 - A*b^3)*x
^6 + 4*B*a^3 - A*a^2*b + 2*(4*B*a^2*b - A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*
x - (-a*b^2)^(1/3)))/(a*b^7*x^6 + 2*a^2*b^6*x^3 + a^3*b^5)]
```

giac [A] time = 0.21, size = 210, normalized size = 0.95

$$\frac{Bx^2}{2\,b^3} - \frac{5\,\sqrt{3}\,(4\,Ba - Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\,(4\,Ba - Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,b^3} + \frac{5\left(4\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} - A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)
^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(1/3)*b^3) + 5/54*(4*B*a - A*b)*log(x^2 + x
*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(1/3)*b^3) + 5/27*(4*B*a*(-a/b)^(1/
3) - A*b*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/
18*(14*B*a*b*x^5 - 8*A*b^2*x^5 + 11*B*a^2*x^2 - 5*A*a*b*x^2)/((b*x^3 + a)^2
*b^3)
```

maple [A] time = 0.06, size = 275, normalized size = 1.24

$$-\frac{4A\,x^5}{9\left(b\,x^3+a\right)^2 b} + \frac{7Ba\,x^5}{9\left(b\,x^3+a\right)^2 b^2} - \frac{5Aa\,x^2}{18\left(b\,x^3+a\right)^2 b^2} + \frac{11B\,a^2x^2}{18\left(b\,x^3+a\right)^2 b^3} + \frac{B\,x^2}{2b^3} + \frac{5\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}} b^3} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 1/2*B/b^3*x^2-4/9/b/(b*x^3+a)^2*A*x^5+7/9/b^2/(b*x^3+a)^2*B*x^5*a-5/18/b^2/
(b*x^3+a)^2*A*x^2*a+11/18/b^3/(b*x^3+a)^2*B*x^2*a^2-5/27/b^3*A/(a/b)^(1/3)*
ln(x+(a/b)^(1/3))+5/54/b^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+
5/27/b^3*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+20/2
```

```
7/b^4*B*a/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-10/27/b^4*B*a/(a/b)^(1/3)*ln(x^2-(a
/b)^(1/3)*x+(a/b)^(2/3))-20/27/b^4*B*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/
2)*(2/(a/b)^(1/3)*x-1))
```

maxima [A] time = 1.32, size = 196, normalized size = 0.88

$$\frac{2\left(7\,Bab-4\,Ab^2\right)x^5+\left(11\,Ba^2-5\,Aab\right)x^2}{18\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}+\frac{Bx^2}{2\,b^3}-\frac{5\sqrt{3}\,(4\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\,(4\,Ba-Ab)\log\left(x^2-\right.}{54\,b^4\left(\frac{a}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(7*B*a*b - 4*A*b^2)*x^5 + (11*B*a^2 - 5*A*a*b)*x^2)/(b^5*x^6 + 2*a*
b^4*x^3 + a^2*b^3) + 1/2*B*x^2/b^3 - 5/27*sqrt(3)*(4*B*a - A*b)*arctan(1/3*
sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(1/3)) - 5/54*(4*B*a -
A*b)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(1/3)) + 5/27*(4*B*a
 - A*b)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(1/3))
```

mupad [B] time = 2.56, size = 187, normalized size = 0.84

$$\frac{x^2\left(\frac{11\,B\,a^2}{18}-\frac{5\,A\,a\,b}{18}\right)-x^5\left(\frac{4\,A\,b^2}{9}-\frac{7\,B\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{B\,x^2}{2\,b^3}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-4\,B\,a\right)}{27\,a^{1/3}\,b^{11/3}}-\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\right.}{27\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (x^2*((11*B*a^2)/18 - (5*A*a*b)/18) - x^5*((4*A*b^2)/9 - (7*B*a*b)/9))/(a^2
*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (B*x^2)/(2*b^3) - (5*log(b^(1/3)*x + a^(1/3
))*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3)) - (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(
1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3
)) + (5*log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1
/2)*(A*b - 4*B*a))/(27*a^(1/3)*b^(11/3))
```

sympy [A] time = 5.85, size = 162, normalized size = 0.73

$$\frac{Bx^2}{2b^3}+\frac{x^5\left(-8Ab^2+14Bab\right)+x^2\left(-5Aab+11Ba^2\right)}{18a^2b^3+36ab^4x^3+18b^5x^6}+\operatorname{RootSum}\left(19683t^3ab^{11}+125A^3b^3-1500A^2Bab^2+6000AB\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x**2/(2*b**3) + (x**5*(-8*A*b**2 + 14*B*a*b) + x**2*(-5*A*a*b + 11*B*a**2
))/(18*a**2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a*b
**11 + 125*A**3*b**3 - 1500*A**2*B*a*b**2 + 6000*A*B**2*a**2*b - 8000*B**3*
a**3, Lambda(_t, _t*log(729*_t**2*a*b**7/(25*A**2*b**2 - 200*A*B*a*b + 400*
B**2*a**2) + x)))
```

3.98 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=220

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}-2$$

```
[Out] -2/9*(A*b-7*B*a)*x/a/b^3+1/6*(A*b-B*a)*x^7/a/b/(b*x^3+a)^2+1/18*(A*b-7*B*a)
*x^4/a/b^2/(b*x^3+a)+2/27*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/b^(10/3
)-1/27*(A*b-7*B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/b^(10/
3)-2/27*(A*b-7*B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/
3)/b^(10/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 288, 321, 200, 31, 634, 617, 204, 628}

$$-\frac{(Ab-7aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{27a^{2/3}b^{10/3}}+\frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}}-\frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}}+\frac{}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (-2*(A*b - 7*a*B)*x)/(9*a*b^3) + ((A*b - a*B)*x^7)/(6*a*b*(a + b*x^3)^2) +
((A*b - 7*a*B)*x^4)/(18*a*b^2*(a + b*x^3)) - (2*(A*b - 7*a*B)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(2/3)*b^(10/3)) + (2*(A*
b - 7*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(2/3)*b^(10/3)) - ((A*b - 7*a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(2/3)*b^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}dx = \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(-Ab+7aB)\int \frac{x^6}{\left(a+bx^3\right)^2}dx}{6ab}$$

$$= \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} - \frac{(2(Ab-7aB))\int \frac{x^3}{a+bx^3}dx}{9ab^2}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{(2(Ab-7aB))\int \frac{1}{a+bx^3}dx}{9b^3}$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{(2(Ab-7aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{2/3}b^3} + \ldots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \ldots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} + \frac{2(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{2/3}b^{10/3}} - \ldots$$

$$= -\frac{2(Ab-7aB)x}{9ab^3} + \frac{(Ab-aB)x^7}{6ab\left(a+bx^3\right)^2} + \frac{(Ab-7aB)x^4}{18ab^2\left(a+bx^3\right)} - \frac{2(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{2/3}b^{10/3}} + \ldots$$

Mathematica [A] time = 0.27, size = 188, normalized size = 0.85

$$\frac{\frac{2(7aB-Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} + \frac{4(Ab-7aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{4\sqrt{3}(7aB-Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{3\sqrt[3]{b}x(7Ab-13aB)}{a+bx^3} + \frac{9a\sqrt[3]{b}x(Ab-\ldots}{\left(a+bx^3\right)^2}\ldots}{54b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (54*b^(1/3)*B*x + (9*a*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 - (3*b^(1/3)*(7
*A*b - 13*a*B)*x)/(a + b*x^3) + (4*Sqrt[3]*(-(A*b) + 7*a*B)*ArcTan[(1 - (2*
b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*(A*b - 7*a*B)*Log[a^(1/3) + b^(1
/3)*x])/a^(2/3) + (2*(-(A*b) + 7*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2])/a^(2/3))/(54*b^(10/3))
```

fricas [B] time = 0.88, size = 789, normalized size = 3.59

$$\left[\frac{54Ba^2b^3x^7 + 21\left(7Ba^3b^2 - Aa^2b^3\right)x^4 - 6\sqrt{\frac{1}{3}}\left(\left(7Ba^2b^3 - Aab^4\right)x^6 + 7Ba^4b - Aa^3b^2 + 2\left(7Ba^3b^2 - Aa^2b^3\right)x^3 \ldots\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(54*B*a^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 6*sqrt(1/3)*((
7*B*a^2*b^3 - A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2
*b^3)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^
2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*
b)^(1/3)/b))/(b*x^3 + a)) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b
+ 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(7*
B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*B*
a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), 1/54*(54*B*a
^2*b^3*x^7 + 21*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 12*sqrt(1/3)*((7*B*a^2*b^3
- A*a*b^4)*x^6 + 7*B*a^4*b - A*a^3*b^2 + 2*(7*B*a^3*b^2 - A*a^2*b^3)*x^3)*s
qrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)
*sqrt((a^2*b)^(1/3)/b)/a^2) + 2*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*
b + 2*(7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*
x + (a^2*b)^(1/3)*a) - 4*((7*B*a*b^2 - A*b^3)*x^6 + 7*B*a^3 - A*a^2*b + 2*(
7*B*a^2*b - A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(7*
B*a^4*b - A*a^3*b^2)*x)/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4)]
```

giac [A] time = 0.20, size = 187, normalized size = 0.85

$$\frac{2\sqrt{3}\,(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}+\frac{(7\,Ba-Ab)\log\left(x^{2}+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^{2}\right)^{\frac{2}{3}}b^{2}}+\frac{Bx}{b^{3}}+\frac{2\,(7\,Ba-Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log}{27\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(2/3)*b^2) + 1/27*(7*B*a - A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(2/3)*b^2) + B*x/b^3 + 2/27*(7*B*a - A*b)*(-a/b)^(1
/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^3) + 1/18*(13*B*a*b*x^4 - 7*A*b^2*x^4 +
 10*B*a^2*x - 4*A*a*b*x)/((b*x^3 + a)^2*b^3)
```

maple [A] time = 0.05, size = 268, normalized size = 1.22

$$-\frac{7A\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}b}+\frac{13Ba\,x^{4}}{18\left(b\,x^{3}+a\right)^{2}b^{2}}-\frac{2Aax}{9\left(b\,x^{3}+a\right)^{2}b^{2}}+\frac{5B\,a^{2}x}{9\left(b\,x^{3}+a\right)^{2}b^{3}}+\frac{2\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2A\ln\left(x+\right.}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] B/b^3*x-7/18/b/(b*x^3+a)^2*A*x^4+13/18/b^2/(b*x^3+a)^2*B*x^4*a-2/9/b^2/(b*x
^3+a)^2*a*A*x+5/9/b^3/(b*x^3+a)^2*a^2*B*x+2/27/b^3*A/(a/b)^(2/3)*ln(x+(a/b)
^(1/3))-1/27/b^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+2/27/b^3*A
/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-14/27/b^4*B*a/
(a/b)^(2/3)*ln(x+(a/b)^(1/3))+7/27/b^4*B*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x
```

```
+(a/b)^(2/3))-14/27/b^4*B*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)
^(1/3)*x-1))
```

maxima [A] time = 1.36, size = 191, normalized size = 0.87

$$\frac{\left(13\,Bab-7\,Ab^2\right)x^4+2\left(5\,Ba^2-2\,Aab\right)x}{18\left(b^5x^6+2\,ab^4x^3+a^2b^3\right)}+\frac{Bx}{b^3}-\frac{2\sqrt{3}\,(7\,Ba-Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,b^4\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{(7\,Ba-Ab)\log\left(x^2-\right.}{27\,b^4\left(\frac{a}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((13*B*a*b - 7*A*b^2)*x^4 + 2*(5*B*a^2 - 2*A*a*b)*x)/(b^5*x^6 + 2*a*b^
4*x^3 + a^2*b^3) + B*x/b^3 - 2/27*sqrt(3)*(7*B*a - A*b)*arctan(1/3*sqrt(3)*
(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) + 1/27*(7*B*a - A*b)*log
(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) - 2/27*(7*B*a - A*b)*
log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

mupad [B] time = 2.60, size = 183, normalized size = 0.83

$$\frac{B\,x}{b^3}-\frac{x^4\left(\frac{7\,A\,b^2}{18}-\frac{13\,B\,a\,b}{18}\right)-x\left(\frac{5\,B\,a^2}{9}-\frac{2\,A\,a\,b}{9}\right)}{a^2\,b^3+2\,a\,b^4\,x^3+b^5\,x^6}+\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b-7\,B\,a\right)}{27\,a^{2/3}\,b^{10/3}}-\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\right.}{27\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (B*x)/b^3 - (x^4*((7*A*b^2)/18 - (13*B*a*b)/18) - x*((5*B*a^2)/9 - (2*A*a*b
)/9))/(a^2*b^3 + b^5*x^6 + 2*a*b^4*x^3) + (2*log(b^(1/3)*x + a^(1/3))*(A*b
- 7*B*a))/(27*a^(2/3)*b^(10/3)) - (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x +
 a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a))/(27*a^(2/3)*b^(10/3)) + (2*
log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - 7*B*a))/(27*a^(2/3)*b^(10/3))
```

sympy [A] time = 3.39, size = 141, normalized size = 0.64

$$\frac{Bx}{b^3}+\frac{x^4\left(-7Ab^2+13Bab\right)+x\left(-4Aab+10Ba^2\right)}{18a^2b^3+36ab^4x^3+18b^5x^6}+\mathrm{RootSum}\left(19683t^3a^2b^{10}-8A^3b^3+168A^2Bab^2-1176AB^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] B*x/b**3 + (x**4*(-7*A*b**2 + 13*B*a*b) + x*(-4*A*a*b + 10*B*a**2))/(18*a**
2*b**3 + 36*a*b**4*x**3 + 18*b**5*x**6) + RootSum(19683*_t**3*a**2*b**10 -
8*A**3*b**3 + 168*A**2*B*a*b**2 - 1176*A*B**2*a**2*b + 2744*B**3*a**3, Lamb
da(_t, _t*log(-27*_t*a*b**3/(-2*A*b + 14*B*a) + x)))
```

3.99 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=201

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB}{18ab^2(}$$

```
[Out] 1/6*(A*b-B*a)*x^5/a/b/(b*x^3+a)^2-1/18*(A*b+5*B*a)*x^2/a/b^2/(b*x^3+a)-1/27
*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/b^(8/3)+1/54*(A*b+5*B*a)*ln(a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(4/3)/b^(8/3)-1/27*(A*b+5*B*a)*arctan(
1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/b^(8/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 288, 292, 31, 634, 617, 204, 628}

$$\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{4/3}b^{8/3}}-\frac{(5aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{4/3}b^{8/3}}-\frac{x^2(5aB}{18ab^2(}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^5)/(6*a*b*(a + b*x^3)^2) - ((A*b + 5*a*B)*x^2)/(18*a*b^2*(a
+ b*x^3)) - ((A*b + 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(9*Sqrt[3]*a^(4/3)*b^(8/3)) - ((A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/(
27*a^(4/3)*b^(8/3)) + ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2])/(54*a^(4/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}+\frac{(Ab+5aB)\int \frac{x^4}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}+\frac{(Ab+5aB)\int \frac{x}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}-\frac{(Ab+5aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{4/3}b^{7/3}}+\frac{(Ab+5aB)\int \frac{\sqrt[3]{a}+\cdots}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{\cdots}}\cdots}{27a^{4/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}-\frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}}+\frac{(Ab+5aB)\int \frac{-\sqrt[3]{\cdots}}{a^{2/3}-\cdots}\cdots}{54a^{4/3}\cdots}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}-\frac{(Ab+5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{4/3}b^{8/3}}+\frac{(Ab+5aB)\log\left(a^{2}\cdots\right.}{54\cdots}$$

$$= \frac{(Ab-aB)x^5}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+5aB)x^2}{18ab^2\left(a+bx^3\right)}-\frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\,\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}b^{8/3}}-\frac{(Ab+5aB)\log\left(\sqrt[3]{\cdots}\right.}{27a^{4/3}b^{8/3}}$$

Mathematica [A] time = 0.29, size = 181, normalized size = 0.90

$$\frac{\frac{(5aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}} - \frac{2(5aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}} - \frac{2\sqrt{3}(5aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{6b^{2/3}x^2(Ab-4aB)}{a\left(a+bx^3\right)} - \frac{9b^{2/3}x^2(Ab-aB)}{\left(a+bx^3\right)^2}}{54b^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(2/3)*(A*b - a*B)*x^2)/(a + b*x^3)^2 + (6*b^(2/3)*(A*b - 4*a*B)*x^2)
/(a*(a + b*x^3)) - (2*Sqrt[3]*(A*b + 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(4/3) - (2*(A*b + 5*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(4/3)
+ ((A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3))/(
54*b^(8/3))
```

fricas [B] time = 0.99, size = 756, normalized size = 3.76

$$\left[-\frac{6\left(4Ba^2b^3 - Aab^4\right)x^5 + 3\left(5Ba^3b^2 + Aa^2b^3\right)x^2 - 3\sqrt{\frac{1}{3}}\left(\left(5Ba^2b^3 + Aab^4\right)x^6 + 5Ba^4b + Aa^3b^2 + 2\left(5Ba^3b^2 + \ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(6*(4*B*a^2*b^3 - A*a*b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 3
*sqrt(1/3)*((5*B*a^2*b^3 + A*a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^
3*b^2 + A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqr
t(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3
)/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^
3 + A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a
*b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 +
A*a^2*b + 2*(5*B*a^2*b + A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1
/3)))/(a^2*b^6*x^6 + 2*a^3*b^5*x^3 + a^4*b^4), -1/54*(6*(4*B*a^2*b^3 - A*a*
b^4)*x^5 + 3*(5*B*a^3*b^2 + A*a^2*b^3)*x^2 - 6*sqrt(1/3)*((5*B*a^2*b^3 + A*
a*b^4)*x^6 + 5*B*a^4*b + A*a^3*b^2 + 2*(5*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(
-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)
^(1/3)/a)/b) - ((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b
+ A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^
(2/3)) + 2*((5*B*a*b^2 + A*b^3)*x^6 + 5*B*a^3 + A*a^2*b + 2*(5*B*a^2*b + A*
a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^2*b^6*x^6 + 2*a^3*
b^5*x^3 + a^4*b^4)]
```

giac [A] time = 0.20, size = 206, normalized size = 1.02

$$\frac{\sqrt{3}\,(5\,Ba + Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{(5\,Ba + Ab)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{1}{3}}ab^2} - \frac{\left(5\,Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}} + Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\ldots\right.}{27\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a*b^2) - 1/54*(5*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a*b^2) - 1/27*(5*B*a*(-a/b)^(1/3) + A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(8*B*a*
b*x^5 - 2*A*b^2*x^5 + 5*B*a^2*x^2 + A*a*b*x^2)/((b*x^3 + a)^2*a*b^2)
```

maple [A] time = 0.05, size = 241, normalized size = 1.20

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{5\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5B\ln}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(A*b-4*B*a)/a/b*x^5-1/18*(A*b+5*B*a)/b^2*x^2)/(b*x^3+a)^2-1/27/b^2/a/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-5/27/b^3/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1/5
4/b^2/a/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+5/54/b^3/(a/b)^(1/3
)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+5/27/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.27, size = 195, normalized size = 0.97

$$-\frac{2\left(4\,Bab-Ab^{2}\right)x^{5}+\left(5\,Ba^{2}+Aab\right)x^{2}}{18\left(ab^{4}x^{6}+2\,a^{2}b^{3}x^{3}+a^{3}b^{2}\right)}+\frac{\sqrt{3}\,(5\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(5\,Ba+Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{54\,ab^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*(2*(4*B*a*b - A*b^2)*x^5 + (5*B*a^2 + A*a*b)*x^2)/(a*b^4*x^6 + 2*a^2*
b^3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(5*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (
a/b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(1/3)) + 1/54*(5*B*a + A*b)*log(x^2 -
 x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(1/3)) - 1/27*(5*B*a + A*b)*log(
x + (a/b)^(1/3))/(a*b^3*(a/b)^(1/3))
```

mupad [B] time = 0.27, size = 175, normalized size = 0.87

$$-\frac{\frac{x^{2}\,(A\,b+5\,B\,a)}{18\,b^{2}}-\frac{x^{5}\,(A\,b-4\,B\,a)}{9\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(A\,b+5\,B\,a)}{27\,a^{4/3}\,b^{8/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(A\,b}{27\,a^{4/3}\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(A*
b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - (log(b^(1/3)*x + a^(1/3))*(A*b + 5*B*a))
```

```
/(27*a^(4/3)*b^(8/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a))/(27*a^(4/3)*b^(8/3)) - ((x^2*(A*b + 5*B
*a))/(18*b^2) - (x^5*(A*b - 4*B*a))/(9*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

sympy [A] time = 4.75, size = 155, normalized size = 0.77

$$\frac{x^5\left(2Ab^2-8Bab\right)+x^2\left(-Aab-5Ba^2\right)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\text{RootSum}\left(19683t^3a^4b^8+A^3b^3+15A^2Bab^2+75AB^2a^2b+125B^3a^3,\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(2*A*b**2 - 8*B*a*b) + x**2*(-A*a*b - 5*B*a**2))/(18*a**3*b**2 + 36*a
**2*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**4*b**8 + A**3*b**3
 + 15*A**2*B*a*b**2 + 75*A*B**2*a**2*b + 125*B**3*a**3, Lambda(_t, _t*log(7
29*_t**2*a**3*b**5/(A**2*b**2 + 10*A*B*a*b + 25*B**2*a**2) + x)))
```

3.100 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=199

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(}{9ab}$$

```
[Out] 1/6*(A*b-B*a)*x^4/a/b/(b*x^3+a)^2-1/9*(A*b+2*B*a)*x/a/b^2/(b*x^3+a)+1/27*(A
*b+2*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/b^(7/3)-1/54*(A*b+2*B*a)*ln(a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/b^(7/3)-1/27*(A*b+2*B*a)*arctan(1/3
*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {457, 288, 200, 31, 634, 617, 204, 628}

$$-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{5/3}b^{7/3}}+\frac{(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}}-\frac{(2aB+Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}b^{7/3}}-\frac{x(}{9ab}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^4)/(6*a*b*(a + b*x^3)^2) - ((A*b + 2*a*B)*x)/(9*a*b^2*(a + b
*x^3)) - ((A*b + 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(9*Sqrt[3]*a^(5/3)*b^(7/3)) + ((A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*
a^(5/3)*b^(7/3)) - ((A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)
*x^2])/(54*a^(5/3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+4aB)\int \frac{x^3}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\int \frac{1}{a+bx^3}\,dx}{9ab^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{5/3}b^2} + \frac{(Ab+2aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b\ldots}}{27a^{5/3}b^2}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}\ldots}{a^{2/3}-\sqrt[3]{a}\ldots}}{54a^{5/3}b^{7/3}}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{5/3}b^{7/3}} - \frac{(Ab+2aB)\log\left(a^{2/3}-\ldots\right.}{54a^{5/3}b\ldots}$$

$$= \frac{(Ab-aB)x^4}{6ab\left(a+bx^3\right)^2} - \frac{(Ab+2aB)x}{9ab^2\left(a+bx^3\right)} - \frac{(Ab+2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{5/3}b^{7/3}} + \frac{(Ab+2aB)\log\left(\sqrt[3]{a}+\ldots\right.}{27a^{5/3}b^{7/3}}$$

Mathematica [A] time = 0.24, size = 178, normalized size = 0.89

$$\frac{-\frac{(2aB+Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2(2aB+Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}(2aB+Ab)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}x(Ab-7aB)}{a(a+bx^3)}-\frac{9\sqrt[3]{b}x(Ab-aB}{(a+bx^3)^2}}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*b^(1/3)*(A*b - a*B)*x)/(a + b*x^3)^2 + (3*b^(1/3)*(A*b - 7*a*B)*x)/(a*
(a + b*x^3)) - (2*Sqrt[3]*(A*b + 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/
Sqrt[3]])/a^(5/3) + (2*(A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - ((
A*b + 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(54*b
^(7/3))
```

fricas [B] time = 1.01, size = 743, normalized size = 3.73

$$\left[-\frac{3\left(7Ba^3b^2-Aa^2b^3\right)x^4-3\sqrt{\frac{1}{3}}\left(\left(2Ba^2b^3+Aab^4\right)x^6+2Ba^4b+Aa^3b^2+2\left(2Ba^3b^2+Aa^2b^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)*x^4 - 3*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b
^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt(-(a
^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*
a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
 + a)) + ((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*
b^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) -
2*((2*B*a*b^2 + A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^
3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/
(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), -1/54*(3*(7*B*a^3*b^2 - A*a^2*b^3)
*x^4 - 6*sqrt(1/3)*((2*B*a^2*b^3 + A*a*b^4)*x^6 + 2*B*a^4*b + A*a^3*b^2 + 2
*(2*B*a^3*b^2 + A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(
a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + ((2*B*a*b^2
+ A*b^3)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/
3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) - 2*((2*B*a*b^2 + A*b^3
)*x^6 + 2*B*a^3 + A*a^2*b + 2*(2*B*a^2*b + A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(
a*b*x + (a^2*b)^(2/3)) + 6*(2*B*a^4*b + A*a^3*b^2)*x)/(a^3*b^5*x^6 + 2*a^4*
b^4*x^3 + a^5*b^3)]
```

giac [A] time = 0.19, size = 187, normalized size = 0.94

$$-\frac{\sqrt{3}\left(2Ba+Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2Ba+Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}ab}-\frac{\left(2Ba+Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x\right.\right.}{27a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a*b) - 1/54*(2*B*a + A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b) - 1/27*(2*B*a + A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^2*b^2) - 1/18*(7*B*a*b*x^4 - A*b^2*x^4 + 4*B*a^2*x
 + 2*A*a*b*x)/((b*x^3 + a)^2*a*b^2)
```

maple [A] time = 0.05, size = 239, normalized size = 1.20

$$\frac{\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}-\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^{2}}+\frac{2\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2B\ln\left(x\right.}{27\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(A*b-7*B*a)/a/b*x^4-1/9*(A*b+2*B*a)/b^2*x)/(b*x^3+a)^2+1/27/b^2/a/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+2/27/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-1/54/
b^2/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/27/b^3/(a/b)^(2/3)*
ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+1/27/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+2/27/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.25, size = 193, normalized size = 0.97

$$-\frac{\left(7\,Bab-Ab^{2}\right)x^{4}+2\left(2\,Ba^{2}+Aab\right)x}{18\left(ab^{4}x^{6}+2\,a^{2}b^{3}x^{3}+a^{3}b^{2}\right)}+\frac{\sqrt{3}\,(2\,Ba+Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,ab^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(2\,Ba+Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right.\right.}{54\,ab^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/18*((7*B*a*b - A*b^2)*x^4 + 2*(2*B*a^2 + A*a*b)*x)/(a*b^4*x^6 + 2*a^2*b^
3*x^3 + a^3*b^2) + 1/27*sqrt(3)*(2*B*a + A*b)*arctan(1/3*sqrt(3)*(2*x - (a/
b)^(1/3))/(a/b)^(1/3))/(a*b^3*(a/b)^(2/3)) - 1/54*(2*B*a + A*b)*log(x^2 - x
*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 1/27*(2*B*a + A*b)*log(x
+ (a/b)^(1/3))/(a*b^3*(a/b)^(2/3))
```

mupad [B] time = 2.56, size = 173, normalized size = 0.87

$$\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(A\,b+2\,B\,a\right)}{27\,a^{5/3}\,b^{7/3}}-\frac{\frac{x\,(A\,b+2\,B\,a)}{9\,b^{2}}-\frac{x^{4}\,(A\,b-7\,B\,a)}{18\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(A\,b+2\,B\,a\right.}{27\,a^{5/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (log(b^(1/3)*x + a^(1/3))*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) - ((x*(A*b +
2*B*a))/(9*b^2) - (x^4*(A*b - 7*B*a))/(18*a*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

```
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(A*b + 2*B*a))/(27*a^(5/3)*b^(7/3))
```

sympy [A] time = 3.25, size = 136, normalized size = 0.68

$$\frac{x^4\left(Ab^2-7Bab\right)+x\left(-2Aab-4Ba^2\right)}{18a^3b^2+36a^2b^3x^3+18ab^4x^6}+\text{RootSum}\left(19683t^3a^5b^7-A^3b^3-6A^2Bab^2-12AB^2a^2b-8B^3a^3,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(A*b**2 - 7*B*a*b) + x*(-2*A*a*b - 4*B*a**2))/(18*a**3*b**2 + 36*a**2
*b**3*x**3 + 18*a*b**4*x**6) + RootSum(19683*_t**3*a**5*b**7 - A**3*b**3 -
6*A**2*B*a*b**2 - 12*A*B**2*a**2*b - 8*B**3*a**3, Lambda(_t, _t*log(27*_t*a
**2*b**2/(A*b + 2*B*a) + x)))
```

3.101 $\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=201

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}}+\frac{x^2(aB+}{9a^2b\,(a}$$

```
[Out] 1/6*(A*b-B*a)*x^2/a/b/(b*x^3+a)^2+1/9*(2*A*b+B*a)*x^2/a^2/b/(b*x^3+a)-1/27*
(2*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/b^(5/3)+1/54*(2*A*b+B*a)*ln(a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/b^(5/3)-1/27*(2*A*b+B*a)*arctan(1
/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/b^(5/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {457, 290, 292, 31, 634, 617, 204, 628}

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}}-\frac{(aB+2Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}}+\frac{x^2(aB+}{9a^2b\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^2)/(6*a*b*(a + b*x^3)^2) + ((2*A*b + a*B)*x^2)/(9*a^2*b*(a +
 b*x^3)) - ((2*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(9*Sqrt[3]*a^(7/3)*b^(5/3)) - ((2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(2
7*a^(7/3)*b^(5/3)) + ((2*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(54*a^(7/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(4Ab+2aB)\int \frac{x}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} + \frac{(2Ab+aB)\int \frac{x}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{7/3}b^{4/3}} + \frac{(2Ab+aB)\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\cdots}\cdots}{27a^{7/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\int \frac{-\sqrt[3]{a}\cdots}{a^{2/3}-\sqrt[3]{a}\cdots}\cdots}{54a^{7/3}b^{5/\cdots}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{7/3}b^{5/3}} + \frac{(2Ab+aB)\log\left(a^{2/3}\cdots\right.}{54a^{7/\cdots}}$$

$$= \frac{(Ab-aB)x^2}{6ab\left(a+bx^3\right)^2} + \frac{(2Ab+aB)x^2}{9a^2b\left(a+bx^3\right)} - \frac{(2Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}b^{5/3}} - \frac{(2Ab+aB)\log\left(\sqrt[3]{a}\cdots\right.}{27a^{7/3}b^{5/3}}$$

Mathematica [A] time = 0.24, size = 178, normalized size = 0.89

$$\frac{(aB+2Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{4/3}b^{2/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{6\sqrt[3]{a}b^{2/3}x^2(aB+2Ab)}{a+bx^3}-2(aB+2Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-}{54a^{7/3}b^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(4/3)*b^(2/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1/3)*b^(2/3)
*(2*A*b + a*B)*x^2)/(a + b*x^3) - 2*Sqrt[3]*(2*A*b + a*B)*ArcTan[(1 - (2*b^
(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*(2*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] + (2*
A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(7/3)*b^(5
/3))
```

fricas [B] time = 0.99, size = 752, normalized size = 3.74

$$\left[\frac{6\left(Ba^2b^3+2\,Aab^4\right)x^5-3\left(Ba^3b^2-7\,Aa^2b^3\right)x^2+3\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3+2\,Aab^4\right)x^6+Ba^4b+2\,Aa^3b^2+2\left(Ba^3b^2+2\,A\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(6*(B*a^2*b^3 + 2*A*a*b^4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 3*
sqrt(1/3)*((B*a^2*b^3 + 2*A*a*b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b
^2 + 2*A*a^2*b^3)*x^3)*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x^3 - a*b + 3*sqrt
(1/3)*(a*b*x + 2*(-a*b^2)^(2/3)*x^2 + (-a*b^2)^(1/3)*a)*sqrt((-a*b^2)^(1/3)
/a) - 3*(-a*b^2)^(2/3)*x)/(b*x^3 + a)) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 +
 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*
b^2)^(1/3)*b*x + (-a*b^2)^(2/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A
*a^2*b + 2*(B*a^2*b + 2*A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/
3)))/(a^3*b^5*x^6 + 2*a^4*b^4*x^3 + a^5*b^3), 1/54*(6*(B*a^2*b^3 + 2*A*a*b^
4)*x^5 - 3*(B*a^3*b^2 - 7*A*a^2*b^3)*x^2 + 6*sqrt(1/3)*((B*a^2*b^3 + 2*A*a*
b^4)*x^6 + B*a^4*b + 2*A*a^3*b^2 + 2*(B*a^3*b^2 + 2*A*a^2*b^3)*x^3)*sqrt(-(
-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(
1/3)/a)/b) + ((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*
A*a*b^2)*x^3)*(-a*b^2)^(2/3)*log(b^2*x^2 + (-a*b^2)^(1/3)*b*x + (-a*b^2)^(2
/3)) - 2*((B*a*b^2 + 2*A*b^3)*x^6 + B*a^3 + 2*A*a^2*b + 2*(B*a^2*b + 2*A*a*
b^2)*x^3)*(-a*b^2)^(2/3)*log(b*x - (-a*b^2)^(1/3)))/(a^3*b^5*x^6 + 2*a^4*b^
4*x^3 + a^5*b^3)]
```

giac [A] time = 0.21, size = 207, normalized size = 1.03

$$\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b}-\frac{(Ba+2\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\,\left(-ab^2\right)^{\frac{1}{3}}\,a^2b}-\frac{\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right.}{27\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^2*b) - 1/54*(B*a + 2*A*b)*log(x^2 + x*(-a/b)^(1/3)
+ (-a/b)^(2/3))/((-a*b^2)^(1/3)*a^2*b) - 1/27*(B*a*(-a/b)^(1/3) + 2*A*b*(-a
/b)^(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(2*B*a*b*
x^5 + 4*A*b^2*x^5 - B*a^2*x^2 + 7*A*a*b*x^2)/((b*x^3 + a)^2*a^2*b)
```

maple [A] time = 0.05, size = 251, normalized size = 1.25

$$\frac{2\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}-\frac{2A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{A\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}b}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{B\ln}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/9*(2*A*b+B*a)/a^2*x^5+1/18*(7*A*b-B*a)/a/b*x^2)/(b*x^3+a)^2-2/27/a^2/b/(
a/b)^(1/3)*ln(x+(a/b)^(1/3))*A-1/27/a/b^2/(a/b)^(1/3)*ln(x+(a/b)^(1/3))*B+1
/27/a^2/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A+1/54/a/b^2/(a/b)^
(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+2/27/a^2/b*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2*3^(1/2)/(a/b)^(1/3)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.40, size = 195, normalized size = 0.97

$$\frac{2\left(Bab+2\,Ab^{2}\right)x^{5}-\left(Ba^{2}-7\,Aab\right)x^{2}}{18\left(a^{2}b^{3}x^{6}+2\,a^{3}b^{2}x^{3}+a^{4}b\right)}+\frac{\sqrt{3}\,(Ba+2\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba+2\,Ab)\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\right.}{54\,a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(2*(B*a*b + 2*A*b^2)*x^5 - (B*a^2 - 7*A*a*b)*x^2)/(a^2*b^3*x^6 + 2*a^3
*b^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 2*A*b)*arctan(1/3*sqrt(3)*(2*x - (a
/b)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3)) + 1/54*(B*a + 2*A*b)*log(x^2
- x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(1/3)) - 1/27*(B*a + 2*A*b)*l
og(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(1/3))
```

mupad [B] time = 0.27, size = 175, normalized size = 0.87

$$\frac{\frac{x^{5}\,(2\,A\,b+B\,a)}{9\,a^{2}}+\frac{x^{2}\,(7\,A\,b-B\,a)}{18\,a\,b}}{a^{2}+2\,a\,b\,x^{3}+b^{2}\,x^{6}}-\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(2\,A\,b+B\,a)}{27\,a^{7/3}\,b^{5/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(2\,A\,b}{27\,a^{7/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^5*(2*A*b + B*a))/(9*a^2) + (x^2*(7*A*b - B*a))/(18*a*b))/(a^2 + b^2*x^6
 + 2*a*b*x^3) - (log(b^(1/3)*x + a^(1/3))*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3
)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 - 1/2
```

```
)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)
*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*A*b + B*a))/(27*a^(7/3)*b^(5/3))
```

sympy [A] time = 2.03, size = 153, normalized size = 0.76

$$\frac{x^5\left(4Ab^2+2Bab\right)+x^2\left(7Aab-Ba^2\right)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\text{RootSum}\left(19683t^3a^7b^5+8A^3b^3+12A^2Bab^2+6AB^2a^2b+B^3a^3,\left(t\mapsto t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**5*(4*A*b**2 + 2*B*a*b) + x**2*(7*A*a*b - B*a**2))/(18*a**4*b + 36*a**3*
b**2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**7*b**5 + 8*A**3*b**
3 + 12*A**2*B*a*b**2 + 6*A*B**2*a**2*b + B**3*a**3, Lambda(_t, _t*log(729*_
t**2*a**5*b**3/(4*A**2*b**2 + 4*A*B*a*b + B**2*a**2) + x)))
```

3.102 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=197

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{x}{18\ldots}$$

```
[Out] 1/6*(A*b-B*a)*x/a/b/(b*x^3+a)^2+1/18*(5*A*b+B*a)*x/a^2/b/(b*x^3+a)+1/27*(5*
A*b+B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(8/3)/b^(4/3)-1/54*(5*A*b+B*a)*ln(a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/b^(4/3)-1/27*(5*A*b+B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/b^(4/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{8/3}b^{4/3}}+\frac{(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}}-\frac{(aB+5Ab)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}}+\frac{x}{18\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x)/(6*a*b*(a + b*x^3)^2) + ((5*A*b + a*B)*x)/(18*a^2*b*(a + b*
x^3)) - ((5*A*b + a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
9*Sqrt[3]*a^(8/3)*b^(4/3)) + ((5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a
^(8/3)*b^(4/3)) - ((5*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2])/(54*a^(8/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)\int \frac{1}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\int \frac{1}{a+bx^3}\,dx}{9a^2b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{27a^{8/3}b} + \frac{(5Ab+aB)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/\cdots}}}{27a^{8/3}b}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+\cdots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x\cdots}}{54a^{8/3}b^{4/3}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{8/3}b^{4/3}} - \frac{(5Ab+aB)\log\left(a^{2/3}-\cdots\right.}{54a^{8/3}b^{4/\cdots}}$$

$$= \frac{(Ab-aB)x}{6ab\left(a+bx^3\right)^2} + \frac{(5Ab+aB)x}{18a^2b\left(a+bx^3\right)} - \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{8/3}b^{4/3}} + \frac{(5Ab+aB)\log\left(\sqrt[3]{a}+\cdots\right.}{27a^{8/3}b^{4/3}}$$

Mathematica [A] time = 0.19, size = 175, normalized size = 0.89

$$\frac{-(aB+5Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)-\frac{9a^{5/3}\sqrt[3]{b}x(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{3a^{2/3}\sqrt[3]{b}x(aB+5Ab)}{a+bx^3}+2(aB+5Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{54a^{8/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^3,x]
```

```
[Out] ((-9*a^(5/3)*b^(1/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2/3)*b^(1/3)*(
5*A*b + a*B)*x)/(a + b*x^3) - 2*Sqrt[3]*(5*A*b + a*B)*ArcTan[(1 - (2*b^(1/3
)*x)/a^(1/3))/Sqrt[3]] + 2*(5*A*b + a*B)*Log[a^(1/3) + b^(1/3)*x] - (5*A*b
+ a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(8/3)*b^(4/3))
```

fricas [B] time = 1.01, size = 743, normalized size = 3.77

$$\left[\frac{3\left(Ba^3b^2+5\,Aa^2b^3\right)x^4+3\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3+5\,Aab^4\right)x^6+Ba^4b+5\,Aa^3b^2+2\left(Ba^3b^2+5\,Aa^2b^3\right)x^3\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\,\mathrm{lo}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x^4 + 3*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^
4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt(-(a^
2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3)*(2*a
*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(b*x^3
+ a)) - ((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b
^2)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2
*((B*a*b^2 + 5*A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3
)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(
a^4*b^4*x^6 + 2*a^5*b^3*x^3 + a^6*b^2), 1/54*(3*(B*a^3*b^2 + 5*A*a^2*b^3)*x
^4 + 6*sqrt(1/3)*((B*a^2*b^3 + 5*A*a*b^4)*x^6 + B*a^4*b + 5*A*a^3*b^2 + 2*(
B*a^3*b^2 + 5*A*a^2*b^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^
2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((B*a*b^2 + 5*
A*b^3)*x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)
*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((B*a*b^2 + 5*A*b^3)*
x^6 + B*a^3 + 5*A*a^2*b + 2*(B*a^2*b + 5*A*a*b^2)*x^3)*(a^2*b)^(2/3)*log(a*
b*x + (a^2*b)^(2/3)) - 6*(B*a^4*b - 4*A*a^3*b^2)*x)/(a^4*b^4*x^6 + 2*a^5*b^
3*x^3 + a^6*b^2)]
```

giac [A] time = 0.20, size = 180, normalized size = 0.91

$$-\frac{\sqrt{3}\left(Ba+5\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{\left(Ba+5\,Ab\right)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(-ab^2\right)^{\frac{2}{3}}a^2}-\frac{\left(Ba+5\,Ab\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x\right.\right.}{27\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^
(1/3))/((-a*b^2)^(2/3)*a^2) - 1/54*(B*a + 5*A*b)*log(x^2 + x*(-a/b)^(1/3) +
 (-a/b)^(2/3))/((-a*b^2)^(2/3)*a^2) - 1/27*(B*a + 5*A*b)*(-a/b)^(1/3)*log(a
bs(x - (-a/b)^(1/3)))/(a^3*b) + 1/18*(B*a*b*x^4 + 5*A*b^2*x^4 - 2*B*a^2*x +
 8*A*a*b*x)/((b*x^3 + a)^2*a^2*b)
```

maple [A] time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3}\,A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{5A\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}-\frac{5A\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{54\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2b}+\frac{\sqrt{3}\,B\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a\,b^2}+\frac{B\ln\left(\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] (1/18*(5*A*b+B*a)/a^2*x^4+1/9*(4*A*b-B*a)/a/b*x)/(b*x^3+a)^2+5/27/a^2/b/(a/
b)^(2/3)*ln(x+(a/b)^(1/3))*A+1/27/a/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*B-5/5
4/a^2/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*A-1/54/a/b^2/(a/b)^(2
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*B+5/27/a^2/b/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*A+1/27/a/b^2/(a/b)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*B
```

maxima [A] time = 1.37, size = 192, normalized size = 0.97

$$\frac{\left(Bab+5\,Ab^2\right)x^4-2\left(Ba^2-4\,Aab\right)x}{18\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}+\frac{\sqrt{3}\,(Ba+5\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{(Ba+5\,Ab)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)\right.}{54\,a^2b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*((B*a*b + 5*A*b^2)*x^4 - 2*(B*a^2 - 4*A*a*b)*x)/(a^2*b^3*x^6 + 2*a^3*b
^2*x^3 + a^4*b) + 1/27*sqrt(3)*(B*a + 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b
)^(1/3))/(a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3)) - 1/54*(B*a + 5*A*b)*log(x^2 -
x*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b^2*(a/b)^(2/3)) + 1/27*(B*a + 5*A*b)*log
(x + (a/b)^(1/3))/(a^2*b^2*(a/b)^(2/3))
```

mupad [B] time = 0.26, size = 173, normalized size = 0.88

$$\frac{\frac{x^4\,(5\,A\,b+B\,a)}{18\,a^2}+\frac{x\,(4\,A\,b-B\,a)}{9\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(5\,A\,b+B\,a)}{27\,a^{8/3}\,b^{4/3}}-\frac{\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,(5\,A\,b+B\,a}{27\,a^{8/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^3,x)
```

```
[Out] ((x^4*(5*A*b + B*a))/(18*a^2) + (x*(4*A*b - B*a))/(9*a*b))/(a^2 + b^2*x^6 +
 2*a*b*x^3) + (log(b^(1/3)*x + a^(1/3))*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
 - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
```

```
(5*A*b + B*a))/(27*a^(8/3)*b^(4/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x
 - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a))/(27*a^(8/3)*b^(4/3))
```

sympy [A] time = 1.51, size = 133, normalized size = 0.68

$$\frac{x^4\left(5Ab^2+Bab\right)+x\left(8Aab-2Ba^2\right)}{18a^4b+36a^3b^2x^3+18a^2b^3x^6}+\mathrm{RootSum}\left(19683t^3a^8b^4-125A^3b^3-75A^2Bab^2-15AB^2a^2b-B^3a^3,\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] (x**4*(5*A*b**2 + B*a*b) + x*(8*A*a*b - 2*B*a**2))/(18*a**4*b + 36*a**3*b**
2*x**3 + 18*a**2*b**3*x**6) + RootSum(19683*_t**3*a**8*b**4 - 125*A**3*b**3
 - 75*A**2*B*a*b**2 - 15*A*B**2*a**2*b - B**3*a**3, Lambda(_t, _t*log(27*_t
*a**3*b/(5*A*b + B*a) + x)))
```

3.103 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=227

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}b^{2/3}}-\frac{2(7\ldots}{9\ldots}$$

```
[Out] -2/9*(7*A*b-B*a)/a^3/b/x+1/6*(A*b-B*a)/a/b/x/(b*x^3+a)^2+1/18*(7*A*b-B*a)/a
^2/b/x/(b*x^3+a)+2/27*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/b^(2/3)-1/
27*(7*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(10/3)/b^(2/3)+2
/27*(7*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/
b^(2/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{(7Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{10/3}b^{2/3}}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{10/3}b^{2/3}}+\frac{\ldots}{18a\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] (-2*(7*A*b - a*B))/(9*a^3*b*x) + (A*b - a*B)/(6*a*b*x*(a + b*x^3)^2) + (7*A
*b - a*B)/(18*a^2*b*x*(a + b*x^3)) + (2*(7*A*b - a*B)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(10/3)*b^(2/3)) + (2*(7*A*b - a*
B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(10/3)*b^(2/3)) - ((7*A*b - a*B)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(10/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^3}dx = \frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{(7Ab-aB)\int \frac{1}{x^2\left(a+bx^3\right)^2}dx}{6ab}$$

$$= \frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{(2(7Ab-aB))\int \frac{1}{x^2\left(a+bx^3\right)}dx}{9a^2b}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}-\frac{(2(7Ab-aB))\int \frac{x}{a+bx^3}dx}{9a^3}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{(2(7Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}dx}{27a^{10/3}\sqrt[3]{b}}-\frac{(2(}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}-\frac{(}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{10/3}b^{2/3}}-\frac{(}{}$$

$$= -\frac{2(7Ab-aB)}{9a^3bx}+\frac{Ab-aB}{6abx\left(a+bx^3\right)^2}+\frac{7Ab-aB}{18a^2bx\left(a+bx^3\right)}+\frac{2(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{10/3}b^{2/3}}+\frac{2}{}$$

Mathematica [A] time = 0.28, size = 193, normalized size = 0.85

$$\frac{\frac{2(aB-7Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}}+\frac{9a^{4/3}x^2(aB-Ab)}{\left(a+bx^3\right)^2}+\frac{4(7Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}}+\frac{4\sqrt{3}(7Ab-aB)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}}+\frac{6\sqrt[3]{a}x^2(2aB-5Ab)}{a+bx^3}}{54a^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(1/3)*A)/x + (9*a^(4/3)*(-(A*b) + a*B)*x^2)/(a + b*x^3)^2 + (6*a^(1
/3)*(-5*A*b + 2*a*B)*x^2)/(a + b*x^3) + (4*Sqrt[3]*(7*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) + (4*(7*A*b - a*B)*Log[a^(1/3) +
 b^(1/3)*x])/b^(2/3) + (2*(-7*A*b + a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2])/b^(2/3))/(54*a^(10/3))
```

fricas [B] time = 0.95, size = 776, normalized size = 3.42

$$\left[\frac{12\left(Ba^2b^3-7Aab^4\right)x^6-54Aa^3b^2+21\left(Ba^3b^2-7Aa^2b^3\right)x^3-6\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3-7Aab^4\right)x^7+2\left(Ba^3b^2-7Aa^2b^3\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*
a^2*b^3)*x^3 - 6*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*
A*a^2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt(-(a*b^2)^(1/3)/a)*log((2*b
^2*x^3 - a*b - 3*sqrt(1/3)*(a*b*x + 2*(a*b^2)^(2/3)*x^2 - (a*b^2)^(1/3)*a)*
sqrt(-(a*b^2)^(1/3)/a) - 3*(a*b^2)^(2/3)*x)/(b*x^3 + a)) + 2*((B*a*b^2 - 7*
A*b^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(
2/3)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x), 1/54
*(12*(B*a^2*b^3 - 7*A*a*b^4)*x^6 - 54*A*a^3*b^2 + 21*(B*a^3*b^2 - 7*A*a^2*b
^3)*x^3 - 12*sqrt(1/3)*((B*a^2*b^3 - 7*A*a*b^4)*x^7 + 2*(B*a^3*b^2 - 7*A*a^
2*b^3)*x^4 + (B*a^4*b - 7*A*a^3*b^2)*x)*sqrt((a*b^2)^(1/3)/a)*arctan(-sqrt(
1/3)*(2*b*x - (a*b^2)^(1/3))*sqrt((a*b^2)^(1/3)/a)/b) + 2*((B*a*b^2 - 7*A*b
^3)*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3
)*log(b^2*x^2 - (a*b^2)^(1/3)*b*x + (a*b^2)^(2/3)) - 4*((B*a*b^2 - 7*A*b^3)
*x^7 + 2*(B*a^2*b - 7*A*a*b^2)*x^4 + (B*a^3 - 7*A*a^2*b)*x)*(a*b^2)^(2/3)*l
og(b*x + (a*b^2)^(1/3)))/(a^4*b^4*x^7 + 2*a^5*b^3*x^4 + a^6*b^2*x)]
```

giac [A] time = 0.20, size = 204, normalized size = 0.90

$$\frac{2\sqrt{3}\,(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3}-\frac{(Ba-7\,Ab)\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{27\left(-ab^2\right)^{\frac{1}{3}}a^3}-\frac{2\left(Ba\left(-\frac{a}{b}\right)^{\frac{1}{3}}-7\,Ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(
1/3))/((-a*b^2)^(1/3)*a^3) - 1/27*(B*a - 7*A*b)*log(x^2 + x*(-a/b)^(1/3) +
(-a/b)^(2/3))/((-a*b^2)^(1/3)*a^3) - 2/27*(B*a*(-a/b)^(1/3) - 7*A*b*(-a/b)^
(1/3))*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 - A/(a^3*x) + 1/18*(4*B*
a*b*x^5 - 10*A*b^2*x^5 + 7*B*a^2*x^2 - 13*A*a*b*x^2)/((b*x^3 + a)^2*a^3)
```

maple [A] time = 0.06, size = 281, normalized size = 1.24

$$-\frac{5A\,b^2x^5}{9\left(b\,x^3+a\right)^2a^3}+\frac{2Bb\,x^5}{9\left(b\,x^3+a\right)^2a^2}-\frac{13Ab\,x^2}{18\left(b\,x^3+a\right)^2a^2}+\frac{7B\,x^2}{18\left(b\,x^3+a\right)^2a}-\frac{14\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{14A\ln}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^3,x)
```

```
[Out] -5/9/a^3/(b*x^3+a)^2*A*x^5*b^2+2/9/a^2/(b*x^3+a)^2*B*x^5*b-13/18/a^2/(b*x^3
+a)^2*A*x^2*b+7/18/a/(b*x^3+a)^2*B*x^2+14/27/a^3*A/(a/b)^(1/3)*ln(x+(a/b)^(
1/3))-7/27/a^3*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*A*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-2/27/a^2*B/b/(a
/b)^(1/3)*ln(x+(a/b)^(1/3))+1/27/a^2*B/b/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(
```

```
a/b)^(2/3))+2/27/a^2*B*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1
/3)*x-1))-A/a^3/x
```

maxima [A] time = 1.41, size = 199, normalized size = 0.88

$$\frac{4\left(Bab-7\,Ab^2\right)x^6+7\left(Ba^2-7\,Aab\right)x^3-18\,Aa^2}{18\left(a^3b^2x^7+2\,a^4bx^4+a^5x\right)}+\frac{2\,\sqrt{3}\,(Ba-7\,Ab)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^3b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{(Ba-7\,Ab)\log\left(x^2-\right.}{27\,a^3b\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(4*(B*a*b - 7*A*b^2)*x^6 + 7*(B*a^2 - 7*A*a*b)*x^3 - 18*A*a^2)/(a^3*b^
2*x^7 + 2*a^4*b*x^4 + a^5*x) + 2/27*sqrt(3)*(B*a - 7*A*b)*arctan(1/3*sqrt(3
)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(1/3)) + 1/27*(B*a - 7*A*b)
*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(1/3)) - 2/27*(B*a - 7
*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(1/3))
```

mupad [B] time = 2.60, size = 185, normalized size = 0.81

$$\frac{2\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(7\,A\,b-B\,a\right)}{27\,a^{10/3}\,b^{2/3}}-\frac{\frac{A}{a}+\frac{7\,x^3\,\left(7\,A\,b-B\,a\right)}{18\,a^2}+\frac{2\,b\,x^6\,\left(7\,A\,b-B\,a\right)}{9\,a^3}}{a^2\,x+2\,a\,b\,x^4+b^2\,x^7}+\frac{2\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1}{2}\right.}{27\,a^{10/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^3),x)
```

```
[Out] (2*log(b^(1/3)*x + a^(1/3))*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (A/a + (
7*x^3*(7*A*b - B*a))/(18*a^2) + (2*b*x^6*(7*A*b - B*a))/(9*a^3))/(a^2*x + b
^2*x^7 + 2*a*b*x^4) + (2*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((
3^(1/2)*1i)/2 - 1/2)*(7*A*b - B*a))/(27*a^(10/3)*b^(2/3)) - (2*log(3^(1/2)*
a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*a))/(
27*a^(10/3)*b^(2/3))
```

sympy [A] time = 1.90, size = 162, normalized size = 0.71

$$\frac{-18Aa^2+x^6\left(-28Ab^2+4Bab\right)+x^3\left(-49Aab+7Ba^2\right)}{18a^5x+36a^4bx^4+18a^3b^2x^7}+\mathrm{RootSum}\left(19683t^3a^{10}b^2-2744A^3b^3+1176A^2Bab^2-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**3,x)
```

```
[Out] (-18*A*a**2 + x**6*(-28*A*b**2 + 4*B*a*b) + x**3*(-49*A*a*b + 7*B*a**2))/(1
8*a**5*x + 36*a**4*b*x**4 + 18*a**3*b**2*x**7) + RootSum(19683*_t**3*a**10*
b**2 - 2744*A**3*b**3 + 1176*A**2*B*a*b**2 - 168*A*B**2*a**2*b + 8*B**3*a**
3, Lambda(_t, _t*log(729*_t**2*a**7*b/(196*A**2*b**2 - 56*A*B*a*b + 4*B**2*
a**2) + x)))
```

3.104 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=227

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+\frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{11/3}\sqrt[3]{b}}-5$$

```
[Out] -5/18*(4*A*b-B*a)/a^3/b/x^2+1/6*(A*b-B*a)/a/b/x^2/(b*x^3+a)^2+1/9*(4*A*b-B*
a)/a^2/b/x^2/(b*x^3+a)-5/27*(4*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(11/3)/b^(1
/3)+5/54*(4*A*b-B*a)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(11/3)/b^(
1/3)+5/27*(4*A*b-B*a)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(
11/3)/b^(1/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{54a^{11/3}\sqrt[3]{b}}+\frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)}-\frac{5(4Ab-aB)}{18a^3bx^2}-\frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{27a^{11/3}\sqrt[3]{b}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] (-5*(4*A*b - a*B))/(18*a^3*b*x^2) + (A*b - a*B)/(6*a*b*x^2*(a + b*x^3)^2) +
 (4*A*b - a*B)/(9*a^2*b*x^2*(a + b*x^3)) + (5*(4*A*b - a*B)*ArcTan[(a^(1/3)
 - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(11/3)*b^(1/3)) - (5*(4*A*
b - a*B)*Log[a^(1/3) + b^(1/3)*x])/(27*a^(11/3)*b^(1/3)) + (5*(4*A*b - a*B)
*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(54*a^(11/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{(8Ab-2aB)\int \frac{1}{x^3\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{(5(4Ab-aB))\int \frac{1}{x^3\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{a+bx^3}\,dx}{9a^3}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{(5(4Ab-aB))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{27a^{11/3}} -$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} - \frac{5(4Ab-aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{11/3}\sqrt[3]{b}}$$

$$= -\frac{5(4Ab-aB)}{18a^3bx^2} + \frac{Ab-aB}{6abx^2\left(a+bx^3\right)^2} + \frac{4Ab-aB}{9a^2bx^2\left(a+bx^3\right)} + \frac{5(4Ab-aB)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{11/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.26, size = 189, normalized size = 0.83

$$\frac{\frac{5(4Ab-aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{\sqrt[3]{b}} + \frac{9a^{5/3}x(aB-Ab)}{\left(a+bx^3\right)^2} + \frac{3a^{2/3}x(5aB-11Ab)}{a+bx^3} - \frac{27a^{2/3}A}{x^2} + \frac{10(aB-4Ab)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt[3]{b}} + \frac{10\sqrt{3}(4Ab-aB)}{}}{54a^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(2/3)*A)/x^2 + (9*a^(5/3)*(-(A*b) + a*B)*x)/(a + b*x^3)^2 + (3*a^(2
/3)*(-11*A*b + 5*a*B)*x)/(a + b*x^3) + (10*Sqrt[3]*(4*A*b - a*B)*ArcTan[(1
- (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) + (10*(-4*A*b + a*B)*Log[a^(1/3)
 + b^(1/3)*x])/b^(1/3) + (5*(4*A*b - a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/b^(1/3))/(54*a^(11/3))
```

fricas [B] time = 1.06, size = 812, normalized size = 3.58

$$\left[\frac{15\left(Ba^3b^2-4\,Aa^2b^3\right)x^6-27\,Aa^4b+24\left(Ba^4b-4\,Aa^3b^2\right)x^3-15\sqrt{\frac{1}{3}}\left(\left(Ba^2b^3-4\,Aab^4\right)x^8+2\left(Ba^3b^2-4\,Aa^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a^4*b - 4*A*a^
3*b^2)*x^3 - 15*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*a^3*b^2 - 4*A
*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt((-a^2*b)^(1/3)/b)*log((2*
a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 5*((B*a*b^
2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(
-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 10*((B*a
*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^5 + (B*a^3 - 4*A*a^2*b)*x^2
)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))/(a^5*b^3*x^8 + 2*a^6*b^2*x^5
+ a^7*b*x^2), 1/54*(15*(B*a^3*b^2 - 4*A*a^2*b^3)*x^6 - 27*A*a^4*b + 24*(B*a
^4*b - 4*A*a^3*b^2)*x^3 + 30*sqrt(1/3)*((B*a^2*b^3 - 4*A*a*b^4)*x^8 + 2*(B*
a^3*b^2 - 4*A*a^2*b^3)*x^5 + (B*a^4*b - 4*A*a^3*b^2)*x^2)*sqrt(-(-a^2*b)^(1
/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2
*b)^(1/3)/b)/a^2) - 5*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)*x^
5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x
- (-a^2*b)^(1/3)*a) + 10*((B*a*b^2 - 4*A*b^3)*x^8 + 2*(B*a^2*b - 4*A*a*b^2)
*x^5 + (B*a^3 - 4*A*a^2*b)*x^2)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)))
/(a^5*b^3*x^8 + 2*a^6*b^2*x^5 + a^7*b*x^2)]
```

giac [A] time = 0.19, size = 209, normalized size = 0.92

$$-\frac{5\,(Ba-4\,Ab)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^4}+\frac{5\,\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}Ba-4\,\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^4b}+\frac{5\left(\left(-ab^2\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -5/27*(B*a - 4*A*b)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^4 + 5/27*sqrt
(3)*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqrt(3)*(2*x + (
-a/b)^(1/3))/(-a/b)^(1/3))/(a^4*b) + 5/54*((-a*b^2)^(1/3)*B*a - 4*(-a*b^2)^
(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b) + 1/18*(5*B*a*b
*x^6 - 20*A*b^2*x^6 + 8*B*a^2*x^3 - 32*A*a*b*x^3 - 9*A*a^2)/((b*x^4 + a*x)^
2*a^3)
```

maple [A] time = 0.05, size = 277, normalized size = 1.22

$$-\frac{11A\,b^2x^4}{18\left(b\,x^3+a\right)^2a^3}+\frac{5Bb\,x^4}{18\left(b\,x^3+a\right)^2a^2}-\frac{7Abx}{9\left(b\,x^3+a\right)^2a^2}+\frac{4Bx}{9\left(b\,x^3+a\right)^2a}-\frac{20\sqrt{3}\;A\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20A\ln\left(x\right.}{27\left(\frac{a}{b}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^3,x)
```

```
[Out] -11/18/a^3/(b*x^3+a)^2*A*x^4*b^2+5/18/a^2/(b*x^3+a)^2*B*x^4*b-7/9/a^2/(b*x^
3+a)^2*b*A*x+4/9/a/(b*x^3+a)^2*B*x-20/27/a^3*A/(a/b)^(2/3)*ln(x+(a/b)^(1/3)
)+10/27/a^3*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-20/27/a^3*A/(a/
b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+5/27/a^2*B/b/(a/b)
```

```
^(2/3)*ln(x+(a/b)^(1/3))-5/54/a^2*B/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b
)^(2/3))+5/27/a^2*B/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)
*x-1))-1/2*A/a^3/x^2
```

maxima [A] time = 1.20, size = 201, normalized size = 0.89

$$\frac{5\left(Bab-4\,Ab^{2}\right)x^{6}+8\left(Ba^{2}-4\,Aab\right)x^{3}-9\,Aa^{2}}{18\left(a^{3}b^{2}x^{8}+2\,a^{4}bx^{5}+a^{5}x^{2}\right)}+\frac{5\sqrt{3}\left(Ba-4\,Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27\,a^{3}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\left(Ba-4\,Ab\right)\log\left(x\right.}{54\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*(5*(B*a*b - 4*A*b^2)*x^6 + 8*(B*a^2 - 4*A*a*b)*x^3 - 9*A*a^2)/(a^3*b^2
*x^8 + 2*a^4*b*x^5 + a^5*x^2) + 5/27*sqrt(3)*(B*a - 4*A*b)*arctan(1/3*sqrt(
3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(a^3*b*(a/b)^(2/3)) - 5/54*(B*a - 4*A*b
)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*(a/b)^(2/3)) + 5/27*(B*a -
4*A*b)*log(x + (a/b)^(1/3))/(a^3*b*(a/b)^(2/3))
```

mupad [B] time = 2.58, size = 188, normalized size = 0.83

$$-\frac{\frac{A}{2\,a}+\frac{4\,x^3\,\left(4\,A\,b-B\,a\right)}{9\,a^2}+\frac{5\,b\,x^6\,\left(4\,A\,b-B\,a\right)}{18\,a^3}}{a^2\,x^2+2\,a\,b\,x^5+b^2\,x^8}-\frac{5\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,\left(4\,A\,b-B\,a\right)}{27\,a^{11/3}\,b^{1/3}}+\frac{5\,\ln\left(a^{1/3}-2\,b^{1/3}\,x+\sqrt{3}\,a^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2}+\right.}{27\,a^{11/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^3),x)
```

```
[Out] (5*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(
4*A*b - B*a))/(27*a^(11/3)*b^(1/3)) - (5*log(b^(1/3)*x + a^(1/3))*(4*A*b -
B*a))/(27*a^(11/3)*b^(1/3)) - (A/(2*a) + (4*x^3*(4*A*b - B*a))/(9*a^2) + (5
*b*x^6*(4*A*b - B*a))/(18*a^3))/(a^2*x^2 + b^2*x^8 + 2*a*b*x^5) - (5*log(3^
(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(4*A*b - B
*a))/(27*a^(11/3)*b^(1/3))
```

sympy [A] time = 1.73, size = 143, normalized size = 0.63

$$\frac{-9Aa^{2}+x^{6}\left(-20Ab^{2}+5Bab\right)+x^{3}\left(-32Aab+8Ba^{2}\right)}{18a^{5}x^{2}+36a^{4}bx^{5}+18a^{3}b^{2}x^{8}}+\operatorname{RootSum}\left(19683t^{3}a^{11}b+8000A^{3}b^{3}-6000A^{2}Bab^{2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**3,x)
```

```
[Out] (-9*A*a**2 + x**6*(-20*A*b**2 + 5*B*a*b) + x**3*(-32*A*a*b + 8*B*a**2))/(18
*a**5*x**2 + 36*a**4*b*x**5 + 18*a**3*b**2*x**8) + RootSum(19683*_t**3*a**1
1*b + 8000*A**3*b**3 - 6000*A**2*B*a*b**2 + 1500*A*B**2*a**2*b - 125*B**3*a
**3, Lambda(_t, _t*log(27*_t*a**4/(-20*A*b + 5*B*a) + x)))
```

3.105 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=246

$$\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{13/3}}-\frac{7\sqrt[3]{b}\,(5Ab-2aB)\tan^{-1}\left(\frac{\sqrt[3]{a}}{\cdots}\right.}{9\sqrt{3}\,a^{13/3}}$$

```
[Out] -7/36*(5*A*b-2*B*a)/a^3/b/x^4+7/9*(5*A*b-2*B*a)/a^4/x+1/6*(A*b-B*a)/a/b/x^4
/(b*x^3+a)^2+1/9*(5*A*b-2*B*a)/a^2/b/x^4/(b*x^3+a)-7/27*b^(1/3)*(5*A*b-2*B*
a)*ln(a^(1/3)+b^(1/3)*x)/a^(13/3)+7/54*b^(1/3)*(5*A*b-2*B*a)*ln(a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(13/3)-7/27*b^(1/3)*(5*A*b-2*B*a)*arctan(1/3*
(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(13/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {457, 290, 325, 292, 31, 634, 617, 204, 628}

$$\frac{7\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{54a^{13/3}}+\frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)}-\frac{7(5Ab-2aB)}{36a^3bx^4}+\frac{7(5Ab-2aB)}{9a^4x}-\frac{7\sqrt[3]{b}\,(5Ab-2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] (-7*(5*A*b - 2*a*B))/(36*a^3*b*x^4) + (7*(5*A*b - 2*a*B))/(9*a^4*x) + (A*b
- a*B)/(6*a*b*x^4*(a + b*x^3)^2) + (5*A*b - 2*a*B)/(9*a^2*b*x^4*(a + b*x^3)
) - (7*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1
/3))])/(9*Sqrt[3]*a^(13/3)) - (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(1/3) + b^(1
/3)*x])/(27*a^(13/3)) + (7*b^(1/3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x + b^(2/3)*x^2])/(54*a^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
```

```
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{(10Ab-4aB)\int \frac{1}{x^5\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7(5Ab-2aB))\int \frac{1}{x^5\left(a+bx^3\right)}\,dx}{9a^2b}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{(7(5Ab-2aB))\int \frac{1}{x^2\left(a+bx^3\right)}\,dx}{9a^3}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} + \frac{(7b(5Ab-2aB}{9a^4}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{\left(7b^{2/3}(5Ab-2\right.}{27}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a}{2}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a}{2}$$

$$= -\frac{7(5Ab-2aB)}{36a^3bx^4} + \frac{7(5Ab-2aB)}{9a^4x} + \frac{Ab-aB}{6abx^4\left(a+bx^3\right)^2} + \frac{5Ab-2aB}{9a^2bx^4\left(a+bx^3\right)} - \frac{7\sqrt[3]{b}\,(5Ab-2a}{9}$$

Mathematica [**A**] time = 0.27, size = 214, normalized size = 0.87

$$\frac{14\sqrt[3]{b}\,(5Ab-2aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right) - \frac{18a^{4/3}bx^2(aB-Ab)}{\left(a+bx^3\right)^2} - \frac{27a^{4/3}A}{x^4} - \frac{12\sqrt[3]{a}\,bx^2(5aB-8Ab)}{a+bx^3} - \frac{108\sqrt[3]{a}\,(aB-3Ab)}{x} + 2}{108a^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^3),x]
```

```
[Out] ((-27*a^(4/3)*A)/x^4 - (108*a^(1/3)*(-3*A*b + a*B))/x - (18*a^(4/3)*b*(-(A*
b) + a*B)*x^2)/(a + b*x^3)^2 - (12*a^(1/3)*b*(-8*A*b + 5*a*B)*x^2)/(a + b*x
^3) - 28*Sqrt[3]*b^(1/3)*(5*A*b - 2*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))
/Sqrt[3]] + 28*b^(1/3)*(-5*A*b + 2*a*B)*Log[a^(1/3) + b^(1/3)*x] + 14*b^(1/
3)*(5*A*b - 2*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(108*a^(
13/3))
```

fricas [**A**] time = 0.98, size = 366, normalized size = 1.49

$$-\frac{84\left(2\,Bab^2-5\,Ab^3\right)x^9+147\left(2\,Ba^2b-5\,Aab^2\right)x^6+27\,Aa^3+54\left(2\,Ba^3-5\,Aa^2b\right)x^3+28\,\sqrt{3}\left(\left(2\,Bab^2-5\,Ab^3\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(84*(2*B*a*b^2 - 5*A*b^3)*x^9 + 147*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 27
*A*a^3 + 54*(2*B*a^3 - 5*A*a^2*b)*x^3 + 28*sqrt(3)*((2*B*a*b^2 - 5*A*b^3)*x
^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/a)^(1/
3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*sqrt(3)) - 14*((2*B*a*b^2 - 5*A*
b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*(-b/
a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1/3)) + 28*((2*B*a*b^2 -
5*A*b^3)*x^10 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^7 + (2*B*a^3 - 5*A*a^2*b)*x^4)*
(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)))/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x
^4)
```

giac [**A**] time = 0.20, size = 254, normalized size = 1.03

$$\frac{7\left(2\,Bab\left(-\frac{a}{b}\right)^{\frac{1}{3}}-5\,Ab^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27\,a^{5}}+\frac{7\,\sqrt{3}\left(2\left(-ab^{2}\right)^{\frac{2}{3}}Ba-5\left(-ab^{2}\right)^{\frac{2}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2\,x\right.}{3\left(\right.}\right.}{27\,a^{5}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 7/27*(2*B*a*b*(-a/b)^(1/3) - 5*A*b^2*(-a/b)^(1/3))*(-a/b)^(1/3)*log(abs(x -
 (-a/b)^(1/3)))/a^5 + 7/27*sqrt(3)*(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)
*A*b)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^5*b) - 7/54*
(2*(-a*b^2)^(2/3)*B*a - 5*(-a*b^2)^(2/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-
a/b)^(2/3))/(a^5*b) - 1/18*(10*B*a*b^2*x^5 - 16*A*b^3*x^5 + 13*B*a^2*b*x^2
- 19*A*a*b^2*x^2)/((b*x^3 + a)^2*a^4) - 1/4*(4*B*a*x^3 - 12*A*b*x^3 + A*a)/
(a^4*x^4)
```

maple [**A**] time = 0.06, size = 299, normalized size = 1.22

$$\frac{8A\,b^{3}x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{4}}-\frac{5B\,b^{2}x^{5}}{9\left(b\,x^{3}+a\right)^{2}a^{3}}+\frac{19A\,b^{2}x^{2}}{18\left(b\,x^{3}+a\right)^{2}a^{3}}-\frac{13Bb\,x^{2}}{18\left(b\,x^{3}+a\right)^{2}a^{2}}+\frac{35\sqrt{3}\;Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{4}}-\frac{35Ab}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^3,x)
```

```
[Out] 8/9/a^4*b^3/(b*x^3+a)^2*x^5*A-5/9/a^3*b^2/(b*x^3+a)^2*x^5*B+19/18/a^3*b^2/(
b*x^3+a)^2*A*x^2-13/18/a^2*b/(b*x^3+a)^2*B*x^2-35/27/a^4*b*A/(a/b)^(1/3)*ln
(x+(a/b)^(1/3))+35/54/a^4*b*A/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))
+35/27/a^4*b*A*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+
14/27/a^3*B/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-7/27/a^3*B/(a/b)^(1/3)*ln(x^2-(a/
b)^(1/3)*x+(a/b)^(2/3))-14/27/a^3*B*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*
(2/(a/b)^(1/3)*x-1))-1/4/a^3*A/x^4+3/a^4/x*A*b-B/a^3/x
```

maxima [**A**] time = 1.42, size = 221, normalized size = 0.90

$$-\frac{28\left(2\,Bab^{2}-5\,Ab^{3}\right)x^{9}+49\left(2\,Ba^{2}b-5\,Aab^{2}\right)x^{6}+9\,Aa^{3}+18\left(2\,Ba^{3}-5\,Aa^{2}b\right)x^{3}}{36\left(a^{4}b^{2}x^{10}+2\,a^{5}bx^{7}+a^{6}x^{4}\right)}-\frac{7\,\sqrt{3}\left(2\,Ba-5\,Ab\right)\mathrm{arctan}}{27\,a^{4}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*(28*(2*B*a*b^2 - 5*A*b^3)*x^9 + 49*(2*B*a^2*b - 5*A*a*b^2)*x^6 + 9*A*
a^3 + 18*(2*B*a^3 - 5*A*a^2*b)*x^3)/(a^4*b^2*x^10 + 2*a^5*b*x^7 + a^6*x^4)
- 7/27*sqrt(3)*(2*B*a - 5*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)
^(1/3))/(a^4*(a/b)^(1/3)) - 7/54*(2*B*a - 5*A*b)*log(x^2 - x*(a/b)^(1/3) +
(a/b)^(2/3))/(a^4*(a/b)^(1/3)) + 7/27*(2*B*a - 5*A*b)*log(x + (a/b)^(1/3))/
(a^4*(a/b)^(1/3))
```

mupad [B] time = 2.64, size = 240, normalized size = 0.98

$$\frac{\frac{x^3\,(5\,A\,b-2\,B\,a)}{2\,a^2}-\frac{A}{4\,a}+\frac{7\,b^2\,x^9\,(5\,A\,b-2\,B\,a)}{9\,a^4}+\frac{49\,b\,x^6\,(5\,A\,b-2\,B\,a)}{36\,a^3}}{a^2\,x^4+2\,a\,b\,x^7+b^2\,x^{10}}+\frac{7\,{(-b)}^{1/3}\,\ln\left(a^{1/3}\,{(-b)}^{8/3}+b^3\,x\right)\,(5\,A\,b-2\,B\,a)}{27\,a^{13/3}}+\frac{7\,{(-b)}^{1/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(5*A*b - 2*B*a))/(2*a^2) - A/(4*a) + (7*b^2*x^9*(5*A*b - 2*B*a))/(9*a
^4) + (49*b*x^6*(5*A*b - 2*B*a))/(36*a^3))/(a^2*x^4 + b^2*x^10 + 2*a*b*x^7)
 + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) + b^3*x)*(5*A*b - 2*B*a))/(27*a^(13
/3)) + (7*(-b)^(1/3)*log(a^(1/3)*(-b)^(8/3) - 2*b^3*x + 3^(1/2)*a^(1/3)*(-b
)^(8/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3)) - (7*(-b)
^(1/3)*log(2*b^3*x - a^(1/3)*(-b)^(8/3) + 3^(1/2)*a^(1/3)*(-b)^(8/3)*1i)*((
3^(1/2)*1i)/2 + 1/2)*(5*A*b - 2*B*a))/(27*a^(13/3))
```

sympy [A] time = 1.91, size = 189, normalized size = 0.77

$$\operatorname{RootSum}\left(19683t^3a^{13}+42875A^3b^4-51450A^2Bab^3+20580AB^2a^2b^2-2744B^3a^3b,\left(t\mapsto t\log\left(\frac{7}{1225A^2b^3-98}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**13 + 42875*A**3*b**4 - 51450*A**2*B*a*b**3 + 20580*A
*B**2*a**2*b**2 - 2744*B**3*a**3*b, Lambda(_t, _t*log(729*_t**2*a**9/(1225*
A**2*b**3 - 980*A*B*a*b**2 + 196*B**2*a**2*b) + x))) + (-9*A*a**3 + x**9*(1
40*A*b**3 - 56*B*a*b**2) + x**6*(245*A*a*b**2 - 98*B*a**2*b) + x**3*(90*A*a
**2*b - 36*B*a**3))/(36*a**6*x**4 + 72*a**5*b*x**7 + 36*a**4*b**2*x**10)
```

3.106 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\,\text{ta}}{9\sqrt{3}\,a^{1}}$$

```
[Out] -4/45*(11*A*b-5*B*a)/a^3/b/x^5+2/9*(11*A*b-5*B*a)/a^4/x^2+1/6*(A*b-B*a)/a/b
/x^5/(b*x^3+a)^2+1/18*(11*A*b-5*B*a)/a^2/b/x^5/(b*x^3+a)+4/27*b^(2/3)*(11*A
*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x)/a^(14/3)-2/27*b^(2/3)*(11*A*b-5*B*a)*ln(a^(
2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(14/3)-4/27*b^(2/3)*(11*A*b-5*B*a)*ar
ctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(14/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {457, 290, 325, 200, 31, 634, 617, 204, 628}

$$-\frac{2b^{2/3}(11Ab-5aB)\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{27a^{14/3}}+\frac{4b^{2/3}(11Ab-5aB)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{27a^{14/3}}-\frac{4b^{2/3}(11Ab-5aB)\,\text{ta}}{9\sqrt{3}\,a^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] (-4*(11*A*b - 5*a*B))/(45*a^3*b*x^5) + (2*(11*A*b - 5*a*B))/(9*a^4*x^2) + (
A*b - a*B)/(6*a*b*x^5*(a + b*x^3)^2) + (11*A*b - 5*a*B)/(18*a^2*b*x^5*(a +
b*x^3)) - (4*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(9*Sqrt[3]*a^(14/3)) + (4*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3
) + b^(1/3)*x])/(27*a^(14/3)) - (2*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(2/3) - a
^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(27*a^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
```

```
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^3}dx = \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{(11Ab-5aB)\int \frac{1}{x^6\left(a+bx^3\right)^2}dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4(11Ab-5aB))\int \frac{1}{x^6\left(a+bx^3\right)}dx}{9a^2b}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{(4(11Ab-5aB))\int \frac{1}{x^3\left(a+bx^3\right)}}{9a^3}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{(4b(11Ab}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} + \frac{4b^{2/3}(11}{}$$

$$= -\frac{4(11Ab-5aB)}{45a^3bx^5} + \frac{2(11Ab-5aB)}{9a^4x^2} + \frac{Ab-aB}{6abx^5\left(a+bx^3\right)^2} + \frac{11Ab-5aB}{18a^2bx^5\left(a+bx^3\right)} - \frac{4b^{2/3}(11}{}$$

Mathematica [A] time = 0.32, size = 210, normalized size = 0.85

$$\frac{20b^{2/3}(5aB-11Ab)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{45a^{5/3}bx(aB-Ab)}{\left(a+bx^3\right)^2} - \frac{15a^{2/3}bx(11aB-17Ab)}{a+bx^3} - \frac{135a^{2/3}(aB-3Ab)}{x^2} - \frac{54a^{5/3}A}{x^5}}{270a^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^3),x]
```

```
[Out] ((-54*a^(5/3)*A)/x^5 - (135*a^(2/3)*(-3*A*b + a*B))/x^2 - (45*a^(5/3)*b*(-(
A*b) + a*B)*x)/(a + b*x^3)^2 - (15*a^(2/3)*b*(-17*A*b + 11*a*B)*x)/(a + b*x
^3) - 40*Sqrt[3]*b^(2/3)*(11*A*b - 5*a*B)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]] + 40*b^(2/3)*(11*A*b - 5*a*B)*Log[a^(1/3) + b^(1/3)*x] + 20*b^(2
/3)*(-11*A*b + 5*a*B)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(270*
a^(14/3))
```

fricas [A] time = 1.01, size = 384, normalized size = 1.56

$$-\frac{60\left(5\,Bab^2-11\,Ab^3\right)x^9+96\left(5\,Ba^2b-11\,Aab^2\right)x^6+54\,Aa^3+27\left(5\,Ba^3-11\,Aa^2b\right)x^3+40\sqrt{3}\left(\left(5\,Bab^2-11\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/270*(60*(5*B*a*b^2 - 11*A*b^3)*x^9 + 96*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 5
4*A*a^3 + 27*(5*B*a^3 - 11*A*a^2*b)*x^3 + 40*sqrt(3)*((5*B*a*b^2 - 11*A*b^3
)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/
a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(
(5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b^2)*x^8 + (5*B*a^3 - 1
1*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(
b^2/a^2)^(2/3)) + 40*((5*B*a*b^2 - 11*A*b^3)*x^11 + 2*(5*B*a^2*b - 11*A*a*b
^2)*x^8 + (5*B*a^3 - 11*A*a^2*b)*x^5)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)))/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^5)
```

giac [A] time = 0.19, size = 229, normalized size = 0.93

$$-\frac{4\sqrt{3}\left(5\left(-ab^2\right)^{\frac{1}{3}}Ba-11\left(-ab^2\right)^{\frac{1}{3}}Ab\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{27a^5}+\frac{4\left(5Bab-11Ab^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{27a^5}-\frac{2(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -4/27*sqrt(3)*(5*(-a*b^2)^(1/3)*B*a - 11*(-a*b^2)^(1/3)*A*b)*arctan(1/3*sqr
t(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/a^5 + 4/27*(5*B*a*b - 11*A*b^2)*(-a
/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/a^5 - 2/27*(5*(-a*b^2)^(1/3)*B*a - 11*
(-a*b^2)^(1/3)*A*b)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/a^5 - 1/18*(11
*B*a*b^2*x^4 - 17*A*b^3*x^4 + 14*B*a^2*b*x - 20*A*a*b^2*x)/((b*x^3 + a)^2*a
^4) - 1/10*(5*B*a*x^3 - 15*A*b*x^3 + 2*A*a)/(a^4*x^5)
```

maple [A] time = 0.06, size = 295, normalized size = 1.20

$$\frac{17Ab^3x^4}{18\left(bx^3+a\right)^2a^4}-\frac{11Bb^2x^4}{18\left(bx^3+a\right)^2a^3}+\frac{10Ab^2x}{9\left(bx^3+a\right)^2a^3}-\frac{7Bbx}{9\left(bx^3+a\right)^2a^2}+\frac{44\sqrt{3}\,Ab\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{a}{b}\right)^{\frac{2}{3}}a^4}+\frac{44Ab\ln(\cdots}{27(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^3,x)
```

```
[Out] 17/18/a^4*b^3/(b*x^3+a)^2*A*x^4-11/18/a^3*b^2/(b*x^3+a)^2*B*x^4+10/9/a^3*b^
2/(b*x^3+a)^2*A*x-7/9/a^2*b/(b*x^3+a)^2*B*x+44/27/a^4*b*A/(a/b)^(2/3)*ln(x+
(a/b)^(1/3))-22/27/a^4*b*A/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+44
/27/a^4*b*A/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-20/
27/a^3*B/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+10/27/a^3*B/(a/b)^(2/3)*ln(x^2-(a/b)
^(1/3)*x+(a/b)^(2/3))-20/27/a^3*B/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(a/b)^(1/3)*x-1))-1/5/a^3*A/x^5+3/2/a^4/x^2*A*b-1/2/a^3/x^2*B
```

maxima [A] time = 1.46, size = 221, normalized size = 0.90

$$-\frac{20\left(5Bab^2-11Ab^3\right)x^9+32\left(5Ba^2b-11Aab^2\right)x^6+18Aa^3+9\left(5Ba^3-11Aa^2b\right)x^3}{90\left(a^4b^2x^{11}+2a^5bx^8+a^6x^5\right)}-\frac{4\sqrt{3}(5Ba-11Ab)\arctan(\cdots}{27a^4\left(\frac{a}{b}\right)^{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/90*(20*(5*B*a*b^2 - 11*A*b^3)*x^9 + 32*(5*B*a^2*b - 11*A*a*b^2)*x^6 + 18
*A*a^3 + 9*(5*B*a^3 - 11*A*a^2*b)*x^3)/(a^4*b^2*x^11 + 2*a^5*b*x^8 + a^6*x^
5) - 4/27*sqrt(3)*(5*B*a - 11*A*b)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(
a/b)^(1/3))/(a^4*(a/b)^(2/3)) + 2/27*(5*B*a - 11*A*b)*log(x^2 - x*(a/b)^(1/
3) + (a/b)^(2/3))/(a^4*(a/b)^(2/3)) - 4/27*(5*B*a - 11*A*b)*log(x + (a/b)^(
1/3))/(a^4*(a/b)^(2/3))
```

mupad [B] time = 2.58, size = 207, normalized size = 0.84

$$\frac{\frac{x^3\,(11\,A\,b-5\,B\,a)}{10\,a^2}-\frac{A}{5\,a}+\frac{2\,b^2\,x^9\,(11\,A\,b-5\,B\,a)}{9\,a^4}+\frac{16\,b\,x^6\,(11\,A\,b-5\,B\,a)}{45\,a^3}}{a^2\,x^5+2\,a\,b\,x^8+b^2\,x^{11}}+\frac{4\,b^{2/3}\,\ln\left(b^{1/3}\,x+a^{1/3}\right)\,(11\,A\,b-5\,B\,a)}{27\,a^{14/3}}-\frac{4\,b^{2/3}\,\ln\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^3),x)
```

```
[Out] ((x^3*(11*A*b - 5*B*a))/(10*a^2) - A/(5*a) + (2*b^2*x^9*(11*A*b - 5*B*a))/(
9*a^4) + (16*b*x^6*(11*A*b - 5*B*a))/(45*a^3))/(a^2*x^5 + b^2*x^11 + 2*a*b*
x^8) + (4*b^(2/3)*log(b^(1/3)*x + a^(1/3))*(11*A*b - 5*B*a))/(27*a^(14/3))
- (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/
2 + 1/2)*(11*A*b - 5*B*a))/(27*a^(14/3)) + (4*b^(2/3)*log(3^(1/2)*a^(1/3)*1
i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a))/(27*a^(
14/3))
```

sympy [A] time = 2.18, size = 173, normalized size = 0.70

$$\operatorname{RootSum}\left(19683t^3a^{14}-85184A^3b^5+116160A^2Bab^4-52800AB^2a^2b^3+8000B^3a^3b^2,\left(t\mapsto t\log\left(-\frac{27}{-44Ab^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**3,x)
```

```
[Out] RootSum(19683*_t**3*a**14 - 85184*A**3*b**5 + 116160*A**2*B*a*b**4 - 52800*
A*B**2*a**2*b**3 + 8000*B**3*a**3*b**2, Lambda(_t, _t*log(-27*_t*a**5/(-44*
A*b**2 + 20*B*a*b) + x))) + (-18*A*a**3 + x**9*(220*A*b**3 - 100*B*a*b**2)
+ x**6*(352*A*a*b**2 - 160*B*a**2*b) + x**3*(99*A*a**2*b - 45*B*a**3))/(90*
a**6*x**5 + 180*a**5*b*x**8 + 90*a**4*b**2*x**11)
```

3.107 $\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2\log(a+bx^3)}{3b^2(bc-ad)}-\frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}+\frac{x^3}{3bd}$$

```
[Out] 1/3*x^3/b/d+1/3*a^2*ln(b*x^3+a)/b^2/(-a*d+b*c)-1/3*c^2*ln(d*x^3+c)/d^2/(-a*
d+b*c)
```

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log(a+bx^3)}{3b^2(bc-ad)}-\frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}+\frac{x^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^3/(3*b*d) + (a^2*Log[a + b*x^3])/(3*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^3
])/(3*d^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx^3)(c+dx^3)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^3\right)\\ &= \frac{x^3}{3bd}+\frac{a^2\log(a+bx^3)}{3b^2(bc-ad)}-\frac{c^2\log(c+dx^3)}{3d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log(a+bx^3)-b\left(dx^3(ad-bc)+bc^2\log(c+dx^3)\right)}{3b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^3] - b*(d*(-(b*c) + a*d)*x^3 + b*c^2*Log[c + d*x^3]))/
(3*b^2*d^2*(b*c - a*d))
```

fricas [A] time = 1.05, size = 72, normalized size = 1.03

$$\frac{a^2d^2\log\left(bx^3+a\right)-b^2c^2\log\left(dx^3+c\right)+\left(b^2cd-abd^2\right)x^3}{3\left(b^3cd^2-ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(a^2*d^2*log(b*x^3 + a) - b^2*c^2*log(d*x^3 + c) + (b^2*c*d - a*b*d^2)*
x^3)/(b^3*c*d^2 - a*b^2*d^3)
```

giac [A] time = 0.18, size = 70, normalized size = 1.00

$$\frac{a^2\log\left(\left|bx^3+a\right|\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(\left|dx^3+c\right|\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a^2*log(abs(b*x^3 + a))/(b^3*c - a*b^2*d) - 1/3*c^2*log(abs(d*x^3 + c))
/(b*c*d^2 - a*d^3) + 1/3*x^3/(b*d)
```

maple [A] time = 0.05, size = 65, normalized size = 0.93

$$-\frac{a^2\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,b^2}+\frac{x^3}{3bd}+\frac{c^2\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*x^3/b/d-1/3*a^2/b^2/(a*d-b*c)*ln(b*x^3+a)+1/3*c^2/d^2/(a*d-b*c)*ln(d*x^
3+c)
```

maxima [A] time = 0.59, size = 68, normalized size = 0.97

$$\frac{a^2\log\left(bx^3+a\right)}{3\left(b^3c-ab^2d\right)}-\frac{c^2\log\left(dx^3+c\right)}{3\left(bcd^2-ad^3\right)}+\frac{x^3}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*a^2*log(b*x^3 + a)/(b^3*c - a*b^2*d) - 1/3*c^2*log(d*x^3 + c)/(b*c*d^2
- a*d^3) + 1/3*x^3/(b*d)
```

mupad [B] time = 2.84, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^3+a\right)}{3\,b^3\,c-3\,a\,b^2\,d}+\frac{c^2\,\ln\left(d\,x^3+c\right)}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{x^3}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (a^2*log(a + b*x^3))/(3*b^3*c - 3*a*b^2*d) + (c^2*log(c + d*x^3))/(3*a*d^3
- 3*b*c*d^2) + x^3/(3*b*d)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.108 $\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=301

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2}\right.}{6d^{5/3}(bc-ad)}$$

```
[Out] 1/2*x^2/b/d-1/3*a^(5/3)*ln(a^(1/3)+b^(1/3)*x)/b^(5/3)/(-a*d+b*c)+1/3*c^(5/3
)*ln(c^(1/3)+d^(1/3)*x)/d^(5/3)/(-a*d+b*c)+1/6*a^(5/3)*ln(a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/b^(5/3)/(-a*d+b*c)-1/6*c^(5/3)*ln(c^(2/3)-c^(1/3)*d^(
1/3)*x+d^(2/3)*x^2)/d^(5/3)/(-a*d+b*c)-1/3*a^(5/3)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*x)/a^(1/3)*3^(1/2))/b^(5/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(5/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 584, 292, 31, 634, 617, 204, 628}

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6b^{5/3}(bc-ad)}-\frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3b^{5/3}(bc-ad)}-\frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2}\right.}{6d^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x^2/(2*b*d) - (a^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^(5/3)*(b*c - a*d)) + (c^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d^(5/3)*(b*c - a*d)) - (a^(5/3)*Log[a^(1/3) + b^(1
/3)*x])/(3*b^(5/3)*(b*c - a*d)) + (c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(
5/3)*(b*c - a*d)) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]
)/(6*b^(5/3)*(b*c - a*d)) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2])/(6*d^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{x^2}{2bd} - \frac{\int \frac{x\left(2ac+2(bc+ad)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{2bd}$$

$$= \frac{x^2}{2bd} - \frac{\int \left(\frac{2a^2dx}{(-bc+ad)\left(a+bx^3\right)} + \frac{2bc^2x}{(bc-ad)\left(c+dx^3\right)}\right)dx}{2bd}$$

$$= \frac{x^2}{2bd} + \frac{a^2\int \frac{x}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{x}{c+dx^3}\,dx}{d(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b^{4/3}(bc-ad)} + \frac{a^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b^{4/3}(bc-ad)} + \frac{c^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d^{4/3}(bc-ad)} - \frac{c^{5/3}\int}{3}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{5/3}(bc-ad)} +$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{5/3}(bc-ad)} + \frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}\right.}{6b^{5/3}(bc-ad)}$$

$$= \frac{x^2}{2bd} - \frac{a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{5/3}(bc-ad)} + \frac{c^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{5/3}(bc-ad)} + \frac{c^{5/3}}{3}$$

Mathematica [A] time = 0.24, size = 242, normalized size = 0.80

$$\frac{\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{5/3}}-\frac{2a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{5/3}}-\frac{2\sqrt{3}a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}}-\frac{3ax^2}{b}-\frac{c^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{5/3}}+\frac{2c^{5/3}\log(\sqrt[3]{}}{d^{5/}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-3*a*x^2)/b + (3*c*x^2)/d - (2*Sqrt[3]*a^(5/3)*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/b^(5/3) + (2*Sqrt[3]*c^(5/3)*ArcTan[(1 - (2*d^(1/3)*x)/c
^(1/3))/Sqrt[3]])/d^(5/3) - (2*a^(5/3)*Log[a^(1/3) + b^(1/3)*x])/b^(5/3) +
(2*c^(5/3)*Log[c^(1/3) + d^(1/3)*x])/d^(5/3) + (a^(5/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/b^(5/3) - (c^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/
3)*x + d^(2/3)*x^2])/d^(5/3))/(6*b*c - 6*a*d)
```

fricas [A] time = 1.03, size = 273, normalized size = 0.91

$$\frac{2\sqrt{3}\,ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3a}\right)-2\sqrt{3}\,bc\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,dx\left(-\frac{c^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,c}{3c}\right)+ad\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*a*d*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a^2/b^2)^(1/3
) - sqrt(3)*a)/a) - 2*sqrt(3)*b*c*(-c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*
x*(-c^2/d^2)^(1/3) + sqrt(3)*c)/c) + a*d*(a^2/b^2)^(1/3)*log(a*x^2 - b*x*(a
^2/b^2)^(2/3) + a*(a^2/b^2)^(1/3)) + b*c*(-c^2/d^2)^(1/3)*log(c*x^2 - d*x*(
-c^2/d^2)^(2/3) - c*(-c^2/d^2)^(1/3)) - 2*a*d*(a^2/b^2)^(1/3)*log(a*x + b*(
a^2/b^2)^(2/3)) - 2*b*c*(-c^2/d^2)^(1/3)*log(c*x + d*(-c^2/d^2)^(2/3)) + 3*
(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2)
```

giac [A] time = 0.21, size = 311, normalized size = 1.03

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^4c-\sqrt{3}\,ab^3d}+\frac{\left(-cd^2\right)^{\frac{2}{3}}c\,\mathrm{arcta}}{\sqrt{3}\,bcd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) - (-a*b^2)^
(2/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^4*
c - sqrt(3)*a*b^3*d) + (-c*d^2)^(2/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-a*b^2)^(2/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^4*c - a*b^3*d) - 1/6*(-c*d^2)^
(2/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^3 - a*d^4) + 1/2*x^
2/(b*d)
```

maple [A] time = 0.05, size = 269, normalized size = 0.89

$$-\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{a^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{a^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{\sqrt{3}\,c^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}d^{2}}-\frac{c^{2}}{3(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/2*x^2/b/d+1/3*a^2/b^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*a^2/b^2
/(a*d-b*c)/(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*a^2/b^2/(a*d-b
*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*c^2/d^2
/(a*d-b*c)/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6*c^2/d^2/(a*d-b*c)/(c/d)^(1/3)*
ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3*c^2/d^2/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)
*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.27, size = 324, normalized size = 1.08

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{3}c-ab^{2}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,c^{2}\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^{2}-ad^{3}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{c^{2}\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right.\right.}{6\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{3}\left(\frac{c}{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
 - a*b^2*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c*d^2 - a*d^3)*(c/d)^(1/3)) + 1/6*a^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3)) - 1/6*c
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d
)^(1/3)) - 1/3*a^2*log(x + (a/b)^(1/3))/(b^3*c*(a/b)^(1/3) - a*b^2*d*(a/b)^
(1/3)) + 1/3*c^2*log(x + (c/d)^(1/3))/(b*c*d^2*(c/d)^(1/3) - a*d^3*(c/d)^(1
/3)) + 1/2*x^2/(b*d)
```

mupad [B] time = 11.36, size = 1751, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((((27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))*(a^5/(b^5*(a*d
- b*c)^3))^(1/3))/3 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2
*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2
+ b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^
2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3) + log(((((27*a^2*b*c^2*d*x*(a^2*d^2 +
b^2*c^2)*(a*d - b*c)^2 + 27*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^
5*(a*d - b*c)^3))^(2/3))*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/3 - (9*(a*b^7*c^
8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d
```

```
- b*c)^3))^(2/3))/9 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*
(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1
/3) - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*c^2*d*x*(a^2*d^
2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)
*(a*d - b*c)^4*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3)
)^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b
^2*d^2))*(a^5/(b^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x*(a^2*d^2 + b^2*c^
2 + a*b*c*d))/(b^2*d^2))*(-a^5/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*
d^2 - 81*a*b^7*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)^2
*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (
27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(a^5/(b^5*(a*d
- b*c)^3))^(2/3))/4)*(a^5/(b^5*(a*d - b*c)^3))^(1/3))/6 - (9*(a*b^7*c^8 + a
^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(a^5/(b^5*(a*d - b*c)
^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-a^5
/(27*b^8*c^3 - 27*a^3*b^5*d^3 + 81*a^2*b^6*c*d^2 - 81*a*b^7*c^2*d))^(1/3)*(
3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(27*a^2*b*
c^2*d*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d
^5*(a*d - b*c)^3))^(1/3))/6 + (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a
^7*b*c^2*d^6))/(b^2*d^2))*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 + (a^4*c^4*x
*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d
^5 + 81*a*b^2*c^2*d^6 - 81*a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(27*a^2*b*c^2*d*x*(a^2*d^2 + b^2*c^2)
*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)
^4*(-c^5/(d^5*(a*d - b*c)^3))^(2/3))/4)*(-c^5/(d^5*(a*d - b*c)^3))^(1/3))/6
 - (9*(a*b^7*c^8 + a^8*c*d^7 - a^2*b^6*c^7*d - a^7*b*c^2*d^6))/(b^2*d^2))*(
-c^5/(d^5*(a*d - b*c)^3))^(2/3))/36 - (a^4*c^4*x*(a^2*d^2 + b^2*c^2 + a*b*c
*d))/(b^2*d^2))*(-c^5/(27*a^3*d^8 - 27*b^3*c^3*d^5 + 81*a*b^2*c^2*d^6 - 81*
a^2*b*c*d^7))^(1/3)*(3^(1/2)*1i - 1))/2 + x^2/(2*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.109 $\int \frac{x^6}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=296

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6d^{4/3}(bc-ad)}$$

```
[Out] x/b/d+1/3*a^(4/3)*ln(a^(1/3)+b^(1/3)*x)/b^(4/3)/(-a*d+b*c)-1/3*c^(4/3)*ln(c
^(1/3)+d^(1/3)*x)/d^(4/3)/(-a*d+b*c)-1/6*a^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/b^(4/3)/(-a*d+b*c)+1/6*c^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/d^(4/3)/(-a*d+b*c)-1/3*a^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*
x)/a^(1/3)*3^(1/2))/b^(4/3)/(-a*d+b*c)*3^(1/2)+1/3*c^(4/3)*arctan(1/3*(c^(1
/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{4/3}(bc-ad)}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)}-\frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6d^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] x/(b*d) - (a^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*b^(4/3)*(b*c - a*d)) + (c^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]
*c^(1/3))])/(Sqrt[3]*d^(4/3)*(b*c - a*d)) + (a^(4/3)*Log[a^(1/3) + b^(1/3)*
x])/(3*b^(4/3)*(b*c - a*d)) - (c^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(4/3)
*(b*c - a*d)) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6
*b^(4/3)*(b*c - a*d)) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*
x^2])/(6*d^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
```

```
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{bd} \\
&= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^3}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^3}\,dx}{d(bc-ad)} \\
&= \frac{x}{bd} + \frac{a^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3b(bc-ad)} + \frac{a^{4/3}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3b(bc-ad)} - \frac{c^{4/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3d(bc-ad)} - \frac{c^{4/3}\int
\\
&= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{4/3}(bc-ad)} -
\\
&= \frac{x}{bd} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - \frac{c^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{4/3}(bc-ad)} - \frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b
\\
&= \frac{x}{bd} - \frac{a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{4/3}(bc-ad)} + \frac{c^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)} + \frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{4/3}(bc-ad)} - c^{4/}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 238, normalized size = 0.80

$$-\frac{\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{4/3}}+\frac{2a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{4/3}}-\frac{2\sqrt{3}a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}}-\frac{6ax}{b}+\frac{c^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{4/3}}-\frac{2c^{4/3}\log\left(\sqrt[3]{c}\cdots\right.}{d^{4/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((-6*a*x)/b + (6*c*x)/d - (2*Sqrt[3]*a^(4/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1
/3))/Sqrt[3]])/b^(4/3) + (2*Sqrt[3]*c^(4/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/
3))/Sqrt[3]])/d^(4/3) + (2*a^(4/3)*Log[a^(1/3) + b^(1/3)*x])/b^(4/3) - (2*c
^(4/3)*Log[c^(1/3) + d^(1/3)*x])/d^(4/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2])/b^(4/3) + (c^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2])/d^(4/3))/(6*b*c - 6*a*d)
```

fricas [A] time = 1.01, size = 228, normalized size = 0.77

$$-\frac{2\sqrt{3}ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}bc\left(\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-ad\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}\cdots\right.}{6\left(b^2\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a*d*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a/b)^(2/3) - s
qrt(3)*a)/a) + 2*sqrt(3)*b*c*(c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c/d)^(2
/3) - sqrt(3)*c)/c) - a*d*(-a/b)^(1/3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2
/3)) - b*c*(c/d)^(1/3)*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)) + 2*a*d*(-a/b
)^(1/3)*log(x - (-a/b)^(1/3)) + 2*b*c*(c/d)^(1/3)*log(x + (c/d)^(1/3)) - 6*
(b*c - a*d)*x)/(b^2*c*d - a*b*d^2)
```

giac [A] time = 0.21, size = 308, normalized size = 1.04

$$-\frac{a^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{c^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}c\arctan\left(\cdots\right.}{\sqrt{3}bcd^2-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c - a^2*b*d) + 1/3*
c^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (-a*b^2)^
(1/3)*a*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*
c - sqrt(3)*a*b^2*d) - (-c*d^2)^(1/3)*c*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) + 1/6*(-a*b^2)^(1/3)*a
*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) - 1/6*(-c*d^2)^
(1/3)*c*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3) + x/(b*d
)
```

maple [A] time = 0.05, size = 266, normalized size = 0.90

$$-\frac{\sqrt{3}\, a^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}-\frac{a^{2}\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{a^{2}\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}}+\frac{\sqrt{3}\, c^{2} \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}} d^{2}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] x/b/d-1/3/b^2*a^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/b^2*a^2/(a*d-
b*c)/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/b^2*a^2/(a*d-b*c)/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/d^2*c^2/(a*d-
b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/d^2*c^2/(a*d-b*c)/(c/d)^(2/3)*ln(x^2
-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/d^2*c^2/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.35, size = 349, normalized size = 1.18

$$\frac{\sqrt{3}\, a^{2} \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\, c^{2} \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{3}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a^{2}\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{3}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-ab^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{c^{2}\log\left(x^{2}-x\left(\frac{c}{d}\cdots\right.\right.}{6\left(bcd^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^3*c
*(a/b)^(1/3) - a*b^2*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*c^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d^2*(c/d)^(1/3) - a*d^3*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*a^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) + 1/6*c^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + 1/3*a^2*log(x +
(a/b)^(1/3))/(b^3*c*(a/b)^(2/3) - a*b^2*d*(a/b)^(2/3)) - 1/3*c^2*log(x + (c
/d)^(1/3))/(b*c*d^2*(c/d)^(2/3) - a*d^3*(c/d)^(2/3)) + x/(b*d)
```

mupad [B] time = 1.83, size = 873, normalized size = 2.95

$$\ln\left(a\,x+b^{2}\,c\left(-\frac{a^{4}}{b^{4}\left(a\,d-b\,c\right)^{3}}\right)^{1/3}-a\,b\,d\left(-\frac{a^{4}}{b^{4}\left(a\,d-b\,c\right)^{3}}\right)^{1/3}\right)\left(\frac{a^{4}}{-27\,a^{3}\,b^{4}\,d^{3}+81\,a^{2}\,b^{5}\,c\,d^{2}-81\,a\,b^{6}\,c^{2}\,d+27\,b\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^2*c*(-a^4/(b^4*(a*d - b*c)^3))^(1/3) - a*b*d*(-a^4/(b^4*(a*d -
b*c)^3))^(1/3))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a
*b^6*c^2*d))^(1/3) + log(c*x + a*d^2*(c^4/(d^4*(a*d - b*c)^3))^(1/3) - b*c*
d*(c^4/(d^4*(a*d - b*c)^3))^(1/3))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a
*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3) + x/(b*d) + (log((3*x*(a^2*b^4*c^6 +
```

```
a^6*c^2*d^4))/(b*d) - (3*a*c^2*(3^(1/2)*1i - 1)*(-a^4/(b^4*(a*d - b*c)^3))^
(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*
c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1
i - 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) + (3*a*c^2*(3^(1/2
)*1i + 1)*(-a^4/(b^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d
 - a^4*b*c*d^4))/(2*d))*(a^4/(27*b^7*c^3 - 27*a^3*b^4*d^3 + 81*a^2*b^5*c*d^
2 - 81*a*b^6*c^2*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((3*x*(a^2*b^4*c^6 + a
^6*c^2*d^4))/(b*d) + (3*a^2*c*(3^(1/2)*1i - 1)*(c^4/(d^4*(a*d - b*c)^3))^(1
/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d - a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^
7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 - 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i
- 1))/2 - (log((3*x*(a^2*b^4*c^6 + a^6*c^2*d^4))/(b*d) - (3*a^2*c*(3^(1/2)*
1i + 1)*(c^4/(d^4*(a*d - b*c)^3))^(1/3)*(a^5*d^5 - b^5*c^5 + a*b^4*c^4*d -
a^4*b*c*d^4))/(2*b))*(c^4/(27*a^3*d^7 - 27*b^3*c^3*d^4 + 81*a*b^2*c^2*d^5 -
 81*a^2*b*c*d^6))^(1/3)*(3^(1/2)*1i + 1))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.110 $\int \frac{x^5}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}-\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)}$$

```
[Out] -1/3*a*ln(b*x^3+a)/b/(-a*d+b*c)+1/3*c*ln(d*x^3+c)/d/(-a*d+b*c)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}-\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(a*Log[a + b*x^3])/(3*b*(b*c - a*d)) + (c*Log[c + d*x^3])/(3*d*(b*c - a*d)
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^3\right)\\ &= -\frac{a\log\left(a+bx^3\right)}{3b(bc-ad)}+\frac{c\log\left(c+dx^3\right)}{3d(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^3\right)-bc\log\left(c+dx^3\right)}{3b^2cd-3abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((a*d*Log[a + b*x^3] - b*c*Log[c + d*x^3])/(3*b^2*c*d - 3*a*b*d^2))
```

fricas [A] time = 0.97, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^3+a\right)-bc\log\left(dx^3+c\right)}{3\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(a*d*log(b*x^3 + a) - b*c*log(d*x^3 + c))/(b^2*c*d - a*b*d^2)
```

giac [A] time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^3+a\right|\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^3+c\right|\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*a*log(abs(b*x^3 + a))/(b^2*c - a*b*d) + 1/3*c*log(abs(d*x^3 + c))/(b*c
*d - a*d^2)
```

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{a\ln\left(bx^3+a\right)}{3\left(ad-bc\right)b}-\frac{c\ln\left(dx^3+c\right)}{3\left(ad-bc\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*a/(a*d-b*c)/b*ln(b*x^3+a)-1/3*c/(a*d-b*c)/d*ln(d*x^3+c)
```

maxima [A] time = 0.46, size = 49, normalized size = 0.92

$$-\frac{a\log\left(bx^3+a\right)}{3\left(b^2c-abd\right)}+\frac{c\log\left(dx^3+c\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*a*log(b*x^3 + a)/(b^2*c - a*b*d) + 1/3*c*log(d*x^3 + c)/(b*c*d - a*d^2
)
```

mupad [B] time = 0.31, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^3+a\right)}{3\,b^2\,c-3\,a\,b\,d}-\frac{c\,\ln\left(d\,x^3+c\right)}{3\,a\,d^2-3\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (a*log(a + b*x^3))/(3*b^2*c - 3*a*b*d) - (c*log(c + d*x^3))/(3*a*d^2 - 3*
b*c*d)
```

sympy [B] time = 6.75, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^3+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{3b\left(ad-bc\right)}-\frac{c\log\left(x^3+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{3d\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] a*log(x**3 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(3*b*(a*d - b*c)) - c*log(x**3 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(3*d*(a*d - b*c))
```

3.111 $\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6d^{2/3}(bc-ad)}$$

```
[Out] 1/3*a^(2/3)*ln(a^(1/3)+b^(1/3)*x)/b^(2/3)/(-a*d+b*c)-1/3*c^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/d^(2/3)/(-a*d+b*c)-1/6*a^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/b^(2/3)/(-a*d+b*c)+1/6*c^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/d^(2/3)/(-a*d+b*c)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/b^(2/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 292, 31, 634, 617, 204, 628}

$$-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}+\frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)}+\frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)}+\frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6d^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(2/3
)*(b*c - a*d)) - (c^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(2/3)*(b*c - a*d)) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
2/3)*(b*c - a*d)) - (c^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(2/3)*(b*c - a*
d)) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(2/3)*(
b*c - a*d)) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{(a+bx^3)(c+dx^3)}\,dx = -\frac{a\int \frac{x}{a+bx^3}\,dx}{bc-ad} + \frac{c\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{a^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{b}(bc-ad)} - \frac{c^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{d}(bc-ad)} + \frac{c^{2/3}\int \frac{\sqrt[3]{}}{c^{2/3}-\sqrt[3]{}}}{3\sqrt[3]{d}(}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6b^{2/3}(bc-ad)} - \frac{a\int}{}$$

$$= \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3d^{2/3}(bc-ad)} - \frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6b^{2/3}(bc-ad)}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{2/3}(bc-ad)} - \frac{c^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}d^{2/3}(bc-ad)} + \frac{a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3b^{2/3}(bc-ad)} - \frac{c^{2/3}\log(}{3d^{2/3}}$$

Mathematica [A] time = 0.13, size = 224, normalized size = 0.78

$$\frac{-\frac{a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{b^{2/3}} + \frac{2a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{b^{2/3}} + \frac{2\sqrt{3}a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{2/3}} + \frac{c^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{d^{2/3}} - \frac{2c^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{}\right)}{d^{2/3}}}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(2/3) -
(2*Sqrt[3]*c^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(2/3) + (
2*a^(2/3)*Log[a^(1/3) + b^(1/3)*x])/b^(2/3) - (2*c^(2/3)*Log[c^(1/3) + d^(1
```

```
/3)*x])/d^(2/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(2/3) + (c^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(2/3))
/(6*b*c - 6*a*d)
```

fricas [A] time = 0.98, size = 244, normalized size = 0.85

$$-\frac{2\sqrt{3}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}a}{3a}\right)-2\sqrt{3}\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{c^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}c}{3c}\right)-\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(ax^2-bx\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}}\cdots\right.}{6\,(bc\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(-a^2/b^2)^(1/3)
 + sqrt(3)*a)/a) - 2*sqrt(3)*(c^2/d^2)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(c^2
/d^2)^(1/3) - sqrt(3)*c)/c) - (-a^2/b^2)^(1/3)*log(a*x^2 - b*x*(-a^2/b^2)^(
2/3) - a*(-a^2/b^2)^(1/3)) - (c^2/d^2)^(1/3)*log(c*x^2 - d*x*(c^2/d^2)^(2/3
) + c*(c^2/d^2)^(1/3)) + 2*(-a^2/b^2)^(1/3)*log(a*x + b*(-a^2/b^2)^(2/3)) +
 2*(c^2/d^2)^(1/3)*log(c*x + d*(c^2/d^2)^(2/3)))/(b*c - a*d)
```

giac [A] time = 0.25, size = 286, normalized size = 0.99

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^3c-\sqrt{3}ab^2d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\cdots\right.}{3\left(-\cdots\right.}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(2/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b
^2*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3)
)/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3) - 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(-a/b)
^(1/3) + (-a/b)^(2/3))/(b^3*c - a*b^2*d) + 1/6*(-c*d^2)^(2/3)*log(x^2 + x*(
-c/d)^(1/3) + (-c/d)^(2/3))/(b*c*d^2 - a*d^3)
```

maple [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}d}+\frac{c\ln\left(x+\cdots\right.}{3\,(ad-bc\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*a/(a*d-b*c)/b/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*a/(a*d-b*c)/b/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)*3^(1/2)/b/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*c/(a*d-b*c)/d/(c/d)^(1/3)*ln(x
```

```
+(c/d)^(1/3))-1/6*c/(a*d-b*c)/d/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3
))-1/3*c/(a*d-b*c)*3^(1/2)/d/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1))
```

maxima [A] time = 1.27, size = 289, normalized size = 1.00

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c-abd\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd-ad^{2}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{a\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{c\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right.\right.}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{2}\left(\frac{c}{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c
- a*b*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c*d - a*d^2)*(c/d)^(1/3)) - 1/6*a*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) + 1/6*c*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3)) + 1/3*
a*log(x + (a/b)^(1/3))/(b^2*c*(a/b)^(1/3) - a*b*d*(a/b)^(1/3)) - 1/3*c*log(
x + (c/d)^(1/3))/(b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1/3))
```

mupad [B] time = 9.05, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(a*x + b^3*c^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3) + a^2*b*d^2*(-a^2/(b^2*(
a*d - b*c)^3))^(2/3) - 2*a*b^2*c*d*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))*(a^2/(
27*b^5*c^3 - 27*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3) + l
og(c*x + a^2*d^3*(c^2/(d^2*(a*d - b*c)^3))^(2/3) + b^2*c^2*d*(c^2/(d^2*(a*d
 - b*c)^3))^(2/3) - 2*a*b*c*d^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3))*(c^2/(27*a
^3*d^5 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3) + (log(
((3^(1/2)*1i - 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(54
*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5*c^5*d
+ 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 27*a^3
*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(-a^2/(b^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1
)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i + 1)^2*
(a*d + b*c)*(a*d - b*c)^4*(-a^2/(b^2*(a*d - b*c)^3))^(2/3))/4)*(-a^2/(b^2*(
a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b^5*c
^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(a^2/(27*b^5*c^3 - 2
7*a^3*b^2*d^3 + 81*a^2*b^3*c*d^2 - 81*a*b^4*c^2*d))^(1/3)*(3^(1/2)*1i + 1))
/2 + (log(((3^(1/2)*1i - 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i - 1
)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d^2
*(a*d - b*c)^3))^(1/3))/6 - 9*a^2*b^4*c^4*d^2 - 9*a^4*b^2*c^2*d^4 + 9*a*b^5
*c^5*d + 9*a^5*b*c*d^5))/36 + a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5 -
 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i - 1
))/2 - (log(((3^(1/2)*1i + 1)^2*(c^2/(d^2*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(54*a^2*b^3*c^2*d^3*x*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2)*1i +
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(c^2/(d^2*(a*d - b*c)^3))^(2/3))/4)*(c^2/(d
^2*(a*d - b*c)^3))^(1/3))/6 + 9*a^2*b^4*c^4*d^2 + 9*a^4*b^2*c^2*d^4 - 9*a*b
```

```
^5*c^5*d - 9*a^5*b*c*d^5))/36 - a^2*b*c^2*d*x*(a*d + b*c))*(c^2/(27*a^3*d^5
 - 27*b^3*c^3*d^2 + 81*a*b^2*c^2*d^3 - 81*a^2*b*c*d^4))^(1/3)*(3^(1/2)*1i +
 1))/2
```

sympy [B] time = 123.98, size = 573, normalized size = 1.99

$$\operatorname{RootSum}\left(t^3\left(27a^3d^5 - 81a^2bcd^4 + 81ab^2c^2d^3 - 27b^3c^3d^2\right) - c^2, \left(t \mapsto t\log\left(x + \frac{243t^5a^6b^2d^8 - 1458t^5a^5b^3cd^7 +}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**5 - 81*a**2*b*c*d**4 + 81*a*b**2*c**2*d**3 - 27*b
**3*c**3*d**2) - c**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 14
58*_t**5*a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**
3*b**5*c**3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*
d**3 + 243*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**
4 + 9*_t**2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*
b**4*c**4*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3)))) + RootSum(_
t**3*(27*a**3*b**2*d**3 - 81*a**2*b**3*c*d**2 + 81*a*b**4*c**2*d - 27*b**5*
c**3) + a**2, Lambda(_t, _t*log(x + (243*_t**5*a**6*b**2*d**8 - 1458*_t**5*
a**5*b**3*c*d**7 + 3645*_t**5*a**4*b**4*c**2*d**6 - 4860*_t**5*a**3*b**5*c*
*3*d**5 + 3645*_t**5*a**2*b**6*c**4*d**4 - 1458*_t**5*a*b**7*c**5*d**3 + 24
3*_t**5*b**8*c**6*d**2 + 9*_t**2*a**5*d**5 - 18*_t**2*a**4*b*c*d**4 + 9*_t*
*2*a**3*b**2*c**2*d**3 + 9*_t**2*a**2*b**3*c**3*d**2 - 18*_t**2*a*b**4*c**4
*d + 9*_t**2*b**5*c**5)/(a**3*c*d**2 + a*b**2*c**3))))
```

3.112 $\int \frac{x^3}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$

```
[Out] -1/3*a^(1/3)*ln(a^(1/3)+b^(1/3)*x)/b^(1/3)/(-a*d+b*c)+1/3*c^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/d^(1/3)/(-a*d+b*c)+1/6*a^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/b^(1/3)/(-a*d+b*c)-1/6*c^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/d^(1/3)/(-a*d+b*c)+1/3*a^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/b^(1/3)/(-a*d+b*c)*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/d^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 200, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(1/3
)*(b*c - a*d)) - (c^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))]
)/(Sqrt[3]*d^(1/3)*(b*c - a*d)) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*b^(
1/3)*(b*c - a*d)) + (c^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*d^(1/3)*(b*c - a*
d)) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*b^(1/3)*(
b*c - a*d)) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*d
^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
```

```
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{a\int \frac{1}{a+bx^3}\,dx}{bc-ad}+\frac{c\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{\sqrt[3]{a}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3(bc-ad)}-\frac{\sqrt[3]{a}\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3(bc-ad)}+\frac{\sqrt[3]{c}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3(bc-ad)}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[3]{c}}{c^{2/3}-\sqrt[3]{c}}}{3(bc}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}-\frac{a^{2/3}\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2(bc-ad)}+\frac{\sqrt[3]{a}\int}{}$$

$$= -\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{d}(bc-ad)}+\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{b}(bc-ad)}-$$

$$= \frac{\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{b}(bc-ad)}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{d}(bc-ad)}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{b}(bc-ad)}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}\right.}{3\sqrt[3]{d}(bc}$$

Mathematica [A] time = 0.12, size = 224, normalized size = 0.78

$$\frac{\frac{\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{b}}-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{b}}+\frac{2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{\sqrt[3]{c}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{d}}+\frac{2\sqrt[3]{c}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{d}}-}{6bc-6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/b^(1/3) -
(2*Sqrt[3]*c^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/d^(1/3) - (
2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x])/b^(1/3) + (2*c^(1/3)*Log[c^(1/3) + d^(1
```

```
/3)*x])/d^(1/3) + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
b^(1/3) - (c^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/d^(1/3))
/(6*b*c - 6*a*d)
```

fricas [A] time = 0.94, size = 199, normalized size = 0.69

$$-\frac{2\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)+2\sqrt{3}\left(-\frac{c}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(-\frac{c}{d}\right)^{\frac{2}{3}}-\sqrt{3}c}{3c}\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x*(a/b)^(2/3) - sqrt(3)
*a)/a) + 2*sqrt(3)*(-c/d)^(1/3)*arctan(1/3*(2*sqrt(3)*d*x*(-c/d)^(2/3) - sq
rt(3)*c)/c) - (a/b)^(1/3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)) - (-c/d)^(
1/3)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3)) + 2*(a/b)^(1/3)*log(x + (a/b)
^(1/3)) + 2*(-c/d)^(1/3)*log(x - (-c/d)^(1/3)))/(b*c - a*d)
```

giac [A] time = 0.20, size = 278, normalized size = 0.97

$$\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}-\frac{c\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c-\sqrt{3}abd}+\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right)}{\sqrt{3}bcd-\sqrt{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*a*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) - 1/3*c*(-c/d
)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(1/3)*arctan(
1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b
*d) + (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/
(sqrt(3)*b*c*d - sqrt(3)*a*d^2) - 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1/
3) + (-a/b)^(2/3))/(b^2*c - a*b*d) + 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)^
(1/3) + (-c/d)^(2/3))/(b*c*d - a*d^2)
```

maple [A] time = 0.05, size = 246, normalized size = 0.85

$$\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{a\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{a\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}d}-\frac{c\ln\left(\right.}{3(ad-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*a/(a*d-b*c)/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6*a/(a*d-b*c)/b/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*a/(a*d-b*c)/b/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*c/(a*d-b*c)/d/(c/d)^(2/3)*ln(x+
(c/d)^(1/3))+1/6*c/(a*d-b*c)/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3)
)-1/3*c/(a*d-b*c)/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1))
```

maxima [A] time = 1.21, size = 317, normalized size = 1.10

$$-\frac{\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{1}{3}}-abd\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,c\arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bcd\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{a\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(b^{2}c\left(\frac{a}{b}\right)^{\frac{2}{3}}-abd\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{c\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\cdots\right.}{6\left(bcd\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad^{2}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b^2*c*
(a/b)^(1/3) - a*b*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*c*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*d*(c/d)^(1/3) - a*d^2*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*a*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^2*c*(a/
b)^(2/3) - a*b*d*(a/b)^(2/3)) - 1/6*c*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c*d*(c/d)^(2/3) - a*d^2*(c/d)^(2/3)) - 1/3*a*log(x + (a/b)^(1/3))/(b^2
*c*(a/b)^(2/3) - a*b*d*(a/b)^(2/3)) + 1/3*c*log(x + (c/d)^(1/3))/(b*c*d*(c/
d)^(2/3) - a*d^2*(c/d)^(2/3))
```

mupad [B] time = 8.12, size = 1265, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(x + a*d*(a/(b*(a*d - b*c)^3))^(1/3) - b*c*(a/(b*(a*d - b*c)^3))^(1/3))*
(-a/(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3)
+ log(x - a*d*(-c/(d*(a*d - b*c)^3))^(1/3) + b*c*(-c/(d*(a*d - b*c)^3))^(1/
3))*(-c/(27*a^3*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1
/3) + (log(((3^(1/2)*1i - 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)
^2*(81*a*b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d
+ b*c)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^
(2/3))/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b
^3*c^2*d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-a/
(27*b^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 -
 (log(((3^(1/2)*1i + 1)*(a/(b*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(8
1*a*b^3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c
)*(a*d - b*c)^4*(a/(b*(a*d - b*c)^3))^(1/3))/2)*(a/(b*(a*d - b*c)^3))^(2/3)
)/36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^
2*d^4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-a/(27*b
^4*c^3 - 27*a^3*b*d^3 + 81*a^2*b^2*c*d^2 - 81*a*b^3*c^2*d))^(1/3))/2 + (log
(((3^(1/2)*1i - 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a*
b^3*c*d^3*x*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a
*d - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/
36 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*
d^4))/6 - 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i - 1)*(-c/(27*a^3
*d^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2 - (log((
(3^(1/2)*1i + 1)*(-c/(d*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a*b^
3*c*d^3*x*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-c/(d*(a*d - b*c)^3))^(1/3))/2)*(-c/(d*(a*d - b*c)^3))^(2/3))/36
 + 9*a*b^5*c^4*d^2 + 9*a^4*b^2*c*d^5 - 9*a^2*b^4*c^3*d^3 - 9*a^3*b^3*c^2*d^
4))/6 + 3*a*b^2*c*d^2*x*(a^2*d^2 + b^2*c^2))*(3^(1/2)*1i + 1)*(-c/(27*a^3*d
^4 - 27*b^3*c^3*d + 81*a*b^2*c^2*d^2 - 81*a^2*b*c*d^3))^(1/3))/2
```

sympy [A] time = 20.20, size = 342, normalized size = 1.19

$$\operatorname{RootSum}\left(t^{3}\left(27a^{3}d^{4}-81a^{2}bcd^{3}+81ab^{2}c^{2}d^{2}-27b^{3}c^{3}d\right)+c,\left(t\mapsto t\log\left(x+\frac{162t^{4}a^{4}bd^{5}-648t^{4}a^{3}b^{2}cd^{4}+972t\cdots}{\cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**3*d**4 - 81*a**2*b*c*d**3 + 81*a*b**2*c**2*d**2 - 27*b
**3*c**3*d) + c, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*d**5 - 648*_t**4*a
**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t**4*a*b**4*c**3*d**
2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d - 3*_t*b**2*c**2)
/(a*d + b*c)))) + RootSum(_t**3*(27*a**3*b*d**3 - 81*a**2*b**2*c*d**2 + 81*
a*b**3*c**2*d - 27*b**4*c**3) - a, Lambda(_t, _t*log(x + (162*_t**4*a**4*b*
d**5 - 648*_t**4*a**3*b**2*c*d**4 + 972*_t**4*a**2*b**3*c**2*d**3 - 648*_t*
*4*a*b**4*c**3*d**2 + 162*_t**4*b**5*c**4*d - 3*_t*a**2*d**2 + 6*_t*a*b*c*d
 - 3*_t*b**2*c**2)/(a*d + b*c))))
```

3.113 $\int \frac{x^2}{\left(a+bx^3\right)\left(c+dx^3\right)} \, dx$

Optimal. Leaf size=45

$$\frac{\log\left(a+bx^3\right)}{3(bc-ad)}-\frac{\log\left(c+dx^3\right)}{3(bc-ad)}$$

[Out] 1/3*ln(b*x^3+a)/(-a*d+b*c)-1/3*ln(d*x^3+c)/(-a*d+b*c)

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log\left(a+bx^3\right)}{3(bc-ad)}-\frac{\log\left(c+dx^3\right)}{3(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((a + b*x^3)*(c + d*x^3)),x]

[Out] Log[a + b*x^3]/(3*(b*c - a*d)) - Log[c + d*x^3]/(3*(b*c - a*d))

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^3\right)\left(c+dx^3\right)} \, dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)} \, dx, x, x^3\right)\\
&= \frac{b \operatorname{Subst}\left(\int \frac{1}{a+bx} \, dx, x, x^3\right)}{3(bc-ad)}-\frac{d \operatorname{Subst}\left(\int \frac{1}{c+dx} \, dx, x, x^3\right)}{3(bc-ad)}\\
&= \frac{\log\left(a+bx^3\right)}{3(bc-ad)}-\frac{\log\left(c+dx^3\right)}{3(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log\left(a+bx^3\right)-\log\left(c+dx^3\right)}{3bc-3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)),x]
[Out] (Log[a + b*x^3] - Log[c + d*x^3])/(3*b*c - 3*a*d)
```

fricas [A] time = 0.85, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^3+a\right)-\log\left(dx^3+c\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
[Out] 1/3*(log(b*x^3 + a) - log(d*x^3 + c))/(b*c - a*d)
```

giac [A] time = 0.18, size = 51, normalized size = 1.13

$$\frac{b\log\left(\left|bx^3+a\right|\right)}{3\left(b^2c-abd\right)}-\frac{d\log\left(\left|dx^3+c\right|\right)}{3\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
[Out] 1/3*b*log(abs(b*x^3 + a))/(b^2*c - a*b*d) - 1/3*d*log(abs(d*x^3 + c))/(b*c*
d - a*d^2)
```

maple [A] time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln\left(b\,x^3+a\right)}{3\,(ad-bc)}+\frac{\ln\left(d\,x^3+c\right)}{3ad-3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c),x)
[Out] -1/3/(a*d-b*c)*ln(b*x^3+a)+1/3/(a*d-b*c)*ln(d*x^3+c)
```

maxima [A] time = 0.48, size = 41, normalized size = 0.91

$$\frac{\log\left(bx^3+a\right)}{3\,(bc-ad)}-\frac{\log\left(dx^3+c\right)}{3\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
[Out] 1/3*log(b*x^3 + a)/(b*c - a*d) - 1/3*log(d*x^3 + c)/(b*c - a*d)
```

mupad [B] time = 0.26, size = 602, normalized size = 13.38

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3\right)1i}{3\,a\,d-3\,b\,c}-\left(\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)-\cdots}{\cdots}\right.\cdots}{\frac{\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)+\frac{x^3\left(54\,a^2\,b^3\,d^5+108\,a\,b^4\,c\,d^4+54\,b^5\,c^2\,d^3\right)+108\,a\,b^4\,c^2\,d^3+108\,a^2\,b^3\,c\,d^4}{3\,a\,d-3\,b\,c}+36\,a\,b^3\,c\,d^3}{3\,a\,d-3\,b\,c}+6\,b^3\,d^3\,x^3}{3\,a\,d-3\,b\,c}+\frac{x^3\left(36\,c\,b^4\,d^3+36\,a\,b^3\,d^4\right)-\cdots}{\cdots}\cdots}\right)\cdots}{3\,a\,d-3\,b\,c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] -(atan(((((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b^
5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*
d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)*1i)/(3*a*d -
3*b*c) - (((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)*1i)/(3*a*d -
 3*b*c))/(((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) + (x^3*(54*a^2*b^3*d^5 + 54*b
^5*c^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a
*d - 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) + 6*b^3*d^3*x^3)/(3*a*d - 3*b
*c) + ((x^3*(36*a*b^3*d^4 + 36*b^4*c*d^3) - (x^3*(54*a^2*b^3*d^5 + 54*b^5*c
^2*d^3 + 108*a*b^4*c*d^4) + 108*a*b^4*c^2*d^3 + 108*a^2*b^3*c*d^4)/(3*a*d -
 3*b*c) + 36*a*b^3*c*d^3)/(3*a*d - 3*b*c) - 6*b^3*d^3*x^3)/(3*a*d - 3*b*c))
)*2i)/(3*a*d - 3*b*c)
```

sympy [B] time = 2.47, size = 138, normalized size = 3.07

$$\frac{\log\left(x^3 + \frac{-\frac{a^2d^2}{ad-bc} + \frac{2abcd}{ad-bc} + ad - \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3\,(ad-bc)} - \frac{\log\left(x^3 + \frac{\frac{a^2d^2}{ad-bc} - \frac{2abcd}{ad-bc} + ad + \frac{b^2c^2}{ad-bc} + bc}{2bd}\right)}{3\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] log(x**3 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(3*(a*d - b*c)) - log(x**3 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(3*(a*d - b*c))
```

3.114 $\int \frac{x}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\cdots\right.}{3\sqrt[3]{c}(bc-ad\cdots}$$

```
[Out] -1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*x)/a^(1/3)/(-a*d+b*c)+1/3*d^(1/3)*ln(c^(1/3
)+d^(1/3)*x)/c^(1/3)/(-a*d+b*c)+1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/a^(1/3)/(-a*d+b*c)-1/6*d^(1/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/c^(1/3)/(-a*d+b*c)-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^
(1/3)*3^(1/2))/a^(1/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(c^(1/3)-2
*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(1/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {482, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6\sqrt[3]{c}(bc-ad)}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\cdots\right.}{3\sqrt[3]{c}(bc-ad\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1
/3)*(b*c - a*d))) + (d^(1/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(1/3)*(b*c - a*d)) - (b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(1/3)*(b*c - a*d)) + (d^(1/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(1/3)*(b*c -
 a*d)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(1/3
)*(b*c - a*d)) - (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(1/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{x}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{x}{c+dx^3}\,dx}{bc-ad}$$

$$= -\frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3\sqrt[3]{a}(bc-ad)} + \frac{d^{2/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3\sqrt[3]{c}(bc-ad)} - \frac{d^{2/3}\int \frac{\sqrt[3]{}}{c^{2/3}-\sqrt[3]{c}}}{3\sqrt[3]{c}(b}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6\sqrt[3]{a}(bc-ad)} + \frac{b^{2/3}\int}{}$$

$$= -\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3\sqrt[3]{c}(bc-ad)} + \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6\sqrt[3]{a}(bc-ad)}$$

$$= -\frac{\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)} - \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3\sqrt[3]{a}(bc-ad)} + \frac{\sqrt[3]{d}\log\left(\right.}{3\sqrt[3]{c}(b}$$

Mathematica [A] time = 0.14, size = 224, normalized size = 0.78

$$\frac{-\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{\sqrt[3]{a}} + \frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}} + \frac{2\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\sqrt[3]{d}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{\sqrt[3]{c}} - \frac{2\sqrt[3]{d}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt[3]{c}}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(1/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(1/3) -
(2*Sqrt[3]*d^(1/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(1/3) + (
2*b^(1/3)*Log[a^(1/3) + b^(1/3)*x])/a^(1/3) - (2*d^(1/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(1/3) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(1/3) + (d^(1/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(1/3))
/(-6*b*c + 6*a*d)
```

fricas [A] time = 0.93, size = 201, normalized size = 0.70

$$\frac{2\sqrt{3}\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(bx^2-ax\left(\right.\right.}{6(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) - 1/3*sqrt(3))
- 2*sqrt(3)*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/c)^(1/3) + 1/3*sqrt(3)) +
 (b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(2/3) + a*(b/a)^(1/3)) + (-d/c)^(1/3)*lo
g(d*x^2 - c*x*(-d/c)^(2/3) - c*(-d/c)^(1/3)) - 2*(b/a)^(1/3)*log(b*x + a*(b
/a)^(2/3)) - 2*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)))/(b*c - a*d)
```

giac [A] time = 0.25, size = 290, normalized size = 1.01

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c-\sqrt{3}a^2bd}+\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\right.}{\sqrt{3}bc^2d-\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) - (-a*b^2)^(2/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b^2*c - sqrt(3)*
a^2*b*d) + (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1
/3))/(sqrt(3)*b*c^2*d - sqrt(3)*a*c*d^2) + 1/6*(-a*b^2)^(2/3)*log(x^2 + x*(
-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c - a^2*b*d) - 1/6*(-c*d^2)^(2/3)*log(x^
2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^2*d - a*c*d^2)
```

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\right.}{6(ad-bc)\left(\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6/(a*d-b*c)/(a/b)^(1/3)*ln(x^
2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3
^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3/(a*d-b*c)/(c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6
/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)*3^(1
/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.29, size = 265, normalized size = 0.92

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3(bc-ad)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}-\frac{1\cdots}{3\left(b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c - a*d
)*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(
1/3))/((b*c - a*d)*(c/d)^(1/3)) + 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3)
)/(b*c*(a/b)^(1/3) - a*d*(a/b)^(1/3)) - 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)
^(2/3))/(b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3)) - 1/3*log(x + (a/b)^(1/3))/(b*c
*(a/b)^(1/3) - a*d*(a/b)^(1/3)) + 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(1/3)
 - a*d*(c/d)^(1/3))
```

mupad [B] time = 5.42, size = 982, normalized size = 3.41

$$\ln\left(b\,x+a^3\,d^2\,\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}+a\,b^2\,c^2\,\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}-2\,a^2\,b\,c\,d\,\left(\frac{b}{a\,(a\,d-b\,c)^3}\right)^{2/3}\right)\left(\frac{\cdots}{27\,a^4\,d^3-81\,a^3\,b\,c\,d^2\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b*x + a^3*d^2*(b/(a*(a*d - b*c)^3))^(2/3) + a*b^2*c^2*(b/(a*(a*d - b*c)
^3))^(2/3) - 2*a^2*b*c*d*(b/(a*(a*d - b*c)^3))^(2/3))*(b/(27*a^4*d^3 - 27*a
*b^3*c^3 + 81*a^2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3) + log(d*x + b^2*c^3*(-
d/(c*(a*d - b*c)^3))^(2/3) + a^2*c*d^2*(-d/(c*(a*d - b*c)^3))^(2/3) - 2*a*b
*c^2*d*(-d/(c*(a*d - b*c)^3))^(2/3))*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81*a^2
*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3) + (log(b^4*d^4*x - (b*(3^(1/2)*1i - 1)^
3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1/2
)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(216
*a*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^2*
b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 - (log(b^4*d^4*x + (b*(3^(1/2)*1i + 1
)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^(1
/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b/(a*(a*d - b*c)^3))^(2/3))/4))/(2
16*a*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(b/(27*a^4*d^3 - 27*a*b^3*c^3 + 81*a^
2*b^2*c^2*d - 81*a^3*b*c*d^2))^(1/3))/2 + (log(b^4*d^4*x + (d*(3^(1/2)*1i -
 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*(3^
(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/4))
/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i - 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 + 81
*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2 - (log(b^4*d^4*x - (d*(3^(1/2)*1
i + 1)^3*(27*b^3*d^3*x*(a^2*d^2 + b^2*c^2)*(a*d - b*c)^2 + (27*a*b^3*c*d^3*
(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d/(c*(a*d - b*c)^3))^(2/3))/
4))/(216*c*(a*d - b*c)^3))*(3^(1/2)*1i + 1)*(d/(27*b^3*c^4 - 27*a^3*c*d^3 +
 81*a^2*b*c^2*d^2 - 81*a*b^2*c^3*d))^(1/3))/2
```

sympy [A] time = 14.51, size = 515, normalized size = 1.79

$$\mathrm{RootSum}\left(t^3\left(27a^4d^3-81a^3bcd^2+81a^2b^2c^2d-27ab^3c^3\right)-b,\left(t\mapsto t\log\left(x+\frac{243t^5a^7cd^6-1458t^5a^6bc^2d^5+\cdots}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**4*d**3 - 81*a**3*b*c*d**2 + 81*a**2*b**2*c**2*d - 27*a
*b**3*c**3) - b, Lambda(_t, _t*log(x + (243*_t**5*a**7*c*d**6 - 1458*_t**5*
a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860*_t**5*a**4*b**3*c*
*4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a**2*b**5*c**6*d + 24
3*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*b*c*d**3 + 18*_t**2
*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*b**4*c**4)/(a*b*d**
2 + b**2*c*d)))) + RootSum(_t**3*(27*a**3*c*d**3 - 81*a**2*b*c**2*d**2 + 81
*a*b**2*c**3*d - 27*b**3*c**4) + d, Lambda(_t, _t*log(x + (243*_t**5*a**7*c
*d**6 - 1458*_t**5*a**6*b*c**2*d**5 + 3645*_t**5*a**5*b**2*c**3*d**4 - 4860
*_t**5*a**4*b**3*c**4*d**3 + 3645*_t**5*a**3*b**4*c**5*d**2 - 1458*_t**5*a*
*2*b**5*c**6*d + 243*_t**5*a*b**6*c**7 + 9*_t**2*a**4*d**4 - 18*_t**2*a**3*
b*c*d**3 + 18*_t**2*a**2*b**2*c**2*d**2 - 18*_t**2*a*b**3*c**3*d + 9*_t**2*
b**4*c**4)/(a*b*d**2 + b**2*c*d))))
```

3.115 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6c^{2/3}(bc-ad)}$$

```
[Out] 1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)-1/3*d^(2/3)*ln(c^(1/3)
+d^(1/3)*x)/c^(2/3)/(-a*d+b*c)-1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/a^(2/3)/(-a*d+b*c)+1/6*d^(2/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/c^(2/3)/(-a*d+b*c)-1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(
1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(c^(1/3)-2*
d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(2/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = $\{391, 200, 31, 634, 617, 204, 628\}$

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6c^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\cdots}}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int \cdots}{\cdots}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} \cdots$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log \cdots}{3c^{2/3}\cdots}$$

Mathematica [A] time = 0.18, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\cdots\right)}{c^{2/3}}\cdots}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3))
/(-6*b*c + 6*a*d)
```

fricas [A] time = 1.10, size = 254, normalized size = 0.88

$$-\frac{2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)\cdots\right.}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

giac [A] time = 0.21, size = 278, normalized size = 0.97

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\cdots\right)}{3\cdots}\right)}{\sqrt{3}bc^2-\sqrt{3}a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

maple [A] time = 0.05, size = 222, normalized size = 0.77

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)\cdots\right.}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/(a*d-b*c)/(a/b)^(2/3)*ln(x
^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/(a*d-b*c)/(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/
6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3/(a*d-b*c)/(c/
d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

maxima [A] time = 1.34, size = 293, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}}-ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}}-ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((b*c*(a/b)
^(1/3) - a*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*
x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c*(c/d)^(1/3) - a*d*(c/d)^(1/3))*(c/d)^(1
/3)) - 1/6*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b*c*(a/b)^(2/3) - a*d*(a
/b)^(2/3)) + 1/6*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c*(c/d)^(2/3) -
a*d*(c/d)^(2/3)) + 1/3*log(x + (a/b)^(1/3))/(b*c*(a/b)^(2/3) - a*d*(a/b)^(2
/3)) - 1/3*log(x + (c/d)^(1/3))/(b*c*(c/d)^(2/3) - a*d*(c/d)^(2/3))
```

mupad [B] time = 9.01, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a
*b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))*(a*d + b*
c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-b^2/
(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3) +
log(((d^2/(c^2*(a*d - b*c)^3))^(1/3)*(9*a^2*b^4*d^6 + 9*b^6*c^2*d^4 - 18*a*
b^5*c*d^5 - 9*b^3*d^3*(x + a*c*(d^2/(c^2*(a*d - b*c)^3))^(1/3))*(a*d + b*c)
*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(2/3)))/3 - 6*b^5*d^5*x)*(-d^2/(27
*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3) + (lo
g(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/
2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*a*b^3*c*d^3*(3^(
1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)
*(-b^2/(a^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*
a*b^5*c*d^5))/6)*(-b^2/(27*a^5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81
*a^4*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x - ((3^(1/2)*1i
+ 1)*(-b^2/(a^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*b^3*d^3*x*(a
*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d
 - b*c)^4*(-b^2/(a^2*(a*d - b*c)^3))^(1/3))/2)*(-b^2/(a^2*(a*d - b*c)^3))^(
2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-b^2/(27*a^
5*d^3 - 27*a^2*b^3*c^3 + 81*a^3*b^2*c^2*d - 81*a^4*b*c*d^2))^(1/3)*(3^(1/2)
*1i + 1))/2 + (log(6*b^5*d^5*x + ((3^(1/2)*1i - 1)*(d^2/(c^2*(a*d - b*c)^3)
)^(1/3)*(((3^(1/2)*1i - 1)^2*(81*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 + (81*
a*b^3*c*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c
)^3))^(1/3))/2)*(d^2/(c^2*(a*d - b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6
*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*
b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(6*b^5*d^5*x -
 ((3^(1/2)*1i + 1)*(d^2/(c^2*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81
*b^3*d^3*x*(a*d + b*c)*(a*d - b*c)^4 - (81*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*(d^2/(c^2*(a*d - b*c)^3))^(1/3))/2)*(d^2/(c^2*(a*d -
 b*c)^3))^(2/3))/36 - 9*a^2*b^4*d^6 - 9*b^6*c^2*d^4 + 18*a*b^5*c*d^5))/6)*(
-d^2/(27*b^3*c^5 - 27*a^3*c^2*d^3 + 81*a^2*b*c^3*d^2 - 81*a*b^2*c^4*d))^(1/
3)*(3^(1/2)*1i + 1))/2
```

sympy [A] time = 133.33, size = 447, normalized size = 1.55

$$\text{RootSum}\left(t^3\left(27a^5d^3-81a^4bcd^2+81a^3b^2c^2d-27a^2b^3c^3\right)+b^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t^4a^6bc^3d^4+16\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

3.116 $\int \frac{1}{x(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] ln(x)/a/c-1/3*b*ln(b*x^3+a)/a/(-a*d+b*c)+1/3*d*ln(d*x^3+c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^3])/(3*a*(b*c - a*d)) + (d*Log[c + d*x^3])/(3
*c*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^3\right)\\
&= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^3\right)}{3a(bc-ad)}+\frac{d\log\left(c+dx^3\right)}{3c(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^3\right)+ad\log\left(c+dx^3\right)-3ad\log(x)+3bc\log(x)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (3*b*c*Log[x] - 3*a*d*Log[x] - b*c*Log[a + b*x^3] + a*d*Log[c + d*x^3])/(3*
a*b*c^2 - 3*a^2*c*d)
```

fricas [A] time = 1.68, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^3+a\right)-ad\log\left(dx^3+c\right)-3\,(bc-ad)\log(x)}{3\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/3*(b*c*log(b*x^3 + a) - a*d*log(d*x^3 + c) - 3*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

giac [A] time = 0.18, size = 71, normalized size = 1.15

$$-\frac{b^2\log\left(\left|bx^3+a\right|\right)}{3\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^3+c\right|\right)}{3\left(bc^2d-acd^2\right)}+\frac{\log\left(|x|\right)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^2*log(abs(b*x^3 + a))/(a*b^2*c - a^2*b*d) + 1/3*d^2*log(abs(d*x^3 +
c))/(b*c^2*d - a*c*d^2) + log(abs(x))/(a*c)
```

maple [A] time = 0.05, size = 59, normalized size = 0.95

$$\frac{b\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a}-\frac{d\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b/a/(a*d-b*c)*ln(b*x^3+a)-1/3*d/c/(a*d-b*c)*ln(d*x^3+c)+ln(x)/a/c
```

maxima [A] time = 0.54, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^3+a\right)}{3\left(abc-a^2d\right)}+\frac{d\log\left(dx^3+c\right)}{3\left(bc^2-acd\right)}+\frac{\log\left(x^3\right)}{3\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b*log(b*x^3 + a)/(a*b*c - a^2*d) + 1/3*d*log(d*x^3 + c)/(b*c^2 - a*c*d
) + 1/3*log(x^3)/(a*c)
```

mupad [B] time = 2.84, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^3+a\right)}{3\,a^2\,d-3\,a\,b\,c}+\frac{d\,\ln\left(d\,x^3+c\right)}{3\,b\,c^2-3\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b*log(a + b*x^3))/(3*a^2*d - 3*a*b*c) + (d*log(c + d*x^3))/(3*b*c^2 - 3*a*
c*d) + log(x)/(a*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.117 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=299

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6c^{4/3}(bc-ad)}$$

```
[Out] -1/a/c/x+1/3*b^(4/3)*ln(a^(1/3)+b^(1/3)*x)/a^(4/3)/(-a*d+b*c)-1/3*d^(4/3)*l
n(c^(1/3)+d^(1/3)*x)/c^(4/3)/(-a*d+b*c)-1/6*b^(4/3)*ln(a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/a^(4/3)/(-a*d+b*c)+1/6*d^(4/3)*ln(c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/c^(4/3)/(-a*d+b*c)+1/3*b^(4/3)*arctan(1/3*(a^(1/3)-2*b^(1/
3)*x)/a^(1/3)*3^(1/2))/a^(4/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(c
^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(4/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}+\frac{d^{4/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right.}{6c^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(4/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/
(Sqrt[3]*a^(4/3)*(b*c - a*d)) - (d^(4/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sq
rt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)) + (b^(4/3)*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(4/3)*(b*c - a*d)) - (d^(4/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^
(4/3)*(b*c - a*d)) - (b^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(6*a^(4/3)*(b*c - a*d)) + (d^(4/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2])/(6*c^(4/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{acx}+\frac{\int \frac{x\left(-bc-ad-bdx^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{ac}$$

$$= -\frac{1}{acx}+\frac{\int \left(-\frac{b^2cx}{(bc-ad)\left(a+bx^3\right)}-\frac{ad^2x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{ac}$$

$$= -\frac{1}{acx}-\frac{b^2\int \frac{x}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{x}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx}+\frac{b^{5/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{4/3}(bc-ad)}-\frac{b^{5/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{4/3}(bc-ad)}-\frac{d^{5/3}\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{4/3}(bc-ad)}+\cdots$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\cdots}{6a^{4/3}(bc-ad)}$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}-\frac{d^{4/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{4/3}(bc-ad)}-\frac{b^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{6a^{4/3}(bc-a\cdots}$$

$$= -\frac{1}{acx}+\frac{b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}(bc-ad)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{4/3}(bc-ad)}+\frac{b^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{4/3}(bc-ad)}\cdots$$

Mathematica [A] time = 0.21, size = 244, normalized size = 0.82

$$\frac{\frac{b^{4/3}x\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{4/3}}-\frac{2b^{4/3}x\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{4/3}}-\frac{2\sqrt{3}b^{4/3}x\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}+\frac{6b}{a}-\frac{d^{4/3}x\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{4/3}}+\frac{2d^{4/3}x\log\left(\cdots\right.}{c^{4/3}}}{6adx-6bcx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (2*Sqrt[3]*b^(4/3)*x*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3)
)/Sqrt[3]])/a^(4/3) + (2*Sqrt[3]*d^(4/3)*x*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3
))/Sqrt[3]])/c^(4/3) - (2*b^(4/3)*x*Log[a^(1/3) + b^(1/3)*x])/a^(4/3) + (2*
d^(4/3)*x*Log[c^(1/3) + d^(1/3)*x])/c^(4/3) + (b^(4/3)*x*Log[a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(4/3) - (d^(4/3)*x*Log[c^(2/3) - c^(1/3)*d^
(1/3)*x + d^(2/3)*x^2])/c^(4/3))/(-6*b*c*x + 6*a*d*x)
```

fricas [A] time = 0.98, size = 238, normalized size = 0.80

$$-\frac{2\sqrt{3}\,bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-2\sqrt{3}\,adx\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-bcx\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3) + 1/3*
sqrt(3)) - 2*sqrt(3)*a*d*x*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(d/c)^(1/3) - 1
/3*sqrt(3)) - b*c*x*(-b/a)^(1/3)*log(b*x^2 - a*x*(-b/a)^(2/3) - a*(-b/a)^(1
/3)) - a*d*x*(d/c)^(1/3)*log(d*x^2 - c*x*(d/c)^(2/3) + c*(d/c)^(1/3)) + 2*b
*c*x*(-b/a)^(1/3)*log(b*x + a*(-b/a)^(2/3)) + 2*a*d*x*(d/c)^(1/3)*log(d*x +
 c*(d/c)^(2/3)) + 6*b*c - 6*a*d)/((a*b*c^2 - a^2*c*d)*x)
```

giac [A] time = 0.25, size = 305, normalized size = 1.02

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\cdots\right.}{3\cdots}\right.}{\sqrt{3}\,bc^3-\sqrt{3}\,a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) + (-a*b^2)^(2/3)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c - s
qrt(3)*a^3*d) - (-c*d^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/
d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(2/3)*log(x^2 +
x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(2/3)*log(x
^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/(a*c*x)
```

maple [A] time = 0.06, size = 257, normalized size = 0.86

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} a}-\frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} a}+\frac{b\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}} a}-\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}} c}+\frac{d\ln\left(\cdots\right.}{3\left(ad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b/a/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b/a/(a*d-b*c)/(a/b)^(1
/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b/a/(a*d-b*c)*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d/c/(a*d-b*c)/(c/d)^(1/3)*ln(x
+(c/d)^(1/3))-1/6*d/c/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3
))-1/3*d/c/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*
x-1))-1/a/c/x
```

maxima [A] time = 1.20, size = 300, normalized size = 1.00

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc-a^{2}d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(2\,x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{2}-acd\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\cdots\right.\right.}{6\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c
- a^2*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sqrt(3)*(2*x - (c/d)^(1/3)
)/(c/d)^(1/3))/((b*c^2 - a*c*d)*(c/d)^(1/3)) - 1/6*b*log(x^2 - x*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) + 1/6*d*log(x^2 -
x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) + 1/3*
b*log(x + (a/b)^(1/3))/(a*b*c*(a/b)^(1/3) - a^2*d*(a/b)^(1/3)) - 1/3*d*log(
x + (c/d)^(1/3))/(b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1/3)) - 1/(a*c*x)
```

mupad [B] time = 3.85, size = 716, normalized size = 2.39

$$\ln\left(b-a^{2}\,d\,x\left(-\frac{b^{4}}{a^{4}\left(a\,d-b\,c\right)^{3}}\right)^{1/3}+a\,b\,c\,x\left(-\frac{b^{4}}{a^{4}\left(a\,d-b\,c\right)^{3}}\right)^{1/3}\right)\left(-\frac{b^{4}}{27\,a^{7}\,d^{3}-81\,a^{6}\,b\,c\,d^{2}+81\,a^{5}\,b^{2}\,c^{2}\,d-27\,a^{\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(b - a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) + a*b*c*x*(-b^4/(a^4*(a*d
- b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 8
1*a^6*b*c*d^2))^(1/3) + log(d - b*c^2*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3) + a
*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^7 - 27*a^3*c^4*d^3
+ 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3) - 1/(a*c*x) - (log(b - 3^(1/2)*
b*1i + 2*a^2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a
*d - b*c)^3))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d
```

```
- 81*a^6*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(b + 3^(1/2)*b*1i + 2*a^
2*d*x*(-b^4/(a^4*(a*d - b*c)^3))^(1/3) - 2*a*b*c*x*(-b^4/(a^4*(a*d - b*c)^3
))^(1/3))*(-b^4/(27*a^7*d^3 - 27*a^4*b^3*c^3 + 81*a^5*b^2*c^2*d - 81*a^6*b*
c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(d - 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/
(c^4*(a*d - b*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d
^4/(27*b^3*c^7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)
*(3^(1/2)*1i + 1))/2 + (log(d + 3^(1/2)*d*1i + 2*b*c^2*x*(d^4/(c^4*(a*d - b
*c)^3))^(1/3) - 2*a*c*d*x*(d^4/(c^4*(a*d - b*c)^3))^(1/3))*(-d^4/(27*b^3*c^
7 - 27*a^3*c^4*d^3 + 81*a^2*b*c^5*d^2 - 81*a*b^2*c^6*d))^(1/3)*(3^(1/2)*1i
- 1))/2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.118 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=301

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^2\right.}{6c^{5/3}(bc-ad)}$$

```
[Out] -1/2/a/c/x^2-1/3*b^(5/3)*ln(a^(1/3)+b^(1/3)*x)/a^(5/3)/(-a*d+b*c)+1/3*d^(5/
3)*ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)+1/6*b^(5/3)*ln(a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/a^(5/3)/(-a*d+b*c)-1/6*d^(5/3)*ln(c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)+1/3*b^(5/3)*arctan(1/3*(a^(1/3)-2*b
^(1/3)*x)/a^(1/3)*3^(1/2))/a^(5/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(5/3)*arctan(1/
3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{5/3}(bc-ad)}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}(bc-ad)}-\frac{d^{5/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^2\right.}{6c^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(2*a*c*x^2) + (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(5/3)*(b*c - a*d)) - (d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)) - (b^(5/3)*Log[a^(1/3) + b
^(1/3)*x])/(3*a^(5/3)*(b*c - a*d)) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*
c^(5/3)*(b*c - a*d)) + (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/(6*a^(5/3)*(b*c - a*d)) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2])/(6*c^(5/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
```

```
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{2acx^2}+\frac{\int \frac{-2(bc+ad)-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{2ac}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{a+bx^3}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^3}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{2acx^2}-\frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{5/3}(bc-ad)}-\frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{5/3}(bc-ad)}+\frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}\,x}\,dx}{3c^{5/3}(bc-ad)}+\frac{d^2\int \cdots}{\cdots}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\cdots}{6a^{5/3}(bc-ad)}$$

$$= -\frac{1}{2acx^2}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)}+\frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{5/3}(bc-ad)}+\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots\right.}{6a^{5/3}(bc-ad\cdots}$$

$$= -\frac{1}{2acx^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}(bc-ad)}-\frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)}-\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{5/3}(bc-ad)}\cdots$$

Mathematica [A] time = 0.22, size = 259, normalized size = 0.86

$$\frac{\frac{2b^{5/3}x^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{b^{5/3}x^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}-\frac{2\sqrt{3}b^{5/3}x^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3b}{a}-\frac{2d^{5/3}x^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{d^{5/3}x^2\log\left(c^{2/3}\cdots\right.}{\cdots}}{6x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (2*Sqrt[3]*b^(5/3)*x^2*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/
3))/Sqrt[3]])/a^(5/3) + (2*Sqrt[3]*d^(5/3)*x^2*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]])/c^(5/3) + (2*b^(5/3)*x^2*Log[a^(1/3) + b^(1/3)*x])/a^(5/3)
 - (2*d^(5/3)*x^2*Log[c^(1/3) + d^(1/3)*x])/c^(5/3) - (b^(5/3)*x^2*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3) + (d^(5/3)*x^2*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(5/3))/(6*(-(b*c) + a*d)*x^2)
```

fricas [A] time = 3.35, size = 301, normalized size = 1.00

$$-\frac{2\sqrt{3}\,bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)+2\sqrt{3}\,adx^2\left(-\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(-\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)-bcx^2\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*b*c*x^2*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(b^2/a^2)
^(2/3) - sqrt(3)*b)/b) + 2*sqrt(3)*a*d*x^2*(-d^2/c^2)^(1/3)*arctan(1/3*(2*s
qrt(3)*c*x*(-d^2/c^2)^(2/3) - sqrt(3)*d)/d) - b*c*x^2*(b^2/a^2)^(1/3)*log(b
^2*x^2 - a*b*x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) - a*d*x^2*(-d^2/c^2)^
(1/3)*log(d^2*x^2 + c*d*x*(-d^2/c^2)^(1/3) + c^2*(-d^2/c^2)^(2/3)) + 2*b*c*
x^2*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)^(1/3)) + 2*a*d*x^2*(-d^2/c^2)^(1/
3)*log(d*x - c*(-d^2/c^2)^(1/3)) + 3*b*c - 3*a*d)/((a*b*c^2 - a^2*c*d)*x^2)
```

giac [A] time = 0.21, size = 309, normalized size = 1.03

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^2bc-\sqrt{3}\,a^3d}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\cdots}{\sqrt{3}\,bc^3-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b*c - a^3*d) - 1/3*d^2
*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^3 - a*c^2*d) - (-a*b^2)^(1/3)
*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b*c -
 sqrt(3)*a^3*d) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/
(-c/d)^(1/3))/(sqrt(3)*b*c^3 - sqrt(3)*a*c^2*d) - 1/6*(-a*b^2)^(1/3)*b*log(
x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b*c - a^3*d) + 1/6*(-c*d^2)^(1/3)
*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^3 - a*c^2*d) - 1/2/(a*c*x^
2)
```

maple [A] time = 0.05, size = 257, normalized size = 0.85

$$\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a}+\frac{b\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a}-\frac{b\ln\left(x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}} x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}} a}-\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}} c}-\frac{d\ln\left(x+\right.}{3\left(ad-bc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3/a*b/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))-1/6/a*b/(a*d-b*c)/(a/b)^(2/
3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3/a*b/(a*d-b*c)/(a/b)^(2/3)*3^(1/2)*
arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/2/a/c/x^2-1/3/c*d/(a*d-b*c)/(c/d)
^(2/3)*ln(x+(c/d)^(1/3))+1/6/c*d/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))-1/3/c*d/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(
c/d)^(1/3)*x-1))
```

maxima [A] time = 1.14, size = 328, normalized size = 1.09

$$-\frac{\sqrt{3}\, b \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(abc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{1}{3}}-acd\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b\log\left(x^{2}-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(abc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^{2}d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}-\frac{d\log\left(x^{2}-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\right.\right.}{6\left(bc^{2}\left(\frac{c}{d}\right)^{\frac{2}{3}}-acd\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a*b*c*
(a/b)^(1/3) - a^2*d*(a/b)^(1/3))*(a/b)^(1/3)) + 1/3*sqrt(3)*d*arctan(1/3*sq
rt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^2*(c/d)^(1/3) - a*c*d*(c/d)^(1
/3))*(c/d)^(1/3)) + 1/6*b*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(a*b*c*(a/
b)^(2/3) - a^2*d*(a/b)^(2/3)) - 1/6*d*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3)
)/(b*c^2*(c/d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/3*b*log(x + (a/b)^(1/3))/(a*b
*c*(a/b)^(2/3) - a^2*d*(a/b)^(2/3)) + 1/3*d*log(x + (c/d)^(1/3))/(b*c^2*(c/
d)^(2/3) - a*c*d*(c/d)^(2/3)) - 1/2/(a*c*x^2)
```

mupad [B] time = 11.83, size = 1829, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^5/(a^5*(a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a
*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))*(b^5/(a^5*(a*d - b*c)^3))^(2/3))/9 + 9
*a^6*b^9*c^11*d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^
6*d^9))/3 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(27*a^8*d^3 - 27*
a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3) + log(((-d^5/(c^5*(
a*d - b*c)^3))^(1/3)*(((81*a^10*b^3*c^10*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^
5/(c^5*(a*d - b*c)^3))^(1/3) - 81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2
+ b^2*c^2 + a*b*c*d))*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/9 + 9*a^6*b^9*c^11*
```

```
d^4 - 9*a^7*b^8*c^10*d^5 - 9*a^10*b^5*c^7*d^8 + 9*a^11*b^4*c^6*d^9))/3 + 3*
a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c^5*d^3 +
81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(b^5/(a^
5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b
*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1
)*(a*d + b*c)*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(
a*d - b*c)^3))^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10
*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*
c^2))*(b^5/(27*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(b^5/(a^5*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^
2 + b^2*c^2 + a*b*c*d) + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)
*(a*d - b*c)^4*(b^5/(a^5*(a*d - b*c)^3))^(1/3))/2)*(b^5/(a^5*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8
- 9*a^11*b^4*c^6*d^9))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(b^5/(2
7*a^8*d^3 - 27*a^5*b^3*c^3 + 81*a^6*b^2*c^2*d - 81*a^7*b*c*d^2))^(1/3)*(3^(
1/2)*1i + 1))/2 + (log(((3^(1/2)*1i - 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(
((3^(1/2)*1i - 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2
+ a*b*c*d) - (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)
^4*(-d^5/(c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/3
6 - 9*a^6*b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b
^4*c^6*d^9))/6 - 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8
- 27*a^3*c^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i -
1))/2 - (log(((3^(1/2)*1i + 1)*(-d^5/(c^5*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*
1i + 1)^2*(81*a^8*b^3*c^8*d^3*x*(a*d - b*c)^4*(a^2*d^2 + b^2*c^2 + a*b*c*d)
 + (81*a^10*b^3*c^10*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-d^5/(
c^5*(a*d - b*c)^3))^(1/3))/2)*(-d^5/(c^5*(a*d - b*c)^3))^(2/3))/36 - 9*a^6*
b^9*c^11*d^4 + 9*a^7*b^8*c^10*d^5 + 9*a^10*b^5*c^7*d^8 - 9*a^11*b^4*c^6*d^9
))/6 + 3*a^6*b^6*c^6*d^6*x*(a^2*d^2 + b^2*c^2))*(d^5/(27*b^3*c^8 - 27*a^3*c
^5*d^3 + 81*a^2*b*c^6*d^2 - 81*a*b^2*c^7*d))^(1/3)*(3^(1/2)*1i + 1))/2 - 1/
(2*a*c*x^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.119 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

```
[Out] -1/3/a/c/x^3-(a*d+b*c)*ln(x)/a^2/c^2+1/3*b^2*ln(b*x^3+a)/a^2/(-a*d+b*c)-1/3
*d^2*ln(d*x^3+c)/c^2/(-a*d+b*c)
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(3*a*c*x^3) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^3])/(3*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx,\right.\\ &= -\frac{1}{3acx^3}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^3\right)}{3a^2(bc-ad)}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^3\right)}{3a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^3\right)}{3c^2(bc-ad)}-\frac{1}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/3*1/(a*c*x^3) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^3])
/(3*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^3])/(3*c^2*(b*c - a*d))
```

fricas [A] time = 5.07, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^3\log\left(bx^3+a\right)-a^2d^2x^3\log\left(dx^3+c\right)-3\left(b^2c^2-a^2d^2\right)x^3\log(x)-abc^2+a^2cd}{3\left(a^2bc^3-a^3c^2d\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*c^2*x^3*log(b*x^3 + a) - a^2*d^2*x^3*log(d*x^3 + c) - 3*(b^2*c^2 -
 a^2*d^2)*x^3*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^3)
```

giac [A] time = 0.19, size = 111, normalized size = 1.28

$$\frac{b^3\log\left(\left|bx^3+a\right|\right)}{3\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(\left|dx^3+c\right|\right)}{3\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(|x|\right)}{a^2c^2}+\frac{bcx^3+adx^3-ac}{3\,a^2c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^3*log(abs(b*x^3 + a))/(a^2*b^2*c - a^3*b*d) - 1/3*d^3*log(abs(d*x^3 +
 c))/(b*c^3*d - a*c^2*d^2) - (b*c + a*d)*log(abs(x))/(a^2*c^2) + 1/3*(b*c*x
^3 + a*d*x^3 - a*c)/(a^2*c^2*x^3)
```

maple [A] time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a^2}+\frac{d^2\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c^2}-\frac{d\ln(x)}{a\,c^2}-\frac{b\ln(x)}{a^2c}-\frac{1}{3ac\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b^2/a^2/(a*d-b*c)*ln(b*x^3+a)+1/3*d^2/c^2/(a*d-b*c)*ln(d*x^3+c)-1/3/a/
c/x^3-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

maxima [A] time = 0.49, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^3+a\right)}{3\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^3+c\right)}{3\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^3\right)}{3\,a^2c^2}-\frac{1}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*b^2*log(b*x^3 + a)/(a^2*b*c - a^3*d) - 1/3*d^2*log(d*x^3 + c)/(b*c^3 -
a*c^2*d) - 1/3*(b*c + a*d)*log(x^3)/(a^2*c^2) - 1/3/(a*c*x^3)
```

mupad [B] time = 3.22, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln\left(b\,x^3+a\right)}{3\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^3+c\right)}{3\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{3\,a\,c\,x^3}-\frac{\ln(x)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] - (b^2*log(a + b*x^3))/(3*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^3))/(3*(b*c
^3 - a*c^2*d)) - 1/(3*a*c*x^3) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.120 $\int \frac{1}{x^5(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=318

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\right.}{6c^{7/3}(bc-}$$

```
[Out] -1/4/a/c/x^4+(a*d+b*c)/a^2/c^2/x-1/3*b^(7/3)*ln(a^(1/3)+b^(1/3)*x)/a^(7/3)/
(-a*d+b*c)+1/3*d^(7/3)*ln(c^(1/3)+d^(1/3)*x)/c^(7/3)/(-a*d+b*c)+1/6*b^(7/3)
*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(7/3)/(-a*d+b*c)-1/6*d^(7/3)*l
n(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(7/3)/(-a*d+b*c)-1/3*b^(7/3)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(7/3)/(-a*d+b*c)*3^(1/2)+1
/3*d^(7/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(7/3)/(-a*d+
b*c)*3^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$\frac{b^{7/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{6a^{7/3}(bc-ad)}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{ad+bc}{a^2c^2x}-\frac{d^{7/3}\log\left(c^{2/3}-\sqrt[3]{c}\right.}{6c^{7/3}(bc-}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(4*a*c*x^4) + (b*c + a*d)/(a^2*c^2*x) - (b^(7/3)*ArcTan[(a^(1/3) - 2*b^(
1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*(b*c - a*d)) + (d^(7/3)*ArcTan
[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d))
- (b^(7/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(7/3)*(b*c - a*d)) + (d^(7/3)*Log
[c^(1/3) + d^(1/3)*x])/(3*c^(7/3)*(b*c - a*d)) + (b^(7/3)*Log[a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(7/3)*(b*c - a*d)) - (d^(7/3)*Log[c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(7/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{4acx^4}+\frac{\int \frac{-4(bc+ad)-4bdx^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4ac}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \frac{x\left(-4\left(b^2c^2+abcd+a^2d^2\right)-4bd(bc+ad)x^3\right)}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{\int \left(-\frac{4b^3c^2x}{(bc-ad)\left(a+bx^3\right)}-\frac{4a^2d^3x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{4a^2c^2}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}+\frac{b^3\int \frac{x}{a+bx^3}\,dx}{a^2(bc-ad)}-\frac{d^3\int \frac{x}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{8/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{7/3}(bc-ad)}+\frac{b^{8/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{7/3}(bc-ad)}+\frac{d^{8/3}\int \frac{}{\sqrt[3]{}}}{3c^{7/3}(}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\int \frac{}{a^{2/3}}}{6a}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{7/3}(bc-ad)}+\frac{d^{7/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{7/3}(bc-ad)}+\frac{b^{7/3}\log(}{}$$

$$= -\frac{1}{4acx^4}+\frac{bc+ad}{a^2c^2x}-\frac{b^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}(bc-ad)}+\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)}-\frac{b^{7/3}\log(}{3a^{7/3}}$$

Mathematica [A] time = 0.26, size = 282, normalized size = 0.89

$$\frac{\frac{4b^{7/3}x^4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{7/3}}+\frac{4\sqrt{3}\,b^{7/3}x^4\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{7/3}}-\frac{2b^{7/3}x^4\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{7/3}}-\frac{12b^2x^3}{a^2}+\frac{3b}{a}-\frac{4d^{7/3}x^4\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{c^{7/3}}-\ -\ 4}{12x^4(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((3*b)/a - (3*d)/c - (12*b^2*x^3)/a^2 + (12*d^2*x^3)/c^2 + (4*Sqrt[3]*b^(7/
3)*x^4*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(7/3) - (4*Sqrt[3]*d^
(7/3)*x^4*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(7/3) + (4*b^(7/3)
*x^4*Log[a^(1/3) + b^(1/3)*x])/a^(7/3) - (4*d^(7/3)*x^4*Log[c^(1/3) + d^(1/
3)*x])/c^(7/3) - (2*b^(7/3)*x^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2])/a^(7/3) + (2*d^(7/3)*x^4*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
])/c^(7/3))/(12*(-(b*c) + a*d)*x^4)
```

fricas [A] time = 3.28, size = 305, normalized size = 0.96

$$\frac{4\sqrt{3}\,b^2c^2x^4\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-4\sqrt{3}\,a^2d^2x^4\left(-\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x\left(-\frac{d}{c}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)+2\,b^2c^2x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b^2*c^2*x^4*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(b/a)^(1/3) -
1/3*sqrt(3)) - 4*sqrt(3)*a^2*d^2*x^4*(-d/c)^(1/3)*arctan(2/3*sqrt(3)*x*(-d/
c)^(1/3) + 1/3*sqrt(3)) + 2*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x^2 - a*x*(b/a)^(
2/3) + a*(b/a)^(1/3)) + 2*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x^2 - c*x*(-d/c)^(
2/3) - c*(-d/c)^(1/3)) - 4*b^2*c^2*x^4*(b/a)^(1/3)*log(b*x + a*(b/a)^(2/3))
 - 4*a^2*d^2*x^4*(-d/c)^(1/3)*log(d*x + c*(-d/c)^(2/3)) - 3*a*b*c^2 + 3*a^2
*c*d + 12*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

giac [A] time = 0.24, size = 328, normalized size = 1.03

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}-\frac{\left(-ab^2\right)^{\frac{2}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^3bc-\sqrt{3}\,a^4d}+\frac{\left(-cd^2\right)^{\frac{2}{3}}d\arctan\left(\right.}{\sqrt{3}\,bc^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) - (-a*b^2)^(2/3
)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*c
- sqrt(3)*a^4*d) + (-c*d^2)^(2/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))
/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(2/3)*b*log
(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2)^(2/3
)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/4*(4*b*c
*x^3 + 4*a*d*x^3 - a*c)/(a^2*c^2*x^4)
```

maple [A] time = 0.05, size = 291, normalized size = 0.92

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{b^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}-\frac{b^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2}+\frac{\sqrt{3}\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^2}-\frac{d^2}{3\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b^2/a^2/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))-1/6*b^2/a^2/(a*d-b*c)/(
a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3*b^2/a^2/(a*d-b*c)*3^(1/2)/
(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))-1/3*d^2/c^2/(a*d-b*c)/(
c/d)^(1/3)*ln(x+(c/d)^(1/3))+1/6*d^2/c^2/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d)
^(1/3)*x+(c/d)^(2/3))+1/3*d^2/c^2/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3*
3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/4/a/c/x^4+1/a/c^2/x*d+1/a^2/c/x*b
```

maxima [A] time = 1.13, size = 341, normalized size = 1.07

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc-a^3d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,d^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3-ac^2d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}+\frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}-\frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right.\right.}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c - a^3*d)*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1/3*sqrt(3)*(2*x - (c/d)^
(1/3))/(c/d)^(1/3))/((b*c^3 - a*c^2*d)*(c/d)^(1/3)) + 1/6*b^2*log(x^2 - x*(
a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3)) - 1/6*d
^2*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d
)^(1/3)) - 1/3*b^2*log(x + (a/b)^(1/3))/(a^2*b*c*(a/b)^(1/3) - a^3*d*(a/b)^
(1/3)) + 1/3*d^2*log(x + (c/d)^(1/3))/(b*c^3*(c/d)^(1/3) - a*c^2*d*(c/d)^(1
/3)) + 1/4*(4*(b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^4)
```

mupad [B] time = 11.37, size = 1734, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b
^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(b^7
/(a^7*(a*d - b*c)^3))^(2/3))*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^
11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13
*d^11))/9 + a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^
8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3) + log(((-d^7/(c^7*(a*d - b*c)^3))^(2/3
)*(((27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + 27*a^19*b^3
*c^19*d^3*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))*(-d^7
/(c^7*(a*d - b*c)^3))^(1/3))/3 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^
5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/9 + a^13*b^9*c^13*d^9*x)*
(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1
/3) - (1/(4*a*c) - (x^3*(a*d + b*c))/(a^2*c^2))/x^4 + (log(((3^(1/2)*1i - 1
)^2*(b^7/(a^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*a^14*b^3*c^14*d^
3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i -
 1)^2*(a*d + b*c)*(a*d - b*c)^4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a
^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^14*b^10*c^19*d^5 -
 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*b^9*c^13*d^9*x)*(b
^7/(27*a^10*d^3 - 27*a^7*b^3*c^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3
)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(b^7/(a^7*(a*d - b*c)^3))^
(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d -
 b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^
4*(b^7/(a^7*(a*d - b*c)^3))^(2/3))/4)*(b^7/(a^7*(a*d - b*c)^3))^(1/3))/6 -
9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b^5*c^14*d^10 - 9*a^20
*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(b^7/(27*a^10*d^3 - 27*a^7*b^3*c
^3 + 81*a^8*b^2*c^2*d - 81*a^9*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2 + (log((
(3^(1/2)*1i - 1)^2*(-d^7/(c^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(27*
a^14*b^3*c^14*d^3*x*(a^6*d^6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d
^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^
(2/3))/4)*(-d^7/(c^7*(a*d - b*c)^3))^(1/3))/6 + 9*a^13*b^11*c^20*d^4 - 9*a^
14*b^10*c^19*d^5 - 9*a^19*b^5*c^14*d^10 + 9*a^20*b^4*c^13*d^11))/36 + a^13*
b^9*c^13*d^9*x)*(d^7/(27*b^3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*
a*b^2*c^9*d))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)^2*(-d^7/(c
^7*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i + 1)*(27*a^14*b^3*c^14*d^3*x*(a^6*d^
6 + b^6*c^6)*(a*d - b*c)^2 + (27*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)^2*(a*d
+ b*c)*(a*d - b*c)^4*(-d^7/(c^7*(a*d - b*c)^3))^(2/3))/4)*(-d^7/(c^7*(a*d -
 b*c)^3))^(1/3))/6 - 9*a^13*b^11*c^20*d^4 + 9*a^14*b^10*c^19*d^5 + 9*a^19*b
^5*c^14*d^10 - 9*a^20*b^4*c^13*d^11))/36 - a^13*b^9*c^13*d^9*x)*(d^7/(27*b^
3*c^10 - 27*a^3*c^7*d^3 + 81*a^2*b*c^8*d^2 - 81*a*b^2*c^9*d))^(1/3)*(3^(1/2
)*1i + 1))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{1}{x^6(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=321

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{\right.}{6c^{8/3}(b}$$

```
[Out] -1/5/a/c/x^5+1/2*(a*d+b*c)/a^2/c^2/x^2+1/3*b^(8/3)*ln(a^(1/3)+b^(1/3)*x)/a^
(8/3)/(-a*d+b*c)-1/3*d^(8/3)*ln(c^(1/3)+d^(1/3)*x)/c^(8/3)/(-a*d+b*c)-1/6*b
^(8/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(8/3)/(-a*d+b*c)+1/6*d^(
8/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(8/3)/(-a*d+b*c)-1/3*b^(8/
3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(8/3)/(-a*d+b*c)*3^(
1/2)+1/3*d^(8/3)*arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(8/3)/
(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{8/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{8/3}(bc-ad)}+\frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{8/3}(bc-ad)}-\frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{8/3}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}+\frac{d^{8/3}\log\left(c^{2/3}-\sqrt[3]{\right.}{6c^{8/3}(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(2*a^2*c^2*x^2) - (b^(8/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Ar
cTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*
d)) + (b^(8/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(8/3)*(b*c - a*d)) - (d^(8/3)
*Log[c^(1/3) + d^(1/3)*x])/(3*c^(8/3)*(b*c - a*d)) - (b^(8/3)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(8/3)*(b*c - a*d)) + (d^(8/3)*Log[c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(8/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
```

```
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{5acx^5} + \frac{\int \frac{-5(bc+ad)-5bdx^3}{x^3\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} - \frac{\int \frac{-10\left(b^2c^2+abcd+a^2d^2\right)-10bd(bc+ad)x^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{10a^2c^2}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^3\int \frac{1}{a+bx^3}\,dx}{a^2(bc-ad)} - \frac{d^3\int \frac{1}{c+dx^3}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^3\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx}{3a^{8/3}(bc-ad)} + \frac{b^3\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx}{3a^{8/3}(bc-ad)} - \frac{d^3\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{\ldots}}}{3c^{8/3}(bc-\ldots}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)} - \frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)} - \frac{b^{8/3}\int \ldots}{6a\ldots}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} + \frac{b^{8/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{3a^{8/3}(bc-ad)} - \frac{d^{8/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{3c^{8/3}(bc-ad)} - \frac{b^{8/3}\log\left(\ldots\right.}{\ldots}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{2a^2c^2x^2} - \frac{b^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\,x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}(bc-ad)} + \frac{d^{8/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{8/3}(bc-ad)} + \frac{b^{8/3}\log \ldots}{3a^{8/3}\ldots}$$

Mathematica [A] time = 0.22, size = 282, normalized size = 0.88

$$\frac{-\frac{10b^{8/3}x^5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{a^{8/3}} + \frac{10\sqrt{3}\,b^{8/3}x^5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{8/3}} + \frac{5b^{8/3}x^5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2\right)}{a^{8/3}} - \frac{15b^2x^3}{a^2} + \frac{6b}{a} + \frac{10d^{8/3}x^5\log\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{c^{8/3}}\ldots}{30x^5(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((6*b)/a - (6*d)/c - (15*b^2*x^3)/a^2 + (15*d^2*x^3)/c^2 + (10*Sqrt[3]*b^(8
/3)*x^5*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(8/3) - (10*Sqrt[3]*
d^(8/3)*x^5*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(8/3) - (10*b^(8
/3)*x^5*Log[a^(1/3) + b^(1/3)*x])/a^(8/3) + (10*d^(8/3)*x^5*Log[c^(1/3) + d
^(1/3)*x])/c^(8/3) + (5*b^(8/3)*x^5*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/a^(8/3) - (5*d^(8/3)*x^5*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/c^(8/3))/(30*(-(b*c) + a*d)*x^5)
```

fricas [A] time = 1.02, size = 356, normalized size = 1.11

$$-\frac{10\sqrt{3}\,b^2c^2x^5\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3b}\right) + 10\sqrt{3}\,a^2d^2x^5\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3d}\right) - 5\,b^2c^2x^5\left(-\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-
b^2/a^2)^(2/3) - sqrt(3)*b)/b) + 10*sqrt(3)*a^2*d^2*x^5*(d^2/c^2)^(1/3)*arc
```

```
tan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3) - sqrt(3)*d)/d) - 5*b^2*c^2*x^5*(-b^
2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) -
 5*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d
^2/c^2)^(2/3)) + 10*b^2*c^2*x^5*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/
3)) + 10*a^2*d^2*x^5*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) + 6*a*b*c
^2 - 6*a^2*c*d - 15*(b^2*c^2 - a^2*d^2)*x^3)/((a^2*b*c^3 - a^3*c^2*d)*x^5)
```

giac [A] time = 0.21, size = 336, normalized size = 1.05

$$-\frac{b^3\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}a^3bc-\sqrt{3}a^4d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}d^2\arctan}{\sqrt{3}bc^4-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^3*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^3*b*c - a^4*d) + 1/3*d^
3*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^4 - a*c^3*d) + (-a*b^2)^(1/3
)*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^3*b*
c - sqrt(3)*a^4*d) - (-c*d^2)^(1/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1
/3))/(-c/d)^(1/3))/(sqrt(3)*b*c^4 - sqrt(3)*a*c^3*d) + 1/6*(-a*b^2)^(1/3)*b
^2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b*c - a^4*d) - 1/6*(-c*d^2
)^(1/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^4 - a*c^3*d) + 1/
10*(5*b*c*x^3 + 5*a*d*x^3 - 2*a*c)/(a^2*c^2*x^5)
```

maple [A] time = 0.05, size = 293, normalized size = 0.91

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}-\frac{b^2\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{b^2\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{2}{3}}a^2}+\frac{\sqrt{3}\,d^2\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{2}{3}}c^2}+\frac{d^2}{3(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/a^2*b^2/(a*d-b*c)/(a/b)^(2/3)*ln(x+(a/b)^(1/3))+1/6/a^2*b^2/(a*d-b*c)/
(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))-1/3/a^2*b^2/(a*d-b*c)/(a/b)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3/c^2*d^2/(a*d-b*c)/
(c/d)^(2/3)*ln(x+(c/d)^(1/3))-1/6/c^2*d^2/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d
)^(1/3)*x+(c/d)^(2/3))+1/3/c^2*d^2/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/5/a/c/x^5+1/2/a/c^2/x^2*d+1/2/a^2/c/x^2*b
```

maxima [A] time = 1.23, size = 369, normalized size = 1.15

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^2bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}\,d^2\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b^2\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^2bc\left(\frac{a}{b}\right)^{\frac{2}{3}}-a^3d\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}+\frac{d^2\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}\right.}{6\left(bc^3\left(\frac{c}{d}\right)^{\frac{2}{3}}-ac^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^2*b
*c*(a/b)^(1/3) - a^3*d*(a/b)^(1/3))*(a/b)^(1/3)) - 1/3*sqrt(3)*d^2*arctan(1
/3*sqrt(3)*(2*x - (c/d)^(1/3))/(c/d)^(1/3))/((b*c^3*(c/d)^(1/3) - a*c^2*d*(
c/d)^(1/3))*(c/d)^(1/3)) - 1/6*b^2*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(
a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) + 1/6*d^2*log(x^2 - x*(c/d)^(1/3)
+ (c/d)^(2/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/3*b^2*log(x +
(a/b)^(1/3))/(a^2*b*c*(a/b)^(2/3) - a^3*d*(a/b)^(2/3)) - 1/3*d^2*log(x + (c
/d)^(1/3))/(b*c^3*(c/d)^(2/3) - a*c^2*d*(c/d)^(2/3)) + 1/10*(5*(b*c + a*d)*
x^3 - 2*a*c)/(a^2*c^2*x^5)
```

mupad [B] time = 11.57, size = 1860, normalized size = 5.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^12*b^12*c
^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*c^16*d^3
*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(-b^8/(a^8*(a*d - b*c)^3))^(1/3) + a^2*
d^2*x + b^2*c^2*x)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*c^12*d
^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*
c^2*d - 81*a^10*b*c*d^2))^(1/3) - (1/(5*a*c) - (x^3*(a*d + b*c))/(2*a^2*c^2
))/x^5 + log(((d^8/(c^8*(a*d - b*c)^3))^(1/3)*(9*a^13*b^11*c^19*d^5 - 9*a^1
2*b^12*c^20*d^4 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12 + 9*a^16*b^3*
c^16*d^3*(a*d + b*c)*(a*d - b*c)^4*(a^3*c^3*(d^8/(c^8*(a*d - b*c)^3))^(1/3)
 + a^2*d^2*x + b^2*c^2*x)*(d^8/(c^8*(a*d - b*c)^3))^(2/3)))/3 + 3*a^12*b^7*
c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^
2*b*c^9*d^2 - 81*a*b^2*c^10*d))^(1/3) + (log(((3^(1/2)*1i - 1)*(-b^8/(a^8*(
a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*
c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^
3*(3^(1/2)*1i - 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(a^8*(a*d - b*c)^3))^(1/
3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13
*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b
^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^3 - 27*a^8*b^3*c^3 + 81
*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/
2)*1i + 1)*(-b^8/(a^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b
^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)
 - (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(-b^8/(
a^8*(a*d - b*c)^3))^(1/3))/2)*(-b^8/(a^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12
*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c
^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4))*(-b^8/(27*a^11*d^
3 - 27*a^8*b^3*c^3 + 81*a^9*b^2*c^2*d - 81*a^10*b*c*d^2))^(1/3)*(3^(1/2)*1i
 + 1))/2 + (log(((3^(1/2)*1i - 1)*(d^8/(c^8*(a*d - b*c)^3))^(1/3)*(((3^(1/2
)*1i - 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*d^3 + b^3*c^3 + a*b^
2*c^2*d + a^2*b*c*d^2) + (81*a^19*b^3*c^19*d^3*(3^(1/2)*1i - 1)*(a*d + b*c)
*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8/(c^8*(a*d - b*c)^3)
)^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d^5 + 9*a^19*b^5*c^13
*d^11 - 9*a^20*b^4*c^12*d^12))/6 + 3*a^12*b^7*c^12*d^7*x*(a^4*d^4 + b^4*c^4
))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d^2 - 81*a*b^2*c^10*d
))^(1/3)*(3^(1/2)*1i - 1))/2 - (log(((3^(1/2)*1i + 1)*(d^8/(c^8*(a*d - b*c)
^3))^(1/3)*(((3^(1/2)*1i + 1)^2*(81*a^16*b^3*c^16*d^3*x*(a*d - b*c)^4*(a^3*
d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2) - (81*a^19*b^3*c^19*d^3*(3^(1/2)
*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*(d^8/(c^8*(a*d - b*c)^3))^(1/3))/2)*(d^8
/(c^8*(a*d - b*c)^3))^(2/3))/36 - 9*a^12*b^12*c^20*d^4 + 9*a^13*b^11*c^19*d
^5 + 9*a^19*b^5*c^13*d^11 - 9*a^20*b^4*c^12*d^12))/6 - 3*a^12*b^7*c^12*d^7*
x*(a^4*d^4 + b^4*c^4))*(-d^8/(27*b^3*c^11 - 27*a^3*c^8*d^3 + 81*a^2*b*c^9*d
^2 - 81*a*b^2*c^10*d))^(1/3)*(3^(1/2)*1i + 1))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)/(d*x**3+c),x)
[Out] Timed out
```

3.122 $\int \frac{1}{x^7(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=119

$$-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/6/a/c/x^6+1/3*(a*d+b*c)/a^2/c^2/x^3+(a^2*d^2+a*b*c*d+b^2*c^2)*ln(x)/a^3/
c^3-1/3*b^3*ln(b*x^3+a)/a^3/(-a*d+b*c)+1/3*d^3*ln(d*x^3+c)/c^3/(-a*d+b*c)
```

Rubi [A] time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*d^
2)*Log[x])/(a^3*c^3) - (b^3*Log[a + b*x^3])/(3*a^3*(b*c - a*d)) + (d^3*Log[
c + d*x^3])/(3*c^3*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(a+bx^3\right)\left(c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(a+bx)(c+dx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{acx^3}+\frac{-bc-ad}{a^2c^2x^2}+\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^4}{a^3(-bc+ad)(a+bx)}+\right.\right.\\ &= -\frac{1}{6acx^6}+\frac{bc+ad}{3a^2c^2x^3}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log(x)}{a^3c^3}-\frac{b^3\log\left(a+bx^3\right)}{3a^3(bc-ad)}+\frac{d^3\log(}{3c^3(b}\end{aligned}$$

Mathematica [A] time = 0.06, size = 119, normalized size = 1.00

$$\frac{b^3\log\left(a+bx^3\right)}{3a^3(ad-bc)}+\frac{ad+bc}{3a^2c^2x^3}+\frac{\log(x)\left(a^2d^2+abcd+b^2c^2\right)}{a^3c^3}+\frac{d^3\log\left(c+dx^3\right)}{3c^3(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/6*1/(a*c*x^6) + (b*c + a*d)/(3*a^2*c^2*x^3) + ((b^2*c^2 + a*b*c*d + a^2*
d^2)*Log[x])/(a^3*c^3) + (b^3*Log[a + b*x^3])/(3*a^3*(-(b*c) + a*d)) + (d^3
*Log[c + d*x^3])/(3*c^3*(b*c - a*d))
```

fricas [A] time = 12.87, size = 127, normalized size = 1.07

$$-\frac{2\,b^3c^3x^6\log\left(bx^3+a\right)-2\,a^3d^3x^6\log\left(dx^3+c\right)-6\left(b^3c^3-a^3d^3\right)x^6\log(x)+a^2bc^3-a^3c^2d-2\left(ab^2c^3-a^3cd^2\right)x^3}{6\left(a^3bc^4-a^4c^3d\right)x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3*x^6*log(b*x^3 + a) - 2*a^3*d^3*x^6*log(d*x^3 + c) - 6*(b^3*
c^3 - a^3*d^3)*x^6*log(x) + a^2*b*c^3 - a^3*c^2*d - 2*(a*b^2*c^3 - a^3*c*d^
2)*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^6)
```

giac [A] time = 0.26, size = 165, normalized size = 1.39

$$-\frac{b^4\log\left(\left|bx^3+a\right|\right)}{3\left(a^3b^2c-a^4bd\right)}+\frac{d^4\log\left(\left|dx^3+c\right|\right)}{3\left(bc^4d-ac^3d^2\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(|x|\right)}{a^3c^3}-\frac{3\,b^2c^2x^6+3\,abcdx^6+3\,a^2d^2x^6-2\,abc^2x^3}{6\,a^3c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^4*log(abs(b*x^3 + a))/(a^3*b^2*c - a^4*b*d) + 1/3*d^4*log(abs(d*x^3
+ c))/(b*c^4*d - a*c^3*d^2) + (b^2*c^2 + a*b*c*d + a^2*d^2)*log(abs(x))/(a^
3*c^3) - 1/6*(3*b^2*c^2*x^6 + 3*a*b*c*d*x^6 + 3*a^2*d^2*x^6 - 2*a*b*c^2*x^3
 - 2*a^2*c*d*x^3 + a^2*c^2)/(a^3*c^3*x^6)
```

maple [A] time = 0.06, size = 124, normalized size = 1.04

$$\frac{b^3\ln\left(b\,x^3+a\right)}{3\,(ad-bc)\,a^3}-\frac{d^3\ln\left(d\,x^3+c\right)}{3\,(ad-bc)\,c^3}+\frac{d^2\ln(x)}{a\,c^3}+\frac{bd\ln(x)}{a^2c^2}+\frac{b^2\ln(x)}{a^3c}+\frac{d}{3a\,c^2x^3}+\frac{b}{3a^2c\,x^3}-\frac{1}{6ac\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] 1/3*b^3/a^3/(a*d-b*c)*ln(b*x^3+a)-1/3*d^3/c^3/(a*d-b*c)*ln(d*x^3+c)-1/6/a/c
/x^6+1/3/a/c^2/x^3*d+1/3/a^2/c/x^3*b+1/a/c^3*ln(x)*d^2+1/a^2/c^2*ln(x)*b*d+
1/a^3/c*ln(x)*b^2
```

maxima [A] time = 0.52, size = 117, normalized size = 0.98

$$-\frac{b^3\log\left(bx^3+a\right)}{3\left(a^3bc-a^4d\right)}+\frac{d^3\log\left(dx^3+c\right)}{3\left(bc^4-ac^3d\right)}+\frac{\left(b^2c^2+abcd+a^2d^2\right)\log\left(x^3\right)}{3\,a^3c^3}+\frac{2\,(bc+ad)x^3-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*b^3*log(b*x^3 + a)/(a^3*b*c - a^4*d) + 1/3*d^3*log(d*x^3 + c)/(b*c^4 -
 a*c^3*d) + 1/3*(b^2*c^2 + a*b*c*d + a^2*d^2)*log(x^3)/(a^3*c^3) + 1/6*(2*(
b*c + a*d)*x^3 - a*c)/(a^2*c^2*x^6)
```

mupad [B] time = 3.21, size = 118, normalized size = 0.99

$$\frac{b^3\,\ln\left(b\,x^3+a\right)}{3\,a^4\,d-3\,a^3\,b\,c}-\frac{\frac{1}{6\,a\,c}-\frac{x^3\,(a\,d+b\,c)}{3\,a^2\,c^2}}{x^6}+\frac{d^3\,\ln\left(d\,x^3+c\right)}{3\,b\,c^4-3\,a\,c^3\,d}+\frac{\ln(x)\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{a^3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] (b^3*log(a + b*x^3))/(3*a^4*d - 3*a^3*b*c) - (1/(6*a*c) - (x^3*(a*d + b*c))
/(3*a^2*c^2))/x^6 + (d^3*log(c + d*x^3))/(3*b*c^4 - 3*a*c^3*d) + (log(x)*(a
^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{1}{x^8(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=352

$$-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+bc}{4a^2c^2x^4}-\frac{a^2d^2+abcd+b^2}{a^3c^3x}$$

```
[Out] -1/7/a/c/x^7+1/4*(a*d+b*c)/a^2/c^2/x^4+(-a^2*d^2-a*b*c*d-b^2*c^2)/a^3/c^3/x
+1/3*b^(10/3)*ln(a^(1/3)+b^(1/3)*x)/a^(10/3)/(-a*d+b*c)-1/3*d^(10/3)*ln(c^(
1/3)+d^(1/3)*x)/c^(10/3)/(-a*d+b*c)-1/6*b^(10/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/a^(10/3)/(-a*d+b*c)+1/6*d^(10/3)*ln(c^(2/3)-c^(1/3)*d^(1/3)
*x+d^(2/3)*x^2)/c^(10/3)/(-a*d+b*c)+1/3*b^(10/3)*arctan(1/3*(a^(1/3)-2*b^(1
/3)*x)/a^(1/3)*3^(1/2))/a^(10/3)/(-a*d+b*c)*3^(1/2)-1/3*d^(10/3)*arctan(1/3
*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(10/3)/(-a*d+b*c)*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 292, 31, 634, 617, 204, 628}

$$-\frac{a^2d^2+abcd+b^2c^2}{a^3c^3x}-\frac{b^{10/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{10/3}(bc-ad)}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{10/3}(bc-ad)}+\frac{ad+}{4a^2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -1/(7*a*c*x^7) + (b*c + a*d)/(4*a^2*c^2*x^4) - (b^2*c^2 + a*b*c*d + a^2*d^2
)/(a^3*c^3*x) + (b^(10/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))]
)/(Sqrt[3]*a^(10/3)*(b*c - a*d)) - (d^(10/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)
/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(10/3)*(b*c - a*d)) + (b^(10/3)*Log[a^(1/3)
 + b^(1/3)*x])/(3*a^(10/3)*(b*c - a*d)) - (d^(10/3)*Log[c^(1/3) + d^(1/3)*x
])/(3*c^(10/3)*(b*c - a*d)) - (b^(10/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b
^(2/3)*x^2])/(6*a^(10/3)*(b*c - a*d)) + (d^(10/3)*Log[c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2])/(6*c^(10/3)*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^8\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = -\frac{1}{7acx^7}+\frac{\int \frac{-7(bc+ad)-7bdx^3}{x^5\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{7ac}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{\int \frac{-28\left(b^2c^2+abcd+a^2d^2\right)-28bd(bc+ad)x^3}{x^2\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{28a^2c^2}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \frac{x\left(-28(bc+ad)\left(b^2c^2+a^2d^2\right)-28bd\left(b^2c^2+abcd+a^2\right.\right.}{\left(a+bx^3\right)\left(c+dx^3\right)}}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{\int \left(-\frac{28b^4c^3x}{(bc-ad)\left(a+bx^3\right)}-\frac{28a^3d^4x}{(-bc+ad)\left(c+dx^3\right)}\right)dx}{28a^3c^3}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}-\frac{b^4\int \frac{x}{a+bx^3}\,dx}{a^3(bc-ad)}+\frac{d^4\int \frac{x}{c+dx^3}\,dx}{c^3(bc-ad)}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{11/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{10/3}(bc-ad)}-\frac{b^{11/3}\int \frac{\sqrt[3]{a}+\sqrt[3]{}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}}}{3a^{10/3}(bc-}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}\right.}{3c^{10/3}(bc}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{10/3}(bc-ad)}-\frac{d^{10/3}\log\left(\sqrt[3]{c}\right.}{3c^{10/3}(bc}$$

$$= -\frac{1}{7acx^7}+\frac{bc+ad}{4a^2c^2x^4}-\frac{b^2c^2+abcd+a^2d^2}{a^3c^3x}+\frac{b^{10/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{10/3}(bc-ad)}-\frac{d^{10/3}\tan^{-1}\left(\right.}{\sqrt{3}\,c^{10/3}(b}$$

Mathematica [A] time = 0.24, size = 304, normalized size = 0.86

$$\frac{-\frac{28b^{10/3}x^7\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{10/3}}-\frac{28\sqrt{3}b^{10/3}x^7\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{10/3}}+\frac{14b^{10/3}x^7\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{10/3}}+\frac{84b^3x^6}{a^3}-\frac{21b^2x^3}{a^2}+\frac{12b}{a}+\frac{28d^{10/3}x^7\,\mathrm{lo}}{c}}{84x^7(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*(a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((12*b)/a - (12*d)/c - (21*b^2*x^3)/a^2 + (21*d^2*x^3)/c^2 + (84*b^3*x^6)/a
^3 - (84*d^3*x^6)/c^3 - (28*Sqrt[3]*b^(10/3)*x^7*ArcTan[(1 - (2*b^(1/3)*x)/
a^(1/3))/Sqrt[3]])/a^(10/3) + (28*Sqrt[3]*d^(10/3)*x^7*ArcTan[(1 - (2*d^(1/
3)*x)/c^(1/3))/Sqrt[3]])/c^(10/3) - (28*b^(10/3)*x^7*Log[a^(1/3) + b^(1/3)*
x])/a^(10/3) + (28*d^(10/3)*x^7*Log[c^(1/3) + d^(1/3)*x])/c^(10/3) + (14*b^
(10/3)*x^7*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(10/3) - (14*d
^(10/3)*x^7*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(10/3))/(84*(
-(b*c) + a*d)*x^7)
```

fricas [A] time = 1.37, size = 332, normalized size = 0.94

$$-\frac{28\sqrt{3}b^3c^3x^7\left(-\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(-\frac{b}{a}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-28\sqrt{3}a^3d^3x^7\left(\frac{d}{c}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x\left(\frac{d}{c}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-14b^3c^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^3*c^3*x^7*(-b/a)^(1/3)*arctan(2/3*sqrt(3)*x*(-b/a)^(1/3
) + 1/3*sqrt(3)) - 28*sqrt(3)*a^3*d^3*x^7*(d/c)^(1/3)*arctan(2/3*sqrt(3)*x*
(d/c)^(1/3) - 1/3*sqrt(3)) - 14*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x^2 - a*x*(-
b/a)^(2/3) - a*(-b/a)^(1/3)) - 14*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x^2 - c*x*(
d/c)^(2/3) + c*(d/c)^(1/3)) + 28*b^3*c^3*x^7*(-b/a)^(1/3)*log(b*x + a*(-b/a
)^(2/3)) + 28*a^3*d^3*x^7*(d/c)^(1/3)*log(d*x + c*(d/c)^(2/3)) + 84*(b^3*c^
3 - a^3*d^3)*x^6 + 12*a^2*b*c^3 - 12*a^3*c^2*d - 21*(a*b^2*c^3 - a^3*c*d^2)
*x^3)/((a^3*b*c^4 - a^4*c^3*d)*x^7)
```

giac [**A**] time = 0.22, size = 377, normalized size = 1.07

$$\frac{b^4\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(a^4bc-a^5d\right)}-\frac{d^4\left(-\frac{c}{d}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^5-ac^4d\right)}+\frac{\left(-ab^2\right)^{\frac{2}{3}}b^2\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,a^4bc-\sqrt{3}\,a^5d}-\frac{\left(-cd^2\right)^{\frac{2}{3}}d^2\,\mathrm{arct}\cdots}{\sqrt{3}\,bc^5\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^4*(-a/b)^(2/3)*log(abs(x - (-a/b)^(1/3)))/(a^4*b*c - a^5*d) - 1/3*d^4
*(-c/d)^(2/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^5 - a*c^4*d) + (-a*b^2)^(2/3)
*b^2*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^4*b*c
 - sqrt(3)*a^5*d) - (-c*d^2)^(2/3)*d^2*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(sqrt(3)*b*c^5 - sqrt(3)*a*c^4*d) - 1/6*(-a*b^2)^(2/3)*b^
2*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^4*b*c - a^5*d) + 1/6*(-c*d^2)
^(2/3)*d^2*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b*c^5 - a*c^4*d) - 1/2
8*(28*b^2*c^2*x^6 + 28*a*b*c*d*x^6 + 28*a^2*d^2*x^6 - 7*a*b*c^2*x^3 - 7*a^2
*c*d*x^3 + 4*a^2*c^2)/(a^3*c^3*x^7)
```

maple [**A**] time = 0.05, size = 334, normalized size = 0.95

$$\frac{\sqrt{3}\,b^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{b^3\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}+\frac{b^3\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}a^3}-\frac{\sqrt{3}\,d^3\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}c^3}+\frac{d\cdots}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3*b^3/a^3/(a*d-b*c)/(a/b)^(1/3)*ln(x+(a/b)^(1/3))+1/6*b^3/a^3/(a*d-b*c)/
(a/b)^(1/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))+1/3*b^3/a^3/(a*d-b*c)*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))+1/3*d^3/c^3/(a*d-b*c)/
(c/d)^(1/3)*ln(x+(c/d)^(1/3))-1/6*d^3/c^3/(a*d-b*c)/(c/d)^(1/3)*ln(x^2-(c/d
)^(1/3)*x+(c/d)^(2/3))-1/3*d^3/c^3/(a*d-b*c)*3^(1/2)/(c/d)^(1/3)*arctan(1/3
*3^(1/2)*(2/(c/d)^(1/3)*x-1))-1/7/a/c/x^7+1/4/a/c^2/x^4*d+1/4/a^2/c/x^4*b-1
/a/c^3/x*d^2-1/a^2/c^2/x*b*d-1/a^3/c/x*b^2
```

maxima [A] time = 1.16, size = 376, normalized size = 1.07

$$-\frac{\sqrt{3}\, b^3 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(a^3bc-a^4d\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\, d^3 \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc^4-ac^3d\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}-\frac{b^3\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(a^3bc\left(\frac{a}{b}\right)^{\frac{1}{3}}-a^4d\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}+\frac{d^3\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\right.\right.}{6\left(bc^4\left(\frac{c}{d}\right)^{\frac{1}{3}}-ac^3d\left(\frac{c}{d}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/((a^3*
b*c - a^4*d)*(a/b)^(1/3)) + 1/3*sqrt(3)*d^3*arctan(1/3*sqrt(3)*(2*x - (c/d)
^(1/3))/(c/d)^(1/3))/((b*c^4 - a*c^3*d)*(c/d)^(1/3)) - 1/6*b^3*log(x^2 - x*
(a/b)^(1/3) + (a/b)^(2/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)^(1/3)) + 1/6*
d^3*log(x^2 - x*(c/d)^(1/3) + (c/d)^(2/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/
d)^(1/3)) + 1/3*b^3*log(x + (a/b)^(1/3))/(a^3*b*c*(a/b)^(1/3) - a^4*d*(a/b)
^(1/3)) - 1/3*d^3*log(x + (c/d)^(1/3))/(b*c^4*(c/d)^(1/3) - a*c^3*d*(c/d)^(
1/3)) - 1/28*(28*(b^2*c^2 + a*b*c*d + a^2*d^2)*x^6 + 4*a^2*c^2 - 7*(a*b*c^2
 + a^2*c*d)*x^3)/(a^3*c^3*x^7)
```

mupad [B] time = 11.91, size = 1814, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] log(((-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8
+ b^8*c^8)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(
-b^10/(a^10*(a*d - b*c)^3))^(2/3))*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/3 -
9*a^19*b^14*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29
*b^4*c^19*d^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3
 - 27*a^10*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3) + log(((d^
10/(c^10*(a*d - b*c)^3))^(2/3)*(((27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8
)*(a*d - b*c)^2 + 27*a^28*b^3*c^28*d^3*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^1
0*(a*d - b*c)^3))^(2/3))*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/3 - 9*a^19*b^14
*c^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d
^14))/9 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c
^10*d^3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3) - (1/(7*a*c) - (x^3*(
a*d + b*c))/(4*a^2*c^2) + (x^6*(a^2*d^2 + b^2*c^2 + a*b*c*d))/(a^3*c^3))/x^
7 - (log(((3^(1/2)*1i + 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*
1i + 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^2
8*b^3*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a
*d - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14
*c^29*d^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d
^14))/36 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10
*b^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i + 1))/2
+ (log(((3^(1/2)*1i - 1)^2*(-b^10/(a^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i
 - 1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*
b^3*c^28*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(-b^10/(a^10*(a*d
 - b*c)^3))^(2/3))/4)*(-b^10/(a^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c
^29*d^4 + 9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^1
4))/36 - a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-b^10/(27*a^13*d^3 - 27*a^10*b
^3*c^3 + 81*a^11*b^2*c^2*d - 81*a^12*b*c*d^2))^(1/3)*(3^(1/2)*1i - 1))/2 -
(log(((3^(1/2)*1i + 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i +
1)*(27*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3
*c^28*d^3*(3^(1/2)*1i + 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b
```

```
*c)^3))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 + 9*a^19*b^14*c^29*d
^4 - 9*a^20*b^13*c^28*d^5 - 9*a^28*b^5*c^20*d^13 + 9*a^29*b^4*c^19*d^14))/3
6 + a^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^
3 + 81*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i + 1))/2 + (log(
((3^(1/2)*1i - 1)^2*(d^10/(c^10*(a*d - b*c)^3))^(2/3)*(((3^(1/2)*1i - 1)*(2
7*a^21*b^3*c^21*d^3*x*(a^8*d^8 + b^8*c^8)*(a*d - b*c)^2 + (27*a^28*b^3*c^28
*d^3*(3^(1/2)*1i - 1)^2*(a*d + b*c)*(a*d - b*c)^4*(d^10/(c^10*(a*d - b*c)^3
))^(2/3))/4)*(d^10/(c^10*(a*d - b*c)^3))^(1/3))/6 - 9*a^19*b^14*c^29*d^4 +
9*a^20*b^13*c^28*d^5 + 9*a^28*b^5*c^20*d^13 - 9*a^29*b^4*c^19*d^14))/36 - a
^19*b^11*c^19*d^11*x*(a*d + b*c))*(-d^10/(27*b^3*c^13 - 27*a^3*c^10*d^3 + 8
1*a^2*b*c^11*d^2 - 81*a*b^2*c^12*d))^(1/3)*(3^(1/2)*1i - 1))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.124 $\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx$

Optimal. Leaf size=148

$$\frac{a^5Ax^{m+1}}{m+1}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{b^4x^{m+16}(5aB+Ab)}{m+16}+\frac{5ab^3x^{m+13}(}{m+}$$

```
[Out] a^5*A*x^(1+m)/(1+m)+a^4*(5*A*b+B*a)*x^(4+m)/(4+m)+5*a^3*b*(2*A*b+B*a)*x^(7+
m)/(7+m)+10*a^2*b^2*(A*b+B*a)*x^(10+m)/(10+m)+5*a*b^3*(A*b+2*B*a)*x^(13+m)/
(13+m)+b^4*(A*b+5*B*a)*x^(16+m)/(16+m)+b^5*B*x^(19+m)/(19+m)
```

Rubi [A] time = 0.10, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{10a^2b^2x^{m+10}(aB+Ab)}{m+10}+\frac{a^4x^{m+4}(aB+5Ab)}{m+4}+\frac{5a^3bx^{m+7}(aB+2Ab)}{m+7}+\frac{a^5Ax^{m+1}}{m+1}+\frac{5ab^3x^{m+13}(2aB+Ab)}{m+13}+\frac{b^4x^{m+16}(}{m}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] (a^5*A*x^(1 + m))/(1 + m) + (a^4*(5*A*b + a*B)*x^(4 + m))/(4 + m) + (5*a^3*
b*(2*A*b + a*B)*x^(7 + m))/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^(10 + m))/(1
0 + m) + (5*a*b^3*(A*b + 2*a*B)*x^(13 + m))/(13 + m) + (b^4*(A*b + 5*a*B)*x
^(16 + m))/(16 + m) + (b^5*B*x^(19 + m))/(19 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m \left(a+bx^3\right)^5 \left(A+Bx^3\right) dx &= \int \left(a^5Ax^m+a^4(5Ab+aB)x^{3+m}+5a^3b(2Ab+aB)x^{6+m}+10a^2b^2(Ab+aB)x^{9+}\right.\\ &= \frac{a^5Ax^{1+m}}{1+m}+\frac{a^4(5Ab+aB)x^{4+m}}{4+m}+\frac{5a^3b(2Ab+aB)x^{7+m}}{7+m}+\frac{10a^2b^2(Ab+aB)x^{10+}}{10+m}\end{aligned}$$

Mathematica [A] time = 0.26, size = 137, normalized size = 0.93

$$x^{m+1}\left(\frac{a^5A}{m+1}+\frac{a^4x^3(aB+5Ab)}{m+4}+\frac{5a^3bx^6(aB+2Ab)}{m+7}+\frac{10a^2b^2x^9(aB+Ab)}{m+10}+\frac{b^4x^{15}(5aB+Ab)}{m+16}+\frac{5ab^3x^{12}(2aB-}{m+13}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^5*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^5*A)/(1 + m) + (a^4*(5*A*b + a*B)*x^3)/(4 + m) + (5*a^3*b*(2*
A*b + a*B)*x^6)/(7 + m) + (10*a^2*b^2*(A*b + a*B)*x^9)/(10 + m) + (5*a*b^3*
(A*b + 2*a*B)*x^12)/(13 + m) + (b^4*(A*b + 5*a*B)*x^15)/(16 + m) + (b^5*B*x
^18)/(19 + m))
```

fricas [B] time = 0.88, size = 851, normalized size = 5.75

$$\frac{\left(\left(Bb^5m^6+51\,Bb^5m^5+1005\,Bb^5m^4+9605\,Bb^5m^3+45474\,Bb^5m^2+95064\,Bb^5m+58240\,Bb^5\right)x^{19}+\left(\left(5\,Bab^4+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^5*m^6 + 51*B*b^5*m^5 + 1005*B*b^5*m^4 + 9605*B*b^5*m^3 + 45474*B*b^5*
m^2 + 95064*B*b^5*m + 58240*B*b^5)*x^19 + ((5*B*a*b^4 + A*b^5)*m^6 + 345800
*B*a*b^4 + 69160*A*b^5 + 54*(5*B*a*b^4 + A*b^5)*m^5 + 1110*(5*B*a*b^4 + A*b
^5)*m^4 + 10940*(5*B*a*b^4 + A*b^5)*m^3 + 52929*(5*B*a*b^4 + A*b^5)*m^2 + 1
12206*(5*B*a*b^4 + A*b^5)*m)*x^16 + 5*((2*B*a^2*b^3 + A*a*b^4)*m^6 + 170240
*B*a^2*b^3 + 85120*A*a*b^4 + 57*(2*B*a^2*b^3 + A*a*b^4)*m^5 + 1233*(2*B*a^2
*b^3 + A*a*b^4)*m^4 + 12671*(2*B*a^2*b^3 + A*a*b^4)*m^3 + 63246*(2*B*a^2*b^
3 + A*a*b^4)*m^2 + 136872*(2*B*a^2*b^3 + A*a*b^4)*m)*x^13 + 10*((B*a^3*b^2
+ A*a^2*b^3)*m^6 + 110656*B*a^3*b^2 + 110656*A*a^2*b^3 + 60*(B*a^3*b^2 + A*
a^2*b^3)*m^5 + 1374*(B*a^3*b^2 + A*a^2*b^3)*m^4 + 14960*(B*a^3*b^2 + A*a^2*
b^3)*m^3 + 78369*(B*a^3*b^2 + A*a^2*b^3)*m^2 + 175380*(B*a^3*b^2 + A*a^2*b^
3)*m)*x^10 + 5*((B*a^4*b + 2*A*a^3*b^2)*m^6 + 158080*B*a^4*b + 316160*A*a^3
*b^2 + 63*(B*a^4*b + 2*A*a^3*b^2)*m^5 + 1533*(B*a^4*b + 2*A*a^3*b^2)*m^4 +
17969*(B*a^4*b + 2*A*a^3*b^2)*m^3 + 102186*(B*a^4*b + 2*A*a^3*b^2)*m^2 + 24
3768*(B*a^4*b + 2*A*a^3*b^2)*m)*x^7 + ((B*a^5 + 5*A*a^4*b)*m^6 + 276640*B*a
^5 + 1383200*A*a^4*b + 66*(B*a^5 + 5*A*a^4*b)*m^5 + 1710*(B*a^5 + 5*A*a^4*b
)*m^4 + 21860*(B*a^5 + 5*A*a^4*b)*m^3 + 140529*(B*a^5 + 5*A*a^4*b)*m^2 + 39
6954*(B*a^5 + 5*A*a^4*b)*m)*x^4 + (A*a^5*m^6 + 69*A*a^5*m^5 + 1905*A*a^5*m^
4 + 26795*A*a^5*m^3 + 201174*A*a^5*m^2 + 757896*A*a^5*m + 1106560*A*a^5)*x)
*x^m/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 + 959070*m^2 + 18644
56*m + 1106560)
```

giac [B] time = 0.29, size = 1331, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^5*m^6*x^19*x^m + 51*B*b^5*m^5*x^19*x^m + 1005*B*b^5*m^4*x^19*x^m + 5*B
*a*b^4*m^6*x^16*x^m + A*b^5*m^6*x^16*x^m + 9605*B*b^5*m^3*x^19*x^m + 270*B*
a*b^4*m^5*x^16*x^m + 54*A*b^5*m^5*x^16*x^m + 45474*B*b^5*m^2*x^19*x^m + 555
0*B*a*b^4*m^4*x^16*x^m + 1110*A*b^5*m^4*x^16*x^m + 95064*B*b^5*m*x^19*x^m +
 10*B*a^2*b^3*m^6*x^13*x^m + 5*A*a*b^4*m^6*x^13*x^m + 54700*B*a*b^4*m^3*x^1
6*x^m + 10940*A*b^5*m^3*x^16*x^m + 58240*B*b^5*x^19*x^m + 570*B*a^2*b^3*m^5
*x^13*x^m + 285*A*a*b^4*m^5*x^13*x^m + 264645*B*a*b^4*m^2*x^16*x^m + 52929*
A*b^5*m^2*x^16*x^m + 12330*B*a^2*b^3*m^4*x^13*x^m + 6165*A*a*b^4*m^4*x^13*x
^m + 561030*B*a*b^4*m*x^16*x^m + 112206*A*b^5*m*x^16*x^m + 10*B*a^3*b^2*m^6
*x^10*x^m + 10*A*a^2*b^3*m^6*x^10*x^m + 126710*B*a^2*b^3*m^3*x^13*x^m + 633
55*A*a*b^4*m^3*x^13*x^m + 345800*B*a*b^4*x^16*x^m + 69160*A*b^5*x^16*x^m +
600*B*a^3*b^2*m^5*x^10*x^m + 600*A*a^2*b^3*m^5*x^10*x^m + 632460*B*a^2*b^3*
m^2*x^13*x^m + 316230*A*a*b^4*m^2*x^13*x^m + 13740*B*a^3*b^2*m^4*x^10*x^m +
 13740*A*a^2*b^3*m^4*x^10*x^m + 1368720*B*a^2*b^3*m*x^13*x^m + 684360*A*a*b
^4*m*x^13*x^m + 5*B*a^4*b*m^6*x^7*x^m + 10*A*a^3*b^2*m^6*x^7*x^m + 149600*B
*a^3*b^2*m^3*x^10*x^m + 149600*A*a^2*b^3*m^3*x^10*x^m + 851200*B*a^2*b^3*x^
13*x^m + 425600*A*a*b^4*x^13*x^m + 315*B*a^4*b*m^5*x^7*x^m + 630*A*a^3*b^2*
m^5*x^7*x^m + 783690*B*a^3*b^2*m^2*x^10*x^m + 783690*A*a^2*b^3*m^2*x^10*x^m
 + 7665*B*a^4*b*m^4*x^7*x^m + 15330*A*a^3*b^2*m^4*x^7*x^m + 1753800*B*a^3*b
^2*m*x^10*x^m + 1753800*A*a^2*b^3*m*x^10*x^m + B*a^5*m^6*x^4*x^m + 5*A*a^4*
b*m^6*x^4*x^m + 89845*B*a^4*b*m^3*x^7*x^m + 179690*A*a^3*b^2*m^3*x^7*x^m +
1106560*B*a^3*b^2*x^10*x^m + 1106560*A*a^2*b^3*x^10*x^m + 66*B*a^5*m^5*x^4*
x^m + 330*A*a^4*b*m^5*x^4*x^m + 510930*B*a^4*b*m^2*x^7*x^m + 1021860*A*a^3*
b^2*m^2*x^7*x^m + 1710*B*a^5*m^4*x^4*x^m + 8550*A*a^4*b*m^4*x^4*x^m + 12188
40*B*a^4*b*m*x^7*x^m + 2437680*A*a^3*b^2*m*x^7*x^m + A*a^5*m^6*x*x^m + 2186
0*B*a^5*m^3*x^4*x^m + 109300*A*a^4*b*m^3*x^4*x^m + 790400*B*a^4*b*x^7*x^m +
 1580800*A*a^3*b^2*x^7*x^m + 69*A*a^5*m^5*x*x^m + 140529*B*a^5*m^2*x^4*x^m
```

```
+ 702645*A*a^4*b*m^2*x^4*x^m + 1905*A*a^5*m^4*x*x^m + 396954*B*a^5*m*x^4*x^
m + 1984770*A*a^4*b*m*x^4*x^m + 26795*A*a^5*m^3*x*x^m + 276640*B*a^5*x^4*x^
m + 1383200*A*a^4*b*x^4*x^m + 201174*A*a^5*m^2*x*x^m + 757896*A*a^5*m*x*x^m
 + 1106560*A*a^5*x*x^m)/(m^7 + 70*m^6 + 1974*m^5 + 28700*m^4 + 227969*m^3 +
 959070*m^2 + 1864456*m + 1106560)
```

maple [B] time = 0.05, size = 1078, normalized size = 7.28

$$\frac{\left(B\,b^5m^6x^{18} + 51B\,b^5m^5x^{18} + 1005B\,b^5m^4x^{18} + A\,b^5m^6x^{15} + 5Ba\,b^4m^6x^{15} + 9605B\,b^5m^3x^{18} + 54A\,b^5m^5x^{15} + 27\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^5*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b^5*m^6*x^18+51*B*b^5*m^5*x^18+1005*B*b^5*m^4*x^18+A*b^5*m^6*x^1
5+5*B*a*b^4*m^6*x^15+9605*B*b^5*m^3*x^18+54*A*b^5*m^5*x^15+270*B*a*b^4*m^5*
x^15+45474*B*b^5*m^2*x^18+1110*A*b^5*m^4*x^15+5550*B*a*b^4*m^4*x^15+95064*B
*b^5*m*x^18+5*A*a*b^4*m^6*x^12+10940*A*b^5*m^3*x^15+10*B*a^2*b^3*m^6*x^12+5
4700*B*a*b^4*m^3*x^15+58240*B*b^5*x^18+285*A*a*b^4*m^5*x^12+52929*A*b^5*m^2
*x^15+570*B*a^2*b^3*m^5*x^12+264645*B*a*b^4*m^2*x^15+6165*A*a*b^4*m^4*x^12+
112206*A*b^5*m*x^15+12330*B*a^2*b^3*m^4*x^12+561030*B*a*b^4*m*x^15+10*A*a^2
*b^3*m^6*x^9+63355*A*a*b^4*m^3*x^12+69160*A*b^5*x^15+10*B*a^3*b^2*m^6*x^9+1
26710*B*a^2*b^3*m^3*x^12+345800*B*a*b^4*x^15+600*A*a^2*b^3*m^5*x^9+316230*A
*a*b^4*m^2*x^12+600*B*a^3*b^2*m^5*x^9+632460*B*a^2*b^3*m^2*x^12+13740*A*a^2
*b^3*m^4*x^9+684360*A*a*b^4*m*x^12+13740*B*a^3*b^2*m^4*x^9+1368720*B*a^2*b^
3*m*x^12+10*A*a^3*b^2*m^6*x^6+149600*A*a^2*b^3*m^3*x^9+425600*A*a*b^4*x^12+
5*B*a^4*b*m^6*x^6+149600*B*a^3*b^2*m^3*x^9+851200*B*a^2*b^3*x^12+630*A*a^3*
b^2*m^5*x^6+783690*A*a^2*b^3*m^2*x^9+315*B*a^4*b*m^5*x^6+783690*B*a^3*b^2*m
^2*x^9+15330*A*a^3*b^2*m^4*x^6+1753800*A*a^2*b^3*m*x^9+7665*B*a^4*b*m^4*x^6
+1753800*B*a^3*b^2*m*x^9+5*A*a^4*b*m^6*x^3+179690*A*a^3*b^2*m^3*x^6+1106560
*A*a^2*b^3*x^9+B*a^5*m^6*x^3+89845*B*a^4*b*m^3*x^6+1106560*B*a^3*b^2*x^9+33
0*A*a^4*b*m^5*x^3+1021860*A*a^3*b^2*m^2*x^6+66*B*a^5*m^5*x^3+510930*B*a^4*b
*m^2*x^6+8550*A*a^4*b*m^4*x^3+2437680*A*a^3*b^2*m*x^6+1710*B*a^5*m^4*x^3+12
18840*B*a^4*b*m*x^6+A*a^5*m^6+109300*A*a^4*b*m^3*x^3+1580800*A*a^3*b^2*x^6+
21860*B*a^5*m^3*x^3+790400*B*a^4*b*x^6+69*A*a^5*m^5+702645*A*a^4*b*m^2*x^3+
140529*B*a^5*m^2*x^3+1905*A*a^5*m^4+1984770*A*a^4*b*m*x^3+396954*B*a^5*m*x^
3+26795*A*a^5*m^3+1383200*A*a^4*b*x^3+276640*B*a^5*x^3+201174*A*a^5*m^2+757
896*A*a^5*m+1106560*A*a^5)/(m+1)/(m+4)/(m+7)/(m+10)/(m+13)/(m+16)/(m+19)
```

maxima [A] time = 0.52, size = 205, normalized size = 1.39

$$\frac{Bb^5x^{m+19}}{m+19} + \frac{5\,Bab^4x^{m+16}}{m+16} + \frac{Ab^5x^{m+16}}{m+16} + \frac{10\,Ba^2b^3x^{m+13}}{m+13} + \frac{5\,Aab^4x^{m+13}}{m+13} + \frac{10\,Ba^3b^2x^{m+10}}{m+10} + \frac{10\,Aa^2b^3x^{m+10}}{m+10} + \frac{5\,Ba^4bx^{m\cdots}}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^5*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^5*x^(m + 19)/(m + 19) + 5*B*a*b^4*x^(m + 16)/(m + 16) + A*b^5*x^(m + 16
)/(m + 16) + 10*B*a^2*b^3*x^(m + 13)/(m + 13) + 5*A*a*b^4*x^(m + 13)/(m + 1
3) + 10*B*a^3*b^2*x^(m + 10)/(m + 10) + 10*A*a^2*b^3*x^(m + 10)/(m + 10) +
5*B*a^4*b*x^(m + 7)/(m + 7) + 10*A*a^3*b^2*x^(m + 7)/(m + 7) + B*a^5*x^(m +
 4)/(m + 4) + 5*A*a^4*b*x^(m + 4)/(m + 4) + A*a^5*x^(m + 1)/(m + 1)
```

mupad [B] time = 3.21, size = 559, normalized size = 3.78

$$\frac{B\,b^5\,x^m\,x^{19}\,\left(m^6+51\,m^5+1005\,m^4+9605\,m^3+45474\,m^2+95064\,m+58240\right)}{m^7+70\,m^6+1974\,m^5+28700\,m^4+227969\,m^3+959070\,m^2+1864456\,m+1106560} + \frac{a^4\,x^m\,x^4\,\left(5\,A\,b+B\,a\right)\,\left(\cdots\right.}{m^7+70\,m^6+1974\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^5,x)
```

```
[Out] (B*b^5*x^m*x^19*(95064*m + 45474*m^2 + 9605*m^3 + 1005*m^4 + 51*m^5 + m^6 +
 58240))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m
^6 + m^7 + 1106560) + (a^4*x^m*x^4*(5*A*b + B*a)*(396954*m + 140529*m^2 + 2
1860*m^3 + 1710*m^4 + 66*m^5 + m^6 + 276640))/(1864456*m + 959070*m^2 + 227
969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (b^4*x^m*x^16*(A
*b + 5*B*a)*(112206*m + 52929*m^2 + 10940*m^3 + 1110*m^4 + 54*m^5 + m^6 + 6
9160))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6
 + m^7 + 1106560) + (A*a^5*x*x^m*(757896*m + 201174*m^2 + 26795*m^3 + 1905*
m^4 + 69*m^5 + m^6 + 1106560))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700
*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (10*a^2*b^2*x^m*x^10*(A*b + B*a
)*(175380*m + 78369*m^2 + 14960*m^3 + 1374*m^4 + 60*m^5 + m^6 + 110656))/(1
864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 +
1106560) + (5*a*b^3*x^m*x^13*(A*b + 2*B*a)*(136872*m + 63246*m^2 + 12671*m^
3 + 1233*m^4 + 57*m^5 + m^6 + 85120))/(1864456*m + 959070*m^2 + 227969*m^3
+ 28700*m^4 + 1974*m^5 + 70*m^6 + m^7 + 1106560) + (5*a^3*b*x^m*x^7*(2*A*b
+ B*a)*(243768*m + 102186*m^2 + 17969*m^3 + 1533*m^4 + 63*m^5 + m^6 + 15808
0))/(1864456*m + 959070*m^2 + 227969*m^3 + 28700*m^4 + 1974*m^5 + 70*m^6 +
m^7 + 1106560)
```

<u>**sympy**</u> **[A]** time = 23.94, size = 5418, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**5*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**5/(18*x**18) - A*a**4*b/(3*x**15) - 5*A*a**3*b**2/(6*x**12
) - 10*A*a**2*b**3/(9*x**9) - 5*A*a*b**4/(6*x**6) - A*b**5/(3*x**3) - B*a**
5/(15*x**15) - 5*B*a**4*b/(12*x**12) - 10*B*a**3*b**2/(9*x**9) - 5*B*a**2*b
**3/(3*x**6) - 5*B*a*b**4/(3*x**3) + B*b**5*log(x), Eq(m, -19)), (-A*a**5/(
15*x**15) - 5*A*a**4*b/(12*x**12) - 10*A*a**3*b**2/(9*x**9) - 5*A*a**2*b**3
/(3*x**6) - 5*A*a*b**4/(3*x**3) + A*b**5*log(x) - B*a**5/(12*x**12) - 5*B*a
**4*b/(9*x**9) - 5*B*a**3*b**2/(3*x**6) - 10*B*a**2*b**3/(3*x**3) + 5*B*a*b
**4*log(x) + B*b**5*x**3/3, Eq(m, -16)), (-A*a**5/(12*x**12) - 5*A*a**4*b/(
9*x**9) - 5*A*a**3*b**2/(3*x**6) - 10*A*a**2*b**3/(3*x**3) + 5*A*a*b**4*log
(x) + A*b**5*x**3/3 - B*a**5/(9*x**9) - 5*B*a**4*b/(6*x**6) - 10*B*a**3*b**
2/(3*x**3) + 10*B*a**2*b**3*log(x) + 5*B*a*b**4*x**3/3 + B*b**5*x**6/6, Eq(
m, -13)), (-A*a**5/(9*x**9) - 5*A*a**4*b/(6*x**6) - 10*A*a**3*b**2/(3*x**3)
 + 10*A*a**2*b**3*log(x) + 5*A*a*b**4*x**3/3 + A*b**5*x**6/6 - B*a**5/(6*x*
*6) - 5*B*a**4*b/(3*x**3) + 10*B*a**3*b**2*log(x) + 10*B*a**2*b**3*x**3/3 +
 5*B*a*b**4*x**6/6 + B*b**5*x**9/9, Eq(m, -10)), (-A*a**5/(6*x**6) - 5*A*a*
*4*b/(3*x**3) + 10*A*a**3*b**2*log(x) + 10*A*a**2*b**3*x**3/3 + 5*A*a*b**4*
x**6/6 + A*b**5*x**9/9 - B*a**5/(3*x**3) + 5*B*a**4*b*log(x) + 10*B*a**3*b*
*2*x**3/3 + 5*B*a**2*b**3*x**6/3 + 5*B*a*b**4*x**9/9 + B*b**5*x**12/12, Eq(
m, -7)), (-A*a**5/(3*x**3) + 5*A*a**4*b*log(x) + 10*A*a**3*b**2*x**3/3 + 5*
A*a**2*b**3*x**6/3 + 5*A*a*b**4*x**9/9 + A*b**5*x**12/12 + B*a**5*log(x) +
5*B*a**4*b*x**3/3 + 5*B*a**3*b**2*x**6/3 + 10*B*a**2*b**3*x**9/9 + 5*B*a*b*
*4*x**12/12 + B*b**5*x**15/15, Eq(m, -4)), (A*a**5*log(x) + 5*A*a**4*b*x**3
/3 + 5*A*a**3*b**2*x**6/3 + 10*A*a**2*b**3*x**9/9 + 5*A*a*b**4*x**12/12 + A
*b**5*x**15/15 + B*a**5*x**3/3 + 5*B*a**4*b*x**6/6 + 10*B*a**3*b**2*x**9/9
+ 5*B*a**2*b**3*x**12/6 + B*a*b**4*x**15/3 + B*b**5*x**18/18, Eq(m, -1)), (
A*a**5*m**6*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 69*A*a**5*m**5*x*x**m/(m**7 + 70*m**6
 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560
) + 1905*A*a**5*m**4*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 26795*A*a**5*m**3*x*x**m/(m*
*7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456
*m + 1106560) + 201174*A*a**5*m**2*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
```

```
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 757896*A*a**5*
m*x*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 1106560*A*a**5*x*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*A*a*
*4*b*m**6*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3
+ 959070*m**2 + 1864456*m + 1106560) + 330*A*a**4*b*m**5*x**4*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 8550*A*a**4*b*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700
*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 109300*A*a**4*b*
m**3*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959
070*m**2 + 1864456*m + 1106560) + 702645*A*a**4*b*m**2*x**4*x**m/(m**7 + 70
*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 11
06560) + 1984770*A*a**4*b*m*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1383200*A*a**4*b*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*A*a**3*b**2*m**6*x**7*x**m/(m**7 + 70*m**6 +
 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
+ 630*A*a**3*b**2*m**5*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 15330*A*a**3*b**2*m**4*
x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m
**2 + 1864456*m + 1106560) + 179690*A*a**3*b**2*m**3*x**7*x**m/(m**7 + 70*m
**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106
560) + 1021860*A*a**3*b**2*m**2*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 2437680*A*a**3
*b**2*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 1580800*A*a**3*b**2*x**7*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 10*A*a**2*b**3*m**6*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 600*A*a**2*b**
3*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
959070*m**2 + 1864456*m + 1106560) + 13740*A*a**2*b**3*m**4*x**10*x**m/(m**
7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*
m + 1106560) + 149600*A*a**2*b**3*m**3*x**10*x**m/(m**7 + 70*m**6 + 1974*m*
*5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 783690
*A*a**2*b**3*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*A*a**2*b**3*m*x**10
*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2
+ 1864456*m + 1106560) + 1106560*A*a**2*b**3*x**10*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
5*A*a*b**4*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 22796
9*m**3 + 959070*m**2 + 1864456*m + 1106560) + 285*A*a*b**4*m**5*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 6165*A*a*b**4*m**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**
5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 63355*A
*a*b**4*m**3*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 316230*A*a*b**4*m**2*x**13*x**m/
(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864
456*m + 1106560) + 684360*A*a*b**4*m*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 425600*A
*a*b**4*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + A*b**5*m**6*x**16*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 54*A*b**5*m**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2
27969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1110*A*b**5*m**4*x**16*x*
*m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1
864456*m + 1106560) + 10940*A*b**5*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m
**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 52929
*A*b**5*m**2*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 112206*A*b**5*m*x**16*x**m/(m**7
```

```
 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
 + 1106560) + 69160*A*b**5*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m
**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + B*a**5*m**6*x**4*x
**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
1864456*m + 1106560) + 66*B*a**5*m**5*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1710*B*a
**5*m**4*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 +
 959070*m**2 + 1864456*m + 1106560) + 21860*B*a**5*m**3*x**4*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 140529*B*a**5*m**2*x**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 396954*B*a**5*m*x
**4*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 276640*B*a**5*x**4*x**m/(m**7 + 70*m**6 + 1974*
m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 5*B*
a**4*b*m**6*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**
3 + 959070*m**2 + 1864456*m + 1106560) + 315*B*a**4*b*m**5*x**7*x**m/(m**7
+ 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m
+ 1106560) + 7665*B*a**4*b*m**4*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 287
00*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 89845*B*a**4*b
*m**3*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95
9070*m**2 + 1864456*m + 1106560) + 510930*B*a**4*b*m**2*x**7*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 1218840*B*a**4*b*m*x**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*
m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 790400*B*a**4*b*x
**7*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m*
*2 + 1864456*m + 1106560) + 10*B*a**3*b**2*m**6*x**10*x**m/(m**7 + 70*m**6
+ 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560)
 + 600*B*a**3*b**2*m**5*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 13740*B*a**3*b**2*m**
4*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 95907
0*m**2 + 1864456*m + 1106560) + 149600*B*a**3*b**2*m**3*x**10*x**m/(m**7 +
70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m +
1106560) + 783690*B*a**3*b**2*m**2*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 +
 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1753800*B*
a**3*b**2*m*x**10*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m*
*3 + 959070*m**2 + 1864456*m + 1106560) + 1106560*B*a**3*b**2*x**10*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 10*B*a**2*b**3*m**6*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5
 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 570*B*a*
*2*b**3*m**5*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + 12330*B*a**2*b**3*m**4*x**13*x**
m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 18
64456*m + 1106560) + 126710*B*a**2*b**3*m**3*x**13*x**m/(m**7 + 70*m**6 + 1
974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) +
632460*B*a**2*b**3*m**2*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4
 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1368720*B*a**2*b**3*m
*x**13*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070
*m**2 + 1864456*m + 1106560) + 851200*B*a**2*b**3*x**13*x**m/(m**7 + 70*m**
6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 110656
0) + 5*B*a*b**4*m**6*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 +
227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 270*B*a*b**4*m**5*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 5550*B*a*b**4*m**4*x**16*x**m/(m**7 + 70*m**6 + 197
4*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 54
700*B*a*b**4*m**3*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227
969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 264645*B*a*b**4*m**2*x**16*
x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 +
 1864456*m + 1106560) + 561030*B*a*b**4*m*x**16*x**m/(m**7 + 70*m**6 + 1974
*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 345
```

```
800*B*a*b**4*x**16*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m
**3 + 959070*m**2 + 1864456*m + 1106560) + B*b**5*m**6*x**19*x**m/(m**7 + 7
0*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1
106560) + 51*B*b**5*m**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**
4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 1005*B*b**5*m**4*x**
19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**
2 + 1864456*m + 1106560) + 9605*B*b**5*m**3*x**19*x**m/(m**7 + 70*m**6 + 19
74*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560) + 4
5474*B*b**5*m**2*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 2279
69*m**3 + 959070*m**2 + 1864456*m + 1106560) + 95064*B*b**5*m*x**19*x**m/(m
**7 + 70*m**6 + 1974*m**5 + 28700*m**4 + 227969*m**3 + 959070*m**2 + 186445
6*m + 1106560) + 58240*B*b**5*x**19*x**m/(m**7 + 70*m**6 + 1974*m**5 + 2870
0*m**4 + 227969*m**3 + 959070*m**2 + 1864456*m + 1106560), True))
```

3.125 $\int x^m \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=71

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

```
[Out] a^2*A*x^(1+m)/(1+m)+a*(2*A*b+B*a)*x^(4+m)/(4+m)+b*(A*b+2*B*a)*x^(7+m)/(7+m)
+b^2*B*x^(10+m)/(10+m)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{a^2Ax^{m+1}}{m+1}+\frac{ax^{m+4}(aB+2Ab)}{m+4}+\frac{bx^{m+7}(2aB+Ab)}{m+7}+\frac{b^2Bx^{m+10}}{m+10}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (a^2*A*x^(1 + m))/(1 + m) + (a*(2*A*b + a*B)*x^(4 + m))/(4 + m) + (b*(A*b +
 2*a*B)*x^(7 + m))/(7 + m) + (b^2*B*x^(10 + m))/(10 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)^2\left(A+Bx^3\right)dx&=\int\left(a^2Ax^m+a(2Ab+aB)x^{3+m}+b(Ab+2aB)x^{6+m}+b^2Bx^{9+m}\right)dx\\&=\frac{a^2Ax^{1+m}}{1+m}+\frac{a(2Ab+aB)x^{4+m}}{4+m}+\frac{b(Ab+2aB)x^{7+m}}{7+m}+\frac{b^2Bx^{10+m}}{10+m}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.93

$$x^{m+1}\left(\frac{a^2A}{m+1}+\frac{bx^6(2aB+Ab)}{m+7}+\frac{ax^3(aB+2Ab)}{m+4}+\frac{b^2Bx^9}{m+10}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a^2*A)/(1 + m) + (a*(2*A*b + a*B)*x^3)/(4 + m) + (b*(A*b + 2*a*
B)*x^6)/(7 + m) + (b^2*B*x^9)/(10 + m))
```

fricas [B] time = 0.94, size = 215, normalized size = 3.03

$$\frac{\left(\left(Bb^2m^3+12\,Bb^2m^2+39\,Bb^2m+28\,Bb^2\right)x^{10}+\left(\left(2\,Bab+Ab^2\right)m^3+80\,Bab+40\,Ab^2+15\left(2\,Bab+Ab^2\right)m^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b^2*m^3 + 12*B*b^2*m^2 + 39*B*b^2*m + 28*B*b^2)*x^10 + ((2*B*a*b + A*b^
2)*m^3 + 80*B*a*b + 40*A*b^2 + 15*(2*B*a*b + A*b^2)*m^2 + 54*(2*B*a*b + A*b
^2)*m)*x^7 + ((B*a^2 + 2*A*a*b)*m^3 + 70*B*a^2 + 140*A*a*b + 18*(B*a^2 + 2*
A*a*b)*m^2 + 87*(B*a^2 + 2*A*a*b)*m)*x^4 + (A*a^2*m^3 + 21*A*a^2*m^2 + 138*
A*a^2*m + 280*A*a^2)*x)*x^m/(m^4 + 22*m^3 + 159*m^2 + 418*m + 280)
```

giac [B] time = 0.20, size = 332, normalized size = 4.68

$$\frac{Bb^2m^3x^{10}x^m + 12\,Bb^2m^2x^{10}x^m + 39\,Bb^2mx^{10}x^m + 2\,Babm^3x^7x^m + Ab^2m^3x^7x^m + 28\,Bb^2x^{10}x^m + 30\,Babm^2x^7x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b^2*m^3*x^10*x^m + 12*B*b^2*m^2*x^10*x^m + 39*B*b^2*m*x^10*x^m + 2*B*a*b
*m^3*x^7*x^m + A*b^2*m^3*x^7*x^m + 28*B*b^2*x^10*x^m + 30*B*a*b*m^2*x^7*x^m
 + 15*A*b^2*m^2*x^7*x^m + 108*B*a*b*m*x^7*x^m + 54*A*b^2*m*x^7*x^m + B*a^2*
m^3*x^4*x^m + 2*A*a*b*m^3*x^4*x^m + 80*B*a*b*x^7*x^m + 40*A*b^2*x^7*x^m + 1
8*B*a^2*m^2*x^4*x^m + 36*A*a*b*m^2*x^4*x^m + 87*B*a^2*m*x^4*x^m + 174*A*a*b
*m*x^4*x^m + A*a^2*m^3*x*x^m + 70*B*a^2*x^4*x^m + 140*A*a*b*x^4*x^m + 21*A*
a^2*m^2*x*x^m + 138*A*a^2*m*x*x^m + 280*A*a^2*x*x^m)/(m^4 + 22*m^3 + 159*m^
2 + 418*m + 280)
```

maple [B] time = 0.05, size = 262, normalized size = 3.69

$$\frac{\left(B\,b^2m^3x^9 + 12B\,b^2m^2x^9 + 39B\,b^2m\,x^9 + A\,b^2m^3x^6 + 2Bab\,m^3x^6 + 28b^2B\,x^9 + 15A\,b^2m^2x^6 + 30Bab\,m^2x^6 + 54A\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b^2*m^3*x^9+12*B*b^2*m^2*x^9+39*B*b^2*m*x^9+A*b^2*m^3*x^6+2*B*a*
b*m^3*x^6+28*B*b^2*x^9+15*A*b^2*m^2*x^6+30*B*a*b*m^2*x^6+54*A*b^2*m*x^6+108
*B*a*b*m*x^6+2*A*a*b*m^3*x^3+40*A*b^2*x^6+B*a^2*m^3*x^3+80*B*a*b*x^6+36*A*a
*b*m^2*x^3+18*B*a^2*m^2*x^3+174*A*a*b*m*x^3+87*B*a^2*m*x^3+A*a^2*m^3+140*A*
a*b*x^3+70*B*a^2*x^3+21*A*a^2*m^2+138*A*a^2*m+280*A*a^2)/(m+10)/(m+7)/(m+4)
/(m+1)
```

maxima [A] time = 0.49, size = 91, normalized size = 1.28

$$\frac{Bb^2x^{m+10}}{m+10} + \frac{2\,Babx^{m+7}}{m+7} + \frac{Ab^2x^{m+7}}{m+7} + \frac{Ba^2x^{m+4}}{m+4} + \frac{2\,Aabx^{m+4}}{m+4} + \frac{Aa^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b^2*x^(m + 10)/(m + 10) + 2*B*a*b*x^(m + 7)/(m + 7) + A*b^2*x^(m + 7)/(m
+ 7) + B*a^2*x^(m + 4)/(m + 4) + 2*A*a*b*x^(m + 4)/(m + 4) + A*a^2*x^(m + 1
)/(m + 1)
```

mupad [B] time = 2.72, size = 177, normalized size = 2.49

$$x^m\left(\frac{B\,b^2\,x^{10}\,\left(m^3+12\,m^2+39\,m+28\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{A\,a^2\,x\,\left(m^3+21\,m^2+138\,m+280\right)}{m^4+22\,m^3+159\,m^2+418\,m+280}+\frac{a\,x^4\,\left(2\,A\,b+B\,a\right)\,\left(m^3+18\right.}{m^4+22\,m^3+159\,m^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^m*((B*b^2*x^10*(39*m + 12*m^2 + m^3 + 28))/(418*m + 159*m^2 + 22*m^3 + m^
4 + 280) + (A*a^2*x*(138*m + 21*m^2 + m^3 + 280))/(418*m + 159*m^2 + 22*m^3
 + m^4 + 280) + (a*x^4*(2*A*b + B*a)*(87*m + 18*m^2 + m^3 + 70))/(418*m + 1
59*m^2 + 22*m^3 + m^4 + 280) + (b*x^7*(A*b + 2*B*a)*(54*m + 15*m^2 + m^3 +
40))/(418*m + 159*m^2 + 22*m^3 + m^4 + 280))
```

sympy [A] time = 6.31, size = 1057, normalized size = 14.89

$$\begin{cases} -\frac{Aa^2}{9x^9} - \frac{Aab}{3x^6} - \frac{Ab^2}{3x^3} - \frac{Ba^2}{6x^6} - \frac{2Bab}{3x^3} + Bb^2\log(x) \\ -\frac{Aa^2}{6x^6} - \frac{2Aab}{3x^3} + Ab^2\log(x) - \frac{Ba^2}{3x^3} + 2Bab\log(x) + \frac{Bb^2x^3}{3} \\ -\frac{Aa^2}{3x^3} + 2Aab\log(x) + \frac{Ab^2x^3}{3} + Ba^2\log(x) + \frac{2Babx^3}{3} + \frac{Bb^2x^6}{6} \\ Aa^2\log(x) + \frac{2Aabx^3}{3} + \frac{Ab^2x^6}{6} + \frac{Ba^2x^3}{3} + \frac{Babx^6}{3} + \frac{Bb^2x^9}{9} \\ \frac{Aa^2m^3xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{21Aa^2m^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{138Aa^2mxx^m}{m^4+22m^3+159m^2+418m+280} + \frac{280Aa^2xx^m}{m^4+22m^3+159m^2+418m+280} + \frac{}{m^4+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a**2/(9*x**9) - A*a*b/(3*x**6) - A*b**2/(3*x**3) - B*a**2/(6*
x**6) - 2*B*a*b/(3*x**3) + B*b**2*log(x), Eq(m, -10)), (-A*a**2/(6*x**6) -
2*A*a*b/(3*x**3) + A*b**2*log(x) - B*a**2/(3*x**3) + 2*B*a*b*log(x) + B*b**
2*x**3/3, Eq(m, -7)), (-A*a**2/(3*x**3) + 2*A*a*b*log(x) + A*b**2*x**3/3 +
B*a**2*log(x) + 2*B*a*b*x**3/3 + B*b**2*x**6/6, Eq(m, -4)), (A*a**2*log(x)
+ 2*A*a*b*x**3/3 + A*b**2*x**6/6 + B*a**2*x**3/3 + B*a*b*x**6/3 + B*b**2*x*
*9/9, Eq(m, -1)), (A*a**2*m**3*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m +
280) + 21*A*a**2*m**2*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 13
8*A*a**2*m*x*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 280*A*a**2*x*
x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 2*A*a*b*m**3*x**4*x**m/(m*
*4 + 22*m**3 + 159*m**2 + 418*m + 280) + 36*A*a*b*m**2*x**4*x**m/(m**4 + 22
*m**3 + 159*m**2 + 418*m + 280) + 174*A*a*b*m*x**4*x**m/(m**4 + 22*m**3 + 1
59*m**2 + 418*m + 280) + 140*A*a*b*x**4*x**m/(m**4 + 22*m**3 + 159*m**2 + 4
18*m + 280) + A*b**2*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 28
0) + 15*A*b**2*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 5
4*A*b**2*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 40*A*b**2*
x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*a**2*m**3*x**4*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 18*B*a**2*m**2*x**4*x**m/(m**4
 + 22*m**3 + 159*m**2 + 418*m + 280) + 87*B*a**2*m*x**4*x**m/(m**4 + 22*m**
3 + 159*m**2 + 418*m + 280) + 70*B*a**2*x**4*x**m/(m**4 + 22*m**3 + 159*m**
2 + 418*m + 280) + 2*B*a*b*m**3*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*
m + 280) + 30*B*a*b*m**2*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280
) + 108*B*a*b*m*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 80*B*
a*b*x**7*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + B*b**2*m**3*x**10
*x**m/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 12*B*b**2*m**2*x**10*x**m
/(m**4 + 22*m**3 + 159*m**2 + 418*m + 280) + 39*B*b**2*m*x**10*x**m/(m**4 +
 22*m**3 + 159*m**2 + 418*m + 280) + 28*B*b**2*x**10*x**m/(m**4 + 22*m**3 +
 159*m**2 + 418*m + 280), True))
```

3.126 $\int x^m \left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=45

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

```
[Out] a*A*x^(1+m)/(1+m)+(A*b+B*a)*x^(4+m)/(4+m)+b*B*x^(7+m)/(7+m)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {448}

$$\frac{x^{m+4}(aB+Ab)}{m+4}+\frac{aAx^{m+1}}{m+1}+\frac{bBx^{m+7}}{m+7}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] (a*A*x^(1 + m))/(1 + m) + ((A*b + a*B)*x^(4 + m))/(4 + m) + (b*B*x^(7 + m))
/(7 + m)
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^m\left(a+bx^3\right)\left(A+Bx^3\right)dx&=\int\left(aAx^m+(Ab+aB)x^{3+m}+bBx^{6+m}\right)dx\\&=\frac{aAx^{1+m}}{1+m}+\frac{(Ab+aB)x^{4+m}}{4+m}+\frac{bBx^{7+m}}{7+m}\end{aligned}$$

Mathematica [A] time = 0.05, size = 42, normalized size = 0.93

$$x^{m+1}\left(\frac{x^3(aB+Ab)}{m+4}+\frac{aA}{m+1}+\frac{bBx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x^3)*(A + B*x^3),x]
```

```
[Out] x^(1 + m)*((a*A)/(1 + m) + ((A*b + a*B)*x^3)/(4 + m) + (b*B*x^6)/(7 + m))
```

fricas [B] time = 1.07, size = 92, normalized size = 2.04

$$\frac{\left(\left(Bbm^2+5\,Bbm+4\,Bb\right)x^7+\left((Ba+Ab)m^2+7\,Ba+7\,Ab+8\,(Ba+Ab)m\right)x^4+\left(Aam^2+11\,Aam+28\,Aa\right)x\right)x^m}{m^3+12\,m^2+39\,m+28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] ((B*b*m^2 + 5*B*b*m + 4*B*b)*x^7 + ((B*a + A*b)*m^2 + 7*B*a + 7*A*b + 8*(B*
a + A*b)*m)*x^4 + (A*a*m^2 + 11*A*a*m + 28*A*a)*x)*x^m/(m^3 + 12*m^2 + 39*m
 + 28)
```

giac [B] time = 0.17, size = 143, normalized size = 3.18

$$\frac{Bbm^2x^7x^m + 5\,Bbmx^7x^m + 4\,Bbx^7x^m + Bam^2x^4x^m + Abm^2x^4x^m + 8\,Bamx^4x^m + 8\,Abmx^4x^m + 7\,Bax^4x^m + 7\,}{m^3 + 12\,m^2 + 39\,m + 28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] (B*b*m^2*x^7*x^m + 5*B*b*m*x^7*x^m + 4*B*b*x^7*x^m + B*a*m^2*x^4*x^m + A*b*
m^2*x^4*x^m + 8*B*a*m*x^4*x^m + 8*A*b*m*x^4*x^m + 7*B*a*x^4*x^m + 7*A*b*x^4
*x^m + A*a*m^2*x*x^m + 11*A*a*m*x*x^m + 28*A*a*x*x^m)/(m^3 + 12*m^2 + 39*m
+ 28)
```

maple [B] time = 0.04, size = 110, normalized size = 2.44

$$\frac{\left(Bb\,m^2x^6 + 5Bbm\,x^6 + 4Bb\,x^6 + Ab\,m^2x^3 + Ba\,m^2x^3 + 8Abm\,x^3 + 8Bam\,x^3 + 7Ab\,x^3 + 7Ba\,x^3 + Aa\,m^2 + 11\right.}{(m+7)\,(m+4)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] x^(m+1)*(B*b*m^2*x^6+5*B*b*m*x^6+4*B*b*x^6+A*b*m^2*x^3+B*a*m^2*x^3+8*A*b*m*
x^3+8*B*a*m*x^3+7*A*b*x^3+7*B*a*x^3+A*a*m^2+11*A*a*m+28*A*a)/(m+7)/(m+4)/(m
+1)
```

maxima [A] time = 0.61, size = 53, normalized size = 1.18

$$\frac{Bbx^{m+7}}{m+7} + \frac{Bax^{m+4}}{m+4} + \frac{Abx^{m+4}}{m+4} + \frac{Aax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] B*b*x^(m + 7)/(m + 7) + B*a*x^(m + 4)/(m + 4) + A*b*x^(m + 4)/(m + 4) + A*a
*x^(m + 1)/(m + 1)
```

mupad [B] time = 2.65, size = 95, normalized size = 2.11

$$x^m\left(\frac{x^4\,\left(A\,b+B\,a\right)\,\left(m^2+8\,m+7\right)}{m^3+12\,m^2+39\,m+28}+\frac{B\,b\,x^7\,\left(m^2+5\,m+4\right)}{m^3+12\,m^2+39\,m+28}+\frac{A\,a\,x\,\left(m^2+11\,m+28\right)}{m^3+12\,m^2+39\,m+28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] x^m*((x^4*(A*b + B*a)*(8*m + m^2 + 7))/(39*m + 12*m^2 + m^3 + 28) + (B*b*x^
7*(5*m + m^2 + 4))/(39*m + 12*m^2 + m^3 + 28) + (A*a*x*(11*m + m^2 + 28))/(
39*m + 12*m^2 + m^3 + 28))
```

sympy [A] time = 2.89, size = 410, normalized size = 9.11

$$\begin{cases} -\frac{Aa}{6x^6} - \frac{Ab}{3x^3} - \frac{Ba}{3x^3} + Bb\log\left(x\right) \\ -\frac{Aa}{3x^3} + Ab\log\left(x\right) + Ba\log\left(x\right) + \frac{Bbx^3}{3} \\ Aa\log\left(x\right) + \frac{Abx^3}{3} + \frac{Bax^3}{3} + \frac{Bbx^6}{6} \\ \frac{Aam^2xx^m}{m^3+12m^2+39m+28} + \frac{11Aamxx^m}{m^3+12m^2+39m+28} + \frac{28Aaxx^m}{m^3+12m^2+39m+28} + \frac{Abm^2x^4x^m}{m^3+12m^2+39m+28} + \frac{8Abmx^4x^m}{m^3+12m^2+39m+28} + \frac{7Abx^4x^m}{m^3+12m^2+39m+28} + \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] Piecewise((-A*a/(6*x**6) - A*b/(3*x**3) - B*a/(3*x**3) + B*b*log(x), Eq(m,
-7)), (-A*a/(3*x**3) + A*b*log(x) + B*a*log(x) + B*b*x**3/3, Eq(m, -4)), (A
*a*log(x) + A*b*x**3/3 + B*a*x**3/3 + B*b*x**6/6, Eq(m, -1)), (A*a*m**2*x*x
**m/(m**3 + 12*m**2 + 39*m + 28) + 11*A*a*m*x*x**m/(m**3 + 12*m**2 + 39*m +
 28) + 28*A*a*x*x**m/(m**3 + 12*m**2 + 39*m + 28) + A*b*m**2*x**4*x**m/(m**
3 + 12*m**2 + 39*m + 28) + 8*A*b*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) +
 7*A*b*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*a*m**2*x**4*x**m/(m**3 +
12*m**2 + 39*m + 28) + 8*B*a*m*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + 7*B
*a*x**4*x**m/(m**3 + 12*m**2 + 39*m + 28) + B*b*m**2*x**7*x**m/(m**3 + 12*m
**2 + 39*m + 28) + 5*B*b*m*x**7*x**m/(m**3 + 12*m**2 + 39*m + 28) + 4*B*b*x
**7*x**m/(m**3 + 12*m**2 + 39*m + 28), True))
```

3.127 $\int \frac{x^m(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=66

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

```
[Out] B*x^(1+m)/b/(1+m)+(A*b-B*a)*x^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b
*x^3/a)/a/b/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {459, 364}

$$\frac{x^{m+1}(Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ab(m+1)}+\frac{Bx^{m+1}}{b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (B*x^(1 + m))/(b*(1 + m)) + ((A*b - a*B)*x^(1 + m)*Hypergeometric2F1[1, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(a*b*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^3\right)}{a+bx^3}\,dx &= \frac{Bx^{1+m}}{b(1+m)}-\frac{(-Ab(1+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{b(1+m)}\\ &= \frac{Bx^{1+m}}{b(1+m)}+\frac{(Ab-aB)x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{ab(1+m)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.83

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\right)}{ab(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (x^(1 + m)*(a*B + (A*b - a*B)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -(
(b*x^3)/a)]))/(a*b*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)x^m}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(B\,x^3+A\right)x^m}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^m}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m\,\left(B\,x^3+A\right)}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3), x)
```

sympy [C] time = 22.06, size = 190, normalized size = 2.88

$$\frac{Amxx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Axx^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{1}{3}\right)\Gamma\left(\frac{m}{3}+\frac{1}{3}\right)}{9a\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)}+\frac{Bmx^4x^m\Phi\left(\frac{bx^3e^{i\pi}}{a},1,\frac{m}{3}+\frac{4}{3}\right)\Gamma\left(\frac{m}{3}+\right.}{9a\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] A*m*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 1/3)*gamma(m/3 + 1/3
)/(9*a*gamma(m/3 + 4/3)) + A*x*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m
/3 + 1/3)*gamma(m/3 + 1/3)/(9*a*gamma(m/3 + 4/3)) + B*m*x**4*x**m*lerchphi(
b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma(m/3 + 4/3)/(9*a*gamma(m/3 + 7
/3)) + 4*B*x**4*x**m*lerchphi(b*x**3*exp_polar(I*pi)/a, 1, m/3 + 4/3)*gamma
(m/3 + 4/3)/(9*a*gamma(m/3 + 7/3))
```

3.128 $\int \frac{x^m(A+Bx^3)}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] 1/3*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)+1/3*(A*b*(2-m)+a*B*(1+m))*x^(1+m)*hyper
geom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^2/b/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+Ab(2-m))\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3a^2b(m+1)}+\frac{x^{m+1}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(3*a*b*(a + b*x^3)) + ((A*b*(2 - m) + a*B*(1 + m))*
x^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a^2*
b*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx &= \frac{(Ab-aB)x^{1+m}}{3ab\left(a+bx^3\right)}+\frac{(-Ab(-2+m)+aB(1+m))\int \frac{x^m}{a+bx^3}\,dx}{3ab}\\ &= \frac{(Ab-aB)x^{1+m}}{3ab\left(a+bx^3\right)}+\frac{(Ab(2-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{3a^2b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{a^2b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^2
*b*(1 + m))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3 + A\right)x^m}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x^m}{\left(bx^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(B\,x^3 + A\right)x^m}{\left(b\,x^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x^m}{\left(bx^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^m\,\left(B\,x^3 + A\right)}{\left(b\,x^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.129 $\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=93

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] 1/6*(A*b-B*a)*x^(1+m)/a/b/(b*x^3+a)^2+1/6*(A*b*(5-m)+a*B*(1+m))*x^(1+m)*hyp
ergeom([2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a^3/b/(1+m)
```

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {457, 364}

$$\frac{x^{m+1}(aB(m+1)+Ab(5-m))\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{6a^3b(m+1)}+\frac{x^{m+1}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(1 + m))/(6*a*b*(a + b*x^3)^2) + ((A*b*(5 - m) + a*B*(1 + m)
)*x^(1 + m)*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(6*a^
3*b*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{x^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{(-Ab(-5+m)+aB(1+m))\int \frac{x^m}{\left(a+bx^3\right)^2}\,dx}{6ab}\\ &= \frac{(Ab-aB)x^{1+m}}{6ab\left(a+bx^3\right)^2}+\frac{(Ab(5-m)+aB(1+m))x^{1+m}\,{}_2F_1\left(2,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{6a^3b(1+m)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.86

$$\frac{x^{m+1}\left((Ab-aB)\,{}_2F_1\left(3,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(2,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{a^3b(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (x^(1 + m)*(a*B*Hypergeometric2F1[2, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)] +
(A*b - a*B)*Hypergeometric2F1[3, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)]))/(a^3
*b*(1 + m))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)x^m}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*x^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)x^m}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int\frac{\left(B\,x^3+A\right)x^m}{\left(b\,x^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] int(x^m*(B*x^3+A)/(b*x^3+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)x^m}{\left(bx^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^m/(b*x^3 + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^m\,\left(B\,x^3+A\right)}{\left(b\,x^3+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] int((x^m*(A + B*x^3))/(a + b*x^3)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.130 $\int \frac{(ex)^m}{(a+bx^3)(c+dx^3)} dx$

Optimal. Leaf size=112

$$\frac{b(ex)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)/a/(-a*d+b*c)/e
/(1+m)-d*(e*x)^(1+m)*hypergeom([1, 1/3+1/3*m],[4/3+1/3*m],-d*x^3/c)/c/(-a*d
+b*c)/e/(1+m)
```

Rubi [A] time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {482, 364}

$$\frac{b(ex)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1} {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/
(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/3
, (4 + m)/3, -((d*x^3)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\left(a+bx^3\right)\left(c+dx^3\right)} dx &= \frac{b\int \frac{(ex)^m}{a+bx^3} dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{c+dx^3} dx}{bc-ad}\\ &= \frac{b(ex)^{1+m} {}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{bx^3}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m} {}_2F_1\left(1,\frac{1+m}{3};\frac{4+m}{3};-\frac{dx^3}{c}\right)}{c(bc-ad)e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 86, normalized size = 0.77

$$\frac{x(ex)^m \left(ad\, {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{dx^3}{c}\right)-bc\, {}_2F_1\left(1,\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((b*x^3)/a)])
 + a*d*Hypergeometric2F1[1, (1 + m)/3, (4 + m)/3, -((d*x^3)/c)]))/(a*c*(-(b
*c) + a*d)*(1 + m))
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(ex)^m}{bdx^6 + (bc + ad)x^3 + ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*d*x^6 + (b*c + a*d)*x^3 + a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^3 + a)(dx^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^3 + a)\,(d\,x^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] int((e*x)^m/(b*x^3+a)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^3 + a)(dx^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^3 + a)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(b\,x^3 + a)\,(d\,x^3 + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^3)*(c + d*x^3)),x)
```

```
[Out] int((e*x)^m/((a + b*x^3)*(c + d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.131 $\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

```
[Out] 2/9*a*A*x^(9/2)+2/15*(A*b+B*a)*x^(15/2)+2/21*b*B*x^(21/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{15}x^{15/2}(aB+Ab)+\frac{2}{9}aAx^{9/2}+\frac{2}{21}bBx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(9/2))/9 + (2*(A*b + a*B)*x^(15/2))/15 + (2*b*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aAx^{7/2}+(Ab+aB)x^{13/2}+bBx^{19/2}\right)\,dx\\ &= \frac{2}{9}aAx^{9/2}+\frac{2}{15}(Ab+aB)x^{15/2}+\frac{2}{21}bBx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{315}x^{9/2}\left(21x^3(aB+Ab)+35aA+15bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(9/2)*(35*a*A + 21*(A*b + a*B)*x^3 + 15*b*B*x^6))/315
```

fricas [A] time = 0.78, size = 32, normalized size = 0.82

$$\frac{2}{315}\left(15\,Bbx^{10}+21\,(Ba+Ab)x^7+35\,Aax^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/315*(15*B*b*x^10 + 21*(B*a + A*b)*x^7 + 35*A*a*x^4)*sqrt(x)
```

giac [A] time = 0.17, size = 29, normalized size = 0.74

$$\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,Bax^{\frac{15}{2}}+\frac{2}{15}\,Abx^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*B*a*x^(15/2) + 2/15*A*b*x^(15/2) + 2/9*A*a*x^(9/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(15Bb\,x^{6}+21Ab\,x^{3}+21Ba\,x^{3}+35Aa\right)x^{\frac{9}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/315*x^(9/2)*(15*B*b*x^6+21*A*b*x^3+21*B*a*x^3+35*A*a)
```

maxima [A] time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{21}\,Bbx^{\frac{21}{2}}+\frac{2}{15}\,(Ba+Ab)x^{\frac{15}{2}}+\frac{2}{9}\,Aax^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/21*B*b*x^(21/2) + 2/15*(B*a + A*b)*x^(15/2) + 2/9*A*a*x^(9/2)
```

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$\frac{2\,x^{9/2}\,\left(35\,A\,a+21\,A\,b\,x^3+21\,B\,a\,x^3+15\,B\,b\,x^6\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(9/2)*(35*A*a + 21*A*b*x^3 + 21*B*a*x^3 + 15*B*b*x^6))/315
```

sympy [A] time = 20.86, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{9}{2}}}{9}+\frac{2Abx^{\frac{15}{2}}}{15}+\frac{2Bax^{\frac{15}{2}}}{15}+\frac{2Bbx^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(9/2)/9 + 2*A*b*x**(15/2)/15 + 2*B*a*x**(15/2)/15 + 2*B*b*x**(21/2
)/21
```

3.132 $\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

```
[Out] 2/7*a*A*x^(7/2)+2/13*(A*b+B*a)*x^(13/2)+2/19*b*B*x^(19/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{13}x^{13/2}(aB+Ab)+\frac{2}{7}aAx^{7/2}+\frac{2}{19}bBx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(7/2))/7 + (2*(A*b + a*B)*x^(13/2))/13 + (2*b*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{5/2}+(Ab+aB)x^{11/2}+bBx^{17/2}\right)\,dx\\&=\frac{2}{7}aAx^{7/2}+\frac{2}{13}(Ab+aB)x^{13/2}+\frac{2}{19}bBx^{19/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2x^{7/2}\left(133x^3(aB+Ab)+247aA+91bBx^6\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(7/2)*(247*a*A + 133*(A*b + a*B)*x^3 + 91*b*B*x^6))/1729
```

fricas [A] time = 0.61, size = 32, normalized size = 0.82

$$\frac{2}{1729}\left(91\,Bbx^9+133\,(Ba+Ab)x^6+247\,Aax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/1729*(91*B*b*x^9 + 133*(B*a + A*b)*x^6 + 247*A*a*x^3)*sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,Bax^{\frac{13}{2}}+\frac{2}{13}\,Abx^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*B*a*x^(13/2) + 2/13*A*b*x^(13/2) + 2/7*A*a*x^(7/2)
```

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{2\left(91Bb\,x^{6}+133Ab\,x^{3}+133Ba\,x^{3}+247Aa\right)x^{\frac{7}{2}}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/1729*x^(7/2)*(91*B*b*x^6+133*A*b*x^3+133*B*a*x^3+247*A*a)
```

maxima [A] time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{19}\,Bbx^{\frac{19}{2}}+\frac{2}{13}\,(Ba+Ab)x^{\frac{13}{2}}+\frac{2}{7}\,Aax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/19*B*b*x^(19/2) + 2/13*(B*a + A*b)*x^(13/2) + 2/7*A*a*x^(7/2)
```

mupad [B] time = 2.56, size = 31, normalized size = 0.79

$$\frac{2\,x^{7/2}\,\left(247\,A\,a+133\,A\,b\,x^3+133\,B\,a\,x^3+91\,B\,b\,x^6\right)}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(7/2)*(247*A*a + 133*A*b*x^3 + 133*B*a*x^3 + 91*B*b*x^6))/1729
```

sympy [A] time = 12.71, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{7}{2}}}{7}+\frac{2Abx^{\frac{13}{2}}}{13}+\frac{2Bax^{\frac{13}{2}}}{13}+\frac{2Bbx^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(7/2)/7 + 2*A*b*x**(13/2)/13 + 2*B*a*x**(13/2)/13 + 2*B*b*x**(19/2
)/19
```

3.133 $\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

```
[Out] 2/5*a*A*x^(5/2)+2/11*(A*b+B*a)*x^(11/2)+2/17*b*B*x^(17/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{11}x^{11/2}(aB+Ab)+\frac{2}{5}aAx^{5/2}+\frac{2}{17}bBx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(5/2))/5 + (2*(A*b + a*B)*x^(11/2))/11 + (2*b*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx&=\int\left(aAx^{3/2}+(Ab+aB)x^{9/2}+bBx^{15/2}\right)\,dx\\&=\frac{2}{5}aAx^{5/2}+\frac{2}{11}(Ab+aB)x^{11/2}+\frac{2}{17}bBx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.85

$$\frac{2}{935}x^{5/2}\left(85x^3(aB+Ab)+187aA+55bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(5/2)*(187*a*A + 85*(A*b + a*B)*x^3 + 55*b*B*x^6))/935
```

fricas [A] time = 0.56, size = 32, normalized size = 0.82

$$\frac{2}{935}\left(55\,Bbx^8+85\,(Ba+Ab)x^5+187\,Aax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="fricas")
[Out] 2/935*(55*B*b*x^8 + 85*(B*a + A*b)*x^5 + 187*A*a*x^2)*sqrt(x)
```

giac [A] time = 0.18, size = 29, normalized size = 0.74

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,Bax^{\frac{11}{2}}+\frac{2}{11}\,Abx^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*B*a*x^(11/2) + 2/11*A*b*x^(11/2) + 2/5*A*a*x^(5/2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.82

$$\frac{2\left(55Bb\,x^{6}+85Ab\,x^{3}+85Ba\,x^{3}+187Aa\right)x^{\frac{5}{2}}}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)*(B*x^3+A),x)
```

```
[Out] 2/935*x^(5/2)*(55*B*b*x^6+85*A*b*x^3+85*B*a*x^3+187*A*a)
```

maxima [A] time = 0.45, size = 27, normalized size = 0.69

$$\frac{2}{17}\,Bbx^{\frac{17}{2}}+\frac{2}{11}\,(Ba+Ab)x^{\frac{11}{2}}+\frac{2}{5}\,Aax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/17*B*b*x^(17/2) + 2/11*(B*a + A*b)*x^(11/2) + 2/5*A*a*x^(5/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{5/2}\,\left(187\,A\,a+85\,A\,b\,x^3+85\,B\,a\,x^3+55\,B\,b\,x^6\right)}{935}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(5/2)*(187*A*a + 85*A*b*x^3 + 85*B*a*x^3 + 55*B*b*x^6))/935
```

sympy [A] time = 6.74, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{5}{2}}}{5}+\frac{2Abx^{\frac{11}{2}}}{11}+\frac{2Bax^{\frac{11}{2}}}{11}+\frac{2Bbx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)*(B*x**3+A),x)
```

```
[Out] 2*A*a*x**(5/2)/5 + 2*A*b*x**(11/2)/11 + 2*B*a*x**(11/2)/11 + 2*B*b*x**(17/2
)/17
```

3.134 $\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=39

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

```
[Out] 2/3*a*A*x^(3/2)+2/9*(A*b+B*a)*x^(9/2)+2/15*b*B*x^(15/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{9}x^{9/2}(aB+Ab)+\frac{2}{3}aAx^{3/2}+\frac{2}{15}bBx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*a*A*x^(3/2))/3 + (2*(A*b + a*B)*x^(9/2))/9 + (2*b*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)\left(A+Bx^3\right)\,dx &= \int \left(aA\sqrt{x}+(Ab+aB)x^{7/2}+bBx^{13/2}\right)dx\\ &= \frac{2}{3}aAx^{3/2}+\frac{2}{9}(Ab+aB)x^{9/2}+\frac{2}{15}bBx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{2}{45}x^{3/2}\left(5x^3(aB+Ab)+15aA+3bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)*(A + B*x^3),x]
[Out] (2*x^(3/2)*(15*a*A + 5*(A*b + a*B)*x^3 + 3*b*B*x^6))/45
```

fricas [A] time = 0.84, size = 30, normalized size = 0.77

$$\frac{2}{45}\left(3\,Bbx^7+5\,(Ba+Ab)x^4+15\,Aax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*B*b*x^7 + 5*(B*a + A*b)*x^4 + 15*A*a*x)*sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.74

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,Bax^{\frac{9}{2}}+\frac{2}{9}\,Abx^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/9*B*a*x^(9/2) + 2/9*A*b*x^(9/2) + 2/3*A*a*x^(3/2)
```

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$\frac{2\left(3Bb\,x^{6}+5Ab\,x^{3}+5Ba\,x^{3}+15Aa\right)x^{\frac{3}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/45*x^(3/2)*(3*B*b*x^6+5*A*b*x^3+5*B*a*x^3+15*A*a)
```

maxima [A] time = 0.47, size = 27, normalized size = 0.69

$$\frac{2}{15}\,Bbx^{\frac{15}{2}}+\frac{2}{9}\,(Ba+Ab)x^{\frac{9}{2}}+\frac{2}{3}\,Aax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b*x^(15/2) + 2/9*(B*a + A*b)*x^(9/2) + 2/3*A*a*x^(3/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{2\,x^{3/2}\,\left(15\,A\,a+5\,A\,b\,x^3+5\,B\,a\,x^3+3\,B\,b\,x^6\right)}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3),x)
```

```
[Out] (2*x^(3/2)*(15*A*a + 5*A*b*x^3 + 5*B*a*x^3 + 3*B*b*x^6))/45
```

sympy [A] time = 3.37, size = 46, normalized size = 1.18

$$\frac{2Aax^{\frac{3}{2}}}{3}+\frac{2Abx^{\frac{9}{2}}}{9}+\frac{2Bax^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a*x**(3/2)/3 + 2*A*b*x**(9/2)/9 + 2*B*a*x**(9/2)/9 + 2*B*b*x**(15/2)/15
```

3.135 $\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

```
[Out] 2/7*(A*b+B*a)*x^(7/2)+2/13*b*B*x^(13/2)+2*a*A*x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{7}x^{7/2}(aB+Ab)+2aA\sqrt{x}+\frac{2}{13}bBx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
[Out] 2*a*A*Sqrt[x] + (2*(A*b + a*B)*x^(7/2))/7 + (2*b*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{\sqrt{x}}\,dx &= \int \left(\frac{aA}{\sqrt{x}}+(Ab+aB)x^{5/2}+bBx^{11/2}\right)dx\\ &= 2aA\sqrt{x}+\frac{2}{7}(Ab+aB)x^{7/2}+\frac{2}{13}bBx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 33, normalized size = 0.89

$$\frac{2}{91}\sqrt{x}\left(13x^3(aB+Ab)+91aA+7bBx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/Sqrt[x],x]
[Out] (2*Sqrt[x]*(91*a*A + 13*(A*b + a*B)*x^3 + 7*b*B*x^6))/91
```

fricas [A] time = 0.80, size = 29, normalized size = 0.78

$$\frac{2}{91}\left(7\,Bbx^6+13\,(Ba+Ab)x^3+91\,Aa\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
[Out] 2/91*(7*B*b*x^6 + 13*(B*a + A*b)*x^3 + 91*A*a)*sqrt(x)
```

giac [**A**] time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{7}\,Bax^{\frac{7}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="giac")
[Out] 2/13*B*b*x^(13/2) + 2/7*B*a*x^(7/2) + 2/7*A*b*x^(7/2) + 2*A*a*sqrt(x)
```

maple [**A**] time = 0.04, size = 32, normalized size = 0.86

$$\frac{2\left(7Bb\,x^{6}+13Ab\,x^{3}+13Ba\,x^{3}+91Aa\right)\sqrt{x}}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(1/2),x)
[Out] 2/91*x^(1/2)*(7*B*b*x^6+13*A*b*x^3+13*B*a*x^3+91*A*a)
```

maxima [**A**] time = 0.50, size = 27, normalized size = 0.73

$$\frac{2}{13}\,Bbx^{\frac{13}{2}}+\frac{2}{7}\,(Ba+Ab)x^{\frac{7}{2}}+2\,Aa\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
[Out] 2/13*B*b*x^(13/2) + 2/7*(B*a + A*b)*x^(7/2) + 2*A*a*sqrt(x)
```

mupad [**B**] time = 2.59, size = 31, normalized size = 0.84

$$\frac{2\,\sqrt{x}\,\left(91\,A\,a+13\,A\,b\,x^3+13\,B\,a\,x^3+7\,B\,b\,x^6\right)}{91}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(1/2),x)
[Out] (2*x^(1/2)*(91*A*a + 13*A*b*x^3 + 13*B*a*x^3 + 7*B*b*x^6))/91
```

sympy [**A**] time = 2.30, size = 44, normalized size = 1.19

$$2Aa\sqrt{x}+\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Bax^{\frac{7}{2}}}{7}+\frac{2Bbx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(1/2),x)
[Out] 2*A*a*sqrt(x) + 2*A*b*x**(7/2)/7 + 2*B*a*x**(7/2)/7 + 2*B*b*x**(13/2)/13
```

3.136 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} \, dx$

Optimal. Leaf size=37

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

```
[Out] 2/5*(A*b+B*a)*x^(5/2)+2/11*b*B*x^(11/2)-2*a*A/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{5}x^{5/2}(aB+Ab)-\frac{2aA}{\sqrt{x}}+\frac{2}{11}bBx^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a*A)/Sqrt[x] + (2*(A*b + a*B)*x^(5/2))/5 + (2*b*B*x^(11/2))/11
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{3/2}} \, dx &= \int \left(\frac{aA}{x^{3/2}}+(Ab+aB)x^{3/2}+bBx^{9/2}\right) dx\\ &= -\frac{2aA}{\sqrt{x}}+\frac{2}{5}(Ab+aB)x^{5/2}+\frac{2}{11}bBx^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.95

$$\frac{2\left(-55aA+11aBx^3+11Abx^3+5bBx^6\right)}{55\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (2*(-55*a*A + 11*A*b*x^3 + 11*a*B*x^3 + 5*b*B*x^6))/(55*Sqrt[x])
```

fricas [A] time = 0.72, size = 29, normalized size = 0.78

$$\frac{2\left(5\,Bbx^6+11\,(Ba+Ab)x^3-55\,Aa\right)}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/55*(5*B*b*x^6 + 11*(B*a + A*b)*x^3 - 55*A*a)/sqrt(x)
```

giac [A] time = 0.15, size = 29, normalized size = 0.78

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,Bax^{\frac{5}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="giac")
[Out] 2/11*B*b*x^(11/2) + 2/5*B*a*x^(5/2) + 2/5*A*b*x^(5/2) - 2*A*a/sqrt(x)
```

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$-\frac{2\left(-5Bb\,x^{6}-11Ab\,x^{3}-11Ba\,x^{3}+55Aa\right)}{55\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(3/2),x)
[Out] -2/55*(-5*B*b*x^6-11*A*b*x^3-11*B*a*x^3+55*A*a)/x^(1/2)
```

maxima [A] time = 0.46, size = 27, normalized size = 0.73

$$\frac{2}{11}\,Bbx^{\frac{11}{2}}+\frac{2}{5}\,(Ba+Ab)x^{\frac{5}{2}}-\frac{2\,Aa}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
[Out] 2/11*B*b*x^(11/2) + 2/5*(B*a + A*b)*x^(5/2) - 2*A*a/sqrt(x)
```

mupad [B] time = 2.60, size = 31, normalized size = 0.84

$$\frac{22\,A\,b\,x^3-110\,A\,a+22\,B\,a\,x^3+10\,B\,b\,x^6}{55\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(3/2),x)
[Out] (22*A*b*x^3 - 110*A*a + 22*B*a*x^3 + 10*B*b*x^6)/(55*x^(1/2))
```

sympy [A] time = 2.77, size = 44, normalized size = 1.19

$$-\frac{2Aa}{\sqrt{x}}+\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bax^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(3/2),x)
[Out] -2*A*a/sqrt(x) + 2*A*b*x**(5/2)/5 + 2*B*a*x**(5/2)/5 + 2*B*b*x**(11/2)/11
```

3.137 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

```
[Out] -2/3*a*A/x^(3/2)+2/3*(A*b+B*a)*x^(3/2)+2/9*b*B*x^(9/2)
```

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$\frac{2}{3}x^{3/2}(aB+Ab)-\frac{2aA}{3x^{3/2}}+\frac{2}{9}bBx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (-2*a*A)/(3*x^(3/2)) + (2*(A*b + a*B)*x^(3/2))/3 + (2*b*B*x^(9/2))/9
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{aA}{x^{5/2}}+(Ab+aB)\sqrt{x}+bBx^{7/2}\right)dx\\ &= -\frac{2aA}{3x^{3/2}}+\frac{2}{3}(Ab+aB)x^{3/2}+\frac{2}{9}bBx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.87

$$\frac{2\left(-3aA+3aBx^3+3Abx^3+bBx^6\right)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(5/2),x]
[Out] (2*(-3*a*A + 3*A*b*x^3 + 3*a*B*x^3 + b*B*x^6))/(9*x^(3/2))
```

fricas [A] time = 0.85, size = 28, normalized size = 0.72

$$\frac{2\left(Bbx^6+3\,(Ba+Ab)x^3-3\,Aa\right)}{9\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
[Out] 2/9*(B*b*x^6 + 3*(B*a + A*b)*x^3 - 3*A*a)/x^(3/2)
```

giac [A] time = 0.16, size = 29, normalized size = 0.74

$$\frac{2}{9} Bbx^{\frac{9}{2}} + \frac{2}{3} Bax^{\frac{3}{2}} + \frac{2}{3} Abx^{\frac{3}{2}} - \frac{2 Aa}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*B*a*x^(3/2) + 2/3*A*b*x^(3/2) - 2/3*A*a/x^(3/2)
```

maple [A] time = 0.04, size = 32, normalized size = 0.82

$$-\frac{2 \left(-Bb x^{6} - 3Ab x^{3} - 3Ba x^{3} + 3Aa\right)}{9x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/9*(-B*b*x^6-3*A*b*x^3-3*B*a*x^3+3*A*a)/x^(3/2)
```

maxima [A] time = 0.48, size = 27, normalized size = 0.69

$$\frac{2}{9} Bbx^{\frac{9}{2}} + \frac{2}{3} (Ba + Ab)x^{\frac{3}{2}} - \frac{2 Aa}{3 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*b*x^(9/2) + 2/3*(B*a + A*b)*x^(3/2) - 2/3*A*a/x^(3/2)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.79

$$\frac{6 A b x^3 - 6 A a + 6 B a x^3 + 2 B b x^6}{9 x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(5/2),x)
```

```
[Out] (6*A*b*x^3 - 6*A*a + 6*B*a*x^3 + 2*B*b*x^6)/(9*x^(3/2))
```

sympy [A] time = 3.74, size = 46, normalized size = 1.18

$$-\frac{2Aa}{3x^{\frac{3}{2}}} + \frac{2Abx^{\frac{3}{2}}}{3} + \frac{2Bax^{\frac{3}{2}}}{3} + \frac{2Bbx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a/(3*x**(3/2)) + 2*A*b*x**(3/2)/3 + 2*B*a*x**(3/2)/3 + 2*B*b*x**(9/2)/
9
```

3.138 $\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=37

$$2\sqrt{x}\,(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

```
[Out] -2/5*a*A/x^(5/2)+2/7*b*B*x^(7/2)+2*(A*b+B*a)*x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {448}

$$2\sqrt{x}\,(aB+Ab)-\frac{2aA}{5x^{5/2}}+\frac{2}{7}bBx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (-2*a*A)/(5*x^(5/2)) + 2*(A*b + a*B)*Sqrt[x] + (2*b*B*x^(7/2))/7
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)(A+Bx^3)}{x^{7/2}}\,dx &= \int\left(\frac{aA}{x^{7/2}}+\frac{Ab+aB}{\sqrt{x}}+bBx^{5/2}\right)dx\\ &= -\frac{2aA}{5x^{5/2}}+2(Ab+aB)\sqrt{x}+\frac{2}{7}bBx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.97

$$\frac{2\left(5bx^3\left(7A+Bx^3\right)-7a\left(A-5Bx^3\right)\right)}{35x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)*(A + B*x^3))/x^(7/2),x]
[Out] (2*(-7*a*(A - 5*B*x^3) + 5*b*x^3*(7*A + B*x^3)))/(35*x^(5/2))
```

fricas [A] time = 0.70, size = 29, normalized size = 0.78

$$\frac{2\left(5\,Bbx^6+35\,(Ba+Ab)x^3-7\,Aa\right)}{35\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
[Out] 2/35*(5*B*b*x^6 + 35*(B*a + A*b)*x^3 - 7*A*a)/x^(5/2)
```

giac [A] time = 0.26, size = 29, normalized size = 0.78

$$\frac{2}{7}Bbx^{\frac{7}{2}} + 2Ba\sqrt{x} + 2Ab\sqrt{x} - \frac{2Aa}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="giac")
[Out] 2/7*B*b*x^(7/2) + 2*B*a*sqrt(x) + 2*A*b*sqrt(x) - 2/5*A*a/x^(5/2)
```

maple [A] time = 0.04, size = 32, normalized size = 0.86

$$-\frac{2\left(-5Bb\,x^{6}-35Ab\,x^{3}-35Ba\,x^{3}+7Aa\right)}{35x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(B*x^3+A)/x^(7/2),x)
[Out] -2/35*(-5*B*b*x^6-35*A*b*x^3-35*B*a*x^3+7*A*a)/x^(5/2)
```

maxima [A] time = 0.45, size = 27, normalized size = 0.73

$$\frac{2}{7}Bbx^{\frac{7}{2}} + 2(Ba + Ab)\sqrt{x} - \frac{2Aa}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
[Out] 2/7*B*b*x^(7/2) + 2*(B*a + A*b)*sqrt(x) - 2/5*A*a/x^(5/2)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.81

$$\frac{2\,A\,b\,x^{3}-\frac{2\,A\,a}{5}+2\,B\,a\,x^{3}+\frac{2\,B\,b\,x^{6}}{7}}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3))/x^(7/2),x)
[Out] (2*A*b*x^3 - (2*A*a)/5 + 2*B*a*x^3 + (2*B*b*x^6)/7)/x^(5/2)
```

sympy [A] time = 4.21, size = 42, normalized size = 1.14

$$-\frac{2Aa}{5x^{\frac{5}{2}}} + 2Ab\sqrt{x} + 2Ba\sqrt{x} + \frac{2Bbx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(B*x**3+A)/x**(7/2),x)
[Out] -2*A*a/(5*x**(5/2)) + 2*A*b*sqrt(x) + 2*B*a*sqrt(x) + 2*B*b*x**(7/2)/7
```

3.139 $\int x^{7/2} \left(a+bx^3\right)^2 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=63

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

```
[Out] 2/9*a^2*A*x^(9/2)+2/15*a*(2*A*b+B*a)*x^(15/2)+2/21*b*(A*b+2*B*a)*x^(21/2)+2
/27*b^2*B*x^(27/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2Ax^{9/2}+\frac{2}{21}bx^{21/2}(2aB+Ab)+\frac{2}{15}ax^{15/2}(aB+2Ab)+\frac{2}{27}b^2Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(9/2))/9 + (2*a*(2*A*b + a*B)*x^(15/2))/15 + (2*b*(A*b + 2*a*B)*
x^(21/2))/21 + (2*b^2*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2Ax^{7/2}+a(2Ab+aB)x^{13/2}+b(Ab+2aB)x^{19/2}+b^2Bx^{25/2}\right)dx\\ &= \frac{2}{9}a^2Ax^{9/2}+\frac{2}{15}a(2Ab+aB)x^{15/2}+\frac{2}{21}b(Ab+2aB)x^{21/2}+\frac{2}{27}b^2Bx^{27/2}\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.84

$$\frac{2}{945}x^{9/2}\left(105a^2A+45bx^6(2aB+Ab)+63ax^3(aB+2Ab)+35b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(105*a^2*A + 63*a*(2*A*b + a*B)*x^3 + 45*b*(A*b + 2*a*B)*x^6 + 3
5*b^2*B*x^9))/945
```

fricas [A] time = 0.82, size = 56, normalized size = 0.89

$$\frac{2}{945}\left(35\,Bb^2x^{13}+45\left(2\,Bab+Ab^2\right)x^{10}+63\left(Ba^2+2\,Aab\right)x^7+105\,Aa^2x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^2*x^13 + 45*(2*B*a*b + A*b^2)*x^10 + 63*(B*a^2 + 2*A*a*b)*x^7
 + 105*A*a^2*x^4)*sqrt(x)
```

giac [A] time = 0.15, size = 53, normalized size = 0.84

$$\frac{2}{27}Bb^2x^{\frac{27}{2}} + \frac{4}{21}Babx^{\frac{21}{2}} + \frac{2}{21}Ab^2x^{\frac{21}{2}} + \frac{2}{15}Ba^2x^{\frac{15}{2}} + \frac{4}{15}Aabx^{\frac{15}{2}} + \frac{2}{9}Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 4/21*B*a*b*x^(21/2) + 2/21*A*b^2*x^(21/2) + 2/15*B*a^
2*x^(15/2) + 4/15*A*a*b*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$\frac{2\left(35b^2Bx^9 + 45Ab^2x^6 + 90Babx^6 + 126Aabx^3 + 63Ba^2x^3 + 105a^2A\right)x^{\frac{9}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/945*x^(9/2)*(35*B*b^2*x^9+45*A*b^2*x^6+90*B*a*b*x^6+126*A*a*b*x^3+63*B*a^
2*x^3+105*A*a^2)
```

maxima [A] time = 0.56, size = 51, normalized size = 0.81

$$\frac{2}{27}Bb^2x^{\frac{27}{2}} + \frac{2}{21}\left(2Bab + Ab^2\right)x^{\frac{21}{2}} + \frac{2}{15}\left(Ba^2 + 2Aab\right)x^{\frac{15}{2}} + \frac{2}{9}Aa^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^2*x^(27/2) + 2/21*(2*B*a*b + A*b^2)*x^(21/2) + 2/15*(B*a^2 + 2*A*a
*b)*x^(15/2) + 2/9*A*a^2*x^(9/2)
```

mupad [B] time = 2.57, size = 51, normalized size = 0.81

$$x^{15/2}\left(\frac{2Ba^2}{15} + \frac{4Aba}{15}\right) + x^{21/2}\left(\frac{2Ab^2}{21} + \frac{4Bab}{21}\right) + \frac{2Aa^2x^{9/2}}{9} + \frac{2Bb^2x^{27/2}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(15/2)*((2*B*a^2)/15 + (4*A*a*b)/15) + x^(21/2)*((2*A*b^2)/21 + (4*B*a*b)
/21) + (2*A*a^2*x^(9/2))/9 + (2*B*b^2*x^(27/2))/27
```

sympy [A] time = 47.81, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{9}{2}}}{9} + \frac{4Aabx^{\frac{15}{2}}}{15} + \frac{2Ab^2x^{\frac{21}{2}}}{21} + \frac{2Ba^2x^{\frac{15}{2}}}{15} + \frac{4Babx^{\frac{21}{2}}}{21} + \frac{2Bb^2x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(9/2)/9 + 4*A*a*b*x**(15/2)/15 + 2*A*b**2*x**(21/2)/21 + 2*B*a*
*2*x**(15/2)/15 + 4*B*a*b*x**(21/2)/21 + 2*B*b**2*x**(27/2)/27
```

3.140 $\int x^{5/2} \left(a+bx^3\right)^2 \left(A+Bx^3\right) dx$

Optimal. Leaf size=63

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

```
[Out] 2/7*a^2*A*x^(7/2)+2/13*a*(2*A*b+B*a)*x^(13/2)+2/19*b*(A*b+2*B*a)*x^(19/2)+2
/25*b^2*B*x^(25/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{5/2}\left(a+bx^3\right)^2\left(A+Bx^3\right)dx &= \int \left(a^2Ax^{5/2}+a(2Ab+aB)x^{11/2}+b(Ab+2aB)x^{17/2}+b^2Bx^{23/2}\right)dx\\ &= \frac{2}{7}a^2Ax^{7/2}+\frac{2}{13}a(2Ab+aB)x^{13/2}+\frac{2}{19}b(Ab+2aB)x^{19/2}+\frac{2}{25}b^2Bx^{25/2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$\frac{2}{7}a^2Ax^{7/2}+\frac{2}{19}bx^{19/2}(2aB+Ab)+\frac{2}{13}ax^{13/2}(aB+2Ab)+\frac{2}{25}b^2Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(7/2))/7 + (2*a*(2*A*b + a*B)*x^(13/2))/13 + (2*b*(A*b + 2*a*B)*
x^(19/2))/19 + (2*b^2*B*x^(25/2))/25
```

fricas [A] time = 0.86, size = 56, normalized size = 0.89

$$\frac{2}{43225}\left(1729\,Bb^2x^{12}+2275\left(2\,Bab+Ab^2\right)x^9+3325\left(Ba^2+2\,Aab\right)x^6+6175\,Aa^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^2*x^12 + 2275*(2*B*a*b + A*b^2)*x^9 + 3325*(B*a^2 + 2*A*a
*b)*x^6 + 6175*A*a^2*x^3)*sqrt(x)
```

giac [A] time = 0.18, size = 53, normalized size = 0.84

$$\frac{2}{25}Bb^2x^{\frac{25}{2}} + \frac{4}{19}Babx^{\frac{19}{2}} + \frac{2}{19}Ab^2x^{\frac{19}{2}} + \frac{2}{13}Ba^2x^{\frac{13}{2}} + \frac{4}{13}Aabx^{\frac{13}{2}} + \frac{2}{7}Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 4/19*B*a*b*x^(19/2) + 2/19*A*b^2*x^(19/2) + 2/13*B*a^
2*x^(13/2) + 4/13*A*a*b*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2\left(1729b^2Bx^9 + 2275Ab^2x^6 + 4550Babx^6 + 6650Aabx^3 + 3325Ba^2x^3 + 6175a^2A\right)x^{\frac{7}{2}}}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/43225*x^(7/2)*(1729*B*b^2*x^9+2275*A*b^2*x^6+4550*B*a*b*x^6+6650*A*a*b*x^
3+3325*B*a^2*x^3+6175*A*a^2)
```

maxima [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{25}Bb^2x^{\frac{25}{2}} + \frac{2}{19}\left(2Bab + Ab^2\right)x^{\frac{19}{2}} + \frac{2}{13}\left(Ba^2 + 2Aab\right)x^{\frac{13}{2}} + \frac{2}{7}Aa^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^2*x^(25/2) + 2/19*(2*B*a*b + A*b^2)*x^(19/2) + 2/13*(B*a^2 + 2*A*a
*b)*x^(13/2) + 2/7*A*a^2*x^(7/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{13/2}\left(\frac{2Ba^2}{13} + \frac{4Aba}{13}\right) + x^{19/2}\left(\frac{2Ab^2}{19} + \frac{4Bab}{19}\right) + \frac{2Aa^2x^{7/2}}{7} + \frac{2Bb^2x^{25/2}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(13/2)*((2*B*a^2)/13 + (4*A*a*b)/13) + x^(19/2)*((2*A*b^2)/19 + (4*B*a*b)
/19) + (2*A*a^2*x^(7/2))/7 + (2*B*b^2*x^(25/2))/25
```

sympy [A] time = 29.39, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{7}{2}}}{7} + \frac{4Aabx^{\frac{13}{2}}}{13} + \frac{2Ab^2x^{\frac{19}{2}}}{19} + \frac{2Ba^2x^{\frac{13}{2}}}{13} + \frac{4Babx^{\frac{19}{2}}}{19} + \frac{2Bb^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(7/2)/7 + 4*A*a*b*x**(13/2)/13 + 2*A*b**2*x**(19/2)/19 + 2*B*a*
*2*x**(13/2)/13 + 4*B*a*b*x**(19/2)/19 + 2*B*b**2*x**(25/2)/25
```

3.141 $\int x^{3/2} \left(a + bx^3\right)^2 \left(A + Bx^3\right) dx$

Optimal. Leaf size=63

$$\frac{2}{5}a^2 Ax^{5/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{11}ax^{11/2}(aB + 2Ab) + \frac{2}{23}b^2 Bx^{23/2}$$

```
[Out] 2/5*a^2*A*x^(5/2)+2/11*a*(2*A*b+B*a)*x^(11/2)+2/17*b*(A*b+2*B*a)*x^(17/2)+2
/23*b^2*B*x^(23/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {448}

$$\frac{2}{5}a^2 Ax^{5/2} + \frac{2}{17}bx^{17/2}(2aB + Ab) + \frac{2}{11}ax^{11/2}(aB + 2Ab) + \frac{2}{23}b^2 Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(5/2))/5 + (2*a*(2*A*b + a*B)*x^(11/2))/11 + (2*b*(A*b + 2*a*B)*
x^(17/2))/17 + (2*b^2*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int x^{3/2} \left(a + bx^3\right)^2 \left(A + Bx^3\right) dx &= \int \left(a^2 Ax^{3/2} + a(2Ab + aB)x^{9/2} + b(Ab + 2aB)x^{15/2} + b^2 Bx^{21/2}\right) dx \\ &= \frac{2}{5}a^2 Ax^{5/2} + \frac{2}{11}a(2Ab + aB)x^{11/2} + \frac{2}{17}b(Ab + 2aB)x^{17/2} + \frac{2}{23}b^2 Bx^{23/2} \end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.84

$$\frac{2x^{5/2}\left(4301a^2 A + 1265bx^6(2aB + Ab) + 1955ax^3(aB + 2Ab) + 935b^2 Bx^9\right)}{21505}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(5/2)*(4301*a^2*A + 1955*a*(2*A*b + a*B)*x^3 + 1265*b*(A*b + 2*a*B)*x^
6 + 935*b^2*B*x^9))/21505
```

fricas [A] time = 0.89, size = 56, normalized size = 0.89

$$\frac{2}{21505}\left(935\,Bb^2x^{11} + 1265\left(2\,Bab + Ab^2\right)x^8 + 1955\left(Ba^2 + 2\,Aab\right)x^5 + 4301\,Aa^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^2*x^11 + 1265*(2*B*a*b + A*b^2)*x^8 + 1955*(B*a^2 + 2*A*a*
b)*x^5 + 4301*A*a^2*x^2)*sqrt(x)
```

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{4}{17}\,Babx^{\frac{17}{2}}+\frac{2}{17}\,Ab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^2x^{\frac{11}{2}}+\frac{4}{11}\,Aabx^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 4/17*B*a*b*x^(17/2) + 2/17*A*b^2*x^(17/2) + 2/11*B*a^
2*x^(11/2) + 4/11*A*a*b*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2\left(935b^2B\,x^9+1265A\,b^2x^6+2530Bab\,x^6+3910Aab\,x^3+1955B\,a^2x^3+4301a^2A\right)x^{\frac{5}{2}}}{21505}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x)
```

```
[Out] 2/21505*x^(5/2)*(935*B*b^2*x^9+1265*A*b^2*x^6+2530*B*a*b*x^6+3910*A*a*b*x^3
+1955*B*a^2*x^3+4301*A*a^2)
```

maxima [A] time = 0.50, size = 51, normalized size = 0.81

$$\frac{2}{23}\,Bb^2x^{\frac{23}{2}}+\frac{2}{17}\left(2\,Bab+Ab^2\right)x^{\frac{17}{2}}+\frac{2}{11}\left(Ba^2+2\,Aab\right)x^{\frac{11}{2}}+\frac{2}{5}\,Aa^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^2*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^2*x^(23/2) + 2/17*(2*B*a*b + A*b^2)*x^(17/2) + 2/11*(B*a^2 + 2*A*a
*b)*x^(11/2) + 2/5*A*a^2*x^(5/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{11/2}\left(\frac{2\,B\,a^2}{11}+\frac{4\,A\,b\,a}{11}\right)+x^{17/2}\left(\frac{2\,A\,b^2}{17}+\frac{4\,B\,a\,b}{17}\right)+\frac{2\,A\,a^2\,x^{5/2}}{5}+\frac{2\,B\,b^2\,x^{23/2}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(11/2)*((2*B*a^2)/11 + (4*A*a*b)/11) + x^(17/2)*((2*A*b^2)/17 + (4*B*a*b)
/17) + (2*A*a^2*x^(5/2))/5 + (2*B*b^2*x^(23/2))/23
```

sympy [A] time = 21.96, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{5}{2}}}{5}+\frac{4Aabx^{\frac{11}{2}}}{11}+\frac{2Ab^2x^{\frac{17}{2}}}{17}+\frac{2Ba^2x^{\frac{11}{2}}}{11}+\frac{4Babx^{\frac{17}{2}}}{17}+\frac{2Bb^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**2*(B*x**3+A),x)
```

```
[Out] 2*A*a**2*x**(5/2)/5 + 4*A*a*b*x**(11/2)/11 + 2*A*b**2*x**(17/2)/17 + 2*B*a*
*2*x**(11/2)/11 + 4*B*a*b*x**(17/2)/17 + 2*B*b**2*x**(23/2)/23
```

3.142 $\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=63

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

```
[Out] 2/3*a^2*A*x^(3/2)+2/9*a*(2*A*b+B*a)*x^(9/2)+2/15*b*(A*b+2*B*a)*x^(15/2)+2/2
1*b^2*B*x^(21/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2Ax^{3/2}+\frac{2}{15}bx^{15/2}(2aB+Ab)+\frac{2}{9}ax^{9/2}(aB+2Ab)+\frac{2}{21}b^2Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*a^2*A*x^(3/2))/3 + (2*a*(2*A*b + a*B)*x^(9/2))/9 + (2*b*(A*b + 2*a*B)*x^
(15/2))/15 + (2*b^2*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^2\left(A+Bx^3\right)\,dx &= \int \left(a^2A\sqrt{x}+a(2Ab+aB)x^{7/2}+b(Ab+2aB)x^{13/2}+b^2Bx^{19/2}\right)dx\\ &= \frac{2}{3}a^2Ax^{3/2}+\frac{2}{9}a(2Ab+aB)x^{9/2}+\frac{2}{15}b(Ab+2aB)x^{15/2}+\frac{2}{21}b^2Bx^{21/2}\end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 0.84

$$\frac{2}{315}x^{3/2}\left(105a^2A+21bx^6(2aB+Ab)+35ax^3(aB+2Ab)+15b^2Bx^9\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^2*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(105*a^2*A + 35*a*(2*A*b + a*B)*x^3 + 21*b*(A*b + 2*a*B)*x^6 + 1
5*b^2*B*x^9))/315
```

fricas [A] time = 0.88, size = 54, normalized size = 0.86

$$\frac{2}{315}\left(15\,Bb^2x^{10}+21\left(2\,Bab+Ab^2\right)x^7+35\left(Ba^2+2\,Aab\right)x^4+105\,Aa^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^2*x^10 + 21*(2*B*a*b + A*b^2)*x^7 + 35*(B*a^2 + 2*A*a*b)*x^4
+ 105*A*a^2*x)*sqrt(x)
```

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{21}Bb^2x^{\frac{21}{2}} + \frac{4}{15}Babx^{\frac{15}{2}} + \frac{2}{15}Ab^2x^{\frac{15}{2}} + \frac{2}{9}Ba^2x^{\frac{9}{2}} + \frac{4}{9}Aabx^{\frac{9}{2}} + \frac{2}{3}Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 4/15*B*a*b*x^(15/2) + 2/15*A*b^2*x^(15/2) + 2/9*B*a^2
*x^(9/2) + 4/9*A*a*b*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

maple [A] time = 0.04, size = 56, normalized size = 0.89

$$\frac{2\left(15b^2Bx^9 + 21Ab^2x^6 + 42Babx^6 + 70Aabx^3 + 35Ba^2x^3 + 105a^2A\right)x^{\frac{3}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/315*x^(3/2)*(15*B*b^2*x^9+21*A*b^2*x^6+42*B*a*b*x^6+70*A*a*b*x^3+35*B*a^2
*x^3+105*A*a^2)
```

maxima [A] time = 0.54, size = 51, normalized size = 0.81

$$\frac{2}{21}Bb^2x^{\frac{21}{2}} + \frac{2}{15}\left(2Bab + Ab^2\right)x^{\frac{15}{2}} + \frac{2}{9}\left(Ba^2 + 2Aab\right)x^{\frac{9}{2}} + \frac{2}{3}Aa^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^2*x^(21/2) + 2/15*(2*B*a*b + A*b^2)*x^(15/2) + 2/9*(B*a^2 + 2*A*a*
b)*x^(9/2) + 2/3*A*a^2*x^(3/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{9/2}\left(\frac{2Ba^2}{9} + \frac{4Aba}{9}\right) + x^{15/2}\left(\frac{2Ab^2}{15} + \frac{4Bab}{15}\right) + \frac{2Aa^2x^{3/2}}{3} + \frac{2Bb^2x^{21/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^2,x)
```

```
[Out] x^(9/2)*((2*B*a^2)/9 + (4*A*a*b)/9) + x^(15/2)*((2*A*b^2)/15 + (4*B*a*b)/15
) + (2*A*a^2*x^(3/2))/3 + (2*B*b^2*x^(21/2))/21
```

sympy [A] time = 5.31, size = 80, normalized size = 1.27

$$\frac{2Aa^2x^{\frac{3}{2}}}{3} + \frac{4Aabx^{\frac{9}{2}}}{9} + \frac{2Ab^2x^{\frac{15}{2}}}{15} + \frac{2Ba^2x^{\frac{9}{2}}}{9} + \frac{4Babx^{\frac{15}{2}}}{15} + \frac{2Bb^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**2*x**(3/2)/3 + 4*A*a*b*x**(9/2)/9 + 2*A*b**2*x**(15/2)/15 + 2*B*a**2
*x**(9/2)/9 + 4*B*a*b*x**(15/2)/15 + 2*B*b**2*x**(21/2)/21
```

3.143 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx$

Optimal. Leaf size=61

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

```
[Out] 2/7*a*(2*A*b+B*a)*x^(7/2)+2/13*b*(A*b+2*B*a)*x^(13/2)+2/19*b^2*B*x^(19/2)+2
*a^2*A*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2A\sqrt{x} + \frac{2}{13}bx^{13/2}(2aB + Ab) + \frac{2}{7}ax^{7/2}(aB + 2Ab) + \frac{2}{19}b^2Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^2*A*Sqrt[x] + (2*a*(2*A*b + a*B)*x^(7/2))/7 + (2*b*(A*b + 2*a*B)*x^(13/
2))/13 + (2*b^2*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^2 (A+Bx^3)}{\sqrt{x}} \, dx &= \int \left(\frac{a^2A}{\sqrt{x}} + a(2Ab+aB)x^{5/2} + b(Ab+2aB)x^{11/2} + b^2Bx^{17/2}\right) dx \\ &= 2a^2A\sqrt{x} + \frac{2}{7}a(2Ab+aB)x^{7/2} + \frac{2}{13}b(Ab+2aB)x^{13/2} + \frac{2}{19}b^2Bx^{19/2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 53, normalized size = 0.87

$$\frac{2\sqrt{x}\left(1729a^2A + 133bx^6(2aB+Ab) + 247ax^3(aB+2Ab) + 91b^2Bx^9\right)}{1729}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(1729*a^2*A + 247*a*(2*A*b + a*B)*x^3 + 133*b*(A*b + 2*a*B)*x^6
+ 91*b^2*B*x^9))/1729
```

fricas [A] time = 0.65, size = 53, normalized size = 0.87

$$\frac{2}{1729}\left(91\,Bb^2x^9 + 133\left(2\,Bab + Ab^2\right)x^6 + 247\left(Ba^2 + 2\,Aab\right)x^3 + 1729\,Aa^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/1729*(91*B*b^2*x^9 + 133*(2*B*a*b + A*b^2)*x^6 + 247*(B*a^2 + 2*A*a*b)*x^
3 + 1729*A*a^2)*sqrt(x)
```

giac [A] time = 0.23, size = 53, normalized size = 0.87

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{4}{13}\,Babx^{\frac{13}{2}}+\frac{2}{13}\,Ab^2x^{\frac{13}{2}}+\frac{2}{7}\,Ba^2x^{\frac{7}{2}}+\frac{4}{7}\,Aabx^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 4/13*B*a*b*x^(13/2) + 2/13*A*b^2*x^(13/2) + 2/7*B*a^2
*x^(7/2) + 4/7*A*a*b*x^(7/2) + 2*A*a^2*sqrt(x)
```

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$\frac{2\left(91b^2B\,x^9+133A\,b^2x^6+266Bab\,x^6+494Aab\,x^3+247B\,a^2x^3+1729a^2A\right)\sqrt{x}}{1729}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/1729*x^(1/2)*(91*B*b^2*x^9+133*A*b^2*x^6+266*B*a*b*x^6+494*A*a*b*x^3+247*
B*a^2*x^3+1729*A*a^2)
```

maxima [A] time = 0.51, size = 51, normalized size = 0.84

$$\frac{2}{19}\,Bb^2x^{\frac{19}{2}}+\frac{2}{13}\left(2\,Bab+Ab^2\right)x^{\frac{13}{2}}+\frac{2}{7}\left(Ba^2+2\,Aab\right)x^{\frac{7}{2}}+2\,Aa^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^2*x^(19/2) + 2/13*(2*B*a*b + A*b^2)*x^(13/2) + 2/7*(B*a^2 + 2*A*a*
b)*x^(7/2) + 2*A*a^2*sqrt(x)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{7/2}\left(\frac{2\,B\,a^2}{7}+\frac{4\,A\,b\,a}{7}\right)+x^{13/2}\left(\frac{2\,A\,b^2}{13}+\frac{4\,B\,a\,b}{13}\right)+2\,A\,a^2\,\sqrt{x}+\frac{2\,B\,b^2\,x^{19/2}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^2)/7 + (4*A*a*b)/7) + x^(13/2)*((2*A*b^2)/13 + (4*B*a*b)/13
) + 2*A*a^2*x^(1/2) + (2*B*b^2*x^(19/2))/19
```

sympy [A] time = 9.22, size = 78, normalized size = 1.28

$$2Aa^2\sqrt{x}+\frac{4Aabx^{\frac{7}{2}}}{7}+\frac{2Ab^2x^{\frac{13}{2}}}{13}+\frac{2Ba^2x^{\frac{7}{2}}}{7}+\frac{4Babx^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**2*sqrt(x) + 4*A*a*b*x**(7/2)/7 + 2*A*b**2*x**(13/2)/13 + 2*B*a**2*x*
*(7/2)/7 + 4*B*a*b*x**(13/2)/13 + 2*B*b**2*x**(19/2)/19
```

3.144 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

```
[Out] 2/5*a*(2*A*b+B*a)*x^(5/2)+2/11*b*(A*b+2*B*a)*x^(11/2)+2/17*b^2*B*x^(17/2)-2
*a^2*A/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{\sqrt{x}}+\frac{2}{11}bx^{11/2}(2aB+Ab)+\frac{2}{5}ax^{5/2}(aB+2Ab)+\frac{2}{17}b^2Bx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^2*A)/Sqrt[x] + (2*a*(2*A*b + a*B)*x^(5/2))/5 + (2*b*(A*b + 2*a*B)*x^(
11/2))/11 + (2*b^2*B*x^(17/2))/17
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{a^2A}{x^{3/2}}+a(2Ab+aB)x^{3/2}+b(Ab+2aB)x^{9/2}+b^2Bx^{15/2}\right)dx\\ &= -\frac{2a^2A}{\sqrt{x}}+\frac{2}{5}a(2Ab+aB)x^{5/2}+\frac{2}{11}b(Ab+2aB)x^{11/2}+\frac{2}{17}b^2Bx^{17/2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 0.98

$$\frac{-374a^2\left(5A-Bx^3\right)+68abx^3\left(11A+5Bx^3\right)+10b^2x^6\left(17A+11Bx^3\right)}{935\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-374*a^2*(5*A - B*x^3) + 68*a*b*x^3*(11*A + 5*B*x^3) + 10*b^2*x^6*(17*A +
11*B*x^3))/(935*Sqrt[x])
```

fricas [A] time = 0.57, size = 53, normalized size = 0.87

$$\frac{2\left(55\,Bb^2x^9+85\left(2\,Bab+Ab^2\right)x^6+187\left(Ba^2+2\,Aab\right)x^3-935\,Aa^2\right)}{935\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/935*(55*B*b^2*x^9 + 85*(2*B*a*b + A*b^2)*x^6 + 187*(B*a^2 + 2*A*a*b)*x^3
- 935*A*a^2)/sqrt(x)
```

giac [A] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{17}Bb^2x^{\frac{17}{2}}+\frac{4}{11}Babx^{\frac{11}{2}}+\frac{2}{11}Ab^2x^{\frac{11}{2}}+\frac{2}{5}Ba^2x^{\frac{5}{2}}+\frac{4}{5}Aabx^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 4/11*B*a*b*x^(11/2) + 2/11*A*b^2*x^(11/2) + 2/5*B*a^2
*x^(5/2) + 4/5*A*a*b*x^(5/2) - 2*A*a^2/sqrt(x)
```

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$-\frac{2\left(-55b^2B\,x^9-85A\,b^2x^6-170Bab\,x^6-374Aab\,x^3-187B\,a^2x^3+935a^2A\right)}{935\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/935*(-55*B*b^2*x^9-85*A*b^2*x^6-170*B*a*b*x^6-374*A*a*b*x^3-187*B*a^2*x^
3+935*A*a^2)/x^(1/2)
```

maxima [A] time = 0.54, size = 51, normalized size = 0.84

$$\frac{2}{17}Bb^2x^{\frac{17}{2}}+\frac{2}{11}\left(2\,Bab+Ab^2\right)x^{\frac{11}{2}}+\frac{2}{5}\left(Ba^2+2\,Aab\right)x^{\frac{5}{2}}-\frac{2\,Aa^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/17*B*b^2*x^(17/2) + 2/11*(2*B*a*b + A*b^2)*x^(11/2) + 2/5*(B*a^2 + 2*A*a*
b)*x^(5/2) - 2*A*a^2/sqrt(x)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$x^{5/2}\left(\frac{2\,B\,a^2}{5}+\frac{4\,A\,b\,a}{5}\right)+x^{11/2}\left(\frac{2\,A\,b^2}{11}+\frac{4\,B\,a\,b}{11}\right)-\frac{2\,A\,a^2}{\sqrt{x}}+\frac{2\,B\,b^2\,x^{17/2}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^2)/5 + (4*A*a*b)/5) + x^(11/2)*((2*A*b^2)/11 + (4*B*a*b)/11
) - (2*A*a^2)/x^(1/2) + (2*B*b^2*x^(17/2))/17
```

sympy [A] time = 7.80, size = 78, normalized size = 1.28

$$-\frac{2Aa^2}{\sqrt{x}}+\frac{4Aabx^{\frac{5}{2}}}{5}+\frac{2Ab^2x^{\frac{11}{2}}}{11}+\frac{2Ba^2x^{\frac{5}{2}}}{5}+\frac{4Babx^{\frac{11}{2}}}{11}+\frac{2Bb^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**2/sqrt(x) + 4*A*a*b*x**(5/2)/5 + 2*A*b**2*x**(11/2)/11 + 2*B*a**2*x
**(5/2)/5 + 4*B*a*b*x**(11/2)/11 + 2*B*b**2*x**(17/2)/17
```

3.145 $\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} \, dx$

Optimal. Leaf size=63

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

```
[Out] -2/3*a^2*A/x^(3/2)+2/3*a*(2*A*b+B*a)*x^(3/2)+2/9*b*(A*b+2*B*a)*x^(9/2)+2/15
*b^2*B*x^(15/2)
```

Rubi [A] time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{3x^{3/2}}+\frac{2}{9}bx^{9/2}(2aB+Ab)+\frac{2}{3}ax^{3/2}(aB+2Ab)+\frac{2}{15}b^2Bx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^2*A)/(3*x^(3/2)) + (2*a*(2*A*b + a*B)*x^(3/2))/3 + (2*b*(A*b + 2*a*B)
*x^(9/2))/9 + (2*b^2*B*x^(15/2))/15
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2 (A+Bx^3)}{x^{5/2}} \, dx &= \int \left(\frac{a^2A}{x^{5/2}}+a(2Ab+aB)\sqrt{x}+b(Ab+2aB)x^{7/2}+b^2Bx^{13/2}\right) dx\\ &= -\frac{2a^2A}{3x^{3/2}}+\frac{2}{3}a(2Ab+aB)x^{3/2}+\frac{2}{9}b(Ab+2aB)x^{9/2}+\frac{2}{15}b^2Bx^{15/2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.90

$$\frac{-30a^2(A-Bx^3)+20abx^3(3A+Bx^3)+2b^2x^6(5A+3Bx^3)}{45x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-30*a^2*(A - B*x^3) + 20*a*b*x^3*(3*A + B*x^3) + 2*b^2*x^6*(5*A + 3*B*x^3)
)/(45*x^(3/2))
```

fricas [A] time = 0.61, size = 53, normalized size = 0.84

$$\frac{2\left(3\,Bb^2x^9+5\left(2\,Bab+Ab^2\right)x^6+15\left(Ba^2+2\,Aab\right)x^3-15\,Aa^2\right)}{45\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^9 + 5*(2*B*a*b + A*b^2)*x^6 + 15*(B*a^2 + 2*A*a*b)*x^3 - 15
*A*a^2)/x^(3/2)
```

giac [A] time = 0.16, size = 53, normalized size = 0.84

$$\frac{2}{15}Bb^2x^{\frac{15}{2}}+\frac{4}{9}Babx^{\frac{9}{2}}+\frac{2}{9}Ab^2x^{\frac{9}{2}}+\frac{2}{3}Ba^2x^{\frac{3}{2}}+\frac{4}{3}Aabx^{\frac{3}{2}}-\frac{2Aa^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 4/9*B*a*b*x^(9/2) + 2/9*A*b^2*x^(9/2) + 2/3*B*a^2*x^(
3/2) + 4/3*A*a*b*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.89

$$-\frac{2\left(-3b^2Bx^9-5Ab^2x^6-10Babx^6-30Aabx^3-15Ba^2x^3+15a^2A\right)}{45x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/45*(-3*B*b^2*x^9-5*A*b^2*x^6-10*B*a*b*x^6-30*A*a*b*x^3-15*B*a^2*x^3+15*A
*a^2)/x^(3/2)
```

maxima [A] time = 0.51, size = 51, normalized size = 0.81

$$\frac{2}{15}Bb^2x^{\frac{15}{2}}+\frac{2}{9}\left(2Bab+Ab^2\right)x^{\frac{9}{2}}+\frac{2}{3}\left(Ba^2+2Aab\right)x^{\frac{3}{2}}-\frac{2Aa^2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*b^2*x^(15/2) + 2/9*(2*B*a*b + A*b^2)*x^(9/2) + 2/3*(B*a^2 + 2*A*a*b)
*x^(3/2) - 2/3*A*a^2/x^(3/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.81

$$x^{3/2}\left(\frac{2Ba^2}{3}+\frac{4Aba}{3}\right)+x^{9/2}\left(\frac{2Ab^2}{9}+\frac{4Bab}{9}\right)-\frac{2Aa^2}{3x^{3/2}}+\frac{2Bb^2x^{15/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^2)/3 + (4*A*a*b)/3) + x^(9/2)*((2*A*b^2)/9 + (4*B*a*b)/9) -
 (2*A*a^2)/(3*x^(3/2)) + (2*B*b^2*x^(15/2))/15
```

sympy [A] time = 11.74, size = 80, normalized size = 1.27

$$-\frac{2Aa^2}{3x^{\frac{3}{2}}}+\frac{4Aabx^{\frac{3}{2}}}{3}+\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{2Ba^2x^{\frac{3}{2}}}{3}+\frac{4Babx^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**2/(3*x**(3/2)) + 4*A*a*b*x**(3/2)/3 + 2*A*b**2*x**(9/2)/9 + 2*B*a**
2*x**(3/2)/3 + 4*B*a*b*x**(9/2)/9 + 2*B*b**2*x**(15/2)/15
```

3.146 $\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

```
[Out] -2/5*a^2*A/x^(5/2)+2/7*b*(A*b+2*B*a)*x^(7/2)+2/13*b^2*B*x^(13/2)+2*a*(2*A*b
+B*a)*x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$-\frac{2a^2A}{5x^{5/2}}+\frac{2}{7}bx^{7/2}(2aB+Ab)+2a\sqrt{x}\,(aB+2Ab)+\frac{2}{13}b^2Bx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^2*A)/(5*x^(5/2)) + 2*a*(2*A*b + a*B)*Sqrt[x] + (2*b*(A*b + 2*a*B)*x^(
7/2))/7 + (2*b^2*B*x^(13/2))/13
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^2(A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{a^2A}{x^{7/2}}+\frac{a(2Ab+aB)}{\sqrt{x}}+b(Ab+2aB)x^{5/2}+b^2Bx^{11/2}\right)dx\\ &= -\frac{2a^2A}{5x^{5/2}}+2a(2Ab+aB)\sqrt{x}+\frac{2}{7}b(Ab+2aB)x^{7/2}+\frac{2}{13}b^2Bx^{13/2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.93

$$\frac{2\left(-91a^2\left(A-5Bx^3\right)+130abx^3\left(7A+Bx^3\right)+5b^2x^6\left(13A+7Bx^3\right)\right)}{455x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^2*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (2*(-91*a^2*(A - 5*B*x^3) + 130*a*b*x^3*(7*A + B*x^3) + 5*b^2*x^6*(13*A + 7
*B*x^3)))/(455*x^(5/2))
```

fricas [A] time = 0.82, size = 53, normalized size = 0.87

$$\frac{2\left(35\,Bb^2x^9+65\left(2\,Bab+Ab^2\right)x^6+455\left(Ba^2+2\,Aab\right)x^3-91\,Aa^2\right)}{455\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/455*(35*B*b^2*x^9 + 65*(2*B*a*b + A*b^2)*x^6 + 455*(B*a^2 + 2*A*a*b)*x^3
- 91*A*a^2)/x^(5/2)
```

giac [A] time = 0.15, size = 53, normalized size = 0.87

$$\frac{2}{13}Bb^2x^{\frac{13}{2}} + \frac{4}{7}Babx^{\frac{7}{2}} + \frac{2}{7}Ab^2x^{\frac{7}{2}} + 2Ba^2\sqrt{x} + 4Aab\sqrt{x} - \frac{2Aa^2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 4/7*B*a*b*x^(7/2) + 2/7*A*b^2*x^(7/2) + 2*B*a^2*sqrt(
x) + 4*A*a*b*sqrt(x) - 2/5*A*a^2/x^(5/2)
```

maple [A] time = 0.05, size = 56, normalized size = 0.92

$$-\frac{2\left(-35b^2Bx^9 - 65Ab^2x^6 - 130Babx^6 - 910Aabx^3 - 455Ba^2x^3 + 91a^2A\right)}{455x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/455*(-35*B*b^2*x^9-65*A*b^2*x^6-130*B*a*b*x^6-910*A*a*b*x^3-455*B*a^2*x^
3+91*A*a^2)/x^(5/2)
```

maxima [A] time = 0.46, size = 51, normalized size = 0.84

$$\frac{2}{13}Bb^2x^{\frac{13}{2}} + \frac{2}{7}\left(2Bab + Ab^2\right)x^{\frac{7}{2}} + 2\left(Ba^2 + 2Aab\right)\sqrt{x} - \frac{2Aa^2}{5x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/13*B*b^2*x^(13/2) + 2/7*(2*B*a*b + A*b^2)*x^(7/2) + 2*(B*a^2 + 2*A*a*b)*s
qrt(x) - 2/5*A*a^2/x^(5/2)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.84

$$\sqrt{x}\,\left(2Ba^2 + 4Aba\right) + x^{7/2}\left(\frac{2Ab^2}{7} + \frac{4Bab}{7}\right) - \frac{2Aa^2}{5x^{5/2}} + \frac{2Bb^2x^{13/2}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^2)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^2 + 4*A*a*b) + x^(7/2)*((2*A*b^2)/7 + (4*B*a*b)/7) - (2*A*a^
2)/(5*x^(5/2)) + (2*B*b^2*x^(13/2))/13
```

sympy [A] time = 18.41, size = 76, normalized size = 1.25

$$-\frac{2Aa^2}{5x^{\frac{5}{2}}} + 4Aab\sqrt{x} + \frac{2Ab^2x^{\frac{7}{2}}}{7} + 2Ba^2\sqrt{x} + \frac{4Babx^{\frac{7}{2}}}{7} + \frac{2Bb^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**2/(5*x**(5/2)) + 4*A*a*b*sqrt(x) + 2*A*b**2*x**(7/2)/7 + 2*B*a**2*s
qrt(x) + 4*B*a*b*x**(7/2)/7 + 2*B*b**2*x**(13/2)/13
```

3.147 $\int x^{7/2} \left(a+bx^3\right)^3 \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=85

$$\frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

```
[Out] 2/9*a^3*A*x^(9/2)+2/15*a^2*(3*A*b+B*a)*x^(15/2)+2/7*a*b*(A*b+B*a)*x^(21/2)+
2/27*b^2*(A*b+3*B*a)*x^(27/2)+2/33*b^3*B*x^(33/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{15}a^2x^{15/2}(aB+3Ab)+\frac{2}{9}a^3Ax^{9/2}+\frac{2}{27}b^2x^{27/2}(3aB+Ab)+\frac{2}{7}abx^{21/2}(aB+Ab)+\frac{2}{33}b^3Bx^{33/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(9/2))/9 + (2*a^2*(3*A*b + a*B)*x^(15/2))/15 + (2*a*b*(A*b + a*B
)*x^(21/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(27/2))/27 + (2*b^3*B*x^(33/2))/33
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^{7/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx &= \int \left(a^3Ax^{7/2}+a^2(3Ab+aB)x^{13/2}+3ab(Ab+aB)x^{19/2}+b^2(Ab+3aB)x^{25/2}\right.\\ &= \frac{2}{9}a^3Ax^{9/2}+\frac{2}{15}a^2(3Ab+aB)x^{15/2}+\frac{2}{7}ab(Ab+aB)x^{21/2}+\frac{2}{27}b^2(Ab+3aB)x\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.84

$$\frac{2x^{9/2}\left(1155a^3A+693a^2x^3(aB+3Ab)+385b^2x^9(3aB+Ab)+1485abx^6(aB+Ab)+315b^3Bx^{12}\right)}{10395}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(9/2)*(1155*a^3*A + 693*a^2*(3*A*b + a*B)*x^3 + 1485*a*b*(A*b + a*B)*x
^6 + 385*b^2*(A*b + 3*a*B)*x^9 + 315*b^3*B*x^12))/10395
```

fricas [A] time = 0.84, size = 78, normalized size = 0.92

$$\frac{2}{10395}\left(315\,Bb^3x^{16}+385\left(3\,Bab^2+Ab^3\right)x^{13}+1485\left(Ba^2b+Aab^2\right)x^{10}+1155\,Aa^3x^4+693\left(Ba^3+3\,Aa^2b\right)x^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^3*x^16 + 385*(3*B*a*b^2 + A*b^3)*x^13 + 1485*(B*a^2*b + A*
a*b^2)*x^10 + 1155*A*a^3*x^4 + 693*(B*a^3 + 3*A*a^2*b)*x^7)*sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{33}Bb^3x^{\frac{33}{2}}+\frac{2}{9}Bab^2x^{\frac{27}{2}}+\frac{2}{27}Ab^3x^{\frac{27}{2}}+\frac{2}{7}Ba^2bx^{\frac{21}{2}}+\frac{2}{7}Aab^2x^{\frac{21}{2}}+\frac{2}{15}Ba^3x^{\frac{15}{2}}+\frac{2}{5}Aa^2bx^{\frac{15}{2}}+\frac{2}{9}Aa^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/9*B*a*b^2*x^(27/2) + 2/27*A*b^3*x^(27/2) + 2/7*B*a^
2*b*x^(21/2) + 2/7*A*a*b^2*x^(21/2) + 2/15*B*a^3*x^(15/2) + 2/5*A*a^2*b*x^(
15/2) + 2/9*A*a^3*x^(9/2)
```

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2\left(315B\,b^3x^{12}+385x^9A\,b^3+1155x^9Ba\,b^2+1485x^6Aa\,b^2+1485x^6B\,a^2b+2079x^3A\,a^2b+693B\,a^3x^3+1155A\,a^3\right.}{10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/10395*x^(9/2)*(315*B*b^3*x^12+385*A*b^3*x^9+1155*B*a*b^2*x^9+1485*A*a*b^2
*x^6+1485*B*a^2*b*x^6+2079*A*a^2*b*x^3+693*B*a^3*x^3+1155*A*a^3)
```

maxima [A] time = 0.57, size = 73, normalized size = 0.86

$$\frac{2}{33}Bb^3x^{\frac{33}{2}}+\frac{2}{27}\left(3\,Bab^2+Ab^3\right)x^{\frac{27}{2}}+\frac{2}{7}\left(Ba^2b+Aab^2\right)x^{\frac{21}{2}}+\frac{2}{9}Aa^3x^{\frac{9}{2}}+\frac{2}{15}\left(Ba^3+3\,Aa^2b\right)x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/33*B*b^3*x^(33/2) + 2/27*(3*B*a*b^2 + A*b^3)*x^(27/2) + 2/7*(B*a^2*b + A*
a*b^2)*x^(21/2) + 2/9*A*a^3*x^(9/2) + 2/15*(B*a^3 + 3*A*a^2*b)*x^(15/2)
```

mupad [B] time = 2.52, size = 69, normalized size = 0.81

$$x^{15/2}\left(\frac{2\,B\,a^3}{15}+\frac{2\,A\,b\,a^2}{5}\right)+x^{27/2}\left(\frac{2\,A\,b^3}{27}+\frac{2\,B\,a\,b^2}{9}\right)+\frac{2\,A\,a^3\,x^{9/2}}{9}+\frac{2\,B\,b^3\,x^{33/2}}{33}+\frac{2\,a\,b\,x^{21/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(15/2)*((2*B*a^3)/15 + (2*A*a^2*b)/5) + x^(27/2)*((2*A*b^3)/27 + (2*B*a*b
^2)/9) + (2*A*a^3*x^(9/2))/9 + (2*B*b^3*x^(33/2))/33 + (2*a*b*x^(21/2)*(A*b
 + B*a))/7
```

sympy [A] time = 92.99, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{9}{2}}}{9}+\frac{2Aa^2bx^{\frac{15}{2}}}{5}+\frac{2Aab^2x^{\frac{21}{2}}}{7}+\frac{2Ab^3x^{\frac{27}{2}}}{27}+\frac{2Ba^3x^{\frac{15}{2}}}{15}+\frac{2Ba^2bx^{\frac{21}{2}}}{7}+\frac{2Bab^2x^{\frac{27}{2}}}{9}+\frac{2Bb^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(9/2)/9 + 2*A*a**2*b*x**(15/2)/5 + 2*A*a*b**2*x**(21/2)/7 + 2*A
*b**3*x**(27/2)/27 + 2*B*a**3*x**(15/2)/15 + 2*B*a**2*b*x**(21/2)/7 + 2*B*a
*b**2*x**(27/2)/9 + 2*B*b**3*x**(33/2)/33
```

3.148 $\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx$

Optimal. Leaf size=85

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

```
[Out] 2/7*a^3*A*x^(7/2)+2/13*a^2*(3*A*b+B*a)*x^(13/2)+6/19*a*b*(A*b+B*a)*x^(19/2)
+2/25*b^2*(A*b+3*B*a)*x^(25/2)+2/31*b^3*B*x^(31/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{7}a^3Ax^{7/2}+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{5/2}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx=\int\left(a^3Ax^{5/2}+a^2(3Ab+aB)x^{11/2}+3ab(Ab+aB)x^{17/2}+b^2(Ab+3aB)x^{23/2}\right.$$
$$=\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2(3Ab+aB)x^{13/2}+\frac{6}{19}ab(Ab+aB)x^{19/2}+\frac{2}{25}b^2(Ab+3aB)$$

Mathematica [A] time = 0.10, size = 85, normalized size = 1.00

$$\frac{2}{7}a^3Ax^{7/2}+\frac{2}{13}a^2x^{13/2}(aB+3Ab)+\frac{2}{25}b^2x^{25/2}(3aB+Ab)+\frac{6}{19}abx^{19/2}(aB+Ab)+\frac{2}{31}b^3Bx^{31/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(7/2))/7 + (2*a^2*(3*A*b + a*B)*x^(13/2))/13 + (6*a*b*(A*b + a*B
)*x^(19/2))/19 + (2*b^2*(A*b + 3*a*B)*x^(25/2))/25 + (2*b^3*B*x^(31/2))/31
```

fricas [A] time = 0.85, size = 78, normalized size = 0.92

$$\frac{2}{1339975}\left(43225\,Bb^3x^{15}+53599\left(3\,Bab^2+Ab^3\right)x^{12}+211575\left(Ba^2b+Aab^2\right)x^9+191425\,Aa^3x^3+103075\,(Ba\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/1339975*(43225*B*b^3*x^15 + 53599*(3*B*a*b^2 + A*b^3)*x^12 + 211575*(B*a^
2*b + A*a*b^2)*x^9 + 191425*A*a^3*x^3 + 103075*(B*a^3 + 3*A*a^2*b)*x^6)*sqr
t(x)
```

giac [A] time = 0.15, size = 77, normalized size = 0.91

$$\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{6}{25}\,Bab^2x^{\frac{25}{2}}+\frac{2}{25}\,Ab^3x^{\frac{25}{2}}+\frac{6}{19}\,Ba^2bx^{\frac{19}{2}}+\frac{6}{19}\,Aab^2x^{\frac{19}{2}}+\frac{2}{13}\,Ba^3x^{\frac{13}{2}}+\frac{6}{13}\,Aa^2bx^{\frac{13}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 6/25*B*a*b^2*x^(25/2) + 2/25*A*b^3*x^(25/2) + 6/19*B*
a^2*b*x^(19/2) + 6/19*A*a*b^2*x^(19/2) + 2/13*B*a^3*x^(13/2) + 6/13*A*a^2*b
*x^(13/2) + 2/7*A*a^3*x^(7/2)
```

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$\frac{2\left(43225B\,b^3x^{12}+53599x^9A\,b^3+160797x^9Ba\,b^2+211575x^6Aa\,b^2+211575x^6B\,a^2b+309225x^3A\,a^2b+103075\right.}{1339975}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/1339975*x^(7/2)*(43225*B*b^3*x^12+53599*A*b^3*x^9+160797*B*a*b^2*x^9+2115
75*A*a*b^2*x^6+211575*B*a^2*b*x^6+309225*A*a^2*b*x^3+103075*B*a^3*x^3+19142
5*A*a^3)
```

maxima [A] time = 0.50, size = 73, normalized size = 0.86

$$\frac{2}{31}\,Bb^3x^{\frac{31}{2}}+\frac{2}{25}\left(3\,Bab^2+Ab^3\right)x^{\frac{25}{2}}+\frac{6}{19}\left(Ba^2b+Aab^2\right)x^{\frac{19}{2}}+\frac{2}{7}\,Aa^3x^{\frac{7}{2}}+\frac{2}{13}\left(Ba^3+3\,Aa^2b\right)x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/31*B*b^3*x^(31/2) + 2/25*(3*B*a*b^2 + A*b^3)*x^(25/2) + 6/19*(B*a^2*b + A
*a*b^2)*x^(19/2) + 2/7*A*a^3*x^(7/2) + 2/13*(B*a^3 + 3*A*a^2*b)*x^(13/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{13/2}\left(\frac{2\,B\,a^3}{13}+\frac{6\,A\,b\,a^2}{13}\right)+x^{25/2}\left(\frac{2\,A\,b^3}{25}+\frac{6\,B\,a\,b^2}{25}\right)+\frac{2\,A\,a^3\,x^{7/2}}{7}+\frac{2\,B\,b^3\,x^{31/2}}{31}+\frac{6\,a\,b\,x^{19/2}\,\left(A\,b+B\,a\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(13/2)*((2*B*a^3)/13 + (6*A*a^2*b)/13) + x^(25/2)*((2*A*b^3)/25 + (6*B*a*
b^2)/25) + (2*A*a^3*x^(7/2))/7 + (2*B*b^3*x^(31/2))/31 + (6*a*b*x^(19/2)*(A
*b + B*a))/19
```

sympy [A] time = 58.70, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{7}{2}}}{7}+\frac{6Aa^2bx^{\frac{13}{2}}}{13}+\frac{6Aab^2x^{\frac{19}{2}}}{19}+\frac{2Ab^3x^{\frac{25}{2}}}{25}+\frac{2Ba^3x^{\frac{13}{2}}}{13}+\frac{6Ba^2bx^{\frac{19}{2}}}{19}+\frac{6Bab^2x^{\frac{25}{2}}}{25}+\frac{2Bb^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(7/2)/7 + 6*A*a**2*b*x**(13/2)/13 + 6*A*a*b**2*x**(19/2)/19 + 2
*A*b**3*x**(25/2)/25 + 2*B*a**3*x**(13/2)/13 + 6*B*a**2*b*x**(19/2)/19 + 6*
B*a*b**2*x**(25/2)/25 + 2*B*b**3*x**(31/2)/31
```

3.149 $\int x^{3/2} \left(a+bx^3\right)^3 \left(A+Bx^3\right) dx$

Optimal. Leaf size=85

$$\frac{2}{5}a^3 A x^{5/2} + \frac{2}{11}a^2 x^{11/2}(aB+3Ab) + \frac{2}{23}b^2 x^{23/2}(3aB+Ab) + \frac{6}{17}abx^{17/2}(aB+Ab) + \frac{2}{29}b^3 B x^{29/2}$$

```
[Out] 2/5*a^3*A*x^(5/2)+2/11*a^2*(3*A*b+B*a)*x^(11/2)+6/17*a*b*(A*b+B*a)*x^(17/2)
+2/23*b^2*(A*b+3*B*a)*x^(23/2)+2/29*b^3*B*x^(29/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{11}a^2 x^{11/2}(aB+3Ab) + \frac{2}{5}a^3 A x^{5/2} + \frac{2}{23}b^2 x^{23/2}(3aB+Ab) + \frac{6}{17}abx^{17/2}(aB+Ab) + \frac{2}{29}b^3 B x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int x^{3/2}\left(a+bx^3\right)^3\left(A+Bx^3\right) dx = \int \left(a^3 A x^{3/2} + a^2(3Ab+aB)x^{9/2} + 3ab(Ab+aB)x^{15/2} + b^2(Ab+3aB)x^{21/2} \right.$$
$$= \frac{2}{5}a^3 A x^{5/2} + \frac{2}{11}a^2(3Ab+aB)x^{11/2} + \frac{6}{17}ab(Ab+aB)x^{17/2} + \frac{2}{23}b^2(Ab+3aB)$$

Mathematica [A] time = 0.08, size = 85, normalized size = 1.00

$$\frac{2}{5}a^3 A x^{5/2} + \frac{2}{11}a^2 x^{11/2}(aB+3Ab) + \frac{2}{23}b^2 x^{23/2}(3aB+Ab) + \frac{6}{17}abx^{17/2}(aB+Ab) + \frac{2}{29}b^3 B x^{29/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(5/2))/5 + (2*a^2*(3*A*b + a*B)*x^(11/2))/11 + (6*a*b*(A*b + a*B
)*x^(17/2))/17 + (2*b^2*(A*b + 3*a*B)*x^(23/2))/23 + (2*b^3*B*x^(29/2))/29
```

fricas [A] time = 0.94, size = 78, normalized size = 0.92

$$\frac{2}{623645}\left(21505\,Bb^3x^{14} + 27115\left(3\,Bab^2 + Ab^3\right)x^{11} + 110055\left(Ba^2b + Aab^2\right)x^8 + 124729\,Aa^3x^2 + 56695\left(Ba^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/623645*(21505*B*b^3*x^14 + 27115*(3*B*a*b^2 + A*b^3)*x^11 + 110055*(B*a^2
*b + A*a*b^2)*x^8 + 124729*A*a^3*x^2 + 56695*(B*a^3 + 3*A*a^2*b)*x^5)*sqrt(
x)
```

giac [A] time = 0.17, size = 77, normalized size = 0.91

$$\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{6}{23}\,Bab^2x^{\frac{23}{2}}+\frac{2}{23}\,Ab^3x^{\frac{23}{2}}+\frac{6}{17}\,Ba^2bx^{\frac{17}{2}}+\frac{6}{17}\,Aab^2x^{\frac{17}{2}}+\frac{2}{11}\,Ba^3x^{\frac{11}{2}}+\frac{6}{11}\,Aa^2bx^{\frac{11}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 6/23*B*a*b^2*x^(23/2) + 2/23*A*b^3*x^(23/2) + 6/17*B*
a^2*b*x^(17/2) + 6/17*A*a*b^2*x^(17/2) + 2/11*B*a^3*x^(11/2) + 6/11*A*a^2*b
*x^(11/2) + 2/5*A*a^3*x^(5/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(21505B\,b^3x^{12}+27115x^9A\,b^3+81345x^9Ba\,b^2+110055x^6Aa\,b^2+110055x^6B\,a^2b+170085x^3A\,a^2b+56695B\,a\right.}{623645}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x)
```

```
[Out] 2/623645*x^(5/2)*(21505*B*b^3*x^12+27115*A*b^3*x^9+81345*B*a*b^2*x^9+110055
*A*a*b^2*x^6+110055*B*a^2*b*x^6+170085*A*a^2*b*x^3+56695*B*a^3*x^3+124729*A
*a^3)
```

maxima [A] time = 0.48, size = 73, normalized size = 0.86

$$\frac{2}{29}\,Bb^3x^{\frac{29}{2}}+\frac{2}{23}\left(3\,Bab^2+Ab^3\right)x^{\frac{23}{2}}+\frac{6}{17}\left(Ba^2b+Aab^2\right)x^{\frac{17}{2}}+\frac{2}{5}\,Aa^3x^{\frac{5}{2}}+\frac{2}{11}\left(Ba^3+3\,Aa^2b\right)x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x^3+a)^3*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/29*B*b^3*x^(29/2) + 2/23*(3*B*a*b^2 + A*b^3)*x^(23/2) + 6/17*(B*a^2*b + A
*a*b^2)*x^(17/2) + 2/5*A*a^3*x^(5/2) + 2/11*(B*a^3 + 3*A*a^2*b)*x^(11/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{11/2}\,\left(\frac{2\,B\,a^3}{11}+\frac{6\,A\,b\,a^2}{11}\right)+x^{23/2}\,\left(\frac{2\,A\,b^3}{23}+\frac{6\,B\,a\,b^2}{23}\right)+\frac{2\,A\,a^3\,x^{5/2}}{5}+\frac{2\,B\,b^3\,x^{29/2}}{29}+\frac{6\,a\,b\,x^{17/2}\,\left(A\,b+B\,a\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(11/2)*((2*B*a^3)/11 + (6*A*a^2*b)/11) + x^(23/2)*((2*A*b^3)/23 + (6*B*a*
b^2)/23) + (2*A*a^3*x^(5/2))/5 + (2*B*b^3*x^(29/2))/29 + (6*a*b*x^(17/2)*(A
*b + B*a))/17
```

sympy [A] time = 39.60, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{5}{2}}}{5}+\frac{6Aa^2bx^{\frac{11}{2}}}{11}+\frac{6Aab^2x^{\frac{17}{2}}}{17}+\frac{2Ab^3x^{\frac{23}{2}}}{23}+\frac{2Ba^3x^{\frac{11}{2}}}{11}+\frac{6Ba^2bx^{\frac{17}{2}}}{17}+\frac{6Bab^2x^{\frac{23}{2}}}{23}+\frac{2Bb^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x**3+a)**3*(B*x**3+A),x)
```

```
[Out] 2*A*a**3*x**(5/2)/5 + 6*A*a**2*b*x**(11/2)/11 + 6*A*a*b**2*x**(17/2)/17 + 2
*A*b**3*x**(23/2)/23 + 2*B*a**3*x**(11/2)/11 + 6*B*a**2*b*x**(17/2)/17 + 6*
B*a*b**2*x**(23/2)/23 + 2*B*b**3*x**(29/2)/29
```

3.150 $\int \sqrt{x} \left(a+bx^3\right)^3 \left(A+Bx^3\right) dx$

Optimal. Leaf size=85

$$\frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

```
[Out] 2/3*a^3*A*x^(3/2)+2/9*a^2*(3*A*b+B*a)*x^(9/2)+2/5*a*b*(A*b+B*a)*x^(15/2)+2/
21*b^2*(A*b+3*B*a)*x^(21/2)+2/27*b^3*B*x^(27/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{9}a^2x^{9/2}(aB+3Ab)+\frac{2}{3}a^3Ax^{3/2}+\frac{2}{21}b^2x^{21/2}(3aB+Ab)+\frac{2}{5}abx^{15/2}(aB+Ab)+\frac{2}{27}b^3Bx^{27/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*a^3*A*x^(3/2))/3 + (2*a^2*(3*A*b + a*B)*x^(9/2))/9 + (2*a*b*(A*b + a*B)*
x^(15/2))/5 + (2*b^2*(A*b + 3*a*B)*x^(21/2))/21 + (2*b^3*B*x^(27/2))/27
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(a+bx^3\right)^3\left(A+Bx^3\right)dx &= \int \left(a^3A\sqrt{x}+a^2(3Ab+aB)x^{7/2}+3ab(Ab+aB)x^{13/2}+b^2(Ab+3aB)x^{19/2}+\right.\\ &= \frac{2}{3}a^3Ax^{3/2}+\frac{2}{9}a^2(3Ab+aB)x^{9/2}+\frac{2}{5}ab(Ab+aB)x^{15/2}+\frac{2}{21}b^2(Ab+3aB)x^{21}\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 0.84

$$\frac{2}{945}x^{3/2}\left(315a^3A+105a^2x^3(aB+3Ab)+45b^2x^9(3aB+Ab)+189abx^6(aB+Ab)+35b^3Bx^{12}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x^3)^3*(A + B*x^3),x]
```

```
[Out] (2*x^(3/2)*(315*a^3*A + 105*a^2*(3*A*b + a*B)*x^3 + 189*a*b*(A*b + a*B)*x^6
 + 45*b^2*(A*b + 3*a*B)*x^9 + 35*b^3*B*x^12))/945
```

fricas [A] time = 0.85, size = 76, normalized size = 0.89

$$\frac{2}{945}\left(35\,Bb^3x^{13}+45\left(3\,Bab^2+Ab^3\right)x^{10}+189\left(Ba^2b+Aab^2\right)x^7+315\,Aa^3x+105\left(Ba^3+3\,Aa^2b\right)x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^3*x^13 + 45*(3*B*a*b^2 + A*b^3)*x^10 + 189*(B*a^2*b + A*a*b^2
)*x^7 + 315*A*a^3*x + 105*(B*a^3 + 3*A*a^2*b)*x^4)*sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{27}Bb^3x^{\frac{27}{2}}+\frac{2}{7}Bab^2x^{\frac{21}{2}}+\frac{2}{21}Ab^3x^{\frac{21}{2}}+\frac{2}{5}Ba^2bx^{\frac{15}{2}}+\frac{2}{5}Aab^2x^{\frac{15}{2}}+\frac{2}{9}Ba^3x^{\frac{9}{2}}+\frac{2}{3}Aa^2bx^{\frac{9}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/7*B*a*b^2*x^(21/2) + 2/21*A*b^3*x^(21/2) + 2/5*B*a^
2*b*x^(15/2) + 2/5*A*a*b^2*x^(15/2) + 2/9*B*a^3*x^(9/2) + 2/3*A*a^2*b*x^(9/
2) + 2/3*A*a^3*x^(3/2)
```

maple [A] time = 0.05, size = 80, normalized size = 0.94

$$\frac{2\left(35B\,b^3x^{12}+45x^9A\,b^3+135x^9Ba\,b^2+189x^6Aa\,b^2+189x^6B\,a^2b+315x^3A\,a^2b+105B\,a^3x^3+315A\,a^3\right)x^{\frac{3}{2}}}{945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x)
```

```
[Out] 2/945*x^(3/2)*(35*B*b^3*x^12+45*A*b^3*x^9+135*B*a*b^2*x^9+189*A*a*b^2*x^6+1
89*B*a^2*b*x^6+315*A*a^2*b*x^3+105*B*a^3*x^3+315*A*a^3)
```

maxima [A] time = 0.67, size = 73, normalized size = 0.86

$$\frac{2}{27}Bb^3x^{\frac{27}{2}}+\frac{2}{21}\left(3\,Bab^2+Ab^3\right)x^{\frac{21}{2}}+\frac{2}{5}\left(Ba^2b+Aab^2\right)x^{\frac{15}{2}}+\frac{2}{3}Aa^3x^{\frac{3}{2}}+\frac{2}{9}\left(Ba^3+3\,Aa^2b\right)x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*B*b^3*x^(27/2) + 2/21*(3*B*a*b^2 + A*b^3)*x^(21/2) + 2/5*(B*a^2*b + A*
a*b^2)*x^(15/2) + 2/3*A*a^3*x^(3/2) + 2/9*(B*a^3 + 3*A*a^2*b)*x^(9/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{9/2}\left(\frac{2\,B\,a^3}{9}+\frac{2\,A\,b\,a^2}{3}\right)+x^{21/2}\left(\frac{2\,A\,b^3}{21}+\frac{2\,B\,a\,b^2}{7}\right)+\frac{2\,A\,a^3\,x^{3/2}}{3}+\frac{2\,B\,b^3\,x^{27/2}}{27}+\frac{2\,a\,b\,x^{15/2}\,\left(A\,b+B\,a\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(A + B*x^3)*(a + b*x^3)^3,x)
```

```
[Out] x^(9/2)*((2*B*a^3)/9 + (2*A*a^2*b)/3) + x^(21/2)*((2*A*b^3)/21 + (2*B*a*b^2
)/7) + (2*A*a^3*x^(3/2))/3 + (2*B*b^3*x^(27/2))/27 + (2*a*b*x^(15/2)*(A*b +
 B*a))/5
```

sympy [A] time = 6.34, size = 114, normalized size = 1.34

$$\frac{2Aa^3x^{\frac{3}{2}}}{3}+\frac{2Aa^2bx^{\frac{9}{2}}}{3}+\frac{2Aab^2x^{\frac{15}{2}}}{5}+\frac{2Ab^3x^{\frac{21}{2}}}{21}+\frac{2Ba^3x^{\frac{9}{2}}}{9}+\frac{2Ba^2bx^{\frac{15}{2}}}{5}+\frac{2Bab^2x^{\frac{21}{2}}}{7}+\frac{2Bb^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)*x**(1/2),x)
```

```
[Out] 2*A*a**3*x**(3/2)/3 + 2*A*a**2*b*x**(9/2)/3 + 2*A*a*b**2*x**(15/2)/5 + 2*A*
b**3*x**(21/2)/21 + 2*B*a**3*x**(9/2)/9 + 2*B*a**2*b*x**(15/2)/5 + 2*B*a*b*
*2*x**(21/2)/7 + 2*B*b**3*x**(27/2)/27
```

3.151 $\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{\sqrt{x}}\,dx$

Optimal. Leaf size=83

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

```
[Out] 2/7*a^2*(3*A*b+B*a)*x^(7/2)+6/13*a*b*(A*b+B*a)*x^(13/2)+2/19*b^2*(A*b+3*B*a
)*x^(19/2)+2/25*b^3*B*x^(25/2)+2*a^3*A*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{7}a^2x^{7/2}(aB+3Ab)+2a^3A\sqrt{x}+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^3\left(A+Bx^3\right)}{\sqrt{x}}\,dx &= \int \left(\frac{a^3A}{\sqrt{x}}+a^2(3Ab+aB)x^{5/2}+3ab(Ab+aB)x^{11/2}+b^2(Ab+3aB)x^{17/2}+b^3Bx^2\right. \\ &= 2a^3A\sqrt{x}+\frac{2}{7}a^2(3Ab+aB)x^{7/2}+\frac{6}{13}ab(Ab+aB)x^{13/2}+\frac{2}{19}b^2(Ab+3aB)x^{19/2}+\end{aligned}$$

Mathematica [A] time = 0.06, size = 83, normalized size = 1.00

$$2a^3A\sqrt{x}+\frac{2}{7}a^2x^{7/2}(aB+3Ab)+\frac{2}{19}b^2x^{19/2}(3aB+Ab)+\frac{6}{13}abx^{13/2}(aB+Ab)+\frac{2}{25}b^3Bx^{25/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/Sqrt[x],x]
```

```
[Out] 2*a^3*A*Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(7/2))/7 + (6*a*b*(A*b + a*B)*x^(1
3/2))/13 + (2*b^2*(A*b + 3*a*B)*x^(19/2))/19 + (2*b^3*B*x^(25/2))/25
```

fricas [A] time = 1.45, size = 75, normalized size = 0.90

$$\frac{2}{43225}\left(1729\,Bb^3x^{12}+2275\left(3\,Bab^2+Ab^3\right)x^9+9975\left(Ba^2b+Aab^2\right)x^6+43225\,Aa^3+6175\left(Ba^3+3\,Aa^2b\right)x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/43225*(1729*B*b^3*x^12 + 2275*(3*B*a*b^2 + A*b^3)*x^9 + 9975*(B*a^2*b + A
*a*b^2)*x^6 + 43225*A*a^3 + 6175*(B*a^3 + 3*A*a^2*b)*x^3)*sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{25}Bb^3x^{\frac{25}{2}}+\frac{6}{19}Bab^2x^{\frac{19}{2}}+\frac{2}{19}Ab^3x^{\frac{19}{2}}+\frac{6}{13}Ba^2bx^{\frac{13}{2}}+\frac{6}{13}Aab^2x^{\frac{13}{2}}+\frac{2}{7}Ba^3x^{\frac{7}{2}}+\frac{6}{7}Aa^2bx^{\frac{7}{2}}+2Aa^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 6/19*B*a*b^2*x^(19/2) + 2/19*A*b^3*x^(19/2) + 6/13*B*
a^2*b*x^(13/2) + 6/13*A*a*b^2*x^(13/2) + 2/7*B*a^3*x^(7/2) + 6/7*A*a^2*b*x^
(7/2) + 2*A*a^3*sqrt(x)
```

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$\frac{2\left(1729B\,b^3x^{12}+2275x^9A\,b^3+6825x^9Ba\,b^2+9975x^6Aa\,b^2+9975x^6B\,a^2b+18525x^3A\,a^2b+6175B\,a^3x^3+432\right.}{43225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x)
```

```
[Out] 2/43225*x^(1/2)*(1729*B*b^3*x^12+2275*A*b^3*x^9+6825*B*a*b^2*x^9+9975*A*a*b
^2*x^6+9975*B*a^2*b*x^6+18525*A*a^2*b*x^3+6175*B*a^3*x^3+43225*A*a^3)
```

maxima [A] time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{25}Bb^3x^{\frac{25}{2}}+\frac{2}{19}\left(3Bab^2+Ab^3\right)x^{\frac{19}{2}}+\frac{6}{13}\left(Ba^2b+Aab^2\right)x^{\frac{13}{2}}+2Aa^3\sqrt{x}+\frac{2}{7}\left(Ba^3+3Aa^2b\right)x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/25*B*b^3*x^(25/2) + 2/19*(3*B*a*b^2 + A*b^3)*x^(19/2) + 6/13*(B*a^2*b + A
*a*b^2)*x^(13/2) + 2*A*a^3*sqrt(x) + 2/7*(B*a^3 + 3*A*a^2*b)*x^(7/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{7/2}\left(\frac{2\,B\,a^3}{7}+\frac{6\,A\,b\,a^2}{7}\right)+x^{19/2}\left(\frac{2\,A\,b^3}{19}+\frac{6\,B\,a\,b^2}{19}\right)+2\,A\,a^3\,\sqrt{x}+\frac{2\,B\,b^3\,x^{25/2}}{25}+\frac{6\,a\,b\,x^{13/2}\,\left(A\,b+B\,a\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(1/2),x)
```

```
[Out] x^(7/2)*((2*B*a^3)/7 + (6*A*a^2*b)/7) + x^(19/2)*((2*A*b^3)/19 + (6*B*a*b^2
)/19) + 2*A*a^3*x^(1/2) + (2*B*b^3*x^(25/2))/25 + (6*a*b*x^(13/2)*(A*b + B*
a))/13
```

sympy [A] time = 23.64, size = 112, normalized size = 1.35

$$2Aa^3\sqrt{x}+\frac{6Aa^2bx^{\frac{7}{2}}}{7}+\frac{6Aab^2x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{19}{2}}}{19}+\frac{2Ba^3x^{\frac{7}{2}}}{7}+\frac{6Ba^2bx^{\frac{13}{2}}}{13}+\frac{6Bab^2x^{\frac{19}{2}}}{19}+\frac{2Bb^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(1/2),x)
```

```
[Out] 2*A*a**3*sqrt(x) + 6*A*a**2*b*x**(7/2)/7 + 6*A*a*b**2*x**(13/2)/13 + 2*A*b*
*3*x**(19/2)/19 + 2*B*a**3*x**(7/2)/7 + 6*B*a**2*b*x**(13/2)/13 + 6*B*a*b**
2*x**(19/2)/19 + 2*B*b**3*x**(25/2)/25
```

3.152 $\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{3/2}} \, dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2x^{5/2}(aB+3Ab)+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

```
[Out] 2/5*a^2*(3*A*b+B*a)*x^(5/2)+6/11*a*b*(A*b+B*a)*x^(11/2)+2/17*b^2*(A*b+3*B*a
)*x^(17/2)+2/23*b^3*B*x^(23/2)-2*a^3*A/x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{5}a^2x^{5/2}(aB+3Ab)-\frac{2a^3A}{\sqrt{x}}+\frac{2}{17}b^2x^{17/2}(3aB+Ab)+\frac{6}{11}abx^{11/2}(aB+Ab)+\frac{2}{23}b^3Bx^{23/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-2*a^3*A)/Sqrt[x] + (2*a^2*(3*A*b + a*B)*x^(5/2))/5 + (6*a*b*(A*b + a*B)*x
^(11/2))/11 + (2*b^2*(A*b + 3*a*B)*x^(17/2))/17 + (2*b^3*B*x^(23/2))/23
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{3/2}}\,dx &= \int \left(\frac{a^3A}{x^{3/2}}+a^2(3Ab+aB)x^{3/2}+3ab(Ab+aB)x^{9/2}+b^2(Ab+3aB)x^{15/2}+b^3Bx^{21}\right.\\ &= -\frac{2a^3A}{\sqrt{x}}+\frac{2}{5}a^2(3Ab+aB)x^{5/2}+\frac{6}{11}ab(Ab+aB)x^{11/2}+\frac{2}{17}b^2(Ab+3aB)x^{17/2}+\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 0.98

$$\frac{-8602a^3\left(5A-Bx^3\right)+2346a^2bx^3\left(11A+5Bx^3\right)+690ab^2x^6\left(17A+11Bx^3\right)+110b^3x^9\left(23A+17Bx^3\right)}{21505\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(3/2),x]
```

```
[Out] (-8602*a^3*(5*A - B*x^3) + 2346*a^2*b*x^3*(11*A + 5*B*x^3) + 690*a*b^2*x^6*
(17*A + 11*B*x^3) + 110*b^3*x^9*(23*A + 17*B*x^3))/(21505*Sqrt[x])
```

fricas [A] time = 1.18, size = 75, normalized size = 0.90

$$\frac{2\left(935\,Bb^3x^{12}+1265\left(3\,Bab^2+Ab^3\right)x^9+5865\left(Ba^2b+Aab^2\right)x^6-21505\,Aa^3+4301\left(Ba^3+3\,Aa^2b\right)x^3\right)}{21505\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21505*(935*B*b^3*x^12 + 1265*(3*B*a*b^2 + A*b^3)*x^9 + 5865*(B*a^2*b + A*
a*b^2)*x^6 - 21505*A*a^3 + 4301*(B*a^3 + 3*A*a^2*b)*x^3)/sqrt(x)
```

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{6}{17}\,Bab^2x^{\frac{17}{2}}+\frac{2}{17}\,Ab^3x^{\frac{17}{2}}+\frac{6}{11}\,Ba^2bx^{\frac{11}{2}}+\frac{6}{11}\,Aab^2x^{\frac{11}{2}}+\frac{2}{5}\,Ba^3x^{\frac{5}{2}}+\frac{6}{5}\,Aa^2bx^{\frac{5}{2}}-\frac{2\,Aa^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 6/17*B*a*b^2*x^(17/2) + 2/17*A*b^3*x^(17/2) + 6/11*B*
a^2*b*x^(11/2) + 6/11*A*a*b^2*x^(11/2) + 2/5*B*a^3*x^(5/2) + 6/5*A*a^2*b*x^
(5/2) - 2*A*a^3/sqrt(x)
```

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$-\frac{2\left(-935B\,b^3x^{12}-1265x^9A\,b^3-3795x^9Ba\,b^2-5865x^6Aa\,b^2-5865x^6B\,a^2b-12903x^3A\,a^2b-4301B\,a^3x^3+21\right.}{21505\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x)
```

```
[Out] -2/21505*(-935*B*b^3*x^12-1265*A*b^3*x^9-3795*B*a*b^2*x^9-5865*A*a*b^2*x^6-
5865*B*a^2*b*x^6-12903*A*a^2*b*x^3-4301*B*a^3*x^3+21505*A*a^3)/x^(1/2)
```

maxima [A] time = 0.56, size = 73, normalized size = 0.88

$$\frac{2}{23}\,Bb^3x^{\frac{23}{2}}+\frac{2}{17}\left(3\,Bab^2+Ab^3\right)x^{\frac{17}{2}}+\frac{6}{11}\left(Ba^2b+Aab^2\right)x^{\frac{11}{2}}-\frac{2\,Aa^3}{\sqrt{x}}+\frac{2}{5}\left(Ba^3+3\,Aa^2b\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*B*b^3*x^(23/2) + 2/17*(3*B*a*b^2 + A*b^3)*x^(17/2) + 6/11*(B*a^2*b + A
*a*b^2)*x^(11/2) - 2*A*a^3/sqrt(x) + 2/5*(B*a^3 + 3*A*a^2*b)*x^(5/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$x^{5/2}\left(\frac{2\,B\,a^3}{5}+\frac{6\,A\,b\,a^2}{5}\right)+x^{17/2}\left(\frac{2\,A\,b^3}{17}+\frac{6\,B\,a\,b^2}{17}\right)-\frac{2\,A\,a^3}{\sqrt{x}}+\frac{2\,B\,b^3\,x^{23/2}}{23}+\frac{6\,a\,b\,x^{11/2}\,\left(A\,b+B\,a\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(3/2),x)
```

```
[Out] x^(5/2)*((2*B*a^3)/5 + (6*A*a^2*b)/5) + x^(17/2)*((2*A*b^3)/17 + (6*B*a*b^2
)/17) - (2*A*a^3)/x^(1/2) + (2*B*b^3*x^(23/2))/23 + (6*a*b*x^(11/2)*(A*b +
B*a))/11
```

sympy [A] time = 19.49, size = 112, normalized size = 1.35

$$-\frac{2Aa^3}{\sqrt{x}}+\frac{6Aa^2bx^{\frac{5}{2}}}{5}+\frac{6Aab^2x^{\frac{11}{2}}}{11}+\frac{2Ab^3x^{\frac{17}{2}}}{17}+\frac{2Ba^3x^{\frac{5}{2}}}{5}+\frac{6Ba^2bx^{\frac{11}{2}}}{11}+\frac{6Bab^2x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(3/2),x)
```

```
[Out] -2*A*a**3/sqrt(x) + 6*A*a**2*b*x**(5/2)/5 + 6*A*a*b**2*x**(11/2)/11 + 2*A*b
**3*x**(17/2)/17 + 2*B*a**3*x**(5/2)/5 + 6*B*a**2*b*x**(11/2)/11 + 6*B*a*b*
*2*x**(17/2)/17 + 2*B*b**3*x**(23/2)/23
```

3.153 $\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2x^{3/2}(aB+3Ab)+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

```
[Out] -2/3*a^3*A/x^(3/2)+2/3*a^2*(3*A*b+B*a)*x^(3/2)+2/3*a*b*(A*b+B*a)*x^(9/2)+2/
15*b^2*(A*b+3*B*a)*x^(15/2)+2/21*b^3*B*x^(21/2)
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$\frac{2}{3}a^2x^{3/2}(aB+3Ab)-\frac{2a^3A}{3x^{3/2}}+\frac{2}{15}b^2x^{15/2}(3aB+Ab)+\frac{2}{3}abx^{9/2}(aB+Ab)+\frac{2}{21}b^3Bx^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (-2*a^3*A)/(3*x^(3/2)) + (2*a^2*(3*A*b + a*B)*x^(3/2))/3 + (2*a*b*(A*b + a*
B)*x^(9/2))/3 + (2*b^2*(A*b + 3*a*B)*x^(15/2))/15 + (2*b^3*B*x^(21/2))/21
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3(A+Bx^3)}{x^{5/2}}\,dx &= \int \left(\frac{a^3A}{x^{5/2}}+a^2(3Ab+aB)\sqrt{x}+3ab(Ab+aB)x^{7/2}+b^2(Ab+3aB)x^{13/2}+b^3Bx^{19/2}\right)\\ &= -\frac{2a^3A}{3x^{3/2}}+\frac{2}{3}a^2(3Ab+aB)x^{3/2}+\frac{2}{3}ab(Ab+aB)x^{9/2}+\frac{2}{15}b^2(Ab+3aB)x^{15/2}+\frac{2}{21}b^3B\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.91

$$\frac{2\left(-35a^3\left(A-Bx^3\right)+35a^2bx^3\left(3A+Bx^3\right)+7ab^2x^6\left(5A+3Bx^3\right)+b^3x^9\left(7A+5Bx^3\right)\right)}{105x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(5/2),x]
```

```
[Out] (2*(-35*a^3*(A - B*x^3) + 35*a^2*b*x^3*(3*A + B*x^3) + 7*a*b^2*x^6*(5*A + 3
*B*x^3) + b^3*x^9*(7*A + 5*B*x^3)))/(105*x^(3/2))
```

fricas [A] time = 1.12, size = 75, normalized size = 0.88

$$\frac{2\left(5\,Bb^3x^{12}+7\left(3\,Bab^2+Ab^3\right)x^9+35\left(Ba^2b+Aab^2\right)x^6-35\,Aa^3+35\left(Ba^3+3\,Aa^2b\right)x^3\right)}{105\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^12 + 7*(3*B*a*b^2 + A*b^3)*x^9 + 35*(B*a^2*b + A*a*b^2)*x^
6 - 35*A*a^3 + 35*(B*a^3 + 3*A*a^2*b)*x^3)/x^(3/2)
```

giac [A] time = 0.16, size = 77, normalized size = 0.91

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{5}\,Bab^2x^{\frac{15}{2}} + \frac{2}{15}\,Ab^3x^{\frac{15}{2}} + \frac{2}{3}\,Ba^2bx^{\frac{9}{2}} + \frac{2}{3}\,Aab^2x^{\frac{9}{2}} + \frac{2}{3}\,Ba^3x^{\frac{3}{2}} + 2\,Aa^2bx^{\frac{3}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/5*B*a*b^2*x^(15/2) + 2/15*A*b^3*x^(15/2) + 2/3*B*a^
2*b*x^(9/2) + 2/3*A*a*b^2*x^(9/2) + 2/3*B*a^3*x^(3/2) + 2*A*a^2*b*x^(3/2) -
 2/3*A*a^3/x^(3/2)
```

maple [A] time = 0.04, size = 80, normalized size = 0.94

$$-\frac{2\left(-5B\,b^3x^{12} - 7x^9A\,b^3 - 21x^9Ba\,b^2 - 35x^6Aa\,b^2 - 35x^6B\,a^2b - 105x^3A\,a^2b - 35B\,a^3x^3 + 35A\,a^3\right)}{105x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x)
```

```
[Out] -2/105*(-5*B*b^3*x^12-7*A*b^3*x^9-21*B*a*b^2*x^9-35*A*a*b^2*x^6-35*B*a^2*b*
x^6-105*A*a^2*b*x^3-35*B*a^3*x^3+35*A*a^3)/x^(3/2)
```

maxima [A] time = 0.64, size = 73, normalized size = 0.86

$$\frac{2}{21}\,Bb^3x^{\frac{21}{2}} + \frac{2}{15}\left(3\,Bab^2 + Ab^3\right)x^{\frac{15}{2}} + \frac{2}{3}\left(Ba^2b + Aab^2\right)x^{\frac{9}{2}} - \frac{2\,Aa^3}{3\,x^{\frac{3}{2}}} + \frac{2}{3}\left(Ba^3 + 3\,Aa^2b\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*B*b^3*x^(21/2) + 2/15*(3*B*a*b^2 + A*b^3)*x^(15/2) + 2/3*(B*a^2*b + A*
a*b^2)*x^(9/2) - 2/3*A*a^3/x^(3/2) + 2/3*(B*a^3 + 3*A*a^2*b)*x^(3/2)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.81

$$x^{3/2}\left(\frac{2\,B\,a^3}{3} + 2\,A\,b\,a^2\right) + x^{15/2}\left(\frac{2\,A\,b^3}{15} + \frac{2\,B\,a\,b^2}{5}\right) - \frac{2\,A\,a^3}{3\,x^{3/2}} + \frac{2\,B\,b^3\,x^{21/2}}{21} + \frac{2\,a\,b\,x^{9/2}\,\left(A\,b + B\,a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(5/2),x)
```

```
[Out] x^(3/2)*((2*B*a^3)/3 + 2*A*a^2*b) + x^(15/2)*((2*A*b^3)/15 + (2*B*a*b^2)/5)
 - (2*A*a^3)/(3*x^(3/2)) + (2*B*b^3*x^(21/2))/21 + (2*a*b*x^(9/2)*(A*b + B*
a))/3
```

sympy [A] time = 28.37, size = 112, normalized size = 1.32

$$-\frac{2Aa^3}{3x^{\frac{3}{2}}} + 2Aa^2bx^{\frac{3}{2}} + \frac{2Aab^2x^{\frac{9}{2}}}{3} + \frac{2Ab^3x^{\frac{15}{2}}}{15} + \frac{2Ba^3x^{\frac{3}{2}}}{3} + \frac{2Ba^2bx^{\frac{9}{2}}}{3} + \frac{2Bab^2x^{\frac{15}{2}}}{5} + \frac{2Bb^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(5/2),x)
```

```
[Out] -2*A*a**3/(3*x**(3/2)) + 2*A*a**2*b*x**(3/2) + 2*A*a*b**2*x**(9/2)/3 + 2*A*
b**3*x**(15/2)/15 + 2*B*a**3*x**(3/2)/3 + 2*B*a**2*b*x**(9/2)/3 + 2*B*a*b**
2*x**(15/2)/5 + 2*B*b**3*x**(21/2)/21
```

3.154 $\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}} \, dx$

Optimal. Leaf size=83

$$-\frac{2a^3A}{5x^{5/2}}+2a^2\sqrt{x}\,(aB+3Ab)+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

```
[Out] -2/5*a^3*A/x^(5/2)+6/7*a*b*(A*b+B*a)*x^(7/2)+2/13*b^2*(A*b+3*B*a)*x^(13/2)+
2/19*b^3*B*x^(19/2)+2*a^2*(3*A*b+B*a)*x^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {448}

$$2a^2\sqrt{x}\,(aB+3Ab)-\frac{2a^3A}{5x^{5/2}}+\frac{2}{13}b^2x^{13/2}(3aB+Ab)+\frac{6}{7}abx^{7/2}(aB+Ab)+\frac{2}{19}b^3Bx^{19/2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-2*a^3*A)/(5*x^(5/2)) + 2*a^2*(3*A*b + a*B)*Sqrt[x] + (6*a*b*(A*b + a*B)*x
^(7/2))/7 + (2*b^2*(A*b + 3*a*B)*x^(13/2))/13 + (2*b^3*B*x^(19/2))/19
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^3 (A+Bx^3)}{x^{7/2}}\,dx &= \int \left(\frac{a^3A}{x^{7/2}}+\frac{a^2(3Ab+aB)}{\sqrt{x}}+3ab(Ab+aB)x^{5/2}+b^2(Ab+3aB)x^{11/2}+b^3Bx^{17/2}\right) \\ &= -\frac{2a^3A}{5x^{5/2}}+2a^2(3Ab+aB)\sqrt{x}+\frac{6}{7}ab(Ab+aB)x^{7/2}+\frac{2}{13}b^2(Ab+3aB)x^{13/2}+\frac{2}{19}b\end{aligned}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.94

$$\frac{-3458a^3\left(A-5Bx^3\right)+7410a^2bx^3\left(7A+Bx^3\right)+570ab^2x^6\left(13A+7Bx^3\right)+70b^3x^9\left(19A+13Bx^3\right)}{8645x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^3*(A + B*x^3))/x^(7/2),x]
```

```
[Out] (-3458*a^3*(A - 5*B*x^3) + 7410*a^2*b*x^3*(7*A + B*x^3) + 570*a*b^2*x^6*(13
*A + 7*B*x^3) + 70*b^3*x^9*(19*A + 13*B*x^3))/(8645*x^(5/2))
```

fricas [A] time = 1.09, size = 75, normalized size = 0.90

$$\frac{2\left(455\,Bb^3x^{12}+665\left(3\,Bab^2+Ab^3\right)x^9+3705\left(Ba^2b+Aab^2\right)x^6-1729\,Aa^3+8645\left(Ba^3+3\,Aa^2b\right)x^3\right)}{8645\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8645*(455*B*b^3*x^12 + 665*(3*B*a*b^2 + A*b^3)*x^9 + 3705*(B*a^2*b + A*a*
b^2)*x^6 - 1729*A*a^3 + 8645*(B*a^3 + 3*A*a^2*b)*x^3)/x^(5/2)
```

giac [A] time = 0.16, size = 77, normalized size = 0.93

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{6}{13}\,Bab^2x^{\frac{13}{2}}+\frac{2}{13}\,Ab^3x^{\frac{13}{2}}+\frac{6}{7}\,Ba^2bx^{\frac{7}{2}}+\frac{6}{7}\,Aab^2x^{\frac{7}{2}}+2\,Ba^3\sqrt{x}+6\,Aa^2b\sqrt{x}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 6/13*B*a*b^2*x^(13/2) + 2/13*A*b^3*x^(13/2) + 6/7*B*a
^2*b*x^(7/2) + 6/7*A*a*b^2*x^(7/2) + 2*B*a^3*sqrt(x) + 6*A*a^2*b*sqrt(x) -
2/5*A*a^3/x^(5/2)
```

maple [A] time = 0.04, size = 80, normalized size = 0.96

$$-\frac{2\left(-455B\,b^3x^{12}-665x^9A\,b^3-1995x^9Ba\,b^2-3705x^6Aa\,b^2-3705x^6B\,a^2b-25935x^3A\,a^2b-8645B\,a^3x^3+172\right.}{8645x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x)
```

```
[Out] -2/8645*(-455*B*b^3*x^12-665*A*b^3*x^9-1995*B*a*b^2*x^9-3705*A*a*b^2*x^6-37
05*B*a^2*b*x^6-25935*A*a^2*b*x^3-8645*B*a^3*x^3+1729*A*a^3)/x^(5/2)
```

maxima [A] time = 0.55, size = 73, normalized size = 0.88

$$\frac{2}{19}\,Bb^3x^{\frac{19}{2}}+\frac{2}{13}\left(3\,Bab^2+Ab^3\right)x^{\frac{13}{2}}+\frac{6}{7}\left(Ba^2b+Aab^2\right)x^{\frac{7}{2}}-\frac{2\,Aa^3}{5\,x^{\frac{5}{2}}}+2\left(Ba^3+3\,Aa^2b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3*(B*x^3+A)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*B*b^3*x^(19/2) + 2/13*(3*B*a*b^2 + A*b^3)*x^(13/2) + 6/7*(B*a^2*b + A*
a*b^2)*x^(7/2) - 2/5*A*a^3/x^(5/2) + 2*(B*a^3 + 3*A*a^2*b)*sqrt(x)
```

mupad [B] time = 0.03, size = 69, normalized size = 0.83

$$\sqrt{x}\,\left(2\,B\,a^3+6\,A\,b\,a^2\right)+x^{13/2}\,\left(\frac{2\,A\,b^3}{13}+\frac{6\,B\,a\,b^2}{13}\right)-\frac{2\,A\,a^3}{5\,x^{5/2}}+\frac{2\,B\,b^3\,x^{19/2}}{19}+\frac{6\,a\,b\,x^{7/2}\,\left(A\,b+B\,a\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^3)/x^(7/2),x)
```

```
[Out] x^(1/2)*(2*B*a^3 + 6*A*a^2*b) + x^(13/2)*((2*A*b^3)/13 + (6*B*a*b^2)/13) -
(2*A*a^3)/(5*x^(5/2)) + (2*B*b^3*x^(19/2))/19 + (6*a*b*x^(7/2)*(A*b + B*a))
/7
```

sympy [A] time = 29.28, size = 110, normalized size = 1.33

$$-\frac{2Aa^3}{5x^{\frac{5}{2}}}+6Aa^2b\sqrt{x}+\frac{6Aab^2x^{\frac{7}{2}}}{7}+\frac{2Ab^3x^{\frac{13}{2}}}{13}+2Ba^3\sqrt{x}+\frac{6Ba^2bx^{\frac{7}{2}}}{7}+\frac{6Bab^2x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3*(B*x**3+A)/x**(7/2),x)
```

```
[Out] -2*A*a**3/(5*x**(5/2)) + 6*A*a**2*b*sqrt(x) + 6*A*a*b**2*x**(7/2)/7 + 2*A*b
**3*x**(13/2)/13 + 2*B*a**3*sqrt(x) + 6*B*a**2*b*x**(7/2)/7 + 6*B*a*b**2*x*
*(13/2)/13 + 2*B*b**3*x**(19/2)/19
```

3.155 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3} \, dx$

Optimal. Leaf size=73

$$-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2x^{3/2}(Ab-aB)}{3b^2}+\frac{2Bx^{9/2}}{9b}$$

```
[Out] 2/3*(A*b-B*a)*x^(3/2)/b^2+2/9*B*x^(9/2)/b-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1
/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 321, 329, 275, 205}

$$\frac{2x^{3/2}(Ab-aB)}{3b^2}-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2Bx^{9/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*x^(3/2))/(3*b^2) + (2*B*x^(9/2))/(9*b) - (2*Sqrt[a]*(A*b - a
*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{9/2}}{9b}-\frac{\left(2\left(-\frac{9Ab}{2}+\frac{9aB}{2}\right)\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{9b}$$
$$=\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(a(Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{b^2}$$
$$=\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{b^2}$$
$$=\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{(2a(Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3b^2}$$
$$=\frac{2(Ab-aB)x^{3/2}}{3b^2}+\frac{2Bx^{9/2}}{9b}-\frac{2\sqrt{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}$$

Mathematica [A] time = 0.12, size = 67, normalized size = 0.92

$$\frac{2\sqrt{a}\,(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3b^{5/2}}+\frac{2x^{3/2}\left(-3aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(3/2)*(3*A*b - 3*a*B + b*B*x^3))/(9*b^2) + (2*Sqrt[a]*(-(A*b) + a*B)*A
rcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*b^(5/2))
```

fricas [A] time = 0.71, size = 143, normalized size = 1.96

$$\left[-\frac{3\,(Ba-Ab)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^3-2\,bx^{\frac{3}{2}}\sqrt{-\frac{a}{b}}-a}{bx^3+a}\right)-2\left(Bbx^4-3\,(Ba-Ab)x\right)\sqrt{x}}{9\,b^2},\ \frac{2\left(3\,(Ba-Ab)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx^{\frac{3}{2}}\sqrt{\frac{a}{b}}}{a}\right)+\right.}{9\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(B*a - A*b)*sqrt(-a/b)*log((b*x^3 - 2*b*x^(3/2)*sqrt(-a/b) - a)/(b
*x^3 + a)) - 2*(B*b*x^4 - 3*(B*a - A*b)*x)*sqrt(x))/b^2, 2/9*(3*(B*a - A*b)
*sqrt(a/b)*arctan(b*x^(3/2)*sqrt(a/b)/a) + (B*b*x^4 - 3*(B*a - A*b)*x)*sqrt
(x))/b^2]
```

giac [A] time = 0.17, size = 64, normalized size = 0.88

$$\frac{2\left(Ba^2-Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2}+\frac{2\left(Bb^2x^{\frac{9}{2}}-3\,Babx^{\frac{3}{2}}+3\,Ab^2x^{\frac{3}{2}}\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b^
2*x^(9/2) - 3*B*a*b*x^(3/2) + 3*A*b^2*x^(3/2))/b^3
```

maple [A] time = 0.05, size = 78, normalized size = 1.07

$$\frac{2B\,x^{\frac{9}{2}}}{9b}-\frac{2Aa\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b}+\frac{2B\,a^{2}\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b^{2}}+\frac{2A\,x^{\frac{3}{2}}}{3b}-\frac{2Ba\,x^{\frac{3}{2}}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/9*B*x^(9/2)/b+2/3/b*A*x^(3/2)-2/3/b^2*B*a*x^(3/2)-2/3*a/b/(a*b)^(1/2)*arc
tan(x^(3/2)*b/(a*b)^(1/2))*A+2/3*a^2/b^2/(a*b)^(1/2)*arctan(x^(3/2)*b/(a*b)
^(1/2))*B
```

maxima [A] time = 1.32, size = 58, normalized size = 0.79

$$\frac{2\left(Ba^{2}-Aab\right)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,b^{2}}+\frac{2\left(Bbx^{\frac{9}{2}}-3\,(Ba-Ab)x^{\frac{3}{2}}\right)}{9\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a^2 - A*a*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/9*(B*b*
x^(9/2) - 3*(B*a - A*b)*x^(3/2))/b^2
```

mupad [B] time = 2.61, size = 111, normalized size = 1.52

$$x^{3/2}\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^{2}}\right)+\frac{2\,B\,x^{9/2}}{9\,b}-\frac{2\,\sqrt{a}\;\mathrm{atan}\left(\frac{72\,b^{3/2}\,x^{3/2}\,\left(A^{2}\,a^{2}\,b^{2}-2\,A\,B\,a^{3}\,b+B^{2}\,a^{4}\right)}{\sqrt{a}\,\left(72\,A\,a^{2}\,b^{2}-72\,B\,a^{3}\,b\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(3/2)*((2*A)/(3*b) - (2*B*a)/(3*b^2)) + (2*B*x^(9/2))/(9*b) - (2*a^(1/2)*
atan((72*b^(3/2)*x^(3/2)*(B^2*a^4 + A^2*a^2*b^2 - 2*A*B*a^3*b))/(a^(1/2)*(7
2*A*a^2*b^2 - 72*B*a^3*b)*(A*b - B*a)))*(A*b - B*a))/(3*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.156 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx$

Optimal. Leaf size=288

$$\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}+\frac{\sqrt[6]{a}\,(Ab}{}$$

```
[Out] 2/7*B*x^(7/2)/b-2/3*a^(1/6)*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/b^(13
/6)-1/3*a^(1/6)*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/
6)-1/3*a^(1/6)*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/b^(13/6)
+1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2)
)/b^(13/6)*3^(1/2)-1/6*a^(1/6)*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/
6)*3^(1/2)*x^(1/2))/b^(13/6)*3^(1/2)+2*(A*b-B*a)*x^(1/2)/b^2
```

Rubi [A] time = 0.52, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {459, 321, 329, 209, 634, 618, 204, 628, 205}

$$\frac{2\sqrt{x}\,(Ab-aB)}{b^2}+\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,b^{13/6}}-\frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{a}+\right.}{2\sqrt{3}\,b^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[x])/b^2 + (2*B*x^(7/2))/(7*b) + (a^(1/6)*(A*b - a*B)*Ar
cTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (a^(1/6)*(A*b -
 a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) - (2*a^(1
/6)*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(3*b^(13/6)) + (a^(1/6)*
(A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*
Sqrt[3]*b^(13/6)) - (a^(1/6)*(A*b - a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
```

```
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{7/2}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+\frac{7aB}{2}\right)\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{7b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(a(Ab-aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{(2a(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{\left(2\sqrt[6]{a}\,(Ab-aB)\right)\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\left(\sqrt[6]{a}\,(Ab-aB)\right)\text{Subst}\left(\int \right.}{2}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} - \frac{2\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} + \frac{\sqrt[6]{a}\,(Ab-aB)\log\left(\sqrt[3]{a}\,-\right.}{2\sqrt{3}\,b}$$

$$= \frac{2(Ab-aB)\sqrt{x}}{b^2} + \frac{2Bx^{7/2}}{7b} + \frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3b^{13/6}} - \frac{\sqrt[6]{a}\,(Ab-aB)\tan^{-1}}{3b^{13/}}$$

Mathematica [C] time = 0.09, size = 54, normalized size = 0.19

$$\frac{2\sqrt{x}\left((7aB-7Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-7aB+7Ab+bBx^3\right)}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[x]*(7*A*b - 7*a*B + b*B*x^3 + (-7*A*b + 7*a*B)*Hypergeometric2F1[1/
6, 1, 7/6, -((b*x^3)/a)]))/(7*b^2)
```

fricas [B] time = 1.31, size = 2433, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/42*(28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*
A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4
*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*
a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a*b^2 - A*b^3)*
sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^
3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6))*b^11*(-(
B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*
B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/6) + 2*sqrt(3)*(B*a*b^1
1 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) - sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4
*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6))/(B^6*a^7 - 6*A*B^
5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*
```

```
A^5*B*a^2*b^5 + A^6*a*b^6)) + 28*sqrt(3)*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 1
5*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b
^5 + A^6*a*b^6)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(b^4*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*
b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6
)/b^13)^(1/6))*b^11*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(5/
6) + 2*sqrt(3)*(B*a*b^11 - A*b^12)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*
A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5
 + A^6*a*b^6)/b^13)^(5/6) + sqrt(3)*(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a
^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*
b^6))/(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 +
15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)) - 7*b^2*(-(B^6*a^7 - 6*A
*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 -
 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^
6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*
B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x +
4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2
 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b
^13)^(1/6)) + 7*b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^
3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/
6)*log(4*b^4*(-(B^6*a^7 - 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a
^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a*b^2 - A*b^3)*sqrt(x)*(-(B^6*a^7
- 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*
b^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)) + 14*b^2*(-(B^6*a^7 - 6*A*B
^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6
*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(b^2*(-(B^6*a^7 - 6*A*B^5*a^6*b
+ 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A^5*B*a^
2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 14*b^2*(-(B^6*a^7 -
 6*A*B^5*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b
^4 - 6*A^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6)*log(-b^2*(-(B^6*a^7 - 6*A*B^5
*a^6*b + 15*A^2*B^4*a^5*b^2 - 20*A^3*B^3*a^4*b^3 + 15*A^4*B^2*a^3*b^4 - 6*A
^5*B*a^2*b^5 + A^6*a*b^6)/b^13)^(1/6) - (B*a - A*b)*sqrt(x)) - 12*(B*b*x^3
- 7*B*a + 7*A*b)*sqrt(x))/b^2
```

giac [A] time = 0.21, size = 289, normalized size = 1.00

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,b^3}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba-\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/b^3 - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/b^3 + 1/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(
x))/(a/b)^(1/6))/b^3 + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(-
(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/b^3 + 2/3*((a*b^5)^(1/6)*B*a
 - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/b^3 + 2/7*(B*b^6*x^(7/2)
- 7*B*a*b^5*sqrt(x) + 7*A*b^6*sqrt(x))/b^7
```

maple [A] time = 0.18, size = 377, normalized size = 1.31

$$\frac{2B\,x^{\frac{7}{2}}}{7b}-\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3b}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/7*B*x^(7/2)/b+2/b*A*x^(1/2)-2/b^2*B*a*x^(1/2)-2/3/b*(a/b)^(1/6)*arctan(x^
(1/2)/(a/b)^(1/6))*A+2/3*a/b^2*(a/b)^(1/6)*arctan(x^(1/2)/(a/b)^(1/6))*B+1/
6/b*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))*A-1/6
*a/b^2*3^(1/2)*(a/b)^(1/6)*ln(3^(1/2)*(a/b)^(1/6)*x^(1/2)-x-(a/b)^(1/3))*B-
1/3/b*(a/b)^(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))*A+1/3*a/b^2*(a/b)^
(1/6)*arctan(-3^(1/2)+2*x^(1/2)/(a/b)^(1/6))*B-1/6/b*3^(1/2)*(a/b)^(1/6)*ln
(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/6*a/b^2*3^(1/2)*(a/b)^(1/6)
*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/3/b*(a/b)^(1/6)*arctan(2
*x^(1/2)/(a/b)^(1/6)+3^(1/2))*A+1/3*a/b^2*(a/b)^(1/6)*arctan(2*x^(1/2)/(a/b
)^(1/6)+3^(1/2))*B
```

maxima [A] time = 1.17, size = 295, normalized size = 1.02

$$\frac{\left(\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa\right.}{}\right.}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))*a/b^2 + 2/7*(B*b*x^(7/2) - 7*(B*a - A*b)*sqrt(x
))/b^2
```

mupad [B] time = 2.89, size = 1933, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] x^(1/2)*((2*A)/b - (2*B*a)/b^2) + (2*B*x^(7/2))/(7*b) + ((-a)^(1/6)*atan(((
(-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*
b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*(A*b - B*a)*(B
^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b
^(13/6)) + ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*
A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*(A
*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/
```

```
6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*
a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(
-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*
b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*(A*b - B*a)*((96*x^(1/2)*(B^4*a
^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^
3 + (96*(-a)^(1/6)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A
^2*B*a^5*b^2))/b^(19/6)))/(3*b^(13/6))))*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-
a)^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*
(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^
3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*
a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((
-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^
4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a
)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2
*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/
2)*1i)/2 - 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2
*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)
*1i)/2 - 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*
a^5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b
 - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a
^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 - 1/2)*(A*b -
 B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))
/(3*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*b^(13/6)) + ((-a)
^(1/6)*atan((((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B
^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3)
)/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^
4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)) + ((-a
)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*
b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^
(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a
^6*b + 3*A^2*B*a^5*b^2))/b^(19/6))*1i)/(3*b^(13/6)))/(((-a)^(1/6)*((3^(1/2)
*1i)/2 + 1/2)*(A*b - B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a
^6*b^2 - 4*A*B^3*a^7*b - 4*A^3*B*a^5*b^3))/b^3 - (96*(-a)^(1/6)*((3^(1/2)*1
i)/2 + 1/2)*(A*b - B*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^
5*b^2))/b^(19/6)))/(3*b^(13/6)) - ((-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*((96*x^(1/2)*(B^4*a^8 + A^4*a^4*b^4 + 6*A^2*B^2*a^6*b^2 - 4*A*B^3*a^7
*b - 4*A^3*B*a^5*b^3))/b^3 + (96*(-a)^(1/6)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B
*a)*(B^3*a^7 - A^3*a^4*b^3 - 3*A*B^2*a^6*b + 3*A^2*B*a^5*b^2))/b^(19/6)))/(
3*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*b^(13/6))
```

sympy [A] time = 177.80, size = 881, normalized size = 3.06

$$\begin{cases} \tilde{\infty}\left(2A\sqrt{x} + \frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{13}{2}}}{13}}{a} \\ \frac{\sqrt[6]{-1}\,A\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b} - \frac{\sqrt[6]{-1}\,A\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3b} + \frac{\sqrt[6]{-1}\,A\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}\right.}{6b} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(7/2)/7), Eq(a, 0) & Eq(b, 0)), ((2*A*
sqrt(x) + 2*B*x**(7/2)/7)/b, Eq(a, 0)), ((2*A*x**(7/2)/7 + 2*B*x**(13/2)/13
)/a, Eq(b, 0)), ((-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1
/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/6)*log
((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) + (-1)**(1/6)*A*a**(1/6
)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*A*a**(1/6)*(1/b)**(1/
6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)
*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*at
an(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*
b) + (-1)**(1/6)*sqrt(3)*A*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(
5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b) + 2*A*sqrt(x)/b - (-1
)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + s
qrt(x))/(3*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(
1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2) - (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)
*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*
(1/b)**(1/3) + 4*x)/(6*b**2) + (-1)**(1/6)*B*a**(7/6)*(1/b)**(1/6)*log(4*(-
1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/
3) + 4*x)/(6*b**2) + (-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(
3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) -
(-1)**(1/6)*sqrt(3)*B*a**(7/6)*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*
sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2) - 2*B*a*sqrt(x)/b**2 +
2*B*x**(7/2)/(7*b), True))
```

3.157 $\int \frac{x^{3/2}(A+Bx^3)}{a+bx^3}\,dx$

Optimal. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\tan^{-1}\left(\cdot\right.}{3\sqrt[6]{a}b^{1}}$$

```
[Out] 2/5*B*x^(5/2)/b+2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11
/6)+1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/
6)+1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(1/6)/b^(11/6)
+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(1/6
)/b^(11/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/
2)*x^(1/2))/a^(1/6)/b^(11/6)*3^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}\sqrt[6]{a}b^{11/6}}-\frac{(Ab-aB)\tan^{-1}\left(\cdot\right.}{3\sqrt[6]{a}b^{1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(5/2))/(5*b) - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt
[x])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(1/6)*b^(11/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(1/6)*b^(11/6)) - ((A*b - a*
B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a
^(1/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{5/2}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{\left(4\left(-\frac{5Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{5b}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3b^{5/3}} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}\cdots}{2}}{\sqrt[3]{a}-\sqrt{3}\cdots}\cdots\right)}{3\sqrt[6]{a}\,b^{5/\cdots}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \cdots\right)}{2\sqrt{3}\,\sqrt[6]{a}\,b^{11/6}}$$

$$= \frac{2Bx^{5/2}}{5b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,\sqrt[6]{a}\,b^{11/6}} - \cdots$$

$$= \frac{2Bx^{5/2}}{5b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3\sqrt[6]{a}\,b^{11/6}} + \frac{2(Ab\cdots}{\cdots}$$

Mathematica [C] time = 0.07, size = 45, normalized size = 0.17

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\right)}{5ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*x^(5/2)*(a*B + (A*b - a*B)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)]
))/(5*a*b)
```

fricas [B] time = 1.30, size = 3635, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/30*(12*B*x^(5/2) - 20*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^1
4)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + (B^1
0*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^
4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3
*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^7*b^7 -
6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3
*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*
B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6
*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^1
1))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 -
 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5
*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A
^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b +
15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^
5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 20*sqrt(3)*b*(-(B^
6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^
2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*arctan(1/3*(sqrt(3)*sq
rt(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^
3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b
+ 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*
b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8
*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5
+ 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B
*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b
^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b
^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2/3))*b^2*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6) + 2*sqrt(3)*(B^5*a^5*b^2
 - 5*A*B^4*a^4*b^3 + 10*A^2*B^3*a^3*b^4 - 10*A^3*B^2*a^2*b^5 + 5*A^4*B*a*b^
6 - A^5*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1
/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^
3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a
^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5
```

```
*B*a*b^5 + A^6*b^6)) + 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^
11))^(1/6)*log(a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5
/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3 +
5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 10*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a*b^11))^(1/6)*log(-a*b^9*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*
a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6
)/(a*b^11))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*
B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) - 5*b*(-(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6)/(a*b^11))^(1/6)*log(4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10
+ 10*A^2*B^3*a^4*b^11 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^
10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A
^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^
3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7
 - 6*A*B^5*a^6*b^8 + 15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*
a^3*b^11 - 6*A^5*B*a^2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a*b^11))^(2/3)) + 5*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a*b^11))^(1/6)*log(-4*(B^5*a^6*b^9 - 5*A*B^4*a^5*b^10 + 10*A^2*B^3*a^4*b^11
 - 10*A^3*B^2*a^3*b^12 + 5*A^4*B*a^2*b^13 - A^5*a*b^14)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^
9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*
A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^
2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^7*b^7 - 6*A*B^5*a^6*b^8 +
15*A^2*B^4*a^5*b^9 - 20*A^3*B^3*a^4*b^10 + 15*A^4*B^2*a^3*b^11 - 6*A^5*B*a^
2*b^12 + A^6*a*b^13)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a*b^11))^(2
/3)))/b
```

giac [A] time = 1.35, size = 280, normalized size = 1.04

$$\frac{2\,Bx^{\frac{5}{2}}}{5\,b}-\frac{2\left(Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}-Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\,ab}+\frac{\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,ab^{6}}-\frac{\sqrt{3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/5*B*x^(5/2)/b - 2/3*(B*a*(a/b)^(5/6) - A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a
/b)^(1/6))/(a*b) + 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^6) - 1/6*sqrt(3)*((a*b^
5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a
/b)^(1/3))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^6) - 1/3*((a*b^5)^(5/6)*B*
a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6
))/(a*b^6)
```

maple [A] time = 0.16, size = 356, normalized size = 1.32

$$\frac{2B\,x^{\frac{5}{2}}}{5b}-\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}} A\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}} A\ln\left(-x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a}+\frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a),x)
```

```
[Out] 2/5*B*x^(5/2)/b+2/3/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A-2/3/b^2/(
a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B*a+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-1/6/b*3^(1/2)*(a/b)^(5/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/3/b/(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)-3^(1/2))*A-1/3/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)-3^(1/2))*B*a-1/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(
a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a
/b)^(1/3))*B+1/3/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A-1/3/
b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B*a
```

maxima [A] time = 1.14, size = 212, normalized size = 0.79

$$\frac{2\,Bx^{\frac{5}{2}}}{5\,b}+\frac{(Ba-Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\,\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\,\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}\cdots}{\cdots}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}\cdots}}\right)}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/5*B*x^(5/2)/b + 1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt
(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)
*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqr
t(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x
))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3
)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

mupad [B] time = 2.85, size = 1640, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(5/2))/(5*b) + (atan((((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 +
96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 +
 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(
1/3)*b^(11/3)) + ((A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5
*b + 96*A^2*B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5
*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3
)))/(((A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*
B*a^4*b^2 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*
A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - ((A*b - B*
a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^
```

```
(1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(
27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*(A*b - B*a)*2i)/(3*(-a)^(
1/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3
*b^3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*
b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/
2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96
*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b
^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-
a)^(1/3)*b^(11/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^
3 - 32*B^3*a^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)
/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3
))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/
2)^2*(A*b - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B
*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 8
64*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*
b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
+ (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a
^6 + 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A
*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^
(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b
 - B*a)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2
+ (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^
5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6)))*1i)/((-a)^(1/3)*b^(11/
3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^3*b^3 - 32*B^3*a^6
+ 96*A*B^2*a^5*b - 96*A^2*B*a^4*b^2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
- B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2 - 1728*A*B*a^4*b^3))/(27*(-a)^(1/
6)*b^(11/6))))/((-a)^(1/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a
)^2*(32*B^3*a^6 - 32*A^3*a^3*b^3 - 96*A*B^2*a^5*b + 96*A^2*B*a^4*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^3*b^4 + 864*B^2*a^5*b^2
- 1728*A*B*a^4*b^3))/(27*(-a)^(1/6)*b^(11/6))))/((-a)^(1/3)*b^(11/3))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(1/6)*b^(11/6))
```

sympy [A] time = 64.30, size = 857, normalized size = 3.17

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{-\frac{2A}{\sqrt{x}}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{11}{2}}}{11}}{a} \\ -\frac{(-1)^{\frac{5}{6}}A\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}\,b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}A\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt[6]{a}\,b\sqrt[6]{\frac{1}{b}}}-\frac{(-1)^{\frac{5}{6}}A\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{6\sqrt[6]{a}\,b\sqrt[6]{\frac{1}{b}}}+\frac{(-1)^{\frac{5}{6}}A}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/sqrt(x) + 2*B*x**(5/2)/5), Eq(a, 0) & Eq(b, 0)), ((-2*
A/sqrt(x) + 2*B*x**(5/2)/5)/b, Eq(a, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(11/2)/
11)/a, Eq(b, 0)), (-(-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) +
sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log((-1)**(1/6)*a**(1/
6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*A*log(
-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)
```

```
**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*A*log(4*(-1)**(1/6
)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x
)/(6*a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 - 2*(-
1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(
1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x
)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*B*a*
*(5/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*(1/b)**(1/
6)) - (-1)**(5/6)*B*a**(5/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x
))/(3*b**2*(1/b)**(1/6)) + (-1)**(5/6)*B*a**(5/6)*log(-4*(-1)**(1/6)*a**(1/
6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**
2*(1/b)**(1/6)) - (-1)**(5/6)*B*a**(5/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)
*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b**2*(1/b)**(
1/6)) + (-1)**(5/6)*sqrt(3)*B*a**(5/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(
3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) - (-1)**(5/6)*s
qrt(3)*B*a**(5/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6
)*(1/b)**(1/6)))/(3*b**2*(1/b)**(1/6)) + 2*B*x**(5/2)/(5*b), True))
```

3.158 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3} dx$

Optimal. Leaf size=53

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

```
[Out] 2/3*B*x^(3/2)/b+2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/
2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 329, 275, 205}

$$\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}+\frac{2Bx^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*B*x^(3/2))/(3*b) + (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*
Sqrt[a]*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{a+bx^3}\,dx = \frac{2Bx^{3/2}}{3b} - \frac{\left(2\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3b}$$
$$= \frac{2Bx^{3/2}}{3b} - \frac{\left(4\left(-\frac{3Ab}{2}+\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b}$$
$$= \frac{2Bx^{3/2}}{3b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b}$$
$$= \frac{2Bx^{3/2}}{3b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{3/2}}$$

Mathematica [A] time = 0.08, size = 52, normalized size = 0.98

$$\frac{2}{3}\left(\frac{Bx^{3/2}}{b} - \frac{(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3),x]
```

```
[Out] (2*((B*x^(3/2))/b - ((-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(Sqr
t[a]*b^(3/2))))/3
```

fricas [A] time = 0.90, size = 108, normalized size = 2.04

$$\left[\frac{2\,Babx^{\frac{3}{2}} + (Ba-Ab)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{3\,ab^2}, \frac{2\left(Babx^{\frac{3}{2}} - (Ba-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)\right)}{3\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(2*B*a*b*x^(3/2) + (B*a - A*b)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^
(3/2) - a)/(b*x^3 + a)))/(a*b^2), 2/3*(B*a*b*x^(3/2) - (B*a - A*b)*sqrt(a*b
)*arctan(sqrt(a*b)*x^(3/2)/a))/(a*b^2)]
```

giac [A] time = 0.17, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b} - \frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

maple [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}} - \frac{2Ba\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b} + \frac{2B\,x^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*B*x^(3/2)/b+2/3/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A-2/3/b/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B*a
```

maxima [A] time = 1.21, size = 39, normalized size = 0.74

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*B*x^(3/2)/b - 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 2.60, size = 93, normalized size = 1.75

$$\frac{2\,B\,x^{3/2}}{3\,b}-\frac{2\,\mathrm{atan}\left(\frac{3\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(24\,A^2\,b^3-48\,A\,B\,a\,b^2+24\,B^2\,a^2\,b\right)}{\left(72\,B\,a^2\,b^2-72\,A\,a\,b^3\right)\,\left(A\,b-B\,a\right)}\right)\,\left(A\,b-B\,a\right)}{3\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3),x)
```

```
[Out] (2*B*x^(3/2))/(3*b) - (2*atan((3*a^(1/2)*b^(3/2)*x^(3/2)*(24*A^2*b^3 + 24*B
^2*a^2*b - 48*A*B*a*b^2))/((72*B*a^2*b^2 - 72*A*a*b^3)*(A*b - B*a)))*(A*b -
 B*a))/(3*a^(1/2)*b^(3/2))
```

sympy [A] time = 22.39, size = 537, normalized size = 10.13

$$\begin{cases}\tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{9}{2}}}{9}}{a} \\ -\frac{iA\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iA\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}+\frac{iA\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3\sqrt{a}\,b\sqrt{\frac{1}{b}}}-\frac{iA\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\cdots}\right.}{3\sqrt{a}\cdots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3), Eq(a, 0) & Eq(b, 0)),
((-2*A/(3*x**(3/2)) + 2*B*x**(3/2)/3)/b, Eq(a, 0)), ((2*A*x**(3/2)/3 + 2*B*
x**(9/2)/9)/a, Eq(b, 0)), (-I*A*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sq
rt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6)
+ sqrt(x))/(3*sqrt(a)*b*sqrt(1/b)) + I*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x
)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sq
rt(1/b)) - I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1
/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) + I*B*sqrt(a)*log(
```

```
-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I*B*sqrt
(a)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b**2*sqrt(1/b)) - I
*B*sqrt(a)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)
*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*b**2*sqrt(1/b)) + I*B*sqrt(a)*log(4*(-1)**
(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
 4*x)/(3*b**2*sqrt(1/b)) + 2*B*x**(3/2)/(3*b), True))
```

3.159 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan}{3a}$$

```
[Out] 2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)
*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)+1/3*(A*b-B*a)*a
rctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(5/6)/b^(7/6)-1/6*(A*b-B*a)*ln(a
^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(7/6)*3^(1/2)+1
/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/
b^(7/6)*3^(1/2)+2*B*x^(1/2)/b
```

Rubi [A] time = 0.47, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {459, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{5/6}b^{7/6}}-\frac{(Ab-aB)\tan}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*B*Sqrt[x])/b - ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)
])/(3*a^(5/6)*b^(7/6)) + ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(5/6)*b^(7/6)) + (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(
1/6)])/(3*a^(5/6)*b^(7/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(
1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b^(7/6)) + ((A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(5/6)*b
^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)}\,dx &= \frac{2B\sqrt{x}}{b} - \frac{\left(2\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{b} \\
&= \frac{2B\sqrt{x}}{b} - \frac{\left(4\left(-\frac{Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b} \\
&= \frac{2B\sqrt{x}}{b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{3a^{5/6}b} + \frac{(2(Ab-aB))\,\text{Subst}\left(\int \right.}{} \\
&= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{x}\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}} \\
&= \frac{2B\sqrt{x}}{b} + \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} - \frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,a^{5/6}b^{7/6}} + \frac{(Ab}{} \\
&= \frac{2B\sqrt{x}}{b} - \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{5/6}b^{7/6}} + \frac{2(Ab-aB}{3}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.16

$$\frac{2\sqrt{x}\left((Ab-aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+aB\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)),x]
```

```
[Out] (2*Sqrt[x]*(a*B + (A*b - a*B)*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])
)/(a*b)
```

fricas [B] time = 0.77, size = 2424, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4
*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^
6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^2*b - A*a*b^
2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3
*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6))*a^4*
b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + 2*sqrt(3)*
(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^5*b^7))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*b*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^2*(-(B^6*a^
6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^
2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + (B^2*a^2 - 2*A*B*a*b +
A^2*b^2)*x - (B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^
2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A
^6*b^6)/(a^5*b^7))^(1/6))*a^4*b^6*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a
^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)
/(a^5*b^7))^(5/6) + 2*sqrt(3)*(B*a^5*b^6 - A*a^4*b^7)*sqrt(x)*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*
a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^
5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - b*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6
 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2
*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b +
 A^2*b^2)*x + 4*(B*a^2*b - A*a*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15
*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
+ A^6*b^6)/(a^5*b^7))^(1/6)) + b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^
4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/
(a^5*b^7))^(1/6)*log(4*a^2*b^2*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^7))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^2*b - A*a*b^2
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)) + 2*b
*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*
A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(a*b*(-(B^6*
a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*
a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqrt(x))
```

```
- 2*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3
+ 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6)*log(-a*b*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^7))^(1/6) - (B*a - A*b)*sqr
t(x)) + 12*B*sqrt(x))/b
```

giac [A] time = 0.21, size = 280, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6ab^2} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\right.}{6ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqr
t(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^2) + 1/6*sqrt(3)*((a*b^5)^
(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(
3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^2) - 1/3*((a*b^5)^(1/6)*B*a -
 (a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/
(a*b^2) - 2/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^
(1/6))/(a*b^2)
```

maple [A] time = 0.15, size = 353, normalized size = 1.32

$$\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3a} + \frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x + \sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)/x^(1/2),x)
```

```
[Out] 2*B*x^(1/2)/b+2/3/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A-2/3/b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-1/6/a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A+1/6/b*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/
6)*x^(1/2)-3^(1/2))*A-1/3/b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2
))*B+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3)
)*A-1/6/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))
*B+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A-1/3/b*(a/b)^(1
/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B
```

maxima [A] time = 1.42, size = 278, normalized size = 1.04

$$\frac{2B\sqrt{x}}{b} - \frac{\frac{\sqrt{3}(Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + 2\left(Ba^{\frac{4}{3}}b\right.}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*B*sqrt(x)/b - 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x
) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(
3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*
```

```
a*b^(1/3) - A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/
3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3
))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3
)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^
(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/b
```

mupad [B] time = 2.88, size = 1915, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)),x)
```

```
[Out] (2*B*x^(1/2))/b + (atan((((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2
*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b
^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6
)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/6)) + ((x^(1/2)*(96*A^4*b^5
+ 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2)
 + ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*
A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b - B*a)*1i)/(3*(-a)^(5/6)*b^(7/
6)))/(((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*
B*a*b^4 - 384*A*B^3*a^3*b^2) - ((A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*(A*b -
B*a))/(3*(-a)^(5/6)*b^(7/6)) - ((x^(1/2)*(96*A^4*b^5 + 96*B^4*a^4*b + 576*A
^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + ((A*b - B*a)*(288*A
^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a
)^(5/6)*b^(7/6)))*(A*b - B*a))/(3*(-a)^(5/6)*b^(7/6))))*(A*b - B*a)*2i)/(3*
(-a)^(5/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(
96*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B
^3*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*
a^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1
i)/(3*(-a)^(5/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(9
6*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^
3*a^3*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a
^4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i
)/(3*(-a)^(5/6)*b^(7/6)))/((((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96
*A^4*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3
*a^3*b^2) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^
4*b^2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3
*(-a)^(5/6)*b^(7/6)) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4
*b^5 + 96*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3
*b^2) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^
2 + 864*A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a
)^(5/6)*b^(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(
7/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 9
6*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) -
(((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*
A*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/
6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96
*B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (
((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A
*B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6)))*1i)/(3*(-a)^(5/6
)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*
B^4*a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) - ((
(3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*
B^2*a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^
(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(x^(1/2)*(96*A^4*b^5 + 96*B^4*
a^4*b + 576*A^2*B^2*a^2*b^3 - 384*A^3*B*a*b^4 - 384*A*B^3*a^3*b^2) + (((3^(
1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a*b^5 - 288*B^3*a^4*b^2 + 864*A*B^2*
```

```
a^3*b^3 - 864*A^2*B*a^2*b^4))/(3*(-a)^(5/6)*b^(7/6))))/(3*(-a)^(5/6)*b^(7/6
))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(5/6)*b^(7/6))
```

sympy [A] time = 23.16, size = 833, normalized size = 3.11

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}+2B\sqrt{x}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{7}{2}}}{7}}{a} \\ -\frac{\sqrt[6]{-1}\,A\sqrt[6]{\frac{1}{b}}\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{5}{6}}}+\frac{\sqrt[6]{-1}\,A\sqrt[6]{\frac{1}{b}}\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{5}{6}}}-\frac{\sqrt[6]{-1}\,A\sqrt[6]{\frac{1}{b}}\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{6a^{\frac{5}{6}}}+\cdot \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*(-2*A/(5*x**(5/2)) + 2*B*sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(5*x**(5/2)) + 2*B*sqrt(x))/b, Eq(a, 0)), ((2*A*sqrt(x) + 2*B*x**(7/2)/
7)/a, Eq(b, 0)), (-(-1)**(1/6)*A*(1/b)**(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/
b)**(1/6) + sqrt(x))/(3*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log((-1)**(1
/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(5/6)) - (-1)**(1/6)*A*(1/b)**(1
/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/
3)*(1/b)**(1/3) + 4*x)/(6*a**(5/6)) + (-1)**(1/6)*A*(1/b)**(1/6)*log(4*(-1)
**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3)
 + 4*x)/(6*a**(5/6)) + (-1)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 -
2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) - (-1
)**(1/6)*sqrt(3)*A*(1/b)**(1/6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt
(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(5/6)) + (-1)**(1/6)*B*a**(1/6)*(1/b)*
*(1/6)*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*b) - (-1)**(1/6
)*B*a**(1/6)*(1/b)**(1/6)*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/
(3*b) + (-1)**(1/6)*B*a**(1/6)*(1/b)**(1/6)*log(-4*(-1)**(1/6)*a**(1/6)*sqr
t(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)
**(1/6)*B*a**(1/6)*(1/b)**(1/6)*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(
1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*b) - (-1)**(1/6)*sqrt(
3)*B*a**(1/6)*(1/b)**(1/6)*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(
3*a**(1/6)*(1/b)**(1/6)))/(3*b) + (-1)**(1/6)*sqrt(3)*B*a**(1/6)*(1/b)**(1/
6)*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6))
)/(3*b) + 2*B*sqrt(x)/b, True))
```

3.160 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=268

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan}{3a}$$

```
[Out] -2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a
)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/3*(A*b-B*a)*
arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(7/6)/b^(5/6)-1/6*(A*b-B*a)*ln(
a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(5/6)*3^(1/2)+
1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)
/b^(5/6)*3^(1/2)-2*A/a/x^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{7/6}b^{5/6}}+\frac{(Ab-aB)\tan}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(a*Sqrt[x]) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1
/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x
])/a^(1/6)])/(3*a^(7/6)*b^(5/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/
a^(1/6)])/(3*a^(7/6)*b^(5/6)) - ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*
b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6)*b^(5/6)) + ((A*b - a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(7/6
)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)}\,dx &= -\frac{2A}{a\sqrt{x}}-\frac{\left(2\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\int \frac{x^{3/2}}{a+bx^3}\,dx}{a}\\
&= -\frac{2A}{a\sqrt{x}}-\frac{\left(4\left(\frac{Ab}{2}-\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a}\\
&= -\frac{2A}{a\sqrt{x}}-\frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\sqrt{x}\right)}{3a^{7/6}b^{2/3}}-\\
&= -\frac{2A}{a\sqrt{x}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}+2\sqrt[3]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\sqrt{x}\right)}{2\sqrt{3}\,a^{7/6}b^{5/6}}\\
&= -\frac{2A}{a\sqrt{x}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,a^{7/6}b^{5/6}}+\\
&= -\frac{2A}{a\sqrt{x}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{7/6}b^{5/6}}-
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.17

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)),x]
```

```
[Out] (2*(-5*a*A + (-(A*b) + a*B)*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a
)]))/(5*a^2*Sqrt[x])
```

fricas [B] time = 1.18, size = 3663, normalized size = 13.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(3)*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3
*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/
6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3
*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B
^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B
^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + (B^10*a^10 - 10*A*
B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4
- 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*
B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - (B^6*a^11*b^3 - 6*A*B^5*a^10*
b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*
B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5
))^(2/3))*a*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*
a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) +
2*sqrt(3)*(B^5*a^6*b - 5*A*B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^
3*b^4 + 5*A^4*B*a^2*b^5 - A^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^
4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b
^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 1
5*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*arctan(1/3*(sqrt(3)*sqrt(-4*(
B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 +
 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2
*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^
6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^
2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A
^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9
+ A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 -
20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-
(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4
*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3))*a*b*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^6*b - 5*A*
B^4*a^5*b^2 + 10*A^2*B^3*a^4*b^3 - 10*A^3*B^2*a^3*b^4 + 5*A^4*B*a^2*b^5 - A
^5*a*b^6)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6
) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*B^5*a^5
```

```
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)) - 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2
- 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b
^5))^(1/6)*log(a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20
*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))
^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 - 10*A^3*B^2*a^2*b^3
 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + 2*a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^5))^(1/6)*log(-a^6*b^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(5/6) - (B^5*a^5 - 5*A*B^4*a^4*b + 10*A^2*B^3*a^3*b^2 -
 10*A^3*B^2*a^2*b^3 + 5*A^4*B*a*b^4 - A^5*b^5)*sqrt(x)) + a*x*(-(B^6*a^6 -
6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^
4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(1/6)*log(4*(B^5*a^11*b^4 - 5*A*B^4
*a^10*b^5 + 10*A^2*B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5
*a^6*b^9)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*
B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6
) + 4*(B^10*a^10 - 10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^
3 + 210*A^4*B^6*a^6*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A
^7*B^3*a^3*b^7 + 45*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^
6*a^11*b^3 - 6*A*B^5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 1
5*A^4*B^2*a^7*b^7 - 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5
*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^5))^(2/3)) - a*x*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A
^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 +
A^6*b^6)/(a^7*b^5))^(1/6)*log(-4*(B^5*a^11*b^4 - 5*A*B^4*a^10*b^5 + 10*A^2*
B^3*a^9*b^6 - 10*A^3*B^2*a^8*b^7 + 5*A^4*B*a^7*b^8 - A^5*a^6*b^9)*sqrt(x)*(
-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^
4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^5))^(5/6) + 4*(B^10*a^10 -
10*A*B^9*a^9*b + 45*A^2*B^8*a^8*b^2 - 120*A^3*B^7*a^7*b^3 + 210*A^4*B^6*a^6
*b^4 - 252*A^5*B^5*a^5*b^5 + 210*A^6*B^4*a^4*b^6 - 120*A^7*B^3*a^3*b^7 + 45
*A^8*B^2*a^2*b^8 - 10*A^9*B*a*b^9 + A^10*b^10)*x - 4*(B^6*a^11*b^3 - 6*A*B^
5*a^10*b^4 + 15*A^2*B^4*a^9*b^5 - 20*A^3*B^3*a^8*b^6 + 15*A^4*B^2*a^7*b^7 -
 6*A^5*B*a^6*b^8 + A^6*a^5*b^9)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4
*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(
a^7*b^5))^(2/3)) - 12*A*sqrt(x))/(a*x)
```

giac [A] time = 0.41, size = 280, normalized size = 1.04

$$-\frac{2A}{a\sqrt{x}}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6a^2b^5}+\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba-\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\ldots\right.\right.}{6a^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2*A/(a*sqrt(x)) - 1/6*sqrt(3)*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(
sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^5) + 1/6*sqrt(3)*((a*
b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^5) + 1/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan
((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^5) + 1/3*((a*b^5)^(5
/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b
)^(1/6))/(a^2*b^5) + 2/3*((a*b^5)^(5/6)*B*a - (a*b^5)^(5/6)*A*b)*arctan(sqr
t(x)/(a/b)^(1/6))/(a^2*b^5)
```

maple [A] time = 0.16, size = 355, normalized size = 1.32

$$-\frac{2A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}-\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{6}}a}+\frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{5}{6}}Ab\ln\left(x+\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right.\right.}{6a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a),x)
```

```
[Out] -2/3/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+2/3/b/(a/b)^(1/6)*arctan
(1/(a/b)^(1/6)*x^(1/2))*B-1/6/a^2*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(
1/6)*x^(1/2)-(a/b)^(1/3))*A*b+1/6/a*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)
^(1/6)*x^(1/2)-(a/b)^(1/3))*B-1/3/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)-3^(1/2))*A+1/3/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B+1/6/
a^2*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A*b-1
/6/a*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B-1/
3/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+1/3/b/(a/b)^(1/6)*a
rctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B-2*A/a/x^(1/2)
```

maxima [A] time = 1.34, size = 212, normalized size = 0.79

$$-\frac{(Ba-Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\frac{2\arctan\left(-\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}-\cdots}{\sqrt{a^{\frac{1}{3}}\cdots}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] -1/6*(B*a - A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
 a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x)
+ b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6
) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)
)) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b
^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^
(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2*A/(a*sqrt(x))
```

mupad [B] time = 2.86, size = 1700, normalized size = 6.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)),x)
```

```
[Out] (atan((((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4
 + 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^1
2*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3
)) + ((A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 -
 96*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/(((A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 9
6*A^2*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^
4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - ((
A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2
```

```
*B*a^10*b^5 + (x^(1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1
728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*(A*b -
B*a)*2i)/(3*(-a)^(7/6)*b^(5/6)) - (2*A)/(a*x^(1/2)) + (atan(((((3^(1/2)*1i)
/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11
*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864
*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/
6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32
*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x
^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*
b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3))
)/((((3^(1/2)*1i)/2 - 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3*a^9*b^
6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)
*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27
*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*
b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4 - 96*A^2*B
*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 +
 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/
3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*2i)/(3*(-a)^(7/6)*b^(5/6))
 + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*a^12*b^3 - 32*A^3
*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2
 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^
5))/(27*(-a)^(7/6)*b^(5/6)))*1i)/((-a)^(7/3)*b^(5/3)) + (((3^(1/2)*1i)/2 +
1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A*B^2*a^11*b^4
- 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*
a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6)))*
1i)/((-a)^(7/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*B^3*
a^12*b^3 - 32*A^3*a^9*b^6 - 96*A*B^2*a^11*b^4 + 96*A^2*B*a^10*b^5 + (x^(1/2
)*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 -
 1728*A*B*a^11*b^5))/(27*(-a)^(7/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3)) - (((3^
(1/2)*1i)/2 + 1/2)^2*(A*b - B*a)^2*(32*A^3*a^9*b^6 - 32*B^3*a^12*b^3 + 96*A
*B^2*a^11*b^4 - 96*A^2*B*a^10*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b -
B*a)*(864*A^2*a^10*b^6 + 864*B^2*a^12*b^4 - 1728*A*B*a^11*b^5))/(27*(-a)^(7
/6)*b^(5/6))))/((-a)^(7/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i
)/(3*(-a)^(7/6)*b^(5/6))
```

sympy [A] time = 59.23, size = 836, normalized size = 3.12

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}} - \frac{2B}{\sqrt{x}}}{b} \\ \frac{-\frac{2A}{\sqrt{x}} + \frac{2Bx^{\frac{5}{2}}}{5}}{a} \\ -\frac{2A}{a\sqrt{x}} + \frac{(-1)^{\frac{5}{6}} A \log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}} - \frac{(-1)^{\frac{5}{6}} A \log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}} + \sqrt{x}\right)}{3a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}} + \frac{(-1)^{\frac{5}{6}} A \log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}} + 4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}} + 4x\right)}{6a^{\frac{7}{6}}\sqrt[6]{\frac{1}{b}}} - \cdots \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/sqrt(x)), Eq(a, 0) & Eq(b, 0)), ((-
2*A/(7*x**(7/2)) - 2*B/sqrt(x))/b, Eq(a, 0)), ((-2*A/sqrt(x) + 2*B*x**(5/2)
/5)/a, Eq(b, 0)), (-2*A/(a*sqrt(x)) + (-1)**(5/6)*A*log(-(-1)**(1/6)*a**(1/
6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log((-
```

```
1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(7/6)*(1/b)**(1/6)) + (-1)
**(5/6)*A*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*
a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6)*A*log(
4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + 4*x)/(6*a**(7/6)*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3
)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*
(1/b)**(1/6)) - (-1)**(5/6)*sqrt(3)*A*atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3
)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(3*a**(7/6)*(1/b)**(1/6)) - (-1)**(5/6
)*B*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(1/6)*b*(1/b)**
(1/6)) + (-1)**(5/6)*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*
a**(1/6)*b*(1/b)**(1/6)) - (-1)**(5/6)*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x
)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(
1/b)**(1/6)) + (-1)**(5/6)*B*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6
) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(6*a**(1/6)*b*(1/b)**(1/6))
- (-1)**(5/6)*sqrt(3)*B*atan(sqrt(3)/3 - 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a
**(1/6)*(1/b)**(1/6)))/(3*a**(1/6)*b*(1/b)**(1/6)) + (-1)**(5/6)*sqrt(3)*B*
atan(sqrt(3)/3 + 2*(-1)**(5/6)*sqrt(3)*sqrt(x)/(3*a**(1/6)*(1/b)**(1/6)))/(
3*a**(1/6)*b*(1/b)**(1/6)), True))
```

3.161 $\int \frac{A+Bx^3}{x^{5/2}(a+bx^3)}\,dx$

Optimal. Leaf size=53

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

```
[Out] -2/3*A/a/x^(3/2)-2/3*(A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 329, 275, 205}

$$-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}-\frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) - (2*(A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3
*a^(3/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)}\,dx = -\frac{2A}{3ax^{3/2}} - \frac{\left(2\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{\left(4\left(\frac{3Ab}{2}-\frac{3aB}{2}\right)\right)\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3a}$$
$$= -\frac{2A}{3ax^{3/2}} - \frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{2(aB-Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}\sqrt{b}} - \frac{2A}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(3*a*x^(3/2)) + (2*(-(A*b) + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])
/(3*a^(3/2)*Sqrt[b])
```

fricas [A] time = 0.94, size = 120, normalized size = 2.26

$$\left[\frac{(Ba-Ab)\sqrt{-ab}\,x^2\log\left(\frac{bx^3+2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\,Aab\sqrt{x}}{3\,a^2bx^2}, \frac{2\left((Ba-Ab)\sqrt{ab}\,x^2\arctan\left(\frac{\sqrt{ab}\,x^{\frac{3}{2}}}{a}\right)-Aab\sqrt{x}\right)}{3\,a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*((B*a - A*b)*sqrt(-a*b)*x^2*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*
x^3 + a)) - 2*A*a*b*sqrt(x))/(a^2*b*x^2), 2/3*((B*a - A*b)*sqrt(a*b)*x^2*ar
ctan(sqrt(a*b)*x^(3/2)/a) - A*a*b*sqrt(x))/(a^2*b*x^2)]
```

giac [A] time = 0.17, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a} - \frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

maple [A] time = 0.06, size = 53, normalized size = 1.00

$$-\frac{2Ab\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,a} + \frac{2B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}} - \frac{2A}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a),x)
```

```
[Out] -2/3*A/a/x^(3/2)-2/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+2/3/
(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

maxima [A] time = 1.40, size = 39, normalized size = 0.74

$$\frac{2\,(Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a}-\frac{2\,A}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 2/3*(B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a) - 2/3*A/(a*x^(3/2
))
```

mupad [B] time = 0.10, size = 102, normalized size = 1.92

$$-\frac{2\,A}{3\,a\,x^{3/2}}-\frac{2\,\mathrm{atan}\left(\frac{3\,a^{3/2}\,\sqrt{b}\,x^{3/2}\,\left(24\,A^2\,a^3\,b^5-48\,A\,B\,a^4\,b^4+24\,B^2\,a^5\,b^3\right)}{\left(A\,b-B\,a\right)\,\left(72\,A\,a^5\,b^4-72\,B\,a^6\,b^3\right)}\right)\,\left(A\,b-B\,a\right)}{3\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(3*a*x^(3/2)) - (2*atan((3*a^(3/2)*b^(1/2)*x^(3/2)*(24*A^2*a^3*b^5
+ 24*B^2*a^5*b^3 - 48*A*B*a^4*b^4))/((A*b - B*a)*(72*A*a^5*b^4 - 72*B*a^6*b
^3)))*(A*b - B*a))/(3*a^(3/2)*b^(1/2))
```

sympy [A] time = 156.58, size = 527, normalized size = 9.94

$$\begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}+\frac{2Bx^{\frac{3}{2}}}{3}}{a} \\ -\frac{2A}{3ax^{\frac{3}{2}}}+\frac{iA\log\left(-\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{iA\log\left(\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt[6]{\frac{1}{b}}+\sqrt{x}\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{iA\log\left(-4\sqrt[6]{-1}\,\sqrt[6]{a}\,\sqrt{x}\,\sqrt[6]{\frac{1}{b}}+4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}+4x\right)}{3a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{iA\log\left(4\sqrt[6]{-1}\,\sqrt[6]{a}\right.}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a),x)
```

```
[Out] Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2))), Eq(a, 0) & Eq(b, 0))
, ((-2*A/(9*x**(9/2)) - 2*B/(3*x**(3/2)))/b, Eq(a, 0)), ((-2*A/(3*x**(3/2))
 + 2*B*x**(3/2)/3)/a, Eq(b, 0)), (-2*A/(3*a*x**(3/2)) + I*A*log(-(-1)**(1/6
)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log((-1)**(
1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*a**(3/2)*sqrt(1/b)) - I*A*log(-4*(
-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1
/3) + 4*x)/(3*a**(3/2)*sqrt(1/b)) + I*A*log(4*(-1)**(1/6)*a**(1/6)*sqrt(x)*
```

```
(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*a**(3/2)*sqrt(
1/b)) - I*B*log(-(-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a)*b*
sqrt(1/b)) + I*B*log((-1)**(1/6)*a**(1/6)*(1/b)**(1/6) + sqrt(x))/(3*sqrt(a
)*b*sqrt(1/b)) + I*B*log(-4*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(
-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)) - I*B*log(4
*(-1)**(1/6)*a**(1/6)*sqrt(x)*(1/b)**(1/6) + 4*(-1)**(1/3)*a**(1/3)*(1/b)**
(1/3) + 4*x)/(3*sqrt(a)*b*sqrt(1/b)), True))
```

3.162 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=270

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\right.}{3a^{11/6}}$$

```
[Out] -2/5*A/a/x^(5/2)-2/3*(A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(
1/6)-1/3*(A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1
/6)-1/3*(A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(1/6
)+1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11
/6)/b^(1/6)*3^(1/2)-1/6*(A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1
/2)*x^(1/2))/a^(11/6)/b^(1/6)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {453, 329, 209, 634, 618, 204, 628, 205}

$$\frac{(Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{2\sqrt{3}a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\tan^{-1}\left(\right.}{3a^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*A)/(5*a*x^(5/2)) + ((A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^
(1/6)])/(3*a^(11/6)*b^(1/6)) - ((A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqr
t[x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) - (2*(A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[
x])/a^(1/6)])/(3*a^(11/6)*b^(1/6)) + ((A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(
1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*a^(11/6)*b^(1/6)) - ((A*b - a
*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(2*Sqrt[3]*
a^(11/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)}\,dx &= -\frac{2A}{5ax^{5/2}}-\frac{\left(2\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{5a}\\
&= -\frac{2A}{5ax^{5/2}}-\frac{\left(4\left(\frac{5Ab}{2}-\frac{5aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx,x,\sqrt{x}\right)}{5a}\\
&= -\frac{2A}{5ax^{5/2}}-\frac{(2(Ab-aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\,\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\sqrt{x}\right)}{3a^{11/6}}-\frac{(2(Ab-aB))\,\text{Subst}}{}\\
&= -\frac{2A}{5ax^{5/2}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx,x,\right.}{6a^{5/3}}\\
&= -\frac{2A}{5ax^{5/2}}-\frac{2(Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}+\frac{(Ab-aB)\log\left(\sqrt[3]{a}-\sqrt{3}\,\sqrt[6]{a}\,\sqrt[6]{b}\,\sqrt{x}+\sqrt[3]{b}\,x\right)}{2\sqrt{3}\,a^{11/6}\sqrt[6]{b}}-\\
&= -\frac{2A}{5ax^{5/2}}+\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{(Ab-aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\,\sqrt{x}}{\sqrt[6]{a}}\right)}{3a^{11/6}\sqrt[6]{b}}-\frac{2(Ab}{}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 0.17

$$\frac{10x^3(aB-Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)-2aA}{5a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)),x]
```

```
[Out] (-2*a*A + 10*(-(A*b) + a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)
])/(5*a^2*x^(5/2))
```

fricas [B] time = 1.19, size = 2424, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/30*(20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 2
0*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))
^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + (B*a^3 - A*a^2*b
)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*
b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6))*a^9*b*
(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A
^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5/6) + 2*sqrt(3)*(B*a^
10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(5/6) - sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B
^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6))/(B^6*a^6 - 6*A*
B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
6*A^5*B*a*b^5 + A^6*b^6)) + 20*sqrt(3)*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 1
5*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5
 + A^6*b^6)/(a^11*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + (B^2*a^2 - 2*A*B*a*b + A^2*b^2)
*x - (B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*
b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a
^11*b))^(1/6))*a^9*b*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A
^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(5
/6) + 2*sqrt(3)*(B*a^10*b - A*a^9*b^2)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b +
 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b
^5 + A^6*b^6)/(a^11*b))^(5/6) + sqrt(3)*(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B
^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*
b^6))/(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 +
15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)) - 5*a*x^3*(-(B^6*a^6 - 6*A*B
^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6
*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x + 4*
(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 -
 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b
))^(1/6)) + 5*a*x^3*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^
3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/
6)*log(4*a^4*(-(B^6*a^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a
^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/3) + 4*
(B^2*a^2 - 2*A*B*a*b + A^2*b^2)*x - 4*(B*a^3 - A*a^2*b)*sqrt(x)*(-(B^6*a^6
- 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*
```

```
b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)) + 10*a*x^3*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(a^2*(-(B^6*a^6 - 6*A*B^5*a^5*
b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 - 6*A^5*B*
a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) - 10*a*x^3*(-(B^6*a
^6 - 6*A*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a
^2*b^4 - 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6)*log(-a^2*(-(B^6*a^6 - 6*A
*B^5*a^5*b + 15*A^2*B^4*a^4*b^2 - 20*A^3*B^3*a^3*b^3 + 15*A^4*B^2*a^2*b^4 -
 6*A^5*B*a*b^5 + A^6*b^6)/(a^11*b))^(1/6) - (B*a - A*b)*sqrt(x)) + 12*A*sqr
t(x))/(a*x^3)
```

giac [**A**] time = 0.21, size = 280, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba - \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{6\,a^2b} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}} Ba - \left(ab^5\right)^{\frac{1}{6}} Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + \cdots\right.}{6\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/
b)^(1/6) + x + (a/b)^(1/3))/(a^2*b) - 1/6*sqrt(3)*((a*b^5)^(1/6)*B*a - (a*b
^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b)
+ 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) +
 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 1/3*((a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b) + 2/3*((
a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b) -
 2/5*A/(a*x^(5/2))
```

maple [**A**] time = 0.18, size = 358, normalized size = 1.33

$$-\frac{2\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{3a^2} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{3a^2} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{3a^2} - \frac{\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} Ab\ln\left(x + \cdots\right.}{6\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a),x)
```

```
[Out] -2/5*A/a/x^(5/2)-2/3/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A*b+2/3/
a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+1/6/a^2*3^(1/2)*(a/b)^(1/6)*l
n(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A*b-1/6/a*3^(1/2)*(a/b)^(1/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B-1/3/a^2*(a/b)^(1/6)*arcta
n(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A*b+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)
*x^(1/2)-3^(1/2))*B-1/6/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^
(1/2)+(a/b)^(1/3))*A*b+1/6/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x
^(1/2)+(a/b)^(1/3))*B-1/3/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1
/2))*A*b+1/3/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*B
```

maxima [**A**] time = 1.42, size = 278, normalized size = 1.03

$$\frac{\frac{\sqrt{3}\,(Ba-Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}\,(Ba-Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\right)\arctan\left(\frac{b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}} + \frac{2\left(Ba^{\frac{4}{3}}b^{\frac{1}{3}}-Aa^{\cdots}\right.}{\cdots}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/6*(sqrt(3)*(B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/
6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) - A*b^
(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(
3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*s
qrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(
sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1
/3)*sqrt(a^(1/3)*b^(1/3))))/a - 2/5*A/(a*x^(5/2))
```

mupad [B] time = 2.91, size = 2023, normalized size = 7.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)),x)
```

```
[Out] - (2*A)/(5*a*x^(5/2)) - (atan((((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 +
 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*b - B*a
)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b
^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + ((x
^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a
^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*
b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
 - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/(((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^
9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - ((A*
b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*
B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) -
 ((x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B
^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + ((A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a
^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*
(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/
6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*
b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(
1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B
^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(
3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 +
 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b
^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b
^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b
- B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(96*A
^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384
*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 28
8*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(
1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1
/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*
b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(A*b - B*a)*(288*A^3*a^7
*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(
11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)
*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^7 - 384*A*B^
3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*(288*A
^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*
(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)) + (((3^(1/2)*
1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^2*B^2*a^7*b^
7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(A*b -
 B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 - 864*A^2*B*a
^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a)*1i)/(3*(-a)^(11/6)*b^(1/6)))/
```

```
((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5 + 576*A^
2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) - (((3^(1/2)*1i)/2 +
 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*a^9*b^6 -
 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)^(11/6)*b
^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(96*A^4*a^5*b^9 + 96*B^4*a^9*b^5
 + 576*A^2*B^2*a^7*b^7 - 384*A*B^3*a^8*b^6 - 384*A^3*B*a^6*b^8) + (((3^(1/2
)*1i)/2 + 1/2)*(A*b - B*a)*(288*A^3*a^7*b^8 - 288*B^3*a^10*b^5 + 864*A*B^2*
a^9*b^6 - 864*A^2*B*a^8*b^7))/(3*(-a)^(11/6)*b^(1/6)))*(A*b - B*a))/(3*(-a)
^(11/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(A*b - B*a)*2i)/(3*(-a)^(11/6)*b^
(1/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] -1/3*(A*b-3*B*a)*x^(3/2)/a/b^2+1/3*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)+1/3*(A*b
-3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 321, 329, 275, 205}

$$-\frac{x^{3/2}(Ab-3aB)}{3ab^2}+\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}+\frac{x^{9/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 3*a*B)*x^(3/2))/(3*a*b^2) + ((A*b - a*B)*x^(9/2))/(3*a*b*(a + b*x^
3)) + ((A*b - 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*Sqrt[a]*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)} + \frac{\left(-\frac{3Ab}{2}+\frac{9aB}{2}\right)\int \frac{x^{7/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-3aB)x^{3/2}}{3ab^2} + \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-3aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2b^2}$$

$$= -\frac{(Ab-3aB)x^{3/2}}{3ab^2} + \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-3aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{(Ab-3aB)x^{3/2}}{3ab^2} + \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-3aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3b^2}$$

$$= -\frac{(Ab-3aB)x^{3/2}}{3ab^2} + \frac{(Ab-aB)x^{9/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3\sqrt{a}\,b^{5/2}}$$

Mathematica [A] time = 0.18, size = 77, normalized size = 0.81

$$\frac{\frac{(Ab-3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{\sqrt{b}\,x^{3/2}\left(3aB-Ab+2bBx^3\right)}{a+bx^3}}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((Sqrt[b]*x^(3/2)*(-(A*b) + 3*a*B + 2*b*B*x^3))/(a + b*x^3) + ((A*b - 3*a*B
)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[a])/(3*b^(5/2))
```

fricas [A] time = 0.94, size = 222, normalized size = 2.34

$$\left[\frac{\left(\left(3\,Bab-Ab^2\right)x^3+3\,Ba^2-Aab\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(2\,Bab^2x^4+\left(3\,Ba^2b-Aab^2\right)x\right)\sqrt{x}}{6\left(ab^4x^3+a^2b^3\right)}, -\frac{\left(\left(3\,Ba\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((3*B*a*b - A*b^2)*x^3 + 3*B*a^2 - A*a*b)*sqrt(-a*b)*log((b*x^3 - 2*s
qrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2*(2*B*a*b^2*x^4 + (3*B*a^2*b - A*a*b
^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3), -1/3*(((3*B*a*b - A*b^2)*x^3 + 3*B*a
^2 - A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*B*a*b^2*x^4 + (3*B*a
^2*b - A*a*b^2)*x)*sqrt(x))/(a*b^4*x^3 + a^2*b^3)]
```

giac [A] time = 0.18, size = 68, normalized size = 0.72

$$\frac{2\,Bx^{\frac{3}{2}}}{3\,b^2} - \frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^2} + \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\left(bx^3+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a - A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b
)*b^2) + 1/3*(B*a*x^(3/2) - A*b*x^(3/2))/((b*x^3 + a)*b^2)
```

maple [A] time = 0.06, size = 93, normalized size = 0.98

$$-\frac{A\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)b}+\frac{Ba\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)b^{2}}+\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b}-\frac{Ba\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{2}}+\frac{2B\,x^{\frac{3}{2}}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2/3*B*x^(3/2)/b^2-1/3/b*x^(3/2)/(b*x^3+a)*A+1/3/b^2*x^(3/2)/(b*x^3+a)*B*a+1
/3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A-1/b^2/(a*b)^(1/2)*arctan
(1/(a*b)^(1/2)*b*x^(3/2))*B*a
```

maxima [A] time = 1.24, size = 68, normalized size = 0.72

$$\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\left(b^{3}x^{3}+ab^{2}\right)}+\frac{2\,Bx^{\frac{3}{2}}}{3\,b^{2}}-\frac{(3\,Ba-Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*x^(3/2)/(b^3*x^3 + a*b^2) + 2/3*B*x^(3/2)/b^2 - 1/3*(3*B*a
- A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 2.65, size = 116, normalized size = 1.22

$$\frac{2\,B\,x^{3/2}}{3\,b^{2}}-\frac{x^{3/2}\,\left(\frac{A\,b}{3}-\frac{B\,a}{3}\right)}{b^{3}\,x^{3}+a\,b^{2}}+\frac{\mathrm{atan}\left(\frac{36\,\sqrt{a}\,b^{3/2}\,x^{3/2}\,\left(A^{2}\,b^{2}-6\,A\,B\,a\,b+9\,B^{2}\,a^{2}\right)}{\left(A\,b-3\,B\,a\right)\,\left(36\,A\,a\,b^{2}-108\,B\,a^{2}\,b\right)}\right)\,\left(A\,b-3\,B\,a\right)}{3\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(3/2))/(3*b^2) - (x^(3/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) + (
atan((36*a^(1/2)*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 - 6*A*B*a*b))/((A*b -
 3*B*a)*(36*A*a*b^2 - 108*B*a^2*b)))*(A*b - 3*B*a))/(3*a^(1/2)*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.164 $\int \frac{x^{5/2}(A+Bx^3)}{(a+bx^3)^2} dx$

Optimal. Leaf size=312

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)}{}$$

```
[Out] 1/3*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)+1/9*(A*b-7*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(5/6)/b^(13/6)+1/18*(A*b-7*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(5/6)/b^(13/6)-1/36*(A*b-7*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)+1/36*(A*b-7*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(5/6)/b^(13/6)*3^(1/2)-1/3*
(A*b-7*B*a)*x^(1/2)/a/b^2
```

Rubi [A] time = 0.50, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 321, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(Ab-7aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}+\frac{(Ab-7aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{5/6}b^{13/6}}-\frac{(Ab-7aB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] -((A*b - 7*a*B)*Sqrt[x])/(3*a*b^2) + ((A*b - a*B)*x^(7/2))/(3*a*b*(a + b*x^
3)) - ((A*b - 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(
5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6
)])/(18*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)
])/(9*a^(5/6)*b^(13/6)) - ((A*b - 7*a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1
/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/6)*b^(13/6)) + ((A*b - 7*a*B)*Lo
g[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(5/
6)*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{\left(-\frac{Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{a+bx^3}\,dx}{3ab}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}\,x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\,x+\sqrt[3]{b}\,x^2}\,dx, x, \sqrt{\ldots}\right.}{9a^{5/6}b^2}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\,\mathrm{Subst}\left(\ldots\right.}{1\ldots}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab-7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{5/6}b^{13/6}} - \frac{(Ab-7aB)\log\left(\sqrt[3]{a}\ldots\right.}{12\sqrt{\ldots}}$$

$$= -\frac{(Ab-7aB)\sqrt{x}}{3ab^2} + \frac{(Ab-aB)x^{7/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab-7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{5/6}b^{13/6}} + \frac{(Ab-7aB)\,\mathrm{t}\ldots}{18\ldots}$$

Mathematica [C] time = 0.09, size = 76, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(Ab-7aB)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a\left(7aB-Ab+6bBx^3\right)\right)}{3ab^2\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[x]*(a*(-(A*b) + 7*a*B + 6*b*B*x^3) + (A*b - 7*a*B)*(a + b*x^3)*Hyperg
eometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(3*a*b^2*(a + b*x^3))
```

fricas [B] time = 1.05, size = 2566, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b +
36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5
*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-
(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3
*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3
) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)
*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*
B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(
1/6))*a^4*b^11*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b
^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)
```

```
/(a^5*b^13))^(5/6) + 2*sqrt(3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(11764
9*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b
^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) - sq
rt(3)*(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B
^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3
+ 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)) + 4*sqrt(3)*(b^3*x^3 + a
*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860
*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^1
3))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B
^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b
^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b
 + A^2*b^2)*x - (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6))*a^4*b^11*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + 2*sqrt(
3)*(7*B*a^5*b^11 - A*a^4*b^12)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5
*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 4
2*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 - 1008
42*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2
*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6))/(117649*B^6*a^6 - 100842*A*B^5*a^5*b
+ 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A
^5*B*a*b^5 + A^6*b^6)) - (b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5
*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4
 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^
6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735
*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^
2 - 14*A*B*a*b + A^2*b^2)*x + (7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6
*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + (b^3*x
^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2
- 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a
^5*b^13))^(1/6)*log(a^2*b^4*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*
A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b
^5 + A^6*b^6)/(a^5*b^13))^(1/3) + (49*B^2*a^2 - 14*A*B*a*b + A^2*b^2)*x - (
7*B*a^2*b^2 - A*a*b^3)*sqrt(x)*(-(117649*B^6*a^6 - 100842*A*B^5*a^5*b + 360
15*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*
a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)) + 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6
 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*
A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(a*b^2*(-(
117649*B^6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*
a^3*b^3 + 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)
 - (7*B*a - A*b)*sqrt(x)) - 2*(b^3*x^3 + a*b^2)*(-(117649*B^6*a^6 - 100842*
A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 735*A^4*B^2*a^
2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6)*log(-a*b^2*(-(117649*B^
6*a^6 - 100842*A*B^5*a^5*b + 36015*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 +
 735*A^4*B^2*a^2*b^4 - 42*A^5*B*a*b^5 + A^6*b^6)/(a^5*b^13))^(1/6) - (7*B*a
 - A*b)*sqrt(x)) + 12*(6*B*b*x^3 + 7*B*a - A*b)*sqrt(x))/(b^3*x^3 + a*b^2)
```

giac [A] time = 0.33, size = 313, normalized size = 1.00

$$\frac{2\,B\sqrt{x}}{b^2} - \frac{\sqrt{3}\left(7\,\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,ab^3} + \frac{\sqrt{3}\left(7\,\left(ab^5\right)^{\frac{1}{6}}Ba - \left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\right.}{36\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2*B*sqrt(x)/b^2 - 1/36*sqrt(3)*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*lo
g(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a*b^3) + 1/36*sqrt(3)*(7*
(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x
 + (a/b)^(1/3))/(a*b^3) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*b^2)
 - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) - 1/18*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^
(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a*b^3) -
 1/9*(7*(a*b^5)^(1/6)*B*a - (a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/
(a*b^3)
```

maple [A] time = 0.17, size = 405, normalized size = 1.30

$$-\frac{A\sqrt{x}}{3\left(bx^3+a\right)b}+\frac{Ba\sqrt{x}}{3\left(bx^3+a\right)b^2}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18ab}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\cdots\right.}{18ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 2*B/b^2*x^(1/2)-1/3/b*x^(1/2)/(b*x^3+a)*A+1/3/b^2*x^(1/2)/(b*x^3+a)*B*a-7/9
/b^2*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+7/36/b^2*B*3^(1/2)*(a/b)^(
1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*
arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-7/36/b^2*B*3^(1/2)*(a/b)^(1/6)*ln(x+3
^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/b^2*B*(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)+3^(1/2))+1/9/b*A/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2
))-1/36/b*A/a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(
1/3))+1/18/b*A/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+1/36/b*A
/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/b
*A/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))
```

maxima [A] time = 1.35, size = 311, normalized size = 1.00

$$\frac{(Ba-Ab)\sqrt{x}}{3\left(b^3x^3+ab^2\right)}+\frac{2B\sqrt{x}}{b^2}-\frac{\frac{\sqrt{3}(7Ba-Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(7Ba-Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(7Bab^{\frac{1}{3}}-Ab^{\frac{4}{3}}\cdots\right.}{a^{\frac{2}{3}}b^{\frac{1}{3}}\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(B*a - A*b)*sqrt(x)/(b^3*x^3 + a*b^2) + 2*B*sqrt(x)/b^2 - 1/36*(sqrt(3)
*(7*B*a - A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) - sqrt(3)*(7*B*a - A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(
x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) - A*b^(4/3))
*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3
)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a
^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(
a^(1/3)*b^(1/3))) + 2*(7*B*a^(4/3)*b^(1/3) - A*a^(1/3)*b^(4/3))*arctan(-(sq
rt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3
)*sqrt(a^(1/3)*b^(1/3))))/b^2
```

mupad [B] time = 2.89, size = 1884, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (2*B*x^(1/2))/b^2 - (x^(1/2)*((A*b)/3 - (B*a)/3))/(a*b^2 + b^3*x^3) - (atan
(((((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^
3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7
*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 +
 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 1
47*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)
)/(18*(-a)^(5/6)*b^(13/6)) - (((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2
*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*(A*b - 7*B
*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a
)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*(A*b - 7*B*a)*
1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*((2*x^(1/2)*(A
^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A^3*B*a
*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^4 - A^
3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6)))*(A
*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((2*x^(1
/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b - 28*A
^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*B^3*a^
4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b^(19/6
)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2)*1i)/2 - 1/2)*((
2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3*b
- 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(343*
B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6)*b
^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)
*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a^3
*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 - 1/2)*(A*b - 7*B*a)*(3
43*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5/6
)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/
2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6)) - (atan(((((3^(1/2)*1i)/2 + 1/
2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A*B^3*a
^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*
(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(-a)^(5
/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)) + (((3^(1/2)*1i)/
2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 - 1372*A
*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)*(A*b - 7
*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))/(27*(
-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a)*1i)/(18*(-a)^(5/6)*b^(13/6)))/((((3^(1/2
)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2 -
1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) - (2*((3^(1/2)*1i)/2 + 1/2)*(A
*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^2))
/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6)) - (((3^(
1/2)*1i)/2 + 1/2)*((2*x^(1/2)*(A^4*b^4 + 2401*B^4*a^4 + 294*A^2*B^2*a^2*b^2
 - 1372*A*B^3*a^3*b - 28*A^3*B*a*b^3))/(27*b^3) + (2*((3^(1/2)*1i)/2 + 1/2)
*(A*b - 7*B*a)*(343*B^3*a^4 - A^3*a*b^3 - 147*A*B^2*a^3*b + 21*A^2*B*a^2*b^
2))/(27*(-a)^(5/6)*b^(19/6)))*(A*b - 7*B*a))/(18*(-a)^(5/6)*b^(13/6))))*((3
^(1/2)*1i)/2 + 1/2)*(A*b - 7*B*a)*1i)/(9*(-a)^(5/6)*b^(13/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.165 $\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=289

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\,t}{1}$$

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9*(A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/
a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2
)/a^(1/6))/a^(7/6)/b^(11/6)+1/18*(A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/
2)/a^(1/6))/a^(7/6)/b^(11/6)+1/36*(A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*
b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)-1/36*(A*b+5*B*a)*ln(a^(1/
3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(7/6)/b^(11/6)*3^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{7/6}b^{11/6}}-\frac{(5aB+Ab)\,t}{1}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(3*a*b*(a + b*x^3)) - ((A*b + 5*a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5*a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(7/6)*b^(11/6)) + ((A*b + 5
*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(7/6)*b^(11/6)) + ((A*b + 5*a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(7/6)*b^(11/6)) - ((A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(7/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\int\frac{x^{3/2}}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{\left(2\left(\frac{Ab}{2}+\frac{5aB}{2}\right)\right)\text{Subst}\left(\int\frac{x^4}{a+bx^6}\,dx,x,\sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\,\text{Subst}\left(\int\frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx,x,\sqrt{x}\right)}{9a^{7/6}b^{5/3}} + \frac{(Ab+5aB)\,\text{Su}}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\,\text{Subst}\left(\int\frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}}\right.}{12\sqrt{3}\,a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\right.}{12\sqrt{3}\,a^{7/6}b^{11/6}}$$

$$= \frac{(Ab-aB)x^{5/2}}{3ab\left(a+bx^3\right)} - \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{7/6}b^{11/6}} + \frac{(Ab+5aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right.}{18a^{7/6}b^{11/6}}$$

Mathematica [C] time = 0.10, size = 62, normalized size = 0.21

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^2*b)
```

fricas [B] time = 1.22, size = 3787, normalized size = 13.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*(B*a - A*b)*x^(5/2) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*
a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375
*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9
*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) +
 (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 937
5000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 1
31250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A
^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 93
75*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*
B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*
```

```
B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 +
 A^6*b^6)/(a^7*b^11))^(2/3))*a*b^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9
375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B
*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*
B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6
 + A^5*a*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a
^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*
b^6)/(a^7*b^11))^(1/6) + sqrt(3)*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*
A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b
^5 + A^6*b^6))/(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) + 4
*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^
2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
 + A^6*b^6)/(a^7*b^11))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^11*b^
9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 2
5*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5
*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30
*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A
*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A
^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^
7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (156
25*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^
3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15
625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b
^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3))*a*b
^2*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B
^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1
/6) - 2*sqrt(3)*(3125*B^5*a^6*b^2 + 3125*A*B^4*a^5*b^3 + 1250*A^2*B^3*a^4*b
^4 + 250*A^3*B^2*a^3*b^5 + 25*A^4*B*a^2*b^6 + A^5*a*b^7)*sqrt(x)*(-(15625*B
^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 +
375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6) - sqrt(3)
*(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a
^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6))/(15625*B^6*a^6 +
18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B
^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)) - 2*(a*b^2*x^3 + a^2*b)*(-(15625*B^
6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 3
75*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(a^6*b^
9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^
3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/
6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^3*B^2*
a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) + 2*(a*b^2*x^3 + a^2*b)*(-(156
25*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-
a^6*b^9*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*
A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11
))^(5/6) + (3125*B^5*a^5 + 3125*A*B^4*a^4*b + 1250*A^2*B^3*a^3*b^2 + 250*A^
3*B^2*a^2*b^3 + 25*A^4*B*a*b^4 + A^5*b^5)*sqrt(x)) - (a*b^2*x^3 + a^2*b)*(-
(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^
3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*l
og((3125*B^5*a^11*b^9 + 3125*A*B^4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*
A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a
^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*
A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^
10*a^10 + 19531250*A*B^9*a^9*b + 17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7
*a^7*b^3 + 3281250*A^4*B^6*a^6*b^4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^
4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 +
 A^10*b^10)*x - (15625*B^6*a^11*b^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a
^9*b^9 + 2500*A^3*B^3*a^8*b^10 + 375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 +
 A^6*a^5*b^13)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2
```

```
+ 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a
^7*b^11))^(2/3)) + (a*b^2*x^3 + a^2*b)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b
 + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A
^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(1/6)*log(-(3125*B^5*a^11*b^9 + 3125*A*B^
4*a^10*b^10 + 1250*A^2*B^3*a^9*b^11 + 250*A^3*B^2*a^8*b^12 + 25*A^4*B*a^7*b
^13 + A^5*a^6*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 18750*A*B^5*a^5*b + 9375*A^2
*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*B^2*a^2*b^4 + 30*A^5*B*a*b^5
+ A^6*b^6)/(a^7*b^11))^(5/6) + (9765625*B^10*a^10 + 19531250*A*B^9*a^9*b +
17578125*A^2*B^8*a^8*b^2 + 9375000*A^3*B^7*a^7*b^3 + 3281250*A^4*B^6*a^6*b^
4 + 787500*A^5*B^5*a^5*b^5 + 131250*A^6*B^4*a^4*b^6 + 15000*A^7*B^3*a^3*b^7
 + 1125*A^8*B^2*a^2*b^8 + 50*A^9*B*a*b^9 + A^10*b^10)*x - (15625*B^6*a^11*b
^7 + 18750*A*B^5*a^10*b^8 + 9375*A^2*B^4*a^9*b^9 + 2500*A^3*B^3*a^8*b^10 +
375*A^4*B^2*a^7*b^11 + 30*A^5*B*a^6*b^12 + A^6*a^5*b^13)*(-(15625*B^6*a^6 +
 18750*A*B^5*a^5*b + 9375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 375*A^4*
B^2*a^2*b^4 + 30*A^5*B*a*b^5 + A^6*b^6)/(a^7*b^11))^(2/3)))/(a*b^2*x^3 + a^
2*b)
```

giac [A] time = 1.38, size = 302, normalized size = 1.04

$$\frac{\left(5\,Ba\left(\frac{a}{b}\right)^{\frac{5}{6}}+Ab\left(\frac{a}{b}\right)^{\frac{5}{6}}\right)\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\,a^2b}-\frac{Bax^{\frac{5}{2}}-Abx^{\frac{5}{2}}}{3\left(bx^3+a\right)ab}-\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{5}{6}}Ba+\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\right.}{36\,a^2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/9*(5*B*a*(a/b)^(5/6) + A*b*(a/b)^(5/6))*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*
b) - 1/3*(B*a*x^(5/2) - A*b*x^(5/2))/((b*x^3 + a)*a*b) - 1/36*sqrt(3)*(5*(a
*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x +
(a/b)^(1/3))/(a^2*b^6) + 1/36*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*
A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^6) + 1/18*(
5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^2*b^6) + 1/18*(5*(a*b^5)^(5/6)*B*a + (a*b^5)^(5/6)*A
*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^6)
```

maple [A] time = 0.16, size = 387, normalized size = 1.34

$$\frac{\left(Ab-Ba\right)x^{\frac{5}{2}}}{3\left(b\,x^3+a\right)ab}+\frac{A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}+\frac{A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}ab}-\frac{\sqrt{3}\left(\frac{a}{b}\right)^{\frac{5}{6}}A\ln\left(x+\sqrt{3}\left(\frac{a}{b}\right.\right.}{36a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)+1/9/b/a/(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))*A+5/9/b^2/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+1/36/a^2*3
^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A+5/36/b/
a*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+1/18
/b/a/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/18/b^2/(a/b)^(1/
6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B-1/36/a^2*3^(1/2)*(a/b)^(5/6)*ln(
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/36/b/a*3^(1/2)*(a/b)^(5/6)*l
n(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+1/18/b/a/(a/b)^(1/6)*arctan(
2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+5/18/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x^(1/2)+3^(1/2))*B
```

maxima [A] time = 1.47, size = 235, normalized size = 0.81

$$-\frac{(Ba-Ab)x^{\frac{5}{2}}}{3\left(ab^2x^3+a^2b\right)}-\frac{(5\,Ba+Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}+2b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}-\right.}{36\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(5/2)/(a*b^2*x^3 + a^2*b) - 1/36*(5*B*a + A*b)*(sqrt(3)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)
) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqr
t(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^
(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(
1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*s
qrt(a^(1/3)*b^(1/3))))/(a*b)
```

mupad [B] time = 2.87, size = 1578, normalized size = 5.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (x^(5/2)*(A*b - B*a))/(3*a*b*(a + b*x^3)) - (atan(((((3^(1/2)*1i)/2 - 1/2)^
2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 +
600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)
^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3
 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((3^(1/2)*
1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^
3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/((((3^(1/2)*1i
)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a
^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A
^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(
324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(A*b + 5*B*a)^2*((4*A^
3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(1/2)*((
3^(1/2)*1i)/2 - 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*
B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*((3^(1/2
)*1i)/2 - 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (atan(((((3^(1/2
)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B
^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(
24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))
)*1i)/(324*(-a)^(7/3)*b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2
*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 + (x^(
1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))
/((((3^(1/2)*1i)/2 + 1/2)^2*(A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/
3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b
 + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6
)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(A*b +
5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^
2 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a
^3*b^2 + 240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11
```

```
/3))))*((3^(1/2)*1i)/2 + 1/2)*(A*b + 5*B*a)*1i)/(9*(-a)^(7/6)*b^(11/6)) - (
atan((((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b +
 20*A^2*B*a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 +
240*A*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)) -
 ((A*b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A
^2*B*a*b^2 + (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A
*B*a^2*b^3))/(18*(-a)^(7/6)*b^(11/6)))*1i)/(324*(-a)^(7/3)*b^(11/3)))/(((A*
b + 5*B*a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*
a*b^2 - (x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^
2*b^3))/(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3)) + ((A*b + 5*B*
a)^2*((4*A^3*b^3)/3 + (500*B^3*a^3)/3 + 100*A*B^2*a^2*b + 20*A^2*B*a*b^2 +
(x^(1/2)*(A*b + 5*B*a)*(24*A^2*a*b^4 + 600*B^2*a^3*b^2 + 240*A*B*a^2*b^3))/
(18*(-a)^(7/6)*b^(11/6))))/(324*(-a)^(7/3)*b^(11/3))))*(A*b + 5*B*a)*1i)/(9
*(-a)^(7/6)*b^(11/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.166 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=71

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3*(A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^
(1/2))/a^(3/2)/b^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {457, 329, 275, 205}

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}+\frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2ab}$$
$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx, x, \sqrt{x}\right)}{ab}$$
$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^{3/2}\right)}{3ab}$$
$$= \frac{(Ab-aB)x^{3/2}}{3ab\left(a+bx^3\right)} + \frac{(Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}}$$

Mathematica [**A**] time = 0.06, size = 71, normalized size = 1.00

$$\frac{(aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{3/2}b^{3/2}} + \frac{x^{3/2}(Ab-aB)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^2,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(3*a*b*(a + b*x^3)) + ((A*b + a*B)*ArcTan[(Sqrt[b]*x^
(3/2))/Sqrt[a]])/(3*a^(3/2)*b^(3/2))
```

fricas [**A**] time = 0.88, size = 190, normalized size = 2.68

$$\left[-\frac{2\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}}+\left(\left(Bab+Ab^2\right)x^3+Ba^2+Aab\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)}{6\left(a^2b^3x^3+a^3b^2\right)}, -\frac{\left(Ba^2b-Aab^2\right)x^{\frac{3}{2}}-\left(\left(Bab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(B*a^2*b - A*a*b^2)*x^(3/2) + ((B*a*b + A*b^2)*x^3 + B*a^2 + A*a*b
)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)))/(a^2*b^3*
x^3 + a^3*b^2), -1/3*((B*a^2*b - A*a*b^2)*x^(3/2) - ((B*a*b + A*b^2)*x^3 +
B*a^2 + A*a*b)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a))/(a^2*b^3*x^3 + a^3*b^
2)]
```

giac [**A**] time = 0.28, size = 63, normalized size = 0.89

$$\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab} - \frac{Bax^{\frac{3}{2}}-Abx^{\frac{3}{2}}}{3\left(bx^3+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/3*(B*a*x^(3
/2) - A*b*x^(3/2))/((b*x^3 + a)*a*b)
```

maple [A] time = 0.06, size = 74, normalized size = 1.04

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;a}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\;b}+\frac{(Ab-Ba)\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)+1/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x^(3/2))*A+1/3/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

maxima [A] time = 1.40, size = 61, normalized size = 0.86

$$-\frac{(Ba-Ab)x^{\frac{3}{2}}}{3\left(ab^{2}x^{3}+a^{2}b\right)}+\frac{(Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*x^(3/2)/(a*b^2*x^3 + a^2*b) + 1/3*(B*a + A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.14, size = 115, normalized size = 1.62

$$\frac{B\,a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,b^2\,x^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,a\,b\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+A\,\sqrt{a}\,b^{3/2}\,x^{3/2}-B\,a^{3/2}\,\sqrt{b}\,x^{3/2}+B\,a\,b\,x^3\,\mathrm{atan}}{3\,a^{5/2}\,b^{3/2}+3\,a^{3/2}\,b^{5/2}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^2,x)
```

```
[Out] (B*a^2*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*b^2*x^3*atan((b^(1/2)*x^(3/2))/a
^(1/2)) + A*a*b*atan((b^(1/2)*x^(3/2))/a^(1/2)) + A*a^(1/2)*b^(3/2)*x^(3/2)
 - B*a^(3/2)*b^(1/2)*x^(3/2) + B*a*b*x^3*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(
3*a^(5/2)*b^(3/2) + 3*a^(3/2)*b^(5/2)*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.167 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=289

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab}{}$$

```
[Out] 1/9*(5*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*A*
b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)+1/18*(5*
A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(7/6)-1/36*(5
*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^
(7/6)*3^(1/2)+1/36*(5*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)
*x^(1/2))/a^(11/6)/b^(7/6)*3^(1/2)+1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)
```

Rubi [A] time = 0.47, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {457, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}+\frac{(aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{11/6}b^{7/6}}-\frac{(aB+5Ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(3*a*b*(a + b*x^3)) - ((5*A*b + a*B)*ArcTan[Sqrt[3] -
 (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*ArcTa
n[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(11/6)*b^(7/6)) + ((5*A*b +
 a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(9*a^(11/6)*b^(7/6)) - ((5*A*b + a
*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]
*a^(11/6)*b^(7/6)) + ((5*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*S
qrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(11/6)*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^2}\,dx = \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{\left(2\left(\frac{5Ab}{2}+\frac{aB}{2}\right)\right)\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{11/6}b} + \frac{(5Ab+aB)\,\text{Su}}{}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\,\text{Subst}\left(\int \frac{-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}+2\sqrt[3]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x}\right.}{12\sqrt{3}\,a^{11/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} + \frac{(5Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{11/6}b^{7/6}} - \frac{(5Ab+aB)\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\right.}{12\sqrt{3}\,a^{11/6}b^{7/6}}$$

$$= \frac{(Ab-aB)\sqrt{x}}{3ab\left(a+bx^3\right)} - \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}} + \frac{(5Ab+aB)\tan^{-1}\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{11/6}b^{7/6}}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.24

$$\frac{\sqrt{x}\left(\left(a+bx^3\right)(aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+a(Ab-aB)\right)}{3a^2b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^2),x]
```

```
[Out] (Sqrt[x]*(a*(A*b - a*B) + (5*A*b + a*B)*(a + b*x^3)*Hypergeometric2F1[1/6,
1, 7/6, -((b*x^3)/a)]))/(3*a^2*b*(a + b*x^3))
```

fricas [B] time = 1.15, size = 2555, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B
^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^
5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^2*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) +
(B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6))*a
^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3
*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sqrt(x)*(-(B^6*a^6 + 30*A*B^
5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4
 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(5/6) + sqrt(3)*(B^6*a^6
```

```
+ 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^
2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(B^6*a^6 + 30*A*B^5*a^5*b +
 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*
A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6 +
30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*
a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x - (B*a^3*b
+ 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 +
2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^7))^(1/6))*a^9*b^6*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*
a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 +
 15625*A^6*b^6)/(a^11*b^7))^(5/6) - 2*sqrt(3)*(B*a^10*b^6 + 5*A*a^9*b^7)*sq
rt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*
b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))
^(5/6) - sqrt(3)*(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6))/(
B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 937
5*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)) + (a*b^2*x^3 + a^2*
b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1
/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a*b + 25*A^2*b^2)*x + (B*a^3*b + 5*A*a
^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^
3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(
a^11*b^7))^(1/6)) - (a*b^2*x^3 + a^2*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A
^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*
a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(a^4*b^2*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/3) + (B^2*a^2 + 10*A*B*a
*b + 25*A^2*b^2)*x - (B*a^3*b + 5*A*a^2*b^2)*sqrt(x)*(-(B^6*a^6 + 30*A*B^5*
a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 +
 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)) + 2*(a*b^2*x^3 + a^2
*b)*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^
3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(
1/6)*log(a^2*b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3
*B^3*a^3*b^3 + 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a
^11*b^7))^(1/6) + (B*a + 5*A*b)*sqrt(x)) - 2*(a*b^2*x^3 + a^2*b)*(-(B^6*a^6
 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3 + 9375*A^4*B
^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/6)*log(-a^2*
b*(-(B^6*a^6 + 30*A*B^5*a^5*b + 375*A^2*B^4*a^4*b^2 + 2500*A^3*B^3*a^3*b^3
+ 9375*A^4*B^2*a^2*b^4 + 18750*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^7))^(1/
6) + (B*a + 5*A*b)*sqrt(x)) - 12*(B*a - A*b)*sqrt(x))/(a*b^2*x^3 + a^2*b)
```

giac [A] time = 0.31, size = 302, normalized size = 1.04

$$\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^2b^2}-\frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}\right.}{36\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x)*
(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^2) - 1/36*sqrt(3)*((a*b^5)^(1/6)*B*a
+ 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/
```

```
(a^2*b^2) - 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a*b) + 1/18*((a*b^
5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x)
)/(a/b)^(1/6))/(a^2*b^2) + 1/18*((a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*a
rctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^2) + 1/9*((a*b
^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^2)
```

maple [A] time = 0.16, size = 387, normalized size = 1.34

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}} A \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}} A \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{18a^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}} A \arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{18a^2}+\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}} A \ln\left(x+\cdots\right.}{36\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x)
```

```
[Out] 1/3*(A*b-B*a)*x^(1/2)/a/b/(b*x^3+a)+5/9/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))*A+1/9/b/a*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-5/36/a^2*3
^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-1/36/b/
a*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+5/18
/a^2*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+1/18/b/a*(a/b)^(1/
6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*B+5/36/a^2*3^(1/2)*(a/b)^(1/6)*ln(
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+1/36/b/a*3^(1/2)*(a/b)^(1/6)*l
n(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/18/a^2*(a/b)^(1/6)*arctan(
2/(a/b)^(1/6)*x^(1/2)+3^(1/2))*A+1/18/b/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*
x^(1/2)+3^(1/2))*B
```

maxima [A] time = 1.20, size = 301, normalized size = 1.04

$$-\frac{(Ba-Ab)\sqrt{x}}{3\left(ab^2x^3+a^2b\right)}+\frac{\frac{\sqrt{3}(Ba+5Ab)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(Ba+5Ab)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}+\frac{4\left(Bab^{\frac{1}{3}}+5Ab^{\frac{4}{3}}\right)\arctan\cdots}{a^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}\cdots}}{36\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(B*a - A*b)*sqrt(x)/(a*b^2*x^3 + a^2*b) + 1/36*(sqrt(3)*(B*a + 5*A*b)*
log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)
) - sqrt(3)*(B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x
+ a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)
*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2
*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6)
+ 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3)
)) + 2*(B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b
^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*
b^(1/3))))/(a*b)
```

mupad [B] time = 2.92, size = 1922, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan(((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^
3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 + B
```

```
^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*
(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^
2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a)*1i)/(18*(-a)^(11/6)*b^(
7/6)))/((((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A
^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*(5*A*b + B*a)*(125*A^3*b^5 +
B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))
*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6)) - (((2*x^(1/2)*(625*A^4*b^5 + B^4*
a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4)
 + (2*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*
a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*(5*A*b + B*a))/(18*(-a)^(11/6)*b^(7/6))
))*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) + (atan(((((3^(1/2)*1i)/2 - 1/
2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3
 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 - 1/2
)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*
b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)) + (((3^(1/2)*1
i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^
2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i
)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A
*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(11/6)*b^(7/6)))/((((
3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 1
50*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) - (2*((3
^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b
^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))
 - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^4*a^4
*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^4) +
(2*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 75*A^2
*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6))))/(18*(-a)^(11/6)*b^
(7/6))))*((3^(1/2)*1i)/2 - 1/2)*(5*A*b + B*a)*1i)/(9*(-a)^(11/6)*b^(7/6)) +
 (atan(((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4*b^5 + B^
4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2))/(27*a^
4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^3*b^2 + 7
5*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(18*(-a)^(
11/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)*(625*A^4
*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^3*a^3*b^2
))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5 + B^3*a^
3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6)))*1i)/(
18*(-a)^(11/6)*b^(7/6)))/((((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x^(1/2)
*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 20*A*B^
3*a^3*b^2))/(27*a^4) - (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A^3*b^5
 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*b^(7/6
))))/(18*(-a)^(11/6)*b^(7/6)) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*((2*x
^(1/2)*(625*A^4*b^5 + B^4*a^4*b + 150*A^2*B^2*a^2*b^3 + 500*A^3*B*a*b^4 + 2
0*A*B^3*a^3*b^2))/(27*a^4) + (2*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*(125*A
^3*b^5 + B^3*a^3*b^2 + 75*A^2*B*a*b^4 + 15*A*B^2*a^2*b^3))/(27*(-a)^(23/6)*
b^(7/6))))/(18*(-a)^(11/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + B*a)*
1i)/(9*(-a)^(11/6)*b^(7/6)) + (x^(1/2)*(A*b - B*a))/(3*a*b*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**2/x**(1/2),x)
```

```
[Out] Timed out
```

3.168 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)}{}$$

```
[Out] -1/9*(7*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7*A
*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/18*(7
*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(5/6)-1/36*(
7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b
^(5/6)*3^(1/2)+1/36*(7*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(5/6)*3^(1/2)+1/3*(-7*A*b+B*a)/a^2/b/x^(1/2)+1/3*(A*b
-B*a)/a/b/(b*x^3+a)/x^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{(7Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{13/6}b^{5/6}}+\frac{(7Ab-aB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] -(7*A*b - a*B)/(3*a^2*b*Sqrt[x]) + (A*b - a*B)/(3*a*b*Sqrt[x]*(a + b*x^3))
+ ((7*A*b - a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*a^(13/6
)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(18*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(
9*a^(13/6)*b^(5/6)) - ((7*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*
Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*b^(5/6)) + ((7*A*b - a*B)*Log[a^
(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(13/6)*
b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} + \frac{\left(\frac{7Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{6a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\,\mathrm{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right)}{9a^{13/6}b^{2/3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\,\mathrm{Subst}\left(\int \right.}{12\sqrt{}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} - \frac{(7Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\log\left(\sqrt[3]{a} -\right.}{12\sqrt{3}}$$

$$= -\frac{7Ab-aB}{3a^2b\sqrt{x}} + \frac{Ab-aB}{3ab\sqrt{x}\left(a+bx^3\right)} + \frac{(7Ab-aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{13/6}b^{5/6}} - \frac{(7Ab-aB)\tan}{18a}$$

Mathematica [C] time = 0.14, size = 70, normalized size = 0.22

$$\frac{2\left(x^3(aB-Ab)\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)-Abx^3\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)-5aA\right)}{5a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x]
```

```
[Out] (2*(-5*a*A - A*b*x^3*Hypergeometric2F1[5/6, 1, 11/6, -((b*x^3)/a)] + (-(A*b
) + a*B)*x^3*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)]))/(5*a^3*Sqrt[x]
)
```

fricas [B] time = 1.22, size = 3798, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B
^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*
b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt((B^5*a^1
6*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 +
12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^
5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 -
100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*
B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a
^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*
```

```
B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249
*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 -
6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 11
7649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*
b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 1
17649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^2
+ 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 16807*
A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 68
60*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^
6*b^6)/(a^13*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*
a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5
 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*
A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b
^6)) + 4*sqrt(3)*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(B^5*a
^16*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7
+ 12005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*
a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4
- 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*
A*B^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6
*a^6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^
7*B^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 2824752
49*A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5
- 6860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 +
117649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)/(a^13*b^5))^(2/3))*a^2*b*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^7*b - 35*A*B^4*a^6*b^
2 + 490*A^2*B^3*a^5*b^3 - 3430*A^3*B^2*a^4*b^4 + 12005*A^4*B*a^3*b^5 - 1680
7*A^5*a^2*b^6)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 -
6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*
A^6*b^6)/(a^13*b^5))^(1/6) + sqrt(3)*(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^
4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b
^5 + 117649*A^6*b^6))/(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 686
0*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6
*b^6)) - 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^
4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 +
 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(a^11*b^4*(-(B^6*a^6 - 42*A*B^5*a^5*b
 + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100
842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a^5 - 35*A*B^4*a
^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^4*B*a*b^4 - 168
07*A^5*b^5)*sqrt(x)) + 2*(a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b +
735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-a^11*b^4*(-(B^6*a^6 -
 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^
2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) - (B^5*a
^5 - 35*A*B^4*a^4*b + 490*A^2*B^3*a^3*b^2 - 3430*A^3*B^2*a^2*b^3 + 12005*A^
4*B*a*b^4 - 16807*A^5*b^5)*sqrt(x)) + (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A
*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2
*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log((B^5*a^16
*b^4 - 35*A*B^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 1
2005*A^4*B*a^12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5
*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 1
00842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B
^9*a^9*b + 2205*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^
```

```
6*b^4 - 4235364*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B
^3*a^3*b^7 + 259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*
A^10*b^10)*x - (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6
860*A^3*B^3*a^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117
649*A^6*a^9*b^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A
^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^
6)/(a^13*b^5))^(2/3)) - (a^2*b*x^4 + a^3*x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 7
35*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*
A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(1/6)*log(-(B^5*a^16*b^4 - 35*A*B
^4*a^15*b^5 + 490*A^2*B^3*a^14*b^6 - 3430*A^3*B^2*a^13*b^7 + 12005*A^4*B*a^
12*b^8 - 16807*A^5*a^11*b^9)*sqrt(x)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*
B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a
*b^5 + 117649*A^6*b^6)/(a^13*b^5))^(5/6) + (B^10*a^10 - 70*A*B^9*a^9*b + 22
05*A^2*B^8*a^8*b^2 - 41160*A^3*B^7*a^7*b^3 + 504210*A^4*B^6*a^6*b^4 - 42353
64*A^5*B^5*a^5*b^5 + 24706290*A^6*B^4*a^4*b^6 - 98825160*A^7*B^3*a^3*b^7 +
259416045*A^8*B^2*a^2*b^8 - 403536070*A^9*B*a*b^9 + 282475249*A^10*b^10)*x
- (B^6*a^15*b^3 - 42*A*B^5*a^14*b^4 + 735*A^2*B^4*a^13*b^5 - 6860*A^3*B^3*a
^12*b^6 + 36015*A^4*B^2*a^11*b^7 - 100842*A^5*B*a^10*b^8 + 117649*A^6*a^9*b
^9)*(-(B^6*a^6 - 42*A*B^5*a^5*b + 735*A^2*B^4*a^4*b^2 - 6860*A^3*B^3*a^3*b^
3 + 36015*A^4*B^2*a^2*b^4 - 100842*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^5)
)^(2/3)) + 12*((B*a - 7*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^4 + a^3*x)
```

giac [A] time = 0.47, size = 307, normalized size = 0.97

$$\frac{Bax^3 - 7\,Abx^3 - 6\,Aa}{3\left(bx^{\frac{7}{2}} + a\sqrt{x}\right)a^2} - \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - 7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b^5} + \frac{\sqrt{3}\left(\left(ab^5\right)^{\frac{5}{6}}Ba - 7\left(ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a*x^3 - 7*A*b*x^3 - 6*A*a)/((b*x^(7/2) + a*sqrt(x))*a^2) - 1/36*sqrt
(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/
6) + x + (a/b)^(1/3))/(a^3*b^5) + 1/36*sqrt(3)*((a*b^5)^(5/6)*B*a - 7*(a*b^
5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^5)
 + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/
6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^5) + 1/18*((a*b^5)^(5/6)*B*a - 7*(a*b^5
)^(5/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^
5) + 1/9*((a*b^5)^(5/6)*B*a - 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/
6))/(a^3*b^5)
```

maple [A] time = 0.16, size = 401, normalized size = 1.26

$$-\frac{Ab\,x^{\frac{5}{2}}}{3\left(b\,x^3 + a\right)a^2} + \frac{B\,x^{\frac{5}{2}}}{3\left(b\,x^3 + a\right)a} - \frac{7A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2} - \frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2} - \frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} + \sqrt{3}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2} + \frac{7\sqrt{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(5/2)/(b*x^3+a)*A*b+1/3/a*x^(5/2)/(b*x^3+a)*B-7/9/a^2*A/(a/b)^(1
/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(-x+3^
(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))+7/36/a^3*A*b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b
)^(1/6)*x^(1/2)+(a/b)^(1/3))-7/18/a^2*A/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^
```

```
(1/2)+3^(1/2))+1/9/a*B/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+1/36/a^2
*B*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))+1/18/
a*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-1/36/a^2*B*3^(1/2)*
(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+1/18/a*B/b/(a/b)^
(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2*A/a^2/x^(1/2)
```

maxima [A] time = 1.16, size = 240, normalized size = 0.75

$$\frac{(Ba-7\,Ab)x^3-6\,Aa}{3\left(a^2bx^{\frac{7}{2}}+a^3\sqrt{x}\right)}-\frac{(Ba-7\,Ab)\left(\frac{\sqrt{3}\,\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\,\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}+2\,b^{\frac{1}{3}}\sqrt{x}}{\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{b^{\frac{2}{3}}\sqrt{a^{\frac{1}{3}}b^{\frac{1}{3}}}}\right.}{36\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 7*A*b)*x^3 - 6*A*a)/(a^2*b*x^(7/2) + a^3*sqrt(x)) - 1/36*(B*a -
 7*A*b)*(sqrt(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)
*x + a^(1/3))/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(
1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*ar
ctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/
(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(
1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

mupad [B] time = 2.91, size = 1757, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^2),x)
```

```
[Out] (atan((((7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*
a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6)))*1i)/((-a)^(13/3)*b^(5/3)) + ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*
B^3*a^18*b^3 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/(((7*A*b - B*a)^2
*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*
a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*
b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5
/3)) - ((7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*
a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*(7*A*b - B*a)*(23147208*A^2*a^17
*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/
6))))/((-a)^(13/3)*b^(5/3))))*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) - (
(2*A)/a + (x^3*(7*A*b - B*a))/(3*a^2))/(a*x^(1/2) + b*x^(7/2)) + (atan(((((
3^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^
6 - 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
 1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*
A*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3
^(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3
 + 1701*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 -
1/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A
*B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^
(1/2)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6
```

```
- 1701*A*B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1
/2)*(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*
B*a^18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2
)*1i)/2 - 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 17
01*A*B^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*
(7*A*b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^
18*b^5))/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)
/2 - 1/2)*(7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)
/2 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*
B^2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*
b - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^
5))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)) + (((3^(1/2)*1i)/
2 + 1/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B
^2*a^17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
 - B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5
))/(5832*(-a)^(13/6)*b^(5/6)))*1i)/((-a)^(13/3)*b^(5/3)))/((((3^(1/2)*1i)/2
 + 1/2)^2*(7*A*b - B*a)^2*(81*B^3*a^18*b^3 - 27783*A^3*a^15*b^6 - 1701*A*B^
2*a^17*b^4 + 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b
- B*a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5)
)/(5832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1
/2)^2*(7*A*b - B*a)^2*(27783*A^3*a^15*b^6 - 81*B^3*a^18*b^3 + 1701*A*B^2*a^
17*b^4 - 11907*A^2*B*a^16*b^5 + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b - B*
a)*(23147208*A^2*a^17*b^6 + 472392*B^2*a^19*b^4 - 6613488*A*B*a^18*b^5))/(5
832*(-a)^(13/6)*b^(5/6))))/((-a)^(13/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(
7*A*b - B*a)*1i)/(9*(-a)^(13/6)*b^(5/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.169 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=96

$$-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{aB-3Ab}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

```
[Out] 1/3*(-3*A*b+B*a)/a^2/b/x^(3/2)+1/3*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)-1/3*(3*A
*b-B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 325, 329, 275, 205}

$$-\frac{3Ab-aB}{3a^2bx^{3/2}}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] -(3*A*b - a*B)/(3*a^2*b*x^(3/2)) + (A*b - a*B)/(3*a*b*x^(3/2)*(a + b*x^3))
- ((3*A*b - a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(3*a^(5/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
```

```
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}+\frac{\left(\frac{9Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$=-\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{2a^2}$$

$$=-\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{a^2}$$

$$=-\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{3a^2}$$

$$=-\frac{3Ab-aB}{3a^2bx^{3/2}}+\frac{Ab-aB}{3abx^{3/2}\left(a+bx^3\right)}-\frac{(3Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{3a^{5/2}\sqrt{b}}$$

Mathematica [A] time = 0.13, size = 79, normalized size = 0.82

$$\frac{\frac{(aB-3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}\left(-2aA+aBx^3-3Abx^3\right)}{x^{3/2}\left(a+bx^3\right)}}{3a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x]
```

```
[Out] ((Sqrt[a]*(-2*a*A - 3*A*b*x^3 + a*B*x^3))/(x^(3/2)*(a + b*x^3)) + ((-3*A*b
+ a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[b])/(3*a^(5/2))
```

fricas [A] time = 0.82, size = 232, normalized size = 2.42

$$\left[\frac{\left(\left(Bab-3\,Ab^2\right)x^5+\left(Ba^2-3\,Aab\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^3+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\left(2\,Aa^2b-\left(Ba^2b-3\,Aab^2\right)x^3\right)\sqrt{x}}{6\left(a^3b^2x^5+a^4bx^2\right)},\ \frac{\left((Ba\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((B*a*b - 3*A*b^2)*x^5 + (B*a^2 - 3*A*a*b)*x^2)*sqrt(-a*b)*log((b*x^3
 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2*(2*A*a^2*b - (B*a^2*b - 3*A*a
*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2), 1/3*(((B*a*b - 3*A*b^2)*x^5
+ (B*a^2 - 3*A*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) - (2*A*a^2*b
 - (B*a^2*b - 3*A*a*b^2)*x^3)*sqrt(x))/(a^3*b^2*x^5 + a^4*b*x^2)]
```

giac [A] time = 0.16, size = 66, normalized size = 0.69

$$\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^2}+\frac{Bax^3-3\,Abx^3-2\,Aa}{3\left(bx^{\frac{9}{2}}+ax^{\frac{3}{2}}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(B*a - 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(B*a*x^
3 - 3*A*b*x^3 - 2*A*a)/((b*x^(9/2) + a*x^(3/2))*a^2)
```

maple [A] time = 0.07, size = 93, normalized size = 0.97

$$-\frac{A b\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)a^{2}}+\frac{B\,x^{\frac{3}{2}}}{3\left(b\,x^{3}+a\right)a}-\frac{A b\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{\sqrt{a b}\;a^{2}}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{a b}}\right)}{3\sqrt{a b}\;a}-\frac{2A}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(3/2)/(b*x^3+a)*A*b+1/3/a*x^(3/2)/(b*x^3+a)*B-1/a^2/(a*b)^(1/2)*
arctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+1/3/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*
b*x^(3/2))*B-2/3*A/a^2/x^(3/2)
```

maxima [A] time = 1.19, size = 67, normalized size = 0.70

$$\frac{(Ba-3\,Ab)x^{3}-2\,Aa}{3\left(a^{2}bx^{\frac{9}{2}}+a^{3}x^{\frac{3}{2}}\right)}+\frac{(Ba-3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{3\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((B*a - 3*A*b)*x^3 - 2*A*a)/(a^2*b*x^(9/2) + a^3*x^(3/2)) + 1/3*(B*a -
3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.15, size = 139, normalized size = 1.45

$$-\frac{2\,A\,a^{3/2}\,\sqrt{b}-B\,a^2\,x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,b^2\,x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+3\,A\,\sqrt{a}\,b^{3/2}\,x^3-B\,a^{3/2}\,\sqrt{b}\,x^3+3\,A\,a\,b\,x^{3/2}\,\mathrm{at}}{3\,a^{7/2}\,\sqrt{b}\,x^{3/2}+3\,a^{5/2}\,b^{3/2}\,x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^2),x)
```

```
[Out] -(2*A*a^(3/2)*b^(1/2) - B*a^2*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A
*b^2*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) + 3*A*a^(1/2)*b^(3/2)*x^3 - B*
a^(3/2)*b^(1/2)*x^3 + 3*A*a*b*x^(3/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)) - B*a
*b*x^(9/2)*atan((b^(1/2)*x^(3/2))/a^(1/2)))/(3*a^(7/2)*b^(1/2)*x^(3/2) + 3*
a^(5/2)*b^(3/2)*x^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.170 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=318

$$\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}+\frac{(11Ab-}{}$$

```
[Out] 1/15*(-11*A*b+5*B*a)/a^2/b/x^(5/2)+1/3*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)-1/9*
(11*A*b-5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(11*A*
b-5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)-1/18*(
11*A*b-5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(1/6)+1/
36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(
17/6)/b^(1/6)*3^(1/2)-1/36*(11*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(1/6)*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 325, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{11Ab-5aB}{15a^2bx^{5/2}}+\frac{(11Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}-\frac{(11Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\right.}{12\sqrt{3}a^{17/6}\sqrt[6]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] -(11*A*b - 5*a*B)/(15*a^2*b*x^(5/2)) + (A*b - a*B)/(3*a*b*x^(5/2)*(a + b*x^
3)) + ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(18*
a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/
a^(1/6)])/(18*a^(17/6)*b^(1/6)) - ((11*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x]
)/a^(1/6)])/(9*a^(17/6)*b^(1/6)) + ((11*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*
a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(12*Sqrt[3]*a^(17/6)*b^(1/6)) - ((11*
A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(1
2*Sqrt[3]*a^(17/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^2}\,dx = \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{\left(\frac{11Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{3ab}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{6a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \right.}{9a^{17/6}}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5aB)\,\mathrm{Sub}}{}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} - \frac{(11Ab-5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{9a^{17/6}\sqrt[6]{b}} + \frac{(11Ab-5aB)\log}{1}$$

$$= -\frac{11Ab-5aB}{15a^2bx^{5/2}} + \frac{Ab-aB}{3abx^{5/2}\left(a+bx^3\right)} + \frac{(11Ab-5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{18a^{17/6}\sqrt[6]{b}} - \frac{(11Ab-5a}{}$$

Mathematica [C] time = 0.11, size = 74, normalized size = 0.23

$$\frac{5x^3(5aB-11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)+\frac{a\left(-6aA+5aBx^3-11Abx^3\right)}{a+bx^3}}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x]
```

```
[Out] ((a*(-6*a*A - 11*A*b*x^3 + 5*a*B*x^3))/(a + b*x^3) + 5*(-11*A*b + 5*a*B)*x^
3*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)])/(15*a^3*x^(5/2))
```

fricas [B] time = 1.15, size = 2584, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] -1/180*(20*sqrt(3)*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^
5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a
^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4
*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*
B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 12
1*A^2*b^2)*x + (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B
^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*
B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*
b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500
```

```
*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(
-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)/(a^17*b))^(5/6) - sqrt(3)*(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11343
75*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 48
31530*A^5*B*a*b^5 + 1771561*A^6*b^6))/(15625*B^6*a^6 - 206250*A*B^5*a^5*b +
 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^
4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b*x^6 + a^3*x
^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 33275
00*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 177156
1*A^6*b^6)/(a^17*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*(-(15625*B^6*a^6
- 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 +
5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^
(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)
*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 -
3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1
771561*A^6*b^6)/(a^17*b))^(1/6))*a^14*b*(-(15625*B^6*a^6 - 206250*A*B^5*a^5
*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^
2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + 2*sqrt(3)*
(5*B*a^15*b - 11*A*a^14*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b
+ 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b
^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(5/6) + sqrt(3)*(1562
5*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*
a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6))
/(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^
3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6
*b^6)) - 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 11
34375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 -
 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^6*(-(15625*B^
6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*
b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^1
7*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 121*A^2*b^2)*x + (5*B*a^4 - 11*A*
a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*
b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b
^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 5*(a^2*b*x^6 + a^3*x^3)*(-(15625*B
^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3
*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^
17*b))^(1/6)*log(a^6*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^
4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5
*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/3) + (25*B^2*a^2 - 110*A*B*a*b + 1
21*A^2*b^2)*x - (5*B*a^4 - 11*A*a^3*b)*sqrt(x)*(-(15625*B^6*a^6 - 206250*A*
B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4
*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)) + 10
*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*
B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A
^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6)*log(a^3*(-(15625*B^6*a^6 - 20
6250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5490
375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(1/6
) - (5*B*a - 11*A*b)*sqrt(x)) - 10*(a^2*b*x^6 + a^3*x^3)*(-(15625*B^6*a^6 -
 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b^2 - 3327500*A^3*B^3*a^3*b^3 + 5
490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b))^(
1/6)*log(-a^3*(-(15625*B^6*a^6 - 206250*A*B^5*a^5*b + 1134375*A^2*B^4*a^4*b
^2 - 3327500*A^3*B^3*a^3*b^3 + 5490375*A^4*B^2*a^2*b^4 - 4831530*A^5*B*a*b^
5 + 1771561*A^6*b^6)/(a^17*b))^(1/6) - (5*B*a - 11*A*b)*sqrt(x)) - 12*((5*B
*a - 11*A*b)*x^3 - 6*A*a)*sqrt(x))/(a^2*b*x^6 + a^3*x^3)
```

giac [A] time = 0.22, size = 313, normalized size = 0.98

$$\frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba - 11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{36\,a^3b} - \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba - 11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\right.}{36\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b) - 1/36*sqrt(3)*(5*(a*b^5)^(1/6)*B
*a - 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b) + 1/3*(B*a*sqrt(x) - A*b*sqrt(x))/((b*x^3 + a)*a^2) + 1/18*(5*(
a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sq
rt(x))/(a/b)^(1/6))/(a^3*b) + 1/18*(5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b) + 1/9*(
5*(a*b^5)^(1/6)*B*a - 11*(a*b^5)^(1/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^
3*b) - 2/5*A/(a^2*x^(5/2))
```

maple [A] time = 0.16, size = 395, normalized size = 1.24

$$-\frac{Ab\sqrt{x}}{3\left(b\,x^3+a\right)a^2} + \frac{B\sqrt{x}}{3\left(b\,x^3+a\right)a} - \frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{9a^3} - \frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}} - \sqrt{3}\right)}{18a^3} - \frac{11\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arcta}{18a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3/a^2*x^(1/2)/(b*x^3+a)*A*b+1/3/a*x^(1/2)/(b*x^3+a)*B-11/9/a^3*A*b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(-
x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan
(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-11/36/a^3*A*b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1
/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-11/18/a^3*A*b*(a/b)^(1/6)*arctan(2/(a/
b)^(1/6)*x^(1/2)+3^(1/2))+5/9/a^2*B*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2
))-5/36/a^2*B*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(
1/3))+5/18/a^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+5/36/a^2
*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))+5/18/a
^2*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2/5/a^2*A/x^(5/2)
```

maxima [A] time = 1.19, size = 312, normalized size = 0.98

$$\frac{(5\,Ba - 11\,Ab)x^3 - 6\,Aa}{15\left(a^2bx^{\frac{11}{2}} + a^3x^{\frac{5}{2}}\right)} + \frac{\frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}(5\,Ba-11\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4\left(5\,Bab^{\frac{1}{3}}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/15*((5*B*a - 11*A*b)*x^3 - 6*A*a)/(a^2*b*x^(11/2) + a^3*x^(5/2)) + 1/36*(
sqrt(3)*(5*B*a - 11*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x +
a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 11*A*b)*log(-sqrt(3)*a^(1/6)*
b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3)
```

```
- 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(
1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3)*b^(4/3)
)*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)
))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 11*A*a^(1/3
)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/
3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^2
```

mupad [B] time = 2.96, size = 2080, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^2),x)
```

```
[Out] - ((2*A)/(5*a) + (x^3*(11*A*b - 5*B*a))/(15*a^2))/(a*x^(5/2) + b*x^(11/2))
- (atan((((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*
A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11
*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^
2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b -
5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)) + ((x^(1/2)*(21346578*A^4*a^10*b^9 + 91
1250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38
811960*A^3*B*a^11*b^8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500
*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a
)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)^(17/6)*b^(1/6)))/(((x^(1/2
)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7
- 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^8) - ((11*A*b - 5*B*a)*(34
930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 4763
2860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^
(17/6)*b^(1/6)) - ((x^(1/2)*(21346578*A^4*a^10*b^9 + 911250*B^4*a^14*b^5 +
26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 38811960*A^3*B*a^11*b^
8) + ((11*A*b - 5*B*a)*(34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 2165
1300*A*B^2*a^15*b^6 - 47632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(
11*A*b - 5*B*a))/(18*(-a)^(17/6)*b^(1/6))))*(11*A*b - 5*B*a)*1i)/(9*(-a)^(1
7/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*
(-a)^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*
b^9 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*
b^6 - 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(
34930764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47
632860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a
)^(17/6)*b^(1/6)) - (((3^(1/2)*1i)/2 - 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
 + 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
 - 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*(349
30764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632
860*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(
17/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)
*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)) + (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
```

```
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a)*1i)/(18*(-a)
^(17/6)*b^(1/6)))/((((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9
+ 911250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6
- 38811960*A^3*B*a^11*b^8) - (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493
0764*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 476328
60*A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(1
7/6)*b^(1/6)) - (((3^(1/2)*1i)/2 + 1/2)*(x^(1/2)*(21346578*A^4*a^10*b^9 + 9
11250*B^4*a^14*b^5 + 26462700*A^2*B^2*a^12*b^7 - 8019000*A*B^3*a^13*b^6 - 3
8811960*A^3*B*a^11*b^8) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*(3493076
4*A^3*a^13*b^8 - 3280500*B^3*a^16*b^5 + 21651300*A*B^2*a^15*b^6 - 47632860*
A^2*B*a^14*b^7))/(18*(-a)^(17/6)*b^(1/6)))*(11*A*b - 5*B*a))/(18*(-a)^(17/6
)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b - 5*B*a)*1i)/(9*(-a)^(17/6)*b^(
1/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.171 $\int \frac{x^{7/2}(A+Bx^3)}{(a+bx^3)^3} dx$

Optimal. Leaf size=104

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2(a+bx^3)}+\frac{x^{9/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

```
[Out] 1/6*(A*b-B*a)*x^(9/2)/a/b/(b*x^3+a)^2-1/12*(A*b+3*B*a)*x^(3/2)/a/b^2/(b*x^3
+a)+1/12*(A*b+3*B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 288, 329, 275, 205}

$$\frac{(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}-\frac{x^{3/2}(3aB+Ab)}{12ab^2(a+bx^3)}+\frac{x^{9/2}(Ab-aB)}{6ab(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(9/2))/(6*a*b*(a + b*x^3)^2) - ((A*b + 3*a*B)*x^(3/2))/(12*a
*b^2*(a + b*x^3)) + ((A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(3/2)*b^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{x^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{3Ab}{2}+\frac{9aB}{2}\right)\int\frac{x^{7/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$
$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\int\frac{\sqrt{x}}{a+bx^3}\,dx}{8ab^2}$$
$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\,\mathrm{Subst}\left(\int\frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4ab^2}$$
$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\,\mathrm{Subst}\left(\int\frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12ab^2}$$
$$= \frac{(Ab-aB)x^{9/2}}{6ab\left(a+bx^3\right)^2}-\frac{(Ab+3aB)x^{3/2}}{12ab^2\left(a+bx^3\right)}+\frac{(Ab+3aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Mathematica [A] time = 0.19, size = 93, normalized size = 0.89

$$\frac{\frac{\sqrt{a}\,\sqrt{b}\,x^{3/2}\left(-3a^2B-ab\left(A+5Bx^3\right)+Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(3aB+Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - a*b*(A + 5*B*x^3)))/(a +
b*x^3)^2 + (A*b + 3*a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(3/2)*b^(
5/2))
```

fricas [A] time = 0.98, size = 314, normalized size = 3.02

$$\left[-\frac{\left(\left(3\,Bab^2+Ab^3\right)x^6+3\,Ba^3+Aa^2b+2\left(3\,Ba^2b+Aab^2\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(\left(5\,Ba^2b^2-Aab^3\right.\right.}{24\left(a^2b^5x^6+2\,a^3b^4x^3+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3 + A*a^2*b + 2*(3*B*a^2*b + A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) + 2
*((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a^2*b^
5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), 1/12*(((3*B*a*b^2 + A*b^3)*x^6 + 3*B*a^3
+ A*a^2*b + 2*(3*B*a^2*b + A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
```

```
/a) - ((5*B*a^2*b^2 - A*a*b^3)*x^4 + (3*B*a^3*b + A*a^2*b^2)*x)*sqrt(x))/(a
^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

giac [A] time = 0.21, size = 84, normalized size = 0.81

$$\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2}-\frac{5\,Babx^{\frac{9}{2}}-Ab^2x^{\frac{9}{2}}+3\,Ba^2x^{\frac{3}{2}}+Aabx^{\frac{3}{2}}}{12\,\left(bx^3+a\right)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/12*(5*
B*a*b*x^(9/2) - A*b^2*x^(9/2) + 3*B*a^2*x^(3/2) + A*a*b*x^(3/2))/((b*x^3 +
a)^2*a*b^2)
```

maple [A] time = 0.07, size = 96, normalized size = 0.92

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab}\;ab}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^2}+\frac{\frac{(Ab-5Ba)x^{\frac{9}{2}}}{12ab}-\frac{(Ab+3Ba)x^{\frac{3}{2}}}{12b^2}}{\left(b\,x^3+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(A*b-5*B*a)/a/b*x^(9/2)-1/8*(A*b+3*B*a)/b^2*x^(3/2))/(b*x^3+a)^2+1
/12/b/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A+1/4/b^2/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

maxima [A] time = 1.43, size = 96, normalized size = 0.92

$$-\frac{\left(5\,Bab-Ab^2\right)x^{\frac{9}{2}}+\left(3\,Ba^2+Aab\right)x^{\frac{3}{2}}}{12\,\left(ab^4x^6+2\,a^2b^3x^3+a^3b^2\right)}+\frac{(3\,Ba+Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/12*((5*B*a*b - A*b^2)*x^(9/2) + (3*B*a^2 + A*a*b)*x^(3/2))/(a*b^4*x^6 +
2*a^2*b^3*x^3 + a^3*b^2) + 1/12*(3*B*a + A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(
sqrt(a*b)*a*b^2)
```

mupad [B] time = 2.76, size = 133, normalized size = 1.28

$$\frac{\mathrm{atan}\left(\frac{9\,b^{3/2}\,x^{3/2}\,\left(A^2\,b^2+6\,A\,B\,a\,b+9\,B^2\,a^2\right)}{\sqrt{a}\,\left(9\,A\,b^2+27\,B\,a\,b\right)\,\left(A\,b+3\,B\,a\right)}\right)\,\left(A\,b+3\,B\,a\right)}{12\,a^{3/2}\,b^{5/2}}-\frac{\frac{x^{3/2}\,\left(A\,b+3\,B\,a\right)}{12\,b^2}-\frac{x^{9/2}\,\left(A\,b-5\,B\,a\right)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(7/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((9*b^(3/2)*x^(3/2)*(A^2*b^2 + 9*B^2*a^2 + 6*A*B*a*b))/(a^(1/2)*(9*A*b
^2 + 27*B*a*b)*(A*b + 3*B*a)))*(A*b + 3*B*a))/(12*a^(3/2)*b^(5/2)) - ((x^(3
/2)*(A*b + 3*B*a))/(12*b^2) - (x^(9/2)*(A*b - 5*B*a))/(12*a*b))/(a^2 + b^2*
x^6 + 2*a*b*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)*(B*x**3+A)/(b*x**3+a)**3,x)
[Out] Timed out
```

3.172 $\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=327

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5Ab}{2}$$

```
[Out] 1/6*(A*b-B*a)*x^(7/2)/a/b/(b*x^3+a)^2+1/108*(5*A*b+7*B*a)*arctan(b^(1/6)*x^
(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(-3^(1/2)+2*b^(1
/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)+1/216*(5*A*b+7*B*a)*arctan(3^(1/2)+2
*b^(1/6)*x^(1/2)/a^(1/6))/a^(11/6)/b^(13/6)-1/432*(5*A*b+7*B*a)*ln(a^(1/3)+
b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6)/b^(13/6)*3^(1/2)+1/432*
(5*A*b+7*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(11/6
)/b^(13/6)*3^(1/2)-1/36*(5*A*b+7*B*a)*x^(1/2)/a/b^2/(b*x^3+a)
```

Rubi [A] time = 0.49, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {457, 288, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{(7aB+5Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}+\frac{(7aB+5Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{11/6}b^{13/6}}-\frac{(7aB+5Ab}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(7/2))/(6*a*b*(a + b*x^3)^2) - ((5*A*b + 7*a*B)*Sqrt[x])/(36
*a*b^2*(a + b*x^3)) - ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(11/6)*b^(13/6)) + ((5*A*b + 7*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(11/6)*b^(13/6)) - ((5*A*b + 7*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11
/6)*b^(13/6)) + ((5*A*b + 7*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(11/6)*b^(13/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{5Ab}{2}+\frac{7aB}{2}\right)\int \frac{x^{5/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36ab^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{x}\right.}{108a^{11/6}b^2}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\,\text{Subst}\left(\right.}{144}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} + \frac{(5Ab+7aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{11/6}b^{13/6}} - \frac{(5Ab+7aB)\log\left(\sqrt[3]{a}\right.}{144\sqrt{}}$$

$$= \frac{(Ab-aB)x^{7/2}}{6ab\left(a+bx^3\right)^2} - \frac{(5Ab+7aB)\sqrt{x}}{36ab^2\left(a+bx^3\right)} - \frac{(5Ab+7aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{11/6}b^{13/6}} + \frac{(5Ab+7aB)\,\text{t}}{216}$$

Mathematica [**C**] time = 0.11, size = 92, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-7a^2B-ab\left(5A+13Bx^3\right)+Ab^2x^3\right)+\left(a+bx^3\right)^2(7aB+5Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^2b^2\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (Sqrt[x]*(a*(-7*a^2*B + A*b^2*x^3 - a*b*(5*A + 13*B*x^3)) + (5*A*b + 7*a*B)
*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^2*b^2*(
a + b*x^3)^2)
```

fricas [**B**] time = 1.18, size = 2714, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(4*sqrt(3)*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 +
504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4593
75*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)
*arctan(1/3*(2*sqrt(3)*sqrt(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
+ 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
+ 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70
*A*B*a*b + 25*A^2*b^2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^
```

6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/6))*a^9*b^11*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*
a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*
b^5 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^
9*b^12)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(5/6) + sqrt(3)*(117649*B^6*a^6 + 504210*A*B
^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2
*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*
B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^
2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)) + 4*sqrt(3)*(a*b^4*x^6 + 2
*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2
*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5
*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^4
b^4(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^
6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B
*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 90
0375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 13
1250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6))*a^9*b^11*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(5/6) - 2*sqrt(3)*(7*B*a^10*b^11 + 5*A*a^9*b^12)*sqrt(x)*(-(117649*B^6*a
^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 +
 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^
(5/6) - sqrt(3)*(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b
^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 +
 15625*A^6*b^6))/(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)) + (a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6
 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 4
59375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1
/6)*log(a^4*b^4*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
 + 15625*A^6*b^6)/(a^11*b^13))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^
2)*x + (7*B*a^3*b^2 + 5*A*a^2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5
*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a
^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)) - (a*b^4*x
^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 9003
75*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 1312
50*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*log(a^4*b^4*(-(117649*B^
6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^
3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13
))^(1/3) + (49*B^2*a^2 + 70*A*B*a*b + 25*A^2*b^2)*x - (7*B*a^3*b^2 + 5*A*a^
2*b^3)*sqrt(x)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*
b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5
+ 15625*A^6*b^6)/(a^11*b^13))^(1/6)) + 2*(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b
^2)*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6
*b^6)/(a^11*b^13))^(1/6)*log(a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b
 + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4
 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)
sqrt(x)) - 2(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2)*(-(117649*B^6*a^6 + 504
210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 459375*
A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15625*A^6*b^6)/(a^11*b^13))^(1/6)*lo
g(-a^2*b^2*(-(117649*B^6*a^6 + 504210*A*B^5*a^5*b + 900375*A^2*B^4*a^4*b^2
+ 857500*A^3*B^3*a^3*b^3 + 459375*A^4*B^2*a^2*b^4 + 131250*A^5*B*a*b^5 + 15
625*A^6*b^6)/(a^11*b^13))^(1/6) + (7*B*a + 5*A*b)*sqrt(x)) - 12*((13*B*a*b

```
- A*b^2)*x^3 + 7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3
*b^2)
```

giac [A] time = 0.23, size = 328, normalized size = 1.00

$$\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^2b^3}-\frac{\sqrt{3}\left(7\left(ab^5\right)^{\frac{1}{6}}Ba+5\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\,\right.}{432\,a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/432*sqrt(3)*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(
x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^2*b^3) - 1/432*sqrt(3)*(7*(a*b^5)^(1/6
)*B*a + 5*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(
1/3))/(a^2*b^3) + 1/216*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(
(sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^2*b^3) + 1/216*(7*(a*b^5)
^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))
/(a/b)^(1/6))/(a^2*b^3) + 1/108*(7*(a*b^5)^(1/6)*B*a + 5*(a*b^5)^(1/6)*A*b)
*arctan(sqrt(x)/(a/b)^(1/6))/(a^2*b^3) - 1/36*(13*B*a*b*x^(7/2) - A*b^2*x^(
7/2) + 7*B*a^2*sqrt(x) + 5*A*a*b*sqrt(x))/((b*x^3 + a)^2*a*b^2)
```

maple [A] time = 0.16, size = 416, normalized size = 1.27

$$\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a^2b}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a^2b}-\frac{5\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x-\sqrt{3}\right.}{432a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(A*b-13*B*a)/a/b*x^(7/2)-1/72*(5*A*b+7*B*a)/b^2*x^(1/2))/(b*x^3+a)^
2+5/108/b/a^2*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+7/108/b^2/a*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x
-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A-7/432/b^2/a*3^(1/2)*(a/b)^(1/6)
*ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))*B+5/432/b/a^2*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/
b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+7/432/b^2/a*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2
)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+5/216/b/a^2*(a/b)^(1/6)*arctan(2/(a/b)
^(1/6)*x^(1/2)+3^(1/2))*A+7/216/b^2/a*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1
/2)+3^(1/2))*B
```

maxima [A] time = 1.40, size = 341, normalized size = 1.04

$$-\frac{\left(13\,Bab-Ab^2\right)x^{\frac{7}{2}}+\left(7\,Ba^2+5\,Aab\right)\sqrt{x}}{36\left(ab^4x^6+2\,a^2b^3x^3+a^3b^2\right)}+\frac{\frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}(7\,Ba+5\,Ab)\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+\right.}{a^{\frac{5}{6}}b^{\frac{1}{6}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] -1/36*((13*B*a*b - A*b^2)*x^(7/2) + (7*B*a^2 + 5*A*a*b)*sqrt(x))/(a*b^4*x^6
 + 2*a^2*b^3*x^3 + a^3*b^2) + 1/432*(sqrt(3)*(7*B*a + 5*A*b)*log(sqrt(3)*a^
```

```
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(7
*B*a + 5*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(
a^(5/6)*b^(1/6)) + 4*(7*B*a*b^(1/3) + 5*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/s
qrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(7*B*a^(4
/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1
/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(
7*B*a^(4/3)*b^(1/3) + 5*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
 - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3
))))/(a*b^2)
```

mupad [B] time = 2.98, size = 1944, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(5/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] (atan((((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 52
5*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 240
1*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(2
79936*a^4*b^3))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) - ((((5*A*b
+ 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(
279936*(-a)^(23/6)*b^(19/6)) + (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*
A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(
5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(((((5*A*b + 7*B*a)*(125*A^3
*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6
)*b^(19/6)) - (x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 +
 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(
216*(-a)^(11/6)*b^(13/6)) + ((((5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 +
 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)) + (x^(1/
2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b +
3500*A^3*B*a*b^3))/(279936*a^4*b^3))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(1
3/6))))*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) - ((x^(1/2)*(5*A*b +
 7*B*a))/(36*b^2) - (x^(7/2)*(A*b - 13*B*a))/(36*a*b))/(a^2 + b^2*x^6 + 2*a
*b*x^3) + (atan(((((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*
a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*
a^4*b^3) - (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a
^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*
A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)) + (((3^(1/2)*1i)/2 - 1/2)*((x^(
1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b
+ 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*
B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(2799
36*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/(
(((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^
2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(
1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^
2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(21
6*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)*1i)/2 - 1/2)*((x^(1/2)*(625*A^4*b^4 +
2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))
/(279936*a^4*b^3) + (((3^(1/2)*1i)/2 - 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 +
343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19
/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6))))*((3^(1/2)*1i)/2 - 1/2)*
(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)*b^(13/6)) + (atan(((((3^(1/2)*1i)/2 +
1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*
B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(
5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b
^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a)*1i)/(216*(-a)^(11/6)*b^
(13/6)) + (((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7
350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3
) + (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 73
```

```
5*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7
*B*a)*1i)/(216*(-a)^(11/6)*b^(13/6)))/((((3^(1/2)*1i)/2 + 1/2)*((x^(1/2)*(6
25*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2 + 6860*A*B^3*a^3*b + 3500*
A^3*B*a*b^3))/(279936*a^4*b^3) - (((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*(1
25*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*A^2*B*a*b^2))/(279936*(-a)
^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11/6)*b^(13/6)) - (((3^(1/2)
*1i)/2 + 1/2)*((x^(1/2)*(625*A^4*b^4 + 2401*B^4*a^4 + 7350*A^2*B^2*a^2*b^2
+ 6860*A*B^3*a^3*b + 3500*A^3*B*a*b^3))/(279936*a^4*b^3) + (((3^(1/2)*1i)/2
 + 1/2)*(5*A*b + 7*B*a)*(125*A^3*b^3 + 343*B^3*a^3 + 735*A*B^2*a^2*b + 525*
A^2*B*a*b^2))/(279936*(-a)^(23/6)*b^(19/6)))*(5*A*b + 7*B*a))/(216*(-a)^(11
/6)*b^(13/6))))*((3^(1/2)*1i)/2 + 1/2)*(5*A*b + 7*B*a)*1i)/(108*(-a)^(11/6)
*b^(13/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.173 $\int \frac{x^{3/2}(A+Bx^3)}{(a+bx^3)^3}\,dx$

Optimal. Leaf size=327

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{13/6}b^{11/6}}-\frac{(5aB+7A}{}$$

```
[Out] 1/6*(A*b-B*a)*x^(5/2)/a/b/(b*x^3+a)^2+1/36*(7*A*b+5*B*a)*x^(5/2)/a^2/b/(b*x
^3+a)+1/108*(7*A*b+5*B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^(11/6)
+1/216*(7*A*b+5*B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/6)/b^
(11/6)+1/216*(7*A*b+5*B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(13/
6)/b^(11/6)+1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2
)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)-1/432*(7*A*b+5*B*a)*ln(a^(1/3)+b^(1/3)
*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(13/6)/b^(11/6)*3^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 295, 634, 618, 204, 628, 205}

$$\frac{(5aB+7Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{13/6}b^{11/6}}-\frac{(5aB+7Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{13/6}b^{11/6}}-\frac{(5aB+7A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(5/2))/(6*a*b*(a + b*x^3)^2) + ((7*A*b + 5*a*B)*x^(5/2))/(36
*a^2*b*(a + b*x^3)) - ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(13/6)*b^(11/6)) + ((7*A*b + 5*a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13
/6)*b^(11/6)) - ((7*A*b + 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(13/6)*b^(11/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{\left(\frac{7Ab}{2}+\frac{5aB}{2}\right)\int \frac{x^{3/2}}{\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\int \frac{x^{3/2}}{a+bx^3}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\,\text{Subst}\left(\int \frac{x^4}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{a}}{2}+\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x,}{108a^{13/6}b^{5/3}}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\,\text{Subs}}{}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} + \frac{(7Ab+5aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{13/6}b^{11/6}} + \frac{(7Ab+5aB)\log(}{14}$$

$$= \frac{(Ab-aB)x^{5/2}}{6ab\left(a+bx^3\right)^2} + \frac{(7Ab+5aB)x^{5/2}}{36a^2b\left(a+bx^3\right)} - \frac{(7Ab+5aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{13/6}b^{11/6}} + \frac{(7Ab+5a}{}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.19

$$\frac{2x^{5/2}\left((Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)+aB\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)\right)}{5a^3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] (2*x^(5/2)*(a*B*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a)] + (A*b - a*B)
*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3)/a)]))/(5*a^3*b)
```

fricas [B] time = 1.29, size = 3951, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/432*(4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9003
75*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*sqrt((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 6
1250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 +
16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 13
```

6718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^
3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*
A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8
+ 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 1
31250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 +
900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(
-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*
B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)
/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375
*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210
*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5*a^7
*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*b^5
+ 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 13125
0*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^
4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) + s
qrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85750
0*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^
6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8575
00*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A
^6*b^6)) + 4*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
+ 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 9
00375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(
1/6)*arctan(1/3*(2*sqrt(3)*sqrt(-(3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10
+ 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^1
3 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10
+ 136718750*A*B^9*a^9*b + 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^
7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431
250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*
b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7
+ 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^
10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^1
3)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*
A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*
b^6)/(a^13*b^11))^(2/3))*a^2*b^2*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 45
9375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 50
4210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6) - 2*sqrt(3)*(3125*B^5
*a^7*b^2 + 21875*A*B^4*a^6*b^3 + 61250*A^2*B^3*a^5*b^4 + 85750*A^3*B^2*a^4*
b^5 + 60025*A^4*B*a^3*b^6 + 16807*A^5*a^2*b^7)*sqrt(x)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
- sqrt(3)*(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 8
57500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 11764
9*A^6*b^6))/(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 +
857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 1176
49*A^6*b^6)) - 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 1
31250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90037
5*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)
*log(a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^
2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 +
117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b + 61
250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*A^5
*b^5)*sqrt(x)) + 2*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 +
131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900
375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/
6)*log(-a^11*b^9*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4
*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5
+ 117649*A^6*b^6)/(a^13*b^11))^(5/6) + (3125*B^5*a^5 + 21875*A*B^4*a^4*b +

```
 61250*A^2*B^3*a^3*b^2 + 85750*A^3*B^2*a^2*b^3 + 60025*A^4*B*a*b^4 + 16807*
A^5*b^5)*sqrt(x)) - (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6
+ 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 90
0375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1
/6)*log((3125*B^5*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^1
1 + 85750*A^3*B^2*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*
sqrt(x)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 85
7500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649
*A^6*b^6)/(a^13*b^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b +
 861328125*A^2*B^8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^
6*a^6*b^4 + 13235512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 123
53145000*A^7*B^3*a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*
b^9 + 282475249*A^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8
+ 459375*A^2*B^4*a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*
b^11 + 504210*A^5*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131
250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*
A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3))
+ (a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5
*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b
^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(1/6)*log(-(3125*B^5
*a^16*b^9 + 21875*A*B^4*a^15*b^10 + 61250*A^2*B^3*a^14*b^11 + 85750*A^3*B^2
*a^13*b^12 + 60025*A^4*B*a^12*b^13 + 16807*A^5*a^11*b^14)*sqrt(x)*(-(15625*
B^6*a^6 + 131250*A*B^5*a^5*b + 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*
b^3 + 900375*A^4*B^2*a^2*b^4 + 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b
^11))^(5/6) + (9765625*B^10*a^10 + 136718750*A*B^9*a^9*b + 861328125*A^2*B^
8*a^8*b^2 + 3215625000*A^3*B^7*a^7*b^3 + 7878281250*A^4*B^6*a^6*b^4 + 13235
512500*A^5*B^5*a^5*b^5 + 15441431250*A^6*B^4*a^4*b^6 + 12353145000*A^7*B^3*
a^3*b^7 + 6485401125*A^8*B^2*a^2*b^8 + 2017680350*A^9*B*a*b^9 + 282475249*A
^10*b^10)*x - (15625*B^6*a^15*b^7 + 131250*A*B^5*a^14*b^8 + 459375*A^2*B^4*
a^13*b^9 + 857500*A^3*B^3*a^12*b^10 + 900375*A^4*B^2*a^11*b^11 + 504210*A^5
*B*a^10*b^12 + 117649*A^6*a^9*b^13)*(-(15625*B^6*a^6 + 131250*A*B^5*a^5*b +
 459375*A^2*B^4*a^4*b^2 + 857500*A^3*B^3*a^3*b^3 + 900375*A^4*B^2*a^2*b^4 +
 504210*A^5*B*a*b^5 + 117649*A^6*b^6)/(a^13*b^11))^(2/3)) - 12*((5*B*a*b +
7*A*b^2)*x^5 - (B*a^2 - 13*A*a*b)*x^2)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^
3 + a^4*b)
```

giac [A] time = 0.37, size = 328, normalized size = 1.00

$$\frac{5\,Babx^{\frac{11}{2}} + 7\,Ab^2x^{\frac{11}{2}} - Ba^2x^{\frac{5}{2}} + 13\,Aabx^{\frac{5}{2}}}{36\left(bx^3+a\right)^2a^2b} - \frac{\sqrt{3}\left(5\left(ab^5\right)^{\frac{5}{6}}Ba + 7\left(ab^5\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}} + x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^3b^6} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/36*(5*B*a*b*x^(11/2) + 7*A*b^2*x^(11/2) - B*a^2*x^(5/2) + 13*A*a*b*x^(5/2
))/((b*x^3 + a)^2*a^2*b) - 1/432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(
5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/
432*sqrt(3)*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*log(-sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7
*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(
a^3*b^6) + 1/216*(5*(a*b^5)^(5/6)*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(-(sqrt(
3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^6) + 1/108*(5*(a*b^5)^(5/6)
*B*a + 7*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^3*b^6)
```

maple [A] time = 0.16, size = 411, normalized size = 1.26

$$\frac{7A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2b}+\frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2b}+\frac{7A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216\left(\frac{a}{b}\right)^{\frac{1}{6}}a^2b}+\frac{7\left(\frac{a}{b}\right)^{\frac{5}{6}}\sqrt{3}\,A\ln\left(x-\sqrt{3}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right.}{432a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x)
```

```
[Out] 2*(1/72*(7*A*b+5*B*a)/a^2*x^(11/2)+1/72*(13*A*b-B*a)/a/b*x^(5/2))/(b*x^3+a)
^2+7/108/a^2/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+5/108/a/b^2/(a/b
)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B+7/432/a^3*(a/b)^(5/6)*3^(1/2)*ln(x-
3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*(a/b)^(5/6)*3^(1/2)*
ln(x-3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arc
tan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2/(a/b)
^(1/6)*x^(1/2)-3^(1/2))*B-7/432/a^3*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^
(1/6)*x^(1/2)+(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(
a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+7/216/a^2/b/(a/b)^(1/6)*arctan(2/(a/b)^(1
/6)*x^(1/2)+3^(1/2))*A+5/216/a/b^2/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)
+3^(1/2))*B
```

maxima [A] time = 1.23, size = 271, normalized size = 0.83

$$\frac{\left(5\,Bab+7\,Ab^2\right)x^{\frac{11}{2}}-\left(Ba^2-13\,Aab\right)x^{\frac{5}{2}}}{36\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}-\frac{(5\,Ba+7\,Ab)\left(\frac{\sqrt{3}\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}}b^{\frac{5}{6}}}-\right.}{432\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(B*x^3+A)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*((5*B*a*b + 7*A*b^2)*x^(11/2) - (B*a^2 - 13*A*a*b)*x^(5/2))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) - 1/432*(5*B*a + 7*A*b)*(sqrt(3)*log(sqrt(3)*a^
(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6)) - sqrt(3)*lo
g(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(5/6))
 - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1
/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*a^(1/6)*b^(1/6)
- 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3)))
 - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^
(1/3))))/(a^2*b)
```

mupad [B] time = 2.89, size = 1672, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(11/2)*(7*A*b + 5*B*a))/(36*a^2) + (x^(5/2)*(13*A*b - B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) + (atan(((((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^
2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B*a)*(49*A^2*b^
4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*(7*A*b + 5
*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)) - (((343*A^3*b^3 + 125*B^3*a^3 + 5
25*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b + 5*B*a)*(49
```

```
*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))*(7*
A*b + 5*B*a)^2*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((343*A^3*b^3 + 125*B^3*
a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*(7*A*b + 5*B
*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6
)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3)) + (((343*A^3*b^3 + 125*
B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*(7*A*b +
 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(
11/6)))*(7*A*b + 5*B*a)^2)/(46656*(-a)^(13/3)*b^(11/3))))*(7*A*b + 5*B*a)*1
i)/(108*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5
*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1
296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25
*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^
(13/3)*b^(11/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^
3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*
((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*
B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((
((3^(1/2)*1i)/2 - 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 52
5*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 - 1/
2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)
^(19/6)*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 - 1/2)^
2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2
*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*(49*
A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46
656*(-a)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 - 1/2)*(7*A*b + 5*B*a)*1i)/(108
*(-a)^(13/6)*b^(11/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2
*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3
) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^
2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*
b^(11/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125
*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) + (x^(1/2)*((3^(1/
2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3
))/(1296*(-a)^(19/6)*b^(11/6)))*1i)/(46656*(-a)^(13/3)*b^(11/3)))/((((3^(1/
2)*1i)/2 + 1/2)^2*(7*A*b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2
*a^2*b + 735*A^2*B*a*b^2)/(1296*a^3) - (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A
*b + 5*B*a)*(49*A^2*b^4 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)
*b^(11/6))))/(46656*(-a)^(13/3)*b^(11/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(7*A*
b + 5*B*a)^2*((343*A^3*b^3 + 125*B^3*a^3 + 525*A*B^2*a^2*b + 735*A^2*B*a*b^
2)/(1296*a^3) + (x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*(49*A^2*b^4
 + 25*B^2*a^2*b^2 + 70*A*B*a*b^3))/(1296*(-a)^(19/6)*b^(11/6))))/(46656*(-a
)^(13/3)*b^(11/3))))*((3^(1/2)*1i)/2 + 1/2)*(7*A*b + 5*B*a)*1i)/(108*(-a)^(
13/6)*b^(11/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(B*x**3+A)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.174 $\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=104

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

```
[Out] 1/6*(A*b-B*a)*x^(3/2)/a/b/(b*x^3+a)^2+1/12*(3*A*b+B*a)*x^(3/2)/a^2/b/(b*x^3
+a)+1/12*(3*A*b+B*a)*arctan(x^(3/2)*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 290, 329, 275, 205}

$$\frac{(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}+\frac{x^{3/2}(aB+3Ab)}{12a^2b\left(a+bx^3\right)}+\frac{x^{3/2}(Ab-aB)}{6ab\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((A*b - a*B)*x^(3/2))/(6*a*b*(a + b*x^3)^2) + ((3*A*b + a*B)*x^(3/2))/(12*a
^2*b*(a + b*x^3)) + ((3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a
^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
```

```
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}\left(A+Bx^3\right)}{\left(a+bx^3\right)^3}\,dx &= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{\left(\frac{9Ab}{2}+\frac{3aB}{2}\right)\int \frac{\sqrt{x}}{\left(a+bx^3\right)^2}\,dx}{6ab}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\,\mathrm{Subst}\left(\int \frac{x^2}{a+bx^6}\,dx,x,\sqrt{x}\right)}{4a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^{3/2}\right)}{12a^2b}\\
&= \frac{(Ab-aB)x^{3/2}}{6ab\left(a+bx^3\right)^2}+\frac{(3Ab+aB)x^{3/2}}{12a^2b\left(a+bx^3\right)}+\frac{(3Ab+aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.90

$$\frac{\frac{\sqrt{a}\sqrt{b}\,x^{3/2}\left(-a^2B+ab\left(5A+Bx^3\right)+3Ab^2x^3\right)}{\left(a+bx^3\right)^2}+(aB+3Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{12a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*(A + B*x^3))/(a + b*x^3)^3,x]
```

```
[Out] ((Sqrt[a]*Sqrt[b]*x^(3/2)*(-(a^2*B) + 3*A*b^2*x^3 + a*b*(5*A + B*x^3)))/(a
+ b*x^3)^2 + (3*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]])/(12*a^(5/2)*b
^(3/2))
```

fricas [A] time = 0.90, size = 313, normalized size = 3.01

$$\left[-\frac{\left(\left(Bab^2+3\,Ab^3\right)x^6+Ba^3+3\,Aa^2b+2\left(Ba^2b+3\,Aab^2\right)x^3\right)\sqrt{-ab}\,\log\left(\frac{bx^3-2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)-2\left(\left(Ba^2b^2+3\,Aab^3\right.\right.}{24\left(a^3b^4x^6+2\,a^4b^3x^3+a^5b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 + 3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b
^2)*x^3)*sqrt(-a*b)*log((b*x^3 - 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 + a)) - 2
*((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a^3*b^
4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2), 1/12*(((B*a*b^2 + 3*A*b^3)*x^6 + B*a^3 +
3*A*a^2*b + 2*(B*a^2*b + 3*A*a*b^2)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)
```

```
/a) + ((B*a^2*b^2 + 3*A*a*b^3)*x^4 - (B*a^3*b - 5*A*a^2*b^2)*x)*sqrt(x))/(a
^3*b^4*x^6 + 2*a^4*b^3*x^3 + a^5*b^2)]
```

giac [A] time = 0.18, size = 84, normalized size = 0.81

$$\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^2b}+\frac{Babx^{\frac{9}{2}}+3\,Ab^2x^{\frac{9}{2}}-Ba^2x^{\frac{3}{2}}+5\,Aabx^{\frac{3}{2}}}{12\,(bx^3+a)^2a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/12*(B*
a*b*x^(9/2) + 3*A*b^2*x^(9/2) - B*a^2*x^(3/2) + 5*A*a*b*x^(3/2))/((b*x^3 +
a)^2*a^2*b)
```

maple [A] time = 0.07, size = 97, normalized size = 0.93

$$\frac{A\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^2}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\sqrt{ab}\;ab}+\frac{\frac{(3Ab+Ba)x^{\frac{9}{2}}}{12a^2}+\frac{(5Ab-Ba)x^{\frac{3}{2}}}{12ab}}{(b\,x^3+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x)
```

```
[Out] 2/3*(1/8*(3*A*b+B*a)/a^2*x^(9/2)+1/8*(5*A*b-B*a)/a/b*x^(3/2))/(b*x^3+a)^2+1
/4/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(3/2))*A+1/12/a/b/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*B
```

maxima [A] time = 1.27, size = 96, normalized size = 0.92

$$\frac{(Bab+3\,Ab^2)x^{\frac{9}{2}}-(Ba^2-5\,Aab)x^{\frac{3}{2}}}{12\,(a^2b^3x^6+2\,a^3b^2x^3+a^4b)}+\frac{(Ba+3\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{12\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*x^(1/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*((B*a*b + 3*A*b^2)*x^(9/2) - (B*a^2 - 5*A*a*b)*x^(3/2))/(a^2*b^3*x^6 +
 2*a^3*b^2*x^3 + a^4*b) + 1/12*(B*a + 3*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(s
qrt(a*b)*a^2*b)
```

mupad [B] time = 2.71, size = 136, normalized size = 1.31

$$\frac{\frac{x^{9/2}\,(3\,A\,b+B\,a)}{12\,a^2}+\frac{x^{3/2}\,(5\,A\,b-B\,a)}{12\,a\,b}}{a^2+2\,a\,b\,x^3+b^2\,x^6}+\frac{\mathrm{atan}\left(\frac{b^{3/2}\,x^{3/2}\,(9\,A^2\,b^3+6\,A\,B\,a\,b^2+B^2\,a^2\,b)}{\sqrt{a}\,(3\,A\,b+B\,a)\,(3\,A\,b^3+B\,a\,b^2)}\right)(3\,A\,b+B\,a)}{12\,a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)*(A + B*x^3))/(a + b*x^3)^3,x)
```

```
[Out] ((x^(9/2)*(3*A*b + B*a))/(12*a^2) + (x^(3/2)*(5*A*b - B*a))/(12*a*b))/(a^2
+ b^2*x^6 + 2*a*b*x^3) + (atan((b^(3/2)*x^(3/2)*(9*A^2*b^3 + B^2*a^2*b + 6*
A*B*a*b^2))/(a^(1/2)*(3*A*b + B*a)*(3*A*b^3 + B*a*b^2)))*(3*A*b + B*a))/(12
*a^(5/2)*b^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*x**(1/2)/(b*x**3+a)**3,x)
[Out] Timed out
```

3.175 $\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=321

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}-\frac{5(aB+11}{}$$

```
[Out] 5/108*(11*A*b+B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/216*(
11*A*b+B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)+5/2
16*(11*A*b+B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(17/6)/b^(7/6)-
5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^
(17/6)/b^(7/6)*3^(1/2)+5/432*(11*A*b+B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1
/6)*3^(1/2)*x^(1/2))/a^(17/6)/b^(7/6)*3^(1/2)+1/6*(A*b-B*a)*x^(1/2)/a/b/(b*
x^3+a)^2+1/36*(11*A*b+B*a)*x^(1/2)/a^2/b/(b*x^3+a)
```

Rubi [A] time = 0.52, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {457, 290, 329, 209, 634, 618, 204, 628, 205}

$$-\frac{5(aB+11Ab)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}+\frac{5(aB+11Ab)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{17/6}b^{7/6}}-\frac{5(aB+11}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] ((A*b - a*B)*Sqrt[x])/(6*a*b*(a + b*x^3)^2) + ((11*A*b + a*B)*Sqrt[x])/(36*
a^2*b*(a + b*x^3)) - (5*(11*A*b + a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])
/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan[Sqrt[3] + (2*b
^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(17/6)*b^(7/6)) + (5*(11*A*b + a*B)*ArcTan
[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(17/6)*b^(7/6)) - (5*(11*A*b + a*B)*Log
[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17
/6)*b^(7/6)) + (5*(11*A*b + a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt
[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(17/6)*b^(7/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{x}\left(a+bx^3\right)^3}\,dx = \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\int \frac{1}{\sqrt{x}\left(a+bx^3\right)^2}\,dx}{12ab}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\int \frac{1}{\sqrt{x}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\,\text{Subst}\left(\int \frac{1}{a+bx^6}\,dx, x, \sqrt{x}\right)}{36a^2b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{(5(11Ab+aB))\,\text{Subst}\left(\int \frac{\sqrt[6]{a}-\frac{1}{2}\sqrt{3}\sqrt[6]{b}x}{\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}x+\sqrt[3]{b}x^2}\,dx, x, \sqrt{}\right.}{108a^{17/6}b}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{(5(11Ab+aB))\,\text{Subs}}{1}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} + \frac{5(11Ab+aB)\tan^{-1}\left(\frac{\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{108a^{17/6}b^{7/6}} - \frac{5(11Ab+aB)\log\left(\sqrt[3]{}\right.}{144}$$

$$= \frac{(Ab-aB)\sqrt{x}}{6ab\left(a+bx^3\right)^2} + \frac{(11Ab+aB)\sqrt{x}}{36a^2b\left(a+bx^3\right)} - \frac{5(11Ab+aB)\tan^{-1}\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt{x}}{\sqrt[6]{a}}\right)}{216a^{17/6}b^{7/6}} + \frac{5(11Ab+aB)}{2}$$

Mathematica [C] time = 0.05, size = 91, normalized size = 0.28

$$\frac{\sqrt{x}\left(a\left(-5a^2B+ab\left(17A+Bx^3\right)+11Ab^2x^3\right)+5\left(a+bx^3\right)^2(aB+11Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)\right)}{36a^3b\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[x]*(a + b*x^3)^3),x]
```

```
[Out] (Sqrt[x]*(a*(-5*a^2*B + 11*A*b^2*x^3 + a*b*(17*A + B*x^3)) + 5*(11*A*b + a*
B)*(a + b*x^3)^2*Hypergeometric2F1[1/6, 1, 7/6, -((b*x^3)/a)]))/(36*a^3*b*(
a + b*x^3)^2)
```

fricas [B] time = 1.18, size = 2674, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/432*(20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B
^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^
2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*arctan(1/3*
(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2
+ 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 177
1561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B*a*b + 121*A^2*b^2)*x +
(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
```

```
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*(-(B^6*a^6 + 66*A*B^5*a^
5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4
 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - 2*sqrt(3)*(B*a
^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4
*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*
b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) + sqrt(3)*(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^
2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5
*B*a*b^5 + 1771561*A^6*b^6)) + 20*sqrt(3)*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^
4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3
*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17
*b^7))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b
 + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + (B^2*a^2 + 22*A*B
*a*b + 121*A^2*b^2)*x - (B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6))*a^14*b^6*
(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3
+ 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7)
)^(5/6) - 2*sqrt(3)*(B*a^15*b^6 + 11*A*a^14*b^7)*sqrt(x)*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(5/6) - sqrt(3)*
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6))/(B^6*a^6 +
66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*
B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)) + 5*(a^2*b^3*x^6 + 2*a
^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26
620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561
*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*B^5*a^5*b + 18
15*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 96630
6*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*a^2 + 22*A*B*a
*b + 121*A^2*b^2)*x + 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(B^6*a^6 + 66*A
*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*
a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)) - 5*(a^2
*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^
4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a
*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(25*a^6*b^2*(-(B^6*a^6 + 66*A*
B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a
^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/3) + 25*(B^2*
a^2 + 22*A*B*a*b + 121*A^2*b^2)*x - 25*(B*a^4*b + 11*A*a^3*b^2)*sqrt(x)*(-(
B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2
19615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(
1/6)) + 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-(B^6*a^6 + 66*A*B^5*a^5*
b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 +
 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6)*log(5*a^3*b*(-(B^6
*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 + 2196
15*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^(1/6
) + 5*(B*a + 11*A*b)*sqrt(x)) - 10*(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*b)*(-
(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 26620*A^3*B^3*a^3*b^3 +
219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*A^6*b^6)/(a^17*b^7))^
(1/6)*log(-5*a^3*b*(-(B^6*a^6 + 66*A*B^5*a^5*b + 1815*A^2*B^4*a^4*b^2 + 266
20*A^3*B^3*a^3*b^3 + 219615*A^4*B^2*a^2*b^4 + 966306*A^5*B*a*b^5 + 1771561*
A^6*b^6)/(a^17*b^7))^(1/6) + 5*(B*a + 11*A*b)*sqrt(x)) + 12*((B*a*b + 11*A*
b^2)*x^3 - 5*B*a^2 + 17*A*a*b)*sqrt(x))/(a^2*b^3*x^6 + 2*a^3*b^2*x^3 + a^4*
b)
```

giac [A] time = 0.24, size = 322, normalized size = 1.00

$$\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432a^3b^2}-\frac{5\sqrt{3}\left(\left(ab^5\right)^{\frac{1}{6}}Ba+11\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(-\sqrt{3}\sqrt{x}\right.}{432a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 5/432*sqrt(3)*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqrt(x
)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^3*b^2) - 5/432*sqrt(3)*((a*b^5)^(1/6)*B
*a + 11*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/
3))/(a^3*b^2) + 5/216*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan((sq
rt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^3*b^2) + 5/216*((a*b^5)^(1/6
)*B*a + 11*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/
b)^(1/6))/(a^3*b^2) + 5/108*((a*b^5)^(1/6)*B*a + 11*(a*b^5)^(1/6)*A*b)*arct
an(sqrt(x)/(a/b)^(1/6))/(a^3*b^2) + 1/36*(B*a*b*x^(7/2) + 11*A*b^2*x^(7/2)
- 5*B*a^2*sqrt(x) + 17*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^2*b)
```

maple [A] time = 0.17, size = 407, normalized size = 1.27

$$\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^3}+\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{216a^3}+\frac{55\left(\frac{a}{b}\right)^{\frac{1}{6}}A\arctan\left(\frac{2\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{216a^3}+\frac{55\sqrt{3}\left(\frac{a}{b}\right)^{\frac{1}{6}}A\ln\left(x+\right.}{43}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x)
```

```
[Out] 2*(1/72*(11*A*b+B*a)/a^2*x^(7/2)+1/72*(17*A*b-5*B*a)/a/b*x^(1/2))/(b*x^3+a)
^2+55/108/a^3*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*A+5/108/a^2/b*(a/b)
^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))*B-55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(-x
+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*A-5/432/a^2/b*3^(1/2)*(a/b)^(1/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2/(a/b
)^(1/6)*x^(1/2)-3^(1/2))*B+55/432/a^3*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b
)^(1/6)*x^(1/2)+(a/b)^(1/3))*A+5/432/a^2/b*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)
*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))*B+55/216/a^3*(a/b)^(1/6)*arctan(2/(a/b)^(
1/6)*x^(1/2)+3^(1/2))*A+5/216/a^2/b*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2
)+3^(1/2))*B
```

maxima [A] time = 1.21, size = 336, normalized size = 1.05

$$\frac{\left(Bab+11\,Ab^2\right)x^{\frac{7}{2}}-\left(5\,Ba^2-17\,Aab\right)\sqrt{x}}{36\left(a^2b^3x^6+2\,a^3b^2x^3+a^4b\right)}+\frac{5\left(\frac{\sqrt{3}\left(Ba+11\,Ab\right)\log\left(\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\left(Ba+11\,Ab\right)\log\left(-\sqrt{3}a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x\right.}{a^{\frac{5}{6}}b^{\frac{1}{6}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/36*((B*a*b + 11*A*b^2)*x^(7/2) - (5*B*a^2 - 17*A*a*b)*sqrt(x))/(a^2*b^3*x
^6 + 2*a^3*b^2*x^3 + a^4*b) + 5/432*(sqrt(3)*(B*a + 11*A*b)*log(sqrt(3)*a^(
```

```
1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) - sqrt(3)*(B*
a + 11*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^
(5/6)*b^(1/6)) + 4*(B*a*b^(1/3) + 11*A*b^(4/3))*arctan(b^(1/3)*sqrt(x)/sqrt
(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^(4/3)*b
^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*
sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))) + 2*(B*a^
(4/3)*b^(1/3) + 11*A*a^(1/3)*b^(4/3))*arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*
b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/
(a^2*b)
```

mupad [B] time = 2.95, size = 1952, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(1/2)*(a + b*x^3)^3),x)
```

```
[Out] ((x^(7/2)*(11*A*b + B*a))/(36*a^2) + (x^(1/2)*(17*A*b - 5*B*a))/(36*a*b))/(
a^2 + b^2*x^6 + 2*a*b*x^3) - (atan(((((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4
*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^
8) - (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33
*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)
^(17/6)*b^(7/6)) + (((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*
a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6)))*(11*A*b + B*a)*5i)/(216*(-a)^(17/6)*b^(7/6)))
/((5*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*
A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*(11*A*b + B*a)*(1331*A
^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(4
7/6)*b^(7/6)))*(11*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6)) - (5*((625*x^(1/2)
*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A
*B^3*a^3*b^2))/(279936*a^8) + (625*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b
^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*(11
*A*b + B*a))/(216*(-a)^(17/6)*b^(7/6))))*(11*A*b + B*a)*5i)/(108*(-a)^(17/6
)*b^(7/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(1
4641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^
3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331
*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^
(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)) + (((3^(1/2)*1i)/2 - 1/2)*(1
1*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3
 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/
2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*
A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6))
)/((5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 +
B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(27
9936*a^8) - (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*
a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))
))/(216*(-a)^(17/6)*b^(7/6)) - (5*((3^(1/2)*1i)/2 - 1/2)*(11*A*b + B*a)*((6
25*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*
b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 - 1/2)*(11*A*b
 + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/
(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2
- 1/2)*(11*A*b + B*a)*5i)/(108*(-a)^(17/6)*b^(7/6)) - (atan(((((3^(1/2)*1i)
/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2
*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((
3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B
*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6)))*5i)/(216*(-a)^(17
/6)*b^(7/6)) + (((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*
A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3
*b^2))/(279936*a^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*
```

```
b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6
)*b^(7/6)))*5i)/(216*(-a)^(17/6)*b^(7/6)))/((5*((3^(1/2)*1i)/2 + 1/2)*(11*A
*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^4*b + 726*A^2*B^2*a^2*b^3 +
5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a^8) - (625*((3^(1/2)*1i)/2 +
 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^2 + 363*A^2*B*a*b^4 + 33*A*B
^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(216*(-a)^(17/6)*b^(7/6)) - (5*
((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*((625*x^(1/2)*(14641*A^4*b^5 + B^4*a^
4*b + 726*A^2*B^2*a^2*b^3 + 5324*A^3*B*a*b^4 + 44*A*B^3*a^3*b^2))/(279936*a
^8) + (625*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*(1331*A^3*b^5 + B^3*a^3*b^
2 + 363*A^2*B*a*b^4 + 33*A*B^2*a^2*b^3))/(279936*(-a)^(47/6)*b^(7/6))))/(21
6*(-a)^(17/6)*b^(7/6))))*((3^(1/2)*1i)/2 + 1/2)*(11*A*b + B*a)*5i)/(108*(-a
)^(17/6)*b^(7/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**3/x**(1/2),x)
```

```
[Out] Timed out
```

3.176 $\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=351

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13A}{}$$

```
[Out] -7/108*(13*A*b-B*a)*arctan(b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/216*
(13*A*b-B*a)*arctan(-3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)-7/
216*(13*A*b-B*a)*arctan(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(19/6)/b^(5/6)
-7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a
^(19/6)/b^(5/6)*3^(1/2)+7/432*(13*A*b-B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(
1/6)*3^(1/2)*x^(1/2))/a^(19/6)/b^(5/6)*3^(1/2)-7/36*(13*A*b-B*a)/a^3/b/x^(1
/2)+1/6*(A*b-B*a)/a/b/(b*x^3+a)^2/x^(1/2)+1/36*(13*A*b-B*a)/a^2/b/(b*x^3+a)
/x^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 295, 634, 618, 204, 628, 205}

$$-\frac{7(13Ab-aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13Ab-aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}a^{19/6}b^{5/6}}+\frac{7(13A}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] (-7*(13*A*b - a*B))/(36*a^3*b*Sqrt[x]) + (A*b - a*B)/(6*a*b*Sqrt[x]*(a + b*
x^3)^2) + (13*A*b - a*B)/(36*a^2*b*Sqrt[x]*(a + b*x^3)) + (7*(13*A*b - a*B)
*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6)*b^(5/6)) - (7
*(13*A*b - a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(19/6
)*b^(5/6)) - (7*(13*A*b - a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1/6)])/(108*a^(1
9/6)*b^(5/6)) - (7*(13*A*b - a*B)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*Sqr
t[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6)*b^(5/6)) + (7*(13*A*b - a*B)*Log[a
^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(19/6
)*b^(5/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{3/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{\left(\frac{13Ab}{2}-\frac{aB}{2}\right)\int \frac{1}{x^{3/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}+\frac{(7(13Ab-aB))\int \frac{1}{x^{3/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\int \frac{x^{3/2}}{a+bx^3}}{72a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\,\mathrm{Subst}\left(\right.}{36a^3}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{(7(13Ab-aB))\,\mathrm{Subst}\left(\right.}{10}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}}{}\right.}{108a^{19/6}b^{5/6}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}-\frac{7(13Ab-aB)\tan^{-1}\left(\frac{\sqrt[6]{b}}{}\right.}{108a^{19/6}b^{5/6}}$$

$$= -\frac{7(13Ab-aB)}{36a^3b\sqrt{x}}+\frac{Ab-aB}{6ab\sqrt{x}\left(a+bx^3\right)^2}+\frac{13Ab-aB}{36a^2b\sqrt{x}\left(a+bx^3\right)}+\frac{7(13Ab-aB)\tan^{-1}\left(\sqrt{3}\right.}{216a^{19/6}b^{5/6}}$$

Mathematica [C] time = 0.13, size = 113, normalized size = 0.32

$$2\left(-\frac{x^{5/2}(Ab-aB)\,{}_2F_1\left(\frac{5}{6},3;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4}-\frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},1;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4}-\frac{Abx^{5/2}\,{}_2F_1\left(\frac{5}{6},2;\frac{11}{6};-\frac{bx^3}{a}\right)}{5a^4}-\frac{A}{a^3\sqrt{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x]
```

```
[Out] 2*(-(A/(a^3*Sqrt[x])) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 1, 11/6, -((b*x
^3)/a)])/(5*a^4) - (A*b*x^(5/2)*Hypergeometric2F1[5/6, 2, 11/6, -((b*x^3)/a
)])/(5*a^4) - ((A*b - a*B)*x^(5/2)*Hypergeometric2F1[5/6, 3, 11/6, -((b*x^3
)/a)])/(5*a^4))
```

fricas [B] time = 1.14, size = 3904, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432*(28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5
*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*
```

b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arctan(1/3*(
2*sqrt(3)*sqrt((B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^6 -
21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*sqrt(
x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b
^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*
b^5))^(5/6) + (B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8*b^2 - 263640*
A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*a^5*b^5 + 1013
629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 36707882445*A^8*B^2*a
^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x - (B^6*a^19*b
^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 4
28415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(
B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 4
28415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^
(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3
*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b
^6)/(a^19*b^5))^(1/6) + 2*sqrt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*
B^3*a^6*b^3 - 21970*A^3*B^2*a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3
*b^6)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6) - sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a
^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b
^5 + 4826809*A^6*b^6))/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)) + 28*sqrt(3)*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 -
78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*
B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*arct
an(1/50421*(2*sqrt(3)*sqrt(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1
690*A^2*B^3*a^19*b^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371
293*A^5*a^16*b^9)*sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^
2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 +
4826809*A^6*b^6)/(a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b
 + 7605*A^2*B^8*a^8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4
- 93565836*A^5*B^5*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^
3*a^3*b^7 + 36707882445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858
491849*A^10*b^10)*x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^
2*B^4*a^17*b^5 - 43940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758
*A^5*B*a^14*b^8 + 4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*
A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*
A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(2/3))*a^3*b*(-(B^6*a^6 - 78*A*B
^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^
2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 33614*sq
rt(3)*(B^5*a^8*b - 65*A*B^4*a^7*b^2 + 1690*A^2*B^3*a^6*b^3 - 21970*A^3*B^2*
a^5*b^4 + 142805*A^4*B*a^4*b^5 - 371293*A^5*a^3*b^6)*sqrt(x)*(-(B^6*a^6 - 7
8*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B
^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6) + 168
07*sqrt(3)*(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3
*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6))
/(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 +
 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)) - 14*(a^3
*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*
a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*
b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(1/6)*log(16807*a^16*b^4*(-(B^6*a^6 - 78
*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^
2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 1680
7*(B^5*a^5 - 65*A*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3
+ 142805*A^4*B*a*b^4 - 371293*A^5*b^5)*sqrt(x)) + 14*(a^3*b^2*x^7 + 2*a^4*b
*x^4 + a^5*x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^
3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*
b^6)/(a^19*b^5))^(1/6)*log(-16807*a^16*b^4*(-(B^6*a^6 - 78*A*B^5*a^5*b + 25

```
35*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 22277
58*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(5/6) - 16807*(B^5*a^5 - 65*A
*B^4*a^4*b + 1690*A^2*B^3*a^3*b^2 - 21970*A^3*B^2*a^2*b^3 + 142805*A^4*B*a*
b^4 - 371293*A^5*b^5)*sqrt(x)) + 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b^
6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*s
qrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a
^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a
^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^8
*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5*
a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 3670788
2445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*x
 - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 43
940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 + 4
826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 4
3940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 48268
09*A^6*b^6)/(a^19*b^5))^(2/3)) - 7*(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)*(-(B
^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*a^3*b^3 + 42
8415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(a^19*b^5))^(
1/6)*log(-282475249*(B^5*a^21*b^4 - 65*A*B^4*a^20*b^5 + 1690*A^2*B^3*a^19*b
^6 - 21970*A^3*B^2*a^18*b^7 + 142805*A^4*B*a^17*b^8 - 371293*A^5*a^16*b^9)*
sqrt(x)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 - 43940*A^3*B^3*
a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826809*A^6*b^6)/(
a^19*b^5))^(5/6) + 282475249*(B^10*a^10 - 130*A*B^9*a^9*b + 7605*A^2*B^8*a^
8*b^2 - 263640*A^3*B^7*a^7*b^3 + 5997810*A^4*B^6*a^6*b^4 - 93565836*A^5*B^5
*a^5*b^5 + 1013629890*A^6*B^4*a^4*b^6 - 7529822040*A^7*B^3*a^3*b^7 + 367078
82445*A^8*B^2*a^2*b^8 - 106044993730*A^9*B*a*b^9 + 137858491849*A^10*b^10)*
x - 282475249*(B^6*a^19*b^3 - 78*A*B^5*a^18*b^4 + 2535*A^2*B^4*a^17*b^5 - 4
3940*A^3*B^3*a^16*b^6 + 428415*A^4*B^2*a^15*b^7 - 2227758*A^5*B*a^14*b^8 +
4826809*A^6*a^13*b^9)*(-(B^6*a^6 - 78*A*B^5*a^5*b + 2535*A^2*B^4*a^4*b^2 -
43940*A^3*B^3*a^3*b^3 + 428415*A^4*B^2*a^2*b^4 - 2227758*A^5*B*a*b^5 + 4826
809*A^6*b^6)/(a^19*b^5))^(2/3)) + 12*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2
- 13*A*a*b)*x^3 - 72*A*a^2)*sqrt(x))/(a^3*b^2*x^7 + 2*a^4*b*x^4 + a^5*x)
```

giac [A] time = 0.48, size = 329, normalized size = 0.94

$$-\frac{2\,A}{a^{3}\sqrt{x}}+\frac{7\,Babx^{\frac{11}{2}}-19\,Ab^{2}x^{\frac{11}{2}}+13\,Ba^{2}x^{\frac{5}{2}}-25\,Aabx^{\frac{5}{2}}}{36\,\left(bx^{3}+a\right)^{2}a^{3}}-\frac{7\,\sqrt{3}\left(\left(ab^{5}\right)^{\frac{5}{6}}Ba-13\,\left(ab^{5}\right)^{\frac{5}{6}}Ab\right)\log\left(\sqrt{3}\,\sqrt{x}\,\left(\frac{a}{b}\right)^{\frac{1}{6}}+\right.}{432\,a^{4}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] -2*A/(a^3*sqrt(x)) + 1/36*(7*B*a*b*x^(11/2) - 19*A*b^2*x^(11/2) + 13*B*a^2*
x^(5/2) - 25*A*a*b*x^(5/2))/((b*x^3 + a)^2*a^3) - 7/432*sqrt(3)*((a*b^5)^(5
/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*log(sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b^5) + 7/432*sqrt(3)*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b
)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b^5) + 7/216*((a
*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan((sqrt(3)*(a/b)^(1/6) + 2*sqr
t(x))/(a/b)^(1/6))/(a^4*b^5) + 7/216*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*
A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x))/(a/b)^(1/6))/(a^4*b^5) + 7/1
08*((a*b^5)^(5/6)*B*a - 13*(a*b^5)^(5/6)*A*b)*arctan(sqrt(x)/(a/b)^(1/6))/(
a^4*b^5)
```

maple [A] time = 0.17, size = 441, normalized size = 1.26

$$-\frac{19 A\,b^{2} x^{\frac{11}{2}}}{36 \left(b\,x^{3}+a\right)^{2} a^{3}}+\frac{7 B b\,x^{\frac{11}{2}}}{36 \left(b\,x^{3}+a\right)^{2} a^{2}}-\frac{25 A b\,x^{\frac{5}{2}}}{36 \left(b\,x^{3}+a\right)^{2} a^{2}}+\frac{13 B\,x^{\frac{5}{2}}}{36 \left(b\,x^{3}+a\right)^{2} a}-\frac{91 A \arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^{3}}-\frac{91 A \arctan\left(\frac{2 \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right.}{216 \left(\frac{a}{b}\right)^{\frac{1}{6}} a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x)
```

```
[Out] -19/36/a^3/(b*x^3+a)^2*x^(11/2)*b^2*A+7/36/a^2/(b*x^3+a)^2*x^(11/2)*B*b-25/
36/a^2/(b*x^3+a)^2*A*x^(5/2)*b+13/36/a/(b*x^3+a)^2*B*x^(5/2)-91/108/a^3*A/(
a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))-91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)
*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*ar
ctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))+91/432/a^4*A*b*3^(1/2)*(a/b)^(5/6)*ln(x
+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-91/216/a^3*A/(a/b)^(1/6)*arctan(2
/(a/b)^(1/6)*x^(1/2)+3^(1/2))+7/108/a^2*B/b/(a/b)^(1/6)*arctan(1/(a/b)^(1/6
)*x^(1/2))+7/432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2
)-(a/b)^(1/3))+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/
2))-7/432/a^3*B*3^(1/2)*(a/b)^(5/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^
(1/3))+7/216/a^2*B/b/(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))-2*A/
a^3/x^(1/2)
```

maxima [A] time = 1.29, size = 273, normalized size = 0.78

$$\frac{7 \left(B a b-13 A b^{2}\right) x^{6}+13 \left(B a^{2}-13 A a b\right) x^{3}-72 A a^{2}}{36 \left(a^{3} b^{2} x^{\frac{13}{2}}+2 a^{4} b x^{\frac{7}{2}}+a^{5} \sqrt{x}\right)}-\frac{7 \left(B a-13 A b\right) \left(\frac{\sqrt{3} \log\left(\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}} \sqrt{x}+b^{\frac{1}{3}} x+a^{\frac{1}{3}}\right)}{a^{\frac{1}{6}} b^{\frac{5}{6}}}-\frac{\sqrt{3} \log\left(-\sqrt{3} a^{\frac{1}{6}} b^{\frac{1}{6}}\right.}{a^{\frac{1}{6}} b^{\frac{5}{6}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(3/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(7*(B*a*b - 13*A*b^2)*x^6 + 13*(B*a^2 - 13*A*a*b)*x^3 - 72*A*a^2)/(a^3
*b^2*x^(13/2) + 2*a^4*b*x^(7/2) + a^5*sqrt(x)) - 7/432*(B*a - 13*A*b)*(sqrt
(3)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a^(1/6)*b^(
5/6)) - sqrt(3)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))
/(a^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))
/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt(a^(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3
)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(b^(2/3)*sqrt
(a^(1/3)*b^(1/3))) - 4*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(b^(2/
3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

mupad [B] time = 2.91, size = 1786, normalized size = 5.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(3/2)*(a + b*x^3)^3),x)
```

```
[Out] (atan((((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^2
1*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343
*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*
B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/
```

6)))*1i)/((-a)^(19/3)*b^(5/3)) + ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 2156456413259366 4*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/(((13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - ((13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) - ((2*A)/a + (13*x^3*(13*A*b - B*a))/(36*a^2) + (7*b*x^6*(13*A*b - B*a))/(36*a^3))/(a^2*x^(1/2) + b^2*x^(13/2) + 2*a*b*x^(7/2)) + (atan(((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 - 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 - 1/2)*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6)) + (atan(((((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)) + (((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6)))*1i)/((-a)^(19/3)*b^(5/3)))/((((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(28229306112*B^3*a^24*b^3 - 62019785528064*A^3*a^21*b^6 - 1100942938368*A*B^2*a^23*b^4 + 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3)) - (((3^(1/2)*1i)/2 + 1/2)^2*(13*A*b - B*a)^2*(62019785528064*A^3*a^21*b^6 - 28229306112*B^3*a^24*b^3 + 1100942938368*A*B^2*a^23*b^4 - 14312258198784*A^2*B*a^22*b^5 + (343*x^(1/2)*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*(140169666861858816*A^2*a^24*b^6 + 829406312792064*B^2*a^26*b^4 - 21564564132593664*A*B*a^25*b^5))/(10077696*(-a)^(19/6)*b^(5/6))))/((-a)^(19/3)*b^(5/3))))*((3^(1/2)*1i)/2 + 1/2)*(13*A*b - B*a)*7i)/(108*(-a)^(19/6)*b^(5/6))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(3/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.177 $\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3} dx$

Optimal. Leaf size=129

$$-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{aB-5Ab}{4a^3bx^{3/2}}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

```
[Out] 1/4*(-5*A*b+B*a)/a^3/b/x^(3/2)+1/6*(A*b-B*a)/a/b/x^(3/2)/(b*x^3+a)^2+1/12*(
5*A*b-B*a)/a^2/b/x^(3/2)/(b*x^3+a)-1/4*(5*A*b-B*a)*arctan(x^(3/2)*b^(1/2)/a
^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 130, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {457, 290, 325, 329, 275, 205}

$$\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{5Ab-aB}{4a^3bx^{3/2}}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] -(5*A*b - a*B)/(4*a^3*b*x^(3/2)) + (A*b - a*B)/(6*a*b*x^(3/2)*(a + b*x^3)^2
) + (5*A*b - a*B)/(12*a^2*b*x^(3/2)*(a + b*x^3)) - ((5*A*b - a*B)*ArcTan[(S
qrt[b]*x^(3/2))/Sqrt[a]])/(4*a^(7/2)*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{5/2}\left(a+bx^3\right)^3}\,dx = \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{15Ab}{2}-\frac{3aB}{2}\right)\int \frac{1}{x^{5/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$= \frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}+\frac{(3(5Ab-aB))\int \frac{1}{x^{5/2}\left(a+bx^3\right)}\,dx}{8a^2b}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\int \frac{\sqrt{x}}{a+bx^3}\,dx}{8a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(3(5Ab-aB))\operatorname{Subst}\left(\int \frac{x^2}{a+bx^6}\,d\right.}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,\right.}{4a^3}$$

$$= -\frac{5Ab-aB}{4a^3bx^{3/2}}+\frac{Ab-aB}{6abx^{3/2}\left(a+bx^3\right)^2}+\frac{5Ab-aB}{12a^2bx^{3/2}\left(a+bx^3\right)}-\frac{(5Ab-aB)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}$$

Mathematica [A] time = 0.19, size = 102, normalized size = 0.79

$$\frac{(aB-5Ab)\tan^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{4a^{7/2}\sqrt{b}}+\frac{a^2\left(5Bx^3-8A\right)+a\left(3bBx^6-25Abx^3\right)-15Ab^2x^6}{12a^3x^{3/2}\left(a+bx^3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x]
```

```
[Out] (-15*A*b^2*x^6 + a^2*(-8*A + 5*B*x^3) + a*(-25*A*b*x^3 + 3*b*B*x^6))/(12*a^
3*x^(3/2)*(a + b*x^3)^2) + ((-5*A*b + a*B)*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a]
])/(4*a^(7/2)*Sqrt[b])
```

fricas [A] time = 0.91, size = 347, normalized size = 2.69

$$\left[\frac{3\left(\left(Bab^2-5\,Ab^3\right)x^8+2\left(Ba^2b-5\,Aab^2\right)x^5+\left(Ba^3-5\,Aa^2b\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^3+2\,\sqrt{-ab}\,x^{\frac{3}{2}}-a}{bx^3+a}\right)+2\left(3\left(Ba^2b^2-5\,A\right.\right.}{24\left(a^4b^3x^8+2\,a^5b^2x^5+a^6bx^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(3*((B*a*b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 -
5*A*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^3 + 2*sqrt(-a*b)*x^(3/2) - a)/(b*x^3 +
a)) + 2*(3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b
^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2*x^5 + a^6*b*x^2), 1/12*(3*((B*a*
b^2 - 5*A*b^3)*x^8 + 2*(B*a^2*b - 5*A*a*b^2)*x^5 + (B*a^3 - 5*A*a^2*b)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x^(3/2)/a) + (3*(B*a^2*b^2 - 5*A*a*b^3)*x^6 - 8
*A*a^3*b + 5*(B*a^3*b - 5*A*a^2*b^2)*x^3)*sqrt(x))/(a^4*b^3*x^8 + 2*a^5*b^2
*x^5 + a^6*b*x^2)]
```

giac [A] time = 0.18, size = 88, normalized size = 0.68

$$\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}-\frac{2\,A}{3\,a^3x^{\frac{3}{2}}}+\frac{3\,Babx^{\frac{9}{2}}-7\,Ab^2x^{\frac{9}{2}}+5\,Ba^2x^{\frac{3}{2}}-9\,Aabx^{\frac{3}{2}}}{12\,\left(bx^3+a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*(B*a - 5*A*b)*arctan(b*x^(3/2)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/3*A/(a^3*
x^(3/2)) + 1/12*(3*B*a*b*x^(9/2) - 7*A*b^2*x^(9/2) + 5*B*a^2*x^(3/2) - 9*A*
a*b*x^(3/2))/((b*x^3 + a)^2*a^3)
```

maple [A] time = 0.07, size = 133, normalized size = 1.03

$$-\frac{7A\,b^2x^{\frac{9}{2}}}{12\left(b\,x^3+a\right)^2a^3}+\frac{Bb\,x^{\frac{9}{2}}}{4\left(b\,x^3+a\right)^2a^2}-\frac{3Ab\,x^{\frac{3}{2}}}{4\left(b\,x^3+a\right)^2a^2}+\frac{5B\,x^{\frac{3}{2}}}{12\left(b\,x^3+a\right)^2a}-\frac{5Ab\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^3}+\frac{B\arctan\left(\frac{b\,x^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x)
```

```
[Out] -7/12/a^3/(b*x^3+a)^2*x^(9/2)*b^2*A+1/4/a^2/(b*x^3+a)^2*x^(9/2)*B*b-3/4/a^2
/(b*x^3+a)^2*A*x^(3/2)*b+5/12/a/(b*x^3+a)^2*B*x^(3/2)-5/4/a^3/(a*b)^(1/2)*a
rctan(1/(a*b)^(1/2)*b*x^(3/2))*A*b+1/4/a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)
*b*x^(3/2))*B-2/3/a^3*A/x^(3/2)
```

maxima [A] time = 1.35, size = 100, normalized size = 0.78

$$\frac{3\left(Bab-5\,Ab^2\right)x^6+5\left(Ba^2-5\,Aab\right)x^3-8\,Aa^2}{12\left(a^3b^2x^{\frac{15}{2}}+2\,a^4bx^{\frac{9}{2}}+a^5x^{\frac{3}{2}}\right)}+\frac{(Ba-5\,Ab)\arctan\left(\frac{bx^{\frac{3}{2}}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(5/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(3*(B*a*b - 5*A*b^2)*x^6 + 5*(B*a^2 - 5*A*a*b)*x^3 - 8*A*a^2)/(a^3*b^2
*x^(15/2) + 2*a^4*b*x^(9/2) + a^5*x^(3/2)) + 1/4*(B*a - 5*A*b)*arctan(b*x^(
3/2)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 2.73, size = 163, normalized size = 1.26

$$-\frac{\frac{2\,A}{3\,a}+\frac{5\,x^3\,(5\,A\,b-B\,a)}{12\,a^2}+\frac{b\,x^6\,(5\,A\,b-B\,a)}{4\,a^3}}{a^2\,x^{3/2}+b^2\,x^{15/2}+2\,a\,b\,x^{9/2}}-\frac{\mathrm{atan}\left(\frac{8\,a^{7/2}\,\sqrt{b}\,x^{3/2}\,\left(86400\,A^2\,a^9\,b^5-34560\,A\,B\,a^{10}\,b^4+3456\,B^2\,a^{11}\,b^3\right)}{(5\,A\,b-B\,a)\,\left(138240\,A\,a^{13}\,b^4-27648\,B\,a^{14}\,b^3\right)}\right)\,(5\,A\,b-B\,a)}{4\,a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(5/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(3*a) + (5*x^3*(5*A*b - B*a))/(12*a^2) + (b*x^6*(5*A*b - B*a))/(4*
a^3))/(a^2*x^(3/2) + b^2*x^(15/2) + 2*a*b*x^(9/2)) - (atan((8*a^(7/2)*b^(1/
2)*x^(3/2)*(86400*A^2*a^9*b^5 + 3456*B^2*a^11*b^3 - 34560*A*B*a^10*b^4))/((
5*A*b - B*a)*(138240*A*a^13*b^4 - 27648*B*a^14*b^3)))*(5*A*b - B*a))/(4*a^(
7/2)*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(5/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.178 $\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx$

Optimal. Leaf size=351

$$\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}+\frac{11(}{}$$

```
[Out] -11/180*(17*A*b-5*B*a)/a^3/b/x^(5/2)+1/6*(A*b-B*a)/a/b/x^(5/2)/(b*x^3+a)^2+
1/36*(17*A*b-5*B*a)/a^2/b/x^(5/2)/(b*x^3+a)-11/108*(17*A*b-5*B*a)*arctan(b^
(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan(-3^(1/
2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)-11/216*(17*A*b-5*B*a)*arctan
(3^(1/2)+2*b^(1/6)*x^(1/2)/a^(1/6))/a^(23/6)/b^(1/6)+11/432*(17*A*b-5*B*a)*
ln(a^(1/3)+b^(1/3)*x-a^(1/6)*b^(1/6)*3^(1/2)*x^(1/2))/a^(23/6)/b^(1/6)*3^(1
/2)-11/432*(17*A*b-5*B*a)*ln(a^(1/3)+b^(1/3)*x+a^(1/6)*b^(1/6)*3^(1/2)*x^(1
/2))/a^(23/6)/b^(1/6)*3^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {457, 290, 325, 329, 209, 634, 618, 204, 628, 205}

$$\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{11(17Ab-5aB)\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt{x}+\sqrt[3]{a}+\sqrt[3]{b}x\right)}{144\sqrt{3}\,a^{23/6}\sqrt[6]{b}}-\frac{11(17Ab-5aB)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] (-11*(17*A*b - 5*a*B))/(180*a^3*b*x^(5/2)) + (A*b - a*B)/(6*a*b*x^(5/2)*(a
+ b*x^3)^2) + (17*A*b - 5*a*B)/(36*a^2*b*x^(5/2)*(a + b*x^3)) + (11*(17*A*b
 - 5*a*B)*ArcTan[Sqrt[3] - (2*b^(1/6)*Sqrt[x])/a^(1/6)])/(216*a^(23/6)*b^(1
/6)) - (11*(17*A*b - 5*a*B)*ArcTan[Sqrt[3] + (2*b^(1/6)*Sqrt[x])/a^(1/6)])/
(216*a^(23/6)*b^(1/6)) - (11*(17*A*b - 5*a*B)*ArcTan[(b^(1/6)*Sqrt[x])/a^(1
/6)])/(108*a^(23/6)*b^(1/6)) + (11*(17*A*b - 5*a*B)*Log[a^(1/3) - Sqrt[3]*a
^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])/(144*Sqrt[3]*a^(23/6)*b^(1/6)) - (11*(
17*A*b - 5*a*B)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*Sqrt[x] + b^(1/3)*x])
/(144*Sqrt[3]*a^(23/6)*b^(1/6))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
```

```
PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^{7/2}\left(a+bx^3\right)^3}\,dx=\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{\left(\frac{17Ab}{2}-\frac{5aB}{2}\right)\int \frac{1}{x^{7/2}\left(a+bx^3\right)^2}\,dx}{6ab}$$

$$=\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{(11(17Ab-5aB))\int \frac{1}{x^{7/2}\left(a+bx^3\right)}\,dx}{72a^2b}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\int -}{72a^3}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\,\mathrm{Sul}}{36}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{(11(17Ab-5aB))\,\mathrm{Sul}}{}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-}}{108a^{23/6}\sqrt[6]{b}}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}-\frac{11(17Ab-5aB)\tan^{-}}{108a^{23/6}\sqrt[6]{b}}$$

$$=-\frac{11(17Ab-5aB)}{180a^3bx^{5/2}}+\frac{Ab-aB}{6abx^{5/2}\left(a+bx^3\right)^2}+\frac{17Ab-5aB}{36a^2bx^{5/2}\left(a+bx^3\right)}+\frac{11(17Ab-5aB)\tan^{-}}{216a^{23/6}}$$

Mathematica [C] time = 0.12, size = 96, normalized size = 0.27

$$\frac{\frac{a\left(a^2\left(85Bx^3-72A\right)+a\left(55bBx^6-289Abx^3\right)-187Ab^2x^6\right)}{\left(a+bx^3\right)^2}+55x^3(5aB-17Ab)\,{}_2F_1\left(\frac{1}{6},1;\frac{7}{6};-\frac{bx^3}{a}\right)}{180a^4x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x]
```

```
[Out] ((a*(-187*A*b^2*x^6 + a^2*(-72*A + 85*B*x^3) + a*(-289*A*b*x^3 + 55*b*B*x^6
)))/(a + b*x^3)^2 + 55*(-17*A*b + 5*a*B)*x^3*Hypergeometric2F1[1/6, 1, 7/6,
 -((b*x^3)/a)])/(180*a^4*x^(5/2))
```

fricas [B] time = 0.78, size = 2690, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2160*(220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6
 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3
+ 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23
*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a
```

```
^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^
2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + (25*
B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x + (5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(
15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3
*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A
^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 270
9375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
- 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt(3)*(5*B
*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27
09375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4
 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) - sqrt(3)*(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 122825
00*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 2413
7569*A^6*b^6)) + 220*sqrt(3)*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625
*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*
a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^
6)/(a^23*b))^(1/6)*arctan(1/3*(2*sqrt(3)*sqrt(a^8*(-(15625*B^6*a^6 - 318750
*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132037
5*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3
) + (25*B^2*a^2 - 170*A*B*a*b + 289*A^2*b^2)*x - (5*B*a^5 - 17*A*a^4*b)*sqr
t(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 1228
2500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24
137569*A^6*b^6)/(a^23*b))^(1/6))*a^19*b*(-(15625*B^6*a^6 - 318750*A*B^5*a^5
*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*
a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + 2*sqrt
(3)*(5*B*a^20*b - 17*A*a^19*b^2)*sqrt(x)*(-(15625*B^6*a^6 - 318750*A*B^5*a^
5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2
*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(5/6) + sqrt(
3)*(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500
*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 241375
69*A^6*b^6))/(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2
- 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^
5 + 24137569*A^6*b^6)) - 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 27093
75*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 -
42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2
- 170*A*B*a*b + 289*A^2*b^2)*x + 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(1562
5*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3
*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b
^6)/(a^23*b))^(1/6)) + 55*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^
6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3
*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/
(a^23*b))^(1/6)*log(121*a^8*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375
*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42
595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/3) + 121*(25*B^2*a^2 -
170*A*B*a*b + 289*A^2*b^2)*x - 121*(5*B*a^5 - 17*A*a^4*b)*sqrt(x)*(-(15625*
B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a
^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6
)/(a^23*b))^(1/6)) + 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6
*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*
b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(
a^23*b))^(1/6)*log(11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A
^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 4259
5710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*
sqrt(x)) - 110*(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)*(-(15625*B^6*a^6 - 318
```

```
750*A*B^5*a^5*b + 2709375*A^2*B^4*a^4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 3132
0375*A^4*B^2*a^2*b^4 - 42595710*A^5*B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(
1/6)*log(-11*a^4*(-(15625*B^6*a^6 - 318750*A*B^5*a^5*b + 2709375*A^2*B^4*a^
4*b^2 - 12282500*A^3*B^3*a^3*b^3 + 31320375*A^4*B^2*a^2*b^4 - 42595710*A^5*
B*a*b^5 + 24137569*A^6*b^6)/(a^23*b))^(1/6) - 11*(5*B*a - 17*A*b)*sqrt(x))
- 12*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2)
*sqrt(x))/(a^3*b^2*x^9 + 2*a^4*b*x^6 + a^5*x^3)
```

giac [A] time = 0.24, size = 334, normalized size = 0.95

$$\frac{11\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\sqrt{3}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{6}}+x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{432\,a^4b}-\frac{11\sqrt{3}\left(5\left(ab^5\right)^{\frac{1}{6}}Ba-17\left(ab^5\right)^{\frac{1}{6}}Ab\right)\log\left(\cdots\right.}{432\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="giac")
```

```
[Out] 11/432*sqrt(3)*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(sqrt(3)*sqr
t(x)*(a/b)^(1/6) + x + (a/b)^(1/3))/(a^4*b) - 11/432*sqrt(3)*(5*(a*b^5)^(1/
6)*B*a - 17*(a*b^5)^(1/6)*A*b)*log(-sqrt(3)*sqrt(x)*(a/b)^(1/6) + x + (a/b)
^(1/3))/(a^4*b) + 11/216*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arcta
n((sqrt(3)*(a/b)^(1/6) + 2*sqrt(x))/(a/b)^(1/6))/(a^4*b) + 11/216*(5*(a*b^5
)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*b)*arctan(-(sqrt(3)*(a/b)^(1/6) - 2*sqrt(x
))/(a/b)^(1/6))/(a^4*b) + 11/108*(5*(a*b^5)^(1/6)*B*a - 17*(a*b^5)^(1/6)*A*
b)*arctan(sqrt(x)/(a/b)^(1/6))/(a^4*b) + 1/36*(11*B*a*b*x^(7/2) - 23*A*b^2*
x^(7/2) + 17*B*a^2*sqrt(x) - 29*A*a*b*sqrt(x))/((b*x^3 + a)^2*a^3) - 2/5*A/
(a^3*x^(5/2))
```

maple [A] time = 0.17, size = 435, normalized size = 1.24

$$-\frac{23A\,b^2x^{\frac{7}{2}}}{36\left(b\,x^3+a\right)^2a^3}+\frac{11Bb\,x^{\frac{7}{2}}}{36\left(b\,x^3+a\right)^2a^2}-\frac{29Ab\sqrt{x}}{36\left(b\,x^3+a\right)^2a^2}+\frac{17B\sqrt{x}}{36\left(b\,x^3+a\right)^2a}-\frac{187\left(\frac{a}{b}\right)^{\frac{1}{6}}Ab\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{6}}}\right)}{108a^4}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x)
```

```
[Out] -23/36/a^3/(b*x^3+a)^2*x^(7/2)*b^2*A+11/36/a^2/(b*x^3+a)^2*x^(7/2)*B*b-29/3
6/a^2/(b*x^3+a)^2*A*x^(1/2)*b+17/36/a/(b*x^3+a)^2*B*x^(1/2)-187/108/a^4*A*b
*(a/b)^(1/6)*arctan(1/(a/b)^(1/6)*x^(1/2))+187/432/a^4*A*b*3^(1/2)*(a/b)^(1
/6)*ln(-x+3^(1/2)*(a/b)^(1/6)*x^(1/2)-(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1
/6)*arctan(2/(a/b)^(1/6)*x^(1/2)-3^(1/2))-187/432/a^4*A*b*3^(1/2)*(a/b)^(1/
6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1/2)+(a/b)^(1/3))-187/216/a^4*A*b*(a/b)^(1/6
)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1/2))+55/108/a^3*B*(a/b)^(1/6)*arctan(1/(
a/b)^(1/6)*x^(1/2))-55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(-x+3^(1/2)*(a/b)^(1
/6)*x^(1/2)-(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1
/2)-3^(1/2))+55/432/a^3*B*3^(1/2)*(a/b)^(1/6)*ln(x+3^(1/2)*(a/b)^(1/6)*x^(1
/2)+(a/b)^(1/3))+55/216/a^3*B*(a/b)^(1/6)*arctan(2/(a/b)^(1/6)*x^(1/2)+3^(1
/2))-2/5*A/a^3/x^(5/2)
```

maxima [A] time = 1.34, size = 346, normalized size = 0.99

$$\frac{11\left(5\,Bab-17\,Ab^{2}\right)x^{6}+17\left(5\,Ba^{2}-17\,Aab\right)x^{3}-72\,Aa^{2}}{180\left(a^{3}b^{2}x^{\frac{17}{2}}+2\,a^{4}bx^{\frac{11}{2}}+a^{5}x^{\frac{5}{2}}\right)}+\frac{11\left(\frac{\sqrt{3}\,(5\,Ba-17\,Ab)\log\left(\sqrt{3}\,a^{\frac{1}{6}}b^{\frac{1}{6}}\sqrt{x}+b^{\frac{1}{3}}x+a^{\frac{1}{3}}\right)}{a^{\frac{5}{6}}b^{\frac{1}{6}}}-\frac{\sqrt{3}\,(5\,Ba-17\,Ab)}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^(7/2)/(b*x^3+a)^3,x, algorithm="maxima")
```

```
[Out] 1/180*(11*(5*B*a*b - 17*A*b^2)*x^6 + 17*(5*B*a^2 - 17*A*a*b)*x^3 - 72*A*a^2
)/(a^3*b^2*x^(17/2) + 2*a^4*b*x^(11/2) + a^5*x^(5/2)) + 11/432*(sqrt(3)*(5*
B*a - 17*A*b)*log(sqrt(3)*a^(1/6)*b^(1/6)*sqrt(x) + b^(1/3)*x + a^(1/3))/(a
^(5/6)*b^(1/6)) - sqrt(3)*(5*B*a - 17*A*b)*log(-sqrt(3)*a^(1/6)*b^(1/6)*sqr
t(x) + b^(1/3)*x + a^(1/3))/(a^(5/6)*b^(1/6)) + 4*(5*B*a*b^(1/3) - 17*A*b^(
4/3))*arctan(b^(1/3)*sqrt(x)/sqrt(a^(1/3)*b^(1/3)))/(a^(2/3)*b^(1/3)*sqrt(a
^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*arctan((s
qrt(3)*a^(1/6)*b^(1/6) + 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3)))/(a*b^(1/
3)*sqrt(a^(1/3)*b^(1/3))) + 2*(5*B*a^(4/3)*b^(1/3) - 17*A*a^(1/3)*b^(4/3))*
arctan(-(sqrt(3)*a^(1/6)*b^(1/6) - 2*b^(1/3)*sqrt(x))/sqrt(a^(1/3)*b^(1/3))
)/(a*b^(1/3)*sqrt(a^(1/3)*b^(1/3))))/a^3
```

mupad [B] time = 2.96, size = 2109, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^(7/2)*(a + b*x^3)^3),x)
```

```
[Out] - ((2*A)/(5*a) + (17*x^3*(17*A*b - 5*B*a))/(180*a^2) + (11*b*x^6*(17*A*b -
5*B*a))/(180*a^3))/(a^2*x^(5/2) + b^2*x^(17/2) + 2*a*b*x^(11/2)) - (atan(((
(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 +
 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5
21928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(
17*A*b - 5*B*a)*11i)/(216*(-a)^(23/6)*b^(1/6)) + ((x^(1/2)*(443639472636450
816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B
^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a
^16*b^8) + (11*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837
801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 45215221495504
8960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a)*11i)/(216
*(-a)^(23/6)*b^(1/6)))/((11*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*(17*A*b
 - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5
 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/
(216*(-a)^(23/6)*b^(1/6)))*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6)) - (1
1*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5
 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 -
 521928791337000960*A^3*B*a^16*b^8) + (11*(17*A*b - 5*B*a)*(51243917694905
5488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^
2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))
*(17*A*b - 5*B*a))/(216*(-a)^(23/6)*b^(1/6))))*(17*A*b - 5*B*a)*11i)/(108*(
-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1
/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 2302
```

```
62702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928
791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*
(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 1329859
45575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^
(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i)/2 - 1/2)*(
17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331981981056000
0*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*
B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1
/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B
^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*
a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)))/((11*
((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^
15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^
7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) -
 (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b
^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4
52152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/(216*(-a)^(2
3/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(44363
9472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23026270206044
1600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219287913370009
60*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)*(512439176
949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298594557501440
0*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(
1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 - 1/2)*(17*A*b - 5*B*a)
*11i)/(108*(-a)^(23/6)*b^(1/6)) - (atan(((((3^(1/2)*1i)/2 + 1/2)*(17*A*b -
5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^1
9*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*
b^6 - 521928791337000960*A^3*B*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A
*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b
^5 + 132985945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7)
)/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(1/6)) + (((3^(1/2)*1i
)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(443639472636450816*A^4*a^15*b^9 + 331
9819810560000*B^4*a^19*b^5 + 230262702060441600*A^2*B^2*a^17*b^7 - 45149549
423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B*a^16*b^8) + (11*((3^(1/
2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(512439176949055488*A^3*a^19*b^8 - 1303783
7801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*a^21*b^6 - 4521522149550
48960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6)))*11i)/(216*(-a)^(23/6)*b^(
1/6)))/((11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^(1/2)*(4436394726364
50816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 230262702060441600*A^2
*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 521928791337000960*A^3*B
*a^16*b^8) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(51243917694905548
8*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 132985945575014400*A*B^2*
a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a)^(23/6)*b^(1/6))))/
(216*(-a)^(23/6)*b^(1/6)) - (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a)*(x^
(1/2)*(443639472636450816*A^4*a^15*b^9 + 3319819810560000*B^4*a^19*b^5 + 23
0262702060441600*A^2*B^2*a^17*b^7 - 45149549423616000*A*B^3*a^18*b^6 - 5219
28791337000960*A^3*B*a^16*b^8) + (11*((3^(1/2)*1i)/2 + 1/2)*(17*A*b - 5*B*a
)*(512439176949055488*A^3*a^19*b^8 - 13037837801472000*B^3*a^22*b^5 + 13298
5945575014400*A*B^2*a^21*b^6 - 452152214955048960*A^2*B*a^20*b^7))/(216*(-a
)^(23/6)*b^(1/6))))/(216*(-a)^(23/6)*b^(1/6))))*((3^(1/2)*1i)/2 + 1/2)*(17*
A*b - 5*B*a)*11i)/(108*(-a)^(23/6)*b^(1/6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**(7/2)/(b*x**3+a)**3,x)
```

```
[Out] Timed out
```

3.179 $\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

```
[Out] 2/9*a^2*(A*b-B*a)*(b*x^3+a)^(3/2)/b^4-2/15*a*(2*A*b-3*B*a)*(b*x^3+a)^(5/2)/
b^4+2/21*(A*b-3*B*a)*(b*x^3+a)^(7/2)/b^4+2/27*B*(b*x^3+a)^(9/2)/b^4
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^4}+\frac{2\left(a+bx^3\right)^{7/2}(Ab-3aB)}{21b^4}-\frac{2a\left(a+bx^3\right)^{5/2}(2Ab-3aB)}{15b^4}+\frac{2B\left(a+bx^3\right)^{9/2}}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b
*x^3)^(5/2))/(15*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(7/2))/(21*b^4) + (2*B
*(a + b*x^3)^(9/2))/(27*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\sqrt{a+bx^3}\left(A+Bx^3\right)dx&=\frac{1}{3}\text{Subst}\left(\int x^2\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)\sqrt{a+bx}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{3/2}}{b^3}+\frac{(Ab-3aB}{b}\right.\right.\\&=\frac{2a^2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right.}{21b^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 75, normalized size = 0.73

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-16a^3B+24a^2b\left(A+Bx^3\right)-6ab^2x^3\left(6A+5Bx^3\right)+5b^3x^6\left(9A+7Bx^3\right)\right)}{945b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(-16*a^3*B + 24*a^2*b*(A + B*x^3) - 6*a*b^2*x^3*(6*A +
 5*B*x^3) + 5*b^3*x^6*(9*A + 7*B*x^3)))/(945*b^4)
```

fricas [A] time = 0.62, size = 99, normalized size = 0.96

$$\frac{2\left(35\,Bb^4x^{12}+5\left(Bab^3+9\,Ab^4\right)x^9-3\left(2\,Ba^2b^2-3\,Aab^3\right)x^6-16\,Ba^4+24\,Aa^3b+4\left(2\,Ba^3b-3\,Aa^2b^2\right)x^3\right)\sqrt{b}}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(B*a*b^3 + 9*A*b^4)*x^9 - 3*(2*B*a^2*b^2 - 3*A*a*b
^3)*x^6 - 16*B*a^4 + 24*A*a^3*b + 4*(2*B*a^3*b - 3*A*a^2*b^2)*x^3)*sqrt(b*x
^3 + a)/b^4
```

giac [A] time = 0.16, size = 104, normalized size = 1.01

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-135\left(bx^3+a\right)^{\frac{7}{2}}Ba+189\left(bx^3+a\right)^{\frac{5}{2}}Ba^2-105\left(bx^3+a\right)^{\frac{3}{2}}Ba^3+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-126\left(bx\right.\right.}{945\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 135*(b*x^3 + a)^(7/2)*B*a + 189*(b*x^3 + a)
^(5/2)*B*a^2 - 105*(b*x^3 + a)^(3/2)*B*a^3 + 45*(b*x^3 + a)^(7/2)*A*b - 126
*(b*x^3 + a)^(5/2)*A*a*b + 105*(b*x^3 + a)^(3/2)*A*a^2*b)/b^4
```

maple [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(35B\,x^9b^3+45A\,b^3x^6-30Ba\,b^2x^6-36Aa\,b^2x^3+24B\,a^2b\,x^3+24A\,a^2b-16B\,a^3\right)}{945b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/945*(b*x^3+a)^(3/2)*(35*B*b^3*x^9+45*A*b^3*x^6-30*B*a*b^2*x^6-36*A*a*b^2*
x^3+24*B*a^2*b*x^3+24*A*a^2*b-16*B*a^3)/b^4
```

maxima [A] time = 0.62, size = 118, normalized size = 1.15

$$\frac{2}{945}\,B\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^4}-\frac{135\left(bx^3+a\right)^{\frac{7}{2}}a}{b^4}+\frac{189\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^4}-\frac{105\left(bx^3+a\right)^{\frac{3}{2}}a^3}{b^4}\right)+\frac{2}{315}\,A\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/945*B*(35*(b*x^3 + a)^(9/2)/b^4 - 135*(b*x^3 + a)^(7/2)*a/b^4 + 189*(b*x^
3 + a)^(5/2)*a^2/b^4 - 105*(b*x^3 + a)^(3/2)*a^3/b^4) + 2/315*A*(15*(b*x^3
+ a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3 + a)^(3/2)*a^2/b^3)
```

mupad [B] time = 2.72, size = 154, normalized size = 1.50

$$\frac{2\,B\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{x^9\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{21\,b}+\frac{8\,a^2\,\left(2\,A\,a-\frac{6\,a\left(2\,A\,b+\frac{2\,B\,a}{9}\right)}{7\,b}\right)\,\sqrt{b\,x^3+a}}{45\,b^3}+\frac{x^6\,\left(2\,A\,a-\frac{6\,a\left(2\,A\,b+\right.}{7\,b}\right.}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^12*(a + b*x^3)^(1/2))/27 + (x^9*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/9
))/(21*b) + (8*a^2*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^(1
/2))/(45*b^3) + (x^6*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x^3)^
(1/2))/(15*b) - (4*a*x^3*(2*A*a - (6*a*(2*A*b + (2*B*a)/9))/(7*b))*(a + b*x
^3)^(1/2))/(45*b^2)
```

sympy [A] time = 4.17, size = 219, normalized size = 2.13

$$\begin{cases} \frac{16Aa^3\sqrt{a+bx^3}}{315b^3} - \frac{8Aa^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2Aax^6\sqrt{a+bx^3}}{105b} + \frac{2Ax^9\sqrt{a+bx^3}}{21} - \frac{32Ba^4\sqrt{a+bx^3}}{945b^4} + \frac{16Ba^3x^3\sqrt{a+bx^3}}{945b^3} - \frac{4Ba^2x^6\sqrt{a+bx^3}}{315b^2} + \frac{2Bax^9\sqrt{a-}}{189b} \\ \sqrt{a}\left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**3*sqrt(a + b*x**3)/(315*b**3) - 8*A*a**2*x**3*sqrt(a + b
*x**3)/(315*b**2) + 2*A*a*x**6*sqrt(a + b*x**3)/(105*b) + 2*A*x**9*sqrt(a +
 b*x**3)/21 - 32*B*a**4*sqrt(a + b*x**3)/(945*b**4) + 16*B*a**3*x**3*sqrt(a
 + b*x**3)/(945*b**3) - 4*B*a**2*x**6*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x
**9*sqrt(a + b*x**3)/(189*b) + 2*B*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (s
qrt(a)*(A*x**9/9 + B*x**12/12), True))
```

3.180 $\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

```
[Out] -2/9*a*(A*b-B*a)*(b*x^3+a)^(3/2)/b^3+2/15*(A*b-2*B*a)*(b*x^3+a)^(5/2)/b^3+2
/21*B*(b*x^3+a)^(7/2)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-2aB)}{15b^3}-\frac{2a\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)
^(5/2))/(15*b^3) + (2*B*(a + b*x^3)^(7/2))/(21*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{a+bx^3}\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x\sqrt{a+bx}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{a(-Ab+aB)\sqrt{a+bx}}{b^2}+\frac{(Ab-2aB)(a+bx)^{3/2}}{b^2}+\frac{B(a+bx)^{5/2}}{b^2}\right)\right.\\ &= -\frac{2a(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\left(a+bx^3\right)^{3/2}\left(8a^2B-2ab\left(7A+6Bx^3\right)+3b^2x^3\left(7A+5Bx^3\right)\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(8*a^2*B + 3*b^2*x^3*(7*A + 5*B*x^3) - 2*a*b*(7*A + 6*
B*x^3)))/(315*b^3)
```

fricas [A] time = 0.86, size = 75, normalized size = 1.03

$$\frac{2\left(15\,Bb^3x^9+3\left(Bab^2+7\,Ab^3\right)x^6+8\,Ba^3-14\,Aa^2b-\left(4\,Ba^2b-7\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 + 3*(B*a*b^2 + 7*A*b^3)*x^6 + 8*B*a^3 - 14*A*a^2*b - (4
*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^3
```

giac [A] time = 0.18, size = 73, normalized size = 1.00

$$\frac{2\left(15\left(bx^3+a\right)^{\frac{7}{2}}B-42\left(bx^3+a\right)^{\frac{5}{2}}Ba+35\left(bx^3+a\right)^{\frac{3}{2}}Ba^2+21\left(bx^3+a\right)^{\frac{5}{2}}Ab-35\left(bx^3+a\right)^{\frac{3}{2}}Aab\right)}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(15*(b*x^3 + a)^(7/2)*B - 42*(b*x^3 + a)^(5/2)*B*a + 35*(b*x^3 + a)^(
3/2)*B*a^2 + 21*(b*x^3 + a)^(5/2)*A*b - 35*(b*x^3 + a)^(3/2)*A*a*b)/b^3
```

maple [A] time = 0.04, size = 53, normalized size = 0.73

$$-\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}\left(-15B\,b^2x^6-21A\,b^2x^3+12Bab\,x^3+14Aab-8B\,a^2\right)}{315b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] -2/315*(b*x^3+a)^(3/2)*(-15*B*b^2*x^6-21*A*b^2*x^3+12*B*a*b*x^3+14*A*a*b-8*
B*a^2)/b^3
```

maxima [A] time = 0.51, size = 84, normalized size = 1.15

$$\frac{2}{315}\,B\left(\frac{15\left(bx^3+a\right)^{\frac{7}{2}}}{b^3}-\frac{42\left(bx^3+a\right)^{\frac{5}{2}}a}{b^3}+\frac{35\left(bx^3+a\right)^{\frac{3}{2}}a^2}{b^3}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^2}-\frac{5\left(bx^3+a\right)^{\frac{3}{2}}a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*B*(15*(b*x^3 + a)^(7/2)/b^3 - 42*(b*x^3 + a)^(5/2)*a/b^3 + 35*(b*x^3
+ a)^(3/2)*a^2/b^3) + 2/45*A*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)
*a/b^2)
```

mupad [B] time = 2.66, size = 114, normalized size = 1.56

$$\frac{2\,B\,x^9\,\sqrt{b\,x^3+a}}{21}+\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a-\frac{4\,a\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)\,\sqrt{b\,x^3+a}}{9\,b^2}+\frac{x^3\,\left(2\,A\,a-\frac{4\,a\left(2\,A\,b+\frac{2\,B\,a}{7}\right)}{5\,b}\right)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (2*B*x^9*(a + b*x^3)^(1/2))/21 + (x^6*(a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/7)
)/(15*b) - (2*a*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2)
)/(9*b^2) + (x^3*(2*A*a - (4*a*(2*A*b + (2*B*a)/7))/(5*b))*(a + b*x^3)^(1/2
))/(9*b)
```

sympy [A] time = 1.82, size = 168, normalized size = 2.30

$$\begin{cases} -\frac{4Aa^2\sqrt{a+bx^3}}{45b^2} + \frac{2Aax^3\sqrt{a+bx^3}}{45b} + \frac{2Ax^6\sqrt{a+bx^3}}{15} + \frac{16Ba^3\sqrt{a+bx^3}}{315b^3} - \frac{8Ba^2x^3\sqrt{a+bx^3}}{315b^2} + \frac{2Bax^6\sqrt{a+bx^3}}{105b} + \frac{2Bx^9\sqrt{a+bx^3}}{21} & \text{for } b \neq \\ \sqrt{a}\left(\frac{Ax^6}{6} + \frac{Bx^9}{9}\right) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*A*a*x**3*sqrt(a + b*x**
3)/(45*b) + 2*A*x**6*sqrt(a + b*x**3)/15 + 16*B*a**3*sqrt(a + b*x**3)/(315*
b**3) - 8*B*a**2*x**3*sqrt(a + b*x**3)/(315*b**2) + 2*B*a*x**6*sqrt(a + b*x
**3)/(105*b) + 2*B*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (sqrt(a)*(A*x**6/6
+ B*x**9/9), True))
```

3.181 $\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

```
[Out] 2/9*(A*b-B*a)*(b*x^3+a)^(3/2)/b^2+2/15*B*(b*x^3+a)^(5/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-aB)}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(3/2))/(9*b^2) + (2*B*(a + b*x^3)^(5/2))/(15*b^2
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx^3}\left(A+Bx^3\right)dx&=\frac{1}{3}\operatorname{Subst}\left(\int\sqrt{a+bx}\,(A+Bx)\,dx,x,x^3\right)\\&=\frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{(Ab-aB)\sqrt{a+bx}}{b}+\frac{B(a+bx)^{3/2}}{b}\right)dx,x,x^3\right)\\&=\frac{2(Ab-aB)\left(a+bx^3\right)^{3/2}}{9b^2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{3/2}\left(-2aB+5Ab+3bBx^3\right)}{45b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(3/2)*(5*A*b - 2*a*B + 3*b*B*x^3))/(45*b^2)
```

fricas [A] time = 0.96, size = 50, normalized size = 1.09

$$\frac{2\left(3\,Bb^{2}x^{6}+\left(Bab+5\,Ab^{2}\right)x^{3}-2\,Ba^{2}+5\,Aab\right)\sqrt{bx^{3}+a}}{45\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 + (B*a*b + 5*A*b^2)*x^3 - 2*B*a^2 + 5*A*a*b)*sqrt(b*x^3 +
 a)/b^2
```

giac [A] time = 0.15, size = 44, normalized size = 0.96

$$\frac{2\left(3\left(bx^{3}+a\right)^{\frac{5}{2}}B-5\left(bx^{3}+a\right)^{\frac{3}{2}}Ba+5\left(bx^{3}+a\right)^{\frac{3}{2}}Ab\right)}{45\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*(b*x^3 + a)^(5/2)*B - 5*(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)
*A*b)/b^2
```

maple [A] time = 0.04, size = 31, normalized size = 0.67

$$\frac{2\left(b\,x^{3}+a\right)^{\frac{3}{2}}\left(3Bb\,x^{3}+5Ab-2Ba\right)}{45b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 2/45*(b*x^3+a)^(3/2)*(3*B*b*x^3+5*A*b-2*B*a)/b^2
```

maxima [A] time = 0.59, size = 49, normalized size = 1.07

$$\frac{2}{45}\,B\left(\frac{3\left(bx^{3}+a\right)^{\frac{5}{2}}}{b^{2}}-\frac{5\left(bx^{3}+a\right)^{\frac{3}{2}}a}{b^{2}}\right)+\frac{2\left(bx^{3}+a\right)^{\frac{3}{2}}A}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^2 - 5*(b*x^3 + a)^(3/2)*a/b^2) + 2/9*(b*x^3 +
 a)^(3/2)*A/b
```

mupad [B] time = 2.60, size = 44, normalized size = 0.96

$$\frac{6\,B\,{\left(b\,x^3+a\right)}^{5/2}+10\,A\,b\,{\left(b\,x^3+a\right)}^{3/2}-10\,B\,a\,{\left(b\,x^3+a\right)}^{3/2}}{45\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] (6*B*(a + b*x^3)^(5/2) + 10*A*b*(a + b*x^3)^(3/2) - 10*B*a*(a + b*x^3)^(3/2
))/(45*b^2)
```

sympy [A] time = 0.74, size = 117, normalized size = 2.54

$$\begin{cases} \frac{2Aa\sqrt{a+bx^3}}{9b} + \frac{2Ax^3\sqrt{a+bx^3}}{9} - \frac{4Ba^2\sqrt{a+bx^3}}{45b^2} + \frac{2Bax^3\sqrt{a+bx^3}}{45b} + \frac{2Bx^6\sqrt{a+bx^3}}{15} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*a*sqrt(a + b*x**3)/(9*b) + 2*A*x**3*sqrt(a + b*x**3)/9 - 4*B
*a**2*sqrt(a + b*x**3)/(45*b**2) + 2*B*a*x**3*sqrt(a + b*x**3)/(45*b) + 2*B
*x**6*sqrt(a + b*x**3)/15, Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**6/6), True)
)
```

3.182 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}\,dx$

Optimal. Leaf size=64

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*
A*(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$\frac{2}{3}A\sqrt{a+bx^3}-\frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2B\left(a+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*A*Sqrt[a + b*x^3])/3 + (2*B*(a + b*x^3)^(3/2))/(9*b) - (2*Sqrt[a]*A*ArcT
anh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x}\,dx, x, x^3\right)\\
&= \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{1}{3}A\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{1}{3}(aA)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} + \frac{(2aA)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}\\
&= \frac{2}{3}A\sqrt{a+bx^3} + \frac{2B\left(a+bx^3\right)^{3/2}}{9b} - \frac{2}{3}\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.94

$$\frac{2}{9}\left(\frac{\sqrt{a+bx^3}\left(B\left(a+bx^3\right)+3Ab\right)}{b} - 3\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x,x]
```

```
[Out] (2*((Sqrt[a + b*x^3]*(3*A*b + B*(a + b*x^3)))/b - 3*Sqrt[a]*A*ArcTanh[Sqrt[
a + b*x^3]/Sqrt[a]]))/9
```

fricas [A] time = 0.97, size = 125, normalized size = 1.95

$$\left[\frac{3\,A\sqrt{a}\,b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(Bbx^3+Ba+3\,Ab\right)\sqrt{bx^3+a}}{9\,b}, \frac{2\left(3\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\left(Bbx^3+\right.\right.}{9\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/9*(3*A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*
(B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b, 2/9*(3*A*sqrt(-a)*b*arctan(sqrt
(b*x^3 + a)*sqrt(-a)/a) + (B*b*x^3 + B*a + 3*A*b)*sqrt(b*x^3 + a))/b]
```

giac [A] time = 0.15, size = 61, normalized size = 0.95

$$\frac{2\,Aa\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}} + \frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^2+3\,\sqrt{bx^3+a}\,Ab^3\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/9*((b*x^3 + a)^(3/2)*
B*b^2 + 3*sqrt(b*x^3 + a)*A*b^3)/b^3
```

maple [A] time = 0.05, size = 50, normalized size = 0.78

$$\left(-\frac{2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}+\frac{2\sqrt{b\,x^3+a}}{3}\right)A+\frac{2\left(b\,x^3+a\right)^{\frac{3}{2}}B}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x,x)
```

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b+A*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/
3*(b*x^3+a)^(1/2))
```

maxima [A] time = 1.20, size = 67, normalized size = 1.05

$$\frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\sqrt{bx^3+a}\right)A+\frac{2\left(bx^3+a\right)^{\frac{3}{2}}B}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) +
 2*sqrt(b*x^3 + a))*A + 2/9*(b*x^3 + a)^(3/2)*B/b
```

mupad [B] time = 2.71, size = 80, normalized size = 1.25

$$\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9}+\frac{\sqrt{b\,x^3+a}\,\left(2\,A\,b+\frac{2\,B\,a}{3}\right)}{3\,b}+\frac{A\,\sqrt{a}\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x,x)
```

```
[Out] (2*B*x^3*(a + b*x^3)^(1/2))/9 + ((a + b*x^3)^(1/2)*(2*A*b + (2*B*a)/3))/(3*
b) + (A*a^(1/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a
^(1/2)))/x^6))/3
```

sympy [A] time = 25.83, size = 76, normalized size = 1.19

$$-\frac{A\left(-\frac{2a\operatorname{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2\sqrt{a+bx^3}\right)}{3}-\frac{B\left(\begin{cases}-\sqrt{a}\,x^3 & \text{for}\ b=0\\ -\frac{2\left(a+bx^3\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x,x)
```

```
[Out] -A*(-2*a*atan(sqrt(a + b*x**3)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*x**3))/3 -
 B*Piecewise((-sqrt(a)*x**3, Eq(b, 0)), (-2*(a + b*x**3)**(3/2)/(3*b), True
))/3
```

3.183 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

```
[Out] -1/3*A*(b*x^3+a)^(3/2)/a/x^3-1/3*(A*b+2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2
))/a^(1/2)+1/3*(A*b+2*B*a)*(b*x^3+a)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(2aB+Ab)}{3a}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(3*a) - (A*(a + b*x^3)^(3/2))/(3*a*x^3) - (
(A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^4}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^2}\,dx,x,x^3\right)\\
&= -\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)}{6a}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{1}{6}(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,\right.\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}+\frac{(Ab+2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a\right.}}{3b}\\
&= \frac{(Ab+2aB)\sqrt{a+bx^3}}{3a}-\frac{A\left(a+bx^3\right)^{3/2}}{3ax^3}-\frac{(Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.75

$$
\frac{1}{3}\left(\frac{\sqrt{a+bx^3}\left(2Bx^3-A\right)}{x^3}-\frac{(2aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^4,x]
```

```
[Out] ((Sqrt[a + b*x^3]*(-A + 2*B*x^3))/x^3 - ((A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x
^3]/Sqrt[a]])/Sqrt[a])/3
```

fricas [A] time = 0.94, size = 143, normalized size = 1.70

$$
\left[\frac{(2\,Ba+Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bax^3-Aa\right)\sqrt{bx^3+a}}{6\,ax^3},\ \frac{(2\,Ba+Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-}}{a}\right.}{3\,ax^3}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*((2*B*a + A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*
a)/x^3) + 2*(2*B*a*x^3 - A*a)*sqrt(b*x^3 + a))/(a*x^3), 1/3*((2*B*a + A*b)*
sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (2*B*a*x^3 - A*a)*sqrt(b*
x^3 + a))/(a*x^3)]
```

giac [A] time = 0.17, size = 68, normalized size = 0.81

$$
\frac{2\,\sqrt{bx^3+a}\,Bb+\frac{\left(2\,Bab+Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx^3+a}\,Ab}{x^3}}{3\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(b*x^3 + a)*B*b + (2*B*a*b + A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(
-a))/sqrt(-a) - sqrt(b*x^3 + a)*A*b/x^3)/b
```

maple [A] time = 0.05, size = 72, normalized size = 0.86

$$\left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}-\frac{\sqrt{b x^3+a}}{3x^3}\right)A+\left(-\frac{2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3}+\frac{2\sqrt{b x^3+a}}{3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x)
```

```
[Out] A*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
+B*(-2/3*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+2/3*(b*x^3+a)^(1/2))
```

maxima [A] time = 1.26, size = 107, normalized size = 1.27

$$\frac{1}{6}\left(\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^3+a}}{x^3}\right)A+\frac{1}{3}\left(\sqrt{a}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\sqrt{bx^3+a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
 - 2*sqrt(b*x^3 + a)/x^3)*A + 1/3*(sqrt(a)*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a))) + 2*sqrt(b*x^3 + a))*B
```

mupad [B] time = 2.93, size = 76, normalized size = 0.90

$$\frac{2B\sqrt{b x^3+a}}{3}-\frac{A\sqrt{b x^3+a}}{3x^3}+\frac{\ln\left(\frac{\left(\sqrt{b x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(\frac{A b}{2}+B a\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^4,x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/3 - (A*(a + b*x^3)^(1/2))/(3*x^3) + (log((((a + b*x
^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*((A*b)/2 + B*a))
/(3*a^(1/2))
```

sympy [A] time = 43.66, size = 134, normalized size = 1.60

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}-\frac{2B\sqrt{a}\ \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba}{3\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2B\sqrt{b}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**4,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*b*asinh(sqrt(a)/(sqrt(b)*x
**(3/2)))/(3*sqrt(a)) - 2*B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/3 + 2
*B*a/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*B*sqrt(b)*x**(3/2)/(3*sq
rt(a/(b*x**3) + 1))
```

3.184 $\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^7}\,dx$

Optimal. Leaf size=88

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}\,(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

```
[Out] -1/6*A*(b*x^3+a)^(3/2)/a/x^6+1/12*b*(A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(
1/2))/a^(3/2)+1/12*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^3
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}+\frac{\sqrt{a+bx^3}\,(Ab-4aB)}{12ax^3}-\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] ((A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a*x^3) - (A*(a + b*x^3)^(3/2))/(6*a*x^6
) + (b*(A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^7}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}\,(A+Bx)}{x^3}\,dx, x, x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} + \frac{\left(-\frac{Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, x^3\right)}{6a}$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} - \frac{(b(Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} - \frac{(Ab-4aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+}\right.}{12a}$$
$$= \frac{(Ab-4aB)\sqrt{a+bx^3}}{12ax^3} - \frac{A\left(a+bx^3\right)^{3/2}}{6ax^6} + \frac{b(Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{3/2}}$$

Mathematica [A] time = 0.08, size = 93, normalized size = 1.06

$$\frac{-\left(a+bx^3\right)\left(2a\left(A+2Bx^3\right)+Abx^3\right)-bx^6\sqrt{\frac{bx^3}{a}+1}\,(4aB-Ab)\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{12ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^7,x]
```

```
[Out] (-((a + b*x^3)*(A*b*x^3 + 2*a*(A + 2*B*x^3))) - b*(-(A*b) + 4*a*B)*x^6*Sqrt
[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]])/(12*a*x^6*Sqrt[a + b*x^3])
```

fricas [A] time = 1.02, size = 172, normalized size = 1.95

$$\left[-\frac{\left(4\,Bab-Ab^2\right)\sqrt{a}\,x^6\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(\left(4\,Ba^2+Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{24\,a^2x^6}, \frac{\left(4\,Bab-Ab^2\right)\sqrt{-a}\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - A*b^2)*sqrt(a)*x^6*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a
) + 2*a)/x^3) + 2*((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x
^6), 1/12*((4*B*a*b - A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a
) - ((4*B*a^2 + A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^2*x^6)]
```

giac [A] time = 0.20, size = 120, normalized size = 1.36

$$\frac{\frac{\left(4\,Bab^2-Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}\,Ba^2b^2+\left(bx^3+a\right)^{\frac{3}{2}}Ab^3+\sqrt{bx^3+a}\,Aab^3}{ab^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*((4*B*a*b^2 - A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - (
4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 + (b*x^3 + a)^(3/
2)*A*b^3 + sqrt(b*x^3 + a)*A*a*b^3)/(a*b^2*x^6))/b
```

maple [A] time = 0.05, size = 96, normalized size = 1.09

$$\left(\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{12 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}\;b}{12 a\,x^{3}}-\frac{\sqrt{b\,x^{3}+a}}{6 x^{6}}\right) A+\left(-\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 \sqrt{a}}-\frac{\sqrt{b\,x^{3}+a}}{3 x^{3}}\right) B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x)
```

```
[Out] B*(-1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)-1/3*(b*x^3+a)^(1/2)/x^3)
+A*(1/12*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/6*(b*x^3+a)^(1/2)/x
^6-1/12*(b*x^3+a)^(1/2)/a*b/x^3)
```

maxima [B] time = 1.32, size = 158, normalized size = 1.80

$$-\frac{1}{24}\left(\frac{b^{2} \log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2\left(\left(bx^{3}+a\right)^{\frac{3}{2}}b^{2}+\sqrt{bx^{3}+a}\,ab^{2}\right)}{\left(bx^{3}+a\right)^{2}a-2\left(bx^{3}+a\right)a^{2}+a^{3}}\right)A+\frac{1}{6}\left(\frac{b \log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\sqrt{bx^{3}+a}}{x^{3}}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/24*(b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(
3/2) + 2*((b*x^3 + a)^(3/2)*b^2 + sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a -
 2*(b*x^3 + a)*a^2 + a^3))*A + 1/6*(b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt
(b*x^3 + a) + sqrt(a)))/sqrt(a) - 2*sqrt(b*x^3 + a)/x^3)*B
```

mupad [B] time = 3.12, size = 93, normalized size = 1.06

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)\left(A\,b-4\,B\,a\right)}{24\,a^{3/2}}-\frac{\left(4\,B\,a^2+A\,b\,a\right)\sqrt{b\,x^3+a}}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^7,x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)
*(A*b - 4*B*a))/(24*a^(3/2)) - ((4*B*a^2 + A*a*b)*(a + b*x^3)^(1/2))/(12*a^
2*x^3) - (A*(a + b*x^3)^(1/2))/(6*x^6)
```

sympy [B] time = 129.14, size = 160, normalized size = 1.82

$$-\frac{Aa}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{A\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{Ab^{\frac{3}{2}}}{12ax^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{Ab^{2}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{12a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{3}}+1}}{3x^{\frac{3}{2}}}-\frac{Bb\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**7,x)
```

```
[Out] -A*a/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*sqrt(b)/(4*x**(9/2)*sqr
t(a/(b*x**3) + 1)) - A*b**(3/2)/(12*a*x**(3/2)*sqrt(a/(b*x**3) + 1)) + A*b*
*2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(12*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x*
*3) + 1)/(3*x**(3/2)) - B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a))
```

3.185 $\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=303

$$-\frac{4\, 3^{3/4}\sqrt{2+\sqrt{3}}\, a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\ldots$$

```
[Out] 2/17*B*x^4*(b*x^3+a)^(3/2)/b+6/935*a*(17*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2
/187*(17*A*b-8*B*a)*x^4*(b*x^3+a)^(1/2)/b-4/935*3^(3/4)*a^2*(17*A*b-8*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {459, 279, 321, 218}

$$-\frac{4\, 3^{3/4}\sqrt{2+\sqrt{3}}\, a^2\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{6ax\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(17*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(935*b^2) + (2*(17*A*b - 8*a*B)*x^
4*Sqrt[a + b*x^3])/(187*b) + (2*B*x^4*(a + b*x^3)^(3/2))/(17*b) - (4*3^(3/4
)*Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+4aB\right)\right)\int x^3\sqrt{a+bx^3}\,dx}{17b}$$

$$= \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b} + \frac{(3a(17Ab-8aB))\int \frac{x^3}{\sqrt{a+bx^3}}}{187b}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

$$= \frac{6a(17Ab-8aB)x\sqrt{a+bx^3}}{935b^2} + \frac{2(17Ab-8aB)x^4\sqrt{a+bx^3}}{187b} + \frac{2Bx^4\left(a+bx^3\right)^{3/2}}{17b}$$

Mathematica [C] time = 0.17, size = 89, normalized size = 0.29

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a(8aB-17Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \left(a+bx^3\right)\left(8aB-17Ab-11bBx^3\right)\right)}{187b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-17*A*b + 8*a*B - 11*b*B*x^3)) + (a*(-
17*A*b + 8*a*B)*Hypergeometric2F1[-1/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (
b*x^3)/a]))/(187*b^2)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^6+Ax^3\right)\sqrt{bx^3+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

maple [B] time = 0.05, size = 658, normalized size = 2.17

$$\left(\frac{2\sqrt{b\,x^3+a}\;x^4}{11}+\frac{4i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\cdots}{55\sqrt{b\,x^3+a}\;b^2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/17*(b*x^3+a)^(1/2)*x^7+6/187*(b*x^3+a)^(1/2)*a/b*x^4-48/935*(b*x^3+a)^
(1/2)*a^2/b^2*x-32/935*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^
(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x
-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^
(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*
b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)/b)^(1/2)))+A*(2/11*(b*x^3+a)^(1/2)*x^4+6/55*(b*x^3+a)^(1/2)*a/b*x+4
/55*I*a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/
(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^
2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(B x^3 + A\right) \sqrt{b x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 3.47, size = 83, normalized size = 0.27

$$\frac{A\sqrt{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}\,x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(7/3)) + B*sqrt(a)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

3.186 $\int \sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=268

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+\cdots}}{\cdots}$$

```
[Out] 2/11*B*x*(b*x^3+a)^(3/2)/b+2/55*(11*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+2/55*3^(
3/4)*a*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^
(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 218}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{55b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(11*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(55*b) + (2*B*x*(a + b*x^3)^(3/2))/(
11*b) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*b^(4/3)*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+aB\right)\right)\int \sqrt{a+bx^3}\,dx}{11b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{(3a(11Ab-2aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b}$$

$$= \frac{2(11Ab-2aB)x\sqrt{a+bx^3}}{55b} + \frac{2Bx\left(a+bx^3\right)^{3/2}}{11b} + \frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,a(11Ab-2aB)\,(}{}$$

Mathematica [C] time = 0.09, size = 75, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(\frac{(11Ab-2aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}+B\left(a+bx^3\right)\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^3]*(A + B*x^3),x]
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3) + ((11*A*b - 2*a*B)*Hypergeometric2F1[-
1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(11*b)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^3+A\right)\sqrt{bx^3+a}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

maple [B] time = 0.05, size = 618, normalized size = 2.31

$$\left(-\frac{2i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\, \sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\; a\,\mathrm{EllipticF}\left(\cdots\right.}{5\sqrt{b\,x^3+a}\;b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/11*(b*x^3+a)^(1/2)*x^4+6/55*(b*x^3+a)^(1/2)*a/b*x+4/55*I*a^2/b^2*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/5*(b*x^3+a)^(1
/2)*x-2/5*I*a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*
b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 3.24, size = 82, normalized size = 0.31

$$\frac{A\sqrt{a}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}\,x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) + B*sqrt(a)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

3.187 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx$

Optimal. Leaf size=269

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\sqrt{a+bx^3}\,(}{1(}$$

```
[Out] -1/2*A*(b*x^3+a)^(3/2)/a/x^2+1/10*(5*A*b+4*B*a)*x*(b*x^3+a)^(1/2)/a+1/10*3^
(3/4)*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1
/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 195, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{10\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x\sqrt{a+bx^3}\,(}{1(}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
```

```
[Out] ((5*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/(10*a) - (A*(a + b*x^3)^(3/2))/(2*a*x^2
) + (3^(3/4)*Sqrt[2 + Sqrt[3]]*(5*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(10*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
+ b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^3}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} - \frac{\left(-\frac{5Ab}{2}-2aB\right)\int \sqrt{a+bx^3}\,dx}{2a}$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} + \frac{1}{20}(3(5Ab+4aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(5Ab+4aB)x\sqrt{a+bx^3}}{10a} - \frac{A\left(a+bx^3\right)^{3/2}}{2ax^2} + \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,(5Ab+4aB)\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{10\sqrt[3]{}}$$

Mathematica [C] time = 0.07, size = 81, normalized size = 0.30

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3(4aB+5Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - A\left(a+bx^3\right)\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^3,x]
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) + ((5*A*b + 4*a*B)*x^3*Hypergeometric2F1
[-1/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x^2)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

maple [B] time = 0.05, size = 596, normalized size = 2.22

$$\left(-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}}{\quad}\right.}{2\sqrt{b\,x^3+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x)
```

```
[Out] B*(2/5*(b*x^3+a)^(1/2)*x-2/5*I*a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*(
(x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
)^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/b)^(1/2)))+A*(-1/2*(b*x^3+a)^(1/2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^3, x)
```

sympy [A] time = 3.38, size = 85, normalized size = 0.32

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{B\sqrt{a}\,x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**3,x)
```

```
[Out] A*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x**2*gamma(1/3)) + B*sqrt(a)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(4/3))
```

3.188 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^6}\,dx$

Optimal. Leaf size=272

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\sqrt{a+b}\cdots$$

```
[Out] -1/5*A*(b*x^3+a)^(3/2)/a/x^5+1/20*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/x^2-1/20*3
^(3/4)*b^(2/3)*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3
)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 277, 218}

$$-\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\sqrt{a+b}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] ((A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a*x^2) - (A*(a + b*x^3)^(3/2))/(5*a*x^
5) - (3^(3/4)*Sqrt[2 + Sqrt[3]]*b^(2/3)*(A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*a*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^6}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{\left(\frac{Ab}{2}-5aB\right)\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{5a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{(3b(Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a}$$

$$= \frac{(Ab-10aB)\sqrt{a+bx^3}}{20ax^2} - \frac{A\left(a+bx^3\right)^{3/2}}{5ax^5} - \frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}(Ab-10aB)\left(\sqrt[3]{a} + \right.}{20}$$

Mathematica [C] time = 0.18, size = 80, normalized size = 0.29

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{Ab}{2}-5aB\right){}_2F_1\left(-\frac{2}{3},-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - 2A\left(a+bx^3\right)\right)}{10ax^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + (((A*b)/2 - 5*a*B)*x^3*Hypergeometric2
F1[-2/3, -1/2, 1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(10*a*x^5)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

maple [B] time = 0.05, size = 616, normalized size = 2.26

$$\left(\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,b\,\mathrm{EllipticF}\left(\sqrt{3}\,\cdots\right.}{20\sqrt{b\,x^{3}+a}\;a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*(b*x^3+a)^(1/2)/a*b/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+B*(-1/2*(b*x^3+a)^(1/
2)/x^2-1/2*I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2
)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)\sqrt{bx^{3}+a}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^{3}+A\right)\,\sqrt{b\,x^{3}+a}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^6, x)
```

sympy [A] time = 3.63, size = 94, normalized size = 0.35

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**6,x)
```

```
[Out] A*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**5*gamma(-2/3)) + B*sqrt(a)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3))
```

3.189 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^9}\,dx$

Optimal. Leaf size=305

$$\frac{3b\sqrt{a+bx^3}\,(7Ab-16aB)}{320a^2x^2}+\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/8*A*(b*x^3+a)^(3/2)/a/x^8+1/80*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^5+3/32
0*b*(7*A*b-16*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/320*3^(3/4)*b^(5/3)*(7*A*b-16*
B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^2
/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 277, 325, 218}

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3b\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] ((7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(80*a*x^5) + (3*b*(7*A*b - 16*a*B)*Sqrt[
a + b*x^3])/(320*a^2*x^2) - (A*(a + b*x^3)^(3/2))/(8*a*x^8) + (3^(3/4)*Sqrt
[2 + Sqrt[3]]*b^(5/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a^2*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^9}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} - \frac{\left(\frac{7Ab}{2}-8aB\right)\int \frac{\sqrt{a+bx^3}}{x^6}\,dx}{8a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} - \frac{(3b(7Ab-16aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{160a}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} + \frac{\left(3b^2(7\right.}{}$$

$$= \frac{(7Ab-16aB)\sqrt{a+bx^3}}{80ax^5} + \frac{3b(7Ab-16aB)\sqrt{a+bx^3}}{320a^2x^2} - \frac{A\left(a+bx^3\right)^{3/2}}{8ax^8} + \frac{3^{3/4}\sqrt{2}}{}$$

Mathematica [C] time = 0.16, size = 80, normalized size = 0.26

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{7Ab}{2}-8aB\right){}_2F_1\left(-\frac{5}{3},-\frac{1}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-5A\left(a+bx^3\right)\right)}{40ax^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^9,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + (((7*A*b)/2 - 8*a*B)*x^3*Hypergeometri
c2F1[-5/3, -1/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*a*x^8)
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

maple [B] time = 0.05, size = 660, normalized size = 2.16

$$\left(-\frac{7i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}}}\, \sqrt{-\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, b^2\, \mathrm{EllipticF}\left(\right.}{320\sqrt{b x^3+a}\;a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x)
```

```
[Out] B*(-1/5*(b*x^3+a)^(1/2)/x^5-3/20*(b*x^3+a)^(1/2)/a*b/x^2+1/20*I*b/a*3^(1/2)
*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-1/8*(b*x^3+a)^(1/
2)/x^8-3/80*(b*x^3+a)^(1/2)/a*b/x^5+21/320*(b*x^3+a)^(1/2)/a^2*b^2/x^2-7/32
0*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)
^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^9, x)
```

sympy [A] time = 3.83, size = 97, normalized size = 0.32

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{8}{3},-\frac{1}{2}\\-\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**9,x)
```

```
[Out] A*sqrt(a)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**8*gamma(-5/3)) + B*sqrt(a)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3))
```

3.190 $\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=581

$$-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+12\sqrt[4]{3}\sqrt{2 \cdots}$$

```
[Out] 2/19*B*x^5*(b*x^3+a)^(3/2)/b+6/1729*a*(19*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b
^2+2/247*(19*A*b-10*B*a)*x^5*(b*x^3+a)^(1/2)/b-24/1729*a^2*(19*A*b-10*B*a)*
(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-8/1729*3^(3/4)*a^(7
/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(
1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3
)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)+12/1729*3^(1/4)*a^(7/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 279, 321, 303, 218, 1877}

$$-\frac{24a^2\sqrt{a+bx^3}(19Ab-10aB)}{1729b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{8\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+(1 \cdots}{\sqrt[3]{b}x+(1 \cdots}\right.\right.}{1729b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (6*a*(19*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b^2) + (2*(19*A*b - 10*a*
B)*x^5*Sqrt[a + b*x^3])/(247*b) - (24*a^2*(19*A*b - 10*a*B)*Sqrt[a + b*x^3]
)/(1729*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^5*(a + b*x^3)
^(3/2))/(19*b) + (12*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 10*a*B)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(
8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 10*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+5aB\right)\right)\int x^4\sqrt{a+bx^3}\,dx}{19b}$$

$$= \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b} + \frac{(3a(19Ab-10aB))\int\frac{x^4}{\sqrt{a+bx^3}}}{247b}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} + \frac{2Bx^5\left(a+bx^3\right)^{3/2}}{19b}$$

$$= \frac{6a(19Ab-10aB)x^2\sqrt{a+bx^3}}{1729b^2} + \frac{2(19Ab-10aB)x^5\sqrt{a+bx^3}}{247b} - \frac{24a^2(19Ab-10}{1729b^{8/3}\left(\left(1+\sqrt{\right.\right.}}$$

Mathematica [C] time = 0.15, size = 91, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(\frac{a(10aB-19Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)\left(10aB-19Ab-13bBx^3\right)\right)}{247b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)*(-19*A*b + 10*a*B - 13*b*B*x^3)) + (a
*(-19*A*b + 10*a*B)*Hypergeometric2F1[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(247*b^2)
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bx^7+Ax^4\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\sqrt{bx^3+a}\,x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)
```

maple [B] time = 0.05, size = 966, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x)

[Out] B*(2/19*(b*x^3+a)^(1/2)*x^8+6/247*(b*x^3+a)^(1/2)*a/b*x^5-60/1729*(b*x^3+a)^(1/2)*a^2/b^2*x^2-80/1729*I*a^3/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(2/13*(b*x^3+a)^(1/2)*x^5+6/91*(b*x^3+a)^(1/2)*a/b*x^2+8/91*I*a^2/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\, x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\, \left(B\, x^3 + A\right)\, \sqrt{b\, x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2),x)

[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(1/2), x)

sympy [A] time = 3.00, size = 83, normalized size = 0.14

$$\frac{A\sqrt{a}\,x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{B\sqrt{a}\,x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{8}{3}\\ \frac{11}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(B*x**3+A)*(b*x**3+a)**(1/2),x)

```
[Out] A*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(8/3)) + B*sqrt(a)*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,),
b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3))
```

3.191 $\int x\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=548

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-}\cdots}{\cdots}$$

```
[Out] 2/13*B*x^2*(b*x^3+a)^(3/2)/b+2/91*(13*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+6/91
*a*(13*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2
/91*3^(3/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-3/91*3^(1/4)*a^(4/3)*(13*A*b-4*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {459, 279, 303, 218, 1877}

$$\frac{2\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{91b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*(13*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(91*b) + (6*a*(13*A*b - 4*a*B)*Sqr
t[a + b*x^3])/(91*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*B*x^2*(
a + b*x^3)^(3/2))/(13*b) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b - 4
*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^
(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(
91*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b - 4*a*B)*
(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3)
+ b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(
5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+2aB\right)\right)\int x\sqrt{a+bx^3}\,dx}{13b}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b} + \frac{(3a(13Ab-4aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{91b}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b} + \frac{(3a(13Ab-4aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\sqrt{a+bx^3}}}{91b^{4/3}}$$

$$= \frac{2(13Ab-4aB)x^2\sqrt{a+bx^3}}{91b} + \frac{6a(13Ab-4aB)\sqrt{a+bx^3}}{91b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{2Bx^2\left(a+bx^3\right)^{3/2}}{13b}$$

Mathematica [C] time = 0.11, size = 75, normalized size = 0.14

$$\frac{x^2\sqrt{a+bx^3}\left(\frac{(13Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+4B\left(a+bx^3\right)\right)}{26b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3) + ((13*A*b - 4*a*B)*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(26*b)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^4 + Ax\right)\sqrt{bx^3 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

maple [B] time = 0.05, size = 926, normalized size = 1.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/13*(b*x^3+a)^(1/2)*x^5+6/91*(b*x^3+a)^(1/2)*a/b*x^2+8/91*I*a^2/b^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(2/7*(b*x^3+a)^(1/2)*x^2-2/7*I*a*3^(1
/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1
/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3
^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/
2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\, x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(Bx^3+A\right)\sqrt{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(1/2),x)
```

```
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.69, size = 83, normalized size = 0.15

$$\frac{A\sqrt{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{B\sqrt{a}\,x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(5/3)) + B*sqrt(a)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3))
```

3.192 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx$

Optimal. Leaf size=545

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

```
[Out] -A*(b*x^3+a)^(3/2)/a/x+1/7*(7*A*b+2*B*a)*x^2*(b*x^3+a)^(1/2)/a+3/7*(7*A*b+2
*B*a)*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/7*3^(3/4)*a
^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/
3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/14*3^(1/4)*a^(1/3)*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipt
icE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {453, 279, 303, 218, 1877}

$$\frac{\sqrt{2}\,3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] ((7*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/(7*a) + (3*(7*A*b + 2*a*B)*Sqrt[a + b
*x^3])/(7*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/
2))/(a*x) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*3^(3/4)*a^(1/3)*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^2}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{ax} - \frac{\left(-\frac{7Ab}{2}-aB\right)\int x\sqrt{a+bx^3}\,dx}{a}$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} + \frac{1}{14}(3(7Ab+2aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} + \frac{(3(7Ab+2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{14\sqrt[3]{b}} + \cdots$$

$$= \frac{(7Ab+2aB)x^2\sqrt{a+bx^3}}{7a} + \frac{3(7Ab+2aB)\sqrt{a+bx^3}}{7b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{ax} - \frac{3\sqrt[4]{3}\cdots}{\cdots}$$

Mathematica [C] time = 0.08, size = 81, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3(2aB+7Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-2A\left(a+bx^3\right)\right)}{2ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-2*A*(a + b*x^3) + ((7*A*b + 2*a*B)*x^3*Hypergeometric2F1
[-1/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(2*a*x)
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

maple [B] time = 0.06, size = 902, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x)
```

```
[Out] B*(2/7*(b*x^3+a)^(1/2)*x^2-2/7*I*a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-
a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2
))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^
2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
+A*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2
)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-
I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
```

```
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)
/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^
(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^2, x)
```

sympy [A] time = 3.05, size = 85, normalized size = 0.16

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}\,x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**2,x)
```

```
[Out] A*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)
/(3*x*gamma(2/3)) + B*sqrt(a)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(5/3))
```

3.193 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx$

Optimal. Leaf size=546

$$\frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{\ldots}}{\ldots}$$

```
[Out] -1/4*A*(b*x^3+a)^(3/2)/a/x^4-1/8*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/x+3/8*b^(1/3
)*(A*b+8*B*a)*(b*x^3+a)^(1/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/8*3^(3/4)
*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/
2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)-3/16*3^(1/4)*b^(1/3)*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
E((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 303, 218, 1877}

$$\frac{3^{3/4}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\;\sqrt[3]{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] -((A*b + 8*a*B)*Sqrt[a + b*x^3])/(8*a*x) + (3*b^(1/3)*(A*b + 8*a*B)*Sqrt[a
+ b*x^3])/(8*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))
/(4*a*x^4) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(2/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3]) + (3^(3/4)*b^(1/3)*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^
(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(
1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^5}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} - \frac{\left(-\frac{Ab}{2}-4aB\right)\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{4a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} + \frac{(3b(Ab+8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} + \frac{\left(3b^{2/3}(Ab+8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{16a}$$

$$= -\frac{(Ab+8aB)\sqrt{a+bx^3}}{8ax} + \frac{3\sqrt[3]{b}\,(Ab+8aB)\sqrt{a+bx^3}}{8a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{A\left(a+bx^3\right)^{3/2}}{4ax^4} - \frac{3\sqrt[4]{3}\sqrt{2-}}{}$$

Mathematica [C] time = 0.09, size = 80, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-\frac{x^3(8aB+Ab)\,_2F_1\left(-\frac{1}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-A\left(a+bx^3\right)\right)}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^5,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((A*b + 8*a*B)*x^3*Hypergeometric2F1[-
1/2, -1/3, 2/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(4*a*x^4)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

maple [B] time = 0.06, size = 920, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x)
```

```
[Out] A*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*(b*x^3+a)^(1/2)/a*b/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I
*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(
1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2))))+B*(-(b*x^3+a)^(1/2)/x-I*3^(1/2)*(-a*b^2)^(1/3)*
(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)
^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*
```

```
b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I
*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3 + A)\,\sqrt{b\,x^3 + a}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^5, x)
```

sympy [A] time = 3.25, size = 92, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{B\sqrt{a}\,\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**5,x)
```

```
[Out] A*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**4*gamma(-1/3)) + B*sqrt(a)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,),
b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3))
```

3.194 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx$

Optimal. Leaf size=581

$$-\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

```
[Out] -1/7*A*(b*x^3+a)^(3/2)/a/x^7+1/56*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a/x^4+3/11
2*b*(5*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x-3/112*b^(4/3)*(5*A*b-14*B*a)*(b*x^
3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/112*3^(3/4)*b^(4/3)*(5*A*b
-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)
^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2
)+3/224*3^(1/4)*b^(4/3)*(5*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$-\frac{3b^{4/3}\sqrt{a+bx^3}\,(5Ab-14aB)}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3^{3/4}b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\cdots}\right.\right.}{56\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] ((5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a*x^4) + (3*b*(5*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(112*a^2*x) - (3*b^(4/3)*(5*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(7*a*x^7)
 + (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3]) - (3^(3/4)*b^(4/3)*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^8}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(\frac{5Ab}{2}-7aB\right)\int \frac{\sqrt{a+bx^3}}{x^5}\,dx}{7a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{(3b(5Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(3b^2(5\right.}{}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{A\left(a+bx^3\right)^{3/2}}{7ax^7} - \frac{\left(3b^{5/3}(\right.}{}$$

$$= \frac{(5Ab-14aB)\sqrt{a+bx^3}}{56ax^4} + \frac{3b(5Ab-14aB)\sqrt{a+bx^3}}{112a^2x} - \frac{3b^{4/3}(5Ab-14aB)\sqrt{a+}}{112a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}$$

Mathematica [C] time = 0.10, size = 80, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{5Ab}{2}-7aB\right){}_2F_1\left(-\frac{4}{3},-\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-4A\left(a+bx^3\right)\right)}{28ax^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-4*A*(a + b*x^3) + (((5*A*b)/2 - 7*a*B)*x^3*Hypergeometri
c2F1[-4/3, -1/2, -1/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(28*a*x^7)
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

maple [B] time = 0.09, size = 964, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)/x^4-3/8*(b*x^3+a)^(1/2)/a*b/x-1/8*I*b/a*3^(1/2)*(-a
*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I
*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(
1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-1/7*(b*x^3+a)^(1/2)/x^7-3/56/a*b*(b*x^3+a)^
(1/2)/x^4+15/112/a^2*b^2*(b*x^3+a)^(1/2)/x+5/112*I*b^2/a^2*3^(1/2)*(-a*b^2)
^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3 + A)\,\sqrt{b\,x^3 + a}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^8, x)
```

sympy [A] time = 3.78, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)} + \frac{B\sqrt{a}\,\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**8,x)
```

```
[Out] A*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**7*gamma(-4/3)) + B*sqrt(a)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3))
```

3.195 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx$

Optimal. Leaf size=614

$$\frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-20aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b}{}$$

```
[Out] -1/10*A*(b*x^3+a)^(3/2)/a/x^10+1/140*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a/x^7+
3/1120*b*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-3/448*b^2*(11*A*b-20*B*a)*
(b*x^3+a)^(1/2)/a^3/x+3/448*b^(7/3)*(11*A*b-20*B*a)*(b*x^3+a)^(1/2)/a^3/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/448*3^(3/4)*b^(7/3)*(11*A*b-20*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/896*3^(1/4)*b
^(7/3)*(11*A*b-20*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$\frac{3b^{7/3}\sqrt{a+bx^3}(11Ab-20aB)}{448a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{3b^2\sqrt{a+bx^3}(11Ab-20aB)}{448a^3x}+\frac{3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2}{224\sqrt{2}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] ((11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(140*a*x^7) + (3*b*(11*A*b - 20*a*B)*Sq
rt[a + b*x^3])/(1120*a^2*x^4) - (3*b^2*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(
448*a^3*x) + (3*b^(7/3)*(11*A*b - 20*a*B)*Sqrt[a + b*x^3])/(448*a^3*((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)) - (A*(a + b*x^3)^(3/2))/(10*a*x^10) - (3*3^(1
/4)*Sqrt[2 - Sqrt[3]]*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
3^(3/4)*b^(7/3)*(11*A*b - 20*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(224*Sqrt[2]*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11}}\,dx = -\frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} - \frac{\left(\frac{11Ab}{2}-10aB\right)\int \frac{\sqrt{a+bx^3}}{x^8}\,dx}{10a}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} - \frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} - \frac{(3b(11Ab-20aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{280a}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{A\left(a+bx^3\right)^{3/2}}{10ax^{10}} + \frac{\left(3b^2(11\right.}{}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+b}}{448a^3x}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+b}}{448a^3x}$$

$$= \frac{(11Ab-20aB)\sqrt{a+bx^3}}{140ax^7} + \frac{3b(11Ab-20aB)\sqrt{a+bx^3}}{1120a^2x^4} - \frac{3b^2(11Ab-20aB)\sqrt{a+b}}{448a^3x}$$

Mathematica [C] time = 0.10, size = 80, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{11Ab}{2}-10aB\right){}_2F_1\left(-\frac{7}{3},-\frac{1}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-7A\left(a+bx^3\right)\right)}{70ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*(-7*A*(a + b*x^3) + (((11*A*b)/2 - 10*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -1/2, -4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(70*a*x^10)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

maple [B] time = 0.08, size = 1006, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x)
```

```
[Out] B*(-1/7*(b*x^3+a)^(1/2)/x^7-3/56*(b*x^3+a)^(1/2)/a*b/x^4+15/112*(b*x^3+a)^(
1/2)/a^2*b^2/x+5/112*I*b^2/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(
-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1
/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)
^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a
*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/
3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-1
/10*(b*x^3+a)^(1/2)/x^10-3/140/a*b*(b*x^3+a)^(1/2)/x^7+33/1120/a^2*b^2*(b*x
^3+a)^(1/2)/x^4-33/448/a^3*b^3*(b*x^3+a)^(1/2)/x-11/448*I/a^3*b^3*3^(1/2)*(
-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*
(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)
^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(
x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/
3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^11, x)
```

sympy [A] time = 5.14, size = 97, normalized size = 0.16

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{10}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{10}{3}, -\frac{1}{2} \\ -\frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)} + \frac{B\sqrt{a}\,\Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{7}{3}, -\frac{1}{2} \\ -\frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^{7}\Gamma\left(-\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**11,x)
```

```
[Out] A*sqrt(a)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi)
/a)/(3*x**10*gamma(-7/3)) + B*sqrt(a)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3))
```

3.196 $\int x^8 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=103

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

```
[Out] 2/15*a^2*(A*b-B*a)*(b*x^3+a)^(5/2)/b^4-2/21*a*(2*A*b-3*B*a)*(b*x^3+a)^(7/2)
/b^4+2/27*(A*b-3*B*a)*(b*x^3+a)^(9/2)/b^4+2/33*B*(b*x^3+a)^(11/2)/b^4
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2a^2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^4}+\frac{2\left(a+bx^3\right)^{9/2}(Ab-3aB)}{27b^4}-\frac{2a\left(a+bx^3\right)^{7/2}(2Ab-3aB)}{21b^4}+\frac{2B\left(a+bx^3\right)^{11/2}}{33b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*a^2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^4) - (2*a*(2*A*b - 3*a*B)*(a +
b*x^3)^(7/2))/(21*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(9/2))/(27*b^4) + (2*
B*(a + b*x^3)^(11/2))/(33*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^8\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int x^2(a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(-\frac{a^2(-Ab+aB)(a+bx)^{3/2}}{b^3}+\frac{a(-2Ab+3aB)(a+bx)^{5/2}}{b^3}+\frac{(A\end{aligned}$$

$$= \frac{2a^2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{7/2}}{21b^4}+\frac{2(Ab-3aB)\left(a\right.}{27b^4}$$

Mathematica [A] time = 0.09, size = 78, normalized size = 0.76

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-48a^3B+8a^2b\left(11A+15Bx^3\right)-10ab^2x^3\left(22A+21Bx^3\right)+35b^3x^6\left(11A+9Bx^3\right)\right)}{10395b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(-48*a^3*B + 35*b^3*x^6*(11*A + 9*B*x^3) + 8*a^2*b*(11
*A + 15*B*x^3) - 10*a*b^2*x^3*(22*A + 21*B*x^3)))/(10395*b^4)
```

fricas [A] time = 0.96, size = 124, normalized size = 1.20

$$\frac{2\left(315\,Bb^5x^{15}+35\left(12\,Bab^4+11\,Ab^5\right)x^{12}+5\left(3\,Ba^2b^3+110\,Aab^4\right)x^9-3\left(6\,Ba^3b^2-11\,Aa^2b^3\right)x^6-48\,Ba^5+88\,A\right.}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/10395*(315*B*b^5*x^15 + 35*(12*B*a*b^4 + 11*A*b^5)*x^12 + 5*(3*B*a^2*b^3
+ 110*A*a*b^4)*x^9 - 3*(6*B*a^3*b^2 - 11*A*a^2*b^3)*x^6 - 48*B*a^5 + 88*A*a
^4*b + 4*(6*B*a^4*b - 11*A*a^3*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

giac [A] time = 0.16, size = 104, normalized size = 1.01

$$\frac{2\left(315\left(bx^3+a\right)^{\frac{11}{2}}B-1155\left(bx^3+a\right)^{\frac{9}{2}}Ba+1485\left(bx^3+a\right)^{\frac{7}{2}}Ba^2-693\left(bx^3+a\right)^{\frac{5}{2}}Ba^3+385\left(bx^3+a\right)^{\frac{9}{2}}Ab-990\left(\right.\right.}{10395\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/10395*(315*(b*x^3 + a)^(11/2)*B - 1155*(b*x^3 + a)^(9/2)*B*a + 1485*(b*x^
3 + a)^(7/2)*B*a^2 - 693*(b*x^3 + a)^(5/2)*B*a^3 + 385*(b*x^3 + a)^(9/2)*A*
b - 990*(b*x^3 + a)^(7/2)*A*a*b + 693*(b*x^3 + a)^(5/2)*A*a^2*b)/b^4
```

maple [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(315B\,x^9b^3+385A\,b^3x^6-210Ba\,b^2x^6-220Aa\,b^2x^3+120B\,a^2b\,x^3+88A\,a^2b-48B\,a^3\right)}{10395b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 2/10395*(b*x^3+a)^(5/2)*(315*B*b^3*x^9+385*A*b^3*x^6-210*B*a*b^2*x^6-220*A*
a*b^2*x^3+120*B*a^2*b*x^3+88*A*a^2*b-48*B*a^3)/b^4
```

maxima [A] time = 0.61, size = 118, normalized size = 1.15

$$\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)A+\frac{2}{3465}\left(\frac{105\left(bx^3+a\right)^{\frac{11}{2}}}{b^4}-\frac{385\left(bx^3+a\right)^{\frac{9}{2}}a}{b^4}+\frac{495\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 +
a)^(5/2)*a^2/b^3)*A + 2/3465*(105*(b*x^3 + a)^(11/2)/b^4 - 385*(b*x^3 + a)^
(9/2)*a/b^4 + 495*(b*x^3 + a)^(7/2)*a^2/b^4 - 231*(b*x^3 + a)^(5/2)*a^3/b^4
)*B
```

mupad [B] time = 2.65, size = 206, normalized size = 2.00

$$\frac{20\,A\,a\,x^9\,\sqrt{b\,x^3+a}}{189}+\frac{2\,A\,b\,x^{12}\,\sqrt{b\,x^3+a}}{27}+\frac{8\,B\,a\,x^{12}\,\sqrt{b\,x^3+a}}{99}+\frac{2\,B\,b\,x^{15}\,\sqrt{b\,x^3+a}}{33}+\frac{16\,A\,a^4\,\sqrt{b\,x^3+a}}{945\,b^3}-\frac{32\,B\,a^5}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (20*A*a*x^9*(a + b*x^3)^(1/2))/189 + (2*A*b*x^12*(a + b*x^3)^(1/2))/27 + (8
*B*a*x^12*(a + b*x^3)^(1/2))/99 + (2*B*b*x^15*(a + b*x^3)^(1/2))/33 + (16*A
*a^4*(a + b*x^3)^(1/2))/(945*b^3) - (32*B*a^5*(a + b*x^3)^(1/2))/(3465*b^4)
 - (8*A*a^3*x^3*(a + b*x^3)^(1/2))/(945*b^2) + (2*A*a^2*x^6*(a + b*x^3)^(1/
2))/(315*b) + (16*B*a^4*x^3*(a + b*x^3)^(1/2))/(3465*b^3) - (4*B*a^3*x^6*(a
 + b*x^3)^(1/2))/(1155*b^2) + (2*B*a^2*x^9*(a + b*x^3)^(1/2))/(693*b)
```

sympy [A] time = 8.44, size = 267, normalized size = 2.59

$$\begin{cases} \frac{16Aa^4\sqrt{a+bx^3}}{945b^3} - \frac{8Aa^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2Aa^2x^6\sqrt{a+bx^3}}{315b} + \frac{20Aax^9\sqrt{a+bx^3}}{189} + \frac{2Abx^{12}\sqrt{a+bx^3}}{27} - \frac{32Ba^5\sqrt{a+bx^3}}{3465b^4} + \frac{16Ba^4x^3\sqrt{a+bx^3}}{3465b^3} - \frac{4B \\ a^{\frac{3}{2}}\left(\frac{Ax^9}{9} + \frac{Bx^{12}}{12}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((16*A*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*A*a**3*x**3*sqrt(a + b
*x**3)/(945*b**2) + 2*A*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*A*a*x**9*sq
rt(a + b*x**3)/189 + 2*A*b*x**12*sqrt(a + b*x**3)/27 - 32*B*a**5*sqrt(a + b
*x**3)/(3465*b**4) + 16*B*a**4*x**3*sqrt(a + b*x**3)/(3465*b**3) - 4*B*a**3
*x**6*sqrt(a + b*x**3)/(1155*b**2) + 2*B*a**2*x**9*sqrt(a + b*x**3)/(693*b)
 + 8*B*a*x**12*sqrt(a + b*x**3)/99 + 2*B*b*x**15*sqrt(a + b*x**3)/33, Ne(b,
 0)), (a**(3/2)*(A*x**9/9 + B*x**12/12), True))
```

3.197 $\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx$

Optimal. Leaf size=73

$$\frac{2 \left(a+bx^3\right)^{7/2} (Ab-2aB)}{21b^3}-\frac{2a \left(a+bx^3\right)^{5/2} (Ab-aB)}{15b^3}+\frac{2B \left(a+bx^3\right)^{9/2}}{27b^3}$$

```
[Out] -2/15*a*(A*b-B*a)*(b*x^3+a)^(5/2)/b^3+2/21*(A*b-2*B*a)*(b*x^3+a)^(7/2)/b^3+
2/27*B*(b*x^3+a)^(9/2)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2 \left(a+bx^3\right)^{7/2} (Ab-2aB)}{21b^3}-\frac{2a \left(a+bx^3\right)^{5/2} (Ab-aB)}{15b^3}+\frac{2B \left(a+bx^3\right)^{9/2}}{27b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (-2*a*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3
)^(7/2))/(21*b^3) + (2*B*(a + b*x^3)^(9/2))/(27*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) \, dx &= \frac{1}{3} \text{Subst}\left(\int x(a+bx)^{3/2}(A+Bx) \, dx, x, x^3\right)\\
&= \frac{1}{3} \text{Subst}\left(\int \left(\frac{a(-Ab+aB)(a+bx)^{3/2}}{b^2}+\frac{(Ab-2aB)(a+bx)^{5/2}}{b^2}+\frac{B(a+bx)^{7/2}}{b^2}\right.\right.\\
&= -\frac{2a(Ab-aB) \left(a+bx^3\right)^{5/2}}{15b^3}+\frac{2(Ab-2aB) \left(a+bx^3\right)^{7/2}}{21b^3}+\frac{2B \left(a+bx^3\right)^{9/2}}{27b^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.78

$$\frac{2 \left(a+bx^3\right)^{5/2} \left(8a^2B-2ab \left(9A+10Bx^3\right)+5b^2x^3 \left(9A+7Bx^3\right)\right)}{945b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(8*a^2*B + 5*b^2*x^3*(9*A + 7*B*x^3) - 2*a*b*(9*A + 10
*B*x^3)))/(945*b^3)
```

fricas [A] time = 0.95, size = 99, normalized size = 1.36

$$\frac{2\left(35\,Bb^4x^{12}+5\left(10\,Bab^3+9\,Ab^4\right)x^9+3\left(Ba^2b^2+24\,Aab^3\right)x^6+8\,Ba^4-18\,Aa^3b-\left(4\,Ba^3b-9\,Aa^2b^2\right)x^3\right)\sqrt{b}}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/945*(35*B*b^4*x^12 + 5*(10*B*a*b^3 + 9*A*b^4)*x^9 + 3*(B*a^2*b^2 + 24*A*a
*b^3)*x^6 + 8*B*a^4 - 18*A*a^3*b - (4*B*a^3*b - 9*A*a^2*b^2)*x^3)*sqrt(b*x^
3 + a)/b^3
```

giac [A] time = 0.18, size = 73, normalized size = 1.00

$$\frac{2\left(35\left(bx^3+a\right)^{\frac{9}{2}}B-90\left(bx^3+a\right)^{\frac{7}{2}}Ba+63\left(bx^3+a\right)^{\frac{5}{2}}Ba^2+45\left(bx^3+a\right)^{\frac{7}{2}}Ab-63\left(bx^3+a\right)^{\frac{5}{2}}Aab\right)}{945\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/945*(35*(b*x^3 + a)^(9/2)*B - 90*(b*x^3 + a)^(7/2)*B*a + 63*(b*x^3 + a)^(
5/2)*B*a^2 + 45*(b*x^3 + a)^(7/2)*A*b - 63*(b*x^3 + a)^(5/2)*A*a*b)/b^3
```

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(-35B\,b^2x^6-45A\,b^2x^3+20Bab\,x^3+18Aab-8B\,a^2\right)}{945b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] -2/945*(b*x^3+a)^(5/2)*(-35*B*b^2*x^6-45*A*b^2*x^3+20*B*a*b*x^3+18*A*a*b-8*
B*a^2)/b^3
```

maxima [A] time = 0.61, size = 84, normalized size = 1.15

$$\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)A+\frac{2}{945}\left(\frac{35\left(bx^3+a\right)^{\frac{9}{2}}}{b^3}-\frac{90\left(bx^3+a\right)^{\frac{7}{2}}a}{b^3}+\frac{63\left(bx^3+a\right)^{\frac{5}{2}}a^2}{b^3}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)^(5/2)*a/b^2)*A + 2/945*(35*(
b*x^3 + a)^(9/2)/b^3 - 90*(b*x^3 + a)^(7/2)*a/b^3 + 63*(b*x^3 + a)^(5/2)*a^
2/b^3)*B
```

mupad [B] time = 2.72, size = 211, normalized size = 2.89

$$\frac{x^6\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{15\,b}-\frac{2\,a\,\left(2\,A\,a^2-\frac{4\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{6\,a\left(2\,A\,b^2+\frac{20\,B\,a\,b}{9}\right)}{7\,b}\right)}{5\,b}\right)\sqrt{b\,x^3+a}}{9\,b^2}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] (x^6*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 + (20*B*a*b)/9))/
(7*b)))/(15*b) - (2*a*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2 +
(20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b^2) + (2*B*b*x^12*(a +
 b*x^3)^(1/2))/27 + (x^3*(2*A*a^2 - (4*a*(2*B*a^2 + 4*A*a*b - (6*a*(2*A*b^2
 + (20*B*a*b)/9))/(7*b)))/(5*b))*(a + b*x^3)^(1/2))/(9*b) + (x^9*(2*A*b^2 +
 (20*B*a*b)/9)*(a + b*x^3)^(1/2))/(21*b)
```

sympy [A] time = 4.75, size = 216, normalized size = 2.96

$$\begin{cases} -\frac{4Aa^3\sqrt{a+bx^3}}{105b^2} + \frac{2Aa^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Aax^6\sqrt{a+bx^3}}{105} + \frac{2Abx^9\sqrt{a+bx^3}}{21} + \frac{16Ba^4\sqrt{a+bx^3}}{945b^3} - \frac{8Ba^3x^3\sqrt{a+bx^3}}{945b^2} + \frac{2Ba^2x^6\sqrt{a+bx^3}}{315b} + \frac{20Bax^9}{1\cdots} \\ a^{\frac{3}{2}}\left(\frac{Ax^6}{6} + \frac{Bx^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((-4*A*a**3*sqrt(a + b*x**3)/(105*b**2) + 2*A*a**2*x**3*sqrt(a + b
*x**3)/(105*b) + 16*A*a*x**6*sqrt(a + b*x**3)/105 + 2*A*b*x**9*sqrt(a + b*x
**3)/21 + 16*B*a**4*sqrt(a + b*x**3)/(945*b**3) - 8*B*a**3*x**3*sqrt(a + b*
x**3)/(945*b**2) + 2*B*a**2*x**6*sqrt(a + b*x**3)/(315*b) + 20*B*a*x**9*sqr
t(a + b*x**3)/189 + 2*B*b*x**12*sqrt(a + b*x**3)/27, Ne(b, 0)), (a**(3/2)*(
A*x**6/6 + B*x**9/9), True))
```

3.198 $\int x^2 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=46

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

```
[Out] 2/15*(A*b-B*a)*(b*x^3+a)^(5/2)/b^2+2/21*B*(b*x^3+a)^(7/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\left(a+bx^3\right)^{5/2}(Ab-aB)}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(A*b - a*B)*(a + b*x^3)^(5/2))/(15*b^2) + (2*B*(a + b*x^3)^(7/2))/(21*b^
2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx &= \frac{1}{3}\text{Subst}\left(\int (a+bx)^{3/2}(A+Bx)\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int\left(\frac{(Ab-aB)(a+bx)^{3/2}}{b}+\frac{B(a+bx)^{5/2}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\left(a+bx^3\right)^{5/2}}{15b^2}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.74

$$\frac{2\left(a+bx^3\right)^{5/2}\left(-2aB+7Ab+5bBx^3\right)}{105b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*(a + b*x^3)^(5/2)*(7*A*b - 2*a*B + 5*b*B*x^3))/(105*b^2)
```

fricas [A] time = 0.88, size = 73, normalized size = 1.59

$$\frac{2\left(5\,Bb^3x^9+\left(8\,Bab^2+7\,Ab^3\right)x^6-2\,Ba^3+7\,Aa^2b+\left(Ba^2b+14\,Aab^2\right)x^3\right)\sqrt{bx^3+a}}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] 2/105*(5*B*b^3*x^9 + (8*B*a*b^2 + 7*A*b^3)*x^6 - 2*B*a^3 + 7*A*a^2*b + (B*a
^2*b + 14*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^2
```

giac [A] time = 0.17, size = 44, normalized size = 0.96

$$\frac{2\left(5\left(bx^3+a\right)^{\frac{7}{2}}B-7\left(bx^3+a\right)^{\frac{5}{2}}Ba+7\left(bx^3+a\right)^{\frac{5}{2}}Ab\right)}{105\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 2/105*(5*(b*x^3 + a)^(7/2)*B - 7*(b*x^3 + a)^(5/2)*B*a + 7*(b*x^3 + a)^(5/2
)*A*b)/b^2
```

maple [A] time = 0.05, size = 31, normalized size = 0.67

$$\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}\left(5Bb\,x^3+7Ab-2Ba\right)}{105b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 2/105*(b*x^3+a)^(5/2)*(5*B*b*x^3+7*A*b-2*B*a)/b^2
```

maxima [A] time = 0.49, size = 49, normalized size = 1.07

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}A}{15\,b}+\frac{2}{105}\left(\frac{5\left(bx^3+a\right)^{\frac{7}{2}}}{b^2}-\frac{7\left(bx^3+a\right)^{\frac{5}{2}}a}{b^2}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*A/b + 2/105*(5*(b*x^3 + a)^(7/2)/b^2 - 7*(b*x^3 + a)
^(5/2)*a/b^2)*B
```

mupad [B] time = 3.35, size = 150, normalized size = 3.26

$$\frac{\left(2\,A\,a^2-\frac{2\,a\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{3\,b}\right)\sqrt{b\,x^3+a}}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{4\,a\left(2\,A\,b^2+\frac{16\,B\,a\,b}{7}\right)}{5\,b}\right)}{9\,b}+\frac{2\,B\,b\,x^9}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] ((2*A*a^2 - (2*a*(2*B*a^2 + 4*A*a*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b))
)/(3*b))*(a + b*x^3)^(1/2))/(3*b) + (x^3*(a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a
*b - (4*a*(2*A*b^2 + (16*B*a*b)/7))/(5*b)))/(9*b) + (2*B*b*x^9*(a + b*x^3)^
(1/2))/21 + (x^6*(2*A*b^2 + (16*B*a*b)/7)*(a + b*x^3)^(1/2))/(15*b)
```

sympy [A] time = 2.52, size = 165, normalized size = 3.59

$$\begin{cases} \frac{2Aa^2\sqrt{a+bx^3}}{15b} + \frac{4Aax^3\sqrt{a+bx^3}}{15} + \frac{2Abx^6\sqrt{a+bx^3}}{15} - \frac{4Ba^3\sqrt{a+bx^3}}{105b^2} + \frac{2Ba^2x^3\sqrt{a+bx^3}}{105b} + \frac{16Bax^6\sqrt{a+bx^3}}{105} + \frac{2Bbx^9\sqrt{a+bx^3}}{21} & \text{for } b \neq \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3} + \frac{Bx^6}{6}\right) & \text{otherw} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Piecewise((2*A*a**2*sqrt(a + b*x**3)/(15*b) + 4*A*a*x**3*sqrt(a + b*x**3)/1
5 + 2*A*b*x**6*sqrt(a + b*x**3)/15 - 4*B*a**3*sqrt(a + b*x**3)/(105*b**2) +
 2*B*a**2*x**3*sqrt(a + b*x**3)/(105*b) + 16*B*a*x**6*sqrt(a + b*x**3)/105
+ 2*B*b*x**9*sqrt(a + b*x**3)/21, Ne(b, 0)), (a**(3/2)*(A*x**3/3 + B*x**6/6
), True))
```

3.199 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x}\,dx$

Optimal. Leaf size=81

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

```
[Out] 2/9*A*(b*x^3+a)^(3/2)+2/15*B*(b*x^3+a)^(5/2)/b-2/3*a^(3/2)*A*arctanh((b*x^3
+a)^(1/2)/a^(1/2))+2/3*a*A*(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2}{3}aA\sqrt{a+bx^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*a*A*Sqrt[a + b*x^3])/3 + (2*A*(a + b*x^3)^(3/2))/9 + (2*B*(a + b*x^3)^(5
/2))/(15*b) - (2*a^(3/2)*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x}\,dx,x,x^3\right)$$
$$= \frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}A\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx,x,x^3\right)$$
$$= \frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}(aA)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,x^3\right)$$
$$= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{1}{3}\left(a^2A\right)\text{Subst}\left(\int \frac{}{x\sqrt{a}}\right.$$
$$= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}+\frac{\left(2a^2A\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{3b}$$
$$= \frac{2}{3}aA\sqrt{a+bx^3}+\frac{2}{9}A\left(a+bx^3\right)^{3/2}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b}-\frac{2}{3}a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.99

$$\frac{2\left(-15a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)+5Ab\left(a+bx^3\right)^{3/2}+15aAb\sqrt{a+bx^3}+3B\left(a+bx^3\right)^{5/2}\right)}{45b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x,x]
```

```
[Out] (2*(15*a*A*b*Sqrt[a + b*x^3] + 5*A*b*(a + b*x^3)^(3/2) + 3*B*(a + b*x^3)^(5
/2) - 15*a^(3/2)*A*b*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]))/(45*b)
```

fricas [A] time = 0.96, size = 172, normalized size = 2.12

$$\left[\frac{15\,Aa^{\frac{3}{2}}b\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Bb^2x^6+\left(6\,Bab+5\,Ab^2\right)x^3+3\,Ba^2+20\,Aab\right)\sqrt{bx^3+a}}{45\,b},\ \frac{2\left(15\,A\sqrt{-a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="fricas")
```

```
[Out] [1/45*(15*A*a^(3/2)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) +
2*(3*B*b^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 +
 a))/b, 2/45*(15*A*sqrt(-a)*a*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*B*b
^2*x^6 + (6*B*a*b + 5*A*b^2)*x^3 + 3*B*a^2 + 20*A*a*b)*sqrt(b*x^3 + a))/b]
```

giac [A] time = 0.16, size = 80, normalized size = 0.99

$$\frac{2\,Aa^2\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}Bb^4+5\left(bx^3+a\right)^{\frac{3}{2}}Ab^5+15\,\sqrt{bx^3+a}\,Aab^5\right)}{45\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="giac")
```

```
[Out] 2/3*A*a^2*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/45*(3*(b*x^3 + a)^(
5/2)*B*b^4 + 5*(b*x^3 + a)^(3/2)*A*b^5 + 15*sqrt(b*x^3 + a)*A*a*b^5)/b^5
```

maple [A] time = 0.05, size = 66, normalized size = 0.81

$$\left(\frac{2\sqrt{b\,x^3+a}\;b\,x^3}{9}-\frac{2a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3}+\frac{8\sqrt{b\,x^3+a}\;a}{9}\right)A+\frac{2\left(b\,x^3+a\right)^{\frac{5}{2}}B}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x,x)
```

```
[Out] 2/15*B*(b*x^3+a)^(5/2)/b+A*(2/9*(b*x^3+a)^(1/2)*b*x^3+8/9*(b*x^3+a)^(1/2)*a
-2/3*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.26, size = 80, normalized size = 0.99

$$\frac{2\left(bx^3+a\right)^{\frac{5}{2}}B}{15\,b}+\frac{1}{9}\left(3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+a\right)^{\frac{3}{2}}+6\sqrt{bx^3+a}\,a\right)A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x,x, algorithm="maxima")
```

```
[Out] 2/15*(b*x^3 + a)^(5/2)*B/b + 1/9*(3*a^(3/2)*log((sqrt(b*x^3 + a) - sqrt(a))
/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3/2) + 6*sqrt(b*x^3 + a)*a)*
A
```

mupad [B] time = 2.79, size = 131, normalized size = 1.62

$$\frac{A\,a^{3/2}\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3}+\frac{\sqrt{b\,x^3+a}\,\left(2\,B\,a^2+4\,A\,a\,b-\frac{2\,a\left(2\,A\,b^2+\frac{12\,B\,a\,b}{5}\right)}{3\,b}\right)}{3\,b}+\frac{2\,B\,b\,x^6\,\sqrt{b\,x^3+a}}{15}+\frac{x^3\,(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x,x)
```

```
[Out] (A*a^(3/2)*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2
)))/x^6))/3 + ((a + b*x^3)^(1/2)*(2*B*a^2 + 4*A*a*b - (2*a*(2*A*b^2 + (12*B
*a*b)/5))/(3*b)))/(3*b) + (2*B*b*x^6*(a + b*x^3)^(1/2))/15 + (x^3*(2*A*b^2
+ (12*B*a*b)/5)*(a + b*x^3)^(1/2))/(9*b)
```

sympy [A] time = 66.50, size = 82, normalized size = 1.01

$$\frac{2Aa^2\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3\sqrt{-a}}+\frac{2Aa\sqrt{a+bx^3}}{3}+\frac{2A\left(a+bx^3\right)^{\frac{3}{2}}}{9}+\frac{2B\left(a+bx^3\right)^{\frac{5}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x,x)
```

```
[Out] 2*A*a**2*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*sqrt(-a)) + 2*A*a*sqrt(a + b*x*
*3)/3 + 2*A*(a + b*x**3)**(3/2)/9 + 2*B*(a + b*x**3)**(5/2)/(15*b)
```

3.200 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^4} \, dx$

Optimal. Leaf size=110

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}\,(2aB+3Ab)-\frac{1}{3}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A(a+bx^3)^{5/2}}{3ax^3}$$

```
[Out] 1/9*(3*A*b+2*B*a)*(b*x^3+a)^(3/2)/a-1/3*A*(b*x^3+a)^(5/2)/a/x^3-1/3*(3*A*b+
2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+1/3*(3*A*b+2*B*a)*(b*x^3+a)
^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {446, 78, 50, 63, 208}

$$\frac{(a+bx^3)^{3/2}(2aB+3Ab)}{9a}+\frac{1}{3}\sqrt{a+bx^3}\,(2aB+3Ab)-\frac{1}{3}\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)-\frac{A(a+bx^3)^{5/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((3*A*b + 2*a*B)*Sqrt[a + b*x^3])/3 + ((3*A*b + 2*a*B)*(a + b*x^3)^(3/2))/(
9*a) - (A*(a + b*x^3)^(5/2))/(3*a*x^3) - (Sqrt[a]*(3*A*b + 2*a*B)*ArcTanh[S
qrt[a + b*x^3]/Sqrt[a]])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^4}dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^2}dx, x, x^3\right)$$
$$= -\frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{\left(\frac{3Ab}{2}+aB\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, x^3\right)}{3a}$$
$$= \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(3Ab+2aB)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\, d\right.$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{1}{6}(a(3Ab$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} + \frac{(a(3Ab}{}$$
$$= \frac{1}{3}(3Ab+2aB)\sqrt{a+bx^3} + \frac{(3Ab+2aB)\left(a+bx^3\right)^{3/2}}{9a} - \frac{A\left(a+bx^3\right)^{5/2}}{3ax^3} - \frac{1}{3}\sqrt{a}\,(3A$$

Mathematica [A] time = 0.09, size = 80, normalized size = 0.73

$$\frac{1}{9}\left(\frac{\sqrt{a+bx^3}\left(-3aA+8aBx^3+6Abx^3+2bBx^6\right)}{x^3} - 3\sqrt{a}\,(2aB+3Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^4,x]
```

```
[Out] ((Sqrt[a + b*x^3]*(-3*a*A + 6*A*b*x^3 + 8*a*B*x^3 + 2*b*B*x^6))/x^3 - 3*Sqr
t[a]*(3*A*b + 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/9
```

fricas [A] time = 0.81, size = 169, normalized size = 1.54

$$\left[\frac{3\,(2\,Ba+3\,Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(2\,Bbx^6+2\,(4\,Ba+3\,Ab)x^3-3\,Aa\right)\sqrt{bx^3+a}}{18\,x^3}, \frac{3\,(2\,Ba+3\,Ab}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="fricas")
```

```
[Out] [1/18*(3*(2*B*a + 3*A*b)*sqrt(a)*x^3*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a)
 + 2*a)/x^3) + 2*(2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a
))/x^3, 1/9*(3*(2*B*a + 3*A*b)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + (2*B*b*x^6 + 2*(4*B*a + 3*A*b)*x^3 - 3*A*a)*sqrt(b*x^3 + a))/x^3]
```

giac [A] time = 0.20, size = 103, normalized size = 0.94

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}Bb+6\,\sqrt{bx^3+a}\,Bab+6\,\sqrt{bx^3+a}\,Ab^2+\frac{3\left(2\,Ba^2b+3\,Aab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3\,\sqrt{bx^3+a}\,Aab}{x^3}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/9*(2*(b*x^3 + a)^(3/2)*B*b + 6*sqrt(b*x^3 + a)*B*a*b + 6*sqrt(b*x^3 + a)*
A*b^2 + 3*(2*B*a^2*b + 3*A*a*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a)
 - 3*sqrt(b*x^3 + a)*A*a*b/x^3)/b
```

maple [A] time = 0.05, size = 101, normalized size = 0.92

$$\left(-\sqrt{a}\, b \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)+\frac{2\sqrt{b x^3+a}\, b}{3}-\frac{\sqrt{b x^3+a}\, a}{3x^3}\right)A+\left(\frac{2\sqrt{b x^3+a}\, b x^3}{9}-\frac{2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x^3+a}}{\sqrt{a}}\right)}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x)
```

```
[Out] A*(-1/3*(b*x^3+a)^(1/2)*a/x^3+2/3*(b*x^3+a)^(1/2)*b-b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2))+B*(2/9*(b*x^3+a)^(1/2)*b*x^3+8/9*(b*x^3+a)^(1/2)*a-2/3
*a^(3/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.44, size = 134, normalized size = 1.22

$$\frac{1}{6}\left(3\sqrt{a}\, b \log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+4\sqrt{bx^3+a}\, b-\frac{2\sqrt{bx^3+a}\, a}{x^3}\right)A+\frac{1}{9}\left(3a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)+2\left(bx^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
)) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a)*a/x^3)*A + 1/9*(3*a^(3/2)*log(
(sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 2*(b*x^3 + a)^(3
/2) + 6*sqrt(b*x^3 + a)*a)*B
```

mupad [B] time = 3.38, size = 111, normalized size = 1.01

$$\frac{\ln\left(\frac{\left(\sqrt{b x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b x^3+a}+\sqrt{a}\right)}{x^6}\right)(3 A b+2 B a)\sqrt{\frac{a}{4}}}{3}+\frac{\left(2 A b^2+\frac{8 B a b}{3}\right)\sqrt{b x^3+a}}{3 b}-\frac{A a\sqrt{b x^3+a}}{3 x^3}+\frac{2 B b x^3\sqrt{b\ldots}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^4,x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(
3*A*b + 2*B*a)*(a/4)^(1/2))/3 + ((2*A*b^2 + (8*B*a*b)/3)*(a + b*x^3)^(1/2))
/(3*b) - (A*a*(a + b*x^3)^(1/2))/(3*x^3) + (2*B*b*x^3*(a + b*x^3)^(1/2))/9
```

sympy [A] time = 58.40, size = 223, normalized size = 2.03

$$-A\sqrt{a}\, b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)-\frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}+\frac{2Aa\sqrt{b}}{3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{2Ab^{\frac{3}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx^3}+1}}-\frac{2Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\, x^{\frac{3}{2}}}\right)}{3}+\frac{2Ba^2}{3\sqrt{b}\, x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**4,x)
```

```
[Out] -A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - A*a*sqrt(b)*sqrt(a/(b*x**3
) + 1)/(3*x**(3/2)) + 2*A*a*sqrt(b)/(3*x**(3/2)*sqrt(a/(b*x**3) + 1)) + 2*A
*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) - 2*B*a**(3/2)*asinh(sqrt(a)/(s
qrt(b)*x**(3/2)))/3 + 2*B*a**2/(3*sqrt(b)*x**(3/2)*sqrt(a/(b*x**3) + 1)) +
2*B*a*sqrt(b)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1)) + B*b*Piecewise((sqrt(a)*x*
*3/3, Eq(b, 0)), (2*(a + b*x**3)**(3/2)/(9*b), True))
```

3.201 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^7} dx$

Optimal. Leaf size=115

$$-\frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A(a+bx^3)^{5/2}}{6ax^6}$$

```
[Out] -1/12*(A*b+4*B*a)*(b*x^3+a)^(3/2)/a/x^3-1/6*A*(b*x^3+a)^(5/2)/a/x^6-1/4*b*(
A*b+4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+1/4*b*(A*b+4*B*a)*(b*x^
3+a)^(1/2)/a
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$-\frac{(a+bx^3)^{3/2}(4aB+Ab)}{12ax^3}+\frac{b\sqrt{a+bx^3}(4aB+Ab)}{4a}-\frac{b(4aB+Ab)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{A(a+bx^3)^{5/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] (b*(A*b + 4*a*B)*Sqrt[a + b*x^3])/(4*a) - ((A*b + 4*a*B)*(a + b*x^3)^(3/2))
/(12*a*x^3) - (A*(a + b*x^3)^(5/2))/(6*a*x^6) - (b*(A*b + 4*a*B)*ArcTanh[Sq
rt[a + b*x^3]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^7}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}(A+Bx)}{x^3}\,dx,x,x^3\right)$$

$$= -\frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(Ab+4aB)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx,x,x^3\right)}{12a}$$

$$= -\frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{(b(Ab+4aB))\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,\right.}{8a}$$

$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{8}(b(Ab+4$$

$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} + \frac{1}{4}(Ab+4a$$

$$= \frac{b(Ab+4aB)\sqrt{a+bx^3}}{4a} - \frac{(Ab+4aB)\left(a+bx^3\right)^{3/2}}{12ax^3} - \frac{A\left(a+bx^3\right)^{5/2}}{6ax^6} - \frac{b(Ab+4a}{}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{5/2}\left(bx^6(4aB+Ab)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx^3}{a}+1\right)-5a^2A\right)}{30a^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^7,x]
```

```
[Out] ((a + b*x^3)^(5/2)*(-5*a^2*A + b*(A*b + 4*a*B)*x^6*Hypergeometric2F1[2, 5/2
, 7/2, 1 + (b*x^3)/a]))/(30*a^3*x^6)
```

fricas [A] time = 0.96, size = 191, normalized size = 1.66

$$\left[\frac{3\left(4\,Bab+Ab^2\right)\sqrt{a}\,x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(8\,Babx^6-\left(4\,Ba^2+5\,Aab\right)x^3-2\,Aa^2\right)\sqrt{bx^3+a}}{24\,ax^6},\frac{3\left(4\,Bab+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(4*B*a*b + A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(
a) + 2*a)/x^3) + 2*(8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt(b
*x^3 + a))/(a*x^6), 1/12*(3*(4*B*a*b + A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^
3 + a)*sqrt(-a)/a) + (8*B*a*b*x^6 - (4*B*a^2 + 5*A*a*b)*x^3 - 2*A*a^2)*sqrt
(b*x^3 + a))/(a*x^6)]
```

giac [A] time = 0.18, size = 131, normalized size = 1.14

$$\frac{8\sqrt{bx^3+a}\,Bb^2 + \frac{3\left(4\,Bab^2+Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\sqrt{bx^3+a}\,Ba^2b^2+5\left(bx^3+a\right)^{\frac{3}{2}}Ab^3-3\sqrt{bx^3+a}\,Aab^3}{b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(b*x^3 + a)*B*b^2 + 3*(4*B*a*b^2 + A*b^3)*arctan(sqrt(b*x^3 + a
)/sqrt(-a))/sqrt(-a) - (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a
^2*b^2 + 5*(b*x^3 + a)^(3/2)*A*b^3 - 3*sqrt(b*x^3 + a)*A*a*b^3)/(b^2*x^6))/
b
```

maple [A] time = 0.05, size = 107, normalized size = 0.93

$$\left(-\frac{b^2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4\sqrt{a}} - \frac{5\sqrt{b\,x^3+a}\;b}{12x^3} - \frac{\sqrt{b\,x^3+a}\;a}{6x^6}\right)A + \left(-\sqrt{a}\;b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right) + \frac{2\sqrt{b\,x^3+a}\;b}{3} - \frac{\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x)
```

```
[Out] B*(-1/3*(b*x^3+a)^(1/2)*a/x^3+2/3*(b*x^3+a)^(1/2)*b-b*arctanh((b*x^3+a)^(1/
2)/a^(1/2))*a^(1/2))+A*(-1/6*a*(b*x^3+a)^(1/2)/x^6-5/12*(b*x^3+a)^(1/2)*b/x
^3-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 1.31, size = 171, normalized size = 1.49

$$\frac{1}{24}\left(\frac{3\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{\sqrt{a}} - \frac{2\left(5\left(bx^3+a\right)^{\frac{3}{2}}b^2 - 3\sqrt{bx^3+a}\,ab^2\right)}{\left(bx^3+a\right)^2 - 2\left(bx^3+a\right)a + a^2}\right)A + \frac{1}{6}\left(3\sqrt{a}\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right) + 4\sqrt{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^7,x, algorithm="maxima")
```

```
[Out] 1/24*(3*b^2*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sq
rt(a) - 2*(5*(b*x^3 + a)^(3/2)*b^2 - 3*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^
2 - 2*(b*x^3 + a)*a + a^2))*A + 1/6*(3*sqrt(a)*b*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a))) + 4*sqrt(b*x^3 + a)*b - 2*sqrt(b*x^3 + a
)*a/x^3)*B
```

mupad [B] time = 3.47, size = 110, normalized size = 0.96

$$\frac{2\,B\,b\,\sqrt{b\,x^3+a}}{3} - \frac{\sqrt{b\,x^3+a}\;\left(4\,B\,a^3+5\,A\,b\,a^2\right)}{12\,a^2\,x^3} - \frac{A\,a\,\sqrt{b\,x^3+a}}{6\,x^6} + \frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\left(A\,b+4\,B\,a\right.}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^7,x)
```

```
[Out] (2*B*b*(a + b*x^3)^(1/2))/3 - ((a + b*x^3)^(1/2)*(4*B*a^3 + 5*A*a^2*b))/(12
*a^2*x^3) - (A*a*(a + b*x^3)^(1/2))/(6*x^6) + (b*log((((a + b*x^3)^(1/2) -
a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)*(A*b + 4*B*a))/(8*a^(1/2))
```

sympy [B] time = 153.11, size = 243, normalized size = 2.11

$$-\frac{Aa^2}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Aa\sqrt{b}}{4x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{3x^{\frac{3}{2}}}-\frac{Ab^{\frac{3}{2}}}{12x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{Ab^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4\sqrt{a}}-B\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**7,x)
```

```
[Out] -A*a**2/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) - A*a*sqrt(b)/(4*x**(9/2
)*sqrt(a/(b*x**3) + 1)) - A*b**(3/2)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) - A*
b**(3/2)/(12*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A*b**2*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(4*sqrt(a)) - B*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2))) - B
*a*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*x**(3/2)) + 2*B*a*sqrt(b)/(3*x**(3/2)*sq
rt(a/(b*x**3) + 1)) + 2*B*b**(3/2)*x**(3/2)/(3*sqrt(a/(b*x**3) + 1))
```

3.202 $\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=336

$$\frac{54a^2x\sqrt{a+bx^3}\,(23Ab-8aB)}{21505b^2}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(23Ab-8aB)F\Bigg(\sin^{-1}\Bigg(\cdots}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/391*(23*A*b-8*B*a)*x^4*(b*x^3+a)^(3/2)/b+2/23*B*x^4*(b*x^3+a)^(5/2)/b+54/
21505*a^2*(23*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+18/4301*a*(23*A*b-8*B*a)*x^4
*(b*x^3+a)^(1/2)/b-36/21505*3^(3/4)*a^3*(23*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*
EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I
*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {459, 279, 321, 218}

$$\frac{54a^2x\sqrt{a+bx^3}\,(23Ab-8aB)}{21505b^2}-\frac{36\,3^{3/4}\sqrt{2+\sqrt{3}}\,a^3\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(23Ab-8aB)F\Bigg(\sin^{-1}\Bigg(\cdots}{21505b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(23*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(21505*b^2) + (18*a*(23*A*b - 8
*a*B)*x^4*Sqrt[a + b*x^3])/(4301*b) + (2*(23*A*b - 8*a*B)*x^4*(a + b*x^3)^(
3/2))/(391*b) + (2*B*x^4*(a + b*x^3)^(5/2))/(23*b) - (36*3^(3/4)*Sqrt[2 + S
qrt[3]]*a^3*(23*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(21505*b^(7/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int x^3\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx=\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}-\frac{\left(2\left(-\frac{23Ab}{2}+4aB\right)\right)\int x^3\left(a+bx^3\right)^{3/2}dx}{23b}$$

$$=\frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b}+\frac{2Bx^4\left(a+bx^3\right)^{5/2}}{23b}+\frac{(9a(23Ab-8aB))\int x^3\sqrt{}}{391b}$$

$$=\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8aB)x^4\left(a+bx^3\right)^{3/2}}{391b}+\frac{2Bx^4\left(a+bx\right.}{23b}$$

$$=\frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2}+\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8a}{}$$

$$=\frac{54a^2(23Ab-8aB)x\sqrt{a+bx^3}}{21505b^2}+\frac{18a(23Ab-8aB)x^4\sqrt{a+bx^3}}{4301b}+\frac{2(23Ab-8a}{}$$

Mathematica [C] time = 0.17, size = 93, normalized size = 0.28

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2(8aB-23Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)^2\left(8aB-23Ab-17bBx^3\right)\right)}{391b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-23*A*b + 8*a*B - 17*b*B*x^3)) + (a^
2*(-23*A*b + 8*a*B)*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1
 + (b*x^3)/a]))/(391*b^2)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bbx^9+(Ba+Ab)x^6+Aax^3\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^9 + (B*a + A*b)*x^6 + A*a*x^3)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

maple [B] time = 0.05, size = 694, normalized size = 2.07

$$\left(\frac{2\sqrt{b\,x^3+a}\;b\,x^7}{17}+\frac{40\sqrt{b\,x^3+a}\;a\,x^4}{187}+\frac{36 i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\;\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/23*(b*x^3+a)^(1/2)*b*x^10+52/391*(b*x^3+a)^(1/2)*a*x^7+54/4301*(b*x^3+
a)^(1/2)*a^2/b*x^4-432/21505*(b*x^3+a)^(1/2)*a^3/b^2*x-288/21505*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/17*b*x^7*
(b*x^3+a)^(1/2)+40/187*(b*x^3+a)^(1/2)*a*x^4+54/935*(b*x^3+a)^(1/2)*a^2/b*x
+36/935*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)
/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-
a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(B x^3 + A\right) \left(b x^3 + a\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^3*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 5.06, size = 172, normalized size = 0.51

$$\frac{Aa^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{A\sqrt{a}\,bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{B\sqrt{a}\,bx^{10}\Gamma\left(\frac{10}{3}\right. }{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(7/3)) + A*sqrt(a)*b*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*a**(3/2)*x**7*gamma(7/3)*h
yper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + B*sq
rt(a)*b*x**10*gamma(10/3)*hyper((-1/2, 10/3), (13/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(13/3))
```

3.203 $\int \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=299

$$\frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}\, a^2 \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(17Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{}$$

```
[Out] 2/187*(17*A*b-2*B*a)*x*(b*x^3+a)^(3/2)/b+2/17*B*x*(b*x^3+a)^(5/2)/b+18/935*
a*(17*A*b-2*B*a)*x*(b*x^3+a)^(1/2)/b+18/935*3^(3/4)*a^2*(17*A*b-2*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(4/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 218}

$$\frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}\, a^2 \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(17Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{935b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(17*A*b - 2*a*B)*x*Sqrt[a + b*x^3])/(935*b) + (2*(17*A*b - 2*a*B)*x*(
a + b*x^3)^(3/2))/(187*b) + (2*B*x*(a + b*x^3)^(5/2))/(17*b) + (18*3^(3/4)*
Sqrt[2 + Sqrt[3]]*a^2*(17*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*E
llipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/
3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} - \frac{\left(2\left(-\frac{17Ab}{2}+aB\right)\right)\int \left(a+bx^3\right)^{3/2}dx}{17b}$$
$$= \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b} + \frac{(9a(17Ab-2aB))\int \sqrt{a+bx^3}$$
$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$
$$= \frac{18a(17Ab-2aB)x\sqrt{a+bx^3}}{935b} + \frac{2(17Ab-2aB)x\left(a+bx^3\right)^{3/2}}{187b} + \frac{2Bx\left(a+bx^3\right)^{5/2}}{17b}$$

Mathematica [C] time = 0.08, size = 77, normalized size = 0.26

$$\frac{2x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^2-\frac{a\left(aB-\frac{17Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{17b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (2*x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2 - (a*((-17*A*b)/2 + a*B)*Hypergeometr
ic2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(17*b)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.04, size = 654, normalized size = 2.19

$$\left\{ \frac{2\sqrt{b\,x^{3}+a}\;b\,x^{4}}{11}-\frac{18 i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\;\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\;\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right.}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{55\sqrt{b\,x^{3}+a}} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/17*(b*x^3+a)^(1/2)*b*x^7+40/187*(b*x^3+a)^(1/2)*a*x^4+54/935*(b*x^3+a)
^(1/2)*a^2/b*x+36/935*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-
(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(
1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/b)^(1/2)))+A*(2/11*(b*x^3+a)^(1/2)*b*x^4+28/55*(b*x^3+a)^(1/2)*a*x-1
8/55*I*a^2*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/
2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2
)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^{3}+A\right)\left(bx^{3}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^{3}+A\right)\,\left(b\,x^{3}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 4.75, size = 170, normalized size = 0.57

$$\frac{Aa^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{A\sqrt{a}\,bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{B\sqrt{a}\,bx^7\Gamma\left(\frac{7}{3}\right){}_2F \cdots}{3\Gamma\left(\cdots \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + A*sqrt(a)*b*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*a**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + B*sqrt(a)*b
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(10/3))
```

3.204 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx$

Optimal. Leaf size=295

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x(a}{}$$

```
[Out] 1/22*(11*A*b+4*B*a)*x*(b*x^3+a)^(3/2)/a-1/2*A*(b*x^3+a)^(5/2)/a/x^2+9/110*(
11*A*b+4*B*a)*x*(b*x^3+a)^(1/2)+9/110*3^(3/4)*a*(11*A*b+4*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x
+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(1/3)/(b*x^3+a)^(1
/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 195, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+11Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{110\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{x(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
```

```
[Out] (9*(11*A*b + 4*a*B)*x*Sqrt[a + b*x^3])/110 + ((11*A*b + 4*a*B)*x*(a + b*x^3
)^(3/2))/(22*a) - (A*(a + b*x^3)^(5/2))/(2*a*x^2) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*a*(11*A*b + 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(110*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + S
qrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^3}\,dx = -\frac{A(a+bx^3)^{5/2}}{2ax^2} - \frac{\left(-\frac{11Ab}{2}-2aB\right)\int (a+bx^3)^{3/2}\,dx}{2a}$$

$$= \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{1}{44}(9(11Ab+4aB))\int \sqrt{a+bx^3}$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{1}{22}$$

$$= \frac{9}{110}(11Ab+4aB)x\sqrt{a+bx^3} + \frac{(11Ab+4aB)x(a+bx^3)^{3/2}}{22a} - \frac{A(a+bx^3)^{5/2}}{2ax^2} + \frac{9}{}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.28

$$-\frac{x\sqrt{a+bx^3}\left(-2aB-\frac{11Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{A(a+bx^3)^{5/2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^3,x]
[Out] -1/2*(A*(a + b*x^3)^(5/2))/(a*x^2) - (((-11*A*b)/2 - 2*a*B)*x*Sqrt[a + b*x^
3]*Hypergeometric2F1[-3/2, 1/3, 4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

maple [B] time = 0.05, size = 629, normalized size = 2.13

$$\left(-\frac{9i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\,a\,\mathrm{Elliptic}}{10\sqrt{b x^{3}+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x)
```

```
[Out] B*(2/11*(b*x^3+a)^(1/2)*b*x^4+28/55*(b*x^3+a)^(1/2)*a*x-18/55*I*a^2*3^(1/2)
*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-1/2*a*(b*x^3+a)
^(1/2)/x^2+2/5*(b*x^3+a)^(1/2)*b*x-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)\left(bx^{3}+a\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^3, x)
```

sympy [A] time = 5.38, size = 172, normalized size = 0.58

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{A\sqrt{a}\,bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{Ba^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{B\sqrt{a}\,bx^4\Gamma\left(\frac{4}{3}\right){}_2F}{3\Gamma(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**3,x)
```

```
[Out] A*a**(3/2)*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x**2*gamma(1/3)) + A*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*a**(3/2)*x*gamma(1/3)*hyper((
-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + B*sqrt(a)*b*
x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(7/3))
```

3.205 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^6}\,dx$

Optimal. Leaf size=297

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{9bx}{\cdots}$$

```
[Out] -1/4*(A*b+2*B*a)*(b*x^3+a)^(3/2)/a/x^2-1/5*A*(b*x^3+a)^(5/2)/a/x^5+9/20*b*(
A*b+2*B*a)*x*(b*x^3+a)^(1/2)/a+9/20*3^(3/4)*b^(2/3)*(A*b+2*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {453, 277, 195, 218}

$$\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{20\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\left(a+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (9*b*(A*b + 2*a*B)*x*Sqrt[a + b*x^3])/(20*a) - ((A*b + 2*a*B)*(a + b*x^3)^(
3/2))/(4*a*x^2) - (A*(a + b*x^3)^(5/2))/(5*a*x^5) + (9*3^(3/4)*Sqrt[2 + Sqr
t[3]]*b^(2/3)*(A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[Ar
cSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x
)], -7 - 4*Sqrt[3]])/(20*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^6}\,dx &= -\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5}+-\frac{\left(-\frac{5Ab}{2}-5aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^3}\,dx}{5a}\\ &= -\frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2}-\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5}+\frac{(9b(Ab+2aB))\int \sqrt{a+bx^3}\,dx}{8a}\\ &= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a}-\frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2}-\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5}+\frac{1}{40}(27b(\\ &= \frac{9b(Ab+2aB)x\sqrt{a+bx^3}}{20a}-\frac{(Ab+2aB)\left(a+bx^3\right)^{3/2}}{4ax^2}-\frac{A\left(a+bx^3\right)^{5/2}}{5ax^5}+\frac{9\,3^{3/4}\sqrt{2}}{}\end{aligned}$$

Mathematica [C] time = 0.08, size = 82, normalized size = 0.28

$$\frac{\sqrt{a+bx^3}\left(-\frac{5x^3(2aB+Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{2}{3};\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{2A\left(a+bx^3\right)^2}{a}\right)}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^6,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-2*A*(a + b*x^3)^2)/a - (5*(A*b + 2*a*B)*x^3*Hypergeomet
ric2F1[-3/2, -2/3, 1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(10*x^5)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

maple [B] time = 0.05, size = 626, normalized size = 2.11

$$-\frac{9i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a b^2\right)^{\frac{1}{3}}}}\, b \,\mathrm{EllipticI}}{20\sqrt{b x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x)
```

```
[Out] A*(-1/5*a*(b*x^3+a)^(1/2)/x^5-13/20*(b*x^3+a)^(1/2)*b/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+B*(-1/2*(b*x^3+a)^(1/2
)*a/x^2+2/5*(b*x^3+a)^(1/2)*b*x-9/10*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)
*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^6, x)
```

sympy [A] time = 6.18, size = 184, normalized size = 0.62

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},-\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^2\Gamma\left(\frac{1}{3}\right)}+\frac{B\sqrt{a}\,bx\Gamma\left(\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**6,x)
```

```
[Out] A*a**(3/2)*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**5*gamma(-2/3)) + A*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) + B*a**(3/2)*gamma(-2/3)*h
yper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**2*gamma(1/3)) +
B*sqrt(a)*b*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*gamma(4/3))
```

3.206 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^9}\,dx$

Optimal. Leaf size=302

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{(a\ldots}{\ldots}$$

```
[Out] 1/80*(A*b-16*B*a)*(b*x^3+a)^(3/2)/a/x^5-1/8*A*(b*x^3+a)^(5/2)/a/x^8+9/320*b
*(A*b-16*B*a)*(b*x^3+a)^(1/2)/a/x^2-9/320*3^(3/4)*b^(5/3)*(A*b-16*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a/(b*x^3+a)^
(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 277, 218}

$$-\frac{9\,3^{3/4}\sqrt{2+\sqrt{3}}\,b^{5/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{320a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{(a\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
```

```
[Out] (9*b*(A*b - 16*a*B)*Sqrt[a + b*x^3])/(320*a*x^2) + ((A*b - 16*a*B)*(a + b*x
^3)^(3/2))/(80*a*x^5) - (A*(a + b*x^3)^(5/2))/(8*a*x^8) - (9*3^(3/4)*Sqrt[2
 + Sqrt[3]]*b^(5/3)*(A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(320*a*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^9}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{\left(\frac{Ab}{2}-8aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^6}\,dx}{8a}$$

$$= \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{(9b(Ab-16aB))\int \frac{\sqrt{a+bx^3}}{x^3}\,dx}{160a}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{\left(27b^2(A\right.}{}$$

$$= \frac{9b(Ab-16aB)\sqrt{a+bx^3}}{320ax^2} + \frac{(Ab-16aB)\left(a+bx^3\right)^{3/2}}{80ax^5} - \frac{A\left(a+bx^3\right)^{5/2}}{8ax^8} - \frac{9\,3^{3/4}\sqrt{}}{}$$

Mathematica [C] time = 0.15, size = 82, normalized size = 0.27

$$\frac{\sqrt{a+bx^3}\left(\frac{x^3\left(\frac{Ab}{2}-8aB\right){}_2F_1\left(-\frac{5}{3},-\frac{3}{2};-\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{5A\left(a+bx^3\right)^2}{a}\right)}{40x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^9,x]
[Out] (Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + (((A*b)/2 - 8*a*B)*x^3*Hypergeom
etric2F1[-5/3, -3/2, -2/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(40*x^8)
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^9},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^9, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

maple [B] time = 0.09, size = 653, normalized size = 2.16

$$\left\{ \frac{9 i \sqrt{3}\, \left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a\,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\, b^{2} \mathrm{EllipticF}}{320 \sqrt{b\,x^{3}+a}\; a} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x)
```

```
[Out] B*(-1/5*(b*x^3+a)^(1/2)*a/x^5-13/20*(b*x^3+a)^(1/2)*b/x^2-9/20*I*b*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-1/8*a*(b*x^3+a)^(1
/2)/x^8-19/80*b*(b*x^3+a)^(1/2)/x^5-27/320*b^2/a*(b*x^3+a)^(1/2)/x^2+9/320*
I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b
*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/
3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3} + A\right)\left(bx^{3} + a\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^{3} + A\right)\,\left(b\,x^{3} + a\right)^{3/2}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^9, x)
```

sympy [A] time = 6.53, size = 196, normalized size = 0.65

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{8}{3}\right){}_2F_1\left(\begin{matrix} -\frac{8}{3}, -\frac{1}{2} \\ -\frac{5}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^8\Gamma\left(-\frac{5}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{5}{3}, -\frac{1}{2} \\ -\frac{2}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**9,x)
```

```
[Out] A*a**(3/2)*gamma(-8/3)*hyper((-8/3, -1/2), (-5/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**8*gamma(-5/3)) + A*sqrt(a)*b*gamma(-5/3)*hyper((-5/3, -1/2), (-2/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)) + B*a**(3/2)*gamma(-5/3)
*hyper((-5/3, -1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**5*gamma(-2/3)
) + B*sqrt(a)*b*gamma(-2/3)*hyper((-2/3, -1/2), (1/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x**2*gamma(1/3))
```

3.207 $\int x^4 \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=614

$$-\frac{72\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{8645b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{108\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] 2/95*(5*A*b-2*B*a)*x^5*(b*x^3+a)^(3/2)/b+2/25*B*x^5*(b*x^3+a)^(5/2)/b+54/86
45*a^2*(5*A*b-2*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+18/1235*a*(5*A*b-2*B*a)*x^5*(b
*x^3+a)^(1/2)/b-216/8645*a^3*(5*A*b-2*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))-72/8645*3^(3/4)*a^(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(
1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1
/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+108/8645*3^(1/4)*a^
(10/3)*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {459, 279, 321, 303, 218, 1877}

$$\frac{54a^2x^2\sqrt{a+bx^3}\,(5Ab-2aB)}{8645b^2}-\frac{216a^3\sqrt{a+bx^3}\,(5Ab-2aB)}{8645b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{72\sqrt{2}\,3^{3/4}a^{10/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}}\cdots}{8645b^{8/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (54*a^2*(5*A*b - 2*a*B)*x^2*Sqrt[a + b*x^3])/(8645*b^2) + (18*a*(5*A*b - 2*
a*B)*x^5*Sqrt[a + b*x^3])/(1235*b) - (216*a^3*(5*A*b - 2*a*B)*Sqrt[a + b*x^
3])/(8645*b^(8/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (2*(5*A*b - 2*a*B)
*x^5*(a + b*x^3)^(3/2))/(95*b) + (2*B*x^5*(a + b*x^3)^(5/2))/(25*b) + (108*
3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^
(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
 - (72*Sqrt[2]*3^(3/4)*a^(10/3)*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(8645*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x^4\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx=\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}-\frac{\left(2\left(-\frac{25Ab}{2}+5aB\right)\right)\int x^4\left(a+bx^3\right)^{3/2}dx}{25b}$$
$$=\frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b}+\frac{2Bx^5\left(a+bx^3\right)^{5/2}}{25b}+\frac{(9a(5Ab-2aB))\int x^4\sqrt{}}{95b}$$
$$=\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}+\frac{2(5Ab-2aB)x^5\left(a+bx^3\right)^{3/2}}{95b}+\frac{2Bx^5\left(a+b\right.}{25b}$$
$$=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}+\frac{2(5Ab-2a}{}$$
$$=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}+\frac{2(5Ab-2a}{}$$
$$=\frac{54a^2(5Ab-2aB)x^2\sqrt{a+bx^3}}{8645b^2}+\frac{18a(5Ab-2aB)x^5\sqrt{a+bx^3}}{1235b}-\frac{216a^3(5A}{8645b^{8/3}\left(\left(\right.\right.}$$

Mathematica [C] time = 0.24, size = 96, normalized size = 0.16

$$\frac{2x^2\sqrt{a+bx^3}\left(\frac{5a^2(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\left(a+bx^3\right)^2\left(10aB-25Ab-19bBx^3\right)\right)}{475b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*x^2*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*(-25*A*b + 10*a*B - 19*b*B*x^3)) +
(5*a^2*(-5*A*b + 2*a*B)*Hypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sq
rt[1 + (b*x^3)/a]))/(475*b^2)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bbx^{10}+(Ba+Ab)x^7+Aax^4\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^10 + (B*a + A*b)*x^7 + A*a*x^4)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

maple [B] time = 0.05, size = 1002, normalized size = 1.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/25*b*x^11*(b*x^3+a)^(1/2)+56/475*(b*x^3+a)^(1/2)*a*x^8+54/6175*(b*x^3+
a)^(1/2)*a^2/b*x^5-108/8645*(b*x^3+a)^(1/2)*a^3/b^2*x^2-144/8645*I*a^4/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(2/19*b*x^8*(b*x^3+a)^(1/2)+44/247
*(b*x^3+a)^(1/2)*a*x^5+54/1729*(b*x^3+a)^(1/2)*a^2/b*x^2+72/1729*I*a^3/b^2*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x^4*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 5.80, size = 172, normalized size = 0.28

$$\frac{Aa^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)} + \frac{A\sqrt{a}\,bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{Ba^{\frac{3}{2}}x^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{3}\right)} + \frac{B\sqrt{a}\,bx^{11}\Gamma\left(\frac{11}{3}\right)}{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(8/3)) + A*sqrt(a)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*a**(3/2)*x**8*gamma(8/3)*h
yper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/3)) + B*sq
rt(a)*b*x**11*gamma(11/3)*hyper((-1/2, 11/3), (14/3,), b*x**3*exp_polar(I*p
i)/a)/(3*gamma(14/3))
```

3.208 $\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=581

$$\frac{18\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

```
[Out] 2/247*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(3/2)/b+2/19*B*x^2*(b*x^3+a)^(5/2)/b+18/
1729*a*(19*A*b-4*B*a)*x^2*(b*x^3+a)^(1/2)/b+54/1729*a^2*(19*A*b-4*B*a)*(b*x
^3+a)^(1/2)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+18/1729*3^(3/4)*a^(7/3)
*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)-27/1729*3^(1/4)*a^(7/3)*(19*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*Ellipti
cE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {459, 279, 303, 218, 1877}

$$\frac{54a^2\sqrt{a+bx^3}(19Ab-4aB)}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{18\sqrt{2}\,3^{3/4}a^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(19Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\cdots\right.}{\sqrt[3]{b}x+\left(1+\cdots\right.}\right.\right.}{1729b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (18*a*(19*A*b - 4*a*B)*x^2*Sqrt[a + b*x^3])/(1729*b) + (54*a^2*(19*A*b - 4*
a*B)*Sqrt[a + b*x^3])/(1729*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) +
(2*(19*A*b - 4*a*B)*x^2*(a + b*x^3)^(3/2))/(247*b) + (2*B*x^2*(a + b*x^3)^(
5/2))/(19*b) - (27*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(7/3)*(19*A*b - 4*a*B)*(a^(1
/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqr
t[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(
1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3]) + (18*Sqrt[2]*3^(3/4)*a^(7/3)*(19*A*b - 4*a*B)*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3
)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*b^(5/3)*S
qrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int x\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b} - \frac{\left(2\left(-\frac{19Ab}{2}+2aB\right)\right)\int x\left(a+bx^3\right)^{3/2}dx}{19b}$$

$$= \frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b} + \frac{2Bx^2\left(a+bx^3\right)^{5/2}}{19b} + \frac{(9a(19Ab-4aB))\int x\sqrt{}}{247b}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b} + \frac{2Bx^2\left(a+\right.}{19b}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{2(19Ab-4aB)x^2\left(a+bx^3\right)^{3/2}}{247b} + \frac{2Bx^2\left(a+\right.}{19b}$$

$$= \frac{18a(19Ab-4aB)x^2\sqrt{a+bx^3}}{1729b} + \frac{54a^2(19Ab-4aB)\sqrt{a+bx^3}}{1729b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{2(19Ab-}{}$$

Mathematica [C] time = 0.25, size = 78, normalized size = 0.13

$$\frac{x^2\sqrt{a+bx^3}\left(\frac{a(19Ab-4aB)\,_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}+4B\left(a+bx^3\right)^2\right)}{38b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x^2*Sqrt[a + b*x^3]*(4*B*(a + b*x^3)^2 + (a*(19*A*b - 4*a*B)*Hypergeometri
c2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(38*b)
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bbx^7+(Ba+Ab)x^4+Aax\right)\sqrt{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^7 + (B*a + A*b)*x^4 + A*a*x)*sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

maple [B] time = 0.04, size = 962, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] B*(2/19*(b*x^3+a)^(1/2)*b*x^8+44/247*(b*x^3+a)^(1/2)*a*x^5+54/1729*(b*x^3+a
)^(1/2)*a^2/b*x^2+72/1729*I*a^3/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^
2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*
((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a
*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)
)+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2
)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+
A*(2/13*b*x^5*(b*x^3+a)^(1/2)+32/91*(b*x^3+a)^(1/2)*a*x^2-18/91*I*a^2*3^(1/
2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
```

```
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{3}{2}} x\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^3)*(a + b*x^3)^(3/2),x)
```

```
[Out] int(x*(A + B*x^3)*(a + b*x^3)^(3/2), x)
```

sympy [A] time = 4.76, size = 172, normalized size = 0.30

$$\frac{Aa^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{A\sqrt{a}\,bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{B\sqrt{a}\,bx^8\Gamma\left(\frac{8}{3}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(5/3)) + A*sqrt(a)*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*a**(3/2)*x**5*gamma(5/3)*hyp
er((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(8/3)) + B*sqrt(a
)*b*x**8*gamma(8/3)*hyper((-1/2, 8/3), (11/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(11/3))
```

3.209 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^2}\,dx$

Optimal. Leaf size=573

$$\frac{9\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+13Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-}}{}$$

```
[Out] 1/13*(13*A*b+2*B*a)*x^2*(b*x^3+a)^(3/2)/a-A*(b*x^3+a)^(5/2)/a/x+9/91*(13*A*
b+2*B*a)*x^2*(b*x^3+a)^(1/2)+27/91*a*(13*A*b+2*B*a)*(b*x^3+a)^(1/2)/b^(2/3)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/91*3^(3/4)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/
3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/182*3^(1/4
)*a^(4/3)*(13*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(
1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2
*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 573, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 279, 303, 218, 1877}

$$\frac{9\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(2aB+13Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] (9*(13*A*b + 2*a*B)*x^2*Sqrt[a + b*x^3])/91 + (27*a*(13*A*b + 2*a*B)*Sqrt[a
 + b*x^3])/(91*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((13*A*b + 2*
a*B)*x^2*(a + b*x^3)^(3/2))/(13*a) - (A*(a + b*x^3)^(5/2))/(a*x) - (27*3^(1
/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(182*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (9
*Sqrt[2]*3^(3/4)*a^(4/3)*(13*A*b + 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^2}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{ax} - \frac{\left(-\frac{13Ab}{2}-aB\right)\int x\left(a+bx^3\right)^{3/2}\,dx}{a}$$

$$= \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax} + \frac{1}{26}(9(13Ab+2aB))\int x\sqrt{a+$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{(13Ab+2aB)x^2\left(a+bx^3\right)^{3/2}}{13a} - \frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

$$= \frac{9}{91}(13Ab+2aB)x^2\sqrt{a+bx^3} + \frac{27a(13Ab+2aB)\sqrt{a+bx^3}}{91b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{(13Ab+2aB)}{1$$

Mathematica [C] time = 0.04, size = 83, normalized size = 0.14

$$-\frac{x^2\sqrt{a+bx^3}\left(-aB-\frac{13Ab}{2}\right){}_2F_1\left(-\frac{3}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{A\left(a+bx^3\right)^{5/2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^2,x]
```

```
[Out] -((A*(a + b*x^3)^(5/2))/(a*x)) - (((-13*A*b)/2 - a*B)*x^2*Sqrt[a + b*x^3]*H
ypergeometric2F1[-3/2, 2/3, 5/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

maple [B] time = 0.06, size = 937, normalized size = 1.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x)
```

```
[Out] B*(2/13*(b*x^3+a)^(1/2)*b*x^5+32/91*(b*x^3+a)^(1/2)*a*x^2-18/91*I*a^2*3^(1/
2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-a*(b*x^3+a)^(1/2)/x+2/7*(b*x^3+a)^(
1/2)*b*x^2-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3
)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
```

```
2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/
b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3+A)\,(b\,x^3+a)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^2, x)
```

sympy [A] time = 5.67, size = 173, normalized size = 0.30

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)}+\frac{A\sqrt{a}\,bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{Ba^{\frac{3}{2}}x^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{5}{3}\right)}+\frac{B\sqrt{a}\,bx^5\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**2,x)
```

```
[Out] A*a**(3/2)*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a
)/(3*x*gamma(2/3)) + A*sqrt(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,),
 b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*a**(3/2)*x**2*gamma(2/3)*hype
r((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(5/3)) + B*sqrt(a)
*b*x**5*gamma(5/3)*hyper((-1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*
gamma(8/3))
```

3.210 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^5}\,dx$

Optimal. Leaf size=578

$$\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

```
[Out] -1/8*(7*A*b+8*B*a)*(b*x^3+a)^(3/2)/a/x-1/4*A*(b*x^3+a)^(5/2)/a/x^4+9/56*b*(
7*A*b+8*B*a)*x^2*(b*x^3+a)^(1/2)/a+27/56*b^(1/3)*(7*A*b+8*B*a)*(b*x^3+a)^(1
/2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/56*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/112*3^(1
/4)*a^(1/3)*b^(1/3)*(7*A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^
(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))^2)^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 279, 303, 218, 1877}

$$\frac{9\,3^{3/4}\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{28\sqrt{2}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{27\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] (9*b*(7*A*b + 8*a*B)*x^2*Sqrt[a + b*x^3])/(56*a) + (27*b^(1/3)*(7*A*b + 8*a
*B)*Sqrt[a + b*x^3])/(56*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((7*A*b + 8
*a*B)*(a + b*x^3)^(3/2))/(8*a*x) - (A*(a + b*x^3)^(5/2))/(4*a*x^4) - (27*3^
(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*Sqrt[(a^(1/3)*(a^(1/3
) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (
9*3^(3/4)*a^(1/3)*b^(1/3)*(7*A*b + 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

& PosQ[a]

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^5}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} - \frac{\left(-\frac{7Ab}{2}-4aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^2}dx}{4a}$$
$$= -\frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{(9b(7Ab+8aB))\int x\sqrt{a+bx^3}\,dx}{16a}$$
$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{1}{112}(\text{2}$$
$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} - \frac{(7Ab+8aB)\left(a+bx^3\right)^{3/2}}{8ax} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4} + \frac{1}{112}($$
$$= \frac{9b(7Ab+8aB)x^2\sqrt{a+bx^3}}{56a} + \frac{27\sqrt[3]{b}(7Ab+8aB)\sqrt{a+bx^3}}{56\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{(7Ab+8aB)\left(a+b\right.}{8ax}$$

Mathematica [C] time = 0.10, size = 85, normalized size = 0.15

$$\frac{\sqrt{a+bx^3}\left(-4aB-\frac{7Ab}{2}\right){}_2F_1\left(-\frac{3}{2},-\frac{1}{3};\frac{2}{3};-\frac{bx^3}{a}\right)}{4x\sqrt{\frac{bx^3}{a}+1}} - \frac{A\left(a+bx^3\right)^{5/2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^5,x]
```

```
[Out] -1/4*(A*(a + b*x^3)^(5/2))/(a*x^4) + (((-7*A*b)/2 - 4*a*B)*Sqrt[a + b*x^3]*
Hypergeometric2F1[-3/2, -1/3, 2/3, -((b*x^3)/a)])/(4*x*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

maple [B] time = 0.05, size = 932, normalized size = 1.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x)
```

```
[Out] A*(-1/4*a*(b*x^3+a)^(1/2)/x^4-11/8*(b*x^3+a)^(1/2)*b/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/b)^(1/2))))+B*(-(b*x^3+a)^(1/2)*a/x+2/7*(b*x^3+a)^(1/2)*b*x^
2-9/7*I*a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/
(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\left(b\,x^3 + a\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^5, x)
```

sympy [A] time = 6.54, size = 182, normalized size = 0.31

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3}, -\frac{1}{2}\\ -\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)} + \frac{A\sqrt{a}\,b\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x\Gamma\left(\frac{2}{3}\right)} + \frac{B\sqrt{a}\,bx^2\Gamma}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**5,x)
```

```
[Out] A*a**(3/2)*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**4*gamma(-1/3)) + A*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,
), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*a**(3/2)*gamma(-1/3)*hype
r((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*x*gamma(2/3)) + B*sqrt
(a)*b*x**2*gamma(2/3)*hyper((-1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(5/3))
```

3.211 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx$

Optimal. Leaf size=576

$$\frac{9\, 3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(14aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 27\sqrt[4]{3}\sqrt{2-}$$

```
[Out] -1/56*(A*b+14*B*a)*(b*x^3+a)^(3/2)/a/x^4-1/7*A*(b*x^3+a)^(5/2)/a/x^7-9/112*
b*(A*b+14*B*a)*(b*x^3+a)^(1/2)/a/x+27/112*b^(4/3)*(A*b+14*B*a)*(b*x^3+a)^(1
/2)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+9/112*3^(3/4)*b^(4/3)*(A*b+14*B*a)*(a
^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-27/224*3^
(1/4)*b^(4/3)*(A*b+14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 576, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 277, 303, 218, 1877}

$$\frac{9\, 3^{3/4} b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(14aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 27\sqrt[4]{3}\sqrt{2-}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (-9*b*(A*b + 14*a*B)*Sqrt[a + b*x^3])/(112*a*x) + (27*b^(4/3)*(A*b + 14*a*B
)*Sqrt[a + b*x^3])/(112*a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - ((A*b + 14
*a*B)*(a + b*x^3)^(3/2))/(56*a*x^4) - (A*(a + b*x^3)^(5/2))/(7*a*x^7) - (27
*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(2/3)*Sqrt[(a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) +
 (9*3^(3/4)*b^(4/3)*(A*b + 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellip
ticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*a^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^8}\,dx = -\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}-\frac{\left(-\frac{Ab}{2}-7aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^5}\,dx}{7a}$$

$$= -\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}+\frac{(9b(Ab+14aB))\int \frac{\sqrt{a+bx^3}}{x^2}\,dx}{112a}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax}-\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}+\frac{\left(27b^2(\right.}{}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax}-\frac{(Ab+14aB)\left(a+bx^3\right)^{3/2}}{56ax^4}-\frac{A\left(a+bx^3\right)^{5/2}}{7ax^7}+\frac{\left(27b^{5/}\right.}{}$$

$$= -\frac{9b(Ab+14aB)\sqrt{a+bx^3}}{112ax}+\frac{27b^{4/3}(Ab+14aB)\sqrt{a+bx^3}}{112a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{(Ab+14aB)\left(a+bx\right.}{56ax^4}$$

Mathematica [C] time = 0.08, size = 82, normalized size = 0.14

$$\frac{\sqrt{a+bx^3}\left(-\frac{x^3(14aB+Ab)\,{}_2F_1\left(-\frac{3}{2},-\frac{4}{3};-\frac{1}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{4A\left(a+bx^3\right)^2}{a}\right)}{28x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^8,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-4*A*(a + b*x^3)^2)/a - ((A*b + 14*a*B)*x^3*Hypergeometr
ic2F1[-3/2, -4/3, -1/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(28*x^7)
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^8, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

maple [B] time = 0.10, size = 957, normalized size = 1.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)*a/x^4-11/8*(b*x^3+a)^(1/2)*b/x-9/8*I*b*3^(1/2)*(-a*
b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/b)^(1/2))))+A*(-1/7*a*(b*x^3+a)^(1/2)/x^7-17/56*b*(b*x^3+a)^
(1/2)/x^4-27/112*b^2/a*(b*x^3+a)^(1/2)/x-9/112*I/a*b^2*3^(1/2)*(-a*b^2)^(1/
3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2
```

```
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\left(b\,x^3 + a\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^8, x)
```

sympy [A] time = 6.06, size = 194, normalized size = 0.34

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^4\Gamma\left(-\frac{1}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**8,x)
```

```
[Out] A*a**(3/2)*gamma(-7/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/
a)/(3*x**7*gamma(-4/3)) + A*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3
,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)) + B*a**(3/2)*gamma(-4/3)
*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**4*gamma(-1/3)
) + B*sqrt(a)*b*gamma(-1/3)*hyper((-1/2, -1/3), (2/3,), b*x**3*exp_polar(I*
pi)/a)/(3*x*gamma(2/3))
```

3.212 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{x^{11}}\,dx$

Optimal. Leaf size=608

$$-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{224\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

```
[Out] 1/28*(A*b-4*B*a)*(b*x^3+a)^(3/2)/a/x^7-1/10*A*(b*x^3+a)^(5/2)/a/x^10+9/224*
b*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/x^4+27/448*b^2*(A*b-4*B*a)*(b*x^3+a)^(1/2)/
a^2/x-27/448*b^(7/3)*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))-9/448*3^(3/4)*b^(7/3)*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((
b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
^2)^(1/2)/a^(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+27/896*3^(1/4)*b^(7/3)*(A*b-4*B*a)*(a^
(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 277, 325, 303, 218, 1877}

$$\frac{27b^2\sqrt{a+bx^3}\,(Ab-4aB)}{448a^2x}-\frac{27b^{7/3}\sqrt{a+bx^3}\,(Ab-4aB)}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{9\,3^{3/4}b^{7/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(Ab-4\cdots}{224\sqrt{2}\,a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\cdots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (9*b*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(224*a*x^4) + (27*b^2*(A*b - 4*a*B)*Sqr
t[a + b*x^3])/(448*a^2*x) - (27*b^(7/3)*(A*b - 4*a*B)*Sqrt[a + b*x^3])/(448
*a^2*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + ((A*b - 4*a*B)*(a + b*x^3)^(3/2
))/(28*a*x^7) - (A*(a + b*x^3)^(5/2))/(10*a*x^10) + (27*3^(1/4)*Sqrt[2 - Sq
rt[3]]*b^(7/3)*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*
b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)], -7 - 4*Sqrt[3]])/(896*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*b^(7/3)*(
A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2
/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt
[3]])/(224*Sqrt[2]*a^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{x^{11}}dx = -\frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{\left(\frac{5Ab}{2}-10aB\right)\int \frac{\left(a+bx^3\right)^{3/2}}{x^8}dx}{10a}$$

$$= \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{(9b(Ab-4aB))\int \frac{\sqrt{a+bx^3}}{x^5}dx}{56a}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3/2}}{28ax^7} - \frac{A\left(a+bx^3\right)^{5/2}}{10ax^{10}} - \frac{\left(27b^2(\ldots\right.}{}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3\ldots}}{28ax^7}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} + \frac{(Ab-4aB)\left(a+bx^3\right)^{3\ldots}}{28ax^7}$$

$$= \frac{9b(Ab-4aB)\sqrt{a+bx^3}}{224ax^4} + \frac{27b^2(Ab-4aB)\sqrt{a+bx^3}}{448a^2x} - \frac{27b^{7/3}(Ab-4aB)\sqrt{a\ldots}}{448a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\ldots\right.}$$

Mathematica [C] time = 0.08, size = 82, normalized size = 0.13

$$\frac{\sqrt{a+bx^3}\left(\frac{5x^3(Ab-4aB)\,{}_2F_1\left(-\frac{7}{3},-\frac{3}{2};-\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt{\frac{bx^3}{a}+1}} - \frac{7A\left(a+bx^3\right)^2}{a}\right)}{70x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/x^11,x]
```

```
[Out] (Sqrt[a + b*x^3]*((-7*A*(a + b*x^3)^2)/a + (5*(A*b - 4*a*B)*x^3*Hypergeomet
ric2F1[-7/3, -3/2, -4/3, -((b*x^3)/a)])/(2*Sqrt[1 + (b*x^3)/a])))/(70*x^10)
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}}{x^{11}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)/x^11, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

maple [B] time = 0.10, size = 1002, normalized size = 1.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x)
```

```
[Out] B*(-1/7*(b*x^3+a)^(1/2)*a/x^7-17/56*(b*x^3+a)^(1/2)*b/x^4-27/112*(b*x^3+a)^
(1/2)/a*b^2/x-9/112*I/a*b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*
b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)
*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1
/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^
2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/
(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-1/10
*a*(b*x^3+a)^(1/2)/x^10-23/140*b*(b*x^3+a)^(1/2)/x^7-27/1120*b^2/a*(b*x^3+a
)^(1/2)/x^4+27/448*b^3/a^2*(b*x^3+a)^(1/2)/x+9/448*I*b^3/a^2*3^(1/2)*(-a*b^
2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(
-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x
+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/x^11,x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/x^11, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\left(b\,x^3 + a\right)^{3/2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11,x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/x^11, x)
```

sympy [A] time = 6.88, size = 199, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{10}{3},-\frac{1}{2}\\-\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{10}\Gamma\left(-\frac{7}{3}\right)}+\frac{A\sqrt{a}\,b\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{Ba^{\frac{3}{2}}\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},-\frac{1}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\sqrt{a}\,b\Gamma\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/x**11,x)
```

```
[Out] A*a**(3/2)*gamma(-10/3)*hyper((-10/3, -1/2), (-7/3,), b*x**3*exp_polar(I*pi
)/a)/(3*x**10*gamma(-7/3)) + A*sqrt(a)*b*gamma(-7/3)*hyper((-7/3, -1/2), (-
4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4/3)) + B*a**(3/2)*gamma(-7
/3)*hyper((-7/3, -1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*x**7*gamma(-4
/3)) + B*sqrt(a)*b*gamma(-4/3)*hyper((-4/3, -1/2), (-1/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*x**4*gamma(-1/3))
```

3.213 $\int \frac{x^8(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=103

$$\frac{2a^2\sqrt{a+bx^3}\,(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

```
[Out] -2/9*a*(2*A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*(A*b-3*B*a)*(b*x^3+a)^(5/2)/b
^4+2/21*B*(b*x^3+a)^(7/2)/b^4+2/3*a^2*(A*b-B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{446, 77\}$

$$\frac{2a^2\sqrt{a+bx^3}\,(Ab-aB)}{3b^4}+\frac{2\left(a+bx^3\right)^{5/2}(Ab-3aB)}{15b^4}-\frac{2a\left(a+bx^3\right)^{3/2}(2Ab-3aB)}{9b^4}+\frac{2B\left(a+bx^3\right)^{7/2}}{21b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*a^2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^4) - (2*a*(2*A*b - 3*a*B)*(a + b*x
^3)^(3/2))/(9*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(5/2))/(15*b^4) + (2*B*(a
 + b*x^3)^(7/2))/(21*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx}}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3\sqrt{a+bx}}+\frac{a(-2Ab+3aB)\sqrt{a+bx}}{b^3}+\frac{(Ab-3aB)(a+bx)^{3/2}}{b^3}+\frac{B(a+\cdots}{b\cdots}\right.\right.$$

$$= \frac{2a^2(Ab-aB)\sqrt{a+bx^3}}{3b^4}-\frac{2a(2Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{5/2}}{15b^4}+\frac{2B\left(a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.08, size = 78, normalized size = 0.76

$$\frac{2\sqrt{a+bx^3}\left(-48a^3B+8a^2b\left(7A+3Bx^3\right)-2ab^2x^3\left(14A+9Bx^3\right)+3b^3x^6\left(7A+5Bx^3\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-48*a^3*B + 8*a^2*b*(7*A + 3*B*x^3) + 3*b^3*x^6*(7*A +
5*B*x^3) - 2*a*b^2*x^3*(14*A + 9*B*x^3)))/(315*b^4)
```

fricas [A] time = 1.00, size = 76, normalized size = 0.74

$$\frac{2\left(15\,Bb^{3}x^{9}-3\left(6\,Bab^{2}-7\,Ab^{3}\right)x^{6}-48\,Ba^{3}+56\,Aa^{2}b+4\left(6\,Ba^{2}b-7\,Aab^{2}\right)x^{3}\right)\sqrt{bx^{3}+a}}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(15*B*b^3*x^9 - 3*(6*B*a*b^2 - 7*A*b^3)*x^6 - 48*B*a^3 + 56*A*a^2*b +
 4*(6*B*a^2*b - 7*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/b^4
```

giac [A] time = 0.16, size = 101, normalized size = 0.98

$$-\frac{2\left(Ba^{3}-Aa^{2}b\right)\sqrt{bx^{3}+a}}{3\,b^{4}}+\frac{2\left(15\left(bx^{3}+a\right)^{\frac{7}{2}}B-63\left(bx^{3}+a\right)^{\frac{5}{2}}Ba+105\left(bx^{3}+a\right)^{\frac{3}{2}}Ba^{2}+21\left(bx^{3}+a\right)^{\frac{5}{2}}Ab-7\cdots\right.}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^3 - A*a^2*b)*sqrt(b*x^3 + a)/b^4 + 2/315*(15*(b*x^3 + a)^(7/2)*B
- 63*(b*x^3 + a)^(5/2)*B*a + 105*(b*x^3 + a)^(3/2)*B*a^2 + 21*(b*x^3 + a)^(
5/2)*A*b - 70*(b*x^3 + a)^(3/2)*A*a*b)/b^4
```

maple [A] time = 0.04, size = 77, normalized size = 0.75

$$\frac{2\sqrt{b\,x^{3}+a}\,\left(15B\,x^{9}b^{3}+21A\,b^{3}x^{6}-18Ba\,b^{2}x^{6}-28Aa\,b^{2}x^{3}+24B\,a^{2}b\,x^{3}+56A\,a^{2}b-48B\,a^{3}\right)}{315b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 2/315*(b*x^3+a)^(1/2)*(15*B*b^3*x^9+21*A*b^3*x^6-18*B*a*b^2*x^6-28*A*a*b^2*
x^3+24*B*a^2*b*x^3+56*A*a^2*b-48*B*a^3)/b^4
```

maxima [A] time = 0.59, size = 118, normalized size = 1.15

$$\frac{2}{105}\,B\left(\frac{5\left(bx^{3}+a\right)^{\frac{7}{2}}}{b^{4}}-\frac{21\left(bx^{3}+a\right)^{\frac{5}{2}}a}{b^{4}}+\frac{35\left(bx^{3}+a\right)^{\frac{3}{2}}a^{2}}{b^{4}}-\frac{35\sqrt{bx^{3}+a}\,a^{3}}{b^{4}}\right)+\frac{2}{45}\,A\left(\frac{3\left(bx^{3}+a\right)^{\frac{5}{2}}}{b^{3}}-\frac{10\left(bx^{3}+\cdots\right.}{b^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*B*(5*(b*x^3 + a)^(7/2)/b^4 - 21*(b*x^3 + a)^(5/2)*a/b^4 + 35*(b*x^3 +
 a)^(3/2)*a^2/b^4 - 35*sqrt(b*x^3 + a)*a^3/b^4) + 2/45*A*(3*(b*x^3 + a)^(5/
2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^3 + a)*a^2/b^3)
```

mupad [B] time = 2.68, size = 104, normalized size = 1.01

$$\frac{8\,a^{2}\,\sqrt{b\,x^{3}+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^{3}}+\frac{x^{6}\,\sqrt{b\,x^{3}+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{15\,b}+\frac{2\,B\,x^{9}\,\sqrt{b\,x^{3}+a}}{21\,b}-\frac{4\,a\,x^{3}\,\sqrt{b\,x^{3}+a}\,\left(2\,A-\frac{12\,B\,a}{7\,b}\right)}{45\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (8*a^2*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^3) + (x^6*(a + b*x^3
)^(1/2)*(2*A - (12*B*a)/(7*b)))/(15*b) + (2*B*x^9*(a + b*x^3)^(1/2))/(21*b)
 - (4*a*x^3*(a + b*x^3)^(1/2)*(2*A - (12*B*a)/(7*b)))/(45*b^2)
```

sympy [A] time = 3.44, size = 175, normalized size = 1.70

$$\begin{cases} \frac{16Aa^2\sqrt{a+bx^3}}{45b^3} - \frac{8Aax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Ax^6\sqrt{a+bx^3}}{15b} - \frac{32Ba^3\sqrt{a+bx^3}}{105b^4} + \frac{16Ba^2x^3\sqrt{a+bx^3}}{105b^3} - \frac{4Bax^6\sqrt{a+bx^3}}{35b^2} + \frac{2Bx^9\sqrt{a+bx^3}}{21b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^9}{9} + \frac{Bx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((16*A*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*A*a*x**3*sqrt(a + b*x**
3)/(45*b**2) + 2*A*x**6*sqrt(a + b*x**3)/(15*b) - 32*B*a**3*sqrt(a + b*x**3
)/(105*b**4) + 16*B*a**2*x**3*sqrt(a + b*x**3)/(105*b**3) - 4*B*a*x**6*sqrt
(a + b*x**3)/(35*b**2) + 2*B*x**9*sqrt(a + b*x**3)/(21*b), Ne(b, 0)), ((A*x
**9/9 + B*x**12/12)/sqrt(a), True))
```

3.214 $\int \frac{x^5(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=73

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}\,(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

```
[Out] 2/9*(A*b-2*B*a)*(b*x^3+a)^(3/2)/b^3+2/15*B*(b*x^3+a)^(5/2)/b^3-2/3*a*(A*b-B
*a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\left(a+bx^3\right)^{3/2}(Ab-2aB)}{9b^3}-\frac{2a\sqrt{a+bx^3}\,(Ab-aB)}{3b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (-2*a*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^3) + (2*(A*b - 2*a*B)*(a + b*x^3)^(
3/2))/(9*b^3) + (2*B*(a + b*x^3)^(5/2))/(15*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2\sqrt{a+bx}}+\frac{(Ab-2aB)\sqrt{a+bx}}{b^2}+\frac{B(a+bx)^{3/2}}{b^2}\right)dx,x,x^3\right)\\
&= -\frac{2a(Ab-aB)\sqrt{a+bx^3}}{3b^3}+\frac{2(Ab-2aB)\left(a+bx^3\right)^{3/2}}{9b^3}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.77

$$\frac{2\sqrt{a+bx^3}\left(8a^2B-2ab\left(5A+2Bx^3\right)+b^2x^3\left(5A+3Bx^3\right)\right)}{45b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(8*a^2*B - 2*a*b*(5*A + 2*B*x^3) + b^2*x^3*(5*A + 3*B*x^
3)))/(45*b^3)
```

fricas [A] time = 0.94, size = 52, normalized size = 0.71

$$\frac{2\left(3\,Bb^2x^6-\left(4\,Bab-5\,Ab^2\right)x^3+8\,Ba^2-10\,Aab\right)\sqrt{bx^3+a}}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^2*x^6 - (4*B*a*b - 5*A*b^2)*x^3 + 8*B*a^2 - 10*A*a*b)*sqrt(b*x^
3 + a)/b^3
```

giac [A] time = 0.16, size = 70, normalized size = 0.96

$$\frac{2\,\sqrt{bx^3+a}\left(Ba^2-Aab\right)}{3\,b^3}+\frac{2\left(3\left(bx^3+a\right)^{\frac{5}{2}}B-10\left(bx^3+a\right)^{\frac{3}{2}}Ba+5\left(bx^3+a\right)^{\frac{3}{2}}Ab\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*(B*a^2 - A*a*b)/b^3 + 2/45*(3*(b*x^3 + a)^(5/2)*B - 10*
(b*x^3 + a)^(3/2)*B*a + 5*(b*x^3 + a)^(3/2)*A*b)/b^3
```

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2\sqrt{b\,x^3+a}\,\left(-3B\,b^2x^6-5A\,b^2x^3+4Bab\,x^3+10Aab-8B\,a^2\right)}{45b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/45*(b*x^3+a)^(1/2)*(-3*B*b^2*x^6-5*A*b^2*x^3+4*B*a*b*x^3+10*A*a*b-8*B*a^
2)/b^3
```

maxima [A] time = 0.45, size = 83, normalized size = 1.14

$$\frac{2}{45}\,B\left(\frac{3\left(bx^3+a\right)^{\frac{5}{2}}}{b^3}-\frac{10\left(bx^3+a\right)^{\frac{3}{2}}a}{b^3}+\frac{15\,\sqrt{bx^3+a}\,a^2}{b^3}\right)+\frac{2}{9}\,A\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^2}-\frac{3\,\sqrt{bx^3+a}\,a}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*B*(3*(b*x^3 + a)^(5/2)/b^3 - 10*(b*x^3 + a)^(3/2)*a/b^3 + 15*sqrt(b*x^
3 + a)*a^2/b^3) + 2/9*A*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2)
```

mupad [B] time = 2.65, size = 52, normalized size = 0.71

$$\frac{2\,\sqrt{b\,x^3+a}\,\left(8\,B\,a^2-4\,B\,a\,b\,x^3-10\,A\,a\,b+3\,B\,b^2\,x^6+5\,A\,b^2\,x^3\right)}{45\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] (2*(a + b*x^3)^(1/2)*(8*B*a^2 + 5*A*b^2*x^3 + 3*B*b^2*x^6 - 10*A*a*b - 4*B*
a*b*x^3))/(45*b^3)
```

sympy [A] time = 1.82, size = 124, normalized size = 1.70

$$\begin{cases} -\frac{4Aa\sqrt{a+bx^3}}{9b^2} + \frac{2Ax^3\sqrt{a+bx^3}}{9b} + \frac{16Ba^2\sqrt{a+bx^3}}{45b^3} - \frac{8Bax^3\sqrt{a+bx^3}}{45b^2} + \frac{2Bx^6\sqrt{a+bx^3}}{15b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((-4*A*a*sqrt(a + b*x**3)/(9*b**2) + 2*A*x**3*sqrt(a + b*x**3)/(9*
b) + 16*B*a**2*sqrt(a + b*x**3)/(45*b**3) - 8*B*a*x**3*sqrt(a + b*x**3)/(45
*b**2) + 2*B*x**6*sqrt(a + b*x**3)/(15*b), Ne(b, 0)), ((A*x**6/6 + B*x**9/9
)/sqrt(a), True))
```

3.215 $\int \frac{x^2(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{a+bx^3}\,(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

[Out] 2/9*B*(b*x^3+a)^(3/2)/b^2+2/3*(A*b-B*a)*(b*x^3+a)^(1/2)/b^2

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2\sqrt{a+bx^3}\,(Ab-aB)}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]

[Out] (2*(A*b - a*B)*Sqrt[a + b*x^3])/(3*b^2) + (2*B*(a + b*x^3)^(3/2))/(9*b^2)

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b}\right)dx,x,x^3\right)\\ &= \frac{2(Ab-aB)\sqrt{a+bx^3}}{3b^2}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\sqrt{a+bx^3}\left(-2aB+3Ab+bBx^3\right)}{9b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^3))/Sqrt[a + b*x^3],x]

[Out] (2*Sqrt[a + b*x^3]*(3*A*b - 2*a*B + b*B*x^3))/(9*b^2)

fricas [A] time = 0.98, size = 29, normalized size = 0.63

$$\frac{2\left(Bbx^3 - 2\,Ba + 3\,Ab\right)\sqrt{bx^3 + a}}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] 2/9*(B*b*x^3 - 2*B*a + 3*A*b)*sqrt(b*x^3 + a)/b^2

giac [A] time = 0.16, size = 38, normalized size = 0.83

$$\frac{2\left(bx^3 + a\right)^{\frac{3}{2}}B}{9\,b^2} - \frac{2\,\sqrt{bx^3 + a}\,(Ba - Ab)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] 2/9*(b*x^3 + a)^(3/2)*B/b^2 - 2/3*sqrt(b*x^3 + a)*(B*a - A*b)/b^2

maple [A] time = 0.04, size = 30, normalized size = 0.65

$$\frac{2\sqrt{b\,x^3 + a}\ \left(Bb\,x^3 + 3Ab - 2Ba\right)}{9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x)

[Out] 2/9*(b*x^3+a)^(1/2)*(B*b*x^3+3*A*b-2*B*a)/b^2

maxima [A] time = 0.46, size = 48, normalized size = 1.04

$$\frac{2}{9}\,B\left(\frac{\left(bx^3 + a\right)^{\frac{3}{2}}}{b^2} - \frac{3\,\sqrt{bx^3 + a}\,a}{b^2}\right) + \frac{2\,\sqrt{bx^3 + a}\,A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^2 - 3*sqrt(b*x^3 + a)*a/b^2) + 2/3*sqrt(b*x^3 + a)*A/b

mupad [B] time = 2.60, size = 29, normalized size = 0.63

$$\frac{2\,\sqrt{b\,x^3 + a}\ \left(B\,b\,x^3 + 3\,A\,b - 2\,B\,a\right)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(1/2),x)

[Out] (2*(a + b*x^3)^(1/2)*(3*A*b - 2*B*a + B*b*x^3))/(9*b^2)

sympy [A] time = 0.98, size = 75, normalized size = 1.63

$$\begin{cases} \frac{2A\sqrt{a+bx^3}}{3b} - \frac{4Ba\sqrt{a+bx^3}}{9b^2} + \frac{2Bx^3\sqrt{a+bx^3}}{9b} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] Piecewise((2*A*sqrt(a + b*x**3)/(3*b) - 4*B*a*sqrt(a + b*x**3)/(9*b**2) + 2
*B*x**3*sqrt(a + b*x**3)/(9*b), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/sqrt(a),
True))
```

3.216 $\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=48

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx}}\,dx,x,x^3\right) \\
&= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{1}{3}A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right) \\
&= \frac{2B\sqrt{a+bx^3}}{3b}+\frac{(2A)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3b} \\
&= \frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 48, normalized size = 1.00

$$
\frac{2B\sqrt{a+bx^3}}{3b}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*B*Sqrt[a + b*x^3])/(3*b) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqr
t[a])
```

fricas [A] time = 1.08, size = 105, normalized size = 2.19

$$
\left[\frac{A\sqrt{a}\,b\log\left(\frac{bx^3-2\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\sqrt{bx^3+a}\,Ba}{3\,ab},\ \frac{2\left(A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx^3+a}\,Ba\right)}{3\,ab}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(A*sqrt(a)*b*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*sq
rt(b*x^3 + a)*B*a)/(a*b), 2/3*(A*sqrt(-a)*b*arctan(sqrt(b*x^3 + a)*sqrt(-a)
/a) + sqrt(b*x^3 + a)*B*a)/(a*b)]
```

giac [A] time = 0.19, size = 40, normalized size = 0.83

$$
\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}+\frac{2\,\sqrt{bx^3+a}\,B}{3\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*x^3 + a)*B/b
```

maple [A] time = 0.04, size = 37, normalized size = 0.77

$$
-\frac{2A\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{b\,x^3+a}\;B}{3b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b
```

maxima [A] time = 1.08, size = 54, normalized size = 1.12

$$\frac{A\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}+\frac{2\sqrt{bx^3+a}\,B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
+ 2/3*sqrt(b*x^3 + a)*B/b
```

mupad [B] time = 2.72, size = 57, normalized size = 1.19

$$\frac{2\,B\,\sqrt{b\,x^3+a}}{3\,b}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(1/2)),x)
```

```
[Out] (2*B*(a + b*x^3)^(1/2))/(3*b) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a
 + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(1/2))
```

sympy [A] time = 11.27, size = 65, normalized size = 1.35

$$\frac{2A\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^3}}\right)}{3a\sqrt{-\frac{1}{a}}}-\frac{B\left(\begin{cases}-\frac{x^3}{\sqrt{a}} & \text{for } b=0\\ -\frac{2\sqrt{a+bx^3}}{b} & \text{otherwise}\end{cases}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*atan(1/(sqrt(-1/a)*sqrt(a + b*x**3)))/(3*a*sqrt(-1/a)) - B*Piecewise((-
x**3/sqrt(a), Eq(b, 0)), (-2*sqrt(a + b*x**3)/b, True))/3
```

3.217 $\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=58

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

```
[Out] 1/3*(A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*A*(b*x^3+a)^(1
/2)/a/x^3
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}-\frac{A\sqrt{a+bx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(3*a*x^3) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqr
t[a]])/(3*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^4\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(-\frac{Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{\left(2\left(-\frac{Ab}{2}+aB\right)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3ab}\\
&= -\frac{A\sqrt{a+bx^3}}{3ax^3}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.98

$$
\frac{1}{3}\left(\frac{(Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A\sqrt{a+bx^3}}{ax^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*Sqrt[a + b*x^3]),x]
```

```
[Out] (-((A*Sqrt[a + b*x^3])/(a*x^3)) + ((A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sq
rt[a]])/a^(3/2))/3
```

fricas [A] time = 0.95, size = 126, normalized size = 2.17

$$
\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^3\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\,\sqrt{bx^3+a}\,Aa}{6\,a^2x^3},\ \frac{(2\,Ba-Ab)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{bx^3+a}\,\sqrt{-a}}{a}\right)-\sqrt{bx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*B*a - A*b)*sqrt(a)*x^3*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2
*a)/x^3) + 2*sqrt(b*x^3 + a)*A*a)/(a^2*x^3), 1/3*((2*B*a - A*b)*sqrt(-a)*x^
3*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^3 + a)*A*a)/(a^2*x^3)]
```

giac [A] time = 0.16, size = 62, normalized size = 1.07

$$
\frac{\frac{\left(2\,Bab-Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}-\frac{\sqrt{bx^3+a}\,Ab}{ax^3}}{3\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*B*a*b - A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - sqrt
(b*x^3 + a)*A*b/(a*x^3))/b
```

maple [A] time = 0.05, size = 62, normalized size = 1.07

$$
-\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{a}}+\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^3+a}}{3a\,x^3}\right)A
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
)-2/3*B*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [B] time = 1.17, size = 109, normalized size = 1.88

$$-\frac{1}{6}A\left(\frac{2\sqrt{bx^3+a}\,b}{(bx^3+a)a-a^2}+\frac{b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{B\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/3*B*log((sqrt(b*x^3 +
a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 2.89, size = 67, normalized size = 1.16

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(A\,b-2\,B\,a)}{6\,a^{3/2}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(1/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
A*b - 2*B*a))/(6*a^(3/2)) - (A*(a + b*x^3)^(1/2))/(3*a*x^3)
```

sympy [A] time = 31.83, size = 80, normalized size = 1.38

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ax^{\frac{3}{2}}}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}-\frac{2B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x^{\frac{3}{2}}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*x**(3/2)) + A*b*asinh(sqrt(a)/(sqrt(b)
*x**(3/2)))/(3*a**(3/2)) - 2*B*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*sqrt(a)
)
```

3.218 $\int \frac{A+Bx^3}{x^7 \sqrt{a+bx^3}} dx$

Optimal. Leaf size=90

$$-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}+\frac{\sqrt{a+bx^3}\,(3Ab-4aB)}{12a^2x^3}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

```
[Out] -1/12*b*(3*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*A*(b*x^3
+a)^(1/2)/a/x^6+1/12*(3*A*b-4*B*a)*(b*x^3+a)^(1/2)/a^2/x^3
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(3Ab-4aB)}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}-\frac{A\sqrt{a+bx^3}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(6*a*x^6) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^3])/(12*a^2*
x^3) - (b*(3*A*b - 4*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(12*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^7\sqrt{a+bx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx}}\,dx,x,x^3\right)\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{\left(-\frac{3Ab}{2}+2aB\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{6a}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(b(3Ab-4aB))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{24a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}+\frac{(3Ab-4aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{12a^2}\\
&= -\frac{A\sqrt{a+bx^3}}{6ax^6}+\frac{(3Ab-4aB)\sqrt{a+bx^3}}{12a^2x^3}-\frac{b(3Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{12a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.29, size = 81, normalized size = 0.90

$$\frac{\sqrt{a+bx^3}\left(b\left(2aB-\frac{3Ab}{2}\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{\sqrt{\frac{bx^3}{a}+1}}-\frac{a}{bx^3}\right)-\frac{a^2A}{x^6}\right)}{6a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*Sqrt[a + b*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*(-((a^2*A)/x^6) + b*((-3*A*b)/2 + 2*a*B)*(-(a/(b*x^3)) + A
rcTanh[Sqrt[1 + (b*x^3)/a]]/Sqrt[1 + (b*x^3)/a])))/(6*a^3)
```

fricas [A] time = 1.09, size = 173, normalized size = 1.92

$$\left[-\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{a}\,x^6\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(\left(4\,Ba^2-3\,Aab\right)x^3+2\,Aa^2\right)\sqrt{bx^3+a}}{24\,a^3x^6},-\frac{\left(4\,Bab-3\,Ab^2\right)\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*((4*B*a*b - 3*A*b^2)*sqrt(a)*x^6*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt
(a) + 2*a)/x^3) + 2*((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a
^3*x^6), -1/12*((4*B*a*b - 3*A*b^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a)*sqr
t(-a)/a) + ((4*B*a^2 - 3*A*a*b)*x^3 + 2*A*a^2)*sqrt(b*x^3 + a))/(a^3*x^6)]
```

giac [A] time = 0.19, size = 121, normalized size = 1.34

$$-\frac{\frac{\left(4\,Bab^2-3\,Ab^3\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab^2-4\,\sqrt{bx^3+a}\,Ba^2b^2-3\left(bx^3+a\right)^{\frac{3}{2}}Ab^3+5\,\sqrt{bx^3+a}\,Aab^3}{a^2b^2x^6}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*((4*B*a*b^2 - 3*A*b^3)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2
) + (4*(b*x^3 + a)^(3/2)*B*a*b^2 - 4*sqrt(b*x^3 + a)*B*a^2*b^2 - 3*(b*x^3 +
 a)^(3/2)*A*b^3 + 5*sqrt(b*x^3 + a)*A*a*b^3)/(a^2*b^2*x^6))/b
```

maple [A] time = 0.08, size = 102, normalized size = 1.13

$$\left(-\frac{b^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{4 a^{\frac{5}{2}}}+\frac{\sqrt{b\,x^{3}+a}\;b}{4 a^{2} x^{3}}-\frac{\sqrt{b\,x^{3}+a}}{6 a\,x^{6}}\right) A+\left(\frac{b \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3 a^{\frac{3}{2}}}-\frac{\sqrt{b\,x^{3}+a}}{3 a\,x^{3}}\right) B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(1/3*b*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)-1/3*(b*x^3+a)^(1/2)/a/x^3
)+A*(-1/4*b^2*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)-1/6*(b*x^3+a)^(1/2)/
a/x^6+1/4*(b*x^3+a)^(1/2)/a^2*b/x^3)
```

maxima [B] time = 1.20, size = 178, normalized size = 1.98

$$-\frac{1}{6} B\left(\frac{2 \sqrt{bx^{3}+a}\,b}{\left(bx^{3}+a\right)a-a^{2}}+\frac{b \log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}\right)+\frac{1}{24} A\left(\frac{3 b^{2} \log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\left(bx^{3}+a\right)^{\frac{3}{2}} b^{2}-5 \sqrt{bx^{3}+a}\,a\right.}{\left(bx^{3}+a\right)^{2} a^{2}-2\left(bx^{3}+a\right)a^{3}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*B*(2*sqrt(b*x^3 + a)*b/((b*x^3 + a)*a - a^2) + b*log((sqrt(b*x^3 + a)
- sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)) + 1/24*A*(3*b^2*log((sqrt(
b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*(b*x^3 +
a)^(3/2)*b^2 - 5*sqrt(b*x^3 + a)*a*b^2)/((b*x^3 + a)^2*a^2 - 2*(b*x^3 + a)*
a^3 + a^4))
```

mupad [B] time = 2.99, size = 95, normalized size = 1.06

$$\frac{\sqrt{b\,x^3+a}\,\left(3\,A\,b-4\,B\,a\right)}{12\,a^2\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a\,x^6}+\frac{b\,\ln\left(\frac{{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}^3\,\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)\,\left(3\,A\,b-4\,B\,a\right)}{24\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(1/2)),x)
```

```
[Out] ((a + b*x^3)^(1/2)*(3*A*b - 4*B*a))/(12*a^2*x^3) - (A*(a + b*x^3)^(1/2))/(6
*a*x^6) + (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1
/2)))/x^6)*(3*A*b - 4*B*a))/(24*a^(5/2))
```

sympy [B] time = 71.18, size = 163, normalized size = 1.81

$$-\frac{A}{6\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{A\sqrt{b}}{12ax^{\frac{9}{2}}\sqrt{\frac{a}{bx^{3}}+1}}+\frac{Ab^{\frac{3}{2}}}{4a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx^{3}}+1}}-\frac{Ab^{2} \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{5}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^{3}}+1}}{3ax^{\frac{3}{2}}}+\frac{Bb \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{3a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(1/2),x)
```

```
[Out] -A/(6*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + A*sqrt(b)/(12*a*x**(9/2)*sq
rt(a/(b*x**3) + 1)) + A*b**(3/2)/(4*a**2*x**(3/2)*sqrt(a/(b*x**3) + 1)) - A
*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(5/2)) - B*sqrt(b)*sqrt(a/(b*
x**3) + 1)/(3*a*x**(3/2)) + B*b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(3*a**(3/
2))
```

3.219 $\int \frac{x^3(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=270

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+}}{}$$

```
[Out] 2/55*(11*A*b-8*B*a)*x*(b*x^3+a)^(1/2)/b^2+2/11*B*x^4*(b*x^3+a)^(1/2)/b-4/16
5*a*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/
2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {459, 321, 218}

$$-\frac{4\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(11*A*b - 8*a*B)*x*Sqrt[a + b*x^3])/(55*b^2) + (2*B*x^4*Sqrt[a + b*x^3])
/(11*b) - (4*Sqrt[2 + Sqrt[3]]*a*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{\left(2\left(-\frac{11Ab}{2}+4aB\right)\right)\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{11b}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{(2a(11Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{55b^2}$$

$$= \frac{2(11Ab-8aB)x\sqrt{a+bx^3}}{55b^2} + \frac{2Bx^4\sqrt{a+bx^3}}{11b} - \frac{4\sqrt{2+\sqrt{3}}\,a(11Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\cdots}}{55\sqrt[4]{3}\,b^{7/3}\sqrt{\cdots}}$$

Mathematica [C] time = 0.13, size = 89, normalized size = 0.33

$$\frac{2x\left(a\sqrt{\frac{bx^3}{a}+1}\,(8aB-11Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(8aB-11Ab-5bBx^3\right)\right)}{55b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x*(-((a + b*x^3)*(-11*A*b + 8*a*B - 5*b*B*x^3)) + a*(-11*A*b + 8*a*B)*Sq
rt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(55*b^2*
Sqrt[a + b*x^3])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^6+Ax^3}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^3}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

maple [B] time = 0.05, size = 624, normalized size = 2.31

$$\left(\frac{4 i\sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a b^{2}\right)^{\frac{1}{3}}}}\;a\,\mathrm{EllipticF}}{15\sqrt{b\,x^{3}+a}\;b^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/11*(b*x^3+a)^(1/2)/b*x^4-16/55*(b*x^3+a)^(1/2)*a/b^2*x-32/165*I*a^2/b^
3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/5*(b*x^
3+a)^(1/2)/b*x+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*
b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)
*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^
(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*
3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)x^{3}}{\sqrt{bx^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}\,\left(B\,x^{3}+A\right)}{\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 3.17, size = 80, normalized size = 0.30

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((1/2, 7/3), (10/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(10/3))
```

3.220 $\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=239

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+b}}{5b}$$

```
[Out] 2/5*B*x*(b*x^3+a)^(1/2)/b+2/15*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {388, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2Bx\sqrt{a+b}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*Sqrt[a + b*x^3])/(5*b) + (2*Sqrt[2 + Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^
(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{a+bx^3}}\,dx = \frac{2Bx\sqrt{a+bx^3}}{5b} - \frac{\left(2\left(-\frac{5Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{2Bx\sqrt{a+bx^3}}{5b} + \frac{2\sqrt{2+\sqrt{3}}\,(5Ab-2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.05, size = 74, normalized size = 0.31

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,(5Ab-2aB)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+2Bx\left(a+bx^3\right)}{5b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x*(a + b*x^3) + (5*A*b - 2*a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F
1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(5*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^3+A}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

maple [B] time = 0.05, size = 586, normalized size = 2.45

$$-\frac{2 i \sqrt{3}\, \left(-a \,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(x +\frac{\left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}-\frac{i \sqrt{3}\, \left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a \,b^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x -\frac{\left(-a \,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3 \left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}}}\, \sqrt{-\frac{i \left(x +\frac{\left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}+\frac{i \sqrt{3}\, \left(-a \,b^{2}\right)^{\frac{1}{3}}}{2 b}\right) \sqrt{3}\, b}{\left(-a \,b^{2}\right)^{\frac{1}{3}}}}\, A \,\mathrm{EllipticF}}{3 \sqrt{b \,x^{3}+a}\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/5*(b*x^3+a)^(1/2)/b*x+4/15*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/b)^(1/2)))-2/3*I*A*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^
2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*
((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.12, size = 78, normalized size = 0.33

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*sqrt(a)*gamma(7/3))
```

3.221 $\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

```
[Out] -1/2*A*(b*x^3+a)^(1/2)/a/x^2-1/6*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(1/3)/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{A\sqrt{a+bx^3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(2*a*x^2) - (Sqrt[2 + Sqrt[3]]*(A*b - 4*a*B)*(a^(1/3)
+ b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)
*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*b^(
1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\left(\frac{Ab}{2}-2aB\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{2a}$$

$$= -\frac{A\sqrt{a+bx^3}}{2ax^2} - \frac{\sqrt{2+\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{2\sqrt[4]{3}\,a\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.32

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(4aB-Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2A\left(a+bx^3\right)}{4ax^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (-(A*b) + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(4*a*x^2*Sqrt[a + b*x^3])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^6+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

maple [B] time = 0.05, size = 587, normalized size = 2.42

$$\left(\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\ \sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\ \sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\ \mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{6\sqrt{b\,x^{3}+a}\;a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*B*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+A*(-
1/2*(b*x^3+a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((
x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))
^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a
*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/
2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^{3} + A}{\sqrt{bx^{3} + a}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^{3} + A}{x^{3}\,\sqrt{b\,x^{3} + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 2.42, size = 82, normalized size = 0.34

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(4/3))
```

3.222 $\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=274

$$\frac{\sqrt{a+bx^3}\,(7Ab-10aB)}{20a^2x^2}+\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

```
[Out] -1/5*A*(b*x^3+a)^(1/2)/a/x^5+1/20*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^2/x^2+1/
60*b^(2/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/
2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^2/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 325, 218}

$$\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\sqrt{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(5*a*x^5) + ((7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(20*a^2
*x^2) + (Sqrt[2 + Sqrt[3]]*b^(2/3)*(7*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*S
qrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(20*3^(1/4)*a^2*Sqrt[(a^(1/3)*
(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^
3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{5ax^5} - \frac{\left(\frac{7Ab}{2}-5aB\right)\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{5a}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{(b(7Ab-10aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{40a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{5ax^5} + \frac{(7Ab-10aB)\sqrt{a+bx^3}}{20a^2x^2} + \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}(7Ab-10aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left((1+\cdots\right.}}}{20\sqrt[4]{3}\,a^2\sqrt{\frac{\sqrt[3]{a}(\cdots}{\left((1+\sqrt{3}\cdots\right.}}}$$

Mathematica [C] time = 0.07, size = 78, normalized size = 0.28

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(7Ab-10aB)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-4A\left(a+bx^3\right)}{20ax^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (7*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/(20*a*x^5*Sqrt[a + b*x^3])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^9+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^9 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

maple [B] time = 0.05, size = 625, normalized size = 2.28

$$-\frac{7i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,b\,\mathrm{Elliptic}}{60\sqrt{b\,x^{3}+a}\;a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/a/x^5+7/20*(b*x^3+a)^(1/2)/a^2*b/x^2-7/60*I/a^2*b*3
^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+B*(-1/2*(b*x^3+
a)^(1/2)/a/x^2+1/6*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^
(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*
(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/
2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^{3}+A}{\sqrt{bx^{3}+a}\,x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^{3}+A}{x^{6}\,\sqrt{b\,x^{3}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 2.87, size = 90, normalized size = 0.33

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{5}{3},\frac{1}{2}\\-\frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 1/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 1/2), (1/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x**2*gamma(1/3))
```

3.223 $\int \frac{x^4(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=548

$$-\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-}}{}$$

```
[Out] 2/91*(13*A*b-10*B*a)*x^2*(b*x^3+a)^(1/2)/b^2+2/13*B*x^5*(b*x^3+a)^(1/2)/b-8
/91*a*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))-8/273*a^(4/3)*(13*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a
^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*
((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/91*3^(1/4)*a^(4/3)*(13*A*b-10*B*a)*(a^(
1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b
^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(8/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 321, 303, 218, 1877}

$$-\frac{8\sqrt{2}\,a^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(13Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{91\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt[4]{3}\sqrt{2-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*(13*A*b - 10*a*B)*x^2*Sqrt[a + b*x^3])/(91*b^2) + (2*B*x^5*Sqrt[a + b*x^
3])/(13*b) - (8*a*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(91*b^(8/3)*((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)) + (4*3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(4/3)*(13*A*b -
 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])
*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]]
)/(91*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*a^(4/3)*(13*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)
*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{\left(2\left(-\frac{13Ab}{2}+5aB\right)\right)\int \frac{x^4}{\sqrt{a+bx^3}}\,dx}{13b}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{91b^2}$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{(4a(13Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{91b^{7/3}} -$$

$$= \frac{2(13Ab-10aB)x^2\sqrt{a+bx^3}}{91b^2} + \frac{2Bx^5\sqrt{a+bx^3}}{13b} - \frac{8a(13Ab-10aB)\sqrt{a+bx^3}}{91b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{4\sqrt[4]{3}\sqrt{2}}{}$$

Mathematica [C] time = 0.12, size = 91, normalized size = 0.17

$$\frac{2x^2\left(a\sqrt{\frac{bx^3}{a}+1}\,(10aB-13Ab)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(10aB-13Ab-7bBx^3\right)\right)}{91b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*x^2*(-((a + b*x^3)*(-13*A*b + 10*a*B - 7*b*B*x^3)) + a*(-13*A*b + 10*a*B
)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(91*
b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^7 + Ax^4}{\sqrt{bx^3 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^7 + A*x^4)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x^4}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

maple [B] time = 0.05, size = 932, normalized size = 1.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/13*(b*x^3+a)^(1/2)/b*x^5-20/91*(b*x^3+a)^(1/2)*a/b^2*x^2-80/273*I*a^2/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3
+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE
(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3
*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(2/7*(b*x^3+a)^(1/2)/b*x^2+8/2
1*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/
(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
```

```
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)x^4}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*x^4/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(B\,x^3 + A)}{\sqrt{b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2),x)
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 3.17, size = 80, normalized size = 0.15

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{3}\\ \frac{8}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{8}{3}\\ \frac{11}{3}\end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] A*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((1/2, 8/3), (11/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*gamma(11/3))
```

3.224 $\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=517

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

```
[Out] 2/7*B*x^2*(b*x^3+a)^(1/2)/b+2/7*(7*A*b-4*B*a)*(b*x^3+a)^(1/2)/b^(5/3)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+2/21*a^(1/3)*(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*E
llipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*
3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/7*3^(1/4)*a^(1/3)*
(7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 517, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {459, 303, 218, 1877}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*x^2*Sqrt[a + b*x^3])/(7*b) + (2*(7*A*b - 4*a*B)*Sqrt[a + b*x^3])/(7*b^
(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(
1/3)*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3]
)*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(7*A*b - 4*
a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(
1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7
*3^(1/4)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2Bx^2\sqrt{a+bx^3}}{7b} - \frac{\left(2\left(-\frac{7Ab}{2}+2aB\right)\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b}$$

$$= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{(7Ab-4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\,(7Ab-4aB)\right)\int \frac{}{\sqrt{a\cdot}}}{7b^{4/3}}$$

$$= \frac{2Bx^2\sqrt{a+bx^3}}{7b} + \frac{2(7Ab-4aB)\sqrt{a+bx^3}}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{7b^{5/3}\sqrt{\cdot}}$$

Mathematica [C] time = 0.06, size = 75, normalized size = 0.15

$$\frac{x^2\left(\sqrt{\frac{bx^3}{a}+1}\,(7Ab-4aB)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+4B\left(a+bx^3\right)\right)}{14b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x^2*(4*B*(a + b*x^3) + (7*A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[1/2, 2/3, 5/3, -((b*x^3)/a)]))/(14*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{Bx^4+Ax}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

maple [B] time = 0.05, size = 892, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(2/7*(b*x^3+a)^(1/2)/b*x^2+8/21*I*a/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*
(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/
2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)
^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(
-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/
2))))-2/3*I*A*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(B\,x^3 + A\right)}{\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(1/2),x)
```

```
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 2.88, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*sq
rt(a)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((1/2, 5/3), (8/3,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(a)*gamma(8/3))
```

3.225 $\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=509

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\right.$$

```
[Out] -A*(b*x^3+a)^(1/2)/a/x+(A*b+2*B*a)*(b*x^3+a)^(1/2)/a/b^(2/3)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))+1/3*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+
a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*(A*b+2*B*a)*(a^(1/3)
+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 303, 218, 1877}

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}a^{2/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\sqrt[4]{3}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] -((A*Sqrt[a + b*x^3])/(a*x)) + ((A*b + 2*a*B)*Sqrt[a + b*x^3])/(a*b^(2/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*
a^(2/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (Sqrt[2]*(A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*b^(2/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{ax} - \frac{\left(-\frac{Ab}{2}-aB\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt[3]{b}} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}(Ab+2aB)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{A\sqrt{a+bx^3}}{ax} + \frac{(Ab+2aB)\sqrt{a+bx^3}}{ab^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/}\cdots}{\left((\cdots\right.}}}{2a^{2/3}b^{2/3}\sqrt{\frac{\cdots}{\left((1\cdots\right.}}}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.15

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(2aB+Ab)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)-4A\left(a+bx^3\right)}{4ax\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*Sqrt[a + b*x^3]),x]
```

```
[Out] (-4*A*(a + b*x^3) + (A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1
[1/2, 2/3, 5/3, -((b*x^3)/a)])/(4*a*x*Sqrt[a + b*x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^5+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

maple [B] time = 0.06, size = 891, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*I*B*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*El
lipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-(b*x^3+a)^(1/2)/a/x-1/
3*I/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x
^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1
/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^2\,\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 2.46, size = 82, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{2}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,x\Gamma\left(\frac{2}{3}\right)}+\frac{Bx^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((1/2, 2/3), (5/3,), b*x**3*exp_p
olar(I*pi)/a)/(3*sqrt(a)*gamma(5/3))
```

3.226 $\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=550

$$-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}}{}$$

```
[Out] -1/4*A*(b*x^3+a)^(1/2)/a/x^4+1/8*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/x-1/8*b^
(1/3)*(5*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^2/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-1/2
4*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1
-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^
(5/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)+1/16*3^(1/4)*b^(1/3)*(5*A*b-8*B*a)*(a^(1/3)+b^(1/3
)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^3+a)^(1/2)
/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 325, 303, 218, 1877}

$$-\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(4*a*x^4) + ((5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*x
) - (b^(1/3)*(5*A*b - 8*a*B)*Sqrt[a + b*x^3])/(8*a^2*((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*a^(5/3)*Sq
rt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*S
qrt[a + b*x^3]) - (b^(1/3)*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*a^(5/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{4ax^4} - \frac{\left(\frac{5Ab}{2}-4aB\right)\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{4a}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{(b(5Ab-8aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{16a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\left(b^{2/3}(5Ab-8aB)\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{16a^2} - \left(\sqrt{\tfrac{1}{2}\left(2 -\right.}\right.$$

$$= -\frac{A\sqrt{a+bx^3}}{4ax^4} + \frac{(5Ab-8aB)\sqrt{a+bx^3}}{8a^2x} - \frac{\sqrt[3]{b}\,(5Ab-8aB)\sqrt{a+bx^3}}{8a^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,(5}{}$$

Mathematica [C] time = 0.05, size = 78, normalized size = 0.14

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(5Ab-8aB)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-2A\left(a+bx^3\right)}{8ax^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*(a + b*x^3) + (5*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/(8*a*x^4*Sqrt[a + b*x^3])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^8+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^8 + a*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

maple [B] time = 0.06, size = 929, normalized size = 1.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x)
```

```
[Out] A*(-1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*(b*x^3+a)^(1/2)/a^2*b/x+5/24*I/a^2*b*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2
)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(
1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)
*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2
)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+B*(-(b*x^3+a)^(1/2)/a/x-1/3*I/a*3^(1/2)*
(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)
*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2
)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
```

```
1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\, x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^5\,\sqrt{b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 2.68, size = 88, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^4 \Gamma\left(-\frac{1}{3}\right)} + \frac{B\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x \Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 1/2), (2/3,), b*x**3*exp
_polar(I*pi)/a)/(3*sqrt(a)*x*gamma(2/3))
```

3.227 $\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=581

$$\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,b}{}$$

```
[Out] -1/7*A*(b*x^3+a)^(1/2)/a/x^7+1/56*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^2/x^4-5
/112*b*(11*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x+5/112*b^(4/3)*(11*A*b-14*B*a)*
(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/336*b^(4/3)*(11*A*b-1
4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*
x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2
)^(1/2)-5/224*3^(1/4)*b^(4/3)*(11*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE
((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 325, 303, 218, 1877}

$$\frac{5b^{4/3}\sqrt{a+bx^3}(11Ab-14aB)}{112a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{5b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\right.}{56\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] -(A*Sqrt[a + b*x^3])/(7*a*x^7) + ((11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(56*a^
2*x^4) - (5*b*(11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*x) + (5*b^(4/3)*(
11*A*b - 14*a*B)*Sqrt[a + b*x^3])/(112*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)) - (5*3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(224*a^(8/3)*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + (5*b^(4/3)*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3
)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*
x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\sqrt{a+bx^3}}\,dx = -\frac{A\sqrt{a+bx^3}}{7ax^7} - \frac{\left(\frac{11Ab}{2}-7aB\right)\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} + \frac{(5b(11Ab-14aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{112a^2}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{\left(5b^2(11Ab-1\right.}{2}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{\left(5b^{5/3}(11Ab-\right.}{}$$

$$= -\frac{A\sqrt{a+bx^3}}{7ax^7} + \frac{(11Ab-14aB)\sqrt{a+bx^3}}{56a^2x^4} - \frac{5b(11Ab-14aB)\sqrt{a+bx^3}}{112a^3x} + \frac{5b^{4/3}(11Ab-1}{112a^3\left(\left(1+\sqrt{\right.\right.}}$$

Mathematica [C] time = 0.04, size = 78, normalized size = 0.13

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(11Ab-14aB)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)-8A\left(a+bx^3\right)}{56ax^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*Sqrt[a + b*x^3]),x]
```

```
[Out] (-8*A*(a + b*x^3) + (11*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometri
c2F1[-4/3, 1/2, -1/3, -((b*x^3)/a)])/(56*a*x^7*Sqrt[a + b*x^3])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{bx^{11}+ax^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b*x^11 + a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

maple [B] time = 0.08, size = 970, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x)
```

```
[Out] B*(-1/4*(b*x^3+a)^(1/2)/a/x^4+5/8*(b*x^3+a)^(1/2)/a^2*b/x+5/24*I/a^2*b*3^(1
/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2
)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(
1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a
*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)
*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2
)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-1/7/a*(b*x^3+a)^(1/2)/x^7+11/56/a^2*
b*(b*x^3+a)^(1/2)/x^4-55/112/a^3*b^2*(b*x^3+a)^(1/2)/x-55/336*I/a^3*b^2*3^(
1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\, x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^8\,\sqrt{b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 3.14, size = 94, normalized size = 0.16

$$\frac{A\Gamma\left(-\frac{7}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{7}{3}, \frac{1}{2} \\ -\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^7 \Gamma\left(-\frac{4}{3}\right)} + \frac{B\Gamma\left(-\frac{4}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\, x^4 \Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 1/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt
(a)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 1/2), (-1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*sqrt(a)*x**4*gamma(-1/3))
```

3.228 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}\,(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

```
[Out] 2/9*(A*b-3*B*a)*(b*x^3+a)^(3/2)/b^4+2/15*B*(b*x^3+a)^(5/2)/b^4-2/3*a^2*(A*b
-B*a)/b^4/(b*x^3+a)^(1/2)-2/3*a*(2*A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\left(a+bx^3\right)^{3/2}(Ab-3aB)}{9b^4}-\frac{2a\sqrt{a+bx^3}\,(2Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(3*b^4*Sqrt[a + b*x^3]) - (2*a*(2*A*b - 3*a*B)*Sqrt[a
+ b*x^3])/(3*b^4) + (2*(A*b - 3*a*B)*(a + b*x^3)^(3/2))/(9*b^4) + (2*B*(a +
 b*x^3)^(5/2))/(15*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{3/2}}+\frac{a(-2Ab+3aB)}{b^3\sqrt{a+bx}}+\frac{(Ab-3aB)\sqrt{a+bx}}{b^3}+\frac{B(a+bx)^{3/2}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{2a^2(Ab-aB)}{3b^4\sqrt{a+bx^3}}-\frac{2a(2Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2(Ab-3aB)\left(a+bx^3\right)^{3/2}}{9b^4}+\frac{2B\left(a+bx^3\right)^{5/2}}{15b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.75

$$\frac{2\left(48a^3B-8a^2b\left(5A-3Bx^3\right)-2ab^2x^3\left(10A+3Bx^3\right)+b^3x^6\left(5A+3Bx^3\right)\right)}{45b^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(48*a^3*B - 8*a^2*b*(5*A - 3*B*x^3) + b^3*x^6*(5*A + 3*B*x^3) - 2*a*b^2*
x^3*(10*A + 3*B*x^3)))/(45*b^4*Sqrt[a + b*x^3])
```

fricas [A] time = 0.81, size = 88, normalized size = 0.85

$$\frac{2\left(3 Bb^3x^9 - \left(6 Bab^2 - 5 Ab^3\right)x^6 + 48 Ba^3 - 40 Aa^2b + 4\left(6 Ba^2b - 5 Aab^2\right)x^3\right)\sqrt{bx^3 + a}}{45\left(b^5x^3 + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(3*B*b^3*x^9 - (6*B*a*b^2 - 5*A*b^3)*x^6 + 48*B*a^3 - 40*A*a^2*b + 4*(
6*B*a^2*b - 5*A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^5*x^3 + a*b^4)
```

giac [A] time = 0.17, size = 114, normalized size = 1.11

$$\frac{2\left(Ba^3 - Aa^2b\right)}{3\sqrt{bx^3 + a}\,b^4} + \frac{2\left(3\left(bx^3 + a\right)^{\frac{5}{2}}Bb^{16} - 15\left(bx^3 + a\right)^{\frac{3}{2}}Bab^{16} + 45\sqrt{bx^3 + a}\,Ba^2b^{16} + 5\left(bx^3 + a\right)^{\frac{3}{2}}Ab^{17} - 30\sqrt{bx^3}\right.}{45\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(B*a^3 - A*a^2*b)/(sqrt(b*x^3 + a)*b^4) + 2/45*(3*(b*x^3 + a)^(5/2)*B*b
^16 - 15*(b*x^3 + a)^(3/2)*B*a*b^16 + 45*sqrt(b*x^3 + a)*B*a^2*b^16 + 5*(b*
x^3 + a)^(3/2)*A*b^17 - 30*sqrt(b*x^3 + a)*A*a*b^17)/b^20
```

maple [A] time = 0.04, size = 77, normalized size = 0.75

$$-\frac{2\left(-3B\,x^9b^3 - 5A\,b^3x^6 + 6Ba\,b^2x^6 + 20Aa\,b^2x^3 - 24B\,a^2b\,x^3 + 40A\,a^2b - 48B\,a^3\right)}{45\sqrt{b\,x^3 + a}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/45/(b*x^3+a)^(1/2)*(-3*B*b^3*x^9-5*A*b^3*x^6+6*B*a*b^2*x^6+20*A*a*b^2*x^
3-24*B*a^2*b*x^3+40*A*a^2*b-48*B*a^3)/b^4
```

maxima [A] time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{15}B\left(\frac{\left(bx^3 + a\right)^{\frac{5}{2}}}{b^4} - \frac{5\left(bx^3 + a\right)^{\frac{3}{2}}a}{b^4} + \frac{15\sqrt{bx^3 + a}\,a^2}{b^4} + \frac{5\,a^3}{\sqrt{bx^3 + a}\,b^4}\right) + \frac{2}{9}A\left(\frac{\left(bx^3 + a\right)^{\frac{3}{2}}}{b^3} - \frac{6\sqrt{bx^3 + a}\,a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3 +}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*B*((b*x^3 + a)^(5/2)/b^4 - 5*(b*x^3 + a)^(3/2)*a/b^4 + 15*sqrt(b*x^3 +
 a)*a^2/b^4 + 5*a^3/(sqrt(b*x^3 + a)*b^4)) + 2/9*A*((b*x^3 + a)^(3/2)/b^3 -
 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3 + a)*b^3))
```

mupad [B] time = 2.77, size = 152, normalized size = 1.48

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(B\,a^2-A\,a\,b\right)}{b^3}-\frac{2\,a\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{3\,b}\right)}{3\,b}+\frac{x^3\,\sqrt{b\,x^3+a}\,\left(\frac{2\,\left(A\,b^2-B\,a\,b\right)}{b^3}-\frac{8\,B\,a}{5\,b^2}\right)}{9\,b}-\frac{a^2\,\left(\frac{2\,A}{3\,b}-\frac{2\,B\,a}{3\,b^2}\right)}{b^2\,\sqrt{b\,x^3+a}}+\frac{2\,B\,x^6\,\sqrt{b\,x^3+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(B*a^2 - A*a*b))/b^3 - (2*a*((2*(A*b^2 - B*a*b))/b^3
 - (8*B*a)/(5*b^2)))/(3*b)))/(3*b) + (x^3*(a + b*x^3)^(1/2)*((2*(A*b^2 - B*
a*b))/b^3 - (8*B*a)/(5*b^2)))/(9*b) - (a^2*((2*A)/(3*b) - (2*B*a)/(3*b^2)))
/(b^2*(a + b*x^3)^(1/2)) + (2*B*x^6*(a + b*x^3)^(1/2))/(15*b^2)
```

sympy [A] time = 3.90, size = 175, normalized size = 1.70

$$\begin{cases} -\frac{16Aa^2}{9b^3\sqrt{a+bx^3}} - \frac{8Aax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Ax^6}{9b\sqrt{a+bx^3}} + \frac{32Ba^3}{15b^4\sqrt{a+bx^3}} + \frac{16Ba^2x^3}{15b^3\sqrt{a+bx^3}} - \frac{4Bax^6}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^9}{15b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-16*A*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*A*a*x**3/(9*b**2*sqrt(a
 + b*x**3)) + 2*A*x**6/(9*b*sqrt(a + b*x**3)) + 32*B*a**3/(15*b**4*sqrt(a +
 b*x**3)) + 16*B*a**2*x**3/(15*b**3*sqrt(a + b*x**3)) - 4*B*a*x**6/(15*b**2
*sqrt(a + b*x**3)) + 2*B*x**9/(15*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**9/
9 + B*x**12/12)/a**(3/2), True))
```

3.229 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sqrt{a+bx^3}\,(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

```
[Out] 2/9*B*(b*x^3+a)^(3/2)/b^3+2/3*a*(A*b-B*a)/b^3/(b*x^3+a)^(1/2)+2/3*(A*b-2*B*
a)*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{2\sqrt{a+bx^3}\,(Ab-2aB)}{3b^3}+\frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(3*b^3*Sqrt[a + b*x^3]) + (2*(A*b - 2*a*B)*Sqrt[a + b*x^3
])/(3*b^3) + (2*B*(a + b*x^3)^(3/2))/(9*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{3/2}}+\frac{Ab-2aB}{b^2\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^2}\right)dx,x,x^3\right)\\ &= \frac{2a(Ab-aB)}{3b^3\sqrt{a+bx^3}}+\frac{2(Ab-2aB)\sqrt{a+bx^3}}{3b^3}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.75

$$\frac{2\left(-8a^2B+a\left(6Ab-4bBx^3\right)+b^2x^3\left(3A+Bx^3\right)\right)}{9b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-8*a^2*B + b^2*x^3*(3*A + B*x^3) + a*(6*A*b - 4*b*B*x^3)))/(9*b^3*Sqrt[
a + b*x^3])
```

fricas [A] time = 0.96, size = 63, normalized size = 0.86

$$\frac{2\left(Bb^2x^6 - \left(4\,Bab - 3\,Ab^2\right)x^3 - 8\,Ba^2 + 6\,Aab\right)\sqrt{bx^3 + a}}{9\left(b^4x^3 + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^2*x^6 - (4*B*a*b - 3*A*b^2)*x^3 - 8*B*a^2 + 6*A*a*b)*sqrt(b*x^3 +
a)/(b^4*x^3 + a*b^3)
```

giac [A] time = 0.18, size = 77, normalized size = 1.05

$$-\frac{2\left(Ba^2 - Aab\right)}{3\sqrt{bx^3 + a}\,b^3} + \frac{2\left(\left(bx^3 + a\right)^{\frac{3}{2}}Bb^6 - 6\sqrt{bx^3 + a}\,Bab^6 + 3\sqrt{bx^3 + a}\,Ab^7\right)}{9\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(B*a^2 - A*a*b)/(sqrt(b*x^3 + a)*b^3) + 2/9*((b*x^3 + a)^(3/2)*B*b^6 -
 6*sqrt(b*x^3 + a)*B*a*b^6 + 3*sqrt(b*x^3 + a)*A*b^7)/b^9
```

maple [A] time = 0.05, size = 52, normalized size = 0.71

$$\frac{\frac{2}{9}B\,b^2x^6 + \frac{2}{3}A\,b^2x^3 - \frac{8}{9}Bab\,x^3 + \frac{4}{3}Aab - \frac{16}{9}B\,a^2}{\sqrt{b\,x^3 + a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(1/2)*(B*b^2*x^6+3*A*b^2*x^3-4*B*a*b*x^3+6*A*a*b-8*B*a^2)/b^3
```

maxima [A] time = 0.65, size = 81, normalized size = 1.11

$$\frac{2}{9}\,B\left(\frac{\left(bx^3 + a\right)^{\frac{3}{2}}}{b^3} - \frac{6\sqrt{bx^3 + a}\,a}{b^3} - \frac{3\,a^2}{\sqrt{bx^3 + a}\,b^3}\right) + \frac{2}{3}\,A\left(\frac{\sqrt{bx^3 + a}}{b^2} + \frac{a}{\sqrt{bx^3 + a}\,b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^3 - 6*sqrt(b*x^3 + a)*a/b^3 - 3*a^2/(sqrt(b*x^3
+ a)*b^3)) + 2/3*A*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2))
```

mupad [B] time = 2.68, size = 60, normalized size = 0.82

$$\frac{2\,B\left(b\,x^3 + a\right)^2 - 6\,B\,a^2 + 6\,A\,b\,\left(b\,x^3 + a\right) - 12\,B\,a\,\left(b\,x^3 + a\right) + 6\,A\,a\,b}{9\,b^3\sqrt{b\,x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] (2*B*(a + b*x^3)^2 - 6*B*a^2 + 6*A*b*(a + b*x^3) - 12*B*a*(a + b*x^3) + 6*A
*a*b)/(9*b^3*(a + b*x^3)^(1/2))
```

sympy [A] time = 1.84, size = 124, normalized size = 1.70

$$\begin{cases} \frac{4Aa}{3b^2\sqrt{a+bx^3}} + \frac{2Ax^3}{3b\sqrt{a+bx^3}} - \frac{16Ba^2}{9b^3\sqrt{a+bx^3}} - \frac{8Bax^3}{9b^2\sqrt{a+bx^3}} + \frac{2Bx^6}{9b\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((4*A*a/(3*b**2*sqrt(a + b*x**3)) + 2*A*x**3/(3*b*sqrt(a + b*x**3)
) - 16*B*a**2/(9*b**3*sqrt(a + b*x**3)) - 8*B*a*x**3/(9*b**2*sqrt(a + b*x**
3)) + 2*B*x**6/(9*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a*
*(3/2), True))
```

3.230 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

```
[Out] -2/3*(A*b-B*a)/b^2/(b*x^3+a)^(1/2)+2/3*B*(b*x^3+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{2B\sqrt{a+bx^3}}{3b^2}-\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(A*b - a*B))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{(a+bx)^{3/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{3/2}}+\frac{B}{b\sqrt{a+bx}}\right)dx,x,x^3\right)\\ &= -\frac{2(Ab-aB)}{3b^2\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$\frac{2\left(2aB-Ab+bBx^3\right)}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(-(A*b) + 2*a*B + b*B*x^3))/(3*b^2*Sqrt[a + b*x^3])
```

fricas [A] time = 0.62, size = 41, normalized size = 0.89

$$\frac{2\left(Bbx^3 + 2Ba - Ab\right)\sqrt{bx^3 + a}}{3\left(b^3x^3 + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(B*b*x^3 + 2*B*a - A*b)*sqrt(b*x^3 + a)/(b^3*x^3 + a*b^2)
```

giac [A] time = 0.18, size = 38, normalized size = 0.83

$$\frac{2\sqrt{bx^3 + a}\,B}{3\,b^2} + \frac{2\,(Ba - Ab)}{3\sqrt{bx^3 + a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x^3 + a)*B/b^2 + 2/3*(B*a - A*b)/(sqrt(b*x^3 + a)*b^2)
```

maple [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2\left(-Bb\,x^3 + Ab - 2Ba\right)}{3\sqrt{b\,x^3 + a}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x)
[Out] -2/3/(b*x^3+a)^(1/2)*(-B*b*x^3+A*b-2*B*a)/b^2
```

maxima [A] time = 0.56, size = 47, normalized size = 1.02

$$\frac{2}{3}\,B\left(\frac{\sqrt{bx^3 + a}}{b^2} + \frac{a}{\sqrt{bx^3 + a}\,b^2}\right) - \frac{2\,A}{3\sqrt{bx^3 + a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*B*(sqrt(b*x^3 + a)/b^2 + a/(sqrt(b*x^3 + a)*b^2)) - 2/3*A/(sqrt(b*x^3 +
 a)*b)
```

mupad [B] time = 2.61, size = 33, normalized size = 0.72

$$\frac{2\,B\,a - 2\,A\,b + 2\,B\,\left(b\,x^3 + a\right)}{3\,b^2\,\sqrt{b\,x^3 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(3/2),x)
[Out] (2*B*a - 2*A*b + 2*B*(a + b*x^3))/(3*b^2*(a + b*x^3)^(1/2))
```

sympy [A] time = 1.08, size = 75, normalized size = 1.63

$$\begin{cases} -\frac{2A}{3b\sqrt{a+bx^3}} + \frac{4Ba}{3b^2\sqrt{a+bx^3}} + \frac{2Bx^3}{3b\sqrt{a+bx^3}} & \text{for}\ b \neq 0 \\ \frac{\frac{Ax^3}{3} + \frac{Bx^6}{6}}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Piecewise((-2*A/(3*b*sqrt(a + b*x**3)) + 4*B*a/(3*b**2*sqrt(a + b*x**3)) +
2*B*x**3/(3*b*sqrt(a + b*x**3)), Ne(b, 0)), ((A*x**3/3 + B*x**6/6)/a**(3/2)
, True))
```

3.231 $\int \frac{A+Bx^3}{x(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=58

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

```
[Out] -2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(3/2)+2/3*(A*b-B*a)/a/b/(b*x^3+a)
^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 78, 63, 208}

$$\frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B))/(3*a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt
[a]])/(3*a^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} + \frac{A\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^3\right)}{3a}$$
$$= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} + \frac{(2A)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^3}\right)}{3ab}$$
$$= \frac{2(Ab-aB)}{3ab\sqrt{a+bx^3}} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}}$$

Mathematica [**A**] time = 0.07, size = 58, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2(Ab-aB)}{ab\sqrt{a+bx^3}} - \frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(3/2)),x]
```

```
[Out] ((2*(A*b - a*B))/(a*b*Sqrt[a + b*x^3]) - (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[
a]])/a^(3/2))/3
```

fricas [**A**] time = 0.93, size = 170, normalized size = 2.93

$$\left[\frac{\left(Ab^2x^3+Aab\right)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)-2\,\sqrt{bx^3+a}\,\left(Ba^2-Aab\right)}{3\,\left(a^2b^2x^3+a^3b\right)}, \frac{2\left(\left(Ab^2x^3+Aab\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+\cdots}}{a}\right.\right.}{3\,\left(a^2b^2x^3+\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3*((A*b^2*x^3 + A*a*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) +
2*a)/x^3) - 2*sqrt(b*x^3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b), 2/3*(
(A*b^2*x^3 + A*a*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) - sqrt(b*x^
3 + a)*(B*a^2 - A*a*b))/(a^2*b^2*x^3 + a^3*b)]
```

giac [**A**] time = 0.16, size = 53, normalized size = 0.91

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a} - \frac{2\,(Ba-Ab)}{3\,\sqrt{bx^3+a}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a) - 2/3*(B*a - A*b)/(sqrt
(b*x^3 + a)*a*b)
```

maple [A] time = 0.05, size = 57, normalized size = 0.98

$$\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{3}+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^{3}+\frac{a}{b}\right)b}\;a}\right)A-\frac{2B}{3\sqrt{b\,x^{3}+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*B/b/(b*x^3+a)^(1/2)+A*(2/3/a/((x^3+a/b)*b)^(1/2)-2/3*arctanh((b*x^3+a)
^(1/2)/a^(1/2))/a^(3/2))
```

maxima [A] time = 1.35, size = 70, normalized size = 1.21

$$\frac{1}{3}\,A\left(\frac{\log\left(\frac{\sqrt{bx^{3}+a}-\sqrt{a}}{\sqrt{bx^{3}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^{3}+a}\,a}\right)-\frac{2\,B}{3\,\sqrt{bx^{3}+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*A*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(3/2)
 + 2/(sqrt(b*x^3 + a)*a)) - 2/3*B/(sqrt(b*x^3 + a)*b)
```

mupad [B] time = 2.77, size = 65, normalized size = 1.12

$$\frac{\frac{2\,A}{3\,a}-\frac{2\,B}{3\,b}}{\sqrt{b\,x^{3}+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^{3}+a}-\sqrt{a}\right)^{3}\,\left(\sqrt{b\,x^{3}+a}+\sqrt{a}\right)}{x^{6}}\right)}{3\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(3/2)),x)
```

```
[Out] ((2*A)/(3*a) - (2*B)/(3*b))/(a + b*x^3)^(1/2) + (A*log((((a + b*x^3)^(1/2)
- a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6))/(3*a^(3/2))
```

sympy [A] time = 20.82, size = 56, normalized size = 0.97

$$\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^{3}}}{\sqrt{-a}}\right)}{3a\sqrt{-a}}-\frac{2\left(-Ab+Ba\right)}{3ab\sqrt{a+bx^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a*sqrt(-a)) - 2*(-A*b + B*a)/(3*a*b*
sqrt(a + b*x**3))
```

3.232 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2aB-3Ab}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

```
[Out] 1/3*(3*A*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(5/2)+1/3*(-3*A*b+2*B*
a)/a^2/(b*x^3+a)^(1/2)-1/3*A/a/x^3/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}-\frac{A}{3ax^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(3*A*b - 2*a*B)/(3*a^2*Sqrt[a + b*x^3]) - A/(3*a*x^3*Sqrt[a + b*x^3]) + ((
3*A*b - 2*a*B)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{3/2}}\,dx,x,x^3\right)\\
&= -\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{\left(-\frac{3Ab}{2}+aB\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{3a}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}-\frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{6a^2}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}-\frac{(3Ab-2aB)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^2b}\\
&= -\frac{3Ab-2aB}{3a^2\sqrt{a+bx^3}}-\frac{A}{3ax^3\sqrt{a+bx^3}}+\frac{(3Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.66

$$\frac{x^3(2aB-3Ab)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)-aA}{3a^2x^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a*A) + (-3*A*b + 2*a*B)*x^3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^3)/
a])/(3*a^2*x^3*Sqrt[a + b*x^3])
```

fricas [A] time = 0.97, size = 233, normalized size = 2.71

$$\left[-\frac{\left(\left(2\,Bab-3\,Ab^2\right)x^6+\left(2\,Ba^2-3\,Aab\right)x^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)-2\,\left(\left(2\,Ba^2-3\,Aab\right)x^3-Aa^2\right)\sqrt{bx^3+a}}{6\,\left(a^3bx^6+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*B*a*b - 3*A*b^2)*x^6 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(a)*log((b*x
^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) - 2*((2*B*a^2 - 3*A*a*b)*x^3 - A
*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3), 1/3*(((2*B*a*b - 3*A*b^2)*x^6
 + (2*B*a^2 - 3*A*a*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (
(2*B*a^2 - 3*A*a*b)*x^3 - A*a^2)*sqrt(b*x^3 + a))/(a^3*b*x^6 + a^4*x^3)]
```

giac [A] time = 0.17, size = 99, normalized size = 1.15

$$\frac{(2\,Ba-3\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^2}+\frac{2\left(bx^3+a\right)Ba-2\,Ba^2-3\left(bx^3+a\right)Ab+2\,Aab}{3\left(\left(bx^3+a\right)^{\frac{3}{2}}-\sqrt{bx^3+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 3*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(
2*(b*x^3 + a)*B*a - 2*B*a^2 - 3*(b*x^3 + a)*A*b + 2*A*a*b)/(((b*x^3 + a)^(3
/2) - sqrt(b*x^3 + a)*a)*a^2)
```

maple [A] time = 0.06, size = 100, normalized size = 1.16

$$\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2b}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^2}-\frac{\sqrt{b\,x^3+a}}{3a^2x^3}\right)A+\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{3}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a}\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x)
```

```
[Out] A*(-1/3*(b*x^3+a)^(1/2)/a^2/x^3-2/3/((x^3+a/b)*b)^(1/2)/a^2*b+b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(5/2))+B*(2/3/((x^3+a/b)*b)^(1/2)/a-2/3*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(3/2))
```

maxima [B] time = 1.39, size = 144, normalized size = 1.67

$$-\frac{1}{6}A\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+a}\,a^3}+\frac{3\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}\right)+\frac{1}{3}B\left(\frac{\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx^3+a}\,a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*A*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a
)*a^3) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a
^(5/2)) + 1/3*B*(log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)
))/a^(3/2) + 2/(sqrt(b*x^3 + a)*a))
```

mupad [B] time = 2.93, size = 131, normalized size = 1.52

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(3\,A\,b-2\,B\,a)}{6\,a^{5/2}}-\frac{\frac{2\,B\,a^2-3\,A\,a\,b}{2\,a^3}-\frac{a\left(\frac{A\,b^2}{3\,a^3}+\frac{5\,b\left(2\,B\,a^2-3\,A\,a\,b\right)}{6\,a^4}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{A\,\sqrt{b\,x^3+a}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(3/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
3*A*b - 2*B*a))/(6*a^(5/2)) - ((2*B*a^2 - 3*A*a*b)/(2*a^3) - (a*((A*b^2)/(3
*a^3) + (5*b*(2*B*a^2 - 3*A*a*b))/(6*a^4)))/b)/(a + b*x^3)^(1/2) - (A*(a +
b*x^3)^(1/2))/(3*a^2*x^3)
```

sympy [B] time = 79.79, size = 264, normalized size = 3.07

$$A\left(-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{\sqrt{b}}{a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{a^{\frac{5}{2}}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^3}{a}}}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}+\frac{a^3\log\left(\frac{bx^3}{a}\right)}{3a^{\frac{9}{2}}+3a^{\frac{7}{2}}bx^3}-\frac{2a^3\log\left(\sqrt{\right.}}{3a^{\frac{9}{2}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(3*a*sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*
sqrt(a/(b*x**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2)) + B*(
2*a**3*sqrt(1 + b*x**3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**3*log(b*x**
3/a)/(3*a**(9/2) + 3*a**(7/2)*b*x**3) - 2*a**3*log(sqrt(1 + b*x**3/a) + 1)/
(3*a**(9/2) + 3*a**(7/2)*b*x**3) + a**2*b*x**3*log(b*x**3/a)/(3*a**(9/2) +
3*a**(7/2)*b*x**3) - 2*a**2*b*x**3*log(sqrt(1 + b*x**3/a) + 1)/(3*a**(9/2)
+ 3*a**(7/2)*b*x**3))
```

3.233 $\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{b(5Ab-4aB)}{4a^3\sqrt{a+bx^3}}+\frac{5Ab-4aB}{12a^2x^3\sqrt{a+bx^3}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

```
[Out] -1/4*b*(5*A*b-4*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/4*b*(5*A*b-
4*B*a)/a^3/(b*x^3+a)^(1/2)-1/6*A/a/x^6/(b*x^3+a)^(1/2)+1/12*(5*A*b-4*B*a)/a
^2/x^3/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{\sqrt{a+bx^3}\,(5Ab-4aB)}{4a^3x^3}-\frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}}-\frac{b(5Ab-4aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{A}{6ax^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(6*a*x^6*Sqrt[a + b*x^3]) - (5*A*b - 4*a*B)/(6*a^2*x^3*Sqrt[a + b*x^3])
+ ((5*A*b - 4*a*B)*Sqrt[a + b*x^3])/(4*a^3*x^3) - (b*(5*A*b - 4*a*B)*ArcTan
h[Sqrt[a + b*x^3]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^7\left(a+bx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{A+Bx}{x^3(a+bx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} + \frac{\left(-\frac{5Ab}{2}+2aB\right)\mathrm{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} - \frac{(5Ab-4aB)\,\mathrm{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx,x,x^3\right)}{4a^2}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} + \frac{(b(5Ab-4aB))\,\mathrm{Subst}\left(\int\right.}{8a^3}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} + \frac{(5Ab-4aB)\,\mathrm{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{4a^3}$$

$$= -\frac{A}{6ax^6\sqrt{a+bx^3}} - \frac{5Ab-4aB}{6a^2x^3\sqrt{a+bx^3}} + \frac{(5Ab-4aB)\sqrt{a+bx^3}}{4a^3x^3} - \frac{b(5Ab-4aB)\tanh^{-1}\left(\sqrt{a}\right.}{4a^{7/2}}$$

Mathematica [C] time = 0.02, size = 60, normalized size = 0.51

$$\frac{bx^6(5Ab-4aB)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx^3}{a}+1\right)-a^2A}{6a^3x^6\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-(a^2*A) + b*(5*A*b - 4*a*B)*x^6*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^
3)/a])/(6*a^3*x^6*Sqrt[a + b*x^3])
```

fricas [A] time = 0.70, size = 289, normalized size = 2.45

$$\left[-\frac{3\left(\left(4\,Bab^2-5\,Ab^3\right)x^9+\left(4\,Ba^2b-5\,Aab^2\right)x^6\right)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(3\left(4\,Ba^2b-5\,Aab^2\right)x^6+2\,Aa^3\right.}{24\left(a^4bx^9+a^5x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^2)*x^6)*sqrt(a)
*log((b*x^3 - 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*(4*B*a^2*b - 5*A
*a*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*
x^9 + a^5*x^6), -1/12*(3*((4*B*a*b^2 - 5*A*b^3)*x^9 + (4*B*a^2*b - 5*A*a*b^
```

```
2)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(4*B*a^2*b - 5*A*a
*b^2)*x^6 + 2*A*a^3 + (4*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b*x^
9 + a^5*x^6)]
```

giac [A] time = 0.18, size = 137, normalized size = 1.16

$$-\frac{\left(4\,Bab-5\,Ab^2\right)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3}-\frac{2\left(Bab-Ab^2\right)}{3\,\sqrt{bx^3+a}\,a^3}-\frac{4\left(bx^3+a\right)^{\frac{3}{2}}Bab-4\,\sqrt{bx^3+a}\,Ba^2b-7\left(bx^3+a\right)^{\frac{3}{2}}Ab^2+ 9}{12\,a^3b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*B*a*b - 5*A*b^2)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) -
2/3*(B*a*b - A*b^2)/(sqrt(b*x^3 + a)*a^3) - 1/12*(4*(b*x^3 + a)^(3/2)*B*a*b
 - 4*sqrt(b*x^3 + a)*B*a^2*b - 7*(b*x^3 + a)^(3/2)*A*b^2 + 9*sqrt(b*x^3 + a
)*A*a*b^2)/(a^3*b^2*x^6)
```

maple [A] time = 0.05, size = 141, normalized size = 1.19

$$\left(-\frac{5b^2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}+\frac{2b^2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^3}+\frac{7\sqrt{b\,x^3+a}\;b}{12a^3x^3}-\frac{\sqrt{b\,x^3+a}}{6a^2x^6}\right)A+\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{}{3\sqrt{\left(x^3\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-1/3*(b*x^3+a)^(1/2)/a^2/x^3-2/3/((x^3+a/b)*b)^(1/2)/a^2*b+b*arctanh((b*
x^3+a)^(1/2)/a^(1/2))/a^(5/2))+A*(-1/6*(b*x^3+a)^(1/2)/a^2/x^6+7/12*(b*x^3+
a)^(1/2)/a^3*b/x^3+2/3/((x^3+a/b)*b)^(1/2)/a^3*b^2-5/4*b^2*arctanh((b*x^3+a
)^(1/2)/a^(1/2))/a^(7/2))
```

maxima [B] time = 1.34, size = 215, normalized size = 1.82

$$\frac{1}{24}A\left(\frac{2\left(15\left(bx^3+a\right)^2b^2-25\left(bx^3+a\right)ab^2+8\,a^2b^2\right)}{\left(bx^3+a\right)^{\frac{5}{2}}a^3-2\left(bx^3+a\right)^{\frac{3}{2}}a^4+\sqrt{bx^3+a}\,a^5}+\frac{15\,b^2\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)-\frac{1}{6}B\left(\frac{2\left(3\left(bx^3+a\right)b-2\,ab\right.}{\left(bx^3+a\right)^{\frac{3}{2}}a^2-\sqrt{bx^3+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^7/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*A*(2*(15*(b*x^3 + a)^2*b^2 - 25*(b*x^3 + a)*a*b^2 + 8*a^2*b^2)/((b*x^3
 + a)^(5/2)*a^3 - 2*(b*x^3 + a)^(3/2)*a^4 + sqrt(b*x^3 + a)*a^5) + 15*b^2*l
og((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) - 1/6*
B*(2*(3*(b*x^3 + a)*b - 2*a*b)/((b*x^3 + a)^(3/2)*a^2 - sqrt(b*x^3 + a)*a^3
) + 3*b*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2
))
```

mupad [B] time = 3.18, size = 167, normalized size = 1.42

$$\frac{b\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)(5\,A\,b-4\,B\,a)}{8\,a^{7/2}}-\frac{\left(4\,B\,a^2-7\,A\,a\,b\right)\sqrt{b\,x^3+a}}{12\,a^4\,x^3}-\frac{A\,\sqrt{b\,x^3+a}}{6\,a^2\,x^6}-\frac{a\left(\frac{7\,A\,b^3-4\,B\,a\,b^2}{12\,a^4}-\frac{5\,b}{}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^7*(a + b*x^3)^(3/2)),x)
```

```
[Out] (b*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)))/x^6)
*(5*A*b - 4*B*a))/(8*a^(7/2)) - ((4*B*a^2 - 7*A*a*b)*(a + b*x^3)^(1/2))/(12
*a^4*x^3) - (A*(a + b*x^3)^(1/2))/(6*a^2*x^6) - ((a*((7*A*b^3 - 4*B*a*b^2)/
(12*a^4) - (5*b^2*(5*A*b - 4*B*a))/(8*a^4)))/b + (3*b*(5*A*b - 4*B*a))/(8*a
^3))/(a + b*x^3)^(1/2)
```

sympy [A] time = 179.01, size = 192, normalized size = 1.63

$$A\left(-\frac{1}{6a\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5\sqrt{b}}{12a^2x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}+\frac{5b^{\frac{3}{2}}}{4a^3x^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}-\frac{5b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{\frac{3}{2}}}\right)}{4a^{\frac{7}{2}}}\right)+B\left(-\frac{1}{3a\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx^3}+1}}-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**7/(b*x**3+a)**(3/2),x)
```

```
[Out] A*(-1/(6*a*sqrt(b)*x**(15/2)*sqrt(a/(b*x**3) + 1)) + 5*sqrt(b)/(12*a**2*x**
(9/2)*sqrt(a/(b*x**3) + 1)) + 5*b**(3/2)/(4*a**3*x**(3/2)*sqrt(a/(b*x**3) +
 1)) - 5*b**2*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/(4*a**(7/2))) + B*(-1/(3*a*
sqrt(b)*x**(9/2)*sqrt(a/(b*x**3) + 1)) - sqrt(b)/(a**2*x**(3/2)*sqrt(a/(b*x
**3) + 1)) + b*asinh(sqrt(a)/(sqrt(b)*x**(3/2)))/a**(5/2))
```

3.234 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{165\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}+\frac{16x\sqrt{\ }}{}$$

```
[Out] -2/33*(11*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(1/2)+2/11*B*x^7/b/(b*x^3+a)^(1/2)+
16/165*(11*A*b-14*B*a)*x*(b*x^3+a)^(1/2)/b^3-32/495*a*(11*A*b-14*B*a)*(a^(1
/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10
/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 288, 321, 218}

$$-\frac{32\sqrt{2+\sqrt{3}}\,a\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(11Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{165\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{2x^4(1}{33b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(11*A*b - 14*a*B)*x^4)/(33*b^2*Sqrt[a + b*x^3]) + (2*B*x^7)/(11*b*Sqrt[
a + b*x^3]) + (16*(11*A*b - 14*a*B)*x*Sqrt[a + b*x^3])/(165*b^3) - (32*Sqrt
[2 + Sqrt[3]]*a*(11*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)], -7 - 4*Sqrt[3]])/(165*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b
^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2Bx^7}{11b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{11Ab}{2}+7aB\right)\right)\int \frac{x^6}{\left(a+bx^3\right)^{3/2}}\,dx}{11b}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{(8(11Ab-14aB))\int \frac{x^3}{\sqrt{a+bx^3}}\,dx}{33b^2}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{(16a(11Ab-14aB))}{165b^3}$$

$$= -\frac{2(11Ab-14aB)x^4}{33b^2\sqrt{a+bx^3}} + \frac{2Bx^7}{11b\sqrt{a+bx^3}} + \frac{16(11Ab-14aB)x\sqrt{a+bx^3}}{165b^3} - \frac{32\sqrt{2+\sqrt{3}}\,a(11Ab}{}$$

Mathematica [C] time = 0.15, size = 103, normalized size = 0.34

$$\frac{2x\left(-112a^2B+8a\sqrt{\frac{bx^3}{a}+1}\,(14aB-11Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a\left(88Ab-42bBx^3\right)+3b^2x^3\left(11A+5Bx^3\right)\right)}{165b^3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-112*a^2*B + 3*b^2*x^3*(11*A + 5*B*x^3) + a*(88*A*b - 42*b*B*x^3) + 8
*a*(-11*A*b + 14*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(165*b^3*Sqrt[a + b*x^3])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^9+Ax^6\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^9 + A*x^6)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)x^6}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.08, size = 666, normalized size = 2.22

$$\left(\frac{2ax}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\, b^2} + \frac{32 i\sqrt{3}\, \left(-a\,b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\, \sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \right.}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{45\sqrt{b\,x^3+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/b^3*a^2*x/((x^3+a/b)*b)^(1/2)+2/11/b^2*x^4*(b*x^3+a)^(1/2)-38/55*a/
b^3*x*(b*x^3+a)^(1/2)-448/495*I*a^2/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/3/((x^3+a/b)*b)^(1/2)*a/b^2*x+2/5*(b*x^3+a)^
(1/2)/b^2*x+32/45*I*a/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^
2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(
-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)
^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^
(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)x^6}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6 \left(B x^3 + A\right)}{\left(b x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 29.49, size = 80, normalized size = 0.27

$$\frac{Ax^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)} + \frac{Bx^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{10}{3} \\ \frac{13}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a
**(3/2)*gamma(10/3)) + B*x**10*gamma(10/3)*hyper((3/2, 10/3), (13/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(13/3))
```

3.235 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=269

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(5Ab-8}{15b^2\sqrt{a+b}}$$

```
[Out] -2/15*(5*A*b-8*B*a)*x/b^2/(b*x^3+a)^(1/2)+2/5*B*x^4/b/(b*x^3+a)^(1/2)+4/45*
(5*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {459, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(5Ab-8}{15b^2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(5*A*b - 8*a*B)*x)/(15*b^2*Sqrt[a + b*x^3]) + (2*B*x^4)/(5*b*Sqrt[a + b
*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(5*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a
^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*b^(7/3)*Sqrt[(a^(1/3)*(a
^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]
)
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2Bx^4}{5b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{5Ab}{2}+4aB\right)\right)\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{(2(5Ab-8aB))\int \frac{1}{\sqrt{a+bx^3}}\,dx}{15b^2}$$

$$= -\frac{2(5Ab-8aB)x}{15b^2\sqrt{a+bx^3}} + \frac{2Bx^4}{5b\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\,(5Ab-8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}}}{15\sqrt[4]{3}\,b^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}}}$$

Mathematica [C] time = 0.10, size = 78, normalized size = 0.29

$$\frac{2x\left(\sqrt{\frac{bx^3}{a}+1}\,(5Ab-8aB)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+8aB-5Ab+3bBx^3\right)}{15b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(-5*A*b + 8*a*B + 3*b*B*x^3 + (5*A*b - 8*a*B)*Sqrt[1 + (b*x^3)/a]*Hype
rgeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(15*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^6+Ax^3\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^3}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.05, size = 627, normalized size = 2.33

$$-\frac{2x}{3\sqrt{\left(x^{3}+\frac{a}{b}\right)b}\,b}-\frac{4i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b\right.}{2b}\cdots}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\cdots}{9\sqrt{b\,x^{3}+a}\;b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/((x^3+a/b)*b)^(1/2)*a/b^2*x+2/5*(b*x^3+a)^(1/2)/b^2*x+32/45*I*a/b^3*
3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-2/3/((x^3+
a/b)*b)^(1/2)/b*x-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)
/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*
b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)
*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^
(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*
3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)x^{3}}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}\,\left(B\,x^{3}+A\right)}{\left(b\,x^{3}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 14.09, size = 80, normalized size = 0.30

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((3/2, 7/3), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(10/3))
```

3.236 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=251

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB}{3ab\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*x/a/b/(b*x^3+a)^(1/2)+2/9*(A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*Ell
ipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/b^(4/3)/(b*x^3+a)^(1/
2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(Ab-aB}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(3*a*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(A*b + 2*a
*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1
/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*
3^(1/4)*a*b^(4/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{\left(2\left(\frac{Ab}{2}+aB\right)\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)x}{3ab\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\,(Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\ldots}{\left(1+\sqrt{3}\right)\sqrt[3]{a}\ldots}\right.\right.}{3\sqrt[4]{3}\,ab^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.29

$$\frac{x\left(\sqrt{\frac{bx^3}{a}+1}\,(2aB+Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-2aB+2Ab\right)}{3ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(2*A*b - 2*a*B + (A*b + 2*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3
, 1/2, 4/3, -((b*x^3)/a)]))/(3*a*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.06, size = 613, normalized size = 2.44

$$\left\{\frac{2x}{3\sqrt{\left(x^{3}+\frac{a}{b}\right)b}\,a}-\frac{2i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)}{2b}\right.}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}}{9\sqrt{b\,x^{3}+a}\;ab}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/((x^3+a/b)*b)^(1/2)/b*x-4/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*
(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/3/((x^3+a/b)*b)^(1/2)/a*x-2/9*I/a*3^(1/2)*
(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*
EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^{3}+A}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^{3}+A}{\left(b\,x^{3}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 10.05, size = 78, normalized size = 0.31

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(3/2)*gamma(7/3))
```

3.237 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=272

$$-\frac{x(7Ab-4aB)}{6a^2\sqrt{a+bx^3}}-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{}\right.}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A/a/x^2/(b*x^3+a)^(1/2)-1/6*(7*A*b-4*B*a)*x/a^2/(b*x^3+a)^(1/2)-1/18*(
7*A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^2/b^(1/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 199, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(7Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{6\sqrt[4]{3}a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{x(7Ab-4a}{6a^2\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(2*a*x^2*Sqrt[a + b*x^3]) - ((7*A*b - 4*a*B)*x)/(6*a^2*Sqrt[a + b*x^3])
- (Sqrt[2 + Sqrt[3]]*(7*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ell
ipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)], -7 - 4*Sqrt[3]])/(6*3^(1/4)*a^2*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3)
 + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{\left(\frac{7Ab}{2}-2aB\right)\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{12a^2}$$

$$= -\frac{A}{2ax^2\sqrt{a+bx^3}} - \frac{(7Ab-4aB)x}{6a^2\sqrt{a+bx^3}} - \frac{\sqrt{2+\sqrt{3}}\,(7Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/\cdots}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}}}{6\sqrt[4]{3}\,a^2\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\cdots\right.}}}$$

Mathematica [C] time = 0.06, size = 86, normalized size = 0.32

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(4aB-7Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-6aA+8aBx^3-14Abx^3}{12a^2x^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-6*a*A - 14*A*b*x^3 + 8*a*B*x^3 + (-7*A*b + 4*a*B)*x^3*Sqrt[1 + (b*x^3)/a]
*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/(12*a^2*x^2*Sqrt[a + b*x^3
])
```

fricas [F] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^9+2\,abx^6+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^9 + 2*a*b*x^6 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="giac")
```

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)

maple [B] time = 0.06, size = 631, normalized size = 2.32

$$\left(-\frac{2bx}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^2}+\frac{7i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\right.}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{18\sqrt{b\,x^3+a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x)

[Out] B*(2/3/((x^3+a/b)*b)^(1/2)/a*x-2/9*I/a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(-2/3/((x^3+a/b)*b)^(1/2)/a^2*b*x-1/2*(b*x^3+a)^(1/2)/a^2/x^2+7/18*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(3/2),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 26.01, size = 82, normalized size = 0.30

$$\frac{A\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}+\frac{Bx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x**2*gamma(1/3)) + B*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(4/3))
```

3.238 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=304

$$\frac{7\sqrt{a+bx^3}\,(13Ab-10aB)}{60a^3x^2}-\frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}}+\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(13Ab-10aB)}{60\sqrt[4]{3}\,a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\sqrt{a}$$

```
[Out] -1/5*A/a/x^5/(b*x^3+a)^(1/2)+1/15*(-13*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(1/2)+
7/60*(13*A*b-10*B*a)*(b*x^3+a)^(1/2)/a^3/x^2+7/180*b^(2/3)*(13*A*b-10*B*a)*
(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^3/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 290, 325, 218}

$$\frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(13Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{60\sqrt[4]{3}\,a^3\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,\sqrt{a+bx^3}}+\frac{7\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(5*a*x^5*Sqrt[a + b*x^3]) - (13*A*b - 10*a*B)/(15*a^2*x^2*Sqrt[a + b*x^3
]) + (7*(13*A*b - 10*a*B)*Sqrt[a + b*x^3])/(60*a^3*x^2) + (7*Sqrt[2 + Sqrt[
3]]*b^(2/3)*(13*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)
*x)], -7 - 4*Sqrt[3]])/(60*3^(1/4)*a^3*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{\left(\frac{13Ab}{2}-5aB\right)\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} - \frac{(7(13Ab-10aB))\int \frac{1}{x^3\sqrt{a+bx^3}}\,dx}{30a^2}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{(7b(13Ab-10aB))\int}{120a^3}$$

$$= -\frac{A}{5ax^5\sqrt{a+bx^3}} - \frac{13Ab-10aB}{15a^2x^2\sqrt{a+bx^3}} + \frac{7(13Ab-10aB)\sqrt{a+bx^3}}{60a^3x^2} + \frac{7\sqrt{2+\sqrt{3}}\,b^{2/3}(13Ab}{}$$

Mathematica [C] time = 0.06, size = 72, normalized size = 0.24

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(13Ab-10aB)\,{}_2F_1\left(-\frac{2}{3},\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-4aA}{20a^2x^5\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (13*A*b - 10*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3,
 3/2, 1/3, -((b*x^3)/a)])/(20*a^2*x^5*Sqrt[a + b*x^3])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^{12}+2\,abx^9+a^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^12 + 2*a*b*x^9 + a^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

maple [B] time = 0.05, size = 667, normalized size = 2.19

$$\left(\frac{2b^2 x}{3\sqrt{\left(x^3+\frac{a}{b}\right) b}\; a^3} - \frac{91 i\sqrt{3}\, \left(-a\, b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\, b}{\left(-a\, b^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{x-\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}}}\, \sqrt{-\frac{i\left(x+\frac{\left(-a\, b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a \cdots\right.}{2b}\right.}{\left(-a\, b^2\right)^{\frac{1}{3}}}} \cdots}{180\sqrt{b\, x^3+a} \cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x)
```

```
[Out] A*(-1/5*(b*x^3+a)^(1/2)/a^2/x^5+17/20*(b*x^3+a)^(1/2)/a^3*b/x^2+2/3*b^2/a^3
*x/((x^3+a/b)*b)^(1/2)-91/180*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*
b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1
/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)/b)^(1/2)))+B*(-2/3/((x^3+a/b)*b)^(1/2)/a^2*b*x-1/2*(b*x^3+a)^(
1/2)/a^2/x^2+7/18*I/a^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)
^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I
*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(
1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1
/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b
)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^6\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 79.85, size = 90, normalized size = 0.30

$$\frac{A\Gamma\left(-\frac{5}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{5}{3},\frac{3}{2}\\-\frac{2}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^5\Gamma\left(-\frac{2}{3}\right)}+\frac{B\Gamma\left(-\frac{2}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{1}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^2\Gamma\left(\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-5/3)*hyper((-5/3, 3/2), (-2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**5*gamma(-2/3)) + B*gamma(-2/3)*hyper((-2/3, 3/2), (1/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x**2*gamma(1/3))
```

3.239 $\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

<u>**Optimal**</u>. Leaf size=547

$$\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{}}{}$$

```
[Out] -2/21*(7*A*b-10*B*a)*x^2/b^2/(b*x^3+a)^(1/2)+2/7*B*x^5/b/(b*x^3+a)^(1/2)+8/
21*(7*A*b-10*B*a)*(b*x^3+a)^(1/2)/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+8
/63*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)-4/21*a^(1/3)*(7*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)
*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(8/3)/(b*x^3+a)^(
1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

<u>**Rubi**</u> **[A]** time = 0.25, antiderivative size = 547, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 303, 218, 1877}

$$\frac{8\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(7Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{21\sqrt[4]{3}\,b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (-2*(7*A*b - 10*a*B)*x^2)/(21*b^2*Sqrt[a + b*x^3]) + (2*B*x^5)/(7*b*Sqrt[a
+ b*x^3]) + (8*(7*A*b - 10*a*B)*Sqrt[a + b*x^3])/(21*b^(8/3)*((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)) - (4*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 10*a*B)*(a^(
1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^
(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*
b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)^2]*Sqrt[a + b*x^3]) + (8*Sqrt[2]*a^(1/3)*(7*A*b - 10*a*B)*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x
)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*b^(8/3
)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

<u>Rule 218</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2Bx^5}{7b\sqrt{a+bx^3}} - \frac{\left(2\left(-\frac{7Ab}{2}+5aB\right)\right)\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{21b^2}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{(4(7Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{21b^{7/3}} + \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right.}\right.\cdots}{\cdots}$$

$$= -\frac{2(7Ab-10aB)x^2}{21b^2\sqrt{a+bx^3}} + \frac{2Bx^5}{7b\sqrt{a+bx^3}} + \frac{8(7Ab-10aB)\sqrt{a+bx^3}}{21b^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{4\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,(7Ab\cdots}{\cdots}$$

Mathematica [C] time = 0.08, size = 79, normalized size = 0.14

$$\frac{2x^2\left(\sqrt{\frac{bx^3}{a}+1}\,(10aB-7Ab)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-10aB+7Ab+bBx^3\right)}{7b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x]
[Out] (2*x^2*(7*A*b - 10*a*B + b*B*x^3 + (-7*A*b + 10*a*B)*Sqrt[1 + (b*x^3)/a]*Hy
pergeometric2F1[2/3, 3/2, 5/3, -((b*x^3)/a)]))/(7*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^7+Ax^4\right)\sqrt{bx^3+a}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)x^4}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.06, size = 937, normalized size = 1.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x)
[Out] B*(2/3/b^2*a*x^2/((x^3+a/b)*b)^(1/2)+2/7*(b*x^3+a)^(1/2)/b^2*x^2+80/63*I*a/
b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3
+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE
(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(
1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3
*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-2/3/b*x^2/((x^3+a/b)*b)^(1/2
)-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*E
```

```
llipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)x^4}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(B\,x^3 + A)}{(b\,x^3 + a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 13.66, size = 80, normalized size = 0.15

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{8}{3}\\\frac{11}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((3/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*gamma(11/3))
```

3.240 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=524

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] 2/3*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(1/2)-2/3*(A*b-4*B*a)*(b*x^3+a)^(1/2)/a/b^(
5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-2/9*(A*b-4*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/3*(A*b-4*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/
4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 524, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {457, 303, 218, 1877}

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(3*a*b*Sqrt[a + b*x^3]) - (2*(A*b - 4*a*B)*Sqrt[a + b*x
^3])/(3*a*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]
*(A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(A*b - 4*a*
B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3
^(1/4)*a^(2/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*
a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx &= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}+\frac{\left(2\left(-\frac{Ab}{2}+2aB\right)\right)\int\frac{x}{\sqrt{a+bx^3}}\,dx}{3ab}\\
&= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}-\frac{(Ab-4aB)\int\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{3ab^{4/3}}-\frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,(Ab-4aB)\right)\int\frac{1}{\sqrt{a+bx^3}}\,dx}{3a^{2/3}b^{4/3}}\\
&= \frac{2(Ab-aB)x^2}{3ab\sqrt{a+bx^3}}-\frac{2(Ab-4aB)\sqrt{a+bx^3}}{3ab^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}+\frac{\sqrt{2-\sqrt{3}}\,(Ab-4aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left((1+\cdots\right.}}\cdots}{3^{3/4}a^{2/3}b^{5/3}\sqrt{\frac{\cdots}{(\cdots}}\cdots}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 71, normalized size = 0.14

$$\frac{x^2\left(\sqrt{\frac{bx^3}{a}+1}\,(Ab-4aB)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+4aB\right)}{2ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x^2*(4*a*B + (A*b - 4*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3/2,
 5/3, -((b*x^3)/a)]))/(2*a*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^{4}+Ax\right)\sqrt{bx^{3}+a}}{b^{2}x^{6}+2\,abx^{3}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)x}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

maple [B] time = 0.05, size = 921, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(-2/3/((x^3+a/b)*b)^(1/2)/b*x^2-8/9*I/b^2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1
/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/
3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(
1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)
*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(
1/2))))+A*(2/3/((x^3+a/b)*b)^(1/2)/a*x^2+2/9*I/a*3^(1/2)*(-a*b^2)^(1/3)/b*(
I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^
(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*
3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^{3}+A\right)x}{\left(bx^{3}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(B x^3 + A\right)}{\left(b x^3 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(3/2),x)
```

```
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 9.53, size = 80, normalized size = 0.15

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{3}{2}\\\frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}+\frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{5}{3}\\\frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(3/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((3/2, 5/3), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(3/2)*gamma(8/3))
```

3.241 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=548

$$\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] -A/a/x/(b*x^3+a)^(1/2)-1/3*(5*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(1/2)+1/3*(5*A*b
-2*B*a)*(b*x^3+a)^(1/2)/a^2/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+1/9*(5*
A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b
^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1
/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)
/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1
+3^(1/2)))^2)^(1/2)-1/6*(5*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 548, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 290, 303, 218, 1877}

$$\frac{\sqrt{a+bx^3}(5Ab-2aB)}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(5Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\cdots\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] -(A/(a*x*Sqrt[a + b*x^3])) - ((5*A*b - 2*a*B)*x^2)/(3*a^2*Sqrt[a + b*x^3])
+ ((5*A*b - 2*a*B)*Sqrt[a + b*x^3])/(3*a^2*b^(2/3)*((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)) - (Sqrt[2 - Sqrt[3]]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(3/4)*a^(5/3)*b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (Sqrt[2]*(5*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*a^(5/3)*b^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3
])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{3/2}}\,dx &= -\frac{A}{ax\sqrt{a+bx^3}}-\frac{\left(\frac{5Ab}{2}-aB\right)\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{a}\\
&= -\frac{A}{ax\sqrt{a+bx^3}}-\frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{6a^2}\\
&= -\frac{A}{ax\sqrt{a+bx^3}}-\frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{6a^2\sqrt[3]{b}}+\frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}(5A\cdots\right.}{3a\cdots}\\
&= -\frac{A}{ax\sqrt{a+bx^3}}-\frac{(5Ab-2aB)x^2}{3a^2\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\sqrt{a+bx^3}}{3a^2b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}-\frac{\sqrt{2-\sqrt{3}}\,(5Ab-2a\cdots}{\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.13

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(2aB-5Ab)\,{}_2F_1\left(\frac{2}{3},\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4aA}{4a^2x\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-4*a*A + (-5*A*b + 2*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 3
/2, 5/3, -((b*x^3)/a)])/(4*a^2*x*Sqrt[a + b*x^3])
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^8+2\,abx^5+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^8 + 2*a*b*x^5 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

maple [B] time = 0.05, size = 939, normalized size = 1.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/((x^3+a/b)*b)^(1/2)/a*x^2+2/9*I/a*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2
*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)
^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1
/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b
)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*
(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1
/2))))+A*(-2/3/((x^3+a/b)*b)^(1/2)/a^2*b*x^2-(b*x^3+a)^(1/2)/a^2/x-5/9*I/a^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a
)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1
/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
```

```
2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3
^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^2\,\left(b\,x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 20.35, size = 82, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{1}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{2}{3} \end{matrix}\middle| {\frac{bx^3 e^{i\pi}}{a}} \right)}{3a^{\frac{3}{2}} x\Gamma\left(\frac{2}{3}\right)} + \frac{Bx^2\Gamma\left(\frac{2}{3}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{3} \end{matrix}\middle| {\frac{bx^3 e^{i\pi}}{a}} \right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*x*gamma(2/3)) + B*x**2*gamma(2/3)*hyper((2/3, 3/2), (5/3,), b*x**3*exp_
polar(I*pi)/a)/(3*a**(3/2)*gamma(5/3))
```

3.242 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}}{}$$

[Out] -1/4*A/a/x^4/(b*x^3+a)^(1/2)+1/12*(-11*A*b+8*B*a)/a^2/x/(b*x^3+a)^(1/2)+5/24*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/x-5/24*b^(1/3)*(11*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^3/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))-5/72*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/48*b^(1/3)*(11*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.30, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$-\frac{5\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{12\sqrt{2}\sqrt[4]{3}a^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{5\sqrt{2-\sqrt{3}}\sqrt[3]{b}}{}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]

[Out] -A/(4*a*x^4*Sqrt[a + b*x^3]) - (11*A*b - 8*a*B)/(12*a^2*x*Sqrt[a + b*x^3]) + (5*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*x) - (5*b^(1/3)*(11*A*b - 8*a*B)*Sqrt[a + b*x^3])/(24*a^3*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (5*Sqrt[2 - Sqrt[3]]*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (5*b^(1/3)*(11*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*a^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{\left(\frac{11Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} - \frac{(5(11Ab-8aB))\int \frac{1}{x^2\sqrt{a+bx^3}}\,dx}{24a^2}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{(5b(11Ab-8aB))\int}{48a^3}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{\left(5b^{2/3}(11Ab-8aB)\right)}{48}$$

$$= -\frac{A}{4ax^4\sqrt{a+bx^3}} - \frac{11Ab-8aB}{12a^2x\sqrt{a+bx^3}} + \frac{5(11Ab-8aB)\sqrt{a+bx^3}}{24a^3x} - \frac{5\sqrt[3]{b}\,(11Ab-8aB)\sqrt{}}{24a^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\right.}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.12

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(11Ab-8aB)\,{}_2F_1\left(-\frac{1}{3},\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-2aA}{8a^2x^4\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x]
[Out] (-2*a*A + (11*A*b - 8*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3,
3/2, 2/3, -((b*x^3)/a)])/(8*a^2*x^4*Sqrt[a + b*x^3])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^{11}+2\,abx^8+a^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^11 + 2*a*b*x^8 + a^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

maple [B] time = 0.05, size = 975, normalized size = 1.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x)
```

```
[Out] A*(2/3/((x^3+a/b)*b)^(1/2)/a^3*b^2*x^2-1/4*(b*x^3+a)^(1/2)/a^2/x^4+13/8*(b*
x^3+a)^(1/2)/a^3*b/x+55/72*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*(
(x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
)^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*
b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)
^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+B
*(-2/3/((x^3+a/b)*b)^(1/2)/a^2*b*x^2-(b*x^3+a)^(1/2)/a^2/x-5/9*I/a^2*3^(1/2
)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*
((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b
^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^5\,{\left(b\,x^3 + a\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(3/2)), x)
```

sympy [A] time = 58.88, size = 88, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}} x^4 \Gamma\left(-\frac{1}{3}\right)} + \frac{B\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}} x \Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**4*gamma(-1/3)) + B*gamma(-1/3)*hyper((-1/3, 3/2), (2/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(3/2)*x*gamma(2/3))
```

3.243 $\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=611

$$\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{55\sqrt{2-\sqrt{3}}\,b^{4/3}\left(\cdots\right.}{\cdots}$$

```
[Out] -1/7*A/a/x^7/(b*x^3+a)^(1/2)+1/21*(-17*A*b+14*B*a)/a^2/x^4/(b*x^3+a)^(1/2)+
11/168*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^3/x^4-55/336*b*(17*A*b-14*B*a)*(b*
x^3+a)^(1/2)/a^4/x+55/336*b^(4/3)*(17*A*b-14*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1
/3)*x+a^(1/3)*(1+3^(1/2)))+55/1008*b^(4/3)*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*
(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-55/672*b^(4/3)
*(17*A*b-14*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(1/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 611, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$\frac{55b^{4/3}\sqrt{a+bx^3}(17Ab-14aB)}{336a^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{55b^{4/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(17Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\cdots\right.}{168\sqrt{2}\sqrt[4]{3}a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
```

```
[Out] -A/(7*a*x^7*Sqrt[a + b*x^3]) - (17*A*b - 14*a*B)/(21*a^2*x^4*Sqrt[a + b*x^3
]) + (11*(17*A*b - 14*a*B)*Sqrt[a + b*x^3])/(168*a^3*x^4) - (55*b*(17*A*b -
 14*a*B)*Sqrt[a + b*x^3])/(336*a^4*x) + (55*b^(4/3)*(17*A*b - 14*a*B)*Sqrt[
a + b*x^3])/(336*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (55*Sqrt[2 - Sq
rt[3]]*b^(4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1
/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipti
cE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)], -7 - 4*Sqrt[3]])/(224*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^
(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (55*b^(
4/3)*(17*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(168*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1
/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^8\left(a+bx^3\right)^{3/2}}\,dx = -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{\left(\frac{17Ab}{2}-7aB\right)\int \frac{1}{x^5\left(a+bx^3\right)^{3/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} - \frac{(11(17Ab-14aB))\int \frac{1}{x^5\sqrt{a+bx^3}}\,dx}{42a^2}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} + \frac{(55b(17Ab-14aB))}{336a^3}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{}}{336a^4x}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{}}{336a^4x}$$

$$= -\frac{A}{7ax^7\sqrt{a+bx^3}} - \frac{17Ab-14aB}{21a^2x^4\sqrt{a+bx^3}} + \frac{11(17Ab-14aB)\sqrt{a+bx^3}}{168a^3x^4} - \frac{55b(17Ab-14aB)\sqrt{}}{336a^4x}$$

Mathematica [C] time = 0.04, size = 72, normalized size = 0.12

$$\frac{x^3\sqrt{\frac{bx^3}{a}+1}\,(17Ab-14aB)\,{}_2F_1\left(-\frac{4}{3},\frac{3}{2};-\frac{1}{3};-\frac{bx^3}{a}\right)-8aA}{56a^2x^7\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x]
[Out] (-8*a*A + (17*A*b - 14*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-4/3,
 3/2, -1/3, -((b*x^3)/a)])/(56*a^2*x^7*Sqrt[a + b*x^3])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^2x^{14}+2\,abx^{11}+a^2x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="fricas")
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^2*x^14 + 2*a*b*x^11 + a^2*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

maple [B] time = 0.10, size = 1018, normalized size = 1.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x)
```

```
[Out] B*(2/3/((x^3+a/b)*b)^(1/2)/a^3*b^2*x^2-1/4*(b*x^3+a)^(1/2)/a^2/x^4+13/8*(b*
x^3+a)^(1/2)/a^3*b/x+55/72*I/a^3*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2
)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*(
(x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
)^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)
/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*
b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))
+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)
^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A
*(-1/7/a^2*(b*x^3+a)^(1/2)/x^7+25/56*b/a^3*(b*x^3+a)^(1/2)/x^4-237/112/a^4*
b^2*(b*x^3+a)^(1/2)/x-2/3*b^3/a^4*x^2/((x^3+a/b)*b)^(1/2)-935/1008*I/a^4*b^
2*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a
)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1
/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/
2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3
^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(
-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} x^8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^8/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^8\,\left(b\,x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/(x^8*(a + b*x^3)^(3/2)), x)
```

sympy [**A**] time = 136.39, size = 94, normalized size = 0.15

$$\frac{A\Gamma\left(-\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{7}{3},\frac{3}{2}\\-\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^7\Gamma\left(-\frac{4}{3}\right)}+\frac{B\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{3}{2}\\-\frac{1}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}x^4\Gamma\left(-\frac{1}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**8/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-7/3)*hyper((-7/3, 3/2), (-4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(
3/2)*x**7*gamma(-4/3)) + B*gamma(-4/3)*hyper((-4/3, 3/2), (-1/3,), b*x**3*e
xp_polar(I*pi)/a)/(3*a**(3/2)*x**4*gamma(-1/3))
```

3.244 $\int \frac{x^8(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=103

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

```
[Out] -2/9*a^2*(A*b-B*a)/b^4/(b*x^3+a)^(3/2)+2/9*B*(b*x^3+a)^(3/2)/b^4+2/3*a*(2*A
*b-3*B*a)/b^4/(b*x^3+a)^(1/2)+2/3*(A*b-3*B*a)*(b*x^3+a)^(1/2)/b^4
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}\,(Ab-3aB)}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*(A*b - a*B))/(9*b^4*(a + b*x^3)^(3/2)) + (2*a*(2*A*b - 3*a*B))/(3*b
^4*Sqrt[a + b*x^3]) + (2*(A*b - 3*a*B)*Sqrt[a + b*x^3])/(3*b^4) + (2*B*(a +
 b*x^3)^(3/2))/(9*b^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(-Ab+aB)}{b^3(a+bx)^{5/2}}+\frac{a(-2Ab+3aB)}{b^3(a+bx)^{3/2}}+\frac{Ab-3aB}{b^3\sqrt{a+bx}}+\frac{B\sqrt{a+bx}}{b^3}\right)dx,x,x^3\right)\\ &= -\frac{2a^2(Ab-aB)}{9b^4\left(a+bx^3\right)^{3/2}}+\frac{2a(2Ab-3aB)}{3b^4\sqrt{a+bx^3}}+\frac{2(Ab-3aB)\sqrt{a+bx^3}}{3b^4}+\frac{2B\left(a+bx^3\right)^{3/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.71

$$\frac{2\left(-16a^3B+8a^2b\left(A-3Bx^3\right)-6ab^2x^3\left(Bx^3-2A\right)+b^3x^6\left(3A+Bx^3\right)\right)}{9b^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(-16*a^3*B + 8*a^2*b*(A - 3*B*x^3) - 6*a*b^2*x^3*(-2*A + B*x^3) + b^3*x^
6*(3*A + B*x^3)))/(9*b^4*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.66, size = 98, normalized size = 0.95

$$\frac{2\left(Bb^3x^9-3\left(2\,Bab^2-Ab^3\right)x^6-16\,Ba^3+8\,Aa^2b-12\left(2\,Ba^2b-Aab^2\right)x^3\right)\sqrt{bx^3+a}}{9\left(b^6x^6+2\,ab^5x^3+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(B*b^3*x^9 - 3*(2*B*a*b^2 - A*b^3)*x^6 - 16*B*a^3 + 8*A*a^2*b - 12*(2*B
*a^2*b - A*a*b^2)*x^3)*sqrt(b*x^3 + a)/(b^6*x^6 + 2*a*b^5*x^3 + a^2*b^4)
```

giac [A] time = 0.17, size = 104, normalized size = 1.01

$$-\frac{2\left(9\left(bx^3+a\right)Ba^2-Ba^3-6\left(bx^3+a\right)Aab+Aa^2b\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}b^4}+\frac{2\left(\left(bx^3+a\right)^{\frac{3}{2}}Bb^8-9\sqrt{bx^3+a}\,Bab^8+3\sqrt{bx^3+a}\,Ab^9\right)}{9\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(9*(b*x^3 + a)*B*a^2 - B*a^3 - 6*(b*x^3 + a)*A*a*b + A*a^2*b)/((b*x^3
+ a)^(3/2)*b^4) + 2/9*((b*x^3 + a)^(3/2)*B*b^8 - 9*sqrt(b*x^3 + a)*B*a*b^8
+ 3*sqrt(b*x^3 + a)*A*b^9)/b^12
```

maple [A] time = 0.05, size = 76, normalized size = 0.74

$$\frac{\frac{2}{9}B\,x^9b^3+\frac{2}{3}A\,b^3x^6-\frac{4}{3}Ba\,b^2x^6+\frac{8}{3}Aa\,b^2x^3-\frac{16}{3}B\,a^2b\,x^3+\frac{16}{9}A\,a^2b-\frac{32}{9}B\,a^3}{\left(b\,x^3+a\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] 2/9/(b*x^3+a)^(3/2)*(B*b^3*x^9+3*A*b^3*x^6-6*B*a*b^2*x^6+12*A*a*b^2*x^3-24*
B*a^2*b*x^3+8*A*a^2*b-16*B*a^3)/b^4
```

maxima [A] time = 0.59, size = 116, normalized size = 1.13

$$\frac{2}{9}B\left(\frac{\left(bx^3+a\right)^{\frac{3}{2}}}{b^4}-\frac{9\sqrt{bx^3+a}\,a}{b^4}-\frac{9\,a^2}{\sqrt{bx^3+a}\,b^4}+\frac{a^3}{\left(bx^3+a\right)^{\frac{3}{2}}b^4}\right)+\frac{2}{9}A\left(\frac{3\sqrt{bx^3+a}}{b^3}+\frac{6\,a}{\sqrt{bx^3+a}\,b^3}-\frac{a^2}{\left(bx^3+a\right)^{\frac{3}{2}}b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*((b*x^3 + a)^(3/2)/b^4 - 9*sqrt(b*x^3 + a)*a/b^4 - 9*a^2/(sqrt(b*x^3
+ a)*b^4) + a^3/((b*x^3 + a)^(3/2)*b^4)) + 2/9*A*(3*sqrt(b*x^3 + a)/b^3 + 6
*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)^(3/2)*b^3))
```

mupad [B] time = 2.80, size = 145, normalized size = 1.41

$$\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,(A\,b-2\,B\,a)}{b^3}-\frac{4\,B\,a}{3\,b^3}\right)}{3\,b}-\frac{\frac{2\,B\,a^2-2\,A\,a\,b}{3\,b^4}-\frac{a\,\left(\frac{2\,A\,b^2-2\,B\,a\,b}{3\,b^4}-\frac{2\,B\,a}{3\,b^3}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{a^2\,\left(\frac{2\,A}{9\,b}-\frac{2\,B\,a}{9\,b^2}\right)}{b^2\,{\left(b\,x^3+a\right)}^{3/2}}+\frac{2\,B\,x^3\,\sqrt{b\,x^3+a}}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] ((a + b*x^3)^(1/2)*((2*(A*b - 2*B*a))/b^3 - (4*B*a)/(3*b^3)))/(3*b) - ((2*B
*a^2 - 2*A*a*b)/(3*b^4) - (a*((2*A*b^2 - 2*B*a*b)/(3*b^4) - (2*B*a)/(3*b^3)
))/b)/(a + b*x^3)^(1/2) - (a^2*((2*A)/(9*b) - (2*B*a)/(9*b^2)))/(b^2*(a + b
*x^3)^(3/2)) + (2*B*x^3*(a + b*x^3)^(1/2))/(9*b^3)
```

sympy [A] time = 5.18, size = 338, normalized size = 3.28

$$\begin{cases} \frac{16Aa^2b}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{24Aab^2x^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} + \frac{6Ab^3x^6}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{32Ba^3}{9ab^4\sqrt{a+bx^3}+9b^5x^3\sqrt{a+bx^3}} - \frac{4}{9ab^4\sqrt{a+b}} \\ \frac{\frac{Ax^9}{9}+\frac{Bx^{12}}{12}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((16*A*a**2*b/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*
x**3)) + 24*A*a*b**2*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) + 6*A*b**3*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
 b*x**3)) - 32*B*a**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a + b*x
**3)) - 48*B*a**2*b*x**3/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a +
b*x**3)) - 12*B*a*b**2*x**6/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)) + 2*B*b**3*x**9/(9*a*b**4*sqrt(a + b*x**3) + 9*b**5*x**3*sqrt(a
 + b*x**3)), Ne(b, 0)), ((A*x**9/9 + B*x**12/12)/a**(5/2), True))
```

3.245 $\int \frac{x^5(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

```
[Out] 2/9*a*(A*b-B*a)/b^3/(b*x^3+a)^(3/2)-2/3*(A*b-2*B*a)/b^3/(b*x^3+a)^(1/2)+2/3
*B*(b*x^3+a)^(1/2)/b^3
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}+\frac{2B\sqrt{a+bx^3}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*a*(A*b - a*B))/(9*b^3*(a + b*x^3)^(3/2)) - (2*(A*b - 2*a*B))/(3*b^3*Sqrt
[a + b*x^3]) + (2*B*Sqrt[a + b*x^3])/(3*b^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{a(-Ab+aB)}{b^2(a+bx)^{5/2}}+\frac{Ab-2aB}{b^2(a+bx)^{3/2}}+\frac{B}{b^2\sqrt{a+bx}}\right)dx,x,x^3\right)\\ &= \frac{2a(Ab-aB)}{9b^3\left(a+bx^3\right)^{3/2}}-\frac{2(Ab-2aB)}{3b^3\sqrt{a+bx^3}}+\frac{2B\sqrt{a+bx^3}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.74

$$\frac{16a^2B-4ab\left(A-6Bx^3\right)+6b^2x^3\left(Bx^3-A\right)}{9b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (16*a^2*B - 4*a*b*(A - 6*B*x^3) + 6*b^2*x^3*(-A + B*x^3))/(9*b^3*(a + b*x^3
)^(3/2))
```

fricas [A] time = 0.60, size = 75, normalized size = 1.03

$$\frac{2 \left(3 Bb^2x^6 + 3 \left(4 Bab - Ab^2\right)x^3 + 8 Ba^2 - 2 Aab\right)\sqrt{bx^3 + a}}{9 \left(b^5x^6 + 2 ab^4x^3 + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*B*b^2*x^6 + 3*(4*B*a*b - A*b^2)*x^3 + 8*B*a^2 - 2*A*a*b)*sqrt(b*x^3
+ a)/(b^5*x^6 + 2*a*b^4*x^3 + a^2*b^3)
```

giac [A] time = 0.20, size = 63, normalized size = 0.86

$$\frac{2 \sqrt{bx^3 + a}\, B}{3 b^3} + \frac{2 \left(6 \left(bx^3 + a\right)Ba - Ba^2 - 3 \left(bx^3 + a\right)Ab + Aab\right)}{9 \left(bx^3 + a\right)^{\frac{3}{2}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x^3 + a)*B/b^3 + 2/9*(6*(b*x^3 + a)*B*a - B*a^2 - 3*(b*x^3 + a)*
A*b + A*a*b)/((b*x^3 + a)^(3/2)*b^3)
```

maple [A] time = 0.05, size = 53, normalized size = 0.73

$$-\frac{2 \left(-3B\, b^2x^6 + 3A\, b^2x^3 - 12Bab\, x^3 + 2Aab - 8B\, a^2\right)}{9 \left(b\, x^3 + a\right)^{\frac{3}{2}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(-3*B*b^2*x^6+3*A*b^2*x^3-12*B*a*b*x^3+2*A*a*b-8*B*a^2
)/b^3
```

maxima [A] time = 0.49, size = 84, normalized size = 1.15

$$\frac{2}{9} B \left(\frac{3 \sqrt{bx^3 + a}}{b^3} + \frac{6 a}{\sqrt{bx^3 + a}\, b^3} - \frac{a^2}{\left(bx^3 + a\right)^{\frac{3}{2}} b^3}\right) - \frac{2}{9} A \left(\frac{3}{\sqrt{bx^3 + a}\, b^2} - \frac{a}{\left(bx^3 + a\right)^{\frac{3}{2}} b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*B*(3*sqrt(b*x^3 + a)/b^3 + 6*a/(sqrt(b*x^3 + a)*b^3) - a^2/((b*x^3 + a)
^(3/2)*b^3)) - 2/9*A*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2))
```

mupad [B] time = 2.76, size = 60, normalized size = 0.82

$$\frac{6 B \left(b\, x^3 + a\right)^2 - 2 B\, a^2 - 6 A\, b \left(b\, x^3 + a\right) + 12 B\, a \left(b\, x^3 + a\right) + 2 A\, a\, b}{9 b^3 \left(b\, x^3 + a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] (6*B*(a + b*x^3)^2 - 2*B*a^2 - 6*A*b*(a + b*x^3) + 12*B*a*(a + b*x^3) + 2*A
*a*b)/(9*b^3*(a + b*x^3)^(3/2))
```

sympy [A] time = 2.42, size = 240, normalized size = 3.29

$$\begin{cases} -\frac{4Aab}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} - \frac{6Ab^2x^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{16Ba^2}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \frac{24Babx^3}{9ab^3\sqrt{a+bx^3}+9b^4x^3\sqrt{a+bx^3}} + \\ \frac{\frac{Ax^6}{6}+\frac{Bx^9}{9}}{a^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-4*A*a*b/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) - 6*A*b**2*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**
3)) + 16*B*a**2/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 24*B*a*b*x**3/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3))
+ 6*B*b**2*x**6/(9*a*b**3*sqrt(a + b*x**3) + 9*b**4*x**3*sqrt(a + b*x**3)),
 Ne(b, 0)), ((A*x**6/6 + B*x**9/9)/a**(5/2), True))
```

3.246 $\int \frac{x^2(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

```
[Out] -2/9*(A*b-B*a)/b^2/(b*x^3+a)^(3/2)-2/3*B/b^2/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b - a*B))/(9*b^2*(a + b*x^3)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*x^3])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{(a+bx)^{5/2}}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{Ab-aB}{b(a+bx)^{5/2}}+\frac{B}{b(a+bx)^{3/2}}\right)dx,x,x^3\right)\\ &= -\frac{2(Ab-aB)}{9b^2\left(a+bx^3\right)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.72

$$-\frac{2\left(2aB+Ab+3bBx^3\right)}{9b^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(A*b + 2*a*B + 3*b*B*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.67, size = 52, normalized size = 1.13

$$-\frac{2\left(3\,Bbx^{3}+2\,Ba+Ab\right)\sqrt{bx^{3}+a}}{9\left(b^{4}x^{6}+2\,ab^{3}x^{3}+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*B*b*x^3 + 2*B*a + A*b)*sqrt(b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2
*b^2)
```

giac [A] time = 0.16, size = 32, normalized size = 0.70

$$-\frac{2\left(3\left(bx^{3}+a\right)B-Ba+Ab\right)}{9\left(bx^{3}+a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*(3*(b*x^3 + a)*B - B*a + A*b)/((b*x^3 + a)^(3/2)*b^2)
```

maple [A] time = 0.05, size = 30, normalized size = 0.65

$$-\frac{2\left(3Bb\,x^{3}+Ab+2Ba\right)}{9\left(b\,x^{3}+a\right)^{\frac{3}{2}}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9/(b*x^3+a)^(3/2)*(3*B*b*x^3+A*b+2*B*a)/b^2
```

maxima [A] time = 0.50, size = 49, normalized size = 1.07

$$-\frac{2}{9}\,B\left(\frac{3}{\sqrt{bx^{3}+a}\,b^{2}}-\frac{a}{\left(bx^{3}+a\right)^{\frac{3}{2}}b^{2}}\right)-\frac{2\,A}{9\left(bx^{3}+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/9*B*(3/(sqrt(b*x^3 + a)*b^2) - a/((b*x^3 + a)^(3/2)*b^2)) - 2/9*A/((b*x^
3 + a)^(3/2)*b)
```

mupad [B] time = 2.68, size = 33, normalized size = 0.72

$$-\frac{2\,A\,b-2\,B\,a+6\,B\,\left(b\,x^{3}+a\right)}{9\,b^{2}\,\left(b\,x^{3}+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] -(2*A*b - 2*B*a + 6*B*(a + b*x^3))/(9*b^2*(a + b*x^3)^(3/2))
```

sympy [A] time = 1.34, size = 144, normalized size = 3.13

$$\begin{cases} -\frac{2Ab}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{4Ba}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} - \frac{6Bbx^3}{9ab^2\sqrt{a+bx^3}+9b^3x^3\sqrt{a+bx^3}} & \text{for } b \neq 0 \\ \frac{\frac{Ax^3}{3}+\frac{Bx^6}{6}}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Piecewise((-2*A*b/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)
) - 4*B*a/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)) - 6*B*
b*x**3/(9*a*b**2*sqrt(a + b*x**3) + 9*b**3*x**3*sqrt(a + b*x**3)), Ne(b, 0)
), ((A*x**3/3 + B*x**6/6)/a**(5/2), True))
```

3.247 $\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

```
[Out] 2/9*(A*b-B*a)/a/b/(b*x^3+a)^(3/2)-2/3*A*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^
(5/2)+2/3*A/a^2/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}+\frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B))/(9*a*b*(a + b*x^3)^(3/2)) + (2*A)/(3*a^2*Sqrt[a + b*x^3]) -
 (2*A*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^3}{x\left(a+bx^3\right)^{5/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{A+Bx}{x(a+bx)^{5/2}}\,dx,x,x^3\right)\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{3a}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{A\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{3a^2}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{(2A)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^2b}\\
&= \frac{2(Ab-aB)}{9ab\left(a+bx^3\right)^{3/2}}+\frac{2A}{3a^2\sqrt{a+bx^3}}-\frac{2A\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 62, normalized size = 0.81

$$\frac{2a(Ab-aB)+6Ab\left(a+bx^3\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^3}{a}+1\right)}{9a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*a*(A*b - a*B) + 6*A*b*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b
*x^3)/a])/(9*a^2*b*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.69, size = 243, normalized size = 3.16

$$\left[\frac{3\left(Ab^3x^6+2\,Aab^2x^3+Aa^2b\right)\sqrt{a}\,\log\left(\frac{bx^3-2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)+2\left(3\,Aab^2x^3-Ba^3+4\,Aa^2b\right)\sqrt{bx^3+a}}{9\left(a^3b^3x^6+2\,a^4b^2x^3+a^5b\right)},\ \frac{2\left(3\left(Ab\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*(A*b^3*x^6 + 2*A*a*b^2*x^3 + A*a^2*b)*sqrt(a)*log((b*x^3 - 2*sqrt(b
*x^3 + a)*sqrt(a) + 2*a)/x^3) + 2*(3*A*a*b^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(
b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 + a^5*b), 2/9*(3*(A*b^3*x^6 + 2*A*
a*b^2*x^3 + A*a^2*b)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*A*a*b
^2*x^3 - B*a^3 + 4*A*a^2*b)*sqrt(b*x^3 + a))/(a^3*b^3*x^6 + 2*a^4*b^2*x^3 +
 a^5*b)]
```

giac [A] time = 0.17, size = 67, normalized size = 0.87

$$\frac{2\,A\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^2}-\frac{2\left(Ba^2-3\left(bx^3+a\right)Ab-Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*A*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 2/9*(B*a^2 - 3*(b*x
^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a^2*b)
```

maple [A] time = 0.09, size = 85, normalized size = 1.10

$$\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}+\frac{2}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^2}+\frac{2\sqrt{b\,x^3+a}}{9\left(x^3+\frac{a}{b}\right)^2a\,b^2}\right)A-\frac{2B}{9\left(b\,x^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*B/b/(b*x^3+a)^(3/2)+A*(2/9/a/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+2/3/a^2/(
(x^3+a/b)*b)^(1/2)-2/3/a^(5/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.17, size = 81, normalized size = 1.05

$$\frac{1}{9}\,A\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3+4\,a\right)}{\left(bx^3+a\right)^{\frac{3}{2}}a^2}\right)-\frac{2\,B}{9\left(bx^3+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*A*(3*log((sqrt(b*x^3 + a) - sqrt(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/
2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)^(3/2)*a^2)) - 2/9*B/((b*x^3 + a)^(3/2)*
b)
```

mupad [B] time = 2.78, size = 80, normalized size = 1.04

$$\frac{\frac{2\,A}{9\,a}-\frac{2\,B}{9\,b}}{\left(b\,x^3+a\right)^{3/2}}+\frac{2\,A}{3\,a^2\,\sqrt{b\,x^3+a}}+\frac{A\,\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)}{x^6}\right)}{3\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x*(a + b*x^3)^(5/2)),x)
```

```
[Out] ((2*A)/(9*a) - (2*B)/(9*b))/(a + b*x^3)^(3/2) + (2*A)/(3*a^2*(a + b*x^3)^(1
/2)) + (A*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2)
))/x^6))/(3*a^(5/2))
```

sympy [A] time = 38.43, size = 76, normalized size = 0.99

$$\frac{2A}{3a^2\sqrt{a+bx^3}}+\frac{2A\,\mathrm{atan}\left(\frac{\sqrt{a+bx^3}}{\sqrt{-a}}\right)}{3a^2\sqrt{-a}}-\frac{2\left(-Ab+Ba\right)}{9ab\left(a+bx^3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x/(b*x**3+a)**(5/2),x)
```

```
[Out] 2*A/(3*a**2*sqrt(a + b*x**3)) + 2*A*atan(sqrt(a + b*x**3)/sqrt(-a))/(3*a**2
*sqrt(-a)) - 2*(-A*b + B*a)/(9*a*b*(a + b*x**3)**(3/2))
```

3.248 $\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=113

$$\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{2aB-5Ab}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

```
[Out] 1/9*(-5*A*b+2*B*a)/a^2/(b*x^3+a)^(3/2)-1/3*A/a/x^3/(b*x^3+a)^(3/2)+1/3*(5*A
*b-2*B*a)*arctanh((b*x^3+a)^(1/2)/a^(1/2))/a^(7/2)+1/3*(-5*A*b+2*B*a)/a^3/(
b*x^3+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(5*A*b - 2*a*B)/(9*a^2*(a + b*x^3)^(3/2)) - A/(3*a*x^3*(a + b*x^3)^(3/2))
- (5*A*b - 2*a*B)/(3*a^3*Sqrt[a + b*x^3]) + ((5*A*b - 2*a*B)*ArcTanh[Sqrt[a
 + b*x^3]/Sqrt[a]])/(3*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^4\left(a+bx^3\right)^{5/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{A+Bx}{x^2(a+bx)^{5/2}}\,dx,x,x^3\right)$$
$$= -\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}+\frac{\left(-\frac{5Ab}{2}+aB\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx,x,x^3\right)}{3a}$$
$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx,x,x^3\right)}{6a^2}$$
$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^3\right)}{6a^3}$$
$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}-\frac{(5Ab-2aB)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^3}\right)}{3a^3b}$$
$$= -\frac{5Ab-2aB}{9a^2\left(a+bx^3\right)^{3/2}}-\frac{A}{3ax^3\left(a+bx^3\right)^{3/2}}-\frac{5Ab-2aB}{3a^3\sqrt{a+bx^3}}+\frac{(5Ab-2aB)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{7/2}}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.50

$$\frac{x^3(2aB-5Ab)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx^3}{a}+1\right)-3aA}{9a^2x^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-3*a*A + (-5*A*b + 2*a*B)*x^3*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x^3)
/a])/(9*a^2*x^3*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.64, size = 351, normalized size = 3.11

$$\left[-\frac{3\left(\left(2\,Bab^2-5\,Ab^3\right)x^9+2\left(2\,Ba^2b-5\,Aab^2\right)x^6+\left(2\,Ba^3-5\,Aa^2b\right)x^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,\sqrt{bx^3+a}\,\sqrt{a}+2\,a}{x^3}\right)-2\left(3\left(2\right.\right.}{18\left(a^4b^2x^9+2\,a^5bx^6+a^6x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B
*a^3 - 5*A*a^2*b)*x^3)*sqrt(a)*log((b*x^3 + 2*sqrt(b*x^3 + a)*sqrt(a) + 2*a
```

```
)/x^3) - 2*(3*(2*B*a^2*b - 5*A*a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*
b)*x^3)*sqrt(b*x^3 + a))/(a^4*b^2*x^9 + 2*a^5*b*x^6 + a^6*x^3), 1/9*(3*((2*
B*a*b^2 - 5*A*b^3)*x^9 + 2*(2*B*a^2*b - 5*A*a*b^2)*x^6 + (2*B*a^3 - 5*A*a^2
*b)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a) + (3*(2*B*a^2*b - 5*A*
a*b^2)*x^6 - 3*A*a^3 + 4*(2*B*a^3 - 5*A*a^2*b)*x^3)*sqrt(b*x^3 + a))/(a^4*b
^2*x^9 + 2*a^5*b*x^6 + a^6*x^3)]
```

giac [A] time = 0.19, size = 101, normalized size = 0.89

$$\frac{(2\,Ba-5\,Ab)\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a^3}+\frac{2\left(3\left(bx^3+a\right)Ba+Ba^2-6\left(bx^3+a\right)Ab-Aab\right)}{9\left(bx^3+a\right)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx^3+a}\,A}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*B*a - 5*A*b)*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2/9*(
3*(b*x^3 + a)*B*a + B*a^2 - 6*(b*x^3 + a)*A*b - A*a*b)/((b*x^3 + a)^(3/2)*a
^3) - 1/3*sqrt(b*x^3 + a)*A/(a^3*x^3)
```

maple [A] time = 0.09, size = 157, normalized size = 1.39

$$\left(\frac{5b\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{7}{2}}}-\frac{4b}{3\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^3}-\frac{2\sqrt{b\,x^3+a}}{9\left(x^3+\frac{a}{b}\right)^2a^2b}-\frac{\sqrt{b\,x^3+a}}{3a^3x^3}\right)A+\left(-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3a^{\frac{5}{2}}}+\frac{2}{3\sqrt{\left(x^3+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x)
```

```
[Out] A*(-1/3/a^3*(b*x^3+a)^(1/2)/x^3-2/9/a^2/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-4/3*b
/a^3/((x^3+a/b)*b)^(1/2)+5/3*b/a^(7/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))+B*
(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/a/b^2+2/3/((x^3+a/b)*b)^(1/2)/a^2-2/3/a^(5
/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 1.25, size = 170, normalized size = 1.50

$$-\frac{1}{18}\,A\left(\frac{2\left(15\left(bx^3+a\right)^2b-10\left(bx^3+a\right)ab-2\,a^2b\right)}{\left(bx^3+a\right)^{\frac{5}{2}}a^3-\left(bx^3+a\right)^{\frac{3}{2}}a^4}+\frac{15\,b\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{7}{2}}}\right)+\frac{1}{9}\,B\left(\frac{3\,\log\left(\frac{\sqrt{bx^3+a}-\sqrt{a}}{\sqrt{bx^3+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\left(3\,bx^3\right.}{\left(bx^3+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^4/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/18*A*(2*(15*(b*x^3 + a)^2*b - 10*(b*x^3 + a)*a*b - 2*a^2*b)/((b*x^3 + a)
^(5/2)*a^3 - (b*x^3 + a)^(3/2)*a^4) + 15*b*log((sqrt(b*x^3 + a) - sqrt(a))/
(sqrt(b*x^3 + a) + sqrt(a)))/a^(7/2)) + 1/9*B*(3*log((sqrt(b*x^3 + a) - sqr
t(a))/(sqrt(b*x^3 + a) + sqrt(a)))/a^(5/2) + 2*(3*b*x^3 + 4*a)/((b*x^3 + a)
^(3/2)*a^2))
```

mupad [B] time = 2.97, size = 198, normalized size = 1.75

$$\frac{\ln\left(\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\sqrt{b\,x^3+a}+\sqrt{a}\right)^3}{x^6}\right)(5\,A\,b-2\,B\,a)}{6\,a^{7/2}}-\frac{\frac{2\,B\,a^2-5\,A\,a\,b}{2\,a^4}-\frac{a\left(\frac{A\,b^2}{3\,a^4}+\frac{5\,b\left(2\,B\,a^2-5\,A\,a\,b\right)}{6\,a^5}\right)}{b}}{\sqrt{b\,x^3+a}}-\frac{\frac{2\,B\,a^3-5\,A\,a^2\,b}{4\,a^4}-\frac{a\left(\frac{13\,b\left(2\,B\,a^3-5\,A\,a^2\,b\right.}{36\,a^5}\right.}{b}}{\left(b\,x^3+a\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^4*(a + b*x^3)^(5/2)),x)
```

```
[Out] (log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6)*(
5*A*b - 2*B*a))/(6*a^(7/2)) - ((2*B*a^2 - 5*A*a*b)/(2*a^4) - (a*((A*b^2)/(3
*a^4) + (5*b*(2*B*a^2 - 5*A*a*b))/(6*a^5)))/b)/(a + b*x^3)^(1/2) - ((2*B*a^
3 - 5*A*a^2*b)/(4*a^4) - (a*((13*b*(2*B*a^3 - 5*A*a^2*b))/(36*a^5) + (A*b^2
)/(3*a^3)))/b)/(a + b*x^3)^(3/2) - (A*(a + b*x^3)^(1/2))/(3*a^3*x^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**4/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.249 $\int \frac{x^6(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=299

$$\frac{32\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{135\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{16x(5Ab-1}{135b^3\sqrt{a+}$$

```
[Out] -2/45*(5*A*b-14*B*a)*x^4/b^2/(b*x^3+a)^(3/2)+2/5*B*x^7/b/(b*x^3+a)^(3/2)-16
/135*(5*A*b-14*B*a)*x/b^3/(b*x^3+a)^(1/2)+32/405*(5*A*b-14*B*a)*(a^(1/3)+b^
(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(10/3)/(b
*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {459, 288, 218}

$$\frac{32\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(5Ab-14aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{135\sqrt[4]{3}\,b^{10/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}-\frac{2x^4(5Ab-1}{45b^2\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(5*A*b - 14*a*B)*x^4)/(45*b^2*(a + b*x^3)^(3/2)) + (2*B*x^7)/(5*b*(a +
b*x^3)^(3/2)) - (16*(5*A*b - 14*a*B)*x)/(135*b^3*Sqrt[a + b*x^3]) + (32*Sqr
t[2 + Sqrt[3]]*(5*A*b - 14*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/
3)*x)], -7 - 4*Sqrt[3]])/(135*3^(1/4)*b^(10/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(
1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{x^6\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}} - \frac{\left(2\left(-\frac{5Ab}{2}+7aB\right)\right)\int \frac{x^6}{\left(a+bx^3\right)^{5/2}}\,dx}{5b}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}} + \frac{(8(5Ab-14aB))\int \frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx}{45b^2}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{(16(5Ab-14aB))\int \frac{1}{\sqrt{a+bx^3}}}{135b^3}$$

$$= -\frac{2(5Ab-14aB)x^4}{45b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^7}{5b\left(a+bx^3\right)^{3/2}} - \frac{16(5Ab-14aB)x}{135b^3\sqrt{a+bx^3}} + \frac{32\sqrt{2+\sqrt{3}}\,(5Ab-14aB)}{}$$

Mathematica [C] time = 0.17, size = 108, normalized size = 0.36

$$\frac{2x\left(112a^2B+8\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(5Ab-14aB)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a\left(154bBx^3-40Ab\right)+b^2x^3\left(27Bx^3-55\right.\right.}{135b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(112*a^2*B + b^2*x^3*(-55*A + 27*B*x^3) + a*(-40*A*b + 154*b*B*x^3) +
8*(5*A*b - 14*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((b*x^3)/a)]))/(135*b^3*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^9+Ax^6\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^9 + A*x^6)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^6}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.10, size = 683, normalized size = 2.28

$$\left(-\frac{22x}{27\sqrt{\left(x^3+\frac{a}{b}\right)b}\,b^2}+\frac{2\sqrt{b\,x^3+a}\;ax}{9\left(x^3+\frac{a}{b}\right)^2 b^4}-\frac{32i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*a^2*x/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+40/27/b^3*a*x/((x^3+a/b)*b)^(
1/2)+2/5/b^3*x*(b*x^3+a)^(1/2)+448/405*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3)*(I*(x
+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3
)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*
(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)
^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(2/9*a*x/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2
-22/27/b^2*x/((x^3+a/b)*b)^(1/2)-32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1
/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*
b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(
1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^6}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^6/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^6*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.250 $\int \frac{x^3(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*x^4/a/b/(b*x^3+a)^(3/2)-2/27*(A*b+8*B*a)*x/a/b^2/(b*x^3+a)^(1
/2)+4/81*(A*b+8*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a/b^(7/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)
/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {457, 288, 218}

$$\frac{4\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(8aB+Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2x(8aB+Ab)}{27ab^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^4)/(9*a*b*(a + b*x^3)^(3/2)) - (2*(A*b + 8*a*B)*x)/(27*a*b
^2*Sqrt[a + b*x^3]) + (4*Sqrt[2 + Sqrt[3]]*(A*b + 8*a*B)*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a*b^(7/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{x^3\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{Ab}{2}+4aB\right)\right)\int\frac{x^3}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{(2(Ab+8aB))\int\frac{1}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^4}{9ab\left(a+bx^3\right)^{3/2}} - \frac{2(Ab+8aB)x}{27ab^2\sqrt{a+bx^3}} + \frac{4\sqrt{2+\sqrt{3}}\,(Ab+8aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}\cdots}{27\sqrt[4]{3}\,ab^{7/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}\cdots}$$

Mathematica [C] time = 0.19, size = 99, normalized size = 0.35

$$\frac{2x\left(-8a^2B+\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(8aB+Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)-ab\left(A+11Bx^3\right)+2Ab^2x^3\right)}{27ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x*(-8*a^2*B + 2*A*b^2*x^3 - a*b*(A + 11*B*x^3) + (A*b + 8*a*B)*(a + b*x^
3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)]))/(27
*a*b^2*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^6+Ax^3\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^6 + A*x^3)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^3}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.08, size = 669, normalized size = 2.36

$$\left(\frac{4x}{27\sqrt{\left(x^3+\frac{a}{b}\right)b}\,ab} - \frac{4i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{-\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}}{81\sqrt{b\,x^3+a}\,a\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2*a/b^4*x-22/27/((x^3+a/b)*b)^(1/2)/b^2*x-
32/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2
)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)
^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*
(-2/9*x/b^3*(b*x^3+a)^(1/2)/(x^3+a/b)^2+4/27/b/a*x/((x^3+a/b)*b)^(1/2)-4/81
*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(
b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1
/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^3}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^3/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

sympy [A] time = 115.03, size = 80, normalized size = 0.28

$$\frac{Ax^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{Bx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(7/3)) + B*x**7*gamma(7/3)*hyper((7/3, 5/2), (10/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(10/3))
```

3.251 $\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=283

$$\frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*x/a/b/(b*x^3+a)^(3/2)+2/27*(7*A*b+2*B*a)*x/a^2/b/(b*x^3+a)^(1
/2)+2/81*(7*A*b+2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2
^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(3/4)/a^2/b^(4/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3
)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 199, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(2aB+7Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2x(2aB+7Ab)}{27a^2b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(7*A*b + 2*a*B)*x)/(27*a^2
*b*Sqrt[a + b*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(7*A*b + 2*a*B)*(a^(1/3) + b^(1/
3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^2*b^(4/3)
*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2
]*Sqrt[a + b*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rubi steps

$$\int \frac{A+Bx^3}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{7Ab}{2}+aB\right)\right)\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{(7Ab+2aB)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(7Ab+2aB)x}{27a^2b\sqrt{a+bx^3}} + \frac{2\sqrt{2+\sqrt{3}}\,(7Ab+2aB)\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\ldots\right.}}}{27\sqrt[4]{3}\,a^2b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}\ldots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}$$

Mathematica [C] time = 0.07, size = 103, normalized size = 0.36

$$\frac{-2a^2Bx+x\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(2aB+7Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+4abx\left(5A+Bx^3\right)+14Ab^2x^4}{27a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*a^2*B*x + 14*A*b^2*x^4 + 4*a*b*x*(5*A + B*x^3) + (7*A*b + 2*a*B)*x*(a +
 b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])
/(27*a^2*b*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3 +
 a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.07, size = 674, normalized size = 2.38

$$\left(\frac{14x}{27\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^2}+\frac{2\sqrt{b\,x^3+a}\;x}{9\left(x^3+\frac{a}{b}\right)^2 a\,b^2}-\frac{14i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/b^3*x+4/27/((x^3+a/b)*b)^(1/2)/a/b*x-4/
81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+A*(
2/9/a*x/b^2*(b*x^3+a)^(1/2)/(x^3+a/b)^2+14/27/a^2*x/((x^3+a/b)*b)^(1/2)-14/
81*I/a^2*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)
/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)/(a + b*x^3)^(5/2), x)
```

sympy [**A**] time = 79.48, size = 78, normalized size = 0.28

$$\frac{Ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{Bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/2)*gamma(4/3)) + B*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/2)*gamma(7/3))
```

3.252 $\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{7x(13Ab-4aB)}{54a^3\sqrt{a+bx^3}}-\frac{x(13Ab-4aB)}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+(}{\sqrt[3]{b}x+(}\right.\right.}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] -1/2*A/a/x^2/(b*x^3+a)^(3/2)-1/18*(13*A*b-4*B*a)*x/a^2/(b*x^3+a)^(3/2)-7/54
*(13*A*b-4*B*a)*x/a^3/(b*x^3+a)^(1/2)-7/162*(13*A*b-4*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^3/b^(1/3)/(b*x
^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 199, 218}

$$-\frac{7\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(13Ab-4aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{54\sqrt[4]{3}a^3\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{7x(13Ab-}{54a^3\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] -A/(2*a*x^2*(a + b*x^3)^(3/2)) - ((13*A*b - 4*a*B)*x)/(18*a^2*(a + b*x^3)^(
3/2)) - (7*(13*A*b - 4*a*B)*x)/(54*a^3*Sqrt[a + b*x^3]) - (7*Sqrt[2 + Sqrt[
3]]*(13*A*b - 4*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((
1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7
- 4*Sqrt[3]])/(54*3^(1/4)*a^3*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^3\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{13Ab}{2}-2aB\right)\int \frac{1}{\left(a+bx^3\right)^{5/2}}\,dx}{2a}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{(7(13Ab-4aB))\int \frac{1}{\left(a+bx^3\right)^{3/2}}\,dx}{36a^2}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}-\frac{(7(13Ab-4aB))\int \frac{1}{\sqrt{a+bx^3}}$$

$$= -\frac{A}{2ax^2\left(a+bx^3\right)^{3/2}}-\frac{(13Ab-4aB)x}{18a^2\left(a+bx^3\right)^{3/2}}-\frac{7(13Ab-4aB)x}{54a^3\sqrt{a+bx^3}}-\frac{7\sqrt{2+\sqrt{3}}\,(13Ab-4aB)}{}$$

Mathematica [C] time = 0.09, size = 116, normalized size = 0.39

$$\frac{a^2\left(80Bx^3-54A\right)+7x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(4aB-13Ab)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a\left(56bBx^6-260Abx^3\right)-182A}{108a^3x^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-182*A*b^2*x^6 + a^2*(-54*A + 80*B*x^3) + a*(-260*A*b*x^3 + 56*b*B*x^6) +
7*(-13*A*b + 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1
/3, 1/2, 4/3, -((b*x^3)/a)])/(108*a^3*x^2*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^{12}+3\,ab^2x^9+3\,a^2bx^6+a^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^12 + 3*a*b^2*x^9 + 3*a^2*b*x^6
+ a^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

maple [B] time = 0.08, size = 689, normalized size = 2.30

$$\left(-\frac{32bx}{27\sqrt{\left(x^3+\frac{a}{b}\right)b}\,a^3}-\frac{2\sqrt{b\,x^3+a}\,x}{9\left(x^3+\frac{a}{b}\right)^2a^2b}+\frac{91i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/a/b^2*x+14/27/((x^3+a/b)*b)^(1/2)/a^2*x-
14/81*I/a^2*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/
(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^
2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+
A*(-2/9/a^2*x/b*(b*x^3+a)^(1/2)/(x^3+a/b)^2-32/27*b/a^3*x/((x^3+a/b)*b)^(1/
2)-1/2/a^3*(b*x^3+a)^(1/2)/x^2+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1
/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^3/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{x^3\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^3*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**3/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.253 $\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=334

$$\frac{91\sqrt{a+bx^3}\,(19Ab-10aB)}{540a^4x^2}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}+\frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\ldots\right.}}\ldots}{540\sqrt[4]{3}\,a^4\ldots}$$

```
[Out] -1/5*A/a/x^5/(b*x^3+a)^(3/2)+1/45*(-19*A*b+10*B*a)/a^2/x^2/(b*x^3+a)^(3/2)-
13/135*(19*A*b-10*B*a)/a^3/x^2/(b*x^3+a)^(1/2)+91/540*(19*A*b-10*B*a)*(b*x^
3+a)^(1/2)/a^4/x^2+91/1620*b^(2/3)*(19*A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*Elli
pticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2
)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^4/(b*x^3+a)^(1/2)/(a^(
1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 290, 325, 218}

$$\frac{91\sqrt{2+\sqrt{3}}\,b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,(19Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{540\sqrt[4]{3}\,a^4\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{91\sqrt{a+\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] -A/(5*a*x^5*(a + b*x^3)^(3/2)) - (19*A*b - 10*a*B)/(45*a^2*x^2*(a + b*x^3)^
(3/2)) - (13*(19*A*b - 10*a*B))/(135*a^3*x^2*Sqrt[a + b*x^3]) + (91*(19*A*b
 - 10*a*B)*Sqrt[a + b*x^3])/(540*a^4*x^2) + (91*Sqrt[2 + Sqrt[3]]*b^(2/3)*(
19*A*b - 10*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*
Sqrt[3]])/(540*3^(1/4)*a^4*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^6\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{\left(\frac{19Ab}{2}-5aB\right)\int \frac{1}{x^3\left(a+bx^3\right)^{5/2}}\,dx}{5a}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{(13(19Ab-10aB))\int \frac{1}{x^3\left(a+bx^3\right)^{3/2}}\,dx}{90a^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}-\frac{(91(19Ab-10aB))\int}{270a^3}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}+\frac{91(19Ab-10aB)\sqrt{a}}{540a^4x^2}$$

$$= -\frac{A}{5ax^5\left(a+bx^3\right)^{3/2}}-\frac{19Ab-10aB}{45a^2x^2\left(a+bx^3\right)^{3/2}}-\frac{13(19Ab-10aB)}{135a^3x^2\sqrt{a+bx^3}}+\frac{91(19Ab-10aB)\sqrt{a}}{540a^4x^2}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.25

$$\frac{x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(\frac{19Ab}{2}-5aB\right){}_2F_1\left(-\frac{2}{3},\frac{5}{2};\frac{1}{3};-\frac{bx^3}{a}\right)-2a^2A}{10a^3x^5\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*a^2*A + ((19*A*b)/2 - 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-2/3, 5/2, 1/3, -((b*x^3)/a)])/(10*a^3*x^5*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^{15}+3\,ab^2x^{12}+3\,a^2bx^9+a^3x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^15 + 3*a*b^2*x^12 + 3*a^2*b*x^9
 + a^3*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

maple [B] time = 0.09, size = 722, normalized size = 2.16

$$\left(\frac{50b^2x}{27\sqrt{\left(x^3+\frac{a}{b}\right)b}\;a^4} + \frac{2\sqrt{b\,x^3+a}\;x}{9\left(x^3+\frac{a}{b}\right)^2 a^3} - \frac{1729 i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}\right)\sqrt{3}\,b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\sqrt{\frac{x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}}{-\frac{3\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{2b}}}\;\sqrt{-\cdots}}{\cdots} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x)
```

```
[Out] A*(-1/5/a^3*(b*x^3+a)^(1/2)/x^5+27/20/a^4*b*(b*x^3+a)^(1/2)/x^2+2/9/a^3*x*(
b*x^3+a)^(1/2)/(x^3+a/b)^2+50/27*b^2/a^4*x/((x^3+a/b)*b)^(1/2)-1729/1620*I/
a^4*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x
^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)
/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+B*(-2/9*
(b*x^3+a)^(1/2)/(x^3+a/b)^2/a^2/b*x-32/27/((x^3+a/b)*b)^(1/2)/a^3*b*x-1/2*(
b*x^3+a)^(1/2)/a^3/x^2+91/162*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^
2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*
((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(
-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^6/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^6\,\left(b\,x^3 + a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^6*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**6/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.254 $\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=577

$$\frac{80\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{189\sqrt[4]{3}\,b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\ \cdots}{\cdots}$$

```
[Out] -2/63*(7*A*b-16*B*a)*x^5/b^2/(b*x^3+a)^(3/2)+2/7*B*x^8/b/(b*x^3+a)^(3/2)-20
/189*(7*A*b-16*B*a)*x^2/b^3/(b*x^3+a)^(1/2)+80/189*(7*A*b-16*B*a)*(b*x^3+a)
^(1/2)/b^(11/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+80/567*a^(1/3)*(7*A*b-16*B*
a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x
+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^
(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(11/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)-40/189*a^(1/3)*(7*A*b-16*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/b^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/
3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 577, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 288, 303, 218, 1877}

$$\frac{80\sqrt{2}\,\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(7Ab-16aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{189\sqrt[4]{3}\,b^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\frac{40\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (-2*(7*A*b - 16*a*B)*x^5)/(63*b^2*(a + b*x^3)^(3/2)) + (2*B*x^8)/(7*b*(a +
b*x^3)^(3/2)) - (20*(7*A*b - 16*a*B)*x^2)/(189*b^3*Sqrt[a + b*x^3]) + (80*(
7*A*b - 16*a*B)*Sqrt[a + b*x^3])/(189*b^(11/3)*((1 + Sqrt[3])*a^(1/3) + b^(
1/3)*x)) - (40*Sqrt[2 - Sqrt[3]]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3
)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3
) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1
+ Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(63*3^(3/4)*b^(11/3)*Sqr
t[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sq
rt[a + b*x^3]) + (80*Sqrt[2]*a^(1/3)*(7*A*b - 16*a*B)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(189*3^(1/4)*b^(11/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
```

```
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^7\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{\left(2\left(-\frac{7Ab}{2}+8aB\right)\right)\int \frac{x^7}{\left(a+bx^3\right)^{5/2}}\,dx}{7b}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} + \frac{(10(7Ab-16aB))\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{63b^2}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{x}{\sqrt{a+bx^3}}\,d}{189b^3}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{(40(7Ab-16aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt{a+b}}}{189b^{10/3}}$$

$$= -\frac{2(7Ab-16aB)x^5}{63b^2\left(a+bx^3\right)^{3/2}} + \frac{2Bx^8}{7b\left(a+bx^3\right)^{3/2}} - \frac{20(7Ab-16aB)x^2}{189b^3\sqrt{a+bx^3}} + \frac{80(7Ab-16aB)\sqrt{a+bx^3}}{189b^{11/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\right.}$$

Mathematica [C] time = 0.11, size = 109, normalized size = 0.19

$$\frac{2x^2\left(-32a^2B+2\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(16aB-7Ab)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)+2ab\left(7A-8Bx^3\right)+b^2x^3\left(7A+Bx^3\right)\right)}{7b^3\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*x^2*(-32*a^2*B + 2*a*b*(7*A - 8*B*x^3) + b^2*x^3*(7*A + B*x^3) + 2*(-7*A
*b + 16*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/
3, -((b*x^3)/a)]))/(7*b^3*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^{10}+Ax^7\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^10 + A*x^7)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*
x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^7}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.10, size = 997, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*a^2*x^2/b^5*(b*x^3+a)^(1/2)/(x^3+a/b)^2+44/27/b^3*a*x^2/((x^3+a/b)*
b)^(1/2)+2/7/b^3*x^2*(b*x^3+a)^(1/2)+1280/567*I*a/b^4*3^(1/2)*(-a*b^2)^(1/3
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1
/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^
2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),
(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/
3)/b)/b)^(1/2))))+A*(2/9*a*x^2/b^4*(b*x^3+a)^(1/2)/(x^3+a/b)^2-26/27/b^2*x^
2/((x^3+a/b)*b)^(1/2)-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)
^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((
x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))
^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/
(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b
^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+
(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^
(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x^7}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^7/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(B\,x^3 + A\right)}{{\left(b\,x^3 + a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^7*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.255 $\int \frac{x^4(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=559

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 2/9*(A*b-B*a)*x^5/a/b/(b*x^3+a)^(3/2)+2/27*(A*b-10*B*a)*x^2/a/b^2/(b*x^3+a)
^(1/2)-8/27*(A*b-10*B*a)*(b*x^3+a)^(1/2)/a/b^(8/3)/(b^(1/3)*x+a^(1/3)*(1+3^
(1/2)))-8/81*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/a^(2/3)/b^(8/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/
3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/27*(A*b-10*B*a)*(a^(1/3)+b^(1/3)*x)*El
lipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(8/3)/(b*x^3
+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 559, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {457, 288, 303, 218, 1877}

$$-\frac{8\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(Ab-10aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{2/3}b^{8/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{4\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^5)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(A*b - 10*a*B)*x^2)/(27*
a*b^2*Sqrt[a + b*x^3]) - (8*(A*b - 10*a*B)*Sqrt[a + b*x^3])/(27*a*b^(8/3)*(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (4*Sqrt[2 - Sqrt[3]]*(A*b - 10*a*B)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/
4)*a^(2/3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1
/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (8*Sqrt[2]*(A*b - 10*a*B)*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*
x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(2/
3)*b^(8/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^
(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4 \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(-\frac{Ab}{2}+5aB\right)\right)\int \frac{x^4}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{(4(Ab-10aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{27ab^{7/3}} - \frac{\left(4\sqrt{2\left(2-\right.}\right.}{}$$

$$= \frac{2(Ab-aB)x^5}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(Ab-10aB)x^2}{27ab^2\sqrt{a+bx^3}} - \frac{8(Ab-10aB)\sqrt{a+bx^3}}{27ab^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{4\sqrt{2-\sqrt{3}}\,(Ab}{}$$

Mathematica [C] time = 0.10, size = 92, normalized size = 0.16

$$\frac{2x^2\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(Ab-10aB)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-aAb+5aB\left(2a+bx^3\right)\right)}{5ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x^2*(-(a*A*b) + 5*a*B*(2*a + b*x^3) + (A*b - 10*a*B)*(a + b*x^3)*Sqrt[1
+ (b*x^3)/a]*Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(5*a*b^2*(a +
 b*x^3)^(3/2))
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^7+Ax^4\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] integral((B*x^7 + A*x^4)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x
^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x^4}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.08, size = 981, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2*a/b^4*x^2-26/27/((x^3+a/b)*b)^(1/2)/b^2*
x^2-80/81*I/b^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^
(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-2/9*x^2/b^3*(b*x
^3+a)^(1/2)/(x^3+a/b)^2+8/27/b/a*x^2/((x^3+a/b)*b)^(1/2)+8/81*I/b^2/a*3^(1/
2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)
*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^(1
/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*
b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2)*
(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)
^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x^4}{\left(bx^3 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x^4/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x^4*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

sympy [A] time = 162.52, size = 80, normalized size = 0.14

$$\frac{Ax^5\Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{5}{3}, \frac{5}{2} \\ \frac{8}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)} + \frac{Bx^8\Gamma\left(\frac{8}{3}\right) {}_{2}F_{1}\left(\left.\begin{matrix} \frac{5}{2}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(8/3)) + B*x**8*gamma(8/3)*hyper((5/2, 8/3), (11/3,), b*x**3*ex
p_polar(I*pi)/a)/(3*a**(5/2)*gamma(11/3))
```

3.256 $\int \frac{x(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=563

$$-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

```
[Out] 2/9*(A*b-B*a)*x^2/a/b/(b*x^3+a)^(3/2)+2/27*(5*A*b+4*B*a)*x^2/a^2/b/(b*x^3+a
)^(1/2)-2/27*(5*A*b+4*B*a)*(b*x^3+a)^(1/2)/a^2/b^(5/3)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))-2/81*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(
1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(
b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(5*A*b+4*B*a)*(a^(1/3)+b^(1/3)
*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(5/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 563, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {457, 290, 303, 218, 1877}

$$-\frac{2\sqrt{a+bx^3}(4aB+5Ab)}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{2\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(4aB+5Ab)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*x^2)/(9*a*b*(a + b*x^3)^(3/2)) + (2*(5*A*b + 4*a*B)*x^2)/(27
*a^2*b*Sqrt[a + b*x^3]) - (2*(5*A*b + 4*a*B)*Sqrt[a + b*x^3])/(27*a^2*b^(5/
3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (Sqrt[2 - Sqrt[3]]*(5*A*b + 4*a*B
)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((
1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3
) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^
(3/4)*a^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (2*Sqrt[2]*(5*A*b + 4*a*B)*(a^(1/
3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt
[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1
/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a
^(5/3)*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3)
+ b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(\frac{5Ab}{2}+2aB\right)\right)\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{(5Ab+4aB)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt{a+bx^3}}\,dx}{27a^2b^{4/3}} - \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,(5\ldots\right.}{2\ldots}$$

$$= \frac{2(Ab-aB)x^2}{9ab\left(a+bx^3\right)^{3/2}} + \frac{2(5Ab+4aB)x^2}{27a^2b\sqrt{a+bx^3}} - \frac{2(5Ab+4aB)\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} + \frac{\sqrt{2-\sqrt{3}}\,(5Ab+\ldots}{\ldots}$$

Mathematica [C] time = 0.08, size = 81, normalized size = 0.14

$$\frac{x^2\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(4aB+5Ab)\,{}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)-4a^2B\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x^2*(-4*a^2*B + (5*A*b + 4*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeome
tric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^4+Ax\right)\sqrt{bx^3+a}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^4 + A*x)*sqrt(b*x^3 + a)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)x}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

maple [B] time = 0.08, size = 986, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(-2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/b^3*x^2+8/27/((x^3+a/b)*b)^(1/2)/a/b*x^
2+8/81*I/b^2/a*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)
/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(
-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(
1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(2/9/a*x^2/b^2*(b*x
^3+a)^(1/2)/(x^3+a/b)^2+10/27/a^2*x^2/((x^3+a/b)*b)^(1/2)+10/81*I/a^2*3^(1/
2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/
2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-
a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*
I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^
2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)x}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*x/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(B\,x^3+A\right)}{{\left(b\,x^3+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^3))/(a + b*x^3)^(5/2),x)
```

```
[Out] int((x*(A + B*x^3))/(a + b*x^3)^(5/2), x)
```

sympy [A] time = 79.72, size = 80, normalized size = 0.14

$$\frac{Ax^2\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{2}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{5}{3}\right)} + \frac{Bx^5\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{5}{3},\frac{5}{2}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{2}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] A*x**2*gamma(2/3)*hyper((2/3, 5/2), (5/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(5/2)*gamma(5/3)) + B*x**5*gamma(5/3)*hyper((5/3, 5/2), (8/3,), b*x**3*exp
_polar(I*pi)/a)/(3*a**(5/2)*gamma(8/3))
```

3.257 $\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=578

$$\frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{5\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}\right.}{}$$

[Out] -A/a/x/(b*x^3+a)^(3/2)-1/9*(11*A*b-2*B*a)*x^2/a^2/(b*x^3+a)^(3/2)-5/27*(11*A*b-2*B*a)*x^2/a^3/(b*x^3+a)^(1/2)+5/27*(11*A*b-2*B*a)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+5/81*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-5/54*(11*A*b-2*B*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.30, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {453, 290, 303, 218, 1877}

$$\frac{5\sqrt{a+bx^3}(11Ab-2aB)}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{5\sqrt{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}(11Ab-2aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]

[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - ((11*A*b - 2*a*B)*x^2)/(9*a^2*(a + b*x^3)^(3/2)) - (5*(11*A*b - 2*a*B)*x^2)/(27*a^3*Sqrt[a + b*x^3]) + (5*(11*A*b - 2*a*B)*Sqrt[a + b*x^3])/(27*a^3*b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (5*Sqrt[2 - Sqrt[3]]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (5*Sqrt[2]*(11*A*b - 2*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*a^(8/3)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], 
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
```

```
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^2\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{11Ab}{2}-aB\right)\int \frac{x}{\left(a+bx^3\right)^{5/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{(5(11Ab-2aB))\int \frac{x}{\left(a+bx^3\right)^{3/2}}\,dx}{18a^2}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{x}{\sqrt{a+bx^3}}\,d}{54a^3}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{(5(11Ab-2aB))\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt{a+b}}}{54a^3\sqrt[3]{b}}$$

$$= -\frac{A}{ax\left(a+bx^3\right)^{3/2}} - \frac{(11Ab-2aB)x^2}{9a^2\left(a+bx^3\right)^{3/2}} - \frac{5(11Ab-2aB)x^2}{27a^3\sqrt{a+bx^3}} + \frac{5(11Ab-2aB)\sqrt{a+bx^3}}{27a^3b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{}\right.}$$

Mathematica [C] time = 0.04, size = 86, normalized size = 0.15

$$-\frac{x^2\sqrt{\frac{bx^3}{a}+1}\left(\frac{11Ab}{2}-aB\right){}_2F_1\left(\frac{2}{3},\frac{5}{2};\frac{5}{3};-\frac{bx^3}{a}\right)}{2a^3\sqrt{a+bx^3}} - \frac{A}{ax\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^3)^(3/2))) - (((11*A*b)/2 - a*B)*x^2*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[2/3, 5/2, 5/3, -((b*x^3)/a)])/(2*a^3*Sqrt[a + b*x^3])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^{11}+3\,ab^2x^8+3\,a^2bx^5+a^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^11 + 3*a*b^2*x^8 + 3*a^2*b*x^5
+ a^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

maple [B] time = 0.10, size = 1001, normalized size = 1.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x)
```

```
[Out] B*(2/9*(b*x^3+a)^(1/2)/(x^3+a/b)^2/a/b^2*x^2+10/27/((x^3+a/b)*b)^(1/2)/a^2*
x^2+10/81*I/a^2*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)
/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b
)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+A*(-2/9/a^2*x^2/b*
(b*x^3+a)^(1/2)/(x^3+a/b)^2-28/27*b/a^3*x^2/((x^3+a/b)*b)^(1/2)-1/a^3*(b*x^
3+a)^(1/2)/x-55/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2
)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-
I*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^
(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)
/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^
(1/3)/b*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*
b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3
/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^2/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^2\,\left(b\,x^3 + a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^2*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**2/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.258 $\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=610

$$-\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\,\sqrt[4]{3}\,a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\ \cdots}{\cdots}$$

```
[Out] -1/4*A/a/x^4/(b*x^3+a)^(3/2)+1/36*(-17*A*b+8*B*a)/a^2/x/(b*x^3+a)^(3/2)-11/
108*(17*A*b-8*B*a)/a^3/x/(b*x^3+a)^(1/2)+55/216*(17*A*b-8*B*a)*(b*x^3+a)^(1
/2)/a^4/x-55/216*b^(1/3)*(17*A*b-8*B*a)*(b*x^3+a)^(1/2)/a^4/(b^(1/3)*x+a^(1
/3)*(1+3^(1/2)))-55/648*b^(1/3)*(17*A*b-8*B*a)*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(11/3)*2^(1/2)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(
1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+55/432*b^(1/3)*(17*A*b-8*B
*a)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*
x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1
/4)/a^(11/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 610, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 290, 325, 303, 218, 1877}

$$-\frac{55\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,(17Ab-8aB)F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\,\sqrt[4]{3}\,a^{11/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{a+bx^3}}+\frac{55\sqrt{2-\sqrt{3}}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] -A/(4*a*x^4*(a + b*x^3)^(3/2)) - (17*A*b - 8*a*B)/(36*a^2*x*(a + b*x^3)^(3/
2)) - (11*(17*A*b - 8*a*B))/(108*a^3*x*Sqrt[a + b*x^3]) + (55*(17*A*b - 8*a
*B)*Sqrt[a + b*x^3])/(216*a^4*x) - (55*b^(1/3)*(17*A*b - 8*a*B)*Sqrt[a + b*
x^3])/(216*a^4*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (55*Sqrt[2 - Sqrt[3]]
*b^(1/3)*(17*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(144*3^(3/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x
))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (55*b^(1/3)*(1
7*A*b - 8*a*B)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sq
rt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sq
rt[3]])/(108*Sqrt[2]*3^(1/4)*a^(11/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{x^5\left(a+bx^3\right)^{5/2}}\,dx = -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{\left(\frac{17Ab}{2}-4aB\right)\int \frac{1}{x^2\left(a+bx^3\right)^{5/2}}\,dx}{4a}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{(11(17Ab-8aB))\int \frac{1}{x^2\left(a+bx^3\right)^{3/2}}\,dx}{72a^2}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} - \frac{(55(17Ab-8aB))\int \frac{}{x^2}}{216a^3}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+b}}{216a^4x}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+b}}{216a^4x}$$

$$= -\frac{A}{4ax^4\left(a+bx^3\right)^{3/2}} - \frac{17Ab-8aB}{36a^2x\left(a+bx^3\right)^{3/2}} - \frac{11(17Ab-8aB)}{108a^3x\sqrt{a+bx^3}} + \frac{55(17Ab-8aB)\sqrt{a+b}}{216a^4x}$$

Mathematica [C] time = 0.06, size = 83, normalized size = 0.14

$$\frac{x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(\frac{17Ab}{2}-4aB\right){}_2F_1\left(-\frac{1}{3},\frac{5}{2};\frac{2}{3};-\frac{bx^3}{a}\right)-a^2A}{4a^3x^4\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-(a^2*A) + ((17*A*b)/2 - 4*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[-1/3, 5/2, 2/3, -((b*x^3)/a)])/(4*a^3*x^4*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{b^3x^{14}+3\,ab^2x^{11}+3\,a^2bx^8+a^3x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/(b^3*x^14 + 3*a*b^2*x^11 + 3*a^2*b*x^8
 + a^3*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

maple [B] time = 0.12, size = 1034, normalized size = 1.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x)
```

```
[Out] A*(-1/4/a^3*(b*x^3+a)^(1/2)/x^4+21/8/a^4*b*(b*x^3+a)^(1/2)/x+2/9/a^3*x^2*(b
*x^3+a)^(1/2)/(x^3+a/b)^2+46/27*b^2/a^4*x^2/((x^3+a/b)*b)^(1/2)+935/648*I/a
^4*b*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^
3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2
)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))+B*(-2/9*(b*x^3+a)^(1/2)/(x^3+a/
b)^2/a^2/b*x^2-28/27/((x^3+a/b)*b)^(1/2)/a^3*b*x^2-(b*x^3+a)^(1/2)/a^3/x-55
/81*I/a^3*3^(1/2)*(-a*b^2)^(1/3)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/
2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/
(b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/x^5/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{x^5\,\left(b\,x^3 + a\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/(x^5*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/x**5/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.259 $\int \frac{x^8 \sqrt{c+dx^3}}{4c+dx^3} \, dx$

Optimal. Leaf size=97

$$-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}+\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] -10/9*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3-32/3*c^(5/2)*arctan(1/
3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d^3*3^(1/2)+32/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 88, 50, 63, 203}

$$\frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (32*c^2*Sqrt[c + d*x^3])/(3*d^3) - (10*c*(c + d*x^3)^(3/2))/(9*d^3) + (2*(c
 + d*x^3)^(5/2))/(15*d^3) - (32*c^(5/2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqr
t[c])])/(Sqrt[3]*d^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c\sqrt{c+dx}}{d^2}+\frac{(c+dx)^{3/2}}{d^2}+\frac{16c^2\sqrt{c+dx}}{d^2(4c+dx)}\right)dx, x, x^3\right)\\
&= -\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(16c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)}{3d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{\left(16c^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x\right.}{d^2}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{\left(32c^3\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx}\right.}{d^3}\\
&= \frac{32c^2\sqrt{c+dx^3}}{3d^3}-\frac{10c\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{32c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.79

$$\frac{2\sqrt{c+dx^3}\left(218c^2-19cdx^3+3d^2x^6\right)-480\sqrt{3}\,c^{5/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(218*c^2 - 19*c*d*x^3 + 3*d^2*x^6) - 480*Sqrt[3]*c^(5/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(45*d^3)
```

fricas [A] time = 0.62, size = 156, normalized size = 1.61

$$\left[\frac{2\left(120\sqrt{3}\sqrt{-c}\,c^2\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\left(3d^2x^6-19cdx^3+218c^2\right)\sqrt{dx^3+c}\right)}{45d^3}, -\frac{2\left(240\sqrt{3}\,c^{\frac{5}{2}}\arctan\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/45*(120*sqrt(3)*sqrt(-c)*c^2*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt
(-c) - 2*c)/(d*x^3 + 4*c)) + (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3
+ c))/d^3, -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sq
rt(c)) - (3*d^2*x^6 - 19*c*d*x^3 + 218*c^2)*sqrt(d*x^3 + c))/d^3]
```

giac [A] time = 0.17, size = 82, normalized size = 0.85

$$-\frac{32\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}-25\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+240\sqrt{dx^3+c}\,c^2d^{12}\right)}{45d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -32/3*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/4
5*(3*(d*x^3 + c)^(5/2)*d^12 - 25*(d*x^3 + c)^(3/2)*c*d^12 + 240*sqrt(d*x^3
+ c)*c^2*d^12)/d^15
```

maple [C] time = 4.46, size = 506, normalized size = 5.22

$$16\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2d^2+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d^2*(d*(2/15*x^6*(d*x^3+c)^(1/2)+2/45*c/d*x^3*(d*x^3+c)^(1/2)-4/45*c^2*(d
*x^3+c)^(1/2)/d^2)-8/9*c/d*(d*x^3+c)^(3/2))+16*c^2/d^2*(2/3*(d*x^3+c)^(1/2)
/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*d*(2*x+1/d*(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)*(d*(x-1/d*(-c*d^2)^(1/3))/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)))^(1/2)*(-1/2*I*d*(2*x+1/d*(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)))/(-c*d^2)^(1/3))^(1/2)/(d*x^3+c)^(1/2
)*(I*(-c*d^2)^(1/3)*_alpha*3^(1/2)*d-I*3^(1/2)*(-c*d^2)^(2/3)+2*_alpha^2*d^
2-(-c*d^2)^(1/3)*_alpha*d-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/
d*(-c*d^2)^(1/3)-1/2*I*3^(1/2)/d*(-c*d^2)^(1/3))*3^(1/2)*d/(-c*d^2)^(1/3))^
(1/2),1/6/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha+I*3^(1/2)*c*d-3*(-c*d^2)^(2/3)*_alpha-3*c*d)/c,(I*3^(1/2)/d*(-c*d^2
)^(1/3)/(-3/2/d*(-c*d^2)^(1/3)+1/2*I*3^(1/2)/d*(-c*d^2)^(1/3)))^(1/2)),_alp
ha=RootOf(_Z^3*d+4*c)))
```

maxima [A] time = 1.15, size = 69, normalized size = 0.71

$$-\frac{2\left(240\sqrt{3}\,c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\left(dx^3+c\right)^{\frac{5}{2}}+25\left(dx^3+c\right)^{\frac{3}{2}}c-240\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/45*(240*sqrt(3)*c^(5/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*
(d*x^3 + c)^(5/2) + 25*(d*x^3 + c)^(3/2)*c - 240*sqrt(d*x^3 + c)*c^2)/d^3
```

mupad [B] time = 4.54, size = 109, normalized size = 1.12

$$\frac{436\,c^2\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\sqrt{d\,x^3+c}}{15\,d}-\frac{38\,c\,x^3\sqrt{d\,x^3+c}}{45\,d^2}+\frac{\sqrt{3}\,c^{5/2}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)16\mathrm{i}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (436*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/(15*d) - (
38*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (3^(1/2)*c^(5/2)*log((2*3^(1/2)*c +
c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*16i)/(3*d^3)
```

sympy [A] time = 38.27, size = 85, normalized size = 0.88

$$\frac{2\left(-\frac{16\sqrt{3}\,c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{16c^2\sqrt{c+dx^3}}{3}-\frac{5c\left(c+dx^3\right)^{\frac{3}{2}}}{9}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-16*sqrt(3)*c**(5/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + 16*c
**2*sqrt(c + d*x**3)/3 - 5*c*(c + d*x**3)**(3/2)/9 + (c + d*x**3)**(5/2)/15
)/d**3
```

3.260 $\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=76

$$\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2+8/3*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1
/2))/d^2*3^(1/2)-8/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 50, 63, 203}

$$\frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}-\frac{8c\sqrt{c+dx^3}}{3d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (-8*c*Sqrt[c + d*x^3])/(3*d^2) + (2*(c + d*x^3)^(3/2))/(9*d^2) + (8*c^(3/2)
*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{(4c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx,x,x^3\right)}{3d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(4c^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx,x,x^3\right)}{d}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(8c^2\right)\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{8c\sqrt{c+dx^3}}{3d^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{8c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.86

$$\frac{24\sqrt{3}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+2\left(dx^3-11c\right)\sqrt{c+dx^3}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(-11*c + d*x^3)*Sqrt[c + d*x^3] + 24*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^2)
```

fricas [A] time = 0.69, size = 129, normalized size = 1.70

$$\left[\frac{2\left(6\sqrt{3}\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+\sqrt{dx^3+c}\left(dx^3-11c\right)\right)}{9d^2},\ \frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\sqrt{dx^3+c}\right.}{9d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [2/9*(6*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 11*c))/d^2, 2/9*(12*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 11*c))/d^2]
```

giac [A] time = 0.19, size = 64, normalized size = 0.84

$$\frac{8\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{3d^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4-12\sqrt{dx^3+c}\,cd^4\right)}{9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 8/3*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^2 + 2/9*(
(d*x^3 + c)^(3/2)*d^4 - 12*sqrt(d*x^3 + c)*c*d^4)/d^6
```

maple [C] time = 0.17, size = 446, normalized size = 5.87

$$-\frac{4\left(\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d^2+\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^2-4*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

maxima [A] time = 1.30, size = 53, normalized size = 0.70

$$\frac{2\left(12\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}-12\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] 2/9*(12*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 12*sqrt(d*x^3 + c)*c)/d^2
```

mupad [B] time = 4.28, size = 88, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}-\frac{22\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,4\mathrm{i}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d) - (22*c*(c + d*x^3)^(1/2))/(9*d^2) + (3^(1/
2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)
/(4*c + d*x^3))*4i)/(3*d^2)
```

sympy [**A**] time = 16.96, size = 68, normalized size = 0.89

$$\frac{2\left(\frac{4\sqrt{3}c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}-\frac{4c\sqrt{c+dx^3}}{3}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(4*sqrt(3)*c**(3/2)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 - 4*c*sq
rt(c + d*x**3)/3 + (c + d*x**3)**(3/2)/9)/d**2
```

3.261 $\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=57

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d}$$

```
[Out] -2/3*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d*3^(1/2)+2/3*(d*x
^3+c)^(1/2)/d
```

Rubi [A] time = 0.05, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 50, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d}-\frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d) - (2*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(Sqrt[3]*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{4c+dx}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - c\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - \frac{(2c)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}$$
$$= \frac{2\sqrt{c+dx^3}}{3d} - \frac{2\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{\sqrt{3}\,d}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.95

$$\frac{2\left(\sqrt{c+dx^3} - \sqrt{3}\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*(Sqrt[c + d*x^3] - Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[
c])]))/(3*d)
```

fricas [A] time = 0.72, size = 110, normalized size = 1.93

$$\left[\frac{\sqrt{3}\,\sqrt{-c}\,\log\left(\frac{dx^3-2\,\sqrt{3}\,\sqrt{dx^3+c}\,\sqrt{-c}-2\,c}{dx^3+4\,c}\right)+2\,\sqrt{dx^3+c}}{3\,d}, -\frac{2\left(\sqrt{3}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c)) + 2*sqrt(d*x^3 + c))/d, -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*
sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*x^3 + c))/d]
```

giac [A] time = 0.16, size = 44, normalized size = 0.77

$$-\frac{2\,\sqrt{3}\,\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{3\,d} + \frac{2\,\sqrt{dx^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d + 2/3*sq
rt(d*x^3 + c)/d
```

maple [C] time = 0.15, size = 425, normalized size = 7.46

$$\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

maxima [A] time = 1.21, size = 42, normalized size = 0.74

$$-\frac{2\left(\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-\sqrt{dx^3+c}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] -2/3*(sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - sqrt(d*
x^3 + c))/d
```

mupad [B] time = 3.87, size = 71, normalized size = 1.25

$$\frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)1\mathrm{i}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(
c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(3*d)
```

sympy [A] time = 6.10, size = 51, normalized size = 0.89

$$\frac{2\left(-\frac{\sqrt{3}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3}+\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] 2*(-sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/3 + sqrt(c +
 d*x**3)/3)/d
```

3.262 $\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)+1/6*arctan(1/3*(d*x^3+c)^(1/2
)*3^(1/2)/c^(1/2))*3^(1/2)/c^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 83, 63, 208, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(2*Sqrt[3]*Sqrt[c]) - ArcTanh[Sqr
t[c + d*x^3]/Sqrt[c]]/(6*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(4c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{12}\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{4}d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{6d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{2\sqrt{3}\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.91

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) - \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(4*c + d*x^3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(6*Sqrt[c])
```

fricas [A] time = 0.74, size = 147, normalized size = 2.26

$$\left[\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + \sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{12c}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) - 2\sqrt{-c}\,\text{ar}}{12c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, -1/12*(sqrt(3)*s
qrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c
)) - 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c]
```

giac [A] time = 0.16, size = 50, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{6\sqrt{c}} + \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/sqrt(c) + 1/6*arcta
n(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c)
```

maple [C] time = 0.19, size = 468, normalized size = 7.20

$$-\frac{\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\,\_Z^3+4c\right)^2d^2+i\left(\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x)
```

```
[Out] -1/4/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(2/3*(d*x^3+c)^(1/2)-2
/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x), x)
```

mupad [B] time = 4.66, size = 93, normalized size = 1.43

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,\sqrt{c}}+\frac{\sqrt{3}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)1\mathrm{i}}{12\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(1/2)) + (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(12*c^(1/2))
```

sympy [A] time = 13.28, size = 66, normalized size = 1.02

$$\frac{2\left(\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}+\frac{\sqrt{3}\,d\,\mathrm{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{12\sqrt{c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(d*x**3+4*c),x)
```

```
[Out] 2*(d*atan(sqrt(c + d*x**3)/sqrt(-c))/(12*sqrt(-c)) + sqrt(3)*d*atan(sqrt(3)
*sqrt(c + d*x**3)/(3*sqrt(c)))/(12*sqrt(c)))/d
```

3.263 $\int \frac{\sqrt{c+dx^3}}{x^4(4c+dx^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

```
[Out] -1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/24*d*arctan(1/3*(d*x^3+c
)^(1/2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {446, 99, 156, 63, 208, 203}

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}-\frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])
/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^4\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(4c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{cd-\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{48c} - \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{16c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{24c} - \frac{d\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{8c}\\
&= -\frac{\sqrt{c+dx^3}}{12cx^3} - \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{8\sqrt{3}\,c^{3/2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{24c^{3/2}} - \frac{\sqrt{c+dx^3}}{12cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c*x^3) - (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(8*Sqrt[3]*c^(3/2)) - (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(24*c^(3/2))
```

fricas [A] time = 0.62, size = 194, normalized size = 2.20

$$\left[-\frac{2\sqrt{3}\sqrt{c}\,dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) - \sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 4\sqrt{dx^3+c}\,c}{48\,c^2x^3}, -\frac{\sqrt{3}\sqrt{-c}\,dx^3\log\left(\frac{dx^3+2\sqrt{}\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] [-1/48*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 - sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 4*sqr
t(d*x^3 + c)*c)/(c^2*x^3), -1/48*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 + 2*sqr
```

t(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) - 2*sqrt(-c)*d*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]

giac [A] time = 0.17, size = 72, normalized size = 0.82

$$-\frac{\sqrt{3}\, d \arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{24\, c^{\frac{3}{2}}} + \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{24\sqrt{-c}\, c} - \frac{\sqrt{dx^3+c}}{12\, cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="giac")

[Out] -1/24*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/24*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/12*sqrt(d*x^3 + c)/(c*x^3)

maple [C] time = 0.19, size = 511, normalized size = 5.81

$$\left(\frac{2\sqrt{d x^3+c}}{3d} + \frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right)^2 d^2+i\left(-\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x)

[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))+1/16*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))-1/16/c^2*d*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^4), x)
```

mupad [B] time = 4.86, size = 113, normalized size = 1.28

$$\frac{d\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{48\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c\,x^3}+\frac{\sqrt{3}\,d\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)1\mathrm{i}}{48\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)
)/(48*c^(3/2)) - (c + d*x^3)^(1/2)/(12*c*x^3) + (3^(1/2)*d*log((2*3^(1/2)*c
 + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(48*c^(
3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4 \left(4c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(4*c + d*x**3)), x)
```

3.264 $\int \frac{x^4 \sqrt{c+dx^3}}{4c+dx^3}\, dx$

Optimal. Leaf size=689

$$-\frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d^{5/3}}-\frac{2\sqrt[3]{2}\,c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\tanh}{3d}$$

```
[Out] -2*2^(1/3)*c^(7/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1
/2))/d^(5/3)+2/3*2^(1/3)*c^(7/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-2
/3*2^(1/3)*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^
3+c)^(1/2))/d^(5/3)*3^(1/2)+2/3*2^(1/3)*c^(7/6)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2/7*x^2*(d*x^3+c)^(1/2)/d-50/7*c*(d*x^3+c)
^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-50/21*c^(4/3)*(c^(1/3)+d^(1/
3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)
*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+25/7*3^(1/4)*
c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {478, 584, 303, 218, 1877, 484}

$$-\frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d^{5/3}}-\frac{2\sqrt[3]{2}\,c^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{2\sqrt[3]{2}\,c^{7/6}\tanh}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*x^2*Sqrt[c + d*x^3])/(7*d) - (50*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (2*2^(1/3)*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c
^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(Sqrt[3]*d^(5/3)) + (2*2^(1/
3)*c^(7/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(Sqrt[3]*d^(5/3)) - (
2*2^(1/3)*c^(7/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c +
d*x^3]])/d^(5/3) + (2*2^(1/3)*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
d^(5/3)) + (25*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5
0*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{4c+dx^3}\,dx = \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \frac{x\left(8c^2+\frac{25}{2}cdx^3\right)}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\int \left(\frac{25cx}{2\sqrt{c+dx^3}} - \frac{42c^2x}{\sqrt{c+dx^3}\,(4c+dx^3)}\right)dx}{7d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(25c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{(12c^2)\int \frac{x}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx}{d}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}} + \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{\sqrt{3}\,d^{5/3}} - \frac{2\sqrt[3]{2}\,c^{7/6}\,\mathrm{ta}}{}$$

$$= \frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{50c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{2\sqrt[3]{2}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{\sqrt{3}\,d^{5/3}} + \frac{2\sqrt[3]{2}\,c}{}$$

Mathematica [C] time = 0.08, size = 133, normalized size = 0.19

$$\frac{-5dx^5\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) - 8cx^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) + 8x^2\left(c+dx^3\right)}{28d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (8*x^2*(c + d*x^3) - 8*c*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] - 5*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(28*d*Sqrt[c + d*x^3])
```

fricas [F] time = 5.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{dx^3+4\,c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^4}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)
```

maple [C] time = 0.26, size = 1309, normalized size = 1.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)

[Out] 1/d*(2/7*x^2*(d*x^3+c)^(1/2)-2/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-4*c/d*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\, x^4}{dx^3 + 4\,c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 + 4*c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)

[Out] int((x^4*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c + dx^3}}{4c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.265 $\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=659

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\cdots}$$

```
[Out] 1/2*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^
(1/3)/d^(2/3)-1/6*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/d^(2/3)+
1/6*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1
/2))*2^(1/3)/d^(2/3)*3^(1/2)-1/6*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)
/c^(1/2))*2^(1/3)/d^(2/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {489, 303, 218, 1877, 484}

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (c^(1/6
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/
(2^(2/3)*Sqrt[3]*d^(2/3)) - (c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
]])])/(2^(2/3)*Sqrt[3]*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(2^(2/3)*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(3*2^(2/3)*d^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/
3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/
3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(
2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{4c+dx^3}\,dx = -\left((3c)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx\right) + \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} - \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} + \frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}d^{2/3}} - \frac{\sqrt[6]{c}\ \mathrm{ta}}{}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\,\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} - \frac{\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{2^{2/3}\sqrt{3}\,d^{2/3}} + \frac{\sqrt[6]{c}\ \mathrm{tan}}{}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.10

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*
x^3)/c])/(8*Sqrt[c + d*x^3])
```

fricas [F] time = 2.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x}{dx^3+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

maple [C] time = 0.23, size = 848, normalized size = 1.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] -2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*
```

```
(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\,x}{dx^3 + 4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 + 4*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{d\,x^3 + c}}{d\,x^3 + 4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c + dx^3}}{4c + dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.266 $\int \frac{\sqrt{c+dx^3}}{x^2(4c+dx^3)}\,dx$

Optimal. Leaf size=697

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{8c^{2/3}}$$

```
[Out] -1/8*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(5/6)+1/24*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6
)-1/24*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)
^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)+1/24*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*3^(
1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c/x+1/4*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*3^(1/4)*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/
c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {475, 584, 303, 218, 1877, 484}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{8c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(4*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d
^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) + (d^(1/3)*ArcTan[
Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(4*2^(2/3)*Sqrt[3]*c^(5/6)) - (d^(1/3)*
ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(4*2^(2/3
)*c^(5/6)) + (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(12*2^(2/3)*c^(5/6)
) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*E
llipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)
*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^
(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d
^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*Sqrt[2]
*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(4c+dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\int \frac{x\left(5cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{3cdx}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\right)dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{1}{4}(3d)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}} - \frac{\sqrt[3]{d}\tanh^{-1}\left(\frac{\sqrt[6]{c}(\ldots}{\ldots}\right.}{4\,2^{2/3}c^{5\ldots}}$$

$$= -\frac{\sqrt{c+dx^3}}{4cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{4\,2^{2/3}\sqrt{3}\,c^{5/6}} + \frac{\sqrt[3]{d}\tan^{-1}\left(\ldots\right.}{4\,2^{2/3}\sqrt{\ldots}}$$

Mathematica [C] time = 0.08, size = 136, normalized size = 0.20

$$\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) + 25cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right) - 40c\left(c+dx^3\right)}{160c^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(4*c + d*x^3)),x]
[Out] (-40*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^2*x*Sqrt[c + d*x^3])
```

fricas [F] time = 1.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{dx^5+4\,cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="fricas")
[Out] integral(sqrt(d*x^3 + c)/(d*x^5 + 4*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

maple [C] time = 0.26, size = 1306, normalized size = 1.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x)
```

```
[Out] -1/4/c*d*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/
d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4
*c)))+1/4/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))
^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d
^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+
(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(dx^3 + 4c)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3 + c}}{x^2\,\left(d\,x^3 + 4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^2*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^2 \left(4c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(4*c + d*x**3)), x)
```

3.267 $\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,-1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(
1+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -(d*x^3)/(4*c), -((d*x^3)/
c)])/(16*c*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.36, size = 236, normalized size = 3.58

$$\frac{x\left(32\left(\frac{64c^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{d\left(4c+dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)}+\frac{c}{d}+x^3\right)-17x^3\sqrt{\frac{dx^3}{c}+1}\,\ldots}{80\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(4*c + d*x^3),x]
```

```
[Out] (x*(-17*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c] + 32*(c/d + x^3 + (64*c^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), -1/4*(d*x^3)/c])/(d*(4*c + d*x^3)*(-16*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3
)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*
x^3)/c]))))))/(80*Sqrt[c + d*x^3])
```

fricas [F] time = 4.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^3}{dx^3+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^3}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

maple [C] time = 0.26, size = 1003, normalized size = 15.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] 1/d*(2/5*x*(d*x^3+c)^(1/2)-2/5*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))-4*c/d*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c
*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
```

```
)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/
2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(
1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/
3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+
I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha
)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^3}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(d*x^3 + 4*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(4*c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(d*x**3+4*c),x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.268 $\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/4*x*AppellF1(1/3,-1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(4*c*Sqrt[1 + (d*x^3)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{4c+dx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.20, size = 165, normalized size = 2.58

$$\frac{16cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\left(4c+dx^3\right)\left(16cF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-2F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(4*c + d*x^3),x]
```

```
[Out] (16*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -1/4
*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^
3)/c] - 2*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{dx^3+4\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

maple [C] time = 0.14, size = 696, normalized size = 10.88

$$-\frac{2i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{(-cd^2)^{\frac{1}{3}}}{d}}{-\frac{3(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{(-cd^2)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,(-cd^2)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{(-cd^2)^{\frac{1}{3}}}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}}{\cdots}\right.}{3\sqrt{d\,x^3+c}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(d*x^3+4*c),x)
```

```
[Out] -2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/
d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
```

```
),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{dx^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(d*x^3+4*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(d*x^3 + 4*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{d\,x^3+4\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(4*c + d*x^3),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(4*c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{4c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(d*x**3+4*c),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(4*c + d*x**3), x)
```

3.269 $\int \frac{\sqrt{c+dx^3}}{x^3(4c+dx^3)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -1/8*AppellF1(-2/3,-1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(d*x^3+c)^(1/2)/c/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)/c)
])/(8*c*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(4c+dx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.14, size = 244, normalized size = 3.70

$$\frac{\frac{2048c^3dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{(4c+dx^3)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)\right)}-d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c}\right.}{256c^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(4*c + d*x^3)),x]
```

```
[Out] (-32*c*(c + d*x^3) - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c] + (2048*c^3*d*x^3*AppellF1[1/3, 1/2, 1, 4/3,
 -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1,
4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(
(d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1
/4*(d*x^3)/c]))))/(256*c^2*x^2*Sqrt[c + d*x^3])
```

fricas [F] time = 2.39, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{dx^6+4\,cx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d*x^6 + 4*c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

maple [C] time = 0.19, size = 1002, normalized size = 15.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x)
```

```
[Out] -1/4/c*d*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2))+1/3*I/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d+4*c)))+1/4/c*(-1/2*(d*x^3+c)^(1/2)/x^2-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3+4\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(d*x^3+4*c),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 + 4*c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^3\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^3\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(d*x**3+4*c),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(4*c + d*x**3)), x)
```

3.270 $\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=78

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+32/9*c^(3/2)*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))/d^3*3^(1/2)-10/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 88, 63, 203}

$$\frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}-\frac{10c\sqrt{c+dx^3}}{3d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-10*c*Sqrt[c + d*x^3])/(3*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (32*c^(3/
2)*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{5c}{d^2\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{d^2} + \frac{16c^2}{d^2\sqrt{c+dx}\,(4c+dx)}\right)dx, x, x^3\right)$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{(16c^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d^2}$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{(32c^2)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}$$
$$= -\frac{10c\sqrt{c+dx^3}}{3d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{32c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^3}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.83

$$\frac{32\sqrt{3}\,c^{3/2}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right) + 2\left(dx^3-14c\right)\sqrt{c+dx^3}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*(-14*c + d*x^3)*Sqrt[c + d*x^3] + 32*Sqrt[3]*c^(3/2)*ArcTan[Sqrt[c + d*x
^3]/(Sqrt[3]*Sqrt[c])])/(9*d^3)
```

fricas [A] time = 0.72, size = 129, normalized size = 1.65

$$\left[\frac{2\left(8\sqrt{3}\sqrt{-c}\,c\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right) + \sqrt{dx^3+c}\left(dx^3-14c\right)\right)}{9d^3}, \frac{2\left(16\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + \sqrt{dx^3+c}\right.}{9d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(8*sqrt(3)*sqrt(-c)*c*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c)
- 2*c)/(d*x^3 + 4*c)) + sqrt(d*x^3 + c)*(d*x^3 - 14*c))/d^3, 2/9*(16*sqrt(3
)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + sqrt(d*x^3 + c)*(d*
x^3 - 14*c))/d^3]
```

giac [A] time = 0.16, size = 64, normalized size = 0.82

$$\frac{32\sqrt{3}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9d^3} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6 - 15\sqrt{dx^3+c}\,cd^6\right)}{9d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 32/9*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d^3 + 2/9*
((d*x^3 + c)^(3/2)*d^6 - 15*sqrt(d*x^3 + c)*c*d^6)/d^9
```

maple [C] time = 0.24, size = 467, normalized size = 5.99

$$\frac{\left(\frac{2\sqrt{d\,x^3+c}\,x^3}{9d}-\frac{4\sqrt{d\,x^3+c}\,c}{9d^2}\right)d-\frac{8\sqrt{d\,x^3+c}\,c}{3d}}{d^2}-\frac{16ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2\cdots\right.}{\cdots}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^2*(d*(2/9/d*x^3*(d*x^3+c)^(1/2)-4/9*c*(d*x^3+c)^(1/2)/d^2)-8/3*c*(d*x^3
+c)^(1/2)/d)-16/9*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d+4*c))
```

maxima [A] time = 1.34, size = 53, normalized size = 0.68

$$\frac{2\left(16\sqrt{3}c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}-15\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(16*sqrt(3)*c^(3/2)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) + (d*x^
3 + c)^(3/2) - 15*sqrt(d*x^3 + c)*c)/d^3
```

mupad [B] time = 5.38, size = 88, normalized size = 1.13

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{28\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{\sqrt{3}\,c^{3/2}\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)16{}\mathrm{i}}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*d^2) - (28*c*(c + d*x^3)^(1/2))/(9*d^3) + (3^(
1/2)*c^(3/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^
3)/(4*c + d*x^3))*16i)/(9*d^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.271 $\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

```
[Out] -8/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))*c^(1/2)/d^2*3^(1/2)+2/3*(d
*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {446, 80, 63, 203}

$$\frac{2\sqrt{c+dx^3}}{3d^2}-\frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^2) - (8*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sq
rt[c])])/(3*Sqrt[3]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(4c)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{3d}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{(8c)\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}$$
$$= \frac{2\sqrt{c+dx^3}}{3d^2} - \frac{8\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\,d^2}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.95

$$\frac{6\sqrt{c+dx^3} - 8\sqrt{3}\sqrt{c}\,\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (6*Sqrt[c + d*x^3] - 8*Sqrt[3]*Sqrt[c]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt
[c])])/(9*d^2)
```

fricas [A] time = 0.74, size = 112, normalized size = 1.90

$$\left[\frac{2\left(2\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)+3\sqrt{dx^3+c}\right)}{9d^2}, -\frac{2\left(4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(2*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) -
2*c)/(d*x^3 + 4*c)) + 3*sqrt(d*x^3 + c))/d^2, -2/9*(4*sqrt(3)*sqrt(c)*arcta
n(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqrt(d*x^3 + c))/d^2]
```

giac [A] time = 0.16, size = 49, normalized size = 0.83

$$-\frac{2\left(\frac{4\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{d}-\frac{3\sqrt{dx^3+c}}{d}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/d - 3*s
qrt(d*x^3 + c)/d)/d
```

maple [C] time = 0.18, size = 425, normalized size = 7.20

$$\frac{2\sqrt{d\,x^3+c}}{3d^2}+\frac{4i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{Roo}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^2+4/9*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

maxima [A] time = 1.46, size = 43, normalized size = 0.73

$$-\frac{2\left(4\sqrt{3}\sqrt{c}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*sqr
t(d*x^3 + c))/d^2
```

mupad [B] time = 4.86, size = 71, normalized size = 1.20

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^2}+\frac{\sqrt{3}\,\sqrt{c}\,\ln\left(\frac{2\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)4\mathrm{i}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (3^(1/2)*c^(1/2)*log((2*3^(1/2)*c + c^(1/2)
*(c + d*x^3)^(1/2)*6i - 3^(1/2)*d*x^3)/(4*c + d*x^3))*4i)/(9*d^2)
```

sympy [A] time = 15.58, size = 65, normalized size = 1.10

$$\begin{cases} \frac{2\left(-\frac{4\sqrt{3}\,\sqrt{c}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d}+\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for}\ d\neq 0 \\ \frac{x^6}{24c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-4*sqrt(3)*sqrt(c)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))
/(9*d) + sqrt(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(24*c**(3/2)), True))
```

3.272 $\int \frac{x^2}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

```
[Out] 2/9*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(1/2))/d*3^(1/2)/c^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {444, 63, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*Sqrt[c]*d)
```

fricas [A] time = 0.70, size = 87, normalized size = 2.18

$$\left[-\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3-2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)}{9cd}, \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/9*sqrt(3)*sqrt(-c)*log((d*x^3 - 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*
c)/(d*x^3 + 4*c))/(c*d), 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqr
t(c))/(sqrt(c)*d)]
```

giac [A] time = 0.18, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

maple [C] time = 0.18, size = 413, normalized size = 10.32

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
```

```
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*d+4*c))
```

maxima [**A**] time = 1.24, size = 29, normalized size = 0.72

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/(sqrt(c)*d)
```

mupad [**B**] time = 5.21, size = 56, normalized size = 1.40

$$\frac{\sqrt{3}\ \ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{2\,d\,x^3+8\,c}\right)\,1\mathrm{i}}{9\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*log((c^(1/2)*(c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(
8*c + 2*d*x^3))*1i)/(9*c^(1/2)*d)
```

sympy [**A**] time = 10.09, size = 37, normalized size = 0.92

$$\frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*sqrt(3)*atan(sqrt(3)*sqrt(c + d*x**3)/(3*sqrt(c)))/(9*sqrt(c)*d)
```

3.273 $\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

```
[Out] -1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/18*arctan(1/3*(d*x^3+c)^(1/
2)*3^(1/2)/c^(1/2))/c^(3/2)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 86, 63, 208, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\,\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])]/(6*Sqrt[3]*c^(3/2)) - ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]]/(6*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\sqrt{c+dx^3}\,(4c+dx^3)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{12c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{6c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{6cd}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{6\sqrt{3}\,c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6c^{3/2}}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.91

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)+3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/18*(Sqrt[3]*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])] + 3*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]])/c^(3/2)
```

fricas [A] time = 0.79, size = 148, normalized size = 2.28

$$\left[-\frac{2\sqrt{3}\sqrt{c}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)-3\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{36c^2}, -\frac{\sqrt{3}\sqrt{-c}\log\left(\frac{dx^3+2\sqrt{3}\sqrt{dx^3+c}\sqrt{-c}-2c}{dx^3+4c}\right)-6\sqrt{-c}}{36c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(2*sqrt(3)*sqrt(c)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c)) - 3*s
qrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, -1/36*(sqrt
(3)*sqrt(-c)*log((d*x^3 + 2*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3
+ 4*c)) - 6*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/c^2]
```

giac [A] time = 0.17, size = 53, normalized size = 0.82

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right)}{18\,c^{\frac{3}{2}}}+\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(3/2) + 1/6*arc
tan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

maple [C] time = 0.19, size = 433, normalized size = 6.66

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{6c^{\frac{3}{2}}}+\frac{i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/36*I/c^2/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d+4*c))-1/6*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^{3}+4\,c\right)\sqrt{dx^{3}+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x), x)
```

mupad [B] time = 5.51, size = 94, normalized size = 1.45

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{12\,c^{3/2}}+\frac{\sqrt{3}\,\ln\left(\frac{2\,\sqrt{3}\,c-\sqrt{3}\,d\,x^3+\sqrt{c}\,\sqrt{d\,x^3+c}\,6{}\mathrm{i}}{d\,x^3+4\,c}\right)\,1{}\mathrm{i}}{36\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(1
2*c^(3/2)) + (3^(1/2)*log((2*3^(1/2)*c + c^(1/2)*(c + d*x^3)^(1/2)*6i - 3^(
1/2)*d*x^3)/(4*c + d*x^3))*1i)/(36*c^(3/2))
```

sympy [A] time = 12.25, size = 63, normalized size = 0.97

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^{3}}}{\sqrt{-c}}\right)}{6c\sqrt{-c}}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}\,\sqrt{c+dx^{3}}}{3\sqrt{c}}\right)}{18c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**3)/sqrt(-c))/(6*c*sqrt(-c)) - sqrt(3)*atan(sqrt(3)*sqrt(
c + d*x**3)/(3*sqrt(c)))/(18*c**(3/2))
```

3.274 $\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=88

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}-\frac{\sqrt{c+dx^3}}{12c^2x^3}$$

```
[Out] 1/8*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/72*d*arctan(1/3*(d*x^3+c)^
(1/2)*3^(1/2)/c^(1/2))/c^(5/2)*3^(1/2)-1/12*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {446, 103, 156, 63, 208, 203}

$$-\frac{\sqrt{c+dx^3}}{12c^2x^3}+\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(12*c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])
])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{3cd+\frac{d^2x}{2}}{x\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{12c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{16c^2} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}\,(4c+dx)}\,dx, x, x^3\right)}{48c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{8c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{3c+x^2}\,dx, x, \sqrt{c+dx^3}\right)}{24c^2}\\
&= -\frac{\sqrt{c+dx^3}}{12c^2x^3} + \frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{24\sqrt{3}\,c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{8c^{5/2}} - \frac{\sqrt{c+dx^3}}{12c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -1/12*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c
])])/(24*Sqrt[3]*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(8*c^(5/2)
)
```

fricas [A] time = 0.66, size = 194, normalized size = 2.20

$$\left[\frac{2\sqrt{3}\sqrt{c}\,dx^3\arctan\left(\frac{\sqrt{3}\sqrt{dx^3+c}}{3\sqrt{c}}\right) + 9\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 12\sqrt{dx^3+c}\,c}{144\,c^3x^3}, -\frac{\sqrt{3}\sqrt{-c}\,dx^3\log\left(\frac{dx^3-2\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/144*(2*sqrt(3)*sqrt(c)*d*x^3*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))
 + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 12*
sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/144*(sqrt(3)*sqrt(-c)*d*x^3*log((d*x^3 - 2
```

```
*sqrt(3)*sqrt(d*x^3 + c)*sqrt(-c) - 2*c)/(d*x^3 + 4*c)) + 18*sqrt(-c)*d*x^3
*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

giac [A] time = 0.16, size = 72, normalized size = 0.82

$$\frac{\sqrt{3}\,d\arctan\left(\frac{\sqrt{3}\,\sqrt{dx^3+c}}{3\,\sqrt{c}}\right)}{72\,c^{\frac{5}{2}}}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{8\,\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{12\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/72*sqrt(3)*d*arctan(1/3*sqrt(3)*sqrt(d*x^3 + c)/sqrt(c))/c^(5/2) - 1/8*d*
arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/12*sqrt(d*x^3 + c)/(c^2
*x^3)
```

maple [C] time = 0.23, size = 477, normalized size = 5.42

$$\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{24c^{\frac{5}{2}}}+\frac{\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}}{4c}-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/4/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-1/144*I/d/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha
*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d+4*c))+1/24*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^4), x)
```

mupad [B] time = 5.72, size = 112, normalized size = 1.27

$$\frac{d\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)}{16\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{12\,c^2\,x^3}+\frac{\sqrt{3}\,d\,\ln\left(\frac{\sqrt{3}\,d\,x^3-2\,\sqrt{3}\,c+\sqrt{c}\,\sqrt{d\,x^3+c}\,6\mathrm{i}}{d\,x^3+4\,c}\right)\,1\mathrm{i}}{144\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (d*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)
)/(16*c^(5/2)) - (c + d*x^3)^(1/2)/(12*c^2*x^3) + (3^(1/2)*d*log((c^(1/2)*(
c + d*x^3)^(1/2)*6i - 2*3^(1/2)*c + 3^(1/2)*d*x^3)/(4*c + d*x^3))*1i)/(144*
c^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.275 $\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=667

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^5}$$

```
[Out] 2/3*2^(1/3)*c^(1/6)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(
1/2))/d^(5/3)-2/9*2^(1/3)*c^(1/6)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+
2/9*2^(1/3)*c^(1/6)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x
^3+c)^(1/2))/d^(5/3)*3^(1/2)-2/9*2^(1/3)*c^(1/6)*arctan(1/3*(d*x^3+c)^(1/2)
*3^(1/2)/c^(1/2))/d^(5/3)*3^(1/2)+2*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))+2/3*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(
1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 667, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {483, 303, 218, 1877, 484}

$$\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{d^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (2*2^(1
/3)*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (2*2^(1/3)*c^(1/6)*ArcTan[Sqrt[c + d*x^3]/(
Sqrt[3]*Sqrt[c])])/(3*Sqrt[3]*d^(5/3)) + (2*2^(1/3)*c^(1/6)*ArcTanh[(c^(1/6
)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*d^(5/3)) - (2*2^(1/3)
*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(9*d^(5/3)) - (3^(1/4)*Sqrt[2 -
Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3
)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c
^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c
 + d*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d} - \frac{(4c)\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{d}$$

$$= \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\sqrt{3}d^{5/3}} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tanh^{-1}\left(\right.}{3d^{5}}$$

$$= \frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{5/3}} - \frac{2\sqrt[3]{2}\sqrt[6]{c}\tan^{-1}}{3\sqrt{3}d^{5}}$$

Mathematica [C] time = 0.04, size = 67, normalized size = 0.10

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{20c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(20*c*Sqrt[c + d*x^3])
```

fricas [F] time = 4.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{d^2x^6+5\,cdx^3+4\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.25, size = 848, normalized size = 1.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
```

```
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*El
lipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+4/9*I/d^4*2^(1/2)*sum(1/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-
3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{c + dx^3}\,\left(4c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**4/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.276 $\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=206

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

```
[Out] -1/6*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2^(1/3)/c
^(5/6)/d^(2/3)+1/18*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)/d^(2/3
)-1/18*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*
2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)+1/18*arctan(1/3*(d*x^3+c)^(1/2)*3^(1/2)/c^(
1/2))*2^(1/3)/c^(5/6)/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3
*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) + ArcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c]
)]/(3*2^(2/3)*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3
)*d^(1/3)*x))/Sqrt[c + d*x^3]]/(3*2^(2/3)*c^(5/6)*d^(2/3)) + ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]]/(9*2^(2/3)*c^(5/6)*d^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{3\ 2^{2/3}\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\ 2^{2/3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{9\ 2^{2/3}c^{5/6}d^{2/3}}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.33

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{8c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(8*c*Sqrt[c + d*x^3])
```

fricas [B] time = 1.83, size = 2274, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt
(1/3)*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c
^5*d^4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^
5*d^4))^(5/6))*sqrt(d*x^3 + c) + (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*
d)*(-1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) + 3*(sqrt(3)*sqrt(1/3)*c^3*
d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(
1/6) - 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5
/6))*sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3
)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/
2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3)
 + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*
d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c
*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c)
)/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) + 1/9*s
qrt(3)*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*arctan(-1/3*(3*(sqrt(3)*sqrt(1/3)
*c^3*d^2*x*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^
4))^(1/6) + 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 + 4*c^5*d^3)*(-1/(c^5*d^4
))^(5/6))*sqrt(d*x^3 + c) - (2*sqrt(3)*(1/2)^(1/3)*(c^2*d^2*x^3 + c^3*d)*(-
1/(c^5*d^4))^(1/3) + sqrt(3)*(d*x^4 + c*x) - 3*(sqrt(3)*sqrt(1/3)*c^3*d^2*x
*sqrt(-1/(c^5*d^4)) + 2*sqrt(3)*(1/432)^(1/6)*c*d*x^2*(-1/(c^5*d^4))^(1/6)
- 24*sqrt(3)*(1/432)^(5/6)*(c^4*d^4*x^3 - 2*c^5*d^3)*(-1/(c^5*d^4))^(5/6))*
sqrt(d*x^3 + c))*sqrt((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(2/3)*(c^
4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*(1/2)^(1
/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1/3) - 12
*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c^3*d^4*x
^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)*(c*d^3*
x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^
3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)))/(d*x^4 + c*x)) - 1/36*(1/43
2)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/
2)^(2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) +
 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4)
)^(1/3) + 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3
)*(c^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(
1/6)*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x
^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/36*(1/432)^(
1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 60*c*d^2*x^6 - 32*c^3 - 24*(1/2)^(
2/3)*(c^4*d^5*x^7 + 5*c^5*d^4*x^4 + 4*c^6*d^3*x)*(-1/(c^5*d^4))^(2/3) + 12*
(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5*d^4))^(1
/3) - 12*(648*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) - sqrt(1/3)*(c
^3*d^4*x^6 - 16*c^4*d^3*x^3 - 8*c^5*d^2)*sqrt(-1/(c^5*d^4)) - (1/432)^(1/6)
*(c*d^3*x^7 + 2*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6))*sqrt(d*x^3 +
 c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) + 1/18*(1/432)^(1/6)
*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d*x^3 - 32*c^3 +
 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-1/(c^5*d^4))^(2
/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(-1/(c^5
*d^4))^(1/3) + 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^4))^(5/6) + sqr
```

```
t(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1/(c^5*d^4)) + 2
*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/(c^5*d^4))^(1/6
))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)) - 1/1
8*(1/432)^(1/6)*(-1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 66*c*d^2*x^6 - 72*c^2*d
*x^3 - 32*c^3 + 48*(1/2)^(2/3)*(c^4*d^5*x^7 - c^5*d^4*x^4 - 2*c^6*d^3*x)*(-
1/(c^5*d^4))^(2/3) + 12*(1/2)^(1/3)*(c^2*d^4*x^8 - 7*c^3*d^3*x^5 - 8*c^4*d^
2*x^2)*(-1/(c^5*d^4))^(1/3) - 6*(1296*(1/432)^(5/6)*c^5*d^5*x^5*(-1/(c^5*d^
4))^(5/6) + sqrt(1/3)*(5*c^3*d^4*x^6 - 20*c^4*d^3*x^3 - 16*c^5*d^2)*sqrt(-1
/(c^5*d^4)) + 2*(1/432)^(1/6)*(c*d^3*x^7 - 16*c^2*d^2*x^4 - 8*c^3*d*x)*(-1/
(c^5*d^4))^(1/6))*sqrt(d*x^3 + c))/(d^3*x^9 + 12*c*d^2*x^6 + 48*c^2*d*x^3 +
 64*c^3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx^3 + 4c)\sqrt{dx^3 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.18, size = 416, normalized size = 2.02

$$i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right.\right.$$

$$-\overline{\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_
alpha=RootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(dx^3 + 4c)\sqrt{dx^3 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

mupad [B] time = 25.80, size = 453, normalized size = 2.20

$$\frac{\sqrt{3}\ 314928^{1/3}\ \ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{3}\ \sqrt{-c}-2^{1/3}\ \sqrt{3}\ (-c)^{1/6}\,d^{1/3}\,x\right)^3\left(54\ \sqrt{d\,x^3+c}-54\ \sqrt{3}\ \sqrt{-c}+54\,2^{1/3}\ \sqrt{3}\ (-c)^{1/6}\,d^{1/3}\,x\right)}{\left(d^{1/3}\,x-2^{2/3}\,(-c)^{1/3}\right)^6}\right)}{2916\,(-c)^{5/6}\,d^{2/3}}+\frac{\sqrt{3}\ 314928^{1/3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] (3^(1/2)*314928^(1/3)*log((((c + d*x^3)^(1/2) + 3^(1/2)*(-c)^(1/2) - 2^(1/3
)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(54*(c + d*x^3)^(1/2) - 54*3^(1/2)*(-c)^(
1/2) + 54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(d^(1/3)*x - 2^(2/3)*(-c)^
(1/3))^6))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)*314928^(1/3)*log(((2*3^(1/2
)*(-c)^(1/2) - 2*(c + d*x^3)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*3i + 2^(1
/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d*x^3)^(1/2) + 108*3^(1/2)*(-
c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i + 54*2^(1/3)*3^(1/2)*(-c)^(1/6
)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3) - 2^(2/3)*3^(1/2)*(-c)^(1/3
)*1i)^6)*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(2916*(-c)^(5/6)*d^(2/3)) + (3^(1/2)
*314928^(1/3)*log(((2*(c + d*x^3)^(1/2) + 2*3^(1/2)*(-c)^(1/2) - 2^(1/3)*(-
c)^(1/6)*d^(1/3)*x*3i + 2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x)^3*(108*(c + d
*x^3)^(1/2) - 108*3^(1/2)*(-c)^(1/2) + 2^(1/3)*(-c)^(1/6)*d^(1/3)*x*162i -
54*2^(1/3)*3^(1/2)*(-c)^(1/6)*d^(1/3)*x))/(2*d^(1/3)*x + 2^(2/3)*(-c)^(1/3)
 + 2^(2/3)*3^(1/2)*(-c)^(1/3)*1i)^6)*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/(2916
*(-c)^(5/6)*d^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.277 $\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=697

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\cdots}}{8c^{5/3}\sqrt{\cdots}}$$

```
[Out] 1/24*d^(1/3)*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*d^(1/3)*x)/(d*x^3+c)^(1/2))*2
^(1/3)/c^(11/6)-1/72*d^(1/3)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*2^(1/3)/c^(11
/6)+1/72*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+2^(1/3)*d^(1/3)*x)*3^(1/2)/(d*x^3+
c)^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/72*d^(1/3)*arctan(1/3*(d*x^3+c)^(1/2)*
3^(1/2)/c^(1/2))*2^(1/3)/c^(11/6)*3^(1/2)-1/4*(d*x^3+c)^(1/2)/c^2/x+1/4*d^(
1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/12*d^(1/3)*(c^(1
/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)
/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/8*
3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)
))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 697, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {480, 584, 303, 218, 1877, 484}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{2\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\cdots}}{8c^{5/3}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(4*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(4*c^2*((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)) + (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + 2^(1/
3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) - (d^(1/3)*A
rcTan[Sqrt[c + d*x^3]/(Sqrt[3]*Sqrt[c])])/(12*2^(2/3)*Sqrt[3]*c^(11/6)) + (
d^(1/3)*ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*d^(1/3)*x))/Sqrt[c + d*x^3]])/(
12*2^(2/3)*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(36*2^(2/
3)*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (
d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x
^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*
c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])
/(2*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{\int \frac{x\left(cd+\frac{d^2x^3}{2}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} - \frac{cdx}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\right)dx}{4c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{8c^2} - \frac{d\int \frac{x}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx}{4c}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\,2^{2/3}\sqrt{3}\,c^{11/6}} - \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{3}\sqrt{c}}\right)}{12\,2^{2/3}\sqrt{3}\,c^{11/6}} + \frac{\sqrt[3]{d}\tanh^{-}\cdots}{1\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{4c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{4c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12\,2^{2/3}\sqrt{3}\,c^{11/6}} - \frac{\sqrt[3]{d}\cdots}{1\cdots}$$

Mathematica [C] time = 0.10, size = 136, normalized size = 0.20

$$\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+5cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-40c\left(c+dx^3\right)}{160c^3x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (-40*c*(c + d*x^3) + 5*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), -1/4*(d*x^3)/c] + d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(160*c^3*x*Sqrt[c + d*x^3])
```

fricas [F] time = 2.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^8+5\,cdx^5+4\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^8 + 5*c*d*x^5 + 4*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

maple [C] time = 0.19, size = 874, normalized size = 1.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/36*I/c^2/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d+4*c))+1/4/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c
*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{c + dx^3}\,\left(4c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.278 $\int \frac{x^3}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

```
[Out] 1/16*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/
(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -(d*x^3)/(4*c), -((d*x^
3)/c)])/(16*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{16c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x
^3)/c])/(16*c*Sqrt[c + d*x^3])
```

fricas [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^3}{d^2x^6+5\,cdx^3+4\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.24, size = 696, normalized size = 10.55

$$-\frac{2i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\text{EllipticF}\left(\cdots\right.}{3\sqrt{d\,x^3+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+4/9*
I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
```

```
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(dx^3+4c)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{c+dx^3}\,(4c+dx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.279 $\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(4*c*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{4c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.07, size = 165, normalized size = 2.58

$$\frac{16cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\sqrt{c+dx^3}\left(4c+dx^3\right)\left(16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)-3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] (16*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/(Sqrt[c +
 d*x^3]*(4*c + d*x^3)*(16*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -1/4*(
d*x^3)/c] - 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/
c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])))
```

fricas [F] time = 1.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^6+5\,cdx^3+4\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^6 + 5*c*d*x^3 + 4*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.15, size = 416, normalized size = 6.50

$$i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\Big(2\,\mathrm{RootOf}\left(d\_Z^3+4c\right.$$

$$-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/9*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d+4*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(1/((c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c+dx^3}\,\left(4c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.280 $\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/8*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,-1/4*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -(d*x^3)/(4*c), -((d*x^3)
/c)])/(8*c*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{c+dx^3}\left(4c+dx^3\right)}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(4c+dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{dx^3}{4c},-\frac{dx^3}{c}\right)}{8cx^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.18, size = 243, normalized size = 3.68

$$\frac{-\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{c^3}+\frac{2048dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)}{\left(4c+dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)+2F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)-16cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{dx^3}{4c}\right)\right)}-32}{256x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[c + d*x^3]*(4*c + d*x^3)),x]
```

```
[Out] ((-32*(c + d*x^3))/c^2 - (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/c^3 + (2048*d*x^3*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c])/((4*c + d*x^3)*(-16*c*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 3*d*x^3*(AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -1/4*(d*x^3)/c] + 2*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -1/4*(d*x^3)/c]))))/(256*x^2*Sqrt[c + d*x^3])
```

fricas [F] time = 3.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^9+5\,cdx^6+4\,c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^9 + 5*c*d*x^6 + 4*c^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

maple [C] time = 0.19, size = 722, normalized size = 10.94

$$\frac{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\cdots\right)\cdots}{\left(-c\cdots\right)}}}{3}\cdots\right)\cdots}{6\sqrt{d\,x^3+c}\;c}\cdots}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/36*I/c^2/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/6*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d+4*c))+1/4/c*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(
```

```
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3 + 4\,c\right)\sqrt{dx^3 + c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(d*x^3+4*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + 4*c)*sqrt(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(d\,x^3+4\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(4*c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{c + dx^3}\,\left(4c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(d*x**3+4*c)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(c + d*x**3)*(4*c + d*x**3)), x)
```

3.281 $\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx$

Optimal. Leaf size=127

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

```
[Out] -1/6*arctanh((1+2^(1/3)*x)/(-x^3+1)^(1/2))*2^(1/3)+1/18*arctanh((-x^3+1)^(1
/2))*2^(1/3)-1/18*arctan((1-2^(1/3)*x)*3^(1/2)/(-x^3+1)^(1/2))*2^(1/3)*3^(1
/2)+1/18*arctan(1/3*(-x^3+1)^(1/2)*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {484}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt[3]{2}x+1}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] -ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]]/(3*2^(2/3)*Sqrt[3]) + ArcT
an[Sqrt[1 - x^3]/Sqrt[3]]/(3*2^(2/3)*Sqrt[3]) - ArcTanh[(1 + 2^(1/3)*x)/Sqr
t[1 - x^3]]/(3*2^(2/3)) + ArcTanh[Sqrt[1 - x^3]]/(9*2^(2/3))
```

Rule 484

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Simp[(q*ArcTanh[Sqrt[c + d*x^3]/Rt[c, 2]])/(9*2^(2/3)*
b*Rt[c, 2]), x] + (-Simp[(q*ArcTanh[(Rt[c, 2]*(1 - 2^(1/3)*q*x))/Sqrt[c + d
*x^3]])/(3*2^(2/3)*b*Rt[c, 2]), x] + Simp[(q*ArcTan[Sqrt[c + d*x^3]/(Sqrt[3
]*Rt[c, 2])])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]), x] - Simp[(q*ArcTan[(Sqrt[3]*
Rt[c, 2]*(1 + 2^(1/3)*q*x))/Sqrt[c + d*x^3]])/(3*2^(2/3)*Sqrt[3]*b*Rt[c, 2]
), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[4*b*c - a*d,
0] && PosQ[c]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^3}\left(4-x^3\right)}\,dx=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}x\right)}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{1+\sqrt[3]{2}x}{\sqrt{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\tanh^{-1}\left(\sqrt{1-x^3}\right)}{9\ 2^{2/3}}$$

Mathematica [C] time = 0.03, size = 28, normalized size = 0.22

$$\frac{1}{8}x^2F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};x^3,\frac{x^3}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^3]*(4 - x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/2, 1, 5/3, x^3, x^3/4])/8
```

fricas [B] time = 0.84, size = 1191, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/31104*432^(5/6)*sqrt(3)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2
/3)*(x^8 - 5*x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 -
 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3
888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) - 1/31104*432
^(5/6)*sqrt(3)*log(36*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*
x^5 + 4*x^2) + (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*
x) - 216*432^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(
x^7 - x^4) - 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3
)*log(144*(36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2)
 - (2592*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432
^(1/6)*sqrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) -
 2304)/(x^9 - 12*x^6 + 48*x^3 - 64)) + 1/31104*432^(5/6)*sqrt(3)*log(36*(36
*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6
- 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3
)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
 12*x^6 + 48*x^3 - 64)) - 1/1944*432^(5/6)*arctan(1/216*sqrt(-x^3 + 1)*(72*
432^(1/6)*x^2 + 432^(5/6)*x + 72*sqrt(3))/(2*x^3 - 1)) + 1/3888*432^(5/6)*a
rctan(-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4
) + 18*432^(1/6)*(x^5 + 8*x^2)) + (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sq
rt(3)*2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) - sqrt(-x^3 + 1)*(432^(5/
6)*(2*x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqr
t((36*x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) + (2592
*x^6 - 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*s
qrt(3)*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(
x^9 - 12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4)) + 1/3888*432^(5/6)*arctan(
-1/648*(6*sqrt(-x^3 + 1)*(432^(5/6)*(x^4 + 2*x) - 36*sqrt(3)*(x^3 - 4) + 18
*432^(1/6)*(x^5 + 8*x^2)) - (108*sqrt(3)*2^(2/3)*(x^5 - x^2) - 216*sqrt(3)*
2^(1/3)*(x^4 - x) - 108*sqrt(3)*(x^6 - x^3) + sqrt(-x^3 + 1)*(432^(5/6)*(2*
x^4 + x) - 36*sqrt(3)*(5*x^3 - 8) - 18*432^(1/6)*(x^5 - 10*x^2)))*sqrt((36*
x^9 - 8208*x^6 + 9504*x^3 - 648*2^(2/3)*(x^8 - 5*x^5 + 4*x^2) - (2592*x^6 -
 2592*x^3 - 432^(5/6)*sqrt(3)*(x^7 - 26*x^4 + 16*x) - 216*432^(1/6)*sqrt(3)
*(7*x^5 - 4*x^2))*sqrt(-x^3 + 1) + 3888*2^(1/3)*(x^7 - x^4) - 2304)/(x^9 -
12*x^6 + 48*x^3 - 64)))/(x^6 + 3*x^3 - 4))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

maple [C] time = 0.80, size = 164, normalized size = 1.29

$$i\sqrt{i\left(2x+1-i\sqrt{3}\right)}\,\sqrt{\frac{x-1}{i\sqrt{3}-3}}\,\sqrt{-\frac{i\left(2x+1+i\sqrt{3}\right)}{2}}\,\left(-2\,\mathrm{RootOf}\left(\_Z^3-4\right)^2+\mathrm{RootOf}\left(\_Z^3-4\right)+1+i\sqrt{3}\,\left(-\mathrm{RootOf}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+4)/(-x^3+1)^(1/2),x)
```

```
[Out] 1/36*I*2^(1/2)*sum(_alpha^2*(1/2*I*(2*x+1-I*3^(1/2)))^(1/2)*((x-1)/(I*3^(1/
2)-3))^(1/2)*(-1/2*I*(2*x+1+I*3^(1/2)))^(1/2)/(-x^3+1)^(1/2)*(-2*_alpha^2+_
alpha+1+I*3^(1/2)*(1-_alpha))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2
))*3^(1/2))^(1/2),1/2*_alpha-1/3*I*_alpha^2*3^(1/2)-1/2+1/6*I*_alpha*3^(1/2
)+1/6*I*3^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2)),_alpha=RootOf(_Z^3-
4))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(x^3-4\right)\sqrt{-x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+4)/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((x^3 - 4)*sqrt(-x^3 + 1)), x)
```

mupad [B] time = 3.42, size = 653, normalized size = 5.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((1 - x^3)^(1/2)*(x^3 - 4)),x)
```

```
[Out] - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1
/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)
/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1
/2)*1i)/2 + 3/2)/(2^(2/3) - 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*(1 - x^3)^(1/2)*(2^
(2/3) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i
)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1
i)/2 + 3/2)*(x^3 - 1)^(1/2)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 -
3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x
- 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(2/
3)*((3^(1/2)*1i)/2 + 1/2) + 1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2
)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 + 1
/2)*(1 - x^3)^(1/2)*(2^(2/3)*((3^(1/2)*1i)/2 + 1/2) + 1)*(((3^(1/2)*1i)/2 -
 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 +
1/2) + 1) + x^3)^(1/2)) - (2^(1/3)*((3^(1/2)*1i)/2 + 3/2)*(x^3 - 1)^(1/2)*(
-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i
)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^
(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(2/3)*((3^(1/2)*1i)/2 - 1/2) -
1), asin((-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/
((3^(1/2)*1i)/2 - 3/2)))/(3*((3^(1/2)*1i)/2 - 1/2)*(1 - x^3)^(1/2)*(2^(2/3)
*((3^(1/2)*1i)/2 - 1/2) - 1)*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2)
 - x*(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) + x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^3\sqrt{1-x^3}-4\sqrt{1-x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+4)/(-x**3+1)**(1/2),x)
```

```
[Out] -Integral(x/(x**3*sqrt(1 - x**3) - 4*sqrt(1 - x**3)), x)
```

3.282 $\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=111

$$\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

```
[Out] -38/3*c^2*(d*x^3+c)^(3/2)/d^4-4/5*c*(d*x^3+c)^(5/2)/d^4-2/21*(d*x^3+c)^(7/2
)/d^4+1024*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-1024/3*c^3*(d*x
^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.10, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 88, 50, 63, 206}

$$-\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}+\frac{1024c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-1024*c^3*Sqrt[c + d*x^3])/(3*d^4) - (38*c^2*(c + d*x^3)^(3/2))/(3*d^4) -
(4*c*(c + d*x^3)^(5/2))/(5*d^4) - (2*(c + d*x^3)^(7/2))/(21*d^4) + (1024*c^
(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2\sqrt{c+dx}}{d^3}+\frac{512c^3\sqrt{c+dx}}{d^3(8c-dx)}-\frac{6c(c+dx)^{3/2}}{d^3}-\frac{(c+dx)^{5/2}}{d^3}\right)dx, x, x^3\right)$$
$$= -\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(512c^3\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right.}{3d^3}$$
$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(1536c^4\right)\text{Sul}}{}$$
$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{\left(3072c^4\right)\text{Su}}{}$$
$$= -\frac{1024c^3\sqrt{c+dx^3}}{3d^4}-\frac{38c^2\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{4c\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^4}+\frac{1024c^{7/2}\tan}{d}$$

Mathematica [A] time = 0.11, size = 81, normalized size = 0.73

$$\frac{107520c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(18632c^3+764c^2dx^3+57cd^2x^6+5d^3x^9\right)}{105d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(18632*c^3 + 764*c^2*d*x^3 + 57*c*d^2*x^6 + 5*d^3*x^9)
+ 107520*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(105*d^4)
```

fricas [A] time = 0.71, size = 169, normalized size = 1.52

$$\left[\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(5\,d^3x^9+57\,cd^2x^6+764\,c^2dx^3+18632\,c^3\right)\sqrt{dx^3+c}\right)}{105\,d^4}, -\frac{2\left(53760\,\sqrt{-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/105*(26880*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^
3 + c))/d^4, -2/105*(53760*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + (5*d^3*x^9 + 57*c*d^2*x^6 + 764*c^2*d*x^3 + 18632*c^3)*sqrt(d*x^3 + c
))/d^4]
```

giac [A] time = 0.16, size = 100, normalized size = 0.90

$$-\frac{1024\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{2\left(5\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+42\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+665\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+17920\sqrt{dx^3+c}\,c\,c^3\right.}{105\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1024*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/105*(5*(d
*x^3 + c)^(7/2)*d^24 + 42*(d*x^3 + c)^(5/2)*c*d^24 + 665*(d*x^3 + c)^(3/2)*
c^2*d^24 + 17920*sqrt(d*x^3 + c)*c^3*d^24)/d^28
```

maple [C] time = 0.35, size = 582, normalized size = 5.24

$$-\frac{512\left(\frac{2\sqrt{d x^3+c}}{3d}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3-8c\right)^2 \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/21*x^9*(d*x^3+c)^(1/2)+2/105*c/d*x^6*(d*x^3+c)^(1/2)-8/315*c^2/d^2*
x^3*(d*x^3+c)^(1/2)+16/315*c^3*(d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/15*(d*x^3+c)
^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3+c)^(1/2)*c^2/d^2)-128/9
*c^2*(d*x^3+c)^(3/2)/d^4-512*c^3/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

maxima [A] time = 1.31, size = 96, normalized size = 0.86

$$-\frac{2\left(26880\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+5\left(dx^3+c\right)^{\frac{7}{2}}+42\left(dx^3+c\right)^{\frac{5}{2}}c+665\left(dx^3+c\right)^{\frac{3}{2}}c^2+17920\sqrt{dx^3+c}\,c^3\right)}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/105*(26880*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 5*(d*x^3 + c)^(7/2) + 42*(d*x^3 + c)^(5/2)*c + 665*(d*x^3 + c
)^(3/2)*c^2 + 17920*sqrt(d*x^3 + c)*c^3)/d^4
```

mupad [B] time = 3.51, size = 118, normalized size = 1.06

$$\frac{512\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{37264\,c^3\,\sqrt{d\,x^3+c}}{105\,d^4}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}-\frac{38\,c\,x^6\,\sqrt{d\,x^3+c}}{35\,d^2}-\frac{1528\,c^2\,x^3\,\sqrt{d\,x^3+}}{105\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (512*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^4 - (37264*c^3*(c + d*x^3)^(1/2))/(105*d^4) - (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) - (38*c*x^6*(c + d*x^3)^(1/2))/(35*d^2) - (1528*c^2*x^3*(c + d*x^3
)^(1/2))/(105*d^3)
```

sympy [A] time = 60.27, size = 99, normalized size = 0.89

$$\frac{2\left(-\frac{512c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{512c^3\sqrt{c+dx^3}}{3}-\frac{19c^2\left(c+dx^3\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+dx^3\right)^{\frac{5}{2}}}{5}-\frac{\left(c+dx^3\right)^{\frac{7}{2}}}{21}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-512*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 512*c**3*sqrt(c
 + d*x**3)/3 - 19*c**2*(c + d*x**3)**(3/2)/3 - 2*c*(c + d*x**3)**(5/2)/5 -
(c + d*x**3)**(7/2)/21)/d**4
```

3.283 $\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=90

$$\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] -14/9*c*(d*x^3+c)^(3/2)/d^3-2/15*(d*x^3+c)^(5/2)/d^3+128*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-128/3*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{128c^2\sqrt{c+dx^3}}{3d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-128*c^2*Sqrt[c + d*x^3])/(3*d^3) - (14*c*(c + d*x^3)^(3/2))/(9*d^3) - (2*
(c + d*x^3)^(5/2))/(15*d^3) + (128*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c\sqrt{c+dx}}{d^2}+\frac{64c^2\sqrt{c+dx}}{d^2(8c-dx)}-\frac{(c+dx)^{3/2}}{d^2}\right)dx,x,x^3\right)\\
&= -\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(64c^2\right)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{3d^2}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(192c^3\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{d^2}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{\left(384c^3\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c\right.}}{d^3}\\
&= -\frac{128c^2\sqrt{c+dx^3}}{3d^3}-\frac{14c\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}+\frac{128c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.78

$$\frac{5760c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(998c^2+41cdx^3+3d^2x^6\right)}{45d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(998*c^2 + 41*c*d*x^3 + 3*d^2*x^6) + 5760*c^(5/2)*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^3)
```

fricas [A] time = 0.71, size = 147, normalized size = 1.63

$$\left[\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(3\,d^2x^6+41\,cdx^3+998\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^3},-\frac{2\left(2880\,\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+}}{3}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1440*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 41*c*d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3, -2/45*(28
80*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 41*c*
d*x^3 + 998*c^2)*sqrt(d*x^3 + c))/d^3]
```

giac [A] time = 0.17, size = 83, normalized size = 0.92

$$-\frac{128\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}+35\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+960\,\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -128*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^12 + 35*(d*x^3 + c)^(3/2)*c*d^12 + 960*sqrt(d*x^3 + c)*c^2*
d^12)/d^15
```

maple [C] time = 0.16, size = 507, normalized size = 5.63

$$-\frac{64\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2 d^2 \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*((2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3+
c)^(1/2)*c^2/d^2)*d+16/9*(d*x^3+c)^(3/2)*c/d)-64*c^2/d^2*(2/3*(d*x^3+c)^(1/
2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.21, size = 82, normalized size = 0.91

$$-\frac{2\left(1440\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+35\left(dx^3+c\right)^{\frac{3}{2}}c+960\sqrt{dx^3+c}\,c^2\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1440*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 35*(d*x^3 + c)^(3/2)*c + 960*sqrt(d*x^3 +
 c)*c^2)/d^3
```

mupad [B] time = 3.40, size = 98, normalized size = 1.09

$$\frac{64\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{1996\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}-\frac{82\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (64*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^3 - (1996*c^2*(c + d*x^3)^(1/2))/(45*d^3) - (2*x^6*(c + d*x^3)^(1/2))/(
15*d) - (82*c*x^3*(c + d*x^3)^(1/2))/(45*d^2)
```

sympy [A] time = 30.23, size = 82, normalized size = 0.91

$$\frac{2\left(-\frac{64c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{64c^2\sqrt{c+dx^3}}{3}-\frac{7c\left(c+dx^3\right)^{\frac{3}{2}}}{9}-\frac{\left(c+dx^3\right)^{\frac{5}{2}}}{15}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-64*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 64*c**2*sqrt(c +
 d*x**3)/3 - 7*c*(c + d*x**3)**(3/2)/9 - (c + d*x**3)**(5/2)/15)/d**3
```

3.284 $\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=69

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2+16*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^
2-16/3*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {446, 80, 50, 63, 206}

$$\frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\sqrt{c+dx^3}}{3d^2}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-16*c*Sqrt[c + d*x^3])/(3*d^2) - (2*(c + d*x^3)^(3/2))/(9*d^2) + (16*c^(3/
2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{c+dx^3}}{8c-dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{(8c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(24c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{d}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{\left(48c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d^2}\\
&= -\frac{16c\sqrt{c+dx^3}}{3d^2} - \frac{2\left(c+dx^3\right)^{3/2}}{9d^2} + \frac{16c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.84

$$\frac{144c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(25c+dx^3\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(25*c + d*x^3) + 144*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^2)
```

fricas [A] time = 0.61, size = 121, normalized size = 1.75

$$\left[\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right) - \left(dx^3+25\,c\right)\sqrt{dx^3+c}\right)}{9\,d^2}, -\frac{2\left(72\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right) + \left(dx^3+25\,c\right)\right.}{9\,d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/9*(36*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2, -2/9*(72*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 25*c)*sqrt(d*x^3 + c))/d^2]
```

giac [A] time = 0.21, size = 65, normalized size = 0.94

$$-\frac{16\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^2} - \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4 + 24\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -16*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/9*((d*x^3 +
 c)^(3/2)*d^4 + 24*sqrt(d*x^3 + c)*c*d^4)/d^6
```

maple [C] time = 0.18, size = 446, normalized size = 6.46

$$-\frac{8\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right)^2d^2+\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^2-8*c/d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum
((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*
(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.30, size = 66, normalized size = 0.96

$$-\frac{2\left(36\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+24\sqrt{dx^3+c}\,c\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/9*(36*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 24*sqrt(d*x^3 + c)*c)/d^2
```

mupad [B] time = 3.51, size = 78, normalized size = 1.13

$$\frac{8\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{50\,c\,\sqrt{d\,x^3+c}}{9\,d^2}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (8*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d^2 - (50*c*(c + d*x^3)^(1/2))/(9*d^2) - (2*x^3*(c + d*x^3)^(1/2))/(9*d)
```

sympy [A] time = 14.91, size = 65, normalized size = 0.94

$$\frac{2\left(-\frac{8c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{8c\sqrt{c+dx^3}}{3}-\frac{\left(c+dx^3\right)^{\frac{3}{2}}}{9}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] 2*(-8*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - 8*c*sqrt(c + d*x*
*3)/3 - (c + d*x**3)**(3/2)/9)/d**2
```

3.285 $\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

```
[Out] 2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d-2/3*(d*x^3+c)^(1/2)/d
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{2\sqrt{c+dx^3}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d) + (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)$$
$$= -\frac{2\sqrt{c+dx^3}}{3d} + (3c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{(6c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{d}$$
$$= -\frac{2\sqrt{c+dx^3}}{3d} + \frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.94

$$-\frac{2\left(\sqrt{c+dx^3}-3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*(Sqrt[c + d*x^3] - 3*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(3*
d)
```

fricas [A] time = 0.65, size = 101, normalized size = 2.02

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-2\sqrt{dx^3+c}}{3d}, -\frac{2\left(3\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+\sqrt{dx^3+c}\right)}{3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 2*sqrt(d*x^3 + c))/d, -2/3*(3*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + sqrt(d*x^3 + c))/d]
```

giac [A] time = 0.16, size = 43, normalized size = 0.86

$$-\frac{2c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{2\sqrt{dx^3+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -2*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/3*sqrt(d*x^3 + c
)/d
```

maple [C] time = 0.15, size = 425, normalized size = 8.50

$$-\frac{2\sqrt{d\,x^3+c}}{3d}-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootC}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d-1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.15, size = 56, normalized size = 1.12

$$-\frac{3\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+2\sqrt{dx^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 2*sqrt(d*x^3 + c))/d
```

mupad [B] time = 3.50, size = 59, normalized size = 1.18

$$\frac{\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] (c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d
 - (2*(c + d*x^3)^(1/2))/(3*d)
```

sympy [A] time = 5.11, size = 46, normalized size = 0.92

$$\frac{2\left(-\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] 2*(-c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/sqrt(-c) - sqrt(c + d*x**3)/3)/d
```

3.286 $\int \frac{\sqrt{c+dx^3}}{x(8c-dx^3)}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

```
[Out] 1/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))/c^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 83, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(4*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/
Sqrt[c]]/(12*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)}\,dx, x, x^3\right)$$
$$= \frac{1}{24}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{8}(3d)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12d}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.91

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)),x]
```

```
[Out] (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(12*Sqrt[c])
```

fricas [A] time = 0.73, size = 138, normalized size = 2.38

$$\left[\frac{3\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + \sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)}{24c}, \frac{\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{c}\right) - 3\sqrt{-c}\arctan\left(\frac{\sqrt{dx}}{}\right.}{12c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c, 1/12*(
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c))/c]
```

giac [A] time = 0.16, size = 48, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/4*arctan(1/3*sqrt(d*x^3
+ c)/sqrt(-c))/sqrt(-c)
```

maple [C] time = 0.16, size = 468, normalized size = 8.07

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\,\_Z^3-8c\right)^2 d^2+i\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8/c*d*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/8/c*(2/3*(d*x^3+c)^(1/2)
-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x), x)
```

mupad [B] time = 4.69, size = 125, normalized size = 2.16

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(6\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^3\left(24\,c^2-24\,c^{3/2}\,\sqrt{d\,x^3+c}+d^2\,x^6-20\,c\,d\,x^3\right)^3}{x^{15}\left(8\,c-d\,x^3\right)^3\left(24\,c-d\,x^3\right)^3}\right)}{24\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))*(6*c + d
*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^3*(24*c^2 - 24*c^(3/2)*(c + d*x^3)^(1/2
```

```
) + d^2*x^6 - 20*c*d*x^3)^3)/(x^15*(8*c - d*x^3)^3*(24*c - d*x^3)^3))/(24*c
^(1/2))
```

sympy [A] time = 8.22, size = 60, normalized size = 1.03

$$\frac{2\left(-\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{8\sqrt{-c}}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{24\sqrt{-c}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c),x)
```

```
[Out] 2*(-d*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(8*sqrt(-c)) + d*atan(sqrt(c + d*
x**3)/sqrt(-c))/(24*sqrt(-c)))/d
```

3.287 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=81

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

```
[Out] 1/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-5/96*d*arctanh((d*x^3+c
)^(1/2)/c^(1/2))/c^(3/2)-1/24*(d*x^3+c)^(1/2)/c/x^3
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{\operatorname{Subst}\left(\int \frac{5cd+\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{(5d)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c}+\frac{\left(3d^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{64c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{5\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c}+\frac{(3d)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{32c}\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32c^{3/2}}-\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}-\frac{\sqrt{c+dx^3}}{24cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3
2*c^(3/2)) - (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*c^(3/2))
```

fricas [A] time = 0.72, size = 186, normalized size = 2.30

$$\left[\frac{3\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+5\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\sqrt{dx^3+c}\,c}{192\,c^2x^3}, \frac{5\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(3*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) + 5*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/
x^3) - 8*sqrt(d*x^3 + c)*c)/(c^2*x^3), 1/96*(5*sqrt(-c)*d*x^3*arctan(sqrt(d
*x^3 + c)*sqrt(-c)/c) - 3*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c
)/c) - 4*sqrt(d*x^3 + c)*c)/(c^2*x^3)]
```

giac [A] time = 0.17, size = 73, normalized size = 0.90

$$\frac{5\, d \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\, \sqrt{-c}\, c} - \frac{d \arctan\left(\frac{\sqrt{dx^3+c}}{3\, \sqrt{-c}}\right)}{32\, \sqrt{-c}\, c} - \frac{\sqrt{dx^3+c}}{24\, cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 5/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/32*d*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/(c*x^3)
```

maple [C] time = 0.18, size = 511, normalized size = 6.31

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}}{3d} + \frac{i\left(-c\,d^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\, \sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\, \left(2\,\mathrm{RootOf}\left(d\,\_Z^3-8c\right)^2 d^2+i\left(\cdots\right.\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))-1/64*d^2/c^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3
)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*d*(2/3*(
d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^4), x)
```

mupad [B] time = 3.75, size = 69, normalized size = 0.85

$$\frac{d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{32\,\sqrt{c^3}}-\frac{5\,d\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}-\frac{\sqrt{d\,x^3+c}}{24\,c\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(32*(c^3)^(1/2)) - (5*d*at
anh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)))/(96*(c^3)^(1/2)) - (c + d*x^3)^(1/2
)/(24*c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

3.288 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

```
[Out] 1/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/256*d^2*arctanh((d
*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/48*(d*x^3+c)^(1/2)/c/x^6-1/64*d*(d*x^3+c)^
(1/2)/c^2/x^3
```

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}-\frac{d\sqrt{c+dx^3}}{64c^2x^3}-\frac{\sqrt{c+dx^3}}{48cx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c*x^6) - (d*Sqrt[c + d*x^3])/(64*c^2*x^3) + (d^2*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(5/2)) + (d^2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/(256*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{48cx^6} + \frac{\operatorname{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$
$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{\operatorname{Subst}\left(\int \frac{6c^2d^2-3cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$
$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{512c^2} + \frac{\left(3d^3\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{512c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} - \frac{d\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{256c^2} + \frac{\left(3d^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\right.}{256c^2}$$
$$= -\frac{\sqrt{c+dx^3}}{48cx^6} - \frac{d\sqrt{c+dx^3}}{64c^2x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{5/2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{5/2}}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.90

$$\frac{3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + 3d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - 4\sqrt{c}\sqrt{c+dx^3}\left(4c+3dx^3\right)}{768c^{5/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 3*d*x^3) + 3*d^2*x^6*ArcTanh[Sqrt[c + d*
x^3]/(3*Sqrt[c])] + 3*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(5/2
)*x^6)
```

fricas [A] time = 0.57, size = 188, normalized size = 1.76

$$\left[\frac{3\sqrt{c}\,d^2x^6\log\left(\frac{d^2x^6+24\,cdx^3+8\left(dx^3+4\,c\right)\sqrt{dx^3+c}\,\sqrt{c}+32\,c^2}{dx^6-8\,cx^3}\right)-8\left(3\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{1536\,c^3x^6},-\frac{3\sqrt{-c}\,d^2x^6\arctan\left(\frac{\left(dx^3+4\,c\right)\sqrt{d}}{4\left(cdx^3\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(3*sqrt(c)*d^2*x^6*log((d^2*x^6 + 24*c*d*x^3 + 8*(d*x^3 + 4*c)*sqrt
(d*x^3 + c)*sqrt(c) + 32*c^2)/(d*x^6 - 8*c*x^3)) - 8*(3*c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^3*x^6), -1/768*(3*sqrt(-c)*d^2*x^6*arctan(1/4*(d*x^3 + 4*
c)*sqrt(d*x^3 + c)*sqrt(-c)/(c*d*x^3 + c^2)) + 4*(3*c*d*x^3 + 4*c^2)*sqrt(d
*x^3 + c))/(c^3*x^6)]
```

giac [A] time = 0.18, size = 100, normalized size = 0.93

$$-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}\,c^2}-\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}\,c^2}-\frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2+\sqrt{dx^3+c}\,cd^2}{192\,c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/256*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/192*(3*(d*x^3 + c)^(3/2
)*d^2 + sqrt(d*x^3 + c)*c*d^2)/(c^2*d^2*x^6)
```

maple [C] time = 0.19, size = 574, normalized size = 5.36

$$-\left(\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^3-8c\right)^2d^2+i\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(1/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*d*(d*x^3+c)^(1/2)/c/x^3+1/12
*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-1/512/c^3*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
```

```
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))+1/512/c^3*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d
*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 3.91, size = 83, normalized size = 0.78

$$\frac{d^2\,\mathrm{atanh}\left(\frac{d^4\,\sqrt{d\,x^3+c}}{2048\,c^{7/2}\,\left(\frac{d^4}{2048\,c^3}+\frac{d^5\,x^3}{8192\,c^4}\right)}\right)}{256\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{192\,c\,x^6}-\frac{{\left(d\,x^3+c\right)}^{3/2}}{64\,c^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((d^4*(c + d*x^3)^(1/2))/(2048*c^(7/2)*(d^4/(2048*c^3) + (d^5*x^3
)/(8192*c^4)))))/(256*c^(5/2)) - (c + d*x^3)^(1/2)/(192*c*x^6) - (c + d*x^3
)^(3/2)/(64*c^2*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^7+dx^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**7 + d*x**10), x)
```

3.289 $\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=648

$$-\frac{32\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{12248\sqrt{2}\,c^{7/3}\left(\sqrt[3]{c}+\right.}{}$$

```
[Out] 32*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8
/3)-32*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32*c^(13/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)-2
14/91*c*x^2*(d*x^3+c)^(1/2)/d^2-2/13*x^5*(d*x^3+c)^(1/2)/d-12248/91*c^2*(d*
x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-12248/273*c^(7/3)*(c^(
1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+612
4/91*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(
1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.89, antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {478, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{32c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{32c^{13/6}\tanh^{-1}}{d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-214*c*x^2*Sqrt[c + d*x^3])/(91*d^2) - (2*x^5*Sqrt[c + d*x^3])/(13*d) - (1
2248*c^2*Sqrt[c + d*x^3])/(91*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x))
- (32*Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(8/3) + (32*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1
/6)*Sqrt[c + d*x^3])])/d^(8/3) - (32*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/d^(8/3) + (6124*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(8/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3]) - (12248*Sqrt[2]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
F[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(91*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{2\int \frac{x^4\left(40c^2+\frac{107}{2}cdx^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{13d}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{4\int \frac{x(856c^3d+1531c^2d^2x^3)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}+\frac{4\int \left(-\frac{1531c^2dx}{\sqrt{c+dx^3}}+\frac{13104c^3dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{(6124c^2)\int \frac{x}{\sqrt{c+dx^3}}dx}{91d^2}+\frac{(576c^3)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}dx}{d^2}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{(48c^2)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{d^3}-\frac{(6124c^2)\int \frac{(1-}{}}{91d}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{6124\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32\sqrt{3}\,c^{13/6}\tan^{-1}\left(\right.}{d^{8/3}}$$

$$= -\frac{214cx^2\sqrt{c+dx^3}}{91d^2}-\frac{2x^5\sqrt{c+dx^3}}{13d}-\frac{12248c^2\sqrt{c+dx^3}}{91d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{32\sqrt{3}\,c^{13/6}\tan^{-1}\left(\right.}{d^{8/3}}$$

Mathematica [C] time = 0.10, size = 150, normalized size = 0.23

$$\frac{2140c^2x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+1531cdx^5\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20\left(107c^2x^2+114cd\right.}{910d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-20*(107*c^2*x^2 + 114*c*d*x^5 + 7*d^2*x^8) + 2140*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 1531*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(910*d^2*Sqrt[c + d*x^3])
```

fricas [F] time = 44.32, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^7}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3 + c}\, x^7}{dx^3 - 8\,c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

maple [C] time = 0.24, size = 1788, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/13*x^5*(d*x^3+c)^(1/2)+6/91*c*x^2*(d*x^3+c)^(1/2)/d+8/91*I*c^2
/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+8*c*(2/7*(d*x^3+c)^(1/2)*x^2-2/
7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))))-64*c^2/d^2*(-2/3*I*3^(1/2)*
(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
```

```
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3
)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}\,x^7}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
[Out] -Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.290 $\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=624

$$-\frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{118\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{7\sqrt[4]{3}\ldots}$$

[Out] 4*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)-4*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2/7*x^2*(d*x^3+c)^(1/2)/d-118/7*c*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-118/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+59/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.75, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {478, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{4c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{4c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{118\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{7\sqrt[4]{3}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d) - (118*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(5/3) + (4*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(5/3) - (4*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(5/3) + (59*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (118*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{8c-dx^3}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \frac{x\left(16c^2+\frac{59}{2}cdx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} + \frac{2\int \left(-\frac{59cx}{2\sqrt{c+dx^3}} + \frac{252c^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(59c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d} + \frac{\left(72c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{(6c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} - \frac{(59c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{7d^{4/3}} + \frac{\left(6c^{4/3}\right)\int}{}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{59\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}}{((1}}}{7d^{5/3}\sqrt{\frac{}{((1}}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/}}{}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d} - \frac{118c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}} + \frac{4c^{7/}}{}$$

Mathematica [C] time = 0.08, size = 130, normalized size = 0.21

$$\frac{x^2\left(59dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80\left(c+dx^3\right)\right)}{280d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-80*(c + d*x^3) + 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 59*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(280*d*Sqrt[c + d*x^3])
```

fricas [F] time = 11.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}\,x^4}{dx^3-8\,c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3 + c}\, x^4}{dx^3 - 8\, c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

maple [C] time = 0.17, size = 1310, normalized size = 2.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/7*(d*x^3+c)^(1/2)*x^2-2/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)))-8*c/d*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+
1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)
/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)
*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2
+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^
2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3 + c}\, x^4}{dx^3 - 8\, c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.291 $\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}$$

```
[Out] 1/2*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-1/2*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-1/2*c^(1/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {489, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[
3]*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(2*d^(2/3)) + (c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c +
 d*x^3])])/(2*d^(2/3)) - (c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*
d^(2/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^
(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt
[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(3^(1/4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{8c-dx^3}\,dx = (9c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx - \int \frac{x}{\sqrt{c+dx^3}}\,dx$$

$$= -\frac{3\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{4d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{\sqrt[3]{d}} + \frac{\left(3\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{d}} - \frac{\left(\sqrt{2\left(2\right.}\right.}{}\cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,E\left(\sin^{-1}\left(\cdots\right.\right.}{d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}\cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}} - \cdots$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.10

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/
(8*c)])/(16*Sqrt[c + d*x^3])
```

fricas [F] time = 2.35, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3+c}\,x}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

maple [C] time = 0.16, size = 848, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x)
```

```
[Out] 2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/3*I/d^3*2^(1/2)*sum(1/_alph
a*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
```

```
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3 + c}\,x}{dx^3 - 8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{d\,x^3 + c}}{8\,c - d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3),x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{c + dx^3}}{-8c + dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.292 $\int \frac{\sqrt{c+dx^3}}{x^2(8c-dx^3)}\,dx$

Optimal. Leaf size=632

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{16c^{2/}}$$

```
[Out] 1/16*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(
5/6)-1/16*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-1/16*d^(1/3)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(5/6
)-1/8*(d*x^3+c)^(1/2)/c/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/
3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipti
cE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {475, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{16c^{2/}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(8*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c*((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]])/(16*c^(5/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(
1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*c^(5/6)) - (d^(1/3)*ArcTanh[Sqr
t[c + d*x^3]/(3*Sqrt[c])])/(16*c^(5/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3
)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3
) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*c
^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c
^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(2/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \frac{x\left(13cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{9cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{1}{8}(9d)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{16c}$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} - \frac{3\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{32c} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{16c} + \frac{\left(3d^{2/3}\right)\int \frac{1+\cdots}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\cdots}}{32c^{2/3}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}}\cdots}{16c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\cdots\right)}{16c^{5}\cdots}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{8cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{16c^{5/6}} + \frac{\sqrt[3]{d}\tanh^{-1}\left(\cdots\right)}{16c^{5}\cdots}\cdots$$

Mathematica [C] time = 0.11, size = 137, normalized size = 0.22

$$\frac{-d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+65cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(c+dx^3\right)}{640c^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-80*c*(c + d*x^3) + 65*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^2*x*Sqrt[c + d*x^3])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^5-8\,cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^5 - 8*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

maple [C] time = 0.17, size = 1306, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)
*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^
2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/8/c*d*
(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_al
pha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c),x)
```

```
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

3.293 $\int \frac{\sqrt{c+dx^3}}{x^5(8c-dx^3)}\,dx$

Optimal. Leaf size=654

$$-\frac{\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}}+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{}}{\left(\left(1+\sqrt{}\right.\right.}}}{8\sqrt{2}\sqrt[4]{3}}$$

```
[Out] 1/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/128*d^
(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c
^(11/6)-1/32*(d*x^3+c)^(1/2)/c/x^4-1/16*d*(d*x^3+c)^(1/2)/c^2/x+1/16*d^(4/3
)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(4/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^
(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {475, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{11/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*c*x^4) - (d*Sqrt[c + d*x^3])/(16*c^2*x) + (d^(4/3)*Sqr
t[c + d*x^3])/(16*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(11/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^
3])])/(128*c^(11/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*
c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^
(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^
(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(
8*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{32cx^4} + \frac{\int \frac{16cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \frac{x\left(-100c^2d^2+8cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{\int \left(-\frac{8cd^2x}{\sqrt{c+dx^3}} - \frac{36c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^2} + \frac{\left(9d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{64c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} - \frac{(3d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c^2} + \frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{32c^2} + -$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{32cx^4} - \frac{d\sqrt{c+dx^3}}{16c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{16c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{128c^{11/6}}$$

Mathematica [C] time = 0.14, size = 153, normalized size = 0.23

$$\frac{125cd^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 4\left(d^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 40c\left(c^2+3cdx^3+2d^2x^6\right)\right)}{5120c^3x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (125*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 3*c*d*x^3 + 2*d^2*x^6) + d^3*x^9*Sqrt[1 +
(d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(5120*
c^3*x^4*Sqrt[c + d*x^3])
```

fricas [F] time = 1.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^8-8\,cx^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^8 - 8*c*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3 + c}}{(dx^3 - 8\,c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

maple [C] time = 0.18, size = 1782, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*d*(d*x^3+c)^(1/2)/c/x-1/8*I*d/c*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-
c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64*d^2/c^2*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
```

```
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

3.294 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=678

$$-\frac{\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{\cdots}}{\left((1+\sqrt{\cdots}\right.}}}{56\sqrt{2}\,\sqrt[4]{3}\cdots}$$

```
[Out] 1/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1024
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-1/56*(d*x^3+c)^(1/2)/c/x^7-19/1792*d*(d*x^3+c)^(1/2)/c^2/x^4+1/1
12*d^2*(d*x^3+c)^(1/2)/c^3/x-1/112*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))-1/336*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/224*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {475, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{17/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{1024c^{17/6}}-\frac{d^{7/3}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(56*c*x^7) - (19*d*Sqrt[c + d*x^3])/(1792*c^2*x^4) + (d^2*
Sqrt[c + d*x^3])/(112*c^3*x) - (d^(7/3)*Sqrt[c + d*x^3])/(112*c^3*((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)) - (Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(17/6)) + (d^(7/3)*ArcTanh[(c^
(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(17/6)) - (d^(7/
3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(17/6)) + (3^(1/4)*Sqrt[2
- Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
+ d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1
- Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
4*Sqrt[3]])/(224*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (d^(7/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(56*Sqrt[2]*3^(1/4)*c^(8/3)*
Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{56cx^7}+\frac{\int \frac{19cd+\frac{11d^2x^3}{2}}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}-\frac{\int \frac{128c^2d^2-\frac{95}{2}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}+\frac{\int \frac{x\left(-260c^3d^3+64c^2d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}+\frac{\int \left(-\frac{64c^2d^3x}{\sqrt{c+dx^3}}+\frac{252c^3d^3x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{224c^3}+\frac{\left(9d^3\right)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{512c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{\left(3d^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^3}-\frac{d^{8/3}\int \frac{(1-}{}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\,d^{7/3}\tan^{-}}{10}$$

$$= -\frac{\sqrt{c+dx^3}}{56cx^7}-\frac{19d\sqrt{c+dx^3}}{1792c^2x^4}+\frac{d^2\sqrt{c+dx^3}}{112c^3x}-\frac{d^{7/3}\sqrt{c+dx^3}}{112c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{\sqrt{3}\,d^{7/3}\tan^{-}}{10}$$

Mathematica [C] time = 0.11, size = 164, normalized size = 0.24

$$\frac{32d^4x^{12}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-325cd^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-160c\left(32c^3+51c^2dx^3+3\right.}{286720c^4x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (-160*c*(32*c^3 + 51*c^2*d*x^3 + 3*c*d^2*x^6 - 16*d^3*x^9) - 325*c*d^3*x^9*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 32*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(286720*c^4*x^7*Sqrt[c + d*x^3])
```

fricas [F] time = 3.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{dx^{11}-8\,cx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d*x^11 - 8*c*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

maple [C] time = 0.18, size = 2280, normalized size = 3.36

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x)
```

```
[Out] 1/64/c^2*d*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*(d*x^3+c)^(1/2)/c*d/x-1/8*I*d/c*3^
(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/x^7-3/56*d
*(d*x^3+c)^(1/2)/c/x^4+15/112*d^2*(d*x^3+c)^(1/2)/c^2/x+5/112*I*d^2/c^2*3^(
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^2*(-(d*x^3+c)^(1/2)/x-I*3^(
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
```

```
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/512/c^3*d^3*(-2/3*I*3^(1/2)*(-c*d^2)^
(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3 + c}}{x^8\,\left(8\,c - d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{c + dx^3}}{-8cx^8 + dx^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c),x)
[Out] -Integral(sqrt(c + d*x**3)/(-8*c*x**8 + d*x**11), x)
```

3.295 $\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=130

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+d\right.}{27c}$$

```
[Out] -1024/9*c^3*(d*x^3+c)^(3/2)/d^4-38/5*c^2*(d*x^3+c)^(5/2)/d^4-4/7*c*(d*x^3+c
)^(7/2)/d^4-2/27*(d*x^3+c)^(9/2)/d^4+9216*c^(9/2)*arctanh(1/3*(d*x^3+c)^(1/
2)/c^(1/2))/d^4-3072*c^4*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.11, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}+\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+\right.}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-3072*c^4*Sqrt[c + d*x^3])/d^4 - (1024*c^3*(c + d*x^3)^(3/2))/(9*d^4) - (3
8*c^2*(c + d*x^3)^(5/2))/(5*d^4) - (4*c*(c + d*x^3)^(7/2))/(7*d^4) - (2*(c
+ d*x^3)^(9/2))/(27*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{57c^2(c+dx)^{3/2}}{d^3}+\frac{512c^3(c+dx)^{3/2}}{d^3(8c-dx)}-\frac{6c(c+dx)^{5/2}}{d^3}-\frac{(c+dx)^{7/2}}{d^3}\right)dx, x, x$$
$$= -\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}+\frac{\left(512c^3\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x\right.}{3d^3}$$
$$= -\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+dx^3\right)^{9/2}}{27d^4}+\frac{\left(1536c^4\right)\text{S}}{}$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+\right.}{27}$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+\right.}{27}$$
$$= -\frac{3072c^4\sqrt{c+dx^3}}{d^4}-\frac{1024c^3\left(c+dx^3\right)^{3/2}}{9d^4}-\frac{38c^2\left(c+dx^3\right)^{5/2}}{5d^4}-\frac{4c\left(c+dx^3\right)^{7/2}}{7d^4}-\frac{2\left(c+\right.}{27}$$

Mathematica [A] time = 0.13, size = 93, normalized size = 0.72

$$\frac{9216c^{9/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}-\frac{2\sqrt{c+dx^3}\left(1509176c^4+61892c^3dx^3+4611c^2d^2x^6+410cd^3x^9+35d^4x^{12}\right)}{945d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1509176*c^4 + 61892*c^3*d*x^3 + 4611*c^2*d^2*x^6 + 410
*c*d^3*x^9 + 35*d^4*x^12))/(945*d^4) + (9216*c^(9/2)*ArcTanh[Sqrt[c + d*x^3
]/(3*Sqrt[c])])/d^4
```

fricas [A] time = 0.46, size = 191, normalized size = 1.47

$$\left[\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(35\,d^4x^{12}+410\,cd^3x^9+4611\,c^2d^2x^6+61892\,c^3dx^3+1509176\,c^4\right)\sqrt{dx\right.}{945\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/945*(2177280*c^(9/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x
^3 - 8*c)) - (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2*d^2*x^6 + 61892*c^3*d*
x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4, -2/945*(4354560*sqrt(-c)*c^4*arcta
```

n(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (35*d^4*x^12 + 410*c*d^3*x^9 + 4611*c^2 *d^2*x^6 + 61892*c^3*d*x^3 + 1509176*c^4)*sqrt(d*x^3 + c))/d^4]

giac [A] time = 0.17, size = 117, normalized size = 0.90

$$-\frac{9216\,c^{5}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^{4}}-\frac{2\left(35\left(dx^{3}+c\right)^{\frac{9}{2}}d^{32}+270\left(dx^{3}+c\right)^{\frac{7}{2}}cd^{32}+3591\left(dx^{3}+c\right)^{\frac{5}{2}}c^{2}d^{32}+53760\left(dx^{3}+\cdots\right.\right.}{945\,d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")

[Out] -9216*c^5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/945*(35*(d*x^3 + c)^(9/2)*d^32 + 270*(d*x^3 + c)^(7/2)*c*d^32 + 3591*(d*x^3 + c)^(5/2)*c^2*d^32 + 53760*(d*x^3 + c)^(3/2)*c^3*d^32 + 1451520*sqrt(d*x^3 + c)*c^4*d^32)/d^36

maple [C] time = 0.25, size = 634, normalized size = 4.88

$$-\frac{512\left(\frac{2\sqrt{d\,x^{3}+c}\,x^{3}}{9}+\frac{56\sqrt{d\,x^{3}+c}\;c}{9d}+\frac{3ic\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right.}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)

[Out] -1/d*(2/27*d*x^12*(d*x^3+c)^(1/2)+20/189*c*x^9*(d*x^3+c)^(1/2)+2/315*c^2/d*x^6*(d*x^3+c)^(1/2)-8/945*c^3/d^2*x^3*(d*x^3+c)^(1/2)+16/945*c^4/d^3*(d*x^3+c)^(1/2))-8*c/d^2*(2/21*d*x^9*(d*x^3+c)^(1/2)+16/105*c*x^6*(d*x^3+c)^(1/2)+2/105*c^2/d*x^3*(d*x^3+c)^(1/2)-4/105*c^3/d^2*(d*x^3+c)^(1/2))-128/15*c^2*(d*x^3+c)^(5/2)/d^4-512*c^3/d^3*(2/9*x^3*(d*x^3+c)^(1/2)+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))

maxima [A] time = 1.29, size = 110, normalized size = 0.85

$$-\frac{2\left(2177280\,c^{\frac{9}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+35\left(dx^3+c\right)^{\frac{9}{2}}+270\left(dx^3+c\right)^{\frac{7}{2}}c+3591\left(dx^3+c\right)^{\frac{5}{2}}c^2+53760\left(dx^3+c\right)^{\frac{3}{2}}c^3\right.}{945\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/945*(2177280*c^(9/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c)
+ 3*sqrt(c))) + 35*(d*x^3 + c)^(9/2) + 270*(d*x^3 + c)^(7/2)*c + 3591*(d*x^
3 + c)^(5/2)*c^2 + 53760*(d*x^3 + c)^(3/2)*c^3 + 1451520*sqrt(d*x^3 + c)*c^
4)/d^4
```

mupad [B] time = 3.53, size = 135, normalized size = 1.04

$$\frac{4608\,c^{9/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}-\frac{2\,x^{12}\,\sqrt{d\,x^3+c}}{27}-\frac{3018352\,c^4\,\sqrt{d\,x^3+c}}{945\,d^4}-\frac{164\,c\,x^9\,\sqrt{d\,x^3+c}}{189\,d}-\frac{123784\,c^3\,x^3\,\sqrt{d}}{945\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (4608*c^(9/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 - (2*x^12*(c + d*x^3)^(1/2))/27 - (3018352*c^4*(c + d*x^3)^(1/2))/(
945*d^4) - (164*c*x^9*(c + d*x^3)^(1/2))/(189*d) - (123784*c^3*x^3*(c + d*x
^3)^(1/2))/(945*d^3) - (3074*c^2*x^6*(c + d*x^3)^(1/2))/(315*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

3.296 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=109

$$\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

```
[Out] -128/9*c^2*(d*x^3+c)^(3/2)/d^3-14/15*c*(d*x^3+c)^(5/2)/d^3-2/21*(d*x^3+c)^(
7/2)/d^3+1152*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3-384*c^3*(d*x
^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 88, 50, 63, 206}

$$-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{1152c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-384*c^3*Sqrt[c + d*x^3])/d^3 - (128*c^2*(c + d*x^3)^(3/2))/(9*d^3) - (14*
c*(c + d*x^3)^(5/2))/(15*d^3) - (2*(c + d*x^3)^(7/2))/(21*d^3) + (1152*c^(7
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{7c(c+dx)^{3/2}}{d^2}+\frac{64c^2(c+dx)^{3/2}}{d^2(8c-dx)}-\frac{(c+dx)^{5/2}}{d^2}\right)dx, x, x^3\right)\\
&= -\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}+\frac{\left(64c^2\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}+\frac{\left(192c^3\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x\right.}{d^2}\\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}+\frac{\left(1728c^4\right)\text{Sul}}{}\\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}+\frac{\left(3456c^4\right)\text{Sul}}{}\\
&= -\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{3/2}}{9d^3}-\frac{14c\left(c+dx^3\right)^{5/2}}{15d^3}-\frac{2\left(c+dx^3\right)^{7/2}}{21d^3}+\frac{1152c^{7/2}\tanh}{d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.74

$$\frac{362880c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(62882c^3+2579c^2dx^3+192cd^2x^6+15d^3x^9\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(62882*c^3 + 2579*c^2*d*x^3 + 192*c*d^2*x^6 + 15*d^3*x^
9) + 362880*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(315*d^3)
```

fricas [A] time = 0.44, size = 169, normalized size = 1.55

$$\left[\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(15\,d^3x^9+192\,cd^2x^6+2579\,c^2dx^3+62882\,c^3\right)\sqrt{dx^3+c}\right)}{315\,d^3},-\frac{2\left(181440\,\sqrt{\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/315*(90720*c^(7/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) - (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d
*x^3 + c))/d^3, -2/315*(181440*sqrt(-c)*c^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt
(-c)/c) + (15*d^3*x^9 + 192*c*d^2*x^6 + 2579*c^2*d*x^3 + 62882*c^3)*sqrt(d*
x^3 + c))/d^3]
```

giac [A] time = 0.16, size = 100, normalized size = 0.92

$$-\frac{1152\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{2\left(15\left(dx^3+c\right)^{\frac{7}{2}}d^{18}+147\left(dx^3+c\right)^{\frac{5}{2}}cd^{18}+2240\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{18}+60480\sqrt{dx^3+\cdots}\right.}{315\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -1152*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/315*(15*(
d*x^3 + c)^(7/2)*d^18 + 147*(d*x^3 + c)^(5/2)*c*d^18 + 2240*(d*x^3 + c)^(3/
2)*c^2*d^18 + 60480*sqrt(d*x^3 + c)*c^3*d^18)/d^21
```

maple [C] time = 0.16, size = 541, normalized size = 4.96

$$-\frac{64\left(\frac{2\sqrt{d\,x^3+c}\;x^3}{9}+\frac{56\sqrt{d\,x^3+c}\;c}{9d}+\frac{3ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)\cdots}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d
*x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)+16/15*c/d*(d*x^3+c)^
(5/2))-64*c^2/d^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d
^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3
*d-8*c)))
```

maxima [A] time = 1.34, size = 96, normalized size = 0.88

$$-\frac{2\left(90720\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+15\left(dx^3+c\right)^{\frac{7}{2}}+147\left(dx^3+c\right)^{\frac{5}{2}}c+2240\left(dx^3+c\right)^{\frac{3}{2}}c^2+60480\sqrt{dx^3+c}\,c^3\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/315*(90720*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) +
3*sqrt(c))) + 15*(d*x^3 + c)^(7/2) + 147*(d*x^3 + c)^(5/2)*c + 2240*(d*x^3
+ c)^(3/2)*c^2 + 60480*sqrt(d*x^3 + c)*c^3)/d^3
```

mupad [B] time = 3.50, size = 115, normalized size = 1.06

$$\frac{576\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}-\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21}-\frac{125764\,c^3\,\sqrt{d\,x^3+c}}{315\,d^3}-\frac{128\,c\,x^6\,\sqrt{d\,x^3+c}}{105\,d}-\frac{5158\,c^2\,x^3\,\sqrt{d\,x^3+}}{315\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (576*c^(7/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 - (2*x^9*(c + d*x^3)^(1/2))/21 - (125764*c^3*(c + d*x^3)^(1/2))/(315
*d^3) - (128*c*x^6*(c + d*x^3)^(1/2))/(105*d) - (5158*c^2*x^3*(c + d*x^3)^(
1/2))/(315*d^2)
```

sympy [A] time = 108.79, size = 110, normalized size = 1.01

$$-\frac{1152c^4\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^3\sqrt{-c}}-\frac{384c^3\sqrt{c+dx^3}}{d^3}-\frac{128c^2\left(c+dx^3\right)^{\frac{3}{2}}}{9d^3}-\frac{14c\left(c+dx^3\right)^{\frac{5}{2}}}{15d^3}-\frac{2\left(c+dx^3\right)^{\frac{7}{2}}}{21d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -1152*c**4*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**3*sqrt(-c)) - 384*c**3*s
qrt(c + d*x**3)/d**3 - 128*c**2*(c + d*x**3)**(3/2)/(9*d**3) - 14*c*(c + d*
x**3)**(5/2)/(15*d**3) - 2*(c + d*x**3)**(7/2)/(21*d**3)
```

3.297 $\int \frac{x^5(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=88

$$\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

```
[Out] -16/9*c*(d*x^3+c)^(3/2)/d^2-2/15*(d*x^3+c)^(5/2)/d^2+144*c^(5/2)*arctanh(1/
3*(d*x^3+c)^(1/2)/c^(1/2))/d^2-48*c^2*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 80, 50, 63, 206}

$$-\frac{48c^2\sqrt{c+dx^3}}{d^2}+\frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}-\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-48*c^2*Sqrt[c + d*x^3])/d^2 - (16*c*(c + d*x^3)^(3/2))/(9*d^2) - (2*(c +
d*x^3)^(5/2))/(15*d^2) + (144*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)$$
$$= -\frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{(8c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{3d}$$
$$= -\frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(24c^2\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(216c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x\right.}{d}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{\left(432c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+\right.}{d^2}$$
$$= -\frac{48c^2\sqrt{c+dx^3}}{d^2} - \frac{16c\left(c+dx^3\right)^{3/2}}{9d^2} - \frac{2\left(c+dx^3\right)^{5/2}}{15d^2} + \frac{144c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.80

$$\frac{6480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 2\sqrt{c+dx^3}\left(1123c^2+46cdx^3+3d^2x^6\right)}{45d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(1123*c^2 + 46*c*d*x^3 + 3*d^2*x^6) + 6480*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^2)
```

fricas [A] time = 0.50, size = 147, normalized size = 1.67

$$\left[\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) - \left(3\,d^2x^6+46\,cdx^3+1123\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^2}, -\frac{2\left(3240\,\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{}}{3\,c}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [2/45*(1620*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - (3*d^2*x^6 + 46*c*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2, -2/45*(3
240*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^2*x^6 + 46*c
*d*x^3 + 1123*c^2)*sqrt(d*x^3 + c))/d^2]
```

giac [A] time = 0.18, size = 83, normalized size = 0.94

$$-\frac{144\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^2} - \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^8 + 40\left(dx^3+c\right)^{\frac{3}{2}}cd^8 + 1080\,\sqrt{dx^3+c}\,c^2d^8\right)}{45\,d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -144*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 2/45*(3*(d*x
^3 + c)^(5/2)*d^8 + 40*(d*x^3 + c)^(3/2)*c*d^8 + 1080*sqrt(d*x^3 + c)*c^2*d
^8)/d^10
```

maple [C] time = 0.18, size = 462, normalized size = 5.25

$$-\frac{8\left(\frac{2\sqrt{d\,x^3+c}\,x^3}{9}+\frac{56\sqrt{d\,x^3+c}\,c}{9d}+\frac{3ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/15*(d*x^3+c)^(5/2)/d^2-8*c/d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/
2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.44, size = 82, normalized size = 0.93

$$-\frac{2\left(1620\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+40\left(dx^3+c\right)^{\frac{3}{2}}c+1080\sqrt{dx^3+c}\,c^2\right)}{45\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -2/45*(1620*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 40*(d*x^3 + c)^(3/2)*c + 1080*sqrt(d*x^3
+ c)*c^2)/d^2
```

mupad [B] time = 3.52, size = 95, normalized size = 1.08

$$\frac{72\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15}-\frac{2246\,c^2\,\sqrt{d\,x^3+c}}{45\,d^2}-\frac{92\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (72*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/d^2 - (2*x^6*(c + d*x^3)^(1/2))/15 - (2246*c^2*(c + d*x^3)^(1/2))/(45*d^2
) - (92*c*x^3*(c + d*x^3)^(1/2))/(45*d)
```

sympy [A] time = 62.74, size = 90, normalized size = 1.02

$$-\frac{144c^3\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{d^2\sqrt{-c}}-\frac{48c^2\sqrt{c+dx^3}}{d^2}-\frac{16c\left(c+dx^3\right)^{\frac{3}{2}}}{9d^2}-\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -144*c**3*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d**2*sqrt(-c)) - 48*c**2*sqr
t(c + d*x**3)/d**2 - 16*c*(c + d*x**3)**(3/2)/(9*d**2) - 2*(c + d*x**3)**(5
/2)/(15*d**2)
```

3.298 $\int \frac{x^2(c+dx^3)^{3/2}}{8c-dx^3} dx$

Optimal. Leaf size=67

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d+18*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d-6*
c*(d*x^3+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 50, 63, 206}

$$\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}-\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-6*c*Sqrt[c + d*x^3])/d - (2*(c + d*x^3)^(3/2))/(9*d) + (18*c^(3/2)*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx,x,x^3\right)\\
&= -\frac{2\left(c+dx^3\right)^{3/2}}{9d}+(3c)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\
&= -\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}+\left(27c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= -\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}+\frac{\left(54c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d}\\
&= -\frac{6c\sqrt{c+dx^3}}{d}-\frac{2\left(c+dx^3\right)^{3/2}}{9d}+\frac{18c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.87

$$\frac{162c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(28c+dx^3\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(28*c + d*x^3) + 162*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d)
```

fricas [A] time = 0.49, size = 121, normalized size = 1.81

$$\left[\frac{81\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-2\left(dx^3+28\,c\right)\sqrt{dx^3+c}}{9\,d},\,-\frac{2\left(81\,\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+\left(dx^3+28\,c\right)\sqrt{dx^3}\right.}{9\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/9*(81*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - 2*(d*x^3 + 28*c)*sqrt(d*x^3 + c))/d, -2/9*(81*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 28*c)*sqrt(d*x^3 + c))/d]
```

giac [A] time = 0.16, size = 65, normalized size = 0.97

$$-\frac{18\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^2+27\,\sqrt{dx^3+c}\,cd^2\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] -18*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 2/9*((d*x^3 + c
)^(3/2)*d^2 + 27*sqrt(d*x^3 + c)*c*d^2)/d^3
```

maple [C] time = 0.16, size = 441, normalized size = 6.58

$$3ic\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-\right.}{}\right.}{2\left(\right.}}$$

$$-\frac{2\sqrt{d\,x^{3}+c}\;x^{3}}{9}-\frac{56\sqrt{d\,x^{3}+c}\;c}{9d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/9*(d*x^3+c)^(1/2)*x^3-56/9*(d*x^3+c)^(1/2)*c/d-3*I*c/d^3*2^(1/2)*sum((-c
*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.39, size = 68, normalized size = 1.01

$$-\frac{81\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^{3}+c}-3\,\sqrt{c}}{\sqrt{dx^{3}+c}+3\,\sqrt{c}}\right)+2\left(dx^{3}+c\right)^{\frac{3}{2}}+54\,\sqrt{dx^{3}+c}\,c}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -1/9*(81*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 54*sqrt(d*x^3 + c)*c)/d
```

mupad [B] time = 3.45, size = 75, normalized size = 1.12

$$\frac{9\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d}-\frac{56\,c\,\sqrt{d\,x^3+c}}{9\,d}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] (9*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/d - (56*c*(c + d*x^3)^(1/2))/(9*d) - (2*x^3*(c + d*x^3)^(1/2))/9
```

sympy [A] time = 29.25, size = 65, normalized size = 0.97

$$-\frac{18c^{2}\operatorname{atan}\left(\frac{\sqrt{c+dx^{3}}}{3\sqrt{-c}}\right)}{d\sqrt{-c}}-\frac{6c\sqrt{c+dx^{3}}}{d}-\frac{2\left(c+dx^{3}\right)^{\frac{3}{2}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -18*c**2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(d*sqrt(-c)) - 6*c*sqrt(c + d*
x**3)/d - 2*(c + d*x**3)**(3/2)/(9*d)
```

3.299 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

```
[Out] 9/4*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)-1/12*arctanh((d*x^3+c)^(1/
2)/c^(1/2))*c^(1/2)-2/3*(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 84, 156, 63, 208, 206}

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)}\,dx,x,x^3\right)\\
&= -\frac{2}{3}\sqrt{c+dx^3}-\frac{\text{Subst}\left(\int \frac{-c^2d-10cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d}\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{1}{24}c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)+\frac{1}{8}(27cd)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,d\right.\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{1}{4}(27c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)+\frac{c\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+d}\right.}{12d}\\
&= -\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.00

$$-\frac{2}{3}\sqrt{c+dx^3}+\frac{9}{4}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-\frac{1}{12}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/3 + (9*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/4
 - (Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/12
```

fricas [A] time = 0.48, size = 152, normalized size = 2.08

$$\left[\frac{9}{8}\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\frac{1}{24}\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-\frac{2}{3}\sqrt{dx^3+c},\frac{1}{12}\sqrt{-c}\,\text{arcta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [9/8*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))
+ 1/24*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2/3*sqr
t(d*x^3 + c), 1/12*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 9/4*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 2/3*sqrt(d*x^3 + c)]
```

giac [A] time = 0.18, size = 61, normalized size = 0.84

$$\frac{c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{9\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}-\frac{2}{3}\sqrt{dx^3+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 1/12*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/4*c*arctan(1/3*sqrt(d*
x^3 + c)/sqrt(-c))/sqrt(-c) - 2/3*sqrt(d*x^3 + c)
```

maple [C] time = 0.19, size = 500, normalized size = 6.85

$$-\frac{\left(\frac{2\sqrt{d\,x^{3}+c}\;x^{3}}{9}+\frac{56\sqrt{d\,x^{3}+c}\;c}{9d}+\frac{3ic\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x)
```

```
[Out] -1/8/c*d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/8/c*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arctanh
((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^{3}+c\right)^{\frac{3}{2}}}{\left(dx^{3}-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x), x)
```

mupad [B] time = 5.89, size = 89, normalized size = 1.22

$$\frac{\sqrt{c}\;\ln\left(\frac{\left(\sqrt{d\,x^{3}+c}-\sqrt{c}\right)^{3}\left(\sqrt{d\,x^{3}+c}+\sqrt{c}\right)\left(10\,c+d\,x^{3}+6\,\sqrt{c}\;\sqrt{d\,x^{3}+c}\right)^{27}}{x^{6}\left(8\,c-d\,x^{3}\right)^{27}}\right)}{24}-\frac{2\,\sqrt{d\,x^{3}+c}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
*(10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))^27)/(x^6*(8*c - d*x^3)^27)))/
24 - (2*(c + d*x^3)^(1/2))/3
```

sympy [A] time = 24.74, size = 73, normalized size = 1.00

$$-\frac{9c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{4\sqrt{-c}}+\frac{c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12\sqrt{-c}}-\frac{2\sqrt{c+dx^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c),x)
```

```
[Out] -9*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(4*sqrt(-c)) + c*atan(sqrt(c + d*x
**3)/sqrt(-c))/(12*sqrt(-c)) - 2*sqrt(c + d*x**3)/3
```

3.300 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=78

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] 9/32*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-13/96*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(1/2)-1/24*(d*x^3+c)^(1/2)/x^3
```

Rubi [A] time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24x^3}+\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*x^3) + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} - \frac{\text{Subst}\left(\int \frac{-13c^2d-\frac{17}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{1}{192}(13d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) + \frac{1}{64}\left(27d^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c}}\right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{13}{96}\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right) + \frac{1}{32}(27d)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \right.$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 1.00

$$-\frac{\sqrt{c+dx^3}}{24x^3} + \frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{13d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/x^3 + (9*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(32*
Sqrt[c]) - (13*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c])
```

fricas [A] time = 0.47, size = 186, normalized size = 2.38

$$\left[\frac{27\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 13\sqrt{c}\,dx^3\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 8\sqrt{dx^3+c}\,c}{192\,cx^3}, \frac{13\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/192*(27*sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 13*sqrt(c)*d*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) - 8*sqrt(d*x^3 + c)*c)/(c*x^3), 1/96*(13*sqrt(-c)*d*x^3*arctan(sqrt(
```

```
d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(
-c)/c) - 4*sqrt(d*x^3 + c)*c)/(c*x^3)]
```

giac [A] time = 0.17, size = 64, normalized size = 0.82

$$\frac{13\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}}-\frac{9\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{32\,\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 13/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) - 9/32*d*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/sqrt(-c) - 1/24*sqrt(d*x^3 + c)/x^3
```

maple [C] time = 0.26, size = 556, normalized size = 7.13

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}\,x^3}{9}+\frac{56\sqrt{d\,x^3+c}\,c}{9d}+\frac{3ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((
d*x^3+c)^(1/2)/c^(1/2)))-1/64*d^2/c^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+
c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*d*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*
x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^4), x)
```

mupad [B] time = 3.52, size = 56, normalized size = 0.72

$$\frac{9\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{3\,\sqrt{c}}\right)}{32\,\sqrt{c}}-\frac{13\,d\,\mathrm{atanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96\,\sqrt{c}}-\frac{\sqrt{d\,x^3+c}}{24\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)),x)
```

```
[Out] (9*d*atanh((c + d*x^3)^(1/2)/(3*c^(1/2))))/(32*c^(1/2)) - (13*d*atanh((c +
d*x^3)^(1/2)/c^(1/2)))/(96*c^(1/2)) - (c + d*x^3)^(1/2)/(24*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{c\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx-\int\frac{dx^3\sqrt{c+dx^3}}{-8cx^4+dx^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**4 + d*x**7), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**4 + d*x**7), x)
```

3.301 $\int \frac{(c+dx^3)^{3/2}}{x^7(8c-dx^3)}\,dx$

Optimal. Leaf size=104

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

```
[Out] 9/256*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-37/768*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/48*(d*x^3+c)^(1/2)/x^6-11/192*d*(d*x^3+c)
^(1/2)/c/x^3
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}-\frac{11d\sqrt{c+dx^3}}{192cx^3}-\frac{\sqrt{c+dx^3}}{48x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*x^6) - (11*d*Sqrt[c + d*x^3])/(192*c*x^3) + (9*d^2*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(256*c^(3/2)) - (37*d^2*ArcTanh[Sqrt[c +
 d*x^3]/Sqrt[c]])/(768*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)}\,dx,x,x^3\right)$$

$$=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{\text{Subst}\left(\int \frac{-22c^2d-\frac{35}{2}cd^2x}{x^2(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{48c}$$

$$=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{\text{Subst}\left(\int \frac{74c^3d^2+11c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^3}$$

$$=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{\left(37d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{1536c}+\frac{\left(27d^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\cdots}\right.}{512c}$$

$$=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{(37d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{768c}+\frac{\left(27d^2\right)\text{Subst}\left(\int \cdots\right.}{2\cdots}$$

$$=-\frac{\sqrt{c+dx^3}}{48x^6}-\frac{11d\sqrt{c+dx^3}}{192cx^3}+\frac{9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{256c^{3/2}}-\frac{37d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{3/2}}$$

Mathematica [A] time = 0.11, size = 96, normalized size = 0.92

$$\frac{27d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-37d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)-4\sqrt{c}\sqrt{c+dx^3}\left(4c+11dx^3\right)}{768c^{3/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x]
```

```
[Out] (-4*Sqrt[c]*Sqrt[c + d*x^3]*(4*c + 11*d*x^3) + 27*d^2*x^6*ArcTanh[Sqrt[c +
d*x^3]/(3*Sqrt[c])] - 37*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(
3/2)*x^6)
```

fricas [A] time = 0.47, size = 218, normalized size = 2.10

$$\left[\frac{27\sqrt{c}\,d^2x^6\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)+37\sqrt{c}\,d^2x^6\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2\,c}{x^3}\right)-8\left(11\,cdx^3+4\,c^2\right)\sqrt{dx^3+c}}{1536\,c^2x^6}, \frac{37 \sqrt{\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] [1/1536*(27*sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 37*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c)
+ 2*c)/x^3) - 8*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^2*x^6), 1/768*(37*
sqrt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 27*sqrt(-c)*d^2*x^6*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(11*c*d*x^3 + 4*c^2)*sqrt(d*x^3 +
 c))/(c^2*x^6)]
```

giac [A] time = 0.19, size = 101, normalized size = 0.97

$$\frac{37\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{768\sqrt{-c}\,c}-\frac{9\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{256\sqrt{-c}\,c}-\frac{11\left(dx^3+c\right)^{\frac{3}{2}}d^2-7\sqrt{dx^3+c}\,cd^2}{192\,cd^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] 37/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 9/256*d^2*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/192*(11*(d*x^3 + c)^(3/2)*d
^2 - 7*sqrt(d*x^3 + c)*c*d^2)/(c*d^2*x^6)
```

maple [C] time = 0.24, size = 617, normalized size = 5.93

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}\,x^3}{9}+\frac{56\sqrt{d\,x^3+c}\,c}{9d}+\frac{3ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(1/2)*d*arct
anh((d*x^3+c)^(1/2)/c^(1/2)))+1/8/c*(-1/6*c*(d*x^3+c)^(1/2)/x^6-5/12*d*(d*x
^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2))-1/512/c^3
*d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*su
```

```
m((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/5
12/c^3*d^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arc
tanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 3.74, size = 87, normalized size = 0.84

$$\frac{7\sqrt{d\,x^3+c}}{192\,x^6}-\frac{37\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{768\,\sqrt{c^3}}+\frac{9\,d^2\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{256\,\sqrt{c^3}}-\frac{11\,\left(d\,x^3+c\right)^{3/2}}{192\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)),x)
```

```
[Out] (7*(c + d*x^3)^(1/2))/(192*x^6) - (37*d^2*atanh((c*(c + d*x^3)^(1/2))/(c^3)
^(1/2)))/(768*(c^3)^(1/2)) + (9*d^2*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1
/2))))/(256*(c^3)^(1/2)) - (11*(c + d*x^3)^(3/2))/(192*c*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c),x)
```

```
[Out] Timed out
```

3.302 $\int \frac{x^7(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=669

$$-\frac{288\sqrt{3}\,c^{19/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{288c^{19/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{288c^{19/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}-\frac{698216\sqrt{2}\,3^{3}\cdots}{}$$

```
[Out] 288*c^(19/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-288*c^(19/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-288*c^(19/6)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3
)-36534/1729*c^2*x^2*(d*x^3+c)^(1/2)/d^2-348/247*c*x^5*(d*x^3+c)^(1/2)/d-2/
19*x^8*(d*x^3+c)^(1/2)-2094648/1729*c^3*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-698216/1729*3^(3/4)*c^(10/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1047324/1729*3^(1/4)*c^(10/3)*(c
^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)
*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 669, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {477, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2}-\frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{288\sqrt{3}\,c^{19/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{288c^{19/6}\tanh^{-1}\cdots}{d^{8}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-36534*c^2*x^2*Sqrt[c + d*x^3])/(1729*d^2) - (348*c*x^5*Sqrt[c + d*x^3])/(
247*d) - (2*x^8*Sqrt[c + d*x^3])/19 - (2094648*c^3*Sqrt[c + d*x^3])/(1729*d
^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (288*Sqrt[3]*c^(19/6)*ArcTan[
(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) + (288*c^
(19/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/3
) - (288*c^(19/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) + (1047324*
3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^
(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellip
ticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)], -7 - 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (698216*Sqrt[
2]*3^(3/4)*c^(10/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x
 + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 -
 4*Sqrt[3]])/(1729*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{8c-dx^3} dx = -\frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2\int \frac{x^7\left(-\frac{147c^2d}{2}-87cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{19d}$$

$$= -\frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{4\int \frac{x^4\left(-3480c^3d^2-\frac{18267}{4}c^2d^3x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}dx}{247d^3}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \frac{x\left(-73068c^4d^3-\frac{261831}{2}c^3d^4x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}}{1729d^5}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{8\int \left(\frac{261831c^3d^3x}{2\sqrt{c+dx^3}} - \frac{1120392c^4}{(8c-dx^3)\sqrt{c}}\right.}{1729d^5}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(1047324c^3)\int \frac{x}{\sqrt{c+dx^3}}dx}{1729d^2} +$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{(432c^3)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}}{\sqrt[3]{}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c}}}{d^3}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= -\frac{36534c^2x^2\sqrt{c+dx^3}}{1729d^2} - \frac{348cx^5\sqrt{c+dx^3}}{247d} - \frac{2}{19}x^8\sqrt{c+dx^3} - \frac{2094648c^3\sqrt{c+dx^3}}{1729d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

Mathematica [C] time = 0.12, size = 163, normalized size = 0.24

$$\frac{365340c^3x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+261831c^2dx^5\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20x^2\left(18267c^3+\right.}{17290d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-20*x^2*(18267*c^3 + 19485*c^2*d*x^3 + 1309*c*d^2*x^6 + 91*d^3*x^9) + 3653
40*c^3*x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*
x^3)/(8*c)] + 261831*c^2*d*x^5*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17290*d^2*Sqrt[c + d*x^3])
```

fricas [F] time = 50.83, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(dx^{10}+cx^{7}\right)\sqrt{dx^{3}+c}}{dx^{3}-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^10 + c*x^7)*sqrt(d*x^3 + c)/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^{3}+c\right)^{\frac{3}{2}}x^{7}}{dx^{3}-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

maple [C] time = 0.28, size = 1840, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d^2*(d*(2/19*d*x^8*(d*x^3+c)^(1/2)+44/247*c*x^5*(d*x^3+c)^(1/2)+54/1729*
c^2/d*x^2*(d*x^3+c)^(1/2)+72/1729*I*c^3/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))))+8*c*(2/13*x^5*d*(d*x^3+c)^(1/2)+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I
*c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-64*c^2/d^2*(2/7*(d*x^3+c)^(1
/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
```

```
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/
d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}} x^7}{dx^3 - 8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(d\,x^3+c\right)}^{3/2}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^7\sqrt{c + dx^3}}{-8c + dx^3}\,dx - \int \frac{dx^{10}\sqrt{c + dx^3}}{-8c + dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x**7*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**10*sq
rt(c + d*x**3)/(-8*c + d*x**3), x)
```

3.303 $\int \frac{x^4(c+dx^3)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=645

$$-\frac{36\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}-\frac{4594\sqrt{2}\,3^{3/4}c^{7/3}\,(}{}$$

```
[Out] 36*c^(13/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-36*c^(13/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-36*c^(13/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)-2
40/91*c*x^2*(d*x^3+c)^(1/2)/d-2/13*x^5*(d*x^3+c)^(1/2)-13782/91*c^2*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-4594/91*3^(3/4)*c^(7/3)*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(
1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+6891/91*3
^(1/4)*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 645, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {477, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{36\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{36c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{36c^{13/6}\tanh}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-240*c*x^2*Sqrt[c + d*x^3])/(91*d) - (2*x^5*Sqrt[c + d*x^3])/13 - (13782*c
^2*Sqrt[c + d*x^3])/(91*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (36*
Sqrt[3]*c^(13/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/d^(5/3) + (36*c^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sq
rt[c + d*x^3])])/d^(5/3) - (36*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/d^(5/3) + (6891*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*
Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^
(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (4594*Sqrt[2]*3^(3/4)*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1
/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)], -7 - 4*Sqrt[3]])/(91*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{13}x^5\sqrt{c+dx^3} - \frac{2\int \frac{x^4\left(-\frac{93c^2d}{2}-60cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{13d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \frac{x\left(-960c^3d^2-\frac{6891}{4}c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{4\int \left(\frac{6891c^2d^2x}{4\sqrt{c+dx^3}} - \frac{14742c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{91d^3}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{\left(6891c^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{91d} + \frac{\left(648c^3\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d}{d}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{\left(54c^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{d^2} - \frac{\left(6891c^2\right)\int \frac{(1-}{}}{91}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{6891\sqrt[4]{3}\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{36\sqrt{3}\,c^{13/6}\tan^{-1}\left(\right.}{d^{5/}}$$

$$= -\frac{240cx^2\sqrt{c+dx^3}}{91d} - \frac{2}{13}x^5\sqrt{c+dx^3} - \frac{13782c^2\sqrt{c+dx^3}}{91d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{36\sqrt{3}\,c^{13/6}\tan^{-1}\left(\right.}{d^{5/}}$$

Mathematica [C] time = 0.11, size = 150, normalized size = 0.23

$$\frac{9600c^2x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+6891cdx^5\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80\left(120c^2x^2+127cdx^5\right.}{3640d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-80*(120*c^2*x^2 + 127*c*d*x^5 + 7*d^2*x^8) + 9600*c^2*x^2*Sqrt[1 + (d*x^3
)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 6891*c*d*x^5
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(3640*d*Sqrt[c + d*x^3])
```

fricas [F] time = 16.09, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(dx^7+cx^4\right)\sqrt{dx^3+c}}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^7 + c*x^4)*sqrt(d*x^3 + c)/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

maple [C] time = 0.16, size = 1344, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -1/d*(2/13*(d*x^3+c)^(1/2)*d*x^5+32/91*(d*x^3+c)^(1/2)*c*x^2-18/91*I*c^2*3^
(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-8*c/d*(2/7*(d*x^3+c)^(1/2)*x^2-44/7*
I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellipti
cE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(d\,x^3+c\right)}^{3/2}}{8\,c-d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^{4}\sqrt{c + dx^{3}}}{-8c + dx^{3}}\, dx - \int \frac{dx^{7}\sqrt{c + dx^{3}}}{-8c + dx^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x**4*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**7*sqr
t(c + d*x**3)/(-8*c + d*x**3), x)
```

3.304 $\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx$

Optimal. Leaf size=627

$$-\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}-\frac{44\sqrt{2}\,3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{\cdot}\right.}{}$$

```
[Out] 9/2*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(2
/3)-9/2*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)-9/2*c^(7/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/3)-2
/7*x^2*(d*x^3+c)^(1/2)-132/7*c*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))-44/7*3^(3/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c
^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2)))^2)^(1/2)+66/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*Elliptic
E((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^
(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {477, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{2d^{2/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2d^{2/3}}-\frac{44\sqrt{2}\,3^{3/4}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{\cdot}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/7 - (132*c*Sqrt[c + d*x^3])/(7*d^(2/3)*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(
1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(2*d^(2/3)) + (9*c^(7/6)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(2*d^(2/3)) - (9*c^(7/6)*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2*d^(2/3)) + (66*3^(1/4)*Sqrt[2 - Sqr
t[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(
2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (44*Sqrt[2]*3^(3/4)*c^(4/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{8c-dx^3}\,dx = -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \frac{x\left(-\frac{39c^2d}{2}-33cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{2\int \left(\frac{33cdx}{\sqrt{c+dx^3}} - \frac{567c^2dx}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{7d}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{1}{7}(66c)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \left(81c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{(27c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{4d} - \frac{(66c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}dx}{7\sqrt[3]{d}} + \frac{\left(27c^{4/3}\right)\ldots}{\ldots}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{66\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\ldots}{\left(\left(1+\ldots\right.\right.}}}{7d^{2/3}\sqrt{\frac{\sqrt[3]{\ldots}}{\left(\left(1+\ldots\right.\right.}}}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\ldots}{\ldots}$$

$$= -\frac{2}{7}x^2\sqrt{c+dx^3} - \frac{132c\sqrt{c+dx^3}}{7d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{2d^{2/3}} + \frac{9c^{7/6}\ldots}{\ldots}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.20

$$\frac{x^2\left(132dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+195c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-160\left(c+dx^3\right)\right)}{560\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x]
```

```
[Out] (x^2*(-160*(c + d*x^3) + 195*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/
3, -((d*x^3)/c), (d*x^3)/(8*c)] + 132*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(560*Sqrt[c + d*x^3])
```

fricas [F] time = 3.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(dx^4+cx\right)\sqrt{dx^3+c}}{dx^3-8\,c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^4 + c*x)*sqrt(d*x^3 + c)/(d*x^3 - 8*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

maple [C] time = 0.18, size = 864, normalized size = 1.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x)
```

```
[Out] -2/7*(d*x^3+c)^(1/2)*x^2+44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-3
*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-
I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)
^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_al
pha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{dx^3-8\,c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(d\,x^3+c\right)^{3/2}}{8\,c-d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3),x)
```

```
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx\sqrt{c+dx^3}}{-8c+dx^3}\,dx-\int \frac{dx^4\sqrt{c+dx^3}}{-8c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*x*sqrt(c + d*x**3)/(-8*c + d*x**3), x) - Integral(d*x**4*sqrt(c
 + d*x**3)/(-8*c + d*x**3), x)
```

3.305 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)}\,dx$

Optimal. Leaf size=626

$$-\frac{5\, 3^{3/4} \sqrt[3]{c} \sqrt[3]{d} \left(\sqrt[3]{c}+\sqrt[3]{d}\, x\right) \sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\, x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\, x\right)^2}}\, F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\, x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\, x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\, x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\, x\right)^2}}\sqrt{c+dx^3}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\sqrt[3]{\cdots}}{\cdots}$$

```
[Out] 9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1
/2))-9/16*c^(1/6)*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))-9/16*c^(1/6)
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)-1/8*(d*x^3+c)^(1/2)/x-15/8*d^(1/3)*(d*x^3+c)^(1/2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))-5/8*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)+15/16*3^(1/4)*c^(1/3)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 626, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {474, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5\, 3^{3/4} \sqrt[3]{c} \sqrt[3]{d} \left(\sqrt[3]{c}+\sqrt[3]{d}\, x\right) \sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\, x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\, x\right)^2}}\, F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\, x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\, x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\, x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\, x\right)^2}}\sqrt{c+dx^3}}+\frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\sqrt[3]{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(8*x) - (15*d^(1/3)*Sqrt[c + d*x^3])/(8*((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)) - (9*Sqrt[3]*c^(1/6)*d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/16 + (9*c^(1/6)*d^(1/3)*ArcTanh[(c^(1
/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/16 - (9*c^(1/6)*d^(1/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/16 + (15*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1
/3)*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/
3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[
3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[
3]])/(16*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*Sqrt[c + d*x^3]) - (5*3^(3/4)*c^(1/3)*d^(1/3)*(c^(1/3) + d^(1/3)*
x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
+ d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*Sqrt[(c^(1/3)*(
c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^2(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{8x} + \frac{\int \frac{x\left(21c^2d+\frac{15}{2}cd^2x^3\right)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} + \frac{\int \left(-\frac{15cdx}{2\sqrt{c+dx^3}} + \frac{81c^2dx}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{1}{16}(15d)\int \frac{x}{\sqrt{c+dx^3}}\,dx + \frac{1}{8}(81cd)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{27}{32}\int \frac{2\sqrt[3]{c}\,d^{2/3} - 2dx - \frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}} + \frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx - \frac{1}{16}\left(15d^{2/3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\ldots}{\sqrt{c+dx^3}}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{15\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}{16\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{\ldots}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\ldots\right.}}}$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right) + \frac{9}{16}\ldots$$

$$= -\frac{\sqrt{c+dx^3}}{8x} - \frac{15\sqrt[3]{d}\sqrt{c+dx^3}}{8\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9}{16}\sqrt{3}\sqrt[6]{c}\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right) + \frac{9}{16}\ldots$$

Mathematica [C] time = 0.10, size = 137, normalized size = 0.22

$$\frac{3d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 21cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 16c\left(c+dx^3\right)}{128cx\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x]
```

```
[Out] (-16*c*(c + d*x^3) + 21*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5
/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(128*c*x*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

maple [C] time = 0.21, size = 1339, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x)
```

```
[Out] 1/8/c*(-c*(d*x^3+c)^(1/2)/x+2/7*d*x^2*(d*x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-c*d
^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2))))-1/8/c*d*(2/7*(d*x^3+c)^(1/2)*x^2-44/7*I*c*3^(1/2)
*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1
/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^2\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^2+dx^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**2 + d*x**5), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**2 + d*x**5), x)
```

3.306 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx$

Optimal. Leaf size=651

$$-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}+\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{8}$$

```
[Out] 9/128*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(5/6)-9/128*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)-9/128*d^(4
/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(
5/6)-1/32*(d*x^3+c)^(1/2)/x^4-3/16*d*(d*x^3+c)^(1/2)/c/x+3/16*d^(4/3)*(d*x^
3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/16*3^(3/4)*d^(4/3)*(c^(1/3)+
d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c
^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/32*3^(1/4)*d^(
4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-
c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(
2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 651, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {474, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{128c^{5/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{128c^{5/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{5/6}}+\frac{3^{3/4}d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*x^4) - (3*d*Sqrt[c + d*x^3])/(16*c*x) + (3*d^(4/3)*Sqr
t[c + d*x^3])/(16*c*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(4/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(128*c^
(5/6)) + (9*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])])/(128*c^(5/6)) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(12
8*c^(5/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) +
(3^(3/4)*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*
Sqrt[3]])/(8*Sqrt[2]*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqr
t[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)}\,dx = -\frac{\sqrt{c+dx^3}}{32x^4} + \frac{\int \frac{48c^2d+\frac{69}{2}cd^2x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} - \frac{\int \frac{x(-516c^3d^2+24c^2d^3x^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} - \frac{\int \left(-\frac{24c^2d^2x}{\sqrt{c+dx^3}} - \frac{324c^3d^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{256c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{1}{64}(81d^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx + \frac{(3d^2)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} - \frac{(27d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{256c} + \frac{(3d^{5/3})\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,d}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right.}{128c^{5/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{32x^4} - \frac{3d\sqrt{c+dx^3}}{16cx} + \frac{3d^{4/3}\sqrt{c+dx^3}}{16c\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right.}{128c^{5/6}}$$

Mathematica [C] time = 0.11, size = 154, normalized size = 0.24

$$\frac{645cd^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 4\left(3d^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 40c\left(c^2+7cdx^3+6d^2x^6\right.\right.}{5120c^2x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x]
```

```
[Out] (645*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*(40*c*(c^2 + 7*c*d*x^3 + 6*d^2*x^6) + 3*d^3*x^9*Sqrt[1
+ (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(512
0*c^2*x^4*Sqrt[c + d*x^3])
```

fricas [F] time = 3.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{dx^8-8\,cx^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="fricas")
[Out] integral(-(d*x^3 + c)^(3/2)/(d*x^8 - 8*c*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

maple [C] time = 0.21, size = 1810, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x)
[Out] 1/8/c*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*d*(d*x^3+c)^(1/2)/x-9/8*I*d*3^(1/2)*
(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*x^3+
c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*
(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1
/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64*d^2/c^2*
(2/7*(d*x^3+c)^(1/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
```

```
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3
*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-
I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)
^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_al
pha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^5\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx-\int \frac{dx^3\sqrt{c+dx^3}}{-8cx^5+dx^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c),x)
```

```
[Out] -Integral(c*sqrt(c + d*x**3)/(-8*c*x**5 + d*x**8), x) - Integral(d*x**3*sqr
t(c + d*x**3)/(-8*c*x**5 + d*x**8), x)
```

3.307 $\int \frac{(c+dx^3)^{3/2}}{x^8(8c-dx^3)}\,dx$

Optimal. Leaf size=675

$$-\frac{9\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{28}$$

[Out] 9/1024*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-9/1024*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/c^(11/6)-1/56*(d*x^3+c)^(1/2)/x^7-75/1792*d*(d*x^3+c)^(1/2)/c/x^4-3/56*d^2*(d*x^3+c)^(1/2)/c^2/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.94, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {474, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{9\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{\cdot}}{}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]

[Out] -Sqrt[c + d*x^3]/(56*x^7) - (75*d*Sqrt[c + d*x^3])/(1792*c*x^4) - (3*d^2*Sqrt[c + d*x^3])/(56*c^2*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)) - (9*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3*3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(112*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)}\,dx = -\frac{\sqrt{c+dx^3}}{56x^7}+\frac{\int \frac{75c^2d+\frac{123}{2}cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{56c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{\int \frac{-768c^3d^2-\frac{375}{2}c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1792c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{\int \frac{x\left(5340c^4d^3-384c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{\int \left(\frac{384c^3d^3x}{\sqrt{c+dx^3}}+\frac{2268c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{14336c^5}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{\left(3d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^2}+\frac{\left(81d^3\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\cdots}{512c}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}-\frac{\left(27d^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2}+\frac{\left(3d^{8/3}\right)\cdots}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{3\sqrt[4]{3}\sqrt{2-\cdots}}{\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{7/3}\tan\cdots}{1\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{56x^7}-\frac{75d\sqrt{c+dx^3}}{1792cx^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^2x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{7/3}\tan\cdots}{1\cdots}$$

Mathematica [C] time = 0.11, size = 167, normalized size = 0.25

$$\frac{6675cd^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(6d^4x^{12}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5c\left(32c^3+107c^2dx^3\cdots\right.\right.}{286720c^3x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x]
```

```
[Out] (6675*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 + 107*c^2*d*x^3 + 171*c*d^2*x^6 + 96*d^3
*x^9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)
/c), (d*x^3)/(8*c)]))/(286720*c^3*x^7*Sqrt[c + d*x^3])
```

fricas [F] time = 5.89, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{dx^{11}-8\,cx^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="fricas")
[Out] integral(-(d*x^3 + c)^(3/2)/(d*x^11 - 8*c*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="giac")
[Out] integrate(-(d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

maple [C] time = 0.20, size = 2306, normalized size = 3.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x)
[Out] 1/64/c^2*d*(-1/4*(d*x^3+c)^(1/2)*c/x^4-11/8*(d*x^3+c)^(1/2)*d/x-9/8*I*d*3^(
1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/8/c*(-1/7*c*(d*x^3+c)^(1/2)/x^7-17/56
*d*(d*x^3+c)^(1/2)/x^4-27/112*d^2/c*(d*x^3+c)^(1/2)/x-9/112*I/c*d^2*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^2*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*
x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*
(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
```

```
d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/
3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2
)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/512/c^3
*d^3*(2/7*(d*x^3+c)^(1/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c),x, algorithm="maxima")
[Out] -integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^8\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c),x)
[Out] Timed out
```

3.308 $\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=90

$$\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

```
[Out] -4/3*c*(d*x^3+c)^(3/2)/d^4-2/15*(d*x^3+c)^(5/2)/d^4+1024/9*c^(5/2)*arctanh(
1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4-38*c^2*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$-\frac{38c^2\sqrt{c+dx^3}}{d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-38*c^2*Sqrt[c + d*x^3])/d^4 - (4*c*(c + d*x^3)^(3/2))/(3*d^4) - (2*(c + d
*x^3)^(5/2))/(15*d^4) + (1024*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])
/(9*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{57c^2}{d^3\sqrt{c+dx}}+\frac{512c^3}{d^3(8c-dx)\sqrt{c+dx}}-\frac{6c\sqrt{c+dx}}{d^3}-\frac{(c+dx)^{3/2}}{d^3}\right)dx,x$$
$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{\left(512c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{3d^3}$$
$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{\left(1024c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\right.}{3d^4}$$
$$= -\frac{38c^2\sqrt{c+dx^3}}{d^4}-\frac{4c\left(c+dx^3\right)^{3/2}}{3d^4}-\frac{2\left(c+dx^3\right)^{5/2}}{15d^4}+\frac{1024c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 0.77

$$\frac{5120c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-6\sqrt{c+dx^3}\left(296c^2+12cdx^3+d^2x^6\right)}{45d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-6*Sqrt[c + d*x^3]*(296*c^2 + 12*c*d*x^3 + d^2*x^6) + 5120*c^(5/2)*ArcTanh
[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4)
```

fricas [A] time = 1.12, size = 146, normalized size = 1.62

$$\left[\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(d^2x^6+12\,cdx^3+296\,c^2\right)\sqrt{dx^3+c}\right)}{45\,d^4},-\frac{2\left(2560\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-}}{3c}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/45*(1280*c^(5/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 -
 8*c)) - 3*(d^2*x^6 + 12*c*d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4, -2/45*(25
60*sqrt(-c)*c^2*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^2*x^6 + 12*c*
d*x^3 + 296*c^2)*sqrt(d*x^3 + c))/d^4]
```

giac [A] time = 0.16, size = 82, normalized size = 0.91

$$-\frac{1024\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^4}-\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+10\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+285\sqrt{dx^3+c}\,c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1024/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 2/15*((d*
x^3 + c)^(5/2)*d^16 + 10*(d*x^3 + c)^(3/2)*c*d^16 + 285*sqrt(d*x^3 + c)*c^2
*d^16)/d^20
```

maple [C] time = 0.28, size = 528, normalized size = 5.87

$$-\frac{8\left(\frac{2\sqrt{d\,x^3+c}\,x^3}{9d}-\frac{4\sqrt{d\,x^3+c}\,c}{9d^2}\right)c}{d^2}-\frac{\frac{2\sqrt{d\,x^3+c}\,x^6}{15d}-\frac{8\sqrt{d\,x^3+c}\,c\,x^3}{45d^2}+\frac{16\sqrt{d\,x^3+c}\,c^2}{45d^3}}{d}-\frac{128\sqrt{d\,x^3+c}\,c^2}{3d^4}-\frac{512ic^2\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x \cdots\right.}{\cdots}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d*(2/15/d*x^6*(d*x^3+c)^(1/2)-8/45*c/d^2*x^3*(d*x^3+c)^(1/2)+16/45*c^2*(
d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*
c/d^2)-128/3*c^2*(d*x^3+c)^(1/2)/d^4-512/27*I*c^2/d^6*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.26, size = 82, normalized size = 0.91

$$-\frac{2\left(1280\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+30\left(dx^3+c\right)^{\frac{3}{2}}c+855\,\sqrt{dx^3+c}\,c^2\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(1280*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + 3*(d*x^3 + c)^(5/2) + 30*(d*x^3 + c)^(3/2)*c + 855*sqrt(d*x^3 +
 c)*c^2)/d^4
```

mupad [B] time = 3.22, size = 98, normalized size = 1.09

$$\frac{512\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}-\frac{592\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}-\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}-\frac{8\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(5/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(9*d^4) - (592*c^2*(c + d*x^3)^(1/2))/(15*d^4) - (2*x^6*(c + d*x^3)^(1/2
))/(15*d^2) - (8*c*x^3*(c + d*x^3)^(1/2))/(5*d^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{11}}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**11/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.309 $\int \frac{x^8}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=71

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^3+128/9*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))
/d^3-14/3*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 88, 63, 206}

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}-\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-14*c*Sqrt[c + d*x^3])/(3*d^3) - (2*(c + d*x^3)^(3/2))/(9*d^3) + (128*c^(3
/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{7c}{d^2\sqrt{c+dx}}+\frac{64c^2}{d^2(8c-dx)\sqrt{c+dx}}-\frac{\sqrt{c+dx}}{d^2}\right)dx,x,x^3\right)$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(64c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d^2}$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{\left(128c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^3}$$
$$= -\frac{14c\sqrt{c+dx^3}}{3d^3}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}+\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.82

$$\frac{128c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-2\sqrt{c+dx^3}\left(22c+dx^3\right)}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3]*(22*c + d*x^3) + 128*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(9*d^3)
```

fricas [A] time = 0.98, size = 121, normalized size = 1.70

$$\left[\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10\,c}{dx^3-8\,c}\right)-\left(dx^3+22\,c\right)\sqrt{dx^3+c}\right)}{9\,d^3}, -\frac{2\left(64\sqrt{-c}\,c\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3\,c}\right)+\left(dx^3+22\,c\right)\sqrt{d\right.}{9\,d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(32*c^(3/2)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*
c)) - (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3, -2/9*(64*sqrt(-c)*c*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d*x^3 + 22*c)*sqrt(d*x^3 + c))/d^3]
```

giac [A] time = 0.19, size = 65, normalized size = 0.92

$$-\frac{128\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^3}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6+21\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -128/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/9*((d*x^
3 + c)^(3/2)*d^6 + 21*sqrt(d*x^3 + c)*c*d^6)/d^9
```

maple [C] time = 0.16, size = 468, normalized size = 6.59

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}\;x^3}{9d}-\frac{4\sqrt{d\,x^3+c}\;c}{9d^2}\right)d+\frac{16\sqrt{d\,x^3+c}\;c}{3d}}{d^2}-\frac{64ic\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d^2*((2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)*d+16/3*(d*x^3
+c)^(1/2)*c/d)-64/27*I*c/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.31, size = 66, normalized size = 0.93

$$-\frac{2\left(32\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+21\,\sqrt{dx^3+c}\,c\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(32*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + (d*x^3 + c)^(3/2) + 21*sqrt(d*x^3 + c)*c)/d^3
```

mupad [B] time = 3.39, size = 78, normalized size = 1.10

$$\frac{64\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}-\frac{44\,c\,\sqrt{d\,x^3+c}}{9\,d^3}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(9*d^3) - (44*c*(c + d*x^3)^(1/2))/(9*d^3) - (2*x^3*(c + d*x^3)^(1/2))/(9
*d^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^8}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x**8/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.310 $\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=52

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

```
[Out] 16/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^2-2/3*(d*x^3+c)^(1/2)/d
^2
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 80, 63, 206}

$$\frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}-\frac{2\sqrt{c+dx^3}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(3*d^2) + (16*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(9*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(8c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{(16c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^2}$$
$$= -\frac{2\sqrt{c+dx^3}}{3d^2} + \frac{16\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.94

$$-\frac{2\left(3\sqrt{c+dx^3}-8\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(3*Sqrt[c + d*x^3] - 8*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]))/(
9*d^2)
```

fricas [A] time = 0.92, size = 103, normalized size = 1.98

$$\left[\frac{2\left(4\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\sqrt{dx^3+c}\right)}{9d^2}, -\frac{2\left(8\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\right)}{9d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/9*(4*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c
)) - 3*sqrt(d*x^3 + c))/d^2, -2/9*(8*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sq
rt(-c)/c) + 3*sqrt(d*x^3 + c))/d^2]
```

giac [A] time = 0.16, size = 48, normalized size = 0.92

$$-\frac{2\left(\frac{8c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}+\frac{3\sqrt{dx^3+c}}{d}\right)}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*(8*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 3*sqrt(d*x^3
+ c)/d)/d
```

maple [C] time = 0.17, size = 425, normalized size = 8.17

$$-\frac{2\sqrt{d\,x^3+c}}{3d^2}-\frac{8i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{R}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*(d*x^3+c)^(1/2)/d^2-8/27*I/d^4*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(
-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/
2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^
2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/
2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.40, size = 56, normalized size = 1.08

$$-\frac{2\left(4\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+3\sqrt{dx^3+c}\right)}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(4*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 3*sqrt(d*x^3 + c))/d^2
```

mupad [B] time = 3.27, size = 60, normalized size = 1.15

$$\frac{8\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}-\frac{2\sqrt{d\,x^3+c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (8*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))
/(9*d^2) - (2*(c + d*x^3)^(1/2))/(3*d^2)
```

sympy [A] time = 16.35, size = 61, normalized size = 1.17

$$\begin{cases}\frac{2\left(-\frac{8c\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}-\frac{\sqrt{c+dx^3}}{3d}\right)}{d} & \text{for } d\neq 0\\ \frac{x^6}{48c^{\frac{3}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] Piecewise((2*(-8*c*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c)) - sqr
t(c + d*x**3)/(3*d))/d, Ne(d, 0)), (x**6/(48*c**(3/2)), True))
```

3.311 $\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

```
[Out] 2/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {444, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*Sqrt[c]*d)
```

fricas [A] time = 0.63, size = 78, normalized size = 2.36

$$\left[\frac{\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)}{9\sqrt{c}\,d}, -\frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{9cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c))/(sqrt(c)
*d), -2/9*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)/(c*d)]
```

giac [A] time = 0.16, size = 27, normalized size = 0.82

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d)
```

maple [C] time = 0.20, size = 413, normalized size = 12.52

$$-\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-8c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
```

```
,-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/
3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=
RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.21, size = 42, normalized size = 1.27

$$-\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/(sqrt
(c)*d)
```

mupad [B] time = 3.23, size = 45, normalized size = 1.36

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(9*c^(1/2)*
d)
```

sympy [A] time = 11.20, size = 32, normalized size = 0.97

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{9d\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(9*d*sqrt(-c))
```

3.312 $\int \frac{1}{x(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=58

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

```
[Out] 1/36*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/12*arctanh((d*x^3+c)^(1
/2)/c^(1/2))/c^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 86, 63, 208, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(36*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]
/Sqrt[c]]/(12*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c} + \frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{12c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12cd}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{3/2}}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{36c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 3*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(36*c^(3/2))
```

fricas [A] time = 0.84, size = 139, normalized size = 2.40

$$\left[\frac{\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 3\,\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{72\,c^2}, \frac{3\,\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{c}\right) - \sqrt{-c}\,\arctan\left(\frac{\sqrt{dx}}{}\right.}{36\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/72*(sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)
) + 3*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/c^2, 1/36
*(3*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c))/c^2]
```

giac [A] time = 0.17, size = 54, normalized size = 0.93

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\,\sqrt{-c}\,c} - \frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{36\,\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/36*arctan(1/3*sqrt(d
*x^3 + c)/sqrt(-c))/(sqrt(-c)*c)
```

maple [C] time = 0.17, size = 433, normalized size = 7.47

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d x^3+c}}{\sqrt{c}}\right)}{12 c^{\frac{3}{2}}}-\frac{i\left(-c d^2\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}} \sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{2\left(-c d^2\right)^{\frac{1}{3}}}} \left(2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216*I/c^2/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))-1/12*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\,\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x), x)
```

mupad [B] time = 3.28, size = 47, normalized size = 0.81

$$-\frac{3\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)-\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{36\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] -(3*atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1/2)) - atanh((c*(c + d*x^3)^(1/2))/
(3*(c^3)^(1/2))))/(36*(c^3)^(1/2))
```

sympy [A] time = 12.22, size = 58, normalized size = 1.00

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{36c\sqrt{-c}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(36*c*sqrt(-c)) + atan(sqrt(c + d*x**3
)/sqrt(-c))/(12*c*sqrt(-c))
```

3.313 $\int \frac{1}{x^4(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=81

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}-\frac{\sqrt{c+dx^3}}{24c^2x^3}$$

```
[Out] 1/288*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+1/32*d*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(5/2)-1/24*(d*x^3+c)^(1/2)/c^2/x^3
```

Rubi [A] time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$-\frac{\sqrt{c+dx^3}}{24c^2x^3}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(24*c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2
88*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\operatorname{Subst}\left(\int \frac{3cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{d\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{64c^2} + \frac{d^2\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} - \frac{\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{32c^2} + \frac{d\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+d\right.}}{96c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.00

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{5/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{32c^{5/2}} - \frac{\sqrt{c+dx^3}}{24c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/24*Sqrt[c + d*x^3]/(c^2*x^3) + (d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(288*c^(5/2)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(32*c^(5/2))
```

fricas [A] time = 0.97, size = 184, normalized size = 2.27

$$\left[\frac{\sqrt{c}\,dx^3\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+9\sqrt{c}\,dx^3\log\left(\frac{dx^3+2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-24\sqrt{dx^3+c}\,c}{576c^3x^3}, -\frac{9\sqrt{-c}\,dx^3\arctan\left(\frac{\sqrt{dx^3+c}}{c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/576*(sqrt(c)*d*x^3*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3
 - 8*c)) + 9*sqrt(c)*d*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^
3) - 24*sqrt(d*x^3 + c)*c)/(c^3*x^3), -1/288*(9*sqrt(-c)*d*x^3*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) + sqrt(-c)*d*x^3*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)
/c) + 12*sqrt(d*x^3 + c)*c)/(c^3*x^3)]
```

giac [A] time = 0.16, size = 73, normalized size = 0.90

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{32\sqrt{-c}\,c^2}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}\,c^2}-\frac{\sqrt{dx^3+c}}{24\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/32*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/288*d*arctan(1/
3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/24*sqrt(d*x^3 + c)/(c^2*x^3)
```

maple [C] time = 0.19, size = 477, normalized size = 5.89

$$-\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{96c^{\frac{5}{2}}}+\frac{\frac{d\operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}}{8c}-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-1/1728*I/d/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c))-1/96*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^4), x)
```

mupad [B] time = 3.42, size = 73, normalized size = 0.90

$$\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{32\,\sqrt{c^5}}+\frac{d\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{288\,\sqrt{c^5}}-\frac{\sqrt{d\,x^3+c}}{24\,c^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d*atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2)))/(32*(c^5)^(1/2)) + (d*atanh(
(c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(288*(c^5)^(1/2)) - (c + d*x^3)^(
1/2)/(24*c^2*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^4\sqrt{c+dx^3}+dx^7\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**4*sqrt(c + d*x**3) + d*x**7*sqrt(c + d*x**3)), x)
```

3.314 $\int \frac{1}{x^7(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=107

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

```
[Out] 1/2304*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-7/256*d^2*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/48*(d*x^3+c)^(1/2)/c^2/x^6+5/192*d*(d*x^3
+c)^(1/2)/c^3/x^3
```

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}}-\frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{192c^3x^3}-\frac{\sqrt{c+dx^3}}{48c^2x^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x^6) + (5*d*Sqrt[c + d*x^3])/(192*c^3*x^3) + (d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2304*c^(7/2)) - (7*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(256*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} - \frac{\text{Subst}\left(\int \frac{10cd-\frac{3d^2x}{2}}{x^2(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\text{Subst}\left(\int \frac{42c^2d^2-5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{384c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{512c^3} + \frac{d^3\,\text{Subst}\left(\int \frac{}{(8c}\right.}{15}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{256c^3} + \frac{d^2\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6} + \frac{5d\sqrt{c+dx^3}}{192c^3x^3} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2304c^{7/2}} - \frac{7d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{256c^{7/2}}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 0.89

$$\frac{d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 63d^2x^6\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) + 12\sqrt{c}\sqrt{c+dx^3}\left(5dx^3-4c\right)}{2304c^{7/2}x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3]*(-4*c + 5*d*x^3) + d^2*x^6*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])] - 63*d^2*x^6*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2304*c^(7/
2)*x^6)
```

fricas [A] time = 0.99, size = 217, normalized size = 2.03

$$\left[\frac{\sqrt{c}\, d^2 x^6 \log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 63\sqrt{c}\, d^2 x^6 \log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) + 24\left(5cdx^3-4c^2\right)\sqrt{dx^3+c}}{4608\, c^4 x^6}, \frac{63\sqrt{-c}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4608*(sqrt(c)*d^2*x^6*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*
x^3 - 8*c)) + 63*sqrt(c)*d^2*x^6*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
*c)/x^3) + 24*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^4*x^6), 1/2304*(63*sq
rt(-c)*d^2*x^6*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(-c)*d^2*x^6*arctan
(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/
(c^4*x^6)]
```

giac [A] time = 0.18, size = 101, normalized size = 0.94

$$\frac{7\, d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{256\sqrt{-c}\, c^3} - \frac{d^2 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2304\sqrt{-c}\, c^3} + \frac{5\left(dx^3+c\right)^{\frac{3}{2}} d^2 - 9\sqrt{dx^3+c}\, cd^2}{192\, c^3 d^2 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 7/256*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2304*d^2*arct
an(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/192*(5*(d*x^3 + c)^(3/2
)*d^2 - 9*sqrt(d*x^3 + c)*c*d^2)/(c^3*d^2*x^6)
```

maple [C] time = 0.21, size = 540, normalized size = 5.05

$$-\frac{d^2 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{768 c^{\frac{7}{2}}} + \frac{-\frac{d^2 \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{4c^{\frac{5}{2}}} + \frac{\sqrt{d\,x^3+c}\; d}{4c^2 x^3} - \frac{\sqrt{d\,x^3+c}}{6c\,x^6}}{8c} + \frac{\left(\frac{d \operatorname{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}} - \frac{\sqrt{d\,x^3+c}}{3c\,x^3}\right) d}{64c^2} - \frac{i\left(-c\,d^2\right)^{\frac{1}{3}} \sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2
))/c^(3/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*d*(d*x^3+c)^(1/2)/c^2/x^3
-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/13824*I/c^4*2^(1/2)*su
m((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
```

```
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))-1/76
8*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 3.50, size = 94, normalized size = 0.88

$$\frac{d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2304\,\sqrt{c^7}}-\frac{7\,d^2\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{256\,\sqrt{c^7}}-\frac{3\,\sqrt{d\,x^3+c}}{64\,c^2\,x^6}+\frac{5\,{\left(d\,x^3+c\right)}^{3/2}}{192\,c^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] (d^2*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2))))/(2304*(c^7)^(1/2)) - (
7*d^2*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(256*(c^7)^(1/2)) - (3*(c
 + d*x^3)^(1/2))/(64*c^2*x^6) + (5*(c + d*x^3)^(3/2))/(192*c^3*x^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^7\sqrt{c+dx^3}+dx^{10}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**7*sqrt(c + d*x**3) + d*x**10*sqrt(c + d*x**3)), x)
```

3.315 $\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=630

$$-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{7}$$

```
[Out] 32/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
8/3)-32/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/9*c^(7/6)
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/2
)-2/7*x^2*(d*x^3+c)^(1/2)/d^2-104/7*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))-104/21*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)
*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+52/7*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {479, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}+\frac{32c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}-\frac{32c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}-\frac{104\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right.}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*x^2*Sqrt[c + d*x^3])/(7*d^2) - (104*c*Sqrt[c + d*x^3])/(7*d^(8/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1
/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(8/3)) + (32*c^(7/6)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) - (32*
c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^(8/3)) + (52*3^(1/4)*Sqr
t[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(7*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (104*Sqrt[2]*c^(4/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(
8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{(8c-dx^3)\sqrt{c+dx^3}}\,dx = -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \frac{x(16c^2+26cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} + \frac{2\int \left(-\frac{26cx}{\sqrt{c+dx^3}} + \frac{224c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{7d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(52c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{7d^2} + \frac{(64c^2)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{(16c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} - \frac{(52c)\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{7d^{7/3}} + \frac{(1}{}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{52\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{7d^{8/3}}$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}} + 3$$

$$= -\frac{2x^2\sqrt{c+dx^3}}{7d^2} - \frac{104c\sqrt{c+dx^3}}{7d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{8/3}} + 3$$

Mathematica [C] time = 0.09, size = 130, normalized size = 0.21

$$\frac{x^2\left(13dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+20c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20\left(c+dx^3\right)\right)}{70d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*(-20*(c + d*x^3) + 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3,
 -((d*x^3)/c), (d*x^3)/(8*c)] + 13*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3,
1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(70*d^2*Sqrt[c + d*x^3])
```

fricas [F] time = 31.99, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^7}{d^2x^6-7\,cdx^3-8\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^7/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.24, size = 1311, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/d^2*(d*(2/7*x^2*(d*x^3+c)^(1/2)/d+8/21*I*c/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))))-16/3*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-
I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^
(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-64/27*I*c/d
^5*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_a
lpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1
/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-
1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**7/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.316 $\int \frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=601

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}$$

```
[Out] 4/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5
/3)-4/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)-4/9*c^(1/6)*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)-2
*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-2/3*c^(1/3)*(c^(1/
3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/
(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+3^(1/
4)*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^
(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {483, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*Sqrt[c + d*x^3])/(d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*c^(
1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sq
rt[3]*d^(5/3)) + (4*c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt
[c + d*x^3])])/(9*d^(5/3)) - (4*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(9*d^(5/3)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*S
qrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3
) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (
2*Sqrt[2]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 -
 Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4
*Sqrt[3]])/(3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
```

```
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{d}+\frac{(8c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= -\frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^2}-\frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{d^{4/3}}+\frac{\left(2\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,d\cdots}{3d^{4/3}}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\cdots\right)}}\cdots}{d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\cdots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\cdots\right)}}\cdots}$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{4\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}+\frac{4\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}\cdots\right)}{3\sqrt[6]{c}\cdots}\right)}{9d^{5/3}}\cdots$$

$$= -\frac{2\sqrt{c+dx^3}}{d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{4\sqrt[6]{c}\,\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}}+\frac{4\sqrt[6]{c}\,\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}\cdots\right)}{3\sqrt[6]{c}\cdots}\right)}{9d^{5/3}}\cdots$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.11

$$\frac{x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{40c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(40*c*Sqrt[c + d*x^3])
```

fricas [F] time = 7.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^4}{d^2x^6-7\,cdx^3-8\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.17, size = 848, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-8/27*I/d^4*2^(1/2)*sum(1/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
```

```
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x**4/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.317 $\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=141

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

```
[Out] 1/18*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
2/3)-1/18*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)-1/18*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^(1/2
)
```

Rubi [A] time = 0.41, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(6*Sqrt[3]
*c^(5/6)*d^(2/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x
^3])]/(18*c^(5/6)*d^(2/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(18*c^(5/
6)*d^(2/3))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{12cd}+\frac{\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{12c^{2/3}\sqrt[3]{d}}-\frac{\sqrt[3]{d}\int \frac{x^2}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{4\sqrt[3]{c}}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{9-cx^2}\,dx,x,\frac{\left(1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)^2}{\sqrt{c+dx^3}}\right)}{6\sqrt[3]{c}\,d^{2/3}}-\frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{12\sqrt[3]{c}}+\frac{d^{4/3}\,\text{Sub}}{}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx}\right.}{6\sqrt[3]{c}\,d^{2/3}}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{6\sqrt{3}\,c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{18c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18c^{5/6}d^{2/3}}$$

Mathematica [C] time = 0.06, size = 67, normalized size = 0.48

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{16c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(16*c*Sqrt[c + d*x^3])
```

fricas [B] time = 4.33, size = 2459, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*
d^4))^(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d
^4))^(5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqr
t(d*x^3 + c) + (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3)
+ 12*sqrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*
sqrt(3)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) + sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5
*x^6 + 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*
d^3*x^4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x
^2)*(1/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 -
1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(
2/3) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6
) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^
3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^
8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^
2*x^6 + 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) - 1/54*
sqrt(3)*(1/(c^5*d^4))^(1/6)*arctan(1/9*((9*sqrt(3)*c*d^2*x^5*(1/(c^5*d^4))^
(1/6) - sqrt(3)*(c^4*d^5*x^6 - 40*c^5*d^4*x^3 - 32*c^6*d^3)*(1/(c^5*d^4))^(
5/6) + 3*sqrt(3)*(5*c^3*d^3*x^4 + 8*c^4*d^2*x)*sqrt(1/(c^5*d^4)))*sqrt(d*x^
3 + c) - (18*sqrt(3)*(c^4*d^4*x^5 + c^5*d^3*x^2)*(1/(c^5*d^4))^(2/3) + 12*s
qrt(3)*(c^2*d^3*x^6 - c^3*d^2*x^3 - 2*c^4*d)*(1/(c^5*d^4))^(1/3) + 3*sqrt(3
)*(d^2*x^7 + 5*c*d*x^4 + 4*c^2*x) - sqrt(d*x^3 + c)*(sqrt(3)*(c^4*d^5*x^6 +
 32*c^5*d^4*x^3 + 40*c^6*d^3)*(1/(c^5*d^4))^(5/6) + 3*sqrt(3)*(7*c^3*d^3*x^
4 + 4*c^4*d^2*x)*sqrt(1/(c^5*d^4)) + 9*sqrt(3)*(c*d^2*x^5 + 2*c^2*d*x^2)*(1
/(c^5*d^4))^(1/6)))*sqrt((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c
^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) -
 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*
(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7
- 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20
*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
+ 192*c^2*d*x^3 - 512*c^3)))/(d^2*x^7 - 7*c*d*x^4 - 8*c^2*x)) + 1/108*(1/(c
^5*d^4))^(1/6)*log((d^3*x^9 + 318*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18
*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4 + 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) + 6*sq
rt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 + 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*
c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 64*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7
+ 80*c^2*d^2*x^4 + 160*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38
*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6
 + 192*c^2*d*x^3 - 512*c^3)) - 1/108*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 + 318
*c*d^2*x^6 + 1200*c^2*d*x^3 + 640*c^3 + 18*(5*c^4*d^5*x^7 + 64*c^5*d^4*x^4
+ 32*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(6*(5*c^5*d^5*x^5 +
 32*c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) + (7*c^3*d^4*x^6 + 152*c^4*d^3*x^3 + 6
4*c^5*d^2)*sqrt(1/(c^5*d^4)) + (c*d^3*x^7 + 80*c^2*d^2*x^4 + 160*c^3*d*x)*(
1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 38*c^3*d^3*x^5 + 64*c^4*d^2*x^2)*(1
/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3)) - 1/
216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 276*c*d^2*x^6 - 1608*c^2*d*x^3 - 108
8*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3
) + 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 + c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) -
 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x
^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 +
 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x
^6 + 192*c^2*d*x^3 - 512*c^3)) + 1/216*(1/(c^5*d^4))^(1/6)*log((d^3*x^9 - 2
76*c*d^2*x^6 - 1608*c^2*d*x^3 - 1088*c^3 - 18*(c^4*d^5*x^7 - 52*c^5*d^4*x^4
 - 80*c^6*d^3*x)*(1/(c^5*d^4))^(2/3) - 6*sqrt(d*x^3 + c)*(24*(c^5*d^5*x^5 +
```

```
c^6*d^4*x^2)*(1/(c^5*d^4))^(5/6) - 4*(c^3*d^4*x^6 + 41*c^4*d^3*x^3 + 40*c^
5*d^2)*sqrt(1/(c^5*d^4)) - (c*d^3*x^7 - 28*c^2*d^2*x^4 - 272*c^3*d*x)*(1/(c
^5*d^4))^(1/6)) + 18*(c^2*d^4*x^8 + 20*c^3*d^3*x^5 - 8*c^4*d^2*x^2)*(1/(c^5
*d^4))^(1/3))/(d^3*x^9 - 24*c*d^2*x^6 + 192*c^2*d*x^3 - 512*c^3))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\sqrt{dx^3+c}\,\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.16, size = 416, normalized size = 2.95

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\,\_Z^3-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\sqrt{dx^3+c}\,\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

mupad [B] time = 40.22, size = 272, normalized size = 1.93

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\left(\sqrt{d\,x^3+c}-\sqrt{c}+2\,c^{1/6}\,d^{1/3}\,x\right)^3}{x^3\,\left(d^{1/3}\,x-2\,c^{1/3}\right)^3}\right)}{54\,c^{5/6}\,d^{2/3}}+\frac{\sqrt{2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(-\sqrt{3}\,c^{1/6}\,d^{1/3}\,x+\sqrt{d\,x^3+c}\,1\mathrm{i}+\sqrt{c}\,1\mathrm{i}+c^{1/6}\,d^{1/3}\,x\,1\mathrm{i}\right)^3}{x^3\,\left(d^{1/3}\,x+c^{1/3}-\sqrt{3}\,c^{1/3}\,1\mathrm{i}\right)^3}\right)\sqrt{-1-}{108\,c^{5/6}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) + c^(1/2))*((c + d*x^3)^(1/2) - c^(1/2) + 2*c^(1/6)
*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 2*c^(1/3))^3))/(54*c^(5/6)*d^(2/3)) + (2^(
1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2)*1i + c^(1/2)*1i
+ c^(1/6)*d^(1/3)*x*1i - 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(d^(1/3)*x - 3^
(1/2)*c^(1/3)*1i + c^(1/3))^3))*(3^(1/2)*1i - 1)^(1/2))/(108*c^(5/6)*d^(2/3
)) + (2^(1/2)*log((((c + d*x^3)^(1/2) + c^(1/2))*(c^(1/2)*1i - (c + d*x^3)^
(1/2)*1i + c^(1/6)*d^(1/3)*x*1i + 3^(1/2)*c^(1/6)*d^(1/3)*x)^3)/(x^3*(3^(1/
2)*c^(1/3)*1i + d^(1/3)*x + c^(1/3))^3))*(3^(1/2)*1i + 1)^(1/2)*1i)/(108*c^
(5/6)*d^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(x/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.318 $\int \frac{1}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=632

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{16c^{5/}}$$

[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*3^(1/2)-1/8*(d*x^3+c)^(1/2)/c^2/x+1/8*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/24*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/16*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.72, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {480, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{16c^{5/}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] -Sqrt[c + d*x^3]/(8*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(8*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqrt[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(144*c^(11/6)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\int \frac{x\left(5cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}}+\frac{cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{8c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{d\int \frac{x}{\sqrt{c+dx^3}}dx}{16c^2}+\frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}-\frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{96c^2}+\frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{16c^2}+\frac{d^{2/3}\int \frac{\ldots}{\left(2-\ldots\right.}}{9\ldots}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\ldots}{\left(\left(1\ldots\right.\right.}}}{16c^{5/3}\sqrt{\ldots}}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\tan\ldots}{\ldots}$$

$$= -\frac{\sqrt{c+dx^3}}{8c^2x}+\frac{\sqrt[3]{d}\sqrt{c+dx^3}}{8c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{11/6}}+\frac{\sqrt[3]{d}\tan\ldots}{\ldots}$$

Mathematica [C] time = 0.09, size = 137, normalized size = 0.22

$$\frac{-d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+25cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(c+dx^3\right)}{640c^3x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(c + d*x^3) + 25*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3
, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(640*c^3*x*Sqrt[c + d*x^3])
```

fricas [F] time = 2.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^8-7\,cdx^5-8\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^8 - 7*c*d*x^5 - 8*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

maple [C] time = 0.17, size = 874, normalized size = 1.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
[Out] 1/8/c*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-
1/216*I/c^2/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^2\sqrt{c+dx^3}+dx^5\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**2*sqrt(c + d*x**3) + d*x**5*sqrt(c + d*x**3)), x)
```

3.319 $\int \frac{1}{x^5(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=654

$$-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\ldots\right.}}}{8\sqrt{2}\sqrt[4]{3}c^{8/3}\ldots}$$

```
[Out] 1/1152*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1152*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1152
*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/
6)*3^(1/2)-1/32*(d*x^3+c)^(1/2)/c^2/x^4+1/16*d*(d*x^3+c)^(1/2)/c^3/x-1/16*d
^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-1/48*d^(4/3)*(c^
(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/
32*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1
/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/
2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 654, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {480, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{384\sqrt{3}c^{17/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1152c^{17/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{17/6}}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(32*c^2*x^4) + (d*Sqrt[c + d*x^3])/(16*c^3*x) - (d^(4/3)*S
qrt[c + d*x^3])/(16*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*Arc
Tan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(384*Sqrt[3]*
c^(17/6)) + (d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])])/(1152*c^(17/6)) - (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1
152*c^(17/6)) + (3^(1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*c^(8/3)*Sqrt[(c^(1/3)*(c^(1
/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) -
 (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3]
)*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]
])/(8*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{\int \frac{-16cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{32c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{\int \frac{x\left(60c^2d^2-8cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{\int \left(\frac{8cd^2x}{\sqrt{c+dx^3}}-\frac{4c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{256c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c^3}+\frac{d^2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{64c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{768c^3}-\frac{d^{5/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}}{32c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,d^{4/3}\left(\sqrt[3]{\right.}}{}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\right.}{\sqrt{c+dx}}\right.}{384\sqrt{3}\,c^{17/6}}$$

$$= -\frac{\sqrt{c+dx^3}}{32c^2x^4}+\frac{d\sqrt{c+dx^3}}{16c^3x}-\frac{d^{4/3}\sqrt{c+dx^3}}{16c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\right.}{\sqrt{c+dx}}\right.}{384\sqrt{3}\,c^{17/6}}$$

Mathematica [C] time = 0.10, size = 152, normalized size = 0.23

$$\frac{4d^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-75cd^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+160c\left(-c^2+cdx^3+2d^2x^6\right)}{5120c^4x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (160*c*(-c^2 + c*d*x^3 + 2*d^2*x^6) - 75*c*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Appe
llF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 4*d^3*x^9*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(5120*c^4
*x^4*Sqrt[c + d*x^3])
```

fricas [F] time = 5.37, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^{11}-7\,cdx^8-8\,c^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^11 - 7*c*d*x^8 - 8*c^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

maple [C] time = 0.20, size = 1351, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/c/x^4+5/8*d*(d*x^3+c)^(1/2)/c^2/x+5/24*I/c^2*d*
3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d*(-(d*x^3+c)^(1/2)/c/x-1/3
*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/1728*I/c^3/d*2^(1/2)*sum(1/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^5\sqrt{c+dx^3}+dx^8\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**5*sqrt(c + d*x**3) + d*x**8*sqrt(c + d*x**3)), x)
```

3.320 $\int \frac{1}{x^8(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=678

$$-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{23/6}}+\frac{3^{3/4}d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left((1+\sqrt{\cdots}\right.}}}{28\sqrt{2}c^{11/\cdots}}$$

```
[Out] 1/9216*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(23/6)-1/9216*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/9216
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/
6)*3^(1/2)-1/56*(d*x^3+c)^(1/2)/c^2/x^7+37/1792*d*(d*x^3+c)^(1/2)/c^3/x^4-3
/56*d^2*(d*x^3+c)^(1/2)/c^4/x+3/56*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+1/56*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-3/112*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 678, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {480, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{23/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{23/6}}-\frac{d^{7/3}\tanh\cdots}{921\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(56*c^2*x^7) + (37*d*Sqrt[c + d*x^3])/(1792*c^3*x^4) - (3*
d^2*Sqrt[c + d*x^3])/(56*c^4*x) + (3*d^(7/3)*Sqrt[c + d*x^3])/(56*c^4*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3)
 + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(23/6)) + (d^(7/3)*ArcTanh
[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(23/6)) - (d
^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(23/6)) - (3*3^(1/4)*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(112*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (3^(3/4)*d^(7/3)*(c^(1/
3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1
/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(28*Sqrt[2]*c
^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx = -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{\int \frac{-37cd+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{56c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{\int \frac{-768c^2d^2+\frac{185}{2}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1792c^4}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \frac{x\left(3100c^3d^3-384c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\int \left(\frac{384c^2d^3x}{\sqrt{c+dx^3}}+\frac{28c^3d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{14336c^6}$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{\left(3d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{112c^4}+\frac{d^3\int \frac{x}{\left(8c-dx^3\right)\cdots}}{512\cdots} \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}-\frac{d^2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6144c^4}+\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{56c^2x^7}+\frac{37d\sqrt{c+dx^3}}{1792c^3x^4}-\frac{3d^2\sqrt{c+dx^3}}{56c^4x}+\frac{3d^{7/3}\sqrt{c+dx^3}}{56c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\cdots$$

Mathematica [C] time = 0.16, size = 167, normalized size = 0.25

$$\frac{3875cd^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-32\left(6d^4x^{12}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5c\left(32c^3-5c^2dx^3+\cdots\right.\right.}{286720c^5x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (3875*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c)
, (d*x^3)/(8*c)] - 32*(5*c*(32*c^3 - 5*c^2*d*x^3 + 59*c*d^2*x^6 + 96*d^3*x^
9) + 6*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c)
, (d*x^3)/(8*c)]))/(286720*c^5*x^7*Sqrt[c + d*x^3])
```

fricas [F] time = 13.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^{14}-7\,cdx^{11}-8\,c^2x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^14 - 7*c*d*x^11 - 8*c^2*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

maple [C] time = 0.20, size = 1849, normalized size = 2.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
[Out] 1/64/c^2*d*(-1/4*(d*x^3+c)^(1/2)/c/x^4+5/8*(d*x^3+c)^(1/2)/c^2*d/x+5/24*I/c
^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/c/x
^7+11/56*d*(d*x^3+c)^(1/2)/c^2/x^4-55/112*d^2*(d*x^3+c)^(1/2)/c^3/x-55/336*
I/c^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^2*(-(d*x^3+c)^
(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
```

```
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/13824*I/c^4*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*
d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c
*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z
^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^8\sqrt{c+dx^3}+dx^{11}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**8*sqrt(c + d*x**3) + d*x**11*sqrt(c + d*x**3)), x)
```

3.321 $\int \frac{x^3}{(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,1/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{32c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(
8*c)])/(32*c*Sqrt[c + d*x^3])
```

fricas [F] time = 1.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^3}{d^2x^6-7\,cdx^3-8\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^3/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.24, size = 696, normalized size = 10.55

$$\frac{2i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}}{\cdots}\right.}{3\sqrt{d\,x^3+c}\;d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-8/27*
I/d^4*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^
```

```
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alp
ha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(x**3/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.322 $\int \frac{1}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,1/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/(d*x
^3+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\
&= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] time = 0.17, size = 166, normalized size = 2.59

$$\frac{32cxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((8*c - d*x^3)*Sqrt[c + d*x^3]),x]

[Out] (32*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*Sqrt[c + d*x^3]*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))

fricas [F] time = 1.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^6-7\,cdx^3-8\,c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^6 - 7*c*d*x^3 - 8*c^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)

maple [C] time = 0.16, size = 416, normalized size = 6.50

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\text{RootOf}\left(d\,\_Z^3-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)

[Out] -1/27*I/d^3/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c\sqrt{c+dx^3}+dx^3\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*sqrt(c + d*x**3) + d*x**3*sqrt(c + d*x**3)), x)
```

3.323 $\int \frac{1}{x^3(8c-dx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,1/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16cx^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.24, size = 242, normalized size = 3.67

$$\frac{\frac{d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{4096dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(dx^3-8c\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-\frac{64(c+}{c^2}}{1024x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3))/c^2 + (d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^3 + (4096*d*x^3*AppellF1[1/3, 1/2, 1,
 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((-8*c + d*x^3)*(32*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)]))))/(1024*x^2*Sqrt[c + d*x^3])
```

fricas [F] time = 4.09, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^9-7\,cdx^6-8\,c^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^9 - 7*c*d*x^6 - 8*c^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

maple [C] time = 0.15, size = 722, normalized size = 10.94

$$\frac{\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\left(-\right.}{2}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{3}\right.}{6\sqrt{d\,x^3+c}\;c}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/2/c*(d*x^3+c)^(1/2)/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)))-1/216*I/c^2/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
```

```
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^3\sqrt{c+dx^3}+dx^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
[Out] -Integral(1/(-8*c*x**3*sqrt(c + d*x**3) + d*x**6*sqrt(c + d*x**3)), x)
```

3.324 $\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,1/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c/x^
5/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c*x^5*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40cx^5\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.25, size = 261, normalized size = 3.95

$$\frac{64c\left(\frac{3264c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-16c^2+7cdx^3+23d^2x^6\right)-23d^3x^9}{40960c^4x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-23*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + 64*c*(-16*c^2 + 7*c*d*x^3 + 23*d^2*x^6 + (3264*c^2*d^2*x^6*
AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32
*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(Appel
lF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1
, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(40960*c^4*x^5*Sqrt[c + d*x^3])
```

fricas [F] time = 11.25, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^2x^{12}-7\,cdx^9-8\,c^2x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^2*x^12 - 7*c*d*x^9 - 8*c^2*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

maple [C] time = 0.20, size = 1047, normalized size = 15.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*(-1/5/c*(d*x^3+c)^(1/2)/x^5+7/20/c^2*d*(d*x^3+c)^(1/2)/x^2-7/60*I/c^2
*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/64/c^2*d*
(-1/2*(d*x^3+c)^(1/2)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)))-1/1728*I/c^3/d*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
```

```
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8cx^6\sqrt{c+dx^3}+dx^9\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(1/2),x)
```

```
[Out] -Integral(1/(-8*c*x**6*sqrt(c + d*x**3) + d*x**9*sqrt(c + d*x**3)), x)
```

3.325 $\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

```
[Out] -2/9*(d*x^3+c)^(3/2)/d^4+1024/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2
))/d^4+2/27*c^2/d^4/(d*x^3+c)^(1/2)-4*c*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 87, 43, 63, 206}

$$\frac{2c^2}{27d^4\sqrt{c+dx^3}}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2\left(c+dx^3\right)^{3/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(27*d^4*Sqrt[c + d*x^3]) - (4*c*Sqrt[c + d*x^3])/d^4 - (2*(c + d*x^
3)^(3/2))/(9*d^4) + (1024*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81
*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3}{(8c-dx)(c+dx)^{3/2}}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{c^2}{9d^3(c+dx)^{3/2}}-\frac{7c}{d^3\sqrt{c+dx}}-\frac{x}{d^2\sqrt{c+dx}}+\frac{512c^2}{9d^3(8c-dx)\sqrt{c+dx}}\right.\right.\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{14c\sqrt{c+dx^3}}{3d^4}+\frac{(512c^2)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d^3}-\operatorname{Subs}\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{14c\sqrt{c+dx^3}}{3d^4}+\frac{(1024c^2)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^4}-\operatorname{Su}\\
&= \frac{2c^2}{27d^4\sqrt{c+dx^3}}-\frac{4c\sqrt{c+dx^3}}{d^4}-\frac{2(c+dx^3)^{3/2}}{9d^4}+\frac{1024c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 66, normalized size = 0.73

$$-\frac{2\left(512c^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-456c^2+60cdx^3+3d^2x^6\right)}{27d^4\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-456*c^2 + 60*c*d*x^3 + 3*d^2*x^6 + 512*c^2*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)]))/(27*d^4*Sqrt[c + d*x^3])
```

fricas [A] time = 1.22, size = 189, normalized size = 2.10

$$\left[\frac{2\left(256\left(cdx^3+c^2\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\left(3d^2x^6+60cdx^3+56c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3+cd^4\right)},-\frac{2\left(512\left(cdx^3+c^2\right)\sqrt{-\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(256*(c*d*x^3 + c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) - 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c)
)/(d^5*x^3 + c*d^4), -2/81*(512*(c*d*x^3 + c^2)*sqrt(-c)*arctan(1/3*sqrt(d*
x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 60*c*d*x^3 + 56*c^2)*sqrt(d*x^3 + c))
/(d^5*x^3 + c*d^4)]
```

giac [A] time = 0.20, size = 82, normalized size = 0.91

$$-\frac{1024\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^4}+\frac{2\,c^2}{27\sqrt{dx^3+c}\,d^4}-\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^8+18\sqrt{dx^3+c}\,cd^8\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1024/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/27*c^2
/(sqrt(d*x^3 + c)*d^4) - 2/9*((d*x^3 + c)^(3/2)*d^8 + 18*sqrt(d*x^3 + c)*c*
d^8)/d^12
```

maple [C] time = 0.30, size = 560, normalized size = 6.22

$$-\frac{512\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(-2/3/d^3*c^2/((x^3+c/d)*d)^(1/2)+2/9/d^2*x^3*(d*x^3+c)^(1/2)-10/9*c*(
d*x^3+c)^(1/2)/d^3)-8*c/d^2*(2/3/d^2*c/((x^3+c/d)*d)^(1/2)+2/3*(d*x^3+c)^(1
/2)/d^2)+128/3*c^2/d^4/(d*x^3+c)^(1/2)-512*c^3/d^3*(2/27/d/c/((x^3+c/d)*d)^
(1/2)+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.25, size = 82, normalized size = 0.91

$$-\frac{2\left(256\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+9\left(dx^3+c\right)^{\frac{3}{2}}+162\sqrt{dx^3+c}\,c-\frac{3c^2}{\sqrt{dx^3+c}}\right)}{81\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(256*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 9*(d*x^3 + c)^(3/2) + 162*sqrt(d*x^3 + c)*c - 3*c^2/sqrt(d*x^3 +
 c))/d^4
```

mupad [B] time = 3.78, size = 95, normalized size = 1.06

$$\frac{512\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}-\frac{38\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{2\,c^2}{27\,d^4\,\sqrt{d\,x^3+c}}-\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (512*c^(3/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/(81*d^4) - (38*c*(c + d*x^3)^(1/2))/(9*d^4) + (2*c^2)/(27*d^4*(c + d*x^3
)^(1/2)) - (2*x^3*(c + d*x^3)^(1/2))/(9*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.326 $\int \frac{x^8}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] 128/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3-2/27*c/d^3/(d*x^3+c
)^(1/2)-2/3*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 87, 63, 206}

$$-\frac{2c}{27d^3\sqrt{c+dx^3}}-\frac{2\sqrt{c+dx^3}}{3d^3}+\frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(27*d^3*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(3*d^3) + (128*Sqrt[c
]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int\left(\frac{c}{9d^2(c+dx)^{3/2}} - \frac{1}{d^2\sqrt{c+dx}} + \frac{64c}{9d^2(8c-dx)\sqrt{c+dx}}\right)dx, x, x^3\right)$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(64c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d^2}$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{(128c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^3}$$
$$= -\frac{2c}{27d^3\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{128\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Mathematica [C] time = 0.05, size = 53, normalized size = 0.75

$$-\frac{2\left(64c\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-54c+9dx^3\right)}{27d^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(-54*c + 9*d*x^3 + 64*c*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*
c)]))/(27*d^3*Sqrt[c + d*x^3])
```

fricas [A] time = 1.08, size = 161, normalized size = 2.27

$$\left[\frac{2\left(32\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-3\left(9dx^3+10c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3+cd^3\right)}, -\frac{2\left(64\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\ \cdots}{3c}\right.\right.}{81\left(d^4x^3 \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(32*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 3*(9*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3), -
2/81*(64*(d*x^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(9
*d*x^3 + 10*c)*sqrt(d*x^3 + c))/(d^4*x^3 + c*d^3)]
```

giac [A] time = 0.17, size = 58, normalized size = 0.82

$$-\frac{128\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,d^3} - \frac{2\sqrt{dx^3+c}}{3\,d^3} - \frac{2\,c}{27\sqrt{dx^3+c}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -128/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/3*sqrt(d*
x^3 + c)/d^3 - 2/27*c/(sqrt(d*x^3 + c)*d^3)
```

maple [C] time = 0.17, size = 501, normalized size = 7.06

$$-\frac{64\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d^2*(d*(2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)-16/3*c/d/
(d*x^3+c)^(1/2))-64*c^2/d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^
(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))
```

maxima [A] time = 1.12, size = 68, normalized size = 0.96

$$-\frac{2\left(32\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+27\sqrt{dx^3+c}+\frac{3c}{\sqrt{dx^3+c}}\right)}{81\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/81*(32*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 27*sqrt(d*x^3 + c) + 3*c/sqrt(d*x^3 + c))/d^3
```

mupad [B] time = 3.71, size = 75, normalized size = 1.06

$$\frac{64\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^3}-\frac{2\,c}{27\,d^3\,\sqrt{d\,x^3+c}}-\frac{2\,\sqrt{d\,x^3+c}}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (64*c^(1/2)*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))
)/(81*d^3) - (2*c)/(27*d^3*(c + d*x^3)^(1/2)) - (2*(c + d*x^3)^(1/2))/(3*d^
3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.327 $\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

```
[Out] 16/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+2/27/d^2/(d*x^3+c)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{2}{27d^2\sqrt{c+dx^3}}+\frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*d^2*Sqrt[c + d*x^3]) + (16*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
Sqrt[c]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{8\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}$$
$$= \frac{2}{27d^2\sqrt{c+dx^3}} + \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{2\left(\frac{3}{\sqrt{c+dx^3}}+\frac{8\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{81d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(3/Sqrt[c + d*x^3] + (8*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/
(81*d^2)
```

fricas [A] time = 0.81, size = 149, normalized size = 2.87

$$\left[\frac{2\left(4\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\sqrt{dx^3+c}\,c\right)}{81\left(cd^3x^3+c^2d^2\right)}, -\frac{2\left(8\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-3\sqrt{dx^3+\cdots}\right.}{81\left(cd^3x^3+c^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/81*(4*(d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) + 3*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 + c^2*d^2), -2/81*(8*(d*x
^3 + c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)
*c)/(c*d^3*x^3 + c^2*d^2)]
```

giac [A] time = 0.17, size = 47, normalized size = 0.90

$$-\frac{2\left(\frac{8\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}-\frac{3}{\sqrt{dx^3+c}\,d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*(8*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 3/(sqrt(d*x^3
+ c)*d))/d
```

maple [C] time = 0.18, size = 456, normalized size = 8.77

$$-8\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)

[Out] 2/3/d^2/(d*x^3+c)^(1/2)-8*c/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3 *2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2) ^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3) +I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+ (-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+ I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1 /2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/ 2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c* d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d -8*c)))

maxima [A] time = 1.21, size = 56, normalized size = 1.08

$$-\frac{2\left(\frac{4\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{3}{\sqrt{dx^3+c}}\right)}{81\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")

[Out] -2/81*(4*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s qrt(c) - 3/sqrt(d*x^3 + c))/d^2

mupad [B] time = 3.68, size = 60, normalized size = 1.15

$$\frac{2}{27\,d^2\,\sqrt{d\,x^3+c}}+\frac{8\,\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)

[Out] 2/(27*d^2*(c + d*x^3)^(1/2)) + (8*log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(1/2)*d^2)

sympy [A] time = 27.57, size = 58, normalized size = 1.12

$$\begin{cases} \frac{2\left(\frac{1}{27d\sqrt{c+dx^3}} - \frac{8\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81d\sqrt{-c}}\right)}{d} & \text{for } d \neq 0 \\ \frac{x^6}{48c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)

[Out] Piecewise((2*(1/(27*d*sqrt(c + d*x**3)) - 8*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*d*sqrt(-c)))/d, Ne(d, 0)), (x**6/(48*c**(5/2)), True))

3.328 $\int \frac{x^2}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

```
[Out] 2/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d-2/27/c/d/(d*x^3+c)^(1/2
)
```

Rubi [A] time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}-\frac{2}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(27*c*d*Sqrt[c + d*x^3]) + (2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*
c^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c}$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}$$
$$= -\frac{2}{27cd\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.78

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{27cd\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(27*c*d*Sqrt[c + d*
x^3])
```

fricas [A] time = 0.71, size = 147, normalized size = 2.67

$$\left[\frac{\left(dx^3+c\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-6\sqrt{dx^3+c}\,c}{81\left(c^2d^2x^3+c^3d\right)}, -\frac{2\left(\left(dx^3+c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)+3\sqrt{dx^3+c}\,c\right)}{81\left(c^2d^2x^3+c^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/81*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(
d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 + c^3*d), -2/81*((d*x^3 +
 c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 + c)*c)/
(c^2*d^2*x^3 + c^3*d)]
```

giac [A] time = 0.18, size = 48, normalized size = 0.87

$$-\frac{2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\,cd} - \frac{2}{27\sqrt{dx^3+c}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 2/27/(sqrt(d*x^
3 + c)*c*d)
```

maple [C] time = 0.21, size = 435, normalized size = 7.91

$$-\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/27/((x^3+c/d)*d)^(1/2)/c/d-1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1
/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^
(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.23, size = 58, normalized size = 1.05

$$-\frac{\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{6}{\sqrt{dx^3+c}\,c}}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/81*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(
3/2) + 6/(sqrt(d*x^3 + c)*c))/d
```

mupad [B] time = 3.63, size = 63, normalized size = 1.15

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,c^{3/2}\,d}-\frac{2}{27\,c\,d\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(81*c^(3/2)
*d) - 2/(27*c*d*(c + d*x^3)^(1/2))
```

sympy [A] time = 25.53, size = 51, normalized size = 0.93

$$-\frac{2}{27cd\sqrt{c+dx^3}}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{81cd\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(27*c*d*sqrt(c + d*x**3)) - 2*atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(81*c*
d*sqrt(-c))
```

3.329 $\int \frac{1}{x(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}+\frac{2}{27c^2\sqrt{c+dx^3}}$$

```
[Out] 1/324*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/12*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(5/2)+2/27/c^2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 85, 156, 63, 208, 206}

$$\frac{2}{27c^2\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*Sqrt[c + d*x^3]) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(324*c^(5
/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(12*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd+d^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27c^2d}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2} + \frac{d\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{108c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{12c^2d}$$

$$= \frac{2}{27c^2\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{324c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{12c^{5/2}}$$

Mathematica [C] time = 0.03, size = 63, normalized size = 0.83

$$\frac{9\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) - {}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{108c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] + 9*Hypergeometric2F1[
-1/2, 1, 1/2, 1 + (d*x^3)/c])/(108*c^2*Sqrt[c + d*x^3])
```

fricas [A] time = 0.82, size = 213, normalized size = 2.80

$$\left[\frac{\left(dx^3+c\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right) + 27\left(dx^3+c\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) + 48\,\sqrt{dx^3+c}\,c}{648\left(c^3dx^3+c^4\right)}, \frac{27\left(dx^3+c\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/648*((d*x^3 + c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/
(d*x^3 - 8*c)) + 27*(d*x^3 + c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt
(c) + 2*c)/x^3) + 48*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c^4), 1/324*(27*(d*x^3
 + c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d*x^3 + c)*sqrt(-c)*ar
ctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 + c
^4)]
```

giac [A] time = 0.16, size = 68, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{12\sqrt{-c}\,c^2}-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{324\sqrt{-c}\,c^2}+\frac{2}{27\sqrt{dx^3+c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/12*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/324*arctan(1/3*sqr
t(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) + 2/27/(sqrt(d*x^3 + c)*c^2)
```

maple [C] time = 0.19, size = 485, normalized size = 6.38

$$-\frac{\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\;cd}+\frac{i\left(-cd^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^2\right)^{\frac{1}{3}}+\left(-cd^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\_Z^3-\right.\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/8/c*d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/8/c*(2/3/c/
((x^3+c/d)*d)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x), x)
```

mupad [B] time = 3.66, size = 68, normalized size = 0.89

$$\frac{2}{27\,c^2\,\sqrt{d\,x^3+c}}-\frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{12\,\sqrt{c^5}}+\frac{\operatorname{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{324\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] 2/(27*c^2*(c + d*x^3)^(1/2)) - atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(
12*(c^5)^(1/2)) + atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))/(324*(c^5)
^(1/2))
```

sympy [A] time = 17.02, size = 78, normalized size = 1.03

$$\frac{2}{27c^2\sqrt{c+dx^3}}-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{-c}}\right)}{324c^2\sqrt{-c}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{12c^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2/(27*c**2*sqrt(c + d*x**3)) - atan(sqrt(c + d*x**3)/(3*sqrt(-c)))/(324*c**
2*sqrt(-c)) + atan(sqrt(c + d*x**3)/sqrt(-c))/(12*c**2*sqrt(-c))
```

3.330 $\int \frac{1}{x^4(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}$$

```
[Out] 1/2592*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+11/96*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(7/2)-25/216*d/c^3/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(d*x^
3+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$-\frac{25d}{216c^3\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\sqrt{c+dx^3}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-25*d)/(216*c^3*Sqrt[c + d*x^3]) - 1/(24*c^2*x^3*Sqrt[c + d*x^3]) + (d*Arc
Tanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2592*c^(7/2)) + (11*d*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(96*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{11cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{99c^2d^2}{2}-\frac{25}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{108c^4d}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{(11d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} + \frac{d^2\,\text{S}\ldots}{\ldots}$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} - \frac{11\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96c^3} + \ldots$$

$$= -\frac{25d}{216c^3\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\sqrt{c+dx^3}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{7/2}} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+d\ldots}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Mathematica [C] time = 0.04, size = 77, normalized size = 0.77

$$\frac{-dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)-99dx^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)-36c}{864c^3x^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-36*c - d*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)] - 99*d*x^
3*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(864*c^3*x^3*Sqrt[c + d*x
^3])
```

fricas [A] time = 1.00, size = 272, normalized size = 2.72

$$\left[\frac{\left(d^2x^6+cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+297\left(d^2x^6+cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(25\,cdx^3+9\right.}{5184\left(c^4dx^6+c^5x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/5184*((d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 297*(d^2*x^6 + c*d*x^3)*sqrt(c)*log((d*x^3 + 2*sq
rt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c)
)/(c^4*d*x^6 + c^5*x^3), -1/2592*(297*(d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + c*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d
*x^3 + c)*sqrt(-c)/c) + 12*(25*c*d*x^3 + 9*c^2)*sqrt(d*x^3 + c))/(c^4*d*x^6
 + c^5*x^3)]
```

giac [A] time = 0.18, size = 100, normalized size = 1.00

$$-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}\,c^3}-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{2592\,\sqrt{-c}\,c^3}-\frac{25\left(dx^3+c\right)d-16\,cd}{216\left(\left(dx^3+c\right)^{\frac{3}{2}}-\sqrt{dx^3+c}\,c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -11/96*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/2592*d*arctan(
1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/216*(25*(d*x^3 + c)*d - 16
*c*d)/(((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)*c^3)
```

maple [C] time = 0.19, size = 549, normalized size = 5.49

$$-\left(\frac{2}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(d\,\_Z^3-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3*d/c^2/((x^3+c/d)*d)^(1/2)+d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/64/c^2*d^2*(2/27/((x^3+c/d)*d)^(1/2)/c
/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))+1/64/c^2*d*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^4), x)
```

mupad [B] time = 3.80, size = 88, normalized size = 0.88

$$\frac{11\,d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)}{96\,\sqrt{c^7}}-\frac{25\,d}{216\,c^3\,\sqrt{d\,x^3+c}}+\frac{d\,\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)}{2592\,\sqrt{c^7}}-\frac{1}{24\,c^2\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (11*d*atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2)))/(96*(c^7)^(1/2)) - (25*d)
/(216*c^3*(c + d*x^3)^(1/2)) + (d*atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1
/2))))/(2592*(c^7)^(1/2)) - 1/(24*c^2*x^3*(c + d*x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^4\sqrt{c+dx^3}-7cdx^7\sqrt{c+dx^3}+d^2x^{10}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**4*sqrt(c + d*x**3) - 7*c*d*x**7*sqrt(c + d*x**3) +
d**2*x**10*sqrt(c + d*x**3)), x)
```

3.331 $\int \frac{1}{x^7(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

```
[Out] 1/20736*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-109/768*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+245/1728*d^2/c^4/(d*x^3+c)^(1/2)-1/48/c^
2/x^6/(d*x^3+c)^(1/2)+3/64*d/c^3/x^3/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{245d^2}{1728c^4\sqrt{c+dx^3}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{20736c^{9/2}}-\frac{109d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{768c^{9/2}}+\frac{3d}{64c^3x^3\sqrt{c+dx^3}}-\frac{1}{48c^2x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (245*d^2)/(1728*c^4*Sqrt[c + d*x^3]) - 1/(48*c^2*x^6*Sqrt[c + d*x^3]) + (3*
d)/(64*c^3*x^3*Sqrt[c + d*x^3]) + (d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(20736*c^(9/2)) - (109*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(768*c^(9/2)
)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{18cd-\frac{5d^2x}{2}}{x^2(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{218c^2d^2-27cd^3x}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{384c^4}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{981c^3d^3}{x(8c-d}\right.}{172}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{\left(109d^2\right)\text{Subst}\left(\right.}{15}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{(109d)\text{Subst}\left(\int\right.}{}$$

$$= \frac{245d^2}{1728c^4\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\sqrt{c+dx^3}} + \frac{3d}{64c^3x^3\sqrt{c+dx^3}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+d}}{3\sqrt{c}}\right.}{20736c^{9/2}}$$

Mathematica [C] time = 0.08, size = 91, normalized size = 0.71

$$\frac{-d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+981d^2x^6\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+36c\left(9dx^3-4c\right)}{6912c^4x^6\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (36*c*(-4*c + 9*d*x^3) - d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3
)/(9*c)] + 981*d^2*x^6*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(691
2*c^4*x^6*Sqrt[c + d*x^3])
```

fricas [A] time = 0.90, size = 303, normalized size = 2.37

$$\left[\frac{\left(d^3x^9+cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2943\left(d^3x^9+cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+24\left(245cd^2x\right.}{41472\left(c^5dx^9+c^6x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/41472*((d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 2943*(d^3*x^9 + c*d^2*x^6)*sqrt(c)*log((d*x^3
- 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(245*c*d^2*x^6 + 81*c^2*d*x^3
- 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6), 1/20736*(2943*(d^3*x^9 +
c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (d^3*x^9 + c*d^2*x
^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(245*c*d^2*x^6 + 8
1*c^2*d*x^3 - 36*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 + c^6*x^6)]
```

giac [A] time = 0.16, size = 118, normalized size = 0.92

$$\frac{109\,d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{768\,\sqrt{-c}\,c^{4}}-\frac{d^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{20736\,\sqrt{-c}\,c^{4}}+\frac{2\,d^{2}}{27\,\sqrt{dx^{3}+c}\,c^{4}}+\frac{13\,\left(dx^{3}+c\right)^{\frac{3}{2}}d^{2}-17\,\sqrt{dx^{3}+c}\,cd^{2}}{192\,c^{4}d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 109/768*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/20736*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) + 2/27*d^2/(sqrt(d*x^3 +
 c)*c^4) + 1/192*(13*(d*x^3 + c)^(3/2)*d^2 - 17*sqrt(d*x^3 + c)*c*d^2)/(c^4
*d^2*x^6)
```

maple [C] time = 0.22, size = 636, normalized size = 4.97

$$-\frac{\left(\frac{2}{27\sqrt{\left(x^{3}+\frac{c}{d}\right)d}\,cd}+\frac{i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\operatorname{RootOf}\left(d\_Z^{3}-8c\right)\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*d*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*ar
ctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/8/c*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6
+7/12*d*(d*x^3+c)^(1/2)/c^3/x^3+2/3*d^2/c^3/((x^3+c/d)*d)^(1/2)-5/4*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2))-1/512/c^3*d^3*(2/27/((x^3+c/d)*d)^(1
/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c)))+1/512/c^3*d^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3
*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{\left(dx^{3}+c\right)^{\frac{3}{2}}\left(dx^{3}-8\,c\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^7), x)
```

mupad [B] time = 4.03, size = 112, normalized size = 0.88

$$\frac{245\,d^2}{1728\,c^4\,\sqrt{d\,x^3+c}}-\frac{109\,d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)}{768\,\sqrt{c^9}}+\frac{d^2\,\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)}{20736\,\sqrt{c^9}}-\frac{1}{48\,c^2\,x^6\,\sqrt{d\,x^3+c}}+\frac{3\,d}{64\,c^3\,x^3\,\sqrt{d\,x^3+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] (245*d^2)/(1728*c^4*(c + d*x^3)^(1/2)) - (109*d^2*atanh((c^4*(c + d*x^3)^(1
/2))/(c^9)^(1/2)))/(768*(c^9)^(1/2)) + (d^2*atanh((c^4*(c + d*x^3)^(1/2))/(
3*(c^9)^(1/2))))/(20736*(c^9)^(1/2)) - 1/(48*c^2*x^6*(c + d*x^3)^(1/2)) + (
3*d)/(64*c^3*x^3*(c + d*x^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.332 $\int \frac{x^7}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=629

$$-\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}{9\ldots}$$

```
[Out] 32/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)-32/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)-32/81*c^(1
/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(
1/2)+2/27*x^2/d^2/(d*x^3+c)^(1/2)-56/27*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))-56/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x
+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+28/27*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellip
ticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1
/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {470, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{56\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}+\frac{28\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\ldots\right.}{9\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*d^2*Sqrt[c + d*x^3]) - (56*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)) - (32*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3
) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) + (32*c^(1/6)*ArcTan
h[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d^(8/3)) - (32*
c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) + (28*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (56*Sqrt[2]*c^(1/3)*(c^(1/3
) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/
3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^
(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule steps

$$\int \frac{x^7}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx = \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \frac{x(16c^2-14cdx^3)}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{14cx}{\sqrt{c+dx^3}} - \frac{96c^2x}{(8c-dx^3)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{28\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{(64c)\int \frac{x}{(8c-dx^3)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{16\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} - \frac{28\int \frac{(1-\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} + \frac{(16}{}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{28\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{9\,3}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{\sqrt{c+dx^3}}\right.}{27\sqrt{3}\,d^{8/3}}$$

$$= \frac{2x^2}{27d^2\sqrt{c+dx^3}} - \frac{56\sqrt{c+dx^3}}{27d^{8/3}\left((1+\sqrt{3})\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{32\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}{\sqrt{c+dx^3}}\right.}{27\sqrt{3}\,d^{8/3}}$$

Mathematica [C] time = 0.12, size = 127, normalized size = 0.20

$$\frac{x^2\left(7dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-20c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+20c\right)}{270cd^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(20*c - 20*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] + 7*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(270*c*d^2*Sqrt[c + d*x^3])
```

fricas [F] time = 16.29, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^7}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^7/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.28, size = 1810, normalized size = 2.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d^2*(d*(-2/3/d*x^2/((x^3+c/d)*d)^(1/2)-8/9*I/d^2*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2))))+8*c*(2/3/c*x^2/((x^3+c/d)*d)^(1/2)+2/9*I/c*3^(1/2)*(-c*d^2)
^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2)))))-64*c^2/d^2*(-2/27/c^2*x^2/((x^3+c/d)*d)^(1/2)-2/8
1*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum
(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
```

```
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*
c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

3.333 $\int \frac{x^4}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=635

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\ldots}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\ldots\right.}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}$$

```
[Out] 4/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^(
5/3)-4/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)-4/81*arctan(
c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/2
)-2/27*x^2/c/d/(d*x^3+c)^(1/2)+2/27*(d*x^3+c)^(1/2)/c/d^(5/3)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))+2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/27*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/
2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 635, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {471, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}c^{5/6}d^{5/3}}+\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}-\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\,F\ldots}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\ldots\right.}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(27*c*d*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(27*c*d^(5/3)*((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*c^(5/6)*d^(5/3)) + (4*ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*c^(5/6)*d^(5/3)) - (4*A
rcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[
3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*
3^(3/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x
)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)
*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(16c-\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\int \left(\frac{x}{2\sqrt{c+dx^3}}+\frac{12cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{8\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}dx}{27cd}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} - \frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{27cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}dx}{27cd^{4/3}} + \frac{2\int -}{}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9\,3^{3/}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}$$

$$= -\frac{2x^2}{27cd\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}$$

Mathematica [C] time = 0.11, size = 126, normalized size = 0.20

$$-\frac{x^2\left(dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80c\right)}{1080c^2d\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/1080*(x^2*(80*c - 80*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -(
(d*x^3)/c), (d*x^3)/(8*c)] + d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1
, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(c^2*d*Sqrt[c + d*x^3])
```

fricas [F] time = 2.27, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}\,x^4}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^4/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.16, size = 1346, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(2/3/((x^3+c/d)*d)^(1/2)/c*x^2+2/9*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2))))-8*c/d*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1/2)*(-c*d
^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1
/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] Timed out
```

3.334 $\int \frac{x}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=632

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}}-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin\right.}{27\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}}}$$

```
[Out] 1/162*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d
^(2/3)-1/162*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-1/162*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/3)*
3^(1/2)+2/27*x^2/c^2/(d*x^3+c)^(1/2)-2/27*(d*x^3+c)^(1/2)/c^2/d^(2/3)/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2)))-2/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+1/27*(c^(1/3)+d^(1/3)*x)*EllipticE
((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+
2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/
2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 632, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {472, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}c^{11/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{162c^{11/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{162c^{11/6}d^{2/3}}-\frac{2\sqrt{2}\left(\sqrt[3]{c}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*x^2)/(27*c^2*Sqrt[c + d*x^3]) - (2*Sqrt[c + d*x^3])/(27*c^2*d^(2/3)*((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)) - ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/
3)*x))/Sqrt[c + d*x^3]]/(54*Sqrt[3]*c^(11/6)*d^(2/3)) + ArcTanh[(c^(1/3) +
d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(162*c^(11/6)*d^(2/3)) - ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])]/(162*c^(11/6)*d^(2/3)) + (Sqrt[2 - Sqrt[3]]*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4
)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(5/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \frac{x\left(\frac{5cd}{2}-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\int \left(\frac{dx}{2\sqrt{c+dx^3}} - \frac{3cdx}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27c^2d}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27c^2} + \frac{\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{9c}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{108c^2d} - \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{27c^2\sqrt[3]{d}} + \frac{\int \frac{1+}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right.}}{108c}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{}}{9\,3^{3/4}}$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}\,c^{11/6}d^{2/3}} +$$

$$= \frac{2x^2}{27c^2\sqrt{c+dx^3}} - \frac{2\sqrt{c+dx^3}}{27c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{54\sqrt{3}\,c^{11/6}d^{2/3}} +$$

Mathematica [C] time = 0.10, size = 124, normalized size = 0.20

$$\frac{x^2\left(2dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-25c\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+160c\right)}{2160c^3\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(160*c - 25*c*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 2*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), (d*x^3)/(8*c)]))/(2160*c^3*Sqrt[c + d*x^3])
```

fricas [F] time = 2.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}\,x}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.17, size = 875, normalized size = 1.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2+2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-8c^2\sqrt{c+dx^3} - 7cdx^3\sqrt{c+dx^3} + d^2x^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(x/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

3.335 $\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=653

$$\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{}}{144\,3^{3/4}c}$$

```
[Out] 1/1296*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-1/1296*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/1296
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/
6)*3^(1/2)+2/27/c^2/x/(d*x^3+c)^(1/2)-43/216*(d*x^3+c)^(1/2)/c^3/x+43/216*d
^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+43/648*d^(1/3)*(
c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(
1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-
43/432*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2))
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{43\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{108\sqrt{2}\sqrt[4]{3}\,c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{43\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{}}{144\,3^{3/4}c}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x*Sqrt[c + d*x^3]) - (43*Sqrt[c + d*x^3])/(216*c^3*x) + (43*d^(1/
3)*Sqrt[c + d*x^3])/(216*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3
)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqr
t[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(1296*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])
])/(1296*c^(17/6)) - (43*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sq
rt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(8/3)*Sqrt[(c^(1
/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
d*x^3]) + (43*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)
*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[(
(1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7
 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3
)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{43cd}{2}+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \frac{x\left(\frac{175c^2d^2}{2}-\frac{43}{4}cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{\int \left(\frac{43cd^2x}{4\sqrt{c+dx^3}} + \frac{3c^2d^2x}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{108c^4d}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{(43d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^3} + \frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{72c^2}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^3} + \frac{\left(43d^{2/3}\right)\int \frac{(}{}}{4}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{43\sqrt{2-}}{}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}}{4}$$

$$= \frac{2}{27c^2x\sqrt{c+dx^3}} - \frac{43\sqrt{c+dx^3}}{216c^3x} + \frac{43\sqrt[3]{d}\,\sqrt{c+dx^3}}{216c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}}{4}$$

Mathematica [C] time = 0.10, size = 140, normalized size = 0.21

$$\frac{-43d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+875cdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(27c+43dx^3\right)}{17280c^4x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(27*c + 43*d*x^3) + 875*c*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/
2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 43*d^2*x^6*Sqrt[1 + (d*x^3)/c]*Ap
pellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(17280*c^4*x*Sqrt[c
+ d*x^3])
```

fricas [F] time = 3.30, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{11}-6\,cd^2x^8-15\,c^2dx^5-8\,c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^11 - 6*c*d^2*x^8 - 15*c^2*d*x^5 - 8*c^3*x^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

maple [C] time = 0.22, size = 1361, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-2/3*d/c^2*x^2/((x^3+c/d)*d)^(1/2)-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/8/c*d*(-2/27/((x^3+c/d)*d)^(1/2)/c^
2*x^2-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/
d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
```

```
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(d\,x^3+c\right)^{3/2}\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^2\sqrt{c+dx^3}-7cdx^5\sqrt{c+dx^3}+d^2x^8\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**2*sqrt(c + d*x**3) - 7*c*d*x**5*sqrt(c + d*x**3) +
d**2*x**8*sqrt(c + d*x**3)), x)
```

3.336 $\int \frac{1}{x^5(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=675

$$-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}\,c^{23/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}}-\frac{113d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}}}{216\sqrt{2}\sqrt[4]{3}\,c^{11/3}\cdots}$$

```
[Out] 1/10368*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(23/6)-1/10368*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/10
368*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
23/6)*3^(1/2)+2/27/c^2/x^4/(d*x^3+c)^(1/2)-91/864*(d*x^3+c)^(1/2)/c^3/x^4+1
13/432*d*(d*x^3+c)^(1/2)/c^4/x-113/432*d^(4/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))-113/1296*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(
1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*
((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+113/864*d^(4/3)*(c^(1/3)+d^(1/3)
*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
)),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] time = 0.92, antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3456\sqrt{3}\,c^{23/6}}+\frac{d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{10368c^{23/6}}-\frac{d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{23/6}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^4*Sqrt[c + d*x^3]) - (91*Sqrt[c + d*x^3])/(864*c^3*x^4) + (113*
d*Sqrt[c + d*x^3])/(432*c^4*x) - (113*d^(4/3)*Sqrt[c + d*x^3])/(432*c^4*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/
3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3456*Sqrt[3]*c^(23/6)) + (d^(4/3)*ArcTa
nh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(10368*c^(23/6)) -
 (d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(23/6)) + (113*Sqr
t[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (113*d^(4/3)*(c^(
1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^
(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2
]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(8c-dx^3\right) \left(c+dx^3\right)^{3/2}} \, dx = \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{91cd}{2}+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}} \, dx}{27c^2d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{\int \frac{-904c^2d^2+\frac{455}{4}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}} \, dx}{432c^4d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \frac{x\left(3610c^3d^3-452c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} \, d}{3456c^6d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\int \left(\frac{452c^2d^3x}{\sqrt{c+dx^3}} - \frac{6c^3d^3x}{\left(8c-dx^3\right)\sqrt{c}}\right.}{3456c^6d}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{\left(113d^2\right)\int \frac{x}{\sqrt{c+dx^3}} \, dx}{864c^4} +$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{d\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+d}}}{6912c^4}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

$$= \frac{2}{27c^2x^4\sqrt{c+dx^3}} - \frac{91\sqrt{c+dx^3}}{864c^3x^4} + \frac{113d\sqrt{c+dx^3}}{432c^4x} - \frac{113d^{4/3}\sqrt{c+dx^3}}{432c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}$$

Mathematica [C] time = 0.10, size = 153, normalized size = 0.23

$$\frac{452d^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 9025cd^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 160c\left(-27c^2+135c\right.}{138240c^5x^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (160*c*(-27*c^2 + 135*c*d*x^3 + 226*d^2*x^6) - 9025*c*d^2*x^6*Sqrt[1 + (d*x
^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 452*d^3*x^
```

```
9*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c
)])/(138240*c^5*x^4*Sqrt[c + d*x^3])
```

fricas [F] time = 9.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{14}-6\,cd^2x^{11}-15\,c^2dx^8-8\,c^3x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^14 - 6*c*d^2*x^11 - 15*c^2*d*x^8 - 8*c^3*x
^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

maple [C] time = 0.22, size = 1864, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*d*(d*x^3+c)^(1/2)/c^3/x+2/3*d^2/c^
3*x^2/((x^3+c/d)*d)^(1/2)+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)))+1/64/c^2*d*(-2/3/((x^3+c/d)*d)^(1/2)/c^2*d*x^2-(d*x^3+c)^(1/2)/c^2/x-5/
9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64/c^2*d^2*(-2/27/((x^3+c/
d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x
-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^
```

```
(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(
-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/24
3*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha
*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d
^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),
_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^5\sqrt{c+dx^3}-7cdx^8\sqrt{c+dx^3}+d^2x^{11}\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
[Out] -Integral(1/(-8*c**2*x**5*sqrt(c + d*x**3) - 7*c*d*x**8*sqrt(c + d*x**3) +
d**2*x**11*sqrt(c + d*x**3)), x)
```

3.337 $\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=699

$$-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}-\frac{d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{29/6}}+\frac{953d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\ldots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}{1512\sqrt{2}\sqrt[4]{3}c^{14/\ldots}}$$

```
[Out] 1/82944*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/
c^(29/6)-1/82944*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-1/82
944*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(
29/6)*3^(1/2)+2/27/c^2/x^7/(d*x^3+c)^(1/2)-139/1512*(d*x^3+c)^(1/2)/c^3/x^7
+6095/48384*d*(d*x^3+c)^(1/2)/c^4/x^4-953/3024*d^2*(d*x^3+c)^(1/2)/c^5/x+95
3/3024*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+953/9072
*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2
/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)*2^(1/2)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)-953/6048*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3
)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-
1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 699, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{953d^{7/3}\sqrt{c+dx^3}}{3024c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{953d^2\sqrt{c+dx^3}}{3024c^5x}-\frac{d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}c^{29/6}}+\frac{d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{29/6}}-\frac{d^{7/3}\tan\ldots}{82\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(27*c^2*x^7*Sqrt[c + d*x^3]) - (139*Sqrt[c + d*x^3])/(1512*c^3*x^7) + (60
95*d*Sqrt[c + d*x^3])/(48384*c^4*x^4) - (953*d^2*Sqrt[c + d*x^3])/(3024*c^5
*x) + (953*d^(7/3)*Sqrt[c + d*x^3])/(3024*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) - (d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
d*x^3]])/(27648*Sqrt[3]*c^(29/6)) + (d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^
2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(82944*c^(29/6)) - (d^(7/3)*ArcTanh[Sqrt[c
+ d*x^3]/(3*Sqrt[c])])/(82944*c^(29/6)) - (953*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2016*3^(
3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (953*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1512*Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt
```

```
[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqr
t[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

```
x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{2\int \frac{-\frac{139cd}{2}+\frac{17d^2x^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27c^2d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{\int \frac{-\frac{6095}{2}c^2d^2+\frac{1529}{4}cd^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{756c^4d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{\int \frac{-60992c^3d^3+\frac{30475}{4}c^2d^4}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}}{24192c^6d}$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} - \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

$$= \frac{2}{27c^2x^7\sqrt{c+dx^3}} - \frac{139\sqrt{c+dx^3}}{1512c^3x^7} + \frac{6095d\sqrt{c+dx^3}}{48384c^4x^4} - \frac{953d^2\sqrt{c+dx^3}}{3024c^5x} + \cdots$$

Mathematica [C] time = 0.12, size = 167, normalized size = 0.24

$$\frac{610025cd^3x^9\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 32\left(953d^4x^{12}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 5c\left(864c^3 - \cdots\right.\right.}{7741440c^6x^7\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (610025*c*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), (d*x^3)/(8*c)] - 32*(5*c*(864*c^3 - 1647*c^2*d*x^3 + 9153*c*d^2*x^6 + 1
5248*d^3*x^9) + 953*d^4*x^12*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))/(7741440*c^6*x^7*Sqrt[c + d*x^3])
```

fricas [F] time = 21.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3x^{17} - 6\,cd^2x^{14} - 15\,c^2dx^{11} - 8\,c^3x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^17 - 6*c*d^2*x^14 - 15*c^2*d*x^11 - 8*c^3*
x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3 + c\right)^{\frac{3}{2}}\left(dx^3 - 8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

maple [C] time = 0.20, size = 2389, normalized size = 3.42

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*d*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*(d*x^3+c)^(1/2)/c^3*d/x+2/3/(
(x^3+c/d)*d)^(1/2)/c^3*d^2*x^2+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2))))+1/8/c*(-1/7*(d*x^3+c)^(1/2)/c^2/x^7+25/56*d*(d*x^3+c)^(1/2)/c^3/x^
4-237/112*d^2*(d*x^3+c)^(1/2)/c^4/x-2/3*d^3/c^4*x^2/((x^3+c/d)*d)^(1/2)-935
/1008*I/c^4*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
```

```
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^2*(-2/3/
((x^3+c/d)*d)^(1/2)/c^2*d*x^2-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d
^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2))))-1/512/c^3*d^3*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-
2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.338 $\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

```
[Out] 1/32*x^4*AppellF1(4/3,3/2,1,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(32*c^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(8c-dx^3)(c+dx^3)^{3/2}} \, dx &= \frac{\sqrt{1+\frac{dx^3}{c}} \int \frac{x^3}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}} \, dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{32c^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 233, normalized size = 3.53

$$\frac{x\left(\frac{64c\left(\frac{256c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-1\right)}{d}+x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}{864c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] + (64*c*(-1 + (256*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d
*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/d)
)/(864*c^2*Sqrt[c + d*x^3])
```

fricas [F] time = 2.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{dx^3+c}\,x^3}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)*x^3/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.24, size = 1038, normalized size = 15.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/d*(2/3/c*x/((x^3+c/d)*d)^(1/2)-2/9*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2)))-8*c/d*(-2/27/c^2*x/((x^3+c/d)*d)^(1/2)+2/81*I
/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
```

```
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/243*I/
c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.339 $\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

```
[Out] 1/8*x*AppellF1(1/3,3/2,1,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(d
*x^3+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(8*c^2*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(8c-dx^3)(c+dx^3)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{8c^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.16, size = 230, normalized size = 3.59

$$\frac{x\left(64\left(\frac{176F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{1}{c^2}\right)-\frac{dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right.}{c^3}}{864\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)])/c^3) + 64*(c^(-2) + (176*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3
)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*
x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]
))))))/(864*Sqrt[c + d*x^3])
```

fricas [F] time = 2.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^9-6\,cd^2x^6-15\,c^2dx^3-8\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^9 - 6*c*d^2*x^6 - 15*c^2*d*x^3 - 8*c^3), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

maple [C] time = 0.16, size = 721, normalized size = 11.27

$$\frac{2x}{27\sqrt{\left(x^3+\frac{c}{d}\right)d}\;c^2}-\frac{2i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right.}{\left(-c\,d^2\right)^{\frac{1}{3}}}}}{81\sqrt{d\,x^3+c}\;c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/27/((x^3+c/d)*d)^(1/2)/c^2*x-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2))-1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2
)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
```

```
(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)
^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2\sqrt{c+dx^3}-7cdx^3\sqrt{c+dx^3}+d^2x^6\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*sqrt(c + d*x**3) - 7*c*d*x**3*sqrt(c + d*x**3) + d**2*
x**6*sqrt(c + d*x**3)), x)
```

3.340 $\int \frac{1}{x^3(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/16*AppellF1(-2/3,3/2,1,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/x
^2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(16*c^2*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{16c^2x^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.15, size = 248, normalized size = 3.76

$$\frac{64c\left(-\frac{7360c^2dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-27c-59dx^3\right)+59d^2x^6\sqrt{\frac{dx^3}{c}}\cdots}{27648c^4x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (59*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d
*x^3)/(8*c)] + 64*c*(-27*c - 59*d*x^3 - (7360*c^2*d*x^3*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*AppellF1[1/3, 1/
2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c)
, (d*x^3)/(8*c)])))))/(27648*c^4*x^2*Sqrt[c + d*x^3])
```

fricas [F] time = 6.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3+c}}{d^3x^{12}-6\,cd^2x^9-15\,c^2dx^6-8\,c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^12 - 6*c*d^2*x^9 - 15*c^2*d*x^6 - 8*c^3*x^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

maple [C] time = 0.20, size = 1053, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*d/c^2*x/((x^3+c/d)*d)^(1/2)+7/18*I/
c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3
+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/8/c*d*(-
2/27/((x^3+c/d)*d)^(1/2)/c^2*x+2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2
)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
```

```
-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)
^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-8c^2x^3\sqrt{c+dx^3}-7cdx^6\sqrt{c+dx^3}+d^2x^9\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] -Integral(1/(-8*c**2*x**3*sqrt(c + d*x**3) - 7*c*d*x**6*sqrt(c + d*x**3) +
d**2*x**9*sqrt(c + d*x**3)), x)
```

3.341 $\int \frac{1}{x^6(8c-dx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/40*AppellF1(-5/3,3/2,1,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^5/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 1, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(40*c^2*x^5*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6(8c-dx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};1,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{40c^2x^5\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.24, size = 261, normalized size = 3.95

$$\frac{64c\left(\frac{382528c^2d^2x^6F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{(8c-dx^3)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-432c^2+1269cdx^3+2981d^2x^6\right)-}{1105920c^5x^5\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2981*d^3*x^9*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 64*c*(-432*c^2 + 1269*c*d*x^3 + 2981*d^2*x^6 + (382528*c^
2*d^2*x^6*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c -
d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*
x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4
/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)])))))/(1105920*c^5*x^5*Sqrt[c
+ d*x^3])
```

fricas [F] time = 18.15, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{dx^3 + c}}{d^3x^{15} - 6\,cd^2x^{12} - 15\,c^2dx^9 - 8\,c^3x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(d*x^3 + c)/(d^3*x^15 - 6*c*d^2*x^12 - 15*c^2*d*x^9 - 8*c^3*x
^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{\left(dx^3 + c\right)^{\frac{3}{2}}\left(dx^3 - 8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

maple [C] time = 0.19, size = 1402, normalized size = 21.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*(-1/5/c^2*(d*x^3+c)^(1/2)/x^5+17/20/c^3*d*(d*x^3+c)^(1/2)/x^2+2/3*d^2
/c^3*x/((x^3+c/d)*d)^(1/2)-91/180*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)))+1/64/c^2*d*(-1/2*(d*x^3+c)^(1/2)/c^2/x^2-2/3/((
x^3+c/d)*d)^(1/2)/c^2*d*x+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))-1/64/c^2*d^2*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x+2/81*I
/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
```

```
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/243*I/
c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,{\left(d\,x^3+c\right)}^{3/2}\,\left(8\,c-d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)),x)
```

```
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.342 $\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=737

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(1
/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4*3
^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*
2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2*3^
(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/
3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/
3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/
(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)
*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1
/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^
(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3
)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a +
 b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt
[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(
```

```
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a+bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}}+\frac{\sqrt[4]{3}\sqrt[6]{a}\tan}{}$$

$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\tan^{-}}{\sqrt{}}$$

Mathematica [C] time = 0.09, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a + 6*Sqrt[3]*a))])/((20 + 12*Sqrt[3])*Sqrt[a + b*x^3])
```

fricas [F] time = 64.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4-6\sqrt{3}\,ax+10\,ax\right)\sqrt{bx^3+a}}{b^2x^6+20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4 - 6*sqrt(3)*a*x + 10*a*x)*sqrt(b*x^3 + a)/(b^2*x^6 + 20*a*b
*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}\,x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

maple [C] time = 0.78, size = 977, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(
b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alp
ha*(2*3^(1/2)+3)*(-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)
*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)*(b*(x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2
)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)
+I*3^(1/2)*(-a*b^2)^(1/3)))/(-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(-3*I*(-a
*b^2)^(1/3)*_alpha*3^(1/2)*b+4*b^2*_alpha^2*3^(1/2)+3*I*(-a*b^2)^(2/3)*3^(1
/2)-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-6*b^2*_al
pha^2-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*(-a*b^2)^(1/3)*_alpha*b
+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6/b*(2*I*3^(1/
2)*(-a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-4*I*(-a*b^2)^(
1/3)*_alpha^2*b+2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b+2*I*(-a*b^2)^
(2/3)*_alpha+2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-2*I*a*b-3*a*b)/a,(I*3^(1
/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b
)^(1/2)),_alpha=RootOf(b*_Z^3+6*a*3^(1/2)+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}\,x}{bx^3+2\,a\left(3\,\sqrt{3}\,+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{b\,x^3+a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}\,+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^3}}{10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5+3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.343 $\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=757

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.$$

```
[Out] 1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/
2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1-3^(
1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/4
*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2)
)*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3+a)^(1/2)
)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3
)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*
x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-
b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 757, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(3/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 - Sqrt[3]
)*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) +
(3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*
x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcT
anh[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a
 - b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
```

```
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\,\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= \frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\,\sqrt[4]{3}\,b^{2/3}}+\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan}{}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}+\frac{3^{3/4}\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}+\frac{\sqrt[6]{a}\,\tan^{-1}}{\sqrt{}}$$

Mathematica [C] time = 0.08, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}\,a+10a}\right)}{\left(20+12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a + 6*Sqrt[3]*a)])/((20 + 12*Sqrt[3])*Sqrt[a - b*x^3])
```

fricas [F] time = 57.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4+6\sqrt{3}\,ax-10\,ax\right)\sqrt{-bx^3+a}}{b^2x^6-20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] integral(-(b*x^4 + 6*sqrt(3)*a*x - 10*a*x)*sqrt(-b*x^3 + a)/(b^2*x^6 - 20*a
*b*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-bx^3+a}\,x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

maple [C] time = 4.35, size = 924, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a
*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)*((x-1/b*(a*b^2)^(1/3))/(-3/2/b*
(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)*(I*(x+1/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)
^(1/2)*((-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*EllipticE(1/3*
3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b
/(a*b^2)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2
*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)
*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)*b/(a*b^2
)^(1/3))^(1/2),(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/3)-1/2*I*3^(1
/2)/b*(a*b^2)^(1/3)))^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)+3)*
(a*b^2)^(1/3)*(-1/2*I*b*(2*x+1/b*(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3)))/(
a*b^2)^(1/3))^(1/2)*(b*(x-1/b*(a*b^2)^(1/3))/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a
*b^2)^(1/3)))^(1/2)*(1/2*I*b*(2*x+1/b*(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/
3)))/(a*b^2)^(1/3))^(1/2)/(-b*x^3+a)^(1/2)*(3*I*(a*b^2)^(1/3)*_alpha*3^(1/2
)*b+4*3^(1/2)*_alpha^2*b^2-3*I*(a*b^2)^(2/3)*3^(1/2)-6*I*(a*b^2)^(1/3)*_alp
ha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*_alpha^2*b^2+6*I*(a*b^2)^(2/3)-2*3^
(1/2)*(a*b^2)^(2/3)+3*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(-I*(x+1/2/b*(a*b^2)^(1/3)+1/2*I*3^(1/2)/b*(a*b^2)^(1/3))*3^(1/2)
*b/(a*b^2)^(1/3))^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1
/2)*(a*b^2)^(2/3)*_alpha+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*(a*
b^2)^(2/3)*_alpha+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*a*b-2*3^(1/2)*a*b-3*(a
*b^2)^(2/3)*_alpha+3*a*b)/a,(-I*3^(1/2)/b*(a*b^2)^(1/3)/(-3/2/b*(a*b^2)^(1/
3)-1/2*I*3^(1/2)/b*(a*b^2)^(1/3)))^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a-
10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-bx^3+a}\,x}{bx^3-2\,a\left(3\,\sqrt{3}\,+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x\,\sqrt{a-b\,x^3}}{b\,x^3-2\,a\,\left(3\,\sqrt{3}\,+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] -int((x*(a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a-bx^3}}{-6\sqrt{3}\,a-10a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

3.344 $\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=774

$$-\frac{2\sqrt{2}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}-\right.}{}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/2
*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))
/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b
^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1/6)*
arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^
(2/3)*2^(1/2)-2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))-2/
3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/
3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 304, 219, 1879, 488}

$$-\frac{2\sqrt{2}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}\,x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}\,x\right)^2}}\,\sqrt{bx^3-a}}+\frac{\sqrt[4]{3}\,\sqrt{2+\sqrt{3}}\,\sqrt[3]{a}\,\left(\sqrt[3]{a}-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (-2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(S
qrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*
x^3])])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2)]*Sqrt[-a + b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(
2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(
```

```
1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{-2\left(5+3\sqrt{3}\right)a+bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx + \int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}}$$

$$= -\frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}}{}$$

Mathematica [C] time = 0.08, size = 87, normalized size = 0.11

$$-\frac{x^2\sqrt{bx^3-a}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{1-\frac{bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(-2*(5 + 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(
10*a + 6*Sqrt[3]*a)])/((5 + 3*Sqrt[3])*a*Sqrt[1 - (b*x^3)/a])
```

fricas [F] time = 57.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^4+6\sqrt{3}\,ax-10\,ax\right)\sqrt{bx^3-a}}{b^2x^6-20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4 + 6*sqrt(3)*a*x - 10*a*x)*sqrt(b*x^3 - a)/(b^2*x^6 - 20*a*b
*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3-a}\,x}{bx^3-2\,a\left(3\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

maple [C] time = 0.36, size = 926, normalized size = 1.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)*(a*b^2)^(1/3)/b*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b
^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3/2*(a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b))^(1/2)*(I*(x+1/2*(a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1
/2)*((-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*EllipticE(1/3*3^(
1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b
^2)^(1/3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(-
I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/
3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
a*b^2)^(1/3)/b)/b)^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)+3)*(a*
b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^
(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/
3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2
)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I
*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2
)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alpha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*
b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(
1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*
_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3
)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^
(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3 - a}\, x}{bx^3 - 2\, a\left(3\sqrt{3} + 5\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\sqrt{b\, x^3 - a}}{b\, x^3 - 2\, a\, \left(3\sqrt{3} + 5\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] int((x*(b*x^3 - a)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) + 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a + bx^3}}{-6\sqrt{3}\, a - 10a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(-a + b*x**3)/(-6*sqrt(3)*a - 10*a + b*x**3), x)
```

3.345 $\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=768

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] 1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/
2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/2*3^(1/4)*a^(1/6)*arctan(1/
2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3
)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)-1/6*a^(1
/6)*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/
4)/b^(2/3)*2^(1/2)-2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))-2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2))
)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(
2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)+3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*
(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/
3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.135, Rules used = {489, 304, 219, 1879, 488}

$$-\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (-2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) + (3^(1
/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/
(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*ArcTan[(3
^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^
3])])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])
*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(
2/3)) - (a^(1/6)*ArcTanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*
Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
 b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)
*Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2)]*Sqrt[-a - b*x^3]) - (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(
```

```
1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1
/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{-2\left(5+3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3+2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx + \int \frac{x}{\sqrt{-a-bx^3}}\,dx$$

$$= \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$

$$= -\frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} + \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{-a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}} + \frac{\sqrt[4]{3}\,\sqrt[6]{}}{}$$

Mathematica [C] time = 0.08, size = 90, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{4\left(5+3\sqrt{3}\right)a\sqrt{\frac{a+bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(-2*(5 + 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a + 6*Sqrt[3]*a))])/((5 + 3*Sqrt[3])*a*Sqrt[(a + b*x^3)/a])
```

fricas [F] time = 58.10, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4-6\sqrt{3}\,ax+10\,ax\right)\sqrt{-bx^3-a}}{b^2x^6+20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] integral(-(b*x^4 - 6*sqrt(3)*a*x + 10*a*x)*sqrt(-b*x^3 - a)/(b^2*x^6 + 20*a
*b*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-bx^3-a}\,x}{bx^3+2\,a\left(3\sqrt{3}+5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

maple [C] time = 0.37, size = 983, normalized size = 1.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(
-b*x^3-a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Ell
ipticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3
^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_al
pha*(2*3^(1/2)+3)*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*
b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(
1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1
/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(1/2)*(4*3^(1/2)*
_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(1/2)*(
-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^2)^(1/3)*_alpha*
b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)+3*
(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*(2*I*3^(1/2)*(-
a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/
2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1/2)*(-a*b^2)^(2/
3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/a/b,(I*3^(1/2)
*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(
1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-bx^3 - a}\,x}{bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(-b*x^3-2*a*(5+3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) + 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{-b\,x^3 - a}}{b\,x^3 + 2\,a\,\left(3\,\sqrt{3} + 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)),x)
```

```
[Out] int(-(x*(- a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) + 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{-a - bx^3}}{10a + 6\sqrt{3}\,a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(-b*x**3-2*a*(5+3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a - b*x**3)/(10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.346 $\int \frac{x\sqrt{a+bx^3}}{2(5-3\sqrt{3})a+bx^3}\,dx$

Optimal. Leaf size=738

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+2*(b*x^3+a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.25, antiderivative size = 738, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{}\right.}{}$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]

[Out] (2*Sqrt[a + b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])]])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])]])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(

```
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a+bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a+bx^3}}\,dx$$
$$= -\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}}$$
$$= \frac{2\sqrt{a+bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}\,x\right)}{\sqrt{2}\,\sqrt{a+bx^3}}\right)}{\sqrt{2}\,b^{2/3}} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\mathrm{t}}{}$$

Mathematica [C] time = 0.19, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a + b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3
)/(10*a - 6*Sqrt[3]*a))])/((20 - 12*Sqrt[3])*Sqrt[a + b*x^3])
```

fricas [F] time = 58.34, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(bx^4+6\sqrt{3}\,ax+10\,ax\right)\sqrt{bx^3+a}}{b^2x^6+20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4 + 6*sqrt(3)*a*x + 10*a*x)*sqrt(b*x^3 + a)/(b^2*x^6 + 20*a*b
*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+a}\,x}{bx^3-2\,a\left(3\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

maple [C] time = 0.74, size = 977, normalized size = 1.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2
)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(
b*x^3+a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alp
ha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b
^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1
/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/
2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(3*I*(-a*b^2)
^(1/3)*_alpha*3^(1/2)*b+4*3^(1/2)*_alpha^2*b^2-3*I*(-a*b^2)^(2/3)*3^(1/2)-2
*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+6*_alpha^2*b^2
-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(-a*b^2)^(1/3)*_alpha*b-3*(-
a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6/b*(2*I*3^(1/2)*(-
a*b^2)^(1/3)*_alpha^2*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+4*I*(-a*b^2)^(1/3)*
_alpha^2*b-2*3^(1/2)*(-a*b^2)^(2/3)*_alpha+I*3^(1/2)*a*b-2*I*(-a*b^2)^(2/3)
*_alpha-2*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha+2*I*a*b-3*a*b)/a,(I*3^(1/2)*(
-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/
2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3 + a}\, x}{bx^3 - 2\, a\left(3\, \sqrt{3} - 5\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(b*x^3 + a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\, \sqrt{b\, x^3 + a}}{b\, x^3 - 2\, a\, \left(3\, \sqrt{3} - 5\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int((x*(a + b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a + bx^3}}{-6\sqrt{3}\, a + 10a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(a + b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

3.347 $\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=758

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}{\ldots}$$

```
[Out] -1/2*3^(1/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(
1/2)))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)-1/4*3^(1/4)*a^(1/6)*arctan
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/
2))/b^(2/3)*2^(1/2)+1/4*3^(3/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3
)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1
/6)*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/
4)/b^(2/3)*2^(1/2)+2*(-b*x^3+a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))+2/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)+a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)
/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*
(1+3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(
1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)
*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(-b*x^3+a)^(1/2)/(a^(1/3)*(a^(1/3)
-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {489, 303, 218, 1877, 487}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[a - b*x^3])/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(1/4
)*a^(1/6)*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*a^(1/6)*ArcTan[(3^(1
/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3]
)])/(Sqrt[2]*b^(2/3)) + (3^(3/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(
1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*b^(2/3))
 + (a^(1/6)*ArcTanh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a
])])/(Sqrt[2]*3^(1/4)*b^(2/3)) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*(a^(1/3
) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[
3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/
3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[
(a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt
[a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) - b^(
```

```
1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) - b^(1/3
)*x))/((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*Sqrt[a - b*x^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{a-bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = -\left(\left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx\right)+\int \frac{x}{\sqrt{a-bx^3}}\,dx$$

$$= -\frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\,\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{\sqrt{2}\,b^{2/3}}$$

$$= \frac{2\sqrt{a-bx^3}}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} - \frac{\sqrt[4]{3}\,\sqrt[6]{a}\,\mathrm{ta}}{}$$

Mathematica **[C]** time = 0.08, size = 80, normalized size = 0.11

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20-12\sqrt{3}\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[a - b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10
*a - 6*Sqrt[3]*a)])/((20 - 12*Sqrt[3])*Sqrt[a - b*x^3])
```

fricas **[F]** time = 46.80, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(bx^4-6\sqrt{3}\,ax-10\,ax\right)\sqrt{-bx^3+a}}{b^2x^6-20\,abx^3-8\,a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="frica
s")
```

```
[Out] integral(-(b*x^4 - 6*sqrt(3)*a*x - 10*a*x)*sqrt(-b*x^3 + a)/(b^2*x^6 - 20*a
*b*x^3 - 8*a^2), x)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-bx^3+a}\,x}{bx^3+2\,a\left(3\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac"
)
```

```
[Out] integrate(-sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

maple **[C]** time = 0.77, size = 924, normalized size = 1.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)/b*(a*b^2)^(1/3)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b
^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3/2*(a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b))^(1/2)*(I*(x+1/2*(a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(
1/2)*((-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*
b^2)^(1/3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1
/3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(a*b^2)^(1/3)/b)/b)^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(a
*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)
^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1
/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^
2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(-3*I*(a*b^2)^(1/3)*_alpha*3^(1/2)*b+4*3
^(1/2)*_alpha^2*b^2+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(1/3)*_alph
a*b-6*I*(a*b^2)^(1/3)*_alpha*b+6*_alpha^2*b^2-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(
a*b^2)^(2/3)-3*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2
)^(1/3)*b)^(1/2),1/6/b*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*(a*
b^2)^(2/3)*_alpha-4*I*(a*b^2)^(1/3)*_alpha^2*b-2*3^(1/2)*(a*b^2)^(2/3)*_alp
ha+I*3^(1/2)*a*b+2*I*(a*b^2)^(2/3)*_alpha+2*3^(1/2)*a*b-3*(a*b^2)^(2/3)*_al
pha+2*I*a*b+3*a*b)/a,(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*
3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-bx^3+a}\,x}{bx^3+2\,a\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3+a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxim
a")
```

```
[Out] -integrate(sqrt(-b*x^3 + a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{a-b\,x^3}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int(-(x*(a - b*x^3)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{a-bx^3}}{-10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3+a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(a - b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.348 $\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx$

Optimal. Leaf size=774

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^(
1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/2*
3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)
))*2^(1/2)/(b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh(1/2
*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))/b
^(2/3)*2^(1/2)+2*(b*x^3-a)^(1/2)/b^(2/3)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))+2
/3*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticF((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2
/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)-b^(1/3)*x)*EllipticE((-b^(1/3)*x+
a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)-b^(1
/3)*x)/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 774, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {489, 304, 219, 1879, 488}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)^2}}\sqrt{bx^3-a}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] (2*Sqrt[-a + b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/
3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sq
rt[-a + b*x^3])])/(Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - b^(1/3)*x)*Sqrt[(a^(2/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) - b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2)]*Sqrt[-a + b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) - b^(1/3)*x)*Sqrt[(a^(2
/3) + a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/((1 - Sqrt[3])*a^(1
```

```
/3) - b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) - b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)^2)]*Sqrt[-a + b*x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a+bx^3}}{2\left(5-3\sqrt{3}\right)a-bx^3}\,dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx - \int \frac{x}{\sqrt{-a+bx^3}}\,dx$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}\,b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\ t}{}$$

$$= \frac{2\sqrt{-a+bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}}{\sqrt{}}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.11

$$-\frac{x^2\sqrt{bx^3-a}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a-10a}\right)}{4\left(3\sqrt{3}-5\right)a\sqrt{\frac{a-bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a + b*x^3])/(2*(5 - 3*Sqrt[3])*a - b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a + b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, (b*x^3)/a, -((b*x^3)
/(-10*a + 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[(a - b*x^3)/a])
```

fricas [F] time = 42.37, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4-6\sqrt{3}ax-10ax\right)\sqrt{bx^3-a}}{b^2x^6-20abx^3-8a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] integral(-(b*x^4 - 6*sqrt(3)*a*x - 10*a*x)*sqrt(b*x^3 - a)/(b^2*x^6 - 20*a*
b*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{bx^3-a}\,x}{bx^3+2a\left(3\sqrt{3}-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

maple [C] time = 0.39, size = 926, normalized size = 1.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] -2/3*I*3^(1/2)*(a*b^2)^(1/3)/b*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*
b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3/2*(a*
b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b))^(1/2)*(I*(x+1/2*(a*b^2)^(1/3)/
b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(
1/2)*((-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*EllipticE(1/3*3^
(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*
b^2)^(1/3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2))+1/b*(a*b^2)^(1/3)*EllipticF(1/3*3^(1/2)*(
-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1
/3)*b)^(1/2),(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(a*b^2)^(1/3)/b)/b)^(1/2)))+1/9*I/b^3*2^(1/2)*sum(1/_alpha*(2*3^(1/2)-3)*(a
*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)
^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1
/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^
2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*
I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^
2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_alpha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1
/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^
(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)
*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a*b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/
3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_alpha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)
^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^
(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{bx^3-a}\,x}{bx^3+2\,a\left(3\,\sqrt{3}\,-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3-a)^(1/2)/(-b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

```
[Out] -integrate(sqrt(b*x^3 - a)*x/(b*x^3 + 2*a*(3*sqrt(3) - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x\,\sqrt{b\,x^3-a}}{b\,x^3+2\,a\,\left(3\,\sqrt{3}\,-5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int(-(x*(b*x^3 - a)^(1/2))/(b*x^3 + 2*a*(3*3^(1/2) - 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{-a+bx^3}}{-10a+6\sqrt{3}\,a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3-a)**(1/2)/(-b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] -Integral(x*sqrt(-a + b*x**3)/(-10*a + 6*sqrt(3)*a + b*x**3), x)
```

3.349 $\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}\,dx$

Optimal. Leaf size=768

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] -1/4*3^(3/4)*a^(1/6)*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1
/2))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/6*a^(1/6)*arctan(1/6*(1+3^
(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*3^(3/4)/b^(2/3)*2^(1/2)+1/
2*3^(1/4)*a^(1/6)*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1
/2)))*2^(1/2)/(-b*x^3-a)^(1/2))/b^(2/3)*2^(1/2)+1/4*3^(1/4)*a^(1/6)*arctanh
(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/
2))/b^(2/3)*2^(1/2)+2*(-b*x^3-a)^(1/2)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2
)))+2/3*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1+3^(1/2)
))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/b^
(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1-
3^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x
+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b^(2/3)/(-b*x^3-a)^(1/2)/(-a^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {489, 304, 219, 1879, 488}

$$\frac{2\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}b^{2/3}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{-a-bx^3}}-\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] (2*Sqrt[-a - b*x^3])/(b^(2/3)*((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)) - (3^(3/
4)*a^(1/6)*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sq
rt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) + (a^(1/6)*ArcTan[((1 + Sqrt[
3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(Sqrt[2]*3^(1/4)*b^(2/3))
 + (3^(1/4)*a^(1/6)*ArcTanh[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(
1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(Sqrt[2]*b^(2/3)) + (3^(1/4)*a^(1/6)*
ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt
[-a - b*x^3])])/(2*Sqrt[2]*b^(2/3)) - (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 -
Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) +
b^(1/3)*x)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(b^(2/3)*
Sqrt[-((a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2)]*Sqrt[-a - b*x^3]) + (2*Sqrt[2]*a^(1/3)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2
/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 - Sqrt[3])*a^(1
```

```
/3) + b^(1/3)*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[-((a^(1/3)*(a^(1/
3) + b^(1/3)*x))/((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)^2)]*Sqrt[-a - b*x^3])
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rule 489

```
Int[((x_)*Sqrt[(a_) + (b_.)*(x_)^3])/((c_) + (d_.)*(x_)^3), x_Symbol] :> Di
st[b/d, Int[x/Sqrt[a + b*x^3], x], x] - Dist[(b*c - a*d)/d, Int[x/((c + d*x
^3)*Sqrt[a + b*x^3]), x], x] /; FreeQ[{c, d, a, b}, x] && NeQ[b*c - a*d, 0]
 && (EqQ[b*c - 4*a*d, 0] || EqQ[b*c + 8*a*d, 0] || EqQ[b^2*c^2 - 20*a*b*c*d
 - 8*a^2*d^2, 0])
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{x\sqrt{-a-bx^3}}{2\left(5-3\sqrt{3}\right)a+bx^3}dx = \left(3\left(3-2\sqrt{3}\right)a\right)\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}dx - \int \frac{x}{\sqrt{-a-bx^3}}dx$$

$$= -\frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{\sqrt{2}\sqrt[4]{3}b^{2/3}} + \frac{\sqrt[4]{3}\sqrt[6]{a}\ \text{t}}{}$$

$$= \frac{2\sqrt{-a-bx^3}}{b^{2/3}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)} - \frac{3^{3/4}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}b^{2/3}} + \frac{\sqrt[6]{a}\tan^{-1}}{\sqrt{}}$$

Mathematica [C] time = 0.08, size = 89, normalized size = 0.12

$$-\frac{x^2\sqrt{-a-bx^3}F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{4\left(3\sqrt{3}-5\right)a\sqrt{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[-a - b*x^3])/(2*(5 - 3*Sqrt[3])*a + b*x^3),x]
```

```
[Out] -1/4*(x^2*Sqrt[-a - b*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((b*x^3)/a), -((b*x
^3)/(10*a - 6*Sqrt[3]*a))])/((-5 + 3*Sqrt[3])*a*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 41.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+6\sqrt{3}ax+10ax\right)\sqrt{-bx^3-a}}{b^2x^6+20abx^3-8a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="fricas
")
```

```
[Out] integral((b*x^4 + 6*sqrt(3)*a*x + 10*a*x)*sqrt(-b*x^3 - a)/(b^2*x^6 + 20*a*
b*x^3 - 8*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^3-a}\,x}{bx^3-2a\left(3\sqrt{3}-5\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

maple [C] time = 0.42, size = 983, normalized size = 1.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x)
```

```
[Out] 2/3*I*3^(1/2)*(-a*b^2)^(1/3)/b*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3/2*
(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)*(-I*(x+1/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2)/(-
b*x^3-a)^(1/2)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^
(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2))+(-a*b^2)^(1/3)/b*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^
(1/2)/(-a*b^2)^(1/3)*b)^(1/2),(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)))-1/9*I/b^3*2^(1/2)*sum(1/_alp
ha*(2*3^(1/2)-3)*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b
^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1
/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/
2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(1/2)*(4*3^(1/2)*_
alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(1/2)*(-
a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2)^(1/3)*_alpha*b
-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-a*b^2)^(2/3)-3*(
-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*(2*I*3^(1/2)*(-a
*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*3^(1/2
)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1/2)*(-a*b^2)^(2/3
)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/a/b,(I*3^(1/2)*
(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1
/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^3 - a}\,x}{bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x^3-a)^(1/2)/(b*x^3+2*a*(5-3*3^(1/2))),x, algorithm="maxima
")
```

```
[Out] integrate(sqrt(-b*x^3 - a)*x/(b*x^3 - 2*a*(3*sqrt(3) - 5)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{-b\,x^3 - a}}{b\,x^3 - 2\,a\,\left(3\,\sqrt{3} - 5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)),x)
```

```
[Out] int((x*(- a - b*x^3)^(1/2))/(b*x^3 - 2*a*(3*3^(1/2) - 5)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-a - bx^3}}{-6\sqrt{3}\,a + 10a + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-b*x**3-a)**(1/2)/(b*x**3+2*a*(5-3*3**(1/2))),x)
```

```
[Out] Integral(x*sqrt(-a - b*x**3)/(-6*sqrt(3)*a + 10*a + b*x**3), x)
```

3.350 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=318

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}{\sqrt{2}\sqrt{a+bx\cdots}}\right.}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b
*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1-3^(1/2))*(b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^
(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*
(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^
(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2
^(1/2)/(b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a+bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}\right.}{\sqrt{2}\sqrt{a+bx\cdots}}\right.}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/
6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 - Sqrt[
3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt{2}\,3^{\cdots}}\right.}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.39, size = 538, normalized size = 1.69

$$i\left(-ab^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-ab^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(-ab^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{\left(-ab^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(-ab^2\right)^{\frac{1}{3}}}}\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3b+\right.\right.$$

$$-\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/
3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/
b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*
b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^
2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
```

```
1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5+3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 + a}\,\left(b\,x^3 + 2\,a\,\left(3\,\sqrt{3} + 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3}\,\left(10a + 6\sqrt{3}\,a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5+3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(10*a + 6*sqrt(3)*a + b*x**3)), x)
```

3.351 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=324

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

```
[Out] -1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-
b*x^3+a)^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6
*(1-3^(1/2))*(-b*x^3+a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/
a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x
)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)
*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))
*2^(1/2)/(-b*x^3+a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - S
qrt[3])*ArcTan[((1 - Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/
(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*(1 -
Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt
[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((1
+ Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3
^(1/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt{a-}}{\sqrt{2}\,3^{3/4}\sqrt{}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.08, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.41, size = 509, normalized size = 1.57

$$i\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3b-6\sqrt{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a*
b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(-
3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*(
a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
```

```
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3-2\,a\left(3\,\sqrt{3}+5\right)\right)\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5+3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] -int(x/((a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-6\sqrt{3}\,a\sqrt{a-bx^3}-10a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5+3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-6*sqrt(3)*a*sqrt(a - b*x**3) - 10*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

3.352 $\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=328

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\ldots\right.}{\ldots}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}\ldots}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*3
^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(
2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/6)
*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3-a)^(1/2))*(2-3^(1/2))*3^(1/
4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(b*x^3-a)^(1/2)*3^(
1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\ldots\right.}{\ldots}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*S
qrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a+bx^3}\left(-2\left(5+3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\ldots}{\ldots}\right.}{3\sqrt{2}\sqrt[4]{3}a\ldots}$$

Mathematica [C] time = 0.12, size = 85, normalized size = 0.26

$$-\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{6\sqrt{3}\,a+10a}\right)}{\left(12\sqrt{3}\,a+20a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a + b*x^3]*(-2*(5 + 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(1
0*a + 6*Sqrt[3]*a)])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a + b*x^3]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.41, size = 510, normalized size = 1.55

$$-\frac{i\left(ab^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(ab^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(ab^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(ab^2\right)^{\frac{1}{3}}+\left(ab^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(ab^2\right)^{\frac{1}{3}}}}\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3b-6\sqrt{3}\ a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3
^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2+3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-6*I*(
a*b^2)^(1/3)*_alpha*b-2*3^(1/2)*(a*b^2)^(1/3)*_alpha*b+3*(a*b^2)^(1/3)*_alp
ha*b-3*I*(a*b^2)^(2/3)*3^(1/2)+6*I*(a*b^2)^(2/3)-2*3^(1/2)*(a*b^2)^(2/3)+3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b+4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b-2*I*a*b-2*3^
(1/2)*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*I*(a*b^2)^(2/3)*_alpha+2*3
```

```
^(1/2)*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b-6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} + 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3-2*a*(5+3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) + 5))*sqrt(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 - a}\,\left(b\,x^3 - 2\,a\,\left(3\,\sqrt{3} + 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(x/((b*x^3 - a)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) + 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a + bx^3}\,\left(-6\sqrt{3}\,a - 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3-2*a*(5+3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a + b*x**3)*(-6*sqrt(3)*a - 10*a + b*x**3)), x)
```

3.353 $\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=330

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)}{\sqrt{2}\sqrt{}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

```
[Out] 1/36*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctan(1/2*
3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1+3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2)
)*(2-3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3-a)^(1/2))*(2-3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/6*(1-3^(1/2))*(-b*x^3-a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2-3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {488}

$$\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)}{\sqrt{2}\sqrt{}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] ((2 - Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 + Sqrt[3])*a^(1/3) - 2*b^(1/3)*x
))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 -
 Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 - Sqrt[3
])*ArcTanh[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*S
qrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 - Sqrt[3])*Arc
Tanh[((1 - Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2
]*3^(3/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(-2\left(5+3\sqrt{3}\right)a-bx^3\right)}\,dx=\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2-\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{}\right)}{\sqrt{}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}}$$

Mathematica [C] time = 0.11, size = 87, normalized size = 0.26

$$-\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{6\sqrt{3}a+10a}\right)}{\left(12\sqrt{3}a+20a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(-2*(5 + 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^
3)/(10*a + 6*Sqrt[3]*a))])/((20*a + 12*Sqrt[3]*a)*Sqrt[-a - b*x^3]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.36, size = 541, normalized size = 1.64

$$i\left(-a b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(-a b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{\left(-a b^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(-a b^2\right)^{\frac{1}{3}}}}\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3 b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(
1/2)*(4*3^(1/2)*_alpha^2*b^2-6*_alpha^2*b^2-3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b+3*(-a*b^
2)^(1/3)*_alpha*b+3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)+3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b-4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
```

```
1/2)*a*b+2*3^(1/2)*a*b-2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha+2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha+2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b+6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}\,+5\right)\right)\sqrt{-bx^3-a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3-2*a*(5+3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) + 5))*sqrt(-b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}\,+5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))),x)
```

```
[Out] int(-x/((- a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) + 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{10a\sqrt{-a-bx^3}\,+6\sqrt{3}\,a\sqrt{-a-bx^3}\,+bx^3\sqrt{-a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3-2*a*(5+3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(10*a*sqrt(-a - b*x**3) + 6*sqrt(3)*a*sqrt(-a - b*x**3) + b*x**
3*sqrt(-a - b*x**3)), x)
```

3.354 $\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=310

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}(1\cdots}{\cdots}\right)}{2\sqrt{2}3^{3/4}a^{5\cdots}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)
/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))
*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1/
6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*x^3+a)^(1/2))*(2+3^(1/2))*3^(
1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(b*x^3+a)^(1/2)*3
^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}(1\cdots}{\cdots}\right)}{2\sqrt{2}3^{3/4}a^{5\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*
x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 +
 Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqr
t[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sq
rt[a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a+bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}(1+\cdots}{\cdots}\right)}{6\sqrt{2}\sqrt[4]{3}a^{5\cdots}}$$

Mathematica [C] time = 0.15, size = 83, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}\,a}\right)}{\left(20a-12\sqrt{3}\,a\right)\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[a + b*x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.42, size = 538, normalized size = 1.74

$$i\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\sqrt{\frac{\left(x-\frac{\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\sqrt{-\frac{i\left(2x+\frac{\left(-a\,b^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(-a\,b^2\right)^{\frac{1}{3}}}}\;\Big(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3b-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1
/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alph
a*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^2
)^(1/3)*_alpha*b-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(-
a*b^2)^(2/3)-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6*
(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(1
```

```
/2)*a*b-2*3^(1/2)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1/
2)*(-a*b^2)^(2/3)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha)
/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2
)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3 + a}\,\left(b\,x^3 - 2\,a\,\left(3\,\sqrt{3} - 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx^3}\,\left(-6\sqrt{3}\,a + 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

3.355 $\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx$

Optimal. Leaf size=316

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right.}{\sqrt{2}}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2}}$$

```
[Out] -1/18*arctan(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*(1-3^(1/2)))*2^(1/2)/
(-b*x^3+a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/36*arctan(1
/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2)
)*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/12*arctanh(1/2*3^(1/4)*a^(1
/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b*x^3+a)^(1/2))*(2+3^(1/2))*3
^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)+1/18*arctanh(1/6*(1+3^(1/2))*(-b*x^3+a)^(1/2
)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {487}

$$-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}+2\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}+\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right.}{\sqrt{2}}\right.}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] -((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 + Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x)
)/(Sqrt[2]*Sqrt[a - b*x^3])])/(6*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[(3^(1/4)*a^(1/6)*((1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqr
t[2]*Sqrt[a - b*x^3])])/(3*Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3]
)*ArcTanh[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sq
rt[a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) + ((2 + Sqrt[3])*ArcTa
nh[((1 + Sqrt[3])*Sqrt[a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])/(3*Sqrt[2]*3
^(3/4)*a^(5/6)*b^(2/3))
```

Rule 487

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, -Simp[(q*(2 - r)
*ArcTan[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[a, 2]*r^(3/2))])/(3*Sqrt[2]*R
t[a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTan[(Rt[a, 2]*Sqrt[r]*(1 + r)
*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[a, 2]*d*r^(3/2)), x]
- Simp[(q*(2 - r)*ArcTanh[(Rt[a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*Sqrt[
a + b*x^3])])/(3*Sqrt[2]*Rt[a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*ArcTanh[
(Rt[a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*Sqrt[2]
*Rt[a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &
& EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && PosQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{a-bx^3}\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)}\,dx=-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{a-bx^3}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\right.\right.}{\sqrt{2}}\right.}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/}}$$

Mathematica [C] time = 0.10, size = 83, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[a - b*x^3]*(2*(5 - 3*Sqrt[3])*a - b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[a - b*x^3])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   catde
f: division by zero
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.37, size = 509, normalized size = 1.61

$$i\left(a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(a\,b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(a\,b^2\right)^{\frac{1}{3}}}}\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3b+6\sqrt{\ }\right.\right.$$

$$-\overline{\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(-b*x^3+a)^(1/2)*(4*
3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^
(1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_al
pha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3
*(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b
```

```
^2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a
*b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_al
pha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2
)^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alp
ha=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3+2\,a\left(3\,\sqrt{3}-5\right)\right)\sqrt{-bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{a-b\,x^3}\,\left(b\,x^3+2\,a\,\left(3\,\sqrt{3}-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((a - b*x^3)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{a-bx^3}+6\sqrt{3}\,a\sqrt{a-bx^3}+bx^3\sqrt{a-bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(a - b*x**3) + 6*sqrt(3)*a*sqrt(a - b*x**3) + b*x**3
*sqrt(a - b*x**3)), x)
```

3.356 $\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx$

Optimal. Leaf size=320

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(b*
x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*(
1+3^(1/2))*(b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a^(
5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(2*b^(1/3)*x+a^(1/3)*
(1-3^(1/2)))*2^(1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2
^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)-b^(1/3)*x)*(1+3^(1/2))*2^(
1/2)/(b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{bx^3-a}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{bx^3-a}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\left(1+\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}\cdots\right.}{\sqrt{2}\sqrt{bx^3-a}}\right)}{6\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a + b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a + b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 +
 Sqrt[3])*a^(1/6)*(a^(1/3) - b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(6*Sq
rt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1/6)*((
1 - Sqrt[3])*a^(1/3) + 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a + b*x^3])])/(3*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\left(2\left(5-3\sqrt{3}\right)a-bx^3\right)\sqrt{-a+bx^3}}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a+bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt{2}\,3\cdots}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.11, size = 84, normalized size = 0.26

$$\frac{x^2\sqrt{1-\frac{bx^3}{a}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};\frac{bx^3}{a},\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((2*(5 - 3*Sqrt[3])*a - b*x^3)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*
a - 6*Sqrt[3]*a)])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a + b*x^3])
```

fricas [B] time = 16.18, size = 5060, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(1/3*(3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3
 - sqrt(3)*(153*a^4*b^4*x^3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sq
rt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 -
7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/3) - sqrt(3)*(b*x^4 - a*x) + 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(2
65*a^4*b^4*x^3 + 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x -
71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sq
rt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt(
(b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(
1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 211
2*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4
*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2
- 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*
a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6
)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^
5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5
*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*
a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1
944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x -
3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(135
1*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 + 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 - a*x)) - 1/6*sqrt(3)*(1/1944)^(1/6
)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(1/3*(3*sqrt(b*x^3 - a)*(1
08*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 1978*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^
3 - 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/6)*(41
*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) +
 (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^
5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 - 7*a^3*b - 4*sqrt(3)*(a^2*b
^2*x^3 - a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - sqrt(3)*(b*x^4
- a*x) - 3*sqrt(b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1448*a^5*
b^3 - sqrt(3)*(153*a^4*b^4*x^3 + 836*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*
```

sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 - 100*a*b^3*x^9
+ 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x
^10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(2
23*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(
1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 49
8*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^1
1 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 23
40)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
 + 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896
*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x
^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/
6)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - s
qrt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2)
)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*
a^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4
))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*
a^4)))/(b*x^4 - a*x)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^
4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x^3 +
64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a^6*b^
4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7 + 56
1*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) +
6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5
*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16
*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(a*b^3
*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) + 2*sqrt(b*x^3 - a)*(1944*(1/1944
)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2
131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 23
40)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6 + 13
68*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*x^6 +
 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3
(1/1944)^(1/6)(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64*a^4*
b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqr
t(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6
+ 320*a^3*b*x^3 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)*log((b^4*x^12 + 68*a*b^3*x^9 + 168*a^2*b^2*x^6 - 544*a^3*b*x
^3 + 64*a^4 + 6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 + 11556*a^5*b^5*x^7 + 7776*a
^6*b^4*x^4 + 1440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 + 834*a^5*b^5*x^7
 + 561*a^6*b^4*x^4 + 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/
3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 6
4*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5
 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 64*sqrt(3)*(
a*b^3*x^9 - 3*a^2*b^2*x^6 + 3*a^3*b*x^3 - a^4) - 2*sqrt(b*x^3 - a)*(1944*(1
/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(
3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3)
 + 2340)/(a^5*b^4))^(5/6) + 4*sqrt(1/6)*(168*a^3*b^5*x^9 + 1845*a^4*b^4*x^6
 + 1368*a^5*b^3*x^3 + 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 + 1065*a^4*b^4*
x^6 + 792*a^5*b^3*x^3 + 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)
) + 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 + 64
*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(135
1*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2
*x^6 + 320*a^3*b*x^3 + 64*a^4)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 + 240*a^2*b^2*x^6 - 832*a
^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 - 12492*a^5*b^5*x^7 -
 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 - 1803*a^
5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5
b^4))^(2/3) + 6(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498*a^3*b^4*x^8 + 384*a^4*b
^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 + 96*a^3*b^4*x^8 + 72*a

```
^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 32
*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3 + 8*a^4) + 2*sqrt(b*x^3
- a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*a^6*b^5*x^5 + 568*a^7*b^
4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6)*(123*a^3*b^5*x^9 + 511
2*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 +
2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2
340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 - 12*a^2*b^3*x^7 - 72*a^3*
b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a^2*b^3*x^7 + 8*a^3*b^2*x
^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 - 40
*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/
6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 - 100*a*b^3*x^9 +
 240*a^2*b^2*x^6 - 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^
10 - 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 - 2112*a^7*b^3*x - 4*sqrt(3)*(22
3*a^4*b^6*x^10 - 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 - 304*a^7*b^3*x))*(-(1
351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) + 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 + 498
*a^3*b^4*x^8 + 384*a^4*b^3*x^5 + 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11
 + 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 + 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/3) + 32*sqrt(3)*(a*b^3*x^9 + 6*a^2*b^2*x^6 - 15*a^3*b*x^3
+ 8*a^4) - 2*sqrt(b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 + 2896*
a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 + 1672*a^6*b^5*x^
5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - 2*sqrt(1/6
)*(123*a^3*b^5*x^9 + 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 + 768*a^6*b^2 - sq
rt(3)*(71*a^3*b^5*x^9 + 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 + 448*a^6*b^2))
*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 -
12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 + 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 - 4*a
^2*b^3*x^7 + 8*a^3*b^2*x^4 - 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4)
)^(1/6)))/(b^4*x^12 - 40*a*b^3*x^9 + 384*a^2*b^2*x^6 + 320*a^3*b*x^3 + 64*a
^4))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.36, size = 510, normalized size = 1.59

$$-\frac{i\left(a b^2\right)^{\frac{1}{3}} \sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(a b^2\right)^{\frac{1}{3}}+\left(a b^2\right)^{\frac{1}{3}}}{b}\right)b}{2\left(a b^2\right)^{\frac{1}{3}}}} \sqrt{\frac{\left(x-\frac{\left(a b^2\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(a b^2\right)^{\frac{1}{3}}-i\sqrt{3}\left(a b^2\right)^{\frac{1}{3}}}} \sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(a b^2\right)^{\frac{1}{3}}+\left(a b^2\right)^{\frac{1}{3}}}{b}\right)b}{\left(a b^2\right)^{\frac{1}{3}}}} \left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^3 b+6\sqrt{3}\ a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x)
```

```
[Out] -1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(a*b^2)^(1/3)*(-1/2*I*(2*x+(I*3^(1/2)*(a
*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)*((x-(a*b^2)^(1/3)/b)/(
-3*(a*b^2)^(1/3)-I*3^(1/2)*(a*b^2)^(1/3))*b)^(1/2)*(1/2*I*(2*x+(-I*3^(1/2)*
(a*b^2)^(1/3)+(a*b^2)^(1/3))/b)/(a*b^2)^(1/3)*b)^(1/2)/(b*x^3-a)^(1/2)*(4*3
```

```
^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2-3*I*(a*b^2)^(1/3)*3^(1/2)*_alpha*b-2*3^(
1/2)*(a*b^2)^(1/3)*_alpha*b-6*I*(a*b^2)^(1/3)*_alpha*b-3*(a*b^2)^(1/3)*_alp
ha*b+3*I*(a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(a*b^2)^(2/3)+6*I*(a*b^2)^(2/3)-3*
(a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(-I*(x+1/2*(a*b^2)^(1/3)/b+1/2*I*3^(1
/2)*(a*b^2)^(1/3)/b)*3^(1/2)/(a*b^2)^(1/3)*b)^(1/2),1/6*(-2*I*3^(1/2)*(a*b^
2)^(1/3)*_alpha^2*b-4*I*(a*b^2)^(1/3)*_alpha^2*b+I*3^(1/2)*a*b+2*3^(1/2)*a*
b+2*I*a*b+3*a*b+I*3^(1/2)*(a*b^2)^(2/3)*_alpha-2*3^(1/2)*(a*b^2)^(2/3)*_alp
ha+2*I*(a*b^2)^(2/3)*_alpha-3*(a*b^2)^(2/3)*_alpha)/a/b,(-I*3^(1/2)*(a*b^2)
^(1/3)/(-3/2*(a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(a*b^2)^(1/3)/b)/b)^(1/2)),_alph
a=RootOf(_Z^3*b+6*3^(1/2)*a-10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{\left(bx^3 + 2\,a\left(3\,\sqrt{3} - 5\right)\right)\sqrt{bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x^3+2*a*(5-3*3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] -integrate(x/((b*x^3 + 2*a*(3*sqrt(3) - 5))*sqrt(b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int -\frac{x}{\sqrt{b\,x^3 - a}\,\left(b\,x^3 + 2\,a\,\left(3\,\sqrt{3} - 5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(-x/((b*x^3 - a)^(1/2)*(b*x^3 + 2*a*(3*3^(1/2) - 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{-10a\sqrt{-a + bx^3} + 6\sqrt{3}\,a\sqrt{-a + bx^3} + bx^3\sqrt{-a + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-b*x**3+2*a*(5-3*3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] -Integral(x/(-10*a*sqrt(-a + b*x**3) + 6*sqrt(3)*a*sqrt(-a + b*x**3) + b*x*
*3*sqrt(-a + b*x**3)), x)
```

3.357 $\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\right.}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

```
[Out] 1/12*arctan(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1-3^(1/2))*2^(1/2)/(-b
*x^3-a)^(1/2))*(2+3^(1/2))*3^(1/4)/a^(5/6)/b^(2/3)*2^(1/2)-1/18*arctan(1/6*
(1+3^(1/2))*(-b*x^3-a)^(1/2)*3^(1/4)*2^(1/2)/a^(1/2))*(2+3^(1/2))*3^(1/4)/a
^(5/6)/b^(2/3)*2^(1/2)-1/18*arctanh(1/2*3^(1/4)*a^(1/6)*(-2*b^(1/3)*x+a^(1/
3)*(1-3^(1/2)))*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/
3)*2^(1/2)-1/36*arctanh(1/2*3^(1/4)*a^(1/6)*(a^(1/3)+b^(1/3)*x)*(1+3^(1/2))
*2^(1/2)/(-b*x^3-a)^(1/2))*(2+3^(1/2))*3^(3/4)/a^(5/6)/b^(2/3)*2^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.028, Rules used = {488}

$$\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-a-bx^3}}{\sqrt{2}\,3^{3/4}\sqrt{a}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tanh^{-1}\left(\frac{\sqrt[4]{3}\sqrt[6]{a}\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-2\right.}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{3\sqrt{2}\sqrt[4]{3}\,a^{5/6}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] ((2 + Sqrt[3])*ArcTan[(3^(1/4)*(1 - Sqrt[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))
/(Sqrt[2]*Sqrt[-a - b*x^3])])/(2*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + S
qrt[3])*ArcTan[((1 + Sqrt[3])*Sqrt[-a - b*x^3])/(Sqrt[2]*3^(3/4)*Sqrt[a])])
/(3*Sqrt[2]*3^(3/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*a^(1
/6)*((1 - Sqrt[3])*a^(1/3) - 2*b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(3*
Sqrt[2]*3^(1/4)*a^(5/6)*b^(2/3)) - ((2 + Sqrt[3])*ArcTanh[(3^(1/4)*(1 + Sqr
t[3])*a^(1/6)*(a^(1/3) + b^(1/3)*x))/(Sqrt[2]*Sqrt[-a - b*x^3])])/(6*Sqrt[2
]*3^(1/4)*a^(5/6)*b^(2/3))
```

Rule 488

```
Int[(x_)/(Sqrt[(a_) + (b_.)*(x_)^3]*((c_) + (d_.)*(x_)^3)), x_Symbol] :> Wi
th[{q = Rt[b/a, 3], r = Simplify[(b*c - 10*a*d)/(6*a*d)]}, Simp[(q*(2 - r)*
ArcTanh[((1 - r)*Sqrt[a + b*x^3])/(Sqrt[2]*Rt[-a, 2]*r^(3/2))])/(3*Sqrt[2]*
Rt[-a, 2]*d*r^(3/2)), x] + (-Simp[(q*(2 - r)*ArcTanh[(Rt[-a, 2]*Sqrt[r]*(1
+ r)*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(2*Sqrt[2]*Rt[-a, 2]*d*r^(3/2))
, x] - Simp[(q*(2 - r)*ArcTan[(Rt[-a, 2]*Sqrt[r]*(1 + r - 2*q*x))/(Sqrt[2]*
Sqrt[a + b*x^3])])/(3*Sqrt[2]*Rt[-a, 2]*d*Sqrt[r]), x] - Simp[(q*(2 - r)*Ar
cTan[(Rt[-a, 2]*(1 - r)*Sqrt[r]*(1 + q*x))/(Sqrt[2]*Sqrt[a + b*x^3])])/(6*S
qrt[2]*Rt[-a, 2]*d*Sqrt[r]), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[b^2*c^2 - 20*a*b*c*d - 8*a^2*d^2, 0] && NegQ[a]
```

Rubi steps

$$\int \frac{x}{\sqrt{-a-bx^3}\left(2\left(5-3\sqrt{3}\right)a+bx^3\right)}\,dx=\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\sqrt[4]{3}\left(1-\sqrt{3}\right)\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt{2}\sqrt{-a-bx^3}}\right)}{2\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}-\frac{\left(2+\sqrt{3}\right)\tan^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{-\right.}{\sqrt{2}\,3^{3/4}}\right)}{3\sqrt{2}\,3^{3/4}a^{5/6}b^{2/3}}$$

Mathematica [C] time = 0.13, size = 86, normalized size = 0.27

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{bx^3}{10a-6\sqrt{3}a}\right)}{\left(20a-12\sqrt{3}a\right)\sqrt{-a-bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[-a - b*x^3]*(2*(5 - 3*Sqrt[3])*a + b*x^3)),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)
/(10*a - 6*Sqrt[3]*a))])/((20*a - 12*Sqrt[3]*a)*Sqrt[-a - b*x^3])
```

fricas [B] time = 15.48, size = 5060, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*sqrt(3)*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan
(-1/3*(3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b
^3 - sqrt(3)*(153*a^4*b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) + sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*
sqrt(3) + 2340)/(a^5*b^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)) + (6*(1/9)^(1/3)*(7*a^2*b^2*x^3
+ 7*a^3*b - 4*sqrt(3)*(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(1/3) + sqrt(3)*(b*x^4 + a*x) + 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)
*(265*a^4*b^4*x^3 - 1448*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))
*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x
 - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5
*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sq
rt((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 -
6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 +
2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*
b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9
)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x
^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b
^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 -
 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^
(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(213
1*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340
)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960
*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2
280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*
(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*
x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-
(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2
*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/6*sqrt(3)*(1/1944)^
(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*arctan(-1/3*(3*sqrt(-b*x^3 -
 a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 + 1978*a^5*b^3 - sqrt(3)*(153*a^4*
b^4*x^3 + 1142*a^5*b^3))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + sqrt(1/
6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b
^4)) - (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a*b*x^2)*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(1/6)) - (6*(1/9)^(1/3)*(7*a^2*b^2*x^3 + 7*a^3*b - 4*sqrt(3)*
(a^2*b^2*x^3 + a^3*b))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + sqrt(3)*(
b*x^4 + a*x) - 3*sqrt(-b*x^3 - a)*(108*(1/1944)^(5/6)*(265*a^4*b^4*x^3 - 14
48*a^5*b^3 - sqrt(3)*(153*a^4*b^4*x^3 - 836*a^5*b^3))*(-(1351*sqrt(3) + 234
0)/(a^5*b^4))^(5/6) - sqrt(1/6)*(41*sqrt(3)*a^3*b^2*x - 71*a^3*b^2*x)*sqrt(
```

```
-(1351*sqrt(3) + 2340)/(a^5*b^4)) + (1/1944)^(1/6)*(5*sqrt(3)*a*b*x^2 - 9*a
*b*x^2)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))*sqrt((b^4*x^12 + 100*a*b
^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^
4*b^6*x^10 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqr
t(3)*(223*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*
x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^
11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*
b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(
3) + 2340)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^
3*b*x^3 - 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^
8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a
^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2
*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6
*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*
a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^
4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x
^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/
(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x
^3 + 64*a^4)))/(b*x^4 + a*x)) - 1/24*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)
/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 + 240*a^2*b^2*x^6 + 832*a^3
*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^10 + 12492*a^5*b^5*x^7 - 1
0512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223*a^4*b^6*x^10 + 1803*a^5*
b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b
^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3
*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4
*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) - 32*s
qrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 - 8*a^4) + 2*sqrt(-b*x^3 -
 a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4
*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(
1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6)*(123*a^3*b^5*x^9 - 5112
*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sqrt(3)*(71*a^3*b^5*x^9 - 2
952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))*sqrt(-(1351*sqrt(3) + 23
40)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 + 12*a^2*b^3*x^7 - 72*a^3*b
^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a^2*b^3*x^7 + 8*a^3*b^2*x^
4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*
a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a^4)) + 1/24*(1/1944)^(1/6
)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log((b^4*x^12 + 100*a*b^3*x^9 +
240*a^2*b^2*x^6 + 832*a^3*b*x^3 + 448*a^4 - 6*(1/9)^(2/3)*(1545*a^4*b^6*x^1
0 + 12492*a^5*b^5*x^7 - 10512*a^6*b^4*x^4 + 2112*a^7*b^3*x - 4*sqrt(3)*(223
*a^4*b^6*x^10 + 1803*a^5*b^5*x^7 - 1518*a^6*b^4*x^4 + 304*a^7*b^3*x))*(-(13
51*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(26*a^2*b^5*x^11 - 498*
a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqrt(3)*(5*a^2*b^5*x^11
- 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(-(1351*sqrt(3) + 2340
)/(a^5*b^4))^(1/3) - 32*sqrt(3)*(a*b^3*x^9 - 6*a^2*b^2*x^6 - 15*a^3*b*x^3 -
 8*a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a^5*b^6*x^8 - 2896*
a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8 - 1672*a^6*b^5*x^
5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(5/6) + 2*sqrt(1/6
)*(123*a^3*b^5*x^9 - 5112*a^4*b^4*x^6 + 3960*a^5*b^3*x^3 - 768*a^6*b^2 - sq
rt(3)*(71*a^3*b^5*x^9 - 2952*a^4*b^4*x^6 + 2280*a^5*b^3*x^3 - 448*a^6*b^2))
*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) - 3*(1/1944)^(1/6)*(5*a*b^4*x^10 +
12*a^2*b^3*x^7 - 72*a^3*b^2*x^4 - 160*a^4*b*x - 3*sqrt(3)*(a*b^4*x^10 + 4*a
^2*b^3*x^7 + 8*a^3*b^2*x^4 + 32*a^4*b*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4)
)^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3 + 64*a
^4)) + 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*log(-(b
^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 + 6*(1/9)
^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1440*a^7
*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4*x^4 -
104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(1/3)*(2
6*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2 - 3*sqr
```

```
t(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*x^2))*(
-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*a^2*b^2
*x^6 + 3*a^3*b*x^3 + a^4) + 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*(3691*a
^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*b^6*x^8
 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^
(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b^3*x^3
- 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5*b^3*x^
3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944)^(1/6)
*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*sqrt(3)
*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2340)/(a
^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^3*b*x^3
 + 64*a^4)) - 1/12*(1/1944)^(1/6)*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/6)*
log(-(b^4*x^12 - 68*a*b^3*x^9 + 168*a^2*b^2*x^6 + 544*a^3*b*x^3 + 64*a^4 +
6*(1/9)^(2/3)*(2799*a^4*b^6*x^10 - 11556*a^5*b^5*x^7 + 7776*a^6*b^4*x^4 - 1
440*a^7*b^3*x - 8*sqrt(3)*(202*a^4*b^6*x^10 - 834*a^5*b^5*x^7 + 561*a^6*b^4
*x^4 - 104*a^7*b^3*x))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(2/3) - 6*(1/9)^(
1/3)*(26*a^2*b^5*x^11 - 498*a^3*b^4*x^8 + 384*a^4*b^3*x^5 - 64*a^5*b^2*x^2
- 3*sqrt(3)*(5*a^2*b^5*x^11 - 96*a^3*b^4*x^8 + 72*a^4*b^3*x^5 - 16*a^5*b^2*
x^2))*(-(1351*sqrt(3) + 2340)/(a^5*b^4))^(1/3) + 64*sqrt(3)*(a*b^3*x^9 + 3*
a^2*b^2*x^6 + 3*a^3*b*x^3 + a^4) - 2*sqrt(-b*x^3 - a)*(1944*(1/1944)^(5/6)*
(3691*a^5*b^6*x^8 - 2896*a^6*b^5*x^5 + 568*a^7*b^4*x^2 - sqrt(3)*(2131*a^5*
b^6*x^8 - 1672*a^6*b^5*x^5 + 328*a^7*b^4*x^2))*(-(1351*sqrt(3) + 2340)/(a^5
*b^4))^(5/6) - 4*sqrt(1/6)*(168*a^3*b^5*x^9 - 1845*a^4*b^4*x^6 + 1368*a^5*b
^3*x^3 - 264*a^6*b^2 - sqrt(3)*(97*a^3*b^5*x^9 - 1065*a^4*b^4*x^6 + 792*a^5
*b^3*x^3 - 152*a^6*b^2))*sqrt(-(1351*sqrt(3) + 2340)/(a^5*b^4)) + 3*(1/1944
)^(1/6)*(5*a*b^4*x^10 - 216*a^2*b^3*x^7 + 120*a^3*b^2*x^4 - 64*a^4*b*x - 3*
sqrt(3)*(a*b^4*x^10 - 40*a^2*b^3*x^7 + 40*a^3*b^2*x^4))*(-(1351*sqrt(3) + 2
340)/(a^5*b^4))^(1/6)))/(b^4*x^12 + 40*a*b^3*x^9 + 384*a^2*b^2*x^6 - 320*a^
3*b*x^3 + 64*a^4))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:index.cc index_m operator + Error: Bad Argument Value
```

maple [C] time = 0.39, size = 541, normalized size = 1.68

$$i\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}-i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)b}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\;\sqrt{\frac{\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)b}{-3\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\;\sqrt{-\frac{i\left(2x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)b}{2\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\;\left(4\sqrt{3}\ \mathrm{RootOf}\left(\_Z^{3}b\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x)
```

```
[Out] 1/27*I/b^3/a*2^(1/2)*sum(1/_alpha*(-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3
)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b
)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b
^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(-b*x^3-a)^(
```

```
1/2)*(4*3^(1/2)*_alpha^2*b^2+6*_alpha^2*b^2+3*I*(-a*b^2)^(1/3)*3^(1/2)*_alp
ha*b-2*3^(1/2)*(-a*b^2)^(1/3)*_alpha*b+6*I*(-a*b^2)^(1/3)*_alpha*b-3*(-a*b^
2)^(1/3)*_alpha*b-3*I*(-a*b^2)^(2/3)*3^(1/2)-2*3^(1/2)*(-a*b^2)^(2/3)-6*I*(
-a*b^2)^(2/3)-3*(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/6
*(2*I*3^(1/2)*(-a*b^2)^(1/3)*_alpha^2*b+4*I*(-a*b^2)^(1/3)*_alpha^2*b+I*3^(
1/2)*a*b-2*3^(1/2)*a*b+2*I*a*b-3*a*b-I*3^(1/2)*(-a*b^2)^(2/3)*_alpha-2*3^(1
/2)*(-a*b^2)^(2/3)*_alpha-2*I*(-a*b^2)^(2/3)*_alpha-3*(-a*b^2)^(2/3)*_alpha
)/a/b,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b-6*3^(1/2)*a+10*a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 - 2\,a\left(3\,\sqrt{3}\,-5\right)\right)\sqrt{-bx^3 - a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+2*a*(5-3*3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(x/((b*x^3 - 2*a*(3*sqrt(3) - 5))*sqrt(-b*x^3 - a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{-b\,x^3-a}\,\left(b\,x^3-2\,a\,\left(3\,\sqrt{3}\,-5\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))),x)
```

```
[Out] int(x/((- a - b*x^3)^(1/2)*(b*x^3 - 2*a*(3*3^(1/2) - 5))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-a - bx^3}\,\left(-6\sqrt{3}\,a + 10a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+2*a*(5-3*3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-a - b*x**3)*(-6*sqrt(3)*a + 10*a + b*x**3)), x)
```

3.358 $\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=125

$$-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

```
[Out] -2/9*(a*d+b*c)*(d*x^3+c)^(3/2)/b^2/d^2+2/15*(d*x^3+c)^(5/2)/b/d^2-2/3*a^2*a
rctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+2
/3*a^2*(d*x^3+c)^(1/2)/b^3
```

Rubi [A] time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2\left(c+dx^3\right)^{3/2}(ad+bc)}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*a^2*Sqrt[c + d*x^3])/(3*b^3) - (2*(b*c + a*d)*(c + d*x^3)^(3/2))/(9*b^2*
d^2) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt{c+dx}}{b^2d}+\frac{a^2\sqrt{c+dx}}{b^2(a+bx)}+\frac{(c+dx)^{3/2}}{bd}\right)dx, x, x^3\right)\\
&= -\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{a^2\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b^2}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+d}}\right.}{3b^3}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}+\frac{\left(2a^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3b^3d}\\
&= \frac{2a^2\sqrt{c+dx^3}}{3b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{3/2}}{9b^2d^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd^2}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 121, normalized size = 0.97

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(c+dx^3\right)+b^2\left(-2c^2+cdx^3+3d^2x^6\right)\right)}{45b^3d^2}-\frac{2a^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 - 5*a*b*d*(c + d*x^3) + b^2*(-2*c^2 + c*d*x^
3 + 3*d^2*x^6)))/(45*b^3*d^2) - (2*a^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

fricas [A] time = 0.51, size = 280, normalized size = 2.24

$$\left[\frac{15\,a^2d^2\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(3\,b^2d^2x^6-2\,b^2c^2-5\,abcd+15\,a^2d^2+\left(b^2cd-5\,abd^2\right)x^3\right)\sqrt{}}{45\,b^3d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/45*(15*a^2*d^2*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(3*b^2*d^2*x^6 - 2*b^2*c^
2 - 5*a*b*c*d + 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^3*d^2), -2/45*(15*a^2*d^2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*s
qrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^2*d^2*x^6 - 2*b^2*c^2 - 5*a*b*c*d +
 15*a^2*d^2 + (b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^3*d^2)]
```

giac [A] time = 0.17, size = 139, normalized size = 1.11

$$\frac{2\left(a^2bc - a^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^3} + \frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^8 - 5\left(dx^3+c\right)^{\frac{3}{2}}b^4cd^8 - 5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^9 + 15\sqrt{dx^3+\dots}\right.}{45\,b^5d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b*c - a^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(
-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^8 - 5*(d*x^3 + c)^(3
/2)*b^4*c*d^8 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^9 + 15*sqrt(d*x^3 + c)*a^2*b^2*
d^10)/(b^5*d^10)
```

maple [C] time = 0.42, size = 514, normalized size = 4.11

$$\frac{2\sqrt{d\,x^3+c}}{3b} + \frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\dots\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3
+c)^(1/2)*c^2/d^2)-2/9*a/d*(d*x^3+c)^(3/2))+a^2/b^2*(2/3*(d*x^3+c)^(1/2)/b+
1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_a
lpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1
/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1
/2*b/d*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_
alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.17, size = 176, normalized size = 1.41

$$\frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b^3}+\frac{2\,{\left(d\,x^3+c\right)}^{5/2}}{15\,b\,d^2}-\frac{2\,a\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b^2\,d}-\frac{2\,c\,{\left(d\,x^3+c\right)}^{3/2}}{9\,b\,d^2}+\frac{a^2\,\ln\left(\frac{a^2\,d^2\,1\mathrm{i}+b^2\,c^2\,2\mathrm{i}-2\,\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}-a\,b\,d^2\,x}{2\,b\,x^3+2\,a}\right.}{3\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*a^2*(c + d*x^3)^(1/2))/(3*b^3) + (2*(c + d*x^3)^(5/2))/(15*b*d^2) - (2*a
*(c + d*x^3)^(3/2))/(9*b^2*d) - (2*c*(c + d*x^3)^(3/2))/(9*b*d^2) + (a^2*lo
g((a^2*d^2*1i + b^2*c^2*2i - 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)
- a*b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c
)^(1/2)*1i)/(3*b^(7/2))
```

sympy [A] time = 30.27, size = 128, normalized size = 1.02

$$\frac{2\left(\frac{a^2d^3\sqrt{c+dx^3}}{3b^3}-\frac{a^2d^3(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{5}{2}}}{15b}+\frac{\left(c+dx^3\right)^{\frac{3}{2}}\left(-ad^2-bcd\right)}{9b^2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(a**2*d**3*sqrt(c + d*x**3)/(3*b**3) - a**2*d**3*(a*d - b*c)*atan(sqrt(c
+ d*x**3)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + d*(c + d*x**3
)**(5/2)/(15*b) + (c + d*x**3)**(3/2)*(-a*d**2 - b*c*d)/(9*b**2))/d**3
```

3.359 $\int \frac{x^5 \sqrt{c+dx^3}}{a+bx^3} \, dx$

Optimal. Leaf size=93

$$\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d+2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-2/3*a*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\sqrt{c+dx^3}}{3b^2}+\frac{2a\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (-2*a*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b*d) + (2*a*Sqrt[
b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{3b}$$

$$= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{(a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} - \frac{(2a(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3b^2d}$$

$$= -\frac{2a\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9bd} + \frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Mathematica [A] time = 0.09, size = 88, normalized size = 0.95

$$\frac{2a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}} + \frac{2\sqrt{c+dx^3}\left(b\left(c+dx^3\right)-3ad\right)}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-3*a*d + b*(c + d*x^3)))/(9*b^2*d) + (2*a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

fricas [A] time = 0.48, size = 195, normalized size = 2.10

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(bdx^3+bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2d}, \frac{2\left(3\,ad\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\ldots}{bc\ldots}\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/9*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
 c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(b*d*x^3 + b*c - 3*a*d)*sqrt(d*
x^3 + c))/(b^2*d), 2/9*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*
b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^3 + b*c - 3*a*d)*sqrt(d*x^3 +
c))/(b^2*d)]
```

giac [A] time = 0.16, size = 96, normalized size = 1.03

$$-\frac{2\left(abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2d^2 - 3\,\sqrt{dx^3+c}\,abd^3\right)}{9\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^3 + c)*a*
b*d^3)/(b^3*d^3)
```

maple [C] time = 0.24, size = 458, normalized size = 4.92

$$-\frac{\left(\frac{2\sqrt{d x^3+c}}{3b}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2 \cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d-a/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.06, size = 136, normalized size = 1.46

$$\frac{2\left(d x^3+c\right)^{3/2}}{9 b d}-\frac{2 a \sqrt{d x^3+c}}{3 b^2}+\frac{a \ln\left(\frac{a^2 d^2 1\mathrm{i}+b^2 c^2 2\mathrm{i}+2\sqrt{b}\sqrt{d x^3+c}\left(a d-b c\right)^{3/2}-a b d^2 x^3 1\mathrm{i}+b^2 c d x^3 1\mathrm{i}-a b c d 3\mathrm{i}}{2 b x^3+2 a}\right)\sqrt{a d-b c}\,1\mathrm{i} \cdots}{3 b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(3/2))/(9*b*d) - (2*a*(c + d*x^3)^(1/2))/(3*b^2) + (a*log((a
^2*d^2*1i + b^2*c^2*2i + 2*b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2) - a*
b*d^2*x^3*1i + b^2*c*d*x^3*1i - a*b*c*d*3i)/(2*a + 2*b*x^3))*(a*d - b*c)^(1
/2)*1i)/(3*b^(5/2))
```

sympy [A] time = 14.67, size = 95, normalized size = 1.02

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^3}}{3b^2}+\frac{ad^2(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^3\right)^{\frac{3}{2}}}{9b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**3)/(3*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**3)**(3
/2)/(9*b))/d**2
```

3.360 $\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=70

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+2/3*(d*x^3+c)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}}{3b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b) - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{a+bx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2\sqrt{c+dx^3}}{3b} + \frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}$$
$$= \frac{2\sqrt{c+dx^3}}{3b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.00

$$\frac{1}{3}\left(\frac{2\sqrt{c+dx^3}}{b} - \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] ((2*Sqrt[c + d*x^3])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3
])/Sqrt[b*c - a*d]])/b^(3/2))/3
```

fricas [A] time = 0.47, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}}{3\,b}, -\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^3+c}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c))/b, -2/3*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^3 + c))/b]
```

giac [A] time = 0.16, size = 66, normalized size = 0.94

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b} + \frac{2\,\sqrt{dx^3+c}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 2/3*sqrt(d*x^3 + c)/b
```

maple [C] time = 0.23, size = 434, normalized size = 6.20

$$\frac{2\sqrt{d x^3+c}}{3b}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{Root}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.16, size = 82, normalized size = 1.17

$$\frac{2\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1\mathrm{i}}{3\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)
*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*b^(3
/2))
```

sympy [A] time = 6.35, size = 68, normalized size = 0.97

$$\frac{2\left(\frac{d\sqrt{c+dx^3}}{3b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**3)/(3*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**3)/sqrt((a*d
 - b*c)/b))/(3*b**2*sqrt((a*d - b*c)/b)))/d
```

3.361 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}-\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (-2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a) + (2*Sqrt[b*c - a*d]*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^3\right)$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$
$$= \frac{(2c)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad} - \frac{(2(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3ad}$$
$$= -\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a} + \frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{b}}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)),x]
```

```
[Out] (2*(-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*a)
```

fricas [A] time = 0.48, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\sqrt{c}\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,a}, \frac{2\,\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^3+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)+}{3\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*
sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
c)*sqrt(c) + 2*c)/x^3))/a, 1/3*(2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^3 + a)))/a, 2/3*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
3 + c)*sqrt(-c)/c))/a]
```

giac [A] time = 0.17, size = 79, normalized size = 0.93

$$-\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a} + \frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(b*c - a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [C] time = 0.26, size = 476, normalized size = 5.60

$$-\frac{\frac{2\sqrt{d x^3+c}}{3b}+\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3 b+a\right)^2 d^2-\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(
a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(2/3*(d*x^3+c)^(
1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x), x)
```

mupad [B] time = 7.94, size = 114, normalized size = 1.34

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\sqrt{a\,d-b\,c}\,1{}\mathrm{i}}{3\,a\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(
1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(3*a*b^(1/2))
```

sympy [**A**] time = 12.25, size = 85, normalized size = 1.00

$$\frac{2\left(\frac{cd\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}+\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*b*sqrt((a*d - b*c)/b)))/d
```

3.362 $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

```
[Out] 1/3*(-a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-2/3*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/3*(
d*x^3+c)^(1/2)/a/x^3
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}-\frac{\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]
])/(3*a^2*Sqrt[c]) - (2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2}-\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{6a^2}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3a^2d}-\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{3a^2d}$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}-\frac{2\sqrt{b}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}-2\sqrt{b}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)-\frac{a\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^3])/x^3) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*a^2)
```

fricas [A] time = 0.50, size = 513, normalized size = 4.46

$$\left[\frac{2\sqrt{b^2c-abd}\,cx^3\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-(2\,bc-ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-2\,\sqrt{dx^3+c}\,ac}{6\,a^2cx^3},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(b^2*c - a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3
 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c
*x^3), 1/6*(4*sqrt(-b^2*c + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^3 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^3*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), -1/3*(
```

```
(2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) + sqrt(d*x^3 + c)*a*c)/(a^2*c*x^3), 1/3*(2*sqrt(-b^2*c
 + a*b*d)*c*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - sqrt(d*
x^3 + c)*a*c)/(a^2*c*x^3)]
```

giac [A] time = 0.19, size = 107, normalized size = 0.93

$$\frac{2\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\sqrt{dx^3+c}}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c - a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/3*(2*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)/(a*x^3)
```

maple [C] time = 0.25, size = 518, normalized size = 4.50

$$\begin{cases}\frac{2\sqrt{d\,x^3+c}}{3b}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^2d^2+i\left(\cdots\right.\right.}{\cdots}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2
))+1/a^2*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a^2*b*(2/3*(
d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^4), x)
```

mupad [B] time = 5.13, size = 137, normalized size = 1.19

$$\frac{\ln\left(\frac{a\,d-2\,b\,c+2\,\sqrt{d\,x^3+c}\,\sqrt{b^2\,c-a\,b\,d}-b\,d\,x^3}{b\,x^3+a}\right)\sqrt{b^2\,c-a\,b\,d}}{3\,a^2}-\frac{\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(a\,d-2\,b\,c)}{6\,a^2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (log((a*d - 2*b*c + 2*(c + d*x^3)^(1/2)*(b^2*c - a*b*d)^(1/2) - b*d*x^3)/(a
 + b*x^3))*(b^2*c - a*b*d)^(1/2))/(3*a^2) - (c + d*x^3)^(1/2)/(3*a*x^3) + (
log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a
*d - 2*b*c))/(6*a^2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^4 \left(a + bx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**4*(a + b*x**3)), x)
```

3.363 $\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.62, size = 241, normalized size = 3.77

$$\frac{x\left(8\left(\frac{8a^2c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+c+dx^3\right)+\frac{x^3\sqrt{\frac{dx^3}{c}+1}\,(3bc-5ad)F\cdots}{\cdots}\right)}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x*(((3*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)])/a + 8*(c + d*x^3 + (8*a^2*c^2*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))))/(20*b*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

maple [C] time = 0.35, size = 1012, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] 1/b*(2/5*(d*x^3+c)^(1/2)*x-2/5*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))-a/b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/
```

```
2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(
1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/
3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+
I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha
)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\, x^3}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\, \sqrt{d\, x^3 + c}}{b\, x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3),x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt{c + dx^3}}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a),x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.364 $\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x
^3/c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};-\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{\frac{c+dx^3}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3),x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, -1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)
])/(2*a*Sqrt[(c + d*x^3)/c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\,x}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

maple [C] time = 0.33, size = 857, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b/d^2*2^(1/2)*sum(1/_a
lpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-
3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b
+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\,x}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3),x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^3}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.365 $\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] x*AppellF1(1/3,1,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c)^
(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.19, size = 161, normalized size = 2.73

$$\frac{8acx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-2bcF_1\left(\frac{4}{3};-\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+8acF_1\left(\frac{1}{3};-\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3),x]
```

```
[Out] (8*a*c*x*Sqrt[c + d*x^3]*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3
)/a)])/((a + b*x^3)*(8*a*c*AppellF1[1/3, -1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(-2*b*c*AppellF1[4/3, -1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

maple [C] time = 0.24, size = 705, normalized size = 11.95

$$-\frac{2i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}}}\,\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}}{\cdots}\right.}{3\sqrt{d\,x^{3}+c}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a),x)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/
b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I
*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(
```

```
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3),x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3), x)
```

3.366 $\int \frac{\sqrt{c+dx^3}}{x^2(a+bx^3)}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -AppellF1(-1/3,1,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3/c
)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2\left(a+bx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.12, size = 139, normalized size = 2.24

$$\frac{5x^3\sqrt{\frac{dx^3}{c}+1}\,(3ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20a\left(c+dx^3\right)}{20a^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + 3*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*x*Sqrt[c +
 d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

maple [C] time = 0.27, size = 1314, normalized size = 21.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x)
```

```
[Out] -1/a*b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*
EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b/d^2*2^(1/2)*
sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootO
f(_Z^3*b+a))+1/a*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
```

```
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{(bx^3 + a)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3 + c}}{x^2\,\left(b\,x^3 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^2\left(a + bx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a),x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)), x)
```

3.367 $\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -1/2*AppellF1(-2/3,1,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(a+bx^3\right)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.25, size = 335, normalized size = 5.23

$$\frac{\frac{a\left(32ac\left(2ac-adx^3+6bcx^3+2bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-bdx^6\sqrt{}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (a*(32*a*c*(2*a*c + 6*b*c*x^3 - a*d*x^3 + 2*b*d*x^6)*AppellF1
[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(a + b*x^3)*(c + d*
x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*
c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*App
ellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2
, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

maple [C] time = 0.33, size = 1010, normalized size = 15.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x)
```

```
[Out] -1/a*b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
```

```
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/2*(d*x^3+c)^(1/2)/x^2-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3 + c}}{x^3\,\left(b\,x^3 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^3\left(a + bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a),x)
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)), x)
```

3.368 $\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=154

$$-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}+\frac{2a^2\sqrt{c+dx^3}\,(bc-ad)}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+d\right.}{21b}$$

```
[Out] 2/9*a^2*(d*x^3+c)^(3/2)/b^3-2/15*(a*d+b*c)*(d*x^3+c)^(5/2)/b^2/d^2+2/21*(d*
x^3+c)^(7/2)/b/d^2-2/3*a^2*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)
/(-a*d+b*c)^(1/2))/b^(9/2)+2/3*a^2*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A] time = 0.15, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 88, 50, 63, 208}

$$\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}+\frac{2a^2\sqrt{c+dx^3}\,(bc-ad)}{3b^4}-\frac{2a^2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{2\left(c+dx^3\right)^{5/2}(ad+bc)}{15b^2d^2}+\frac{2\left(c+dx\right.}{21bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*a^2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^4) + (2*a^2*(c + d*x^3)^(3/2))/(9*
b^3) - (2*(b*c + a*d)*(c + d*x^3)^(5/2))/(15*b^2*d^2) + (2*(c + d*x^3)^(7/2
))/(21*b*d^2) - (2*a^2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{a+bx}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)(c+dx)^{3/2}}{b^2d}+\frac{a^2(c+dx)^{3/2}}{b^2(a+bx)}+\frac{(c+dx)^{5/2}}{bd}\right)dx,x,x^3\right)$$
$$= -\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^3\right)}{3b^2}$$
$$= \frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+\frac{\left(a^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{\sqrt{}}{}\right.}{3b^3}$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}+$$
$$= \frac{2a^2(bc-ad)\sqrt{c+dx^3}}{3b^4}+\frac{2a^2\left(c+dx^3\right)^{3/2}}{9b^3}-\frac{2(bc+ad)\left(c+dx^3\right)^{5/2}}{15b^2d^2}+\frac{2\left(c+dx^3\right)^{7/2}}{21bd^2}-$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.94

$$\frac{2\left(105a^2(bc-ad)\left(\frac{\sqrt{c+dx^3}}{b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)+35a^2\left(c+dx^3\right)^{3/2}-\frac{21b\left(c+dx^3\right)^{5/2}(ad+bc)}{d^2}+\frac{15b^2\left(c+dx^3\right)^{7/2}}{d^2}\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(35*a^2*(c + d*x^3)^(3/2) - (21*b*(b*c + a*d)*(c + d*x^3)^(5/2))/d^2 + (
15*b^2*(c + d*x^3)^(7/2))/d^2 + 105*a^2*(b*c - a*d)*(Sqrt[c + d*x^3]/b - (S
qrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))
))/(315*b^3)
```

fricas [A] time = 0.90, size = 410, normalized size = 2.66

$$\left[-\frac{105\left(a^2bcd^2-a^3d^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(15\,b^3d^3x^9+3\left(8\,b^3cd^2-7\,ab^2d^3\right)x^6-6\,b^3\right.}{315\,b^4d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/315*(105*(a^2*b*c*d^2 - a^3*d^3)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b
*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(15*b^
3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21*a*b^2*c^2*d
+ 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^2 + 35*a^2*b*
d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2), -2/315*(105*(a^2*b*c*d^2 - a^3*d^3)*s
qrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a
*d)) - (15*b^3*d^3*x^9 + 3*(8*b^3*c*d^2 - 7*a*b^2*d^3)*x^6 - 6*b^3*c^3 - 21
*a*b^2*c^2*d + 140*a^2*b*c*d^2 - 105*a^3*d^3 + (3*b^3*c^2*d - 42*a*b^2*c*d^
2 + 35*a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*d^2)]
```

giac [A] time = 0.17, size = 193, normalized size = 1.25

$$\frac{2\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^4}+\frac{2\left(15\left(dx^3+c\right)^{\frac{7}{2}}b^6d^{12}-21\left(dx^3+c\right)^{\frac{5}{2}}b^6cd^{12}-21\left(dx^3+c\right)^{\frac{5}{2}}ab^5 d \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^
2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) + 2/315*(15*(d*x^3 + c)^(7/2)*b^6*
d^12 - 21*(d*x^3 + c)^(5/2)*b^6*c*d^12 - 21*(d*x^3 + c)^(5/2)*a*b^5*d^13 +
35*(d*x^3 + c)^(3/2)*a^2*b^4*d^14 + 105*sqrt(d*x^3 + c)*a^2*b^4*c*d^14 - 10
5*sqrt(d*x^3 + c)*a^3*b^3*d^15)/(b^7*d^14)
```

maple [C] time = 0.36, size = 605, normalized size = 3.93

$$\frac{\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{9b}+\frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^2}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2 \cdots\right.}{\cdots}}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 1/b^2*(b*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d*
x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)-2/15*a/d*(d*x^3+c)^(5
/2))+a^2/b^2*(2/9*d/b*x^3*(d*x^3+c)^(1/2)+2/3*(-d*(a*d-2*b*c)/b^2-2/3*d/b*c
)/d*(d*x^3+c)^(1/2)+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/
(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
```

```
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.12, size = 330, normalized size = 2.14

$$\frac{2\,d\,x^9\,\sqrt{d\,x^3+c}}{21\,b}-\frac{\left(\frac{2\,a\,\left(\frac{c^2}{b}+\frac{a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{4\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{16\,c\,d}{7\,b}\right)}{5\,d}\right)}{3\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d}+\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (2*d*x^9*(c + d*x^3)^(1/2))/(21*b) - (((2*a*(c^2/b + (a*((a*d^2)/b^2 - (2*c
*d)/b))/b))/b + (2*c*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4*c*
((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(3*d))*(c + d*x^3)^(1/2))/(3*d) +
 (x^3*(c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (4
*c*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(5*d)))/(9*d) - (x^6*(c + d*x^3)^(1/2)
*((2*a*d^2)/b^2 - (16*c*d)/(7*b)))/(15*d) + (a^2*log((a^2*d^2 + 2*b^2*c^2 -
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*b^(9/2))
```

sympy [A] time = 129.92, size = 153, normalized size = 0.99

$$\frac{2a^2\left(c+dx^3\right)^{\frac{3}{2}}}{9b^3}+\frac{2a^2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^5\sqrt{\frac{ad-bc}{b}}}+\frac{2\left(c+dx^3\right)^{\frac{7}{2}}}{21bd^2}+\frac{\left(c+dx^3\right)^{\frac{5}{2}}\left(-2ad-2bc\right)}{15b^2d^2}+\frac{\sqrt{c+dx^3}\left(-2a^3d+2a\cdots\right)}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*a**2*(c + d*x**3)**(3/2)/(9*b**3) + 2*a**2*(a*d - b*c)**2*atan(sqrt(c + d
*x**3)/sqrt((a*d - b*c)/b))/(3*b**5*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**
(7/2)/(21*b*d**2) + (c + d*x**3)**(5/2)*(-2*a*d - 2*b*c)/(15*b**2*d**2) + s
qrt(c + d*x**3)*(-2*a**3*d + 2*a**2*b*c)/(3*b**4)
```

3.369 $\int \frac{x^5(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=120

$$\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

```
[Out] -2/9*a*(d*x^3+c)^(3/2)/b^2+2/15*(d*x^3+c)^(5/2)/b/d+2/3*a*(-a*d+b*c)^(3/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7/2)-2/3*a*(-a*d+b*c)*
(d*x^3+c)^(1/2)/b^3
```

Rubi [A] time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2}-\frac{2a\sqrt{c+dx^3}\,(bc-ad)}{3b^3}+\frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{2\left(c+dx^3\right)^{5/2}}{15bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (-2*a*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^3) - (2*a*(c + d*x^3)^(3/2))/(9*b^2
) + (2*(c + d*x^3)^(5/2))/(15*b*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5 \left(c+dx^3\right)^{3/2}}{a+bx^3} dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{a+bx} dx, x, x^3\right)$$
$$= \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{a\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx} dx, x, x^3\right)}{3b}$$
$$= -\frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx} dx, x, x^3\right)}{3b^2}$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(a(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{(a+}\right.}{3b^3}$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} - \frac{\left(2a(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{a}\right.}{3b^3}$$
$$= -\frac{2a(bc-ad)\sqrt{c+dx^3}}{3b^3} - \frac{2a\left(c+dx^3\right)^{3/2}}{9b^2} + \frac{2\left(c+dx^3\right)^{5/2}}{15bd} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.}{3b^{7/2}}$$

Mathematica [A] time = 0.12, size = 111, normalized size = 0.92

$$\frac{2\sqrt{c+dx^3}\left(15a^2d^2-5abd\left(4c+dx^3\right)+3b^2\left(c+dx^3\right)^2\right)}{45b^3d} + \frac{2a(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(15*a^2*d^2 + 3*b^2*(c + d*x^3)^2 - 5*a*b*d*(4*c + d*x^3
)))/(45*b^3*d) + (2*a*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(7/2))
```

fricas [A] time = 1.76, size = 297, normalized size = 2.48

$$\left[-\frac{15\left(abcd-a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(3\,b^2d^2x^6+3\,b^2c^2-20\,abcd+15\,a^2d^2+\left(6\,b^2\right.\right.}{45\,b^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/45*(15*(a*b*c*d - a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a
*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(3*b^2*d^2*x
^6 + 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqr
t(d*x^3 + c))/(b^3*d), 2/45*(15*(a*b*c*d - a^2*d^2)*sqrt(-(b*c - a*d)/b)*ar
ctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (3*b^2*d^2*x^6
```

```
+ 3*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 + (6*b^2*c*d - 5*a*b*d^2)*x^3)*sqrt(d
*x^3 + c))/(b^3*d)]
```

giac [A] time = 0.19, size = 151, normalized size = 1.26

$$-\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^4d^4-5\left(dx^3+c\right)^{\frac{3}{2}}ab^3d^5-15\sqrt{dx^3+c}\,ab^3cd^5+\cdots\right.}{45\,b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] -2/3*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2
*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/45*(3*(d*x^3 + c)^(5/2)*b^4*d^4
 - 5*(d*x^3 + c)^(3/2)*a*b^3*d^5 - 15*sqrt(d*x^3 + c)*a*b^3*c*d^5 + 15*sqrt
(d*x^3 + c)*a^2*b^2*d^6)/(b^5*d^5)
```

maple [C] time = 0.32, size = 531, normalized size = 4.42

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}\;d\,x^3}{9b}+\frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^2}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\;\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\;\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b/d-a/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(
a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a
*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.13, size = 215, normalized size = 1.79

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,c^2}{b}+\frac{2\,a\,\left(\frac{a\,d^2}{b^2}-\frac{2\,c\,d}{b}\right)}{b}+\frac{2\,c\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{3\,d}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{12\,c\,d}{5\,b}\right)}{9\,d}+\frac{a\,\ln\left(\frac{a^2\,d^2+2\,b}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*c^2)/b + (2*a*((a*d^2)/b^2 - (2*c*d)/b))/b + (2*c*((
2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(3*d)))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))
/(15*b) - (x^3*(c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (12*c*d)/(5*b)))/(9*d) +
(a*log((a^2*d^2 + 2*b^2*c^2 + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2
i - a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1
i)/(3*b^(7/2))
```

sympy [A] time = 70.60, size = 116, normalized size = 0.97

$$-\frac{2a\left(c+dx^3\right)^{\frac{3}{2}}}{9b^2}-\frac{2a\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^4\sqrt{\frac{ad-bc}{b}}}+\frac{2\left(c+dx^3\right)^{\frac{5}{2}}}{15bd}+\frac{\sqrt{c+dx^3}\left(2a^2d-2abc\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] -2*a*(c + d*x**3)**(3/2)/(9*b**2) - 2*a*(a*d - b*c)**2*atan(sqrt(c + d*x**3
)/sqrt((a*d - b*c)/b))/(3*b**4*sqrt((a*d - b*c)/b)) + 2*(c + d*x**3)**(5/2)
/(15*b*d) + sqrt(c + d*x**3)*(2*a**2*d - 2*a*b*c)/(3*b**3)
```

3.370 $\int \frac{x^2(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=96

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\sqrt{c+dx^3}\,(bc-ad)}{3b^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b-2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/
(-a*d+b*c)^(1/2))/b^(5/2)+2/3*(-a*d+b*c)*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{2\sqrt{c+dx^3}\,(bc-ad)}{3b^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x^3])/(3*b^2) + (2*(c + d*x^3)^(3/2))/(9*b) - (2*
(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^
(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2 \left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx,x,x^3\right)$$
$$= \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx,x,x^3\right)}{3b}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} + \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}$$
$$= \frac{2(bc-ad)\sqrt{c+dx^3}}{3b^2} + \frac{2\left(c+dx^3\right)^{3/2}}{9b} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 0.89

$$\frac{2\sqrt{c+dx^3}\left(-3ad+4bc+bdx^3\right)}{9b^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(4*b*c - 3*a*d + b*d*x^3))/(9*b^2) - (2*(b*c - a*d)^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2))
```

fricas [A] time = 1.34, size = 204, normalized size = 2.12

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(bdx^3+4\,bc-3\,ad\right)\sqrt{dx^3+c}}{9\,b^2}, -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc}{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqr
t(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) - 2*(b*d*x^3 + 4*b*c - 3*a
*d)*sqrt(d*x^3 + c))/b^2, -2/9*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-
sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x^3 + 4*b*c - 3*
a*d)*sqrt(d*x^3 + c))/b^2]
```

giac [A] time = 0.17, size = 113, normalized size = 1.18

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2+3\,\sqrt{dx^3+c}\,b^2c-3\,\sqrt{dx^3+c}\,abd\right)}{9\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2 + 3*sqrt(d*
x^3 + c)*b^2*c - 3*sqrt(d*x^3 + c)*a*b*d)/b^3
```

maple [C] time = 0.25, size = 507, normalized size = 5.28

$$\frac{2\sqrt{d\,x^3+c}\;d\,x^3}{9b}+\frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^2}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1
/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*
d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2
)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.91, size = 143, normalized size = 1.49

$$\frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,a\,d^2}{b^2}-\frac{8\,c\,d}{3\,b}\right)}{3\,d}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,{\left(a\,d-b\,c\right)}^{3\cdots}}{3\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] (log((a^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i
- a*b*d^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)
/(3*b^(5/2)) - ((c + d*x^3)^(1/2)*((2*a*d^2)/b^2 - (8*c*d)/(3*b)))/(3*d) +
(2*d*x^3*(c + d*x^3)^(1/2))/(9*b)
```

sympy [A] time = 35.29, size = 90, normalized size = 0.94

$$\frac{2\left(c+dx^3\right)^{\frac{3}{2}}}{9b}+\frac{\sqrt{c+dx^3}\,\left(-2ad+2bc\right)}{3b^2}+\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] 2*(c + d*x**3)**(3/2)/(9*b) + sqrt(c + d*x**3)*(-2*a*d + 2*b*c)/(3*b**2) +
2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b**3*sqrt((a
*d - b*c)/b))
```

3.371 $\int \frac{(c+dx^3)^{3/2}}{x(a+bx^3)}\,dx$

Optimal. Leaf size=104

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a+2/3*(-a*d+b*c)^(3/2)*arctan
h(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/b^(3/2)+2/3*d*(d*x^3+c)^(1/2)
/b
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 84, 156, 63, 208}

$$\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2d\sqrt{c+dx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*d*Sqrt[c + d*x^3])/(3*b) - (2*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/
(3*a) + (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a
*d]])/(3*a*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)}\,dx,x,x^3\right)\\ &= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{\operatorname{Subst}\left(\int \frac{bc^2+d(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b}\\ &= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{c^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}-\frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3ab}\\ &= \frac{2d\sqrt{c+dx^3}}{3b}+\frac{\left(2c^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}-\frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3abd}\\ &= \frac{2d\sqrt{c+dx^3}}{3b}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a}+\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3ab^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 1.01

$$\frac{2\left(a\sqrt{b}\,d\sqrt{c+dx^3}+(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)-b^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)\right)}{3ab^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)),x]
```

```
[Out] (2*(a*Sqrt[b]*d*Sqrt[c + d*x^3] - b^(3/2)*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/S
qrt[c]] + (b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*
d]]))/(3*a*b^(3/2))
```

fricas [A] time = 1.42, size = 486, normalized size = 4.67

$$\left[\frac{bc^{\frac{3}{2}}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)+2\sqrt{dx^3+c}\,ad-(bc-ad)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)}{3ab}, \frac{bc^{\frac{3}{2}}\log\left(\frac{dx}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/3*(b*c^(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)))/(a*b), 1/3*(b*c^
(3/2)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c
)*a*d + 2*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-
(b*c - a*d)/b)/(b*c - a*d)))/(a*b), 1/3*(2*b*sqrt(-c)*c*arctan(sqrt(d*x^3 +
 c)*sqrt(-c)/c) + 2*sqrt(d*x^3 + c)*a*d - (b*c - a*d)*sqrt((b*c - a*d)/b)*l
og((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3
 + a)))/(a*b), 2/3*(b*sqrt(-c)*c*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(
```

```
d*x^3 + c)*a*d + (b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b
*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

giac [A] time = 0.18, size = 112, normalized size = 1.08

$$\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}+\frac{2\sqrt{dx^3+c}\,d}{3\,b}-\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)) + 2/3*sqrt(d*x^3 + c)
*d/b - 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b
^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

maple [C] time = 0.26, size = 565, normalized size = 5.43

$$-\frac{\left(\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{9b}+\frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^2}\right)\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\cdots}\cdots}{\cdots}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x
^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b
*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+
a))+1/a*(2/9*d*x^3*(d*x^3+c)^(1/2)+8/9*c*(d*x^3+c)^(1/2)-2/3*c^(3/2)*arcta
nh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x), x)
```

mupad [B] time = 7.88, size = 155, normalized size = 1.49

$$\frac{c^{3/2}\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a}+\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d+\sqrt{b}\,\sqrt{d\,x^3+c}\,{\left(a\,d-b\,c\right)}^{3/2}\,2{}\mathrm{i}}{b\,x^3+a}\right)}{3\,a\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a) + (2*d*(c + d*x^3)^(1/2))/(3*b) + (log((a^2*d^2 + 2*b^2*c^2 +
 b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d^2*x^3 + b^2*c*d*x^3
 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a*b^(3/2))
```

sympy [A] time = 39.29, size = 102, normalized size = 0.98

$$\frac{2d\sqrt{c+dx^3}}{3b}+\frac{2c^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}-\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3ab^2\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a),x)
```

```
[Out] 2*d*sqrt(c + d*x**3)/(3*b) + 2*c**2*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sq
rt(-c)) - 2*(a*d - b*c)**2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*
b**2*sqrt((a*d - b*c)/b))
```

3.372 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=116

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

```
[Out] -2/3*(-a*d+b*c)^(3/2)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2
/b^(1/2)+1/3*(-3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2-1/
3*c*(d*x^3+c)^(1/2)/a/x^3
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}+\frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}-\frac{c\sqrt{c+dx^3}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] -(c*Sqrt[c + d*x^3])/(3*a*x^3) + (Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c +
d*x^3]/Sqrt[c]])/(3*a^2) - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*a^2*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}c(2bc-3ad)+\frac{1}{2}d(bc-2ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2} + \frac{(bc-ad)^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+\ldots}}\right.}{3a^2}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} - \frac{(c(2bc-3ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} + \frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int\ldots\right.}{3\ldots}$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3} + \frac{\sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} - \frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}}$$

Mathematica [A] time = 0.13, size = 108, normalized size = 0.93

$$\frac{-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} + \sqrt{c}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{ac\sqrt{c+dx^3}}{x^3}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x]
```

```
[Out] (-((a*c*Sqrt[c + d*x^3])/x^3) + Sqrt[c]*(2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*
x^3]/Sqrt[c]] - (2*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b])/(3*a^2)
```

fricas [A] time = 1.21, size = 538, normalized size = 4.64

$$\left[-\frac{2\,(bc-ad)x^3\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right) + (2\,bc-3\,ad)\sqrt{c}\,x^3\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) + 2\,\sqrt{dx\ldots}}{6\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2
*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + (2*b*c - 3*a*d)*sqrt
(c)*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 +
 c)*a*c)/(a^2*x^3), -1/6*(4*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sq
rt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(c)
```

```
*x^3*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*((2*b*c - 3*a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c) + (b*c - a*d)*x^3*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*
d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + sqrt(d*x^3 + c)
*a*c)/(a^2*x^3), -1/3*(2*(b*c - a*d)*x^3*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (2*b*c - 3*a*d)*sqrt(-c)*x
^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 + c)*a*c)/(a^2*x^3)]
```

giac [A] time = 0.17, size = 121, normalized size = 1.04

$$\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2} - \frac{\left(2\,bc^2 - 3\,acd\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}} - \frac{\sqrt{dx^3+c}\,c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 2/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) - 1/3*(2*b*c^2 - 3*a*c*d)*arctan(sqrt(d*
x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*sqrt(d*x^3 + c)*c/(a*x^3)
```

maple [C] time = 0.28, size = 620, normalized size = 5.34

$$\left(\frac{2\sqrt{d\,x^3+c}\,d\,x^3}{9b} + \frac{2\left(-\frac{2cd}{3b}-\frac{(ad-2bc)d}{b^2}\right)\sqrt{d\,x^3+c}}{3d} + \frac{i\left(-a^2d^2+2abcd-b^2c^2\right)\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2\cdots\right.}{\cdots}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x)
```

```
[Out] 1/a*(-1/3*c*(d*x^3+c)^(1/2)/x^3+2/3*d*(d*x^3+c)^(1/2)-c^(1/2)*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2)))+1/a^2*b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c
*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2
+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-
I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)),_alpha=RootOf(_Z^3*b+a)))-1/a^2*b*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^
3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^4), x)
```

mupad [B] time = 9.52, size = 167, normalized size = 1.44

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-2\,b\,c)}{6\,a^2}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a\,x^3}+\frac{\ln\left(\frac{a^2\,d^2+2\,b^2\,c^2-a\,b\,d^2\,x^3+b^2\,c\,d\,x^3-3\,a\,b\,c\,d-\sqrt{b}\,\sqrt{d\,x^3+c}}{b\,x^3+a}\right.}{3\,a^2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 2*b*c))/(6*a^2) - (c*(c + d*x^3)^(1/2))/(3*a*x^3) + (log((a
^2*d^2 + 2*b^2*c^2 - b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(3/2)*2i - a*b*d
^2*x^3 + b^2*c*d*x^3 - 3*a*b*c*d)/(a + b*x^3))*(a*d - b*c)^(3/2)*1i)/(3*a^2
*b^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x^4\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(x**4*(a + b*x**3)), x)
```

3.373 $\int \frac{x^3(c+dx^3)^{3/2}}{a+bx^3}\,dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,1,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 1, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};1,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.57, size = 280, normalized size = 4.31

$$\frac{x\left(\frac{x^3\sqrt{\frac{dx^3}{c}+1}\left(55a^2d^2-88abcd+27b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}-\frac{64a^2c^2(11ad-14bc)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF\right.}\right.}{220b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x*(8*(c + d*x^3)*(14*b*c - 11*a*d + 5*b*d*x^3) + ((27*b^2*c^2 - 88*a*b*c*d
 + 55*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)])/a - (64*a^2*c^2*(-14*b*c + 11*a*d)*AppellF1[1/3, 1/2, 1
, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7
/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/
c), -((b*x^3)/a)])))))/(220*b^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

maple [C] time = 0.38, size = 1101, normalized size = 16.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 1/b*(2/11*d*x^4*(d*x^3+c)^(1/2)+28/55*c*x*(d*x^3+c)^(1/2)-18/55*I*c^2*3^(1/
2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-a/b*(2/5*d/b*x*(d
*x^3+c)^(1/2)-2/3*I*(-(a*d-2*b*c)/b^2*d-2/5/b*c*d)*3^(1/2)*(-c*d^2)^(1/3)/d
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
```

```
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2
*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^3}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(d\,x^3+c\right)}^{3/2}}{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3),x)
```

```
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(c+dx^3\right)^{\frac{3}{2}}}{a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.374 $\int \frac{x(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/2*c*x^2*AppellF1(2/3,1,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 1, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^3\right)^{3/2}}{a+bx^3}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};1,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 149, normalized size = 2.29

$$\frac{x^2\left(2dx^3\sqrt{\frac{dx^3}{c}+1}\,(10bc-7ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5c\sqrt{\frac{dx^3}{c}+1}\,(7bc-4ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2\right.}{70ab\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3),x]
```

```
[Out] (x^2*(20*a*d*(c + d*x^3) + 5*c*(7*b*c - 4*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1
[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(10*b*c - 7*a*d)*x^3*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))
/(70*a*b*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

maple [C] time = 0.37, size = 930, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/7*d/b*x^2*(d*x^3+c)^(1/2)-2/3*I*(-(a*d-2*b*c)/b^2*d-4/7/b*c*d)*3^(1/2)*(-
c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d
-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}} x}{bx^3 + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(d\,x^3 + c\right)^{3/2}}{b\,x^3 + a} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3),x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c + dx^3\right)^{\frac{3}{2}}}{a + bx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a),x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.375 $\int \frac{(c+dx^3)^{3/2}}{a+bx^3} dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] c*x*AppellF1(1/3,1,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/(1+d*x^3/c
)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 1, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a*Sqrt[1 + (d*x^3)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^3)^{3/2}}{a+bx^3}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{a+bx^3}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\
&= \frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};1,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{1+\frac{dx^3}{c}}}
\end{aligned}$$

Mathematica [B] time = 0.36, size = 351, normalized size = 5.85

$$\frac{x\left(\frac{8\left(3dx^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(2ad^2x^3+b\left(5c^2+2cdx^3+2d^2x^6\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{(a+bx^3)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+\right.}{20b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3),x]
```

```
[Out] (x*((d*(8*b*c - 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-4*a*c*(2*a*d^2*x^3 + b*(5*c^2 + 2*c*d*
x^3 + 2*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
3*d*x^3*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)
/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(20*b
*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

maple [C] time = 0.26, size = 776, normalized size = 12.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a),x)
```

```
[Out] 2/5*(d*x^3+c)^(1/2)/b*d*x-2/3*I*(-2/5/b*c*d-(a*d-2*b*c)/b^2*d)*3^(1/2)*(-c*
d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/b^2/d^2*2^(1/2)*sum
((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}}{bx^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3 + c\right)^{3/2}}{b\,x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c + dx^3\right)^{\frac{3}{2}}}{a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a),x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3), x)
```

3.376 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)} dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -c*AppellF1(-1/3,1,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 1, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};1,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.15, size = 148, normalized size = 2.35

$$\frac{5cx^3\sqrt{\frac{dx^3}{c}+1}(5ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2dx^6\sqrt{\frac{dx^3}{c}+1}(2ad+bc)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20ac\left(c\right.}{20a^2x\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x]
```

```
[Out] (-20*a*c*(c + d*x^3) + 5*c*(-2*b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*d*(b*c + 2*a*d)*x^6*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(2
0*a^2*x*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

maple [C] time = 0.26, size = 1404, normalized size = 22.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/7*(d*x^3+c)^(1/2)/b*d*x^2-2/3*I*(-4/7/b*c*d-(a*d-2*b*c)/b^2*d)*3^
(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3
*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2
*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
```

```
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(-(d*x^3+c)^(1/2)*c/x+2/7*d*x^2*(d*
x^3+c)^(1/2)-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x
+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3
)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(d\,x^3+c\right)}^{3/2}}{x^2\,\left(b\,x^3+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{x^2\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**2*(a + b*x**3)), x)
```

3.377 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)} dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -1/2*c*AppellF1(-2/3,1,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 1, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};1,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.33, size = 343, normalized size = 5.28

$$-\frac{dx^6\sqrt{\frac{dx^3}{c}+1}\,(bc-4ad)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+\frac{8ac\left(3x^3(a+bx^3)(c+dx^3)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\right.\right.\right.}{(a+bx^3)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3}\right.\right.\right.}}{16a^2x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x]
```

```
[Out] -1/16*(d*(b*c - 4*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)] + (8*a*c*(-4*a*c*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3
*(a + b*x^3)*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a
*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a^2*x^2*Sqrt
[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

maple [C] time = 0.30, size = 1096, normalized size = 16.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x)
```

```
[Out] -1/a*b*(2/5*(d*x^3+c)^(1/2)/b*d*x-2/3*I*(-2/5/b*c*d-(a*d-2*b*c)/b^2*d)*3^(1
/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/b^2/d^2*2^(
1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/
2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(
```

```
1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/
3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+
I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha
)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a*(-1/2*c*(d*x^
3+c)^(1/2)/x^2+2/5*d*x*(d*x^3+c)^(1/2)-9/10*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3+c)^{\frac{3}{2}}}{(bx^3+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a),x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(d\,x^3+c)^{3/2}}{x^3\,(b\,x^3+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)),x)
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx^3)^{\frac{3}{2}}}{x^3\,(a+bx^3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a),x)
[Out] Integral((c + d*x**3)**(3/2)/(x**3*(a + b*x**3)), x)
```

3.378 $\int \frac{x^8}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=104

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}\,(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b/d^2-2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-2/3*(a*d+b*c)*(d*x^3+c)^(1/2)/b^2/d^2
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{2\sqrt{c+dx^3}\,(ad+bc)}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*(b*c + a*d)*Sqrt[c + d*x^3])/(3*b^2*d^2) + (2*(c + d*x^3)^(3/2))/(9*b*d
^2) - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)
*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}}+\frac{a^2}{b^2(a+bx)\sqrt{c+dx}}+\frac{\sqrt{c+dx}}{bd}\right)dx,x,x^3\right)$$
$$= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2}$$
$$= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d}$$
$$= -\frac{2(bc+ad)\sqrt{c+dx^3}}{3b^2d^2}+\frac{2\left(c+dx^3\right)^{3/2}}{9bd^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.22, size = 91, normalized size = 0.88

$$\frac{2\sqrt{c+dx^3}\left(-3ad-2bc+bdx^3\right)}{9b^2d^2}-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-2*b*c - 3*a*d + b*d*x^3))/(9*b^2*d^2) - (2*a^2*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

fricas [A] time = 1.09, size = 289, normalized size = 2.78

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^3\right)\sqrt{dx^3+c}}{9\left(b^4cd^2-ab^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/9*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x
^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a^
2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^
3), 2/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c
+ a*b*d)/(b*d*x^3 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*d
 - a*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

giac [A] time = 0.17, size = 106, normalized size = 1.02

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^2d^4-3\,\sqrt{dx^3+c}\,b^2cd^4-3\,\sqrt{dx^3+c}\,abd^5\right)}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 2/9*((d*x^3 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^3 + c)*b^2*c*d^4 - 3*s
qrt(d*x^3 + c)*a*b*d^5)/(b^3*d^6)
```

maple [C] time = 0.34, size = 488, normalized size = 4.69

$$-\frac{i a^{2} \left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i \left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right) d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right) d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \sqrt{-\frac{i \left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right) d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\, \Big(2\,\mathrm{RootOf}\left(\_Z^{3} b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/b^2*(b*(2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)-2/3*a/d*(d*x
^3+c)^(1/2))-1/3*I*a^2/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*
I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/
2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d
-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticP
i(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*
3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/
(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.43, size = 121, normalized size = 1.16

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b\,d}-\frac{\left(\frac{2\,a}{b^2}+\frac{4\,c}{3\,b\,d}\right)\,\sqrt{d\,x^3+c}}{3\,d}+\frac{a^2\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b*d) - (((2*a)/b^2 + (4*c)/(3*b*d))*(c + d*x^3
)^(1/2))/(3*d) + (a^2*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b
*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.379 $\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=74

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

```
[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+2/3*(d*x^3+c)^(1/2)/b/d
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{2\sqrt{c+dx^3}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b*d) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b
*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^3)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd}$$
$$= \frac{2\sqrt{c+dx^3}}{3bd} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.12, size = 74, normalized size = 1.00

$$\frac{2\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{\sqrt{b}\sqrt{c+dx^3}}{d}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*((Sqrt[b]*Sqrt[c + d*x^3])/d + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt
[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*b^(3/2))
```

fricas [A] time = 1.29, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{3\left(b^3cd-ab^2d^2\right)}, -\frac{2\left(\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{}}{bdx}\right.\right.}{3\left(b^3cd-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c
)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -2/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^3 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

giac [A] time = 0.17, size = 64, normalized size = 0.86

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^3+c}}{b}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^3 + c)/b)/d
```

maple [C] time = 0.23, size = 448, normalized size = 6.05

$$ia\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b/d+1/3*I*a/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.10, size = 86, normalized size = 1.16

$$\frac{2\sqrt{d\,x^3+c}}{3\,b\,d}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(
1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a+bx^{3}\right)\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.380 $\int \frac{x^2}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=51

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*Sqrt[b*c
 - a*d])
```

fricas [A] time = 1.12, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)}{3\,\sqrt{b^2c-abd}},\ \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-b^2c+abd}}{bdx^3+bc}\right)}{3\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b
*x^3 + a))/sqrt(b^2*c - a*b*d), 2/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c))/(b^2*c - a*b*d)]
```

giac [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [C] time = 0.28, size = 426, normalized size = 8.35

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+a\right)^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alp
```

```
ha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*
(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 5.89, size = 70, normalized size = 1.37

$$\frac{\ln\left(\frac{a\,d\,1\mathrm{i}-b\,c\,2\mathrm{i}+2\,\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}-b\,d\,x^3\,1\mathrm{i}}{b\,x^3+a}\right)\,1\mathrm{i}}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((a*d*1i - b*c*2i + 2*(c + d*x^3)^(1/2)*(a*b*d - b^2*c)^(1/2) - b*d*x^3
*1i)/(a + b*x^3))*1i)/(3*(a*b*d - b^2*c)^(1/2))
```

sympy [**A**] time = 10.26, size = 39, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

3.381 $\int \frac{1}{x(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=85

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(1/2)+2/3*arctanh(b^(1/2)*(d*x^3+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*Sqrt[c]) + (2*Sqrt[b]*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a}$$
$$=\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3ad}$$
$$=-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*(-(ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d]))/(3*a)
```

fricas [A] time = 0.84, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)}{3\,ac},\frac{2\,c\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{\sqrt{dx^3+c}\,(bc}{bd}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*(c*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt
(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*c), 1/3*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-
c)/c))/(a*c), 2/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^3 + c)*
sqrt(-c)/c))/(a*c)]
```

giac [A] time = 0.17, size = 71, normalized size = 0.84

$$-\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [C] time = 0.24, size = 453, normalized size = 5.33

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^{3}+c}}{\sqrt{c}}\right)}{3a\sqrt{c}}+\frac{i b\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
a/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)\sqrt{dx^{3}+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x), x)
```

mupad [B] time = 7.31, size = 114, normalized size = 1.34

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^{3}+c}-\sqrt{c}\right)^{3}\left(\sqrt{d\,x^{3}+c}+\sqrt{c}\right)}{x^{6}}\right)}{3\,a\,\sqrt{c}}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^{3}+\sqrt{b}\,\sqrt{d\,x^{3}+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^{3}+a}\right)1\mathrm{i}}{3\,a\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(1/2)) + (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d -
b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*1i)/(3*a*(a*d - b*c)^(1/2))
```

sympy [A] time = 19.39, size = 70, normalized size = 0.82

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] -2*atan(sqrt(c + d*x**3)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + 2
*atan(sqrt(c + d*x**3)/sqrt(-c))/(3*a*sqrt(-c))
```

3.382 $\int \frac{1}{x^4(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=117

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

```
[Out] 1/3*(a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-2/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/3*(d
*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}-\frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*a*c*x^3) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a^2*c^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*
c - a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\right.}{6a^2c}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int\right.}{3}$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.13, size = 151, normalized size = 1.29

$$\frac{2b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} - \frac{\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(a*c*x^3) + (2*b*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*
a^2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a*c^(3/2)) + (2*b^(3
/2)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
a^2*(-(b*c) + a*d))
```

fricas [A] time = 1.21, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^2x^3\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right) + (2\,bc+ad)\sqrt{c}\,x^3\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right) - 2\,\sqrt{dx^3+}}{6\,a^2c^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d
*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + (2*b*c + a*d)*sqr
t(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*sqrt(d*x^3
+ c)*a*c)/(a^2*c^2*x^3), -1/6*(4*b*c^2*x^3*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^3*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*sqrt
```

```
(d*x^3 + c)*a*c)/(a^2*c^2*x^3), 1/3*(b*c^2*x^3*sqrt(b/(b*c - a*d))*log((b*d
*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^3 + a)) - (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) -
 sqrt(d*x^3 + c)*a*c)/(a^2*c^2*x^3), -1/3*(2*b*c^2*x^3*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^3*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^
3 + c)*a*c)/(a^2*c^2*x^3)]
```

giac [A] time = 0.19, size = 104, normalized size = 0.89

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{3\,acx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/3*(2*b*c + a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/3*sqrt(d*x^3 + c)/(a*c*x^3)
```

maple [C] time = 0.25, size = 498, normalized size = 4.26

$$\frac{2b\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{\frac{d\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^3+c}}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}-\frac{\sqrt{d\,x^3+c}}{3c\,x^3}}{a}-\frac{ib^2\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3
/2))-1/3*I/a^2*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c
)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+2/3/a^2*b*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^4), x)
```

mupad [B] time = 8.42, size = 142, normalized size = 1.21

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(a\,d+2\,b\,c)}{6\,a^2\,c^{3/2}}-\frac{\sqrt{d\,x^3+c}}{3\,a\,c\,x^3}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)1\mathrm{i}}{3\,a^2\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
a*d + 2*b*c))/(6*a^2*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a*c*x^3) + (b^(3/2)*lo
g((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/
(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.383 $\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^3)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{4a\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)
/a)])/(4*a*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.37, size = 719, normalized size = 11.23

$$ia\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\Big(2\,\mathrm{RootOf}\left(\_Z^3b+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*
I*a/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/
d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3 + a)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{(b\,x^3 + a)\,\sqrt{d\,x^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + bx^3)\,\sqrt{c + dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.384 $\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^
3+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{2a\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)
/a)])/(2*a*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.32, size = 429, normalized size = 6.70

$$i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3 b+a\right)\right.$$

$$-\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3
*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d
,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.385 $\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/(d*x^3+c)^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.05, size = 161, normalized size = 2.73

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{(a+bx^3)\sqrt{c+dx^3}\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)])/((a + b*x
^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.26, size = 429, normalized size = 7.27

$$-\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.386 $\int \frac{1}{x^2(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x/(d*x^3+c)
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*x*Sqrt[c + d*x^3]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}$$
$$= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ax\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.12, size = 141, normalized size = 2.27

$$\frac{5x^3\sqrt{\frac{dx^3}{c}+1}\,(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20a\left(c+dx^3\right)}{20a^2cx\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*a*(c + d*x^3) + 5*(-2*b*c + a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^6*Sqrt[1 + (d*x^3)/c]*A
ppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(20*a^2*c*x*Sqrt[c +
 d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

maple [C] time = 0.26, size = 890, normalized size = 14.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a*b/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*
d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*
b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-(d*x^3+c)^(1/2)/c/x-1/3*
I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3
+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3
*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2 \left(b x^3 + a\right) \sqrt{d x^3 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^3\right) \sqrt{c + dx^3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.387 $\int \frac{1}{x^3(a+bx^3)\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/x^2/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ax^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.32, size = 339, normalized size = 5.30

$$\frac{\frac{8a\left(3x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(2ac+3adx^3+6bcx^3+2bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}-bdx^6\cdots}{16a^2cx^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*Sqrt[c + d*x^3]),x]
```

```
[Out] (-(b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)]) + (8*a*(-4*a*c*(2*a*c + 6*b*c*x^3 + 3*a*d*x^3 + 2*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(a + b*x^3)*(c +
 d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d
*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*
a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*A
ppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3
/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(16*a^2*c*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

maple [C] time = 0.25, size = 738, normalized size = 11.53

$$ib\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\,\Big(2\,\mathrm{RootOf}\left(\_Z^3b+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a*b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c
)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
```

```
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a*(-1/2/c*(d*x^3+c)^(1/2)/
x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)
^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(b\,x^3+a\right)\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.388 $\int \frac{x^8}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}\,(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

```
[Out] -2/3*a^2*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*
c)^(3/2)+2/3*c^2/d^2/(-a*d+b*c)/(d*x^3+c)^(1/2)+2/3*(d*x^3+c)^(1/2)/b/d^2
```

Rubi [A] time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 87, 63, 208}

$$-\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}+\frac{2c^2}{3d^2\sqrt{c+dx^3}\,(bc-ad)}+\frac{2\sqrt{c+dx^3}}{3bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*c^2)/(3*d^2*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*Sqrt[c + d*x^3])/(3*b*d^2)
 - (2*a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b
*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{c^2}{d(-bc+ad)(c+dx)^{3/2}} + \frac{1}{bd\sqrt{c+dx}} + \frac{a^2}{b(bc-ad)(a+bx)\sqrt{c+dx}}\right)\right)$$
$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3b(bc-ad)}$$
$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3bd(bc-ad)}$$
$$= \frac{2c^2}{3d^2(bc-ad)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{3bd^2} - \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.08, size = 100, normalized size = 0.93

$$\frac{2\left(-a^2d^2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a^2d^2+abd\left(c+dx^3\right)+b^2(-c)\left(2c+dx^3\right)\right)}{3b^2d^2\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*(a^2*d^2 + a*b*d*(c + d*x^3) - b^2*c*(2*c + d*x^3) - a^2*d^2*Hypergeomet
ric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]))/(3*b^2*d^2*(-(b*c) + a*
d)*Sqrt[c + d*x^3])
```

fricas [B] time = 1.68, size = 440, normalized size = 4.11

$$\left[-\frac{\left(a^2d^3x^3+a^2cd^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\,b^3c^3-3\,ab^2c^2d+a^2bcd^2+\left(b^3c^2d-2\,ab\right.\right.}{3\left(b^4c^3d^2-2\,ab^3c^2d^3+a^2b^2cd^4+\left(b^4c^2d^3-2\,ab^3cd^4+a^2b^2d^5\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a^2*d^3*x^3 + a^2*c*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c -
 a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*(2*b^3*c^3 -
 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^3)
*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b^2*c*d^4 + (b^4*c^2
*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3), 2/3*((a^2*d^3*x^3 + a^2*c*d^2)*sq
rt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b
*c)) + (2*b^3*c^3 - 3*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^
2 + a^2*b*d^3)*x^3)*sqrt(d*x^3 + c))/(b^4*c^3*d^2 - 2*a*b^3*c^2*d^3 + a^2*b
^2*c*d^4 + (b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^3)]
```

giac [A] time = 0.18, size = 103, normalized size = 0.96

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^2c-abd\right)\sqrt{-b^2c+abd}}+\frac{2\,c^2}{3\left(bcd^2-ad^3\right)\sqrt{dx^3+c}}+\frac{2\,\sqrt{dx^3+c}}{3\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*a^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqr
t(-b^2*c + a*b*d)) + 2/3*c^2/((b*c*d^2 - a*d^3)*sqrt(d*x^3 + c)) + 2/3*sqrt
(d*x^3 + c)/(b*d^2)
```

maple [C] time = 0.34, size = 527, normalized size = 4.93

$$-\left(\frac{i b \left(-c d^2\right)^{\frac{1}{3}} \sqrt{\frac{i \left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{\left(-c d^2\right)^{\frac{1}{3}}}}\, \sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\, \sqrt{-\frac{i \left(2x+\frac{i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right) d}{2\left(-c d^2\right)^{\frac{1}{3}}}}\, \left(2\, \mathrm{RootOf}\left(\_Z^3 b+a\right)^2 d^2+i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b^2*(b*(2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)+2/3*a/d/(d
*x^3+c)^(1/2))+a^2/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1
/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*
d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.46, size = 115, normalized size = 1.07

$$\frac{2\sqrt{d\,x^3+c}}{3\,b\,d^2}-\frac{2\,c^2}{3\,d^2\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{a^2\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)1{}\mathrm{i}}{3\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b*d^2) - (2*c^2)/(3*d^2*(c + d*x^3)^(1/2)*(a*d - b
*c)) + (a^2*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*
2i - b*d*x^3)/(a + b*x^3))*1i)/(3*b^(3/2)*(a*d - b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.389 $\int \frac{x^5}{(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=82

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

[Out] 2/3*a*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^(1/2)-2/3*c/d/(-a*d+b*c)/(d*x^3+c)^(1/2)

Rubi [A] time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}(bc-ad)^{3/2}}-\frac{2c}{3d\sqrt{c+dx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*c)/(3*d*(b*c - a*d)*Sqrt[c + d*x^3]) + (2*a*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}$$
$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}$$
$$= -\frac{2c}{3d(bc-ad)\sqrt{c+dx^3}} + \frac{2a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 1.07

$$\frac{2\left(\frac{c(ad-bc)}{d\sqrt{c+dx^3}} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((c*(-(b*c) + a*d))/(d*Sqrt[c + d*x^3]) + (a*Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b]))/(3*(b*c - a*d)^2)
```

fricas [B] time = 1.54, size = 326, normalized size = 3.98

$$\left[-\frac{\left(ad^2x^3+acd\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(b^2c^2-abcd\right)\sqrt{dx^3+c}}{3\left(b^3c^3d-2\,ab^2c^2d^2+a^2bcd^3+\left(b^3c^2d^2-2\,ab^2cd^3+a^2bd^4\right)x^3\right)}, -\frac{2\left(\left(ad^2x^3+acd\right)\sqrt{-}\right.}{3\left(b^3c^3d-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((a*d^2*x^3 + a*c*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(b^2*c^2 - a*b*c*d
)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3 + (b^3*c^2*d^
2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3), -2/3*((a*d^2*x^3 + a*c*d)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (b^
2*c^2 - a*b*c*d)*sqrt(d*x^3 + c))/(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^
3 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)]
```

giac [A] time = 0.19, size = 78, normalized size = 0.95

$$-\frac{2\left(\frac{ad\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)} + \frac{c}{\sqrt{dx^3+c}\,(bc-ad)}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(a*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*(b*c - a*d)) + c/(sqrt(d*x^3 + c)*(b*c - a*d)))/d
```

maple [C] time = 0.27, size = 487, normalized size = 5.94

$$-\frac{\left(-\frac{ib\left(-cd^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-cd^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-cd^{2}\right)^{\frac{1}{3}}+\left(-cd^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-cd^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-cd^{2}\right)^{\frac{1}{3}}\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b/d/(d*x^3+c)^(1/2)-a/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^
2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.99, size = 94, normalized size = 1.15

$$\frac{2\,c}{3\,d\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{a\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*c)/(3*d*(c + d*x^3)^(1/2)*(a*d - b*c)) + (a*log((2*b*c - a*d + b^(1/2)*(
c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(3*b^(1/2
)*(a*d - b*c)^(3/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^3)(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.390 $\int \frac{x^2}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{2}{3\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

```
[Out] -2/3*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(
3/2)+2/3/(-a*d+b*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{2}{3\sqrt{c+dx^3}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*
x^3])/Sqrt[b*c - a*d]])/(3*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^3)(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3(bc-ad)}$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3d(bc-ad)}$$
$$= \frac{2}{3(bc-ad)\sqrt{c+dx^3}} - \frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.03, size = 52, normalized size = 0.68

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(dx^3+c)}{bc-ad}\right)}{3\sqrt{c+dx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)])/(3*(-(b*c
) + a*d)*Sqrt[c + d*x^3])
```

fricas [A] time = 1.24, size = 236, normalized size = 3.06

$$\left[-\frac{(dx^3+c)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}}{3\left((bcd-ad^2)x^3+bc^2-acd\right)}, -\frac{2\left((dx^3+c)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^3+c}\,(\ldots}{\ldots}\right.\right.}{3\left((bcd-ad^2)x^3+b\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*((d*x^3 + c)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c
))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d), -2/3*((d*x^3 + c)*sqrt(-b/(b*c -
a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b
*c)) - sqrt(d*x^3 + c))/((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)]
```

giac [A] time = 0.19, size = 73, normalized size = 0.95

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,(bc-ad)} + \frac{2}{3\,\sqrt{dx^3+c}\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
(b*c - a*d)) + 2/3/(sqrt(d*x^3 + c)*(b*c - a*d))
```

maple [C] time = 0.23, size = 463, normalized size = 6.01

$$-\frac{ib\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*
d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3
*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.85, size = 89, normalized size = 1.16

$$-\frac{2}{3\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{3\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
- b*d*x^3)/(a + b*x^3))*1i)/(3*(a*d - b*c)^(3/2)) - 2/(3*(c + d*x^3)^(1/2)*
(a*d - b*c))
```

sympy [A] time = 32.78, size = 66, normalized size = 0.86

$$-\frac{2}{3\sqrt{c+dx^{3}}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx^{3}}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] -2/(3*sqrt(c + d*x**3)*(a*d - b*c)) - 2*atan(sqrt(c + d*x**3)/sqrt((a*d - b
*c)/b))/(3*sqrt((a*d - b*c)/b)*(a*d - b*c))
```

3.391 $\int \frac{1}{x(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=114

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a/c^(3/2)+2/3*b^(3/2)*arctanh(b^(1/2)
*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a/(-a*d+b*c)^(3/2)-2/3*d/c/(-a*d+b*c)/(d
*x^3+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 85, 156, 63, 208}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}} - \frac{2d}{3c\sqrt{c+dx^3}\,(bc-ad)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d)/(3*c*(b*c - a*d)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[
c]])/(3*a*c^(3/2)) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*a*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad-bdx}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3c(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,d\right.}{3a(bc-ad)}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3acd} - \frac{\left(2b^2\right)\text{Subst}\left(\int -\right.}{3ac}$$

$$= -\frac{2d}{3c(bc-ad)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3ac^{3/2}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.05, size = 89, normalized size = 0.78

$$-\frac{2\left(bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+(ad-bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)\right)}{3ac\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(b*c*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)] + (-(
b*c) + a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a*c*(b*c -
a*d)*Sqrt[c + d*x^3])
```

fricas [B] time = 1.19, size = 790, normalized size = 6.93

$$\left[-\frac{2\sqrt{dx^3+c}\,acd+\left(bc^2dx^3+bc^3\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-\left(\left(bcd-ad^2\right)x^3+bc^2-acd\right)\sqrt{\right.}{3\left(abc^4-a^2c^3d+\left(abc^3d-a^2c^2d^2\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(2*sqrt(d*x^3 + c)*a*c*d + (b*c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*
log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*
d)))/(b*x^3 + a)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log((d*x^
3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*b*c^3*
d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*(b*c^2*d*x^3 + b*c
^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c -
```

```
a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*sqrt(c)*log
((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a*b*c^4 - a^2*c^3*d + (a*
b*c^3*d - a^2*c^2*d^2)*x^3), -1/3*(2*sqrt(d*x^3 + c)*a*c*d - 2*((b*c*d - a*
d^2)*x^3 + b*c^2 - a*c*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (b*
c^2*d*x^3 + b*c^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(
d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(a*b*c^4 - a^2*c^
3*d + (a*b*c^3*d - a^2*c^2*d^2)*x^3), -2/3*(sqrt(d*x^3 + c)*a*c*d - (b*c^2*
d*x^3 + b*c^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqr
t(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((b*c*d - a*d^2)*x^3 + b*c^2 - a*c*d)*
sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c))/(a*b*c^4 - a^2*c^3*d + (a*b*c^
3*d - a^2*c^2*d^2)*x^3)]
```

giac [A] time = 0.17, size = 111, normalized size = 0.97

$$-\frac{2\,b^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{3\left(abc-a^{2}d\right)\sqrt{-b^{2}c+abd}}-\frac{2\,d}{3\,\sqrt{dx^{3}+c}\left(bc^{2}-acd\right)}+\frac{2\,\arctan\left(\frac{\sqrt{dx^{3}+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*b^2*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a*b*c - a^2*d)*sq
rt(-b^2*c + a*b*d)) - 2/3*d/(sqrt(d*x^3 + c)*(b*c^2 - a*c*d)) + 2/3*arctan(
sqrt(d*x^3 + c)/sqrt(-c))/(a*sqrt(-c)*c)
```

maple [C] time = 0.29, size = 512, normalized size = 4.49

$$-\frac{\left(-\frac{ib\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-c\,d^{2}\right)^{\frac{1}{3}}\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+
b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
```

```
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*b+a)))+1/a*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x), x)
```

mupad [B] time = 8.44, size = 139, normalized size = 1.22

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a\,c^{3/2}}+\frac{2\,d}{3\,c\,\sqrt{d\,x^3+c}\,\left(a\,d-b\,c\right)}+\frac{b^{3/2}\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)1\mathrm{i}}{3\,a\,\left(a\,d-b\,c\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a*c^(3/2)) + (2*d)/(3*c*(c + d*x^3)^(1/2)*(a*d - b*c)) + (b^(3/2)*log((2*b
*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b
*x^3))*1i)/(3*a*(a*d - b*c)^(3/2))
```

sympy [A] time = 24.63, size = 104, normalized size = 0.91

$$\frac{2d}{3c\sqrt{c+dx^3}\,(ad-bc)}+\frac{2b\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}+\frac{2\operatorname{atan}\left(\frac{\sqrt{c+dx^3}}{\sqrt{-c}}\right)}{3ac\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] 2*d/(3*c*sqrt(c + d*x**3)*(a*d - b*c)) + 2*b*atan(sqrt(c + d*x**3)/sqrt((a*
d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)*(a*d - b*c)) + 2*atan(sqrt(c + d*x**3
)/sqrt(-c))/(3*a*c*sqrt(-c))
```

3.392 $\int \frac{1}{x^4(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

```
[Out] 1/3*(3*a*d+2*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(5/2)-2/3*b^(5/2)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(3/2)-1/3*
d*(-3*a*d+b*c)/a/c^2/(-a*d+b*c)/(d*x^3+c)^(1/2)-1/3/a/c/x^3/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}+\frac{(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}}-\frac{d(bc-3ad)}{3ac^2\sqrt{c+dx^3}\,(bc-ad)}-\frac{1}{3acx^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(b*c - 3*a*d))/(3*a*c^2*(b*c - a*d)*Sqrt[c + d*x^3]) - 1/(3*a*c*x^3*Sqr
t[c + d*x^3]) + ((2*b*c + 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^2*c
^(5/2)) - (2*b^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3
*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+3ad)+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}(bc-ad)(2bc+3ad)-\frac{1}{4}bd(b}{x(a+bx)\sqrt{c+dx}}\right.}{3ac^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{3a^2d(bc-ad)}$$

$$= -\frac{d(bc-3ad)}{3ac^2(bc-ad)\sqrt{c+dx^3}} - \frac{1}{3acx^3\sqrt{c+dx^3}} + \frac{(2bc+3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{5/2}} - \frac{2}{-}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.74

$$\frac{2b^2c^2x^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+(ad-bc)\left(x^3(3ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+ac\right)}{3a^2c^2x^3\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b^2*c^2*x^3*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x^3))/(b*c - a*d)]
 + (-(b*c) + a*d)*(a*c + (2*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/2, 1, 1/2
, 1 + (d*x^3)/c]))/(3*a^2*c^2*(b*c - a*d)*x^3*Sqrt[c + d*x^3])
```

fricas [B] time = 1.59, size = 1120, normalized size = 7.09

$$\left[-\frac{2\left(b^2c^3dx^6+b^2c^4x^3\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-\left(\left(2\,b^2c^2d+abcd^2-3\,a^2d^3\right)x^6+\left(2\,b^2c\right.\right.}{6\left(\left(a^2bc^4d-a^3c^3d^2\right)x^6+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2
*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)
) - ((2*b^2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3
*a^2*c*d^2)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3)
 + 2*(a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))
/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/6*(4*(
b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(
b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((2*b^2*c^2*d + a*b*c*d^
2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(c)*log
((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a*b*c^3 - a^2*c^2*d +
(a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2
)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3), -1/3*(((2*b^2*c^2*d + a*b*c*d^2 - 3*a
^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(b/(b*c - a*d
))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c -
 a*d)))/(b*x^3 + a)) + (a*b*c^3 - a^2*c^2*d + (a*b*c^2*d - 3*a^2*c*d^2)*x^3
)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^2)*x^6 + (a^2*b*c^5 - a^3*c^4*
d)*x^3), -1/3*(2*(b^2*c^3*d*x^6 + b^2*c^4*x^3)*sqrt(-b/(b*c - a*d))*arctan(
-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + ((2*b^
2*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*x^6 + (2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b*c^3 - a^2*c^2*d +
 (a*b*c^2*d - 3*a^2*c*d^2)*x^3)*sqrt(d*x^3 + c))/((a^2*b*c^4*d - a^3*c^3*d^
2)*x^6 + (a^2*b*c^5 - a^3*c^4*d)*x^3)]
```

giac [A] time = 0.17, size = 173, normalized size = 1.09

$$\frac{2\,b^3\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}-\frac{\left(dx^3+c\right)bcd-3\left(dx^3+c\right)ad^2+2\,acd^2}{3\left(abc^3-a^2c^2d\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}-\sqrt{dx^3+c}\,c\right)}-\frac{(2\,bc+3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*b^3*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b*c - a^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/3*((d*x^3 + c)*b*c*d - 3*(d*x^3 + c)*a*d^2 + 2*a*c
*d^2)/((a*b*c^3 - a^2*c^2*d)*((d*x^3 + c)^(3/2) - sqrt(d*x^3 + c)*c)) - 1/3
*(2*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c^2)
```

maple [C] time = 0.41, size = 575, normalized size = 3.64

$$-\frac{ib\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)^{2}d^{2}+i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{3}\ \mathrm{Rc}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((
d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+1/a^2*b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1
/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1
/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d
-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a^2*b*(2/3/((x^3+c/d)
*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)\left(dx^{3}+c\right)^{\frac{3}{2}}x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^4), x)
```

mupad [B] time = 10.47, size = 597, normalized size = 3.78

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)^3}{x^6}\right)(3\,a\,d+2\,b\,c)}{6\,a^2\,c^{5/2}}-\frac{\sqrt{d\,x^3+c}}{3\,a\,c^2\,x^3}-\frac{c\left(\frac{c\left(\frac{c\left(\frac{c\left(\frac{3\,a^2\,d^4+24\,a\,b\,c\,d^3+15\,b^2\,c^2\,d^2}{8\,a^3\,c^5}+\frac{c\left(\frac{c\left(\frac{3\,b^2\,d^4}{8\,a^3\,c^5}+\frac{b^2\,d^4\,(5\,a\,d-3\,b\,c)}{8\,a^3\,c^4\,\left(b\,c^2-a\,c\,d\right)}-\frac{b\,d^4\cdots}{\cdots}\right)}{d}\cdots}{\cdots}\right)}{\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((((c + d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(
3*a*d + 2*b*c))/(6*a^2*c^(5/2)) - (c + d*x^3)^(1/2)/(3*a*c^2*x^3) - ((c*((c
*((c*((3*a^2*d^4 + 15*b^2*c^2*d^2 + 24*a*b*c*d^3)/(8*a^3*c^5) + (c*((c*((3*
b^2*d^4)/(8*a^3*c^5) + (b^2*d^4*(5*a*d - 3*b*c))/(8*a^3*c^4*(b*c^2 - a*c*d)
) - (b*d^4*(a*d + 2*b*c)*(5*a*d - 3*b*c))/(4*a^3*c^5*(b*c^2 - a*c*d))))/d -
 (3*b*d^3*(a*d + 2*b*c))/(4*a^3*c^5) + (d*(5*a*d - 3*b*c)*(3*a^2*d^4 + 15*b
^2*c^2*d^2 + 24*a*b*c*d^3))/(24*a^3*c^5*(b*c^2 - a*c*d))))/d - (d^2*(5*a*d
- 3*b*c)*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(12*a^3*c^4*(b*c^2 - a*c*d))
))/d - (d*(6*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(4*a^3*c^4) + (d^2*(5*a*d -
 3*b*c)*(13*a*d + 18*b*c))/(24*a^2*c^3*(b*c^2 - a*c*d))))/d + (d*(13*a*d +
18*b*c))/(8*a^2*c^3) - (d*(3*a*d + 2*b*c)*(5*a*d - 3*b*c))/(6*a^2*c^2*(b*c^
2 - a*c*d))))/d - (3*a*d + 2*b*c)/(2*a^2*c^2))/(c + d*x^3)^(1/2) + (b^(5/2)
*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^
3)/(a + b*x^3))*1i)/(3*a^2*(a*d - b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.393 $\int \frac{x^3}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx=\frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}$$

$$=\frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};1,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4ac\sqrt{c+dx^3}}$$

Mathematica [B] time = 0.21, size = 231, normalized size = 3.45

$$\frac{x\Bigg(-\frac{64a^2cF_1\big(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\big)}{(a+bx^3)\Big(3x^3\Big(2bcF_1\big(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\big)+adF_1\big(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\big)\Big)-8acF_1\big(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\big)\Big)}-\frac{bx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\big(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}}{a}}{12\sqrt{c+dx^3}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-8 - (b*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])/a - (64*a^2*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((
b*x^3)/a)])/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(
12*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.39, size = 1069, normalized size = 15.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(2/3/((x^3+c/d)*d)^(1/2)/c*x-2/9*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)))-a/b*(2/3*d/c*x/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2
/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
```

```
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(x^3/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.394 $\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^3)(c+dx^3)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};1,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.12, size = 142, normalized size = 2.12

$$\frac{x^2\left(2bdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5\sqrt{\frac{dx^3}{c}+1}\,(ad+3bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-20ad\right)}{30ac\sqrt{c+dx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*(-20*a*d + 5*(3*b*c + a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a*c*(b*c - a*d)*Sqrt[c +
d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.36, size = 907, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3*d/c*x^2/(a*d-b*c)/((x^3+c/d)*d)^(1/2)+2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha*(-c*d^2
)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3+a)(dx^3+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(b\,x^3+a)\,(d\,x^3+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(x/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.395 $\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/(d*x^3+c)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a*c*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};1,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.33, size = 338, normalized size = 5.45

$$\frac{x\left(\frac{bdx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a(ad-bc)}+\frac{32ac\left(3ad-3bc+2bdx^3\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24dx^3\left(a+bx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1 \cdots\right.\right.}{\left(a+bx^3\right)(bc-ad)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\cdots\right.\right.}\right)}{12c\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((b*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/(a*(-(b*c) + a*d)) + (32*a*c*(-3*b*c + 3*a*d + 2*b*d*x^3)*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*d*x^3*(a + b*x^3)*
(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF
1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/((b*c - a*d)*(a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(12*c*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)\left(dx^3 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.26, size = 753, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] 2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)/c*d*x-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*d^2)^
(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b
*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/((a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.396 $\int \frac{1}{x^2(a+bx^3)(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x/(d*x^3+
c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a*c*x*Sqrt[c + d*x^3]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};1,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.24, size = 193, normalized size = 2.97

$$\frac{-5x^3\sqrt{\frac{dx^3}{c}+1}\left(5a^2d^2-3abcd+6b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2bdx^6\sqrt{\frac{dx^3}{c}+1}\,(3bc-5ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\right.}{60a^2c^2x\sqrt{c+dx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (20*a*(-3*b*c*(c + d*x^3) + a*d*(3*c + 5*d*x^3)) - 5*(6*b^2*c^2 - 3*a*b*c*d
 + 5*a^2*d^2)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/
c), -((b*x^3)/a)] + 2*b*d*(3*b*c - 5*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[
5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^2*c^2*(b*c - a*d)*x*Sq
rt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)(dx^3 + c)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

maple [C] time = 0.25, size = 1392, normalized size = 21.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a*b*(2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)/c*d*x^2+2/9*I/c/(a*d-b*c)*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alpha
*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
)+1/a*(-2/3/((x^3+c/d)*d)^(1/2)/c^2*d*x^2-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3
```

```
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^3\right)\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.397 $\int \frac{1}{x^3(a+bx^3)(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a/c/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a*c*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(a+bx^3)\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};1,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2acx^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.73, size = 425, normalized size = 6.34

$$\frac{\frac{8a\left(3x^3\left(a^2d\left(3c+7dx^3\right)+ab\left(7d^2x^6-3c^2\right)-3b^2cx^3\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-4ac\left(3a^2d\left(2c+7dx^3\right)+ab\left(-6c^2-3cdx\right.\right.\right.}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.\right.}}{48a^2c^2x^2\sqrt{c+dx^3}\,(ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b*d*(3*b*c - 7*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (8*a*(-4*a*c*(-6*b^2*c*x^3*(3*c + d*x^3) + 3*a^2
*d*(2*c + 7*d*x^3) + a*b*(-6*c^2 - 3*c*d*x^3 + 14*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(-3*b^2*c*x^3*(c + d*x^3) +
 a^2*d*(3*c + 7*d*x^3) + a*b*(-3*c^2 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2
, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x
^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x
^3)/a)]))))/(48*a^2*c^2*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)\left(dx^3 + c\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

maple [C] time = 0.23, size = 1084, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a*b*(2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)/c*d*x-2/9*I/c/(a*d-b*c)*3^(1/2)*(
-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(
1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_al
pha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/
2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/
2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
```

```
lpha=RootOf(_Z^3*b+a))+1/a*(-1/2/c^2*(d*x^3+c)^(1/2)/x^2-2/3*d/c^2*x/((x^3
+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(
-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)*(d*x^3 + c)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(b\,x^3+a\right)\,{\left(d\,x^3+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)\left(c+dx^{3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)*(c + d*x**3)**(3/2)), x)
```

3.398 $\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

```
[Out] -3968/9*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+7/15*x^6*(d*x^3+c)
^(1/2)/d^2+1/3*x^9*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+2/15*c*(47*d*x^3+1146*c)*
(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 97, 153, 147, 63, 206}

$$-\frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}+\frac{7x^6\sqrt{c+dx^3}}{15d^2}+\frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4}+\frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*x^6*Sqrt[c + d*x^3])/(15*d^2) + (x^9*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)
) + (2*c*Sqrt[c + d*x^3]*(1146*c + 47*d*x^3))/(15*d^4) - (3968*c^(5/2)*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3c+\frac{7dx}{2}\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x\left(-56c^2d-\frac{141}{2}cd^2x\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{15d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(1984c^3\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\cdots}\right.}{3d^3}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{\left(3968c^3\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,d\cdots\right.}{3d^4}$$

$$= \frac{7x^6\sqrt{c+dx^3}}{15d^2} + \frac{x^9\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{2c\sqrt{c+dx^3}\left(1146c+47dx^3\right)}{15d^4} - \frac{3968c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^4}$$

Mathematica [A] time = 0.11, size = 101, normalized size = 0.86

$$\frac{19840c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+6\sqrt{c+dx^3}\left(-9168c^3+770c^2dx^3+19cd^2x^6+d^3x^9\right)}{45d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-9168*c^3 + 770*c^2*d*x^3 + 19*c*d^2*x^6 + d^3*x^9) + 1
9840*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^4*(-
8*c + d*x^3))
```

fricas [A] time = 0.91, size = 219, normalized size = 1.87

$$\left[\frac{2\left(4960\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(d^3x^9+19cd^2x^6+770c^2dx^3-9168c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^5x^3-8cd^4\right)}, 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(4960*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 3*(d^3*x^9 + 19*c*d^2*x^6 + 770*c^2*d*x^3 - 91
68*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/45*(9920*(c^2*d*x^3 - 8*c^3
)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(d^3*x^9 + 19*c*d^2*x
^6 + 770*c^2*d*x^3 - 9168*c^3)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

giac [A] time = 0.16, size = 110, normalized size = 0.94

$$\frac{3968\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^4}-\frac{512\sqrt{dx^3+c}\,c^3}{3\left(dx^3-8c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{5}{2}}d^{16}+25\left(dx^3+c\right)^{\frac{3}{2}}cd^{16}+960\sqrt{dx^3+c}\,c^2d^{16}\right)}{15\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3968/9*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/3*sqrt
(d*x^3 + c)*c^3/((d*x^3 - 8*c)*d^4) + 2/15*((d*x^3 + c)^(5/2)*d^16 + 25*(d*
x^3 + c)^(3/2)*c*d^16 + 960*sqrt(d*x^3 + c)*c^2*d^16)/d^20
```

maple [C] time = 0.28, size = 952, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/15*(d*x^3+c)^(1/2)*x^6+2/45*(d*x^3+c)^(1/2)*c/d*x^3-4/45*(d*x^3
+c)^(1/2)*c^2/d^2)+32/9*c/d*(d*x^3+c)^(3/2))+512*c^3/d^3*(-1/3/d*(d*x^3+c)^
(1/2)/(d*x^3-8*c)+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*
d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+192*c^2/d^3*(2/3*(d*x^3+c)^(1/2)/d+1/
3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha
^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*
```

```
(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*d-8*c)))
```

maxima [A] time = 1.24, size = 107, normalized size = 0.91

$$\frac{2\left(4960\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+75\left(dx^3+c\right)^{\frac{3}{2}}c+2880\,\sqrt{dx^3+c}\,c^2-\frac{3840\,\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(4960*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 75*(d*x^3 + c)^(3/2)*c + 2880*sqrt(d*x^3 +
 c)*c^2 - 3840*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^4
```

mupad [B] time = 4.09, size = 127, normalized size = 1.09

$$\frac{1984\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^4}+\frac{1972\,c^2\,\sqrt{d\,x^3+c}}{15\,d^4}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d^2}+\frac{18\,c\,x^3\,\sqrt{d\,x^3+c}}{5\,d^3}+\frac{512\,c^3\,\sqrt{d\,x^3+c}}{3\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (1984*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/(9*d^4) + (1972*c^2*(c + d*x^3)^(1/2))/(15*d^4) + (2*x^6*(c + d*x^3)^(1
/2))/(15*d^2) + (18*c*x^3*(c + d*x^3)^(1/2))/(5*d^3) + (512*c^3*(c + d*x^3)
^(1/2))/(3*d^4*(8*c - d*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.399 $\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64/27*c*(d*x^3+c)^(3/2)/d^3/(-d*x^3+8*c)-352/9*c^(3
/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+352/27*c*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 89, 80, 50, 63, 206}

$$-\frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}+\frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)}+\frac{352c\sqrt{c+dx^3}}{27d^3}+\frac{2\left(c+dx^3\right)^{3/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (352*c*Sqrt[c + d*x^3])/(27*d^3) + (2*(c + d*x^3)^(3/2))/(9*d^3) + (64*c*(c
 + d*x^3)^(3/2))/(27*d^3*(8*c - d*x^3)) - (352*c^(3/2)*ArcTanh[Sqrt[c + d*x
^3]/(3*Sqrt[c])])/(9*d^3)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
```

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}\left(104c^2d+9cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}\\
&= \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{(176c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{27d^2}\\
&= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(176c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3d^2}\\
&= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(352c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d^3}\\
&= \frac{352c\sqrt{c+dx^3}}{27d^3} + \frac{2\left(c+dx^3\right)^{3/2}}{9d^3} + \frac{64c\left(c+dx^3\right)^{3/2}}{27d^3\left(8c-dx^3\right)} - \frac{352c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.88

$$\frac{352c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\left(-488c^2+41cdx^3+d^2x^6\right)}{9d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-488*c^2 + 41*c*d*x^3 + d^2*x^6) + 352*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^3*(-8*c + d*x^3))
```

fricas [A] time = 0.85, size = 191, normalized size = 1.87

$$\left[\frac{2\left(88\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(d^2x^6+41cdx^3-488c^2\right)\sqrt{dx^3+c}\right)}{9\left(d^4x^3-8cd^3\right)}, \frac{2\left(176\left(cdx^3-8c^2\right)\sqrt{\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/9*(88*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) +
 10*c)/(d*x^3 - 8*c)) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(
d^4*x^3 - 8*c*d^3), 2/9*(176*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x
^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 41*c*d*x^3 - 488*c^2)*sqrt(d*x^3 + c))/(d^
4*x^3 - 8*c*d^3)]
```

giac [A] time = 0.18, size = 93, normalized size = 0.91

$$\frac{352\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{9\sqrt{-c}\,d^3}-\frac{64\sqrt{dx^3+c}\,c^2}{3\left(dx^3-8c\right)d^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^6+48\sqrt{dx^3+c}\,cd^6\right)}{9\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 352/9*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 64/3*sqrt(d
*x^3 + c)*c^2/((d*x^3 - 8*c)*d^3) + 2/9*((d*x^3 + c)^(3/2)*d^6 + 48*sqrt(d*
x^3 + c)*c*d^6)/d^9
```

maple [C] time = 0.21, size = 892, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/d^3+64*c^2/d^2*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54
*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alph
a^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)
*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/1
8*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootO
f(_Z^3*d-8*c))+16*c/d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d
^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2
)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(
2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^
2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.22, size = 91, normalized size = 0.89

$$\frac{2\left(88\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{3}{2}}+48\,\sqrt{dx^3+c}\,c-\frac{96\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}\right)}{9\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/9*(88*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + (d*x^3 + c)^(3/2) + 48*sqrt(d*x^3 + c)*c - 96*sqrt(d*x^3 + c)*c^2/(
d*x^3 - 8*c))/d^3
```

mupad [B] time = 4.01, size = 107, normalized size = 1.05

$$\frac{98\,c\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{176\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^3}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{64\,c^2\,\sqrt{d\,x^3+c}}{3\,d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (98*c*(c + d*x^3)^(1/2))/(9*d^3) + (176*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^3) + (2*x^3*(c + d*x^3)^(1/2))/(
9*d^2) + (64*c^2*(c + d*x^3)^(1/2))/(3*d^3*(8*c - d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^8\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**8*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.400 $\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=82

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

```
[Out] 8/27*(d*x^3+c)^(3/2)/d^2/(-d*x^3+8*c)-26/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1
/2))*c^(1/2)/d^2+26/27*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}+\frac{26\sqrt{c+dx^3}}{27d^2}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (26*Sqrt[c + d*x^3])/(27*d^2) + (8*(c + d*x^3)^(3/2))/(27*d^2*(8*c - d*x^3)
) - (26*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{(8c-dx)^2}\,dx,x,x^3\right)\\ &= \frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{13\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)}{27d}\\ &= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{(13c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3d}\\ &= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{(26c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{3d^2}\\ &= \frac{26\sqrt{c+dx^3}}{27d^2}+\frac{8\left(c+dx^3\right)^{3/2}}{27d^2\left(8c-dx^3\right)}-\frac{26\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2}\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.96

$$\frac{6\sqrt{c+dx^3}\left(dx^3-12c\right)+26\sqrt{c}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (6*(-12*c + d*x^3)*Sqrt[c + d*x^3] + 26*Sqrt[c]*(8*c - d*x^3)*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

fricas [A] time = 0.87, size = 165, normalized size = 2.01

$$\left[\frac{13\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+6\,\sqrt{dx^3+c}\left(dx^3-12\,c\right)}{9\left(d^3x^3-8\,cd^2\right)},\ \frac{2\left(13\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)\right.}{9\left(d^3x^3-8\,cd^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) + 6*sqrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2),
2/9*(13*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*s
qrt(d*x^3 + c)*(d*x^3 - 12*c))/(d^3*x^3 - 8*c*d^2)]
```

giac [A] time = 0.16, size = 69, normalized size = 0.84

$$\frac{26\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}\,d^2}+\frac{2\,\sqrt{dx^3+c}}{3\,d^2}-\frac{8\,\sqrt{dx^3+c}\,c}{3\left(dx^3-8\,c\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 26/9*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) + 2/3*sqrt(d*x^3
 + c)/d^2 - 8/3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^2)
```

maple [C] time = 0.17, size = 874, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/3*(d*x
^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.25, size = 79, normalized size = 0.96

$$\frac{13\,\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+6\,\sqrt{dx^3+c}-\frac{24\,\sqrt{dx^3+c}\,c}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(13*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt
(c))) + 6*sqrt(d*x^3 + c) - 24*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^2
```

mupad [B] time = 3.99, size = 87, normalized size = 1.06

$$\frac{2\,\sqrt{d\,x^3+c}}{3\,d^2}+\frac{13\,\sqrt{c}\;\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\;\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{9\,d^2}+\frac{8\,c\,\sqrt{d\,x^3+c}}{3\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^2) + (13*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(
c + d*x^3)^(1/2))/(8*c - d*x^3)))/(9*d^2) + (8*c*(c + d*x^3)^(1/2))/(3*d^2*
(8*c - d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**5*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.401 $\int \frac{x^2 \sqrt{c+dx^3}}{\left(8c-dx^3\right)^2} \, dx$

Optimal. Leaf size=64

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

```
[Out] -1/9*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d/c^(1/2)+1/3*(d*x^3+c)^(1/2)/d/(
-d*x^3+8*c)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 47, 63, 206}

$$\frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(3*d*(8*c - d*x^3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/
(9*Sqrt[c]*d)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(8c-dx)^2}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{1}{6}\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{3d}$$
$$= \frac{\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9\sqrt{c}\,d}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.95

$$\frac{\frac{3\sqrt{c+dx^3}}{8c-dx^3} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] ((3*Sqrt[c + d*x^3])/(8*c - d*x^3) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/S
qrt[c])/(9*d)
```

fricas [A] time = 0.87, size = 149, normalized size = 2.33

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}\,c}{18\left(cd^2x^3-8\,c^2d\right)}, \frac{\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)-3\,\sqrt{dx^3+c}\,c}{9\left(cd^2x^3-8\,c^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)
/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c*d^2*x^3 - 8*c^2*d), 1/9*((d*x^3 -
 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*sqrt(d*x^3 + c)*c
)/(c*d^2*x^3 - 8*c^2*d)]
```

giac [A] time = 0.16, size = 53, normalized size = 0.83

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{9\,\sqrt{-c}\,d} - \frac{\sqrt{dx^3+c}}{3\left(dx^3-8\,c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/9*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 1/3*sqrt(d*x^3 + c)
/((d*x^3 - 8*c)*d)
```

maple [C] time = 0.18, size = 439, normalized size = 6.86

$$i\left(-c\,d^{2}\right)^{\frac{1}{3}} \sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\right.$$

$$-\frac{\sqrt{d\,x^{3}+c}}{3\left(d\,x^{3}-8c\right)d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.39, size = 66, normalized size = 1.03

$$\frac{\frac{\log\left(\frac{\sqrt{dx^{3}+c}-3\sqrt{c}}{\sqrt{dx^{3}+c}+3\sqrt{c}}\right)}{\sqrt{c}}-\frac{6\sqrt{dx^{3}+c}}{dx^{3}-8c}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sqrt
(c) - 6*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d
```

mupad [B] time = 3.93, size = 72, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^{3}-6\,\sqrt{c}\,\sqrt{d\,x^{3}+c}}{8\,c-d\,x^{3}}\right)}{18\,\sqrt{c}\,d}+\frac{\sqrt{d\,x^{3}+c}}{3\,d\,\left(8\,c-d\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(18*c^(1/2)
*d) + (c + d*x^3)^(1/2)/(3*d*(8*c - d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{c+dx^{3}}}{\left(-8c+dx^{3}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.402 $\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=88

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

```
[Out] 5/288*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(3/2)+1/24*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 99, 156, 63, 208, 206}

$$\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}+\frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*(8*c - d*x^3)) + (5*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
]])])/(288*c^(3/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(8c-dx)^2}\,dx, x, x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c} + \frac{(5d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{192c}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{96c} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{96cd}\\
&= \frac{\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{288c^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 102, normalized size = 1.16

$$\frac{12\sqrt{c}\sqrt{c+dx^3}+5\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+3\left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{288c^{3/2}\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (12*Sqrt[c]*Sqrt[c + d*x^3] + 5*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])] + 3*(-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(288*c^(3/2)*(
8*c - d*x^3))
```

fricas [A] time = 0.97, size = 226, normalized size = 2.57

$$\left[\frac{5\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\,\sqrt{dx^3+c}\,c}{576\left(c^2dx^3-8\,c^3\right)}, \frac{3\left(dx^3-8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/576*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)
*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^2*d*x^3 - 8*c^3), 1/288*(3*
(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 5*(d*x^3 - 8*c)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)*c)/(c
^2*d*x^3 - 8*c^3)]
```

giac [A] time = 0.16, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c}-\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{288\sqrt{-c}\,c}-\frac{\sqrt{dx^3+c}}{24\left(dx^3-8c\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 5/288*arctan(1/3*sqrt(
d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/24*sqrt(d*x^3 + c)/((d*x^3 - 8*c)*c)
```

maple [C] time = 0.18, size = 912, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/8/c*d*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((-c*d^
2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2
)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/64/c^2*d*
(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(2/3*(d*x^3+c)^(1/2)-2/3*a
rctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x), x)
```

mupad [B] time = 3.98, size = 76, normalized size = 0.86

$$\frac{5\,\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)}{288\,\sqrt{c^3}}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)}{96\,\sqrt{c^3}}+\frac{\sqrt{d\,x^3+c}}{8\,c\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (5*atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2))))/(288*(c^3)^(1/2)) - atanh(
(c*(c + d*x^3)^(1/2))/(c^3)^(1/2))/(96*(c^3)^(1/2)) + (c + d*x^3)^(1/2)/(8*
c*(24*c - 3*d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x\left(-8c + dx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(-8*c + d*x**3)**2), x)
```

3.403 $\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=124

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}+\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

```
[Out] 7/1152*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/128*d*arctanh((d*x^
3+c)^(1/2)/c^(1/2))/c^(5/2)+1/96*d*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)-1/24*(d
*x^3+c)^(1/2)/c/x^3/(-d*x^3+8*c)
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (d*Sqrt[c + d*x^3])/(96*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c*x^3*(8*c
 - d*x^3)) + (7*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1152*c^(5/2)) - (d
*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(128*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^4\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(8c-dx)^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{6cd+\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}\\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}-\frac{\text{Subst}\left(\int \frac{-54c^2d^2-9cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d}\\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{256c^2}+\frac{\left(7d^2\right)\text{Subst}\left(\int \frac{}{(8c-}\right.}{768}\\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{128c^2}+\frac{(7d)\,\text{Subst}\left(\int\right.}{}\\
&= \frac{d\sqrt{c+dx^3}}{96c^2\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24cx^3\left(8c-dx^3\right)}+\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1152c^{5/2}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{128c^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.78

$$\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)+\frac{12\sqrt{c}\sqrt{c+dx^3}\left(4c-dx^3\right)}{dx^6-8cx^3}}{1152c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*(4*c - d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 7*d*ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(115
2*c^(5/2))
```

fricas [A] time = 0.87, size = 278, normalized size = 2.24

$$\left[\frac{7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(cdx^3-4\,c\right.}{2304\left(c^3dx^6-8\,c^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/2304*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 -
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^3*d*x^6 - 8*c^4*x^3), 1/1152*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan
(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*
sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^3*d*
x^6 - 8*c^4*x^3)]
```

giac [A] time = 0.17, size = 113, normalized size = 0.91

$$\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{128\,\sqrt{-c}\,c^2}-\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{1152\,\sqrt{-c}\,c^2}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}d-5\,\sqrt{dx^3+c}\,cd}{96\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/128*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 7/1152*d*arctan(1
/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/96*((d*x^3 + c)^(3/2)*d - 5
*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^2)
```

maple [C] time = 0.21, size = 957, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*d^2*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum((
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c
^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
)-1/256/c^3*d^2*(2/3*(d*x^3+c)^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)
*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*
d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
```

```
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(2/3*(d
*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8c\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^4), x)
```

mupad [B] time = 4.21, size = 117, normalized size = 0.94

$$\frac{\frac{5d\sqrt{dx^3+c}}{32c}-\frac{d\left(dx^3+c\right)^{3/2}}{32c^2}}{3\left(dx^3+c\right)^2-30c\left(dx^3+c\right)+27c^2}+\frac{d\left(\mathrm{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{\sqrt{c^5}}\right)1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\sqrt{dx^3+c}}{3\sqrt{c^5}}\right)7\mathrm{i}}{9}\right)1\mathrm{i}}{128\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((5*d*(c + d*x^3)^(1/2))/(32*c) - (d*(c + d*x^3)^(3/2))/(32*c^2))/(3*(c + d
*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c^2*(c + d*x^3)^(1/2))/(c
^5)^(1/2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*7i)/9)*1i)/
(128*(c^5)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.404 $\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=164

$$\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}+\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

```
[Out] 23/18432*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/1536*d^2*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8
*c)-1/48*(d*x^3+c)^(1/2)/c/x^6/(-d*x^3+8*c)-7/384*d*(d*x^3+c)^(1/2)/c^2/x^3
/(-d*x^3+8*c)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 99, 151, 156, 63, 208, 206}

$$\frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{7/2}}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d^2*Sqrt[c + d*x^3])/(1536*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c*x^
6*(8*c - d*x^3)) - (7*d*Sqrt[c + d*x^3])/(384*c^2*x^3*(8*c - d*x^3)) + (23*
d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(18432*c^(7/2)) - (d^2*ArcTanh[Sq
rt[c + d*x^3]/Sqrt[c]])/(2048*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{x^7\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3(8c-dx)^2}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}+\frac{\mathrm{Subst}\left(\int \frac{7cd+\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}\\
&= -\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}-\frac{\mathrm{Subst}\left(\int \frac{-6c^2d^2-\frac{21}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}\\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}+\frac{\mathrm{Subst}\left(\int \frac{54c^3d^3+45c^2d^4x}{x(8c-dx)\sqrt{c+dx}}\,dx,\right.}{27648c^5d}\\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}+\frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x\right.}{4096c^3}\\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}+\frac{d\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c}\right.}{2048c^3}\\
&= \frac{5d^2\sqrt{c+dx^3}}{1536c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48cx^6\left(8c-dx^3\right)}-\frac{7d\sqrt{c+dx^3}}{384c^2x^3\left(8c-dx^3\right)}+\frac{23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{18432c^{7/2}}-\frac{d^2}{}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(32c^2+28cdx^3-5d^2x^6\right)}{dx^9-8cx^6}+23d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-9d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{18432c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 28*c*d*x^3 - 5*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 23*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 9*d^2*ArcTanh[Sqrt[
c + d*x^3]/Sqrt[c]])/(18432*c^(7/2))
```

fricas [A] time = 0.93, size = 310, normalized size = 1.89

$$\left[\frac{23\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+9\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,cd^2x\right.}{36864\left(c^4dx^9-8\,c^5x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/36864*(23*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 9*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*
x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d^2*x^6 - 28*c^2*d*x^
3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6), 1/18432*(9*(d^3*x^9 -
 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 23*(d^3*x^9 - 8
*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*(5*c*d^2*x
^6 - 28*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^4*d*x^9 - 8*c^5*x^6)]
```

giac [A] time = 0.18, size = 105, normalized size = 0.64

$$\frac{d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}\,c^3}-\frac{23\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{18432\sqrt{-c}\,c^3}-\frac{\sqrt{dx^3+c}\,d^2}{1536\left(dx^3-8\,c\right)c^3}-\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{384\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 23/18432*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/1536*sqrt(d*x^3 + c)
*d^2/((d*x^3 - 8*c)*c^3) - 1/384*(d*x^3 + c)^(3/2)/(c^3*x^6)
```

maple [C] time = 0.19, size = 1020, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/512/c^3*d^3*(-1/3*(d*x^3+c)^(1/2)/(d*x^3-8*c)/d+1/54*I/d^3/c*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
```

```
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256
/c^3*d*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
1/2))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/x^6-1/12*(d*x^3+c)^(1/2)/c*d/x^3+1/12*
d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))-3/4096/c^4*d^3*(2/3*(d*x^3+c)
^(1/2)/d+1/3*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d
-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_a
lpha=RootOf(_Z^3*d-8*c)))+3/4096/c^4*d^2*(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((
d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^7), x)
```

mupad [B] time = 4.45, size = 154, normalized size = 0.94

$$\frac{\frac{d^2\sqrt{d\,x^3+c}}{512\,c}-\frac{19\,d^2\left(d\,x^3+c\right)^{3/2}}{256\,c^2}+\frac{5\,d^2\left(d\,x^3+c\right)^{5/2}}{512\,c^3}}{33\,c\left(d\,x^3+c\right)^2-57\,c^2\left(d\,x^3+c\right)-3\left(d\,x^3+c\right)^3+27\,c^3}+\frac{d^2\left(\operatorname{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)1\mathrm{i}-\frac{\operatorname{atanh}\left(\frac{c^3\sqrt{d\,x^3+c}}{3\sqrt{c^7}}\right)23\mathrm{i}}{9}\right)1\mathrm{i}}{2048\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^7*(8*c - d*x^3)^2),x)
[Out] ((d^2*(c + d*x^3)^(1/2))/(512*c) - (19*d^2*(c + d*x^3)^(3/2))/(256*c^2) + (
5*d^2*(c + d*x^3)^(5/2))/(512*c^3))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3
) - 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(
1/2))*1i - (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*23i)/9)*1i)/(204
8*(c^7)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**7/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

3.405 $\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=663

$$\frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-\frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{21}$$

```
[Out] -76/9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(8/3)+76/9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+76/9*c^(7/6
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^(1/
2)+13/21*x^2*(d*x^3+c)^(1/2)/d^2+1/3*x^5*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+746
/21*c*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+746/63*c^(4/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-373/21*3^(1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*
(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/
2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 663, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {467, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}d^{8/3}}-\frac{76c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9d^{8/3}}+\frac{76c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9d^{8/3}}+\frac{746\sqrt{2}c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{21}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d^2) + (746*c*Sqrt[c + d*x^3])/(21*d^(8/3)*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^5*Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3
)) + (76*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*
x^3]])/(3*Sqrt[3]*d^(8/3)) - (76*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3
*c^(1/6)*Sqrt[c + d*x^3])])/(9*d^(8/3)) + (76*c^(7/6)*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(9*d^(8/3)) - (373*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^
(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(3/4)*d^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (746*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3)
 - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(21*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
 + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x^4\left(5c+\frac{13dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{2\int \frac{x\left(104c^2d+\frac{373}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{2\int \left(-\frac{373cdx}{2\sqrt{c+dx^3}} + \frac{1596c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{21d^3}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{(373c)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{21d^2} - \frac{\left(152c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{(38c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{3d^3} + \frac{(373c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\ldots}{\sqrt{c+dx^3}}}{21d^{7/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{373\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\sqrt[3]{c}+\ldots\right.}{\ldots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\ldots\right.}{\sqrt{c+dx^3}}\right.}{3\sqrt{3}\,d^{8/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d^2} + \frac{746c\sqrt{c+dx^3}}{21d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{76c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\ldots\right.}{\sqrt{c+dx^3}}\right.}{3\sqrt{3}\,d^{8/3}}$$

Mathematica [C] time = 0.20, size = 176, normalized size = 0.27

$$-\frac{373dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+520cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80\ldots}{840d^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] -1/840*(80*(52*c^2*x^2 + 49*c*d*x^5 - 3*d^2*x^8) + 520*c*x^2*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)
] + 373*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3,
 -((d*x^3)/c), (d*x^3)/(8*c)])/(d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 24.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^7}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^7/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^3+c}\,x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.25, size = 2198, normalized size = 3.32

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^2*(2/7*(d*x^3+c)^(1/2)*x^2-2/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
))))+64*c^2/d^2*(-1/24/c*x^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/72*I/c*3^(1/2)*(
-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
```

```
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+16*c/d^2*(-2/3
*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elliptic
E(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^7/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\sqrt{d\,x^3+c}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x^7*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**7*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.406 $\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=641

$$\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x}{2\,3}$$

```
[Out] -5/9*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(
5/3)+5/9*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(5/3)+5/9*c^(1/6)*a
rctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(5/3)*3^(1/2)+
1/3*x^2*(d*x^3+c)^(1/2)/d/(-d*x^3+8*c)+7/3*(d*x^3+c)^(1/2)/d^(5/3)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))+7/9*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-7/6*3^(1/4)*c^(1/3)*(c^(1/3)+d^(1/3)*x
)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c
^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {467, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{7\sqrt{2}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{3\sqrt[4]{3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x}{2\,3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (7*Sqrt[c + d*x^3])/(3*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*
Sqrt[c + d*x^3])/(3*d*(8*c - d*x^3)) + (5*c^(1/6)*ArcTan[(Sqrt[3]*c^(1/6)*(
c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3*Sqrt[3]*d^(5/3)) - (5*c^(1/6)*Ar
cTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]])/(9*d^(5/3)) + (
5*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]])/(9*d^(5/3)) - (7*Sqrt[2 - S
qrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d
^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - S
qrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*S
qrt[3]])/(2*3^(3/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt
[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (7*Sqrt[2]*c^(1/3)*(c^(1/3)
 + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3
])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3
)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3*3^(1/4)*d^(5
/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x
)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{7dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{\int \left(-\frac{7x}{2\sqrt{c+dx^3}} + \frac{30cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{3d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{7\int \frac{x}{\sqrt{c+dx^3}}\,dx}{6d} - \frac{(10c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6d^2} + \frac{7\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{6d^{4/3}} - \frac{\left(5\sqrt[3]{c}\right)\int \frac{1+\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}\,x}{\sqrt[3]{c}}\right)\sqrt{c-}}}{6d^{4/3}}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} - \frac{7\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{}\right.}}}{2\,3^{3/4}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}} - \frac{5\sqrt[6]{c}\tanh^{-1}}{9}$$

$$= \frac{7\sqrt{c+dx^3}}{3d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{3d\left(8c-dx^3\right)} + \frac{5\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{3\sqrt{3}\,d^{5/3}} - \frac{5\sqrt[6]{c}\tanh^{-1}}{9}$$

Mathematica [C] time = 0.15, size = 167, normalized size = 0.26

$$\frac{7dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(c\right.}{240cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(240*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 4.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^4}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.18, size = 1740, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-1/24*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c*x^2-1/72*I/c*3^(1/2)*(-c*d^2)^(1
/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alph
a)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(-2/3*I*3^(1/2)*(-c*
d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(
1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)
^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
```

```
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_a
lpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\,x^4}{\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^4/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^3 + c}}{\left(8\,c - d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
[Out] int((x^4*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{c + dx^3}}{\left(-8c + dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x**4*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.407 $\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=644

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}}}$$

```
[Out] -1/144*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(2/3)+1/144*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(2/3)+1/144*arc
tan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(2/3)*3^
(1/2)+1/24*x^2*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)+1/24*(d*x^3+c)^(1/2)/c/d^(2/3
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/72*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)*3^(3/4)/c^(2/3)/d^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1
/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/48*(c^(1/3)+d^(1/3)*x)*El
lipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(2/3)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.72, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {469, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}c^{5/6}d^{2/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{144c^{5/6}d^{2/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{144c^{5/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\right.\right.}{12\sqrt{2}\sqrt[4]{3}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/(24*c*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(24*c*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d
^(1/3)*x))/Sqrt[c + d*x^3]]/(48*Sqrt[3]*c^(5/6)*d^(2/3)) - ArcTanh[(c^(1/3)
 + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(144*c^(5/6)*d^(2/3)) + ArcTan
h[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(144*c^(5/6)*d^(2/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*3^(3
/4)*c^(2/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(12*Sqrt[2]*3^(1/4)*c^(2/3)*d^(2/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^3}}{\left(8c-dx^3\right)^2}\,dx = \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-c+\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{3cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{24c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{1}{8}\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{48c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96cd} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}\,dx}{48c\sqrt[3]{d}} - \frac{\int \frac{1+\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}}{\left(2-\frac{\sqrt[3]{d}x}{\sqrt[3]{c}}\right)\sqrt{c+dx^3}}\,dx}{96c^{2/3}\sqrt[3]{d}} +$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\right.}}}{16\,3^{3/4}c^{2/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}}{\left(\left(1+\right.\right.}}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}\sqrt{}}\right.}{144c^{5/6}d^{2}}$$

$$= \frac{\sqrt{c+dx^3}}{24cd^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)} + \frac{x^2\sqrt{c+dx^3}}{24c\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{48\sqrt{3}\,c^{5/6}d^{2/3}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\right.}{3\sqrt[6]{c}\sqrt{}}\right.}{144c^{5/6}d^{2}}$$

Mathematica [C] time = 0.10, size = 164, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 5cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 80cx^2\left(c+\right.}{1920c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(8*c - d*x^3)^2,x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 5*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[
2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqrt[
1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(19
20*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.16, size = 882, normalized size = 1.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -1/24*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c*x^2-1/72*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1
/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{d\,x^3+c}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2,x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(8*c - d*x^3)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^3}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(-8*c + d*x**3)**2, x)
```

3.408 $\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=665

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{24\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2}\cdots}{\left(\left(\cdots\right.\right.}}}{32\,3^{3/4}c^{5}\cdots}$$

```
[Out] 1/144*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(11/6)-1/144*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-1/144*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)*
3^(1/2)-1/48*(d*x^3+c)^(1/2)/c^2/x+1/24*(d*x^3+c)^(1/2)/c/x/(-d*x^3+8*c)+1/
48*d^(1/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/144*d^(1/3
)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2
)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c
)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/
2)-1/96*3^(1/4)*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1
-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*
2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.82, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)\middle|-7-4\sqrt{3}\right)}{24\sqrt{2}\,\sqrt[4]{3}\,c^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2}\cdots}{\left(\left(\cdots\right.\right.}}}{32\,3^{3/4}c^{5}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -Sqrt[c + d*x^3]/(48*c^2*x) + (d^(1/3)*Sqrt[c + d*x^3])/(48*c^2*((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x*(8*c - d*x^3)) - (d^(1/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(48*Sqr
t[3]*c^(11/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c
 + d*x^3])])/(144*c^(11/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]
)/(144*c^(11/6)) - (Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(32*3^(3/4)*c^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(24*Sqrt[2]*3^(1/4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\int \frac{-4c-\frac{5dx^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{\int \frac{x\left(40c^2d-2cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{192c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{\int \left(\frac{2cdx}{\sqrt{c+dx^3}} + \frac{24c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{192c^3}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{8c}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{96c^2} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{96c^2} + \cdots$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\,\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\cdots\right.}{32\cdots}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\cdots\right.}{\sqrt{c+dx^3}}\right.}{48\sqrt{3}\,c^{11/6}}\cdots$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{48c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx\left(8c-dx^3\right)} - \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\cdots\right.}{\sqrt{c+dx^3}}\right.}{48\sqrt{3}\,c^{11/6}}\cdots$$

Mathematica [C] time = 0.17, size = 179, normalized size = 0.27

$$\frac{d^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+50cdx^3\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c\left(6c^{\cdots}\right.}{3840c^3\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] (-80*c*(6*c^2 + 5*c*d*x^3 - d^2*x^6) + 50*c*d*x^3*(8*c - d*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d^2*x^6*
(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/(3840*c^3*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^8-16\,cdx^5+64\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^8 - 16*c*d*x^5 + 64*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

maple [C] time = 0.21, size = 2193, normalized size = 3.30

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/8/c*d*(-1/24*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c*x^2-1/72*I/c*3^(1/2)*(-c*d^2)^
(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)))+1/216*I/d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^
(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-(d*x^3+c)^(
1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Elli
pticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Elliptic
```

```
F(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64/c^2*d*(-2/3*I*3^(1/2)*
(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3
)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(dx^3 - 8\,c\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3 + c}}{x^2\,\left(8\,c - d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(1/2)/(x^2*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^2\left(-8c + dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(-d*x**3+8*c)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(-8*c + d*x**3)**2), x)
```

3.409 $\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}+\frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{9216c^{17/6}}+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/}}{((1}}}{48\sqrt{2}}$$

[Out] 17/9216*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)-17/9216*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-17/9216*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*3^(1/2)-7/768*(d*x^3+c)^(1/2)/c^2/x^4-1/96*d*(d*x^3+c)^(1/2)/c^3/x+1/24*(d*x^3+c)^(1/2)/c/x^4/(-d*x^3+8*c)+1/96*d^(4/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/288*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/192*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.94, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{17d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{3072\sqrt{3}c^{17/6}}+\frac{17d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{9216c^{17/6}}-\frac{17d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+d}}{3\sqrt{}}\right)}{9216c^{17/6}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(768*c^2*x^4) - (d*Sqrt[c + d*x^3])/(96*c^3*x) + (d^(4/3)*Sqrt[c + d*x^3])/(96*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^4*(8*c - d*x^3)) - (17*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(3072*Sqrt[3]*c^(17/6)) + (17*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(9216*c^(17/6)) - (17*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9216*c^(17/6)) - (Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(64*3^(3/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(48*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^5\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{-7c-\frac{11dx^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{\int \frac{64c^2d+\frac{35}{2}cd^2x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{768c^3}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-460c^3d^2+32c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\int \left(-\frac{32c^2d^2x}{\sqrt{c+dx^3}} - \frac{204c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{6144c^5}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{192c^3} + \frac{\left(17d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}}{512c^2}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{(17d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{6144c^3} + \frac{d^{5/3}}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{17d^{4/3}\,\mathrm{ta}}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{768c^2x^4} - \frac{d\sqrt{c+dx^3}}{96c^3x} + \frac{d^{4/3}\sqrt{c+dx^3}}{96c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+dx^3}}{24cx^4\left(8c-dx^3\right)} - \frac{17d^{4/3}\,\mathrm{ta}}{}$$

Mathematica [C] time = 0.14, size = 199, normalized size = 0.29

$$-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{7680c^4\sqrt{c+dx^3}} + \frac{115d^2x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{24576c^3\sqrt{c+dx^3}} + \sqrt{c+dx^3}\left(-\frac{d^2x^2}{1536c^3\left(dx^3-8c\right)} -\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c^2*x^4) - (5*d)/(512*c^3*x) - (d^2*x^2)/(1536*c
^3*(-8*c + d*x^3))) + (115*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
```

```
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(24576*c^3*Sqrt[c + d*x^3]) - (d^3*x^5
*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(7680*c^4*Sqrt[c + d*x^3])
```

fricas [F] time = 3.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^{11}-16\,cdx^8+64\,c^2x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^11 - 16*c*d*x^8 + 64*c^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

maple [C] time = 0.18, size = 2671, normalized size = 3.89

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x)
[Out] 1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*(d*x^3+c)^(1/2)/c*d/x-1/8*I*d/c*3^(1
/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2
)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d^2*(-1/24*(d*x^3+c)^(1/2)/(d*x
^3-8*c)/c*x^2-1/72*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-
I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^
(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/216*I/d^3/
c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_al
```

```
pha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/
2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1
/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))+1/256/c^3*d*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2))))-1/256/c^3*d^2*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^5\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^5*(8*c - d*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**5/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.410 $\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=711

$$-\frac{13d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}}+\frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}}-\frac{13d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{36864c^{23/6}}+\frac{d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{}}{\left((1+\sqrt{}\right.}}}{2688\sqrt{2}\sqrt[4]{}}$$

```
[Out] 13/36864*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-13/36864*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-13
/36864*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-5/672*(d*x^3+c)^(1/2)/c^2/x^7-53/21504*d*(d*x^3+c)^(1/2)/c
^3/x^4-1/5376*d^2*(d*x^3+c)^(1/2)/c^4/x+1/24*(d*x^3+c)^(1/2)/c/x^7/(-d*x^3+
8*c)+1/5376*d^(7/3)*(d*x^3+c)^(1/2)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/1
6128*3^(3/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^
(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)*2^(1
/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)-1/10752*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/
3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1
/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x
+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(11/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d
^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {469, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{d^2\sqrt{c+dx^3}}{5376c^4x}-\frac{13d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{12288\sqrt{3}c^{23/6}}+\frac{13d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{36864c^{23/6}}-\frac{13d^{7/3}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-5*Sqrt[c + d*x^3])/(672*c^2*x^7) - (53*d*Sqrt[c + d*x^3])/(21504*c^3*x^4)
 - (d^2*Sqrt[c + d*x^3])/(5376*c^4*x) + (d^(7/3)*Sqrt[c + d*x^3])/(5376*c^4
*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(24*c*x^7*(8*c - d*
x^3)) - (13*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c +
 d*x^3]])/(12288*Sqrt[3]*c^(23/6)) + (13*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)
*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(36864*c^(23/6)) - (13*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(36864*c^(23/6)) - (Sqrt[2 - Sqrt[3]]*d^(7/3)
*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3)
 + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(3584*
3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c
^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c
^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(2688*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[
```

```
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

```
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-10c-\frac{17dx^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{106c^2d+55cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1344c^3}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\int \frac{-64c^3d^2-265c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{43008c^5}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \frac{x\left(2440c^4d^3-32c^3d^4x^3\right.}{\left(8c-dx^3\right)\sqrt{c+dx^3}}}{344064c^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{\int \left(\frac{32c^3d^3x}{\sqrt{c+dx^3}} + \frac{218}{\left(8c-dx^3\right.}\right.}{344064c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} + \frac{d^3\int \frac{x}{\sqrt{c+dx^3}}\,dx}{10752c^4} + \cdots$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{\sqrt{c+dx^3}}{24cx^7\left(8c-dx^3\right)} - \frac{\left(13d^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-\cdots}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^2}{\cdots}\right.}}{24576c}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{\cdots}}{24cx^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{\cdots}}{24cx^7}$$

$$= -\frac{5\sqrt{c+dx^3}}{672c^2x^7} - \frac{53d\sqrt{c+dx^3}}{21504c^3x^4} - \frac{d^2\sqrt{c+dx^3}}{5376c^4x} + \frac{d^{7/3}\sqrt{c+dx^3}}{5376c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{\cdots}}{24cx^7}$$

Mathematica [C] time = 0.17, size = 209, normalized size = 0.29

$$\frac{1525cd^3x^9\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 8\left(d^4x^{12}\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right.\cdots}{3440640c^5x^7\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (1525*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3
, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(384*c^4 + 648*c^3*d*x^3 + 243*c^2
*d^2*x^6 - 25*c*d^3*x^9 - 4*d^4*x^12) + d^4*x^12*(8*c - d*x^3)*Sqrt[1 + (d*
x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3440640*
c^5*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 8.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^2x^{14}-16\,cdx^{11}+64\,c^2x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^2*x^14 - 16*c*d*x^11 + 64*c^2*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

maple [C] time = 0.19, size = 3169, normalized size = 4.46

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/x^4-3/8*(d*x^3+c)^(1/2)/c*d/x-1/8*I*d/c*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^3*(-1/24*(d*x^3+c)^(1/2)/
(d*x^3-8*c)/c*x^2-1/72*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2
)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d
^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/216*I/
```

```
d^3/c*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)/x^7-3/56*(d*x^3+c)^(1/
2)/c*d/x^4+15/112*(d*x^3+c)^(1/2)/c^2*d^2/x+5/112*I*d^2/c^2*3^(1/2)*(-c*d^2
)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*
(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2))))+3/4096/c^4*d^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^
2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2))))-3/4096/c^4*d^3*(-2/3*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)))+1/3*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(dx^3-8\,c\right)^2 x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((d*x^3 - 8*c)^2*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^3+c}}{x^8\,{\left(8\,c-d\,x^3\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^8*(8*c - d*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**8/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.411 $\int \frac{x^{11}(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{1664c^3\sqrt{c+dx^3}}{d^4}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

```
[Out] 3/7*x^6*(d*x^3+c)^(3/2)/d^2+1/3*x^9*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)+2/21*c*(
d*x^3+c)^(3/2)*(51*d*x^3+694*c)/d^4-4992*c^(7/2)*arctanh(1/3*(d*x^3+c)^(1/2
)/c^(1/2))/d^4+1664*c^3*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {446, 97, 153, 147, 50, 63, 206}

$$\frac{1664c^3\sqrt{c+dx^3}}{d^4}-\frac{4992c^{7/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^4}+\frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2}+\frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4}+\frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (1664*c^3*Sqrt[c + d*x^3])/d^4 + (3*x^6*(c + d*x^3)^(3/2))/(7*d^2) + (x^9*(
c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (2*c*(c + d*x^3)^(3/2)*(694*c + 51*
d*x^3))/(21*d^4) - (4992*c^(7/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
```

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}\left(3c+\frac{9dx}{2}\right)}{8c-dx}\,dx, x, x^3\right)}{3d}$$

$$= \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}\left(-72c^2d-\frac{255}{2}cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{21d^3}$$

$$= \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^3\right)}{21d^4} - \frac{\left(832c^3\right)\text{Subst}\,(}{}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^{\cdot}\right.}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^{\cdot}\right.}{21d^4}$$

$$= \frac{1664c^3\sqrt{c+dx^3}}{d^4} + \frac{3x^6\left(c+dx^3\right)^{3/2}}{7d^2} + \frac{x^9\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2c\left(c+dx^3\right)^{3/2}\left(694c+51dx^{\cdot}\right.}{21d^4}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.83

$$\frac{2\left(52416c^{7/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \sqrt{c+dx^3}\left(-145328c^4+12206c^3dx^3+301c^2d^2x^6+16cd^3x^9+d^4x^1\right.\right.}{21d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(Sqrt[c + d*x^3]*(-145328*c^4 + 12206*c^3*d*x^3 + 301*c^2*d^2*x^6 + 16*c
*d^3*x^9 + d^4*x^12) + 52416*c^(7/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])]))/(21*d^4*(-8*c + d*x^3))
```

fricas [A] time = 0.91, size = 239, normalized size = 1.78

$$\left[\frac{2\left(26208\left(c^3dx^3-8c^4\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + \left(d^4x^{12}+16cd^3x^9+301c^2d^2x^6+12206c^3dx^3-14532\right.\right.}{21\left(d^5x^3-8cd^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/21*(26208*(c^3*d*x^3 - 8*c^4)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqr
t(c) + 10*c)/(d*x^3 - 8*c)) + (d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 +
12206*c^3*d*x^3 - 145328*c^4)*sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4), 2/21*(5
2416*(c^3*d*x^3 - 8*c^4)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) +
```

```
(d^4*x^12 + 16*c*d^3*x^9 + 301*c^2*d^2*x^6 + 12206*c^3*d*x^3 - 145328*c^4)*
sqrt(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

giac [A] time = 0.17, size = 127, normalized size = 0.95

$$\frac{4992\,c^4\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^4}-\frac{1536\sqrt{dx^3+c}\,c^4}{\left(dx^3-8\,c\right)d^4}+\frac{2\left(\left(dx^3+c\right)^{\frac{7}{2}}d^{24}+21\left(dx^3+c\right)^{\frac{5}{2}}cd^{24}+448\left(dx^3+c\right)^{\frac{3}{2}}c^2d^{24}+1568\right.}{21\,d^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 4992*c^4*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 1536*sqrt(d*
x^3 + c)*c^4/((d*x^3 - 8*c)*d^4) + 2/21*((d*x^3 + c)^(7/2)*d^24 + 21*(d*x^3
 + c)^(5/2)*c*d^24 + 448*(d*x^3 + c)^(3/2)*c^2*d^24 + 15680*sqrt(d*x^3 + c)
*c^3*d^24)/d^28
```

maple [C] time = 0.28, size = 998, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^3*(d*(2/21*(d*x^3+c)^(1/2)*d*x^9+16/105*(d*x^3+c)^(1/2)*c*x^6+2/105*(d*
x^3+c)^(1/2)*c^2/d*x^3-4/105*(d*x^3+c)^(1/2)*c^3/d^2)+32/15*c/d*(d*x^3+c)^(
5/2))+512*c^3/d^3*(-3*c/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/3*(d*x^3+c)^(1/2)/d
+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*
d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d
^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_al
pha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/
2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1
/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)
*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.18, size = 119, normalized size = 0.89

$$\frac{2\left(26208\,c^{\frac{7}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+\left(dx^3+c\right)^{\frac{7}{2}}+21\left(dx^3+c\right)^{\frac{5}{2}}c+448\left(dx^3+c\right)^{\frac{3}{2}}c^2+15680\sqrt{dx^3+c}\,c^3-\frac{16128\sqrt{dx^3}}{dx^3-8}\right.}{21\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/21*(26208*c^(7/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*
sqrt(c))) + (d*x^3 + c)^(7/2) + 21*(d*x^3 + c)^(5/2)*c + 448*(d*x^3 + c)^(3
/2)*c^2 + 15680*sqrt(d*x^3 + c)*c^3 - 16128*sqrt(d*x^3 + c)*c^4/(d*x^3 - 8*
c))/d^4
```

mupad [B] time = 4.10, size = 147, normalized size = 1.10

$$\frac{2496\,c^{7/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^4}+\frac{32300\,c^3\,\sqrt{d\,x^3+c}}{21\,d^4}+\frac{2\,x^9\,\sqrt{d\,x^3+c}}{21\,d}+\frac{16\,c\,x^6\,\sqrt{d\,x^3+c}}{7\,d^2}+\frac{986\,c^2\,x^3\,\sqrt{d\,x^3+}}{21\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2496*c^(7/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3
)))/d^4 + (32300*c^3*(c + d*x^3)^(1/2))/(21*d^4) + (2*x^9*(c + d*x^3)^(1/2)
)/(21*d) + (16*c*x^6*(c + d*x^3)^(1/2))/(7*d^2) + (986*c^2*x^3*(c + d*x^3)^
(1/2))/(21*d^3) + (1536*c^4*(c + d*x^3)^(1/2))/(d^4*(8*c - d*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.412 $\int \frac{x^8(c+dx^3)^{3/2}}{(8c-dx^3)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}+\frac{160c^2\sqrt{c+dx^3}}{d^3}+\frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)}+\frac{160c\left(c+dx^3\right)^{3/2}}{27d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

```
[Out] 160/27*c*(d*x^3+c)^(3/2)/d^3+2/15*(d*x^3+c)^(5/2)/d^3+64/27*c*(d*x^3+c)^(5/
2)/d^3/(-d*x^3+8*c)-480*c^(5/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3+16
0*c^2*(d*x^3+c)^(1/2)/d^3
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {446, 89, 80, 50, 63, 206}

$$\frac{160c^2\sqrt{c+dx^3}}{d^3}-\frac{480c^{5/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^3}+\frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)}+\frac{160c\left(c+dx^3\right)^{3/2}}{27d^3}+\frac{2\left(c+dx^3\right)^{5/2}}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (160*c^2*Sqrt[c + d*x^3])/d^3 + (160*c*(c + d*x^3)^(3/2))/(27*d^3) + (2*(c
+ d*x^3)^(5/2))/(15*d^3) + (64*c*(c + d*x^3)^(5/2))/(27*d^3*(8*c - d*x^3))
- (480*c^(5/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
```

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$

$$= \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(168c^2d+9cd^2x\right)}{8c-dx}\,dx, x, x^3\right)}{27cd^3}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{(80c)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d^2}$$

$$= \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(80c^2\right)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x\right.}{d^2}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(720c^3\right)\,\text{Sub}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{\left(1440c^3\right)\,\text{Su}}{}$$

$$= \frac{160c^2\sqrt{c+dx^3}}{d^3} + \frac{160c\left(c+dx^3\right)^{3/2}}{27d^3} + \frac{2\left(c+dx^3\right)^{5/2}}{15d^3} + \frac{64c\left(c+dx^3\right)^{5/2}}{27d^3\left(8c-dx^3\right)} - \frac{480c^{5/2}\,\text{tanh}}{d}$$

Mathematica [A] time = 0.13, size = 102, normalized size = 0.86

$$\frac{21600c^{5/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\left(-29944c^3+2515c^2dx^3+62cd^2x^6+3d^3x^9\right)}{45d^3\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*Sqrt[c + d*x^3]*(-29944*c^3 + 2515*c^2*d*x^3 + 62*c*d^2*x^6 + 3*d^3*x^9)
 + 21600*c^(5/2)*(8*c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(45*d^
3*(-8*c + d*x^3))
```

fricas [A] time = 0.72, size = 219, normalized size = 1.84

$$\left[\frac{2\left(5400\left(c^2dx^3-8c^3\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+\left(3d^3x^9+62cd^2x^6+2515c^2dx^3-29944c^3\right)\sqrt{dx^3+c}\right)}{45\left(d^4x^3-8cd^3\right)}, \frac{2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [2/45*(5400*(c^2*d*x^3 - 8*c^3)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + (3*d^3*x^9 + 62*c*d^2*x^6 + 2515*c^2*d*x^3 - 2
9944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3), 2/45*(10800*(c^2*d*x^3 - 8*
c^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (3*d^3*x^9 + 62*c*d^
2*x^6 + 2515*c^2*d*x^3 - 29944*c^3)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

giac [A] time = 0.20, size = 111, normalized size = 0.93

$$\frac{480\,c^3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d^3}-\frac{192\sqrt{dx^3+c}\,c^3}{\left(dx^3-8c\right)d^3}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}d^{12}+80\left(dx^3+c\right)^{\frac{3}{2}}cd^{12}+3120\sqrt{dx^3+c}\,c^2d^{12}\right)}{45\,d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 480*c^3*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 192*sqrt(d*x^
3 + c)*c^3/((d*x^3 - 8*c)*d^3) + 2/45*(3*(d*x^3 + c)^(5/2)*d^12 + 80*(d*x^3
 + c)^(3/2)*c*d^12 + 3120*sqrt(d*x^3 + c)*c^2*d^12)/d^15
```

maple [C] time = 0.19, size = 920, normalized size = 7.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/d^3+64*c^2/d^2*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3
*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+16*c/d^2*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*
(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
```

```
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.39, size = 107, normalized size = 0.90

$$\frac{2\left(5400\,c^{\frac{5}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+3\left(dx^3+c\right)^{\frac{5}{2}}+80\left(dx^3+c\right)^{\frac{3}{2}}c+3120\,\sqrt{dx^3+c}\,c^2-\frac{4320\,\sqrt{dx^3+c}\,c^3}{dx^3-8\,c}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 2/45*(5400*c^(5/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 3*(d*x^3 + c)^(5/2) + 80*(d*x^3 + c)^(3/2)*c + 3120*sqrt(d*x^3 +
 c)*c^2 - 4320*sqrt(d*x^3 + c)*c^3/(d*x^3 - 8*c))/d^3
```

mupad [B] time = 4.05, size = 127, normalized size = 1.07

$$\frac{240\,c^{5/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^3}+\frac{6406\,c^2\,\sqrt{d\,x^3+c}}{45\,d^3}+\frac{2\,x^6\,\sqrt{d\,x^3+c}}{15\,d}+\frac{172\,c\,x^3\,\sqrt{d\,x^3+c}}{45\,d^2}+\frac{192\,c^3\,\sqrt{d\,x^3+c}}{d^3\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (240*c^(5/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)
))/d^3 + (6406*c^2*(c + d*x^3)^(1/2))/(45*d^3) + (2*x^6*(c + d*x^3)^(1/2))/
(15*d) + (172*c*x^3*(c + d*x^3)^(1/2))/(45*d^2) + (192*c^3*(c + d*x^3)^(1/2
))/(d^3*(8*c - d*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.413 $\int \frac{x^5(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal. Leaf size=97

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

```
[Out] 14/27*(d*x^3+c)^(3/2)/d^2+8/27*(d*x^3+c)^(5/2)/d^2/(-d*x^3+8*c)-42*c^(3/2)*
arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2+14*c*(d*x^3+c)^(1/2)/d^2
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 50, 63, 206}

$$-\frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}+\frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)}+\frac{14\left(c+dx^3\right)^{3/2}}{27d^2}+\frac{14c\sqrt{c+dx^3}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (14*c*Sqrt[c + d*x^3])/d^2 + (14*(c + d*x^3)^(3/2))/(27*d^2) + (8*(c + d*x^
3)^(5/2))/(27*d^2*(8*c - d*x^3)) - (42*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*S
qrt[c])])/d^2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(8c-dx)^2}\,dx, x, x^3\right)$$
$$= \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{7\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{8c-dx}\,dx, x, x^3\right)}{9d}$$
$$= \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{(7c)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx, x, x^3\right)}{d}$$
$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(63c^2\right)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x\right.}{d}$$
$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{\left(126c^2\right)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c}\right.}{d^2}$$
$$= \frac{14c\sqrt{c+dx^3}}{d^2} + \frac{14\left(c+dx^3\right)^{3/2}}{27d^2} + \frac{8\left(c+dx^3\right)^{5/2}}{27d^2\left(8c-dx^3\right)} - \frac{42c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^2}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{378c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+2\sqrt{c+dx^3}\left(-524c^2+44cdx^3+d^2x^6\right)}{9d^2\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
[Out] (2*Sqrt[c + d*x^3]*(-524*c^2 + 44*c*d*x^3 + d^2*x^6) + 378*c^(3/2)*(8*c - d
*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(9*d^2*(-8*c + d*x^3))
```

fricas [A] time = 0.62, size = 192, normalized size = 1.98

$$\left[\frac{189\left(cdx^3-8c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(d^2x^6+44cdx^3-524c^2\right)\sqrt{dx^3+c}}{9\left(d^3x^3-8cd^2\right)}, \frac{2\left(189\left(cdx^3-8c^2\right)\sqrt{-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(189*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
+ 10*c)/(d*x^3 - 8*c)) + 2*(d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c)
)/(d^3*x^3 - 8*c*d^2), 2/9*(189*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(
d*x^3 + c)*sqrt(-c)/c) + (d^2*x^6 + 44*c*d*x^3 - 524*c^2)*sqrt(d*x^3 + c))/
(d^3*x^3 - 8*c*d^2)]
```

giac [A] time = 0.17, size = 93, normalized size = 0.96

$$\frac{42\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{\sqrt{-c}\,d^2}-\frac{24\,\sqrt{dx^3+c}\,c^2}{\left(dx^3-8\,c\right)d^2}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}d^4+51\,\sqrt{dx^3+c}\,cd^4\right)}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 42*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^2) - 24*sqrt(d*x^3
+ c)*c^2/((d*x^3 - 8*c)*d^2) + 2/9*((d*x^3 + c)^(3/2)*d^4 + 51*sqrt(d*x^3 +
 c)*c*d^4)/d^6
```

maple [C] time = 0.17, size = 902, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2
^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^
(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c)))+1/d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
)
```

maxima [A] time = 1.33, size = 93, normalized size = 0.96

$$\frac{189\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+2\left(dx^3+c\right)^{\frac{3}{2}}+102\,\sqrt{dx^3+c}\,c-\frac{216\,\sqrt{dx^3+c}\,c^2}{dx^3-8\,c}}{9\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(189*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 2*(d*x^3 + c)^(3/2) + 102*sqrt(d*x^3 + c)*c - 216*sqrt(d*x^3 + c)*
c^2/(d*x^3 - 8*c))/d^2
```

mupad [B] time = 4.04, size = 107, normalized size = 1.10

$$\frac{104\,c\,\sqrt{d\,x^3+c}}{9\,d^2}+\frac{21\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{d^2}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d}+\frac{24\,c^2\,\sqrt{d\,x^3+c}}{d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (104*c*(c + d*x^3)^(1/2))/(9*d^2) + (21*c^(3/2)*log((10*c + d*x^3 - 6*c^(1/
2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/d^2 + (2*x^3*(c + d*x^3)^(1/2))/(9*d)
 + (24*c^2*(c + d*x^3)^(1/2))/(d^2*(8*c - d*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.414 $\int \frac{x^2(c+dx^3)^{3/2}}{(8c-dx^3)^2} dx$

Optimal. Leaf size=77

$$\frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{\sqrt{c+dx^3}}{d} - \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

```
[Out] 1/3*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-3*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c
^(1/2)/d+(d*x^3+c)^(1/2)/d
```

Rubi [A] time = 0.06, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = $\{444, 47, 50, 63, 206\}$

$$\frac{(c+dx^3)^{3/2}}{3d(8c-dx^3)} + \frac{\sqrt{c+dx^3}}{d} - \frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] Sqrt[c + d*x^3]/d + (c + d*x^3)^(3/2)/(3*d*(8*c - d*x^3)) - (3*Sqrt[c]*ArcT
anh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{(8c-dx)^2}\,dx,x,x^3\right)\\
&= \frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{8c-dx}\,dx,x,x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{1}{2}(9c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{(9c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{d}\\
&= \frac{\sqrt{c+dx^3}}{d}+\frac{\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)}-\frac{3\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.56

$$\frac{2\left(c+dx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{dx^3+c}{9c}\right)}{1215c^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (2*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (c + d*x^3)/(9*c)])/(12
15*c^2*d)
```

fricas [A] time = 0.79, size = 162, normalized size = 2.10

$$\left[\frac{9\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+2\left(2dx^3-25c\right)\sqrt{dx^3+c}}{6\left(d^2x^3-8cd\right)},\ \frac{9\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)}{3\left(d^2x^3-8cd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) + 2*(2*d*x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d), 1
/3*(9*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*d*
x^3 - 25*c)*sqrt(d*x^3 + c))/(d^2*x^3 - 8*c*d)]
```

giac [A] time = 0.17, size = 69, normalized size = 0.90

$$\frac{3c\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d}+\frac{2\sqrt{dx^3+c}}{3d}-\frac{3\sqrt{dx^3+c}\,c}{\left(dx^3-8c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 3*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) + 2/3*sqrt(d*x^3 + c)
/d - 3*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d)
```

maple [C] time = 0.18, size = 451, normalized size = 5.86

$$-\frac{3\sqrt{d\,x^3+c}\,c}{\left(d\,x^3-8c\right)d}+\frac{2\sqrt{d\,x^3+c}}{3d}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d\right.}{d}\right.}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3*2^(1/2)*
sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.31, size = 79, normalized size = 1.03

$$\frac{9\sqrt{c}\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)+4\sqrt{dx^3+c}-\frac{18\sqrt{dx^3+c}\,c}{dx^3-8c}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(9*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(
c))) + 4*sqrt(d*x^3 + c) - 18*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d
```

mupad [B] time = 3.99, size = 87, normalized size = 1.13

$$\frac{2\sqrt{d\,x^3+c}}{3\,d}+\frac{3\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{2\,d}+\frac{3\,c\sqrt{d\,x^3+c}}{d\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d) + (3*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*(c +
 d*x^3)^(1/2))/(8*c - d*x^3)))/(2*d) + (3*c*(c + d*x^3)^(1/2))/(d*(8*c - d*
x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.415 $\int \frac{(c+dx^3)^{3/2}}{x(8c-dx^3)^2} \, dx$

Optimal. Leaf size=85

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

```
[Out] -3/32*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)-1/96*arctanh((d*x^3+c)^(
1/2)/c^(1/2))/c^(1/2)+3/8*(d*x^3+c)^(1/2)/(-d*x^3+8*c)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 98, 156, 63, 208, 206}

$$\frac{3\sqrt{c+dx^3}}{8(8c-dx^3)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (3*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) - (3*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[
c])])/(32*Sqrt[c]) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(8c-dx)^2}\,dx, x, x^3\right)$$
$$= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c^2d+\frac{7}{2}cd^2x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{24cd}$$
$$= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} + \frac{1}{192}\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right) - \frac{1}{64}(9d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+d}}\right.$$
$$= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{9}{32}\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right) + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right.}{96d}$$
$$= \frac{3\sqrt{c+dx^3}}{8\left(8c-dx^3\right)} - \frac{3\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{32\sqrt{c}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 100, normalized size = 1.18

$$\frac{36\sqrt{c}\sqrt{c+dx^3} + \left(9dx^3-72c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) + \left(dx^3-8c\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96\sqrt{c}\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x]
```

```
[Out] (36*Sqrt[c]*Sqrt[c + d*x^3] + (-72*c + 9*d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*
Sqrt[c])] + (-8*c + d*x^3)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(96*Sqrt[c]*(8
*c - d*x^3))
```

fricas [A] time = 0.63, size = 220, normalized size = 2.59

$$\left[\frac{9\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + \left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 72\sqrt{dx^3+c}\,c}{192\left(cdx^3-8c^2\right)}, \frac{\left(dx^3-8\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/192*(9*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + (d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*s
qrt(c) + 2*c)/x^3) - 72*sqrt(d*x^3 + c)*c)/(c*d*x^3 - 8*c^2), 1/96*((d*x^3
- 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 9*(d*x^3 - 8*c)*sqrt(-
c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 36*sqrt(d*x^3 + c)*c)/(c*d*x^3
- 8*c^2)]
```

giac [A] time = 0.17, size = 70, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}}+\frac{3\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{32\sqrt{-c}}-\frac{3\sqrt{dx^3+c}}{8\left(dx^3-8c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/sqrt(-c) + 3/32*arctan(1/3*sqrt(d*x^3
 + c)/sqrt(-c))/sqrt(-c) - 3/8*sqrt(d*x^3 + c)/(d*x^3 - 8*c)
```

maple [C] time = 0.17, size = 956, normalized size = 11.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/8/c*d*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*I/d^3
*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c)))-1/64/c^2*d*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/
d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d
^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*
d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*d-8*c))+1/64/c^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^
(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x), x)
```

mupad [B] time = 4.72, size = 101, normalized size = 1.19

$$\frac{3\sqrt{d\,x^3+c}}{8\,\left(8\,c-d\,x^3\right)}+\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)\,\left(10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}\right)^9}{x^6\,\left(8\,c-d\,x^3\right)^9}\right)}{192\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(8*c - d*x^3)^2),x)
```

```
[Out] (3*(c + d*x^3)^(1/2))/(8*(8*c - d*x^3)) + log((((c + d*x^3)^(1/2) - c^(1/2)
)^3*((c + d*x^3)^(1/2) + c^(1/2))*(10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/
2))^9)/(x^6*(8*c - d*x^3)^9))/(192*c^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.416 $\int \frac{(c+dx^3)^{3/2}}{x^4(8c-dx^3)^2}\,dx$

Optimal. Leaf size=121

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

```
[Out] 3/128*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)-7/384*d*arctanh((d*x^3
+c)^(1/2)/c^(1/2))/c^(3/2)+5/96*d*(d*x^3+c)^(1/2)/c/(-d*x^3+8*c)-1/24*(d*x^
3+c)^(1/2)/x^3/(-d*x^3+8*c)
```

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}}-\frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}+\frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(96*c*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*x^3*(8*c -
 d*x^3)) + (3*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(128*c^(3/2)) - (7*d*
ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-14c^2d-\frac{19}{2}cd^2x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{126c^3d^2+45c^2d^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^3d}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{(7d)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c} + \frac{\left(9d^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{7\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c} + \frac{(9d)\,\text{Subst}\left(\right.}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{96c\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24x^3\left(8c-dx^3\right)} + \frac{3d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{128c^{3/2}} - \frac{7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{3/2}}$$

Mathematica [A] time = 0.16, size = 97, normalized size = 0.80

$$\frac{9d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 7d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) + \frac{4\sqrt{c}\sqrt{c+dx^3}\left(4c-5dx^3\right)}{dx^6-8cx^3}}{384c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*(4*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 9*d*ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 7*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(38
4*c^(3/2))
```

fricas [A] time = 0.60, size = 280, normalized size = 2.31

$$\left[\frac{9\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+7\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-8\left(5\,cdx^3-4\,c^2\right)}{768\left(c^2dx^6-8\,c^3x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(9*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt
(c) + 10*c)/(d*x^3 - 8*c)) + 7*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 - 2
*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c
))/(c^2*d*x^6 - 8*c^3*x^3), 1/384*(7*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(
sqrt(d*x^3 + c)*sqrt(-c)/c) - 9*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) - 4*(5*c*d*x^3 - 4*c^2)*sqrt(d*x^3 + c))/(c^2*d*
x^6 - 8*c^3*x^3)]
```

giac [A] time = 0.19, size = 114, normalized size = 0.94

$$\frac{7\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c}-\frac{3\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{128\,\sqrt{-c}\,c}-\frac{5\left(dx^3+c\right)^{\frac{3}{2}}d-9\,\sqrt{dx^3+c}\,cd}{96\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 7/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 3/128*d*arctan(1/3*
sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c) - 1/96*(5*(d*x^3 + c)^(3/2)*d - 9*sq
rt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c)
```

maple [C] time = 0.19, size = 1014, normalized size = 8.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/64/c^2*d^2*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2*
I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(
1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))-1/256/c^3*d^2*(2/9*(d*x^3+c)^(1/2)
*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(
```

```
2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*(
(x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*
(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-
c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(2/9*(d*x^3+c)^(
1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2
)))
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^4), x)
```

mupad [**B**] time = 4.24, size = 110, normalized size = 0.91

$$\frac{\frac{9\,d\,\sqrt{d\,x^3+c}}{32}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{32\,c}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}+\frac{d\,\left(\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{\sqrt{c^3}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^3}}\right)\,9{}\mathrm{i}}{7}\right)\,7{}\mathrm{i}}{384\,\sqrt{c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(8*c - d*x^3)^2),x)
```

```
[Out] ((9*d*(c + d*x^3)^(1/2))/32 - (5*d*(c + d*x^3)^(3/2))/(32*c))/(3*(c + d*x^3
)^2 - 30*c*(c + d*x^3) + 27*c^2) + (d*(atanh((c*(c + d*x^3)^(1/2))/(c^3)^(1
/2))*1i - (atanh((c*(c + d*x^3)^(1/2))/(3*(c^3)^(1/2)))*9i)/7)*7i)/(384*(c^
3)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}+\frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)}-\frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)}$$

```
[Out] 15/2048*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-17/2048*d^2*arctan
h((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)+7/512*d^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*
c)-1/48*(d*x^3+c)^(1/2)/x^6/(-d*x^3+8*c)-23/384*d*(d*x^3+c)^(1/2)/c/x^3/(-d
*x^3+8*c)
```

Rubi [A] time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 98, 151, 156, 63, 208, 206}

$$\frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)}+\frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}}-\frac{17d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{5/2}}-\frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] (7*d^2*Sqrt[c + d*x^3])/(512*c^2*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*x^6*(
8*c - d*x^3)) - (23*d*Sqrt[c + d*x^3])/(384*c*x^3*(8*c - d*x^3)) + (15*d^2*
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(2048*c^(5/2)) - (17*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(2048*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^7\left(8c-dx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(8c-dx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-23c^2d-\frac{37}{2}cd^2x}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c}$$

$$= -\frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{102c^3d^2+\frac{69}{2}c^2d^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{384c^3}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-918c^4d^3-189c^3d^4x}{x(8c-dx)\sqrt{c+dx}}\,d\right.}{27648c^5d}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{\left(17d^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx\right.}{4096c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{(17d)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x\right.}{2048c^2}$$

$$= \frac{7d^2\sqrt{c+dx^3}}{512c^2\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48x^6\left(8c-dx^3\right)} - \frac{23d\sqrt{c+dx^3}}{384cx^3\left(8c-dx^3\right)} + \frac{15d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2048c^{5/2}} - \frac{17}{}$$

Mathematica [A] time = 0.24, size = 112, normalized size = 0.70

$$\frac{\frac{4\sqrt{c}\sqrt{c+dx^3}(32c^2+92cdx^3-21d^2x^6)}{dx^9-8cx^6}+45d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-51d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x]
```

```
[Out] ((4*Sqrt[c]*Sqrt[c + d*x^3]*(32*c^2 + 92*c*d*x^3 - 21*d^2*x^6))/(-8*c*x^6 +
 d*x^9) + 45*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 51*d^2*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(6144*c^(5/2))
```

fricas [A] time = 0.67, size = 310, normalized size = 1.93

$$\left[\frac{45\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+51\left(d^3x^9-8cd^2x^6\right)\sqrt{c}\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right)-8\left(21cd^2x^6-\right.}{12288\left(c^3dx^9-8c^4x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] [1/12288*(45*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)
*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 51*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d
*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 8*(21*c*d^2*x^6 - 92*c^2*d*x
^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6), 1/6144*(51*(d^3*x^9
- 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 45*(d^3*x^9 -
8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 4*(21*c*d^2*
x^6 - 92*c^2*d*x^3 - 32*c^3)*sqrt(d*x^3 + c))/(c^3*d*x^9 - 8*c^4*x^6)]
```

giac [A] time = 0.18, size = 129, normalized size = 0.80

$$\frac{17\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\sqrt{-c}\,c^2}-\frac{15\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2048\sqrt{-c}\,c^2}-\frac{3\sqrt{dx^3+c}\,d^2}{512\left(dx^3-8c\right)c^2}-\frac{3\left(dx^3+c\right)^{\frac{3}{2}}d^2-2\sqrt{dx^3+c}\,cd^2}{384\,c^2d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] 17/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 15/2048*d^2*a
rctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 3/512*sqrt(d*x^3 + c)*
d^2/((d*x^3 - 8*c)*c^2) - 1/384*(3*(d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 + c
)*c*d^2)/(c^2*d^2*x^6)
```

maple [C] time = 0.19, size = 1075, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/512/c^3*d^3*(-3*(d*x^3+c)^(1/2)/(d*x^3-8*c)*c/d+2/3*(d*x^3+c)^(1/2)/d+1/2
*I/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
```

```
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d
-c^(1/2)*d*arctanh((d*x^3+c)^(1/2)/c^(1/2)))+1/64/c^2*(-1/6*c*(d*x^3+c)^(1/
2)/x^6-5/12*d*(d*x^3+c)^(1/2)/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/
c^(1/2))-3/4096/c^4*d^3*(2/9*(d*x^3+c)^(1/2)*x^3+56/9*(d*x^3+c)^(1/2)*c/d+3
*I*c/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^
2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2
)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alph
a^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)
*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/1
8*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/
3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootO
f(_Z^3*d-8*c)))+3/4096/c^4*d^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/
2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^7/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^7), x)
```

mupad [B] time = 4.62, size = 151, normalized size = 0.94

$$\frac{\frac{81\,d^2\,\sqrt{d\,x^3+c}}{512}-\frac{67\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{256\,c}+\frac{21\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{512\,c^2}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)\,15{}\mathrm{i}}{17}\right)\,17{}\mathrm{i}}{2048\,\sqrt{c^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^7*(8*c - d*x^3)^2),x)
```

```
[Out] ((81*d^2*(c + d*x^3)^(1/2))/512 - (67*d^2*(c + d*x^3)^(3/2))/(256*c) + (21*
d^2*(c + d*x^3)^(5/2))/(512*c^2))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
- 3*(c + d*x^3)^3 + 27*c^3) + (d^2*(atanh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/
2))*1i - (atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2)))*15i)/17)*17i)/(204
8*(c^5)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**7/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.418 $\int \frac{x^7\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=681

$$\frac{108\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{8/3}}+\frac{5906\sqrt{2}\,c^{7/3}\left(\sqrt[3]{c}\right.\cdots}{\cdots}$$

```
[Out] 1/3*x^5*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-108*c^(13/6)*arctanh(1/3*(c^(1/3)+d^
(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(8/3)+108*c^(13/6)*arctanh(1/3*(d*x^3
+c)^(1/2)/c^(1/2))/d^(8/3)+108*c^(13/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*
3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(8/3)+103/13*c*x^2*(d*x^3+c)^(1/2)/d^2+1
9/39*x^5*(d*x^3+c)^(1/2)/d+5906/13*c^2*(d*x^3+c)^(1/2)/d^(8/3)/(d^(1/3)*x+c
^(1/3)*(1+3^(1/2)))+5906/39*c^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/
2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-2953/13*3^(1/4)*c^(7/3)*(c^(1/3)+d^(1/
3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d
^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(8/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 681, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {467, 581, 582, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}+\frac{108\sqrt{3}\,c^{13/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{8/3}}-\frac{108c^{13/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{8/3}}+\frac{108c^{13/6}\tanh\cdots}{d^{8/}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (103*c*x^2*Sqrt[c + d*x^3])/(13*d^2) + (19*x^5*Sqrt[c + d*x^3])/(39*d) + (5
906*c^2*Sqrt[c + d*x^3])/(13*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 (x^5*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)) + (108*Sqrt[3]*c^(13/6)*ArcTan
[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/d^(8/3) - (108*c
^(13/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/d^(8/
3) + (108*c^(13/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/d^(8/3) - (2953*3^
(1/4)*Sqrt[2 - Sqrt[3]]*c^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Elliptic
E[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)], -7 - 4*Sqrt[3]])/(13*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (5906*Sqrt[2]*c^(7
/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/
((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1
/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(13
*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7 \left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2} dx = \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x^4\sqrt{c+dx^3}\left(5c+\frac{19dx^3}{2}\right)}{8c-dx^3} dx}{3d}$$

$$= \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \frac{x^4\left(-\frac{825c^2d}{2}-\frac{2163}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} dx}{39d^2}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{4\int \frac{x\left(-8652c^3d^2-\frac{62013}{4}c^2d^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}} dx}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{4\int \left(\frac{62013c^2d^2x}{4\sqrt{c+dx^3}} - \frac{132678c^3d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right.}{273d^4}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{\left(2953c^2\right)\int \frac{x}{\sqrt{c+dx^3}} dx}{13d^2} - \frac{(1944c}{}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{\left(162c^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}}{d^3}$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} -$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} -$$

$$= \frac{103cx^2\sqrt{c+dx^3}}{13d^2} + \frac{19x^5\sqrt{c+dx^3}}{39d} + \frac{5906c^2\sqrt{c+dx^3}}{13d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^5 \left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} -$$

Mathematica [C] time = 0.17, size = 191, normalized size = 0.28

$$\frac{4120c^2x^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) + 2953cdx^5\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{520d^2\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (80*x^2*(-412*c^3 - 388*c^2*d*x^3 + 25*c*d^2*x^6 + d^3*x^9) + 4120*c^2*x^2*
(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 2953*c*d*x^5*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/
3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(520*d^2*(-8*c + d*x^3)*Sqrt[
c + d*x^3])
```

fricas [F] time = 50.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^{10} + cx^7\right)\sqrt{dx^3 + c}}{d^2x^6 - 16\,cdx^3 + 64\,c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^10 + c*x^7)*sqrt(d*x^3 + c)/(d^2*x^6 - 16*c*d*x^3 + 64*c^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3 + c\right)^{\frac{3}{2}}x^7}{\left(dx^3 - 8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.24, size = 2223, normalized size = 3.26

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/d^2*(2/13*(d*x^3+c)^(1/2)*d*x^5+32/91*c*x^2*(d*x^3+c)^(1/2)-18/91*I*c^2*3
^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+64*c^2/d^2*(-3/8*x^2*(d*x^3+c)^(1/2
)/(d*x^3-8*c)-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I
*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(
1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3/8*I/d^3*2^(
```

```
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+16*c/d^2*(2/7*(d*x^3+c)^(1/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1
/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(
2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*(
(x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*
(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-
c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}} x^7}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^7/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x^7*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.419 $\int \frac{x^4\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=657

$$\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{265\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{7\cdots}$$

```
[Out] 1/3*x^2*(d*x^3+c)^(3/2)/d/(-d*x^3+8*c)-9*c^(7/6)*arctanh(1/3*(c^(1/3)+d^(1/
3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^(5/3)+9*c^(7/6)*arctanh(1/3*(d*x^3+c)^(1
/2)/c^(1/2))/d^(5/3)+9*c^(7/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(
d*x^3+c)^(1/2))*3^(1/2)/d^(5/3)+13/21*x^2*(d*x^3+c)^(1/2)/d+265/7*c*(d*x^3+
c)^(1/2)/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+265/21*c^(4/3)*(c^(1/3)+d^
(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^
(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1
/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-265/14*3^(
1/4)*c^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((
c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {467, 581, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{d^{5/3}}-\frac{9c^{7/6}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{d^{5/3}}+\frac{9c^{7/6}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{d^{5/3}}+\frac{265\sqrt{2}\,c^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{7\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (13*x^2*Sqrt[c + d*x^3])/(21*d) + (265*c*Sqrt[c + d*x^3])/(7*d^(5/3)*((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*(c + d*x^3)^(3/2))/(3*d*(8*c - d*x^3)
) + (9*Sqrt[3]*c^(7/6)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[
c + d*x^3]])/d^(5/3) - (9*c^(7/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6
)*Sqrt[c + d*x^3])])/d^(5/3) + (9*c^(7/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c
])])/d^(5/3) - (265*3^(1/4)*Sqrt[2 - Sqrt[3]]*c^(4/3)*(c^(1/3) + d^(1/3)*x)
*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) +
d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sq
rt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(14*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (265*Sqrt[2]*c^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(7*3^(1/4)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/(
(1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 581

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*(g*x)^(m + 1)*(a +
 b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*g*(m + n*(p + q + 1) + 1)), x] + Dist[1/(
b*(m + n*(p + q + 1) + 1)), Int[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*S
imp[c*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) +
f*n*q*(b*c - a*d) + b*e*d*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, g, m, p}, x] && IGtQ[n, 0] && GtQ[q, 0] &&  !(EqQ[q, 1] && Simple
rQ[e + f*x^n, c + d*x^n])
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx = \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{\int \frac{x\sqrt{c+dx^3}\left(2c+\frac{13dx^3}{2}\right)}{8c-dx^3}dx}{3d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \frac{x\left(-111c^2d-\frac{795}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{2\int \left(\frac{795cdx}{4\sqrt{c+dx^3}} - \frac{1701c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{21d^2}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{(265c)\int \frac{x}{\sqrt{c+dx^3}}dx}{14d} - \frac{\left(162c^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{d}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{(27c)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{2d^2} + \frac{(265c)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}\cdots}{\sqrt{c+dx^3}\cdots}}{14d^{4/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} - \frac{265\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,c^{4/3}\left(\cdots\right.}{\cdots}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\cdots\right.}{\sqrt{c}\cdots}\right.}{d^{5/3}}$$

$$= \frac{13x^2\sqrt{c+dx^3}}{21d} + \frac{265c\sqrt{c+dx^3}}{7d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\left(c+dx^3\right)^{3/2}}{3d\left(8c-dx^3\right)} + \frac{9\sqrt{3}\,c^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\cdots\right.}{\sqrt{c}\cdots}\right.}{d^{5/3}}$$

Mathematica [C] time = 0.18, size = 176, normalized size = 0.27

$$-\frac{53dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+74cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+16x^2\left(\cdots\right.}{112d\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] -1/112*(16*x^2*(37*c^2 + 35*c*d*x^3 - 2*d^2*x^6) + 74*c*x^2*(-8*c + d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)]
 + 53*d*x^5*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(d*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 14.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(dx^{7}+cx^{4}\right)\sqrt{dx^{3}+c}}{d^{2}x^{6}-16\,cdx^{3}+64\,c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^7 + c*x^4)*sqrt(d*x^3 + c)/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(dx^{3}+c\right)^{\frac{3}{2}}x^{4}}{\left(dx^{3}-8\,c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.18, size = 1747, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] 8*c/d*(-3/8*(d*x^3+c)^(1/2)/(d*x^3-8*c)*x^2-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/
d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(
-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/
2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^
2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/
2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/d*(2/7*(d*x^3+c)^(1/2)*x^2-44/
7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
```

```
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alph
a*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^4}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^4/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(d\,x^3+c\right)^{3/2}}{\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
```

```
[Out] int((x^4*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(c+dx^3\right)^{\frac{3}{2}}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
```

```
[Out] Integral(x**4*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.420 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=638

$$\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{16\ldots}$$

```
[Out] -9/16*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d^
(2/3)+9/16*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(2/3)+9/16*c^(1/6
)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2)/d^(2/
3)+3/8*x^2*(d*x^3+c)^(1/2)/(-d*x^3+8*c)+19/8*(d*x^3+c)^(1/2)/d^(2/3)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))+19/24*c^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1
/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2
^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/
2)))^2)^(1/2)*3^(3/4)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/
(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-19/16*3^(1/4)*c^(1/3)*(c^(1/3)+d^(
1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x
+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/d^(2/3)/(d*x^3+c)^(1
/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 638, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {468, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{19\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{4\sqrt{2}\sqrt[4]{3}\,d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\right.}{16\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (19*Sqrt[c + d*x^3])/(8*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*x
^2*Sqrt[c + d*x^3])/(8*(8*c - d*x^3)) + (9*Sqrt[3]*c^(1/6)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(16*d^(2/3)) - (9*c^(1/6)*
ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(16*d^(2/3))
+ (9*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(16*d^(2/3)) - (19*3^(1/
4)*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[A
rcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)], -7 - 4*Sqrt[3]])/(16*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*c^(1/3)*(c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(4*Sqrt[2]*3^(1/
4)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(8c-dx^3\right)^2}dx = \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{\int \frac{x\left(-15c^2d-\frac{57}{2}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{\int \left(\frac{57cdx}{2\sqrt{c+dx^3}}-\frac{243c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{24cd}$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{19}{16}\int \frac{x}{\sqrt{c+dx^3}}dx-\frac{1}{8}(81c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}dx$$

$$= \frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{27\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}dx}{32d}+\frac{19\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x}{\sqrt{c+dx^3}}dx}{16\sqrt[3]{d}}-\frac{\left(27\sqrt[3]{c}\right)\int \frac{1\cdots}{\left(2-\frac{\sqrt[3]{d}\cdots}{\sqrt[3]{\cdots}}\right)\cdots}}{32\sqrt[3]{\cdots}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}-\frac{19\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{3}\cdots\right.\right.}}}{16d^{2/3}\sqrt{\frac{\sqrt[3]{c}\cdots}{\left(\left(1+\sqrt{\cdots}\right.\right.}}}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}-\frac{9\sqrt[6]{c}\,\mathrm{ta}\cdots}{\cdots}$$

$$= \frac{19\sqrt{c+dx^3}}{8d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{3x^2\sqrt{c+dx^3}}{8\left(8c-dx^3\right)}+\frac{9\sqrt{3}\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{16d^{2/3}}-\frac{9\sqrt[6]{c}\,\mathrm{ta}\cdots}{\cdots}$$

Mathematica [C] time = 0.20, size = 141, normalized size = 0.22

$$\frac{x^2\left(-\frac{19dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}-25\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+\frac{240\left(c+dx^3\right)}{8c-dx^3}\right)}{640\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x]
```

```
[Out] (x^2*((240*(c + d*x^3))/(8*c - d*x^3) - 25*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3
, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - (19*d*x^3*Sqrt[1 + (d*x^3)/c]
*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c))/(640*Sqrt[c +
 d*x^3])
```

fricas [F] time = 3.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^4+cx\right)\sqrt{dx^3+c}}{d^2x^6-16\,cdx^3+64\,c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c*x)*sqrt(d*x^3 + c)/(d^2*x^6 - 16*c*d*x^3 + 64*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

maple [C] time = 0.19, size = 873, normalized size = 1.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x)
```

```
[Out] -3/8*(d*x^3+c)^(1/2)/(d*x^3-8*c)*x^2-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(d*x^3 - 8*c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d\,x^3+c\right)}^{3/2}}{{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(8*c - d*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^3\right)^{\frac{3}{2}}}{\left(-8c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(-d*x**3+8*c)**2,x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(-8*c + d*x**3)**2, x)
```

3.421 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=522

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{32c^{2/}}$$

```
[Out] -1/16*(d*x^3+c)^(1/2)/c/x+3/8*(d*x^3+c)^(1/2)/x/(-d*x^3+8*c)+1/16*d^(1/3)*(
d*x^3+c)^(1/2)/c/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/48*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3
)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/32*3^(1/4)
*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2
/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
/c^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {468, 21, 325, 303, 218, 1877}

$$\frac{\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{8\sqrt{2}\sqrt[4]{3}c^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{32c^{2/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] -Sqrt[c + d*x^3]/(16*c*x) + (d^(1/3)*Sqrt[c + d*x^3])/(16*c*((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x*(8*c - d*x^3)) - (3^(1/4)*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(32*c^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + S
qrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(8*Sqrt[2]*3^(1/4)*c^(
2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*
x)^2]*Sqrt[c + d*x^3])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^2\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)} + \frac{\int \frac{12c^2d-\frac{3}{2}cd^2x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)} + \frac{1}{16}\int \frac{1}{x^2\sqrt{c+dx^3}}\,dx$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)} + \frac{d\int \frac{x}{\sqrt{c+dx^3}}\,dx}{32c}$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)} + \frac{d^{2/3}\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{32c} + \frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,d^{2/3}\right)\int \frac{1}{\sqrt{c+dx^3}}}{16c^{2/3}}$$

$$= -\frac{\sqrt{c+dx^3}}{16cx} + \frac{\sqrt[3]{d}\sqrt{c+dx^3}}{16c\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x\left(8c-dx^3\right)} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\right.}{}$$

Mathematica [C] time = 0.70, size = 242, normalized size = 0.46

$$\frac{\left(2c-dx^3\right)\sqrt{c+dx^3}}{16cx\left(dx^3-8c\right)} - \frac{\sqrt[6]{-1}\sqrt[3]{-d}\sqrt{(-1)^{5/6}\left(\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}-1\right)}\sqrt{\frac{(-d)^{2/3}x^2}{c^{2/3}}+\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}+1}\left(\sqrt[3]{-1}F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-d}\,x}{\sqrt[3]{c}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\right|\right.}{16\sqrt[4]{3}\sqrt[3]{c}\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x]
```

```
[Out] ((2*c - d*x^3)*Sqrt[c + d*x^3])/(16*c*x*(-8*c + d*x^3)) - ((-1)^(1/6)*(-d)^
(1/3)*Sqrt[(-1)^(5/6)*(-1 + ((-d)^(1/3)*x)/c^(1/3))]*Sqrt[1 + ((-d)^(1/3)*x
)/c^(1/3) + ((-d)^(2/3)*x^2)/c^(2/3)]*((-I)*Sqrt[3]*EllipticE[ArcSin[Sqrt[-
(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)] + (-1)^(1/3)*E
llipticF[ArcSin[Sqrt[-(-1)^(5/6) - (I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)
^(1/3)]))/(16*3^(1/4)*c^(1/3)*Sqrt[c + d*x^3])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{d^2x^8-16\,cdx^5+64\,c^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^(3/2)/(d^2*x^8 - 16*c*d*x^5 + 64*c^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="giac")

[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)

maple [C] time = 0.19, size = 2217, normalized size = 4.25

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x)

[Out] 1/8/c*d*(-3/8*(d*x^3+c)^(1/2)/(d*x^3-8*c)*x^2-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3/8*I/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*x^3+c)^(1/2)*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64/c^2*d*(2/7*(d*x^3+c)^(1/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x

```
+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)
*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3
*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^2\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^2*(8*c - d*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

3.422 $\int \frac{(c+dx^3)^{3/2}}{x^5(8c-dx^3)^2}\,dx$

Optimal. Leaf size=684

$$-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1024c^{11/6}}+\frac{d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left((1+\cdots\right.}}}{16\sqrt{2}\sqrt[4]{3}\cdots}$$

```
[Out] 9/1024*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(11/6)-9/1024*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)-9/1024
*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(11/6)-13/256*(d*x^3+c)^(1/2)/c/x^4-1/32*d*(d*x^3+c)^(1/2)/c^2/x+3/8*(d
*x^3+c)^(1/2)/x^4/(-d*x^3+8*c)+1/32*d^(4/3)*(d*x^3+c)^(1/2)/c^2/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2)))+1/96*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+
c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/
3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/c^(5/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/64*3^(1/4)*d^(4/3)*(c^(1/3)+d^(1/3)*x)
*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),
I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)/c^(5/3)/(d*x^3+c)^(1/2)/(c^
(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 684, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {468, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{9\sqrt{3}\,d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{1024c^{11/6}}+\frac{9d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1024c^{11/6}}-\frac{9d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\cdots\right.}{1024c^{11/6}}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] (-13*Sqrt[c + d*x^3])/(256*c*x^4) - (d*Sqrt[c + d*x^3])/(32*c^2*x) + (d^(4/
3)*Sqrt[c + d*x^3])/(32*c^2*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[
c + d*x^3])/(8*x^4*(8*c - d*x^3)) - (9*Sqrt[3]*d^(4/3)*ArcTan[(Sqrt[3]*c^(1
/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1024*c^(11/6)) + (9*d^(4/3)*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1024*c^(11/6)
) - (9*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1024*c^(11/6)) - (3^(
1/4)*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE
[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)], -7 - 4*Sqrt[3]])/(64*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (d^(4/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(16*Sqrt[2]*3^(1/
4)*c^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^5\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \frac{39c^2d+\frac{51}{2}cd^2x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{\int \frac{-192c^3d^2-\frac{195}{2}c^2d^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{768c^3d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \frac{x\left(1740c^4d^3-96c^3d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{\int \left(\frac{96c^3d^3x}{\sqrt{c+dx^3}} + \frac{972c^4d^3x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{6144c^5d}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} + \frac{d^2\int \frac{x}{\sqrt{c+dx^3}}\,dx}{64c^2} + \frac{\left(81d^2\right)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+ \ldots}}}{512c}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{(27d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2048c^2} + \frac{d^{5/\ldots}}{\ldots}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{\sqrt[4]{3}\sqrt{2 \ldots}}{\ldots}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{9\sqrt{3}\,d^{4 \ldots}}{\ldots}$$

$$= -\frac{13\sqrt{c+dx^3}}{256cx^4} - \frac{d\sqrt{c+dx^3}}{32c^2x} + \frac{d^{4/3}\sqrt{c+dx^3}}{32c^2\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c+dx^3}}{8x^4\left(8c-dx^3\right)} - \frac{9\sqrt{3}\,d^{4 \ldots}}{\ldots}$$

Mathematica [C] time = 0.15, size = 199, normalized size = 0.29

$$-\frac{d^3x^5\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{2560c^3\sqrt{c+dx^3}} + \frac{145d^2x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{8192c^2\sqrt{c+dx^3}} + \sqrt{c+dx^3}\left(-\frac{3d^2x^2}{512c^2\left(dx^3-8c\right)} \ldots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/256*1/(c*x^4) - (13*d)/(512*c^2*x) - (3*d^2*x^2)/(512*c
^2*(-8*c + d*x^3))) + (145*d^2*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1
```

```
, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(8192*c^2*Sqrt[c + d*x^3]) - (d^3*x^5*
Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(2560*c^3*Sqrt[c + d*x^3])
```

fricas [F] time = 3.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{d^2x^{11}-16\,cdx^8+64\,c^2x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="fricas")
[Out] integral((d*x^3 + c)^(3/2)/(d^2*x^11 - 16*c*d*x^8 + 64*c^2*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="giac")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

maple [C] time = 0.20, size = 2690, normalized size = 3.93

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x)
[Out] 1/64/c^2*(-1/4*c*(d*x^3+c)^(1/2)/x^4-11/8*(d*x^3+c)^(1/2)*d/x-9/8*I*d*3^(1/
2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d^2*(-3/8*(d*x^3+c)^(1/2)/(d*x^3
-8*c)*x^2-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+
1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)
/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3/8*I/d^3*2^(1/2)
*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3
```

```
)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2
+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^
2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1
/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
d-8*c))+1/256/c^3*d*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*x^3+c)^(1/2)*d*x^2-9/7*I*
c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/256/c^3*d^2*(2/7*(d*x^3+c)^(1/2)
*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1/2)*sum
(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*
c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2 x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^5/(-d*x^3+8*c)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^5\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^5*(8*c - d*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**5/(-d*x**3+8*c)**2,x)
```

```
[Out] Timed out
```

3.423 $\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx$

Optimal. Leaf size=708

$$-\frac{9\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\frac{9d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{4096c^{17/6}}+\frac{19d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\cdots}}{896\cdots}$$

```
[Out] 9/4096*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(17/6)-9/4096*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-9/4096
*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))*3^(1/2
)/c^(17/6)-11/224*(d*x^3+c)^(1/2)/c/x^7-83/7168*d*(d*x^3+c)^(1/2)/c^2/x^4-1
9/1792*d^2*(d*x^3+c)^(1/2)/c^3/x+3/8*(d*x^3+c)^(1/2)/x^7/(-d*x^3+8*c)+19/17
92*d^(7/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+19/5376*d^(7
/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*
x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x
^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d*x^3
+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(
1/2)-19/3584*3^(1/4)*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/
3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)
-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(
1+3^(1/2)))^2)^(1/2)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {468, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}-\frac{19d^2\sqrt{c+dx^3}}{1792c^3x}-\frac{9\sqrt{3}\,d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{4096c^{17/6}}+\frac{9d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{4096c^{17/6}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] (-11*Sqrt[c + d*x^3])/(224*c*x^7) - (83*d*Sqrt[c + d*x^3])/(7168*c^2*x^4) -
 (19*d^2*Sqrt[c + d*x^3])/(1792*c^3*x) + (19*d^(7/3)*Sqrt[c + d*x^3])/(1792
*c^3*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (3*Sqrt[c + d*x^3])/(8*x^7*(8*c
 - d*x^3)) - (9*Sqrt[3]*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*
x))/Sqrt[c + d*x^3]])/(4096*c^(17/6)) + (9*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/
3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(4096*c^(17/6)) - (9*d^(7/3)*ArcTanh[
Sqrt[c + d*x^3]/(3*Sqrt[c])])/(4096*c^(17/6)) - (19*3^(1/4)*Sqrt[2 - Sqrt[3
]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(3584*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (19*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqr
t[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3
```

```
])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(896*Sqrt[2]*3^(1/4)*c^(8/3)*Sqr
t[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sq
rt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
```

```
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^8\left(8c-dx^3\right)^2}\,dx = \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{66c^2d+\frac{105}{2}cd^2x^3}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{24cd}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{-498c^3d^2-363c^2d^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1344c^3d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\int \frac{3648c^4d^3+1245c^3d^4x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{43008c^5d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \frac{x\left(-28200c^5d^4+1824c^4d^5\right.}{\left(8c-dx^3\right)\sqrt{c+dx^3}}}{344064c^7d}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\int \left(-\frac{1824c^4d^4x}{\sqrt{c+dx^3}} - \frac{136}{\left(8c-dx\right.}\right.}{344064c^7}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} + \frac{\left(19d^3\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{3584c^3} +$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{3\sqrt{c+dx^3}}{8x^7\left(8c-dx^3\right)} - \frac{\left(27d^2\right)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2d}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}}{c^{2/3}}\right.}}{8192c^3}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c}}{8x^7\left(8\right.}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c}}{8x^7\left(8\right.}$$

$$= -\frac{11\sqrt{c+dx^3}}{224cx^7} - \frac{83d\sqrt{c+dx^3}}{7168c^2x^4} - \frac{19d^2\sqrt{c+dx^3}}{1792c^3x} + \frac{19d^{7/3}\sqrt{c+dx^3}}{1792c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{3\sqrt{c}}{8x^7\left(8\right.}$$

Mathematica [C] time = 0.15, size = 212, normalized size = 0.30

$$-\frac{19d^4x^5\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{143360c^4\sqrt{c+dx^3}} + \frac{1175d^3x^2\sqrt{\frac{c+dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{229376c^3\sqrt{c+dx^3}} + \sqrt{c+dx^3}\left(-\frac{3d^3x^2}{4096c^3\left(dx^3-8c\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]*(-1/448*1/(c*x^7) - (41*d)/(7168*c^2*x^4) - (283*d^2)/(2867
2*c^3*x) - (3*d^3*x^2)/(4096*c^3*(-8*c + d*x^3))) + (1175*d^3*x^2*Sqrt[(c +
 d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(229376
*c^3*Sqrt[c + d*x^3]) - (19*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2,
1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(143360*c^4*Sqrt[c + d*x^3])
```

fricas [F] time = 5.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(dx^3+c\right)^{\frac{3}{2}}}{d^2x^{14}-16\,cdx^{11}+64\,c^2x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^(3/2)/(d^2*x^14 - 16*c*d*x^11 + 64*c^2*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

maple [C] time = 0.18, size = 3186, normalized size = 4.50

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x)
```

```
[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)*c/x^4-11/8*(d*x^3+c)^(1/2)*d/x-9/8*I*d*3^
(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^3*(-3/8*(d*x^3+c)^(1/2)/(d
*x^3-8*c)*x^2-19/24*I*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I
*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(
1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
```

```
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3/8*I/d^3*2^(
1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c)))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)*c/x^7-17/56*d*(d*x^3+c)^(1/2)/x
^4-27/112*d^2/c*(d*x^3+c)^(1/2)/x-9/112*I/c*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2))))+3/4096/c^4*d^2*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*x^3+c)^(1/2)*d*x^2-
9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellip
ticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-3/4096/c^4*d^3*(2/7*(d*x^3+c
)^(1/2)*x^2-44/7*I*c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*
(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1
/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+3*I*c/d^3*2^(1
/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*
d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c
*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z
^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^8/(-d*x^3+8*c)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((d*x^3 - 8*c)^2*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^8\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^8*(8*c - d*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**8/(-d*x**3+8*c)**2,x)
[Out] Timed out
```

3.424 $\int \frac{x^{11}}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=95

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)}$$

```
[Out] -2944/81*c^(3/2)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^4+8/27*x^6*(d*x^3+c
)^(1/2)/d^2/(-d*x^3+8*c)+2/27*(7*d*x^3+170*c)*(d*x^3+c)^(1/2)/d^4
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 147, 63, 206}

$$-\frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}+\frac{8x^6\sqrt{c+dx^3}}{27d^2\,(8c-dx^3)}+\frac{2\sqrt{c+dx^3}\,(170c+7dx^3)}{27d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*x^6*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (2*Sqrt[c + d*x^3]*(170*c
+ 7*d*x^3))/(27*d^4) - (2944*c^(3/2)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/
(81*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+21cdx\right)}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(1472c^2\right)\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\right.}{27d^3}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{\left(2944c^2\right)\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \right.}{27d^4}$$

$$= \frac{8x^6\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{2\sqrt{c+dx^3}\left(170c+7dx^3\right)}{27d^4} - \frac{2944c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^4}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.96

$$\frac{2944c^{3/2}\left(8c-dx^3\right)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+6\sqrt{c+dx^3}\left(-1360c^2+114cdx^3+3d^2x^6\right)}{81d^4\left(dx^3-8c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (6*Sqrt[c + d*x^3]*(-1360*c^2 + 114*c*d*x^3 + 3*d^2*x^6) + 2944*c^(3/2)*(8*
c - d*x^3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^4*(-8*c + d*x^3))
```

fricas [A] time = 0.64, size = 195, normalized size = 2.05

$$\left[\frac{2\left(736\left(cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(3\,d^2x^6+114\,cdx^3-1360\,c^2\right)\sqrt{dx^3+c}\right)}{81\left(d^5x^3-8\,cd^4\right)}, \frac{2\left(1472\left(cdx^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(736*(c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c)
 + 10*c)/(d*x^3 - 8*c)) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt(d*x^3
 + c))/(d^5*x^3 - 8*c*d^4), 2/81*(1472*(c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/
3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(3*d^2*x^6 + 114*c*d*x^3 - 1360*c^2)*sqrt
(d*x^3 + c))/(d^5*x^3 - 8*c*d^4)]
```

giac [A] time = 0.19, size = 93, normalized size = 0.98

$$\frac{2944\,c^{2}\arctan\left(\frac{\sqrt{dx^{3}+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\,d^{4}}-\frac{512\,\sqrt{dx^{3}+c}\,c^{2}}{27\,\left(dx^{3}-8\,c\right)d^{4}}+\frac{2\left(\left(dx^{3}+c\right)^{\frac{3}{2}}d^{8}+45\,\sqrt{dx^{3}+c}\,cd^{8}\right)}{9\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2944/81*c^2*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) - 512/27*sq
rt(d*x^3 + c)*c^2/((d*x^3 - 8*c)*d^4) + 2/9*((d*x^3 + c)^(3/2)*d^8 + 45*sqr
t(d*x^3 + c)*c*d^8)/d^12
```

maple [C] time = 0.30, size = 916, normalized size = 9.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/d^3*((2/9*(d*x^3+c)^(1/2)/d*x^3-4/9*(d*x^3+c)^(1/2)*c/d^2)*d+32/3*(d*x^3+
c)^(1/2)*c/d)+512*c^3/d^3*(-1/27/c/d*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/486*I/c^
2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))+64/9*I*c/d^6*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.34, size = 93, normalized size = 0.98

$$\frac{2\left(736\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{dx^{3}+c}-3\,\sqrt{c}}{\sqrt{dx^{3}+c}+3\,\sqrt{c}}\right)+9\,\left(dx^{3}+c\right)^{\frac{3}{2}}+405\,\sqrt{dx^{3}+c}\,c-\frac{768\,\sqrt{dx^{3}+c}\,c^{2}}{dx^{3}-8\,c}\right)}{81\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(736*c^(3/2)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sq
rt(c))) + 9*(d*x^3 + c)^(3/2) + 405*sqrt(d*x^3 + c)*c - 768*sqrt(d*x^3 + c)
*c^2/(d*x^3 - 8*c))/d^4
```

mupad [B] time = 4.06, size = 107, normalized size = 1.13

$$\frac{92\,c\,\sqrt{d\,x^3+c}}{9\,d^4}+\frac{1472\,c^{3/2}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,d^4}+\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,d^3}+\frac{512\,c^2\,\sqrt{d\,x^3+c}}{27\,d^4\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (92*c*(c + d*x^3)^(1/2))/(9*d^4) + (1472*c^(3/2)*log((10*c + d*x^3 - 6*c^(1
/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^4) + (2*x^3*(c + d*x^3)^(1/2))
/(9*d^3) + (512*c^2*(c + d*x^3)^(1/2))/(27*d^4*(8*c - d*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

3.425 $\int \frac{x^8}{\left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

```
[Out] -224/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^3+2/3*(d*x^3+c)^(1/2
)/d^3+64/27*c*(d*x^3+c)^(1/2)/d^3/(-d*x^3+8*c)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 80, 63, 206}

$$\frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)}+\frac{2\sqrt{c+dx^3}}{3d^3}-\frac{224\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) + (64*c*Sqrt[c + d*x^3])/(27*d^3*(8*c - d*x^3))
 - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(8c-dx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{40c^2d+9cd^2x}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27cd^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(112c)\operatorname{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,x,x^3\right)}{27d^2}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{(224c)\operatorname{Subst}\left(\int \frac{1}{9c-x^2}\,dx,x,\sqrt{c+dx^3}\right)}{27d^3}\\
&= \frac{2\sqrt{c+dx^3}}{3d^3} + \frac{64c\sqrt{c+dx^3}}{27d^3\left(8c-dx^3\right)} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 82, normalized size = 0.99

$$-\frac{64c\sqrt{c+dx^3}}{27d^3\left(dx^3-8c\right)} + \frac{2\sqrt{c+dx^3}}{3d^3} - \frac{224\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*d^3) - (64*c*Sqrt[c + d*x^3])/(27*d^3*(-8*c + d*x^3)
) - (224*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^3)
```

fricas [A] time = 0.64, size = 167, normalized size = 2.01

$$\left[\frac{2\left(56\left(dx^3-8c\right)\sqrt{c}\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)+3\left(9dx^3-104c\right)\sqrt{dx^3+c}\right)}{81\left(d^4x^3-8cd^3\right)}, \frac{2\left(112\left(dx^3-8c\right)\sqrt{-c}\arctan\left(\right.\right.}{81\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/81*(56*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10
*c)/(d*x^3 - 8*c)) + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^
3), 2/81*(112*(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c)
 + 3*(9*d*x^3 - 104*c)*sqrt(d*x^3 + c))/(d^4*x^3 - 8*c*d^3)]
```

giac [A] time = 0.17, size = 69, normalized size = 0.83

$$\frac{224\, c \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{81\sqrt{-c}\, d^3} + \frac{2\sqrt{dx^3+c}}{3\, d^3} - \frac{64\sqrt{dx^3+c}\, c}{27\left(dx^3-8\, c\right) d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 224/81*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) + 2/3*sqrt(d*x
^3 + c)/d^3 - 64/27*sqrt(d*x^3 + c)*c/((d*x^3 - 8*c)*d^3)
```

maple [C] time = 0.20, size = 874, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/d^3+64*c^2/d^2*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1
/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c)))+16/27*I/d^5*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.56, size = 79, normalized size = 0.95

$$\frac{2\left(56\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right) + 27\sqrt{dx^3+c} - \frac{96\sqrt{dx^3+c}\, c}{dx^3-8\, c}\right)}{81\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/81*(56*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqr
t(c))) + 27*sqrt(d*x^3 + c) - 96*sqrt(d*x^3 + c)*c/(d*x^3 - 8*c))/d^3
```

mupad [B] time = 4.00, size = 87, normalized size = 1.05

$$\frac{2\sqrt{d\, x^3+c}}{3\, d^3} + \frac{112\sqrt{c}\,\ln\left(\frac{10\, c+d\, x^3-6\sqrt{c}\,\sqrt{d\, x^3+c}}{8\, c-d\, x^3}\right)}{81\, d^3} + \frac{64\, c\sqrt{d\, x^3+c}}{27\, d^3\,\left(8\, c-d\, x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^3) + (112*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*d^3) + (64*c*(c + d*x^3)^(1/2))/(27*
d^3*(8*c - d*x^3))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.426 $\int \frac{x^5}{\left(8c-dx^3\right)^2 \sqrt{c+dx^3}} \, dx$

Optimal. Leaf size=64

$$\frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

```
[Out] -10/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^2/c^(1/2)+8/27*(d*x^3+c)^(1/2
)/d^2/(-d*x^3+8*c)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {446, 78, 63, 206}

$$\frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)}-\frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (8*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{5\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{27d}$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{10\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27d^2}$$
$$= \frac{8\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 0.98

$$-\frac{8\sqrt{c+dx^3}}{27d^2\left(dx^3-8c\right)} - \frac{10\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81\sqrt{c}\,d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*Sqrt[c + d*x^3])/(27*d^2*(-8*c + d*x^3)) - (10*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(81*Sqrt[c]*d^2)
```

fricas [A] time = 0.72, size = 155, normalized size = 2.42

$$\left[\frac{5\left(dx^3-8c\right)\sqrt{c}\,\log\left(\frac{dx^3-6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right)-24\sqrt{dx^3+c}\,c}{81\left(cd^3x^3-8c^2d^2\right)}, \frac{2\left(5\left(dx^3-8c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\sqrt{-c}}{3c}\right)-12\sqrt{dx\right.}{81\left(cd^3x^3-8c^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/81*(5*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 + c)*sqrt(c) + 10*
c)/(d*x^3 - 8*c)) - 24*sqrt(d*x^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2), 2/81*(5*
(d*x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x
^3 + c)*c)/(c*d^3*x^3 - 8*c^2*d^2)]
```

giac [A] time = 0.16, size = 58, normalized size = 0.91

$$\frac{2\left(\frac{5\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,d} - \frac{12\sqrt{dx^3+c}}{\left(dx^3-8c\right)d}\right)}{81\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/81*(5*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d) - 12*sqrt(d*x^3 +
 c)/((d*x^3 - 8*c)*d))/d
```

maple [C] time = 0.17, size = 861, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 8*c/d*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-
c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*
d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/27*I/
d^4/c*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^
(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2
*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-
c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_
Z^3*d-8*c))
```

maxima [A] time = 1.32, size = 67, normalized size = 1.05

$$\frac{\frac{5 \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{24\sqrt{dx^3+c}}{dx^3-8c}}{81\, d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/81*(5*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/sq
rt(c) - 24*sqrt(d*x^3 + c)/(d*x^3 - 8*c))/d^2
```

mupad [B] time = 4.01, size = 72, normalized size = 1.12

$$\frac{5\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{81\,\sqrt{c}\,d^2}+\frac{8\,\sqrt{d\,x^3+c}}{27\,d^2\,\left(8\,c-d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (5*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(81*c^(
1/2)*d^2) + (8*(c + d*x^3)^(1/2))/(27*d^2*(8*c - d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.427 $\int \frac{x^2}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

```
[Out] 1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d+1/27*(d*x^3+c)^(1/2)/c/
d/(-d*x^3+8*c)
```

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}+\frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d*(8*c - d*x^3)) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c]
)]/(81*c^(3/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{54c}$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{27cd}$$
$$= \frac{\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.96

$$\frac{\frac{3\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + \tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((3*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + ArcTanh[Sqrt[c + d*x^3]/(3*Sqr
t[c])])/(81*c^(3/2)*d)
```

fricas [A] time = 0.54, size = 153, normalized size = 2.28

$$\left[\frac{\left(dx^3-8\,c\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,\sqrt{dx^3+c}\,c}{162\,\left(c^2d^2x^3-8\,c^3d\right)}, -\frac{\left(dx^3-8\,c\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{dx^3+c}\,\sqrt{-c}}{3\,c}\right)+3\,\sqrt{dx^3+c}\,c}{81\,\left(c^2d^2x^3-8\,c^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/162*((d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c
)/(d*x^3 - 8*c)) - 6*sqrt(d*x^3 + c)*c)/(c^2*d^2*x^3 - 8*c^3*d), -1/81*((d*
x^3 - 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*sqrt(d*x^3 +
 c)*c)/(c^2*d^2*x^3 - 8*c^3*d)]
```

giac [A] time = 0.16, size = 59, normalized size = 0.88

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{81\,\sqrt{-c}\,cd} - \frac{\sqrt{dx^3+c}}{27\,\left(dx^3-8\,c\right)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/81*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) - 1/27*sqrt(d*x^3
 + c)/((d*x^3 - 8*c)*c*d)
```

maple [C] time = 0.19, size = 442, normalized size = 6.60

$$-\frac{\sqrt{d\,x^{3}+c}}{27\left(d\,x^{3}-8c\right)cd}-\frac{i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{R}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.39, size = 72, normalized size = 1.07

$$-\frac{\frac{6\sqrt{dx^{3}+c}}{\left(dx^{3}+c\right)c-9\,c^{2}}+\frac{\log\left(\frac{\sqrt{dx^{3}+c}-3\sqrt{c}}{\sqrt{dx^{3}+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{162\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/162*(6*sqrt(d*x^3 + c)/((d*x^3 + c)*c - 9*c^2) + log((sqrt(d*x^3 + c) -
3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d
```

mupad [B] time = 3.98, size = 75, normalized size = 1.12

$$\frac{\ln\left(\frac{10\,c+d\,x^{3}+6\,\sqrt{c}\,\sqrt{d\,x^{3}+c}}{8\,c-d\,x^{3}}\right)}{162\,c^{3/2}\,d}+\frac{\sqrt{d\,x^{3}+c}}{27\,c\,d\,\left(8\,c-d\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(162*c^(3/2
)*d) + (c + d*x^3)^(1/2)/(27*c*d*(8*c - d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\left(-8c+dx^{3}\right)^{2}\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**2/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.428 $\int \frac{1}{x(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}+\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}$$

```
[Out] 13/2592*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)-1/96*arctanh((d*x^3+c)
^(1/2)/c^(1/2))/c^(5/2)+1/216*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)
```

Rubi [A] time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {446, 103, 156, 63, 208, 206}

$$\frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)}+\frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*(8*c - d*x^3)) + (13*ArcTanh[Sqrt[c + d*x^3]/(3*Sq
rt[c])])/(2592*c^(5/2)) - ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]]/(96*c^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{d^2x}{2}}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{216c^2d}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^2} + \frac{(13d)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,\right.}{1728c^2}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{864c^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,\right.}{96c^2d}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{2592c^{5/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{5/2}}$$

Mathematica [A] time = 0.10, size = 83, normalized size = 0.94

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}}{8c-dx^3} + 13\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right) - 27\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2592c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3])/(8*c - d*x^3) + 13*ArcTanh[Sqrt[c + d*x^3]/(3
*Sqrt[c])] - 27*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(2592*c^(5/2))
```

fricas [A] time = 0.77, size = 226, normalized size = 2.57

$$\left[\frac{13\left(dx^3-8c\right)\sqrt{c}\,\log\left(\frac{dx^3+6\sqrt{dx^3+c}\sqrt{c}+10c}{dx^3-8c}\right) + 27\left(dx^3-8c\right)\sqrt{c}\,\log\left(\frac{dx^3-2\sqrt{dx^3+c}\sqrt{c}+2c}{x^3}\right) - 24\sqrt{dx^3+c}\,c}{5184\left(c^3dx^3-8c^4\right)}, \frac{27\left(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/5184*(13*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*sqrt(c) +
10*c)/(d*x^3 - 8*c)) + 27*(d*x^3 - 8*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 +
```

```
c)*sqrt(c) + 2*c)/x^3) - 24*sqrt(d*x^3 + c)*c)/(c^3*d*x^3 - 8*c^4), 1/2592
*(27*(d*x^3 - 8*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 13*(d*x^3
- 8*c)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 12*sqrt(d*x^3 + c)
*c)/(c^3*d*x^3 - 8*c^4)]
```

giac [A] time = 0.19, size = 79, normalized size = 0.90

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\sqrt{-c}\,c^2} - \frac{13\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{2592\sqrt{-c}\,c^2} - \frac{\sqrt{dx^3+c}}{216\left(dx^3-8c\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 13/2592*arctan(1/3*s
qrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2) - 1/216*sqrt(d*x^3 + c)/((d*x^3 - 8
*c)*c^2)
```

maple [C] time = 0.19, size = 880, normalized size = 10.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*d*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)*sum(
(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-
c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d
^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))-1/172
8*I/c^3/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c
*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_a
lpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1
/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-
1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Ro
otOf(_Z^3*d-8*c))-1/96*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x), x)
```

mupad [B] time = 4.01, size = 80, normalized size = 0.91

$$\frac{13\,\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^5}}\right)}{2592\,\sqrt{c^5}}-\frac{\mathrm{atanh}\left(\frac{c^2\,\sqrt{d\,x^3+c}}{\sqrt{c^5}}\right)}{96\,\sqrt{c^5}}+\frac{\sqrt{d\,x^3+c}}{72\,c^2\,\left(24\,c-3\,d\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (13*atanh((c^2*(c + d*x^3)^(1/2))/(3*(c^5)^(1/2))))/(2592*(c^5)^(1/2)) - at
anh((c^2*(c + d*x^3)^(1/2))/(c^5)^(1/2))/(96*(c^5)^(1/2)) + (c + d*x^3)^(1/
2)/(72*c^2*(24*c - 3*d*x^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.429 $\int \frac{1}{x^4(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=124

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}+\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}$$

```
[Out] 11/10368*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)+1/384*d*arctanh((d*
x^3+c)^(1/2)/c^(1/2))/c^(7/2)+5/864*d*(d*x^3+c)^(1/2)/c^3/(-d*x^3+8*c)-1/24
*(d*x^3+c)^(1/2)/c^2/x^3/(-d*x^3+8*c)
```

Rubi [A] time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$\frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)}+\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (5*d*Sqrt[c + d*x^3])/(864*c^3*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(24*c^2*x^3
*(8*c - d*x^3)) + (11*d*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(10368*c^(7/2
)) + (d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(384*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{2cd-\frac{3d^2x}{2}}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{-18c^2d^2+5cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{1728c^4d}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{768c^3} + \frac{(11d}{}$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{384c^3} +$$

$$= \frac{5d\sqrt{c+dx^3}}{864c^3\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{24c^2x^3\left(8c-dx^3\right)} + \frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{10368c^{7/2}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{}}{}\right.}{384c^{7/}}$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.78

$$\frac{11d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)+27d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)+\frac{12\sqrt{c}\sqrt{c+dx^3}\left(36c-5dx^3\right)}{dx^6-8cx^3}}{10368c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*(36*c - 5*d*x^3)*Sqrt[c + d*x^3])/(-8*c*x^3 + d*x^6) + 11*d*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] + 27*d*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(10368*c^(7/2))
```

fricas [A] time = 0.62, size = 280, normalized size = 2.26

$$\left[\frac{11\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+27\left(d^2x^6-8\,cdx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)-24\left(5\,cdx^3-3\right.}{20736\left(c^4dx^6-8\,c^5x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/20736*(11*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c)*s
qrt(c) + 10*c)/(d*x^3 - 8*c)) + 27*(d^2*x^6 - 8*c*d*x^3)*sqrt(c)*log((d*x^3
 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 24*(5*c*d*x^3 - 36*c^2)*sqrt(d*x
^3 + c))/(c^4*d*x^6 - 8*c^5*x^3), -1/10368*(27*(d^2*x^6 - 8*c*d*x^3)*sqrt(-
c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 11*(d^2*x^6 - 8*c*d*x^3)*sqrt(-c)*a
rctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(5*c*d*x^3 - 36*c^2)*sqrt(d*x^3
+ c))/(c^4*d*x^6 - 8*c^5*x^3)]
```

giac [A] time = 0.16, size = 114, normalized size = 0.92

$$-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c^3}-\frac{11\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{10368\,\sqrt{-c}\,c^3}-\frac{5\left(dx^3+c\right)^{\frac{3}{2}}d-41\,\sqrt{dx^3+c}\,cd}{864\left(\left(dx^3+c\right)^2-10\left(dx^3+c\right)c+9\,c^2\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 11/10368*d*arcta
n(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 1/864*(5*(d*x^3 + c)^(3/2)
*d - 41*sqrt(d*x^3 + c)*c*d)/(((d*x^3 + c)^2 - 10*(d*x^3 + c)*c + 9*c^2)*c^
3)
```

maple [C] time = 0.20, size = 926, normalized size = 7.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*d^2*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2)
*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+
1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))
/c^(3/2))-1/6912*I/c^4/d*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
```

```
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2)),_alpha=RootOf(_Z^3*d-8*c))-1/384*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(
7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^4), x)
```

mupad [B] time = 4.11, size = 117, normalized size = 0.94

$$\frac{\frac{41\,d\,\sqrt{d\,x^3+c}}{288\,c^2}-\frac{5\,d\,{\left(d\,x^3+c\right)}^{3/2}}{288\,c^3}}{3\,{\left(d\,x^3+c\right)}^2-30\,c\,\left(d\,x^3+c\right)+27\,c^2}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1{}\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,11{}\mathrm{i}}{27}\right)\,1{}\mathrm{i}}{384\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((41*d*(c + d*x^3)^(1/2))/(288*c^2) - (5*d*(c + d*x^3)^(3/2))/(288*c^3))/(3
*(c + d*x^3)^2 - 30*c*(c + d*x^3) + 27*c^2) - (d*(atanh((c^3*(c + d*x^3)^(1
/2))/(c^7)^(1/2))*1i + (atanh((c^3*(c + d*x^3)^(1/2))/(3*(c^7)^(1/2)))*11i)
/27)*1i)/(384*(c^7)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.430 $\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=164

$$\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

```
[Out] 31/165888*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-19/6144*d^2*arct
anh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/13824*d^2*(d*x^3+c)^(1/2)/c^4/(-d*x
^3+8*c)-1/48*(d*x^3+c)^(1/2)/c^2/x^6/(-d*x^3+8*c)+3/128*d*(d*x^3+c)^(1/2)/c
^3/x^3/(-d*x^3+8*c)
```

Rubi [A] time = 0.13, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 151, 156, 63, 208, 206}

$$-\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)}+\frac{31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{165888c^{9/2}}-\frac{19d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{6144c^{9/2}}+\frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)}-\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-35*d^2*Sqrt[c + d*x^3])/(13824*c^4*(8*c - d*x^3)) - Sqrt[c + d*x^3]/(48*c
^2*x^6*(8*c - d*x^3)) + (3*d*Sqrt[c + d*x^3])/(128*c^3*x^3*(8*c - d*x^3)) +
 (31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(165888*c^(9/2)) - (19*d^2*A
rcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(6144*c^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int \frac{9cd-\frac{5d^2x}{2}}{x^2(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\text{Subst}\left(\int \frac{38c^2d^2-\frac{27}{2}cd^3x}{x(8c-dx)^2\sqrt{c+dx}}\,dx, x, x^3\right.}{384c^4}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{\left(19d^2\right)\text{Su}}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{(19d)\text{Sub}}{}$$

$$= -\frac{35d^2\sqrt{c+dx^3}}{13824c^4\left(8c-dx^3\right)} - \frac{\sqrt{c+dx^3}}{48c^2x^6\left(8c-dx^3\right)} + \frac{3d\sqrt{c+dx^3}}{128c^3x^3\left(8c-dx^3\right)} + \frac{31d^2\tanh}{1658}$$

Mathematica [A] time = 0.21, size = 112, normalized size = 0.68

$$\frac{\frac{12\sqrt{c}\sqrt{c+dx^3}\left(288c^2-324cdx^3+35d^2x^6\right)}{dx^9-8cx^6}+31d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)-513d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{165888c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((12*Sqrt[c]*Sqrt[c + d*x^3]*(288*c^2 - 324*c*d*x^3 + 35*d^2*x^6))/(-8*c*x^
6 + d*x^9) + 31*d^2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])] - 513*d^2*ArcTanh[
Sqrt[c + d*x^3]/Sqrt[c]])/(165888*c^(9/2))
```

fricas [A] time = 0.78, size = 310, normalized size = 1.89

$$\left[\frac{31\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+513\left(d^3x^9-8\,cd^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+24\left(35\,cd^2x\right.}{331776\left(c^5dx^9-8\,c^6x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/331776*(31*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 513*(d^3*x^9 - 8*c*d^2*x^6)*sqrt(c)*log(
(d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(35*c*d^2*x^6 - 324*c^2
*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6*x^6), 1/165888*(513*(
d^3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 31*(d^
3*x^9 - 8*c*d^2*x^6)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(
35*c*d^2*x^6 - 324*c^2*d*x^3 + 288*c^3)*sqrt(d*x^3 + c))/(c^5*d*x^9 - 8*c^6
*x^6)]
```

giac [A] time = 0.18, size = 128, normalized size = 0.78

$$\frac{19\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{6144\,\sqrt{-c}\,c^4}-\frac{31\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{165888\,\sqrt{-c}\,c^4}-\frac{\sqrt{dx^3+c}\,d^2}{13824\left(dx^3-8\,c\right)c^4}+\frac{\left(dx^3+c\right)^{\frac{3}{2}}d^2-2\,\sqrt{dx^3+c}\,cd^2}{384\,c^4d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 19/6144*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 31/165888*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/13824*sqrt(d*x^3 +
 c)*d^2/((d*x^3 - 8*c)*c^4) + 1/384*((d*x^3 + c)^(3/2)*d^2 - 2*sqrt(d*x^3 +
 c)*c*d^2)/(c^4*d^2*x^6)
```

maple [C] time = 0.20, size = 989, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/512/c^3*d^3*(-1/27*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c/d-1/486*I/c^2/d^3*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
```

```
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1
/2))/c^(3/2))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c/x^6+1/4*(d*x^3+c)^(1/2)/c^2*
d/x^3-1/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1/36864*I/c^5*2^(1/
2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(
1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3
)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_al
pha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
-1/2048*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^7), x)
```

mupad [B] time = 4.37, size = 155, normalized size = 0.95

$$-\frac{\frac{647\,d^2\,\sqrt{d\,x^3+c}}{4608\,c^2}-\frac{197\,d^2\,{\left(d\,x^3+c\right)}^{3/2}}{2304\,c^3}+\frac{35\,d^2\,{\left(d\,x^3+c\right)}^{5/2}}{4608\,c^4}}{33\,c\,{\left(d\,x^3+c\right)}^2-57\,c^2\,\left(d\,x^3+c\right)-3\,{\left(d\,x^3+c\right)}^3+27\,c^3}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,31{}\mathrm{i}}{513}\right)\,19{}\mathrm{i}}{6144\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (d^2*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i - (atanh((c^4*(c + d*x^
3)^(1/2))/(3*(c^9)^(1/2)))*31i)/513)*19i)/(6144*(c^9)^(1/2)) - ((647*d^2*(c
 + d*x^3)^(1/2))/(4608*c^2) - (197*d^2*(c + d*x^3)^(3/2))/(2304*c^3) + (35*
d^2*(c + d*x^3)^(5/2))/(4608*c^4))/(33*c*(c + d*x^3)^2 - 57*c^2*(c + d*x^3)
 - 3*(c + d*x^3)^3 + 27*c^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.431 $\int \frac{x^7}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=641

$$\frac{62\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9\,3^{3/}}$$

```
[Out] -44/81*c^(1/6)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/d
^(8/3)+44/81*c^(1/6)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^(8/3)+44/81*c^(
1/6)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/d^(8/3)*3^
(1/2)+8/27*x^2*(d*x^3+c)^(1/2)/d^2/(-d*x^3+8*c)+62/27*(d*x^3+c)^(1/2)/d^(8/
3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+62/81*c^(1/3)*(c^(1/3)+d^(1/3)*x)*Ellipt
icF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/
2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/27*c^(1/3)*(c^(1/3)+d^
(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*
x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/d^(8/3)/(d*
x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 641, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {470, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{62\sqrt{2}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{9\,3^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (62*Sqrt[c + d*x^3])/(27*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (8*
x^2*Sqrt[c + d*x^3])/(27*d^2*(8*c - d*x^3)) + (44*c^(1/6)*ArcTan[(Sqrt[3]*c
^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27*Sqrt[3]*d^(8/3)) - (44*
c^(1/6)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(81*d
^(8/3)) + (44*c^(1/6)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(81*d^(8/3)) -
(31*Sqrt[2 - Sqrt[3]]*c^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(9*3^(3/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (62*Sqrt[2]*c
^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^
2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c
^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/
(27*3^(1/4)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\int \frac{x\left(16c^2+31cdx^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{\int \left(-\frac{31cx}{\sqrt{c+dx^3}} + \frac{264c^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{31\int \frac{x}{\sqrt{c+dx^3}}\,dx}{27d^2} - \frac{(88c)\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{9d^2}$$

$$= \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{22\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{27d^3} + \frac{31\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{27d^{7/3}} - \frac{(2\ \ldots}{\ldots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} - \frac{31\sqrt{2-\sqrt{3}}\,\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\ \ldots\right.}{9\ \ldots}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\ \ldots\right.}{\sqrt{c+dx^3}}\ldots\right.}{27\sqrt{3}\,d^{8/3}}$$

$$= \frac{62\sqrt{c+dx^3}}{27d^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{8x^2\sqrt{c+dx^3}}{27d^2\left(8c-dx^3\right)} + \frac{44\sqrt[6]{c}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\ \ldots\right.}{\sqrt{c+dx^3}}\ldots\right.}{27\sqrt{3}\,d^{8/3}}$$

Mathematica [C] time = 0.19, size = 167, normalized size = 0.26

$$\frac{31dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+320c\ \ldots}{1080cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (320*c*x^2*(c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appell
F1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 31*d*x^5*(-8*c + d*x^3)
*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(1080*c*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 9.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^7}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^7/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.31, size = 1737, normalized size = 2.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*El
lipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+64*c^2/d^2*(-1/216/c^2*x^2
*(d*x^3+c)^(1/2)/(d*x^3-8*c)-1/648*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+16/27*I/d^5*2^(1/2)*sum(1/_alpha*(-c
*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
```

```
^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^7/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^7/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**7/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.432 $\int \frac{x^4}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=647

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\right.}}}$$

```
[Out] -1/81*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d^
(5/3)+1/81*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(5/3)+1/81*arctan
(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(5/3)*3^(1/
2)+1/27*x^2*(d*x^3+c)^(1/2)/c/d/(-d*x^3+8*c)+1/27*(d*x^3+c)^(1/2)/c/d^(5/3)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/81*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3
)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(
1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)
))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/
3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/54*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(2/3)/d^(5/3)/(d*x^3+
c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)
```

Rubi [A] time = 0.71, antiderivative size = 647, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {471, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}{3\sqrt[6]{c}\,\sqrt{c+dx^3}}\right)}{81c^{5/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{81c^{5/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\right.\right.}{27\sqrt[4]{3}\,c^{2/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\right.}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(27*c*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^2*S
qrt[c + d*x^3])/(27*c*d*(8*c - d*x^3)) + ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) +
 d^(1/3)*x))/Sqrt[c + d*x^3]]/(27*Sqrt[3]*c^(5/6)*d^(5/3)) - ArcTanh[(c^(1/
3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])]/(81*c^(5/6)*d^(5/3)) + ArcTa
nh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(81*c^(5/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(
c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) +
 d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(18*3^(3
/4)*c^(2/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(
1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt
[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*c^(2/3)*d^(5/3)*Sqrt[
(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt
[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule steps

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\int \frac{x\left(2c+\frac{dx^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\int \left(-\frac{x}{2\sqrt{c+dx^3}} + \frac{6cx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{27cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{9d} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{54cd}$$

$$= \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{54cd^2} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{54cd^{4/3}} - \frac{\int \frac{1+\cdots}{\left(2-\frac{\sqrt[3]{d}\cdots}{\sqrt[3]{c}}\cdots\right)\cdots}\cdots}{54c\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\cdots}}{18\,3^{3/4}\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}} - \cdots$$

$$= \frac{\sqrt{c+dx^3}}{27cd^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{27cd\left(8c-dx^3\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\sqrt{c+dx^3}}\right)}{27\sqrt{3}\,c^{5/6}d^{5/3}} - \cdots$$

Mathematica [C] time = 0.10, size = 166, normalized size = 0.26

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(\cdots\right.}{2160c^2d\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 10*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
2160*c^2*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.18, size = 1304, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 8*c/d*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x^2-1/648*I/c^2*3^(1/2)*(-c*d
^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(
1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/
3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/
3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*
d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2
)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))
*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2
/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/27*I/d^4/c*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**4/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.433 $\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=644

$$-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin\right.}{108\sqrt{2}\sqrt[4]{3}c^{5/3}d^{2/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{}\right.}{\left(\left(1+\sqrt{3}\right)\right.}}}$$

```
[Out] 7/1296*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/
d^(2/3)-7/1296*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(2/3)-7/1296
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(2/
3)*3^(1/2)+1/216*x^2*(d*x^3+c)^(1/2)/c^2/(-d*x^3+8*c)+1/216*(d*x^3+c)^(1/2)
/c^2/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/648*(c^(1/3)+d^(1/3)*x)*Elli
pticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(
1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(
c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/432*(c^(1/3)+
d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3
)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(5/3)/d
^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3
^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 644, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {472, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}c^{11/6}d^{2/3}}+\frac{7\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{1296c^{11/6}d^{2/3}}-\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{1296c^{11/6}d^{2/3}}+\frac{\left(\sqrt[3]{c}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] Sqrt[c + d*x^3]/(216*c^2*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + (x^
2*Sqrt[c + d*x^3])/(216*c^2*(8*c - d*x^3)) - (7*ArcTan[(Sqrt[3]*c^(1/6)*(c^
(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(432*Sqrt[3]*c^(11/6)*d^(2/3)) + (7*A
rcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(1296*c^(11/6)
*d^(2/3)) - (7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(1296*c^(11/6)*d^(2/3)
) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[
((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -
7 - 4*Sqrt[3]])/(144*3^(3/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/
3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + ((c^(1/3)
+ d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)
*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(108*Sqrt[2]*3^(
1/4)*c^(5/3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rule steps

$$\int \frac{x}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \frac{x\left(25cd-\frac{d^2x^3}{2}\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \left(\frac{dx}{2\sqrt{c+dx^3}} + \frac{21cdx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{216c^2d}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{432c^2} + \frac{7\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{72c}$$

$$= \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{864c^2d} + \frac{\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x}{\sqrt{c+dx^3}}\,dx}{432c^2\sqrt[3]{d}} + \frac{7\int \cdots}{\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{\sqrt{2-\sqrt{3}}\,\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\cdots}{144\cdots}$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\cdots}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{11/6}d^{2/3}}\cdots$$

$$= \frac{\sqrt{c+dx^3}}{216c^2d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{x^2\sqrt{c+dx^3}}{216c^2\left(8c-dx^3\right)} - \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\cdots}{\sqrt{c+dx^3}}\right)}{432\sqrt{3}\,c^{11/6}d^{2/3}}\cdots$$

Mathematica [C] time = 0.09, size = 164, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+125cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\cdots}{17280c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (80*c*x^2*(c + d*x^3) + 125*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF
1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(
17280*c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 1.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.19, size = 882, normalized size = 1.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x^2-1/648*I/c^2*3^(1/2)*(-c*d^2)^(1/
3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c
*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1
/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^
(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipt
icPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.434 $\int \frac{1}{x^2 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=665

$$\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/}\cdots}{\left(\left(1\cdots\right.\right.}}}{288\,3^{3/4}c^{8/}\cdots}$$

```
[Out] 1/648*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^
(17/6)-1/648*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)-1/648*d^
(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)*
3^(1/2)-7/432*(d*x^3+c)^(1/2)/c^3/x+1/216*(d*x^3+c)^(1/2)/c^2/x/(-d*x^3+8*c
)+7/432*d^(1/3)*(d*x^3+c)^(1/2)/c^3/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+7/1296*
d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1
/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/
3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)*2^(1/2)/(d
*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)-7/864*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{7\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{216\sqrt{2}\sqrt[4]{3}c^{8/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}\sqrt{c+dx^3}}-\frac{7\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/}\cdots}{\left(\left(1\cdots\right.\right.}}}{288\,3^{3/4}c^{8/}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-7*Sqrt[c + d*x^3])/(432*c^3*x) + (7*d^(1/3)*Sqrt[c + d*x^3])/(432*c^3*((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*c^2*x*(8*c - d*x^3
)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3
]])/(216*Sqrt[3]*c^(17/6)) + (d^(1/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(648*c^(17/6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(648*c^(17/6)) - (7*Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1
/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1
/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(288*3^(3/4)*c^(8/3)*S
qrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*
Sqrt[c + d*x^3]) + (7*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)
*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)
*x)], -7 - 4*Sqrt[3]])/(216*Sqrt[2]*3^(1/4)*c^(8/3)*Sqrt[(c^(1/3)*(c^(1/3)
+ d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}+\frac{\int \frac{28cd+\frac{5d^2x^3}{2}}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\int \frac{x\left(-160c^2d^2+14cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\int \left(-\frac{14cd^2x}{\sqrt{c+dx^3}}-\frac{48c^2d^2x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)dx}{1728c^4d}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}+\frac{(7d)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{864c^3}+\frac{d\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,d}{36c^2}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{432c^3}+\frac{\left(7d^{2/3}\right)\int}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{7\sqrt{2-}}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\sqrt[3]{d}\tan}{}$$

$$= -\frac{7\sqrt{c+dx^3}}{432c^3x}+\frac{7\sqrt[3]{d}\sqrt{c+dx^3}}{432c^3\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}+\frac{\sqrt{c+dx^3}}{216c^2x\left(8c-dx^3\right)}-\frac{\sqrt[3]{d}\tan}{}$$

Mathematica [C] time = 0.17, size = 180, normalized size = 0.27

$$\frac{7d^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+200cdx^3\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80}{34560c^4\sqrt{c+dx^3}\left(8cx-dx^4\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-80*c*(54*c^2 + 47*c*d*x^3 - 7*d^2*x^6) + 200*c*d*x^3*(8*c - d*x^3)*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 7*d
^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/(34560*c^4*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

fricas [F] time = 1.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{11}-15\,cd^2x^8+48\,c^2dx^5+64\,c^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^11 - 15*c*d^2*x^8 + 48*c^2*d*x^5 + 64*c^3*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

maple [C] time = 0.22, size = 1761, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*d*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x^2-1/648*I/c^2*3^(1/2)*(-c
*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-(d
*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(
-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)
```

```
^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/1728*I/c
^3/d^2*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*
3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c
*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alph
a=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^2*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.435 $\int \frac{1}{x^5\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=687

$$-\frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}\,c^{23/6}}+\frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{82944c^{23/6}}-\frac{5d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\cdots}{\left((1+\cdots\right.}}}{432\sqrt{2}\sqrt[4]{\cdots}}$$

```
[Out] 25/82944*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(23/6)-25/82944*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-25
/82944*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(23/6)*3^(1/2)-31/6912*(d*x^3+c)^(1/2)/c^3/x^4+5/864*d*(d*x^3+c)^(1/2)/c^
4/x+1/216*(d*x^3+c)^(1/2)/c^2/x^4/(-d*x^3+8*c)-5/864*d^(4/3)*(d*x^3+c)^(1/2
)/c^4/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-5/2592*d^(4/3)*(c^(1/3)+d^(1/3)*x)*El
lipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3
^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/
3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)+5/1728*d^(4/3)*(c^(1
/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)
*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^
(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11
/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{25d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{27648\sqrt{3}\,c^{23/6}}+\frac{25d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{82944c^{23/6}}-\frac{25d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+\cdots}}{3\sqrt{\cdots}}\right)}{82944c^{23/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-31*Sqrt[c + d*x^3])/(6912*c^3*x^4) + (5*d*Sqrt[c + d*x^3])/(864*c^4*x) -
(5*d^(4/3)*Sqrt[c + d*x^3])/(864*c^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) +
 Sqrt[c + d*x^3]/(216*c^2*x^4*(8*c - d*x^3)) - (25*d^(4/3)*ArcTan[(Sqrt[3]*
c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(27648*Sqrt[3]*c^(23/6)) +
 (25*d^(4/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/
(82944*c^(23/6)) - (25*d^(4/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(82944
*c^(23/6)) + (5*Sqrt[2 - Sqrt[3]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/
3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2
]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(576*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(
1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
 - (5*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(432*Sqrt[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((
1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5 \left(8c-dx^3\right)^2 \sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} + \frac{\int \frac{31cd+\frac{11d^2x^3}{2}}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} - \frac{\int \frac{320c^2d^2-\frac{155}{2}cd^3x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{6912c^4d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} + \frac{\int \frac{x\left(-980c^3d^3+160c^2d^4x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}}{55296c^6d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} + \frac{\int \left(-\frac{160c^2d^3x}{\sqrt{c+dx^3}} + \frac{300c}{\left(8c-dx^3\right)}\right.}{55296c^6d}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} - \frac{\left(5d^2\right)\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1728c^4} + -$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} + \frac{\sqrt{c+dx^3}}{216c^2x^4\left(8c-dx^3\right)} - \frac{(25d)\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)}}{55296c^4}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+d}}{216c^2x^4\left(8c\right.}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+d}}{216c^2x^4\left(8c\right.}$$

$$= -\frac{31\sqrt{c+dx^3}}{6912c^3x^4} + \frac{5d\sqrt{c+dx^3}}{864c^4x} - \frac{5d^{4/3}\sqrt{c+dx^3}}{864c^4\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)} + \frac{\sqrt{c+d}}{216c^2x^4\left(8c\right.}$$

Mathematica [C] time = 0.20, size = 196, normalized size = 0.29

$$\frac{245cd^2x^6\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right) - 16\left(d^3x^9\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right.}{221184c^5x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

[Out] (245*c*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(2*c*(216*c^3 - 135*c^2*d*x^3 - 311*c*d^2*x^6 + 40*d^3*x^9) + d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(221184*c^5*x^4*(8*c - d*x^3)*Sqrt[c + d*x^3])

fricas [F] time = 6.38, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{14}-15\,cd^2x^{11}+48\,c^2dx^8+64\,c^3x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^3 + c)/(d^3*x^14 - 15*c*d^2*x^11 + 48*c^2*d*x^8 + 64*c^3*x^5), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)

maple [C] time = 0.24, size = 2240, normalized size = 3.26

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

[Out] 1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/c/x^4+5/8*(d*x^3+c)^(1/2)/c^2*d/x+5/24*I/c^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/64/c^2*d^2*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x^2-1/648*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/

```
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(
1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3
+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*
_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*
c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-(d*x^3+c)^(1/2)/c/x-1/3*I/c
*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/6912*I/c^4/d*2^(1/2)*sum(1/_alpha
*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*
(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^5*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.436 $\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=711

$$-\frac{17d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}}+\frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}-\frac{17d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{331776c^{29/6}}+\frac{289d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{24192\ldots}$$

```
[Out] 17/331776*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-17/331776*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
17/331776*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)-17/6048*(d*x^3+c)^(1/2)/c^3/x^7+391/193536*d*(d*x^3+c)^
(1/2)/c^4/x^4-289/48384*d^2*(d*x^3+c)^(1/2)/c^5/x+1/216*(d*x^3+c)^(1/2)/c^2
/x^7/(-d*x^3+8*c)+289/48384*d^(7/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*
(1+3^(1/2)))+289/145152*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^
(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)
-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/c^(14/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-289/96768*d^(7/3)*(c^(1/3)+d^(1/3)*x)*Ell
ipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^
(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^
2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2
)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 711, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {472, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{289d^2\sqrt{c+dx^3}}{48384c^5x}-\frac{17d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{110592\sqrt{3}c^{29/6}}+\frac{17d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{331776c^{29/6}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-17*Sqrt[c + d*x^3])/(6048*c^3*x^7) + (391*d*Sqrt[c + d*x^3])/(193536*c^4*
x^4) - (289*d^2*Sqrt[c + d*x^3])/(48384*c^5*x) + (289*d^(7/3)*Sqrt[c + d*x^
3])/(48384*c^5*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) + Sqrt[c + d*x^3]/(216*
c^2*x^7*(8*c - d*x^3)) - (17*d^(7/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(
1/3)*x))/Sqrt[c + d*x^3]])/(110592*Sqrt[3]*c^(29/6)) + (17*d^(7/3)*ArcTanh[
(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(331776*c^(29/6)) - (
17*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(331776*c^(29/6)) - (289*S
qrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1
/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSi
n[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)],
 -7 - 4*Sqrt[3]])/(32256*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*
x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (289*d^(7/3)*
(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3)
+ d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(24192*
```

```
Sqrt[2]*3^(1/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3]
)*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

```
x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \frac{\int \frac{34cd+\frac{17d^2x^3}{2}}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{216c^2d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \frac{\int \frac{782c^2d^2-187cd^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{12096c^4d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \frac{\int \frac{18496c^3d^3-1955c^2d^4x^3}{x^2\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{387072c^6d}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \cdots$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \cdots$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} + \cdots$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{\sqrt{c+dx^3}}{216c^2x^7\left(8c-dx^3\right)} - \cdots$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}$$

$$= -\frac{17\sqrt{c+dx^3}}{6048c^3x^7} + \frac{391d\sqrt{c+dx^3}}{193536c^4x^4} - \frac{289d^2\sqrt{c+dx^3}}{48384c^5x} + \frac{289d^{7/3}\sqrt{c+dx^3}}{48384c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\cdots\right.}$$

Mathematica [C] time = 0.23, size = 212, normalized size = 0.30

$$-\frac{289d^4x^5\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3870720c^6\sqrt{c+dx^3}}+\frac{9605d^3x^2\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{6193152c^5\sqrt{c+dx^3}}+\sqrt{c+dx^3}\left(-\frac{d^3x^2}{110592c^5\left(dx^3\cdots\right.}\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^8*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] Sqrt[c + d*x^3]*(-1/448*1/(c^3*x^7) + (15*d)/(7168*c^4*x^4) - (171*d^2)/(28672*c^5*x) - (d^3*x^2)/(110592*c^5*(-8*c + d*x^3))) + (9605*d^3*x^2*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(6193152*c^5*Sqrt[c + d*x^3]) - (289*d^4*x^5*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(3870720*c^6*Sqrt[c + d*x^3])

fricas [F] time = 15.15, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{17}-15\,cd^2x^{14}+48\,c^2dx^{11}+64\,c^3x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(d*x^3 + c)/(d^3*x^17 - 15*c*d^2*x^14 + 48*c^2*d*x^11 + 64*c^3*x^8), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)

maple [C] time = 0.19, size = 2738, normalized size = 3.85

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)

[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/c/x^4+5/8*(d*x^3+c)^(1/2)/c^2*d/x+5/24*I/c^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+1/512/c^3*d^3*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x^2-1/648*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/

```
2)))-7/1944*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*
d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)/c/x^7+
11/56*(d*x^3+c)^(1/2)/c^2*d/x^4-55/112*(d*x^3+c)^(1/2)/c^3*d^2/x-55/336*I/c
^3*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*Ellipt
icE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))+3/4096/c^4*d^2*(-(d*x^3+c)^(1
/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/36864*I/c^5*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3
*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^8*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \left(-8c + dx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.437 $\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

```
[Out] 1/448*x^7*AppellF1(7/3,1/2,2,10/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c
^2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 1/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(8c-dx^3)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{(8c-dx^3)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{7}{3};2,\frac{1}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.33, size = 239, normalized size = 3.62

$$\frac{x\left(\frac{256\left(-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+c+dx^3\right)}{8c-dx^3}-\frac{23dx^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c}\right)}{864d^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((-23*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
 (d*x^3)/(8*c)])/c + (256*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(864*d^2*Sqrt[c + d*x^3])
```

fricas [F] time = 1.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^6}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^6/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.28, size = 1431, normalized size = 21.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -2/3*I/d^3*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+64*c
^2/d^2*(-1/216/c^2*x*(d*x^3+c)^(1/2)/(d*x^3-8*c)+1/648*I/c^2*3^(1/2)*(-c*d^
2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2
```

```
*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum
(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)
^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-
8*c))+16/27*I/d^5*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^6/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**6/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.438 $\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,1/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.23, size = 237, normalized size = 3.59

$$\frac{x\left(x^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-\frac{64c\left(-\frac{32c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+c+dx^3\right)}{d\left(dx^3-8c\right)}\right)}{1728c^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*(x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3
)/(8*c)] - (64*c*(c + d*x^3 - (32*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/
c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(d*(-8*c + d
*x^3))))/(1728*c^2*Sqrt[c + d*x^3])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^3}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.26, size = 1150, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 8*c/d*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x+1/648*I/c^2*3^(1/2)*(-c*d^2
)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Ellipti
cF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(
1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I
```

```
*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*
(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^
(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)
*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8
*c))+1/27*I/d^4/c*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{dx^3+c}\,\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(x^3/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(x**3/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.439 $\int \frac{1}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,1/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^2*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.19, size = 237, normalized size = 3.70

$$\frac{x\left(\frac{dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{64\left(\frac{832F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+\frac{c+dx^3}{c^2}\right)}{8c-dx^3}\right)}{13824\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (64*((c + d*x^3)/c^2 + (832*AppellF1[1/3, 1/2, 1, 4/3, -
((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
 (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)
/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(8*
c - d*x^3)))/(13824*Sqrt[c + d*x^3])
```

fricas [F] time = 2.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^9-15\,cd^2x^6+48\,c^2dx^3+64\,c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^9 - 15*c*d^2*x^6 + 48*c^2*d*x^3 + 64*c^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.18, size = 728, normalized size = 11.38

$$-\frac{\sqrt{d\,x^3+c}\,x}{216\left(d\,x^3-8c\right)c^2}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}-\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\sqrt{3}\,d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}}{-\frac{3\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}}}\sqrt{-\frac{i\left(x+\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}+\frac{i\sqrt{3}\,\left(-c\,d^2\right)^{\frac{1}{3}}}{2d}\right)\cdots}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\cdots}{648\sqrt{d\,x^3+c}\;c^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x+1/648*I/c^2*3^(1/2)*(-c*d^2)^(1/3)
/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1/_alph
a^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
```

```
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/((c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/((-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.440 $\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,1/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^2*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^2x^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 266, normalized size = 4.03

$$\frac{\frac{29d^2x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^4}-\frac{4096dxF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}-64(}{221184\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((-64*(c + d*x^3)*(-216*c + 29*d*x^3))/(c^3*x^2*(-8*c + d*x^3)) + (29*d^2*x
^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*
c)])/c^4 - (4096*d*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)
])/(c*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/
(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] -
 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))/(221184*Sqrt
[c + d*x^3])
```

fricas [F] time = 4.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{12}-15\,cd^2x^9+48\,c^2dx^6+64\,c^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^12 - 15*c*d^2*x^9 + 48*c^2*d*x^6 + 64*c^3*x
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

maple [C] time = 0.20, size = 1455, normalized size = 22.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/8/c*d*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x+1/648*I/c^2*3^(1/2)*(-c*d
^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Ellip
ticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-5/972*I/c^2/d^3*2^(1/2)*su
m(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)
+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+
(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2
)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d
-8*c))+1/64/c^2*(-1/2*(d*x^3+c)^(1/2)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)
```

```
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/1728*I/c^3/d^2*2^(1/2)*sum(1/_alph
a^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2
)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2
)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(
-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{d\,x^3+c}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^3*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.441 $\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,1/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^2
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 1/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^2*x^5*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};2,\frac{1}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^2x^5\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.29, size = 279, normalized size = 4.23

$$\frac{-\frac{119d^3x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^5}+\frac{1404928d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^2\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{64}{}}{2211840\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((64*(c + d*x^3)*(864*c^2 - 1080*c*d*x^3 + 119*d^2*x^6))/(c^4*x^5*(-8*c + d
*x^3)) - (119*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x
^3)/c), (d*x^3)/(8*c)])/c^5 + (1404928*d^2*x*AppellF1[1/3, 1/2, 1, 4/3, -((
d*x^3)/c), (d*x^3)/(8*c)])/(c^2*(8*c - d*x^3)*(32*c*AppellF1[1/3, 1/2, 1, 4
/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d
*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^
3)/(8*c)]))))/(2211840*Sqrt[c + d*x^3])
```

fricas [F] time = 9.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^3x^{15}-15\,cd^2x^{12}+48\,c^2dx^9+64\,c^3x^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^3*x^15 - 15*c*d^2*x^12 + 48*c^2*d*x^9 + 64*c^3*
x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

maple [C] time = 0.18, size = 1782, normalized size = 27.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/64/c^2*(-1/5/c*(d*x^3+c)^(1/2)/x^5+7/20/c^2*d*(d*x^3+c)^(1/2)/x^2-7/60*I/
c^2*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/64/c^2
*d^2*(-1/216*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2*x+1/648*I/c^2*3^(1/2)*(-c*d^2)
^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-5/972*I/c^2/d^3*2^(1/2)*sum(1
/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*
3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c
```

```
*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(
2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*
c))+1/256/c^3*d*(-1/2*(d*x^3+c)^(1/2)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/6912*I/c^4/d*2^(1/2)*sum(1/_alpha^
2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3
*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^3+c}\left(dx^3-8\,c\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^3 + c)*(d*x^3 - 8*c)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^6\,\sqrt{d\,x^3+c}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^6*(c + d*x^3)^(1/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-8c+dx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*sqrt(c + d*x**3)), x)
```

3.442 $\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}+\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[Out] -640/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/d^4+8/27*x^6/d^2/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)+2/81*(39*d*x^3+38*c)/d^4/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 98, 146, 63, 206}

$$\frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}}-\frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (8*x^6)/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (2*(38*c + 39*d*x^3))/(81*
d^4*Sqrt[c + d*x^3]) - (640*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(
243*d^4)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
```

```
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{x\left(16c^2+13cdx\right)}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^2}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{(320c)\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx,\right.}{81d^3}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{(640c)\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c\right.}}{81d^4}$$

$$= \frac{8x^6}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\left(38c+39dx^3\right)}{81d^4\sqrt{c+dx^3}} - \frac{640\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243d^4}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.93

$$\frac{6\left(752c^2-198cdx^3+9d^2x^6\right)-640c\left(8c-dx^3\right)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{81d^4\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (6*(752*c^2 - 198*c*d*x^3 + 9*d^2*x^6) - 640*c*(8*c - d*x^3)*Hypergeometric
2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)])/(81*d^4*(-8*c + d*x^3)*Sqrt[c + d*x^3
])
```

fricas [A] time = 0.62, size = 233, normalized size = 2.45

$$\left[\frac{2\left(160\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+3\left(27\,d^2x^6-274\,cdx^3-304\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(d^6x^6-7\,cd^5x^3-8\,c^2d^4\right)}, \frac{2\left(320\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(160*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 3*(27*d^2*x^6 - 274*c*d*x^3 - 304*c^
2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^3 - 8*c^2*d^4), 2/243*(320*(d^2*x^
6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*
(27*d^2*x^6 - 274*c*d*x^3 - 304*c^2)*sqrt(d*x^3 + c))/(d^6*x^6 - 7*c*d^5*x^
3 - 8*c^2*d^4)]
```

giac [A] time = 0.19, size = 88, normalized size = 0.93

$$\frac{640\,c\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{243\,\sqrt{-c}\,d^4}+\frac{2\,\sqrt{dx^3+c}}{3\,d^4}-\frac{2\left(85\left(dx^3+c\right)c+3\,c^2\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 640/243*c*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^4) + 2/3*sqrt(d*
x^3 + c)/d^4 - 2/81*(85*(d*x^3 + c)*c + 3*c^2)/(((d*x^3 + c)^(3/2) - 9*sqrt
(d*x^3 + c)*c)*d^4)
```

maple [C] time = 0.29, size = 970, normalized size = 10.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^3*((2/3/((x^3+c/d)*d)^(1/2)*c/d^2+2/3*(d*x^3+c)^(1/2)/d^2)*d-32/3/(d*x^
3+c)^(1/2)*c/d)+512*c^3/d^3*(-1/243/d/c^2*(d*x^3+c)^(1/2)/(d*x^3-8*c)-2/243
/d/c^2/((x^3+c/d)*d)^(1/2)-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d
+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alph
a)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+192*c^2/d^3*(2/27/((x^3+
c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)
*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.21, size = 98, normalized size = 1.03

$$\frac{2\left(160\,\sqrt{c}\,\log\left(\frac{\sqrt{dx^3+c}-3\,\sqrt{c}}{\sqrt{dx^3+c}+3\,\sqrt{c}}\right)+81\,\sqrt{dx^3+c}-\frac{3\left(85\left(dx^3+c\right)c+3\,c^2\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c}\right)}{243\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(160*sqrt(c)*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*s
qrt(c))) + 81*sqrt(d*x^3 + c) - 3*(85*(d*x^3 + c)*c + 3*c^2)/((d*x^3 + c)^(
3/2) - 9*sqrt(d*x^3 + c)*c))/d^4
```

mupad [B] time = 4.38, size = 111, normalized size = 1.17

$$\frac{2\sqrt{d\,x^3+c}}{3\,d^4}+\frac{320\,\sqrt{c}\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,d^4}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{176\,c^2}{81\,d^4}+\frac{170\,c\,x^3}{81\,d^3}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*d^4) + (320*c^(1/2)*log((10*c + d*x^3 - 6*c^(1/2)*
(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*d^4) + ((c + d*x^3)^(1/2)*((176*c^2
)/(81*d^4) + (170*c*x^3)/(81*d^3)))/(8*c^2 - d^2*x^6 + 7*c*d*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.443 $\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

```
[Out] -32/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/d^3/c^(1/2)-22/81/d^3/(d*x^3+c
)^(1/2)+64/27*c/d^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 89, 78, 63, 206}

$$\frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}}-\frac{22}{81d^3\sqrt{c+dx^3}}-\frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -22/(81*d^3*Sqrt[c + d*x^3]) + (64*c)/(27*d^3*(8*c - d*x^3)*Sqrt[c + d*x^3]
) - (32*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*Sqrt[c]*d^3)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-24c^2d+9cd^2x}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{27cd^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\,\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^{3}\right)}{81d^2}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81d^3}$$

$$= -\frac{22}{81d^3\sqrt{c+dx^3}} + \frac{64c}{27d^3(8c-dx^3)\sqrt{c+dx^3}} - \frac{32\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243\sqrt{c}\,d^3}$$

Mathematica [A] time = 0.11, size = 71, normalized size = 0.86

$$\frac{2\left(\frac{3(8c+11dx^3)}{(8c-dx^3)\sqrt{c+dx^3}} - \frac{16\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{\sqrt{c}}\right)}{243d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*((3*(8*c + 11*d*x^3))/((8*c - d*x^3)*Sqrt[c + d*x^3]) - (16*ArcTanh[Sqrt
[c + d*x^3]/(3*Sqrt[c])])/Sqrt[c]))/(243*d^3)
```

fricas [A] time = 0.62, size = 223, normalized size = 2.69

$$\left[\frac{2\left(8\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3-6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-3\left(11\,cdx^3+8\,c^2\right)\sqrt{dx^3+c}\right)}{243\left(cd^5x^6-7\,c^2d^4x^3-8\,c^3d^3\right)}, \frac{2\left(16\left(d^2x^6-7\,cdx^3-8\,c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [2/243*(8*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 - 6*sqrt(d*x^3 +
 c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 3*(11*c*d*x^3 + 8*c^2)*sqrt(d*x^3 + c)
)/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3), 2/243*(16*(d^2*x^6 - 7*c*d*x^3 -
```

```
8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) - 3*(11*c*d*x^3 + 8
*c^2)*sqrt(d*x^3 + c))/(c*d^5*x^6 - 7*c^2*d^4*x^3 - 8*c^3*d^3)]
```

giac [A] time = 0.17, size = 67, normalized size = 0.81

$$\frac{32 \arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{243\sqrt{-c}\,d^3} - \frac{2\left(11\,dx^3+8\,c\right)}{81\left(\left(dx^3+c\right)^{\frac{3}{2}} - 9\sqrt{dx^3+c}\,c\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 32/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*d^3) - 2/81*(11*d*x^3
 + 8*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*d^3)
```

maple [C] time = 0.18, size = 926, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/d^3/(d*x^3+c)^(1/2)+64*c^2/d^2*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2
/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1
/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3
))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*
EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+16*c/d^2*(2/27/(
(x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^
(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/
d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [A] time = 1.41, size = 81, normalized size = 0.98

$$\frac{2\left(\frac{8 \log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{\sqrt{c}} - \frac{3\left(11\,dx^3+8\,c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c}\right)}{243\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/243*(8*log((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/s
qrt(c) - 3*(11*d*x^3 + 8*c)/((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c))/d^3
```

mupad [B] time = 4.30, size = 94, normalized size = 1.13

$$\frac{16\,\ln\left(\frac{10\,c+d\,x^3-6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,\sqrt{c}\,d^3}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{16\,c}{81\,d^3}+\frac{22\,x^3}{81\,d^2}\right)}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] (16*log((10*c + d*x^3 - 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3)))/(243*c
^(1/2)*d^3) + ((c + d*x^3)^(1/2)*((16*c)/(81*d^3) + (22*x^3)/(81*d^2)))/(8*
c^2 - d^2*x^6 + 7*c*d*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**8/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.444 $\int \frac{x^5}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

```
[Out] 2/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(3/2)/d^2-2/81/c/d^2/(d*x^3+c)
^(1/2)+8/27/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {446, 78, 51, 63, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}+\frac{8}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2}{81cd^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -2/(81*c*d^2*Sqrt[c + d*x^3]) + 8/(27*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
(2*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(243*c^(3/2)*d^2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{9d}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{81cd}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81cd^2}$$

$$= -\frac{2}{81cd^2\sqrt{c+dx^3}} + \frac{8}{27d^2(8c-dx^3)\sqrt{c+dx^3}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{3/2}d^2}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.80

$$-\frac{2\left((dx^3-8c)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+12c\right)}{81cd^2(dx^3-8c)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*(12*c + (-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)]))/(81*c*d^2*(-8*c + d*x^3)*Sqrt[c + d*x^3])
```

fricas [A] time = 0.59, size = 223, normalized size = 2.62

$$\left[\frac{(d^2x^6-7\,cdx^3-8\,c^2)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\,(cdx^3+4\,c^2)\sqrt{dx^3+c}}{243\,(c^2d^4x^6-7\,c^3d^3x^3-8\,c^4d^2)}, -\frac{2\left((d^2x^6-7\,cdx^3-8\,c^2)\sqrt{-c}\,\text{ar}\right.}{243\,(c^2d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 + 4*c^2)*sqrt(d*x^3 + c))/(c^
2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2), -2/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^
2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 + 4*c^2)*sq
rt(d*x^3 + c))/(c^2*d^4*x^6 - 7*c^3*d^3*x^3 - 8*c^4*d^2)]
```

giac [A] time = 0.19, size = 76, normalized size = 0.89

$$-\frac{2\left(\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\sqrt{-c}}\right)}{\sqrt{-c}\,cd}+\frac{3\left(dx^3+4c\right)}{\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\sqrt{dx^3+c}\,c\right)cd}\right)}{243\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2/243*(arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c*d) + 3*(d*x^3 + 4*
c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c*d))/d
```

maple [C] time = 0.18, size = 908, normalized size = 10.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 8*c/d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c
^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*d-8*c)))+1/d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/
d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1
/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/
3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d
^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*
d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*
I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^
(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*d-8*c)))
```

maxima [A] time = 1.23, size = 83, normalized size = 0.98

$$-\frac{\frac{6\left(dx^3+4c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}c-9\sqrt{dx^3+c}\,c^2}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{3}{2}}}}{243\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/243*(6*(d*x^3 + 4*c)/((d*x^3 + c)^(3/2)*c - 9*sqrt(d*x^3 + c)*c^2) + log
((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(3/2))/d^2
```

mupad [B] time = 4.26, size = 96, normalized size = 1.13

$$\frac{\left(\frac{8}{81\,d^2}+\frac{2\,x^3}{81\,c\,d}\right)\,\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}+\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{243\,c^{3/2}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((8/(81*d^2) + (2*x^3)/(81*c*d))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7*c*
d*x^3) + log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(2
43*c^(3/2)*d^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**5/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.445 $\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=88

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[Out] 1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/2)/d-1/81/c^2/d/(d*x^3+c)^(
1/2)+1/27/c/d/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {444, 51, 63, 206}

$$-\frac{1}{81c^2d\sqrt{c+dx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}+\frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/(81*c^2*d*Sqrt[c + d*x^3]) + 1/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]) +
ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c^(5/2)*d)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{18c}$$

$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{162c^2}$$

$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{81c^2d}$$

$$= -\frac{1}{81c^2d\sqrt{c+dx^3}} + \frac{1}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/2}d}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.49

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{dx^3+c}{9c}\right)}{243c^2d\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (c + d*x^3)/(9*c)])/(243*c^2*d*Sqrt[c +
 d*x^3])
```

fricas [A] time = 0.66, size = 219, normalized size = 2.49

$$\left[\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)-6\left(cdx^3-5\,c^2\right)\sqrt{dx^3+c}}{486\left(c^3d^3x^6-7\,c^4d^2x^3-8\,c^5d\right)}, -\frac{\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{-c}\,\text{arcta}}{243\left(c^3d^3x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/486*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3 + c
)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) - 6*(c*d*x^3 - 5*c^2)*sqrt(d*x^3 + c))/(c^
3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d), -1/243*((d^2*x^6 - 7*c*d*x^3 - 8*c^2)
*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 3*(c*d*x^3 - 5*c^2)*sqrt
(d*x^3 + c))/(c^3*d^3*x^6 - 7*c^4*d^2*x^3 - 8*c^5*d)]
```

giac [A] time = 0.16, size = 72, normalized size = 0.82

$$-\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{243\,\sqrt{-c}\,c^2d} - \frac{dx^3-5\,c}{81\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/243*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^2*d) - 1/81*(d*x^3
- 5*c)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^2*d)
```

maple [C] time = 0.19, size = 463, normalized size = 5.26

$$-\frac{\sqrt{d x^3+c}}{243\left(d x^3-8c\right)c^2 d}-\frac{2}{243\sqrt{\left(x^3+\frac{c}{d}\right)d}\; c^2 d}-\frac{i\left(-c d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}+\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c d^2\right)^{\frac{1}{3}}}}\sqrt{-\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)/c^2/d-1/
1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^
2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*d-8*c))
```

maxima [A] time = 1.27, size = 85, normalized size = 0.97

$$-\frac{\frac{6\left(dx^3-5c\right)}{\left(dx^3+c\right)^{\frac{3}{2}}c^2-9\sqrt{dx^3+c}\,c^3}+\frac{\log\left(\frac{\sqrt{dx^3+c}-3\sqrt{c}}{\sqrt{dx^3+c}+3\sqrt{c}}\right)}{c^{\frac{5}{2}}}}{486\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/486*(6*(d*x^3 - 5*c)/((d*x^3 + c)^(3/2)*c^2 - 9*sqrt(d*x^3 + c)*c^3) + l
og((sqrt(d*x^3 + c) - 3*sqrt(c))/(sqrt(d*x^3 + c) + 3*sqrt(c)))/c^(5/2))/d
```

mupad [B] time = 4.26, size = 97, normalized size = 1.10

$$\frac{\ln\left(\frac{10\,c+d\,x^3+6\,\sqrt{c}\,\sqrt{d\,x^3+c}}{8\,c-d\,x^3}\right)}{486\,c^{5/2}\,d}-\frac{\left(\frac{5}{81\,c\,d}-\frac{x^3}{81\,c^2}\right)\sqrt{d\,x^3+c}}{8\,c^2+7\,c\,d\,x^3-d^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] log((10*c + d*x^3 + 6*c^(1/2)*(c + d*x^3)^(1/2))/(8*c - d*x^3))/(486*c^(5/2
)*d) - ((5/(81*c*d) - x^3/(81*c^2))*(c + d*x^3)^(1/2))/(8*c^2 - d^2*x^6 + 7
*c*d*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**2/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.446 $\int \frac{1}{x(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[Out] 7/7776*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(7/2)-1/96*arctanh((d*x^3+c)^
(1/2)/c^(1/2))/c^(7/2)+5/648/c^3/(d*x^3+c)^(1/2)+1/216/c^2/(-d*x^3+8*c)/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {446, 103, 152, 156, 63, 208, 206}

$$\frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5}{648c^3\sqrt{c+dx^3}}+\frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*Sqrt[c + d*x^3]) + 1/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (
7*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(7776*c^(7/2)) - ArcTanh[Sqrt[c + d
*x^3]/Sqrt[c]]/(96*c^(7/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-9cd-\frac{3d^2x}{2}}{x(8c-dx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{216c^2d}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{-\frac{81}{2}c^2d^2+\frac{15}{4}cd^3x}{x(8c-dx)\sqrt{c+dx}}\,dx, x, x^3\right)}{972c^4d^2}$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{192c^3} + \cdots$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\,\text{Subst}\left(\int \frac{1}{9c-x^2}\,dx, x, \sqrt{c+dx^3}\right)}{2592c^3} \cdots$$

$$= \frac{5}{648c^3\sqrt{c+dx^3}} + \frac{1}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{7776c^{7/2}} - \frac{\tanh^{-1}\left(\cdots\right)}{96c\cdots}$$

Mathematica [C] time = 0.05, size = 97, normalized size = 0.92

$$\frac{\left(7dx^3-56c\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+27\left(8c-dx^3\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c}{2592c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c + (-56*c + 7*d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c)
] + 27*(8*c - d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(2592*
c^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [A] time = 0.72, size = 316, normalized size = 2.98

$$\left[\frac{7\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+81\left(d^2x^6-7\,cdx^3-8\,c^2\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3+c}\,\sqrt{c}+2\,c}{x^3}\right)+}{15552\left(c^4d^2x^6-7\,c^5dx^3-8\,c^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/15552*(7*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(c)*log((d*x^3 + 6*sqrt(d*x^3
 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 81*(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqr
t(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 24*(5*c*d*x^3 - 4
3*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 - 8*c^6), 1/7776*(81*(d^
2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - 7*
(d^2*x^6 - 7*c*d*x^3 - 8*c^2)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/
c) + 12*(5*c*d*x^3 - 43*c^2)*sqrt(d*x^3 + c))/(c^4*d^2*x^6 - 7*c^5*d*x^3 -
8*c^6)]
```

giac [A] time = 0.17, size = 93, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{96\,\sqrt{-c}\,c^3}-\frac{7\,\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{7776\,\sqrt{-c}\,c^3}+\frac{5\,dx^3-43\,c}{648\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/96*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) - 7/7776*arctan(1/3*sq
rt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^3) + 1/648*(5*d*x^3 - 43*c)/(((d*x^3 +
c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^3)
```

maple [C] time = 0.17, size = 953, normalized size = 8.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*d*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^(1/2)
/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)),_alpha=RootOf(_Z^3*d-8*c))-1/64/c^2*d*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/2
43*I/c^2/d^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
```

```
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*d-8*c))+1/64/c^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+
c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x), x)
```

mupad [B] time = 4.33, size = 101, normalized size = 0.95

$$-\frac{\frac{5\left(d\,x^3+c\right)}{216\,c^3}-\frac{2}{9\,c^2}}{27\,c\,\sqrt{d\,x^3+c}-3\left(d\,x^3+c\right)^{3/2}}+\frac{\left(\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{\sqrt{c^7}}\right)\,1\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^3\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^7}}\right)\,7\mathrm{i}}{81}\right)\,1\mathrm{i}}{96\,\sqrt{c^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((atanh((c^3*(c + d*x^3)^(1/2))/(c^7)^(1/2))*1i - (atanh((c^3*(c + d*x^3)^(
1/2))/(3*(c^7)^(1/2)))*7i)/81)*1i)/(96*(c^7)^(1/2)) - ((5*(c + d*x^3))/(216
*c^3) - 2/(9*c^2))/(27*c*(c + d*x^3)^(1/2) - 3*(c + d*x^3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.447 $\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}+\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

```
[Out] 5/31104*d*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(9/2)+5/384*d*arctanh((d*x
^3+c)^(1/2)/c^(1/2))/c^(9/2)-35/2592*d/c^4/(d*x^3+c)^(1/2)+5/864*d/c^3/(-d*
x^3+8*c)/(d*x^3+c)^(1/2)-1/24/c^2/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$\frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{35d}{2592c^4\sqrt{c+dx^3}}-\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{31104c^{9/2}}+\frac{5d\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{384c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-35*d)/(2592*c^4*Sqrt[c + d*x^3]) + (5*d)/(864*c^3*(8*c - d*x^3)*Sqrt[c +
d*x^3]) - 1/(24*c^2*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (5*d*ArcTanh[Sqrt[
c + d*x^3]/(3*Sqrt[c])])/(31104*c^(9/2)) + (5*d*ArcTanh[Sqrt[c + d*x^3]/Sqr
t[c]])/(384*c^(9/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{10cd-\frac{5d^2x}{2}}{x(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{24c^2}$$

$$= \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\operatorname{Subst}\left(\int \frac{-90}{x(8c}\right.}{1}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+}}$$

$$= -\frac{35d}{2592c^4\sqrt{c+dx^3}} + \frac{5d}{864c^3\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{24c^2x^3\left(8c-dx^3\right)\sqrt{c+}}$$

Mathematica [C] time = 0.06, size = 117, normalized size = 0.82

$$\frac{5dx^3\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right)+135dx^3\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right)+12c\left(5dx^3-36c\right)}{10368c^4x^3\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (12*c*(-36*c + 5*d*x^3) + 5*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1,
 1/2, (c + d*x^3)/(9*c)] + 135*d*x^3*(-8*c + d*x^3)*Hypergeometric2F1[-1/2,
 1, 1/2, 1 + (d*x^3)/c])/(10368*c^4*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [A] time = 0.65, size = 368, normalized size = 2.57

$$\left[\frac{5\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+405\left(d^3x^9-7\,cd^2x^6-8\,c^2dx^3\right)\sqrt{c}\,\log\left(\frac{dx^3+2\,\sqrt{dx^3+}}{x^3}\right.}{62208\left(c^5d^2x^9-7\,c^6dx^6-8\,c^7x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/62208*(5*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 6*sq
rt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 405*(d^3*x^9 - 7*c*d^2*x^6 -
 8*c^2*d*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) -
24*(35*c*d^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 -
 7*c^6*d*x^6 - 8*c^7*x^3), -1/31104*(405*(d^3*x^9 - 7*c*d^2*x^6 - 8*c^2*d*x
```

```
^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + 5*(d^3*x^9 - 7*c*d^2*x^6
- 8*c^2*d*x^3)*sqrt(-c)*arctan(1/3*sqrt(d*x^3 + c)*sqrt(-c)/c) + 12*(35*c*d
^2*x^6 - 265*c^2*d*x^3 - 108*c^3)*sqrt(d*x^3 + c))/(c^5*d^2*x^9 - 7*c^6*d*x
^6 - 8*c^7*x^3)]
```

giac [A] time = 0.17, size = 129, normalized size = 0.90

$$-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{384\,\sqrt{-c}\,c^4}-\frac{5\,d\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{31104\,\sqrt{-c}\,c^4}-\frac{35\,\left(dx^3+c\right)^2d-335\,\left(dx^3+c\right)cd+192\,c^2d}{2592\,\left(\left(dx^3+c\right)^{\frac{5}{2}}-10\,\left(dx^3+c\right)^{\frac{3}{2}}c+9\,\sqrt{dx^3+c}\,c^2\right)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -5/384*d*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 5/31104*d*arctan
(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^4) - 1/2592*(35*(d*x^3 + c)^2*d
- 335*(d*x^3 + c)*c*d + 192*c^2*d)/(((d*x^3 + c)^(5/2) - 10*(d*x^3 + c)^(3/
2)*c + 9*sqrt(d*x^3 + c)*c^2)*c^4)
```

maple [C] time = 0.20, size = 1019, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*d^2*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)^
(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-
2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))-1
/256/c^3*d^2*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*sum((-c*
d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^
2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d
)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^
(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d
^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3
*d*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^4), x)
```

mupad [B] time = 4.56, size = 133, normalized size = 0.93

$$-\frac{\frac{2\,d}{9\,c^2}+\frac{35\,d\,{\left(d\,x^3+c\right)}^2}{864\,c^4}-\frac{335\,d\,\left(d\,x^3+c\right)}{864\,c^3}}{3\,{\left(d\,x^3+c\right)}^{5/2}-30\,c\,{\left(d\,x^3+c\right)}^{3/2}+27\,c^2\,\sqrt{d\,x^3+c}}-\frac{d\,\left(\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{\sqrt{c^9}}\right)\,1{}\mathrm{i}+\frac{\mathrm{atanh}\left(\frac{c^4\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^9}}\right)\,1{}\mathrm{i}}{81}\right)\,5{}\mathrm{i}}{384\,\sqrt{c^9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] - ((2*d)/(9*c^2) + (35*d*(c + d*x^3)^2)/(864*c^4) - (335*d*(c + d*x^3))/(86
4*c^3))/(3*(c + d*x^3)^(5/2) - 30*c*(c + d*x^3)^(3/2) + 27*c^2*(c + d*x^3)^
(1/2)) - (d*(atanh((c^4*(c + d*x^3)^(1/2))/(c^9)^(1/2))*1i + (atanh((c^4*(c
 + d*x^3)^(1/2))/(3*(c^9)^(1/2)))*1i)/81)*5i)/(384*(c^9)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(-8c + dx^{3}\right)^{2} \left(c + dx^{3}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.448 $\int \frac{1}{x^7(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}-\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{1\text{?}}{384c^3x^3\left(8c-\right.}$$

```
[Out] 13/497664*d^2*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)-33/2048*d^2*arc
tanh((d*x^3+c)^(1/2)/c^(1/2))/c^(11/2)+665/41472*d^2/c^5/(d*x^3+c)^(1/2)-71
/13824*d^2/c^4/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-1/48/c^2/x^6/(-d*x^3+8*c)/(d*x^
3+c)^(1/2)+17/384*d/c^3/x^3/(-d*x^3+8*c)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {446, 103, 151, 152, 156, 63, 208, 206}

$$-\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}+\frac{665d^2}{41472c^5\sqrt{c+dx^3}}+\frac{13d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{497664c^{11/2}}-\frac{33d^2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{2048c^{11/2}}+\frac{}{384c^3x^3\left(8c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (665*d^2)/(41472*c^5*Sqrt[c + d*x^3]) - (71*d^2)/(13824*c^4*(8*c - d*x^3)*S
qrt[c + d*x^3]) - 1/(48*c^2*x^6*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (17*d)/(38
4*c^3*x^3*(8*c - d*x^3)*Sqrt[c + d*x^3]) + (13*d^2*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(497664*c^(11/2)) - (33*d^2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])
/(2048*c^(11/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^3(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\text{Subst}\left(\int \frac{17cd-\frac{7d^2x}{2}}{x^2(8c-dx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{48c^2}$$

$$= -\frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{17d}{384c^3x^3\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{19}{x(8}\right.}{}$$

$$= -\frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{}{384c^3x^3\left(8c\right.}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+d}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+d}}$$

$$= \frac{665d^2}{41472c^5\sqrt{c+dx^3}} - \frac{71d^2}{13824c^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{1}{48c^2x^6\left(8c-dx^3\right)\sqrt{c+d}}$$

Mathematica [C] time = 0.06, size = 135, normalized size = 0.73

$$\frac{13d^2x^6\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3+c}{9c}\right) - 3\left(4c\left(288c^2-612cdx^3+71d^2x^6\right)+891d^2x^6\left(dx^3-8c\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}\right.\right.}{165888c^5x^6\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (13*d^2*x^6*(-8*c + d*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (c + d*x^3)/(9*c
)] - 3*(4*c*(288*c^2 - 612*c*d*x^3 + 71*d^2*x^6) + 891*d^2*x^6*(-8*c + d*x^
3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(165888*c^5*x^6*(8*c -
d*x^3)*Sqrt[c + d*x^3])
```

fricas [A] time = 0.65, size = 398, normalized size = 2.15

$$\left[\frac{13\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3+6\,\sqrt{dx^3+c}\,\sqrt{c}+10\,c}{dx^3-8\,c}\right)+8019\left(d^4x^{12}-7\,cd^3x^9-8\,c^2d^2x^6\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{d}}{}\right.}{995328\left(c^6d^2x^{12}-7\,c^7dx^9-8\,c^8x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/995328*(13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 +
 6*sqrt(d*x^3 + c)*sqrt(c) + 10*c)/(d*x^3 - 8*c)) + 8019*(d^4*x^12 - 7*c*d^
3*x^9 - 8*c^2*d^2*x^6)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c
)/x^3) + 24*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^3*d*x^3 + 864*c^4)*s
qrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6), 1/497664*(8019*(d
^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt
(-c)/c) - 13*(d^4*x^12 - 7*c*d^3*x^9 - 8*c^2*d^2*x^6)*sqrt(-c)*arctan(1/3*s
qrt(d*x^3 + c)*sqrt(-c)/c) + 12*(665*c*d^3*x^9 - 5107*c^2*d^2*x^6 - 1836*c^
3*d*x^3 + 864*c^4)*sqrt(d*x^3 + c))/(c^6*d^2*x^12 - 7*c^7*d*x^9 - 8*c^8*x^6
)]
```

giac [A] time = 0.18, size = 149, normalized size = 0.81

$$\frac{33\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{2048\,\sqrt{-c}\,c^5}-\frac{13\,d^2\arctan\left(\frac{\sqrt{dx^3+c}}{3\,\sqrt{-c}}\right)}{497664\,\sqrt{-c}\,c^5}+\frac{341\,\left(dx^3+c\right)d^2-3072\,cd^2}{41472\left(\left(dx^3+c\right)^{\frac{3}{2}}-9\,\sqrt{dx^3+c}\,c\right)c^5}+\frac{3\,\left(dx^3+c\right)^{\frac{3}{2}}d^2-4\,\sqrt{dx}}{384\,c^5d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 33/2048*d^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) - 13/497664*d^2
*arctan(1/3*sqrt(d*x^3 + c)/sqrt(-c))/(sqrt(-c)*c^5) + 1/41472*(341*(d*x^3
+ c)*d^2 - 3072*c*d^2)/(((d*x^3 + c)^(3/2) - 9*sqrt(d*x^3 + c)*c)*c^5) + 1/
384*(3*(d*x^3 + c)^(3/2)*d^2 - 4*sqrt(d*x^3 + c)*c*d^2)/(c^5*d^2*x^6)
```

maple [C] time = 0.21, size = 1106, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/512/c^3*d^3*(-1/243*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^2/d-2/243/((x^3+c/d)*d)
^(1/2)/c^2/d-1/1458*I/d^3/c^3*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-1/3*(d*x^3+c)^(1/2)/c^2/
x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2
))+1/64/c^2*(-1/6*(d*x^3+c)^(1/2)/c^2/x^6+7/12*(d*x^3+c)^(1/2)/c^3*d/x^3+2/
3/((x^3+c/d)*d)^(1/2)/c^3*d^2-5/4*d^2*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(7
/2))-3/4096/c^4*d^3*(2/27/((x^3+c/d)*d)^(1/2)/c/d+1/243*I/c^2/d^3*2^(1/2)*s
um((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1
/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-
3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+3/
```

```
4096/c^4*d^2*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2)
)/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^7), x)
```

mupad [B] time = 4.76, size = 171, normalized size = 0.92

$$\frac{\frac{2\,d^2}{9\,c^2}-\frac{10373\,d^2\,\left(d\,x^3+c\right)}{13824\,c^3}+\frac{3551\,d^2\,\left(d\,x^3+c\right)^2}{6912\,c^4}-\frac{665\,d^2\,\left(d\,x^3+c\right)^3}{13824\,c^5}}{33\,c\,\left(d\,x^3+c\right)^{5/2}-3\,\left(d\,x^3+c\right)^{7/2}+27\,c^3\,\sqrt{d\,x^3+c}-57\,c^2\,\left(d\,x^3+c\right)^{3/2}}+\frac{d^2\,\left(\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{\sqrt{c^{11}}}\right)\,1{}\mathrm{i}-\frac{\mathrm{atanh}\left(\frac{c^5\,\sqrt{d\,x^3+c}}{3\,\sqrt{c^{11}}}\right)\,\dots}{8019}\right.}{2048\,\sqrt{c^{11}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] ((2*d^2)/(9*c^2) - (10373*d^2*(c + d*x^3))/(13824*c^3) + (3551*d^2*(c + d*x
^3)^2)/(6912*c^4) - (665*d^2*(c + d*x^3)^3)/(13824*c^5))/(33*c*(c + d*x^3)^
(5/2) - 3*(c + d*x^3)^(7/2) + 27*c^3*(c + d*x^3)^(1/2) - 57*c^2*(c + d*x^3)
^(3/2)) + (d^2*(atanh((c^5*(c + d*x^3)^(1/2))/(c^11)^(1/2))*1i - (atanh((c^
5*(c + d*x^3)^(1/2))/(3*(c^11)^(1/2)))*13i)/8019)*33i)/(2048*(c^11)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**7*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.449 $\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=668

$$\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}}-\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}(}{\left(\left(1+\sqrt{\right.\right.}}}}$$

```
[Out] -4/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(5/6)/d
^(8/3)+4/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(5/6)/d^(8/3)+4/243*arc
tan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(5/6)/d^(8/3)*3^
(1/2)-2/81*x^2/c/d^2/(d*x^3+c)^(1/2)+8/27*x^2/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1
/2)+2/81*(d*x^3+c)^(1/2)/c/d^(8/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+2/243*(c
^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1
/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)
*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(2/3)/d^(8/3)/(d*x
^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)-1/81*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(
d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c
^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1
/2)*3^(1/4)/c^(2/3)/d^(8/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d
^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {470, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{4\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{5/6}d^{8/3}}-\frac{4\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{5/6}d^{8/3}}+\frac{4\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{5/6}d^{8/3}}+\frac{2\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}}{81\sqrt[4]{3}c^{2/3}d^{8/3}\sqrt{\frac{\sqrt[3]{c}(}{\left(\left(1+\sqrt{\right.\right.}}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*x^2)/(81*c*d^2*Sqrt[c + d*x^3]) + (8*x^2)/(27*d^2*(8*c - d*x^3)*Sqrt[c
+ d*x^3]) + (2*Sqrt[c + d*x^3])/(81*c*d^(8/3)*((1 + Sqrt[3])*c^(1/3) + d^(1
/3)*x)) + (4*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]
])/(81*Sqrt[3]*c^(5/6)*d^(8/3)) - (4*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(
1/6)*Sqrt[c + d*x^3])])/(243*c^(5/6)*d^(8/3)) + (4*ArcTanh[Sqrt[c + d*x^3]/
(3*Sqrt[c])])/(243*c^(5/6)*d^(8/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)
*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3)
 + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 +
 Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(27*3^(3/4)*c^(2/3)*d^(8/
3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)
^2]*Sqrt[c + d*x^3]) + (2*Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(81*3^(1/4)*c^(2/3)*d^(8/3)*Sqrt[(c^(1/3)*(c^(1/
3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(16c^2+7cdx^3\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd^2}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(-72c^3d-\frac{9}{2}c^2d^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9c^2dx}{2\sqrt{c+dx^3}} - \frac{108c^3dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)}{729c^3d^3}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{8\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{27d^2} + \frac{\int \frac{x}{\sqrt{c+dx^3}}}{81cd}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{81cd^3} +$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{2x^2}{81cd^2\sqrt{c+dx^3}} + \frac{8x^2}{27d^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\sqrt{c+dx^3}}{81cd^{8/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Mathematica [C] time = 0.14, size = 168, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+40cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(4c\right.}{3240c^2d^2\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(4*c + d*x^3) + 40*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(3240*c^2*d^2*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^7}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^7/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.26, size = 2255, normalized size = 3.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^2*(2/3/((x^3+c/d)*d)^(1/2)/c*x^2+2/9*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))))+64*c^2/d^2*(-1/1944*x^2/c^3*(d*x^3+c)^(1/2)/(d*x^3-8*c)+2/243*x^
2/c^3/((x^3+c/d)*d)^(1/2)+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-
c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2
)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2
*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*
x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/
3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)
```

```
*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+16*c/d^2*(-2/27/((x^3+c/d)*d)^(1/2)/c^
2*x^2-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/
d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^7/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^7/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**7/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.450 $\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=671

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\text{si}\ldots\right.}{81\sqrt[4]{3}c^{5/3}d^{5/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\ldots\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}\ldots\right.}}}$$

```
[Out] 1/243*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(11/6)/d
^(5/3)-1/243*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(11/6)/d^(5/3)-1/243*ar
ctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(11/6)/d^(5/3)*
3^(1/2)-1/81*x^2/c^2/d/(d*x^3+c)^(1/2)+1/27*x^2/c/d/(-d*x^3+8*c)/(d*x^3+c)^
(1/2)+1/81*(d*x^3+c)^(1/2)/c^2/d^(5/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+1/24
3*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+
c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(
2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(5/3)/d^(5/3)/
(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)^2)^(1/2)-1/162*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2
)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2)
)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^
2)^(1/2)*3^(1/4)/c^(5/3)/d^(5/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*
x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 671, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.482, Rules used = {471, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{81\sqrt{3}c^{11/6}d^{5/3}}+\frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{243c^{11/6}d^{5/3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{243c^{11/6}d^{5/3}}+\frac{\sqrt{2}\left(\sqrt[3]{c}+\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -x^2/(81*c^2*d*Sqrt[c + d*x^3]) + x^2/(27*c*d*(8*c - d*x^3)*Sqrt[c + d*x^3]
) + Sqrt[c + d*x^3]/(81*c^2*d^(5/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) -
ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]]/(81*Sqrt[3]
*c^(11/6)*d^(5/3)) + ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*
x^3])]/(243*c^(11/6)*d^(5/3)) - ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])]/(243*c
^(11/6)*d^(5/3)) - (Sqrt[2 - Sqrt[3]]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) -
 c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*El
lipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) +
 d^(1/3)*x)], -7 - 4*Sqrt[3]])/(54*3^(3/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c
^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]
) + (Sqrt[2]*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2
/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt
[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt
[3]])/(81*3^(1/4)*c^(5/3)*d^(5/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1
+ Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(2c-\frac{5dx^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \frac{x\left(45c^2d-\frac{9}{4}cd^2x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{729c^3d^2}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{2\int \left(\frac{9cdx}{4\sqrt{c+dx^3}} + \frac{27c^2dx}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\right)$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x}{\sqrt{c+dx^3}}\,dx}{162c^2d} + \frac{2\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+d}}}{27cd}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{162c^2d^2} + \frac{\int}{}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

$$= -\frac{x^2}{81c^2d\sqrt{c+dx^3}} + \frac{x^2}{27cd\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{c+dx^3}}{81c^2d^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}$$

Mathematica [C] time = 0.13, size = 169, normalized size = 0.25

$$\frac{dx^5\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+50cx^2\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+80cx^2\left(dx\right.}{6480c^3d\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (80*c*x^2*(-5*c + d*x^3) + 50*c*x^2*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(-8*c + d*x^3)*S
qrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)])
/(6480*c^3*d*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 1.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^4}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^4/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.19, size = 1788, normalized size = 2.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 8*c/d*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x^2+2/243/((x^3+c/d)*d)^(1/2
)/c^3*x^2+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-
I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^
(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-5/5832*I/c^3
/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2
)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/
3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=
RootOf(_Z^3*d-8*c)))+1/d*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1
/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
```

```
-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-
c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*
d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*
d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2
)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3
^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c
*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(x^4/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**4/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.451 $\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=665

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}}+\frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}}F\left(\cdots\right.}{324\sqrt{2}\sqrt[4]{3}c^{8/3}d^{2/3}\sqrt{\cdots}}$$

```
[Out] 5/3888*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c^(17/6)/
d^(2/3)-5/3888*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(17/6)/d^(2/3)-5/3888
*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(17/6)/d^(2/
3)*3^(1/2)+5/648*x^2/c^3/(d*x^3+c)^(1/2)+1/216*x^2/c^2/(-d*x^3+8*c)/(d*x^3+
c)^(1/2)-5/648*(d*x^3+c)^(1/2)/c^3/d^(2/3)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))-
5/1944*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/
3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3
)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(8/3)/d^(2/3)*2^(
1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(
1/2)))^2)^(1/2)+5/1296*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-
3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2
^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2)))^2)^(1/2)*3^(1/4)/c^(8/3)/d^(2/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^
(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 665, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {472, 579, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{1296\sqrt{3}c^{17/6}d^{2/3}}+\frac{5\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{3888c^{17/6}d^{2/3}}-\frac{5\left(\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (5*x^2)/(648*c^3*Sqrt[c + d*x^3]) + x^2/(216*c^2*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (5*Sqrt[c + d*x^3])/(648*c^3*d^(2/3)*((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)) - (5*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])
/(1296*Sqrt[3]*c^(17/6)*d^(2/3)) + (5*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^
(1/6)*Sqrt[c + d*x^3])])/(3888*c^(17/6)*d^(2/3)) - (5*ArcTanh[Sqrt[c + d*x^
3]/(3*Sqrt[c])])/(3888*c^(17/6)*d^(2/3)) + (5*Sqrt[2 - Sqrt[3]]*(c^(1/3) +
d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*
c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x
)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(432*3^(3/4)*c^(8/
3)*d^(2/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^
(1/3)*x)^2]*Sqrt[c + d*x^3]) - (5*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(
1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Ellipt
icF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(
1/3)*x)], -7 - 4*Sqrt[3]])/(324*Sqrt[2]*3^(1/4)*c^(8/3)*d^(2/3)*Sqrt[(c^(1/
3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d
*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{x}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{x\left(25cd+\frac{5d^2x^3}{2}\right)}{\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{x\left(\frac{45c^2d^2}{2}-\frac{45}{4}cd^3x^3\right)}{\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \left(\frac{45cd^2x}{4\sqrt{c+dx^3}} - \frac{135c^2d^2x}{2\left(8c-dx^3\right)\sqrt{c+dx^3}}\right) d$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{x}{\sqrt{c+dx^3}}\,dx}{1296c^3} + \frac{5\int \frac{x}{\left(8c-dx^3\right)\sqrt{c+}}}{216c^2}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\int \frac{2\sqrt[3]{c}\,d^{2/3}-2dx-\frac{d^{4/3}x^2}{\sqrt[3]{c}}}{\left(4+\frac{2\sqrt[3]{d}\,x}{\sqrt[3]{c}}+\frac{d^{2/3}x^2}{c^{2/3}}\right)\sqrt{c+dx^3}}\,dx}{2592c^3d} - $$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}$$

$$= \frac{5x^2}{648c^3\sqrt{c+dx^3}} + \frac{x^2}{216c^2\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{5\sqrt{c+dx^3}}{648c^3d^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right.}$$

Mathematica [C] time = 0.14, size = 167, normalized size = 0.25

$$\frac{dx^5\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+5cx^2\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+16cx^2\left(43c\right.}{10368c^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (16*c*x^2*(43*c - 5*d*x^3) + 5*c*x^2*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*App
ellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] + d*x^5*(8*c - d*x^3)*
Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]
)/(10368*c^4*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 2.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+cx}}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 112*
c^3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.18, size = 903, normalized size = 1.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x^2+2/243/((x^3+c/d)*d)^(1/2)/c^3*x
^2+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)
/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-5/5832*I/c^3/d^3*2^
(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*
(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)
*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/
2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(
_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.452 $\int \frac{1}{x^2\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=686

$$\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{864\,3^{3}\cdots}$$

```
[Out] 1/3888*d^(1/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))/c
^(23/6)-1/3888*d^(1/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(23/6)-1/3888
*d^(1/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/c^(23/
6)*3^(1/2)+5/648/c^3/x/(d*x^3+c)^(1/2)+1/216/c^2/x/(-d*x^3+8*c)/(d*x^3+c)^(
1/2)-31/1296*(d*x^3+c)^(1/2)/c^4/x+31/1296*d^(1/3)*(d*x^3+c)^(1/2)/c^4/(d^(
1/3)*x+c^(1/3)*(1+3^(1/2)))+31/3888*d^(1/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((
d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*
I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))
^2)^(1/2)*3^(3/4)/c^(11/3)*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3
)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-31/2592*d^(1/3)*(c^(1/3)+d^(1
/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1
/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+
d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(11/3)/(d*x
^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)
^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{31\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{648\sqrt{2}\sqrt[4]{3}\,c^{11/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\sqrt{c+dx^3}}-\frac{31\sqrt{2-\sqrt{3}}\sqrt[3]{d}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{864\,3^{3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x*Sqrt[c + d*x^3]) + 1/(216*c^2*x*(8*c - d*x^3)*Sqrt[c + d*x^3])
 - (31*Sqrt[c + d*x^3])/(1296*c^4*x) + (31*d^(1/3)*Sqrt[c + d*x^3])/(1296*c
^4*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (d^(1/3)*ArcTan[(Sqrt[3]*c^(1/6)*
(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(1296*Sqrt[3]*c^(23/6)) + (d^(1/3)
*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(3888*c^(23/
6)) - (d^(1/3)*ArcTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(3888*c^(23/6)) - (31*
Sqrt[2 - Sqrt[3]]*d^(1/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(
1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcS
in[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)]
, -7 - 4*Sqrt[3]])/(864*3^(3/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x
))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) + (31*d^(1/3)*(c
^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 +
Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) +
d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(648*Sqrt
[2]*3^(1/4)*c^(11/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^
(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
```

```
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^2(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx = \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} + \frac{\int \frac{28cd+\frac{11d^2x^3}{2}}{x^2(8c-dx^3)(c+dx^3)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{\int \frac{-558c^2d^2+\frac{225}{4}cd^3x^3}{x^2(8c-dx^3)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \frac{x(2340c^3d^3}{(8c-dx^3}}{2332}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{\int \left(\frac{279c^2d^3x}{\sqrt{c+dx^3}}\right.}{2}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{(31d)\int \frac{}{\sqrt{c+}}}{2592c}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} - \frac{\int \frac{2\sqrt[3]{c}\,d^{2/3}}{\left(4+\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}+\right.}}{25}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{}}{1296c^4((1}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{}}{1296c^4((1}$$

$$= \frac{5}{648c^3x\sqrt{c+dx^3}} + \frac{1}{216c^2x(8c-dx^3)\sqrt{c+dx^3}} - \frac{31\sqrt{c+dx^3}}{1296c^4x} + \frac{31\sqrt[3]{}}{1296c^4((1}$$

Mathematica [C] time = 0.14, size = 180, normalized size = 0.26

$$\frac{31d^2x^6(dx^3-8c)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+650cdx^3(8c-dx^3)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-80c}{103680c^5\sqrt{c+dx^3}(8cx-dx^4)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-80*c*(162*c^2 + 227*c*d*x^3 - 31*d^2*x^6) + 650*c*d*x^3*(8*c - d*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), (d*x^3)/(8*c)] +
31*d^2*x^6*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -(
(d*x^3)/c), (d*x^3)/(8*c)])/(103680*c^5*Sqrt[c + d*x^3]*(8*c*x - d*x^4))
```

fricas [F] time = 2.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{14}-14\,cd^3x^{11}+33\,c^2d^2x^8+112\,c^3dx^5+64\,c^4x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^14 - 14*c*d^3*x^11 + 33*c^2*d^2*x^8 + 112*c
^3*d*x^5 + 64*c^4*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

maple [C] time = 0.20, size = 2269, normalized size = 3.31

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*d*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x^2+2/243/((x^3+c/d)*d)^(1
/2)/c^3*x^2+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/
d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*
(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/
3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2
)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-5/5832*I/c
^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3
*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(
2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*
3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c
*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d
^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alph
a=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-2/3/((x^3+c/d)*d)^(1/2)/c^2*d*x^2-(d*x^3+
c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2
```

```
)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-
I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^
(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-1/64/c^2*d*
(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(
-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/
2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^
2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/
2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(d\,x^3+c\right)^{3/2}\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
[Out] int(1/(x^2*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**2*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.453 $\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=708

$$-\frac{11d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}+\frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}}-\frac{11d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{248832c^{29/6}}-\frac{77d^{4/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\frac{c^{2/3}\ldots}{\left(\left(1+\ldots\right.\right.}}}{1296\sqrt{2}\ldots}$$

```
[Out] 11/248832*d^(4/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2)
)/c^(29/6)-11/248832*d^(4/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(29/6)-
11/248832*d^(4/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2
))/c^(29/6)*3^(1/2)+5/648/c^3/x^4/(d*x^3+c)^(1/2)+1/216/c^2/x^4/(-d*x^3+8*c
)/(d*x^3+c)^(1/2)-253/20736*(d*x^3+c)^(1/2)/c^4/x^4+77/2592*d*(d*x^3+c)^(1/
2)/c^5/x-77/2592*d^(4/3)*(d*x^3+c)^(1/2)/c^5/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))
)-77/7776*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/
2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/
3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(14/3)
*2^(1/2)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2)))^2)^(1/2)+77/5184*d^(4/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*
x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^
(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/c^(14/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1
/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 708, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$-\frac{77d^{4/3}\sqrt{c+dx^3}}{2592c^5\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{11d^{4/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{82944\sqrt{3}c^{29/6}}+\frac{11d^{4/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{248832c^{29/6}}-\frac{11d^{4/3}\tanh^{-1}\left(\frac{\sqrt{c+\ldots}}{3\ldots}\right)}{248832c^{29/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^4*Sqrt[c + d*x^3]) + 1/(216*c^2*x^4*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (253*Sqrt[c + d*x^3])/(20736*c^4*x^4) + (77*d*Sqrt[c + d*x^3])/(2592
*c^5*x) - (77*d^(4/3)*Sqrt[c + d*x^3])/(2592*c^5*((1 + Sqrt[3])*c^(1/3) + d
^(1/3)*x)) - (11*d^(4/3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqr
t[c + d*x^3]])/(82944*Sqrt[3]*c^(29/6)) + (11*d^(4/3)*ArcTanh[(c^(1/3) + d^
(1/3)*x)^2/(3*c^(1/6)*Sqrt[c + d*x^3])])/(248832*c^(29/6)) - (11*d^(4/3)*Ar
cTanh[Sqrt[c + d*x^3]/(3*Sqrt[c])])/(248832*c^(29/6)) + (77*Sqrt[2 - Sqrt[3
]]*d^(4/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3
)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3
])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3
]])/(1728*3^(3/4)*c^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[
3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c + d*x^3]) - (77*d^(4/3)*(c^(1/3) + d^(1/
3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/
3) + d^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1
 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(1296*Sqrt[2]*3^(1/4)*c
```

```
^(14/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/
3)*x)^2]*Sqrt[c + d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
```

```
x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^5\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{31cd+\frac{17d^2x^3}{2}}{x^5\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-\frac{2277}{2}c^2d^2+\frac{495}{4}cd^3x^3}{x^5\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{\int \frac{-2}{}}{}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

$$= \frac{5}{648c^3x^4\sqrt{c+dx^3}} + \frac{1}{216c^2x^4\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{253\sqrt{c+dx^3}}{20736c^4x^4} + \frac{77d}{25}$$

Mathematica [C] time = 0.20, size = 198, normalized size = 0.28

$$\frac{-16\left(77d^3x^9\left(dx^3-8c\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)+10c\left(648c^3-2997c^2dx^3-4565cd^2x^6+616d^3x^9\right)\right)}{3317760c^6x^4\left(8c-dx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-24475*c*d^2*x^6*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 16*(10*c*(648*c^3 - 2997*c^2*d*x^3 - 456
5*c*d^2*x^6 + 616*d^3*x^9) + 77*d^3*x^9*(-8*c + d*x^3)*Sqrt[1 + (d*x^3)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x^3)/(8*c)]))/(3317760*c^6*x^4*
(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 8.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{17}-14\,cd^3x^{14}+33\,c^2d^2x^{11}+112\,c^3dx^8+64\,c^4x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^17 - 14*c*d^3*x^14 + 33*c^2*d^2*x^11 + 112*
c^3*d*x^8 + 64*c^4*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

maple [C] time = 0.19, size = 2774, normalized size = 3.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*(d*x^3+c)^(1/2)/c^3*d/x+2/3*d^2
/c^3*x^2/((x^3+c/d)*d)^(1/2)+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2))))+1/64/c^2*d^2*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x^2+2/243/((x^
3+c/d)*d)^(1/2)/c^3*x^2+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c
*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*
```

```
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
)-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x
+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)
*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3
*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/256/c^3*d*(-2/3/((x^3+c/d)*d)^(1/2)/c^
2*d*x^2-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)))-1/256/c^3*d^2*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1/2)*(-c
*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^5*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.454 $\int \frac{1}{x^8(8c-dx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=732

$$-\frac{7d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}\,c^{35/6}}+\frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}-\frac{7d^{7/3}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{3\sqrt{c}}\right)}{995328c^{35/6}}+\frac{5179d^{7/3}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)\sqrt{\ldots}}{72576\sqrt{2}\ldots}$$

```
[Out] 7/995328*d^(7/3)*arctanh(1/3*(c^(1/3)+d^(1/3)*x)^2/c^(1/6)/(d*x^3+c)^(1/2))
/c^(35/6)-7/995328*d^(7/3)*arctanh(1/3*(d*x^3+c)^(1/2)/c^(1/2))/c^(35/6)-7/
995328*d^(7/3)*arctan(c^(1/6)*(c^(1/3)+d^(1/3)*x)*3^(1/2)/(d*x^3+c)^(1/2))/
c^(35/6)*3^(1/2)+5/648/c^3/x^7/(d*x^3+c)^(1/2)+1/216/c^2/x^7/(-d*x^3+8*c)/(
d*x^3+c)^(1/2)-191/18144*(d*x^3+c)^(1/2)/c^4/x^7+8257/580608*d*(d*x^3+c)^(1
/2)/c^5/x^4-5179/145152*d^2*(d*x^3+c)^(1/2)/c^6/x+5179/145152*d^(7/3)*(d*x^
3+c)^(1/2)/c^6/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))+5179/435456*d^(7/3)*(c^(1/3)
+d^(1/3)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1
+3^(1/2))),I*3^(1/2)+2*I)*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)
*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c^(17/3)*2^(1/2)/(d*x^3+c)^(1/2)/(
c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)-5179/2
90304*d^(7/3)*(c^(1/3)+d^(1/3)*x)*EllipticE((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))
/(d^(1/3)*x+c^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*(
(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)*3^(1/4)/c^(17/3)/(d*x^3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3
)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 732, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 14, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.518, Rules used = {472, 579, 583, 584, 303, 218, 1877, 486, 444, 63, 206, 2138, 2145, 205}

$$\frac{5179d^{7/3}\sqrt{c+dx^3}}{145152c^6\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}x\right)}-\frac{5179d^2\sqrt{c+dx^3}}{145152c^6x}-\frac{7d^{7/3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{\sqrt{c+dx^3}}\right)}{331776\sqrt{3}\,c^{35/6}}+\frac{7d^{7/3}\tanh^{-1}\left(\frac{\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)^2}{3\sqrt[6]{c}\sqrt{c+dx^3}}\right)}{995328c^{35/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] 5/(648*c^3*x^7*Sqrt[c + d*x^3]) + 1/(216*c^2*x^7*(8*c - d*x^3)*Sqrt[c + d*x
^3]) - (191*Sqrt[c + d*x^3])/(18144*c^4*x^7) + (8257*d*Sqrt[c + d*x^3])/(58
0608*c^5*x^4) - (5179*d^2*Sqrt[c + d*x^3])/(145152*c^6*x) + (5179*d^(7/3)*S
qrt[c + d*x^3])/(145152*c^6*((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)) - (7*d^(7/
3)*ArcTan[(Sqrt[3]*c^(1/6)*(c^(1/3) + d^(1/3)*x))/Sqrt[c + d*x^3]])/(331776
*Sqrt[3]*c^(35/6)) + (7*d^(7/3)*ArcTanh[(c^(1/3) + d^(1/3)*x)^2/(3*c^(1/6)*
Sqrt[c + d*x^3])])/(995328*c^(35/6)) - (7*d^(7/3)*ArcTanh[Sqrt[c + d*x^3]/(
3*Sqrt[c])])/(995328*c^(35/6)) - (5179*Sqrt[2 - Sqrt[3]]*d^(7/3)*(c^(1/3) +
 d^(1/3)*x)*Sqrt[(c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])
*c^(1/3) + d^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*
x)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(96768*3^(3/4)*c^
(17/3)*Sqrt[(c^(1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3
)*x)^2]*Sqrt[c + d*x^3]) + (5179*d^(7/3)*(c^(1/3) + d^(1/3)*x)*Sqrt[(c^(2/3
) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2)/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]
```

```
*EllipticF[ArcSin[((1 - Sqrt[3])*c^(1/3) + d^(1/3)*x)/((1 + Sqrt[3])*c^(1/3
) + d^(1/3)*x)], -7 - 4*Sqrt[3]])/(72576*Sqrt[2]*3^(1/4)*c^(17/3)*Sqrt[(c^(
1/3)*(c^(1/3) + d^(1/3)*x))/((1 + Sqrt[3])*c^(1/3) + d^(1/3)*x)^2]*Sqrt[c +
 d*x^3])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 486

```
Int[(x_)/(((a_) + (b_.)*(x_)^3)*Sqrt[(c_) + (d_.)*(x_)^3]), x_Symbol] :> Wi
```

```
th[{q = Rt[d/c, 3]}, Dist[(d*q)/(4*b), Int[x^2/((8*c - d*x^3)*Sqrt[c + d*x^
3]), x], x] + (-Dist[q^2/(12*b), Int[(1 + q*x)/((2 - q*x)*Sqrt[c + d*x^3]),
 x], x] + Dist[1/(12*b*c), Int[(2*c*q^2 - 2*d*x - d*q*x^2)/((4 + 2*q*x + q^
2*x^2)*Sqrt[c + d*x^3]), x], x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0] && EqQ[8*b*c + a*d, 0]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2138

```
Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_
Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/S
qrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] &&
 EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*
Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h -
(b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; Free
Q[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*
a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{1}{x^8 \left(8c-dx^3\right)^2 \left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} + \frac{\int \frac{34cd+\frac{23d^2x^3}{2}}{x^8\left(8c-dx^3\right)\left(c+dx^3\right)^{3/2}}\,dx}{216c^2d}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{\int \frac{-1719c^2d^2+\frac{765}{4}cd^3x^3}{x^8\left(8c-dx^3\right)\sqrt{c+dx^3}}\,dx}{2916c^4d^2}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{\int \frac{-74313\cdots}{x^5(8\cdots}}{1\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

$$= \frac{5}{648c^3x^7\sqrt{c+dx^3}} + \frac{1}{216c^2x^7\left(8c-dx^3\right)\sqrt{c+dx^3}} - \frac{191\sqrt{c+dx^3}}{18144c^4x^7} + \frac{8257d\sqrt{\cdots}}{58060\cdots}$$

Mathematica [C] time = 0.23, size = 210, normalized size = 0.29

$$\frac{829375cd^3x^9\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-8\left(5179d^4x^{12}\left(8c-dx^3\right)\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{d}{}\right.\right.}{92897280c^7x^7\left(8c-dx^3\right)\sqrt{c}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^8*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (829375*c*d^3*x^9*(8*c - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5
/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 8*(20*c*(10368*c^4 - 18792*c^3*d*x^3 + 1
01817*c^2*d^2*x^6 + 153269*c*d^3*x^9 - 20716*d^4*x^12) + 5179*d^4*x^12*(8*c
 - d*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), (d*x
^3)/(8*c)]))/(92897280*c^7*x^7*(8*c - d*x^3)*Sqrt[c + d*x^3])
```

fricas [F] time = 15.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{20}-14\,cd^3x^{17}+33\,c^2d^2x^{14}+112\,c^3dx^{11}+64\,c^4x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^20 - 14*c*d^3*x^17 + 33*c^2*d^2*x^14 + 112*
c^3*d*x^11 + 64*c^4*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

maple [C] time = 0.23, size = 3299, normalized size = 4.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/256/c^3*d*(-1/4*(d*x^3+c)^(1/2)/c^2/x^4+13/8*(d*x^3+c)^(1/2)/c^3*d/x+2/3/
((x^3+c/d)*d)^(1/2)/c^3*d^2*x^2+55/72*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2))))+1/512/c^3*d^3*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x^2+2/243/
((x^3+c/d)*d)^(1/2)/c^3*x^2+5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2
```

```
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)))-5/5832*I/c^3/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c))+1/64/c^2*(-1/7*(d*x^3+c)^(1/2)/c^2/x
^7+25/56*(d*x^3+c)^(1/2)/c^3*d/x^4-237/112*(d*x^3+c)^(1/2)/c^4*d^2/x-2/3*d^
3/c^4*x^2/((x^3+c/d)*d)^(1/2)-935/1008*I/c^4*d^2*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2))))+3/4096/c^4*d^2*(-2/3/((x^3+c/d)*d)^(1/2)/c^2*d*x^2-(d*x^3+c)^(
1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x
+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3
)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))-3/4096/c^4*d^3*
(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x^2-2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)
^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(
1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/
d)^(1/2)))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+
(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(
-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/
```

```
2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^
2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/
2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^8\left(d\,x^3+c\right)^{3/2}\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^8*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**8*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.455 $\int \frac{x^6}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{2x\left(4c+dx^3\right)}{81cd^2\left(8c-dx^3\right)\sqrt{c+dx^3}}-\frac{2\sqrt{2+\sqrt{3}}\,\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)\sqrt{\frac{c^{2/3}-\sqrt[3]{c}\,\sqrt[3]{d}\,x+d^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{d}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{c}}{\sqrt[3]{d}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{c}}\right)|-7-4\sqrt{3}\right)}{81\sqrt[4]{3}\,cd^{7/3}\sqrt{\frac{\sqrt[3]{c}\left(\sqrt[3]{c}+\sqrt[3]{d}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{c}+\sqrt[3]{d}\,x\right)^2}}\,\sqrt{c+dx^3}}$$

```
[Out] 2/81*x*(d*x^3+4*c)/c/d^2/(-d*x^3+8*c)/(d*x^3+c)^(1/2)-2/243*(c^(1/3)+d^(1/3
)*x)*EllipticF((d^(1/3)*x+c^(1/3)*(1-3^(1/2)))/(d^(1/3)*x+c^(1/3)*(1+3^(1/2
))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((c^(2/3)-c^(1/3)*d^(1/3)*x+d^
(2/3)*x^2)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/c/d^(7/3)/(d*x^
3+c)^(1/2)/(c^(1/3)*(c^(1/3)+d^(1/3)*x)/(d^(1/3)*x+c^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^7\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^7*Sqrt[1 + (d*x^3)/c]*AppellF1[7/3, 2, 3/2, 10/3, (d*x^3)/(8*c), -((d*x^
3)/c)])/(448*c^3*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^6}{\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}} = \frac{x^7\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{7}{3};2,\frac{3}{2};\frac{10}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{448c^3\sqrt{c+dx^3}}$$

Mathematica [C] time = 0.67, size = 189, normalized size = 0.74

$$-\frac{6\sqrt[3]{-d}\,x\left(4c+dx^3\right)+2i3^{3/4}\sqrt[3]{c}\sqrt{\frac{(-1)^{5/6}\left(\sqrt[3]{-d}\,x-\sqrt[3]{c}\right)}{\sqrt[3]{c}}}\sqrt{\frac{(-d)^{2/3}x^2}{c^{2/3}}+\frac{\sqrt[3]{-d}\,x}{\sqrt[3]{c}}+1}\left(dx^3-8c\right)F\left(\sin^{-1}\left(\frac{\sqrt{-\frac{i\sqrt[3]{-d}\,x}{\sqrt[3]{c}}-(-1)^{5/6}}}{\sqrt[4]{3}}\right)\right.}{243c(-d)^{7/3}\left(dx^3-8c\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/243*(6*(-d)^(1/3)*x*(4*c + d*x^3) + (2*I)*3^(3/4)*c^(1/3)*Sqrt[((-1)^(5/
6)*(-c^(1/3) + (-d)^(1/3)*x))/c^(1/3)]*Sqrt[1 + ((-d)^(1/3)*x)/c^(1/3) + ((
-d)^(2/3)*x^2)/c^(2/3)]*(-8*c + d*x^3)*EllipticF[ArcSin[Sqrt[-(-1)^(5/6) -
(I*(-d)^(1/3)*x)/c^(1/3)]/3^(1/4)], (-1)^(1/3)])/(c*(-d)^(7/3)*(-8*c + d*x^
3)*Sqrt[c + d*x^3])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^6}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^6/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.32, size = 1791, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/d^2*(2/3/c*x/((x^3+c/d)*d)^(1/2)-2/9*I/c*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+64*c^2/d^2*(-1/1944/c^3*x*(d*x^3+c)^(1/2)/(d
*x^3-8*c)+2/243/c^3*x/((x^3+c/d)*d)^(1/2)-5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/
3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_al
pha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
+16*c/d^2*(-2/27/c^2*x/((x^3+c/d)*d)^(1/2)+2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3
)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3
*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alp
ha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/
2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)
^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-
(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alph
a-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x^6/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**6/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.456 $\int \frac{x^3}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

```
[Out] 1/256*x^4*AppellF1(4/3,3/2,2,7/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^
3/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, (d*x^3)/(8*c), -((d*x^3
)/c)])/(256*c^3*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{256c^3\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.32, size = 242, normalized size = 3.67

$$\frac{x\left(\frac{3x^3\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3}+\frac{192\left(\frac{160F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}+\frac{dx^3-5c}{c^2}\right)}{d(8c-dx^3)}\right)}{15552\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*((3*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)])/c^3 + (192*((-5*c + d*x^3)/c^2 + (160*AppellF1[1/3, 1/2, 1, 4/
3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)
/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), (d*
x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)]))))
/(d*(8*c - d*x^3))))/(15552*Sqrt[c + d*x^3])
```

fricas [F] time = 3.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}\,x^3}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)*x^3/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 11
2*c^3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.23, size = 1478, normalized size = 22.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 8*c/d*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x+2/243/((x^3+c/d)*d)^(1/2)/
c^3*x-5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x
+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
```

```
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/d*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*
x+2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d
^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+
1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(
1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3
+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*
_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*
c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(x^3/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(x**3/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.457 $\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

```
[Out] 1/64*x*AppellF1(1/3,3/2,2,4/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(64*c^3*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{64c^3\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.33, size = 253, normalized size = 3.95

$$\frac{x\left(192c\left(\frac{1216c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+\frac{5dx^3-43c}{dx^3-8c}\right)-15dx^3\sqrt{\frac{dx^3}{c}+1}\right.}{124416c^4\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(-15*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c),
(d*x^3)/(8*c)] + 192*c*((-43*c + 5*d*x^3)/(-8*c + d*x^3) + (1216*c^2*Appell
F1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/((8*c - d*x^3)*(32*c*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/
3, 1/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3,
 -((d*x^3)/c), (d*x^3)/(8*c)]))))))/(124416*c^4*Sqrt[c + d*x^3])
```

fricas [F] time = 3.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{12}-14\,cd^3x^9+33\,c^2d^2x^6+112\,c^3dx^3+64\,c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^12 - 14*c*d^3*x^9 + 33*c^2*d^2*x^6 + 112*c^
3*d*x^3 + 64*c^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

maple [C] time = 0.17, size = 747, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x+2/243/((x^3+c/d)*d)^(1/2)/c^3*x-5
/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-1
/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2)),_alpha=RootOf(_Z^3*d-8*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(d\,x^3+c\right)^{3/2}\,\left(8\,c-d\,x^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/((c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/((-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.458 $\int \frac{1}{x^3(8c-dx^3)^2(c+dx^3)^{3/2}} dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

```
[Out] -1/128*AppellF1(-2/3,3/2,2,1/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3/
x^2/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, (d*x^3)/(8*c), -((d*x^3)/
c)])/(128*c^3*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3(8c-dx^3)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{128c^3x^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 259, normalized size = 3.92

$$\frac{\frac{64c\left(-\frac{19648c^2dx^3F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}-648c^2-1249cdx^3+167d^2x^6\right)}{8c-dx^3}+167d^2x^6\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3}\right.}{663552c^5x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (167*d^2*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), (
d*x^3)/(8*c)] + (64*c*(-648*c^2 - 1249*c*d*x^3 + 167*d^2*x^6 - (19648*c^2*d
*x^3*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(32*c*AppellF
1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1
/2, 2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((
d*x^3)/c), (d*x^3)/(8*c)]))))/(8*c - d*x^3))/(663552*c^5*x^2*Sqrt[c + d*x^3
])
```

fricas [F] time = 7.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{15}-14\,cd^3x^{12}+33\,c^2d^2x^9+112\,c^3dx^6+64\,c^4x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^15 - 14*c*d^3*x^12 + 33*c^2*d^2*x^9 + 112*c
^3*d*x^6 + 64*c^4*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

maple [C] time = 0.17, size = 1805, normalized size = 27.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/8/c*d*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3-8*c)/c^3*x+2/243/((x^3+c/d)*d)^(1/2
)/c^3*x-5/1944*I/c^3*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*
(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
```

```
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))+1/64/c^2*(-1/2/c^2*(d*x^3+c)^(1/2
)/x^2-2/3*d/c^2*x/((x^3+c/d)*d)^(1/2)+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/64/c^2*d*(-2/27/((x^3+c/d)*d)^(1/2)/c^
2*x+2/81*I/c^2*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
)+1/243*I/c^2/d^3*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*d-8*c)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,{\left(d\,x^3+c\right)}^{3/2}\,{\left(8\,c-d\,x^3\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^3*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-8c+dx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.459 $\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

```
[Out] -1/320*AppellF1(-5/3,3/2,2,-2/3,-d*x^3/c,1/8*d*x^3/c)*(1+d*x^3/c)^(1/2)/c^3
/x^5/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-5/3, 2, 3/2, -2/3, (d*x^3)/(8*c), -((d*x^3)
/c)])/(320*c^3*x^5*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^6\left(8c-dx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{5}{3};2,\frac{3}{2};-\frac{2}{3};\frac{dx^3}{8c},-\frac{dx^3}{c}\right)}{320c^3x^5\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.28, size = 283, normalized size = 4.29

$$\frac{-\frac{2027d^3x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^6}+\frac{16789504d^2xF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)}{c^3\left(8c-dx^3\right)\left(3dx^3\left(F_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)-4F_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)+32cF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},\frac{dx^3}{8c}\right)\right)}+ \cdots}{6635520\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(8*c - d*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] ((64*(2592*c^3 - 7128*c^2*d*x^3 - 15373*c*d^2*x^6 + 2027*d^3*x^9))/(c^5*x^5
*(-8*c + d*x^3)) - (2027*d^3*x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1,
7/3, -((d*x^3)/c), (d*x^3)/(8*c)])/c^6 + (16789504*d^2*x*AppellF1[1/3, 1/2,
 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)])/(c^3*(8*c - d*x^3)*(32*c*AppellF1[1/
3, 1/2, 1, 4/3, -((d*x^3)/c), (d*x^3)/(8*c)] + 3*d*x^3*(AppellF1[4/3, 1/2,
2, 7/3, -((d*x^3)/c), (d*x^3)/(8*c)] - 4*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^
3)/c), (d*x^3)/(8*c)]))))/(6635520*Sqrt[c + d*x^3])
```

fricas [F] time = 21.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^3+c}}{d^4x^{18}-14\,cd^3x^{15}+33\,c^2d^2x^{12}+112\,c^3dx^9+64\,c^4x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^3 + c)/(d^4*x^18 - 14*c*d^3*x^15 + 33*c^2*d^2*x^12 + 112*
c^3*d*x^9 + 64*c^4*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^3+c\right)^{\frac{3}{2}}\left(dx^3-8\,c\right)^2 x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

maple [C] time = 0.18, size = 2156, normalized size = 32.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/64/c^2*(-1/5/c^2*(d*x^3+c)^(1/2)/x^5+17/20/c^3*d*(d*x^3+c)^(1/2)/x^2+2/3*
d^2/c^3*x/((x^3+c/d)*d)^(1/2)-91/180*I/c^3*d*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/64/c^2*d^2*(-1/1944*(d*x^3+c)^(1/2)/(d*x^3
-8*c)/c^3*x+2/243/((x^3+c/d)*d)^(1/2)/c^3*x-5/1944*I/c^3*3^(1/2)*(-c*d^2)^(
1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))-1/972*I/c^3/d^3*2^(1/2)*sum(1/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
```

```
))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/
3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)
))+1/256/c^3*d*(-1/2*(d*x^3+c)^(1/2)/c^2/x^2-2/3/((x^3+c/d)*d)^(1/2)/c^2*d*
x+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/
2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2
)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1
/256/c^3*d^2*(-2/27/((x^3+c/d)*d)^(1/2)/c^2*x+2/81*I/c^2*3^(1/2)*(-c*d^2)^(
1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/243*I/c^2/d^3*2^(1/2)*sum(1/_
alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/
3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),-1/18*(2*I*(-c*d^
2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_a
lpha-3*(-c*d^2)^(2/3)*_alpha)/c/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*d-8*c)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(dx^{3}+c\right)^{\frac{3}{2}}\left(dx^{3}-8\,c\right)^{2}x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-d*x^3+8*c)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^3 + c)^(3/2)*(d*x^3 - 8*c)^2*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{6}\left(d\,x^{3}+c\right)^{3/2}\left(8\,c-d\,x^{3}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2),x)
```

```
[Out] int(1/(x^6*(c + d*x^3)^(3/2)*(8*c - d*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6 \left(-8c + dx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-d*x**3+8*c)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**6*(-8*c + d*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.460 $\int \frac{x^8\sqrt{c+dx^3}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=161

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}-\frac{a\sqrt{c+dx^3}\,(4bc-5ad)}{3b^3(bc-ad)}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-1/3*a^2*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)/(b*x^3+a)+
1/3*a*(-5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(7
/2)/(-a*d+b*c)^(1/2)-1/3*a*(-5*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^3/(-a*d+b*c)
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\sqrt{c+dx^3}\,(4bc-5ad)}{3b^3(bc-ad)}+\frac{a(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}\sqrt{bc-ad}}+\frac{2\left(c+dx^3\right)^{3/2}}{9b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(
3/2))/(9*b^2*d) - (a^2*(c + d*x^3)^(3/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) +
 (a*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*
b^(7/2)*Sqrt[b*c - a*d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{c+dx}\left(-\frac{1}{2}a(2bc-3ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subs}}{}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-5ad))\,\text{Subs}}{}$$

$$= -\frac{a(4bc-5ad)\sqrt{c+dx^3}}{3b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{3/2}}{9b^2d} - \frac{a^2\left(c+dx^3\right)^{3/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{a(4bc-5ad)\tanh^{-1}}{3b^{7/2}\sqrt{bc-}}$$

Mathematica [A] time = 0.29, size = 147, normalized size = 0.91

$$\frac{-\frac{a^2\left(c+dx^3\right)^{3/2}}{a+bx^3} + \frac{a(5ad-4bc)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}} + \frac{2\left(c+dx^3\right)^{3/2}(bc-ad)}{3d}}{3b^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*(b*c - a*d)*(c + d*x^3)^(3/2))/(3*d) - (a^2*(c + d*x^3)^(3/2))/(a + b*x
^3) + (a*(-4*b*c + 5*a*d)*(Sqrt[b]*Sqrt[c + d*x^3] - Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/b^(3/2))/(3*b^2*(b*c - a*d))
```

fricas [A] time = 0.84, size = 469, normalized size = 2.91

$$\left[-\frac{3\left(4\,a^2bcd-5\,a^3d^2+\left(4\,ab^2cd-5\,a^2bd^2\right)x^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(2\left(b^4cd-ab^3d^2\right.\right.}{18\left(ab^5cd-a^2b^4d^2+\left(b^6cd-ab^5d\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(
b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c -
a*b*d))/(b*x^3 + a)) - 2*(2*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^
2*b^2*c*d + 15*a^3*b*d^2 + 2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*s
qrt(d*x^3 + c))/(a*b^5*c*d - a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3), -1/9
*(3*(4*a^2*b*c*d - 5*a^3*d^2 + (4*a*b^2*c*d - 5*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*(b^4*c*d - a*b^3*d^2)*x^6 + 2*a*b^3*c^2 - 17*a^2*b^2*c*d + 15*a^3*b*d^2 +
2*(b^4*c^2 - 6*a*b^3*c*d + 5*a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c*d
- a^2*b^4*d^2 + (b^6*c*d - a*b^5*d^2)*x^3)]
```

giac [A] time = 0.18, size = 136, normalized size = 0.84

$$-\frac{\sqrt{dx^3+c}\,a^2d}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^3}-\frac{\left(4\,abc-5\,a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^3}+\frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^4d^2-6\,\sqrt{dx^3+c}\,ab^3d^3\right)}{9\,b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/(((d*x^3 + c)*b - b*c + a*d)*b^3) - 1/3*(4*a*b*c
 - 5*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b^3) + 2/9*((d*x^3 + c)^(3/2)*b^4*d^2 - 6*sqrt(d*x^3 + c)*a*b^3*d^3)/
(b^6*d^3)
```

maple [C] time = 0.36, size = 917, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/9*(d*x^3+c)^(3/2)/b^2/d-2*a/b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2
)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
b+a)))+a^2/b^2*(-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)-1/6*I/d/b*2^(1/2)*sum(1/(a
*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c
```

```
*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/
3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.84, size = 202, normalized size = 1.25

$$\frac{2\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{4\,c}{3\,b^2}-\frac{2\,b^2\,c-2\,a\,b\,d}{b^4}+\frac{2\,a\,d}{b^3}\right)}{3\,d}+\frac{a^2\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b^2\,\left(b\,x^3+a\right)}+\frac{a\,\ln\left(\frac{2\,b\,c\cdots}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((4*c)/(3*b^2) - (2*
b^2*c - 2*a*b*d)/b^4 + (2*a*d)/b^3))/(3*d) + (a*log((2*b*c - a*d + b^(1/2)*
(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b
*c)*1i)/(6*b^(7/2)*(a*d - b*c)^(1/2)) + (a^2*((2*a*d)/(3*(2*b^2*c - 2*a*b*d
)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b^2*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.461 $\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^3}\,(2bc-3ad)}{3b^2(bc-ad)}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

```
[Out] 1/3*a*(d*x^3+c)^(3/2)/b/(-a*d+b*c)/(b*x^3+a)-1/3*(-3*a*d+2*b*c)*arctanh(b^(
1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)+1/3*(-3*a*d
+2*b*c)*(d*x^3+c)^(1/2)/b^2/(-a*d+b*c)
```

Rubi [A] time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 78, 50, 63, 208}

$$\frac{\sqrt{c+dx^3}\,(2bc-3ad)}{3b^2(bc-ad)}-\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}+\frac{a\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 3*a*d)*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(3/2
))/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt{c+dx}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x\right.}{6b^2}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-3ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c\right.}{3b^2d}$$

$$= \frac{(2bc-3ad)\sqrt{c+dx^3}}{3b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{3/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.15, size = 117, normalized size = 0.86

$$\frac{\frac{(2bc-3ad)\left(\sqrt{b}\sqrt{c+dx^3}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)}{b^{3/2}}+\frac{a\left(c+dx^3\right)^{3/2}}{a+bx^3}}{3b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] ((a*(c + d*x^3)^(3/2))/(a + b*x^3) + ((2*b*c - 3*a*d)*(Sqrt[b]*Sqrt[c + d*x
^3] - Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]]))/
b^(3/2))/(3*b*(b*c - a*d))
```

fricas [A] time = 0.84, size = 334, normalized size = 2.46

$$\left[-\frac{\left(\left(2\,b^2c-3\,abd\right)x^3+2\,abc-3\,a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(3\,ab^2c-3\,a^2bd+2\left(b\right.\right.}{6\left(ab^4c-a^2b^3d+\left(b^5c-ab^4d\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - 3*a*b*d)*x^3 + 2*a*b*c - 3*a^2*d)*sqrt(b^2*c - a*b*d)*lo
g((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 +
a)) - 2*(3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d)*x^3)*sqrt(d*x^3 + c))/
(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3), 1/3*(((2*b^2*c - 3*a*b*d)*x^
3 + 2*a*b*c - 3*a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^
```

```
2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c - 3*a^2*b*d + 2*(b^3*c - a*b^2*d
)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c - a^2*b^3*d + (b^5*c - a*b^4*d)*x^3)]
```

giac [A] time = 0.17, size = 102, normalized size = 0.75

$$\frac{\sqrt{dx^3+c}\,ad}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^2}+\frac{(2\,bc-3\,ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2}+\frac{2\,\sqrt{dx^3+c}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*a*d/(((d*x^3 + c)*b - b*c + a*d)*b^2) + 1/3*(2*b*c - 3*
a*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b
^2) + 2/3*sqrt(d*x^3 + c)/b^2
```

maple [C] time = 0.26, size = 897, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2
*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((
x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(
-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c
*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/
3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2
)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1
/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d
-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(d*x^3+c)^(1/
2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.09, size = 152, normalized size = 1.12

$$\frac{2\sqrt{d\,x^3+c}}{3\,b^2}-\frac{a\,\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,\left(b\,x^3+a\right)}+\frac{\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(3\,a\,d-2\,b\,c\right)}{6\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*(c + d*x^3)^(1/2))/(3*b^2) + (log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/
2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*b^(5
/2)*(a*d - b*c)^(1/2)) - (a*((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(
2*b^2*c - 2*a*b*d)))*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.462 $\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

```
[Out] -1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)
^(1/2)-1/3*(d*x^3+c)^(1/2)/b/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*b*(a + b*x^3)) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/S
qrt[b*c - a*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx,x,x^3\right)$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b}$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b}$$
$$= -\frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)} - \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.11, size = 80, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{3b^{3/2}\sqrt{ad-bc}} - \frac{\sqrt{c+dx^3}}{3b\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] -1/3*Sqrt[c + d*x^3]/(b*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x^3])/
Sqrt[-(b*c) + a*d]])/(3*b^(3/2)*Sqrt[-(b*c) + a*d])
```

fricas [A] time = 0.77, size = 255, normalized size = 3.19

$$\left[\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x^3\right)}, \frac{\left(bdx^3+ad\right)\sqrt{-b^2c+abd}}{3\left(a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sq
rt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) - 2*sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x^3), 1/3*((b*d*x^3 + a*
d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^
3 + b*c)) - sqrt(d*x^3 + c)*(b^2*c - a*b*d))/(a*b^3*c - a^2*b^2*d + (b^4*c
- a*b^3*d)*x^3)]
```

giac [A] time = 0.20, size = 79, normalized size = 0.99

$$\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b} - \frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*b)
```

maple [C] time = 0.27, size = 453, normalized size = 5.66

$$i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\,\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\,\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+a\right)\right.$$

$$-\overline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)
^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(
1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-
c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3
))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2
/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.69, size = 125, normalized size = 1.56

$$\frac{\left(\frac{2\,a\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{2\,b\,c}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)\,\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] (((2*a*d)/(3*(2*b^2*c - 2*a*b*d)) - (2*b*c)/(3*(2*b^2*c - 2*a*b*d)))*(c + d
*x^3)^(1/2))/(a + b*x^3) + (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*1i)/(6*b^(3/2)*(a*d - b*c)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{c + dx^{3}}}{\left(a + bx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.463 $\int \frac{\sqrt{c+dx^3}}{x(a+bx^3)^2}\,dx$

Optimal. Leaf size=121

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a(a+bx^3)}$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^2+1/3*(-a*d+2*b*c)*arctanh(
b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/b^(1/2)/(-a*d+b*c)^(1/2)+1/3*
(d*x^3+c)^(1/2)/a/(b*x^3+a)
```

Rubi [A] time = 0.12, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}-\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}}{3a(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] Sqrt[c + d*x^3]/(3*a*(a + b*x^3)) - (2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt
[c]])/(3*a^2) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
 a*d]])/(3*a^2*Sqrt[b]*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{-c-\frac{dx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^2}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} + \frac{(2c)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{3a^2d}$$

$$= \frac{\sqrt{c+dx^3}}{3a\left(a+bx^3\right)} - \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{b}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.29, size = 112, normalized size = 0.93

$$\frac{\frac{a\sqrt{c+dx^3}}{a+bx^3} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}} - 2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3])/(a + b*x^3) - 2*Sqrt[c]*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c
]] + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(Sq
rt[b]*Sqrt[b*c - a*d]))/(3*a^2)
```

fricas [B] time = 0.89, size = 856, normalized size = 7.07

$$\left[-\frac{\left(\left(2\,b^2c-abd\right)x^3+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)-2\left(ab^2c-a^2bd+\left(b^3c-ab^2\right.\right.}{6\left(a^3b^2c-a^4bd+\left(a^2b^3c-a^3b^2d\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a))
- 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt
(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
```

```
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) - (a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3
)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - (a*b^2*c - a
^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3
), 1/6*(4*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*arctan(sqrt(
d*x^3 + c)*sqrt(-c)/c) - ((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2
*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b
*d))/(b*x^3 + a)) + 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a^3*b^2*c - a^4
*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3), -1/3*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c
 - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/
(b*d*x^3 + b*c)) - 2*(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-c)*a
rctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a
^3*b^2*c - a^4*b*d + (a^2*b^3*c - a^3*b^2*d)*x^3)]
```

giac [A] time = 0.17, size = 114, normalized size = 0.94

$$\frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)a}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2}+\frac{2\,c\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*a) - 1/3*(2*b*c - a*d)*a
rctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*a^2) +
2/3*c*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c))
```

maple [C] time = 0.28, size = 934, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-1/a*b*(-1/3*(d*x^3
+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2
*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/
2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1
/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*
d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elliptic
Pi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I
*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)
/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(2/3*(d*x^3+
c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x), x)
```

mupad [B] time = 8.28, size = 182, normalized size = 1.50

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}-\frac{\left(\frac{b\,d}{3\left(b^2\,c-a\,b\,d\right)}-\frac{b^2\,c}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{d\,x^3+c}\,\sqrt{a\,b\,d-b^2\,c}\,2}{b\,x^3+a}\right.}{6\,a^2\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) - (((b*d)/(3*(b^2*c - a*b*d)) - (b^2*c)/(3*a*(b^2*c - a*b*d
)))*(c + d*x^3)^(1/2))/(a + b*x^3) + (log((2*b*c - a*d + (c + d*x^3)^(1/2)*
(a*b*d - b^2*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*a^2*(
a*b*d - b^2*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x*(a + b*x**3)**2), x)
```

3.464 $\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2} \, dx$

Optimal. Leaf size=161

$$-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

```
[Out] 1/3*(-a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(1/2)-1/3*(-3*a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a^3/(-a*d+b
*c)^(1/2)-2/3*b*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*(d*x^3+c)^(1/2)/a/x^3/(b*
x^3+a)
```

Rubi [A] time = 0.22, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 99, 151, 156, 63, 208}

$$-\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)}-\frac{\sqrt{b}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{bc-ad}}+\frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}}-\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-2*b*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - Sqrt[c + d*x^3]/(3*a*x^3*(a +
b*x^3)) + ((4*b*c - a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^3*Sqrt[c])
- (Sqrt[b]*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d
]])/(3*a^3*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^3}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4bc+ad)-\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(4bc-ad)-bd(bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2(bc-ad)}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} - \frac{(4b}{}$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(b(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^3d} -$$

$$= -\frac{2b\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3\sqrt{c}} - \frac{\sqrt{b}\,(4bc-3ad)\tanh^{-}}{3a^3\sqrt{bc-}}$$

Mathematica [A] time = 0.28, size = 190, normalized size = 1.18

$$\frac{\sqrt{c}\left(a\left(a+2bx^3\right)\sqrt{c+dx^3}\,(bc-ad)+\sqrt{b}\,x^3\left(a+bx^3\right)(4bc-3ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)\right)-x^3\left(a+bx^{3}\right.}{3a^3\sqrt{c}\,x^3\left(a+bx^3\right)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^4*(a + b*x^3)^2),x]
```

```
[Out] (-((4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*x^3*(a + b*x^3)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]]) + Sqrt[c]*(a*(b*c - a*d)*(a + 2*b*x^3)*Sqrt[c + d*x^3] + Sqrt[b
]*(4*b*c - 3*a*d)*Sqrt[b*c - a*d]*x^3*(a + b*x^3)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]]))/(3*a^3*Sqrt[c]*(-(b*c) + a*d)*x^3*(a + b*x^3))
```

fricas [A] time = 0.82, size = 870, normalized size = 5.40

$$\left[-\frac{\left(\left(4\,b^2c^2-3\,abcd\right)x^6+\left(4\,abc^2-3\,a^2cd\right)x^3\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)+\left(\left(4\,b^2c-abd\right)x^6+\right.}{6\left(a^3bcx^6+a^4cx^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(
b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt
(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*
x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*
c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*
c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c - a*d))*ar
ctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) + (
(4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(
d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(
a^3*b*c*x^6 + a^4*c*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d
)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c^2 - 3*a*b*c*
d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*
b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a))
 + 2*(2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c*x^6 + a^4*c*x^3), -1/3
*(((4*b^2*c^2 - 3*a*b*c*d)*x^6 + (4*a*b*c^2 - 3*a^2*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-c)*arctan(sq
rt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*c
*x^6 + a^4*c*x^3)]
```

giac [A] time = 0.22, size = 183, normalized size = 1.14

$$\frac{\left(4\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^3}-\frac{\left(4\,bc-ad\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}bd-2\,\sqrt{dx^3+c}\,bcd+\sqrt{dx^3}}{3\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqr
t(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c - a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))
/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*d - 2*sqrt(d*x^3 + c)*b*c*d +
sqrt(d*x^3 + c)*a*d^2)/((((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x
^3 + c)*a*d - a*c*d)*a^2)
```

maple [C] time = 0.26, size = 978, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/x^3-1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(1
/2))+2/a^3*b^2*(2/3*(d*x^3+c)^(1/2)/b+1/3*I/b/d^2*2^(1/2)*sum((-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(-
1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/b-1/6*I/d/b*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*
(2/3*(d*x^3+c)^(1/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^4), x)
```

mupad [B] time = 9.69, size = 438, normalized size = 2.72

$$\frac{\left(\frac{a\left(\frac{a\left(\frac{a\left(\frac{b^2 d^2}{2a^3c^2}-\frac{b^2d^2(3ad-4bc)}{6a^2c^2\left(a^2d-abc\right)}+\frac{b^2d(2ad-bc)(3ad-4bc)}{6a^3c^2\left(a^2d-abc\right)}\right)}{b}-\frac{bd(2ad-bc)}{2a^3c^2}+\frac{b(3ad-4bc)\left(-a^2d^2+2abcd+2b^2c^2\right)}{6a^3c^2\left(a^2d-abc\right)}\right)}{b}-\frac{-a^2d^2+2abcd+2b^2c^2}{2a^3c^2}+\frac{b(ad-4bc)(3ad-4\cdots}{6a^2c\left(a^2d-abc\cdots\right.}\cdots}{b}\cdots\right.}{bx^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (((a*((a*((a*((b^2*d^2)/(2*a^3*c^2) - (b^2*d^2*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d*(2*a*d - b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b - (b*d*(2*a*d - b*c))/(2*a^3*c^2) + (b*(3*a*d - 4*b*c)*(2*b^2
*c^2 - a^2*d^2 + 2*a*b*c*d))/(6*a^3*c^2*(a^2*d - a*b*c))))/b - (2*b^2*c^2 -
```

```
a^2*d^2 + 2*a*b*c*d)/(2*a^3*c^2) + (b*(a*d - 4*b*c)*(3*a*d - 4*b*c))/(6*a^
2*c*(a^2*d - a*b*c))))/b - (a*d - 4*b*c)/(2*a^2*c))*(c + d*x^3)^(1/2))/(a +
 b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*x^3) + (log((((c + d*x^3)^(1/2) - c^(1/2
))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)*(a*d - 4*b*c))/(6*a^3*c^(1/2)) + (
b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(3*a*d - 4*b*c)*1i)/(6*a^3*(a*d - b*c)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.465 $\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,-1/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{x^3\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.27, size = 235, normalized size = 3.67

$$\frac{x\left(\frac{8\left(\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}-c-dx^3\right)}{a+bx^3}+\frac{5dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}\right)}{24b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x*((5*d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -
((b*x^3)/a)])/a + (8*(-c - d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d
*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -(
(b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^
3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a
+ b*x^3)))/(24*b*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^3}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

maple [C] time = 0.36, size = 1468, normalized size = 22.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1
/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
```

```
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(1/3/a*x*(d*x^3+c)^(1/2)/(b*x^3+a)-1
/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I
/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(
2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*(
(x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*
(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-
c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(
1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*
d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x^3}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)*x^3/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\sqrt{d\,x^3+c}}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(x**3*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.466 $\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,-1/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d
*x^3/c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {511, 510}

$$\frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{x\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{x^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 153, normalized size = 2.39

$$\frac{-dx^5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5cx^2\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+10ax^2}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[c + d*x^3])/(a + b*x^3)^2,x]
```

```
[Out] (10*a*x^2*(c + d*x^3) + 5*c*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*x^5*(a + b*x^3)*Sqrt[1 + (d
*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2*(a
 + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}\,x}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

maple [C] time = 0.36, size = 908, normalized size = 14.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3/a*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_alpha/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
```

```
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}\,x}{\left(bx^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)*x/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{d\,x^3 + c}}{\left(b\,x^3 + a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2,x)
```

```
[Out] int((x*(c + d*x^3)^(1/2))/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c + dx^3}}{\left(a + bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral(x*sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.467 $\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] x*AppellF1(1/3,2,-1/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3/c
)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])
/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\
&= \frac{x\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}
\end{aligned}$$

Mathematica [B] time = 0.25, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a^2}+\frac{8\left(\frac{16c^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+\frac{c+dx^3}{a}\right)}{a+bx^3}\right)}{24\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(a + b*x^3)^2,x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a^2 + (8*((c + d*x^3)/a + (16*c^2*AppellF1[1/3, 1/2, 1, 4/3, -(
(d*x^3)/c), -((b*x^3)/a)])/(8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(
a + b*x^3)))/(24*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

maple [C] time = 0.27, size = 753, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/a*x-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alpha^
2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I
```

```
*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)
^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^3 + c)/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(a + b*x^3)^2,x)
[Out] int((c + d*x^3)^(1/2)/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(a + b*x**3)**2, x)
```

3.468 $\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -AppellF1(-1/3,2,-1/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x^3
/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -1/2, 2/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^2\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.16, size = 172, normalized size = 2.77

$$\frac{5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(9ad-8bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8bdx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{b}{}\right.}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(3*a + 4*b*x^3)*(c + d*x^3) + 5*(-8*b*c + 9*a*d)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 8
*b*d*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^
3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

maple [C] time = 0.37, size = 2227, normalized size = 35.92

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/
d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b/d^2*2^(1/2
)*sum(1/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^
2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/
3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3
)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^
2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d
^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
```

```
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(
-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/
2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Roo
tOf(_Z^3*b+a))-1/a*b*(1/3*(d*x^3+c)^(1/2)/(b*x^3+a)/a*x^2+1/9*I/a/b*3^(1/2
)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*
((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/b/d^2*2^(1/2)*sum((-a*d-2*b*c)/_al
pha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3
)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2
+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^
2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-
c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2
)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(
-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=Root
Of(_Z^3*b+a)))+1/a^2*(-(d*x^3+c)^(1/2)/x-I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*
(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1
/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3+c}}{x^2\,\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^2*(a + b*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^3}}{x^2\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**2/(b*x**3+a)**2,x)
[Out] Integral(sqrt(c + d*x**3)/(x**2*(a + b*x**3)**2), x)
```

3.469 $\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -1/2*AppellF1(-2/3,2,-1/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)])
/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx^3}}{x^3\left(a+bx^3\right)^2}\,dx &= \frac{\sqrt{c+dx^3}\int \frac{\sqrt{1+\frac{dx^3}{c}}}{x^3\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.36, size = 338, normalized size = 5.28

$$\frac{\frac{a\left(32ac\left(6ac-3adx^3+30bcx^3+10bdx^6\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(3a+5bx^3\right)\left(c+dx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^3]/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-5*b*d*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -(
(b*x^3)/a)] + (a*(32*a*c*(6*a*c + 30*b*c*x^3 - 3*a*d*x^3 + 10*b*d*x^6)*Appe
llF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(3*a + 5*b*x^3)
*(c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]
+ a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3
)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2
*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[
4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^
3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3+c}}{\left(bx^3+a\right)^2 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

maple [C] time = 0.26, size = 1768, normalized size = 27.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(-2/3*I/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2))+1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/
2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(
1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+
(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3
```

```
+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*
_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I
*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*
d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I
*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/2*(d*x^3+c)^(1/2)/x^2-1/2
*I*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-
3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/a*b*(1/3*
(d*x^3+c)^(1/2)/(b*x^3+a)/a*x-1/9*I/a/b*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-
c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^
(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/b/d^2*2^(1/2)*sum((a*d-4*b*c)/_alpha^2/(a
*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3
))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^
(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d
^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c
*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/
3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alp
ha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^
3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^3 + c}}{\left(bx^3 + a\right)^2 x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(1/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^3 + c)/((b*x^3 + a)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{d\,x^3 + c}}{x^3\left(b\,x^3 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(1/2)/(x^3*(a + b*x^3)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^3}}{x^3\left(a + bx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(1/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x**3)/(x**3*(a + b*x**3)**2), x)
```

3.470 $\int \frac{x^8(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2\left(a+bx^3\right)(bc-ad)}+\frac{a(4bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{9/2}}-\frac{a\sqrt{c+dx^3}\,(4bc-7ad)}{3b^4}-\frac{a\left(c+dx^3\right)^{3/2}(4bc \cdots}{9b^3(bc-ad \cdots}$$

```
[Out] -1/9*a*(-7*a*d+4*b*c)*(d*x^3+c)^(3/2)/b^3/(-a*d+b*c)+2/15*(d*x^3+c)^(5/2)/b
^2/d-1/3*a^2*(d*x^3+c)^(5/2)/b^2/(-a*d+b*c)/(b*x^3+a)+1/3*a*(-7*a*d+4*b*c)*
arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(9/2)-
1/3*a*(-7*a*d+4*b*c)*(d*x^3+c)^(1/2)/b^4
```

Rubi [A] time = 0.24, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 89, 80, 50, 63, 208}

$$-\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2\left(a+bx^3\right)(bc-ad)}-\frac{a\left(c+dx^3\right)^{3/2}(4bc-7ad)}{9b^3(bc-ad)}-\frac{a\sqrt{c+dx^3}\,(4bc-7ad)}{3b^4}+\frac{a(4bc-7ad)\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b} \cdots}{\sqrt{b \cdots}}\right)}{3b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] -(a*(4*b*c - 7*a*d)*Sqrt[c + d*x^3])/(3*b^4) - (a*(4*b*c - 7*a*d)*(c + d*x^
3)^(3/2))/(9*b^3*(b*c - a*d)) + (2*(c + d*x^3)^(5/2))/(15*b^2*d) - (a^2*(c
+ d*x^3)^(5/2))/(3*b^2*(b*c - a*d)*(a + b*x^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b
*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{(c+dx)^{3/2}\left(-\frac{1}{2}a(2bc-5ad)+b(bc-ad)x\right)}{a+bx}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}\,dx, x, x^3\right)}{6b^2(bc-ad)}$$

$$= -\frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right)^{5/2}}{3b^2(bc-ad)\left(a+bx^3\right)} - \frac{(a(4bc-7ad))}{}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

$$= -\frac{a(4bc-7ad)\sqrt{c+dx^3}}{3b^4} - \frac{a(4bc-7ad)\left(c+dx^3\right)^{3/2}}{9b^3(bc-ad)} + \frac{2\left(c+dx^3\right)^{5/2}}{15b^2d} - \frac{a^2\left(c+dx^3\right.}{3b^2(bc-ad)\left(a\right.}$$

Mathematica [A] time = 0.19, size = 162, normalized size = 0.86

$$\frac{\sqrt{c+dx^3}\left(105a^3d^2+5a^2bd\left(14dx^3-19c\right)+2ab^2\left(3c^2-34cdx^3-7d^2x^6\right)+6b^3x^3\left(c+dx^3\right)^2\right)}{45b^4d\left(a+bx^3\right)} + \frac{a(4bc-7ad)\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(105*a^3*d^2 + 6*b^3*x^3*(c + d*x^3)^2 + 5*a^2*b*d*(-19*c
+ 14*d*x^3) + 2*a*b^2*(3*c^2 - 34*c*d*x^3 - 7*d^2*x^6)))/(45*b^4*d*(a + b*x
^3)) + (a*(4*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*b^(9/2))
```

fricas [A] time = 1.32, size = 443, normalized size = 2.34

$$\left[-\frac{15\left(4a^2bcd-7a^3d^2+\left(4ab^2cd-7a^2bd^2\right)x^3\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad-2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(6b^3d^2x^9+2\left(6b^3cd\right.\right.}{90\left(b^5dx^3+ab^4d\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/90*(15*(4*a^2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt
((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*b*sqrt((b*c
- a*d)/b))/(b*x^3 + a)) - 2*(6*b^3*d^2*x^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^
6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d
+ 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b^5*d*x^3 + a*b^4*d), 1/45*(15*(4*a^
2*b*c*d - 7*a^3*d^2 + (4*a*b^2*c*d - 7*a^2*b*d^2)*x^3)*sqrt(-(b*c - a*d)/b)
*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (6*b^3*d^2*x
^9 + 2*(6*b^3*c*d - 7*a*b^2*d^2)*x^6 + 6*a*b^2*c^2 - 95*a^2*b*c*d + 105*a^3
*d^2 + 2*(3*b^3*c^2 - 34*a*b^2*c*d + 35*a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(b
^5*d*x^3 + a*b^4*d)]
```

giac [A] time = 0.18, size = 211, normalized size = 1.12

$$-\frac{\left(4ab^2c^2-11a^2bcd+7a^3d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,b^4}-\frac{\sqrt{dx^3+c}\,a^2bcd-\sqrt{dx^3+c}\,a^3d^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^4}+\frac{2\left(3\left(dx^3+c\right)^{\frac{5}{2}}b^8d^4-10\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b^2*c^2 - 11*a^2*b*c*d + 7*a^3*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt
(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^4) - 1/3*(sqrt(d*x^3 + c)*a^2*b*c
*d - sqrt(d*x^3 + c)*a^3*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^4) + 2/45*(3*(
d*x^3 + c)^(5/2)*b^8*d^4 - 10*(d*x^3 + c)^(3/2)*a*b^7*d^5 - 30*sqrt(d*x^3 +
 c)*a*b^7*c*d^5 + 45*sqrt(d*x^3 + c)*a^2*b^6*d^6)/(b^10*d^5)
```

maple [C] time = 0.37, size = 1003, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 2/15*(d*x^3+c)^(5/2)/b^2/d-2*a/b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b
*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*
d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
```

```
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(1/3*(a*d-b*c)/b^2*(d*x^3+c)^(1/2)
/(b*x^3+a)+2/3*d*(d*x^3+c)^(1/2)/b^2+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)
*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*
d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ell
ipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^
2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_a
lpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.75, size = 331, normalized size = 1.75

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,{\left(a\,d-b\,c\right)}^2}{b^4}+\frac{2\,c\,\left(\frac{2\,d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{8\,c\,d}{5\,b^2}\right)}{3\,d}+\frac{2\,a\,\left(\frac{d\,\left(a\,d-2\,b\,c\right)}{b^3}+\frac{a\,d^2}{b^3}\right)}{b}\right)}{3\,d}+\frac{2\,d\,x^6\,\sqrt{d\,x^3+c}}{15\,b^2}-\frac{x^3\,\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,(a\,d-2\,b}{b^3}\right.}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] ((c + d*x^3)^(1/2)*((2*(a*d - b*c)^2)/b^4 + (2*c*((2*d*(a*d - 2*b*c))/b^3 +
 (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(3*d) + (2*a*((d*(a*d - 2*b*c))/b^3 + (a
*d^2)/b^3))/b))/(3*d) + (2*d*x^6*(c + d*x^3)^(1/2))/(15*b^2) - (x^3*(c + d*
x^3)^(1/2)*((2*d*(a*d - 2*b*c))/b^3 + (2*a*d^2)/b^3 + (8*c*d)/(5*b^2)))/(9*
d) + (a*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i -
 b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(7*a*d - 4*b*c)*1i)/(6*b^(9/2)) -
(a^2*((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b
*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)*(c + d*x^3)^(1/2))/(b^2*(a +
b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.471 $\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{\sqrt{c+dx^3}\,(2bc-5ad)}{3b^3}+\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-a}$$

```
[Out] 1/9*(-5*a*d+2*b*c)*(d*x^3+c)^(3/2)/b^2/(-a*d+b*c)+1/3*a*(d*x^3+c)^(5/2)/b/(
-a*d+b*c)/(b*x^3+a)-1/3*(-5*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*
d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(7/2)+1/3*(-5*a*d+2*b*c)*(d*x^3+c)^(1/2)/b
^3
```

Rubi [A] time = 0.14, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 50, 63, 208}

$$\frac{\left(c+dx^3\right)^{3/2}(2bc-5ad)}{9b^2(bc-ad)}+\frac{\sqrt{c+dx^3}\,(2bc-5ad)}{3b^3}-\frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}+\frac{a\left(c+dx^3\right)^{5/2}}{3b\left(a+bx^3\right)(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] ((2*b*c - 5*a*d)*Sqrt[c + d*x^3])/(3*b^3) + ((2*b*c - 5*a*d)*(c + d*x^3)^(3
/2))/(9*b^2*(b*c - a*d)) + (a*(c + d*x^3)^(5/2))/(3*b*(b*c - a*d)*(a + b*x^
3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sq
rt[b*c - a*d]])/(3*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}dx = \frac{1}{3}\text{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(a+bx)^2}dx, x, x^3\right)$$
$$= \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{a+bx}dx, x, x^3\right)}{6b(bc-ad)}$$
$$= \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{(2bc-5ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}dx, x, x\right.}{6b^2}$$
$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5a}{}$$
$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} + \frac{((2bc-5a}{}$$
$$= \frac{(2bc-5ad)\sqrt{c+dx^3}}{3b^3} + \frac{(2bc-5ad)\left(c+dx^3\right)^{3/2}}{9b^2(bc-ad)} + \frac{a\left(c+dx^3\right)^{5/2}}{3b(bc-ad)\left(a+bx^3\right)} - \frac{(2bc-5ad}{}$$

Mathematica [A] time = 0.14, size = 125, normalized size = 0.77

$$\frac{\sqrt{c+dx^3}\left(-15a^2d+ab\left(11c-10dx^3\right)+2b^2x^3\left(4c+dx^3\right)\right)}{9b^3\left(a+bx^3\right)} - \frac{(2bc-5ad)\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (Sqrt[c + d*x^3]*(-15*a^2*d + a*b*(11*c - 10*d*x^3) + 2*b^2*x^3*(4*c + d*x^
3)))/(9*b^3*(a + b*x^3)) - ((2*b*c - 5*a*d)*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(7/2))
```

fricas [A] time = 1.20, size = 314, normalized size = 1.93

$$\left[-\frac{3\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2bc-ad+2\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)-2\left(2b^2dx^6+2\left(4b^2c-5abc\right.\right.}{18\left(b^4x^3+ab^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/18*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*sqrt((b*c - a*d)/b)
*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x
^3 + a)) - 2*(2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*c - 15*a^2*d
)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3), -1/9*(3*((2*b^2*c - 5*a*b*d)*x^3 + 2*
a*b*c - 5*a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c
- a*d)/b)/(b*c - a*d)) - (2*b^2*d*x^6 + 2*(4*b^2*c - 5*a*b*d)*x^3 + 11*a*b*
c - 15*a^2*d)*sqrt(d*x^3 + c))/(b^4*x^3 + a*b^3)]
```

giac [A] time = 0.19, size = 173, normalized size = 1.06

$$\frac{\left(2\,b^2c^2 - 7\,abcd + 5\,a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^3} + \frac{\sqrt{dx^3+c}\,abcd - \sqrt{dx^3+c}\,a^2d^2}{3\left(\left(dx^3+c\right)b - bc + ad\right)b^3} + \frac{2\left(\left(dx^3+c\right)^{\frac{3}{2}}b^4 + 3\,\sqrt{dx^3+c}\,b^4 \cdots\right.}{9\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*c^2 - 7*a*b*c*d + 5*a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 1/3*(sqrt(d*x^3 + c)*a*b*c*d - sqr
t(d*x^3 + c)*a^2*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^3) + 2/9*((d*x^3 + c)^
(3/2)*b^4 + 3*sqrt(d*x^3 + c)*b^4*c - 6*sqrt(d*x^3 + c)*a*b^3*d)/b^6
```

maple [C] time = 0.27, size = 983, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+
c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)
*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(
-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/
3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*
d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*
3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-
c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
)-a/b*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*
d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(
1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(
-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*
_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^
(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d
^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)
,1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^
(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
,_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.38, size = 229, normalized size = 1.40

$$\frac{2\,d\,x^3\,\sqrt{d\,x^3+c}}{9\,b^2}-\frac{\sqrt{d\,x^3+c}\,\left(\frac{2\,d\,(a\,d-2\,b\,c)}{b^3}+\frac{2\,a\,d^2}{b^3}+\frac{4\,c\,d}{3\,b^2}\right)}{3\,d}+\frac{a\,\left(\frac{2\,b\,c^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\left(\frac{2\,a\,d^2}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\,\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\,\sqrt{d\,x^3}}{b\,\left(b\,x^3+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*x^3*(c + d*x^3)^(1/2))/(9*b^2) - ((c + d*x^3)^(1/2)*((2*d*(a*d - 2*b*c
))/b^3 + (2*a*d^2)/b^3 + (4*c*d)/(3*b^2)))/(3*d) + (log((2*b*c - a*d + b^(1
/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(a*d - b
*c)^(1/2)*(5*a*d - 2*b*c)*1i)/(6*b^(7/2)) + (a*((2*b*c^2)/(3*(2*b^2*c - 2*a
*b*d)) + (a*((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*
a*b*d))))/b)*(c + d*x^3)^(1/2))/(b*(a + b*x^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.472 $\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=94

$$-\frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

```
[Out] -1/3*(d*x^3+c)^(3/2)/b/(b*x^3+a)-d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*
c)^(1/2))*(-a*d+b*c)^(1/2)/b^(5/2)+d*(d*x^3+c)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 47, 50, 63, 208}

$$-\frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)}+\frac{d\sqrt{c+dx^3}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (d*Sqrt[c + d*x^3])/b^2 - (c + d*x^3)^(3/2)/(3*b*(a + b*x^3)) - (d*Sqrt[b*c
 - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx, x, x^3\right)\\
&= -\frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^3\right)}{2b}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(d(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{2b^2}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{b^2}\\
&= \frac{d\sqrt{c+dx^3}}{b^2} - \frac{\left(c+dx^3\right)^{3/2}}{3b\left(a+bx^3\right)} - \frac{d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.57

$$\frac{2d\left(c+dx^3\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{15(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (2*d*(c + d*x^3)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x^3))/(-(
b*c) + a*d))])/(15*(-(b*c) + a*d)^2)
```

fricas [A] time = 0.98, size = 234, normalized size = 2.49

$$\left[\frac{3\left(bdx^3+ad\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(2\,bdx^3-bc+3\,ad\right)\sqrt{dx^3+c}}{6\left(b^3x^3+ab^2\right)}, -\frac{3\left(bdx^3+ad\right)\sqrt{-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d*x^3 + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d - 2*
sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a)) + 2*(2*b*d*x^3 - b*c +
3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2), -1/3*(3*(b*d*x^3 + a*d)*sqrt(-(b
*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) -
(2*b*d*x^3 - b*c + 3*a*d)*sqrt(d*x^3 + c))/(b^3*x^3 + a*b^2)]
```

giac [A] time = 0.17, size = 122, normalized size = 1.30

$$\frac{2\sqrt{dx^3+c}\,d}{3\,b^2}+\frac{\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx^3+c}\,bcd-\sqrt{dx^3+c}\,ad^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*sqrt(d*x^3 + c)*d/b^2 + (b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-
b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - 1/3*(sqrt(d*x^3 + c)*b*c*d - s
qrt(d*x^3 + c)*a*d^2)/(((d*x^3 + c)*b - b*c + a*d)*b^2)
```

maple [C] time = 0.27, size = 466, normalized size = 4.96

$$\frac{2\sqrt{d\,x^3+c}\;d}{3b^2}+\frac{i\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootC}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*d+1/2*I
/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-
c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)
^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^
2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(
-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2
*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3
^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1
/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha
=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.35, size = 170, normalized size = 1.81

$$\frac{2\,d\,\sqrt{d\,x^3+c}}{3\,b^2}-\frac{\left(\frac{2\,b\,c^2}{3\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\left(\frac{2\,a\,d^2}{3\left(2\,b^2\,c-2\,a\,b\,d\right)}-\frac{4\,b\,c\,d}{3\left(2\,b^2\,c-2\,a\,b\,d\right)}\right)}{b}\right)\sqrt{d\,x^3+c}}{b\,x^3+a}+\frac{d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\sqrt{}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
```

```
[Out] (2*d*(c + d*x^3)^(1/2))/(3*b^2) - (((2*b*c^2)/(3*(2*b^2*c - 2*a*b*d)) + (a*
((2*a*d^2)/(3*(2*b^2*c - 2*a*b*d)) - (4*b*c*d)/(3*(2*b^2*c - 2*a*b*d))))/b)
*(c + d*x^3)^(1/2))/(a + b*x^3) + (d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)
^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*1i)/(
2*b^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.473 $\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\left(a+bx^3\right)}$$

```
[Out] -2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2+1/3*(a*d+2*b*c)*arctanh(b
^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^2/b^(3/2)+1/3*(
-a*d+b*c)*(d*x^3+c)^(1/2)/a/b/(b*x^3+a)
```

Rubi [A] time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 156, 63, 208}

$$\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}-\frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}+\frac{\sqrt{c+dx^3}\,(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d*x^3])/(3*a*b*(a + b*x^3)) - (2*c^(3/2)*ArcTanh[Sqrt
[c + d*x^3]/Sqrt[c]])/(3*a^2) + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x(a+bx)^2}\,dx, x, x^3\right)$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{\text{Subst}\left(\int \frac{bc^2+\frac{1}{2}d(bc+ad)x}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3ab}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{((bc-ad)(2bc+ad))\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{6a^2b}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{((bc-ad)(2bc+ad))\,\text{Sub}}{}$$

$$= \frac{(bc-ad)\sqrt{c+dx^3}}{3ab\left(a+bx^3\right)} - \frac{2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2} + \frac{\sqrt{bc-ad}\,(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2b^{3/2}}$$

Mathematica [A] time = 0.19, size = 122, normalized size = 0.93

$$\frac{\frac{\sqrt{bc-ad}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{3/2}} + \frac{a\sqrt{c+dx^3}\,(bc-ad)}{b\left(a+bx^3\right)} - 2c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x]
```

```
[Out] ((a*(b*c - a*d)*Sqrt[c + d*x^3])/(b*(a + b*x^3)) - 2*c^(3/2)*ArcTanh[Sqrt[c
 + d*x^3]/Sqrt[c]] + (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/b^(3/2))/(3*a^2)
```

fricas [A] time = 1.26, size = 686, normalized size = 5.24

$$\left[\frac{\left(\left(2\,b^2c+abd\right)x^3+2\,abc+a^2d\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+2\left(b^2cx^3+abc\right)\sqrt{c}\,\log\left(\frac{dx^3-2\,\sqrt{dx^3}}{x^3}\right.}{6\left(a^2b^2x^3+a^3b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt((b*c - a*d)/b)*log((b*
d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^3 + a))
 + 2*(b^2*c*x^3 + a*b*c)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2
```

```
*c)/x^3) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/3*((
(2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(
d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b^2*c*x^3 + a*b*c)*sqrt(c
)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + sqrt(d*x^3 + c)*(a*b
*c - a^2*d))/(a^2*b^2*x^3 + a^3*b), 1/6*(4*(b^2*c*x^3 + a*b*c)*sqrt(-c)*arc
tan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)
*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt(
(b*c - a*d)/b))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(a*b*c - a^2*d))/(a^2*b^2*
x^3 + a^3*b), 1/3*(((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*sqrt(-(b*c - a
*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + 2*(b^2
*c*x^3 + a*b*c)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + sqrt(d*x^3 +
c)*(a*b*c - a^2*d))/(a^2*b^2*x^3 + a^3*b)]
```

giac [A] time = 0.17, size = 155, normalized size = 1.18

$$\frac{2\,c^2\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}-\frac{\left(2\,b^2c^2-abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^2b}+\frac{\sqrt{dx^3+c}\,bcd-\sqrt{dx^3+c}\,ad^2}{3\left(\left(dx^3+c\right)b-bc+ad\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 2/3*c^2*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)) - 1/3*(2*b^2*c^2 -
a*b*c*d - a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^
2*c + a*b*d)*a^2*b) + 1/3*(sqrt(d*x^3 + c)*b*c*d - sqrt(d*x^3 + c)*a*d^2)/(
((d*x^3 + c)*b - b*c + a*d)*a*b)
```

maple [C] time = 0.27, size = 1036, normalized size = 7.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b*c*d-(a*d-2*b*c)/b^2*d)*(d
*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/(a*d
-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))
/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1
/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2
)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d
^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)
-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(
1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha
-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*
b+a)))-1/a*b*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/
2)/b^2*d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha
*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/
2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d
)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-
c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(2/9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x
^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)^(1/2)/c^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x), x)
```

mupad [B] time = 9.14, size = 214, normalized size = 1.63

$$\frac{c^{3/2}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2}+\frac{\sqrt{d\,x^3+c}\,\left(\frac{a\left(\frac{b\,d^2}{3\left(b^2\,c-a\,b\,d\right)}-\frac{2\,b^2\,c\,d}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b}+\frac{b^2\,c^2}{3\,a\left(b^2\,c-a\,b\,d\right)}\right)}{b\,x^3+a}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{\cdots}}{b\,x^3+\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x*(a + b*x^3)^2),x)
```

```
[Out] (c^(3/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6))/(3*a^2) + ((c + d*x^3)^(1/2)*((a*((b*d^2)/(3*(b^2*c - a*b*d)) - (2*
b^2*c*d)/(3*a*(b^2*c - a*b*d))))/b + (b^2*c^2)/(3*a*(b^2*c - a*b*d))))/(a +
 b*x^3) + (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i + b*d*x^3)/(a + b*x^3))*(a*d - b*c)^(1/2)*(a*d + 2*b*c)*1i)/(6*a^2*b^(3/2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.474 $\int \frac{(c+dx^3)^{3/2}}{x^4(a+bx^3)^2}\,dx$

Optimal. Leaf size=170

$$\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2(a+bx^3)}-\frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

```
[Out] 1/3*(-3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))*c^(1/2)/a^3-1/3*(-a*d+4
*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a^
3/b^(1/2)-1/3*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/(b*x^3+a)-1/3*c*(d*x^3+c)^(1
/2)/a/x^3/(b*x^3+a)
```

Rubi [A] time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 98, 151, 156, 63, 208}

$$-\frac{\sqrt{c+dx^3}(2bc-ad)}{3a^2(a+bx^3)}+\frac{\sqrt{c}(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3}-\frac{\sqrt{bc-ad}(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3\sqrt{b}}-\frac{c\sqrt{c+dx^3}}{3ax^3(a+bx^3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] -((2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*(a + b*x^3)) - (c*Sqrt[c + d*x^3])/
(3*a*x^3*(a + b*x^3)) + (Sqrt[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sq
rt[c]])/(3*a^3) - (Sqrt[b*c - a*d]*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c +
d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
```

```
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{3/2}}{x^4\left(a+bx^3\right)^2}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(a+bx)^2}\,dx, x, x^3\right)$$

$$= -\frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)+\frac{1}{2}d(3bc-2ad)x}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3a}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}c(4bc-3ad)(bc-ad)+\frac{1}{2}d(bc-ad)(2bc-ad)x}{x(a+bx)\sqrt{c+dx}}\,dx,\right.}{3a^2(bc-ad)}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{6a^3} + \ldots$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} - \frac{(c(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right.}{3a^3d}$$

$$= -\frac{(2bc-ad)\sqrt{c+dx^3}}{3a^2\left(a+bx^3\right)} - \frac{c\sqrt{c+dx^3}}{3ax^3\left(a+bx^3\right)} + \frac{\sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3} - \frac{\sqrt{bc-ad}\,(\ldots}{}$$

Mathematica [A] time = 0.33, size = 142, normalized size = 0.84

$$\frac{\frac{a\sqrt{c+dx^3}\left(-ac+adx^3-2bcx^3\right)}{x^3\left(a+bx^3\right)} + \sqrt{c}\,(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right) - \frac{\sqrt{bc-ad}\,(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{\sqrt{b}}}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(-(a*c) - 2*b*c*x^3 + a*d*x^3))/(x^3*(a + b*x^3)) + Sqr
t[c]*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]] - (Sqrt[b*c - a*d]*(4
*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/Sqrt[b])/(3
*a^3)
```

fricas [A] time = 1.17, size = 838, normalized size = 4.93

$$\left[-\frac{\left(\left(4\,b^2c-abd\right)x^6+\left(4\,abc-a^2d\right)x^3\right)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^3+a}\right)+\left(\left(4\,b^2c-3\,abd\right)x^6+\left(4\,abc-3\right.\right.}{6\left(a^3bx^6+a^4x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3)*sqrt((b*c - a*d)/b)*
log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt((b*c - a*d)/b))/(b*x^
3 + a)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(c)*log((
d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*((2*a*b*c - a^2*d)*x^3 +
a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1/6*(2*((4*b^2*c - a*b*d)*x
^6 + (4*a*b*c - a^2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*
sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3
*a^2*d)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2
*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3), -1
/6*(2*((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(-c)*arctan(s
qrt(d*x^3 + c)*sqrt(-c)/c) + ((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^2*d)*x^3
)*sqrt((b*c - a*d)/b)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*b*sqrt
((b*c - a*d)/b))/(b*x^3 + a)) + 2*((2*a*b*c - a^2*d)*x^3 + a^2*c)*sqrt(d*x^
3 + c))/(a^3*b*x^6 + a^4*x^3), -1/3*(((4*b^2*c - a*b*d)*x^6 + (4*a*b*c - a^
2*d)*x^3)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^3 + c)*b*sqrt(-(b*c - a*d)/
b)/(b*c - a*d)) + ((4*b^2*c - 3*a*b*d)*x^6 + (4*a*b*c - 3*a^2*d)*x^3)*sqrt(
-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + ((2*a*b*c - a^2*d)*x^3 + a^2*c)*sq
rt(d*x^3 + c))/(a^3*b*x^6 + a^4*x^3)]
```

giac [A] time = 0.22, size = 216, normalized size = 1.27

$$\frac{\left(4\,b^2c^2-5\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a^3}-\frac{\left(4\,bc^2-3\,acd\right)\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^3\sqrt{-c}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}bcd-2\,\sqrt{dx^3+c}\,bc^2d}{3\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] 1/3*(4*b^2*c^2 - 5*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c
+ a*b*d))/(sqrt(-b^2*c + a*b*d)*a^3) - 1/3*(4*b*c^2 - 3*a*c*d)*arctan(sqrt(
d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)) - 1/3*(2*(d*x^3 + c)^(3/2)*b*c*d - 2*sq
rt(d*x^3 + c)*b*c^2*d - (d*x^3 + c)^(3/2)*a*d^2 + 2*sqrt(d*x^3 + c)*a*c*d^2
)/(((d*x^3 + c)^2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)*
a^2)
```

maple [C] time = 0.25, size = 1093, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)*c/x^3+2/3*(d*x^3+c)^(1/2)*d-c^(1/2)*d*arctanh((
d*x^3+c)^(1/2)/c^(1/2)))+2/a^3*b^2*(2/9*(d*x^3+c)^(1/2)/b*d*x^3+2/3*(-2/3/b
*c*d-(a*d-2*b*c)/b^2*d)*(d*x^3+c)^(1/2)/d+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d
^2+2*a*b*c*d-b^2*c^2)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*
d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*
d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c
*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2
)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^
(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(
b*x^3+a)/b^2+2/3*(d*x^3+c)^(1/2)/b^2*d+1/2*I/d/b^2*2^(1/2)*sum((-c*d^2)^(1/
3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3)
)*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^
2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*E
llipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alph
a^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*
_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*(2/
9*(d*x^3+c)^(1/2)*d*x^3+8/9*(d*x^3+c)^(1/2)*c-2/3*c^(3/2)*arctanh((d*x^3+c)
^(1/2)/c^(1/2)))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^4), x)
```

mupad [B] time = 10.82, size = 531, normalized size = 3.12

$$\frac{\sqrt{c}\,\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)(3\,a\,d-4\,b\,c)}{6\,a^3}-\frac{c\,\sqrt{d\,x^3+c}}{3\,a^2\,x^3}-\frac{\sqrt{d\,x^3+c}\left(\frac{3\,a\,d-4\,b\,c}{2\,a^2}-\frac{a\left(\frac{a\left(\frac{a\left(\frac{b\,d^2\,(a\,d+b\,c)}{a^3\,c^2}-\frac{a\left(\frac{b^2\,d^3}{2\,a^3\,c^2}-\frac{b^2\,d^3}{6\,a^2\,c\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^4*(a + b*x^3)^2),x)
```

```
[Out] (c^(1/2)*log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2))
)/x^6)*(3*a*d - 4*b*c))/(6*a^3) - (c*(c + d*x^3)^(1/2))/(3*a^2*x^3) - ((c +
 d*x^3)^(1/2)*((3*a*d - 4*b*c)/(2*a^2) - (a*((a*((a*((b*d^2*(a*d + b*c))/(a
^3*c^2) - (a*((b^2*d^3)/(2*a^3*c^2) - (b^2*d^3*(3*a*d - 4*b*c))/(6*a^2*c^2*
(a^2*d - a*b*c)) + (b^2*d^2*(a*d + b*c)*(3*a*d - 4*b*c))/(3*a^3*c^2*(a^2*d
- a*b*c))))/b + (b*(3*a*d - 4*b*c)*(a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2))/(6*
a^3*c^2*(a^2*d - a*b*c))))/b - (a^2*d^3 - b^2*c^2*d + 4*a*b*c*d^2)/(2*a^3*c
^2) + (b*(3*a*d - 4*b*c)*(2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d))/(6*a^3*c^
2*(a^2*d - a*b*c))))/b - (2*b^2*c^3 - 4*a^2*c*d^2 + 2*a*b*c^2*d)/(2*a^3*c^2
) + (b*(3*a*d - 4*b*c)^2)/(6*a^2*(a^2*d - a*b*c))))/b))/(a + b*x^3) + (log(
(2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
 + b*x^3))*(a*d - b*c)^(1/2)*(a*d - 4*b*c)*1i)/(6*a^3*b^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**4/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.475 $\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/4*c*x^4*AppellF1(4/3,2,-3/2,7/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^4*Sqrt[c + d*x^3]*AppellF1[4/3, 2, -3/2, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x^3\left(1+\frac{dx^3}{c}\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\
&= \frac{cx^4\sqrt{c+dx^3}\,F_1\left(\frac{4}{3};2,-\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{1+\frac{dx^3}{c}}}
\end{aligned}$$

Mathematica [B] time = 0.48, size = 338, normalized size = 5.20

$$\frac{x^4\left(\frac{d\sqrt{\frac{dx^3}{c}+1}(43bc-55ad)F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}+\frac{8\left(-8acd\left(11ad+b\left(c+6dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3\left(c+dx^3\right)\left(-11ad+5bc-6bdx^3\right)\left(2bcF_1\left(\frac{4}{3};\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8\right.}\right.}{120b^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^4*((d*(43*b*c - 55*a*d)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -
((d*x^3)/c), -((b*x^3)/a)])/a + (8*(-8*a*c*d*(11*a*d + b*(c + 6*d*x^3))*App
ellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*(c + d*x^3)*(5*b*c
- 11*a*d - 6*b*d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((
a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] +
 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*
AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(120*b^2*Sqrt[c
 + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x^3}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

maple [C] time = 0.35, size = 1587, normalized size = 24.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] 1/b*(2/5*(d*x^3+c)^(1/2)/b*d*x-2/3*I*(-2/5/b*c*d-(a*d-2*b*c)/b^2*d)*3^(1/2)
*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/b^2/d^2*2^(1/2
)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I
*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
```

```
*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2
)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-
(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi
(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3
^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(
a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-a/b*(-1/3*(a*d-b*c)/
a/b*x*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(d^2/b^2-1/6/b^2*d*(a*d-b*c)/a)*3^(1/
2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/
2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/b^2/d^2*2
^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*
(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)
^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d
)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_al
pha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Elli
pticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2
*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_al
pha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}} x^3}{\left(bx^3+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x^3/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3 \left(d\,x^3+c\right)^{3/2}}{\left(b\,x^3+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x^3*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(c+dx^3\right)^{\frac{3}{2}}}{\left(a+bx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral(x**3*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.476 $\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] 1/2*c*x^2*AppellF1(2/3,2,-3/2,5/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1
+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (c*x^2*Sqrt[c + d*x^3]*AppellF1[2/3, 2, -3/2, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(c+dx^3\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{x\left(1+\frac{dx^3}{c}\right)^{3/2}}{\left(a+bx^3\right)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{cx^2\sqrt{c+dx^3}\,F_1\left(\frac{2}{3};2,-\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.26, size = 177, normalized size = 2.72

$$\frac{x^2\left(-dx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(bc-7ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5c\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(2ad+bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\right.\right.}{30a^2b\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x]
```

```
[Out] (x^2*(-10*a*(-(b*c) + a*d)*(c + d*x^3) + 5*c*(b*c + 2*a*d)*(a + b*x^3)*Sqrt
[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - d*
(b*c - 7*a*d)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3
, -((d*x^3)/c), -((b*x^3)/a)]))/(30*a^2*b*(a + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}x}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

maple [C] time = 0.36, size = 955, normalized size = 14.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-2/3*I*(1/b^2*d^2+1/6*(a*d-
b*c)/a/b^2*d)*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/b^2/d^2*2^(1
/2)*sum((7*a^2*d^2-5*a*b*c*d-2*b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/
2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(
1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/
3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
```

```
*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellipti
cPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+
I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha
)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^{3}+c\right)^{\frac{3}{2}} x}{\left(bx^{3}+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)*x/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\left(d\,x^{3}+c\right)^{3/2}}{\left(b\,x^{3}+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2,x)
[Out] int((x*(c + d*x^3)^(3/2))/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(c+dx^{3}\right)^{\frac{3}{2}}}{\left(a+bx^{3}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
[Out] Integral(x*(c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.477 $\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=60

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] c*x*AppellF1(1/3,2,-3/2,4/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/(1+d*x^3
/c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (c*x*Sqrt[c + d*x^3]*AppellF1[1/3, 2, -3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(a^2*Sqrt[1 + (d*x^3)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{(a+bx^3)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= \frac{cx\sqrt{c+dx^3}\,F_1\left(\frac{1}{3};2,-\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.43, size = 339, normalized size = 5.65

$$\frac{x\left(\frac{a\left(24x^3\left(c+dx^3\right)(bc-ad)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-64ac\left(bc\left(3c+dx^3\right)-ad^2x^3\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}+dx^3\cdots}{24a^2b\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(a + b*x^3)^2,x]
```

```
[Out] (x*(d*(b*c + 5*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d
*x^3)/c), -((b*x^3)/a)] + (a*(-64*a*c*(-(a*d^2*x^3) + b*c*(3*c + d*x^3))*Ap
pellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*(b*c - a*d)*x^3*(
c + d*x^3)*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] +
a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*
(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b
*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/
3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))))/(24*a^2*b*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

maple [C] time = 0.26, size = 801, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/a/b*x-2/3*I*(1/b^2*d^2-1/6*(a*d-b*
c)/a/b^2*d)*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^
2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1
/18*I/a/b^2/d^2*2^(1/2)*sum((5*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c
)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*
(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1
```

```
/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^
(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c
*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2
)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(
-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/(b*x^3 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(a + b*x^3)^2,x)
```

```
[Out] int((c + d*x^3)^(3/2)/(a + b*x^3)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c+dx^3\right)^{\frac{3}{2}}}{\left(a+bx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/(b*x**3+a)**2,x)
```

```
[Out] Integral((c + d*x**3)**(3/2)/(a + b*x**3)**2, x)
```

3.478 $\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx$

Optimal. Leaf size=63

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -c*AppellF1(-1/3,2,-3/2,2/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x/(1+d*x
^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] -((c*Sqrt[c + d*x^3]*AppellF1[-1/3, 2, -3/2, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*x*Sqrt[1 + (d*x^3)/c]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^2(a+bx^3)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^2(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{1}{3};2,-\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 190, normalized size = 3.02

$$\frac{5cx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(11ad-8bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+2dx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(4bc-ad)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};\cdots\right.}{60a^3x\left(a+bx^3\right)\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x]
```

```
[Out] (-20*a*(c + d*x^3)*(3*a*c + 4*b*c*x^3 - a*d*x^3) + 5*c*(-8*b*c + 11*a*d)*x^
3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c),
-((b*x^3)/a)] + 2*d*(4*b*c - a*d)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*Appel
lF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*x*(a + b*x^3)*Sq
rt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

maple [C] time = 0.26, size = 2364, normalized size = 37.52

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/7*(d*x^3+c)^(1/2)/b*d*x^2-2/3*I*(-4/7/b*c*d-(a*d-2*b*c)/b^2*d)*
3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)
/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1
/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3
^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I/b^2/d^2*2^(1/2)*sum((-a^2*d^2
+2*a*b*c*d-b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
```

```
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-1/a*b*(-1/3*(a*d-b*c)*(d*x^3+c)^(1/2)
/(b*x^3+a)/a/b*x^2-2/3*I*(1/b^2*d^2+1/6*(a*d-b*c)/a/b^2*d)*3^(1/2)*(-c*d^2)
^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2
*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)
^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d
^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/b^2/d^2*2^(1/2)*sum((7*a^2*d^2-5*a*b*c*d-2
*b^2*c^2)/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-(d*x^3+c)^(1/2)*c/x+2/7*(d*x^3+c)^(1/2)
*d*x^2-9/7*I*c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)
/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c
*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*El
lipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2
)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^2\,\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2),x)
```

```
[Out] int((c + d*x^3)^(3/2)/(x^2*(a + b*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**2/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.479 $\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2} \, dx$

Optimal. Leaf size=65

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

```
[Out] -1/2*c*AppellF1(-2/3,2,-3/2,1/3,-b*x^3/a,-d*x^3/c)*(d*x^3+c)^(1/2)/a^2/x^2/
(1+d*x^3/c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{\frac{dx^3}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] -(c*Sqrt[c + d*x^3]*AppellF1[-2/3, 2, -3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*a^2*x^2*Sqrt[1 + (d*x^3)/c])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^3)^{3/2}}{x^3(a+bx^3)^2}\,dx &= \frac{\left(c\sqrt{c+dx^3}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^{3/2}}{x^3(a+bx^3)^2}\,dx}{\sqrt{1+\frac{dx^3}{c}}}\\ &= -\frac{c\sqrt{c+dx^3}\,F_1\left(-\frac{2}{3};2,-\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{1+\frac{dx^3}{c}}}\end{aligned}$$

Mathematica [B] time = 0.38, size = 370, normalized size = 5.69

$$\frac{\frac{8a\left(4ac\left(a\left(6c^2-15cdx^3-4d^2x^6\right)+10bcx^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(c+dx^3\right)\left(3ac-2adx^3+5bcx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3}\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}}{48a^3x^2\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x]
```

```
[Out] (-(d*(5*b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)]) + (8*a*(4*a*c*(10*b*c*x^3*(3*c + d*x^3) + a*(6*c^
2 - 15*c*d*x^3 - 4*d^2*x^6))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*
x^3)/a)] - 3*x^3*(c + d*x^3)*(3*a*c + 5*b*c*x^3 - 2*a*d*x^3)*(2*b*c*AppellF
1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2
, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2,
7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)
/c), -((b*x^3)/a)]))))/(48*a^3*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

maple [C] time = 0.24, size = 1902, normalized size = 29.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x)
```

```
[Out] -1/a^2*b*(2/5*(d*x^3+c)^(1/2)/b*d*x-2/3*I*(-2/5/b*c*d-(a*d-2*b*c)/b^2*d)*3^
(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^
(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2
*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I/b^2/d^2*2
^(1/2)*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(
1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)
```

```
^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alp
ha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/2*(d*
x^3+c)^(1/2)*c/x^2+2/5*(d*x^3+c)^(1/2)*d*x-9/10*I*c*3^(1/2)*(-c*d^2)^(1/3)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))-1/a*b*(-1/3*(a*d-b*c)*(d*x^3+c)^(1/2)
/(b*x^3+a)/a/b*x-2/3*I*(1/b^2*d^2-1/6*(a*d-b*c)/a/b^2*d)*3^(1/2)*(-c*d^2)^(
1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(
-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/b^2/d^2*2^(1/2)*sum((5
*a^2*d^2-a*b*c*d-4*b^2*c^2)/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c
)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(dx^3+c\right)^{\frac{3}{2}}}{\left(bx^3+a\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(3/2)/x^3/(b*x^3+a)^2,x, algorithm="maxima")
[Out] integrate((d*x^3 + c)^(3/2)/((b*x^3 + a)^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(d\,x^3+c\right)^{3/2}}{x^3\,\left(b\,x^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2),x)
[Out] int((c + d*x^3)^(3/2)/(x^3*(a + b*x^3)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(3/2)/x**3/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

3.480 $\int \frac{x^8}{(a+bx^3)^2 \sqrt{c+dx^3}} \, dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2(a+bx^3)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

```
[Out] 1/3*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+2/3*(d*x^3+c)^(1/2)/b^2/d-1/3*a^2*(d*x^3+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^3}}{3b^2(a+bx^3)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}+\frac{2\sqrt{c+dx^3}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3])/(3*b^2*d) - (a^2*Sqrt[c + d*x^3])/(3*b^2*(b*c - a*d)*(a
 + b*x^3)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c
- a*d]])/(3*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= -\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{3b^2(bc-ad)}\\
&= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b^2(bc-ad)}\\
&= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3b^2d(bc-ad)}\\
&= \frac{2\sqrt{c+dx^3}}{3b^2d}-\frac{a^2\sqrt{c+dx^3}}{3b^2(bc-ad)\left(a+bx^3\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{5/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.28, size = 107, normalized size = 0.87

$$\frac{1}{3}\left(\frac{\sqrt{c+dx^3}\left(\frac{a^2}{\left(a+bx^3\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((Sqrt[c + d*x^3]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^3))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/3
```

fricas [B] time = 1.39, size = 475, normalized size = 3.86

$$\left[\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(2\,ab^3c^2-5\,a^2b^2cd\right.}{6\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(b^2*
c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*
d))/(b*x^3 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3), -1
/3*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^3)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^3)]
```

giac [A] time = 0.23, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^3+c}\,a^2d}{3\left(b^3c-ab^2d\right)\left(\left(dx^3+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{2\sqrt{dx^3+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d*x^3 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^3 + c)*b - b*c + a*d))
- 1/3*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 2/3*sqrt(d*x^3 + c)/(b^2*d)
```

maple [C] time = 0.42, size = 911, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 2/3*(d*x^3+c)^(1/2)/b^2/d+2/3*I*a/b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^
(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1
/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1
/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/
2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3)
)*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_a
lpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/
3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+a^2/b^2*(
1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.29, size = 160, normalized size = 1.30

$$\frac{2\sqrt{d\,x^3+c}\,\left(2\,b^2\,c-2\,a\,b\,d\right)}{3\,d\,\left(2\,b^4\,c-2\,a\,b^3\,d\right)}-\frac{2\,a^2\,\sqrt{d\,x^3+c}}{3\,b\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{a\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(3\,a\,d-4\,b$$

$$6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (2*(c + d*x^3)^(1/2)*(2*b^2*c - 2*a*b*d))/(3*d*(2*b^4*c - 2*a*b^3*d)) - (2*
a^2*(c + d*x^3)^(1/2))/(3*b*(a + b*x^3)*(2*b^2*c - 2*a*b*d)) + (a*log((a*d
- 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i - b*d*x^3)/(a + b*
x^3))*(3*a*d - 4*b*c)*1i)/(6*b^(5/2)*(a*d - b*c)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

3.481 $\int \frac{x^5}{(a+bx^3)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/3*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/3*a*(d*x^3+c)^(1/2)/b/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^3}}{3b(a+bx^3)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (a*Sqrt[c + d*x^3])/(3*b*(b*c - a*d)*(a + b*x^3)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx=\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^3\right)$$

$$=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^3\right)}{6b(bc-ad)}$$

$$=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^3}\right)}{3bd(bc-ad)}$$

$$=\frac{a\sqrt{c+dx^3}}{3b(bc-ad)\left(a+bx^3\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*b^(
3/2))
```

fricas [A] time = 0.98, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^3+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\left(ab^2c-a^2bd\right)\sqrt{dx^3+c}}{6\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3\right)},\ \frac{\left(\left(2\,b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c - a*b*d)*x^3 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) +
 2*(a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^3), 1/3*(((2*b^2*c - a*b*d)
*x^3 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^3 + b*c)) + (a*b^2*c - a^2*b*d)*sqrt(d*x^3 + c))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^3)]
```

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^3+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^3+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(sqrt(d*x^3 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^3 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

maple [C] time = 0.27, size = 892, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)
*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/
3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c
)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_a
lpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(
x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/
3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-
I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*
x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(
1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*
x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x
^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3
)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*
c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,
(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.85, size = 111, normalized size = 1.12

$$\frac{2\,a\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,\left(a\,d-2\,b\,c\right)\,1{}\mathrm{i}}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d - 2*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(3/2)) + (2*a*(c +
 d*x^3)^(1/2))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

3.482 $\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

```
[Out] 1/3*d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/3*(d*x^3+c)^(1/2)/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^3}}{3\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -Sqrt[c + d*x^3]/(3*(b*c - a*d)*(a + b*x^3)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{6(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3(bc-ad)}\\
&= -\frac{\sqrt{c+dx^3}}{3(bc-ad)\left(a+bx^3\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$
\frac{1}{3}\left(\frac{\sqrt{c+dx^3}}{\left(a+bx^3\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[c + d*x^3]/((-(b*c) + a*d)*(a + b*x^3)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^3])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/3
```

fricas [B] time = 0.90, size = 302, normalized size = 3.47

$$
\left[-\frac{\left(bdx^3+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)+2\,\sqrt{dx^3+c}\left(b^2c-abd\right)}{6\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^3\right)}, -\frac{\left(bdx^3+ad\right)\sqrt{-b^2c+ab}}{3\left(ab^3c^2-2\,a^2b\right.}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*d*x^3 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*s
qrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*sqrt(d*x^3 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^3), -1/3*((b*d*x^3 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + sqrt(d*x^3 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^3)]
```

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$
-\frac{d\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,(bc-ad)} - \frac{\sqrt{dx^3+c}\,d}{3\left(\left(dx^3+c\right)b-bc+ad\right)(bc-ad)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/3*sqrt(d*x^3 + c)*d/(((d*x^3 + c)*b - b*c + a*d)*(b*c - a
*d))
```

maple [C] time = 0.26, size = 457, normalized size = 5.25

$$-\frac{i\left(-c\,d^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^{2}\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^{2}\right)^{\frac{1}{3}}+\left(-c\,d^{2}\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^{2}\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^{3}b+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(
-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c
*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c
*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3)
)/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/
3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^
2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*
d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 6.37, size = 104, normalized size = 1.20

$$-\frac{2\,b\,\sqrt{d\,x^3+c}}{3\,\left(b\,x^3+a\right)\,\left(2\,b^2\,c-2\,a\,b\,d\right)}+\frac{d\,\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2{}\mathrm{i}}{b\,x^3+a}\right)\,1{}\mathrm{i}}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] (d*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*
x^3)/(a + b*x^3))*1i)/(6*b^(1/2)*(a*d - b*c)^(3/2)) - (2*b*(c + d*x^3)^(1/2
))/(3*(a + b*x^3)*(2*b^2*c - 2*a*b*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Timed out
```

3.483 $\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

```
[Out] 1/3*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/3
*b*(d*x^3+c)^(1/2)/a/(-a*d+b*c)/(b*x^3+a)
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^3}}{3a\left(a+bx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[c + d*x^3])/(3*a*(b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x
^3]/Sqrt[c]])/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*S
qrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^3\right)}{3a^2} - \frac{(b(2bc-3ad))\operatorname{Subst}\left(\int \frac{\ldots}{(a+\ldots}\right.}{6a^2(bc-a\ldots}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} + \frac{2\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^3}\right)}{3a^2d} - \frac{(b(2bc-3ad))\operatorname{Subs}\ldots}{3\ldots}$$

$$= \frac{b\sqrt{c+dx^3}}{3a(bc-ad)\left(a+bx^3\right)} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.26, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^3}}{\left(a+bx^3\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^3])/((b*c - a*d)*(a + b*x^3)) - (2*ArcTanh[Sqrt[c + d*x^
3]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(3*a^2)
```

fricas [A] time = 1.15, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^3+c}\,abc + \left(2\,abc^2 - 3\,a^2cd + \left(2\,b^2c^2 - 3\,abcd\right)x^3\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right) + 2\left(\left(b^2c\ldots\right.\right.}{6\left(a^3bc^2 - a^4cd + \left(a^2b^2c^2 - a^3bcd\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*sqrt(d*x^3 + c)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b
*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*((b^2*c - a*b*d)*x^3 +
```

```
a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c
)*a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3
 + b*c)) + ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(c)*log((d*x^3 - 2*sqr
t(d*x^3 + c)*sqrt(c) + 2*c)/x^3))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^3), 1/6*(2*sqrt(d*x^3 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^3 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (2*a*b*c^2 - 3*a^2*
c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c
 - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3), 1/3*(sqrt(d*x^3 + c)*
a*b*c + (2*a*b*c^2 - 3*a^2*c*d + (2*b^2*c^2 - 3*a*b*c*d)*x^3)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 +
 b*c)) + 2*((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^3
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

giac [A] time = 0.16, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^3+c}\,bd}{3\left(abc-a^2d\right)\left(\left(dx^3+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{2\,\arctan\left(\frac{\sqrt{dx^3+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(d*x^3 + c)*b*d/((a*b*c - a^2*d)*((d*x^3 + c)*b - b*c + a*d)) - 1/3
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

maple [C] time = 0.28, size = 915, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a^2*b/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1
/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^
(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x
+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^
3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)
*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(
I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^
(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c
*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a*b*(1/3/(a*d-b*c)*(d*x^3+c)^(1/2
)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
```

```
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2/3*arctanh((d*x^3+c)^(1/2)/
c^(1/2))/a^2/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x), x)
```

mupad [B] time = 9.65, size = 162, normalized size = 1.23

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,\sqrt{c}}+\frac{b^2\,\sqrt{d\,x^3+c}}{3\,a\,\left(b\,x^3+a\right)\,\left(b^2\,c-a\,b\,d\right)}+\frac{\sqrt{b}\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)\,(3\,a\,d-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(1/2)) + (b^2*(c + d*x^3)^(1/2))/(3*a*(a + b*x^3)*(b^2*c - a*b*d)) +
 (b^(1/2)*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i
 - b*d*x^3)/(a + b*x^3))*(3*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.484 $\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}\,(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

```
[Out] 1/3*(a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/3*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/3*b*(-a*d+2*b*c)*(d*x^3+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^3+a)-1/3*
(d*x^3+c)^(1/2)/a/c/x^3/(b*x^3+a)
```

Rubi [A] time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}}-\frac{b\sqrt{c+dx^3}\,(2bc-ad)}{3a^2c\left(a+bx^3\right)(bc-ad)}-\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^3])/(3*a^2*c*(b*c - a*d)*(a + b*x^3)) - Sqrt
[c + d*x^3]/(3*a*c*x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3
]/Sqrt[c]])/(3*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{3a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{6a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{3a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^3}}{3a^2c(bc-ad)\left(a+bx^3\right)} - \frac{\sqrt{c+dx^3}}{3acx^3\left(a+bx^3\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{3/2}} - \frac{b^{3/2}}{}$$

Mathematica [A] time = 0.61, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^3}\left(a^2d+ab\left(dx^3-c\right)-2b^2cx^3\right)}{x^3\left(a+bx^3\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] ((a*Sqrt[c + d*x^3]*(a^2*d - 2*b^2*c*x^3 + a*b*(-c + d*x^3)))/((b*c - a*d)*
x^3*(a + b*x^3)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/Sqrt[c]
```

```
+ (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c -
a*d]])/(b*c - a*d)^(3/2))/(3*a^3*c)
```

fricas [A] time = 1.28, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)
*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*log((d*x^3 +
2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a
^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(c)*l
og((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/6*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^3)*sqrt(-c)*arcta
n(sqrt(d*x^3 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^6 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d -
 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/3*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^6 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^3)*sqrt(d*x^3 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3)]
```

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^3bc-a^4d\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)^{\frac{3}{2}}b^2cd-2\sqrt{dx^3+c}\,b^2c^2d-\left(dx^3+c\right)^{\frac{3}{2}}abd^2+2\sqrt{dx^3+c}\,abcd}{3\left(a^2bc^2-a^3cd\right)\left(\left(dx^3+c\right)^2b-2\left(dx^3+c\right)bc+bc^2+\left(dx^3+c\right)a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^3 + c)*b^2*c^2*d - (d*x^3 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^3 + c
)*a*b*c*d^2 - sqrt(d*x^3 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^3 + c)^
2*b - 2*(d*x^3 + c)*b*c + b*c^2 + (d*x^3 + c)*a*d - a*c*d)) - 1/3*(4*b*c +
a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

maple [C] time = 0.29, size = 961, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/c/x^3+1/3*d*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^
(3/2))-2/3*I/a^3*b^2/d^2*2^(1/2)*sum(1/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*b^2*(1/3/(a*d-b*c)*(
d*x^3+c)^(1/2)/(b*x^3+a)-1/6*I/d*2^(1/2)*sum(1/(a*d-b*c)^2*(-c*d^2)^(1/3)*(
1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^
(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)
^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alp
ha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*Ellip
ticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*
d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alp
ha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^
(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+4/3*b/a^3*arcta
nh((d*x^3+c)^(1/2)/c^(1/2))/c^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^4), x)
```

mupad [B] time = 11.55, size = 355, normalized size = 1.92

$$\frac{\sqrt{d\,x^3+c}\,\left(\frac{d\,a^2+4\,b\,c\,a}{2\,a^3\,c^2}-\frac{a\,\left(\frac{2\,c\,b^2+2\,a\,d\,b}{2\,a^3\,c^2}-\frac{a\,\left(\frac{b^2\,d}{2\,a^3\,c^2}+\frac{b\,\left(2\,c\,b^2+2\,a\,d\,b\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}-\frac{b^2\,d\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^2\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}+\frac{b\,\left(d\,a^2+4\,b\,c\,a\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,a^3\,c^2\,\left(a^2\,d-a\,b\,c\right)}\right)}{b}\right)}{b\,x^3+a}-\frac{\sqrt{d\,x^3+c}}{3\,a^2\,c\,x^3}+\ln\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*((a^2*d + 4*a*b*c)/(2*a^3*c^2) - (a*((2*b^2*c + 2*a*b*d)
/(2*a^3*c^2) - (a*((b^2*d)/(2*a^3*c^2) + (b*(2*b^2*c + 2*a*b*d)*(3*a*d - 4*
b*c))/(6*a^3*c^2*(a^2*d - a*b*c)) - (b^2*d*(3*a*d - 4*b*c))/(6*a^2*c^2*(a^2
*d - a*b*c))))/b + (b*(a^2*d + 4*a*b*c)*(3*a*d - 4*b*c))/(6*a^3*c^2*(a^2*d
- a*b*c))))/b))/(a + b*x^3) - (c + d*x^3)^(1/2)/(3*a^2*c*x^3) + (log((((c +
 d*x^3)^(1/2) - c^(1/2))*((c + d*x^3)^(1/2) + c^(1/2))^3)/x^6)*(a*d + 4*b*c
))/(6*a^3*c^(3/2)) + (b^(3/2)*log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*
(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5*a*d - 4*b*c)*1i)/(6*a^3*(a*
d - b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^2 \sqrt{c + dx^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.485 $\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 238, normalized size = 3.72

$$\frac{x\left(\frac{8\left(\frac{8ac^2F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}+c+dx^3\right)}{a+bx^3}+\frac{dx^3\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{a}\right)}{24\sqrt{c+dx^3}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*((d*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((
b*x^3)/a)])/a + (8*(c + d*x^3 + (8*a*c^2*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^
3)/c), -((b*x^3)/a)])/(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b
*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)
/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(a +
b*x^3)))/(24*(-(b*c) + a*d)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.37, size = 1207, normalized size = 18.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3*I/b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-a/b*(-1/3/a*b/(a*d-b*c)*x*(d*
x^3+c)^(1/2)/(b*x^3+a)+1/9*I/(a*d-b*c)/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(
```

```
-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(
1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3
^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-
c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/d^2*2^(1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)
^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)
^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.486 $\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*
x^3+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.16, size = 172, normalized size = 2.69

$$\frac{-bdx^5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^2\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,(bc-3ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx}{a}\right.}{30a^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]

[Out] (10*a*b*x^2*(c + d*x^3) + 5*(b*c - 3*a*d)*x^2*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] - b*d*x^5*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(30*a^2*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")

[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)

maple [C] time = 0.40, size = 923, normalized size = 14.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)

[Out] -1/3/a*b/(a*d-b*c)*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)-1/9*I/(a*d-b*c)/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(a*d-b*c)^2/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^2\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.487 $\int \frac{1}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}} \, dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/(d*x^3+c)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.31, size = 392, normalized size = 6.64

$$\frac{3bx^4\left(dx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8a\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2}\right.\right.}{24a^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)]*(8*a*(3*b*
c - 3*a*d + b*d*x^3) + b*d*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3
, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + 3*b*x^4*(8*a*(c + d*x^3) + d*
x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c)
, -((b*x^3)/a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/
a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))/(24*a^2*
(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3,
-((d*x^3)/c), -((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^2 \sqrt{dx^3 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

maple [C] time = 0.28, size = 769, normalized size = 13.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] -1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/a*b*x+1/9*I/(a*d-b*c)/a*3^(1/2)*(-
c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(
1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/d^2*2^(1/2)*sum(
(-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*
(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3
)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)
^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_al
pha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x
+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I
*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^
(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2 \sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^2 \sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/((a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.488 $\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x/(d*x^3+
c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*x*Sqrt[c + d*x^3]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2x\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.25, size = 226, normalized size = 3.65

$$\frac{-5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(3a^2d^2-15abcd+8b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+20a\left(c+dx^3\right)\left(3a^2d-3ab\left(c-\right.\right.}{60a^3cx\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (20*a*(c + d*x^3)*(3*a^2*d - 4*b^2*c*x^3 - 3*a*b*(c - d*x^3)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^6*(a
+ b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*
x^3)/a)])/(60*a^3*c*(b*c - a*d)*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

maple [C] time = 0.27, size = 1818, normalized size = 29.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a^2*b/d^2*2^(1/2)*sum(1/_alpha/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(
-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2
*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2
)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2
)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^
(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-
c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*
c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c
)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))-1/a*b*(-1/3/(a*d-b*c)*(d*x^3
+c)^(1/2)/(b*x^3+a)/a*b*x^2-1/9*I/(a*d-b*c)/a*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+
1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)
*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(
1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3
^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/
2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)
^(1/2)))+1/18*I/a/d^2*2^(1/2)*sum((-5*a*d+2*b*c)/(a*d-b*c)^2/_alpha*(-c*d^2
```

```
)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^
(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^
(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(
-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(
1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/
3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*
_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(
2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d
+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*
(-(d*x^3+c)^(1/2)/c/x-1/3*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/
2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(
-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^
(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*
d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)
*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(b\,x^3+a\right)^2\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.489 $\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/x^2/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{c+dx^3}}\\
&= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2x^2\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] time = 0.56, size = 411, normalized size = 6.42

$$\frac{a\left(32ac\left(3a^2d\left(2c+3dx^3\right)+3ab\left(-2c^2+7cdx^3+2d^2x^6\right)-10b^2cx^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+24x^3\left(c+dx^3\right)\left(-3a^2d+3ab\left(c-dx^3\right)+5b^2cx^3\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},\right.\right.\right.}{\frac{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}{48a^3cx^2\sqrt{c+dx^3}\,(ad-bc}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*(5*b*c - 3*a*d)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((
d*x^3)/c), -((b*x^3)/a)] + (a*(32*a*c*(-10*b^2*c*x^3*(3*c + d*x^3) + 3*a^2*
d*(2*c + 3*d*x^3) + 3*a*b*(-2*c^2 + 7*c*d*x^3 + 2*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] + 24*x^3*(c + d*x^3)*(-3*a^2*d + 5*
b^2*c*x^3 + 3*a*b*(c - d*x^3))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/
c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x
^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a
)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(48*a^3
*c*(-(b*c) + a*d)*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^2 \sqrt{dx^3 + c}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

maple [C] time = 0.26, size = 1512, normalized size = 23.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x)
```

```
[Out] 1/3*I/a^2*b/d^2*2^(1/2)*sum(1/_alpha^2/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x
+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-
(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1
/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d
^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2
)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b
*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*
d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-1/2*(d*x^3+c)^(1/2
)/c/x^2+1/6*I/c*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(
1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^
(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*
d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))
```

```
)-1/a*b*(-1/3/(a*d-b*c)*(d*x^3+c)^(1/2)/(b*x^3+a)/a*b*x+1/9*I/(a*d-b*c)/a*3
^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/
3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^
(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+
1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*
(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/d^2*2^(
1/2)*sum((-7*a*d+4*b*c)/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I
*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*
d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I
*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/
(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^
(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*
d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*
b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*sqrt(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^2\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**2*sqrt(c + d*x**3)), x)
```

3.490 $\int \frac{x^8}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=150

$$\frac{-a^2d^2-2b^2c^2}{3b^2d\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

```
[Out] 1/3*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2
)/(-a*d+b*c)^(5/2)+1/3*(-a^2*d^2-2*b^2*c^2)/b^2/d/(-a*d+b*c)^2/(d*x^3+c)^(1
/2)-1/3*a^2/b^2/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = $\{446, 89, 78, 63, 208\}$

$$-\frac{a^2d^2+2b^2c^2}{3b^2d\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{a^2}{3b^2\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3b^{3/2}(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(2*b^2*c^2 + a^2*d^2)/(3*b^2*d*(b*c - a*d)^2*Sqrt[c + d*x^3]) - a^2/(3*b^2
*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3]) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[
b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc+ad)+b(bc-ad)x}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{3b^2(bc-ad)}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\,\text{Sul}}{6}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(a(4bc-ad))\,\text{Sul}}{}$$

$$= -\frac{2b^2c^2+a^2d^2}{3b^2d(bc-ad)^2\sqrt{c+dx^3}} - \frac{a^2}{3b^2(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{a(4bc-ad)\tanh}{3b^{3/2}(bc-}$$

Mathematica [A] time = 0.30, size = 134, normalized size = 0.89

$$\frac{\frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{\sqrt{b}\left(a^2d\left(c+dx^3\right)+2abc^2+2b^2c^2x^3\right)}{d\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-((Sqrt[b]*(2*a*b*c^2 + 2*b^2*c^2*x^3 + a^2*d*(c + d*x^3)))/(d*(b*c - a*d)
^2*(a + b*x^3)*Sqrt[c + d*x^3])) + (a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*Sqrt[c
 + d*x^3])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2))/(3*b^(3/2))
```

fricas [B] time = 1.35, size = 746, normalized size = 4.97

$$\left[-\frac{\left(\left(4\,ab^2cd^2-a^2bd^3\right)x^6+4\,a^2bc^2d-a^3cd^2+\left(4\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)x^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3}}{bx^3+a}\right.}{6\left(ab^5c^4d-3\,a^2b^4c^3d^2+3\,a^3b^3c^2d^3-a^4b^2cd^4+\left(b^6c^3d^2-3\,ab^5c^2d^3+3\,a^2b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3
+ 2*b*c - a*d - 2*sqrt(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(2*
a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d + a^2*
b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^4*c^3*d
^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*d^3 + 3
*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2*a^3*b^
3*c*d^4 - a^4*b^2*d^5)*x^3), -1/3*(((4*a*b^2*c*d^2 - a^2*b*d^3)*x^6 + 4*a^2
*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)
) + (2*a*b^3*c^3 - a^2*b^2*c^2*d - a^3*b*c*d^2 + (2*b^4*c^3 - 2*a*b^3*c^2*d
 + a^2*b^2*c*d^2 - a^3*b*d^3)*x^3)*sqrt(d*x^3 + c))/(a*b^5*c^4*d - 3*a^2*b^
4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - a^4*b^2*c*d^4 + (b^6*c^3*d^2 - 3*a*b^5*c^2*
d^3 + 3*a^2*b^4*c*d^4 - a^3*b^3*d^5)*x^6 + (b^6*c^4*d - 2*a*b^5*c^3*d^2 + 2
*a^3*b^3*c*d^4 - a^4*b^2*d^5)*x^3)]
```

giac [A] time = 0.19, size = 195, normalized size = 1.30

$$-\frac{\left(4\,abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)b^2c^2-2\,b^2c^3+2\,abc^2d+\left(dx^3+c\right)a^2d^2}{3\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(4*a*b*c - a^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3 + c)*
b^2*c^2 - 2*b^2*c^3 + 2*a*b*c^2*d + (d*x^3 + c)*a^2*d^2)/((b^3*c^2*d - 2*a*
b^2*c*d^2 + a^2*b*d^3)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*
x^3 + c)*a*d))
```

maple [C] time = 0.36, size = 978, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -2/3/b^2/d/(d*x^3+c)^(1/2)-2*a/b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*
I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3
^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^
2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(
2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d
*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1
/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2
)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^
2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-
3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/
d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(
1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+a^2/b^2*(-1/3*b/(a*d-b*c)^2*(d*
x^3+c)^(1/2)/(b*x^3+a)-2/3*d/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)+1/2*I*b/d*2^(1
/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)
+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d
^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^
2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alp
ha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2
)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^
(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2
```

```
*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3
)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)
^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_al
pha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.90, size = 367, normalized size = 2.45

$$\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\,\left(a\,d+b\,c\right)}{b\,d}+\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)+\frac{a\,c\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\cdots\right.}{b\,d}\cdots}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] ((c + d*x^3)^(1/2)*(x^3*((((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2))*(a*d + b*c))/(b*d) + (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d
 - 2*a*b^2*c*d^2)) + (a*c*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^
2*d - 2*a*b^2*c*d^2)))/(b*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (a*log((
2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a
+ b*x^3))*(a*d - 4*b*c)*1i)/(6*b^(3/2)*(a*d - b*c)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.491 $\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

```
[Out] -1/3*(a*d+2*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*
c)^(5/2)/b^(1/2)+1/3*(a*d+2*b*c)/b/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*a/b/(-a
*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 78, 51, 63, 208}

$$\frac{a}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{ad+2bc}{3b\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\,(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (2*b*c + a*d)/(3*b*(b*c - a*d)^2*Sqrt[c + d*x^3]) + a/(3*b*(b*c - a*d)*(a +
 b*x^3)*Sqrt[c + d*x^3]) - ((2*b*c + a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])
/Sqrt[b*c - a*d]])/(3*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\mathrm{Subst}\left(\int \ldots\right.}{6(bc-\ldots}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{(2bc+ad)\,\mathrm{Subst}\left(\int \ldots\right.}{3d(b\ldots}$$

$$= \frac{2bc+ad}{3b(bc-ad)^2\sqrt{c+dx^3}} + \frac{a}{3b(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(2bc+ad)\tanh^{-1}\left(\ldots\right.}{3\sqrt{b}\,(bc-ad)^{5\ldots}}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.68

$$\frac{\left(a+bx^3\right)(ad+2bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)+a(bc-ad)}{3b\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (a*(b*c - a*d) + (2*b*c + a*d)*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2,
(b*(c + d*x^3))/(b*c - a*d)])/(3*b*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3
])
```

fricas [B] time = 1.39, size = 630, normalized size = 4.70

$$\left[\frac{\left(\left(2\,b^2cd+abd^2\right)x^6+2\,abc^2+a^2cd+\left(2\,b^2c^2+3\,abcd+a^2d^2\right)x^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^3+2\,bc-ad-2\,\sqrt{dx^3+c}\,\sqrt{b^2c-abd}}{bx^3+a}\right)\ldots}{6\left(ab^4c^4-3\,a^2b^3c^3d+3\,a^3b^2c^2d^2-a^4bcd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^6+\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^2 + a^2*c*d + (2*b^2*c^2 + 3*a*b
*c*d + a^2*d^2)*x^3)*sqrt(b^2*c - a*b*d)*log((b*d*x^3 + 2*b*c - a*d - 2*sqr
t(d*x^3 + c)*sqrt(b^2*c - a*b*d))/(b*x^3 + a)) + 2*(3*a*b^2*c^2 - 3*a^2*b*c
*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(a*b^4*c^4 -
```

```
3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d - 3*a*b^4*c
^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4*c^3*d + 2*
a^3*b^2*c*d^3 - a^4*b*d^4)*x^3), 1/3*(((2*b^2*c*d + a*b*d^2)*x^6 + 2*a*b*c^
2 + a^2*c*d + (2*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^3)*sqrt(-b^2*c + a*b*d)*a
rctan(sqrt(d*x^3 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^3 + b*c)) + (3*a*b^2*c^2
- 3*a^2*b*c*d + (2*b^3*c^2 - a*b^2*c*d - a^2*b*d^2)*x^3)*sqrt(d*x^3 + c))/(
a*b^4*c^4 - 3*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 - a^4*b*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (b^5*c^4 - 2*a*b^4
*c^3*d + 2*a^3*b^2*c*d^3 - a^4*b*d^4)*x^3)]
```

giac [A] time = 0.20, size = 181, normalized size = 1.35

$$\frac{\frac{\left(2\,bcd+ad^2\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{2\left(dx^3+c\right)bcd-2\,bc^2d+\left(dx^3+c\right)ad^2+2\,acd^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*b*c*d + a*d^2)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c + a*b*d)) + (2*(d*x^3 + c)*b*c*d -
2*b*c^2*d + (d*x^3 + c)*a*d^2 + 2*a*c*d^2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + sqrt(d*x^3 + c)*a*d)))/d
```

maple [C] time = 0.27, size = 958, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c
)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^
(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+
I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)
^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*
d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*
_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3
/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf
(_Z^3*b+a)))-a/b*(-1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c
)^2/((x^3+c/d)*d)^(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3
)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*
d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))
*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2
)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_
alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*El
lipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha
^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_
alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.70, size = 247, normalized size = 1.84

$$-\frac{\sqrt{d\,x^3+c}\,\left(x^3\,\left(\frac{3\,b\,d\,\left(a\,d+b\,c\right)-b\,d\,\left(a\,d+2\,b\,c\right)}{3\,\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}-\frac{b\,d\,\left(a\,d+b\,c\right)}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)-\frac{a\,b\,c\,d}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+\frac{\ln\left(\frac{2\,b\,c-a\,d+b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3}}{b\,x^3+a}\right.}{6\,\sqrt{b}\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (log((2*b*c - a*d + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^
3)/(a + b*x^3))*(a*d + 2*b*c)*1i)/(6*b^(1/2)*(a*d - b*c)^(5/2)) - ((c + d*x
^3)^(1/2)*(x^3*((3*b*d*(a*d + b*c) - b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3
*c^2*d - 2*a*b^2*c*d^2)) - (b*d*(a*d + b*c))/(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) - (a*b*c*d)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)))/(a*c + x^3
*(a*d + b*c) + b*d*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.492 $\int \frac{x^2}{(a+bx^3)^2 (c+dx^3)^{3/2}} \, dx$

Optimal. Leaf size=108

$$-\frac{d}{\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

```
[Out] d*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/(-a*d+b*c)^(5/2
)-d/(-a*d+b*c)^2/(d*x^3+c)^(1/2)-1/3/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {444, 51, 63, 208}

$$-\frac{d}{\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{1}{3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d/((b*c - a*d)^2*Sqrt[c + d*x^3])) - 1/(3*(b*c - a*d)*(a + b*x^3)*Sqrt[c
+ d*x^3]) + (Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/
(b*c - a*d)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule steps

$$\int \frac{x^2}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx, x, x^3\right)}{2(bc-ad)}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{(bd)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+d}}\right.}{2(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,\right.}{(bc-ad)^2}$$

$$= -\frac{d}{(bc-ad)^2\sqrt{c+dx^3}} - \frac{1}{3(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right.}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.50

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b\left(dx^3+c\right)}{ad-bc}\right)}{3\sqrt{c+dx^3}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x^3))/(-(b*c) + a*d))])/(
3*(-(b*c) + a*d)^2*Sqrt[c + d*x^3])
```

fricas [B] time = 1.42, size = 450, normalized size = 4.17

$$\left[\frac{3\left(bd^2x^6+\left(bcd+ad^2\right)x^3+acd\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^3+2\,bc-ad+2\,\sqrt{dx^3+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^3+a}\right)-2\left(3\,bdx^3+bc+2\,ad\right)\sqrt{dx^3+c}}{6\left(\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^6+ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^6 + (b*c*d + a*d^2)*x^3 + a*c*d)*sqrt(b/(b*c - a*d))*log((
b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/
(b*x^3 + a)) - 2*(3*b*d*x^3 + b*c + 2*a*d)*sqrt(d*x^3 + c))/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x^3), 1/3*(3*(b*d^2*x^6 + (b*c
*d + a*d^2)*x^3 + a*c*d)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c
- a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c)) - (3*b*d*x^3 + b*c + 2*a*d)*sq
rt(d*x^3 + c))/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*
x^3)]
```

giac [A] time = 0.21, size = 153, normalized size = 1.42

$$-\frac{bd\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\left(dx^3+c\right)bd-2\,bcd+2\,ad^2}{3\left(b^2c^2-2\,abcd+a^2d^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*d*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(3*(d*x^3 + c)*b*d - 2*b*c*d + 2*a*d^
2)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*
b*c + sqrt(d*x^3 + c)*a*d))
```

maple [C] time = 0.25, size = 485, normalized size = 4.49

$$ib\left(-c\,d^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(2x+\frac{-i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{\frac{\left(x-\frac{\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{-3\left(-c\,d^2\right)^{\frac{1}{3}}+i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(2x+\frac{i\sqrt{3}\left(-c\,d^2\right)^{\frac{1}{3}}+\left(-c\,d^2\right)^{\frac{1}{3}}}{d}\right)d}{2\left(-c\,d^2\right)^{\frac{1}{3}}}}\left(2\,\mathrm{RootOf}\left(\_Z^3b+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^
(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*
3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d
^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*
(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(
d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(
1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/
2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d
^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d
-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b
/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^
(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 7.43, size = 199, normalized size = 1.84

$$-\frac{\left(\frac{3\,b\,d\,(a\,d+b\,c)-b\,d\,(a\,d+2\,b\,c)}{3\left(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d\right)}+\frac{b^2\,d^2\,x^3}{a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d}\right)\sqrt{d\,x^3+c}}{b\,d\,x^6+(a\,d+b\,c)\,x^3+a\,c}+\frac{\sqrt{b}\,d\,\ln\left(\frac{a\,d-2\,b\,c-b\,d\,x^3+\sqrt{b}\,\sqrt{d\,x^3+c}\,\sqrt{a\,d-b\,c}\,2\mathrm{i}}{b\,x^3+a}\right)1\mathrm{i}}{2\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (b^(1/2)*d*log((a*d - 2*b*c + b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2
i - b*d*x^3)/(a + b*x^3))*1i)/(2*(a*d - b*c)^(5/2)) - (((3*b*d*(a*d + b*c)
- b*d*(a*d + 2*b*c))/(3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (b^2*d^2
*x^3)/(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2))*(c + d*x^3)^(1/2))/(a*c + x^
3*(a*d + b*c) + b*d*x^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Timed out
```

3.493 $\int \frac{1}{x(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=172

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}\,(bc-a$$

```
[Out] -2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^2/c^(3/2)+1/3*b^(3/2)*(-5*a*d+2*b*c
)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(5/2)+1/
3*d*(2*a*d+b*c)/a/c/(-a*d+b*c)^2/(d*x^3+c)^(1/2)+1/3*b/a/(-a*d+b*c)/(b*x^3+
a)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 152, 156, 63, 208}

$$\frac{b^{3/2}(2bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^2(bc-ad)^{5/2}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}+\frac{b}{3a\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)}+\frac{d(2ad+bc)}{3ac\sqrt{c+dx^3}\,(bc-a$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (d*(b*c + 2*a*d))/(3*a*c*(b*c - a*d)^2*Sqrt[c + d*x^3]) + b/(3*a*(b*c - a*d
)*(a + b*x^3)*Sqrt[c + d*x^3]) - (2*ArcTanh[Sqrt[c + d*x^3]/Sqrt[c]])/(3*a^
2*c^(3/2)) + (b^(3/2)*(2*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^3])/Sqr
t[b*c - a*d]])/(3*a^2*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)^{3/2}}\,dx,x,x^3\right)$$

$$= \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{3bdx}{2}}{x(a+bx)(c+dx)^{3/2}}\,dx,x,x^3\right)}{3a(bc-ad)}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-a}{x(a}\right.}{3ac(}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,d\right.}{3a^2c}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{3a^2c}$$

$$= \frac{d(bc+2ad)}{3ac(bc-ad)^2\sqrt{c+dx^3}} + \frac{b}{3a(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^2c^{3/2}}$$

Mathematica [C] time = 0.13, size = 123, normalized size = 0.72

$$\frac{\frac{b(2bc-5ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)}{a(ad-bc)} + \left(\frac{2b}{a}-\frac{2d}{c}\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{dx^3}{c}+1\right) + \frac{b}{a+bx^3}}{3a\sqrt{c+dx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b/(a + b*x^3) + (b*(2*b*c - 5*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c +
 d*x^3))/(b*c - a*d)])/(a*(-(b*c) + a*d)) + ((2*b)/a - (2*d)/c)*Hypergeomet
ric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c])/(3*a*(b*c - a*d)*Sqrt[c + d*x^3])
```

fricas [B] time = 1.76, size = 1819, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 +
 (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log
((b*d*x^3 + 2*b*c - a*d - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d))
)/(b*x^3 + a)) - 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) - 2*(a*
b^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))
/(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^
3*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
 a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*
b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(
-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b
*d*x^3 + b*c)) + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3 -
 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3
)*x^3)*sqrt(c)*log((d*x^3 - 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + (a*b^2*
c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^
3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^
2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5
*c^2*d^3)*x^3), 1/6*(4*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a
^3*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) - (2*a*b^2*c^4 - 5
*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c
^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d
 - 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a*b
^2*c^3 + 2*a^3*c*d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/
(a^3*b^2*c^5 - 2*a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3
*d^2 + a^4*b*c^2*d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 +
a^5*c^2*d^3)*x^3), 1/3*((2*a*b^2*c^4 - 5*a^2*b*c^3*d + (2*b^3*c^3*d - 5*a*b
^2*c^2*d^2)*x^6 + (2*b^3*c^4 - 3*a*b^2*c^3*d - 5*a^2*b*c^2*d^2)*x^3)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^3 + b*c)) + 2*((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^6 + a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^
3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a*b^2*c^3 + 2*a^3*c*
d^2 + (a*b^2*c^2*d + 2*a^2*b*c*d^2)*x^3)*sqrt(d*x^3 + c))/(a^3*b^2*c^5 - 2*
a^4*b*c^4*d + a^5*c^3*d^2 + (a^2*b^3*c^4*d - 2*a^3*b^2*c^3*d^2 + a^4*b*c^2*
d^3)*x^6 + (a^2*b^3*c^5 - a^3*b^2*c^4*d - a^4*b*c^3*d^2 + a^5*c^2*d^3)*x^3)
]
```

giac [A] time = 0.19, size = 226, normalized size = 1.31

$$-\frac{\left(2\,b^3c-5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)\sqrt{-b^2c+abd}}+\frac{\left(dx^3+c\right)b^2cd+2\left(dx^3+c\right)abd^2-2\,abcd^2+2\,a^2d^3}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left(\left(dx^3+c\right)^{\frac{3}{2}}b-\sqrt{dx^3+c}\,bc+\sqrt{dx^3+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/(
(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b^2*c + a*b*d)) + 1/3*((d*x^3 +
 c)*b^2*c*d + 2*(d*x^3 + c)*a*b*d^2 - 2*a*b*c*d^2 + 2*a^2*d^3)/((a*b^2*c^3
```

```
- 2*a^2*b*c^2*d + a^3*c*d^2)*((d*x^3 + c)^(3/2)*b - sqrt(d*x^3 + c)*b*c + s
qrt(d*x^3 + c)*a*d)) + 2/3*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
)
```

maple [C] time = 0.27, size = 1002, normalized size = 5.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a^2*b*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*
d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*
d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(
1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*
d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c
*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)
/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I
*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(
1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3
)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=R
ootOf(_Z^3*b+a))-1/a*b*(-1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(
a*d-b*c)^2/((x^3+c/d)*d)^(1/2)*d+1/2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^
2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2
*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*arctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x), x)
```

mupad [B] time = 12.18, size = 288, normalized size = 1.67

$$\frac{\ln\left(\frac{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3\,\left(\sqrt{d\,x^3+c}+\sqrt{c}\right)}{x^6}\right)}{3\,a^2\,c^{3/2}}+\frac{\left(\frac{{\left(2\,a\,d+b\,c\right)}^4+{\left(2\,a\,d+b\,c\right)}^2\,\left(\left(a\,d+2\,b\,c\right)\,\left(2\,a\,d+b\,c\right)-9\,a\,b\,c\,d\right)}{9\,a\,c\,{\left(2\,a\,d+b\,c\right)}^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x^3\,\left(2\,a\,d+b\,c\right)}{3\,a\,c\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)\,\sqrt{d\,x^3+c}}{b\,d\,x^6+\left(a\,d+b\,c\right)\,x^3+a\,c}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] log((((c + d*x^3)^(1/2) - c^(1/2))^3*((c + d*x^3)^(1/2) + c^(1/2)))/x^6)/(3
*a^2*c^(3/2)) + ((((2*a*d + b*c)^4 + (2*a*d + b*c)^2*((a*d + 2*b*c)*(2*a*d
```

```
+ b*c) - 9*a*b*c*d))/(9*a*c*(2*a*d + b*c)^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
) + (b*d*x^3*(2*a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(c + d
*x^3)^(1/2))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + (b^(3/2)*log((2*b*c - a*d
+ b^(1/2)*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2)*2i + b*d*x^3)/(a + b*x^3))*(5
*a*d - 2*b*c)*1i)/(6*a^2*(a*d - b*c)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.494 $\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{b(2b}{3a^2c\left(a+bx^3\right)}$$

```
[Out] 1/3*(3*a*d+4*b*c)*arctanh((d*x^3+c)^(1/2)/c^(1/2))/a^3/c^(5/2)-1/3*b^(5/2)*
(-7*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^3+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+
b*c)^(5/2)-1/3*d*(3*a^2*d^2-2*a*b*c*d+2*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(d*x^
3+c)^(1/2)-1/3*b*(-a*d+2*b*c)/a^2/c/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)^(1/2)-1/
3/a/c/x^3/(b*x^3+a)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {446, 103, 151, 152, 156, 63, 208}

$$-\frac{d\left(3a^2d^2-2abcd+2b^2c^2\right)}{3a^2c^2\sqrt{c+dx^3}\,(bc-ad)^2}-\frac{b^{5/2}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^3}}{\sqrt{bc-ad}}\right)}{3a^3(bc-ad)^{5/2}}+\frac{(3ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^3}}{\sqrt{c}}\right)}{3a^3c^{5/2}}-\frac{b(2b}{3a^2c\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(d*(2*b^2*c^2 - 2*a*b*c*d + 3*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*Sqrt[c +
d*x^3]) - (b*(2*b*c - a*d))/(3*a^2*c*(b*c - a*d)*(a + b*x^3)*Sqrt[c + d*x^3
]) - 1/(3*a*c*x^3*(a + b*x^3)*Sqrt[c + d*x^3]) + ((4*b*c + 3*a*d)*ArcTanh[S
qrt[c + d*x^3]/Sqrt[c]])/(3*a^3*c^(5/2)) - (b^(5/2)*(4*b*c - 7*a*d)*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x^3])/Sqrt[b*c - a*d]])/(3*a^3*(b*c - a*d)^(5/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
```

```
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+3ad)+\frac{5bdx}{2}}{x(a+bx)^2(c+dx)^{3/2}}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{1}{3acx^3\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{\operatorname{Subst}\left(\int \frac{1}{2}\right.}{}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

$$= -\frac{d\left(2b^2c^2-2abcd+3a^2d^2\right)}{3a^2c^2(bc-ad)^2\sqrt{c+dx^3}} - \frac{b(2bc-ad)}{3a^2c(bc-ad)\left(a+bx^3\right)\sqrt{c+dx^3}} - \frac{}{3acx^3\left(a+\right.}$$

Mathematica [C] time = 0.16, size = 189, normalized size = 0.78

$$\frac{b^2c^2x^3\left(a+bx^3\right)(4bc-7ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\left(dx^3+c\right)}{bc-ad}\right)-(ad-bc)\left(x^3\left(a+bx^3\right)\left(3a^2d^2+abcd-4b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2}\right.\right.}{3a^3c^2x^3\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (b^2*c^2*(4*b*c - 7*a*d)*x^3*(a + b*x^3)*Hypergeometric2F1[-1/2, 1, 1/2, (b
*(c + d*x^3))/(b*c - a*d)] - (-(b*c) + a*d)*(a*c*(a^2*d - 2*b^2*c*x^3 + a*b
*(-c + d*x^3)) + (-4*b^2*c^2 + a*b*c*d + 3*a^2*d^2)*x^3*(a + b*x^3)*Hyperge
ometric2F1[-1/2, 1, 1/2, 1 + (d*x^3)/c]))/(3*a^3*c^2*(b*c - a*d)^2*x^3*(a +
 b*x^3)*Sqrt[c + d*x^3])
```

fricas [B] time = 2.07, size = 2384, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d -
7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(b/(b*c -
 a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(b/(b
*c - a*d)))/(b*x^3 + a)) - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^
3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b
*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2
```

```
+ 3*a^4*c*d^3)*x^3)*sqrt(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x
^3) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2
*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^
2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*
d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a
^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2
*((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3*c^4*d - 7*a^2*
b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^3 + b*c))
 - ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (
4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6
+ (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt
(c)*log((d*x^3 + 2*sqrt(d*x^3 + c)*sqrt(c) + 2*c)/x^3) + 2*(a^2*b^2*c^4 - 2
*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c
*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3
)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9
 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b
^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3), -1/6*(2*((4*b^4*c^3*d - 5*a*b^3
*c^2*d^2 - 2*a^2*b^2*c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d -
7*a^2*b^2*c^2*d^2 + a^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2
*c^3*d - 2*a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c
)*sqrt(-c)/c) + ((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*c^5 - 3*a*b^3
*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4*d)*x^3)*sqrt
(b/(b*c - a*d))*log((b*d*x^3 + 2*b*c - a*d + 2*sqrt(d*x^3 + c)*(b*c - a*d)*
sqrt(b/(b*c - a*d)))/(b*x^3 + a)) + 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^
2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*
c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4*c*d^3)*x^3)*sqrt(d*x^3 + c))/((
a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4
*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*
d + a^6*c^4*d^2)*x^3), -1/3*(((4*b^4*c^4*d - 7*a*b^3*c^3*d^2)*x^9 + (4*b^4*
c^5 - 3*a*b^3*c^4*d - 7*a^2*b^2*c^3*d^2)*x^6 + (4*a*b^3*c^5 - 7*a^2*b^2*c^4
*d)*x^3)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^3 + c)*(b*c - a*d)*sqrt(-b/(
b*c - a*d))/(b*d*x^3 + b*c)) + ((4*b^4*c^3*d - 5*a*b^3*c^2*d^2 - 2*a^2*b^2*
c*d^3 + 3*a^3*b*d^4)*x^9 + (4*b^4*c^4 - a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 + a
^3*b*c*d^3 + 3*a^4*d^4)*x^6 + (4*a*b^3*c^4 - 5*a^2*b^2*c^3*d - 2*a^3*b*c^2*
d^2 + 3*a^4*c*d^3)*x^3)*sqrt(-c)*arctan(sqrt(d*x^3 + c)*sqrt(-c)/c) + (a^2*
b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (2*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 3*a^3*b*c*d^3)*x^6 + (2*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 3*a^4
*c*d^3)*x^3)*sqrt(d*x^3 + c))/((a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c
^3*d^3)*x^9 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
^6 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^3)]
```

giac [A]   time = 0.22, size = 367, normalized size = 1.52

$$\frac{\left(4\,b^4c-7\,ab^3d\right)\arctan\left(\frac{\sqrt{dx^3+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)\sqrt{-b^2c+abd}}-\frac{2\left(dx^3+c\right)^2b^3c^2d-2\left(dx^3+c\right)b^3c^3d-2\left(dx^3+c\right)^2ab^2cd^2+3\left(dx^3\right.}{3\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(\left(dx^3+c\right)^{\frac{5}{2}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*b^4*c - 7*a*b^3*d)*arctan(sqrt(d*x^3 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*sqrt(-b^2*c + a*b*d)) - 1/3*(2*(d*x^3
+ c)^2*b^3*c^2*d - 2*(d*x^3 + c)*b^3*c^3*d - 2*(d*x^3 + c)^2*a*b^2*c*d^2 +
3*(d*x^3 + c)*a*b^2*c^2*d^2 + 3*(d*x^3 + c)^2*a^2*b*d^3 - 7*(d*x^3 + c)*a^2
*b*c*d^3 + 2*a^2*b*c^2*d^3 + 3*(d*x^3 + c)*a^3*d^4 - 2*a^3*c*d^4)/((a^2*b^2
*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*((d*x^3 + c)^(5/2)*b - 2*(d*x^3 + c)^(3
/2)*b*c + sqrt(d*x^3 + c)*b*c^2 + (d*x^3 + c)^(3/2)*a*d - sqrt(d*x^3 + c)*a
```

```
*c*d)) - 1/3*(4*b*c + 3*a*d)*arctan(sqrt(d*x^3 + c)/sqrt(-c))/(a^3*sqrt(-c)
*c^2)
```

maple [C] time = 0.25, size = 1067, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/a^2*(-1/3*(d*x^3+c)^(1/2)/c^2/x^3-2/3/((x^3+c/d)*d)^(1/2)/c^2*d+d*arctanh
((d*x^3+c)^(1/2)/c^(1/2))/c^(5/2))+2/a^3*b^2*(-2/3/(a*d-b*c)/((x^3+c/d)*d)^
(1/2)-1/3*I*b/d^2*2^(1/2)*sum(1/(-a*d+b*c)/(a*d-b*c)*(-c*d^2)^(1/3)*(1/2*I*
(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*
((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)
*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d
)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(
-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^
(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a
*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*
(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))+1/a^2*b^2*(-1/3/(a*d-
b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)*b-2/3/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)*d+1/
2*I*b/d*2^(1/2)*sum(1/(a*d-b*c)^3*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-
c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/
d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^
(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(
1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alph
a*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1
/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*
d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(
-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2)),_alpha=RootOf(_Z^3*b+a)))-2*b/a^3*(2/3/((x^3+c/d)*d)^(1/2)/c-2/3*a
rctanh((d*x^3+c)^(1/2)/c^(1/2))/c^(3/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^4), x)
```

mupad [B] time = 19.63, size = 18847, normalized size = 78.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
```

```
[Out] (2*b*log(1/x^6))/(3*a^3*c^(3/2)) - (c + d*x^3)^(1/2)/(3*a^2*c^2*x^3) + (d*l
og(1/x^6))/(2*a^2*c^(5/2)) + (2*b*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(
c + d*x^3)^(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2))
)/(3*a^3*c^(3/2)) + (d*log(c^(3/2)*(c + d*x^3)^(1/2) - c^(1/2)*(c + d*x^3)^
```

(3/2) + d^2*x^6 + 2*c*d*x^3 + 3*c^(1/2)*d*x^3*(c + d*x^3)^(1/2)))/(2*a^2*c^(5/2)) - (b^7*c^9*x^4*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 + a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) - (5*a^9*d^7*x^4*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (3*a^2*d^2*x*(c + d*x^3)^(1/2))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) + (2*b^2*c^2*x*(c + d*x^3)^(1/2))/(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7) - (b^(7/2)*c*log((a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*2i)/(3*a^3*(a*d - b*c)^(5/2)) + (b^(5/2)*d*log((a^6*b^(15/2)*c^10*36i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^9*d*198i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^12*b^(3/2)*c^4*d^6*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^5*d^5*126i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^6*d^4*360i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^7*d^3*540i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^8*d^2*450i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^6*b^(15/2)*c^9*d*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^11*b^(5/2)*c^4*d^6*x^3*18i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^10*b^(7/2)*c^5*d^5*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^9*b^(9/2)*c^6*d^4*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (a^8*b^(11/2)*c^7*d^3*x^3*180i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (a^7*b^(13/2)*c^8*d^2*x^3*90i)/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (36*a^6*b^7*c^9*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (360*a^8*b^5*c^7*d^2*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (360*a^9*b^4*c^6*d^3*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) + (180*a^10*b^3*c^5*d^4*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (36*a^11*b^2*c^4*d^5*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)) - (180*a^7*b^6*c^8*d*(c + d*x^3)^(1/2)*(a*d - b*c)^(1/2))/(a*(a*d - b*c)^(1/2) + b*x^3*(a*d - b*c)^(1/2)))*7i)/(6*a^2*(a*d - b*c)^(5/2)) + (5*a^4*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^4*b

^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (65*a^3*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (8*b^3*c^3*x^4*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*b^3*c^4*x^4*(c + d*x^3)^(1/2))/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) - (5*a^7*c^2*d^5*x*(c + d*x^3)^(1/2))/(2*(a^5*b^4*c^9*x + a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^2*c^6*d^3*x^4 - 3*a^6*b^3*c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6*d^3*x - 3*a^7*b^2*c^7*d^2*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^4*b^5*c^8*d*x^7)) - (5*a^7*c*d^6*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^4*c^9*x + a^4*b^5*c^9*x^4 - 3*a^6*b^3*c^7*d^2*x^4 - a^7*b^2*c^6*d^3*x^4 - 3*a^6*b^3*c^6*d^3*x^7 + 2*a^7*b^2*c^5*d^4*x^7 + 2*a^8*b*c^6*d^3*x - 3*a^7*b^2*c^7*d^2*x + a^5*b^4*c^8*d*x^4 + 2*a^8*b*c^5*d^4*x^4 + a^4*b^5*c^8*d*x^7)) - (3*a^8*c^2*d^5*x*(c + d*x^3)^(1/2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b^3*c^7*d^2*x^4 - a^8*b^2*c^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^5*d^4*x^7 + 2*a^9*b*c^6*d^3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2*a^9*b*c^5*d^4*x^4 + a^5*b^5*c^8*d*x^7)) - (3*a^8*c*d^6*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^4*c^9*x + a^5*b^5*c^9*x^4 - 3*a^7*b^3*c^7*d^2*x^4 - a^8*b^2*c^6*d^3*x^4 - 3*a^7*b^3*c^6*d^3*x^7 + 2*a^8*b^2*c^5*d^4*x^7 + 2*a^9*b*c^6*d^3*x - 3*a^8*b^2*c^7*d^2*x + a^6*b^4*c^8*d*x^4 + 2*a^9*b*c^5*d^4*x^4 + a^5*b^5*c^8*d*x^7)) + (23*a^9*c^3*d^5*x*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x + a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) - (a*b^6*c^9*x*(c + d*x^3)^(1/2))/(2*(2*a^9*c^6*d^5*x + 2*a^9*c^5*d^6*x^4 + a^5*b^4*c^9*d^2*x^4 + a^6*b^3*c^8*d^3*x^4 - 3*a^7*b^2*c^7*d^4*x^4 + a^5*b^4*c^8*d^3*x^7 - 3*a^7*b^2*c^6*d^5*x^7 - 3*a^8*b*c^7*d^4*x + a^6*b^3*c^9*d^2*x - a^8*b*c^6*d^5*x^4 + 2*a^8*b*c^5*d^6*x^7)) + (3*a^2*b^6*c^9*x^4*(c + d*x^3)^(1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b^3*c^9*d^2*x^4 - 3*a^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^7*d^4*x^7 + a^7*b^3*c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9*b*c^7*d^4*x^4 + 2*a^9*b*c^6*d^5*x^7)) - (5*a^6*c^2*d^3*x*(c + d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x + a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*d*x^7)) - (5*a^6*c*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^6*b^2*c^7*x + a^5*b^3*c^7*x^4 + 2*a^7*b*c^6*d*x + 2*a^6*b^2*c^5*d^2*x^7 + 3*a^6*b^2*c^6*d*x^4 + 2*a^7*b*c^5*d^2*x^4 + a^5*b^3*c^6*d*x^7)) + (4*a^2*b^4*c^6*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (8*a*b^5*c^6*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (14*a^2*b^4*c^7*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7) - (14*a*b^5*c^7*x^4*(c + d*x^3)^(1/2))/(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7) + (a^3*b^4*c^7*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^4*b^4*c^8*x*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2 *x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (65*a^6*c^2*d^4*x*(c + d*x^3)^(1/2))/(8*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (65*a^6*c*d^5*x^4*(c + d*x^3)^(1/2))/(24*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (41*a^6*c^3*d^4*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^3*d^4*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (47*a^8*c^4*d^4*x*(c + d*x^3)^(1/2))/(6*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^3*b^2*c^5*x*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) + (47*a^3*c^2*d^2*x*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) + (38*a^3*c*d^3*x^4*(c + d*x^3)^(1/2))/(3*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (257*a^5*c^3*d^2*x*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (5*a^9*c*d^6*x*(c + d*x^3)^(1/2))/(4*(a^6*b^5*c^9*x + a^5*b^6*c^9*x^4 - 3*a^7*b^4*c^7*d^2*x^4 - a^8*b^3*c^6*d^3*x^4 + 2*a^9*b^2*c^5*d^4*x^4 - 3*a^7*b^4*c^6*d^3*x^7 + 2*a^8*b^3*c^5*d^4*x^7 - 3*a^8*b^3*c^7*d^2*x + 2*a^9*b^2*c^6*d^3*x + a^6*b^5*c^8*d*x^4 + a^5*b^6*c^8*d*x^7)) + (23*a^9*c^2*d^6*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^4*c^10*x + a^6*b^5*c^10*x^4 - 3*a^8*b^3*c^8*d^2*x^4 - a^9*b^2*c^7*d^3*x^4 - 3*a^8*b^3*c^7*d^3*x^7 + 2*a^9*b^2*c^6*d^4*x^7 + 2*a^10*b*c^7*d^3*x - 3*a^9*b^2*c^8*d^2*x + a^7*b^4*c^9*d*x^4 + 2*a^10*b*c^6*d^4*x^4 + a^6*b^5*c^9*d*x^7)) + (a^2*b^6*c^10*x*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a*b^7*c^10*x^4*(c + d*x^3)^(1/2))/(2*(2*a^10*c^7*d^5*x + 2*a^10*c^6*d^6*x^4 + a^6*b^4*c^10*d^2*x^4 + a^7*b^3*c^9*d^3*x^4 - 3*a^8*b^2*c^8*d^4*x^4 + a^6*b^4*c^9*d^3*x^7 - 3*a^8*b^2*c^7*d^5*x^7 - 3*a^9*b*c^8*d^4*x + a^7*b^3*c^10*d^2*x - a^9*b*c^7*d^5*x^4 + 2*a^9*b*c^6*d^6*x^7)) + (a^2*b^5*c^7*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (269*a^3*b^5*c^8*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (26*a^6*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (5*a^7*c^2*d^5*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (79*a^8*c^3*d^5*x^4*(c + d*x^3)^(1/2))/(12*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^

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2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x
^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (34*a^
2*b^3*c^5*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 +
a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6
*b*c^5*d^2*x^7)) - (239*a^5*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^5*b^2*c^7
*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4
+ a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (3*a^2*b^5*c^8*x*(c + d*x^3)^
(1/2))/(4*(2*a^9*c^6*d^4*x + 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^
7*b^2*c^7*d^3*x^4 + a^5*b^4*c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c
^9*d*x - 3*a^8*b*c^7*d^3*x + a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*
b*c^5*d^5*x^7)) - (3*a*b^6*c^8*x^4*(c + d*x^3)^(1/2))/(4*(2*a^9*c^6*d^4*x +
 2*a^9*c^5*d^5*x^4 + a^6*b^3*c^8*d^2*x^4 - 3*a^7*b^2*c^7*d^3*x^4 + a^5*b^4*
c^8*d^2*x^7 - 3*a^7*b^2*c^6*d^4*x^7 + a^6*b^3*c^9*d*x - 3*a^8*b*c^7*d^3*x +
 a^5*b^4*c^9*d*x^4 - a^8*b*c^6*d^4*x^4 + 2*a^8*b*c^5*d^5*x^7)) + (3*a^3*b^5
*c^9*x*(c + d*x^3)^(1/2))/(4*(2*a^10*c^7*d^4*x + 2*a^10*c^6*d^5*x^4 + a^7*b
^3*c^9*d^2*x^4 - 3*a^8*b^2*c^8*d^3*x^4 + a^6*b^4*c^9*d^2*x^7 - 3*a^8*b^2*c^
7*d^4*x^7 + a^7*b^3*c^10*d*x - 3*a^9*b*c^8*d^3*x + a^6*b^4*c^10*d*x^4 - a^9
*b*c^7*d^4*x^4 + 2*a^9*b*c^6*d^5*x^7)) + (5*a^4*c*d^3*x*(c + d*x^3)^(1/2))/
(2*(a^4*b^2*c^6*x + a^3*b^3*c^6*x^4 + 2*a^5*b*c^5*d*x + 2*a^4*b^2*c^4*d^2*x
^7 + 3*a^4*b^2*c^5*d*x^4 + 2*a^5*b*c^4*d^2*x^4 + a^3*b^3*c^5*d*x^7)) - (8*a
*b^4*c^5*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4
*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^
7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6
*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (5*a^5*c*d^4*x*(c + d*x^3)^(1/2))/(a^4*b^3
*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*
c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4
 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) + (5*a^10*c
^2*d^6*x*(c + d*x^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b
^4*c^8*d^2*x^4 - a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c
^7*d^3*x^7 + 2*a^9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d
^3*x + a^7*b^5*c^9*d*x^4 + a^6*b^6*c^9*d*x^7)) + (5*a^10*c*d^7*x^4*(c + d*x
^3)^(1/2))/(4*(a^7*b^5*c^10*x + a^6*b^6*c^10*x^4 - 3*a^8*b^4*c^8*d^2*x^4 -
a^9*b^3*c^7*d^3*x^4 + 2*a^10*b^2*c^6*d^4*x^4 - 3*a^8*b^4*c^7*d^3*x^7 + 2*a^
9*b^3*c^6*d^4*x^7 - 3*a^9*b^3*c^8*d^2*x + 2*a^10*b^2*c^7*d^3*x + a^7*b^5*c^
9*d*x^4 + a^6*b^6*c^9*d*x^7)) - (11*a*b^2*c^3*x*(c + d*x^3)^(1/2))/(3*(a^3*
b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4
*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (14*a*b^2*c^4*x*(c + d
*x^3)^(1/2))/(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5
*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7) + (11*a
*b*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3
*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*
c^2*d^2*x^7)) - (143*a^3*c*d^2*x*(c + d*x^3)^(1/2))/(24*(a^3*b^2*c^5*x^4 +
2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b
^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (22*b^2*c*d*x^4*(c + d*x^3)^(1/2))/(
3*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^
3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) + (11*a^3*b^2*c^3
*d^2*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7
*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 -
3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x
^7 + 2*a^6*b*c^3*d^4*x^7)) - (31*a^4*b^2*c^4*d^2*x*(c + d*x^3)^(1/2))/(2*(a
^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^
6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7
*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (2
8*a^2*b^4*c^5*d*x^4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x
+ a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c
^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^
4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (29*a^5*b*c^2*d^4*x^4*(c + d*x^3)
^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d
^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x
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+ a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d ^4*x^7)) + (23*a^4*b^2*c^5*d^2*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a ^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6 *d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) + (7*a^2*b^4*c^6*d*x^4 *(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7* b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2 *a^7*b*c^5*d^4*x^7)) - (7*a^5*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(4*(a^5*b^3* c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c ^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^5*b^ 2*c^5*d^2*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^ 4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x ^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^ 8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^3*b^4*c^6*d*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3 *c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^6*b*c^3*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3 *x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^ 2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (23*a^6*b^2*c^6*d^2*x*(c + d*x ^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^ 10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^ 8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9 *b*c^6*d^4*x^7)) - (31*a^4*b^4*c^7*d*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^1 0*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2* c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (21*a^7* b*c^4*d^4*x^4*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^ 6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7 *d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6* b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (209*a^2*b*c^2*d^2*x^4*(c + d*x^3)^ (1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d *x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7)) - (89*a^ 3*b^2*c^4*d*x^4*(c + d*x^3)^(1/2))/(6*(a^5*b^2*c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^4 + a^5*b^2*c^6*d*x^7 + 2*a ^6*b*c^5*d^2*x^7)) - (109*a^4*b*c^3*d^2*x^4*(c + d*x^3)^(1/2))/(12*(a^5*b^2 *c^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d* x^4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) - (33*a^2*b*c^2*d*x*(c + d* x^3)^(1/2))/(2*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c ^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) + (3 29*a^2*b*c^3*d*x*(c + d*x^3)^(1/2))/(12*(a^3*b^2*c^6*x^4 + 2*a^5*c^4*d^2*x^ 4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x^4 + a^3*b^2*c^5*d*x^7 + 2 *a^4*b*c^4*d^2*x^7)) - (205*a^4*b*c^4*d*x*(c + d*x^3)^(1/2))/(12*(a^5*b^2*c ^7*x^4 + 2*a^7*c^5*d^2*x^4 + a^6*b*c^7*x + 2*a^7*c^6*d*x + 3*a^6*b*c^6*d*x^ 4 + a^5*b^2*c^6*d*x^7 + 2*a^6*b*c^5*d^2*x^7)) + (6*a^2*b^3*c^3*d^2*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c ^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^ 2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c ^3*d^4*x^7) - (a^3*b^2*c^2*d^3*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^ 7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4* d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (19*a^3*b^3*c^4*d^2*x^ 4*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7 *b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) + (10*a^4*b^2*c^3*d^3*x^4*(c + d*x^3)^(1/2))/(3*(a^5* b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (a^3*b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(2*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (97*a^4*b^2*c^4*d^3*x^4*(c + d*x^3)^(1/2))/(6*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^4*b^3*c^5*d^2*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^5*b^2*c^4*d^3*x^4*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) + (31*a^5*b^3*c^6*d^2*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (415*a^6*b^2*c^5*d^3*x^4*(c + d*x^3)^(1/2))/(24*(a^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) + (11*a*b*c*d*x*(c + d*x^3)^(1/2))/(2*(a^2*b^2*c^4*x^4 + 2*a^4*c^2*d^2*x^4 + a^3*b*c^4*x + 2*a^4*c^3*d*x + 3*a^3*b*c^3*d*x^4 + a^2*b^2*c^3*d*x^7 + 2*a^3*b*c^2*d^2*x^7)) - (10*a^2*b^3*c^4*d*x*(c + d*x^3)^(1/2))/(3*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (19*a^4*b*c^2*d^3*x*(c + d*x^3)^(1/2))/(6*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) - (8*a*b^4*c^4*d*x^4*(c + d*x^3)^(1/2))/(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7) - (15*a^4*b*c*d^4*x^4*(c + d*x^3)^(1/2))/(2*(a^4*b^3*c^7*x + 2*a^7*c^4*d^3*x + a^3*b^4*c^7*x^4 + 2*a^7*c^3*d^4*x^4 - 3*a^5*b^2*c^5*d^2*x^4 - 3*a^5*b^2*c^4*d^3*x^7 - 3*a^6*b*c^5*d^2*x + a^4*b^3*c^6*d*x^4 - a^6*b*c^4*d^3*x^4 + a^3*b^4*c^6*d*x^7 + 2*a^6*b*c^3*d^4*x^7)) + (12*a^3*b^3*c^5*d*x*(c + d*x^3)^(1/2))/(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7) + (51*a^5*b*c^3*d^3*x*(c + d*x^3)^(1/2))/(4*(a^5*b^3*c^8*x + 2*a^8*c^5*d^3*x + a^4*b^4*c^8*x^4 + 2*a^8*c^4*d^4*x^4 - 3*a^6*b^2*c^6*d^2*x^4 - 3*a^6*b^2*c^5*d^3*x^7 - 3*a^7*b*c^6*d^2*x + a^5*b^3*c^7*d*x^4 - a^7*b*c^5*d^3*x^4 + a^4*b^4*c^7*d*x^7 + 2*a^7*b*c^4*d^4*x^7)) - (14*a^3*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(3*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (169*a^5*b*c^4*d^3*x*(c + d*x^3)^(1/2))/(12*(a^5*b^3*c^9*x + 2*a^8*c^6*d^3*x + a^4*b^4*c^9*x^4 + 2*a^8*c^5*d^4*x^4 - 3*a^6*b^2*c^7*d^2*x^4 - 3*a^6*b^2*c^6*d^3*x^7 - 3*a^7*b*c^7*d^2*x + a^5*b^3*c^8*d*x^4 - a^7*b*c^6*d^3*x^4 + a^4*b^4*c^8*d*x^7 + 2*a^7*b*c^5*d^4*x^7)) - (9*a^4*b^3*c^6*d*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (9*a^6*b*

```
c^4*d^3*x*(c + d*x^3)^(1/2))/(8*(a^6*b^3*c^9*x + 2*a^9*c^6*d^3*x + a^5*b^4*
c^9*x^4 + 2*a^9*c^5*d^4*x^4 - 3*a^7*b^2*c^7*d^2*x^4 - 3*a^7*b^2*c^6*d^3*x^7
 - 3*a^8*b*c^7*d^2*x + a^6*b^3*c^8*d*x^4 - a^8*b*c^6*d^3*x^4 + a^5*b^4*c^8*
d*x^7 + 2*a^8*b*c^5*d^4*x^7)) - (7*a^5*b^3*c^7*d*x*(c + d*x^3)^(1/2))/(8*(a
^7*b^3*c^10*x + 2*a^10*c^7*d^3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 -
3*a^8*b^2*c^8*d^2*x^4 - 3*a^8*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3
*c^9*d*x^4 - a^9*b*c^7*d^3*x^4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7))
+ (77*a^7*b*c^5*d^3*x*(c + d*x^3)^(1/2))/(8*(a^7*b^3*c^10*x + 2*a^10*c^7*d^
3*x + a^6*b^4*c^10*x^4 + 2*a^10*c^6*d^4*x^4 - 3*a^8*b^2*c^8*d^2*x^4 - 3*a^8
*b^2*c^7*d^3*x^7 - 3*a^9*b*c^8*d^2*x + a^7*b^3*c^9*d*x^4 - a^9*b*c^7*d^3*x^
4 + a^6*b^4*c^9*d*x^7 + 2*a^9*b*c^6*d^4*x^7)) - (41*a*b^2*c^2*d*x^4*(c + d*
x^3)^(1/2))/(3*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^4 + a^4*b*c^5*x + 2*a^5*c
^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2*a^4*b*c^3*d^2*x^7)) - (7
1*a^2*b*c*d^2*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*c^5*x^4 + 2*a^5*c^3*d^2*x^
4 + a^4*b*c^5*x + 2*a^5*c^4*d*x + 3*a^4*b*c^4*d*x^4 + a^3*b^2*c^4*d*x^7 + 2
*a^4*b*c^3*d^2*x^7)) + (133*a*b^2*c^3*d*x^4*(c + d*x^3)^(1/2))/(6*(a^3*b^2*
c^6*x^4 + 2*a^5*c^4*d^2*x^4 + a^4*b*c^6*x + 2*a^5*c^5*d*x + 3*a^4*b*c^5*d*x
^4 + a^3*b^2*c^5*d*x^7 + 2*a^4*b*c^4*d^2*x^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^2 \left(c + dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.495 $\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,2,3/2,7/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{x^4\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)])/(4*a^2*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x^3}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^4\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};2,\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4a^2c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.36, size = 381, normalized size = 5.69

$$-\frac{x^4\left(\left(3bdx^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+8a\left(2ad+b\left(c+3dx^3\right)\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right.\right.\right.}{8a\left(a+bx^3\right)\sqrt{c+dx^3}\left(bc-ad\right)^2\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/8*(x^4*(-8*a*b*c*d*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
]*(8*a + (a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^
3)/c), -((b*x^3)/a)]) + (8*a*(2*a*d + b*(c + 3*d*x^3)) + 3*b*d*x^3*(a + b*x
^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/
a)])*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*Ap
pellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))/(a*(b*c - a*d)^2*(a
 + b*x^3)*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c),
-((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*
x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.34, size = 1593, normalized size = 23.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/b*(2/3*d/c*x/(a*d-b*c)/((x^3+c/d)*d)^(1/2)-2/9*I/c/(a*d-b*c)*3^(1/2)*(-c*
d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/
2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2
*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1
/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I*b/d^2*2^(1/2)*sum(1/(
a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(
-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2
)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)
^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha
^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*
(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1
/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(
2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*
3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(
1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alph
a=RootOf(_Z^3*b+a)))-a/b*(1/3/a*b^2/(a*d-b*c)^2*x*(d*x^3+c)^(1/2)/(b*x^3+a)
```

```
+2/3*d^2/c*x/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)-2/3*I*(1/6*b*d/(a*d-b*c)^2/a+1
/3/c*d^2/(a*d-b*c)^2)*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)
^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I
*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(
1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3
)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1
/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d
)^(1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c
*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d
^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d
^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/
d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)
*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)
^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c
*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1
/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^
2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/
3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)^2\,\left(d\,x^3+c\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x^3/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(a + bx^{3}\right)^{2} \left(c + dx^{3}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x**3/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.496 $\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,2,3/2,5/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(
d*x^3+c)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 2, 3/2, 5/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*a^2*c*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{x}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x^2\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};2,\frac{3}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.30, size = 216, normalized size = 3.22

$$-\frac{x^2\left(5\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(a^2d^2+6abcd-b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-10a\left(2a^2d^2+2abd^2x^3+b^2c\left(c+d\right.\right.\right.}{30a^2c\left(a+bx^3\right)\sqrt{c+dx^3}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -1/30*(x^2*(-10*a*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3)) + 5*(-(b^
2*c^2) + 6*a*b*c*d + a^2*d^2)*(a + b*x^3)*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + b*d*(b*c + 2*a*d)*x^3*(a + b*x^
3)*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a
)]))/(a^2*c*(b*c - a*d)^2*(a + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)^2 \left(dx^3 + c\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.35, size = 986, normalized size = 14.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/3/a*b^2/(a*d-b*c)^2*x^2*(d*x^3+c)^(1/2)/(b*x^3+a)+2/3*d^2/c*x^2/(a*d-b*c)
^2/((x^3+c/d)*d)^(1/2)-2/3*I*(-1/6/(a*d-b*c)^2/a*b*d-1/3/(a*d-b*c)^2/c*d^2)
*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d
^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(
1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*
3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/
(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/
2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*sum((11*a*d
-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^
(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*
(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(
-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2
*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3
^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)
^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c
```

```
*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)
),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^3+a)^2(dx^3+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(b\,x^3+a)^2\,(d\,x^3+c)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(x/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^3)^2\,(c+dx^3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(x/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.497 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,2,3/2,4/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/(d*x^3+
c)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{430, 429\}$

$$\frac{x\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 2, 3/2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/(a^2*c*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= \frac{x\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};2,\frac{3}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.55, size = 381, normalized size = 6.15

$$\frac{x\left(\frac{a\left(64ac\left(3a^2d^2+2abd\left(dx^3-3c\right)+b^2c\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(2a^2d^2+2abd^2x^3+b^2c\left(c+dx^3\right)\right)\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1\right.\right.}{\left(a+bx^3\right)\left(8acF_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)\right)}\right)}{24a^2c\sqrt{c+dx^3}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (x*(b*d*(b*c + 2*a*d)*x^3*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1, 7/3, -(
(d*x^3)/c), -((b*x^3)/a)] + (a*(64*a*c*(3*a^2*d^2 + 2*a*b*d*(-3*c + d*x^3)
+ b^2*c*(3*c + d*x^3))*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a
)] - 24*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + b^2*c*(c + d*x^3))*(2*b*c*AppellF1
[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1,
7/3, -((d*x^3)/c), -((b*x^3)/a)])))/((a + b*x^3)*(8*a*c*AppellF1[1/3, 1/2,
1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/
3, -((d*x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c
), -((b*x^3)/a)])))))/(24*a^2*c*(b*c - a*d)^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^2 \left(dx^3 + c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

maple [C] time = 0.26, size = 830, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] 1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)/a*b^2*x+2/3/(a*d-b*c)^2/((x^3+c/d
)*d)^(1/2)/c*d^2*x-2/3*I*(1/6/(a*d-b*c)^2/a*b*d+1/3/(a*d-b*c)^2/c*d^2)*3^(1
/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(
1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1
/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)
^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(
-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/18*I/a/d^2*b*2^
(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c
*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*
I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)
/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)
^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(
1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c
*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c
*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)
```

```
*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2
)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^3+a\right)^2\,\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/((a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/((a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.498 $\int \frac{1}{x^2(a+bx^3)^2(c+dx^3)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,2,3/2,2/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x/(d*x^
3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -((Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 2, 3/2, 2/3, -((b*x^3)/a), -((d*x^3)/
c)])/(a^2*c*x*Sqrt[c + d*x^3]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^2\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{1}{3};2,\frac{3}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2cx\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.43, size = 308, normalized size = 4.74

$$2bdx^6\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(5a^2d^2-6abcd+4b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+5x^3\left(a+bx^3\right)\sqrt{\frac{dx^3}{c}+1}\left(5a^3d^3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-20*a*(4*b^3*c^2*x^3*(c + d*x^3) + a^3*d^2*(3*c + 5*d*x^3) + 3*a*b^2*c*(c^
2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*d*(-6*c^2 - 3*c*d*x^3 + 5*d^2*x^6)) + 5*(-
8*b^3*c^3 + 21*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 5*a^3*d^3)*x^3*(a + b*x^3)*Sqr
t[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^3)/c), -((b*x^3)/a)] + 2
*b*d*(4*b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^6*(a + b*x^3)*Sqrt[1 + (d*x^3)/c
]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^3)/c), -((b*x^3)/a)])/(60*a^3*c^2*(b*c
- a*d)^2*x*(a + b*x^3)*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^2 \left(dx^3 + c\right)^{\frac{3}{2}} x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

maple [C] time = 0.30, size = 2383, normalized size = 36.66

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a^2*b*(2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)/c*d*x^2+2/9*I/c/(a*d-b*c)*3^(1/
2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d
)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3
)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)
*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1
/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*
(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)
^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1
/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/3*I*b/d^2*2^(1/2)*sum(1/(a*d-b*c)^2/_alp
ha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)
/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)
*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(
1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)
^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-
c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3
)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*
(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
```

```
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a
)))-1/a*b*(1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)/a*b^2*x^2+2/3/(a*d-b*c
)^2/((x^3+c/d)*d)^(1/2)/c*d^2*x^2-2/3*I*(-1/6/(a*d-b*c)^2/a*b*d-1/3/(a*d-b*
c)^2/c*d^2)*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/
2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/
(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-
c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1
/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)
*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1
/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*
d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+(-c*d^2)^(1/3)/d*Ell
ipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)
^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))+1/18*I/a/d^2*b*2^(1/2)*
sum((11*a*d-2*b*c)/(a*d-b*c)^3/_alpha*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2
)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(
I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+
c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_
alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*
(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1
/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d
-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*
3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/
d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a))+1/a^2*(-2/3/((x^3+c/d)*d)^(1/2)/c^2*
d*x^2-(d*x^3+c)^(1/2)/c^2/x-5/9*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*
d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2
)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)
/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1
/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*((-3/2*(-c*d^2)^(1/3)/d+1/2*I*3
^(1/2)*(-c*d^2)^(1/3)/d)*EllipticE(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1
/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(
-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/
2))+(-c*d^2)^(1/3)/d*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I
*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d
^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)))
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^2\,{\left(d\,x^3+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^2*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+bx^3\right)^2 \left(c+dx^3\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.499 $\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,2,3/2,1/3,-b*x^3/a,-d*x^3/c)*(1+d*x^3/c)^(1/2)/a^2/c/x^2
/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^3}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] -(Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 2, 3/2, 1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(2*a^2*c*x^2*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^3}{c}}\int \frac{1}{x^3\left(a+bx^3\right)^2\left(1+\frac{dx^3}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^3}}\\ &= -\frac{\sqrt{1+\frac{dx^3}{c}}\,F_1\left(-\frac{2}{3};2,\frac{3}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2a^2cx^2\sqrt{c+dx^3}}\end{aligned}$$

Mathematica [B] time = 0.96, size = 515, normalized size = 7.69

$$\frac{a\left(32ac\left(3a^3d^2\left(2c+7dx^3\right)+2a^2bd\left(-6c^2-6cdx^3+7d^2x^6\right)+3ab^2c\left(2c^2-13cdx^3-4d^2x^6\right)+10b^3c^2x^3\left(3c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-24x^3\left(a^3d^2\left(3c+7dx\right.\right.\right.}{\left(a+bx^3\right)\left(3x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{2},2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)+adF_1\left(\frac{4}{3};\frac{3}{2},1;\frac{7}{3}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x]
```

```
[Out] (-(b*d*(5*b^2*c^2 - 6*a*b*c*d + 7*a^2*d^2)*x^6*Sqrt[1 + (d*x^3)/c]*AppellF1
[4/3, 1/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]) + (a*(32*a*c*(10*b^3*c^2*x^
3*(3*c + d*x^3) + 3*a^3*d^2*(2*c + 7*d*x^3) + 3*a*b^2*c*(2*c^2 - 13*c*d*x^3
 - 4*d^2*x^6) + 2*a^2*b*d*(-6*c^2 - 6*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)] - 24*x^3*(5*b^3*c^2*x^3*(c + d*x^3)
 + a^3*d^2*(3*c + 7*d*x^3) + 3*a*b^2*c*(c^2 - c*d*x^3 - 2*d^2*x^6) + a^2*b*
d*(-6*c^2 - 3*c*d*x^3 + 7*d^2*x^6))*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*
x^3)/c), -((b*x^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*
x^3)/a)])))/((a + b*x^3)*(-8*a*c*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^3)/c), -
((b*x^3)/a)] + 3*x^3*(2*b*c*AppellF1[4/3, 1/2, 2, 7/3, -((d*x^3)/c), -((b*x
^3)/a)] + a*d*AppellF1[4/3, 3/2, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))))/(4
8*a^3*c^2*(b*c - a*d)^2*x^2*Sqrt[c + d*x^3])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2 \left(dx^3+c\right)^{\frac{3}{2}} x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

maple [C] time = 0.26, size = 1919, normalized size = 28.64

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x)
```

```
[Out] -1/a^2*b*(2/3/(a*d-b*c)/((x^3+c/d)*d)^(1/2)/c*d*x-2/9*I/c/(a*d-b*c)*3^(1/2)
*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*
3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*
3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*E
llipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^
2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2))+1/3*I*b/d^2*2^(1/2)*su
m(1/(a*d-b*c)^2/_alpha^2*(-c*d^2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1
/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-
c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c
*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_
alpha^2*d^2+I*(-c*d^2)^(1/3)*3^(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(
1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^
2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),
1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(
```

```
2/3)*3^(1/2)*_alpha-3*(-c*d^2)^(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d
^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),
_alpha=RootOf(_Z^3*b+a)))+1/a^2*(-1/2*(d*x^3+c)^(1/2)/c^2/x^2-2/3/((x^3+c/d
)*d)^(1/2)/c^2*d*x+7/18*I/c^2*3^(1/2)*(-c*d^2)^(1/3)*(I*(x+1/2*(-c*d^2)^(1/
3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-
c*d^2)^(1/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/
2)*(-I*(x+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*
d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2
)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(
I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3
)/d)/d)^(1/2)))-1/a*b*(1/3/(a*d-b*c)^2*(d*x^3+c)^(1/2)/(b*x^3+a)/a*b^2*x+2/
3/(a*d-b*c)^2/((x^3+c/d)*d)^(1/2)/c*d^2*x-2/3*I*(1/6/(a*d-b*c)^2/a*b*d+1/3/
(a*d-b*c)^2/c*d^2)*3^(1/2)*(-c*d^2)^(1/3)/d*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*
I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1
/3)/d)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d))^(1/2)*(-I*(x
+1/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3
)*d)^(1/2)/(d*x^3+c)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d
-1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),(I*3^(1/2)
*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(
1/2))+1/18*I/a/d^2*b*2^(1/2)*sum((13*a*d-4*b*c)/(a*d-b*c)^3/_alpha^2*(-c*d^
2)^(1/3)*(1/2*I*(2*x+(-I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/(-c*d^2)
^(1/3)*d)^(1/2)*((x-(-c*d^2)^(1/3)/d)/(-3*(-c*d^2)^(1/3)+I*3^(1/2)*(-c*d^2)
^(1/3))*d)^(1/2)*(-1/2*I*(2*x+(I*3^(1/2)*(-c*d^2)^(1/3)+(-c*d^2)^(1/3))/d)/
(-c*d^2)^(1/3)*d)^(1/2)/(d*x^3+c)^(1/2)*(2*_alpha^2*d^2+I*(-c*d^2)^(1/3)*3^
(1/2)*_alpha*d-(-c*d^2)^(1/3)*_alpha*d-I*3^(1/2)*(-c*d^2)^(2/3)-(-c*d^2)^(2
/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-c*d^2)^(1/3)/d-1/2*I*3^(1/2)*(-c*d^
2)^(1/3)/d)*3^(1/2)/(-c*d^2)^(1/3)*d)^(1/2),1/2*(2*I*(-c*d^2)^(1/3)*3^(1/2)
*_alpha^2*d+I*3^(1/2)*c*d-3*c*d-I*(-c*d^2)^(2/3)*3^(1/2)*_alpha-3*(-c*d^2)^
(2/3)*_alpha)/(a*d-b*c)*b/d,(I*3^(1/2)*(-c*d^2)^(1/3)/(-3/2*(-c*d^2)^(1/3)/
d+1/2*I*3^(1/2)*(-c*d^2)^(1/3)/d)/d)^(1/2)),_alpha=RootOf(_Z^3*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^2\left(dx^3+c\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^2/(d*x^3+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^2*(d*x^3 + c)^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^2\left(d\,x^3+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^2*(c + d*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^3\right)^2\left(c+dx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**2/(d*x**3+c)**(3/2),x)
[Out] Integral(1/(x**3*(a + b*x**3)**2*(c + d*x**3)**(3/2)), x)
```

3.500 $\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=134

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(7/2)/b/e/(23+2*m)-a^2*(2*a*B*(1+m)-A*b*(23+2*m))
*(e*x)^(1+m)*hypergeom([-5/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1
/2)/b/e/(1+m)/(23+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{7/2}(ex)^{m+1}}{be(2m+23)}-\frac{a^2\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+23))\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+23)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(7/2))/(b*e*(23 + 2*m)) - (a^2*(2*a*B*(1 + m
) - A*b*(23 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-5/2, (
1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(23 + 2*m)*Sqrt[1 + (b*x^3
)/a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\int (ex)^m \left(a+bx^3\right)^{5/2} d}{b\left(\frac{23}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{\left(a^2\left(aB(1+m)-Ab\left(\frac{23}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^{m}}{b\left(\frac{23}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{7/2}}{be(23+2m)} - \frac{a^2(2aB(1+m)-Ab(23+2m))(ex)^{1+m}\sqrt{a+bx^3}\ 2}{be(1+m)(23+2m)\sqrt{1+\frac{bx}{a}}}$$

Mathematica [A] time = 0.12, size = 113, normalized size = 0.84

$$\frac{a^2x\sqrt{a+bx^3}\,(ex)^m\left(A(m+4)\,{}_2F_1\left(-\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(-\frac{5}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^2*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-5/2, (1 + m)/3
, (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-5/2, (4 + m)/
3, (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2x^9+\left(2\,Bab+Ab^2\right)x^6+\left(Ba^2+2\,Aab\right)x^3+Aa^2\right)\sqrt{bx^3+a}\,(ex)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{5}{2}}\left(B\,x^3+A\right)(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3 + A\right)\,(e\,x)^m\,\left(b\,x^3 + a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(5/2), x)
```

sympy [C] time = 44.37, size = 388, normalized size = 2.90

$$\frac{Aa^{\frac{5}{2}}e^m x x^m \Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{2Aa^{\frac{3}{2}}be^m x^4 x^m \Gamma\left(\frac{m}{3} + \frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3} + \frac{4}{3} \\ \frac{m}{3} + \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3} + \frac{7}{3}\right)} + \frac{A\sqrt{a}\,b^2 e^m x^7 x^m}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] A*a**(5/2)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + 2*A*a**(3/2)*b*e**m*x*
*4*x**m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_
polar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + A*sqrt(a)*b**2*e**m*x**7*x**m*gamma(m
/3 + 7/3)*hyper((-1/2, m/3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(m/3 + 10/3)) + B*a**(5/2)*e**m*x**4*x**m*gamma(m/3 + 4/3)*hyper((
-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 7/
3)) + 2*B*a**(3/2)*b*e**m*x**7*x**m*gamma(m/3 + 7/3)*hyper((-1/2, m/3 + 7/3
), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3)) + B*sqrt(
a)*b**2*e**m*x**10*x**m*gamma(m/3 + 10/3)*hyper((-1/2, m/3 + 10/3), (m/3 +
13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 13/3))
```

3.501 $\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=132

$$\frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(5/2)/b/e/(17+2*m)-a*(2*a*B*(1+m)-A*b*(17+2*m))*(
e*x)^(1+m)*hypergeom([-3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2
)/b/e/(1+m)/(17+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{5/2}(ex)^{m+1}}{be(2m+17)}-\frac{a\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+17))\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+17)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(5/2))/(b*e*(17 + 2*m)) - (a*(2*a*B*(1 + m)
- A*b*(17 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-3/2, (1
+ m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(17 + 2*m)*Sqrt[1 + (b*x^3)/
a])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\int (ex)^m \left(a+bx^3\right)^{3/2}}{b\left(\frac{17}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{\left(a\left(aB(1+m)-Ab\left(\frac{17}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex}{b\left(\frac{17}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{5/2}}{be(17+2m)} - \frac{a(2aB(1+m)-Ab(17+2m))(ex)^{1+m}\sqrt{a+bx^3}}{be(1+m)(17+2m)\sqrt{1+}}$$

Mathematica [A] time = 0.09, size = 111, normalized size = 0.84

$$\frac{ax\sqrt{a+bx^3}\,(ex)^m\left(A(m+4)\,{}_2F_1\left(-\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(-\frac{3}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^3)^(3/2)*(A + B*x^3),x]
[Out] (a*x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-3/2, (1 + m)/3,
(4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\,(ex)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \left(b\,x^3+a\right)^{\frac{3}{2}}\left(B\,x^3+A\right)(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
[Out] int((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3 + A\right)\,(e\,x)^m\,\left(b\,x^3 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(3/2), x)
```

sympy [C] time = 21.29, size = 252, normalized size = 1.91

$$\frac{Aa^{\frac{3}{2}}e^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{A\sqrt{a}\, b e^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)} + \frac{Ba^{\frac{3}{2}}e^m x^4 x^m \Gamma\left(\frac{m}{3}+\right.}{3\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3
,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + A*sqrt(a)*b*e**m*x**4*
x**m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(m/3 + 7/3)) + B*a**(3/2)*e**m*x**4*x**m*gamma(m/3 + 4/
3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(m/3 + 7/3)) + B*sqrt(a)*b*e**m*x**7*x**m*gamma(m/3 + 7/3)*hyper((-1/2, m/
3 + 7/3), (m/3 + 10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 10/3))
```

3.502 $\int (ex)^m \sqrt{a+bx^3} \left(A+Bx^3\right) dx$

Optimal. Leaf size=131

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(3/2)/b/e/(11+2*m)-(2*a*B*(1+m)-A*b*(11+2*m))*(e*
x)^(1+m)*hypergeom([-1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(b*x^3+a)^(1/2)/
b/e/(1+m)/(11+2*m)/(1+b*x^3/a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\left(a+bx^3\right)^{3/2}(ex)^{m+1}}{be(2m+11)}-\frac{\sqrt{a+bx^3}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+11))\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+11)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (2*B*(e*x)^(1 + m)*(a + b*x^3)^(3/2))/(b*e*(11 + 2*m)) - ((2*a*B*(1 + m) -
A*b*(11 + 2*m))*(e*x)^(1 + m)*Sqrt[a + b*x^3]*Hypergeometric2F1[-1/2, (1 +
m)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(11 + 2*m)*Sqrt[1 + (b*x^3)/a]
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^m \sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\int (ex)^m\sqrt{a+bx^3}\,dx}{b\left(\frac{11}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{11}{2}+m\right)\right)\sqrt{a+bx^3}\right)\int (ex)^m\sqrt{1+}}{b\left(\frac{11}{2}+m\right)\sqrt{1+\frac{bx^3}{a}}}$$

$$= \frac{2B(ex)^{1+m}\left(a+bx^3\right)^{3/2}}{be(11+2m)} - \frac{(2aB(1+m)-Ab(11+2m))(ex)^{1+m}\sqrt{a+bx^3}\,{}_2F_1\left(-\right.}{be(1+m)(11+2m)\sqrt{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.09, size = 110, normalized size = 0.84

$$\frac{x\sqrt{a+bx^3}(ex)^m\left(A(m+4)\,{}_2F_1\left(-\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(-\frac{1}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^m*Sqrt[a + b*x^3]*(A*(4 + m)*Hypergeometric2F1[-1/2, (1 + m)/3, (4
 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[-1/2, (4 + m)/3, (
7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.21, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \sqrt{b\,x^3+a}\,\left(B\,x^3+A\right)(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

```
[Out] int((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^3+a)^(1/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3 + A\right)\,(e\,x)^m\,\sqrt{b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^m*(a + b*x^3)^(1/2), x)
```

sympy [C] time = 5.81, size = 122, normalized size = 0.93

$$\frac{A\sqrt{a}\,e^m x x^m \Gamma\left(\frac{m}{3}+\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{1}{3} \\ \frac{m}{3}+\frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{4}{3}\right)} + \frac{B\sqrt{a}\,e^m x^4 x^m \Gamma\left(\frac{m}{3}+\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{m}{3}+\frac{4}{3} \\ \frac{m}{3}+\frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{m}{3}+\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**3+a)**(1/2)*(B*x**3+A),x)
```

```
[Out] A*sqrt(a)*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((-1/2, m/3 + 1/3), (m/3 + 4/3,
), b*x**3*exp_polar(I*pi)/a)/(3*gamma(m/3 + 4/3)) + B*sqrt(a)*e**m*x**4*x**
m*gamma(m/3 + 4/3)*hyper((-1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(
I*pi)/a)/(3*gamma(m/3 + 7/3))
```

3.503 $\int \frac{(ex)^m(A+Bx^3)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=131

$$\frac{2B\sqrt{a+bx^3}\,(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

```
[Out] 2*B*(e*x)^(1+m)*(b*x^3+a)^(1/2)/b/e/(5+2*m)-(2*a*B*(1+m)-A*b*(5+2*m))*(e*x)
^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)^(1/2)/b
/e/(1+m)/(5+2*m)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{2B\sqrt{a+bx^3}\,(ex)^{m+1}}{be(2m+5)}-\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)-Ab(2m+5))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{be(m+1)(2m+5)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (2*B*(e*x)^(1 + m)*Sqrt[a + b*x^3])/(b*e*(5 + 2*m)) - ((2*a*B*(1 + m) - A*b
*(5 + 2*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1 + m
)/3, (4 + m)/3, -((b*x^3)/a)])/(b*e*(1 + m)*(5 + 2*m)*Sqrt[a + b*x^3])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m \left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{b\left(\frac{5}{2}+m\right)}$$

$$= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{\left(\left(aB(1+m)-Ab\left(\frac{5}{2}+m\right)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{b\left(\frac{5}{2}+m\right)\sqrt{a+bx^3}}$$

$$= \frac{2B(ex)^{1+m}\sqrt{a+bx^3}}{be(5+2m)} - \frac{(2aB(1+m)-Ab(5+2m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3}\right.}{be(1+m)(5+2m)\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 0.84

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,(ex)^m\left(A(m+4)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[1/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[1/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/((1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

fricas [F] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)(ex)^m}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^m}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(B\,x^3+A\right)(ex)^m}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^m/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3 + A\right)\,(e\,x)^m}{\sqrt{b\,x^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(1/2), x)
```

sympy [C] time = 4.87, size = 119, normalized size = 0.91

$$\frac{Ae^m x x^m \Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{Be^m x^4 x^m \Gamma\left(\frac{m}{3} + \frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{m}{3} + \frac{4}{3} \\ \frac{m}{3} + \frac{7}{3} \end{matrix}\right| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{m}{3} + \frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(1/2),x)
[Out] A*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((1/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3
*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(m/3 + 4/3)) + B*e**m*x**4*x**m*gamma(m
/3 + 4/3)*hyper((1/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/(
3*sqrt(a)*gamma(m/3 + 7/3))
```

3.504 $\int \frac{(ex)^m\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(1/2)+1/3*(2*a*B*(1+m)+A*(-2*b*m+
b))*(e*x)^(1+m)*hypergeom([1/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a
)^(1/2)/a/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+A(b-2bm))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{3abe(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*a*B*(1 + m) +
 A*(b - 2*b*m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, (1
 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(3*a*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))\int \frac{(ex)^m}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{\left((2aB(1+m)+A(b-2bm))\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{bx^3}{a}}}\,dx}{3ab\sqrt{a+bx^3}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{3abe\sqrt{a+bx^3}} + \frac{(2aB(1+m)+A(b-2bm))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{3};\frac{4+m}{3};-\frac{b}{}\right.}{3abe(1+m)\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.11, size = 113, normalized size = 0.85

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,(ex)^m\left(A(m+4)\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{3}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{a(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[3/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[3/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

fricas [F] time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}\,(ex)^m}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^2*x^6 + 2*a*b*x^3 + a^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(B\,x^3+A)(ex)^m}{(b\,x^3+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^m}{(bx^3 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3 + A)\,(e\,x)^m}{(b\,x^3 + a)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(3/2), x)
```

sympy [C] time = 88.24, size = 119, normalized size = 0.89

$$\frac{Ae^m x x^m \Gamma\left(\frac{m}{3} + \frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{3} + \frac{1}{3} \\ \frac{m}{3} + \frac{4}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3} + \frac{4}{3}\right)} + \frac{Be^m x^4 x^m \Gamma\left(\frac{m}{3} + \frac{4}{3}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{3} + \frac{4}{3} \\ \frac{m}{3} + \frac{7}{3} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\Gamma\left(\frac{m}{3} + \frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(3/2),x)
[Out] A*e**m*x*x**m*gamma(m/3 + 1/3)*hyper((3/2, m/3 + 1/3), (m/3 + 4/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(3/2)*gamma(m/3 + 4/3)) + B*e**m*x**4*x**m*gamma(
m/3 + 4/3)*hyper((3/2, m/3 + 4/3), (m/3 + 7/3,), b*x**3*exp_polar(I*pi)/a)/
(3*a**(3/2)*gamma(m/3 + 7/3))
```

3.505 $\int \frac{(ex)^m \left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}} \, dx$

Optimal. Leaf size=133

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1+m)/a/b/e/(b*x^3+a)^(3/2)+1/9*(A*b*(7-2*m)+2*a*B*(1+m
))*(e*x)^(1+m)*hypergeom([3/2, 1/3+1/3*m],[4/3+1/3*m],-b*x^3/a)*(1+b*x^3/a)
^(1/2)/a^2/b/e/(1+m)/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{bx^3}{a}+1}\,(ex)^{m+1}(2aB(m+1)+Ab(7-2m))\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)}{9a^2be(m+1)\sqrt{a+bx^3}}+\frac{2(ex)^{m+1}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(1 + m))/(9*a*b*e*(a + b*x^3)^(3/2)) + ((A*b*(7 - 2*m)
 + 2*a*B*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[3/2,
(1 + m)/3, (4 + m)/3, -((b*x^3)/a)])/(9*a^2*b*e*(1 + m)*Sqrt[a + b*x^3])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^m (A+Bx^3)}{(a+bx^3)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{1+m}}{9abe(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\right)\int \frac{(ex)^m}{(a+bx^3)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe(a+bx^3)^{3/2}} + \frac{\left(2\left(-Ab\left(-\frac{7}{2}+m\right)+aB(1+m)\right)\sqrt{1+\frac{bx^3}{a}}\right)\int \frac{(ex)^m}{\left(1+\frac{bx^3}{a}\right)^{3/2}}\,dx}{9a^2b\sqrt{a+bx^3}}$$

$$= \frac{2(Ab-aB)(ex)^{1+m}}{9abe(a+bx^3)^{3/2}} + \frac{(Ab(7-2m)+2aB(1+m))(ex)^{1+m}\sqrt{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{3}{2},\frac{1+m}{3};\frac{4+m}{3};\right.}{9a^2be(1+m)\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.13, size = 113, normalized size = 0.85

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,(ex)^m \left(A(m+4)\,{}_2F_1\left(\frac{5}{2},\frac{m+1}{3};\frac{m+4}{3};-\frac{bx^3}{a}\right)+B(m+1)x^3\,{}_2F_1\left(\frac{5}{2},\frac{m+4}{3};\frac{m+7}{3};-\frac{bx^3}{a}\right)\right)}{a^2(m+1)(m+4)\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (b*x^3)/a]*(A*(4 + m)*Hypergeometric2F1[5/2, (1 + m)/3,
 (4 + m)/3, -((b*x^3)/a)] + B*(1 + m)*x^3*Hypergeometric2F1[5/2, (4 + m)/3,
 (7 + m)/3, -((b*x^3)/a)]))/(a^2*(1 + m)*(4 + m)*Sqrt[a + b*x^3])
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Bx^3+A)\sqrt{bx^3+a}\,(ex)^m}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2
*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3+A)(ex)^m}{(bx^3+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(B\,x^3+A)(ex)^m}{(b\,x^3+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
[Out] int((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^m}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^m/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3 + A\right)\,(e\,x)^m}{\left(b\,x^3 + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2),x)
[Out] int(((A + B*x^3)*(e*x)^m)/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(B*x**3+A)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

3.506 $\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

```
[Out] -1/3*(a*d+b*c)*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(
3/2)/d^(3/2)+1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/b/d
```

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*b*d) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*b^(3/2)*d^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^3\right)}{6bd}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^3}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b^2d}\\
&= \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3bd}-\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3b^{3/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 123, normalized size = 1.40

$$\frac{b\sqrt{d}\sqrt{a+bx^3}\left(c+dx^3\right)-\sqrt{bc-ad}\,(ad+bc)\sqrt{\frac{b\left(c+dx^3\right)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3b^2d^{3/2}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^3]*(c + d*x^3) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^3))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*
d]])/(3*b^2*d^(3/2)*Sqrt[c + d*x^3])
```

fricas [A] time = 0.87, size = 256, normalized size = 2.91

$$\left[\frac{4\sqrt{bx^3+a}\sqrt{dx^3+c}\,bd+(bc+ad)\sqrt{bd}\log\left(8b^2d^2x^6+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^3-4\left(2bdx^3+\right.\right.}{12b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*
b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^3 - 4
*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(b*d)))/(b^2*d
^2), 1/6*(2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*b*d + (b*c + a*d)*sqrt(-b*d)*ar
ctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)
/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d + a*b*d^2)*x^3)))/(b^2*d^2)]
```

giac [A] time = 0.21, size = 104, normalized size = 1.18

$$\frac{\frac{(bc+ad)\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{bx^3+a}\sqrt{b^2c+\left(bx^3+a\right)bd-abd}}{bd}}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*c + a*d)*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 +
 a)*b*d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b*x^3 + a)*sqrt(b^2*c + (b*x^3 + a)
*b*d - a*b*d)/(b*d))/abs(b)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 9.25, size = 283, normalized size = 3.22

$$\frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{d^3\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3\left(\frac{2\,a\,d}{3}+\frac{2\,b\,c}{3}\right)}{b\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{3\,d^2\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^4}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{b}\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}\right)(a\,d+b\,c)}{3\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((2*a*d)/3 + (2*b*c)/3))/(d^3*((c + d*x^3)^
(1/2) - c^(1/2))) + (((a + b*x^3)^(1/2) - a^(1/2))^3*((2*a*d)/3 + (2*b*c)/3
))/(b*d^2*((c + d*x^3)^(1/2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x^3
)^(1/2) - a^(1/2))^2)/(3*d^2*((c + d*x^3)^(1/2) - c^(1/2))^2))/(((a + b*x^3
)^(1/2) - a^(1/2))^4/((c + d*x^3)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a +
 b*x^3)^(1/2) - a^(1/2))^2)/(d*((c + d*x^3)^(1/2) - c^(1/2))^2)) - (2*atanh
((d^(1/2)*((a + b*x^3)^(1/2) - a^(1/2)))/(b^(1/2)*((c + d*x^3)^(1/2) - c^(1
/2))))*(a*d + b*c))/(3*b^(3/2)*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.507 $\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

```
[Out] 2/3*arctanh(d^(1/2)*(b*x^3+a)^(1/2)/b^(1/2)/(d*x^3+c)^(1/2))/b^(1/2)/d^(1/2
)
```

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {444, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^3])/(Sqrt[b]*Sqrt[c + d*x^3])])/(3*Sqrt[b]
*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= \frac{2\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^3}\right)}{3b}$$
$$= \frac{2\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3b}$$
$$= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{b}\sqrt{c+dx^3}}\right)}{3\sqrt{b}\sqrt{d}}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 1.77

$$\frac{2\sqrt{c+dx^3}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^3}}{\sqrt{bc-ad}}\right)}{3\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx^3)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (2*Sqrt[c + d*x^3]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^3])/Sqrt[b*c - a*d]])/(3*S
qrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^3))/(b*c - a*d)])
```

fricas [B] time = 0.89, size = 194, normalized size = 4.04

$$\left[\frac{\sqrt{bd}\log\left(8b^2d^2x^6+b^2c^2+6abcd+a^2d^2+8\left(b^2cd+abd^2\right)x^3+4\left(2bdx^3+bc+ad\right)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{bd}\right.}{6bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(b*d)*log(8*b^2*d^2*x^6 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c
*d + a*b*d^2)*x^3 + 4*(2*b*d*x^3 + b*c + a*d)*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*sqrt(b*d))/(b*d), -1/3*sqrt(-b*d)*arctan(1/2*(2*b*d*x^3 + b*c + a*d)*sqr
t(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-b*d)/(b^2*d^2*x^6 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x^3))/(b*d)]
```

giac [A] time = 0.18, size = 54, normalized size = 1.12

$$-\frac{2b\log\left(\left|-\sqrt{bx^3+a}\sqrt{bd}+\sqrt{b^2c+\left(bx^3+a\right)bd-abd}\right|\right)}{3\sqrt{bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*b*log(abs(-sqrt(b*x^3 + a)*sqrt(b*d) + sqrt(b^2*c + (b*x^3 + a)*b*d -
a*b*d)))/(sqrt(b*d)*abs(b))
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 5.04, size = 49, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}\right)}{3\,\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x^3)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x^3)^(1/2)
 - a^(1/2)))))/(3*(-b*d)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{a + bx^{3}}\,\sqrt{c + dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.508 $\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

```
[Out] -2/3*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(1/2)/c^(1/
2)
```

Rubi [A] time = 0.05, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {446, 93, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*Sqrt[a
]*Sqrt[c])
```

fricas [B] time = 0.91, size = 204, normalized size = 4.25

$$\left[\frac{\sqrt{ac}\log\left(\frac{(b^2c^2+6abcd+a^2d^2)x^6+8a^2c^2+8(abc^2+a^2cd)x^3-4((bc+ad)x^3+2ac)\sqrt{bx^3+a}\sqrt{dx^3+c}\sqrt{ac}}{x^6}\right)}{6ac}, \frac{\sqrt{-ac}\arctan\left(\frac{((bc+ad)x^3+2ac)\sqrt{bx^3+a}\cdots}{2(abcdx^6+a^2c^2+\cdots}\right)}{3ac}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*sqrt(a*c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 + 8*a^2*c^2 + 8*(a*
b*c^2 + a^2*c*d)*x^3 - 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x
^3 + c)*sqrt(a*c))/x^6)/(a*c), 1/3*sqrt(-a*c)*arctan(1/2*((b*c + a*d)*x^3 +
 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 + a^2*c^2 +
 (a*b*c^2 + a^2*c*d)*x^3))/(a*c)]
```

giac [B] time = 0.19, size = 89, normalized size = 1.85

$$-\frac{2\sqrt{bd}\,b\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{3\sqrt{-abcd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*d)*b*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 + a)*sqrt(b*d) -
sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b))/(sqrt(-a*b*c*
d)*abs(b))
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 7.57, size = 136, normalized size = 2.83

$$-\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)-\ln\left(\frac{\left(\sqrt{c}\,\sqrt{b\,x^3+a}-\sqrt{a}\,\sqrt{d\,x^3+c}\right)\left(b\,\sqrt{c}-\frac{\sqrt{a}\,d\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)}{3\,\sqrt{a}\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] -(log(((a + b*x^3)^(1/2) - a^(1/2))/((c + d*x^3)^(1/2) - c^(1/2))) - log(((
c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*c^(1/2) - (a^(1/2
)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^(1/2))))/((c + d*
x^3)^(1/2) - c^(1/2))))/(3*a^(1/2)*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.509 $\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=91

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

```
[Out] 1/3*(a*d+b*c)*arctanh(c^(1/2)*(b*x^3+a)^(1/2)/a^(1/2)/(d*x^3+c)^(1/2))/a^(3
/2)/c^(3/2)-1/3*(b*x^3+a)^(1/2)*(d*x^3+c)^(1/2)/a/c/x^3
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {446, 96, 93, 208}

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}-\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(3*a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sqrt
[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^3\right)}{6ac}$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} - \frac{(bc+ad)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+bx^3}}{\sqrt{c+dx^3}}\right)}{3ac}$$
$$= -\frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3} + \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.07, size = 91, normalized size = 1.00

$$\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx^3}}{\sqrt{a}\sqrt{c+dx^3}}\right)}{3a^{3/2}c^{3/2}} - \frac{\sqrt{a+bx^3}\sqrt{c+dx^3}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -1/3*(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])/(a*c*x^3) + ((b*c + a*d)*ArcTanh[(Sq
rt[c]*Sqrt[a + b*x^3])/(Sqrt[a]*Sqrt[c + d*x^3])])/(3*a^(3/2)*c^(3/2))
```

fricas [A] time = 0.99, size = 278, normalized size = 3.05

$$\left[\frac{\sqrt{ac}\,(bc+ad)x^3\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^6+8\,a^2c^2+8\left(abc^2+a^2cd\right)x^3+4\left((bc+ad)x^3+2\,ac\right)\sqrt{bx^3+a}\,\sqrt{dx^3+c}\,\sqrt{ac}}{x^6}\right)-4\sqrt{bx^3+a}\,\sqrt{dx}}{12\,a^2c^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(sqrt(a*c)*(b*c + a*d)*x^3*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^6 +
 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^3 + 4*((b*c + a*d)*x^3 + 2*a*c)*sqrt(b
*x^3 + a)*sqrt(d*x^3 + c)*sqrt(a*c))/x^6) - 4*sqrt(b*x^3 + a)*sqrt(d*x^3 +
c)*a*c)/(a^2*c^2*x^3), -1/6*(sqrt(-a*c)*(b*c + a*d)*x^3*arctan(1/2*((b*c +
a*d)*x^3 + 2*a*c)*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*sqrt(-a*c)/(a*b*c*d*x^6 +
 a^2*c^2 + (a*b*c^2 + a^2*c*d)*x^3)) + 2*sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*a*
c)/(a^2*c^2*x^3)]
```

giac [B] time = 0.24, size = 413, normalized size = 4.54

$$\frac{\sqrt{bd}\,b^4d\left(\frac{(bc+ad)\arctan\left(-\frac{b^2c+abd-\left(\sqrt{bx^3+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2}{2\sqrt{-abcd}\,b}\right)}{\sqrt{-abcd}\,ab^3cd}-\frac{2\left(b^3c^2-2\,ab^2cd+a^2bd^2-\left(\sqrt{bx^3+a}\,\sqrt{bd}-\right.\right.}{\left(b^4c^2-2\,ab^3cd+a^2b^2d^2-2\left(\sqrt{bx^3+a}\,\sqrt{bd}-\sqrt{b^2c+(bx^3+a)bd-abd}\right)^2 b\right.}\right.}{3\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*d)*b^4*d*((b*c + a*d)*arctan(-1/2*(b^2*c + a*b*d - (sqrt(b*x^3 +
 a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2)/(sqrt(-a*b*c*d)*b
))/(sqrt(-a*b*c*d)*a*b^3*c*d) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2 - (sqr
t(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*b*c - (sq
rt(b*x^3 + a)*sqrt(b*d) - sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*d)/((b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sqrt(b^
2*c + (b*x^3 + a)*b*d - a*b*d))^2*b^2*c - 2*(sqrt(b*x^3 + a)*sqrt(b*d) - sq
rt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^2*a*b*d + (sqrt(b*x^3 + a)*sqrt(b*d) -
 sqrt(b^2*c + (b*x^3 + a)*b*d - a*b*d))^4)*a*b^2*c*d))/abs(b)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^{3}+a}\,\sqrt{d\,x^{3}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 10.77, size = 481, normalized size = 5.29

$$\frac{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)\left(\frac{c\,b^2}{12}+\frac{a\,d\,b}{12}\right)}{a^{3/2}\,c^{3/2}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{b^2}{12\,a\,c\,d}+\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2\left(\frac{a^2\,d^2}{12}-\frac{a\,b\,c\,d}{4}+\frac{b^2\,c^2}{12}\right)}{a^2\,c^2\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^3}{\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^3}+\frac{b\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)}{d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)}-\frac{\left(\sqrt{b\,x^3+a}-\sqrt{a}\right)^2(a\,d+b\,c)}{\sqrt{a}\,\sqrt{c}\,d\left(\sqrt{d\,x^3+c}-\sqrt{c}\right)^2}}+\frac{\ln\left(\frac{\sqrt{b\,x^3+a}-\sqrt{a}}{\sqrt{d\,x^3+c}-\sqrt{c}}\right)\left(\sqrt{a}\,b\,c^{3/2}+a^{3/2}\,\sqrt{c}\,d\right)}{6\,a^2\,c^2}-\frac{\ln\left(\left(\sqrt{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] ((((a + b*x^3)^(1/2) - a^(1/2))*((b^2*c)/12 + (a*b*d)/12))/(a^(3/2)*c^(3/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))) - b^2/(12*a*c*d) + (((a + b*x^3)^(1/2) -
a^(1/2))^2*((a^2*d^2)/12 + (b^2*c^2)/12 - (a*b*c*d)/4))/(a^2*c^2*d*((c + d*
x^3)^(1/2) - c^(1/2))^2))/((((a + b*x^3)^(1/2) - a^(1/2))^3/((c + d*x^3)^(1/
2) - c^(1/2))^3 + (b*((a + b*x^3)^(1/2) - a^(1/2)))/(d*((c + d*x^3)^(1/2) -
 c^(1/2))) - (((a + b*x^3)^(1/2) - a^(1/2))^2*(a*d + b*c))/(a^(1/2)*c^(1/2)
*d*((c + d*x^3)^(1/2) - c^(1/2))^2)) + (log(((a + b*x^3)^(1/2) - a^(1/2))/(
(c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2)*d))/(6*a
^2*c^2) - (log(((c^(1/2)*(a + b*x^3)^(1/2) - a^(1/2)*(c + d*x^3)^(1/2))*(b*
c^(1/2) - (a^(1/2)*d*((a + b*x^3)^(1/2) - a^(1/2)))/((c + d*x^3)^(1/2) - c^
(1/2))))/((c + d*x^3)^(1/2) - c^(1/2)))*(a^(1/2)*b*c^(3/2) + a^(3/2)*c^(1/2
)*d))/(6*a^2*c^2) - (d*((a + b*x^3)^(1/2) - a^(1/2)))/(12*a*c*((c + d*x^3)^
(1/2) - c^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.510 $\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] 1/5*x^5*AppellF1(5/3,1/2,1/2,8/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^4}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\
&= \frac{x^5\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3, -
((b*x^3)/a), -((d*x^3)/c)])/(5*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^4}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^4/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^4}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.511 $\int \frac{x^3}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] 1/4*x^4*AppellF1(4/3,1/2,1/2,7/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^4\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x^3}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\\
&= \frac{x^4\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\,\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(4*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^3}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3/(b*d*x^6 + (b*c + a*d)*x^3 + a
*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^3}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.512 $\int \frac{x}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

```
[Out] 1/2*x^2*AppellF1(2/3,1/2,1/2,5/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*
x^3/c)^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{x}{\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\\
&= \frac{x^2\sqrt{1+\frac{bx^3}{a}}\,\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{a+bx^3}{a}}\sqrt{\frac{c+dx^3}{c}}F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x^2*Sqrt[(a + b*x^3)/a]*Sqrt[(c + d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -
((b*x^3)/a), -((d*x^3)/c)])/(2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

fricas [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x}{bdx^6+(bc+ad)x^3+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x/(b*d*x^6 + (b*c + a*d)*x^3 + a*c
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+a}\sqrt{dx^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.513 $\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] x*AppellF1(1/3,1/2,1/2,4/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (x*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[1/3, 1/2, 1/2, 4/3, -((
b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\
&= \frac{x\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] time = 0.33, size = 170, normalized size = 2.05

$$-\frac{8acxF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\sqrt{a+bx^3}\sqrt{c+dx^3}\left(3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-8*a*c*x*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(Sqrt[a
 + b*x^3]*Sqrt[c + d*x^3]*(-8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 3*x^3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((
d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3 + a}\,\sqrt{dx^3 + c}}{bdx^6 + (bc + ad)x^3 + ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^6 + (b*c + a*d)*x^3 + a*c),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + a}\,\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^3 + a}\,\sqrt{d\,x^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + a}\,\sqrt{dx^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\,x^3 + a}\,\sqrt{d\,x^3 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.514 $\int \frac{1}{x^2 \sqrt{a+bx^3} \sqrt{c+dx^3}} dx$

Optimal. Leaf size=86

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -AppellF1(-1/3,1/2,1/2,2/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^3/c)
^(1/2)/x/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-1/3, 1/2, 1/2, 2/3, -(
(b*x^3)/a), -((d*x^3)/c)]])/(x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^2\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\
&= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] time = 0.26, size = 189, normalized size = 2.20

$$\frac{5x^3\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,(ad+bc)F_1\left(\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+8bdx^6\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},\frac{1}{2};\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{20acx\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (-20*(a + b*x^3)*(c + d*x^3) + 5*(b*c + a*d)*x^3*Sqrt[1 + (b*x^3)/a]*Sqrt[1
 + (d*x^3)/c]*AppellF1[2/3, 1/2, 1/2, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 8*
b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[5/3, 1/2, 1/2, 8/3
, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c*x*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}}{bdx^8+(bc+ad)x^5+acx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^8 + (b*c + a*d)*x^5 + a*c*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+bx^3}\,\sqrt{c+dx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.515 $\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx$

Optimal. Leaf size=88

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

```
[Out] -1/2*AppellF1(-2/3,1/2,1/2,1/3,-b*x^3/a,-d*x^3/c)*(1+b*x^3/a)^(1/2)*(1+d*x^
3/c)^(1/2)/x^2/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] -(Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[-2/3, 1/2, 1/2, 1/3, -((
b*x^3)/a), -((d*x^3)/c)])/(2*x^2*Sqrt[a + b*x^3]*Sqrt[c + d*x^3])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx^3}\sqrt{c+dx^3}}\,dx &= \frac{\sqrt{1+\frac{bx^3}{a}}\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{c+dx^3}}\,dx}{\sqrt{a+bx^3}}\\
&= \frac{\left(\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}\right)\int \frac{1}{x^3\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}}\,dx}{\sqrt{a+bx^3}\sqrt{c+dx^3}}\\
&= -\frac{\sqrt{1+\frac{bx^3}{a}}\sqrt{1+\frac{dx^3}{c}}F_1\left(-\frac{2}{3};\frac{1}{2},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2x^2\sqrt{a+bx^3}\sqrt{c+dx^3}}
\end{aligned}$$

Mathematica [B] time = 0.27, size = 365, normalized size = 4.15

$$\frac{bdx^6\sqrt{\frac{bx^3}{a}+1}\sqrt{\frac{dx^3}{c}+1}F_1\left(\frac{4}{3};\frac{1}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4\left(3x^3(a+bx^3)(c+dx^3)\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bcF_1\left(\frac{4}{3};\frac{3}{2},\frac{1}{2};\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-\right.}{8acF_1\left(\frac{1}{3};\frac{1}{2},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3x^3\left(adF_1\left(\frac{4}{3};\frac{1}{2},\frac{3}{2};\frac{7}{3};-\frac{bx^3}{a},-d\right.\right.}}{8acx^2\sqrt{a+bx^3}\sqrt{c+dx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x^3]*Sqrt[c + d*x^3]),x]
```

```
[Out] (b*d*x^6*Sqrt[1 + (b*x^3)/a]*Sqrt[1 + (d*x^3)/c]*AppellF1[4/3, 1/2, 1/2, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*(-4*a*c*(2*a*c + 3*b*c*x^3 + 3*a*d*x^3
+ 2*b*d*x^6)*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + 3*x
^3*(a + b*x^3)*(c + d*x^3)*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + b*c*AppellF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/(8*a*c*AppellF1[1/3, 1/2, 1/2, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 3*x^
3*(a*d*AppellF1[4/3, 1/2, 3/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + b*c*Appel
lF1[4/3, 3/2, 1/2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(8*a*c*x^2*Sqrt[a +
b*x^3]*Sqrt[c + d*x^3])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+a}\sqrt{dx^3+c}}{bdx^9+(bc+ad)x^6+acx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)/(b*d*x^9 + (b*c + a*d)*x^6 + a*c*x
^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+a}\sqrt{dx^3+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/2)/(d*x^3+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a)*sqrt(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{b\,x^3+a}\,\sqrt{d\,x^3+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^3)^(1/2)*(c + d*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{a+bx^{3}}\,\sqrt{c+dx^{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/2)/(d*x**3+c)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + b*x**3)*sqrt(c + d*x**3)), x)
```

3.516 $\int (ex)^{7/2}\sqrt{a+bx^3}\,(A+Bx^3)\,dx$

Optimal. Leaf size=161

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}(a}{9b}$$

```
[Out] 1/9*B*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e-1/24*a^2*(2*A*b-B*a)*e^(7/2)*arctanh(
(e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/24*a*(2*A*b-B*a)*e^2
*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/12*(2*A*b-B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/
2)/b/e
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$-\frac{a^2e^{7/2}(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{5/2}}+\frac{ae^2(ex)^{3/2}\sqrt{a+bx^3}(2Ab-aB)}{24b^2}+\frac{(ex)^{9/2}\sqrt{a+bx^3}(2Ab-aB)}{12be}+\frac{B(ex)^{9/2}(a}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (a*(2*A*b - a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b^2) + ((2*A*b - a*B)
*(e*x)^(9/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(3/2))/
(9*b*e) - (a^2*(2*A*b - a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)
*Sqrt[a + b*x^3])])/(24*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} - \frac{\left(-9Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\sqrt{a+bx^3}\,dx}{9b}$$

$$= \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{9be} + \frac{(a(2Ab-aB))\int \frac{(ex}{\sqrt{a+}}}{8b}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}(}{9}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}(}{9}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}(}{9}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}(}{9}$$

$$= \frac{a(2Ab-aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{24b^2} + \frac{(2Ab-aB)(ex)^{9/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{9/2}(}{9}$$

Mathematica [A] time = 0.24, size = 145, normalized size = 0.90

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(3a^{3/2}(aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-3a^2B+2ab\left(3A+Bx^3\right)+4b^2x^3\left(3A\right.\right.\right.}{72b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-3*a^2
*B + 2*a*b*(3*A + B*x^3) + 4*b^2*x^3*(3*A + 2*B*x^3)) + 3*a^(3/2)*(-2*A*b +
```

```
a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*
x^3)/a])
```

fricas [A] time = 1.87, size = 295, normalized size = 1.83

$$\left[-\frac{3\left(Ba^3-2\,Aa^2b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(8\,Bb^2e^3x^7+2\right.}{288\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(
8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)*
sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/144*(3*(B*a^3 - 2*A*a^2*b)*e^3*sqrt(-e/b
)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*
(8*B*b^2*e^3*x^7 + 2*(B*a*b + 6*A*b^2)*e^3*x^4 - 3*(B*a^2 - 2*A*a*b)*e^3*x)
*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

giac [A] time = 0.75, size = 251, normalized size = 1.56

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2\,x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Ax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4\,x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3\,a^2e^{(-1)}}{b^2}\right)Bx^{\frac{3}{2}}e^{\frac{5}{2}}-\frac{\left(B^2a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*x^(3/2)*e^(5/2)
+ 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a
^2*e^(-1)/b^2)*B*x^(3/2)*e^(5/2) - 1/24*(B^2*a^6*e^7 - 4*A*B*a^5*b*e^7 + 4*
A^2*a^4*b^2*e^7)*e^(-1/2)*log(abs(-(B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3
/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt(B^2*a^7*e^12 - 4*A*B*a^6*b*e^12 + 4*A^
2*a^5*b^2*e^12 + (B*a^3*x^(3/2)*e^(11/2) - 2*A*a^2*b*x^(3/2)*e^(11/2))^2*b*
e)))/(b^(5/2)*abs(-B*a^3*e^3 + 2*A*a^2*b*e^3))
```

maple [C] time = 8.02, size = 7293, normalized size = 45.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B x^3 + A\right) (e x)^{7/2} \sqrt{b x^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(1/2), x)
```

sympy [B] time = 130.86, size = 292, normalized size = 1.81

$$\frac{A a^{\frac{3}{2}} e^{\frac{7}{2}} x^{\frac{3}{2}}}{12 b \sqrt{1 + \frac{b x^3}{a}}} + \frac{A \sqrt{a}\, e^{\frac{7}{2}} x^{\frac{9}{2}}}{4 \sqrt{1 + \frac{b x^3}{a}}} - \frac{A a^2 e^{\frac{7}{2}} \operatorname{asinh}\left(\frac{\sqrt{b}\, x^{\frac{3}{2}}}{\sqrt{a}}\right)}{12 b^{\frac{3}{2}}} + \frac{A b e^{\frac{7}{2}} x^{\frac{15}{2}}}{6 \sqrt{a} \sqrt{1 + \frac{b x^3}{a}}} - \frac{B a^{\frac{5}{2}} e^{\frac{7}{2}} x^{\frac{3}{2}}}{24 b^2 \sqrt{1 + \frac{b x^3}{a}}} - \frac{B a^{\frac{3}{2}} e^{\frac{7}{2}} x^{\frac{9}{2}}}{72 b \sqrt{1 + \frac{b x^3}{a}}} + \frac{5 B \sqrt{a}\, e^{\frac{7}{2}} x^{\frac{15}{2}}}{36 \sqrt{1 + \frac{b x^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*e**(7/2)*x**(3/2)/(12*b*sqrt(1 + b*x**3/a)) + A*sqrt(a)*e**(7/2)
*x**(9/2)/(4*sqrt(1 + b*x**3/a)) - A*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/s
qrt(a))/(12*b**(3/2)) + A*b*e**(7/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a
)) - B*a**(5/2)*e**(7/2)*x**(3/2)/(24*b**2*sqrt(1 + b*x**3/a)) - B*a**(3/2)
*e**(7/2)*x**(9/2)/(72*b*sqrt(1 + b*x**3/a)) + 5*B*sqrt(a)*e**(7/2)*x**(15/
2)/(36*sqrt(1 + b*x**3/a)) + B*a**3*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a)
)/(24*b**(5/2)) + B*b*e**(7/2)*x**(21/2)/(9*sqrt(a)*sqrt(1 + b*x**3/a))
```

3.517 $\int (ex)^{5/2}\sqrt{a+bx^3}\,\left(A+Bx^3\right)\,dx$

Optimal. Leaf size=324

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3ae^2\sqrt{ex}}{}$$

```
[Out] 1/8*B*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/80*(16*A*b-7*B*a)*(e*x)^(7/2)*(b*x^
3+a)^(1/2)/b/e+3/320*a*(16*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-1
/640*3^(3/4)*a^(5/3)*(16*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 225}

$$-\frac{3^{3/4}a^{5/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{3ae^2\sqrt{ex}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (3*a*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b^2) + ((16*A*b -
 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(80*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^
(3/2))/(8*b*e) - (3^(3/4)*a^(5/3)*(16*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b^2*Sqrt[(b
^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt
[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be} - \frac{\left(-8Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\sqrt{a+bx^3}\,dx}{8b}$$

$$= \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{8be} + \frac{(3a(16Ab-7aB))}{160b}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)}{}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)}{}$$

$$= \frac{3a(16Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{320b^2} + \frac{(16Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{80be} + \frac{B(ex)}{}$$

Mathematica [C] time = 0.16, size = 112, normalized size = 0.35

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(a(7aB-16Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\left(7aB-16Ab-10bBx^3\right)\right)}{80b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)*Sqrt[1 + (b*x^3)/a]*(-16*A*b
+ 7*a*B - 10*b*B*x^3)) + a*(-16*A*b + 7*a*B)*Hypergeometric2F1[-1/2, 1/6, 7
/6, -((b*x^3)/a)]))/(80*b^2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Be^2x^5 + Ae^2x^2\right)\sqrt{bx^3 + a}\,\sqrt{ex}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

maple [C] time = 1.12, size = 4175, normalized size = 12.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
```

```
[Out] 1/320*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(-42*I*B*3^(1/2)*(
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(2/3)*a^3*e+64*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*b^3
+96*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*b*e+40*I*B*3^(1/2)*(1/b^2*e*x*
(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a
*b^2)^(1/3)*x^6*b^3+84*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b*e+96*I*
A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*x^2*a^2*b^3*e-96*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-
```

```
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^3*e+12*I*B*3^(1/
2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3
+a))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b^2+42*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^2*e-120*B*(1/b
^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1
/2)*(-a*b^2)^(1/3)*x^6*b^3+48*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*a*b^2+192*
A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(-a*b^2)^(1/3)*x*a^2*b^2*e-192*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
1/3)*x*a^2*b^2*e-84*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b*e-96*A*(-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(2/3)*a^2*b*e+42*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^3*e-192*A*(1/b^2*e*x*(-
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b
^2)^(1/3)*x^3*b^3-36*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b^2-21*I*B*3^(1/2)*(1/b
^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1
/2)*(-a*b^2)^(1/3)*a^2*b-42*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^2*e-144*A*(1/b^
```

```
2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/
2)*(-a*b^2)^(1/3)*a*b^2+63*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*a^2*b)/(e*x*(b*x^3+a))^
(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,(e\,x)^{5/2}\,\sqrt{b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(1/2), x)
```

sympy [C] time = 45.35, size = 97, normalized size = 0.30

$$\frac{A\sqrt{a}\,e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}\,e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{13}{6} \\ \frac{19}{6} \end{matrix}\right| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(13/6)) + B*sqrt(a)*e**(5/2)*x**(13/2)*gamma(13/6
)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

3.518 $\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=581

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/7*B*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/56*(14*A*b-5*B*a)*(e*x)^(5/2)*(b*x^
3+a)^(1/2)/b/e+3/112*a*(14*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(
1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/112*3^(1/4)*a^(4/3)*(14*A*b-
5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*
(1+3^(1/2)))^2)^(1/2)-1/224*3^(3/4)*a^(4/3)*(14*A*b-5*B*a)*e*(a^(1/3)+b^(1/
3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elli
pticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^
2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)
*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right.}{224b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((14*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*b*e) + (3*(1 + Sqrt[3])*
a*(14*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(5/3)*(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)) + (B*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7*b*e) - (3*3^(
1/4)*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(112*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(
3/4)*(1 - Sqrt[3])*a^(4/3)*(14*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(5/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{\left(-7Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{7b}$$
$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))}{112b}$$
$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} + \frac{(3a(14Ab-5aB))}{}$$
$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{7be} - \frac{(3a(14Ab-5aB))}{}$$
$$= \frac{(14Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{56be} + \frac{3\left(1+\sqrt{3}\right)a(14Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{112b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

Mathematica [C] time = 0.11, size = 94, normalized size = 0.16

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left((14Ab-5aB)\,{}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)\right)}{35b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (14*A
*b - 5*a*B)*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)]))/(35*b*Sqrt[1
 + (b*x^3)/a])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bex^4+Aex\right)\sqrt{bx^3+a}\,\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

maple [C] time = 1.06, size = 5358, normalized size = 9.22

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\sqrt{bx^3 + a}\,(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)*(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,{\left(e\,x\right)}^{3/2}\,\sqrt{b\,x^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(1/2), x)
```

sympy [C] time = 14.98, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{B\sqrt{a}\,e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)*(b*x**3+a)**(1/2),x)
[Out] A*sqrt(a)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*e
xp_polar(I*pi)/a)/(3*gamma(11/6)) + B*sqrt(a)*e**(3/2)*x**(11/2)*gamma(11/6
)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
```

3.519 $\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx$

Optimal. Leaf size=121

$$\frac{a\sqrt{e}(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

```
[Out] 1/6*B*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/12*a*(4*A*b-B*a)*arctanh((e*x)^(3/2
)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+1/12*(4*A*b-B*a)*(e*x)^(
3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a\sqrt{e}(4Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{3/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}(4Ab-aB)}{12be}+\frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] ((4*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b*e) + (B*(e*x)^(3/2)*(a +
b*x^3)^(3/2))/(6*b*e) + (a*(4*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/
2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\sqrt{a+bx^3}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} - \frac{\left(-6Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{6b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,d}{8b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int\right.}{4be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int\right.}{12b}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{(a(4Ab-aB))\,\mathrm{Subst}\left(\int\right.}{12be}$$

$$= \frac{(4Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{12be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6be} + \frac{a(4Ab-aB)\sqrt{e}\tanh^{-1}(}{12b^{3/2}}$$

Mathematica [A] time = 0.15, size = 119, normalized size = 0.98

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}\,x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(B\left(a+2bx^3\right)+4Ab\right)-\sqrt{a}\,(aB-4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)\right)}{12b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*Sqrt[a + b*x^3]*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(4*A*b + B*
(a + 2*b*x^3)) - Sqrt[a]*(-4*A*b + a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])
)/(12*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

fricas [A] time = 1.89, size = 221, normalized size = 1.83

$$\left[-\frac{\left(Ba^2-4\,Aab\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(2\,Bbx^4+(Ba+4\,A\right.}{48\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((B*a^2 - 4*A*a*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e
- 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B*b*x^4
 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/24*((B*a^2 - 4*A*a*b)*s
qrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*
e)) + 2*(2*B*b*x^4 + (B*a + 4*A*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b]
```

giac [A] time = 0.42, size = 137, normalized size = 1.13

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2\,x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Bx^{\frac{3}{2}}e^{\left(-\frac{1}{2}\right)}+\frac{Ba^2e^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{12\,b^{\frac{3}{2}}}+\frac{1}{3}\left(\sqrt{bx^3e^4+ae^4}\,x^{\frac{3}{2}}e^{\frac{3}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*B*x^(3/2)*e^(-1/2)
 + 1/12*B*a^2*e^(1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4
)))/b^(3/2) + 1/3*(sqrt(b*x^3*e^4 + a*e^4)*x^(3/2)*e^(3/2) - a*e^(7/2)*log(
abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/sqrt(b))*A*e^(-3)
```

maple [C] time = 1.09, size = 6858, normalized size = 56.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)*(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3+A\right)\,\sqrt{e\,x}\,\sqrt{b\,x^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(1/2), x)
```

sympy [A] time = 9.67, size = 201, normalized size = 1.66

$$\frac{A\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e}+\frac{Aa\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}}+\frac{Ba^{\frac{3}{2}}\,(ex)^{\frac{3}{2}}}{12be\sqrt{1+\frac{bx^3}{a}}}+\frac{B\sqrt{a}\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}}-\frac{Ba^2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{12b^{\frac{3}{2}}}+\frac{B}{6\sqrt{a}\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)*(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*a**(3/2)*(e*x)**(3/2)/(12
*b*e*sqrt(1 + b*x**3/a)) + B*sqrt(a)*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**3/a
)) - B*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(12*b**(
3/2)) + B*b*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a))
```

3.520 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=286

$$\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+}}{}$$

```
[Out] 1/5*B*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/20*(10*A*b-B*a)*(e*x)^(1/2)*(b*x^3+
a)^(1/2)/b/e+1/40*3^(3/4)*a^(2/3)*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3
)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6
^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] ((10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b*e) + (B*Sqrt[e*x]*(a + b*x
^3)^(3/2))/(5*b*e) + (3^(3/4)*a^(2/3)*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*b*e*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} - \frac{\left(-5Ab+\frac{aB}{2}\right)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,d}{40b}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{(3a(10Ab-aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+}}\right.}{20be}$$

$$= \frac{(10Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{20be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5be} + \frac{3^{3/4}a^{2/3}(10Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.07, size = 93, normalized size = 0.33

$$\frac{x\sqrt{a+bx^3}\left((10Ab-aB)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)\right)}{5b\sqrt{ex}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a] + (10*A*b - a*B)*Hype
rgeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*b*Sqrt[e*x]*Sqrt[1 + (b*x^
3)/a])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

maple [C] time = 1.21, size = 3721, normalized size = 13.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(60*I*A*3^(1/2)*(-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*
b^3*e-6*I*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*3^(1/2)*x^2*a^2*b^2*e-4*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^
(1/2)*x^3*b^2-3*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*a*b-6*I*B*(-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1
/2)*(-a*b^2)^(2/3)*a^2*e-60*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^3*e-10*I*A*3^(1/2)*(1/b^2*e
*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b
*x^3+a))^(1/2)*b^2+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2*e+120*A*(-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
```

```
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/
3)*x*a*b^2*e+12*I*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*(-a*b^2)^(1/3)*x*a^2*b*e-12*B*(-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*(-a*b^2)^(1/3)*x*a^2*b*e-60*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a*b*e+6*B*(-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*(-a*b^2)^(2/3)*a^2*e+12*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*b^2+60*I*A*3^(1/2)*
(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*(-a*b^2)^(2/3)*a*b*e-120*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a
*b^2*e+30*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-
a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*b^2+9*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))
*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*a*b)/(e*x
)^(1/2)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(1/2), x)
```

sympy [C] time = 4.84, size = 97, normalized size = 0.34

$$\frac{A\sqrt{a}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}\,x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*
pi)/a)/(3*sqrt(e)*gamma(7/6)) + B*sqrt(a)*x**(7/2)*gamma(7/6)*hyper((-1/2,
7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
```

3.521 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=580

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{\ldots}}{\ldots}$$

```
[Out] -2*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(1/2)+1/4*(8*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^
(1/2)/a/e^4+3/8*(8*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)
/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3/8*3^(1/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/
3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1
/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b
*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)^2)^(1/2)-1/16*3^(3/4)*a^(1/3)*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3
)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)
/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {453, 279, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - \frac{3\sqrt[4]{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] ((8*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*a*e^4) + (3*(1 + Sqrt[3])*(8
*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(a*e*Sqrt[e*x]) - (3*3^(1/4)*a^(1
/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(8*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sq
rt[3])*a^(1/3)*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(8Ab+aB)\int (ex)^{3/2}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{8e^3}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} + \frac{(3(8Ab+aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,d\right.}{4e^4}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{ae\sqrt{ex}} - \frac{(3(8Ab+aB))\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^2}{\sqrt{a-}}\right.}{8b^{2/3}e^4}$$

$$= \frac{(8Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{4ae^4} + \frac{3\left(1+\sqrt{3}\right)(8Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{8b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \frac{2A\left(a+bx^3\right.}{ae\sqrt{ex}}$$

Mathematica [C] time = 0.07, size = 98, normalized size = 0.17

$$-\frac{4x^4\sqrt{a+bx^3}\left(-\frac{aB}{2}-4Ab\right){}_2F_1\left(-\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5a(ex)^{3/2}\sqrt{\frac{bx^3}{a}+1}} - \frac{2Ax\left(a+bx^3\right)^{3/2}}{a(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (-2*A*x*(a + b*x^3)^(3/2))/(a*(e*x)^(3/2)) - (4*(-4*A*b - (a*B)/2)*x^4*Sqrt
[a + b*x^3]*Hypergeometric2F1[-1/2, 5/6, 11/6, -((b*x^3)/a)])/(5*a*(e*x)^(3
/2)*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

maple [C] time = 1.02, size = 5736, normalized size = 9.89

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(3/2), x)
```

sympy [C] time = 5.50, size = 100, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{B\sqrt{a}\,x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(3/2),x)
[Out] A*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + B*sqrt(a)*x**(5/2)*gamma(5/6)*hyper((-1/
2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
```

3.522 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+2Ab)}{3ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

```
[Out] -2/3*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(3/2)+1/3*(2*A*b+B*a)*arctanh((e*x)^(3/2)*
b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)+1/3*(2*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(1/2)/a/e^4
```

Rubi [A] time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+2Ab)}{3ae^4}+\frac{(aB+2Ab)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((2*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*a*e^4) - (2*A*(a + b*x^3)^(3
/2))/(3*a*e*(e*x)^(3/2)) + ((2*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^
(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \sqrt{ex}\sqrt{a+bx^3}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2e^3}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,d\right.}{e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,d\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,\right.}{3e^4}$$

$$= \frac{(2Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{3ae^4} - \frac{2A\left(a+bx^3\right)^{3/2}}{3ae(ex)^{3/2}} + \frac{(2Ab+aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right.}{3\sqrt{b}\,e^{5/2}}$$

Mathematica [A] time = 0.21, size = 87, normalized size = 0.74

$$\frac{x\sqrt{a+bx^3}\left(\frac{x^{3/2}(aB+2Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{bx^3}{a}+1}} - 2A + Bx^3\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(-2*A + B*x^3 + ((2*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*
x^(3/2))/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x^3)/a])))/(3*(e*x)^(5/2))
```

fricas [A] time = 1.90, size = 207, normalized size = 1.75

$$\left[\frac{(Ba+2\,Ab)\sqrt{be}\,x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)+4\left(Bbx^3-2\,Ab\right)\sqrt{bx^3}}{12\,be^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*((B*a + 2*A*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e -
 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(B*b*x^3 - 2*A*
b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/6*((B*a + 2*A*b)*sqrt(-b*e)*x
^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(
B*b*x^3 - 2*A*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

maple [C] time = 1.04, size = 6668, normalized size = 56.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{bx^3 + a}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3 + A)\,\sqrt{b\,x^3 + a}}{(e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(5/2), x)
```

sympy [A] time = 9.72, size = 160, normalized size = 1.36

$$-\frac{2A\sqrt{a}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1 + \frac{bx^3}{a}}} + \frac{2A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3e^{\frac{5}{2}}} - \frac{2Abx^{\frac{3}{2}}}{3\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1 + \frac{bx^3}{a}}} + \frac{B\sqrt{a}\,x^{\frac{3}{2}}\sqrt{1 + \frac{bx^3}{a}}}{3e^{\frac{5}{2}}} + \frac{Ba\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(5/2),x)
```

```
[Out] -2*A*sqrt(a)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*sqrt(b)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*e**(5/2)) - 2*A*b*x**(3/2)/(3*sqrt(a)*e**(5/2)*
sqrt(1 + b*x**3/a)) + B*sqrt(a)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) +
B*a*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

3.523 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=283

$$\frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{10a}$$

```
[Out] -2/5*A*(b*x^3+a)^(3/2)/a/e/(e*x)^(5/2)+1/10*(4*A*b+5*B*a)*(e*x)^(1/2)*(b*x^
3+a)^(1/2)/a/e^4+1/20*3^(3/4)*(4*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(1/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*
(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{3^{3/4}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{20\sqrt[3]{a}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\sqrt{a+bx^3}}{10a}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] ((4*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(10*a*e^4) - (2*A*(a + b*x^3)^(
3/2))/(5*a*e*(e*x)^(5/2)) + (3^(3/4)*(4*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(20*a^(1/3)*e^4*Sq
rt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]
*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(4Ab+5aB)\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{5ae^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(3(4Ab+5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{20e^3}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{(3(4Ab+5aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,d\right.}{10e^4}$$

$$= \frac{(4Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{10ae^4}-\frac{2A\left(a+bx^3\right)^{3/2}}{5ae(ex)^{5/2}}+\frac{3^{3/4}(4Ab+5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{20\sqrt[3]{a}}$$

Mathematica [C] time = 0.07, size = 97, normalized size = 0.34

$$\frac{2x\sqrt{a+bx^3}\left(x^3(5aB+4Ab)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-A\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\right)}{5a(ex)^{7/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]) + (4*A*b + 5*a*B
)*x^3*Hypergeometric2F1[-1/2, 1/6, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sq
rt[1 + (b*x^3)/a])
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

maple [C] time = 1.02, size = 3512, normalized size = 12.41

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x)
```

```
[Out] -1/10*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b*(-60*I*B*3^(1/2)*(-(I*3^(1/2)-3)
*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b
^2)^(1/3)*x^4*a*b*e-5*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*b+24*I*A*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*x^5*b^3*e-24*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b^3*e+48*A*(-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/
3)*x^4*b^2*e-24*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*b*e+30*I*B*3^(1/2)*(-(I*3^(1
/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a*b^2)^(2/3)*x^3*a*e-12*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
```

```
^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*b+24*I*A*3^(1/2)*(-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(2/3)*x^3*b*e-48*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*b^2*
e+30*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^2*e-30*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^2*e+60*B*(-(I
*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*(-a*b^2)^(1/3)*x^4*a*b*e-30*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3
^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*a*e+4*I*A*
3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^
2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*b+15*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*b)/e^3/(e*
x)^(1/2)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3
))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{(ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{(e\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/(e*x)^(7/2), x)
```

sympy [C] time = 25.04, size = 100, normalized size = 0.35

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{B\sqrt{a}\,\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/(e*x)**(7/2),x)
```

```
[Out] A*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)
/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(a)*sqrt(x)*gamma(1/6)*hyper((-1/
2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6))
```

3.524 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{9/2}}\,dx$

Optimal. Leaf size=564

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 3$$

```
[Out] -2/7*A*(b*x^3+a)^(3/2)/a/x^(7/2)-2/7*(2*A*b+7*B*a)*(b*x^3+a)^(1/2)/a/x^(1/2
)+3/7*b^(1/3)*(2*A*b+7*B*a)*(1+3^(1/2))*x^(1/2)*(b*x^3+a)^(1/2)/a/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))-3/7*3^(1/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/
3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/14*3^(3
/4)*b^(1/3)*(2*A*b+7*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
1-3^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/
3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 564, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {453, 277, 329, 308, 225, 1881}

$$-\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{b}\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14a^{2/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}} - 3$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (-2*(2*A*b + 7*a*B)*Sqrt[a + b*x^3])/(7*a*Sqrt[x]) + (3*(1 + Sqrt[3])*b^(1/
3)*(2*A*b + 7*a*B)*Sqrt[x]*Sqrt[a + b*x^3])/(7*a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) - (2*A*(a + b*x^3)^(3/2))/(7*a*x^(7/2)) - (3*3^(1/4)*b^(1/3)*(2*
A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a
^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
+ Sqrt[3])/4])/(7*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (3^(3/4)*(1 - Sqrt[3])*b^(1
/3)*(2*A*b + 7*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(14*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\,(A+Bx^3)}{x^{9/2}}\,dx = -\frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(2\left(-Ab-\frac{7aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{3/2}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(3b(2Ab+7aB))\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} + \frac{(6b(2Ab+7aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,d\right.}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} - \frac{2A(a+bx^3)^{3/2}}{7ax^{7/2}} - \frac{\left(3\sqrt[3]{b}\,(2Ab+7aB)\right)\text{Subst}\left(\int \frac{(-1+\sqrt{3}}{\sqrt{}}\right.}{7a}$$

$$= -\frac{2(2Ab+7aB)\sqrt{a+bx^3}}{7a\sqrt{x}} + \frac{3\left(1+\sqrt{3}\right)\sqrt[3]{b}\,(2Ab+7aB)\sqrt{x}\,\sqrt{a+bx^3}}{7a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \frac{2A\,(a+b}{7ax^{7/2}}$$

Mathematica [C] time = 0.10, size = 81, normalized size = 0.14

$$\frac{2\sqrt{a+bx^3}\left(-\frac{x^3(7aB+2Ab)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - A\left(a+bx^3\right)\right)}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(9/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)) - ((2*A*b + 7*a*B)*x^3*Hypergeometric2
F1[-1/2, -1/6, 5/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(7*a*x^(7/2))
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

maple [C] time = 1.73, size = 5911, normalized size = 10.48

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{x^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(9/2), x)
```

sympy [C] time = 22.00, size = 97, normalized size = 0.17

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{7}{6}, -\frac{1}{2} \\ -\frac{1}{6} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3x^{\frac{7}{2}}\Gamma\left(-\frac{1}{6}\right)} + \frac{B\sqrt{a}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(9/2),x)
```

```
[Out] A*sqrt(a)*gamma(-7/6)*hyper((-7/6, -1/2), (-1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*x**(7/2)*gamma(-1/6)) + B*sqrt(a)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6
,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(x)*gamma(5/6))
```

3.525 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

```
[Out] -2/9*A*(b*x^3+a)^(3/2)/a/x^(9/2)+2/3*B*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a)^(1
/2))*b^(1/2)-2/3*B*(b*x^3+a)^(1/2)/x^(3/2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {451, 277, 329, 275, 217, 206}

$$-\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}}-\frac{2B\sqrt{a+bx^3}}{3x^{3/2}}+\frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (-2*B*Sqrt[a + b*x^3])/(3*x^(3/2)) - (2*A*(a + b*x^3)^(3/2))/(9*a*x^(9/2))
+ (2*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{11/2}}\,dx &= -\frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + B\int \frac{\sqrt{a+bx^3}}{x^{5/2}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + (bB)\int \frac{\sqrt{x}}{\sqrt{a+bx^3}}\,dx \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + (2bB)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, x^{3/2}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{1}{3}(2bB)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{a+bx^3}}\right) \\
&= -\frac{2B\sqrt{a+bx^3}}{3x^{3/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{9ax^{9/2}} + \frac{2}{3}\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 87, normalized size = 1.10

$$
\frac{2\sqrt{a+bx^3}\left(\frac{3\sqrt{a}\sqrt{b}\,B\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{a\left(A+3Bx^3\right)+Abx^3}{x^{9/2}}\right)}{9a}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(11/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-((A*b*x^3 + a*(A + 3*B*x^3))/x^(9/2)) + (3*Sqrt[a]*Sqr
t[b]*B*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]])/Sqrt[1 + (b*x^3)/a]))/(9*a)
```

fricas [A] time = 1.37, size = 180, normalized size = 2.28

$$
\left[\frac{3\,Ba\sqrt{b}\,x^5\log\left(-8\,b^2x^6 - 8\,abx^3 - 4\left(2\,bx^4 + ax\right)\sqrt{bx^3+a}\,\sqrt{b}\,\sqrt{x} - a^2\right) - 4\left((3\,Ba + Ab)x^3 + Aa\right)\sqrt{bx^3+a}\,\sqrt{}}{18\,ax^5}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*B*a*sqrt(b)*x^5*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*(2*b*x^4 + a*x)*sqr
t(b*x^3 + a)*sqrt(b)*sqrt(x) - a^2) - 4*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^
3 + a)*sqrt(x))/(a*x^5), -1/9*(3*B*a*sqrt(-b)*x^5*arctan(2*sqrt(b*x^3 + a)*
sqrt(-b)*x^(3/2)/(2*b*x^3 + a)) + 2*((3*B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 +
a)*sqrt(x))/(a*x^5)]
```

giac [A] time = 0.25, size = 109, normalized size = 1.38

$$-\frac{2\,Bb\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\left(3\,Bab\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+3\,Ba\sqrt{-b}\,\sqrt{b}\,+A\sqrt{-b}\,b^{\frac{3}{2}}\right)}{9\,a\sqrt{-b}}-\frac{2\left(3\,Ba^3\sqrt{b+\frac{a}{x^3}}\,+Aa^2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}\right)}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="giac")
```

```
[Out] -2/3*B*b*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/9*(3*B*a*b*arctan(sq
rt(b)/sqrt(-b)) + 3*B*a*sqrt(-b)*sqrt(b) + A*sqrt(-b)*b^(3/2))/(a*sqrt(-b))
 - 2/9*(3*B*a^3*sqrt(b + a/x^3) + A*a^2*(b + a/x^3)^(3/2))/a^3
```

maple [C] time = 1.00, size = 3759, normalized size = 47.58

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x)
```

```
[Out] -2/9*(b*x^3+a)^(1/2)/x^(9/2)/b*(18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5
*a-3*A*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3)))^(1/2)*x^3*b^2+I*A*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*b^2-3*A*(x*(b*x^3+a))^(1/2)*(1/b^2*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b-36*I*B*3^(1/2)*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(1/3)*x^6*a*b-18*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1
/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-
2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-
1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(-a*b^2)^(2/3)*x^5*a+I*A*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*x*(-b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a*b-18*I*B*3^(1/2)*(-(I*3^(1/2)
-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+3*I*B*(x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b
^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a*b+36*I*B*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
```

```
x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-18*B*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*x^7*a*b^2+18*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^7*a*b^2+36*B*(
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(1/3)*x^6*a*b-36*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*
3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*(-a*b^2)^(1/3)*x^6*a*b-18*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a+18*B*(-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^5*a+18*I*B*3^(1/2)*(-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^7*a*b^2-9*B*(x*(b*x^3+a))^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*x^3*a*b)/(x*(b*x^3+a))^(1/2)/a/(I*3^(1/2)-3)/(1/b^2*x*(-b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [A] time = 1.22, size = 81, normalized size = 1.03

$$-\frac{1}{3}\left(\sqrt{b}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)+\frac{2\sqrt{bx^3+a}}{x^{\frac{3}{2}}}\right)B-\frac{2\left(bx^3+a\right)^{\frac{3}{2}}A}{9\,ax^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(11/2),x, algorithm="maxima")
[Out] -1/3*(sqrt(b)*log(-(sqrt(b) - sqrt(b*x^3 + a)/x^(3/2))/(sqrt(b) + sqrt(b*x^
```

```
3 + a)/x^(3/2))) + 2*sqrt(b*x^3 + a)/x^(3/2))*B - 2/9*(b*x^3 + a)^(3/2)*A/(
a*x^(9/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\sqrt{b\,x^3+a}}{x^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(11/2), x)
```

sympy [A] time = 61.86, size = 131, normalized size = 1.66

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{9x^3}-\frac{2Ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^3}+1}}{9a}-\frac{2B\sqrt{a}}{3x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2B\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3}-\frac{2Bbx^{\frac{3}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(11/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(9*x**3) - 2*A*b**(3/2)*sqrt(a/(b*x**3) +
 1)/(9*a) - 2*B*sqrt(a)/(3*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*B*sqrt(b)*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/3 - 2*B*b*x**(3/2)/(3*sqrt(a)*sqrt(1 + b*x**3/a)
)
```

3.526 $\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{13/2}}\,dx$

Optimal. Leaf size=269

$$-\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-11aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{55}$$

```
[Out] -2/11*A*(b*x^3+a)^(3/2)/a/x^(11/2)+2/55*(2*A*b-11*B*a)*(b*x^3+a)^(1/2)/a/x^
(5/2)-1/55*3^(3/4)*b*(2*A*b-11*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*x^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)/a^(4/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)
*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {453, 277, 329, 225}

$$-\frac{3^{3/4}b\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-11aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{a+bx^3}}{55}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*(2*A*b - 11*a*B)*Sqrt[a + b*x^3])/(55*a*x^(5/2)) - (2*A*(a + b*x^3)^(3/2
))/(11*a*x^(11/2)) - (3^(3/4)*b*(2*A*b - 11*a*B)*Sqrt[x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(55*a^(4/3)*Sqrt[(b^(1/3
)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^3}\left(A+Bx^3\right)}{x^{13/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{\left(2\left(Ab-\frac{11aB}{2}\right)\right)\int \frac{\sqrt{a+bx^3}}{x^{7/2}}\,dx}{11a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{(3b(2Ab-11aB))\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{(6b(2Ab-11aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,d\right.}{55a}$$

$$= \frac{2(2Ab-11aB)\sqrt{a+bx^3}}{55ax^{5/2}} - \frac{2A\left(a+bx^3\right)^{3/2}}{11ax^{11/2}} - \frac{3^{3/4}b(2Ab-11aB)\sqrt{x}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{55a^{4/3}}$$

Mathematica [C] time = 0.09, size = 80, normalized size = 0.30

$$\frac{2\sqrt{a+bx^3}\left(\frac{x^3(2Ab-11aB)\,{}_2F_1\left(-\frac{5}{6},-\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}-5A\left(a+bx^3\right)\right)}{55ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x^3]*(A + B*x^3))/x^(13/2),x]
```

```
[Out] (2*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3) + ((2*A*b - 11*a*B)*x^3*Hypergeometric
2F1[-5/6, -1/2, 1/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(55*a*x^(11/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

maple [C] time = 1.10, size = 3690, normalized size = 13.72

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x)
[Out] 2/55*(b*x^3+a)^(1/2)/x^(11/2)/(-a*b^2)^(1/3)*(12*I*A*3^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1
/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^8*
b^3+12*I*A*3^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*x^6*b-11*I*B*3^(1/2)*(1/b^2*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^(1/2)*(-a*b^
2)^(1/3)*x^3*a-24*I*A*3^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^7*b^2-5*I*A*3^(1/2
)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^
(1/2)*(-a*b^2)^(1/3)*a-12*A*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^8*b^3+132*I*B*3^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a
*b^2)^(1/3)*x^7*a*b+66*B*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^8*a*b^2+24*A*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
```

```
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*
x^7*b^2-132*B*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^7*a*b-12*A*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3
)*x^6*b+66*B*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*x^6*a-66*I*B*3^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^8*a*
b^2-66*I*B*3^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(2/3)*x^6*a+9*A*(1/b^2*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^
3*b+33*B*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x
^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*a-3*I*A*3^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*b+15
*A*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(x*(b*x^3+a))
^(1/2)*(-a*b^2)^(1/3)*a)/(x*(b*x^3+a))^(1/2)/a/(I*3^(1/2)-3)/(1/b^2*x*(-b*x
+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{bx^3 + a}}{x^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(b*x^3+a)^(1/2)/x^(13/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(b*x^3 + a)/x^(13/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{b\,x^3 + a}}{x^{13/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(1/2))/x^(13/2), x)
```

sympy [C] time = 172.41, size = 97, normalized size = 0.36

$$\frac{A\sqrt{a}\,\Gamma\left(-\frac{11}{6}\right){}_2F_1\left(\begin{matrix}-\frac{11}{6},-\frac{1}{2}\\ -\frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{11}{2}}\Gamma\left(-\frac{5}{6}\right)}+\frac{B\sqrt{a}\,\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(b*x**3+a)**(1/2)/x**(13/2),x)
[Out] A*sqrt(a)*gamma(-11/6)*hyper((-11/6, -1/2), (-5/6,), b*x**3*exp_polar(I*pi)
/a)/(3*x**(11/2)*gamma(-5/6)) + B*sqrt(a)*gamma(-5/6)*hyper((-5/6, -1/2), (
1/6,), b*x**3*exp_polar(I*pi)/a)/(3*x**(5/2)*gamma(1/6))
```

3.527 $\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=201

$$-\frac{a^3e^{7/2}(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}+\frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{192b^2}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be}+\frac{a(\ldots}{}$$

```
[Out] 1/72*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/12*B*(e*x)^(9/2)*(b*x^
3+a)^(5/2)/b/e-1/192*a^3*(8*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/
e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/192*a^2*(8*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b
*x^3+a)^(1/2)/b^2+1/96*a*(8*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] time = 0.14, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^2e^2(ex)^{3/2}\sqrt{a+bx^3}\,(8Ab-3aB)}{192b^2}-\frac{a^3e^{7/2}(8Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{5/2}}+\frac{(ex)^{9/2}\left(a+bx^3\right)^{3/2}(8Ab-3aB)}{72be}+\frac{a(ex\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a^2*(8*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b^2) + (a*(8*A*b
 - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(96*b*e) + ((8*A*b - 3*a*B)*(e*x)^(9
/2)*(a + b*x^3)^(3/2))/(72*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(5/2))/(12*b*e
) - (a^3*(8*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqr
t[a + b*x^3])])/(192*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} - \frac{\left(-12Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{3/2}dx}{12b}$$

$$= \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{12be} + \frac{(a(8Ab-3aB \cdots}{\cdots}$$

$$= \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{72be} + \frac{B(e \cdots}{\cdots}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8 \cdots}{\cdots}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8 \cdots}{\cdots}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8 \cdots}{\cdots}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8 \cdots}{\cdots}$$

$$= \frac{a^2(8Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{192b^2} + \frac{a(8Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{96be} + \frac{(8 \cdots}{\cdots}$$

Mathematica [A] time = 0.33, size = 167, normalized size = 0.83

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(3a^{5/2}(3aB-8Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-9a^3B+6a^2b\left(4A+Bx^3\right)+8ab^2x^3(1 \cdots\right.\right.}{576b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-9*a^3
*B + 6*a^2*b*(4*A + B*x^3) + 16*b^3*x^6*(4*A + 3*B*x^3) + 8*a*b^2*x^3*(14*A
 + 9*B*x^3)) + 3*a^(5/2)*(-8*A*b + 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(576*b^(5/2)*Sqrt[x]*Sqrt[1 + (b*x^3)/a])
```

fricas [A] time = 1.96, size = 355, normalized size = 1.77

$$\left[-\frac{3\left(3\,Ba^4-8\,Aa^3b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(48\,Bb^3e^3x^{10}\right.}{2304\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/2304*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*
x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) -
4*(48*B*b^3*e^3*x^10 + 8*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*
A*a*b^2)*e^3*x^4 - 3*(3*B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x)
)/b^2, -1/1152*(3*(3*B*a^4 - 8*A*a^3*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3
+ a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(48*B*b^3*e^3*x^10 + 8
*(9*B*a*b^2 + 8*A*b^3)*e^3*x^7 + 2*(3*B*a^2*b + 56*A*a*b^2)*e^3*x^4 - 3*(3*
B*a^3 - 8*A*a^2*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2]
```

giac [B] time = 1.45, size = 387, normalized size = 1.93

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2\,x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Aax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4\,x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3\,a^2e^{(-1)}}{b^2}\right)Bax^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*a*x^(3/2)*e^(5/2
) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3
*a^2*e^(-1)/b^2)*B*a*x^(3/2)*e^(5/2) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x
^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a^2*e^(-1)/b^2)*A*b*x^(3/2)*e^(5/2) + 1
/576*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^3 - 5*
a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*B*b*x^(3/2)*e^(5/2) - 1/192*(9
*B^2*a^8*e^7 - 48*A*B*a^7*b*e^7 + 64*A^2*a^6*b^2*e^7)*e^(-1/2)*log(abs(-(3*
B*a^4*x^(3/2)*e^(11/2) - 8*A*a^3*b*x^(3/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt
(9*B^2*a^9*e^12 - 48*A*B*a^8*b*e^12 + 64*A^2*a^7*b^2*e^12 + (3*B*a^4*x^(3/2
)*e^(11/2) - 8*A*a^3*b*x^(3/2)*e^(11/2))^2*b*e)))/(b^(5/2)*abs(-3*B*a^4*e^3
 + 8*A*a^3*b*e^3))
```

maple [C] time = 1.14, size = 7705, normalized size = 38.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

3.528 $\int (ex)^{5/2} \left(a+bx^3\right)^{3/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=364

$$-\frac{9\, 3^{3/4} a^{8/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (22Ab-7aB) F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{14080 b^2 \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}+\frac{27a^2e^2}{}$$

```
[Out] 1/176*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/11*B*(e*x)^(7/2)*(b*
x^3+a)^(5/2)/b/e+9/1760*a*(22*A*b-7*B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+27
/7040*a^2*(22*A*b-7*B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-9/14080*3^(3/4
)*a^(8/3)*(22*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2)
)*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {459, 279, 321, 329, 225}

$$\frac{27a^2e^2\sqrt{ex}\sqrt{a+bx^3}(22Ab-7aB)}{7040b^2}-\frac{9\, 3^{3/4} a^{8/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (22Ab-7aB) F\left(\cos^{-1}\left(\frac{(1-}{(1+}\right.\right.}{14080 b^2 \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(7040*b^2) + (9*a*(
22*A*b - 7*a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1760*b*e) + ((22*A*b - 7*a*B)
*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(176*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(5/2
))/(11*b*e) - (9*3^(3/4)*a^(8/3)*(22*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) +
b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(14080*b^2*Sqrt[(
b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqr
t[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} - \frac{\left(-11Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{3/2}dx}{11b}$$

$$= \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} + \frac{B(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{11be} + \frac{(9a(22Ab-$$

$$= \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be} + \frac{(22Ab-7aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{176be} +$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}$$

$$= \frac{27a^2(22Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{7040b^2} + \frac{9a(22Ab-7aB)(ex)^{7/2}\sqrt{a+bx^3}}{1760be}$$

Mathematica [C] time = 0.17, size = 116, normalized size = 0.32

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(a^2(7aB-22Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)^2\sqrt{\frac{bx^3}{a}+1}\left(7aB-22Ab-16bBx^3\right)\right)}{176b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^2*Sqrt[1 + (b*x^3)/a]*(-22*A*
b + 7*a*B - 16*b*B*x^3)) + a^2*(-22*A*b + 7*a*B)*Hypergeometric2F1[-3/2, 1/
6, 7/6, -((b*x^3)/a)]))/(176*b^2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bbe^2x^8 + (Ba + Ab)e^2x^5 + Aae^2x^2\right)\sqrt{bx^3 + a}\,\sqrt{ex}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e^2*x^8 + (B*a + A*b)*e^2*x^5 + A*a*e^2*x^2)*sqrt(b*x^3 + a)*
sqrt(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

maple [C] time = 1.32, size = 4619, normalized size = 12.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
```

```
[Out] 1/7040*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/(-a*b^2)^(1/3)/b^3*(378*B*(-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(2/3)*a^4*e-1782*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*a^2*b^2+567*B*(1/b^2*e*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x
^3+a))^(1/2)*a^3*b-1920*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^9*b^4-2640*A*(1/b^2*e*x*
(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^
3+a))^(1/2)*x^6*b^4+880*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^6*b^4+594*I*A*
3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^
2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*a^2*b^2-189*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/
```

```
2)*a^3*b-2376*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b^2*e+756*I*B*3^(1
/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^4*b*e+1188*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b
^3*e-378*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^4*b^2*e+1188*I*A*3^(1/2)*(-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(2/3)*a^3*b*e-1188*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2
))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^3*b*e+378*B*(-(I*3^(1
/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^
(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))
*x^2*a^4*b^2*e-1188*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^3*e+1672*I*A*3^(1/2)*(1/b^2*e*x*(
-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3
+a))^(1/2)*x^3*a*b^3+108*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*a^2*b^2+100
0*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*
(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^6*a*b^3-3000*B*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2
)*x^6*a*b^3-5016*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*a*b^3-324*B*(1/b^2*e*x*(-b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
```

```
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^
(1/2)*x^3*a^2*b^2+640*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^9*b^4-756*B*(-(I
*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*(-a*b^2)^(1/3)*x*a^4*b*e+2376*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b^2*e-
378*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^4*e)/(e*x*(b*x^3+a))^(1/2)/(I*3^
(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,(e\,x)^{5/2}\,\left(b\,x^3 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(3/2), x)
```

sympy [C] time = 125.43, size = 199, normalized size = 0.55

$$\frac{Aa^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{6}\right)} + \frac{A\sqrt{a}be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{19}{6}\right)} + \frac{B\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
```

```
[Out] A*a**(3/2)*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(13/6)) + A*sqrt(a)*b*e**(5/2)*x**(13/2)*gamma(1
3/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6))
```

```
 + B*a**(3/2)*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(19/6)) + B*sqrt(a)*b*e**(5/2)*x**(19/2)*g
amma(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
25/6))
```

3.529 $\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=621

$$-\frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-$$

```
[Out] 1/28*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/10*B*(e*x)^(5/2)*(b*x^3+
a)^(5/2)/b/e+9/224*a*(4*A*b-B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+27/448*a^2
*(4*A*b-B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))-27/448*3^(1/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x
)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Elliptic
E((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/
3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b
^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/896
*3^(3/4)*a^(7/3)*(4*A*b-B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1
/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 308, 225, 1881}

$$\frac{27\left(1+\sqrt{3}\right)a^2e\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{9\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB}{896b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (9*a*(4*A*b - a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*b*e) + (27*(1 + Sqrt[3
])*a^2*(4*A*b - a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(5/3)*(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)) + ((4*A*b - a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(2
8*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*b*e) - (27*3^(1/4)*a^(7/3)*(
4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCo
s[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)],
 (2 + Sqrt[3])/4])/(448*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(
1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt[
3])*a^(7/3)*(4*A*b - a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(896*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(
1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} - \frac{\left(-10Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{3/2}dx}{10b}$$

$$= \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10be} + \frac{(9a(4Ab-aB))\int(}{5}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}}{1}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}}{1}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{(4Ab-aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28be} + \frac{B(ex)^{5/2}}{1}$$

$$= \frac{9a(4Ab-aB)(ex)^{5/2}\sqrt{a+bx^3}}{224be} + \frac{27\left(1+\sqrt{3}\right)a^2(4Ab-aB)e\sqrt{ex}\sqrt{a+bx^3}}{448b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

Mathematica [C] time = 0.13, size = 96, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(a(4Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^2\right)}{10b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(4*
A*b - a*B)*Hypergeometric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)]))/(10*b*Sqrt[1
+ (b*x^3)/a])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(Bbex^7+(Ba+Ab)ex^4+Aaex\right)\sqrt{bx^3+a}\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b*e*x^7 + (B*a + A*b)*e*x^4 + A*a*e*x)*sqrt(b*x^3 + a)*sqrt(e*x
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

maple [C] time = 0.96, size = 5790, normalized size = 9.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(3/2)*(B*x^3+A),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,{(e\,x)}^{3/2}\,{\left(b\,x^3 + a\right)}^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2),x)
[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(3/2), x)
```

sympy [C] time = 45.76, size = 199, normalized size = 0.32

$$\frac{Aa^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{A\sqrt{a}\,be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{Ba^{\frac{3}{2}}e^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(b*x**3+a)**(3/2)*(B*x**3+A),x)
[Out] A*a**(3/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*
exp_polar(I*pi)/a)/(3*gamma(11/6)) + A*sqrt(a)*b*e**(3/2)*x**(11/2)*gamma(1
1/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6))
 + B*a**(3/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + B*sqrt(a)*b*e**(3/2)*x**(17/2)*g
amma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(
23/6))
```

3.530 $\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx$

Optimal. Leaf size=161

$$\frac{a^2\sqrt{e}\,(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}\,(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a\right.}{9b}$$

```
[Out] 1/36*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/9*B*(e*x)^(3/2)*(b*x^3+a
)^(5/2)/b/e+1/24*a^2*(6*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3
+a)^(1/2))*e^(1/2)/b^(3/2)+1/24*a*(6*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b
/e
```

Rubi [A] time = 0.11, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{a^2\sqrt{e}\,(6Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24b^{3/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(6Ab-aB)}{36be}+\frac{a(ex)^{3/2}\sqrt{a+bx^3}\,(6Ab-aB)}{24be}+\frac{B(ex)^{3/2}\left(a\right.}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (a*(6*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*b*e) + ((6*A*b - a*B)*(e*
x)^(3/2)*(a + b*x^3)^(3/2))/(36*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(5/2))/(9
*b*e) + (a^2*(6*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*S
qrt[a + b*x^3])])/(24*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} - \frac{\left(-9Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}dx}{9b}$$
$$= \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9be} + \frac{(a(6Ab-aB))\int}{8}$$
$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$
$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$
$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$
$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$
$$= \frac{a(6Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{24be} + \frac{(6Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36be} + \frac{B(ex)^{3/2}}{}$$

Mathematica [A] time = 0.18, size = 143, normalized size = 0.89

$$\frac{\sqrt{ex}\sqrt{a+bx^3}\left(\sqrt{b}\,x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(3a^2B+2ab\left(15A+7Bx^3\right)+4b^2x^3\left(3A+2Bx^3\right)\right)-3a^{3/2}(aB-6Ab)\sinh^{-1}\left(\right.\right.}{72b^{3/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(3/2)*(A + B*x^3),x]
```

```
[Out] (Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(3*a^2*B +
4*b^2*x^3*(3*A + 2*B*x^3) + 2*a*b*(15*A + 7*B*x^3)) - 3*a^(3/2)*(-6*A*b + a
*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*b^(3/2)*Sqrt[x]*Sqrt[1 + (b*x^
3)/a])
```

fricas [A] time = 1.90, size = 273, normalized size = 1.70

$$\left[-\frac{3\left(Ba^3-6\,Aa^2b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(8\,Bb^2x^7+2\left(7\,B\right.\right.}{288\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/288*(3*(B*a^3 - 6*A*a^2*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(8*B*
b^2*x^7 + 2*(7*B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 +
a)*sqrt(e*x))/b, 1/144*(3*(B*a^3 - 6*A*a^2*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^
3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(8*B*b^2*x^7 + 2*(7*
B*a*b + 6*A*b^2)*x^4 + 3*(B*a^2 + 10*A*a*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b
]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,
[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%
}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at paramet
ers values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%%{-2,[
1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%
}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2
,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters values [66,6.82230772497,-23,7
9]Warning, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+
%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,
0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at
parameters values [6,94.9264369817,-8,31]2*B*b*exp(1)/exp(3)*2*((17740800*b
^10*exp(1)^4/638668800/b^10/exp(1)^11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*
exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a
/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqr
t(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-6652800*b^8*exp(1)^10*a^2/6386688
00/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(
1)^4+b*(x*exp(1))^3*exp(1))+2*A*b*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/
exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x
*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1)
)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*
a*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*e
xp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*
exp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*
sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*A*a*exp(1)/exp(3)/exp(1)/3*(1/2*sq
rt(x*exp(1))*x*exp(1)*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4
/sqrt(b*exp(1))*ln(abs(sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1)
)*sqrt(x*exp(1))*x*exp(1))))-1/2*exp(1)/4/exp(1)*exp(3)/b/exp(3)/abs(2*b*a^
2*A+B*a^3)/(4*exp(1)*b^2*a^4*A^2+4*exp(1)*b*B*a^5*A+exp(1)*B^2*a^6)^-1/3/sq
rt(b*exp(1))*ln(abs(sqrt(4*A^2*a^5*b^2*exp(1)^6+4*A*B*a^6*b*exp(1)^6+B^2*a^
7*exp(1)^6+9*b*(-B*a^3*exp(1)*sqrt(x*exp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sq
rt(x*exp(1))*x*exp(1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-B*a^3*exp(1)*sqrt(x*e
xp(1))*x*exp(1)/3-2*A*a^2*b*exp(1)*sqrt(x*exp(1))*x*exp(1)/3)))
```

maple [C] time = 1.06, size = 7290, normalized size = 45.28

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (Bx^3 + A)(bx^3 + a)^{\frac{3}{2}} \sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)*sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(B\,x^3 + A\right)\,\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(3/2), x)
```

sympy [B] time = 25.46, size = 335, normalized size = 2.08

$$\frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{3}{2}}(ex)^{\frac{3}{2}}}{12e\sqrt{1+\frac{bx^3}{a}}} + \frac{A\sqrt{a}\,b\,(ex)^{\frac{9}{2}}}{4e^4\sqrt{1+\frac{bx^3}{a}}} + \frac{Aa^2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{4\sqrt{b}} + \frac{Ab^2(ex)^{\frac{15}{2}}}{6\sqrt{a}\,e^7\sqrt{1+\frac{bx^3}{a}}} + \frac{Ba^{\frac{5}{2}}(ex)^{\frac{3}{2}}}{24be\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)*(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(3/2)*(e*x)**(3/2)/
(12*e*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*(e*x)**(9/2)/(4*e**4*sqrt(1 + b*x**
3/a)) + A*a**2*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(4*sq
rt(b)) + A*b**2*(e*x)**(15/2)/(6*sqrt(a)*e**7*sqrt(1 + b*x**3/a)) + B*a**(5
/2)*(e*x)**(3/2)/(24*b*e*sqrt(1 + b*x**3/a)) + 17*B*a**(3/2)*(e*x)**(9/2)/(
72*e**4*sqrt(1 + b*x**3/a)) + 11*B*sqrt(a)*b*(e*x)**(15/2)/(36*e**7*sqrt(1
+ b*x**3/a)) - B*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2))
)/(24*b**(3/2)) + B*b**2*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a))
```

3.531 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=324

$$\frac{9\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+b\ldots}{}$$

```
[Out] 1/80*(16*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/8*B*(b*x^3+a)^(5/2)*(e*
x)^(1/2)/b/e+9/320*a*(16*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+9/640*3^(
3/4)*a^(5/3)*(16*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1
/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{9\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(16Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (9*a*(16*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*b*e) + ((16*A*b - a*B)*
Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(5/2))/(8*
b*e) + (9*3^(3/4)*a^(5/3)*(16*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sq
rt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*b*e*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} - \frac{\left(-8Ab+\frac{aB}{2}\right)\int\frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{8b}$$

$$= \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{5/2}}{8be} + \frac{(9a(16Ab-aB))\int\frac{\sqrt{a+b}}{\sqrt{ex}}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+b\right.}{8be}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+b\right.}{8be}$$

$$= \frac{9a(16Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{320be} + \frac{(16Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80be} + \frac{B\sqrt{ex}\left(a+b\right.}{8be}$$

Mathematica [C] time = 0.08, size = 96, normalized size = 0.30

$$\frac{x\sqrt{a+bx^3}\left(a(16Ab-aB)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)+B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^2\right)}{8b\sqrt{ex}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^2*Sqrt[1 + (b*x^3)/a] + a*(16*A*b - a*B)*
Hypergeometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)]))/(8*b*Sqrt[e*x]*Sqrt[1 + (
b*x^3)/a])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6+(Ba+Ab)x^3+Aa\right)\sqrt{bx^3+a}\sqrt{ex}}{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(bx^3 + a)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

maple [C] time = 1.17, size = 4173, normalized size = 12.88

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*x/(-a*b^2)^(1/3)/b^2*(-40*I*B*3^(1/2)*(-a*b^2)^(1/3)
*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a
))^(1/2)*x^6*b^3-54*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^2*e+108*I*B*3^(1/2)*(-a
*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*x*a^3*b*e+864*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^3*e-76*
I*B*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^3*a*b^2+864*I*A*3^(1/2)*(-a*b^2)^(2/3)
*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*a^2*b*e-864*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^3*e-27*I*B*3^(1/2)*(-a*b^2)^(1
/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
```

2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*a^2*b+120*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^6*b^3+54*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^2*e-208*I*A*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*a*b^2+1728*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^2*b^2*e-54*I*B*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*e-108*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b*e-864*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*b*e+54*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^3*e+192*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*b^3+228*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b^2-1728*I*A*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a^2*b^2*e-64*I*A*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^3*b^3+624*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*a*b^2+81*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*a^2*b)/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-

```
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\left(b\,x^3 + a\right)^{3/2}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(1/2), x)
```

sympy [C] time = 14.83, size = 199, normalized size = 0.61

$$\frac{Aa^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}\,bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)} + \frac{B\sqrt{a}\,bx^{\frac{13}{2}}\Gamma\left(1\cdots\right.}{3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(1/2),x)
```

```
[Out] A*a**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + A*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/
2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + B*a**
(3/2)*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi
)/a)/(3*sqrt(e)*gamma(13/6)) + B*sqrt(a)*b*x**(13/2)*gamma(13/6)*hyper((-1/
2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6))
```

3.532 $\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=614

$$-\frac{9\, 3^{3/4}\left(1-\sqrt{3}\right) a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,(aB+14Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 1/7*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/a/e^4-2*A*(b*x^3+a)^(5/2)/a/e/
(e*x)^(1/2)+9/56*(14*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+27/112*a*(14*
A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))-27/112*3^(1/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*
x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(
b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-9/22
4*3^(3/4)*a^(4/3)*(14*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3
^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/
2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 308, 225, 1881}

$$-\frac{9\, 3^{3/4}\left(1-\sqrt{3}\right) a^{4/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,(aB+14Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{224b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (9*(14*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(56*e^4) + (27*(1 + Sqrt[3])
*a*(14*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(112*b^(2/3)*e^2*(a^(1/3) + (1
 + Sqrt[3])*b^(1/3)*x)) + ((14*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(3/2))/(7
*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(a*e*Sqrt[e*x]) - (27*3^(1/4)*a^(4/3)*(14
*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(
a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2
 + Sqrt[3])/4])/(112*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (9*3^(3/4)*(1 - Sqrt
[3])*a^(4/3)*(14*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*El
lipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)], (2 + Sqrt[3])/4])/(224*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{3/2}}\,dx = -\frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}} + \frac{(14Ab+aB)\int (ex)^{3/2}(a+bx^3)^{3/2}\,dx}{ae^3}$$

$$= \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a+bx^3)^{5/2}}{ae\sqrt{ex}} + \frac{(9(14Ab+aB))\int (ex)^{3/2}\cdot}{14e^3}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a+bx}{ae\sqrt{ex}}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a+bx}{ae\sqrt{ex}}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{(14Ab+aB)(ex)^{5/2}(a+bx^3)^{3/2}}{7ae^4} - \frac{2A(a+bx}{ae\sqrt{ex}}$$

$$= \frac{9(14Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{56e^4} + \frac{27(1+\sqrt{3})a(14Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{112b^{2/3}e^2\left(\sqrt[3]{a}+(1+\sqrt{3})\sqrt[3]{b}x\right)} + \cdots$$

Mathematica [C] time = 0.12, size = 84, normalized size = 0.14

$$\frac{2x\sqrt{a+bx^3}\left(\frac{x^3(aB+14Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{5A(a+bx^3)^2}{a}\right)}{5(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*((-5*A*(a + b*x^3)^2)/a + ((14*A*b + a*B)*x^3*Hypergeo
metric2F1[-3/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(3
/2))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(Bbx^6+(Ba+Ab)x^3+Aa)\sqrt{bx^3+a}\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3+A)(bx^3+a)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

maple [C] time = 1.14, size = 6142, normalized size = 10.00

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(3/2), x)
```

sympy [C] time = 15.71, size = 202, normalized size = 0.33

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{A\sqrt{a}\,bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{Ba^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{B\sqrt{a}\,bx^{\frac{11}{2}}\Gamma\left(\frac{1}{6}\right.}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(3/2),x)
```

```
[Out] A*a**(3/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + A*sqrt(a)*b*x**(5/2)*gamma(5/6)*hyper((
-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B
*a**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*e**(3/2)*gamma(11/6)) + B*sqrt(a)*b*x**(11/2)*gamma(11/6)*hyper
((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6))
```

3.533 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+4Ab)}{4e^4}-\frac{2A\left(a+bx^3\right)}{3ae(ex)^{3/2}}$$

```
[Out] 1/6*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/a/e^4-2/3*A*(b*x^3+a)^(5/2)/a/e
/(e*x)^(3/2)+1/4*a*(4*A*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a
)^(1/2))/e^(5/2)/b^(1/2)+1/4*(4*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+4Ab)}{6ae^4}+\frac{(ex)^{3/2}\sqrt{a+bx^3}\,(aB+4Ab)}{4e^4}+\frac{a(aB+4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{4\sqrt{b}\,e^{5/2}}-\frac{2A\left(a+bx^3\right)}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] ((4*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(4*e^4) + ((4*A*b + a*B)*(e*x)^
(3/2)*(a + b*x^3)^(3/2))/(6*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(3*a*e*(e*x)^(
3/2)) + (a*(4*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*
x^3])])/(4*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(4Ab+aB)\int \sqrt{ex}\left(a+bx^3\right)^{3/2}\,dx}{ae^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}} + \frac{(3(4Ab+aB))\int \sqrt{ex}\sqrt{a+b}}{4e^3}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

$$= \frac{(4Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{4e^4} + \frac{(4Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{6ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{3ae(ex)^{3/2}}$$

Mathematica [A] time = 0.15, size = 126, normalized size = 0.83

$$\frac{x\sqrt{a+bx^3}\left(3\sqrt{a}\,x^{3/2}(aB+4Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\left(-8aA+5aBx^3+4Abx^3+2bBx^6\right)\right)}{12\sqrt{b}\,(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(-8*a*A + 4*A*b*x^3 + 5*a*B
*x^3 + 2*b*B*x^6) + 3*Sqrt[a]*(4*A*b + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2
))/Sqrt[a]]))/(12*Sqrt[b]*(e*x)^(5/2)*Sqrt[1 + (b*x^3)/a])
```

fricas [A] time = 1.98, size = 255, normalized size = 1.68

$$\left[\frac{3\left(Ba^2+4\,Aab\right)\sqrt{be}\,x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)+4\left(2\,Bb^2x^6+\left(5\,B\right.\right.}{48\,be^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(B*a^2 + 4*A*a*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a
^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(2*B*b^2*
x^6 + (5*B*a*b + 4*A*b^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*
x^2), -1/24*(3*(B*a^2 + 4*A*a*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sq
rt(-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) - 2*(2*B*b^2*x^6 + (5*B*a*b + 4*A*b
^2)*x^3 - 8*A*a*b)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

maple [C] time = 1.18, size = 7108, normalized size = 46.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{3}{2}}}{\left(ex\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{3/2}}{\left(e\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(5/2), x)
```

sympy [B] time = 24.32, size = 289, normalized size = 1.90

$$-\frac{2Aa^{\frac{3}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{2A\sqrt{a}\,bx^{\frac{3}{2}}}{3e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{Aa\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}+\frac{Ba^{\frac{3}{2}}x^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(3/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b*x**(3/
2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 2*A*sqrt(a)*b*x**(3/2)/(3*e**(5/2)*sqr
t(1 + b*x**3/a)) + A*a*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/e**(5/2) + B
*a**(3/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(3/2)*x**(3/2)/(1
2*e**(5/2)*sqrt(1 + b*x**3/a)) + B*sqrt(a)*b*x**(9/2)/(4*e**(5/2)*sqrt(1 +
b*x**3/a)) + B*a**2*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*sqrt(b)*e**(5/2)) +
B*b**2*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

3.534 $\int \frac{(a+bx^3)^{3/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=314

$$\frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+}{}$$

```
[Out] -2/5*A*(b*x^3+a)^(5/2)/a/e/(e*x)^(5/2)+1/5*(2*A*b+B*a)*(b*x^3+a)^(3/2)*(e*x
)^(1/2)/a/e^4+9/20*(2*A*b+B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+9/40*3^(3/4)
*a^(2/3)*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))^2)^(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{9\,3^{3/4}a^{2/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (9*(2*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*e^4) + ((2*A*b + a*B)*Sqrt[
e*x]*(a + b*x^3)^(3/2))/(5*a*e^4) - (2*A*(a + b*x^3)^(5/2))/(5*a*e*(e*x)^(5
/2)) + (9*3^(3/4)*a^(2/3)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*e^4*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/2}\left(A+Bx^3\right)}{(ex)^{7/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}} + \frac{(2Ab+aB)\int \frac{\left(a+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx}{ae^3}$$

$$= \frac{(2Ab+aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}} + \frac{(9(2Ab+aB))\int \frac{\sqrt{a+bx^3}}{\sqrt{ex}}\,dx}{10e^3}$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}} + \cdots$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}} + \cdots$$

$$= \frac{9(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{20e^4} + \frac{(2Ab+aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{5ae^4} - \frac{2A\left(a+bx^3\right)^{5/2}}{5ae(ex)^{5/2}} + \cdots$$

Mathematica [C] time = 0.08, size = 85, normalized size = 0.27

$$\frac{2x\sqrt{a+bx^3}\left(\frac{5x^3(aB+2Ab)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - \frac{A\left(a+bx^3\right)^2}{a}\right)}{5(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(3/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-((A*(a + b*x^3)^2)/a) + (5*(2*A*b + a*B)*x^3*Hyperge
ometric2F1[-3/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*(e*x)^(7
/2))
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bbx^6 + (Ba + Ab)x^3 + Aa\right)\sqrt{bx^3 + a}\sqrt{ex}}{e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b*x^6 + (B*a + A*b)*x^3 + A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x
^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

maple [C] time = 0.98, size = 3966, normalized size = 12.63

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

```
[Out] -1/20*(b*x^3+a)^(1/2)*(-108*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^3*e-54*B*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2*
b^2*e-54*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a^2*e-13*I*B*3^(1/2)*(-a*b^2)^(1/3)
*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a
))^(1/2)*x^3*a*b+30*A*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^3*b^2+12*B*(-a*b^2)^(1/3)*(1
/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^
(1/2)*x^6*b^2-24*A*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*a*b+216*A*(-a*b^2)^(1/3)*(-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
```

```
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*x^4*a*b^2*e+108*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b*e-108*A*(-a*b^2)^(2/
3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*x^3*a*b*e+108*I*A*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a*b*e+1
08*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^3*e-108*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*x^4*a^2*b*e-4*I*B*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^6*b^2+54*I*B*3^(1/2)*
(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*x^5*a^2*b^2*e-10*I*A*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^3*b^2+54*I*
B*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a^2*e-216*I*A*3^(1/2)*(-a*b^2)^(1/3)
*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*x^4*a*b^2*e+8*I*A*3^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*a*b+39*B*(-a*b^
2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*
(b*x^3+a))^(1/2)*x^3*a*b)/x^2/(-a*b^2)^(1/3)/b/e^3/(e*x)^(1/2)/(e*x*(b*x^3+
```

```
a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3
)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{3}{2}}}{(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(3/2)/(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(b\,x^3+a\right)}^{3/2}}{{\left(e\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(3/2))/(e*x)^(7/2), x)
```

sympy [C] time = 41.46, size = 202, normalized size = 0.64

$$\frac{Aa^{\frac{3}{2}}\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{6},-\frac{1}{2}\\ \frac{1}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)}+\frac{A\sqrt{a}\,b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{Ba^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}+\frac{B\sqrt{a}\,bx^{\frac{7}{2}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(3/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + A*sqrt(a)*b*sqrt(x)*gamma(1/6)*hyper((
-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) + B*a
**(3/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi
)/a)/(3*e**(7/2)*gamma(7/6)) + B*sqrt(a)*b*x**(7/2)*gamma(7/6)*hyper((-1/2,
 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(13/6))
```

3.535 $\int (ex)^{7/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=241

$$-\frac{a^4 e^{7/2}(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}+\frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{384b^2}+\frac{a^2(ex)^{9/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{192be}+$$

```
[Out] 1/144*a*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(3/2)/b/e+1/120*(10*A*b-3*B*a)
*(e*x)^(9/2)*(b*x^3+a)^(5/2)/b/e+1/15*B*(e*x)^(9/2)*(b*x^3+a)^(7/2)/b/e-1/3
84*a^4*(10*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)
^(1/2))/b^(5/2)+1/384*a^3*(10*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^
2+1/192*a^2*(10*A*b-3*B*a)*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] time = 0.16, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {459, 279, 321, 329, 275, 217, 206}

$$\frac{a^3e^2(ex)^{3/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{384b^2}-\frac{a^4 e^{7/2}(10Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{384b^{5/2}}+\frac{a^2(ex)^{9/2}\sqrt{a+bx^3}\,(10Ab-3aB)}{192be}+\frac{(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (a^3*(10*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(384*b^2) + (a^2*(10
*A*b - 3*a*B)*(e*x)^(9/2)*Sqrt[a + b*x^3])/(192*b*e) + (a*(10*A*b - 3*a*B)*
(e*x)^(9/2)*(a + b*x^3)^(3/2))/(144*b*e) + ((10*A*b - 3*a*B)*(e*x)^(9/2)*(a
 + b*x^3)^(5/2))/(120*b*e) + (B*(e*x)^(9/2)*(a + b*x^3)^(7/2))/(15*b*e) - (
a^4*(10*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
+ b*x^3])])/(384*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be} - \frac{\left(-15Ab+\frac{9aB}{2}\right)\int (ex)^{7/2}\left(a+bx^3\right)^{5/2}dx}{15b}$$

$$= \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be} + \frac{B(ex)^{9/2}\left(a+bx^3\right)^{7/2}}{15be} + \frac{(a(10Ab-3a$$

$$= \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be} + \frac{(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{5/2}}{120be} +$$

$$= \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} + \frac{a(10Ab-3aB)(ex)^{9/2}\left(a+bx^3\right)^{3/2}}{144be} +$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} +$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} +$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} +$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} +$$

$$= \frac{a^3(10Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{384b^2} + \frac{a^2(10Ab-3aB)(ex)^{9/2}\sqrt{a+bx^3}}{192be} +$$

Mathematica [A] time = 0.31, size = 188, normalized size = 0.78

$$\frac{e^3\sqrt{ex}\sqrt{a+bx^3}\left(15a^{7/2}(3aB-10Ab)\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(-45a^4B+30a^3b\left(5A+Bx^3\right)+4a^2b^2x^3\right.\right.}{5760b^{5/2}\sqrt{x}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(7/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^3*Sqrt[e*x]*Sqrt[a + b*x^3]*(Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]*(-45*a^
4*B + 30*a^3*b*(5*A + B*x^3) + 96*b^4*x^9*(5*A + 4*B*x^3) + 16*a*b^3*x^6*(8
5*A + 63*B*x^3) + 4*a^2*b^2*x^3*(295*A + 186*B*x^3)) + 15*a^(7/2)*(-10*A*b
+ 3*a*B)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(5760*b^(5/2)*Sqrt[x]*Sqrt[1
+ (b*x^3)/a])
```

fricas [A] time = 1.81, size = 409, normalized size = 1.70

$$\left[-\frac{15\left(3Ba^5-10Aa^4b\right)e^3\sqrt{\frac{e}{b}}\log\left(-8b^2ex^6-8abex^3-a^2e+4\left(2b^2x^4+abx\right)\sqrt{bx^3+a}\sqrt{ex}\sqrt{\frac{e}{b}}\right)-4\left(384Bb^4e^3x\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] [-1/23040*(15*(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b
*e*x^3 - a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b))
 - 4*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 + 10*A*b^4)*e^3*x^10 + 8*(93*B*a^
2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b + 118*A*a^2*b^2)*e^3*x^4 - 15*
(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/11520*(15*
(3*B*a^5 - 10*A*a^4*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*
x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(384*B*b^4*e^3*x^13 + 48*(21*B*a*b^3 +
10*A*b^4)*e^3*x^10 + 8*(93*B*a^2*b^2 + 170*A*a*b^3)*e^3*x^7 + 10*(3*B*a^3*b
 + 118*A*a^2*b^2)*e^3*x^4 - 15*(3*B*a^4 - 10*A*a^3*b)*e^3*x)*sqrt(b*x^3 + a
)*sqrt(e*x))/b^2]
```

giac [B] time = 1.49, size = 563, normalized size = 2.34

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(2x^3e^{(-1)}+\frac{ae^{(-1)}}{b}\right)Aa^2x^{\frac{3}{2}}e^{\frac{5}{2}}+\frac{1}{72}\sqrt{bx^3e^4+ae^4}\left(2\left(4x^3e^{(-4)}+\frac{ae^{(-4)}}{b}\right)x^3e^3-\frac{3a^2e^{(-1)}}{b^2}\right)Ba^2x^{\frac{3}{2}}e^{\frac{5}{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*x^3*e^(-1) + a*e^(-1)/b)*A*a^2*x^(3/2)*e^(5
/2) + 1/72*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 -
 3*a^2*e^(-1)/b^2)*B*a^2*x^(3/2)*e^(5/2) + 1/36*sqrt(b*x^3*e^4 + a*e^4)*(2*
(4*x^3*e^(-4) + a*e^(-4)/b)*x^3*e^3 - 3*a^2*e^(-1)/b^2)*A*a*b*x^(3/2)*e^(5/
2) + 1/288*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^
3 - 5*a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*B*a*b*x^(3/2)*e^(5/2) +
1/576*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*x^3*e^(-7) + a*e^(-7)/b)*x^3*e^3 - 5
*a^2*e^(-4)/b^2)*x^3*e^3 + 15*a^3*e^(-1)/b^3)*A*b^2*x^(3/2)*e^(5/2) + 1/576
0*sqrt(b*x^3*e^4 + a*e^4)*(2*(4*(6*(8*x^3*e^(-10) + a*e^(-10)/b)*x^3*e^3 -
7*a^2*e^(-7)/b^2)*x^3*e^3 + 35*a^3*e^(-4)/b^3)*x^3*e^3 - 105*a^4*e^(-1)/b^4
```

```
)*B*b^2*x^(3/2)*e^(5/2) - 1/384*(9*B^2*a^10*e^7 - 60*A*B*a^9*b*e^7 + 100*A^
2*a^8*b^2*e^7)*e^(-1/2)*log(abs(-(3*B*a^5*x^(3/2)*e^(11/2) - 10*A*a^4*b*x^(
3/2)*e^(11/2))*sqrt(b)*e^(1/2) + sqrt(9*B^2*a^11*e^12 - 60*A*B*a^10*b*e^12
+ 100*A^2*a^9*b^2*e^12 + (3*B*a^5*x^(3/2)*e^(11/2) - 10*A*a^4*b*x^(3/2)*e^(
11/2))^2*b*e)))/(b^(5/2)*abs(-3*B*a^5*e^3 + 10*A*a^4*b*e^3))
```

maple [C] time = 1.06, size = 8117, normalized size = 33.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3 + a\right)}^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2),x)
```

```
[Out] int((A + B*x^3)*(e*x)^(7/2)*(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
```

```
[Out] Timed out
```

3.536 $\int (ex)^{5/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=404

$$-\frac{27\, 3^{3/4} a^{11/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (4Ab-aB) F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{11264 b^2 \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}+\frac{81a^3e^2 \cdots}{\cdots}$$

[Out] 15/704*a*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(3/2)/b/e+1/44*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(5/2)/b/e+1/14*B*(e*x)^(7/2)*(b*x^3+a)^(7/2)/b/e+27/1408*a^2*(4*A*b-B*a)*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+81/5632*a^3*(4*A*b-B*a)*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^2-27/11264*3^(3/4)*a^(11/3)*(4*A*b-B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)

Rubi [A] time = 0.34, antiderivative size = 404, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 279, 321, 329, 225}

$$\frac{81a^3e^2\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{5632b^2}-\frac{27\, 3^{3/4} a^{11/3} e^2 \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} (4Ab-aB) F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right) \cdots}{\left(1+\sqrt{3}\right) \cdots}\right.\right.}{11264 b^2 \sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}} \sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]

[Out] (81*a^3*(4*A*b - a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(5632*b^2) + (27*a^2*(4*A*b - a*B)*(e*x)^(7/2)*Sqrt[a + b*x^3])/(1408*b*e) + (15*a*(4*A*b - a*B)*(e*x)^(7/2)*(a + b*x^3)^(3/2))/(704*b*e) + ((4*A*b - a*B)*(e*x)^(7/2)*(a + b*x^3)^(5/2))/(44*b*e) + (B*(e*x)^(7/2)*(a + b*x^3)^(7/2))/(14*b*e) - (27*3^(3/4)*a^(11/3)*(4*A*b - a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(11264*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x]
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx=\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}-\frac{\left(-14Ab+\frac{7aB}{2}\right)\int (ex)^{5/2}\left(a+bx^3\right)^{5/2}dx}{14b}$$

$$=\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B(ex)^{7/2}\left(a+bx^3\right)^{7/2}}{14be}+\frac{(15a(4Ab-aB$$

$$=\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}+\frac{(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{5/2}}{44be}+\frac{B}{}$$

$$=\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{15a(4Ab-aB)(ex)^{7/2}\left(a+bx^3\right)^{3/2}}{704be}+$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{1}{}$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{1}{}$$

$$=\frac{81a^3(4Ab-aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{5632b^2}+\frac{27a^2(4Ab-aB)(ex)^{7/2}\sqrt{a+bx^3}}{1408be}+\frac{1}{}$$

Mathematica [C] time = 0.19, size = 116, normalized size = 0.29

$$\frac{e^2\sqrt{ex}\sqrt{a+bx^3}\left(7a^3(aB-4Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)^3\sqrt{\frac{bx^3}{a}+1}\left(7aB-28Ab-22bBx^3\right)\right)}{308b^2\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(5/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (e^2*Sqrt[e*x]*Sqrt[a + b*x^3]*(-((a + b*x^3)^3*Sqrt[1 + (b*x^3)/a]*(-28*A*
b + 7*a*B - 22*b*B*x^3)) + 7*a^3*(-4*A*b + a*B)*Hypergeometric2F1[-5/2, 1/6
, 7/6, -((b*x^3)/a)]))/(308*b^2*Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 1.02, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2e^2x^{11}+\left(2\,Bab+Ab^2\right)e^2x^8+\left(Ba^2+2\,Aab\right)e^2x^5+Aa^2e^2x^2\right)\sqrt{bx^3+a}\,\sqrt{ex}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*e^2*x^11 + (2*B*a*b + A*b^2)*e^2*x^8 + (B*a^2 + 2*A*a*b)*e^
2*x^5 + A*a^2*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

maple [C] time = 1.09, size = 5063, normalized size = 12.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}\,{\left(b\,x^3+a\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(5/2)*(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)
[Out] Timed out
```

3.537 $\int (ex)^{3/2} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=661

$$-\frac{27\, 3^{3/4} \left(1-\sqrt{3}\right) a^{10/3} e \sqrt{ex} \left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right) \sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x\right)^2}}\,(26Ab-5aB) F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\, x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.}{23296 b^{5/3} \sqrt{\frac{\sqrt[3]{b}\, x\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 3/728*a*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/b/e+1/260*(26*A*b-5*B*a)
*(e*x)^(5/2)*(b*x^3+a)^(5/2)/b/e+1/13*B*(e*x)^(5/2)*(b*x^3+a)^(7/2)/b/e+27/
5824*a^2*(26*A*b-5*B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+81/11648*a^3*(26*A*
b-5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))-81/11648*3^(1/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/
(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/
23296*3^(3/4)*a^(10/3)*(26*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 661, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 279, 329, 308, 225, 1881}

$$\frac{81\left(1+\sqrt{3}\right) a^3 e \sqrt{ex}\sqrt{a+bx^3}\,(26Ab-5aB)}{11648 b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x\right)}-\frac{27\, 3^{3/4}\left(1-\sqrt{3}\right) a^{10/3} e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\, x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x\right)^2}}\,(26Ab}{23296 b^{5/3}\sqrt{\frac{\sqrt[3]{b}\, x\left(\sqrt[3]{a}+\sqrt[3]{b}\, x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\, x\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (27*a^2*(26*A*b - 5*a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(5824*b*e) + (81*(1 +
 Sqrt[3])*a^3*(26*A*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(11648*b^(5/3)*
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*a*(26*A*b - 5*a*B)*(e*x)^(5/2)*(a
 + b*x^3)^(3/2))/(728*b*e) + ((26*A*b - 5*a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2
))/(260*b*e) + (B*(e*x)^(5/2)*(a + b*x^3)^(7/2))/(13*b*e) - (81*3^(1/4)*a^(
10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(11648*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/
4)*(1 - Sqrt[3])*a^(10/3)*(26*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(23296*b^(5/3)*Sqrt[(b^(1/
```

```
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} - \frac{\left(-13Ab+\frac{5aB}{2}\right)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}dx}{13b}$$

$$= \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} + \frac{B(ex)^{5/2}\left(a+bx^3\right)^{7/2}}{13be} + \frac{(3a(26Ab-5aB)}{}$$

$$= \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} + \frac{(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{260be} +$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} +$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} +$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{3a(26Ab-5aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{728be} +$$

$$= \frac{27a^2(26Ab-5aB)(ex)^{5/2}\sqrt{a+bx^3}}{5824be} + \frac{81\left(1+\sqrt{3}\right)a^3(26Ab-5aB)e\sqrt{ex}\sqrt{}}{11648b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\right.}$$

Mathematica [C] time = 0.18, size = 99, normalized size = 0.15

$$\frac{x(ex)^{3/2}\sqrt{a+bx^3}\left(a^2(26Ab-5aB)\,_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\sqrt{\frac{bx^3}{a}+1}\left(a+bx^3\right)^3\right)}{65b\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^(3/2)*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*(e*x)^(3/2)*Sqrt[a + b*x^3]*(5*B*(a + b*x^3)^3*Sqrt[1 + (b*x^3)/a] + a^2
*(26*A*b - 5*a*B)*Hypergeometric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)]))/(65*b*
Sqrt[1 + (b*x^3)/a])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bb^2ex^{10}+\left(2\,Bab+Ab^2\right)ex^7+\left(Ba^2+2\,Aab\right)ex^4+Aa^2ex\right)\sqrt{bx^3+a}\,\sqrt{ex},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*e*x^10 + (2*B*a*b + A*b^2)*e*x^7 + (B*a^2 + 2*A*a*b)*e*x^4
+ A*a^2*e*x)*sqrt(b*x^3 + a)*sqrt(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="giac")

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)

maple [C] time = 1.04, size = 6202, normalized size = 9.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)^(3/2)*(b*x^3+a)^(5/2)*(B*x^3+A),x, algorithm="maxima")

[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*(e*x)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}\,{\left(b\,x^3+a\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2),x)

[Out] int((A + B*x^3)*(e*x)^(3/2)*(a + b*x^3)^(5/2), x)

sympy [C] time = 104.34, size = 308, normalized size = 0.47

$$\frac{Aa^{\frac{5}{2}}e^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{11}{6}\right)} + \frac{2Aa^{\frac{3}{2}}be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{6} \\ \frac{17}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{17}{6}\right)} + \frac{A\sqrt{a}\,b^2e^{\frac{3}{2}}x^{\frac{17}{2}}\Gamma\left(\frac{17}{6}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{17}{6} \\ \frac{23}{6} \end{matrix}\middle| \frac{bx^3e^{i}}{a}\right.}{3\Gamma\left(\frac{23}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**(3/2)*(b*x**3+a)**(5/2)*(B*x**3+A),x)

[Out] A*a**(5/2)*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(11/6)) + 2*A*a**(3/2)*b*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + A*sqrt(a)*b**2*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*a**(5/2)*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(17/6)) + 2*B*a**(3/2)*b*e**(3/2)*x**(17/2)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(23/6)) + B*sqrt(a)*b**2*e**(3/2)*x**(23/2)*gamma(23/6)*hyper((-1/2, 23/6), (29/6,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(29/6))

3.538 $\int \sqrt{ex} \left(a+bx^3\right)^{5/2} \left(A+Bx^3\right) dx$

Optimal. Leaf size=201

$$\frac{5a^3\sqrt{e}(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/}}{}$$

```
[Out] 5/288*a*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/b/e+1/72*(8*A*b-B*a)*(e*x)^
(3/2)*(b*x^3+a)^(5/2)/b/e+1/12*B*(e*x)^(3/2)*(b*x^3+a)^(7/2)/b/e+5/192*a^3*
(8*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^
(3/2)+5/192*a^2*(8*A*b-B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] time = 0.13, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {459, 279, 329, 275, 217, 206}

$$\frac{5a^3\sqrt{e}(8Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{192b^{3/2}}+\frac{5a^2(ex)^{3/2}\sqrt{a+bx^3}(8Ab-aB)}{192be}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(8Ab-aB)}{72be}+\frac{5a(ex)^{3/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (5*a^2*(8*A*b - a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(192*b*e) + (5*a*(8*A*b -
 a*B)*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(288*b*e) + ((8*A*b - a*B)*(e*x)^(3/2)
*(a + b*x^3)^(5/2))/(72*b*e) + (B*(e*x)^(3/2)*(a + b*x^3)^(7/2))/(12*b*e) +
 (5*a^3*(8*A*b - a*B)*Sqrt[e]*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a
 + b*x^3])])/(192*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)dx = \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} - \frac{\left(-12Ab+\frac{3aB}{2}\right)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}dx}{12b}$$

$$= \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex)^{3/2}\left(a+bx^3\right)^{7/2}}{12be} + \frac{(5a(8Ab-aB))}{}$$

$$= \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{72be} + \frac{B(ex}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

$$= \frac{5a^2(8Ab-aB)(ex)^{3/2}\sqrt{a+bx^3}}{192be} + \frac{5a(8Ab-aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{288be} + \frac{(8A}{}$$

Mathematica [A] time = 0.43, size = 146, normalized size = 0.73

$$\frac{x\sqrt{ex}\sqrt{a+bx^3}\left(\frac{(8Ab-aB)\left(15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}\left(33a^2+26abx^3+8b^2x^6\right)\right)}{48\sqrt{b}x^{3/2}\sqrt{\frac{bx^3}{a}+1}}+B\left(a+bx^3\right)^3\right)}{12b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*x]*(a + b*x^3)^(5/2)*(A + B*x^3),x]
```

```
[Out] (x*Sqrt[e*x]*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 + ((8*A*b - a*B)*(Sqrt[b]*x^(
3/2)*Sqrt[1 + (b*x^3)/a]*(33*a^2 + 26*a*b*x^3 + 8*b^2*x^6) + 15*a^(5/2)*Arc
Sinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(48*Sqrt[b]*x^(3/2)*Sqrt[1 + (b*x^3)/a]))
)/(12*b)
```

fricas [A] time = 2.25, size = 323, normalized size = 1.61

$$\left[-\frac{15\left(Ba^4-8\,Aa^3b\right)\sqrt{\frac{e}{b}}\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(48\,Bb^3x^{10}+8\left(\right.\right.}{2304\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2304*(15*(B*a^4 - 8*A*a^3*b)*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
 a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(48
*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 + 2*(59*B*a^2*b + 104*A*a*b^2)*x
^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b, 1/1152*(15*(
B*a^4 - 8*A*a^3*b)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-
e/b)/(2*b*e*x^3 + a*e)) + 2*(48*B*b^3*x^10 + 8*(17*B*a*b^2 + 8*A*b^3)*x^7 +
 2*(59*B*a^2*b + 104*A*a*b^2)*x^4 + 3*(5*B*a^3 + 88*A*a^2*b)*x)*sqrt(b*x^3
+ a)*sqrt(e*x))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,
[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%
}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at paramet
ers values [91,88.2886286299,-21,88]Warning, choosing root of [1,0,%%%{-2,[
1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%
}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2
,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at parameters values [66,6.82230772497,-23,7
9]Warning, choosing root of [1,0,%%%{-2,[1,0,0,4]%%%}+%%%{-2,[0,1,6,1]%%%}+
%%%{-2,[0,1,0,1]%%%},0,%%%{1,[2,0,0,8]%%%}+%%%{2,[1,1,6,5]%%%}+%%%{-2,[1,1,
0,5]%%%}+%%%{1,[0,2,12,2]%%%}+%%%{-2,[0,2,6,2]%%%}+%%%{1,[0,2,0,2]%%%}] at
parameters values [6,94.9264369817,-8,31]2*B*b^2*exp(1)/exp(3)*2*(((2634721
6896000*b^16*exp(1)^7/1264666411008000/b^16/exp(1)^17*sqrt(x*exp(1))*sqrt(x
*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+439120
2816000*b^15*exp(1)^10*a/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(1))*sq
rt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))-54
89003520000*b^14*exp(1)^13*a^2/1264666411008000/b^16/exp(1)^17)*sqrt(x*exp(
1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(
1))+8233505280000*b^13*exp(1)^16*a^3/1264666411008000/b^16/exp(1)^17)*sqrt(
x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(
1))+2*A*b^2*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^
11*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1
))*sqrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*e
xp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*e
xp(1))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*s
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+2*B*a^2
*exp(1)/exp(3)*2*(240*b^4*exp(1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*ex
p(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*e
xp(1)^4*a/5760/b^4/exp(1)^5)*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*s
qrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))+4*A*a*b*exp(1)/exp(3)*2*(240*b^4*exp(
1)/5760/b^4/exp(1)^5*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*ex
p(1))*sqrt(x*exp(1))*sqrt(x*exp(1))+120*b^3*exp(1)^4*a/5760/b^4/exp(1)^5)*s
```

```
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*
exp(1))+2*A*a^2*exp(1)/exp(3)/exp(1)/3*(1/2*sqrt(x*exp(1))*x*exp(1)*sqrt(a*
exp(1)^4+b*(x*exp(1))^3*exp(1))-2*a*exp(1)^4/4/sqrt(b*exp(1))*ln(abs(sqrt(a
*exp(1)^4+b*(x*exp(1))^3*exp(1))-sqrt(b*exp(1))*sqrt(x*exp(1))*x*exp(1))))+
4*B*a*b*exp(1)/exp(3)*2*((17740800*b^10*exp(1)^4/638668800/b^10/exp(1)^11*s
qrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*s
qrt(x*exp(1))+4435200*b^9*exp(1)^7*a/638668800/b^10/exp(1)^11)*sqrt(x*exp(1
))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1))*sqrt(x*exp(1
))-6652800*b^8*exp(1)^10*a^2/638668800/b^10/exp(1)^11)*sqrt(x*exp(1))*sqrt(
x*exp(1))*sqrt(x*exp(1))*sqrt(a*exp(1)^4+b*(x*exp(1))^3*exp(1))-1/2*exp(1)/
32/exp(1)*exp(3)/b/exp(3)/abs(24*b*a^3*A+5*B*a^4)/(576*exp(1)*b^2*a^6*A^2+2
40*exp(1)*b*B*a^7*A+25*exp(1)*B^2*a^8)^-1/3/sqrt(b*exp(1))*ln(abs(sqrt(576*
A^2*a^7*b^2*exp(1)^6+240*A*B*a^8*b*exp(1)^6+25*B^2*a^9*exp(1)^6+9*b*(-5*B*a
^4*exp(1)*sqrt(x*exp(1))*x*exp(1)/3-24*A*a^3*b*exp(1)*sqrt(x*exp(1))*x*exp(
1)/3)^2*exp(1))-sqrt(9*b*exp(1))*(-5*B*a^4*exp(1)*sqrt(x*exp(1))*x*exp(1)/3
-24*A*a^3*b*exp(1)*sqrt(x*exp(1))*x*exp(1)/3)))
```

maple [C] time = 1.10, size = 7702, normalized size = 38.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}} \sqrt{ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)*(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)*sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(B\,x^3 + A\right)\,\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2),x)
[Out] int((A + B*x^3)*(e*x)^(1/2)*(a + b*x^3)^(5/2), x)
```

sympy [B] time = 56.28, size = 413, normalized size = 2.05

$$\frac{Aa^{\frac{5}{2}}\left(ex\right)^{\frac{3}{2}}\sqrt{1 + \frac{bx^3}{a}}}{3e} + \frac{Aa^{\frac{5}{2}}\left(ex\right)^{\frac{3}{2}}}{8e\sqrt{1 + \frac{bx^3}{a}}} + \frac{35Aa^{\frac{3}{2}}b\left(ex\right)^{\frac{9}{2}}}{72e^4\sqrt{1 + \frac{bx^3}{a}}} + \frac{17A\sqrt{a}\,b^2\left(ex\right)^{\frac{15}{2}}}{36e^7\sqrt{1 + \frac{bx^3}{a}}} + \frac{5Aa^3\sqrt{e}\operatorname{asinh}\left(\frac{\sqrt{b}\left(ex\right)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{24\sqrt{b}} + \frac{Ab^3\left(ex\right)}{9\sqrt{a}\,e^{10}\sqrt{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)*(e*x)**(1/2),x)
[Out] A*a**(5/2)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*e) + A*a**(5/2)*(e*x)**(3/2)/
(8*e*sqrt(1 + b*x**3/a)) + 35*A*a**(3/2)*b*(e*x)**(9/2)/(72*e**4*sqrt(1 + b
*x**3/a)) + 17*A*sqrt(a)*b**2*(e*x)**(15/2)/(36*e**7*sqrt(1 + b*x**3/a)) +
```

```
5*A*a**3*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(24*sqrt(b)
) + A*b**3*(e*x)**(21/2)/(9*sqrt(a)*e**10*sqrt(1 + b*x**3/a)) + 5*B*a**(7/2
)*(e*x)**(3/2)/(192*b*e*sqrt(1 + b*x**3/a)) + 133*B*a**(5/2)*(e*x)**(9/2)/(
576*e**4*sqrt(1 + b*x**3/a)) + 127*B*a**(3/2)*b*(e*x)**(15/2)/(288*e**7*sqr
t(1 + b*x**3/a)) + 23*B*sqrt(a)*b**2*(e*x)**(21/2)/(72*e**10*sqrt(1 + b*x**
3/a)) - 5*B*a**4*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(19
2*b**(3/2)) + B*b**3*(e*x)**(27/2)/(12*sqrt(a)*e**13*sqrt(1 + b*x**3/a))
```

3.539 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{\sqrt{ex}}\,dx$

Optimal. Leaf size=364

$$\frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2\sqrt{e}\cdots}{}$$

```
[Out] 3/352*a*(22*A*b-B*a)*(b*x^3+a)^(3/2)*(e*x)^(1/2)/b/e+1/176*(22*A*b-B*a)*(b*
x^3+a)^(5/2)*(e*x)^(1/2)/b/e+1/11*B*(b*x^3+a)^(7/2)*(e*x)^(1/2)/b/e+27/1408
*a^2*(22*A*b-B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+27/2816*3^(3/4)*a^(8/3)*(
22*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a
^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/b/e/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 279, 329, 225}

$$\frac{27\,3^{3/4}a^{8/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(22Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2816be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{27a^2\sqrt{e}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (27*a^2*(22*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(1408*b*e) + (3*a*(22*A*b
 - a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(352*b*e) + ((22*A*b - a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(176*b*e) + (B*Sqrt[e*x]*(a + b*x^3)^(7/2))/(11*b*e) + (
27*3^(3/4)*a^(8/3)*(22*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2816*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) +
b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{\sqrt{ex}}\,dx = \frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be} - \frac{\left(-11Ab+\frac{aB}{2}\right)\int \frac{\left(a+bx^3\right)^{5/2}}{\sqrt{ex}}\,dx}{11b}$$

$$= \frac{(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{176be} + \frac{B\sqrt{ex}\left(a+bx^3\right)^{7/2}}{11be} + \frac{(15a(22Ab-aB))\int \frac{\left(a+bx^3\right)}{\sqrt{ex}}}{352b}$$

$$= \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{176be} + \frac{B\sqrt{ex}\left(a+b\right.}{11be}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-a}{}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-a}{}$$

$$= \frac{27a^2(22Ab-aB)\sqrt{ex}\sqrt{a+bx^3}}{1408be} + \frac{3a(22Ab-aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{352be} + \frac{(22Ab-a}{}$$

Mathematica [C] time = 0.07, size = 84, normalized size = 0.23

$$\frac{x\sqrt{a+bx^3}\left(B\left(a+bx^3\right)^3 - \frac{a^2(aB-22Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}}\right)}{11b\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/Sqrt[e*x],x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(B*(a + b*x^3)^3 - (a^2*(-22*A*b + a*B)*Hypergeometric2F
1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(11*b*Sqrt[e*x])
```

fricas [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2x^9 + \left(2\,Bab + Ab^2\right)x^6 + \left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\right)\sqrt{bx^3 + a}\,\sqrt{ex}}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

maple [C] time = 1.06, size = 4617, normalized size = 12.68

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x)
```

```
[Out] -1/1408*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(162*B*(-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3
)*a^4*e+3102*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*a^2*b^2+243*B*(1/b^2*e*x*(-b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*
a^3*b+384*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-
a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^9*b^4+528*A*(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^6
*b^4-162*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^4*b^2*e+3564*I*A*3^(1/2)*(-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
```

```
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2
*a^3*b^3*e-3564*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^3*b*e+162*B*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^4*
b^2*e-3564*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*x^2*a^3*b^3*e-162*I*B*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*a^4*e+1128*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^6*a*b^3+1848*A*(1/b^2*e*x*(-b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1
/2)*x^3*a*b^3+1068*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*a^2*b^2-324*B*(-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a
*b^2)^(1/3)*x*a^4*b*e+7128*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^3*b^2*e-616*I*A*(
-a*b^2)^(1/3)*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a*b^3-376*I*B*(-a*b^2)^(1/3)*3^(1/2)*(e*x
*(b*x^3+a))^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)*x^6*a*b^3-356*I*B*(-a*b^2)^(1/3)*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(1/b^2*e
*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a^2*b^2+3564*I*A*(
-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^3*b*e-81*I*B*(-a*b^2)^(1/3)*3^(1/2)*(e*x*
(b*x^3+a))^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/
2)*a^3*b-176*I*A*(-a*b^2)^(1/3)*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(1/b^2*e*x*(-
```

```
b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^6*b^4-1034*I*A*(-a*b^2)^(
1/3)*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1
/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3)))^(1/2)*a^2*b^2-128*I*B*(-a*b^2)^(1/3)*3^(1/2)*(e*x*(b*x^3+a))
^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^9*b^4
+324*I*B*(-a*b^2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a^4*b*e-7128*I*A*(-a*b^2)^(1/3)
*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*x*a^3*b^2*e)/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2
)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b
^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{5/2}}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(1/2), x)
```

sympy [C] time = 40.00, size = 308, normalized size = 0.85

$$\frac{Aa^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}+\frac{A\sqrt{a}\,b^2x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{e}\,\Gamma\left(\frac{19}{6}\right)}+\frac{Ba^{\frac{5}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(1/2),x)
[Out] A*a**(5/2)*sqrt(x)*gamma(1/6)*hyper((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I
*pi)/a)/(3*sqrt(e)*gamma(7/6)) + 2*A*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((
-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6)) + A*
```

```
sqrt(a)*b**2*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_
polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*a**(5/2)*x**(7/2)*gamma(7/6)*hyp
er((-1/2, 7/6), (13/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(13/6))
+ 2*B*a**(3/2)*b*x**(13/2)*gamma(13/6)*hyper((-1/2, 13/6), (19/6,), b*x**3*
exp_polar(I*pi)/a)/(3*sqrt(e)*gamma(19/6)) + B*sqrt(a)*b**2*x**(19/2)*gamma
(19/6)*hyper((-1/2, 19/6), (25/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(e)*ga
mma(25/6))
```

3.540 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=650

$$-\frac{27\, 3^{3/4}\left(1-\sqrt{3}\right) a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+20Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{896 b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

```
[Out] 3/28*(20*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(3/2)/e^4+1/10*(20*A*b+B*a)*(e*x)^(
5/2)*(b*x^3+a)^(5/2)/a/e^4-2*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(1/2)+27/224*a*(20
*A*b+B*a)*(e*x)^(5/2)*(b*x^3+a)^(1/2)/e^4+81/448*a^2*(20*A*b+B*a)*(1+3^(1/2
))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-
81/448*3^(1/4)*a^(7/3)*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x
*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*
2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)
+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-27/896*3^(3/4)*a^(7/3)
*(20*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(
e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x
)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 308, 225, 1881}

$$\frac{81\left(1+\sqrt{3}\right)a^2\sqrt{ex}\sqrt{a+bx^3}\,(aB+20Ab)}{448 b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}-\frac{27\, 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(aB+2}{896 b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (27*a*(20*A*b + a*B)*(e*x)^(5/2)*Sqrt[a + b*x^3])/(224*e^4) + (81*(1 + Sqrt
[3])*a^2*(20*A*b + a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(448*b^(2/3)*e^2*(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)) + (3*(20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(
3/2))/(28*e^4) + ((20*A*b + a*B)*(e*x)^(5/2)*(a + b*x^3)^(5/2))/(10*a*e^4)
- (2*A*(a + b*x^3)^(7/2))/(a*e*Sqrt[e*x]) - (81*3^(1/4)*a^(7/3)*(20*A*b + a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(448*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
(1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (27*3^(3/4)*(1 - Sqrt[3])*a^
(7/3)*(20*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3
)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF
[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3
```

```
)*x)], (2 + Sqrt[3])/4])/(896*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{3/2}}dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} + \frac{(20Ab+aB)\int (ex)^{3/2}\left(a+bx^3\right)^{5/2}dx}{ae^3}$$

$$= \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{ae\sqrt{ex}} + \frac{(3(20Ab+aB))\int (ex)^{3/2}}{4e^3}$$

$$= \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{5/2}}{10ae^4} - \frac{2A\left(a+\right.}{ae\surd}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab}{}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab}{}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{3(20Ab+aB)(ex)^{5/2}\left(a+bx^3\right)^{3/2}}{28e^4} + \frac{(20Ab}{}$$

$$= \frac{27a(20Ab+aB)(ex)^{5/2}\sqrt{a+bx^3}}{224e^4} + \frac{81\left(1+\sqrt{3}\right)a^2(20Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{448b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}$$

Mathematica [C] time = 0.05, size = 87, normalized size = 0.13

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2x^3(aB+20Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - 5A\left(a+bx^3\right)^3\right)}{5a(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-5*A*(a + b*x^3)^3 + (a^2*(20*A*b + a*B)*x^3*Hypergeo
metric2F1[-5/2, 5/6, 11/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)^
(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bb^2x^9 + \left(2\,Bab + Ab^2\right)x^6 + \left(Ba^2 + 2\,Aab\right)x^3 + Aa^2\right)\sqrt{bx^3+a}\,\sqrt{ex}}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

maple [C] time = 1.15, size = 6530, normalized size = 10.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{5/2}}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2),x)
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(3/2), x)
```

sympy [C] time = 39.74, size = 311, normalized size = 0.48

$$\frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{1}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}+\frac{A\sqrt{a}\,b^2x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{3}{2}}\Gamma\left(\frac{17}{6}\right)}+\frac{Ba^{\frac{5}{2}}x^{\frac{5}{2}}\Gamma}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(3/2),x)
[Out] A*a**(5/2)*gamma(-1/6)*hyper((-1/2, -1/6), (5/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(3/2)*sqrt(x)*gamma(5/6)) + 2*A*a**(3/2)*b*x**(5/2)*gamma(5/6)*hype
r((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(11/6))
+ A*sqrt(a)*b**2*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b*x**3*
```

```
exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*a**(5/2)*x**(5/2)*gamma(5/6
)*hyper((-1/2, 5/6), (11/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(1
1/6)) + 2*B*a**(3/2)*b*x**(11/2)*gamma(11/6)*hyper((-1/2, 11/6), (17/6,), b
*x**3*exp_polar(I*pi)/a)/(3*e**(3/2)*gamma(17/6)) + B*sqrt(a)*b**2*x**(17/2
)*gamma(17/6)*hyper((-1/2, 17/6), (23/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**
(3/2)*gamma(23/6))
```

3.541 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=188

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\cdots}{}$$

```
[Out] 5/36*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(3/2)/e^4+1/9*(6*A*b+B*a)*(e*x)^(3/2
)*(b*x^3+a)^(5/2)/a/e^4-2/3*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(3/2)+5/24*a^2*(6*A
*b+B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2
)+5/24*a*(6*A*b+B*a)*(e*x)^(3/2)*(b*x^3+a)^(1/2)/e^4
```

Rubi [A] time = 0.13, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 279, 329, 275, 217, 206}

$$\frac{5a^2(aB+6Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{24\sqrt{b}\,e^{5/2}}+\frac{(ex)^{3/2}\left(a+bx^3\right)^{5/2}(aB+6Ab)}{9ae^4}+\frac{5(ex)^{3/2}\left(a+bx^3\right)^{3/2}(aB+6Ab)}{36e^4}+\frac{5a(ex)^{3/2}\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (5*a*(6*A*b + a*B)*(e*x)^(3/2)*Sqrt[a + b*x^3])/(24*e^4) + (5*(6*A*b + a*B)
*(e*x)^(3/2)*(a + b*x^3)^(3/2))/(36*e^4) + ((6*A*b + a*B)*(e*x)^(3/2)*(a +
b*x^3)^(5/2))/(9*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(3*a*e*(e*x)^(3/2)) + (5*
a^2*(6*A*b + a*B)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])
/(24*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{5/2}}\,dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}+\frac{(6Ab+aB)\int \sqrt{ex}\left(a+bx^3\right)^{5/2}\,dx}{ae^3}$$

$$= \frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4}-\frac{2A\left(a+bx^3\right)^{7/2}}{3ae(ex)^{3/2}}+\frac{(5(6Ab+aB))\int \sqrt{ex}\left(a+\right.}{6e^3}$$

$$= \frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{5/2}}{9ae^4}-\frac{2A\left(a+b\right.}{3ae(ex)}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB}{}$$

$$= \frac{5a(6Ab+aB)(ex)^{3/2}\sqrt{a+bx^3}}{24e^4}+\frac{5(6Ab+aB)(ex)^{3/2}\left(a+bx^3\right)^{3/2}}{36e^4}+\frac{(6Ab+aB}{}$$

Mathematica [A] time = 0.21, size = 150, normalized size = 0.80

$$\frac{x\sqrt{a+bx^3}\left(15a^{3/2}x^{3/2}(aB+6Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{\frac{bx^3}{a}+1}\,\left(a^2\left(33Bx^3-48A\right)+a\left(54Abx^3+26bBx^6\right)\right.\right.}{72\sqrt{b}\,(ex)^{5/2}\sqrt{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(5/2),x]
```

```
[Out] (x*Sqrt[a + b*x^3]*(Sqrt[b]*Sqrt[1 + (b*x^3)/a]*(4*b^2*x^6*(3*A + 2*B*x^3)
+ a^2*(-48*A + 33*B*x^3) + a*(54*A*b*x^3 + 26*b*B*x^6)) + 15*a^(3/2)*(6*A*b
```

```
 + a*B)*x^(3/2)*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(72*Sqrt[b]*(e*x)^(5/2
)*Sqrt[1 + (b*x^3)/a])
```

fricas [A] time = 2.04, size = 309, normalized size = 1.64

$$\left[\frac{15\left(Ba^3+6\,Aa^2b\right)\sqrt{be}\,x^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)+4\left(8\,Bb^3x^9+2\left(13\right.\right.}{288\,be^3x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/288*(15*(B*a^3 + 6*A*a^2*b)*sqrt(b*e)*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3
 - a^2*e - 4*(2*b*x^4 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) + 4*(8*B*
b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b + 3*(11*B*a^2*b + 18*A*
a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2), -1/144*(15*(B*a^3 + 6*A
*a^2*b)*sqrt(-b*e)*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(-b*e)*sqrt(e*x)*x/(2*b
*e*x^3 + a*e)) - 2*(8*B*b^3*x^9 + 2*(13*B*a*b^2 + 6*A*b^3)*x^6 - 48*A*a^2*b
 + 3*(11*B*a^2*b + 18*A*a*b^2)*x^3)*sqrt(b*x^3 + a)*sqrt(e*x))/(b*e^3*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

maple [C] time = 1.16, size = 7544, normalized size = 40.13

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,\left(b\,x^3+a\right)^{5/2}}{(e\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(5/2), x)
```

sympy [B] time = 63.94, size = 403, normalized size = 2.14

$$-\frac{2Aa^{\frac{5}{2}}}{3e^{\frac{5}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{2Aa^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3e^{\frac{5}{2}}}-\frac{7Aa^{\frac{3}{2}}bx^{\frac{3}{2}}}{12e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{A\sqrt{a}\,b^2x^{\frac{9}{2}}}{4e^{\frac{5}{2}}\sqrt{1+\frac{bx^3}{a}}}+\frac{5Aa^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4e^{\frac{5}{2}}}+\frac{Ab^3x^{\frac{15}{2}}}{6\sqrt{a}\,e^{\frac{5}{2}}\sqrt{1+\dots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(5/2),x)
```

```
[Out] -2*A*a**(5/2)/(3*e**(5/2)*x**(3/2)*sqrt(1 + b*x**3/a)) + 2*A*a**(3/2)*b*x**
(3/2)*sqrt(1 + b*x**3/a)/(3*e**(5/2)) - 7*A*a**(3/2)*b*x**(3/2)/(12*e**(5/2
)*sqrt(1 + b*x**3/a)) + A*sqrt(a)*b**2*x**(9/2)/(4*e**(5/2)*sqrt(1 + b*x**3
/a)) + 5*A*a**2*sqrt(b)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*e**(5/2)) + A*b*
*3*x**(15/2)/(6*sqrt(a)*e**(5/2)*sqrt(1 + b*x**3/a)) + B*a**(5/2)*x**(3/2)*
sqrt(1 + b*x**3/a)/(3*e**(5/2)) + B*a**(5/2)*x**(3/2)/(8*e**(5/2)*sqrt(1 +
b*x**3/a)) + 35*B*a**(3/2)*b*x**(9/2)/(72*e**(5/2)*sqrt(1 + b*x**3/a)) + 17
*B*sqrt(a)*b**2*x**(15/2)/(36*e**(5/2)*sqrt(1 + b*x**3/a)) + 5*B*a**3*asinh
(sqrt(b)*x**(3/2)/sqrt(a))/(24*sqrt(b)*e**(5/2)) + B*b**3*x**(21/2)/(9*sqrt
(a)*e**(5/2)*sqrt(1 + b*x**3/a))
```

3.542 $\int \frac{(a+bx^3)^{5/2}(A+Bx^3)}{(ex)^{7/2}}\,dx$

Optimal. Leaf size=352

$$\frac{27\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+\ldots}{}$$

```
[Out] -2/5*A*(b*x^3+a)^(7/2)/a/e/(e*x)^(5/2)+3/80*(16*A*b+5*B*a)*(b*x^3+a)^(3/2)*
(e*x)^(1/2)/e^4+1/40*(16*A*b+5*B*a)*(b*x^3+a)^(5/2)*(e*x)^(1/2)/a/e^4+27/32
0*a*(16*A*b+5*B*a)*(e*x)^(1/2)*(b*x^3+a)^(1/2)/e^4+27/640*3^(3/4)*a^(5/3)*(
16*A*b+5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*(
(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 279, 329, 225}

$$\frac{27\,3^{3/4}a^{5/3}\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+16Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{640e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{\sqrt{ex}\,(a+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (27*a*(16*A*b + 5*a*B)*Sqrt[e*x]*Sqrt[a + b*x^3])/(320*e^4) + (3*(16*A*b +
5*a*B)*Sqrt[e*x]*(a + b*x^3)^(3/2))/(80*e^4) + ((16*A*b + 5*a*B)*Sqrt[e*x]*
(a + b*x^3)^(5/2))/(40*a*e^4) - (2*A*(a + b*x^3)^(7/2))/(5*a*e*(e*x)^(5/2))
 + (27*3^(3/4)*a^(5/3)*(16*A*b + 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(640*e^4*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
```

```
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/2}\left(A+Bx^3\right)}{(ex)^{7/2}}dx = -\frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}} + \frac{(16Ab+5aB)\int \frac{\left(a+bx^3\right)^{5/2}}{\sqrt{ex}}dx}{5ae^3}$$

$$= \frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4} - \frac{2A\left(a+bx^3\right)^{7/2}}{5ae(ex)^{5/2}} + \frac{(3(16Ab+5aB))\int \frac{\left(a+bx^3\right.}{\sqrt{ex}}}{16e^3}$$

$$= \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{5/2}}{40ae^4} - \frac{2A\left(a+\right.}{5ae(e}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab}{}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab}{}$$

$$= \frac{27a(16Ab+5aB)\sqrt{ex}\sqrt{a+bx^3}}{320e^4} + \frac{3(16Ab+5aB)\sqrt{ex}\left(a+bx^3\right)^{3/2}}{80e^4} + \frac{(16Ab}{}$$

Mathematica [C] time = 0.05, size = 88, normalized size = 0.25

$$\frac{2x\sqrt{a+bx^3}\left(\frac{a^2x^3(5aB+16Ab)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{6};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{\frac{bx^3}{a}+1}} - A\left(a+bx^3\right)^3\right)}{5a(ex)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^3)^(5/2)*(A + B*x^3))/(e*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*(-(A*(a + b*x^3)^3) + (a^2*(16*A*b + 5*a*B)*x^3*Hyperg
eometric2F1[-5/2, 1/6, 7/6, -((b*x^3)/a)])/Sqrt[1 + (b*x^3)/a]))/(5*a*(e*x)
^(7/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bb^2x^9+\left(2\,Bab+Ab^2\right)x^6+\left(Ba^2+2\,Aab\right)x^3+Aa^2\right)\sqrt{bx^3+a}\,\sqrt{ex}}{e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((B*b^2*x^9 + (2*B*a*b + A*b^2)*x^6 + (B*a^2 + 2*A*a*b)*x^3 + A*a^2
)*sqrt(b*x^3 + a)*sqrt(e*x)/(e^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)\left(bx^3+a\right)^{\frac{5}{2}}}{\left(ex\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

maple [C] time = 1.10, size = 4422, normalized size = 12.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x)
```

```
[Out] -1/320*(b*x^3+a)^(1/2)*(2592*I*A*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a^2*b
*e-64*I*A*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a
*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^6*b^3-810*B*(-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^3*b^
2*e-810*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a^3*e+810*I*B*3^(1/2)*(-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*
```

a^3*b^2*e-2592*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2*b^3*e+120*B*(e*x*(b*x^3+a))^(1/2)*(-a*b^
2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^9*b
^3+192*A*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^6*b^3-384*A*(e*x*(b*x^3+a))^(1/2)*(-a*b^2
)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a^2*b+
5184*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b^2*e+1620*B*(-a*b^2)^(1/3)*(-(I*3^
(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2
))*x^4*a^3*b*e-2592*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a^2*b*e+810*I*B*3^(1/2)*
(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a
b^2)^(1/3)))^(1/2)((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2
))/(I*3^(1/2)-3))^(1/2))*x^3*a^3*e+128*I*A*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-
a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*a
^2*b-1620*I*B*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^3*b*e+1104*A*(e*x*(b*x
^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*x^3*a*b^2+705*B*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e
x(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I
3^(1/2)(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a^2*b+420*B*(e*x*(
b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3)))^(1/2)*x^6*a*b^2-40*I*B*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1
/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^9*b^3-51
84*I*A*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)

```
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a^2*b^2*e-235*I*B*3^(1/2)*(e*x*
(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*x^3*a^2*b-368*I*A*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^
(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^3*a*b^
2+2592*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a^2*b^3*e-140*I*B*3^(1/2)*(e*x*(b*x^3+a))^
(1/2)*(-a*b^2)^(1/3)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*x^6*a*b^2)/x^2/b/(-a*b^2)^(1/3)/e^3/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/2
)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\left(bx^3 + a\right)^{\frac{5}{2}}}{(ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/2)*(B*x^3+A)/(e*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(b*x^3 + a)^(5/2)/(e*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,\left(b\,x^3 + a\right)^{5/2}}{(e\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2),x)
```

```
[Out] int(((A + B*x^3)*(a + b*x^3)^(5/2))/(e*x)^(7/2), x)
```

sympy [C] time = 85.78, size = 311, normalized size = 0.88

$$\frac{Aa^{\frac{5}{2}}\Gamma\left(-\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{5}{6}, -\frac{1}{2} \\ \frac{1}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{2Aa^{\frac{3}{2}}b\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)} + \frac{A\sqrt{a}\,b^2x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3e^{\frac{7}{2}}\Gamma\left(\frac{13}{6}\right)} + \frac{Ba^{\frac{5}{2}}\sqrt{x}\,\Gamma}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/2)*(B*x**3+A)/(e*x)**(7/2),x)
```

```
[Out] A*a**(5/2)*gamma(-5/6)*hyper((-5/6, -1/2), (1/6,), b*x**3*exp_polar(I*pi)/a
)/(3*e**(7/2)*x**(5/2)*gamma(1/6)) + 2*A*a**(3/2)*b*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
A*sqrt(a)*b**2*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*a**(5/2)*sqrt(x)*gamma(1/6)*hype
r((-1/2, 1/6), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma(7/6)) +
```

```
2*B*a**(3/2)*b*x**(7/2)*gamma(7/6)*hyper((-1/2, 7/6), (13/6,), b*x**3*exp_p
olar(I*pi)/a)/(3*e**(7/2)*gamma(13/6)) + B*sqrt(a)*b**2*x**(13/2)*gamma(13/
6)*hyper((-1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*e**(7/2)*gamma
(19/6))
```

3.543 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=121

$$-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-3aB)}{12b^2}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

[Out] -1/12*a*(4*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)+1/12*(4*A*b-3*B*a)*e^2*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b^2+1/6*B*(e*x)^(9/2)*(b*x^3+a)^(1/2)/b/e

Rubi [A] time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 321, 329, 275, 217, 206}

$$\frac{e^2(ex)^{3/2}\sqrt{a+bx^3}\,(4Ab-3aB)}{12b^2}-\frac{ae^{7/2}(4Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{12b^{5/2}}+\frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be}$$

Antiderivative was successfully verified.

[In] Int[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]

[Out] ((4*A*b - 3*a*B)*e^2*(e*x)^(3/2)*Sqrt[a + b*x^3])/(12*b^2) + (B*(e*x)^(9/2)*Sqrt[a + b*x^3])/(6*b*e) - (a*(4*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(12*b^(5/2))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(-6Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\sqrt{a+bx^3}}\,dx}{6b}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,d}{8b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int\right.}{4b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int\right.}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{\left(a(4Ab-3aB)e^2\right)\text{Subst}\left(\int\right.}{12b^2}$$

$$= \frac{(4Ab-3aB)e^2(ex)^{3/2}\sqrt{a+bx^3}}{12b^2} + \frac{B(ex)^{9/2}\sqrt{a+bx^3}}{6be} - \frac{a(4Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{}}{e^{3/2}}\right.}{12b^{5/2}}$$

Mathematica [A] time = 0.17, size = 97, normalized size = 0.80

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\sqrt{a+bx^3}\left(-3aB+4Ab+2bBx^3\right)+a(3aB-4Ab)\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)\right)}{12b^{5/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*Sqrt[a + b*x^3]*(4*A*b - 3*a*B + 2*b*B*x^3)
 + a*(-4*A*b + 3*a*B)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(12*b^(5
/2)*Sqrt[x])
```

fricas [A] time = 2.10, size = 245, normalized size = 2.02

$$\left[-\frac{\left(3\,Ba^2-4\,Aab\right)e^3\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e+4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(2\,Bbe^3x^4-(\right.}{48\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*((3*B*a^2 - 4*A*a*b)*e^3*sqrt(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 -
a^2*e + 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e*x)*sqrt(e/b)) - 4*(2*B
*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/b^2, -1/24*(
(3*B*a^2 - 4*A*a*b)*e^3*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*s
qrt(-e/b)/(2*b*e*x^3 + a*e)) - 2*(2*B*b*e^3*x^4 - (3*B*a - 4*A*b)*e^3*x)*sq
rt(b*x^3 + a)*sqrt(e*x))/b^2]
```

giac [A] time = 0.36, size = 114, normalized size = 0.94

$$\frac{1}{12}\sqrt{bx^3e^4+ae^4}\left(\frac{2\,Bx^3e^{(-2)}}{b}-\frac{\left(3\,Bab^3e^5-4\,Ab^4e^5\right)e^{(-7)}}{b^5}\right)x^{\frac{3}{2}}e^{\frac{7}{2}}-\frac{\left(3\,Ba^2b^3e^9-4\,Aab^4e^9\right)e^{\left(-\frac{11}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{\right.}\right.}{12\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(b*x^3*e^4 + a*e^4)*(2*B*x^3*e^(-2)/b - (3*B*a*b^3*e^5 - 4*A*b^4*e
^5)*e^(-7)/b^5)*x^(3/2)*e^(7/2) - 1/12*(3*B*a^2*b^3*e^9 - 4*A*a*b^4*e^9)*e^
(-11/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)
```

maple [C] time = 1.01, size = 6861, normalized size = 56.70

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 95.30, size = 194, normalized size = 1.60

$$\frac{A\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}{3b}-\frac{Aae^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{Ba^{\frac{3}{2}}e^{\frac{7}{2}}x^{\frac{3}{2}}}{4b^2\sqrt{1+\frac{bx^3}{a}}}-\frac{B\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{9}{2}}}{12b\sqrt{1+\frac{bx^3}{a}}}+\frac{Ba^2e^{\frac{7}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{Be^{\frac{7}{2}}x^{\frac{15}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*e**(7/2)*x**(3/2)*sqrt(1 + b*x**3/a)/(3*b) - A*a*e**(7/2)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - B*a**(3/2)*e**(7/2)*x**(3/2)/(4*b**
2*sqrt(1 + b*x**3/a)) - B*sqrt(a)*e**(7/2)*x**(9/2)/(12*b*sqrt(1 + b*x**3/a
)) + B*a**2*e**(7/2)*asinh(sqrt(b)*x**(3/2)/sqrt(a))/(4*b**(5/2)) + B*e**(7
/2)*x**(15/2)/(6*sqrt(a)*sqrt(1 + b*x**3/a))
```

3.544 $\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=286

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e^2\sqrt{ex}\sqrt{a+}}{}$$

```
[Out] 1/5*B*(e*x)^(7/2)*(b*x^3+a)^(1/2)/b/e+1/20*(10*A*b-7*B*a)*e^2*(e*x)^(1/2)*(
b*x^3+a)^(1/2)/b^2-1/120*a^(2/3)*(10*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),
1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^2/(b*x^3+a)^(1/2)/(b^
(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 321, 329, 225}

$$-\frac{a^{2/3}e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e^2\sqrt{ex}\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] ((10*A*b - 7*a*B)*e^2*Sqrt[e*x]*Sqrt[a + b*x^3])/(20*b^2) + (B*(e*x)^(7/2)*
Sqrt[a + b*x^3])/(5*b*e) - (a^(2/3)*(10*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^
2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(-5Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\sqrt{a+bx^3}}\,dx}{5b}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+b}}}{40b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{\left(a(10Ab-7aB)e^2\right)\text{Subst}\Bigg(\int}{20b^2}$$

$$= \frac{(10Ab-7aB)e^2\sqrt{ex}\sqrt{a+bx^3}}{20b^2} + \frac{B(ex)^{7/2}\sqrt{a+bx^3}}{5be} - \frac{a^{2/3}(10Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] time = 0.15, size = 98, normalized size = 0.34

$$\frac{e^2\sqrt{ex}\left(a\sqrt{\frac{bx^3}{a}+1}\,(7aB-10Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-\left(a+bx^3\right)\left(7aB-10Ab-4bBx^3\right)\right)}{20b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (e^2*Sqrt[e*x]*(-((a + b*x^3)*(-10*A*b + 7*a*B - 4*b*B*x^3)) + a*(-10*A*b +
 7*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])
)/(20*b^2*Sqrt[a + b*x^3])
```

fricas [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Be^2x^5+Ae^2x^2\right)\sqrt{ex}}{\sqrt{bx^3+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

maple [C] time = 1.03, size = 3723, normalized size = 13.02

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
[Out] 1/20*e^2*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b^3/(-a*b^2)^(1/3)*(-40*I*A*3^(1/2)*(-
(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*(-a*b^2)^(1/3)*x*a*b^2*e+4*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1
/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^3*b
^2+20*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^3*e-7*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*
b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2
)*a*b-14*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2*e-20*A*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^3*e+28*
I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^2*b*e+14*B*(-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a^2*b^2*e
+40*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
```

```
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a*b^2*e+10*I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-
a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1
/2)*b^2-28*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))
/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a^2*b*e-20*A*(-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/
3)*a*b*e+14*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*e-12*B*(1/b^2*e*x*(-b*x+(-a*b^2)
^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*x^
3*b^2-14*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*a^2*e+20*I*A*3^(1/2)*(-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*(-a*b^2)^(2/3)*a*b*e-30*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*b^2+21*B*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a)
)^(1/2)*a*b)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^
(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^
(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,{\left(e\,x\right)}^{5/2}}{\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(1/2), x)
```

sympy [C] time = 32.07, size = 94, normalized size = 0.33

$$\frac{Ae^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{13}{6}\right)}+\frac{Be^{\frac{5}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\Gamma\left(\frac{19}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(5/2)*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(13/6)) + B*e**(5/2)*x**(13/2)*gamma(13/6)*hyper((
1/2, 13/6), (19/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(19/6))
```

3.545 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=543

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\ \sqrt[4]{}}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

```
[Out] 1/4*B*(e*x)^(5/2)*(b*x^3+a)^(1/2)/b/e+1/8*(8*A*b-5*B*a)*e*(1+3^(1/2))*(e*x)
^(1/2)*(b*x^3+a)^(1/2)/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-1/8*3^(1/4)*
a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2))
)*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2
)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*
x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/48*a^(1/3)*(8*A*b-5*B*a)*e*(a^(1/3)+
b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^
(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/
4)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {459, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\ \sqrt[4]{}}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(5/2)*Sqrt[a + b*x^3])/(4*b*e) + ((1 + Sqrt[3])*(8*A*b - 5*a*B)*e*
Sqrt[e*x]*Sqrt[a + b*x^3])/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x))
- (3^(1/4)*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (
(1 - Sqrt[3])*a^(1/3)*(8*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a +
 b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx = \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{\left(-4Ab+\frac{5aB}{2}\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{4b}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{(8Ab-5aB)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{4be}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} - \frac{(8Ab-5aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{8b^{5/3}e} - \frac{\left(\left(1-\right.\right.}{}$$

$$= \frac{B(ex)^{5/2}\sqrt{a+bx^3}}{4be} + \frac{\left(1+\sqrt{3}\right)(8Ab-5aB)e\sqrt{ex}\sqrt{a+bx^3}}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\sqrt[3]{a}\,(8Ab-5aB)e}{}$$

Mathematica [C] time = 0.13, size = 80, normalized size = 0.15

$$\frac{x(ex)^{3/2}\left(\sqrt{\frac{bx^3}{a}+1}\,(8Ab-5aB)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+5B\left(a+bx^3\right)\right)}{20b\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (x*(e*x)^(3/2)*(5*B*(a + b*x^3) + (8*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hyper
geometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(20*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bex^4+Aex\right)\sqrt{ex}}{\sqrt{bx^3+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

maple [C] time = 1.09, size = 4914, normalized size = 9.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x)
```

```
[Out] 1/4*e*(e*x)^(1/2)*(b*x^3+a)^(1/2)*(15*B*(-a*b^2)^(1/3)*x^2*a*b*e+8*I*A*3^(1
/2)*x^3*b^3*e+8*I*A*3^(1/2)*(-a*b^2)^(1/3)*x^2*b^2*e+8*I*A*3^(1/2)*(-a*b^2)
^(2/3)*x*b*e-5*I*B*3^(1/2)*(-a*b^2)^(2/3)*x*a*e+I*B*3^(1/2)*(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*x^2*b^2
-5*I*B*3^(1/2)*x^3*a*b^2*e-24*A*x^3*b^3*e+48*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*b
*e+20*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*a*e-30*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*a*e+
16*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1
/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*b^2*e-24*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
E((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*b
^2*e-32*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*b*e+8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(
1/3)*x^2*b^2*e-16*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x*b*e+10*I*B*3^(1/2)
*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-
3))^(1/2))*(-a*b^2)^(2/3)*x*a*e-10*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
```

```
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*a*b*e+15*
B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(-a*b^2)^(1/3)*x^2*a*b*e-8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e+5*I*B*
3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*a^2*b*e+10*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)
-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2
)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e-15*B*(-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+
(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*
b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticE((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b*e-16*
A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*a*b^2*e+24*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b^2*e-5*I*B*3^(1/2)*(-a*b^2)^(1/3)*x^2
*a*b*e-5*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*a*b*e+15*B*x^3*a*b^2*e-24
*A*(-a*b^2)^(1/3)*x^2*b^2*e-24*A*(-a*b^2)^(2/3)*x*b*e+15*B*(-a*b^2)^(2/3)*x
*a*e-3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(e*x*
(b*x^3+a))^(1/2)*x^2*b^2)/b^3/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*
x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*
3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{3}{2}}}{\sqrt{bx^3 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{3/2}}{\sqrt{b\,x^3+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(1/2), x)
```

sympy [C] time = 10.95, size = 94, normalized size = 0.17

$$\frac{Ae^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|{\frac{bx^{3}e^{i\pi}}{a}}\right)}{3\sqrt{a}\,\Gamma\left(\frac{11}{6}\right)}+\frac{Be^{\frac{3}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{6}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{11}{6}\\ \frac{17}{6}\end{matrix}\middle|{\frac{bx^{3}e^{i\pi}}{a}}\right)}{3\sqrt{a}\,\Gamma\left(\frac{17}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*e**(3/2)*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (11/6,), b*x**3*exp_polar(
I*pi)/a)/(3*sqrt(a)*gamma(11/6)) + B*e**(3/2)*x**(11/2)*gamma(11/6)*hyper((
1/2, 11/6), (17/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*gamma(17/6))
```

3.546 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=83

$$\frac{\sqrt{e}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

```
[Out] 1/3*(2*A*b-B*a)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2
)/b^(3/2)+1/3*B*(e*x)^(3/2)*(b*x^3+a)^(1/2)/b/e
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {459, 329, 275, 217, 206}

$$\frac{\sqrt{e}\,(2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}+\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (B*(e*x)^(3/2)*Sqrt[a + b*x^3])/(3*b*e) + ((2*A*b - a*B)*Sqrt[e]*ArcTanh[(S
qrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\sqrt{a+bx^3}}\,dx=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}-\frac{\left(-3Ab+\frac{3aB}{2}\right)\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{3b}$$

$$=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}$$

$$=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}$$

$$=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\,\text{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}$$

$$=\frac{B(ex)^{3/2}\sqrt{a+bx^3}}{3be}+\frac{(2Ab-aB)\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Mathematica [**A**] time = 0.06, size = 78, normalized size = 0.94

$$\frac{\sqrt{ex}\left((2Ab-aB)\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)+\sqrt{b}\,Bx^{3/2}\sqrt{a+bx^3}\right)}{3b^{3/2}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/Sqrt[a + b*x^3],x]
```

```
[Out] (Sqrt[e*x]*(Sqrt[b]*B*x^(3/2)*Sqrt[a + b*x^3] + (2*A*b - a*B)*ArcTanh[(Sqrt
[b]*x^(3/2))/Sqrt[a + b*x^3]]))/(3*b^(3/2)*Sqrt[x])
```

fricas [**A**] time = 1.89, size = 184, normalized size = 2.22

$$\left[\frac{4\sqrt{bx^3+a}\,\sqrt{ex}\,Bx-(Ba-2\,Ab)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)}{12\,b},\ \frac{2\,\sqrt{bx}\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^3 + a)*sqrt(e*x)*B*x - (B*a - 2*A*b)*sqrt(e/b)*log(-8*b^2
*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sqrt(e
*x)*sqrt(e/b)))/b, 1/6*(2*sqrt(b*x^3 + a)*sqrt(e*x)*B*x + (B*a - 2*A*b)*sqr
t(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)
))/b]
```

giac [**A**] time = 0.36, size = 72, normalized size = 0.87

$$\frac{\sqrt{bx^3e^4+ae^4}\,Bx^{\frac{3}{2}}e^{\left(-\frac{3}{2}\right)}}{3\,b}+\frac{\left(Bae^5-2\,Abe^5\right)e^{\left(-\frac{9}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3*e^4 + a*e^4)*B*x^(3/2)*e^(-3/2)/b + 1/3*(B*a*e^5 - 2*A*b*e^5
)*e^(-9/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(3/2)
```

maple [C] time = 1.15, size = 6424, normalized size = 77.40

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)\sqrt{ex}}{\sqrt{bx^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*sqrt(e*x)/sqrt(b*x^3 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(B\,x^3 + A)\,\sqrt{e\,x}}{\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(1/2), x)
```

sympy [A] time = 6.81, size = 107, normalized size = 1.29

$$\frac{2A\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3\sqrt{b}} + \frac{B\sqrt{a}\,(ex)^{\frac{3}{2}}\sqrt{1 + \frac{bx^3}{a}}}{3be} - \frac{Ba\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,(ex)^{\frac{3}{2}}}{\sqrt{a}\,e^{\frac{3}{2}}}\right)}{3b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] 2*A*sqrt(e)*asinh(sqrt(b)*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*sqrt(b)) + B*
sqrt(a)*(e*x)**(3/2)*sqrt(1 + b*x**3/a)/(3*b*e) - B*a*sqrt(e)*asinh(sqrt(b)
*(e*x)**(3/2)/(sqrt(a)*e**(3/2)))/(3*b**(3/2))
```

3.547 $\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=249

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

```
[Out] 1/2*B*(e*x)^(1/2)*(b*x^3+a)^(1/2)/b/e+1/12*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)
*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b/e/(b*x^3+a
)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A] time = 0.18, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {459, 329, 225}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*Sqrt[e*x]*Sqrt[a + b*x^3])/(2*b*e) + ((4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) +
 b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1
+ Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*
x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*a^(1/
3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\sqrt{a+bx^3}}\,dx = \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} - \frac{\left(-2Ab+\frac{aB}{2}\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{2b}$$

$$= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{2be}$$

$$= \frac{B\sqrt{ex}\sqrt{a+bx^3}}{2be} + \frac{(4Ab-aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\right.}{\sqrt[3]{a}+\left(1+\right.}\right.\right.}{4\sqrt[4]{3}\sqrt[3]{a}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.10, size = 80, normalized size = 0.32

$$\frac{x\sqrt{\frac{bx^3}{a}+1}\,(4Ab-aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+Bx\left(a+bx^3\right)}{2b\sqrt{ex}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*Sqrt[a + b*x^3]),x]
```

```
[Out] (B*x*(a + b*x^3) + (4*A*b - a*B)*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/
6, 1/2, 7/6, -((b*x^3)/a)])/(2*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\sqrt{ex}}{bex^4+aex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e*x^4 + a*e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

maple [C] time = 1.09, size = 3275, normalized size = 13.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^3+a)^(1/2)*x/b^2/(-a*b^2)^(1/3)*(8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*b^3*
e-2*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/
2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2*e-16*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/
3)*x*b^2*e+4*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x*a*b*e+8*I*A*3^(1/2)*(-(
I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(-a*b^2)^(2/3)*b*e-8*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(
1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*b^3*e-2*I*B*3^(1/2)*(-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
-a*b^2)^(2/3)*a*e+2*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2*e+16*A*(-a*b^2)^(1/3)*(-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x
*b^2*e-4*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
```

```
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x
*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b*e-8*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)
-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*b*
e+2*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*e-I*B*3^(1/2)*(e*x*(b*x^3+a))^(1/2)*(1/b^2
*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*b+3*B
*(e*x*(b*x^3+a))^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
))^(1/2)*(-a*b^2)^(1/3)*b)/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/
(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(1/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*sqrt(e*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{\sqrt{e\,x}\,\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(1/2)), x)
```

sympy [C] time = 3.54, size = 94, normalized size = 0.38

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6}, \frac{1}{2} \\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)} + \frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{7}{6} \\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(1/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*sqrt(a)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((1/2, 7/6), (13/
6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*sqrt(e)*gamma(13/6))
```

3.548 $\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=542

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{ex}\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

```
[Out] -2*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(1/2)+(2*A*b+B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b
*x^3+a)^(1/2)/a/b^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-3^(1/4)*(2*A*b+
B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*
x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-
a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(
2/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*(2*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)
+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/
3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^
(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b^(2/3)/e
^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {453, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}a^{2/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{\sqrt[4]{3}\sqrt{ex}\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(a*e*Sqrt[e*x]) + ((1 + Sqrt[3])*(2*A*b + a*B)*Sqrt[
e*x]*Sqrt[a + b*x^3])/(a*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) -
 (3^(1/4)*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(
1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellipt
icE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(
1/3)*x)], (2 + Sqrt[3])/4])/(a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) +
 b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1
- Sqrt[3])*(2*A*b + a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ellip
ticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^
(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*a^(2/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x
^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2Ab+aB)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{(2(2Ab+aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} - \frac{(2Ab+aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{ab^{2/3}e^4} - \frac{\left(\left(1-\sqrt{3}\right)(2A\right.}{}$$

$$= -\frac{2A\sqrt{a+bx^3}}{ae\sqrt{ex}} + \frac{\left(1+\sqrt{3}\right)(2Ab+aB)\sqrt{ex}\sqrt{a+bx^3}}{ab^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \frac{\sqrt[4]{3}\,(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{a^{2/3}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 0.15

$$\frac{x\left(2x^3\sqrt{\frac{bx^3}{a}+1}\,(aB+2Ab)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)-10A\left(a+bx^3\right)\right)}{5a(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (x*(-10*A*(a + b*x^3) + 2*(2*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeomet
ric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)]))/(5*a*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{be^2x^5+ae^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^2*x^5 + a*e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

maple [C] time = 1.02, size = 5385, normalized size = 9.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{(e\,x)^{3/2}\,\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(1/2)), x)
```

sympy [C] time = 4.27, size = 97, normalized size = 0.18

$$\frac{A\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{6},\frac{1}{2}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 1/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((1/2, 5/6), (
11/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(3/2)*gamma(11/6))
```

3.549 $\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

```
[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/e^(5/2)/b^(1/2)-
2/3*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(3/2)
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {451, 329, 275, 217, 206}

$$\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}-\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(3*a*e*(e*x)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*(e*x)^(3
/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*Sqrt[b]*e^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{B\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{e^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{(2B)\,\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3e^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{3ae(ex)^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3\sqrt{b}\,e^{5/2}}$$

Mathematica [A] time = 0.06, size = 65, normalized size = 0.87

$$\frac{2x\left(\frac{Bx^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a+bx^3}}\right)}{\sqrt{b}}-\frac{A\sqrt{a+bx^3}}{a}\right)}{3(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*x*(-((A*Sqrt[a + b*x^3])/a) + (B*x^(3/2)*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[
a + b*x^3]])/Sqrt[b]))/(3*(e*x)^(5/2))
```

fricas [A] time = 1.50, size = 183, normalized size = 2.44

$$\left[\frac{\sqrt{be}\,Bax^2\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,bx^4+ax\right)\sqrt{bx^3+a}\,\sqrt{be}\,\sqrt{ex}\right)-4\,\sqrt{bx^3+a}\,\sqrt{ex}\,Ab}{6\,abe^3x^2},-\frac{\sqrt{-be}\,Ba\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b*e)*B*a*x^2*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b*x^4
 + a*x)*sqrt(b*x^3 + a)*sqrt(b*e)*sqrt(e*x)) - 4*sqrt(b*x^3 + a)*sqrt(e*x)*
A*b)/(a*b*e^3*x^2), -1/3*(sqrt(-b*e)*B*a*x^2*arctan(2*sqrt(b*x^3 + a)*sqrt(
-b*e)*sqrt(e*x)*x/(2*b*e*x^3 + a*e)) + 2*sqrt(b*x^3 + a)*sqrt(e*x)*A*b)/(a*
b*e^3*x^2)]
```

giac [A] time = 0.26, size = 110, normalized size = 1.47

$$-\frac{2}{3}\left(\left(\frac{B\arctan\left(\frac{\sqrt{be+\frac{ae}{x^3}}}{\sqrt{-be}}\right)}{\sqrt{-be}}+\frac{\sqrt{be+\frac{ae}{x^3}}\,Ae^{(-1)}}{a}\right)e^{(-1)}-\frac{\left(Ba\arctan\left(\frac{\sqrt{b}\,e^{\frac{1}{2}}}{\sqrt{-be}}\right)e+\sqrt{-be}\,A\sqrt{b}\,e^{\frac{1}{2}}\right)e^{(-2)}}{\sqrt{-be}\,a}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*((B*arctan(sqrt(b*e + a*e/x^3)/sqrt(-b*e))/sqrt(-b*e) + sqrt(b*e + a*e
/x^3)*A*e^(-1)/a)*e^(-1) - (B*a*arctan(sqrt(b)*e^(1/2)/sqrt(-b*e))*e + sqrt
(-b*e)*A*sqrt(b)*e^(1/2))*e^(-2)/(sqrt(-b*e)*a))*e^(-1)
```

maple [C] time = 1.00, size = 3397, normalized size = 45.29

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)^(1/2)/x/b^2*(6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e-6*I*B*3^(1/
2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/
2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4*a*b^2*e-12*I*B*3^(1/2)*(-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(1/3)*x^3*a*b*e+12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticP
i((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)
-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2
*a*e-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^
2*a*e-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*x^4*a*b^2*e+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3
```

```
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
x^4*a*b^2*e+12*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e-12*B*(-(I*3^(1/2)-3)*x*
b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b
*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(
-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1
/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^3*a*b*e-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*
x^2*a*e+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^2*a*e+I
*A*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)*b^2-3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*
b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(
1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*b^2)/e^2/(e*x)^(1/2)/(e*x*(b*x^3+a))^(1/
2)/a/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{B\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}{\sqrt{b}+\frac{\sqrt{bx^3+a}}{x^{\frac{3}{2}}}}\right)}{3\sqrt{b}}}{e^{\frac{5}{2}}} - \frac{2\left(b\sqrt{e}x^4 + a\sqrt{e}x\right)A}{3\sqrt{bx^3+a}\,ae^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*integrate(sqrt(x)/sqrt(b*x^3 + a), x)/e^(5/2) - 2/3*(b*sqrt(e)*x^4 + a*sq
rt(e)*x)*A/(sqrt(b*x^3 + a)*a*e^3*x^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{5/2}\,\sqrt{b\,x^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(1/2)), x)
```

sympy [A] time = 10.11, size = 60, normalized size = 0.80

$$-\frac{2A\sqrt{b}\sqrt{\frac{a}{bx^3}+1}}{3ae^{\frac{5}{2}}}+\frac{2B\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3\sqrt{b}\,e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] -2*A*sqrt(b)*sqrt(a/(b*x**3) + 1)/(3*a*e**(5/2)) + 2*B*asinh(sqrt(b)*x**(3/
2)/sqrt(a))/(3*sqrt(b)*e**(5/2))
```

3.550 $\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

```
[Out] -2/5*A*(b*x^3+a)^(1/2)/a/e/(e*x)^(5/2)-1/15*(2*A*b-5*B*a)*(a^(1/3)+b^(1/3)*
x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1
/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipti
cF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^
(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2
/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/e^4/(b*x^
3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2
)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 329, 225}

$$-\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*A*Sqrt[a + b*x^3])/(5*a*e*(e*x)^(5/2)) - ((2*A*b - 5*a*B)*Sqrt[e*x]*(a^
(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b
^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4
)*a^(4/3)*e^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\sqrt{a+bx^3}}\,dx = -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{5ae^3}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2(2Ab-5aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{5ae^4}$$

$$= -\frac{2A\sqrt{a+bx^3}}{5ae(ex)^{5/2}} - \frac{(2Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right.}{\sqrt[3]{a}+\left(1+\sqrt{3}\right.}\right.\right.}{5\sqrt[4]{3}\,a^{4/3}e^4\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.06, size = 82, normalized size = 0.33

$$-\frac{2x\left(x^3\sqrt{\frac{bx^3}{a}+1}\,(2Ab-5aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+A\left(a+bx^3\right)\right)}{5a(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*Sqrt[a + b*x^3]),x]
```

```
[Out] (-2*x*(A*(a + b*x^3) + (2*A*b - 5*a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometr
ic2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(5*a*(e*x)^(7/2)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{be^4x^7+ae^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b*e^4*x^7 + a*e^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\sqrt{bx^3+a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="giac")
```

[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)

maple [C] time = 1.07, size = 3303, normalized size = 13.43

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x)

[Out] 2/5*(b*x^3+a)^(1/2)/x^2/(-a*b^2)^(1/3)/b/a*(10*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^2*e+10*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*a*e-4*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b^3*e+3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*(e*x*(b*x^3+a))^(1/2)*b-4*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*b*e+8*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*b^2*e-20*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*a*b*e+4*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b^3*e-10*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*a*b^2*e-8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))

```
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-
a*b^2)^(1/3)*x^4*b^2*e+20*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/
(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x
-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I
*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^4*a*b*e+4*
I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3*b*e-10*I*B*3^(1/2)*(-(I*3^(1/2)-3)
*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b
^2)^(2/3)*x^3*a*e-I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(
-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2
)^(1/3)))^(1/2)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*b)/e^3/(e*x)^(1/2)/(e*
x*(b*x^3+a))^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\sqrt{bx^3 + a}\,(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/(sqrt(b*x^3 + a)*(e*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{(e\,x)^{7/2}\,\sqrt{b\,x^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(1/2)), x)
```

sympy [C] time = 32.02, size = 97, normalized size = 0.39

$$\frac{A\Gamma\left(-\frac{5}{6}\right){}_2F_1\left(\left.\begin{matrix}-\frac{5}{6}, \frac{1}{2}\\ \frac{1}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{1}{6}\right)} + \frac{B\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{6}, \frac{1}{2}\\ \frac{7}{6}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt{a}\,e^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(1/2),x)
```

```
[Out] A*gamma(-5/6)*hyper((-5/6, 1/2), (1/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(
a)*e**(7/2)*x**(5/2)*gamma(1/6)) + B*sqrt(x)*gamma(1/6)*hyper((1/6, 1/2), (
7/6,), b*x**3*exp_polar(I*pi)/a)/(3*sqrt(a)*e**(7/2)*gamma(7/6))
```

3.551 $\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

```
[Out] 1/3*(2*A*b-3*B*a)*e^(7/2)*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/
2))/b^(5/2)-1/3*(2*A*b-3*B*a)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a)^(1/2)+1/3*B*(e*
x)^(9/2)/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {459, 288, 329, 275, 217, 206}

$$-\frac{e^2(ex)^{3/2}(2Ab-3aB)}{3b^2\sqrt{a+bx^3}}+\frac{e^{7/2}(2Ab-3aB)\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}+\frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -((2*A*b - 3*a*B)*e^2*(e*x)^(3/2))/(3*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(9/2)
)/(3*b*e*Sqrt[a + b*x^3]) + ((2*A*b - 3*a*B)*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)
^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} - \frac{\left(-3Ab+\frac{9aB}{2}\right)\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{3b}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{2b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx, x, (ex)^{3/2}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{\left((2Ab-3aB)e^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx, x, \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}$$

$$= -\frac{(2Ab-3aB)e^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}} + \frac{B(ex)^{9/2}}{3be\sqrt{a+bx^3}} + \frac{(2Ab-3aB)e^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Mathematica [A] time = 0.18, size = 109, normalized size = 0.91

$$\frac{e^3\sqrt{ex}\left(\sqrt{b}\,x^{3/2}\left(3aB-2Ab+bBx^3\right)-\sqrt{a}\sqrt{\frac{bx^3}{a}+1}\,(3aB-2Ab)\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{x}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-2*A*b + 3*a*B + b*B*x^3) - Sqrt[a]*(-2*A*
b + 3*a*B)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*b^(5
/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

fricas [A] time = 1.99, size = 307, normalized size = 2.56

$$\left[-\frac{\left(\left(3\,Bab-2\,Ab^2\right)e^3x^3+\left(3\,Ba^2-2\,Aab\right)e^3\right)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\right.}{12\left(b^3x^3+ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3*B*a^2 - 2*A*a*b)*e^3)*sqrt(e/b)*l
og(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 +
a)*sqrt(e*x)*sqrt(e/b)) - 4*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(b*x^
3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2), 1/6*(((3*B*a*b - 2*A*b^2)*e^3*x^3 + (3
*B*a^2 - 2*A*a*b)*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a)*sqrt(e*x)*b*x*sq
rt(-e/b)/(2*b*e*x^3 + a*e)) + 2*(B*b*e^3*x^4 + (3*B*a - 2*A*b)*e^3*x)*sqrt(
b*x^3 + a)*sqrt(e*x))/(b^3*x^3 + a*b^2)]
```

giac [A] time = 0.37, size = 107, normalized size = 0.89

$$\frac{\left(\frac{Bx^3e^4}{b} + \frac{3\,Bab^3e^4 - 2\,Ab^4e^4}{b^5}\right)x^{\frac{3}{2}}e^{\frac{3}{2}}}{3\,\sqrt{bx^3e^4 + ae^4}} + \frac{\left(3\,Bab^3e^4 - 2\,Ab^4e^4\right)e^{\left(-\frac{1}{2}\right)}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2 + \sqrt{bx^3e^4 + ae^4}\right|\right)}{3\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3*e^4/b + (3*B*a*b^3*e^4 - 2*A*b^4*e^4)/b^5)*x^(3/2)*e^(3/2)/sqrt(
b*x^3*e^4 + a*e^4) + 1/3*(3*B*a*b^3*e^4 - 2*A*b^4*e^4)*e^(-1/2)*log(abs(-sq
rt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(11/2)
```

maple [C] time = 1.13, size = 7016, normalized size = 58.47

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{7}{2}}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3 + A\right)\,{(e\,x)}^{7/2}}{{\left(b\,x^3 + a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

3.552 $\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{e^2\sqrt{ex}(4Ab-7a}{6b^2\sqrt{a+bx^3}}$$

```
[Out] 1/2*B*(e*x)^(7/2)/b/e/(b*x^3+a)^(1/2)-1/6*(4*A*b-7*B*a)*e^2*(e*x)^(1/2)/b^2
/(b*x^3+a)^(1/2)+1/36*(4*A*b-7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3)/b^2/(b*x^3+a)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {459, 288, 329, 225}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-7aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{12\sqrt[4]{3}\sqrt[3]{a}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{e^2\sqrt{ex}(4Ab-7a}{6b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] -((4*A*b - 7*a*B)*e^2*Sqrt[e*x])/(6*b^2*Sqrt[a + b*x^3]) + (B*(e*x)^(7/2))/
(2*b*e*Sqrt[a + b*x^3]) + ((4*A*b - 7*a*B)*e^2*Sqrt[e*x]*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(12*3^(1/4)*a^(1/3)*b^2*
Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^
2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} - \frac{\left(-2Ab+\frac{7aB}{2}\right)\int \frac{(ex)^{5/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{2b}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{\left((4Ab-7aB)e^3\right)\int \frac{1}{\sqrt{ex}\,\sqrt{a+bx^3}}\,dx}{12b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{\left((4Ab-7aB)e^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{6b^2}$$

$$= -\frac{(4Ab-7aB)e^2\sqrt{ex}}{6b^2\sqrt{a+bx^3}} + \frac{B(ex)^{7/2}}{2be\sqrt{a+bx^3}} + \frac{(4Ab-7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x \cdots}{\left(\sqrt[3]{a}+\left(1+\sqrt{3} \cdots\right.\right.}}}{12\sqrt[4]{3}\,\sqrt[3]{a}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\cdots\right.}{\left(\sqrt[3]{a}+\left(1 \cdots\right.\right.}}}$$

Mathematica [C] time = 0.17, size = 87, normalized size = 0.30

$$\frac{e^2\sqrt{ex}\left(\sqrt{\frac{bx^3}{a}+1}\,(4Ab-7aB)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+7aB-4Ab+3bBx^3\right)}{6b^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (e^2*Sqrt[e*x]*(-4*A*b + 7*a*B + 3*b*B*x^3 + (4*A*b - 7*a*B)*Sqrt[1 + (b*x^
3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(6*b^2*Sqrt[a + b*x^
3])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Be^2x^5+Ae^2x^2\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b
*x^3 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^{\frac{5}{2}}}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

maple [C] time = 1.08, size = 3760, normalized size = 13.15

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
```

```
[Out] -1/6*e^2/x*(e*x)^(1/2)*(-14*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(
I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^2
*a*b^2-3*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(
1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))
^(1/2)*(-a*b^2)^(1/3)*x^4*b^2+8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF
((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)
+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)
*(-a*b^2)^(2/3)*b-7*I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)
*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*a*b+8*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+
a))^(1/2)*x^2*b^3-8*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^2*b^3-14*I*B*3^(1/2
)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(2/3)*a+4*I*A*3^(1/2)*(1/b^2*e*x
```

```
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*b^2+14*
B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2
)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^2*a*b^2+16*A*(-a*b^2)^(1/3)*(-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(
e*x*(b*x^3+a))^(1/2)*x*b^2-28*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2
)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1
/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1
/2)*x*a*b-8*A*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*b+14*B*(-a*b^2
)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*
3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3
^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*a+9*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^4*b^2+28*I*B*3^(1/2)*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x*a*b-16*I*A*3^(1/2)*(-(I*3^(1/2)
-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(
1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e
*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x*b^2-12*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*b^2+21*B*(1/b^2*e*x*(-b*x+
(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x*a*b)/(b*x^3+a)
^(1/2)/b^3/(-a*b^2)^(1/3)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I
*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b
*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{5/2}}{{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

3.553 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e\sqrt{ex}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(1/2)-1/3*(2*A*b-5*B*a)*e*(1+3^(1
/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+
1/3*(2*A*b-5*B*a)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(
1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2)*3^(1/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/18*(2*A*b-5*B*a)*e*(a^(1
/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/a^(2/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1
/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {457, 329, 308, 225, 1881}

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(2Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{6\sqrt[4]{3}a^{2/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{e\sqrt{ex}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(3*a*b*e*Sqrt[a + b*x^3]) - ((1 + Sqrt[3])*(2*A
*b - 5*a*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(3*a*b^(5/3)*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)) + ((2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(3/4)*a^(2/3)*b^(5/3)*Sqrt[(b^(
1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
 + b*x^3]) + ((1 - Sqrt[3])*(2*A*b - 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(6*3^(1/4)*a^(2/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(-Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{(2(2Ab-5aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab-5aB)\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3ab^{5/3}e} + \frac{\left(\left(1-\sqrt{3}\right.\right.}{}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{3abe\sqrt{a+bx^3}} - \frac{\left(1+\sqrt{3}\right)(2Ab-5aB)e\sqrt{ex}\,\sqrt{a+bx^3}}{3ab^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} + \frac{(2Ab-5aB)e\sqrt{ex}\,\left(\sqrt[3]{a}+\right.}{}$$

Mathematica [C] time = 0.12, size = 77, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(\sqrt{\frac{bx^3}{a}+1}\,(2Ab-5aB)\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)+5aB\right)}{5ab\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(5*a*B + (2*A*b - 5*a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2
F1[5/6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a*b*Sqrt[a + b*x^3])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bex^4+Aex\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*x^6 + 2*a*b*x^3 +
 a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{3}{2}}}{\left(bx^3+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

maple [C] time = 1.03, size = 5392, normalized size = 9.75

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Bx^3 + A)(ex)^{\frac{3}{2}}}{(bx^3 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(B\,x^3 + A)\,(e\,x)^{3/2}}{(b\,x^3 + a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(3/2),x)
[Out] Timed out
```

3.554 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

```
[Out] 2/3*B*arctanh((e*x)^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))*e^(1/2)/b^(3/2)+
2/3*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {452, 329, 275, 217, 206}

$$\frac{2(ex)^{3/2}(Ab-aB)}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(3*a*b*e*Sqrt[a + b*x^3]) + (2*B*Sqrt[e]*ArcTan
h[(Sqrt[b]*(e*x)^(3/2))/(e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{3/2}}\,dx=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{B\int\frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{(2B)\,\text{Subst}\left(\int\frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3be}$$

$$=\frac{2(Ab-aB)(ex)^{3/2}}{3abe\sqrt{a+bx^3}}+\frac{2B\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{3/2}}$$

Mathematica [A] time = 0.10, size = 93, normalized size = 1.09

$$\frac{2\sqrt{ex}\left(a^{3/2}B\sqrt{\frac{bx^3}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}(Ab-aB)\right)}{3ab^{3/2}\sqrt{x}\,\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[e*x]*(Sqrt[b]*(A*b - a*B)*x^(3/2) + a^(3/2)*B*Sqrt[1 + (b*x^3)/a]*A
rcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]]))/(3*a*b^(3/2)*Sqrt[x]*Sqrt[a + b*x^3])
```

fricas [A] time = 1.30, size = 234, normalized size = 2.75

$$\left[-\frac{4\sqrt{bx^3+a}\,(Ba-Ab)\sqrt{ex}\,x-\left(Babx^3+Ba^2\right)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\right.}{6\left(ab^2x^3+a^2b\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(4*sqrt(b*x^3 + a)*(B*a - A*b)*sqrt(e*x)*x - (B*a*b*x^3 + B*a^2)*sqrt
(e/b)*log(-8*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b
*x^3 + a)*sqrt(e*x)*sqrt(e/b)))/(a*b^2*x^3 + a^2*b), -1/3*(2*sqrt(b*x^3 + a
)*(B*a - A*b)*sqrt(e*x)*x + (B*a*b*x^3 + B*a^2)*sqrt(-e/b)*arctan(2*sqrt(b*
x^3 + a)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)))/(a*b^2*x^3 + a^2*b)]
```

giac [A] time = 0.62, size = 75, normalized size = 0.88

$$-\frac{2\,Be^{\frac{1}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{3\,b^{\frac{3}{2}}}-\frac{2\,(Bae-Abe)x^{\frac{3}{2}}e^{\frac{3}{2}}}{3\,\sqrt{bx^3e^4+ae^4}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*B*e^(1/2)*log(abs(-sqrt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(
3/2) - 2/3*(B*a*e - A*b*e)*x^(3/2)*e^(3/2)/(sqrt(b*x^3*e^4 + a*e^4)*a*b)
```

maple [C] time = 1.00, size = 3654, normalized size = 42.99

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/3*(e*x)^(1/2)/(b*x^3+a)^(1/2)/b^3*(-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^
2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*E
llipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I
*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a
))^(1/2)*x^2*a*b^2-12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I
*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a
*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(
1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e
*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x*a*b+12*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*
x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-
a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x
^3+a))^(1/2)*(-a*b^2)^(1/3)*x*a*b-6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(
1/2)*(-a*b^2)^(2/3)*a+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3
^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(e*x*(b*x^3+a))^(1/2)*x^2*a*b^2+I*A*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)
-2*b*x-(-a*b^2)^(1/3)))^(1/2)*x^2*b^3+6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x
^2*a*b^2-6*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x
^2*a*b^2-12*B*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1
```

```
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*a*b+12*B*(-a
*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*
a*b-I*B*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*x^2*a*b^2+6*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^
3+a))^(1/2)*(-a*b^2)^(2/3)*a+6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(
1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellip
ticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(
1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(
1/2)*a-6*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2)
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)*
x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^
3+a))^(1/2)*a-3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3
)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1
/2)*x^2*b^3+3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+
2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2
)*x^2*a*b^2)/x/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{ex}}{\left(bx^3 + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(B\,x^3 + A\right)\,\sqrt{e\,x}}{\left(b\,x^3 + a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2),x)
[Out] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(3/2), x)
```

sympy [A] time = 33.62, size = 95, normalized size = 1.12

$$\frac{2A\sqrt{e}\,x^{\frac{3}{2}}}{3a^{\frac{3}{2}}\sqrt{1+\frac{bx^3}{a}}}+B\left(\frac{2\sqrt{e}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x^{\frac{3}{2}}}{\sqrt{a}}\right)}{3b^{\frac{3}{2}}}-\frac{2\sqrt{e}\,x^{\frac{3}{2}}}{3\sqrt{a}\,b\sqrt{1+\frac{bx^3}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] 2*A*sqrt(e)*x**(3/2)/(3*a**(3/2)*sqrt(1 + b*x**3/a)) + B*(2*sqrt(e)*asinh(s
qrt(b)*x**(3/2)/sqrt(a))/(3*b**(3/2)) - 2*sqrt(e)*x**(3/2)/(3*sqrt(a)*b*sqr
t(1 + b*x**3/a)))
```

3.555 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

```
[Out] 2/3*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(1/2)+1/9*(2*A*b+B*a)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b/e
/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {457, 329, 225}

$$\frac{\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(Ab-aB)}{3abe\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(3*a*b*e*Sqrt[a + b*x^3]) + ((2*A*b + a*B)*Sqrt[e
*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqr
t[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3
*3^(1/4)*a^(4/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
 Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{\left(2\left(Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{3ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2(2Ab+aB))\,\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right)}{3abe}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{3abe\sqrt{a+bx^3}} + \frac{(2Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\right.}{\sqrt[3]{a}+\left(1+\right.}\right.\right.}{3\sqrt[4]{3}\,a^{4/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Mathematica [C] time = 0.06, size = 79, normalized size = 0.31

$$\frac{2x\left(\sqrt{\frac{bx^3}{a}+1}\,(aB+2Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-aB+Ab\right)}{3ab\sqrt{ex}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(3/2)),x]
```

```
[Out] (2*x*(A*b - a*B + (2*A*b + a*B)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6,
1/2, 7/6, -((b*x^3)/a)]))/(3*a*b*Sqrt[e*x]*Sqrt[a + b*x^3])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^2ex^7+2\,abex^4+a^2ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e*x^7 + 2*a*b*e*x^4 + a
^2*e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

maple [C] time = 1.03, size = 3565, normalized size = 13.82

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x)
```

```
[Out] -2/3/(b*x^3+a)^(1/2)/b^2/(-a*b^2)^(1/3)/a*(4*I*A*(-a*b^2)^(2/3)*3^(1/2)*(-(
I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a
*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*(e*x*(b*x^3+a))^(1/2)*b+2*I*B*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2
)*x^2*a*b^2-4*I*B*(-a*b^2)^(1/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3
))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*
b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-
(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)
*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*
a*b+I*B*(-a*b^2)^(1/3)*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^
2)^(1/3)))^(1/2)*x*a*b+4*I*A*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^2*b^
3-4*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^2*b^3-8*I*A*(-a*b^2)^(1/3)*3^(1/2)*
(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*
(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3
))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*b^2-2*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2
*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((
-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3
)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x
^2*a*b^2+8*A*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^
(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)
*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1
)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*b^2+4*B*(-a*b
```

```
^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x*a*b-4*A*(-a*b^2)^(2/3)*(-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*(e*x*(b*x^3+a))^(1/2)*b-2*B*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3
)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elliptic
F((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2
)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2
)*a+2*I*B*(-a*b^2)^(2/3)*3^(1/2)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*a-I*A*(-a*
b^2)^(1/3)*3^(1/2)*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/
3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(
1/2)*x*b^2+3*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
*(-a*b^2)^(1/3)*x*b^2-3*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1
/3)))^(1/2)*(-a*b^2)^(1/3)*x*a*b)/(e*x)^(1/2)/(I*3^(1/2)-3)/(1/b^2*e*x*(-b*
x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2
)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} \sqrt{ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*sqrt(e*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)),x)
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(3/2)), x)
```

sympy [C] time = 72.36, size = 94, normalized size = 0.36

$$\frac{A\sqrt{x}\,\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{3}{2}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{7}{6}\right)}+\frac{Bx^{\frac{7}{2}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{7}{6},\frac{3}{2}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}\sqrt{e}\,\Gamma\left(\frac{13}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(3/2)/(e*x)**(1/2),x)
```

```
[Out] A*sqrt(x)*gamma(1/6)*hyper((1/6, 3/2), (7/6,), b*x**3*exp_polar(I*pi)/a)/(3
*a**(3/2)*sqrt(e)*gamma(7/6)) + B*x**(7/2)*gamma(7/6)*hyper((7/6, 3/2), (13
/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*sqrt(e)*gamma(13/6))
```

3.556 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=585

$$-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}\left(\sqrt[3]{a}\right.\cdots}{\cdots}$$

```
[Out] -2/3*(4*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(1/2)-2*A/a/e/(e*x)^(1/2)/(b
*x^3+a)^(1/2)+2/3*(4*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^2/b
^(2/3)/e^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))-2/3*(4*A*b-B*a)*(a^(1/3)+b^(1/3)
*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*Ellipt
icE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(
2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(5/3)/b^(2/3)/
e^2/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)-1/9*(4*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1
-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(
1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(
1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b^(2/3)/e^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] time = 0.57, antiderivative size = 585, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 308, 225, 1881}

$$\frac{2\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(4Ab-aB)}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(4Ab-aB)F\left(\cos^{-1}\left(\frac{(1-\cdots}{(1+\cdots}\right.\right.}{3\sqrt[4]{3}a^{5/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*Sqrt[a + b*x^3]) - (2*(4*A*b - a*B)*(e*x)^(5/2))/(3*a
^2*e^4*Sqrt[a + b*x^3]) + (2*(1 + Sqrt[3])*(4*A*b - a*B)*Sqrt[e*x]*Sqrt[a +
 b*x^3])/(3*a^2*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (2*(4*A*
b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
+ b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(
1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 +
Sqrt[3])/4])/(3^(3/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3
)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - ((1 - Sqrt[
3])*(4*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(3*3^(1/4)*a^(5/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/
3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}} - \frac{(4Ab-aB)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(2(4Ab-aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{3a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{(4(4Ab-aB))\,\text{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right.}{3a^2e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} - \frac{(2(4Ab-aB))\,\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-2b^{2/3}x}{\sqrt{a+\frac{bx^6}{e^3}}}\right.}{3a^2b^{2/3}e^4}$$

$$= -\frac{2A}{ae\sqrt{ex}\,\sqrt{a+bx^3}} - \frac{2(4Ab-aB)(ex)^{5/2}}{3a^2e^4\sqrt{a+bx^3}} + \frac{2\left(1+\sqrt{3}\right)(4Ab-aB)\sqrt{ex}\,\sqrt{a+bx^3}}{3a^2b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)} - \frac{2}{}$$

Mathematica [C] time = 0.05, size = 77, normalized size = 0.13

$$\frac{x\left(2x^3\sqrt{\frac{bx^3}{a}+1}\,(aB-4Ab)\,{}_2F_1\left(\frac{5}{6},\frac{3}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-10aA\right)}{5a^2(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-10*a*A + 2*(-4*A*b + a*B)*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[5/
6, 3/2, 11/6, -((b*x^3)/a)]))/(5*a^2*(e*x)^(3/2)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^2e^2x^8+2\,abe^2x^5+a^2e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e^2*x^8 + 2*a*b*e^2*x^5
 + a^2*e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

maple [C] time = 1.08, size = 5563, normalized size = 9.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{(e\,x)^{3/2}\,\left(b\,x^3 + a\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(3/2)), x)
```

sympy [C] time = 144.00, size = 97, normalized size = 0.17

$$\frac{A\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{3}{2} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{5}{6}\right)} + \frac{Bx^{\frac{5}{2}}\Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{3}{2} \\ \frac{11}{6} \end{matrix}\middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{3}{2}}e^{\frac{3}{2}}\Gamma\left(\frac{11}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] A*gamma(-1/6)*hyper((-1/6, 3/2), (5/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3
/2)*e**(3/2)*sqrt(x)*gamma(5/6)) + B*x**(5/2)*gamma(5/6)*hyper((5/6, 3/2),
(11/6,), b*x**3*exp_polar(I*pi)/a)/(3*a**(3/2)*e**(3/2)*gamma(11/6))
```

3.557 $\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(1/2)-2/3*(2*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {453, 264}

$$-\frac{2(ex)^{3/2}(2Ab-aB)}{3a^2e^4\sqrt{a+bx^3}}-\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*Sqrt[a + b*x^3]) - (2*(2*A*b - a*B)*(e*x)^(3/2))/
(3*a^2*e^4*Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{3/2}}\,dx &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{(2Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{ae^3}\\ &= -\frac{2A}{3ae(ex)^{3/2}\sqrt{a+bx^3}}-\frac{2(2Ab-aB)(ex)^{3/2}}{3a^2e^4\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{x\left(-2aA+2aBx^3-4Abx^3\right)}{3a^2(ex)^{5/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*a*A - 4*A*b*x^3 + 2*a*B*x^3))/(3*a^2*(e*x)^(5/2)*Sqrt[a + b*x^3])
```

fricas [A] time = 0.91, size = 57, normalized size = 0.85

$$\frac{2\left((Ba-2\,Ab)x^3-Aa\right)\sqrt{bx^3+a}\,\sqrt{ex}}{3\left(a^2be^3x^5+a^3e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*((B*a - 2*A*b)*x^3 - A*a)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b*e^3*x^5 + a^
3*e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

maple [A] time = 0.05, size = 39, normalized size = 0.58

$$-\frac{2\left(2A\,x^3b-Ba\,x^3+Aa\right)x}{3\sqrt{b\,x^3+a}\;(ex)^{\frac{5}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] -2/3*x*(2*A*b*x^3-B*a*x^3+A*a)/(b*x^3+a)^(1/2)/a^2/(e*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(5/2)), x)
```

mupad [B] time = 4.75, size = 70, normalized size = 1.04

$$-\frac{\left(\frac{2\,A}{3\,a\,b\,e^2}+\frac{x^3\,(4\,A\,b-2\,B\,a)}{3\,a^2\,b\,e^2}\right)\sqrt{b\,x^3+a}}{x^4\,\sqrt{e\,x}+\frac{a\,x\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] -(((2*A)/(3*a*b*e^2) + (x^3*(4*A*b - 2*B*a))/(3*a^2*b*e^2))*(a + b*x^3)^(1/
2))/(x^4*(e*x)^(1/2) + (a*x*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

3.558 $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=283

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}(8Ab-5a}{15a^2e^4\sqrt{a+b}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(1/2)-2/15*(8*A*b-5*B*a)*(e*x)^(1/2)/a^2/e
^4/(b*x^3+a)^(1/2)-2/45*(8*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 225}

$$-\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(8Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{15\sqrt[4]{3}a^{7/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2\sqrt{ex}(8Ab-5a}{15a^2e^4\sqrt{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x])/
(15*a^2*e^4*Sqrt[a + b*x^3]) - (2*(8*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1
/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqr
t[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(15*3^(1/4)*a^(7/3)*e
^4*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*
x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{3/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{(8Ab-5aB)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(2(8Ab-5aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{15a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{(4(8Ab-5aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \right.}{15a^2e^4}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}}{15a^2e^4\sqrt{a+bx^3}} - \frac{2(8Ab-5aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-}{\left(\sqrt[3]{a}+\right.}}}{15\sqrt[4]{3}\,a^{7/3}e^4\sqrt{}}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.34

$$\frac{x\left(4x^3\sqrt{\frac{bx^3}{a}+1}\,(5aB-8Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-2\left(3aA-5aBx^3+8Abx^3\right)\right)}{15a^2(ex)^{7/2}\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x]
```

```
[Out] (x*(-2*(3*a*A + 8*A*b*x^3 - 5*a*B*x^3) + 4*(-8*A*b + 5*a*B)*x^3*Sqrt[1 + (b
*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(15*a^2*(e*x)^(7/
2)*Sqrt[a + b*x^3])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^2e^4x^{10}+2\,abe^4x^7+a^2e^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^2*e^4*x^10 + 2*a*b*e^4*x^
7 + a^2*e^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

maple [C] time = 1.09, size = 3783, normalized size = 13.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x)
```

```
[Out] 2/15/x^2*(32*I*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*x^4*b^3-3*I*A*(1/b
^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(
1/3)*a*b-8*I*A*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2
*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
*3^(1/2)*(-a*b^2)^(1/3)*x^3*b^2-20*I*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))
/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*
x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I
*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(
I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*3^(1
/2)*(-a*b^2)^(2/3)*x^2*a+32*I*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b
^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*(-a
*b^2)^(2/3)*x^2*b-32*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+
I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*x^4*b^3-20*I*B*(-(I*
3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b
^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*x^4*a*b^2+20*B*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
```

```
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))
^(1/2)*x^4*a*b^2+64*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-
b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))
/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I
*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*b^
2-40*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3
^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-
b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^
(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b-32*A*(-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I
*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*(e*x*(b*x^3+a))^(1/2)*(-a*b^2)^(2/3)*x^2*b+20*B*(-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)
^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))
^(1/2)*(-a*b^2)^(2/3)*x^2*a+40*I*B*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-
a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(
1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^
(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*
(-a*b^2)^(1/3)*x^3*a*b+5*I*B*(1/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-
a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)
^(1/3)))^(1/2)*3^(1/2)*(-a*b^2)^(1/3)*x^3*a*b+24*A*(1/b^2*e*x*(-b*x+(-a*b^2
)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2
)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^3*b^2-15*B*(1/b^2*e*x
*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))*(I*3
^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*x^3*a*b-6
4*I*A*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((I*3^
(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*((I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(e*x*(b*x^3+a))^(1/2)*3^(1/2)*(-a*b^2)^(1/3)*x^3*b^2+9*A*(1
/b^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/
3))*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)*(-a*b^2)^(1/3)*a
*b)/(b*x^3+a)^(1/2)/(-a*b^2)^(1/3)/b/a^2/e^3/(e*x)^(1/2)/(I*3^(1/2)-3)/(1/b
^2*e*x*(-b*x+(-a*b^2)^(1/3))*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(1/3)
)*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{3}{2}} (ex)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(3/2)*(e*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{{\left(e\,x\right)}^{7/2}\,{\left(b\,x^3+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(3/2),x)
```

```
[Out] Timed out
```

3.559 $\int \frac{(ex)^{7/2}(A+Bx^3)}{(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(9/2)/a/b/e/(b*x^3+a)^(3/2)+2/3*B*e^(7/2)*arctanh((e*x)
^(3/2)*b^(1/2)/e^(3/2)/(b*x^3+a)^(1/2))/b^(5/2)-2/3*B*e^2*(e*x)^(3/2)/b^2/(
b*x^3+a)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {452, 288, 329, 275, 217, 206}

$$\frac{2(ex)^{9/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(9/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*B*e^2*(e*x)^(3
/2))/(3*b^2*Sqrt[a + b*x^3]) + (2*B*e^(7/2)*ArcTanh[(Sqrt[b]*(e*x)^(3/2))/(
e^(3/2)*Sqrt[a + b*x^3])])/(3*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^{7/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{B\int \frac{(ex)^{7/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{b}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(Be^3\right)\int \frac{\sqrt{ex}}{\sqrt{a+bx^3}}\,dx}{b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,x,\sqrt{ex}\right)}{b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^2}{e^3}}}\,dx,x,(ex)^{3/2}\right)}{3b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{\left(2Be^2\right)\mathrm{Subst}\left(\int \frac{1}{1-\frac{bx^2}{e^3}}\,dx,x,\frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^2}\\
&= \frac{2(Ab-aB)(ex)^{9/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2Be^2(ex)^{3/2}}{3b^2\sqrt{a+bx^3}}+\frac{2Be^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,(ex)^{3/2}}{e^{3/2}\sqrt{a+bx^3}}\right)}{3b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 119, normalized size = 1.04

$$\frac{2e^3\sqrt{ex}\left(3a^{3/2}B\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}\left(-3a^2B-4abBx^3+Ab^2x^3\right)\right)}{9ab^{5/2}\sqrt{x}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(7/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^3*Sqrt[e*x]*(Sqrt[b]*x^(3/2)*(-3*a^2*B + A*b^2*x^3 - 4*a*b*B*x^3) + 3*
a^(3/2)*B*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*ArcSinh[(Sqrt[b]*x^(3/2))/Sqrt[a]
]))/(9*a*b^(5/2)*Sqrt[x]*(a + b*x^3)^(3/2))
```

fricas [A] time = 1.35, size = 345, normalized size = 3.03

$$\left[\frac{3\left(Bab^2e^3x^6+2\,Ba^2be^3x^3+Ba^3e^3\right)\sqrt{\frac{e}{b}}\,\log\left(-8\,b^2ex^6-8\,abex^3-a^2e-4\left(2\,b^2x^4+abx\right)\sqrt{bx^3+a}\,\sqrt{ex}\,\sqrt{\frac{e}{b}}\right)-4\left(\right.}{18\left(ab^4x^6+2\,a^2b^3x^3+a^3b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*(B*a*b^2*e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(e/b)*log(-8
*b^2*e*x^6 - 8*a*b*e*x^3 - a^2*e - 4*(2*b^2*x^4 + a*b*x)*sqrt(b*x^3 + a)*sq
rt(e*x)*sqrt(e/b)) - 4*((4*B*a*b - A*b^2)*e^3*x^4 + 3*B*a^2*e^3*x)*sqrt(b*x
^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 + a^3*b^2), -1/9*(3*(B*a*b^2*
e^3*x^6 + 2*B*a^2*b*e^3*x^3 + B*a^3*e^3)*sqrt(-e/b)*arctan(2*sqrt(b*x^3 + a
)*sqrt(e*x)*b*x*sqrt(-e/b)/(2*b*e*x^3 + a*e)) + 2*((4*B*a*b - A*b^2)*e^3*x^
4 + 3*B*a^2*e^3*x)*sqrt(b*x^3 + a)*sqrt(e*x))/(a*b^4*x^6 + 2*a^2*b^3*x^3 +
a^3*b^2)]
```

giac [A] time = 0.48, size = 102, normalized size = 0.89

$$-\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Bae^8}{b^2}+\frac{\left(4\,Ba^5b^6e^{24}-Aa^4b^7e^{24}\right)x^3e^{(-16)}}{a^5b^7}\right)e^{\frac{3}{2}}}{9\left(bx^3e^4+ae^4\right)^{\frac{3}{2}}}-\frac{2\,Be^{\frac{7}{2}}\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}e^2+\sqrt{bx^3e^4+ae^4}\right|\right)}{3\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/9*x^(3/2)*(3*B*a*e^8/b^2 + (4*B*a^5*b^6*e^24 - A*a^4*b^7*e^24)*x^3*e^(-1
6)/(a^5*b^7))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2) - 2/3*B*e^(7/2)*log(abs(-sq
rt(b)*x^(3/2)*e^2 + sqrt(b*x^3*e^4 + a*e^4)))/b^(5/2)
```

maple [C] time = 1.05, size = 7081, normalized size = 62.11

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(Bx^3+A\right)(ex)^{\frac{7}{2}}}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(7/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(7/2)/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(B\,x^3+A\right)\,{\left(e\,x\right)}^{7/2}}{{\left(b\,x^3+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(7/2))/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(7/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.560 $\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=299

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2e^2\sqrt{ex}(7aB+2A}{27ab^2\sqrt{a+bx^3}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(7/2)/a/b/e/(b*x^3+a)^(3/2)-2/27*(2*A*b+7*B*a)*e^2*(e*x
)^(1/2)/a/b^2/(b*x^3+a)^(1/2)+1/81*(2*A*b+7*B*a)*e^2*(a^(1/3)+b^(1/3)*x)*((
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(
a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1
-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
,1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x
^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/b^2/(b*x^3+a)^
(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2)
```

Rubi [A] time = 0.23, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 288, 329, 225}

$$\frac{e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(7aB+2Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{4/3}b^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{2e^2\sqrt{ex}(7aB+2A}{27ab^2\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(7/2))/(9*a*b*e*(a + b*x^3)^(3/2)) - (2*(2*A*b + 7*a*B
)*e^2*Sqrt[e*x])/(27*a*b^2*Sqrt[a + b*x^3]) + ((2*A*b + 7*a*B)*e^2*Sqrt[e*x
]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a
^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[
3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*
3^(1/4)*a^(4/3)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^{5/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx=\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(Ab+\frac{7aB}{2}\right)\right)\int \frac{(ex)^{5/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$=\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+\frac{\left((2Ab+7aB)e^3\right)\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27ab^2}$$

$$=\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+\frac{\left(2(2Ab+7aB)e^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx,\right.}{27ab^2}$$

$$=\frac{2(Ab-aB)(ex)^{7/2}}{9abe\left(a+bx^3\right)^{3/2}}-\frac{2(2Ab+7aB)e^2\sqrt{ex}}{27ab^2\sqrt{a+bx^3}}+\frac{(2Ab+7aB)e^2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}}{\left(\sqrt[3]{a}\right.}}}{27\sqrt[4]{3}\,a^{4/3}b^2}$$

Mathematica [C] time = 0.23, size = 108, normalized size = 0.36

$$\frac{2e^2\sqrt{ex}\left(-7a^2B+\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(7aB+2Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-2ab\left(A+5Bx^3\right)+Ab^2x^3\right)}{27ab^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(5/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*e^2*Sqrt[e*x]*(-7*a^2*B + A*b^2*x^3 - 2*a*b*(A + 5*B*x^3) + (2*A*b + 7*a
*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x
^3)/a)]))/(27*a*b^2*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Be^2x^5 + Ae^2x^2\right)\sqrt{bx^3 + a}\,\sqrt{ex}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e^2*x^5 + A*e^2*x^2)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*x^9 + 3*a*b
^2*x^6 + 3*a^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

maple [C] time = 1.00, size = 7083, normalized size = 23.69

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{5}{2}}}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(5/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(5/2)/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,{(e\,x)}^{5/2}}{{\left(b\,x^3 + a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(5/2))/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(5/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.561 $\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=596

$$\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2e\sqrt{ex}}{}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(5/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(4*A*b+5*B*a)*(e*x)^(5
/2)/a^2/b/e/(b*x^3+a)^(1/2)-2/27*(4*A*b+5*B*a)*e*(1+3^(1/2))*(e*x)^(1/2)*(b
*x^3+a)^(1/2)/a^2/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))+2/27*(4*A*b+5*B*a
)*e*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a
^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1
/4)/a^(5/3)/b^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+1/81*(4*A*b+5*B*a)*e*(a^(1/3)+b^(1/3)*x)*(
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/
(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((
1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b
^(5/3)/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {457, 290, 329, 308, 225, 1881}

$$-\frac{2\left(1+\sqrt{3}\right)e\sqrt{ex}\sqrt{a+bx^3}(5aB+4Ab)}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}+\frac{\left(1-\sqrt{3}\right)e\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(5aB+4Ab)F\left(\mathrm{co}\right.}{27\sqrt[4]{3}a^{5/3}b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(5/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(4*A*b + 5*a*B
)*(e*x)^(5/2))/(27*a^2*b*e*Sqrt[a + b*x^3]) - (2*(1 + Sqrt[3])*(4*A*b + 5*a
*B)*e*Sqrt[e*x]*Sqrt[a + b*x^3])/(27*a^2*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)) + (2*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sq
rt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(9*3^(3/4)*a^(5/3)*b^(5/3)*Sqrt[(b^(1
/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a
+ b*x^3]) + ((1 - Sqrt[3])*(4*A*b + 5*a*B)*e*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x
)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])
*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(5/3)*b^(5/3
)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{(ex)^{3/2}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(2Ab+\frac{5aB}{2}\right)\right)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(2(4Ab+5aB))\int \frac{(ex)^{3/2}}{\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{(4(4Ab+5aB))\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \right.}{27a^2be}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(4Ab+5aB))\,\mathrm{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}e^2-}{\sqrt{a+\frac{bx^6}{e^3}}}\right.}{27a^2b^{5/3}e}$$

$$= \frac{2(Ab-aB)(ex)^{5/2}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(4Ab+5aB)(ex)^{5/2}}{27a^2be\sqrt{a+bx^3}} - \frac{2\left(1+\sqrt{3}\right)(4Ab+5aB)e\sqrt{ex}\sqrt{a+bx^3}}{27a^2b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}$$

Mathematica [C] time = 0.12, size = 86, normalized size = 0.14

$$\frac{x(ex)^{3/2}\left(\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(5aB+4Ab)\,{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-5a^2B\right)}{10a^2b\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^(3/2)*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (x*(e*x)^(3/2)*(-5*a^2*B + (4*A*b + 5*a*B)*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*
Hypergeometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(10*a^2*b*(a + b*x^3)^(3/
2))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bex^4+Aex\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*e*x^4 + A*e*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*x^9 + 3*a*b^2*x^6
 + 3*a^2*b*x^3 + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3+A\right)(ex)^{\frac{3}{2}}}{\left(bx^3+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

maple [C] time = 1.07, size = 10786, normalized size = 18.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)(ex)^{\frac{3}{2}}}{\left(bx^3 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(B*x^3+A)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)*(e*x)^(3/2)/(b*x^3 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(B\,x^3 + A\right)\,{(e\,x)}^{3/2}}{{\left(b\,x^3 + a\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] int(((A + B*x^3)*(e*x)^(3/2))/(a + b*x^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(B*x**3+A)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.562 $\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(3/2)/a/b/e/(b*x^3+a)^(3/2)+2/9*(2*A*b+B*a)*(e*x)^(3/2)
/a^2/b/e/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {457, 264}

$$\frac{2(ex)^{3/2}(aB+2Ab)}{9a^2be\sqrt{a+bx^3}}+\frac{2(ex)^{3/2}(Ab-aB)}{9abe\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*(e*x)^(3/2))/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(2*A*b + a*B)*
(e*x)^(3/2))/(9*a^2*b*e*Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ex}\left(A+Bx^3\right)}{\left(a+bx^3\right)^{5/2}}\,dx &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{\left(2\left(3Ab+\frac{3aB}{2}\right)\right)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{9ab}\\ &= \frac{2(Ab-aB)(ex)^{3/2}}{9abe\left(a+bx^3\right)^{3/2}}+\frac{2(2Ab+aB)(ex)^{3/2}}{9a^2be\sqrt{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.56

$$\frac{2x\sqrt{ex}\left(3aA+aBx^3+2Abx^3\right)}{9a^2\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[e*x]*(A + B*x^3))/(a + b*x^3)^(5/2),x]
[Out] (2*x*Sqrt[e*x]*(3*a*A + 2*A*b*x^3 + a*B*x^3))/(9*a^2*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.59, size = 59, normalized size = 0.75

$$\frac{2\left((Ba + 2\,Ab)x^4 + 3\,Aax\right)\sqrt{bx^3 + a}\,\sqrt{ex}}{9\left(a^2b^2x^6 + 2\,a^3bx^3 + a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
[Out] 2/9*((B*a + 2*A*b)*x^4 + 3*A*a*x)*sqrt(b*x^3 + a)*sqrt(e*x)/(a^2*b^2*x^6 +
2*a^3*b*x^3 + a^4)
```

giac [A] time = 0.32, size = 64, normalized size = 0.81

$$\frac{2\,x^{\frac{3}{2}}\left(\frac{3\,Ae^5}{a} + \frac{\left(Ba^5b^5e^{21} + 2\,Aa^4b^6e^{21}\right)x^3e^{(-16)}}{a^6b^5}\right)e^{\frac{3}{2}}}{9\left(bx^3e^4 + ae^4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] 2/9*x^(3/2)*(3*A*e^5/a + (B*a^5*b^5*e^21 + 2*A*a^4*b^6*e^21)*x^3*e^(-16)/(a
^6*b^5))*e^(3/2)/(b*x^3*e^4 + a*e^4)^(3/2)
```

maple [A] time = 0.05, size = 39, normalized size = 0.49

$$\frac{2\left(2A\,x^3b + Ba\,x^3 + 3Aa\right)\sqrt{ex}\;x}{9\left(b\,x^3 + a\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x)
[Out] 2/9*x*(2*A*b*x^3+B*a*x^3+3*A*a)*(e*x)^(1/2)/(b*x^3+a)^(3/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Bx^3 + A\right)\sqrt{ex}}{\left(bx^3 + a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)*(e*x)^(1/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)*sqrt(e*x)/(b*x^3 + a)^(5/2), x)
```

mupad [B] time = 4.63, size = 73, normalized size = 0.92

$$\frac{\left(\frac{2\,A\,x\,\sqrt{e\,x}}{3\,a\,b^2} + \frac{x^4\,\sqrt{e\,x}\,\left(4\,A\,b+2\,B\,a\right)}{9\,a^2\,b^2}\right)\sqrt{b\,x^3 + a}}{x^6 + \frac{a^2}{b^2} + \frac{2\,a\,x^3}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^3)*(e*x)^(1/2))/(a + b*x^3)^(5/2),x)
```

```
[Out] (((2*A*x*(e*x)^(1/2))/(3*a*b^2) + (x^4*(e*x)^(1/2)*(4*A*b + 2*B*a))/(9*a^2*
b^2))*(a + b*x^3)^(1/2))/(x^6 + a^2/b^2 + (2*a*x^3)/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)*(e*x)**(1/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.563 $\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=297

$$\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+$$

```
[Out] 2/9*(A*b-B*a)*(e*x)^(1/2)/a/b/e/(b*x^3+a)^(3/2)+2/27*(8*A*b+B*a)*(e*x)^(1/2
)/a^2/b/e/(b*x^3+a)^(1/2)+2/81*(8*A*b+B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(7/3)/b/e/(b*x^3+a)^(1/2)/(b^(
1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi **[A]** time = 0.22, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {457, 290, 329, 225}

$$\frac{2\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(aB+8Ab)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{7/3}be\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}+\frac{2\sqrt{ex}(aB+8Ab)}{27a^2be\sqrt{a+bx^3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[e*x])/(9*a*b*e*(a + b*x^3)^(3/2)) + (2*(8*A*b + a*B)*Sq
rt[e*x])/(27*a^2*b*e*Sqrt[a + b*x^3]) + (2*(8*A*b + a*B)*Sqrt[e*x]*(a^(1/3)
 + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (
1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3
)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^
(7/3)*b*e*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{A+Bx^3}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx = \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{\left(2\left(4Ab+\frac{aB}{2}\right)\right)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,dx}{9ab}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(2(8Ab+aB))\int \frac{1}{\sqrt{ex}\sqrt{a+bx^3}}\,dx}{27a^2b}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{(4(8Ab+aB))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+\frac{bx^6}{e^3}}}\,dx, x, \sqrt{ex}\right.}{27a^2be}$$

$$= \frac{2(Ab-aB)\sqrt{ex}}{9abe\left(a+bx^3\right)^{3/2}} + \frac{2(8Ab+aB)\sqrt{ex}}{27a^2be\sqrt{a+bx^3}} + \frac{2(8Ab+aB)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\cdots}{\left(\sqrt[3]{a}+\left(1+\sqrt{\cdots}\right.\right.}}}{27\sqrt[4]{3}\,a^{7/3}be\sqrt{\frac{\sqrt[3]{b}\cdots}{\left(\sqrt[3]{a}\cdots\right.}}}$$

Mathematica [C] time = 0.11, size = 107, normalized size = 0.36

$$\frac{2x\left(-2a^2B+2\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(aB+8Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+ab\left(11A+Bx^3\right)+8Ab^2x^3\right)}{27a^2b\sqrt{ex}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/(Sqrt[e*x]*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-2*a^2*B + 8*A*b^2*x^3 + a*b*(11*A + B*x^3) + 2*(8*A*b + a*B)*(a + b*
x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(
27*a^2*b*Sqrt[e*x]*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^3ex^{10}+3\,ab^2ex^7+3\,a^2bex^4+a^3ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e*x^10 + 3*a*b^2*e*x^7
+ 3*a^2*b*e*x^4 + a^3*e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

maple [C] time = 1.11, size = 7077, normalized size = 23.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(b*x^3+a)^(5/2)/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*sqrt(e*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{\sqrt{e\,x}\,\left(b\,x^3 + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(1/2)*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(b*x**3+a)**(5/2)/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

3.564 $\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx$

Optimal. Leaf size=624

$$-\frac{4\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{8\sqrt{ex}\cdots}{\cdots}$$

```
[Out] -2/9*(10*A*b-B*a)*(e*x)^(5/2)/a^2/e^4/(b*x^3+a)^(3/2)-2*A/a/e/(b*x^3+a)^(3/
2)/(e*x)^(1/2)-8/27*(10*A*b-B*a)*(e*x)^(5/2)/a^3/e^4/(b*x^3+a)^(1/2)+8/27*(
10*A*b-B*a)*(1+3^(1/2))*(e*x)^(1/2)*(b*x^3+a)^(1/2)/a^3/b^(2/3)/e^2/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))-8/27*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^
(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+
b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1
/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
-4/81*(10*A*b-B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^
(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/
2))*(e*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b^(2/3)/e^2/(b*x^3+a)^(1/2)/(b^(1/3)
*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {453, 290, 329, 308, 225, 1881}

$$\frac{8\left(1+\sqrt{3}\right)\sqrt{ex}\sqrt{a+bx^3}(10Ab-aB)}{27a^3b^{2/3}e^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)}-\frac{4\left(1-\sqrt{3}\right)\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(10Ab-aB)F\left(\cos^{-1}\cdots\right.}{27\sqrt[4]{3}a^{8/3}b^{2/3}e^2\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(a*e*Sqrt[e*x]*(a + b*x^3)^(3/2)) - (2*(10*A*b - a*B)*(e*x)^(5/2))/(
9*a^2*e^4*(a + b*x^3)^(3/2)) - (8*(10*A*b - a*B)*(e*x)^(5/2))/(27*a^3*e^4*S
qrt[a + b*x^3]) + (8*(1 + Sqrt[3])*(10*A*b - a*B)*Sqrt[e*x]*Sqrt[a + b*x^3]
)/(27*a^3*b^(2/3)*e^2*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)) - (8*(10*A*b - a
*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3)
+ (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[
3])/4])/(9*3^(3/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x
))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3]) - (4*(1 - Sqrt[3
])*(10*A*b - a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(27*3^(1/4)*a^(8/3)*b^(2/3)*e^2*Sqrt[(b^(1/3)*x*(a^(1
/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{3/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{(10Ab-aB)\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{5/2}}dx}{ae^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(4(10Ab-aB))\int \frac{(ex)^{3/2}}{\left(a+bx^3\right)^{3/2}}dx}{9a^2e^3}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(8(10Ab-a}{2}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{(16(10Ab-}{}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} - \frac{(8(10Ab-a}{}$$

$$= -\frac{2A}{ae\sqrt{ex}\left(a+bx^3\right)^{3/2}} - \frac{2(10Ab-aB)(ex)^{5/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{8(10Ab-aB)(ex)^{5/2}}{27a^3e^4\sqrt{a+bx^3}} + \frac{8\left(1+\sqrt{3}\right)}{27a^3b^{2/3}e}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 0.14

$$\frac{2x\left(x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(aB-10Ab)\,{}_2F_1\left(\frac{5}{6},\frac{5}{2};\frac{11}{6};-\frac{bx^3}{a}\right)-5a^2A\right)}{5a^3(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (2*x*(-5*a^2*A + (-10*A*b + a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[5/6, 5/2, 11/6, -((b*x^3)/a)]))/(5*a^3*(e*x)^(3/2)*(a + b*x^3)^
(3/2))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^3e^2x^{11}+3\,ab^2e^2x^8+3\,a^2be^2x^5+a^3e^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e^2*x^11 + 3*a*b^2*e^2*
x^8 + 3*a^2*b*e^2*x^5 + a^3*e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

maple [C] time = 1.14, size = 10961, normalized size = 17.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x)
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(3/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3 + A}{(e\,x)^{3/2}\,\left(b\,x^3 + a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)),x)
[Out] int((A + B*x^3)/((e*x)^(3/2)*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(3/2)/(b*x**3+a)**(5/2),x)
[Out] Timed out
```

3.565 $\int \frac{A+Bx^3}{(ex)^{5/2}(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=104

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

```
[Out] -2/3*A/a/e/(e*x)^(3/2)/(b*x^3+a)^(3/2)-2/9*(4*A*b-B*a)*(e*x)^(3/2)/a^2/e^4/
(b*x^3+a)^(3/2)-4/9*(4*A*b-B*a)*(e*x)^(3/2)/a^3/e^4/(b*x^3+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {453, 273, 264}

$$-\frac{4(ex)^{3/2}(4Ab-aB)}{9a^3e^4\sqrt{a+bx^3}}-\frac{2(ex)^{3/2}(4Ab-aB)}{9a^2e^4\left(a+bx^3\right)^{3/2}}-\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(3*a*e*(e*x)^(3/2)*(a + b*x^3)^(3/2)) - (2*(4*A*b - a*B)*(e*x)^(3/2)
)/(9*a^2*e^4*(a + b*x^3)^(3/2)) - (4*(4*A*b - a*B)*(e*x)^(3/2))/(9*a^3*e^4*
Sqrt[a + b*x^3])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 273

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{5/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}} - \frac{(4Ab-aB)\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{5/2}}\,dx}{ae^3}$$
$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}} - \frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(2(4Ab-aB))\int \frac{\sqrt{ex}}{\left(a+bx^3\right)^{3/2}}\,dx}{3a^2e^3}$$
$$= -\frac{2A}{3ae(ex)^{3/2}\left(a+bx^3\right)^{3/2}} - \frac{2(4Ab-aB)(ex)^{3/2}}{9a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{4(4Ab-aB)(ex)^{3/2}}{9a^3e^4\sqrt{a+bx^3}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 0.62

$$\frac{x\left(-6a^2\left(A-Bx^3\right)+4abx^3\left(Bx^3-6A\right)-16Ab^2x^6\right)}{9a^3(ex)^{5/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-16*A*b^2*x^6 - 6*a^2*(A - B*x^3) + 4*a*b*x^3*(-6*A + B*x^3)))/(9*a^3*(
e*x)^(5/2)*(a + b*x^3)^(3/2))
```

fricas [A] time = 0.86, size = 93, normalized size = 0.89

$$\frac{2\left(2\left(Bab-4\,Ab^2\right)x^6+3\left(Ba^2-4\,Aab\right)x^3-3\,Aa^2\right)\sqrt{bx^3+a}\,\sqrt{ex}}{9\left(a^3b^2e^3x^8+2\,a^4be^3x^5+a^5e^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9*(2*(B*a*b - 4*A*b^2)*x^6 + 3*(B*a^2 - 4*A*a*b)*x^3 - 3*A*a^2)*sqrt(b*x^
3 + a)*sqrt(e*x)/(a^3*b^2*e^3*x^8 + 2*a^4*b*e^3*x^5 + a^5*e^3*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

maple [A] time = 0.05, size = 62, normalized size = 0.60

$$-\frac{2\left(8A\,b^2x^6-2Bab\,x^6+12Aab\,x^3-3B\,a^2x^3+3A\,a^2\right)x}{9\left(b\,x^3+a\right)^{\frac{3}{2}}(ex)^{\frac{5}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] -2/9*x*(8*A*b^2*x^6-2*B*a*b*x^6+12*A*a*b*x^3-3*B*a^2*x^3+3*A*a^2)/(b*x^3+a)
^(3/2)/a^3/(e*x)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3 + A}{\left(bx^3 + a\right)^{\frac{5}{2}} (ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(5/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(5/2)), x)
```

mupad [B] time = 4.80, size = 115, normalized size = 1.11

$$-\frac{\sqrt{b\,x^3+a}\,\left(\frac{2\,A}{3\,a\,b^2\,e^2}-\frac{x^3\,\left(6\,B\,a^2-24\,A\,a\,b\right)}{9\,a^3\,b^2\,e^2}+\frac{x^6\,\left(16\,A\,b^2-4\,B\,a\,b\right)}{9\,a^3\,b^2\,e^2}\right)}{x^7\,\sqrt{e\,x}+\frac{a^2\,x\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^4\,\sqrt{e\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(5/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] -((a + b*x^3)^(1/2)*((2*A)/(3*a*b^2*e^2) - (x^3*(6*B*a^2 - 24*A*a*b))/(9*a^
3*b^2*e^2) + (x^6*(16*A*b^2 - 4*B*a*b))/(9*a^3*b^2*e^2)))/(x^7*(e*x)^(1/2)
+ (a^2*x*(e*x)^(1/2))/b^2 + (2*a*x^4*(e*x)^(1/2))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(5/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.566 $\int \frac{A+Bx^3}{(ex)^{7/2}(a+bx^3)^{5/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{16\sqrt{ex}(14Ab-}{135a^3e^4\sqrt{a+b}}$$

```
[Out] -2/5*A/a/e/(e*x)^(5/2)/(b*x^3+a)^(3/2)-2/45*(14*A*b-5*B*a)*(e*x)^(1/2)/a^2/
e^4/(b*x^3+a)^(3/2)-16/135*(14*A*b-5*B*a)*(e*x)^(1/2)/a^3/e^4/(b*x^3+a)^(1/
2)-16/405*(14*A*b-5*B*a)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(e
*x)^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)*3^(3/4)/a^(10/3)/e^4/(b*x^3+a)^(1/2)/(b^(1/3)*x*(a^(1/3)+b
^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {453, 290, 329, 225}

$$-\frac{16\sqrt{ex}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}(14Ab-5aB)F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{135\sqrt[4]{3}a^{10/3}e^4\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{a+bx^3}}-\frac{16\sqrt{ex}(14Ab-}{135a^3e^4\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (-2*A)/(5*a*e*(e*x)^(5/2)*(a + b*x^3)^(3/2)) - (2*(14*A*b - 5*a*B)*Sqrt[e*x
])/(45*a^2*e^4*(a + b*x^3)^(3/2)) - (16*(14*A*b - 5*a*B)*Sqrt[e*x])/(135*a^
3*e^4*Sqrt[a + b*x^3]) - (16*(14*A*b - 5*a*B)*Sqrt[e*x]*(a^(1/3) + b^(1/3)*
x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(135*3^(1/4)*a^(10/3)*e^4
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a + b*x^3])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
```

```
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^3}{(ex)^{7/2}\left(a+bx^3\right)^{5/2}}\,dx = -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{(14Ab-5aB)\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{5/2}}\,dx}{5ae^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{(8(14Ab-5aB))\int \frac{1}{\sqrt{ex}\left(a+bx^3\right)^{3/2}}\,d}{45a^2e^3}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(16(14Ab}{}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{(32(14Ab}{}$$

$$= -\frac{2A}{5ae(ex)^{5/2}\left(a+bx^3\right)^{3/2}} - \frac{2(14Ab-5aB)\sqrt{ex}}{45a^2e^4\left(a+bx^3\right)^{3/2}} - \frac{16(14Ab-5aB)\sqrt{ex}}{135a^3e^4\sqrt{a+bx^3}} - \frac{16(14Ab}{}$$

Mathematica [C] time = 0.09, size = 121, normalized size = 0.38

$$\frac{x\left(a^2\left(110Bx^3-54A\right)+32x^3\left(a+bx^3\right)\sqrt{\frac{bx^3}{a}+1}\,(5aB-14Ab)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a\left(80bBx^6-308Abx^3\right)-2}{135a^3(ex)^{7/2}\left(a+bx^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x]
```

```
[Out] (x*(-224*A*b^2*x^6 + a^2*(-54*A + 110*B*x^3) + a*(-308*A*b*x^3 + 80*b*B*x^6
) + 32*(-14*A*b + 5*a*B)*x^3*(a + b*x^3)*Sqrt[1 + (b*x^3)/a]*Hypergeometric
2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(135*a^3*(e*x)^(7/2)*(a + b*x^3)^(3/2))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Bx^3+A\right)\sqrt{bx^3+a}\,\sqrt{ex}}{b^3e^4x^{13}+3\,ab^2e^4x^{10}+3\,a^2be^4x^7+a^3e^4x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*x^3 + A)*sqrt(b*x^3 + a)*sqrt(e*x)/(b^3*e^4*x^13 + 3*a*b^2*e^4*
x^10 + 3*a^2*b*e^4*x^7 + a^3*e^4*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

maple [C] time = 1.06, size = 7299, normalized size = 22.81

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Bx^3+A}{\left(bx^3+a\right)^{\frac{5}{2}}(ex)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^3+A)/(e*x)^(7/2)/(b*x^3+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^3 + A)/((b*x^3 + a)^(5/2)*(e*x)^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^3+A}{(e\,x)^{7/2}\,\left(b\,x^3+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)),x)
```

```
[Out] int((A + B*x^3)/((e*x)^(7/2)*(a + b*x^3)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**3+A)/(e*x)**(7/2)/(b*x**3+a)**(5/2),x)
```

```
[Out] Timed out
```

3.567 $\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=220

$$\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx\right.}{4b^4d}$$

```
[Out] -a^3*(b*x^3+a)^(1/3)/b^4/d-1/4*a^2*(b*x^3+a)^(4/3)/b^4/d+1/7*a*(b*x^3+a)^(7
/3)/b^4/d-1/10*(b*x^3+a)^(10/3)/b^4/d+1/6*a^(10/3)*ln(-b*x^3+a)*2^(1/3)/b^4
/d-1/2*a^(10/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^4/d+1/3*2^(1/
3)*a^(10/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b
^4/d*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}\,b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^3*(a + b*x^3)^(1/3))/(b^4*d)) - (a^2*(a + b*x^3)^(4/3))/(4*b^4*d) + (a
*(a + b*x^3)^(7/3))/(7*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) + (2^(1/3)*a^
(10/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(10/3)*Log[a - b*x^3])/(3*2^(2/3)*b^4*d) - (a^(10/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2\sqrt[3]{a+bx}}{b^3d}+\frac{a(a+bx)^{4/3}}{b^3d}-\frac{(a+bx)^{7/3}}{b^3d}+\frac{a^3\sqrt[3]{a+bx}}{b^3(ad-bdx)}\right)dx,x,x^3\right)\\
&= -\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}+\frac{a^3\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)}{3b^3}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}+\frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(\cdots}\right.}{3b^3}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}+\frac{a^{1\cdots}}{\cdots}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}+\frac{a^{10/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^4d}-\frac{a^{1\cdots}}{\cdots}\\
&= -\frac{a^3\sqrt[3]{a+bx^3}}{b^4d}-\frac{a^2\left(a+bx^3\right)^{4/3}}{4b^4d}+\frac{a\left(a+bx^3\right)^{7/3}}{7b^4d}-\frac{\left(a+bx^3\right)^{10/3}}{10b^4d}+\frac{\sqrt[3]{2}\,a^{10/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a\cdots}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^4d}
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 230, normalized size = 1.05

$$
-\frac{140\sqrt[3]{2}\,a^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-70\sqrt[3]{2}\,a^{10/3}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-140\sqrt[3]{2}\sqrt{\cdots}}{420b^4\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/420*(507*a^3*(a + b*x^3)^(1/3) + 111*a^2*b*x^3*(a + b*x^3)^(1/3) + 66*a*
b^2*x^6*(a + b*x^3)^(1/3) + 42*b^3*x^9*(a + b*x^3)^(1/3) - 140*2^(1/3)*Sqrt
[3]*a^(10/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 14
0*2^(1/3)*a^(10/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 70*2^(1/3)*a^
(10/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3
)^(2/3)])/(b^4*d)
```

fricas [A] time = 0.90, size = 186, normalized size = 0.85

$$-\frac{140\,\sqrt{3}\,2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a^3\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+70\cdot 2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a^3\log\left(2^{\frac{2}{3}}\,(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^3*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 70*2^(1/3)*(-a)^(1/3)*a^3*log(2^
(2/3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)
) - 140*2^(1/3)*(-a)^(1/3)*a^3*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3))
+ 3*(14*b^3*x^9 + 22*a*b^2*x^6 + 37*a^2*b*x^3 + 169*a^3)*(b*x^3 + a)^(1/3))
/(b^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootofa^3*(2*a)^(1/3)*1/6/b^4/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^
3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a^3*(2*a)^(1/3)/sqrt(3)/b^4/d*atan(((a+b*
x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^4*b^40*d^10*(2*a)^(1
/3)*1/6/a/b^44/d^11*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/10*(a+b*x^3)^(1
/3)*(a+b*x^3)^3*b^36*d^9-1/7*(a+b*x^3)^(1/3)*(a+b*x^3)^2*a*b^36*d^9+1/4*(a+
b*x^3)^(1/3)*(a+b*x^3)*a^2*b^36*d^9+(a+b*x^3)^(1/3)*a^3*b^36*d^9)/b^40/d^10
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{11}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.55, size = 183, normalized size = 0.83

$$\frac{\frac{140\,\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{10}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{70\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}-\frac{140\cdot 2^{\frac{1}{3}}a^{\frac{10}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}-\frac{3\left(\cdots\right)}{\cdots}}{420\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/420*(140*sqrt(3)*2^(1/3)*a^(10/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(
1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 70*2^(1/3)*a^(10/3)*log(2^(2/3)*a^
(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 140*2^(1
/3)*a^(10/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*(14*(b*x^3 + a
)^(10/3) - 20*(b*x^3 + a)^(7/3)*a + 35*(b*x^3 + a)^(4/3)*a^2 + 140*(b*x^3 +
 a)^(1/3)*a^3)/d)/b^4
```

mupad [B] time = 4.81, size = 240, normalized size = 1.09

$$\frac{a\left(b\,x^{3}+a\right)^{7/3}}{7\,b^{4}\,d}-\frac{a^{3}\left(b\,x^{3}+a\right)^{1/3}}{b^{4}\,d}-\frac{a^{2}\left(b\,x^{3}+a\right)^{4/3}}{4\,b^{4}\,d}-\frac{\left(b\,x^{3}+a\right)^{10/3}}{10\,b^{4}\,d}-\frac{2^{1/3}\,a^{10/3}\,\ln\left(\left(b\,x^{3}+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^{4}\,d}-\frac{2^{1/3}\,a^{10/3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(7/3))/(7*b^4*d) - (a^3*(a + b*x^3)^(1/3))/(b^4*d) - (a^2*(a
 + b*x^3)^(4/3))/(4*b^4*d) - (a + b*x^3)^(10/3)/(10*b^4*d) - (2^(1/3)*a^(10
/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b^4*d) - (2^(1/3)*a^(10/3)
*log((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) - (6*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/
2 - 1/2))/(b^4*d))*((3^(1/2)*1i)/2 - 1/2))/(3*b^4*d) + (2^(1/3)*a^(10/3)*lo
g((6*a^4*(a + b*x^3)^(1/3))/(b^4*d) + (18*2^(1/3)*a^(13/3)*((3^(1/2)*1i)/6
+ 1/6))/(b^4*d))*((3^(1/2)*1i)/6 + 1/6))/(b^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{11}\sqrt[3]{a+bx^{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**11*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.568 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=174

$$\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

```
[Out] -a^2*(b*x^3+a)^(1/3)/b^3/d-1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*a^(7/3)*ln(-b*x^3+
a)*2^(1/3)/b^3/d-1/2*a^(7/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^
3/d+1/3*2^(1/3)*a^(7/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3
)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {446, 88, 50, 57, 617, 204, 31}

$$-\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a^2*(a + b*x^3)^(1/3))/(b^3*d)) - (a + b*x^3)^(7/3)/(7*b^3*d) + (2^(1/3)
*a^(7/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(7/3)*Log[a - b*x^3])/(3*2^(2/3)*b^3*d) - (a^(7/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{(a+bx)^{4/3}}{b^2d}+\frac{a^2\sqrt[3]{a+bx}}{b^2(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3b^2}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}+\frac{a^{7/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right.}{2^{2/3}b^3d}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^3d}-\frac{(\sqrt[3]{2}}{}$$

$$= -\frac{a^2\sqrt[3]{a+bx^3}}{b^3d}-\frac{\left(a+bx^3\right)^{7/3}}{7b^3d}+\frac{\sqrt[3]{2}\,a^{7/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{7/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^3d}-\frac{a^{7/3}\,\text{lo}}{}$$

Mathematica [A]   time = 0.14, size = 207, normalized size = 1.19

$$\frac{1}{3}\left(-\frac{-\frac{2\sqrt[3]{2}\,a^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{bd}-\frac{\sqrt[3]{2}\,a^{7/3}\left(\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)\right)}{bd}}{2b^2}-\frac{3a^2\sqrt[3]{a+b}}{b^3d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-3*a^2*(a + b*x^3)^(1/3))/(b^3*d) - (3*(a + b*x^3)^(7/3))/(7*b^3*d) - ((2
*2^(1/3)*a^(7/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b*d) - (2^(1/3)
*a^(7/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a
^(1/3))] + Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/(b*d))/(2*b^2))/3
```

fricas [A] time = 0.87, size = 174, normalized size = 1.00

$$-\frac{14\sqrt{3}\,2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^{2}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+7\cdot 2^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}a^{2}\log\left(2^{\frac{2}{3}}\left(-a\right)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(bx^{3}\cdots\right.\right.}{42\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/42*(14*sqrt(3)*2^(1/3)*(-a)^(1/3)*a^2*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3
 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 7*2^(1/3)*(-a)^(1/3)*a^2*log(2^(2/
3)*(-a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) -
 14*2^(1/3)*(-a)^(1/3)*a^2*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*
(b^2*x^6 + 2*a*b*x^3 + 8*a^2)*(b*x^3 + a)^(1/3))/(b^3*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a^2*(2*a)^(1/3)/sqrt(3)/b^3/d*atan(((a+b*
```

```
x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^3*b^21*d^7*(2*a)^(1/
3)*1/6/a/b^24/d^8*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/7*(a+b*x^3)^(1/3)
*(a+b*x^3)^2*b^18*d^6+(a+b*x^3)^(1/3)*a^2*b^18*d^6)/b^21/d^7
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^8}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
[Out] int(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.41, size = 155, normalized size = 0.89

$$\frac{\frac{14\,\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{7}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{7\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{14\cdot 2^{\frac{1}{3}}a^{\frac{7}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(\left(b\right.\right.}{}}{42\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] 1/42*(14*sqrt(3)*2^(1/3)*a^(7/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 7*2^(1/3)*a^(7/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 14*2^(1/3)*a^
(7/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*((b*x^3 + a)^(7/3) +
7*(b*x^3 + a)^(1/3)*a^2)/d)/b^3
```

mupad [B] time = 4.65, size = 219, normalized size = 1.26

$$\frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(6\,a^3\,\left(b\,x^3+a\right)^{1/3}-6\,2^{1/3}\,(-a)^{10/3}\right)}{3\,b^3\,d}-\frac{a^2\,\left(b\,x^3+a\right)^{1/3}}{b^3\,d}-\frac{\left(b\,x^3+a\right)^{7/3}}{7\,b^3\,d}-\frac{2^{1/3}\,(-a)^{7/3}\,\ln\left(\frac{6\,a^3\,\left(b\,x^3+a\right)^{1/3}}{b^3\,d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] (2^(1/3)*(-a)^(7/3)*log(6*a^3*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(10/3)))/(
3*b^3*d) - (a^2*(a + b*x^3)^(1/3))/(b^3*d) - (a + b*x^3)^(7/3)/(7*b^3*d) -
(2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) + (6*2^(1/3)*(-a)
^(10/3)*((3^(1/2)*1i)/2 + 1/2))/(b^3*d))*((3^(1/2)*1i)/2 + 1/2))/(3*b^3*d)
+ (2^(1/3)*(-a)^(7/3)*log((6*a^3*(a + b*x^3)^(1/3))/(b^3*d) - (18*2^(1/3)*(
-a)^(10/3)*((3^(1/2)*1i)/6 - 1/6))/(b^3*d))*((3^(1/2)*1i)/6 - 1/6))/(b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^8\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**8*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.569 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=172

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

```
[Out] -a*(b*x^3+a)^(1/3)/b^2/d-1/4*(b*x^3+a)^(4/3)/b^2/d+1/6*a^(4/3)*ln(-b*x^3+a)
*2^(1/3)/b^2/d-1/2*a^(4/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b^2/
d+1/3*2^(1/3)*a^(4/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*
3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 50, 57, 617, 204, 31}

$$\frac{a^{4/3}\log\left(a-bx^3\right)}{3\ 2^{2/3}b^2d}-\frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d}+\frac{\sqrt[3]{2}a^{4/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^2d}-\frac{a\sqrt[3]{a+bx^3}}{b^2d}-\frac{\left(a+bx^3\right)^{4/3}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3))/(b^2*d)) - (a + b*x^3)^(4/3)/(4*b^2*d) + (2^(1/3)*a
^(4/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(4/3)*Log[a - b*x^3])/(3*2^(2/3)*b^2*d) - (a^(4/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx,x,x^3\right)}{3b}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx,x,x^3\right)}{3b}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} + \frac{a^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right.}{2^{2/3}b^2d}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^2d} - \frac{\left(\sqrt[3]{2}\right.}{}$$

$$= -\frac{a\sqrt[3]{a+bx^3}}{b^2d} - \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{\sqrt[3]{2}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d} + \frac{a^{4/3}\log\left(a-bx^3\right)}{3\,2^{2/3}b^2d} - \frac{a^{4/3}\log}{}$$

Mathematica [A] time = 0.10, size = 186, normalized size = 1.08

$$-\frac{4\sqrt[3]{2}\,a^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-2\sqrt[3]{2}\,a^{4/3}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-4\sqrt[3]{2}\sqrt{3}\,a^{4/3}}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/12*(15*a*(a + b*x^3)^(1/3) + 3*b*x^3*(a + b*x^3)^(1/3) - 4*2^(1/3)*Sqrt[
3]*a^(4/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 4*2^
```

```
(1/3)*a^(4/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] - 2*2^(1/3)*a^(4/3)*
Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)
])/(b^2*d)
```

fricas [A] time = 0.64, size = 157, normalized size = 0.91

$$-\frac{4\sqrt{3}\,2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\cdot 2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\,a\log\left(2^{\frac{2}{3}}\,(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right.}{12\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*2^(1/3)*(-a)^(1/3)*a*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 +
a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2*2^(1/3)*(-a)^(1/3)*a*log(2^(2/3)*(-
a)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 4*2^
(1/3)*(-a)^(1/3)*a*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 3*(b*x^3 +
 5*a)*(b*x^3 + a)^(1/3))/(b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+a*(2*a)^(1/3)/sqrt(3)/b^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a^2*b^8*d^4*(2*a)^(1/3)*1/
```

```
6/a/b^10/d^5*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/4*(a+b*x^3)^(1/3)*(a+b
*x^3)*b^6*d^3+(a+b*x^3)^(1/3)*a*b^6*d^3)/b^8/d^4
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^5}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.43, size = 153, normalized size = 0.89

$$\frac{\frac{4\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}+\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{4\cdot 2^{\frac{1}{3}}a^{\frac{4}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-3\left((bx^3 \cdots\right.}{12\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/12*(4*sqrt(3)*2^(1/3)*a^(4/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2*2^(1/3)*a^(4/3)*log(2^(2/3)*a^(2/3)
+ 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 4*2^(1/3)*a^(4
/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 3*((b*x^3 + a)^(4/3) + 4*
(b*x^3 + a)^(1/3)*a)/d)/b^2
```

mupad [B] time = 4.66, size = 200, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\frac{a\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^2\,d}-\frac{2^{1/3}\,a^{4/3}\,\ln\left(\frac{6\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d}-\frac{6\,2^{1/3}\,a^{7/3}\,\left(-\frac{1}{2}+\cdots\right)}{b^2\,d}\cdots\right)}{3\,b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d) + (18*2^(1/3)*a^(7/3
)*((3^(1/2)*1i)/6 + 1/6))/(b^2*d))*((3^(1/2)*1i)/6 + 1/6))/(b^2*d) - (a*(a
+ b*x^3)^(1/3))/(b^2*d) - (2^(1/3)*a^(4/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*
a^(1/3)))/(3*b^2*d) - (2^(1/3)*a^(4/3)*log((6*a^2*(a + b*x^3)^(1/3))/(b^2*d
) - (6*2^(1/3)*a^(7/3)*((3^(1/2)*1i)/2 - 1/2))/(b^2*d))*((3^(1/2)*1i)/2 - 1
/2))/(3*b^2*d) - (a + b*x^3)^(4/3)/(4*b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^5\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.570 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{ad-bdx^3} dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\, 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}$$

```
[Out] -(b*x^3+a)^(1/3)/b/d+1/6*a^(1/3)*ln(-b*x^3+a)*2^(1/3)/b/d-1/2*a^(1/3)*ln(2^
(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/b/d+1/3*2^(1/3)*a^(1/3)*arctan(1/3*(
a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 57, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}}{bd}+\frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\, 2^{2/3}bd}-\frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd}+\frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(b*d)) + (2^(1/3)*a^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
+ b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(1/3)*Log[a - b*x^3]
)/(3*2^(2/3)*b*d) - (a^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
2/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} + \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} + \frac{a^{2/3}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}bd} - \frac{\left(\sqrt[3]{2}\sqrt[3]{a}\right)\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{bd} + \frac{\sqrt[3]{2}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd} + \frac{\sqrt[3]{a}\log\left(a-bx^3\right)}{3\,2^{2/3}bd} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+\right.}}{2^{2/3}bd}$$

Mathematica [A] time = 0.05, size = 167, normalized size = 1.11

$$\frac{\sqrt[3]{2}\sqrt[3]{a}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-6\sqrt[3]{a+bx^3}-2\sqrt[3]{2}\sqrt[3]{a}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-6*(a + b*x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2^(2/3)*(a +
 b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(1/3)*a^(1/3)*Log[2^(1/3)*a^(1/3) -
(a + b*x^3)^(1/3)] + 2^(1/3)*a^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(6*b*d)
```

fricas [A] time = 0.93, size = 144, normalized size = 0.96

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2^{\frac{1}{3}}\,(-a)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}\,(-a)^{\frac{2}{3}}-2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*(-a)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x^3 + a)^
(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + 2^(1/3)*(-a)^(1/3)*log(2^(2/3)*(-a)^(2/3
) - 2^(1/3)*(b*x^3 + a)^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(2/3)) - 2*2^(1/3)*(
-a)^(1/3)*log(2^(1/3)*(-a)^(1/3) + (b*x^3 + a)^(1/3)) + 6*(b*x^3 + a)^(1/3)
)/(b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootof(2*a)^(1/3)*1/6/b/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/
3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)/sqrt(3)/b/d*atan(((a+b*x^3)^(1/3)+1
/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*a*b*d*(2*a)^(1/3)*1/6/a/b^2/d^2*ln
(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(a+b*x^3)^(1/3)/b/d
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{2}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.27, size = 139, normalized size = 0.93

$$\frac{\frac{2\sqrt{3}\,2^{\frac{1}{3}}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}+\frac{2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}-\frac{2\cdot 2^{\frac{1}{3}}a^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}-\frac{6\left(bx^3+a\right.}{d}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*2^(1/3)*a^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
+ 2*(b*x^3 + a)^(1/3))/a^(1/3))/d + 2^(1/3)*a^(1/3)*log(2^(2/3)*a^(2/3) + 2
^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d - 2*2^(1/3)*a^(1/3)
*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d - 6*(b*x^3 + a)^(1/3)/d)/b
```

mupad [B] time = 4.64, size = 194, normalized size = 1.29

$$\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(6\,a\,\left(b\,x^3+a\right)^{1/3}-6\,2^{1/3}\,(-a)^{4/3}\right)}{3\,b\,d}-\frac{\left(b\,x^3+a\right)^{1/3}}{b\,d}+\frac{2^{1/3}\,(-a)^{1/3}\,\ln\left(\frac{6\,a\,\left(b\,x^3+a\right)^{1/3}}{b\,d}-\frac{6\,2^{1/3}\,(-a)^{4/3}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right.}{b\,d}\right.}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] (2^(1/3)*(-a)^(1/3)*log(6*a*(a + b*x^3)^(1/3) - 6*2^(1/3)*(-a)^(4/3)))/(3*b
*d) - (a + b*x^3)^(1/3)/(b*d) + (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1
/3))/(b*d) - (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 - 1/2))/(b*d))*((3^(1/2)
*1i)/2 - 1/2))/(3*b*d) - (2^(1/3)*(-a)^(1/3)*log((6*a*(a + b*x^3)^(1/3))/(b
*d) + (6*2^(1/3)*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/(b*d))*((3^(1/2)*1i)/2
+ 1/2))/(3*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.571 $\int \frac{\sqrt[3]{a+bx^3}}{x(ad-bdx^3)}\,dx$

Optimal. Leaf size=214

$$\frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}$$

```
[Out] -1/2*ln(x)/a^(2/3)/d+1/6*ln(-b*x^3+a)*2^(1/3)/a^(2/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(2/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(2
/3)/d-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/d
*3^(1/2)+1/3*2^(1/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(2/3)/d*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 57, 617, 204, 31}

$$\frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}+\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*d)) + (2^(1/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(2/3)*d) - Log[x]/(2*a^(2/3)*d) + Log[a - b*x^3]/(3*2^(2/
3)*a^(2/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(2/3)*d) - Log[2^(1/3
)*a^(1/3) - (a + b*x^3)^(1/3)]/(2^(2/3)*a^(2/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(ad-bdx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right) + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx, x,\right.}{2^{2/3}a^{2/3}d}$$

$$= -\frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}d} - \frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{2/3}d} + \text{Subs}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d} + \frac{\sqrt[3]{2}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}d} - \frac{\log(x)}{2a^{2/3}d} + \frac{\log\left(a-bx^3\right)}{3\,2^{2/3}a^{2/3}d} + \frac{\log\left(\sqrt[3]{a}-\right.}{2a^{2/}}$$

Mathematica [A] time = 0.11, size = 233, normalized size = 1.09

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-\sqrt[3]{2}\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}\right.}{6a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(
1/3)*Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*
Log[a^(1/3) - (a + b*x^3)^(1/3)] + 2*2^(1/3)*Log[2^(1/3)*a^(1/3) - (a + b*x
^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] -
 2^(1/3)*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x
^3)^(2/3)])/(a^(2/3)*d)
```

fricas [B] time = 0.74, size = 1046, normalized size = 4.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/
3)*arctan(1/12*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a
*d^2)*sqrt(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^
4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))*(-
(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(
3)*a^3*d^5*sqrt(1/(a^4*d^6)) - 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*(-(3*a^2*
d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 1/3*sqrt(3)) + 2/3*sqrt(3)*(1
/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*arctan(1/6*(1
/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)) + 3*sqrt(3)*a*d^2)*sqrt((1/2)^
(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3
))^(1/3)*sqrt(1/(a^4*d^6)) + (1/2)^(2/3)*a^2*d^2*((3*a^2*d^3*sqrt(1/(a^4*d^
6)) - 1)/(a^2*d^3))^(2/3) + (b*x^3 + a)^(2/3))*((3*a^2*d^3*sqrt(1/(a^4*d^6)
) - 1)/(a^2*d^3))^(2/3) - 1/6*(1/2)^(2/3)*(sqrt(3)*a^3*d^5*sqrt(1/(a^4*d^6)
) + 3*sqrt(3)*a*d^2)*(b*x^3 + a)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(
a^2*d^3))^(2/3) - 1/3*sqrt(3)) - 1/6*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d
^6)) + 1)/(a^2*d^3))^(1/3)*log(-4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*(-(
3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/
2)^(2/3)*a^2*d^2*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(2/3) + 4*(
b*x^3 + a)^(2/3)) - 1/6*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2
*d^3))^(1/3)*log(4*(1/2)^(1/3)*(b*x^3 + a)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(1
/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + 4*(1/2)^(2/3)*a^2*d^2
*((3*a^2*d^3*sqrt(1/(a^4*d^6)) - 1)/(a^2*d^3))^(2/3) + 4*(b*x^3 + a)^(2/3))
 + 1/3*(1/2)^(1/3)*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*log
((1/2)^(1/3)*a^3*d^4*(-(3*a^2*d^3*sqrt(1/(a^4*d^6)) + 1)/(a^2*d^3))^(1/3)*s
qrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3)) + 1/3*(1/2)^(1/3)*((3*a^2*d^3*sqrt(1/
(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*log(-(1/2)^(1/3)*a^3*d^4*((3*a^2*d^3*sqrt(
1/(a^4*d^6)) - 1)/(a^2*d^3))^(1/3)*sqrt(1/(a^4*d^6)) + (b*x^3 + a)^(1/3))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootof1/3/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(
1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)
+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*1/6/a/d*ln(abs((a+b*
x^3)^(1/3)-(2*a)^(1/3)))-a^(1/3)*1/6/a/d*ln(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+
b*x^3)^(1/3)+a^(1/3)*a^(1/3))-a^(1/3)/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)+1/2
*a^(1/3))/sqrt(3)*2/a^(1/3))+a^(1/3)*1/3/a/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)
))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x), x)
```

mupad [B] time = 5.10, size = 345, normalized size = 1.61

$$\ln\left(\left(b\,x^{3}+a\right)^{1/3}-a\,d\left(\frac{1}{a^{2}\,d^{3}}\right)^{1/3}\right)\left(\frac{1}{27\,a^{2}\,d^{3}}\right)^{1/3}+\ln\left(\left(b\,x^{3}+a\right)^{1/3}+2^{1/3}\,a\,d\left(-\frac{1}{a^{2}\,d^{3}}\right)^{1/3}\right)\left(-\frac{2}{27\,a^{2}\,d^{3}}\right)^{1/3}-\ln\left(2^{1/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3) - a*d*(1/(a^2*d^3))^(1/3))*(1/(27*a^2*d^3))^(1/3) + l
og((a + b*x^3)^(1/3) + 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3))*(-2/(27*a^2*d^3))^
(1/3) - log(2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) - 2*(a + b*x^3)^(1/3) + 2^(1/3
)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-2/(27*a^2*d
^3))^(1/3) + log(2*(a + b*x^3)^(1/3) - 2^(1/3)*a*d*(-1/(a^2*d^3))^(1/3) + 2
^(1/3)*3^(1/2)*a*d*(-1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-2/(27*
a^2*d^3))^(1/3) + log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) - 3^(1/
2)*a*d*(1/(a^2*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*d^3))^(1/3
) - log(2*(a + b*x^3)^(1/3) + a*d*(1/(a^2*d^3))^(1/3) + 3^(1/2)*a*d*(1/(a^2
*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*d^3))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^{3}}}{-ax+bx^{4}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x + b*x**4), x)/d
```

3.572 $\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=268

$$\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}d}+\frac{\sqrt[3]{2}\,b\tan^{-1}\left(\right.}{\sqrt{}}$$

```
[Out] 1/3*b*(b*x^3+a)^(1/3)/a^2/d-1/3*(b*x^3+a)^(4/3)/a^2/d/x^3-2/3*b*ln(x)/a^(5/
3)/d+1/6*b*ln(-b*x^3+a)*2^(1/3)/a^(5/3)/d+2/3*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(5/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/3)/a^(5/3)/d-4/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/d*3^(1/2)
+1/3*2^(1/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 57, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3}+\frac{b\sqrt[3]{a+bx^3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d}+\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{5/3}d}-\frac{4b\tan^{-1}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/(3*a^2*d) - (a + b*x^3)^(4/3)/(3*a^2*d*x^3) - (4*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)
*d) + (2^(1/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(5/3)*d) - (2*b*Log[x])/(3*a^(5/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(2/3)*a^(5/3)*d) + (2*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(3*a^(5/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(2/3)*a^(5/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd+\frac{1}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(4b)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9a^2d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(4b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \right.}{3a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{2b\log(x)}{3a^{5/3}d} + \frac{b\log\left(a-bx^3\right)}{3\,2^{2/3}a^{5/3}d} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{3a^{5/3}d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{3a^2d} - \frac{\left(a+bx^3\right)^{4/3}}{3a^2dx^3} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}d} + \frac{\sqrt[3]{2}\,b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}d} - \frac{2b\log}{3a^{5/}}$$

Mathematica [A] time = 0.12, size = 280, normalized size = 1.04

$$-\frac{6a^{2/3}\sqrt[3]{a+bx^3} + 4bx^3\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right) - 3\sqrt[3]{2}\,bx^3\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] -1/18*(6*a^(2/3)*(a + b*x^3)^(1/3) + 8*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*
x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 6*2^(1/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3
)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 8*b*x^3*Log[a^(1/3) - (a + b*x^3)^
(1/3)] + 6*2^(1/3)*b*x^3*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 4*b*x^3
*Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 3*2^(1/3)*b
*x^3*Log[2^(2/3)*a^(2/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)])/(a^(5/3)*d*x^3)
```

fricas [A] time = 1.02, size = 321, normalized size = 1.20

$$-\frac{6\sqrt{3}\,2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right) + 3\cdot 2^{\frac{1}{3}}a^2bx^3\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}} -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 3*2^(1/3)*a^2*b*x^3*(
```

```
-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3)*a*
(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 6*2^(1/3)*a^2*b*x^3*(-1/a^2)^(1/3)*lo
g(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 8*sqrt(3)*(a^2)^(1/6)*a*b
*x^3*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^
(1/3)*(a^2)^(2/3))/a^2) + 4*(a^2)^(2/3)*b*x^3*log((b*x^3 + a)^(2/3)*a + (a^
2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 8*(a^2)^(2/3)*b*x^3*log((b*x^
3 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2)/(a^3*d*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootofb/3/a/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)*b/sqrt(3)/a^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*b*(2*a)^(1/3)*1/6/a^2/d*ln
(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-2*a^(1/3)*b*1/9/a^2/d*ln(((a+b*x^3)^(1/3
))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))-4*a^(1/3)*b/3/sqrt(3)/a^2/d*a
tan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+4*b*a^(1/3)*1/9/a^2/d*
ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-(a+b*x^3)^(1/3)*b/3/a/d/(a+b*x^3-a)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(bdx^3 - ad\right)x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^4), x)
```

mupad [B] time = 5.35, size = 455, normalized size = 1.70

$$\frac{4\,\ln\left(b\,\left(b\,x^3+a\right)^{1/3}-a^2\,d\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(\frac{b^3}{a^5\,d^3}\right)^{1/3}}{9}+\ln\left(b\,\left(b\,x^3+a\right)^{1/3}+2^{1/3}\,a^2\,d\,\left(-\frac{b^3}{a^5\,d^3}\right)^{1/3}\right)\,\left(-\frac{2\,b^3}{27\,a^5\,d^3}\right)^{1/3}+\ln\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(a*d - b*d*x^3)),x)
[Out] (4*log(b*(a + b*x^3)^(1/3) - a^2*d*(b^3/(a^5*d^3))^(1/3))*(b^3/(a^5*d^3))^(
1/3))/9 + log(b*(a + b*x^3)^(1/3) + 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3))*(
-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(a^5*
d^3))^(1/3) - 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2
)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2*b*(a + b*x^3)^(1/3) + a^2*d*(b^3/(
a^5*d^3))^(1/3) + 3^(1/2)*a^2*d*(b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*((64*b^3)/(729*a^5*d^3))^(1/3) - log(2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(
1/3) - 2*b*(a + b*x^3)^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(-b^3/(a^5*d^3))^(1/3)
*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1/3) + log(2*b*(a + b*
x^3)^(1/3) - 2^(1/3)*a^2*d*(-b^3/(a^5*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^2*d*(
-b^3/(a^5*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(-(2*b^3)/(27*a^5*d^3))^(1
/3) - (b*(a + b*x^3)^(1/3))/(3*a*(d*(a + b*x^3) - a*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^4+bx^7}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**4 + b*x**7), x)/d
```

3.573 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(ad-bdx^3)}\,dx$

Optimal. Leaf size=283

$$\frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan\ldots}{\ldots}$$

```
[Out] -2/9*b*(b*x^3+a)^(1/3)/a^2/d/x^3-1/6*(b*x^3+a)^(4/3)/a^2/d/x^6-11/18*b^2*ln
(x)/a^(8/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(1/3)/a^(8/3)/d+11/18*b^2*ln(a^(1/3)-(
b*x^3+a)^(1/3))/a^(8/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(1/
3)/a^(8/3)/d-11/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/
2))/a^(8/3)/d*3^(1/2)+1/3*2^(1/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/a^(8/3)/d*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {446, 103, 149, 156, 57, 617, 204, 31}

$$\frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d}+\frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{8/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^{8/3}d}-\frac{11b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{8/3}d}+\frac{\sqrt[3]{2}\,b^2\tan\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-2*b*(a + b*x^3)^(1/3))/(9*a^2*d*x^3) - (a + b*x^3)^(4/3)/(6*a^2*d*x^6) -
(11*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[
3]*a^(8/3)*d) + (2^(1/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)*d) - (11*b^2*Log[x])/(18*a^(8/3)*d) + (
b^2*Log[a - b*x^3])/(3*2^(2/3)*a^(8/3)*d) + (11*b^2*Log[a^(1/3) - (a + b*x^
3)^(1/3)])/(18*a^(8/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/
(2^(2/3)*a^(8/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{4}{3}abd-\frac{2}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{\text{Subst}\left(\int \frac{-\frac{22}{9}a^2b^2d^2-\frac{14}{9}ab^3d^2x}{x(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{\left(11b^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d} - \frac{\left(11b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,d\right.}{18a^{8/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\log(x)}{18a^{8/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\,2^{2/3}a^{8/3}d} + \frac{11b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right.}{18a^{8/3}d}$$

$$= -\frac{2b\sqrt[3]{a+bx^3}}{9a^2dx^3} - \frac{\left(a+bx^3\right)^{4/3}}{6a^2dx^6} - \frac{11b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{8/3}d} + \frac{\sqrt[3]{2}\,b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}d} - \frac{11}{}$$

Mathematica [A] time = 0.14, size = 314, normalized size = 1.11

$$-\frac{11b^2x^6\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-9\sqrt[3]{2}\,b^2x^6\log\left(2^{2/3}a^{2/3}+\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)+2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] -1/54*(9*a^(5/3)*(a + b*x^3)^(1/3) + 21*a^(2/3)*b*x^3*(a + b*x^3)^(1/3) + 2
2*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 18*
2^(1/3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 22*b^2*x^6*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 18*2^(1/3)*b^2*x^6*L
og[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)] + 11*b^2*x^6*Log[a^(2/3) + a^(1/3)*
(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)] - 9*2^(1/3)*b^2*x^6*Log[2^(2/3)*a^(2
/3) + 2^(1/3)*a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)])/(a^(8/3)*d*x^
6)
```

fricas [A] time = 0.94, size = 345, normalized size = 1.22

$$-\frac{18\sqrt{3}\,2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}+\frac{1}{3}\sqrt{3}\right)+9\cdot 2^{\frac{1}{3}}a^2b^2x^6\left(-\frac{1}{a^2}\right)^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}a^2\left(-\frac{1}{a^2}\right)^{\frac{2}{3}}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^(1/3)*arctan(1/3*sqrt(3)*2^(
2/3)*(b*x^3 + a)^(1/3)*a*(-1/a^2)^(2/3) + 1/3*sqrt(3)) + 9*2^(1/3)*a^2*b^2*
x^6*(-1/a^2)^(1/3)*log(2^(2/3)*a^2*(-1/a^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/
3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(2/3)) - 18*2^(1/3)*a^2*b^2*x^6*(-1/a^2)^
(1/3)*log(2^(1/3)*a*(-1/a^2)^(1/3) + (b*x^3 + a)^(1/3)) + 22*sqrt(3)*(a^2)^
(1/6)*a*b^2*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) + 11*(a^2)^(2/3)*b^2*x^6*log((b*x^3 + a)
^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 22*(a^2)^(2/3)*
b^2*x^6*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x^3 + 3*a^3)*(b
*x^3 + a)^(1/3))/(a^4*d*x^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootofb^2/3/a^2/((2*a)^(1/3))^2/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*
x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))+(2*a)^(1/3)*b^2/sqrt(3)/a^3/d*atan(((a+
b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*b^2*(2*a)^(1/3)*1/6/
a^3/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-11*a^(1/3)*b^2*1/54/a^3/d*ln(((a
+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))-11*a^(1/3)*b^2/9/
sqrt(3)/a^3/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+11*b^2*
a^(1/3)*1/27/a^3/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-(7*(a+b*x^3)^(1/3)*(a+b
*x^3)*b^2-4*(a+b*x^3)^(1/3)*a*b^2)/18/a^2/d/(a+b*x^3-a)^2
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(-bd\,x^{3}+ad\right)x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^7), x)
```

mupad [B] time = 5.44, size = 490, normalized size = 1.73

$$\frac{\frac{2\,b^2\,{\left(b\,x^3+a\right)}^{1/3}}{9\,a}-\frac{7\,b^2\,{\left(b\,x^3+a\right)}^{4/3}}{18\,a^2}}{d\,{\left(b\,x^3+a\right)}^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\frac{11\,\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}-a^3\,d\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}\right)\,{\left(\frac{b^6}{a^8\,d^3}\right)}^{1/3}}{27}+\ln\left(b^2\,{\left(b\,x^3+a\right)}^{1/3}+2^{1/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(a*d - b*d*x^3)),x)
```

```
[Out] ((2*b^2*(a + b*x^3)^(1/3))/(9*a) - (7*b^2*(a + b*x^3)^(4/3))/(18*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + (11*log(b^2*(a + b*x^3)^(1/3) -
 a^3*d*(b^6/(a^8*d^3))^(1/3))*(b^6/(a^8*d^3))^(1/3))/27 + log(b^2*(a + b*x^
3)^(1/3) + 2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3))*(-(2*b^6)/(27*a^8*d^3))^(1
/3) - log(2^(1/3)*a^3*d*(-b^6/(a^8*d^3))^(1/3) - 2*b^2*(a + b*x^3)^(1/3) +
2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(-(
2*b^6)/(27*a^8*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) - 2^(1/3)*a^3*d*(-
b^6/(a^8*d^3))^(1/3) + 2^(1/3)*3^(1/2)*a^3*d*(-b^6/(a^8*d^3))^(1/3)*1i)*((3
^(1/2)*1i)/2 - 1/2)*(-(2*b^6)/(27*a^8*d^3))^(1/3) + (11*log(2*b^2*(a + b*x^
3)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) - 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3
)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^8*d^3))^(1/3))/54 - (11*log(2*b^2*(a + b*x^3
)^(1/3) + a^3*d*(b^6/(a^8*d^3))^(1/3) + 3^(1/2)*a^3*d*(b^6/(a^8*d^3))^(1/3)
*1i)*(3^(1/2)*1i + 1)*(b^6/(a^8*d^3))^(1/3))/54
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^7+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**7 + b*x**10), x)/d
```

3.574 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=268

$$\frac{a^2\log\left(ad-bdx^3\right)}{3\ 2^{2/3}b^{8/3}d}+\frac{11a^2\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{8/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{8/3}d}+\frac{11a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{8/3}d}-\sqrt[3]{2}\ldots$$

```
[Out] -7/18*a*x^2*(b*x^3+a)^(1/3)/b^2/d-1/6*x^5*(b*x^3+a)^(1/3)/b/d+1/6*a^2*ln(-b
*d*x^3+a*d)*2^(1/3)/b^(8/3)/d+11/18*a^2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(8/
3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(8/3)/d+11/27*
a^2*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)-1
/3*2^(1/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/
b^(8/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a
])/(8*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.21, size = 177, normalized size = 0.66

$$\frac{22abx^5\left(1-\frac{b^2x^6}{a^2}\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2\left(\left(1-\frac{bx^3}{a}\right)^{2/3}\left(7a^2+10abx^3+3b^2x^6\right)-7a^2\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\right.\right.}{90b^2d\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (22*a*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/
a), (b*x^3)/a] - 5*x^2*((1 - (b*x^3)/a)^(2/3)*(7*a^2 + 10*a*b*x^3 + 3*b^2*x
^6) - 7*a^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^
3)/(a - b*x^3)]))/(90*b^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

fricas [A] time = 1.07, size = 362, normalized size = 1.35

$$-\frac{18\sqrt{3}\,2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-18\cdot 2^{\frac{1}{3}}a^2b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+9\cdot 2^{\frac{1}{3}}a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*2^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3
)*(b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 18*2^(1/3)*a^2*b^2*(
-1/b^2)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 9*2
^(1/3)*a^2*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)
*(b*x^3 + a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 22*sqrt(3
)*a^2*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3
+ a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 22*a^2*(b^2)^(2/3)*log(-((b^
2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 11*a^2*(b^2)^(2/3)*log(((b^2)^(1/3)*
b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*
b^3*x^5 + 7*a*b^2*x^2)*(b*x^3 + a)^(1/3))/(b^4*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^7}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x^7/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^7\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**7*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.575 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=233

$$\frac{a \log \left(ad-bdx^3\right)}{3\,2^{2/3} b^{5/3} d}+\frac{2a \log \left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{3b^{5/3} d}-\frac{a \log \left(\sqrt[3]{2}\,\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3} b^{5/3} d}+\frac{4a \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3} d}-\frac{\sqrt[3]{2}\,a \tan^{-1}\left(\cdots\right)}{\sqrt{3}\,b^{5\cdots}}$$

```
[Out] -1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(1/3)/b^(5/3)/d+2/3*a
*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d-1/2*a*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(1/3)/b^(5/3)/d+4/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3)
)*3^(1/2))/b^(5/3)/d*3^(1/2)-1/3*2^(1/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.28, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^5 \sqrt[3]{a+bx^3}\, F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]
)/(5*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4 \sqrt[3]{a+bx^3}}{ad-bdx^3} \, dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{x^4 \sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^5 \sqrt[3]{a+bx^3}\, F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.17, size = 160, normalized size = 0.69

$$\frac{4bx^5\left(1-\frac{b^2x^6}{a^2}\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5x^2\left(\left(a+bx^3\right)\left(1-\frac{bx^3}{a}\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)\right)}{15bd\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (4*b*x^5*(1 - (b^2*x^6)/a^2)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a),
 (b*x^3)/a] - 5*x^2*((a + b*x^3)*(1 - (b*x^3)/a)^(2/3) - a*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a - b*x^3)]))/(15*b*d*(a
 + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

fricas [A] time = 1.02, size = 338, normalized size = 1.45

$$-\frac{6\sqrt{3}\,2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-6\cdot 2^{\frac{1}{3}}ab^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+3\cdot 2^{\frac{1}{3}}ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*2^(1/3)*a*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(
b*x^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 6*2^(1/3)*a*b^2*(-1/b^2
)^(1/3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 3*2^(1/3)
*a*b^2*(-1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3
+ a)^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 6*(b*x^3 + a)^(1/
3)*b^2*x^2 + 8*sqrt(3)*a*(b^2)^(1/6)*b*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x
+ 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 8*a*(b^2)
^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 4*a*(b^2)^(2/3)*log(
((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)
/x^2))/(b^3*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^4}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3+a)^{\frac{1}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^4/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(b\,x^3+a)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^4\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.576 $\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=201

$$\frac{\log\left(ad-bdx^3\right)}{3\,2^{2/3}b^{2/3}d}+\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}b^{2/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{\sqrt[3]{2}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(1/3)/b^(2/3)/d+1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^
(2/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/b^(2/3)/d+1/3*arc
tan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)-1/3*2^(1
/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3
^(1/2)
```

Rubi [C] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 0.33, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a]
)/(2*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.31

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(2/3))
```

fricas [B] time = 0.97, size = 313, normalized size = 1.56

$$-\frac{2\sqrt{3}\,2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}b\left(-\frac{1}{b^2}\right)^{\frac{2}{3}}+\sqrt{3}\,x}{3\,x}\right)-2\cdot2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}\log\left(\frac{2^{\frac{1}{3}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{1}{3}}b^2\left(-\frac{1}{b^2}\right)^{\frac{1}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*2^(1/3)*b^2*(-1/b^2)^(1/3)*arctan(1/3*(sqrt(3)*2^(2/3)*(b*x
^3 + a)^(1/3)*b*(-1/b^2)^(2/3) + sqrt(3)*x)/x) - 2*2^(1/3)*b^2*(-1/b^2)^(1/
3)*log((2^(1/3)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(1/3))/x) + 2^(1/3)*b^2*(-
1/b^2)^(1/3)*log((2^(2/3)*b^2*x^2*(-1/b^2)^(2/3) - 2^(1/3)*(b*x^3 + a)^(1/3
)*b*x*(-1/b^2)^(1/3) + (b*x^3 + a)^(2/3))/x^2) + 2*sqrt(3)*(b^2)^(1/6)*b*ar
ctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3)
)*(b^2)^(1/6)/(b^2*x)) - 2*(b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1
/3)*b)/x) + (b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b^2)^(2
/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int-\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)*x/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.577 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(ad-bdx^3)}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt[3]{a+bx^3}}{adx}+\frac{\sqrt[3]{b}\log\left(ad-bdx^3\right)}{3\,2^{2/3}ad}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}ad}-\frac{\sqrt[3]{2}\sqrt[3]{b}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,ad}$$

```
[Out] -(b*x^3+a)^(1/3)/a/d/x+1/6*b^(1/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a/d-1/2*b^(1/3)
*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a/d-1/3*2^(1/3)*b^(1/3)*arct
an(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 - (b*x^3)/a)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, (-2*b*x^3)/(a - b*x^3)])/(a*d*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(ad-bdx^3\right)}\,dx&=\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\&=-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1-\frac{bx^3}{a}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{2bx^3}{a-bx^3}\right)}{adx\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 45, normalized size = 0.29

$$-\frac{\sqrt[3]{a+bx^3}\ {}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{2bx^3}{bx^3+a}\right)}{adx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x]
[Out] -(((a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1, 2/3, (2*b*x^3)/(a + b*x^3)]
)/(a*d*x))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^2*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**2 + b*x**5), x)/d
```

3.578 $\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=183

$$\frac{b^{4/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^2d}-\frac{b^{4/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^2d}-\frac{\sqrt[3]{2}\,b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}-\frac{5b\sqrt[3]{a+bx^3}}{4a^2dx}-\frac{\sqrt[3]{a+bx^3}}{4adx^4}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/d/x^4-5/4*b*(b*x^3+a)^(1/3)/a^2/d/x+1/6*b^(4/3)*ln(-
b*d*x^3+a*d)*2^(1/3)/a^2/d-1/2*b^(4/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3)
)*2^(1/3)/a^2/d-1/3*2^(1/3)*b^(4/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^
3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [C] time = 0.42, antiderivative size = 117, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{a^2-bx^3\left(a+3bx^3\right)\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+3bx^3\left(a-bx^3\right)\,_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+4abx^3+3b^2x^6}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -(a^2 + 4*a*b*x^3 + 3*b^2*x^6 - b*x^3*(a + 3*b*x^3)*Hypergeometric2F1[2/3,
1, 5/3, (2*b*x^3)/(a + b*x^3)] + 3*b*x^3*(a - b*x^3)*Hypergeometric2F1[2/3,
 2, 5/3, (2*b*x^3)/(a + b*x^3)])/(4*a^2*d*x^4*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(ad-bdx^3\right)}\,dx=\frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=-\frac{a^2+4abx^3+3b^2x^6-bx^3\left(a+3bx^3\right)\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)+3bx^3\left(a-bx^3\right)\,_2F_1\left(\frac{2}{3},2;\frac{5}{3}\right.}{4a^2dx^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 5.10, size = 125, normalized size = 0.68

$$\frac{b^2x^2\left(\frac{a+bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a\left(1-\frac{bx^3}{a}\right)}\right)}{a^2d\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}-\frac{\left(\frac{5b}{4a^2x}+\frac{1}{4ax^4}\right)\sqrt[3]{a+bx^3}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -((((1/(4*a*x^4) + (5*b)/(4*a^2*x))*(a + b*x^3)^(1/3))/d) + (b^2*x^2*((a + b
*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)/(a*(1 - (b*x^3)/
a))])/(a^2*d*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**5 + b*x**8), x)/d
```

3.579 $\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)} dx$

Optimal. Leaf size=210

$$\frac{b^{7/3}\log\left(ad-bdx^3\right)}{3\ 2^{2/3}a^3d}-\frac{b^{7/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^3d}-\frac{\sqrt[3]{2}b^{7/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}a^3d}-\frac{8b^2\sqrt[3]{a+bx^3}}{7a^3dx}-\frac{2b\sqrt[3]{a+bx^3}}{7a^2dx^4}-\sqrt[3]{}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/a/d/x^7-2/7*b*(b*x^3+a)^(1/3)/a^2/d/x^4-8/7*b^2*(b*x^3
+a)^(1/3)/a^3/d/x+1/6*b^(7/3)*ln(-b*d*x^3+a*d)*2^(1/3)/a^3/d-1/2*b^(7/3)*ln
(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(1/3)/a^3/d-1/3*2^(1/3)*b^(7/3)*arcta
n(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [C] time = 19.78, antiderivative size = 244, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-9bx^3\left(a-bx^3\right)^2 {}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-2bx^3\left(2a^2+3abx^3+9b^2x^6\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+15a^2bx^3{}_2F_1\left(\frac{2}{3},2;\frac{5}{3}\right.}{28a^3dx^7\left(a+b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
```

```
[Out] -(4*a^3 + 10*a^2*b*x^3 + 24*a*b^2*x^6 + 18*b^3*x^9 - 2*b*x^3*(2*a^2 + 3*a*b
*x^3 + 9*b^2*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 1
5*a^2*b*x^3*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^
2*x^6*Hypergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 27*b^3*x^9*Hy
pergeometric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 9*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)])/(28*a^3*d
*x^7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(ad-bdx^3\right)}dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(ad-bdx^3\right)}dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{4a^3+10a^2bx^3+24ab^2x^6+18b^3x^9-2bx^3\left(2a^2+3abx^3+9b^2x^6\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)}{}$$

Mathematica [C] time = 5.12, size = 135, normalized size = 0.64

$$\frac{7b^3x^9\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)-\left(1-\frac{bx^3}{a}\right)^{2/3}\left(a^3+3a^2bx^3+10ab^2x^6+8b^3x^9\right)}{7a^3dx^7\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x]
[Out] (-((1 - (b*x^3)/a)^(2/3)*(a^3 + 3*a^2*b*x^3 + 10*a*b^2*x^6 + 8*b^3*x^9)) +
7*b^3*x^9*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x^3)
/(a - b*x^3)])/(7*a^3*d*x^7*(a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^8*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^8+bx^{11}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**8 + b*x**11), x)/d
```

3.580 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=237

$$\frac{b^{10/3}\log\left(ad-bdx^3\right)}{3\,2^{2/3}a^4d}-\frac{b^{10/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2^{2/3}a^4d}-\frac{\sqrt[3]{2}\,b^{10/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}-\frac{169b^3\sqrt[3]{a+bx^3}}{140a^4dx}-\frac{37b^2\sqrt[3]{a}}{140a}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/a/d/x^10-11/70*b*(b*x^3+a)^(1/3)/a^2/d/x^7-37/140*b^2
*(b*x^3+a)^(1/3)/a^3/d/x^4-169/140*b^3*(b*x^3+a)^(1/3)/a^4/d/x+1/6*b^(10/3)
*ln(-b*d*x^3+a*d)*2^(1/3)/a^4/d-1/2*b^(10/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)
^(1/3))*2^(1/3)/a^4/d-1/3*2^(1/3)*b^(10/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*
x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [C] time = 29.21, antiderivative size = 423, normalized size of antiderivative = 1.78, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(2a+3bx^3\right)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)+27bx^3\left(a-bx^3\right)^3\,{}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{5}{3};\frac{2bx^3}{bx^3+a}\right)-36a^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] -(28*a^4 + 64*a^3*b*x^3 + 90*a^2*b^2*x^6 + 216*a*b^3*x^9 + 162*b^4*x^12 - 2
8*a^3*b*x^3*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 36*a^2*
b^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 54*a*b^3*x^
9*Hypergeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] - 162*b^4*x^12*Hype
rgeometric2F1[2/3, 1, 5/3, (2*b*x^3)/(a + b*x^3)] + 117*a^3*b*x^3*Hypergeom
etric2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 99*a^2*b^2*x^6*Hypergeometri
c2F1[2/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] + 81*a*b^3*x^9*Hypergeometric2F1[2
/3, 2, 5/3, (2*b*x^3)/(a + b*x^3)] - 297*b^4*x^12*Hypergeometric2F1[2/3, 2,
 5/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(2*a + 3*b*x^3)*Hyper
geometricPFQ[{2/3, 2, 2}, {1, 5/3}, (2*b*x^3)/(a + b*x^3)] + 27*b*x^3*(a -
b*x^3)^3*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, (2*b*x^3)/(a + b*x^
3)])/(280*a^4*d*x^10*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{28a^4+64a^3bx^3+90a^2b^2x^6+216ab^3x^9+162b^4x^{12}-28a^3bx^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2bx^3}{a+bx^3}\right)-36a}{}$$

Mathematica [C] time = 5.20, size = 130, normalized size = 0.55

$$\frac{-14a^4-36a^3bx^3-59a^2b^2x^6+\frac{140b^4x^{12}\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};-\frac{2bx^3}{a-bx^3}\right)}{\left(1-\frac{bx^3}{a}\right)^{2/3}}-206ab^3x^9-169b^4x^{12}}{140a^4dx^{10}\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x]
```

```
[Out] (-14*a^4 - 36*a^3*b*x^3 - 59*a^2*b^2*x^6 - 206*a*b^3*x^9 - 169*b^4*x^12 + (
140*b^4*x^12*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, (-2*b*x
^3)/(a - b*x^3)])/(1 - (b*x^3)/a)^(2/3))/(140*a^4*d*x^10*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(bdx^{3}-ad\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^{11}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(a*d - b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(-b*d*x**3+a*d),x)
[Out] Timed out
```

3.581 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=521

$$-\frac{a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}b^{7/3}d}+\frac{a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}b^{7/3}d}-\frac{\sqrt[3]{2}\,a^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{7/3}d}+\frac{a^{5/3}\log}{}$$

```
[Out] -3/5*a*x*(b*x^3+a)^(1/3)/b^2/d-1/5*x^4*(b*x^3+a)^(1/3)/b/d-2/5*a^2*x*(1+b*x
^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(2/3)-1/6*
a^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d+
1/6*a^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*ln(1
+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(7/3)/d+1/12*a^(5/3)*ln(2*2
^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b
*x^3+a)^(1/3))*2^(1/3)/b^(7/3)/d-1/3*2^(1/3)*a^(5/3)*arctan(1/3*(1-2*2^(1/3
)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)-1/6*a^(5/
3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1
/3)/b^(7/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a
])/(7*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^6\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.38, size = 234, normalized size = 0.45

$$\frac{\frac{48a^4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+7abx^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\ldots}{20b^2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-4*(a + b*x^3)*(3*a*x + b*x^4) + 7*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (48*a^4*x*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/
3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)
/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))
/(20*b^2*d*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^6}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

[Out] int(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")

[Out] -integrate((b*x^3 + a)^(1/3)*x^6/(b*d*x^3 - a*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)

[Out] int((x^6*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^6\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)

[Out] -Integral(x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d

3.582 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=494

$$-\frac{a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}b^{4/3}d}+\frac{a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}b^{4/3}d}-\frac{\sqrt[3]{2}\,a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{4/3}d}+\frac{a^{2/3}}{}$$

```
[Out] -1/2*x*(b*x^3+a)^(1/3)/b/d-1/2*a*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[
4/3],-b*x^3/a)/b/d/(b*x^3+a)^(2/3)-1/6*a^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)
*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d+1/6*a^(2/3)*ln(1+2^(2/3)*(a^(1/3)+b^
(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(
1/3)/b^(4/3)/d-1/3*2^(1/3)*a^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+
a)^(1/3))/b^(4/3)/d+1/12*a^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+
a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/b^(4/3)/d-1/3
*2^(1/3)*a^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3
))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/6*a^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^
(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/b^(4/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$\frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]
)/(4*a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx=\frac{\sqrt[3]{a+bx^3}\int\frac{x^3\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=\frac{x^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.21, size = 225, normalized size = 0.46

$$\frac{x\left(\frac{4\left(\frac{4a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}-a-bx^3\right)}{b}+3x^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{b}{}\right.\right.}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x*(3*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b
*x^3)/a] + (4*(-a - b*x^3 + (4*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
 (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*
x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*
AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/b))/(8*d*(a + b*x^
3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^3}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)*x^3/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^3*(a + b*x^3)^(1/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.583 $\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=416

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\cdots\right)}{6\,2^{2/3}\cdots}$$

```
[Out] -1/6*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/
3)/d+1/6*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3
)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*ln(1+2^
(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(1/3)/b^(1/3)/d+1/12*ln(2*2^(1
/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))*2^(1/3)/a^(1/3)/b^(1/3)/d-1/3*2^(1/3)*arctan(1/3*(1-2*2^(1/3)*(
a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(1/3)/b^(1/3)/d*3^(1/2)-1/6*
arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)
/a^(1/3)/b^(1/3)/d*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{ad-bdx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{ad-bdx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.15, size = 154, normalized size = 0.37

$$\frac{4ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a*d - b*d*x^3),x]
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF
1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3),x)
[Out] int((a + b*x^3)^(1/3)/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)/d
```

3.584 $\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=496

$$-\frac{b^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{4/3}d}+\frac{b^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{4/3}d}-\frac{\sqrt[3]{2}\,b^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{4/3}d}+\frac{b^{2/3}}{}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)/a/d/x^2+1/2*b*x*(1+b*x^3/a)^(2/3)*hypergeom([1/3, 2/3]
,[4/3],-b*x^3/a)/a/d/(b*x^3+a)^(2/3)-1/6*b^(2/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/
3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d+1/6*b^(2/3)*ln(1+2^(2/3)*(a^(1/3)+
b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2
^(1/3)/a^(4/3)/d-1/3*2^(1/3)*b^(2/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^
3+a)^(1/3))/a^(4/3)/d+1/12*b^(2/3)*ln(2*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^
3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(4/3)/d-1
/3*2^(1/3)*b^(2/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1
/3))*3^(1/2))/a^(4/3)/d*3^(1/2)-1/6*b^(2/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(1/3)/a^(4/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2adx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), (b*x^3)/a])/
(2*a*d*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2adx^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.15, size = 231, normalized size = 0.47

$$\frac{\frac{48a^3bx^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)} + b^2x^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8a^2dx^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] (-4*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*b*x^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*
x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)
/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)
/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))))/(8*a^2*d*x^
2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^3*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^3+bx^6}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**3 + b*x**6), x)/d
```

3.585 $\int \frac{\sqrt[3]{a+bx^3}}{x^6(ad-bdx^3)}\,dx$

Optimal. Leaf size=523

$$-\frac{b^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\,2^{2/3}a^{7/3}d}+\frac{b^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\,2^{2/3}a^{7/3}d}-\frac{\sqrt[3]{2}\,b^{5/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{7/3}d}+\frac{b^{5/3}\log}{}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)/a/d/x^5-3/5*b*(b*x^3+a)^(1/3)/a^2/d/x^2+2/5*b^2*x*(1+b
*x^3/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(2/3)-1/
6*b^(5/3)*ln(2^(2/3)+(-a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/
d+1/6*b^(5/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(7/3)/d+1/12*b^(5/3)*ln(2
*2^(1/3)+(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)+2^(2/3)*(a^(1/3)+b^(1/3)*x)/
(b*x^3+a)^(1/3))*2^(1/3)/a^(7/3)/d-1/3*2^(1/3)*b^(5/3)*arctan(1/3*(1-2*2^(1
/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(7/3)/d*3^(1/2)-1/6*b^(
5/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^
(1/3)/a^(7/3)/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5adx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), (b*x^3)/a])
/(5*a*d*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(ad-bdx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6\left(ad-bdx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5adx^5\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.17, size = 243, normalized size = 0.46

$$\frac{\frac{3b^3x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}-\frac{4\left(a^2+4abx^3+3b^2x^6\right)}{a^2x^5}+\frac{112b^2xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1\right.\right.}}{20d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-4*(a^2 + 4*a*b*x^3 + 3*b^2*x^6))/(a^2*x^5) + (3*b^3*x^4*(1 + (b*x^3)/a)^
(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/a^3 + (112*b^2*x
*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a]))))/(20*d*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(-bd\,x^3+ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(bdx^3 - ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/((b*d*x^3 - a*d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3 + a\right)^{1/3}}{x^6\,\left(a\,d - b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^6*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt[3]{a+bx^3}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a*x**6 + b*x**9), x)/d
```

3.586 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=223

$$\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^4d}-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+\right.}{5b}$$

```
[Out] -1/2*a^3*(b*x^3+a)^(2/3)/b^4/d-1/5*a^2*(b*x^3+a)^(5/3)/b^4/d+1/8*a*(b*x^3+a
)^(8/3)/b^4/d-1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*a^(11/3)*ln(-b*x^3+a)*2^(2/3)
/b^4/d-1/2*a^(11/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b^4/d-1/3*2
^(2/3)*a^(11/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2
))/b^4/d*3^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}-\frac{a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+}}{\sqrt{3}}\right.}{\sqrt{3}\,b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*(a + b*x^3)^(5/3))/(5*b^4*d) + (a
*(a + b*x^3)^(8/3))/(8*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) - (2^(2/3)*a^
(11/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^4*d) + (a^(11/3)*Log[a - b*x^3])/(3*2^(1/3)*b^4*d) - (a^(11/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^4*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{a^2(a+bx)^{2/3}}{b^3d}+\frac{a(a+bx)^{5/3}}{b^3d}-\frac{(a+bx)^{8/3}}{b^3d}+\frac{a^3(a+bx)^{2/3}}{b^3(ad-bdx)}\right)dx, x, x^3\right)$$

$$= -\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^3\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3b^3}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{\left(2a^4\right)\operatorname{Subst}\left(\int \frac{}{\sqrt[3]{a}} \right.}{3}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}+\frac{a^{11/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^4d}$$

$$= -\frac{a^3\left(a+bx^3\right)^{2/3}}{2b^4d}-\frac{a^2\left(a+bx^3\right)^{5/3}}{5b^4d}+\frac{a\left(a+bx^3\right)^{8/3}}{8b^4d}-\frac{\left(a+bx^3\right)^{11/3}}{11b^4d}-\frac{2^{2/3}a^{11/3}\tan^{-1}\left(\frac{1+}{}\right.}{\sqrt{3}\,b^4d}$$

Mathematica [A] time = 0.28, size = 163, normalized size = 0.73

$$-\frac{-220\,2^{2/3}a^{11/3}\log\left(a-bx^3\right)+660\,2^{2/3}a^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+440\,2^{2/3}\sqrt{3}\,a^{11/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)+}{1320b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -1/1320*(3*(a + b*x^3)^(2/3)*(293*a^3 + 98*a^2*b*x^3 + 65*a*b^2*x^6 + 40*b^
3*x^9) + 440*2^(2/3)*Sqrt[3]*a^(11/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 220*2^(2/3)*a^(11/3)*Log[a - b*x^3] + 660*2^(2/3)*a^
(11/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(b^4*d)
```

fricas [A] time = 1.28, size = 209, normalized size = 0.94

$$-\frac{440 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} a^{3} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} - \sqrt{3} a}{3 a}\right) + 220 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a^{3} \log\left(4^{\frac{2}{3}} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}} + 2 \left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^3*arctan(1/3*(4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 220*4^(1/3)*(-a^2)^(1/3)*a^
3*log(4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^
(1/3)*(-a^2)^(1/3)*a) - 440*4^(1/3)*(-a^2)^(1/3)*a^3*log(-4^(2/3)*(-a^2)^(2
/3) + 2*(b*x^3 + a)^(1/3)*a) + 3*(40*b^3*x^9 + 65*a*b^2*x^6 + 98*a^2*b*x^3
+ 293*a^3)*(b*x^3 + a)^(2/3))/(b^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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```
rootofa^3*((2*a)^(1/3))^2*1/6/b^4/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+
b*x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a^3*((2*a)^(1/3))^2/sqrt(3)/b^4/d*ata
n(((a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^
4*b^44*d^11*(2*a)^(1/3)*1/6/a/b^48/d^12*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3))
)-(1/11*((a+b*x^3)^(1/3))^2*(a+b*x^3)^3*b^40*d^10-1/8*((a+b*x^3)^(1/3))^2*(
a+b*x^3)^2*a*b^40*d^10+1/5*((a+b*x^3)^(1/3))^2*(a+b*x^3)*a^2*b^40*d^10+1/2*
((a+b*x^3)^(1/3))^2*a^3*b^40*d^10)/b^44/d^11
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{11}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.25, size = 183, normalized size = 0.82

$$-\frac{\frac{440\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{11}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{220\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}+\frac{440\cdot 2^{\frac{2}{3}}a^{\frac{11}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}+}{1320\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/1320*(440*sqrt(3)*2^(2/3)*a^(11/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a
^(1/3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 220*2^(2/3)*a^(11/3)*log(2^(2/3)
*a^(2/3) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 440*2
^(2/3)*a^(11/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(40*(b*x^3
+ a)^(11/3) - 55*(b*x^3 + a)^(8/3)*a + 88*(b*x^3 + a)^(5/3)*a^2 + 220*(b*x^
3 + a)^(2/3)*a^3)/d)/b^4
```

mupad [B] time = 4.85, size = 261, normalized size = 1.17

$$\frac{a\left(b\,x^3+a\right)^{8/3}}{8\,b^4\,d}-\frac{a^3\left(b\,x^3+a\right)^{2/3}}{2\,b^4\,d}-\frac{a^2\left(b\,x^3+a\right)^{5/3}}{5\,b^4\,d}-\frac{\left(b\,x^3+a\right)^{11/3}}{11\,b^4\,d}+\frac{4^{1/3}\,(-a)^{11/3}\,\ln\left(4\,a^8\left(b\,x^3+a\right)^{1/3}+4\,2^{1/3}\,(-a)^{25/}\right.}{3\,b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (a*(a + b*x^3)^(8/3))/(8*b^4*d) - (a^3*(a + b*x^3)^(2/3))/(2*b^4*d) - (a^2*
(a + b*x^3)^(5/3))/(5*b^4*d) - (a + b*x^3)^(11/3)/(11*b^4*d) + (4^(1/3)*(-a
)^(11/3)*log(4*a^8*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(25/3)))/(3*b^4*d) -
(4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (2*4^(2/3)*(
-a)^(25/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^8*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*
b^4*d) + (4^(1/3)*(-a)^(11/3)*log((4*a^8*(a + b*x^3)^(1/3))/(b^8*d^2) + (18
*4^(2/3)*(-a)^(25/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^8*d^2))*((3^(1/2)*1i)/6 -
 1/6))/(b^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] Timed out
```

3.587 $\int \frac{x^8(a+bx^3)^{2/3}}{ad-bdx^3} dx$

Optimal. Leaf size=177

$$\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

```
[Out] -1/2*a^2*(b*x^3+a)^(2/3)/b^3/d-1/8*(b*x^3+a)^(8/3)/b^3/d+1/6*a^(8/3)*ln(-b*
x^3+a)*2^(2/3)/b^3/d-1/2*a^(8/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3
)/b^3/d-1/3*2^(2/3)*a^(8/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^
(1/3)*3^(1/2))/b^3/d*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 50, 55, 617, 204, 31}

$$-\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d) - (2^(2/3)
*a^(8/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*b^3*d) + (a^(8/3)*Log[a - b*x^3])/(3*2^(1/3)*b^3*d) - (a^(8/3)*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^3*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{(a+bx)^{5/3}}{b^2d}+\frac{a^2(a+bx)^{2/3}}{b^2(ad-bdx)}\right)dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^2\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)}{3b^2}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}+\frac{a^{8/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a}\right.}{\sqrt[3]{2}\,b^3d}\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-\frac{a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^3d}+\\
&= -\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^3d}-\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{2^{2/3}a^{8/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^3d}+\frac{a^{8/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^3d}-
\end{aligned}$$

Mathematica [A] time = 0.17, size = 153, normalized size = 0.86

$$\frac{4\,2^{2/3}a^{8/3}\log\left(a-bx^3\right)-8\,2^{2/3}\sqrt{3}\,a^{8/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\left(4\,2^{2/3}a^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right.\right.}{24b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-8*2^(2/3)*Sqrt[3]*a^(8/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 4*2^(2/3)*a^(8/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(5*a^2
 + 2*a*b*x^3 + b^2*x^6) + 4*2^(2/3)*a^(8/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^
3)^(1/3)]))/(24*b^3*d)
```

fricas [A] time = 1.15, size = 197, normalized size = 1.11

$$-\frac{8 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} a^{2} \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}}-\sqrt{3} a}{3 a}\right)+4 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a^{2} \log\left(4^{\frac{2}{3}} \left(bx^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}}+2 \left(bx^{3}+a\right)^{\frac{2}{3}} \cdots\right.}{24 b \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/24*(8*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a^2*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4*4^(1/3)*(-a^2)^(1/3)*a^2*log(
4^(2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*
(-a^2)^(1/3)*a) - 8*4^(1/3)*(-a^2)^(1/3)*a^2*log(-4^(2/3)*(-a^2)^(2/3) + 2*
(b*x^3 + a)^(1/3)*a) + 3*(b^2*x^6 + 2*a*b*x^3 + 5*a^2)*(b*x^3 + a)^(2/3))/(
b^3*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootofa^2*((2*a)^(1/3))^2*1/6/b^3/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+
b*x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a^2*((2*a)^(1/3))^2/sqrt(3)/b^3/d*ata
```

```
n(((a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^
3*b^24*d^8*(2*a)^(1/3)*1/6/a/b^27/d^9*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-
(1/8*((a+b*x^3)^(1/3))^2*(a+b*x^3)^2*b^21*d^7+1/2*((a+b*x^3)^(1/3))^2*a^2*b
^21*d^7)/b^24/d^8
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{8}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.17, size = 155, normalized size = 0.88

$$-\frac{\frac{8\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{8}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{4\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}+\frac{8\cdot 2^{\frac{2}{3}}a^{\frac{8}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}+3\Big((b}{24\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/24*(8*sqrt(3)*2^(2/3)*a^(8/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3
) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 4*2^(2/3)*a^(8/3)*log(2^(2/3)*a^(2/3)
 + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 8*2^(2/3)*a^(
8/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*((b*x^3 + a)^(8/3) + 4
*(b*x^3 + a)^(2/3)*a^2)/d)/b^3
```

mupad [B] time = 4.91, size = 206, normalized size = 1.16

$$-\frac{\left(b\,x^3+a\right)^{8/3}}{8\,b^3\,d}-\frac{a^2\,\left(b\,x^3+a\right)^{2/3}}{2\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b^3\,d}-\frac{4^{1/3}\,a^{8/3}\,\ln\left(\frac{4\,a^6\,\left(b\,x^3+a\right)^{1/3}}{b^6\,d^2}-\frac{2\,4^{2/3}\,a^{19/3}\,\left(-\frac{1}{2}\right.}{b^6\,d^2}\right.}{3\,b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1/3))/(b^6*d^2) - (18*4^(2/3)*a^(1
9/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^6*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b^3*d) -
 (a^2*(a + b*x^3)^(2/3))/(2*b^3*d) - (4^(1/3)*a^(8/3)*log((a + b*x^3)^(1/3)
 - 2^(1/3)*a^(1/3)))/(3*b^3*d) - (4^(1/3)*a^(8/3)*log((4*a^6*(a + b*x^3)^(1
/3))/(b^6*d^2) - (2*4^(2/3)*a^(19/3)*((3^(1/2)*1i)/2 - 1/2)^2)/(b^6*d^2))*(
(3^(1/2)*1i)/2 - 1/2))/(3*b^3*d) - (a + b*x^3)^(8/3)/(8*b^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{8}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**8*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.588 $\int \frac{x^5 (a+bx^3)^{2/3}}{ad-bdx^3} \, dx$

Optimal. Leaf size=175

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

```
[Out] -1/2*a*(b*x^3+a)^(2/3)/b^2/d-1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*a^(5/3)*ln(-b*x^
3+a)*2^(2/3)/b^2/d-1/2*a^(5/3)*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
b^2/d-1/3*2^(2/3)*a^(5/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1
/3)*3^(1/2))/b^2/d*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 50, 55, 617, 204, 31}

$$\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^2d}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) - (2^(2/3)*a
^(5/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sq
rt[3]*b^2*d) + (a^(5/3)*Log[a - b*x^3])/(3*2^(1/3)*b^2*d) - (a^(5/3)*Log[2^
(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*b^2*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{a\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)}{3b}\\
&= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)}{3b}\\
&= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}+\frac{a^{5/3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a}\right.}{\sqrt[3]{2}\,b^2d}\\
&= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-\frac{a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^2d}+\\
&= -\frac{a\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}-\frac{2^{2/3}a^{5/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^2d}+\frac{a^{5/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,b^2d}-
\end{aligned}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.82

$$\frac{5\,2^{2/3}a^{5/3}\log\left(a-bx^3\right)-3\left(5\,2^{2/3}a^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\left(7a+2bx^3\right)\right)-10\,2^{2/3}\sqrt{3}\,a^{5/3}\,\text{ta}}{30b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-10*2^(2/3)*Sqrt[3]*a^(5/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3
))/Sqrt[3]] + 5*2^(2/3)*a^(5/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3)*(7*a
+ 2*b*x^3) + 5*2^(2/3)*a^(5/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(
30*b^2*d)
```

fricas [A] time = 1.22, size = 181, normalized size = 1.03

$$-\frac{10 \cdot 4^{\frac{1}{3}} \sqrt{3} \left(-a^{2}\right)^{\frac{1}{3}} a \arctan\left(\frac{4^{\frac{1}{3}} \sqrt{3} \left(b x^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}}-\sqrt{3} a}{3 a}\right)+5 \cdot 4^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{1}{3}} a \log\left(4^{\frac{2}{3}} \left(b x^{3}+a\right)^{\frac{1}{3}} \left(-a^{2}\right)^{\frac{2}{3}}+2 \left(b x^{3}+a\right)^{\frac{2}{3}}\right.}{30 b^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/30*(10*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*a*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3
 + a)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 5*4^(1/3)*(-a^2)^(1/3)*a*log(4^(
2/3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a
^2)^(1/3)*a) - 10*4^(1/3)*(-a^2)^(1/3)*a*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x
^3 + a)^(1/3)*a) + 3*(2*b*x^3 + 7*a)*(b*x^3 + a)^(2/3))/(b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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x^3)^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-a*((2*a)^(1/3))^2/sqrt(3)/b^2/d*atan(((
a+b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a^2*b^
```

```
10*d^5*(2*a)^(1/3)*1/6/a/b^12/d^6*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(1/5
*((a+b*x^3)^(1/3))^2*(a+b*x^3)*b^8*d^4+1/2*((a+b*x^3)^(1/3))^2*a*b^8*d^4)/b
^10/d^5
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{5}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.12, size = 155, normalized size = 0.89

$$-\frac{\frac{10\,\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{5}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\,\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{6\,a^{\frac{1}{3}}}\right)}{d}-\frac{5\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^{3}+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{2}{3}}\right)}{d}+\frac{10\cdot 2^{\frac{2}{3}}a^{\frac{5}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(\cdots\right.}{\cdots}}{30\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/30*(10*sqrt(3)*2^(2/3)*a^(5/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/
3) + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 5*2^(2/3)*a^(5/3)*log(2^(2/3)*a^(2/3
) + 2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 10*2^(2/3)*a
^(5/3)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(2*(b*x^3 + a)^(5/3)
 + 5*(b*x^3 + a)^(2/3)*a)/d)/b^2
```

mupad [B] time = 4.84, size = 221, normalized size = 1.26

$$\frac{4^{1/3}\,{\left(-a\right)}^{5/3}\,\ln\left(4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}+4\,2^{1/3}\,{\left(-a\right)}^{13/3}\right)}{3\,b^2\,d}-\frac{a\,{\left(b\,x^3+a\right)}^{2/3}}{2\,b^2\,d}-\frac{{\left(b\,x^3+a\right)}^{5/3}}{5\,b^2\,d}-\frac{4^{1/3}\,{\left(-a\right)}^{5/3}\,\ln\left(\frac{4\,a^4\,{\left(b\,x^3+a\right)}^{1/3}}{b^4\,d^2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*(-a)^(5/3)*log(4*a^4*(a + b*x^3)^(1/3) + 4*2^(1/3)*(-a)^(13/3)))/(
3*b^2*d) - (a*(a + b*x^3)^(2/3))/(2*b^2*d) - (a + b*x^3)^(5/3)/(5*b^2*d) -
(4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (2*4^(2/3)*(-
a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/(b^4*d^2))*((3^(1/2)*1i)/2 + 1/2))/(3*b
^2*d) + (4^(1/3)*(-a)^(5/3)*log((4*a^4*(a + b*x^3)^(1/3))/(b^4*d^2) + (18*4
^(2/3)*(-a)^(13/3)*((3^(1/2)*1i)/6 - 1/6)^2)/(b^4*d^2))*((3^(1/2)*1i)/6 - 1
/6))/(b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{5}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**5*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.589 $\int \frac{x^2(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=153

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/b/d+1/6*a^(2/3)*ln(-b*x^3+a)*2^(2/3)/b/d-1/2*a^(2/3)*l
n(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/b/d-1/3*2^(2/3)*a^(2/3)*arctan(1
/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/b/d*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {444, 50, 55, 617, 204, 31}

$$\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,bd}-\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*b*d) - (2^(2/3)*a^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a
 + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b*d) + (a^(2/3)*Log[a - b*x^3]
)/(3*2^(1/3)*b*d) - (a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(
1/3)*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}+\frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}-\frac{a\,\text{Subs}}{}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,bd}+\frac{\left(2^{2/3}a^{2/3}\right)\text{Subst}}{}\\
&= -\frac{\left(a+bx^3\right)^{2/3}}{2bd}-\frac{2^{2/3}a^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,bd}+\frac{a^{2/3}\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,bd}-\frac{a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\right.}{\sqrt[3]{2}\,bd}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 130, normalized size = 0.85

$$\frac{2^{2/3}a^{2/3}\log\left(a-bx^3\right)-3\left(2^{2/3}a^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+\left(a+bx^3\right)^{2/3}\right)-2\,2^{2/3}\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right.}{6bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-2*2^(2/3)*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3)
)/Sqrt[3]] + 2^(2/3)*a^(2/3)*Log[a - b*x^3] - 3*((a + b*x^3)^(2/3) + 2^(2/3
)*a^(2/3)*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(6*b*d)
```

fricas [A] time = 1.44, size = 167, normalized size = 1.09

$$-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}-\sqrt{3}\,a}{3\,a}\right)+4^{\frac{1}{3}}\left(-a^2\right)^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}+2\left(bx^3+a\right)^{\frac{2}{3}}a-\right.}{6\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] -1/6*(2*4^(1/3)*sqrt(3)*(-a^2)^(1/3)*arctan(1/3*(4^(1/3)*sqrt(3)*(b*x^3 + a
)^(1/3)*(-a^2)^(1/3) - sqrt(3)*a)/a) + 4^(1/3)*(-a^2)^(1/3)*log(4^(2/3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(2/3)*a - 2*4^(1/3)*(-a^2)^(1/3
)*a) - 2*4^(1/3)*(-a^2)^(1/3)*log(-4^(2/3)*(-a^2)^(2/3) + 2*(b*x^3 + a)^(1/
3)*a) + 3*(b*x^3 + a)^(2/3))/(b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2/sqrt(3)/b/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*a*b^2*d^2*(2*a
)^(1/3)*1/6/a/b^3/d^3*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-1/2*((a+b*x^3)^(
1/3))^2*b*d/b^2/d^2
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{2}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [A] time = 1.31, size = 140, normalized size = 0.92

$$-\frac{\frac{2\sqrt{3}\,2^{\frac{2}{3}}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}a^{\frac{1}{3}}+2\left(bx^3+a\right)^{\frac{1}{3}}\right)}{6a^{\frac{1}{3}}}\right)}{d}-\frac{2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}a^{\frac{2}{3}}+2^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{2}{3}}\right)}{d}+\frac{2\cdot 2^{\frac{2}{3}}a^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}a^{\frac{1}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\right)}{d}+\frac{3\left(bx^3\cdots\right.}{d}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -1/6*(2*sqrt(3)*2^(2/3)*a^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3)*a^(1/3)
 + 2*(b*x^3 + a)^(1/3))/a^(1/3))/d - 2^(2/3)*a^(2/3)*log(2^(2/3)*a^(2/3) +
2^(1/3)*(b*x^3 + a)^(1/3)*a^(1/3) + (b*x^3 + a)^(2/3))/d + 2*2^(2/3)*a^(2/3
)*log(-2^(1/3)*a^(1/3) + (b*x^3 + a)^(1/3))/d + 3*(b*x^3 + a)^(2/3)/d)/b
```

mupad [B] time = 4.83, size = 186, normalized size = 1.22

$$-\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}-2^{1/3}\,a^{1/3}\right)}{3\,b\,d}-\frac{4^{1/3}\,a^{2/3}\,\ln\left(\frac{4\,a^2\,\left(b\,x^3+a\right)^{1/3}}{b^2\,d^2}-\frac{2\,4^{2/3}\,a^{7/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{b^2\,d^2}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{\cdots}}{}\right.}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] (4^(1/3)*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (18*4^(2/3)*a^(7
/3)*((3^(1/2)*1i)/6 + 1/6)^2)/(b^2*d^2))*((3^(1/2)*1i)/6 + 1/6))/(b*d) - (4
^(1/3)*a^(2/3)*log((a + b*x^3)^(1/3) - 2^(1/3)*a^(1/3)))/(3*b*d) - (4^(1/3)
*a^(2/3)*log((4*a^2*(a + b*x^3)^(1/3))/(b^2*d^2) - (2*4^(2/3)*a^(7/3)*((3^(
1/2)*1i)/2 - 1/2)^2)/(b^2*d^2))*((3^(1/2)*1i)/2 - 1/2))/(3*b*d) - (a + b*x^
3)^(2/3)/(2*b*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^2\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**2*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.590 $\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=214

$$\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}\sqrt[3]{a}\,d}$$

```
[Out] -1/2*ln(x)/a^(1/3)/d+1/6*ln(-b*x^3+a)*2^(2/3)/a^(1/3)/d+1/2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(1/3)/d-1/2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(1
/3)/d+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(1/3)/d
*3^(1/2)-1/3*2^(2/3)*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3
^(1/2))/a^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {446, 83, 55, 617, 204, 31}

$$\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{a+bx^3}+\sqrt[3]{}}{\sqrt{3}\sqrt[3]{a}}\right.}{\sqrt{3}\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
d) - (2^(2/3)*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3)
)])/(Sqrt[3]*a^(1/3)*d) - Log[x]/(2*a^(1/3)*d) + Log[a - b*x^3]/(3*2^(1/3)*
a^(1/3)*d) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(1/3)*d) - Log[2^(1/3)*a
^(1/3) - (a + b*x^3)^(1/3)]/(2^(1/3)*a^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(ad-bdx)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}(2b)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx,x,x^3\right)+\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,x,x^3\right)}{3d}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\operatorname{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx^3}\right)}{2d}-\frac{\operatorname{Subst}\left(\int \frac{1}{2^{2/3}a^{2/3}+\sqrt[3]{}}\right.}{}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{a}\,d}-\frac{\text{Subs}}{}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{2^{2/3}\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,d}-\frac{\log(x)}{2\sqrt[3]{a}\,d}+\frac{\log\left(a-bx^3\right)}{3\sqrt[3]{2}\sqrt[3]{a}\,d}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{}\right.}{2\sqrt[3]{a}}$$

Mathematica [A] time = 0.08, size = 164, normalized size = 0.77

$$\frac{2^{2/3}\log\left(a-bx^3\right)+3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-2\,2}{6\sqrt[3]{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*2^(2/3)*
Sqrt[3]*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*Log[x
] + 2^(2/3)*Log[a - b*x^3] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)] - 3*2^(2/3)
*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*d)
```

fricas [A] time = 1.57, size = 530, normalized size = 2.48

$$\left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,a\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-3\sqrt{\frac{1}{3}}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^3+\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 +
 a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log(
(2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a -
 a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) + 4^(1
/3)*a*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)
*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2
/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a
^(1/3)))/(a*d), -1/6*(2*4^(1/3)*sqrt(3)*a*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*s
qrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 4^(1/3)*a*(-1/a)^(1/
3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3)
+ 2*(b*x^3 + a)^(2/3)) - 2*4^(1/3)*a*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/
3) + 2*(b*x^3 + a)^(1/3)) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3
+ a)^(1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a
)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a
*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootof1/3/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/3)
+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2/sqrt(3)/a/d*atan(((a+b*x^3)^(1/3)
+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*(2*a)^(1/3)*1/6/a/d*
ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-(a^(1/3))^2*1/6/a/d*ln(((a+b*x^3)^(1/3
))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+1/sqrt(3)/a^(1/3)/d*atan(((a+
b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+a^(1/3)*a^(1/3)*1/3/a/d*ln(abs
((a+b*x^3)^(1/3)-a^(1/3)))
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(-bd\,x^{3}+ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x), x)
```

mupad [B] time = 5.86, size = 369, normalized size = 1.72

$$\ln\left(2\left(b\,x^{3}+a\right)^{1/3}-2\,2^{1/3}\,a\,d^{2}\left(-\frac{1}{a\,d^{3}}\right)^{2/3}\right)\left(-\frac{4}{27\,a\,d^{3}}\right)^{1/3}+\ln\left(\left(b\,x^{3}+a\right)^{1/3}-a\,d^{2}\left(\frac{1}{a\,d^{3}}\right)^{2/3}\right)\left(\frac{1}{27\,a\,d^{3}}\right)^{1/3}-\ln\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x*(a*d - b*d*x^3)),x)
```

```
[Out] log(2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3))*(-4/(27*a*d^3
))^(1/3) + log((a + b*x^3)^(1/3) - a*d^2*(1/(a*d^3))^(2/3))*(1/(27*a*d^3))^
(1/3) - log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) - 2^(1
/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-4/(27*a*d
^3))^(1/3) + log(4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a*d^2*(-1/(a*d^3))^(2/3) +
 2^(1/3)*3^(1/2)*a*d^2*(-1/(a*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-4/(2
7*a*d^3))^(1/3) - log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) - 3^(1/
2)*a*d^2*(1/(a*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*d^3))^(1/3)
+ log(2*(a + b*x^3)^(1/3) + a*d^2*(1/(a*d^3))^(2/3) + 3^(1/2)*a*d^2*(1/(a*d
^3))^(2/3)*1i)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a*d^3))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax+bx^4}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x + b*x**4), x)/d
```

3.591 $\int \frac{(a+bx^3)^{2/3}}{x^4(ad-bdx^3)}\,dx$

Optimal. Leaf size=269

$$\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}d}-\frac{2^{2/3}b\tan^{-1}}{\sqrt{}}$$

```
[Out] 1/3*b*(b*x^3+a)^(2/3)/a^2/d-1/3*(b*x^3+a)^(5/3)/a^2/d/x^3-5/6*b*ln(x)/a^(4/
3)/d+1/6*b*ln(-b*x^3+a)*2^(2/3)/a^(4/3)/d+5/6*b*ln(a^(1/3)-(b*x^3+a)^(1/3))
/a^(4/3)/d-1/2*b*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/a^(4/3)/d+5/9*
b*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/d*3^(1/2)
-1/3*2^(2/3)*b*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(4/3)/d*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 156, 50, 55, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3}+\frac{b\left(a+bx^3\right)^{2/3}}{3a^2d}+\frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}d}-\frac{b\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{4/3}d}+\frac{5b\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(2/3))/(3*a^2*d) - (a + b*x^3)^(5/3)/(3*a^2*d*x^3) + (5*b*Ar
cTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)
*d) - (2^(2/3)*b*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1
/3))])/(Sqrt[3]*a^(4/3)*d) - (5*b*Log[x])/(6*a^(4/3)*d) + (b*Log[a - b*x^3]
)/(3*2^(1/3)*a^(4/3)*d) + (5*b*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(4/3)
*d) - (b*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^(1/3)*a^(4/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd+\frac{2}{3}b^2dx\right)}{x(ad-bdx)}\,dx, x, x^3\right)}{3a^2d}$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{b^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{ad-bdx}\,dx, x, x^3\right)}{3a^2} + \frac{(5b)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx, x, x^3\right)}{9a^2d}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3a} + \frac{(5b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d} - \frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x\right.}{6a^{4/3}d}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} - \frac{5b\log(x)}{6a^{4/3}d} + \frac{b\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{4/3}d} + \frac{5b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right.}{6a^{4/3}d}$$

$$= \frac{b\left(a+bx^3\right)^{2/3}}{3a^2d} - \frac{\left(a+bx^3\right)^{5/3}}{3a^2dx^3} + \frac{5b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}d} - \frac{2^{2/3}b\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}d} - \frac{5b}{6}$$

Mathematica [A] time = 0.09, size = 213, normalized size = 0.79

$$\frac{10\sqrt{3}\,bx^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 3\left(2\sqrt[3]{a}\left(a+bx^3\right)^{2/3} - 2^{2/3}bx^3\log\left(a-bx^3\right) - 5bx^3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 3\,2^{2}\right.}{18a^{4/3}dx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x]
```

```
[Out] (10*Sqrt[3]*b*x^3*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3*(
2*a^(1/3)*(a + b*x^3)^(2/3) + 2*2^(2/3)*Sqrt[3]*b*x^3*ArcTan[(1 + (2^(2/3)*
(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] + 5*b*x^3*Log[x] - 2^(2/3)*b*x^3*Log[a
 - b*x^3] - 5*b*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] + 3*2^(2/3)*b*x^3*Log[
2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)]))/(18*a^(4/3)*d*x^3)
```

fricas [A] time = 1.65, size = 612, normalized size = 2.28

$$-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}\,abx^3\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) - 15\sqrt{\frac{1}{3}}\,abx^3\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(
b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 15*sqrt(1/3)*a*b*x^3*sqrt(-1
/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b*x^3
+ a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3) + 3*
a)/x^3) + 3*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-
1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*
b*x^3*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) + 5*a
^(2/3)*b*x^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) -
 10*a^(2/3)*b*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)
/(a^2*d*x^3), -1/18*(6*4^(1/3)*sqrt(3)*a*b*x^3*(-1/a)^(1/3)*arctan(1/3*4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) + 3*4^(1/3)*a*b*x
^3*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*
(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 6*4^(1/3)*a*b*x^3*(-1/a)^(1/3)*log(-4
^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1/3)) - 30*sqrt(1/3)*a^(2/3)*b*x^3*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3)) + 5*a^(2/3)*b*x^3*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 10*a^(2/3)*b
*x^3*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*(b*x^3 + a)^(2/3)*a)/(a^2*d*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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aic extensions not allowed in a rootofAlgebraic extensions not allowed in a
 rootofb/3/a/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)^(1/
3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2*b/sqrt(3)/a^2/d*atan(((a+b*x^3)
^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*b*(2*a)^(1/3)*
1/6/a^2/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-5*(a^(1/3))^2*b*1/18/a^2/d*l
```

```
n(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+5*b/3/sqrt(3
)/a/a^(1/3)/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a^(1/3))+5*a^(1/
3)*b*a^(1/3)*1/9/a^2/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))-((a+b*x^3)^(1/3))^2
*b/3/a/d/(a+b*x^3-a)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(-bd\,x^{3}+ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}}}{\left(bdx^{3}-ad\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^4), x)
```

mupad [B] time = 5.56, size = 490, normalized size = 1.82

$$\ln\left(2\,b^2\,\left(b\,x^3+a\right)^{1/3}-2\,2^{1/3}\,a^3\,d^2\,\left(-\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\,\left(-\frac{4\,b^3}{27\,a^4\,d^3}\right)^{1/3}+\frac{5\,\ln\left(b^2\,\left(b\,x^3+a\right)^{1/3}-a^3\,d^2\,\left(\frac{b^3}{a^4\,d^3}\right)^{2/3}\right)\,\left(\frac{b^3}{a^4\,d^3}\right)^{1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(a*d - b*d*x^3)),x)
[Out] log(2*b^2*(a + b*x^3)^(1/3) - 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3))*(-(
4*b^3)/(27*a^4*d^3))^(1/3) + (5*log(b^2*(a + b*x^3)^(1/3) - a^3*d^2*(b^3/(a
^4*d^3))^(2/3))*(b^3/(a^4*d^3))^(1/3))/9 - log(4*b^2*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) - 2^(1/3)*3^(1/2)*a^3*d^2*(-b^3/(a^4
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 + 1/2)*(-(4*b^3)/(27*a^4*d^3))^(1/3) + log
(4*b^2*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3) + 2^(1/
3)*3^(1/2)*a^3*d^2*(-b^3/(a^4*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b
^3)/(27*a^4*d^3))^(1/3) - log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*(b^3/(a^4*d
^3))^(2/3) - 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)*1i)/2 + 1/
2)*((125*b^3)/(729*a^4*d^3))^(1/3) + log(2*b^2*(a + b*x^3)^(1/3) + a^3*d^2*
(b^3/(a^4*d^3))^(2/3) + 3^(1/2)*a^3*d^2*(b^3/(a^4*d^3))^(2/3)*1i)*((3^(1/2)
*1i)/2 - 1/2)*((125*b^3)/(729*a^4*d^3))^(1/3) - (b*(a + b*x^3)^(2/3))/(3*a*
(d*(a + b*x^3) - a*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^{3}\right)^{\frac{2}{3}}}{-ax^{4}+bx^{7}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**4 + b*x**7), x)/d
```

3.592 $\int \frac{(a+bx^3)^{2/3}}{x^7(ad-bdx^3)}\,dx$

Optimal. Leaf size=284

$$\frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}\,a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}d}-\frac{2^{2/3}b^2\tan}{}$$

```
[Out] -5/18*b*(b*x^3+a)^(2/3)/a^2/d/x^3-1/6*(b*x^3+a)^(5/3)/a^2/d/x^6-7/9*b^2*ln(
x)/a^(7/3)/d+1/6*b^2*ln(-b*x^3+a)*2^(2/3)/a^(7/3)/d+7/9*b^2*ln(a^(1/3)-(b*x
^3+a)^(1/3))/a^(7/3)/d-1/2*b^2*ln(2^(1/3)*a^(1/3)-(b*x^3+a)^(1/3))*2^(2/3)/
a^(7/3)/d+14/27*b^2*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))
/a^(7/3)/d*3^(1/2)-1/3*2^(2/3)*b^2*arctan(1/3*(a^(1/3)+2^(2/3)*(b*x^3+a)^(1
/3))/a^(1/3)*3^(1/2))/a^(7/3)/d*3^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {446, 103, 149, 156, 55, 617, 204, 31}

$$\frac{b^2\log(a-bx^3)}{3\sqrt[3]{2}\,a^{7/3}d}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{9a^{7/3}d}-\frac{b^2\log\left(\sqrt[3]{2}\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^{7/3}d}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{7/3}d}-\frac{2^{2/3}b^2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*b*(a + b*x^3)^(2/3))/(18*a^2*d*x^3) - (a + b*x^3)^(5/3)/(6*a^2*d*x^6) +
 (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt
[3]*a^(7/3)*d) - (2^(2/3)*b^2*ArcTan[(a^(1/3) + 2^(2/3)*(a + b*x^3)^(1/3))/
(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)*d) - (7*b^2*Log[x])/(9*a^(7/3)*d) + (b
^2*Log[a - b*x^3])/(3*2^(1/3)*a^(7/3)*d) + (7*b^2*Log[a^(1/3) - (a + b*x^3)
^(1/3)])/(9*a^(7/3)*d) - (b^2*Log[2^(1/3)*a^(1/3) - (a + b*x^3)^(1/3)])/(2^
(1/3)*a^(7/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(ad-bdx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(ad-bdx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}\left(-\frac{5}{3}abd-\frac{1}{3}b^2dx\right)}{x^2(ad-bdx)}\,dx, x, x^3\right)}{6a^2d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{28}{9}a^2b^2d^2-\frac{8}{9}ab^3d^2x}{x\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{6a^3d^2}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{\left(2b^3\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(ad-bdx)}\,dx, x, x^3\right)}{3a^2} + \frac{\left(14b^2\right)\operatorname{Subst}}{}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d} - \frac{\left(7b^2\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,d\right.}{9a^{7/3}d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} - \frac{7b^2\log(x)}{9a^{7/3}d} + \frac{b^2\log\left(a-bx^3\right)}{3\sqrt[3]{2}\,a^{7/3}d} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{9a^{7/3}d}$$

$$= -\frac{5b\left(a+bx^3\right)^{2/3}}{18a^2dx^3} - \frac{\left(a+bx^3\right)^{5/3}}{6a^2dx^6} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{7/3}d} - \frac{2^{2/3}b^2\tan^{-1}\left(\frac{1+\frac{2^{2/3}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}d} -$$

Mathematica [**A**] time = 0.16, size = 247, normalized size = 0.87

$$\frac{28\sqrt{3}\,b^2x^6\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 3\left(3a^{4/3}\left(a+bx^3\right)^{2/3} - 3\,2^{2/3}b^2x^6\log\left(a-bx^3\right) - 14b^2x^6\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right) + 9\right.}{54a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x]
```

```
[Out] (28*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3
*(3*a^(4/3)*(a + b*x^3)^(2/3) + 8*a^(1/3)*b*x^3*(a + b*x^3)^(2/3) + 6*2^(2/
3)*Sqrt[3]*b^2*x^6*ArcTan[(1 + (2^(2/3)*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]
] + 14*b^2*x^6*Log[x] - 3*2^(2/3)*b^2*x^6*Log[a - b*x^3] - 14*b^2*x^6*Log[a
^(1/3) - (a + b*x^3)^(1/3)] + 9*2^(2/3)*b^2*x^6*Log[2^(1/3)*a^(1/3) - (a +
b*x^3)^(1/3)]))/(54*a^(7/3)*d*x^6)
```

fricas [**A**] time = 0.88, size = 660, normalized size = 2.32

$$\left[-\frac{18\cdot 4^{\frac{1}{3}}\sqrt{3}\,ab^2x^6\left(-\frac{1}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{1}{3}\cdot 4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) - 42\sqrt{\frac{1}{3}}\,ab^2x^6\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\,\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a*b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3
)*(b*x^3 + a)^(1/3)*(-1/a)^(1/3) - 1/3*sqrt(3)) - 42*sqrt(1/3)*a*b^2*x^6*sq
rt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^(2/3) - (b
*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/3)*a^(2/3)
 + 3*a)/x^3) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 + a)^(1/
3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) - 18*4^
(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 + a)^(1
/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3)
 + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(8*a*
b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6), -1/54*(18*4^(1/3)*sqrt(3)*a*
b^2*x^6*(-1/a)^(1/3)*arctan(1/3*4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/a)^(1
/3) - 1/3*sqrt(3)) + 9*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(4^(2/3)*(b*x^3 +
a)^(1/3)*a*(-1/a)^(2/3) - 2*4^(1/3)*a*(-1/a)^(1/3) + 2*(b*x^3 + a)^(2/3)) -
 18*4^(1/3)*a*b^2*x^6*(-1/a)^(1/3)*log(-4^(2/3)*a*(-1/a)^(2/3) + 2*(b*x^3 +
 a)^(1/3)) - 84*sqrt(1/3)*a^(2/3)*b^2*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + 14*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(2/3) + (b*x^
3 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*a^(2/3)*b^2*x^6*log((b*x^3 + a)^(1/3)
- a^(1/3)) + 3*(8*a*b*x^3 + 3*a^2)*(b*x^3 + a)^(2/3))/(a^3*d*x^6)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Algeb
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 rootofb^2/3/a^2/(2*a)^(1/3)/d*ln(((a+b*x^3)^(1/3))^2+(2*a)^(1/3)*(a+b*x^3)
^(1/3)+(2*a)^(1/3)*(2*a)^(1/3))-((2*a)^(1/3))^2*b^2/sqrt(3)/a^3/d*atan(((a+
b*x^3)^(1/3)+1/2*(2*a)^(1/3))/sqrt(3)*2/(2*a)^(1/3))-2*(2*a)^(1/3)*b^2*(2*a
```

```
)^(1/3)*1/6/a^3/d*ln(abs((a+b*x^3)^(1/3)-(2*a)^(1/3)))-7*(a^(1/3))^2*b^2*1/
27/a^3/d*ln(((a+b*x^3)^(1/3))^2+a^(1/3)*(a+b*x^3)^(1/3)+a^(1/3)*a^(1/3))+14
*b^2/9/sqrt(3)/a^2/a^(1/3)/d*atan(((a+b*x^3)^(1/3)+1/2*a^(1/3))/sqrt(3)*2/a
^(1/3))+14*a^(1/3)*b^2*a^(1/3)*1/27/a^3/d*ln(abs((a+b*x^3)^(1/3)-a^(1/3)))+
(-8*((a+b*x^3)^(1/3))^2*(a+b*x^3)*b^2+5*((a+b*x^3)^(1/3))^2*a*b^2)/18/a^2/d
/(a+b*x^3-a)^2
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^7), x)
```

mupad [B] time = 5.45, size = 513, normalized size = 1.81

$$\frac{\frac{5\,b^2\,\left(b\,x^3+a\right)^{2/3}}{18\,a}-\frac{4\,b^2\,\left(b\,x^3+a\right)^{5/3}}{9\,a^2}}{d\,\left(b\,x^3+a\right)^2+a^2\,d-2\,a\,d\,\left(b\,x^3+a\right)}+\ln\left(2\,b^4\,\left(b\,x^3+a\right)^{1/3}-2\,2^{1/3}\,a^5\,d^2\,\left(-\frac{b^6}{a^7\,d^3}\right)^{2/3}\right)\,\left(-\frac{4\,b^6}{27\,a^7\,d^3}\right)^{1/3}+\frac{14\,\ln\left(b^4\,\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^7*(a*d - b*d*x^3)),x)
[Out] ((5*b^2*(a + b*x^3)^(2/3))/(18*a) - (4*b^2*(a + b*x^3)^(5/3))/(9*a^2))/(d*(
a + b*x^3)^2 + a^2*d - 2*a*d*(a + b*x^3)) + log(2*b^4*(a + b*x^3)^(1/3) - 2
*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3))*(-(4*b^6)/(27*a^7*d^3))^(1/3) + (1
4*log(b^4*(a + b*x^3)^(1/3) - a^5*d^2*(b^6/(a^7*d^3))^(2/3))*(b^6/(a^7*d^3)
)^(1/3))/27 - log(4*b^4*(a + b*x^3)^(1/3) + 2*2^(1/3)*a^5*d^2*(-b^6/(a^7*d^
3))^(2/3) - 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3)*2i)*((3^(1/2)*1i
)/2 + 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) + log(4*b^4*(a + b*x^3)^(1/3) + 2*
2^(1/3)*a^5*d^2*(-b^6/(a^7*d^3))^(2/3) + 2^(1/3)*3^(1/2)*a^5*d^2*(-b^6/(a^7
*d^3))^(2/3)*2i)*((3^(1/2)*1i)/2 - 1/2)*(-(4*b^6)/(27*a^7*d^3))^(1/3) - (7*
log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) - 3^(1/2)*a^5*d
^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i + 1)*(b^6/(a^7*d^3))^(1/3))/27 + (
7*log(2*b^4*(a + b*x^3)^(1/3) + a^5*d^2*(b^6/(a^7*d^3))^(2/3) + 3^(1/2)*a^5
*d^2*(b^6/(a^7*d^3))^(2/3)*1i)*(3^(1/2)*1i - 1)*(b^6/(a^7*d^3))^(1/3))/27
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^7+bx^{10}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**7 + b*x**10), x)/d
```

3.593 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=264

$$\frac{a^2\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{7/3}d}-\frac{a^2\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{7/3}d}+\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{9b^{7/3}d}-\frac{14a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{7/3}d}+\frac{2^{2/3}a^2\tan\ldots}{}$$

```
[Out] -4/9*a*x*(b*x^3+a)^(2/3)/b^2/d-1/6*x^4*(b*x^3+a)^(2/3)/b/d+1/6*a^2*ln(-b*d*
x^3+a*d)*2^(2/3)/b^(7/3)/d-1/2*a^2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^
(2/3)/b^(7/3)/d+7/9*a^2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d-14/27*a^2*
arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d*3^(1/2)+1/3*2
^(2/3)*a^2*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7
/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), (b*x^3)/a
])/(7*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3} \int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^7 \left(a+bx^3\right)^{2/3} F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{7ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.30, size = 244, normalized size = 0.92

$$\frac{2\, 2^{2/3} a^2 \sqrt[3]{a+bx^3} \left(\log\left(\frac{2^{2/3} b^{2/3} x^2}{\left(ax^3+b\right)^{2/3}} + \frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}} + 1\right) - 2\log\left(1 - \frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right) + 21ab^{4/3}x^4 \sqrt[3]{\cdots}}{54 b^{7/3} d \sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-3*b^(1/3)*(a + b*x^3)*(8*a*x + 3*b*x^4) + 21*a*b^(4/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*2^(2/3)*a^2
*(a + b*x^3)^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)
^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1
 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3
)^(1/3)]))/(54*b^(7/3)*d*(a + b*x^3)^(1/3))
```

fricas [A] time = 0.77, size = 701, normalized size = 2.66

$$\left[-\frac{18 \cdot 4^{\frac{1}{3}} \sqrt{3}\, a^2 b \left(-\frac{1}{b}\right)^{\frac{1}{3}} \arctan\left(-\frac{\sqrt{3}\,x - 4^{\frac{1}{3}} \sqrt{3} \left(bx^3+a\right)^{\frac{1}{3}} \left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3x}\right) - 42\sqrt{\frac{1}{3}}\, a^2 b \sqrt{-\frac{1}{b^{\frac{2}{3}}}} \log\left(3bx^3 - 3\left(bx^3+a\right)^{\frac{1}{3}} b^{\frac{2}{3}} x^2 - \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1
/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 42*sqrt(1/3)*a^2*b*sqrt(-1
/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4
/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/
b^(2/3)) + 2*a) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2
/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-1/b)^(1/3)*log(-(2*4^(1/3
)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^
3 + a)^(2/3))/x^2) - 28*a^2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x)
 + 14*a^2*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d), -1/54
*(18*4^(1/3)*sqrt(3)*a^2*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 18*4^(1/3)*a^2*b*(-1/b)^(1/3)*lo
g(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 9*4^(1/3)*a^2*b*(-
1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b
*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 84*sqrt(1/3)*a^2*b^(2/3)*arct
```

```
an(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 28*a^2*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + 14*a^2*b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 + 8
*a*b*x)*(b*x^3 + a)^(2/3))/(b^3*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^6}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^6}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^6/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^6\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**6*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.594 $\int \frac{x^3(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=229

$$\frac{a\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,b^{4/3}d}-\frac{a\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,b^{4/3}d}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d}-\frac{5a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d}+\frac{2^{2/3}a\tan^{-1}\dots}{\sqrt{3}\dots}$$

```
[Out] -1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*a*ln(-b*d*x^3+a*d)*2^(2/3)/b^(4/3)/d-1/2*a*l
n(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/b^(4/3)/d+5/6*a*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/b^(4/3)/d-5/9*a*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))
*3^(1/2))/b^(4/3)/d*3^(1/2)+1/3*2^(2/3)*a*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 66, normalized size of antiderivative = 0.29, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]
)/(4*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx&=\frac{\left(a+bx^3\right)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\&=\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.22, size = 216, normalized size = 0.94

$$\frac{\frac{15x^4\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\sqrt[3]{a+bx^3}}+\frac{2^{2/3}a\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{b^{4/3}}-\frac{12x(a+bx^3)^{2/3}}{b}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] ((-12*x*(a + b*x^3)^(2/3))/b + (15*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3,
1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])/(a + b*x^3)^(1/3) + (2^(2/3)*a*(2*Sq
rt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log
[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)
/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/b^(4/3))/(36*
d)
```

fricas [A] time = 1.29, size = 653, normalized size = 2.85

$$\left[-\frac{6\cdot 4^{\frac{1}{3}}\sqrt{3}\,ab\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,x-4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-15\sqrt{\frac{1}{3}}\,ab\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\,\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)
*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 15*sqrt(1/3)*a*b*sqrt(-1/b^(2
/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x
^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/
3)) + 2*a) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*
(b*x^3 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-
1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/
3))/x^2) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
+ a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(b^2*d), -1/18*(6*4^(1/3)*sqrt(3)*a*b*(-1/b)^(1
/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3)
)/x) - 6*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3
 + a)^(1/3))/x) + 3*4^(1/3)*a*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(
1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^
2) - 30*sqrt(1/3)*a*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/
3))/(b^(1/3)*x)) + 6*(b*x^3 + a)^(2/3)*b*x - 10*a*b^(2/3)*log(-(b^(1/3)*x -
 (b*x^3 + a)^(1/3))/x) + 5*a*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{3}}{-bd\,x^{3}+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^{3}+a\right)^{\frac{2}{3}} x^{3}}{bdx^{3}-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^3/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}\,\left(b\,x^{3}+a\right)^{2/3}}{a\,d-b\,d\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}}{-a+bx^{3}}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**3*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.595 $\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=200

$$\frac{\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\sqrt[3]{b}\,d}-\frac{\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d}+\frac{2^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d}$$

```
[Out] 1/6*ln(-b*d*x^3+a*d)*2^(2/3)/b^(1/3)/d-1/2*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(
1/3))*2^(2/3)/b^(1/3)/d+1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d-1/3*ar
ctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*3^(1/2)+1/3*2^(
2/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d*
3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 0.30, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/
(a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{ad\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.16, size = 156, normalized size = 0.78

$$\frac{4ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{d\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+2F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(a*d - b*d*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/(d*(a - b*x^3)*(4*a*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + 2*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

fricas [A] time = 1.13, size = 611, normalized size = 3.06

$$\left[-\frac{2\cdot 4^{\frac{1}{3}}\sqrt{3}\,b\left(-\frac{1}{b}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,x-4^{\frac{1}{3}}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{1}{b}\right)^{\frac{1}{3}}}{3\,x}\right)-3\sqrt{\frac{1}{3}}\,b\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] [-1/6*(2*4^(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sq
rt(3)*(b*x^3 + a)^(1/3)*(-1/b)^(1/3))/x) - 3*sqrt(1/3)*b*sqrt(-1/b^(2/3))*l
og(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (
b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) +
2*a) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 +
 a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(1/3)*b*x^2*(-1/b)^(1/3) -
 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b*x^3 + a)^(2/3))/x^2) - 2
*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2
 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(2*4^
(1/3)*sqrt(3)*b*(-1/b)^(1/3)*arctan(-1/3*(sqrt(3)*x - 4^(1/3)*sqrt(3)*(b*x^
3 + a)^(1/3)*(-1/b)^(1/3))/x) - 2*4^(1/3)*b*(-1/b)^(1/3)*log(-(4^(2/3)*b*x*
(-1/b)^(2/3) - 2*(b*x^3 + a)^(1/3))/x) + 4^(1/3)*b*(-1/b)^(1/3)*log(-(2*4^(
1/3)*b*x^2*(-1/b)^(1/3) - 4^(2/3)*(b*x^3 + a)^(1/3)*b*x*(-1/b)^(2/3) - 2*(b
*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*b^(2/3)*arctan(sqrt(1/3)*(b^(1/3)*x + 2
*(b*x^3 + a)^(1/3))/(b^(1/3)*x)) - 2*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.596 $\int \frac{(a+bx^3)^{2/3}}{x^3(ad-bdx^3)}\,dx$

Optimal. Leaf size=157

$$\frac{b^{2/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,ad}-\frac{b^{2/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,ad}+\frac{2^{2/3}b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,ad}-\frac{\left(a+bx^3\right)^{2/3}}{2adx^2}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/d/x^2+1/6*b^(2/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a/d-1/2*b
^(2/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a/d+1/3*2^(2/3)*b^(2/3
)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 0.50, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 - (b*x^3)/a)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, (-2*b*x^3)/(a - b*x^3)])/(2*a*d*x^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}\left(1-\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{2bx^3}{a-bx^3}\right)}{2adx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 47, normalized size = 0.30

$$-\frac{\left(a+bx^3\right)^{2/3}\ {}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};\frac{2bx^3}{bx^3+a}\right)}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1, 1/3, (2*b*x^3)/(a + b*x^
3)])/(a*d*x^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^3*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^3+bx^6}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**3 + b*x**6), x)/d
```

3.597 $\int \frac{(a+bx^3)^{2/3}}{x^6(ad-bdx^3)}\,dx$

Optimal. Leaf size=182

$$\frac{b^{5/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^2d}-\frac{b^{5/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^2d}+\frac{2^{2/3}b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^2d}-\frac{7b\left(a+bx^3\right)^{2/3}}{10a^2dx^2}-\frac{\left(a+bx^3\right)^{2/3}}{5adx^5}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/d/x^5-7/10*b*(b*x^3+a)^(2/3)/a^2/d/x^2+1/6*b^(5/3)*l
n(-b*d*x^3+a*d)*2^(2/3)/a^2/d-1/2*b^(5/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1
/3))*2^(2/3)/a^2/d+1/3*2^(2/3)*b^(5/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b
*x^3+a)^(1/3))*3^(1/2))/a^2/d*3^(1/2)
```

Rubi [C] time = 0.43, antiderivative size = 121, normalized size of antiderivative = 0.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$-\frac{2a^2-4bx^3\left(2a+3bx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+12bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+5abx^3+3b^2x^6}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] -(2*a^2 + 5*a*b*x^3 + 3*b^2*x^6 - 4*b*x^3*(2*a + 3*b*x^3)*Hypergeometric2F1
[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*b*x^3*(a - b*x^3)*Hypergeometric2
F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)])/(10*a^2*d*x^5*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(ad-bdx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=-\frac{2a^2+5abx^3+3b^2x^6-4bx^3\left(2a+3bx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+12bx^3\left(a-bx^3\right)\,{}_2F_1\left(\frac{1}{3},2;\right.}{10a^2dx^5\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 5.20, size = 170, normalized size = 0.93

$$\frac{5\,2^{2/3}b^{5/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)-\frac{3\left(a+bx^3\right)^{2/3}\left(2a+7bx^3\right)}{x^5}}{30a^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(2*a + 7*b*x^3))/x^5 + 5*2^(2/3)*b^(5/3)*(2*Sqrt[3]*
ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (
2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b +
a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(30*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^6\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^6*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^6+bx^9}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**6 + b*x**9), x)/d
```

3.598 $\int \frac{(a+bx^3)^{2/3}}{x^9(ad-bdx^3)}\,dx$

Optimal. Leaf size=209

$$\frac{b^{8/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^3d}-\frac{b^{8/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^3d}+\frac{2^{2/3}b^{8/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^3d}-\frac{5b^2\left(a+bx^3\right)^{2/3}}{8a^3dx^2}-\frac{b\left(a+bx^3\right.}{4a^2dx^5}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/d/x^8-1/4*b*(b*x^3+a)^(2/3)/a^2/d/x^5-5/8*b^2*(b*x^3
+a)^(2/3)/a^3/d/x^2+1/6*b^(8/3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^3/d-1/2*b^(8/3)*
ln(2^(1/3)*b^(1/3)*x-(b*x^3+a)^(1/3))*2^(2/3)/a^3/d+1/3*2^(2/3)*b^(8/3)*arc
tan(1/3*(1+2*2^(1/3)*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^3/d*3^(1/2)
```

Rubi [C] time = 10.68, antiderivative size = 244, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{-18bx^3\left(a-bx^3\right)^2\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-4bx^3\left(5a^2+6abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+42a^2bx^3\,{}_2F_1\left(\frac{1}{3}\right.}{40a^3dx^8\sqrt[3]{}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] -(5*a^3 + 11*a^2*b*x^3 + 15*a*b^2*x^6 + 9*b^3*x^9 - 4*b*x^3*(5*a^2 + 6*a*b*
x^3 + 9*b^2*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 42
*a^2*b*x^3*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 12*a*b^2
*x^6*Hypergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b^3*x^9*Hyp
ergeometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 18*b*x^3*(a - b*x^3)^2
*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)])/(40*a^3*d
*x^8*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^9\left(ad-bdx^3\right)}dx=\frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9\left(ad-bdx^3\right)}dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=-\frac{5a^3+11a^2bx^3+15ab^2x^6+9b^3x^9-4bx^3\left(5a^2+6abx^3+9b^2x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)+42}{}$$

Mathematica [A] time = 5.27, size = 179, normalized size = 0.86

$$\frac{4\,2^{2/3}b^{8/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)-\frac{3\left(a+bx^3\right)^{2/3}\left(a^2+2abx^3+5b^2x^6\right)}{x^8}}{24a^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(a^2 + 2*a*b*x^3 + 5*b^2*x^6))/x^8 + 4*2^(2/3)*b^(8/
3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]]
 - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(1/3)*x)/(b + a*x^3)^(1/3)]))/(24*a
^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^9\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(a*d - b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(-b*d*x**3+a*d),x)
[Out] Timed out
```

3.599 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx$

Optimal. Leaf size=236

$$\frac{b^{11/3}\log\left(ad-bdx^3\right)}{3\sqrt[3]{2}\,a^4d}-\frac{b^{11/3}\log\left(\sqrt[3]{2}\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{\sqrt[3]{2}\,a^4d}+\frac{2^{2/3}b^{11/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,a^4d}-\frac{293b^3\left(a+bx^3\right)^{2/3}}{440a^4dx^2}-\frac{49b^2\left(a\right.}{220\ldots}$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/a/d/x^11-13/88*b*(b*x^3+a)^(2/3)/a^2/d/x^8-49/220*b^2
*(b*x^3+a)^(2/3)/a^3/d/x^5-293/440*b^3*(b*x^3+a)^(2/3)/a^4/d/x^2+1/6*b^(11/
3)*ln(-b*d*x^3+a*d)*2^(2/3)/a^4/d-1/2*b^(11/3)*ln(2^(1/3)*b^(1/3)*x-(b*x^3+
a)^(1/3))*2^(2/3)/a^4/d+1/3*2^(2/3)*b^(11/3)*arctan(1/3*(1+2*2^(1/3)*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/a^4/d*3^(1/2)
```

Rubi [C] time = 17.60, antiderivative size = 391, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{511, 510\}$

$$-\frac{-54bx^3\left(a-bx^3\right)^2\left(5a+6bx^3\right)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)+54bx^3\left(a-bx^3\right)^3\,{}_4F_3\left(\frac{1}{3},2,2,2;1,1,\frac{4}{3};\frac{2bx^3}{bx^3+a}\right)-180a^2b\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] -(40*a^4 + 85*a^3*b*x^3 + 99*a^2*b^2*x^6 + 135*a*b^3*x^9 + 81*b^4*x^12 - 16
0*a^3*b*x^3*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 180*a^2
*b^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 216*a*b^3*
x^9*Hypergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] - 324*b^4*x^12*Hy
pergeometric2F1[1/3, 1, 4/3, (2*b*x^3)/(a + b*x^3)] + 396*a^3*b*x^3*Hyperge
ometric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] + 198*a^2*b^2*x^6*Hypergeome
tric2F1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 594*b^4*x^12*Hypergeometric2F
1[1/3, 2, 4/3, (2*b*x^3)/(a + b*x^3)] - 54*b*x^3*(a - b*x^3)^2*(5*a + 6*b*x
^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, (2*b*x^3)/(a + b*x^3)] + 54*b*
x^3*(a - b*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2}, {1, 1, 4/3}, (2*b*x^3)/
(a + b*x^3)])/(440*a^4*d*x^11*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^{12}(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40a^4+85a^3bx^3+99a^2b^2x^6+135ab^3x^9+81b^4x^{12}-160a^3bx^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2bx^3}{a+bx^3}\right)-}{}$$

Mathematica [A] time = 5.35, size = 196, normalized size = 0.83

$$\frac{\frac{2^{2/3}b^{11/3}\left(\log\left(\frac{2^{2/3}b^{2/3}x^2}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1\right)-2\log\left(1-\frac{\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}\sqrt[3]{b}x}{\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{a^4}-\frac{3(a+bx^3)^{2/3}(40a^3+65a^2bx^3+98ab^2x^6+293b^3x^9)}{220a^4x^{11}}}{6d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x]
```

```
[Out] ((-3*(a + b*x^3)^(2/3)*(40*a^3 + 65*a^2*b*x^3 + 98*a*b^2*x^6 + 293*b^3*x^9)
)/(220*a^4*x^11) + (2^(2/3)*b^(11/3)*(2*Sqrt[3]*ArcTan[(1 + (2*2^(1/3)*b^(1
/3)*x)/(b + a*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (2^(1/3)*b^(1/3)*x)/(b + a*x
^3)^(1/3)] + Log[1 + (2^(2/3)*b^(2/3)*x^2)/(b + a*x^3)^(2/3) + (2^(1/3)*b^(
1/3)*x)/(b + a*x^3)^(1/3)]))/a^4)/(6*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{(-bd\,x^3+ad)\,x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^12*(a*d - b*d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(-b*d*x**3+a*d),x)
[Out] Timed out
```

3.600 $\int \frac{x^7(a+bx^3)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=512

$$\frac{a^{7/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{(a+bx^3)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,b^{8/3}d}-\frac{2^{2/3}a^{7/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{8/3}d}-\frac{a^{7/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a}}{\sqrt[3]{a}}\right.}{2\sqrt[3]{2}\,b^{8/3}d}$$

```
[Out] -9/28*a*x^2*(b*x^3+a)^(2/3)/b^2/d-1/7*x^5*(b*x^3+a)^(2/3)/b/d-19/28*a^2*x^2
*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/b^2/d/(b*x^3+a)^(1/
3)+1/12*a^(7/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(
8/3)/d+1/6*a^(7/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/
3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(8/3)/d-1/3*2^(2/3)*a^(7/
3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(8/3)/d-1/4*a^(7/3)*
ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1
/3))*2^(2/3)/b^(8/3)/d+1/3*2^(2/3)*a^(7/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)
+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(8/3)/d*3^(1/2)+1/6*a^(7/3)*arctan(
1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(8/3
)/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^8\left(a+bx^3\right)^{2/3}F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), (b*x^3)/a
])/(8*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{ad-bdx^3} dx = \frac{\left(a+bx^3\right)^{2/3} \int \frac{x^7 \left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3} dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^8 \left(a+bx^3\right)^{2/3} F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{8ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.11, size = 147, normalized size = 0.29

$$\frac{45a^2x^2\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5\left(9a^2x^2+13abx^5+4b^2x^8\right)+38abx^5\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{140b^2d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (-5*(9*a^2*x^2 + 13*a*b*x^5 + 4*b^2*x^8) + 45*a^2*x^2*(1 + (b*x^3)/a)^(1/3)
*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] + 38*a*b*x^5*(1 + (b*x
^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(140*b^2*
d*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{bdx^3-ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^7}{-bd\,x^3+ad} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x^7/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^7\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x**7*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.601 $\int \frac{x^4\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=485

$$\frac{a^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,b^{5/3}d}-\frac{2^{2/3}a^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{5/3}d}-\frac{a^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right.}{2\sqrt[3]{2}\,b^{5/3}d}$$

```
[Out] -1/4*x^2*(b*x^3+a)^(2/3)/b/d-3/4*a*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/
3],[5/3],-b*x^3/a)/b/d/(b*x^3+a)^(1/3)+1/12*a^(4/3)*ln((a^(1/3)-b^(1/3)*x)^
2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/b^(5/3)/d+1/6*a^(4/3)*ln(1+2^(2/3)*(a^(1/3
)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))
*2^(2/3)/b^(5/3)/d-1/3*2^(2/3)*a^(4/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*
x^3+a)^(1/3))/b^(5/3)/d-1/4*a^(4/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-
2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/b^(5/3)/d+1/3*2^(2/3)*a^(4
/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b
^(5/3)/d*3^(1/2)+1/6*a^(4/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x
^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^(5/3)/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$\frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]
)/(5*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3} \int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^5 \left(a+bx^3\right)^{2/3} F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{5ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.17, size = 127, normalized size = 0.26

$$\frac{x^2\left(6bx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+5a\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-5\left(a+bx^3\right)\right)}{20bd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
[Out] (x^2*(-5*(a + b*x^3) + 5*a*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3,
 -((b*x^3)/a), (b*x^3)/a] + 6*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3
, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(20*b*d*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^4}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(2/3)*x^4/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{2/3}}{a\,d-b\,d\,x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x^{4} \left(a + b x^{3}\right)^{\frac{2}{3}}}{- a + b x^{3}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
```

```
[Out] -Integral(x**4*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.602 $\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx$

Optimal. Leaf size=457

$$\frac{\sqrt[3]{a}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}b^{2/3}d}-\frac{2^{2/3}\sqrt[3]{a}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3b^{2/3}d}-\frac{\sqrt[3]{a}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a\cdots}}{\sqrt[3]{a}}\right.}{2\sqrt[3]{2}b^{2/3}d}$$

```
[Out] -1/2*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/d/(b*x^3+a)
^(1/3)+1/12*a^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)
/b^(2/3)/d+1/6*a^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2
^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/b^(2/3)/d-1/3*2^(2/3)*a
^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/b^(2/3)/d-1/4*a^(1
/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/
a^(1/3))*2^(2/3)/b^(2/3)/d+1/3*2^(2/3)*a^(1/3)*arctan(1/3*(1-2*2^(1/3)*(a^(
1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d*3^(1/2)+1/6*a^(1/3)*arc
tan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/b^
(2/3)/d*3^(1/2)
```

Rubi [C] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a]
)/(2*a*d*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^3\right)^{2/3}}{ad-bdx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{ad-bdx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^2\left(a+bx^3\right)^{2/3} F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2ad\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.04, size = 63, normalized size = 0.14

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{2d\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x]
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), (b*x^3
)/a])/(2*d*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x}{-bd\,x^3+ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
[Out] int(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{bdx^3-ad}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)*x/(b*d*x^3 - a*d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(b\,x^3+a\right)^{2/3}}{a\,d-b\,d\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3),x)
[Out] int((x*(a + b*x^3)^(2/3))/(a*d - b*d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x\left(a+bx^3\right)^{\frac{2}{3}}}{-a+bx^3}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(-b*d*x**3+a*d),x)
[Out] -Integral(x*(a + b*x**3)**(2/3)/(-a + b*x**3), x)/d
```

3.603 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt[3]{b}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,a^{2/3}d}-\frac{2^{2/3}\sqrt[3]{b}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{2/3}d}-\frac{\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}\right.}{2\sqrt[3]{2}\,a^{2/3}d}$$

```
[Out] -(b*x^3+a)^(2/3)/a/d/x+1/2*b*x^2*(1+b*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/
3],-b*x^3/a)/a/d/(b*x^3+a)^(1/3)+1/12*b^(1/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(
1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(2/3)/d+1/6*b^(1/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1
/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/
3)/a^(2/3)/d-1/3*2^(2/3)*b^(1/3)*ln(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)
^(1/3))/a^(2/3)/d-1/4*b^(1/3)*ln(b^(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3
)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*2^(2/3)/a^(2/3)/d+1/3*2^(2/3)*b^(1/3)*ar
ctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(2/3)
/d*3^(1/2)+1/6*b^(1/3)*arctan(1/3*(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^
(1/3))*3^(1/2))*2^(2/3)/a^(2/3)/d*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{adx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), (b*x^3)/a])
/(a*d*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(ad-bdx^3\right)}dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2\left(ad-bdx^3\right)}dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{adx\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.11, size = 136, normalized size = 0.28

$$\frac{15abx^3\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2\left(b^2x^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+5a\left(a+bx^3\right)\right)}{10a^2dx\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x]
[Out] (15*a*b*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a),
(b*x^3)/a] - 2*(5*a*(a + b*x^3) + b^2*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/
3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a]))/(10*a^2*d*x*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="giac")
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(-bd\,x^3+ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
[Out] int((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^2\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^2*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^2+bx^5}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**2 + b*x**5), x)/d
```

3.604 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(ad-bdx^3\right)}\,dx$

Optimal. Leaf size=512

$$\frac{b^{4/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{2}\,a^{5/3}d}-\frac{2^{2/3}b^{4/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3a^{5/3}d}-\frac{b^{4/3}\log\left(\frac{\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a}}-\frac{2^{2/3}\sqrt[3]{b}\sqrt[3]{a\cdots}}{\sqrt[3]{a}}\right.}{2\sqrt[3]{2}\,a^{5/3}d}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)/a/d/x^4-3/2*b*(b*x^3+a)^(2/3)/a^2/d/x+3/4*b^2*x^2*(1+b
*x^3/a)^(1/3)*hypergeom([1/3, 2/3],[5/3],-b*x^3/a)/a^2/d/(b*x^3+a)^(1/3)+1/
12*b^(4/3)*ln((a^(1/3)-b^(1/3)*x)^2*(a^(1/3)+b^(1/3)*x)/a)*2^(2/3)/a^(5/3)/
d+1/6*b^(4/3)*ln(1+2^(2/3)*(a^(1/3)+b^(1/3)*x)^2/(b*x^3+a)^(2/3)-2^(1/3)*(a
^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*2^(2/3)/a^(5/3)/d-1/3*2^(2/3)*b^(4/3)*ln
(1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))/a^(5/3)/d-1/4*b^(4/3)*ln(b^
(1/3)*(a^(1/3)+b^(1/3)*x)/a^(1/3)-2^(2/3)*b^(1/3)*(b*x^3+a)^(1/3)/a^(1/3))*
2^(2/3)/a^(5/3)/d+1/3*2^(2/3)*b^(4/3)*arctan(1/3*(1-2*2^(1/3)*(a^(1/3)+b^(1
/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))/a^(5/3)/d*3^(1/2)+1/6*b^(4/3)*arctan(1/3*(
1+2^(1/3)*(a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/3))*3^(1/2))*2^(2/3)/a^(5/3)/d*3
^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4adx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), (b*x^3)/a])
/(4*a*d*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^5(ad-bdx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5(ad-bdx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= -\frac{(a+bx^3)^{2/3}\,F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{4adx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.09, size = 148, normalized size = 0.29

$$\frac{-5a\left(a^2+7abx^3+6b^2x^6\right)-6b^3x^9\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+35ab^2x^6\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{20a^3dx^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x]
```

```
[Out] (-5*a*(a^2 + 7*a*b*x^3 + 6*b^2*x^6) + 35*a*b^2*x^6*(1 + (b*x^3)/a)^(1/3)*Ap
pellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), (b*x^3)/a] - 6*b^3*x^9*(1 + (b*x^3)/
a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), (b*x^3)/a])/(20*a^3*d*x^4
*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3+a)^{\frac{2}{3}}}{(bdx^3-ad)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{(-bd\,x^3+ad)\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(bdx^3-ad\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(-b*d*x^3+a*d),x, algorithm="maxima")
[Out] -integrate((b*x^3 + a)^(2/3)/((b*d*x^3 - a*d)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^5\,\left(a\,d-b\,d\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^5*(a*d - b*d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{-ax^5+bx^8}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(-b*d*x**3+a*d),x)
[Out] -Integral((a + b*x**3)**(2/3)/(-a*x**5 + b*x**8), x)/d
```

3.605 $\int \frac{x^{14}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=127

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 2/5*(-x^3+1)^(5/3)-1/4*(-x^3+1)^(8/3)+1/11*(-x^3+1)^(11/3)-1/12*ln(x^3+1)*2
^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x
^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*(1 - x^3)^(5/3))/5 - (1 - x^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + ArcTan[(1
 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(6*2^
(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{14}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x^4}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\mathrm{Subst}\left(\int \left(-2(1-x)^{2/3}+2(1-x)^{5/3}-(1-x)^{8/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx, x, x^3\right)\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{}{2^{2/3}+\sqrt[3]{}}\right.\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= \frac{2}{5}\left(1-x^3\right)^{5/3}-\frac{1}{4}\left(1-x^3\right)^{8/3}+\frac{1}{11}\left(1-x^3\right)^{11/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 113, normalized size = 0.89

$$\frac{1}{660}\left(-55\,2^{2/3}\log\left(x^3+1\right)+165\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+110\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)+3\left(1-x^3\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(2/3)*(53 - 38*x^3 + 5*x^6 - 20*x^9) + 110*2^(2/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 55*2^(2/3)*Log[1 + x^3] + 165*
2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/660
```

fricas [A] time = 0.75, size = 118, normalized size = 0.93

$$-\frac{1}{220}\left(20\,x^9-5\,x^6+38\,x^3-53\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\,\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\,\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/220*(20*x^9 - 5*x^6 + 38*x^3 - 53)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6
)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12
*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2
^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

giac [A] time = 0.17, size = 134, normalized size = 1.06

$$-\frac{1}{11}\left(x^{3}-1\right)^{3}\left(-x^{3}+1\right)^{\frac{2}{3}}-\frac{1}{4}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/11*(x^3 - 1)^3*(-x^3 + 1)^(2/3) - 1/4*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/6
*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3)))
 + 2/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/
3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 9.32, size = 792, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/220*(20*x^9-5*x^6+38*x^3-53)*(x^3-1)/(-x^3+1)^(1/3)+1/6*RootOf(_Z^3-4)*ln
((-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*
x^3-90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4
)^2*x^3-3*RootOf(_Z^3-4)*x^3-90*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-126*RootOf(_Z^3-4)*(-x^3+1
)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2
/3)+7*RootOf(_Z^3-4)+210*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2))/(x+1)/(x^2-x+1))-1/6*ln((-3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+RootOf(_Z^3-4)*x^3-24*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-1
26*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)+42*(-x^3+1)^(2/3)-7*RootOf(_Z^3-4)+168*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln((-3*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+Root
Of(_Z^3-4)*x^3-24*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-
21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)-126*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-7*RootOf(_
Z^3-4)+168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2
-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [A] time = 1.24, size = 119, normalized size = 0.94

$$\frac{1}{11}\left(-x^{3}+1\right)^{\frac{11}{3}}-\frac{1}{4}\left(-x^{3}+1\right)^{\frac{8}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/11*(-x^3 + 1)^(11/3) - 1/4*(-x^3 + 1)^(8/3) + 1/6*sqrt(3)*2^(2/3)*arctan(
1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 2/5*(-x^3 + 1)^(5/3)
- 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) +
 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 5.03, size = 133, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2\left(1-x^3\right)^{5/3}}{5}-\frac{\left(1-x^3\right)^{8/3}}{4}+\frac{\left(1-x^3\right)^{11/3}}{11}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{4}\right)\left(-1+\sqrt{\right.}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2*(1 - x^3)^(5/3))/5 - (1 - x
^3)^(8/3)/4 + (1 - x^3)^(11/3)/11 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)
*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3)
 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.606 $\int \frac{x^{11}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=128

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/5*(-x^3+1)^(5/3)-1/8*(-x^3+1)^(8/3)+1/12*ln(x^3+1)*2^
(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$-\frac{1}{8}\left(1-x^3\right)^{8/3}+\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 + (1 - x^3)^(5/3)/5 - (1 - x^3)^(8/3)/8 - ArcTan[(1 + 2^
(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)
) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3} + (1-x)^{5/3} - \frac{1}{\sqrt[3]{1-x}\,(1+x)}\right) dx, x, x^3\right)$$

$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3} + \sqrt[3]{\ldots}}\right.$$

$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

$$= -\frac{1}{2}\left(1-x^3\right)^{2/3} + \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}$$

Mathematica [A] time = 0.06, size = 128, normalized size = 1.00

$$\frac{1}{120}\left(6\left(1-x^3\right)^{2/3}x^3 - 51\left(1-x^3\right)^{2/3} + 10\,2^{2/3}\log\left(x^3+1\right) - 30\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 20\,2^{2/3}\sqrt{3}\tan^{-1}\left(\ldots\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-51*(1 - x^3)^(2/3) + 6*x^3*(1 - x^3)^(2/3) - 15*x^6*(1 - x^3)^(2/3) - 20*
2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 10*2^(2/3)*
Log[1 + x^3] - 30*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/120
```

fricas [A] time = 0.95, size = 137, normalized size = 1.07

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} - \sqrt{6}\,2^{\frac{1}{3}}\right)\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} - 2^{\frac{2}{3}}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/40*(5*x^6
- 2*x^3 + 17)*(-x^3 + 1)^(2/3)
```

giac [A] time = 0.18, size = 127, normalized size = 0.99

$$-\frac{1}{8}\left(x^{3}-1\right)^{2}\left(-x^{3}+1\right)^{\frac{2}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2/3)
```

maple [C] time = 8.41, size = 683, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/40*(5*x^6-2*x^3+17)*(x^3-1)/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_Z*R
ootOf(_Z^3+4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+3
6*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)
+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+5*RootOf(_Z^3+4)*x^3+6*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42
*(-x^3+1)^(2/3)-35*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z
^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln((-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*R
ootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)*x^3-18*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*
(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(RootOf(_Z^3+4)
^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((-12*Ro
otOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*
RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3
+12*RootOf(_Z^3+4)*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3
+4)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1
))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

maxima [A] time = 1.25, size = 119, normalized size = 0.93

$$-\frac{1}{8}\left(-x^{3}+1\right)^{\frac{8}{3}}-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}+\frac{1}{12}\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^3 + 1)^(8/3) - 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/5*(-x^3 + 1)^(5/3) + 1/12*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(2/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

mupad [B] time = 4.73, size = 133, normalized size = 1.04

$$\frac{\left(1-x^{3}\right)^{5/3}}{5}-\frac{\left(1-x^{3}\right)^{2/3}}{2}-\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^{3}\right)^{8/3}}{8}-\frac{2^{2/3}\ln\left(\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(5/3)/5 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1
/3)))/6 - (1 - x^3)^(8/3)/8 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1
/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 + (2^(2/3)*log((1 - x^3)^(1/3) - (2^
(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.607 $\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=97

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/5*(-x^3+1)^(5/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 55, 617, 204, 31}

$$\frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(5/3)/5 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-(1-x)^{2/3}+\frac{1}{\sqrt[3]{1-x}\,(1+x)}\right)dx, x, x^3\right)\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)-\frac{\text{Subs}}{}\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\right.}{\sqrt[3]{2}}\\
&= \frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 96, normalized size = 0.99

$$\frac{1}{60}\left(12\left(1-x^3\right)^{5/3}-5\,2^{2/3}\log\left(x^3+1\right)+15\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+10\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (12*(1 - x^3)^(5/3) + 10*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 5*2^(2/3)*Log[1 + x^3] + 15*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(
1/3)])/60
```

fricas [A] time = 1.28, size = 106, normalized size = 1.09

$$-\frac{1}{5}\left(x^3-1\right)\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/5*(x^3 - 1)*(-x^3 + 1)^(2/3) + 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(s
qrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 + 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) +
2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (
-x^3 + 1)^(1/3))
```

giac [A] time = 0.20, size = 98, normalized size = 1.01

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)
))
```

maple [C] time = 6.73, size = 673, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/5*(x^3-1)^2/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Root
Of(_Z^3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3
-5*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+
42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/
(x+1)/(x^2-x+1))-1/6*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42
*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*
RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln((18*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [A] time = 1.19, size = 97, normalized size = 1.00

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/5*(-x^3 + 1)^(5/3) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^
(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 4.65, size = 111, normalized size = 1.14

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{\left(1-x^3\right)^{5/3}}{5}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (1 - x^3)^(5/3)/5 + (2^(2/3)*l
og((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 -
 (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
 + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.608 $\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)} \, dx$

Optimal. Leaf size=98

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/2*(-x^3+1)^(2/3)+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2
^(2/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 55, 617, 204, 31}

$$-\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/2 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)
*Sqrt[3]) + Log[1 + x^3]/(6*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^
(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{1-x^3}\right)+\frac{\text{Sub}}{}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2}\right.}{\sqrt[3]{2}}\\
&= -\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 0.97

$$\frac{1}{12}\left(-6\left(1-x^3\right)^{2/3}+2^{2/3}\log\left(x^3+1\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-6*(1 - x^3)^(2/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/12
```

fricas [A] time = 1.30, size = 125, normalized size = 1.28

$$-\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-2^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(2*sqrt(6)*(-1)^(1/3)*(-
x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) - 1/12*2^(2/3)*(-1)^(1/3)*log(2^(1/3)*(-
1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^
(2/3)*(-1)^(1/3)*log(-2^(1/3)*(-1)^(2/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 +
1)^(2/3)
```

giac [A] time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/2*(-x^3 + 1)^(2
/3)
```

maple [C] time = 3.82, size = 671, normalized size = 6.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*ln((15*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^
3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf
(_Z^3+4)^2*x^3+5*RootOf(_Z^3+4)*x^3+6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-35
*RootOf(_Z^3+4)-42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x
+1)/(x^2-x+1))-1/6*ln((-12*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)*x^3-18*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(_Z^3+4)^2*(-x^3+1)^(1/3)+42*
(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((-12*RootOf(RootOf(_Z^3+4
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+18*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+12*RootOf(_Z^3+4)
*x^3-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3+21*RootOf(
_Z^3+4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-28*RootOf(_Z^3+4)+42*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_
Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

maxima [A] time = 1.21, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 1/2*(-x^3 + 1)^(2/3)
```

mupad [B] time = 4.69, size = 111, normalized size = 1.13

$$-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{2/3}}{2}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i
+ 1))/12 - (1 - x^3)^(2/3)/2 - (2^(2/3)*log((1 - x^3)^(1/3) - (2^(1/3)*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x^3)^(1/3) - 2^
(1/3)))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.609 $\int \frac{x^2}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=82

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {444, 55, 617, 204, 31}

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x
^3]/(6*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,\right.}{2\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x^3}\right)}{\sqrt[3]{2}}$$

$$= \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 0.89

$$\frac{-\log\left(x^3+1\right)+3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + x^3] + 3
*Log[2^(1/3) - (1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [A] time = 1.28, size = 90, normalized size = 1.10

$$\frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)*(-x^3 +
 1)^(1/3))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 +
 1)^(2/3)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

giac [A] time = 0.18, size = 87, normalized size = 1.06

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 5.59, size = 476, normalized size = 5.80

$$\text{RootOf}\left(36\_Z^2+6\_Z\,\text{RootOf}\left(\_Z^3-4\right)+\text{RootOf}\left(\_Z^3-4\right)^2\right)\ln\left(\frac{72x^3\,\text{RootOf}\left(36\_Z^2+6\_Z\,\text{RootOf}\left(\_Z^3\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(_Z
^3-4)*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+126*R
ootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)+42*(-x^3+1)^(2/3)-35*RootOf(_Z^3-4)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-4)*ln(-(6*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+45*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-2*R
ootOf(_Z^3-4)*x^3-15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x
^3-63*RootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)-21*(-x^3+1)^(2/3)+14*RootOf(_Z^3-4)+105*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [A] time = 1.30, size = 86, normalized size = 1.05

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 4.89, size = 100, normalized size = 1.22

$$\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-2^{1/3}\right)}{6}+\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}-\frac{2^{2/3}\ln\left(\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3) - 2^(1/3)))/6 + (2^(2/3)*log((1 - x^3)^(1/3) -
 (2^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((1 - x
^3)^(1/3) - (2^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.610 $\int \frac{1}{x\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=137

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 55, 618, 204, 31, 617}

$$\frac{\log(x^3+1)}{6\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - ArcTan[(1 + 2^(2/3)*(1 -
x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(1/3)
) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\right.\\
&= -\frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}} + \frac{\operatorname{Subst}\left(\int \frac{}{-3}\right.}{}\\
&= \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x}\right.
\end{aligned}$$

Mathematica [A] time = 0.04, size = 133, normalized size = 0.97

$$\frac{1}{12}\left(2^{2/3}\log\left(x^3+1\right)+6\log\left(1-\sqrt[3]{1-x^3}\right)-3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+4\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)-2\,2^{2/3}\sqrt{\vphantom{3}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] - 2*2^(2/3)*Sqrt[3]*ArcT
an[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 6*Log[x] + 2^(2/3)*Log[1 + x^3]
 + 6*Log[1 - (1 - x^3)^(1/3)] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)])/1
2
```

fricas [C] time = 2.46, size = 410, normalized size = 2.99

$$\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(\frac{1}{8}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3-\frac{3}{4}\cdot 2^{\frac{1}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2+3\left(-x^3+1\right)^{\frac{1}{3}}+1\right)-\frac{1}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(1/8*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^3 - 3/4*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 3
*(-x^3 + 1)^(1/3) + 1) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))
- 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2))*log(3/8
*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)
^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2)
)*log(-3/8*2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/
3))^2)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 3/8*2^(1/3)*(I*sqrt(3)*(-1)^(1
/3) - (-1)^(1/3))^2 + 3*(-x^3 + 1)^(1/3)) + 1/3*sqrt(3)*arctan(2/3*sqrt(3)*
(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 1/3*log(-1/24*(I*sqrt(3)*(-1)^(1/3) - (-1
)^(1/3))^3 + (-x^3 + 1)^(1/3) - 4/3) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 + 1
)^(1/3) + 1)
```

giac [A] time = 0.20, size = 149, normalized size = 1.09

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) - 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arcta
n(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3
+ 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [F] time = 2.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x), x)
```

mupad [B] time = 4.80, size = 256, normalized size = 1.87

$$\frac{\ln\left(6-6\left(1-x^3\right)^{1/3}\right)}{3}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^3\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)^2-135\left(1-x^3\right)^{1/3}\right)-\left(1-x^3\right)^{1/3}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] log(6 - 6*(1 - x^3)^(1/3))/3 + log(((3^(1/2)*1i)/6 - 1/6)^3*(1458*((3^(1/2)
*1i)/6 - 1/6)^2 - 135*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 -
 1/6) - log(- ((3^(1/2)*1i)/6 + 1/6)^3*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - 135
*(1 - x^3)^(1/3)) - (1 - x^3)^(1/3))*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log(
(3*(1 - x^3)^(1/3))/2 - (3*2^(1/3))/2))/6 + ((-1)^(1/3)*2^(2/3)*log((3*(1 -
 x^3)^(1/3))/2 - (3*(-1)^(2/3)*2^(1/3))/2))/6 - ((-1)^(1/3)*2^(2/3)*log(- (
(3^(1/2)*1i + 1)^3*(135*(1 - x^3)^(1/3) - (81*(-1)^(2/3)*2^(1/3)*(3^(1/2)*1
i + 1)^2)/4))/432 - (1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.611 $\int \frac{1}{x^4 \sqrt[3]{1-x^3} \left(1+x^3\right)} \, dx$

Optimal. Leaf size=157

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/3*(-x^3+1)^(2/3)/x^3+1/3*ln(x)-1/12*ln(x^3+1)*2^(2/3)-1/3*ln(1-(-x^3+1)^
(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-2/9*arctan(1/3*(1+2*(-x^3+1)^
(1/3))*3^(1/2))*3^(1/2)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 55, 618, 204, 31, 617}

$$-\frac{\left(1-x^3\right)^{2/3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}-\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(3*x^3) - (2*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]])/(3*S
qrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[x]/3 - Log[1 + x^3]/(6*2^(1/3)) - Log[1 - (1 - x^3)^(1/3)]/3 + Log[2^(
1/3) - (1 - x^3)^(1/3)]/(2*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x^2(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{\frac{2}{3}-\frac{x}{3}}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{2}{9}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right.\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} + \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{1}{3}\,\text{Subst}\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} - \frac{1}{3}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\sqrt[3]{2}}\\
&= -\frac{\left(1-x^3\right)^{2/3}}{3x^3} - \frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log(x)}{3} - \frac{\log\left(1+x^3\right)}{6\sqrt[3]{2}} -\end{aligned}$$

Mathematica [A] time = 0.22, size = 153, normalized size = 0.97

$$\frac{1}{36}\left(3\left(-\frac{4\left(1-x^3\right)^{2/3}}{x^3} - 2^{2/3}\log\left(x^3+1\right) - 4\log\left(1-\sqrt[3]{1-x^3}\right) + 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2}{}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-8*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 3*((-4*(1 - x^3)^(2/3
))/x^3 + 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] +
4*Log[x] - 2^(2/3)*Log[1 + x^3] - 4*Log[1 - (1 - x^3)^(1/3)] + 3*2^(2/3)*Lo
g[2^(1/3) - (1 - x^3)^(1/3)]))/36
```

fricas [A] time = 0.85, size = 187, normalized size = 1.19

$$\frac{6\sqrt{6}\,2^{\frac{1}{6}}x^3\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-3\cdot 2^{\frac{2}{3}}x^3\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+6\cdot 2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*(6*sqrt(6)*2^(1/6)*x^3*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*sqrt(6)
*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*x^3*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3)
+ (-x^3 + 1)^(2/3)) + 6*2^(2/3)*x^3*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - 8*sq
rt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) + 4*x^3*log((-
x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*x^3*log((-x^3 + 1)^(1/3) - 1) -
12*(-x^3 + 1)^(2/3))/x^3
```

giac [A] time = 0.19, size = 163, normalized size = 1.04

$$\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/12*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) + 1/6*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 2/9*sqrt(3)*arctan
(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/3*(-x^3 + 1)^(2/3)/x^3 + 1/9*log
((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 2/9*log(abs((-x^3 + 1)^(1/3) -
1))
```

maple [F] time = 2.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^4), x)
```

mupad [B] time = 4.86, size = 382, normalized size = 2.43

$$\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(81\,2^{1/3}-75\,(1-x^3)^{1/3}\right)}{6}-\frac{38}{3}\right)}{18}+\frac{16\,(1-x^3)^{1/3}}{27}\right)}{6}-\frac{(1-x^3)^{2/3}}{3\,x^3}-\frac{2\,\ln\left(\frac{344\,(1-x^3)^{1/3}}{243}-\frac{344}{243}\right)}{9}+\ln\left(\left(\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2\left(\left(\frac{1}{9}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((2^(1/3)*((2^(2/3)*(81*2^(1/3) - 75*(1 - x^3)^(1/3)))/6 - 38/3
))/18 + (16*(1 - x^3)^(1/3))/27))/6 - (1 - x^3)^(2/3)/(3*x^3) - (2*log((344
*(1 - x^3)^(1/3))/243 - 344/243))/9 + log(((3^(1/2)*1i)/9 + 1/9)^2*(((3^(1/
2)*1i)/9 + 1/9)*(1458*((3^(1/2)*1i)/9 + 1/9)^2 - 75*(1 - x^3)^(1/3)) - 38/3
) + (16*(1 - x^3)^(1/3))/27)*((3^(1/2)*1i)/9 + 1/9) - log((16*(1 - x^3)^(1/
3))/27 - ((3^(1/2)*1i)/9 - 1/9)^2*(((3^(1/2)*1i)/9 - 1/9)*(1458*((3^(1/2)*1
i)/9 - 1/9)^2 - 75*(1 - x^3)^(1/3)) + 38/3))*((3^(1/2)*1i)/9 - 1/9) + (2^(2
/3)*log((16*(1 - x^3)^(1/3))/27 + (2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^
(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2)*1i - 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12
- 38/3))/72)*(3^(1/2)*1i - 1))/12 - (2^(2/3)*log((16*(1 - x^3)^(1/3))/27 -
(2^(1/3)*(3^(1/2)*1i + 1)^2*((2^(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2
)*1i + 1)^2)/4 - 75*(1 - x^3)^(1/3)))/12 + 38/3))/72)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.612 $\int \frac{x^6}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=154

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{1}{3}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2}{\dots}}{\dots}\right)}{\sqrt[3]{2}\sqrt{\dots}}$$

```
[Out] -1/3*x*(-x^3+1)^(2/3)-1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/
3))*2^(2/3)-1/3*ln(x+(-x^3+1)^(1/3))+2/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*
3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 226, normalized size of antiderivative = 1.47, number of steps used = 15, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 522, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{3}\left(1-x^3\right)^{2/3}x+\frac{1}{9}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{2}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\dots\right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(x*(1 - x^3)^(2/3))/3 + (2*ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]])/(3
*Sqrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sq
rt[3]) + Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/9 - (2*Log[1 + x/
(1 - x^3)^(1/3)])/9 - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(
1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
```

```
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{1-x^3}\left(1+x^3\right)}dx = \text{Subst}\left(\int \frac{x^6}{\left(1+x^3\right)^2\left(1+2x^3\right)}dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{3}\text{Subst}\left(\int \frac{1-x^3}{\left(1+x^3\right)\left(1+2x^3\right)}dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{3}\text{Subst}\left(\int \frac{1}{1+x^3}dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \text{Subst}\left(\int \frac{1}{1+2x^3}dx, x, \sqrt[3]{1}\right.$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\text{Subst}\left(\int \frac{1}{1+x}dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}dx, x\right.$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{9}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\right.$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{2}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log(}{}$$
$$= -\frac{1}{3}x\left(1-x^3\right)^{2/3} + \frac{2\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{1}{9}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \right.$$

Mathematica [C] time = 0.16, size = 144, normalized size = 0.94

$$\frac{1}{36}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right) - 12\left(1-x^3\right)^{2/3}x + 2^{2/3}\left(2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right) - \log\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (-12*x*(1 - x^3)^(2/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(2
/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - Log[
1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 2*Log[
1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/36
```

fricas [A] time = 0.79, size = 201, normalized size = 1.31

$$-\frac{1}{3}\left(-x^3+1\right)^{\frac{2}{3}}x - \frac{1}{6}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x - 2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right) + \frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{2^{\frac{1}{3}}x + \left(-x^3+1\right)^{\frac{1}{3}}}{x}\right) - \frac{1}{12}\cdot 2^{\frac{2}{3}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(2/3)*x - 1/6*sqrt(6)*2^(1/6)*arctan(-1/6*2^(1/6)*(sqrt(6)*
2^(1/3)*x - 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) + 1/6*2^(2/3)*log((2^(1/3)*x + (
-x^3 + 1)^(1/3))/x) - 1/12*2^(2/3)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^(1
/3)*x + (-x^3 + 1)^(2/3))/x^2) + 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqr
t(3)*(-x^3 + 1)^(1/3))/x) - 2/9*log((x + (-x^3 + 1)^(1/3))/x) + 1/9*log((x^
2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 1.92, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(-x^3+1)^{\frac{1}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^6/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.613 $\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=135

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/2*ln(x+(
-x^3+1)^(1/3))-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6*a
rctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.53, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 200, 31, 634, 618, 204, 628, 617}

$$-\frac{1}{6}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\text{tan}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) - Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 + Log[1 + x/(1 - x^3)^(1/3)]/3 + Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^3}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Sub}$$
$$= \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) -$$
$$= -\frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}} - \frac{}{\sqrt[3]{}}\right.}{6\sqrt[3]{2}}$$
$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\log\Big($$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.19

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(1/3)*(1 + x^3)),x]
[Out] (x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3])/4
```

fricas [C] time = 2.93, size = 452, normalized size = 3.35

$$\frac{1}{12}\cdot 2^{\frac{2}{3}}\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)\log\left(-\frac{x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^3 - 6\cdot 2^{\frac{1}{3}}x\left(i\sqrt{3}\,(-1)^{\frac{1}{3}}-(-1)^{\frac{1}{3}}\right)^2 + 8x - 24\left(-x^3+1\right)}{8x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/12*2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))*log(-1/8*(x*(I*sqrt(3)*(-1
)^(1/3) - (-1)^(1/3))^3 - 6*2^(1/3)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
 + 8*x - 24*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(-1)^(1/3) - (-
1)^(1/3)) - 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2
))*log(-3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) - (-1)^(
1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) + 2^(1/3)*x*(I*sqrt(3)*(-1)^
(1/3) - (-1)^(1/3))^2 - 8*(-x^3 + 1)^(1/3))/x) - 1/24*(2^(2/3)*(I*sqrt(3)*(
-1)^(1/3) - (-1)^(1/3)) + 2*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(1/3) -
 (-1)^(1/3))^2))*log(3/8*(2^(2/3)*sqrt(3/2)*sqrt(-2^(1/3)*(I*sqrt(3)*(-1)^(
1/3) - (-1)^(1/3))^2)*x*(I*sqrt(3)*(-1)^(1/3) - (-1)^(1/3)) - 2^(1/3)*x*(I*
sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^2 + 8*(-x^3 + 1)^(1/3))/x) - 1/3*sqrt(3)*a
rctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/3*log(1/24*(x*(I
```

```
*sqrt(3)*(-1)^(1/3) - (-1)^(1/3))^3 + 32*x + 24*(-x^3 + 1)^(1/3))/x) - 1/6*
log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 1.74, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-x^3+1)^{\frac{1}{3}}(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^3/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.614 $\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=88

$$-\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(2/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3])) -
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right. \\
&= \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ldots}{1-\sqrt[3]{2}x+2\ldots}\right.}{6\ldots} \\
&= -\frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{\sqrt[3]{2}} \\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 112, normalized size = 1.27

$$\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}+1\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]] - Log[1 + (
2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)] + 2*Log[1 + (2^
(1/3)*x)/(1 - x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 5.06, size = 253, normalized size = 2.88

$$-\frac{1}{18}\sqrt{6}\,2^{\frac{1}{6}}\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-\ldots\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(6)*2^(1/6)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7 + 4*x^4
- x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3 - 1) + 1
2*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105*x^6 + 3*
x^3 + 1)) + 1/18*2^(2/3)*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2^(2/3)*(x^3
 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/36*2^(2/3)*log((3*2^(2/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [C] time = 4.94, size = 931, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(
2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*
x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3
+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+10*x*(-x^3+1)^
(2/3)-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^
2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)
^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^
3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf
(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+
1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-2*x*(-x^3+1)^(2/3)+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.615 $\int \frac{1}{x^3 \sqrt[3]{1-x^3}\left(1+x^3\right)} dx$

Optimal. Leaf size=105

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{2x^2}$$

```
[Out] -1/2*(-x^3+1)^(2/3)/x^2+1/12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(
1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3
)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^3\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\right.$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) + $$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\right.}{\sqrt[3]{2}}$$

$$= -\frac{\left(1-x^3\right)^{2/3}}{2x^2} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}} - \frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

Mathematica [C] time = 0.60, size = 82, normalized size = 0.78

$$-\frac{\left(-6x^6+4x^3+2\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-3x^3\left(x^3+1\right)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{2x^3}{x^3-1}\right)}{4x^2\left(1-x^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/4*((2 + 4*x^3 - 6*x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)
] - 3*x^3*(1 + x^3)*Hypergeometric2F1[4/3, 2, 7/3, (2*x^3)/(-1 + x^3)])/(x^
2*(1 - x^3)^(4/3))
```

fricas [B] time = 4.99, size = 307, normalized size = 2.92

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^2\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33\cdots}{6(109x^9-105x^6+3x^3+1)}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^2*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/
3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3)*
(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^
6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1/3)*x^
2*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x^3
+ 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*x^2*log(-(3*2^
(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(19*x^6
- 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 18*
(-x^3 + 1)^(2/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

maple [C] time = 4.49, size = 929, normalized size = 8.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(1/3)-1/6*ln(-(-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*Ro
otOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3+3*RootOf(RootOf(_Z^3+4)^2+6*_Z
*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-30*(-x^3+1)^(1
/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2
-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf
(_Z^3+4)+36*_Z^2)*x^3-3*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-12*RootOf(Ro
otOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4))/(x+1)/(x^2-x+1)
)*RootOf(_Z^3+4)-ln(-(-36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)^2*RootOf(_Z^3+4)^2*x^3+3*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36
*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*
RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x-30*(-x^3+1)^(1/3)*RootOf(_Z^3+4)
*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*Ro
```

```
otOf(_Z^3+4)^2*x^2+36*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*
x^3-3*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+1/6*RootOf(_Z^3+4)*ln((-54*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-3*RootOf(R
ootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+
1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4
)^2*x+6*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*x^2+5*(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2-18*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-RootOf(_Z^3+4)*x^3+2*(-x^3+1)^(2
/3)*x+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+RootOf(_Z^3+4
))/(x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.616 $\int \frac{1}{x^6 \sqrt[3]{1-x^3} \left(1+x^3\right)} \, dx$

Optimal. Leaf size=124

$$-\frac{\log \left(x^3+1\right)}{6 \sqrt[3]{2}}+\frac{\log \left(-\sqrt[3]{1-x^3}-\sqrt[3]{2} x\right)}{2 \sqrt[3]{2}}-\frac{\tan ^{-1}\left(\frac{1-\frac{2 \sqrt[3]{2} x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2} \sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{5 x^5}+\frac{\left(1-x^3\right)^{2/3}}{5 x^2}$$

```
[Out] -1/5*(-x^3+1)^(2/3)/x^5+1/5*(-x^3+1)^(2/3)/x^2-1/12*ln(x^3+1)*2^(2/3)+1/4*l
n(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{5/3}}{5 x^5}-\frac{\log \left(\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}+1\right)}{6 \sqrt[3]{2}}+\frac{\log \left(\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}+1\right)}{3 \sqrt[3]{2}}-\frac{\tan ^{-1}\left(\frac{1-\frac{2 \sqrt[3]{2} x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2} \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(5/3)/(5*x^5) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^6\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^6\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^6}+\frac{1}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\text{Subst}\left(\int \frac{1}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{3}\,\text{Subst}\left(\int \frac{2-\sqrt[3]{2}\,x}{1-\sqrt[3]{2}\,x+2^{2/3}\ldots}\right.\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)-\ldots\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x\ldots\right.}{\sqrt[3]{2}}\\
&= -\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 5.25, size = 123, normalized size = 0.99

$$\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{(x^3-1)^{2/3}}+1\right)}{6\sqrt[3]{2}}-\frac{(1-x^3)^{5/3}}{5x^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/5*(1 - x^3)^(5/3)/x^5 + (2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)
^(1/3))/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1
 + x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)])/(6*2^(1/3))
```

fricas [B] time = 4.61, size = 283, normalized size = 2.28

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}x^5\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+33x^3-1)+12\sqrt{6}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}\right)}{6(109x^9-105x^6+3x^3+1)}\right)-10\cdot}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/180*(10*sqrt(6)*2^(1/6)*x^5*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5*x^7
 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33*x^3
 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 - 105
*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*x^5*log((6*2^(1/3)*(-x^3 + 1)^(1/3)*x^2 + 2
^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*x^5*log((3*
2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^3 + 1) - 12*(
2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 36*(x^3 - 1)*(-x^3 + 1)
^(2/3))/x^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

maple [C] time = 2.75, size = 955, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] -1/5*(x^6-2*x^3+1)/x^5/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x
^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
```

```
ootOf(_Z^3-4)*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*x^3+10*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-6*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/6*ln(-(6*RootO
f(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12
*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf
(_Z^3-4)^2*x+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z
^3-4)*x^3+6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^
3+1)^(2/3)*x+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-ln(-(6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*Root
Of(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x+2*RootO
f(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^
2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^6*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.617 $\int \frac{1}{x^9 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=141

$$\frac{\log\left(x^3+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{8x^8}+\frac{\left(1-x^3\right)^{2/3}}{20x^5}-\frac{17\left(1-x^3\right)^{2/3}}{40x^2}$$

```
[Out] -1/8*(-x^3+1)^(2/3)/x^8+1/20*(-x^3+1)^(2/3)/x^5-17/40*(-x^3+1)^(2/3)/x^2+1/
12*ln(x^3+1)*2^(2/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1
/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 175, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{8/3}}{8x^8}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{2/3}}{2x^2}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(2/3)/(2*x^2) - (1 - x^3)^(5/3)/(5*x^5) - (1 - x^3)^(8/3)/(8*x^8
) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(1/3)*Sqrt[3]) +
 Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^
(1/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^9 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx &= \text{Subst}\left(\int \frac{\left(1+x^3\right)^3}{x^9\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{x^9}+\frac{1}{x^6}+\frac{1}{x^3}+\frac{1}{-1-2x^3}\right) dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\text{Subst}\left(\int \frac{1}{-1-2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1-\sqrt[3]{2}\,x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)+\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}-\frac{\log\left(1+\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}\,x+\right.\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}-\frac{\log\left(1+\frac{\sqrt[3]{2}}{\sqrt[3]{1-}\right.}{3\sqrt[3]{2}}\\
&= -\frac{\left(1-x^3\right)^{2/3}}{2x^2}-\frac{\left(1-x^3\right)^{5/3}}{5x^5}-\frac{\left(1-x^3\right)^{8/3}}{8x^8}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}}{\sqrt[3]{1-}\right.}{6\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [A] time = 5.11, size = 133, normalized size = 0.94

$$\frac{-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{(x^3-1)^{2/3}}+1\right)}{6\sqrt[3]{2}}-\frac{(1-x^3)^{2/3}(17x^6-2x^3+5)}{40x^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -1/40*((1 - x^3)^(2/3)*(5 - 2*x^3 + 17*x^6))/x^8 + (-2*Sqrt[3]*ArcTan[(-1 +
 (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/3)*x^2)/(-1 + x^3
)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^
(1/3)])/(6*2^(1/3))
```

fricas [B] time = 4.95, size = 320, normalized size = 2.27

$$-\frac{20\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}x^8\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}(5x^7+4x^4-x)(-x^3+1)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}(19x^8-16x^5+x^2)(-x^3+1)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}(71x^9-111x^6+\right.}{6(109x^9-105x^6+3x^3+1)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(20*sqrt(6)*2^(1/6)*(-1)^(1/3)*x^8*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(
2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^(1/3
)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*
x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 20*2^(2/3)*(-1)^(1/3)
*x^8*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)^(1/3)*(x
^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 10*2^(2/3)*(-1)^(1/3)*x^8*log(
-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(1
9*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
 + 9*(17*x^6 - 2*x^3 + 5)*(-x^3 + 1)^(2/3))/x^8
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

maple [C] time = 4.07, size = 963, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] 1/40*(17*x^9-19*x^6+7*x^3-5)/x^8/(-x^3+1)^(1/3)+RootOf(RootOf(_Z^3+4)^2+6*_
Z*RootOf(_Z^3+4)+36*_Z^2)*ln(-(-18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3
+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-9*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_
Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4
```

```
)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2*x+24*(-x^3+1)^(1/3)*RootO
f(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2+5*(-x^3+
1)^(1/3)*RootOf(_Z^3+4)^2*x^2+18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4
)+36*_Z^2)*x^3+9*RootOf(_Z^3+4)*x^3-10*(-x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^
3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)-3*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))-1/6*
ln((18*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4
)^2*x^3-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+
4)^3*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_
Z^2)*RootOf(_Z^3+4)^2*x+24*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3
+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+
6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)
*x^3+2*(-x^3+1)^(2/3)*x-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z
^2)+2*RootOf(_Z^3+4))/(x+1)/(x^2-x+1))*RootOf(_Z^3+4)-ln((18*RootOf(RootOf(
_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)^2*RootOf(_Z^3+4)^2*x^3-6*RootOf(Root
Of(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^3*x^3+12*(-x^3+1)^
(2/3)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*RootOf(_Z^3+4)^2
*x+24*(-x^3+1)^(1/3)*RootOf(_Z^3+4)*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^
3+4)+36*_Z^2)*x^2-(-x^3+1)^(1/3)*RootOf(_Z^3+4)^2*x^2+6*RootOf(RootOf(_Z^3+
4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)*x^3-2*RootOf(_Z^3+4)*x^3+2*(-x^3+1)^(2/3)
*x-6*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)+2*RootOf(_Z^3+4))
/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+4)^2+6*_Z*RootOf(_Z^3+4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^9*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\sqrt[3]{-\left(x-1\right)\left(x^2+x+1\right)}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.618 $\int \frac{x^7}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=271

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\cdots\right)}{\sqrt[3]{\cdots}}$$

```
[Out] -1/4*x^2*(-x^3+1)^(2/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/24*ln((1-
x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/8*
ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-
x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.10, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^8*AppellF1[8/3, 1/3, 1, 11/3, x^3, -x^3])/8
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.04, size = 40, normalized size = 0.15

$$\frac{1}{4}x^2\left(F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};x^3,-x^3\right)-\left(1-x^3\right)^{2/3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(-(1 - x^3)^(2/3) + AppellF1[2/3, -2/3, 1, 5/3, x^3, -x^3]))/4
```

fricas [F] time = 4.39, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^7}{x^6-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(2/3)*x^7/(x^6 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-x^3+1)^{\frac{1}{3}}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x^7/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(1-x^3)^{1/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x^7/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.619 $\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=254

$$\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-}{-}\right.}{\sqrt[3]{2}}$$

```
[Out] 1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+x)^2)*2^(2/3)-1/12
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+x+2^(2/3)*(-x^3+1)
^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*
2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2
^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.10, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.10

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

fricas [F] time = 4.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^4}{x^6-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)*x^4/(x^6 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(x^4/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(1/3)/(x**3+1),x)
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.620 $\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx$

Optimal. Leaf size=233

$$\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\cdots}{\sqrt{\cdots}}\right)}{2\sqrt[3]{2}\sqrt{\cdots}}$$

```
[Out] 1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(
1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2
^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.11, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt[3]{1-x^3}\,(1+x^3)}\,dx = \frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.02, size = 26, normalized size = 0.11

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3])/2
```

fricas [B] time = 3.17, size = 373, normalized size = 1.60

$$-\frac{1}{36}\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{2^{\frac{1}{6}}\left(24\sqrt{6}\,2^{\frac{2}{3}}\,(-1)^{\frac{2}{3}}\left(x^{14}-2x^{11}-6x^{8}-2x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{2}{3}}+12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(x^{16}-\cdots\right.\right.}{6\left(x^{18}-102x^{15}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(6)*2^(1/6)*(-1)^(1/3)*arctan(1/6*2^(1/6)*(24*sqrt(6)*2^(2/3)*(-1
)^(2/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) + 12*sqrt(6)
*(-1)^(1/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(
1/3) + sqrt(6)*2^(1/3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*
x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) -
 1/72*2^(2/3)*(-1)^(1/3)*log(-(12*2^(2/3)*(-1)^(1/3)*(x^8 - 4*x^5 + x^2)*(-
x^3 + 1)^(2/3) - 2^(1/3)*(-1)^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) -
 6*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4
*x^3 + 1)) + 1/36*2^(2/3)*(-1)^(1/3)*log(-(12*(-x^3 + 1)^(2/3)*x^2 - 6*2^(1
/3)*(-1)^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3)*(x^6 + 2*x^3
 + 1))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

maple [F] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

```
[Out] int(x/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(1/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.621 $\int \frac{1}{x^2 \sqrt[3]{1-x^3} \left(1+x^3\right)} \, dx$

Optimal. Leaf size=270

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{4\sqrt[3]{2}}$$

```
[Out] -(-x^3+1)^(2/3)/x-1/2*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/24*ln((1-x)*(1+
x)^2)*2^(2/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+
1)^(1/3))*2^(2/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/8*ln(-1+
x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)
^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -(AppellF1[-1/3, 1/3, 1, 2/3, x^3, -x^3]/x)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2 \sqrt[3]{1-x^3} \left(1+x^3\right)} \, dx = -\frac{F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] time = 0.05, size = 67, normalized size = 0.25

$$-\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(2/3)/x) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - (x^5*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/5
```

fricas [F] time = 3.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^8-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)/(x^8 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}}\left(x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(1-x^3\right)^{1/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.622 $\int \frac{1}{x^5 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=289

$$\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\left(1-x^3\right)^{2/3}}{2x}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right.}{4\sqrt[3]{2}}$$

```
[Out] -1/4*(-x^3+1)^(2/3)/x^4+1/2*(-x^3+1)^(2/3)/x+1/4*x^2*hypergeom([1/3, 2/3],[
5/3],x^3)+1/24*ln((1-x)*(1+x)^2)*2^(2/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)
^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1
)^(1/3))*2^(2/3)-1/8*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/6*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/12*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-4/3, 1/3, 1, -1/3, x^3, -x^3]/(4*x^4)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5 \sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx=-\frac{F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] time = 0.06, size = 76, normalized size = 0.26

$$\frac{2x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)+15x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+5\left(1-x^3\right)^{2/3}\left(2x^3-1\right)}{20x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(1/3)*(1 + x^3)),x]
```

```
[Out] (5*(1 - x^3)^(2/3)*(-1 + 2*x^3) + 15*x^6*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] + 2*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/(20*x^4)
```

fricas [F] time = 3.33, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{11}-x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(2/3)/(x^11 - x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{1}{3}} \left(x^3+1\right) x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
[Out] int(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{1}{3}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(1/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(1/3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(1-x^3\right)^{1/3} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(1/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt[3]{-(x-1)\left(x^2+x+1\right)}\,(x+1)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(1/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(1/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.623 $\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/4*(-x^3+1)^(4/3)-1/7*(-x^3+1)^(7/3)+1/12*ln(x^3+1)*2^(1/3
)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)
^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$-\frac{1}{7}\left(1-x^3\right)^{7/3}+\frac{1}{4}\left(1-x^3\right)^{4/3}-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + (1 - x^3)^(4/3)/4 - (1 - x^3)^(7/3)/7 + ArcTan[(1 + 2^(2
/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^3]/(6*2^(2/3))
- Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{(1-x)^{2/3}} - \sqrt[3]{1-x} + (1-x)^{4/3} - \frac{1}{(1-x)^{2/3}(1+x)}\right) dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{\ldots}\right.}{2\ 2^{2/3}}$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \ldots$$
$$= -\sqrt[3]{1-x^3} + \frac{1}{4}\left(1-x^3\right)^{4/3} - \frac{1}{7}\left(1-x^3\right)^{7/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \ldots$$

Mathematica [A] time = 0.11, size = 151, normalized size = 1.21

$$\frac{1}{84}\left(3\sqrt[3]{1-x^3}\,x^3 - 75\sqrt[3]{1-x^3} - 14\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + 7\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 14\sqrt[3]{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-75*(1 - x^3)^(1/3) + 3*x^3*(1 - x^3)^(1/3) - 12*x^6*(1 - x^3)^(1/3) + 14*
2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 14*2^(1/3)*
Log[2^(1/3) - (1 - x^3)^(1/3)] + 7*2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3)
+ (1 - x^3)^(2/3)])/84
```

fricas [A] time = 0.65, size = 142, normalized size = 1.14

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/28*(4*x^6 - x^3 + 25)*(-x^3 + 1)^(1/3)
```

giac [A] time = 0.18, size = 127, normalized size = 1.02

$$-\frac{1}{7}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{1}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/7*(x^3 - 1)^2*(-x^3 + 1)^(1/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

maple [C] time = 7.45, size = 1579, normalized size = 12.63

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/28*(4*x^6-x^3+25)*(x^3-1)/(-x^3+1)^(2/3)+(1/6*RootOf(_Z^3+2)*ln((36*RootO
f(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-24*R
ootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-36
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^
3+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*
x^3+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-4*RootOf(_Z^
3+2)*x^6-144*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2
)+36*_Z^2)*RootOf(_Z^3+2)*x^3-9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90
*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)-48*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3
+32*RootOf(_Z^3+2)*x^3+144*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z
*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1
/3)-48*(x^6-2*x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+3
6*_Z^2)-28*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))-1/6*ln(-(90*Roo
tOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+12
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-
90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*
x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^
3*x^3-45*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-6*RootOf(
_Z^3+2)*x^6+27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+12*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3
+45*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)+150*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
*x^3+20*RootOf(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-12*RootOf(_Z^3+2)^2*(x^6-2*x^3+1
)^(1/3)+9*(x^6-2*x^3+1)^(2/3)-105*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+
2)+36*_Z^2)-14*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3
+2)-ln(-(90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_
Z^3+2)^2*x^6+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf
(_Z^3+2)^3*x^6-90*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*Ro
otOf(_Z^3+2)^2*x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*
RootOf(_Z^3+2)^3*x^3-45*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)*x^6-6*RootOf(_Z^3+2)*x^6+27*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+12*RootOf(_Z^3+2)^2*(x^6-2*x
^3+1)^(1/3)*x^3+45*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+150*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)*x^3+20*RootOf(_Z^3+2)*x^3-27*(x^6-2*x^3+1)^(1/3)*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-12*RootOf(_Z^3+2)
^2*(x^6-2*x^3+1)^(1/3)+9*(x^6-2*x^3+1)^(2/3)-105*RootOf(RootOf(_Z^3+2)^2+6*
```

```
_Z*RootOf(_Z^3+2)+36*_Z^2)-14*RootOf(_Z^3+2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+
1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(-x^3+1)^(2/3)*((
x^3-1)^2)^(1/3)
```

maxima [A] time = 1.52, size = 119, normalized size = 0.95

$$-\frac{1}{7}\left(-x^3+1\right)^{\frac{7}{3}}+\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/7*(-x^3 + 1)^(7/3) + 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) + 1/4*(-x^3 + 1)^(4/3) + 1/12*2^(1/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)*log(-2^(1/3
) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

mupad [B] time = 5.18, size = 135, normalized size = 1.08

$$\frac{\left(1-x^3\right)^{4/3}}{4}-\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\left(1-x^3\right)^{1/3}\right)}{6}-\frac{\left(1-x^3\right)^{7/3}}{7}-\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2}\right)\left(-\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (1 - x^3)^(4/3)/4 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^
(1/3)))/6 - (1 - x^3)^(7/3)/7 - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)
*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 + (2^(1/3)*log((3*2^(1/3)*(3^(1/
2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.624 $\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=98

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] 1/4*(-x^3+1)^(4/3)-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(1/3)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 88, 57, 617, 204, 31}

$$\frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3)^(4/3)/4 - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*
Sqrt[3]) - Log[1 + x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(-\sqrt[3]{1-x}+\frac{1}{(1-x)^{2/3}(1+x)}\right)dx, x, x^3\right)\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\ldots}\right.}{2\ldots}\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\ldots\right.}{2^{2/3}}\\
&= \frac{1}{4}\left(1-x^3\right)^{4/3}-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 135, normalized size = 1.38

$$\frac{1}{12}\left(-3\sqrt[3]{1-x^3}\,x^3+3\sqrt[3]{1-x^3}+2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)-\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)-2\sqrt[3]{2}\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (3*(1 - x^3)^(1/3) - 3*x^3*(1 - x^3)^(1/3) - 2*2^(1/3)*Sqrt[3]*ArcTan[(1 +
2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
] - 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

fricas [A] time = 0.59, size = 114, normalized size = 1.16

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3)) - 1
/4*(x^3 - 1)*(-x^3 + 1)^(1/3)
```

giac [A] time = 0.18, size = 98, normalized size = 1.00

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3
)))
```

maple [C] time = 7.46, size = 1584, normalized size = 16.16

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/4*(x^3-1)^2/(-x^3+1)^(2/3)+(RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^2*x^6-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Roo
tOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^
2*RootOf(_Z^3-2)^2*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^
2)*RootOf(_Z^3-2)^3*x^3+72*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_
Z^2)*x^6-3*RootOf(_Z^3-2)*x^6-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^
2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+9*RootOf(_Z^3-2)^2*(x^6-2
*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-240*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)*x^3+10*RootOf(_Z^3-2)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(R
ootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-9*RootOf(_Z^3-2
)^2*(x^6-2*x^3+1)^(1/3)-18*(x^6-2*x^3+1)^(2/3)+168*RootOf(RootOf(_Z^3-2)^2+
6*_Z*RootOf(_Z^3-2)+36*_Z^2)-7*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x
+1))-1/6*ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Roo
tOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*R
ootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+3
6*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+24*RootOf(_Z^3-2)^2*(x
^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2)^2*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+192*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)*x^3+40*RootOf(_Z^3-2)*x^3-90*(x^6-2*x^3+1)^(1/3)*Root
Of(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)-24*RootOf(_
Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*RootOf(RootOf(_Z^3-
2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2))/(x-1)/(x+1)/(x^2+x+1)/
(x^2-x+1))*RootOf(_Z^3-2)-ln((144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-
2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+30*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6-144*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootO
f(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-30*RootOf(RootOf(_Z^3-2)^2+6*_Z*R
ootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^3-24*RootOf(RootOf(_Z^3-2)^2+6*_Z
*RootOf(_Z^3-2)+36*_Z^2)*x^6-5*RootOf(_Z^3-2)*x^6+90*(x^6-2*x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+24*Ro
otOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)*x^3-90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3-2
)^2*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+192*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+40*RootOf(_Z^3-2)*x^3-90*(x^6-2*
x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-24*RootOf(_Z^3-2)^2*(x^6-2*x^3+1)^(1/3)-48*(x^6-2*x^3+1)^(2/3)-168*Ro
```

```
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-35*RootOf(_Z^3-2))/(x-1)
/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*
_Z^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [A] time = 1.19, size = 97, normalized size = 0.99

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{4}\left(-x^3+1\right)^{\frac{4}{3}}-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/4*(-x^3 + 1)^(4/3) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)
^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 4.98, size = 113, normalized size = 1.15

$$\frac{2^{1/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}+\frac{\left(1-x^3\right)^{4/3}}{4}+\frac{2^{1/3}\,\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{12}-\frac{2^{1/3}\,\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\right.}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((1 - x^3)^(1/3)/2 - 2^(1/3)/2))/6 + (1 - x^3)^(4/3)/4 + (2^(1/
3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/12 - (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^
(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.625 $\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=95

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/
3)+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 80, 57, 617, 204, 31}

$$-\sqrt[3]{1-x^3}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*S
qrt[3]) + Log[1 + x^3]/(6*2^(2/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx, x, x^3\right)$$
$$= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} + \frac{\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+ \cdots}\right.}{2\sqrt[3]{\cdots}}$$
$$= -\sqrt[3]{1-x^3} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}} - \frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3} \cdots\right.}{2^{2/3}}$$
$$= -\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+x^3\right)}{6\ 2^{2/3}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 1.24

$$\frac{1}{12}\left(-12\sqrt[3]{1-x^3} - 2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) + \sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) + 2\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3} \cdots}{\cdots}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-12*(1 - x^3)^(1/3) + 2*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3
))/Sqrt[3]] - 2*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3
) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])/12
```

fricas [A] time = 0.64, size = 130, normalized size = 1.37

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}}\sqrt{3}\right)\right) - \frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 (-x^3 + 1)^(1/3)
```

giac [A] time = 0.23, size = 98, normalized size = 1.03

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}\right) - \frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}} \cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - (-x^3 + 1)^(1/3)
```

maple [C] time = 5.10, size = 1582, normalized size = 16.65

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] (x^3-1)/(-x^3+1)^(2/3)+(-1/6*ln((30*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6+144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(
_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-30*RootOf(RootOf(_Z^3+2)^2+6*_Z*Roo
tOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-144*RootOf(RootOf(_Z^3+2)^2+6*_Z*
RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+5*RootOf(_Z^3+2)*x^6+24*Root
Of(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6+24*RootOf(_Z^3+2)^2*(x
^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^
3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)-40*RootOf(_Z^3+
2)*x^3-192*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3-24*Root
Of(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-48*(x^6-2*x^3+1)^(2/3)+35*
RootOf(_Z^3+2)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(x
-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(_Z^3+2)-ln((30*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6+144*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6-30*RootOf(RootOf(_
Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-144*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+5*RootOf(_Z
^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6+24*Ro
otOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf
(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+90*(x^6-2*x^3+1)
^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)-40*RootOf(_Z^3+2)*x^3-192*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*
_Z^2)*x^3-24*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)-90*(x^6-2*x^3+1)^(1/3)*Ro
otOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-48*(x^6-2
*x^3+1)^(2/3)+35*RootOf(_Z^3+2)+168*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_
Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z
^2)*ln(-(6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3
+2)^3*x^6-144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf
(_Z^3+2)^2*x^6-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootO
f(_Z^3+2)^3*x^3+144*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*
RootOf(_Z^3+2)^2*x^3-3*RootOf(_Z^3+2)*x^6+72*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*x^6-9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3)*x^3+90*(x
^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Root
Of(_Z^3+2)*x^3+90*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2
)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+10*RootOf(_Z^3+2)*x^3-240*RootOf(RootOf(_Z
^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^3+9*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(
1/3)-90*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*
_Z^2)*RootOf(_Z^3+2)+18*(x^6-2*x^3+1)^(2/3)-7*RootOf(_Z^3+2)+168*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2))/(x-1)/(x+1)/(x^2+x+1)/(x^2-x+1)
))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [A] time = 1.38, size = 97, normalized size = 1.02

$$\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) + 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3
)) - 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) - (-x^3 + 1)^(1/3)
```

mupad [B] time = 4.89, size = 113, normalized size = 1.19

$$-\frac{2^{1/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{2}-\frac{2^{1/3}}{2}\right)}{6}-\left(1-x^3\right)^{1/3}-\frac{2^{1/3}\,\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{12}+\frac{2^{1/3}\,\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{\ }\right.}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1
i + 1))/12 - (1 - x^3)^(1/3) - (2^(1/3)*log(3*(1 - x^3)^(1/3) - (3*2^(1/3)*
(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log((1 - x^3)^(1/3)/2
- 2^(1/3)/2))/6
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.626 $\int \frac{x^2}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=83

$$-\frac{\log(x^3+1)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1
/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {444, 57, 617, 204, 31}

$$-\frac{\log(x^3+1)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) - Log[1
+ x^3]/(6*2^(2/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)$$
$$=-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx,x,\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx,x,\sqrt[3]{1}\right.}{2\sqrt[3]{2}}$$
$$=-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}\sqrt[3]{1-x^3}\right)}{2^{2/3}}$$
$$=-\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}$$

Mathematica [A] time = 0.03, size = 94, normalized size = 1.13

$$-\frac{-2\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 2*Log[2^(1/
3) - (1 - x^3)^(1/3)] + Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)])
/2^(2/3)
```

fricas [A] time = 0.55, size = 98, normalized size = 1.18

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{2}{3}}+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1)^(1/3) +
 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*log(4^(2/3)*(-x^3 + 1)^(1/3) + 2*(-x^3 +
1)^(2/3) + 2*4^(1/3)) + 1/12*4^(2/3)*log(-4^(2/3) + 2*(-x^3 + 1)^(1/3))
```

giac [A] time = 0.20, size = 87, normalized size = 1.05

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3)))
```

maple [C] time = 6.28, size = 529, normalized size = 6.37

$$\text{RootOf}\left(36\_Z^2+6\_Z\,\text{RootOf}\left(\_Z^3-2\right)+\text{RootOf}\left(\_Z^3-2\right)^2\right)\ln\left(\frac{144x^3\,\text{RootOf}\left(36\_Z^2+6\_Z\,\text{RootOf}\left(\_Z\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*ln((144*RootOf(RootOf(
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3-6*RootOf(Root
Of(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3-24*RootOf(Ro
otOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+RootOf(_Z^3-
2)^2*x^3+168*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z
^3-2)-252*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/
3)-7*RootOf(_Z^3-2)^2-42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3))/(
x+1)/(x^2-x+1))+1/6*RootOf(_Z^3-2)*ln(-(180*RootOf(RootOf(_Z^3-2)^2+6*_Z*Ro
otOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^3*x^3+6*RootOf(RootOf(_Z^3-2)^2+6*_Z
*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^4*x^3+90*RootOf(RootOf(_Z^3-2)^2+6*
_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)*x^3+3*RootOf(_Z^3-2)^2*x^3-210*Ro
otOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)+252*RootO
f(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*(-x^3+1)^(1/3)-7*RootOf(_Z^
3-2)^2+42*RootOf(_Z^3-2)*(-x^3+1)^(1/3)-42*(-x^3+1)^(2/3))/(x+1)/(x^2-x+1))
```

maxima [A] time = 1.18, size = 86, normalized size = 1.04

$$-\frac{1}{6}\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/12*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/6*2^(1/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
```

mupad [B] time = 5.05, size = 102, normalized size = 1.23

$$\frac{2^{1/3}\ln\left(3\,2^{1/3}-3\left(1-x^3\right)^{1/3}\right)}{6}+\frac{2^{1/3}\ln\left(3\left(1-x^3\right)^{1/3}-\frac{3\,2^{1/3}\left(-1+\sqrt{3}\,1i\right)}{2}\right)\left(-1+\sqrt{3}\,1i\right)}{12}-\frac{2^{1/3}\ln\left(\frac{3\,2^{1/3}\left(1+\sqrt{3}\,1i\right)}{2}+3\left(\right.\right.}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log(3*2^(1/3) - 3*(1 - x^3)^(1/3)))/6 + (2^(1/3)*log(3*(1 - x^3)^(
1/3) - (3*2^(1/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/12 - (2^(1/3)*log(
(3*2^(1/3)*(3^(1/2)*1i + 1))/2 + 3*(1 - x^3)^(1/3))*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.627 $\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=137

$$\frac{\log\left(x^3+1\right)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

```
[Out] -1/2*ln(x)+1/12*ln(x^3+1)*2^(1/3)+1/2*ln(1-(-x^3+1)^(1/3))-1/4*ln(2^(1/3)-(
-x^3+1)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)
+1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {446, 86, 57, 618, 204, 31, 617}

$$\frac{\log\left(x^3+1\right)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1
 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) - Log[x]/2 + Log[1 + x^3]/(6*2^(2
/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (1 - x^3)^(1/3)]/(2*2^(2/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x(1+x)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx,x,x^3\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,x,x^3\right)\\ &= -\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{1-x^3}\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x+\right.\\ &= -\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}-\frac{\operatorname{Subst}\left(\int\right.}{}\\ &= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\log(x)}{2}+\frac{\log\left(1+x^3\right)}{6\,2^{2/3}}+\frac{1}{2}\log\left(1-\sqrt[3]{\right.\end{aligned}$$

Mathematica [A] time = 0.05, size = 179, normalized size = 1.31

$$\frac{1}{12}\left(4\log\left(1-\sqrt[3]{1-x^3}\right)-2\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)+\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3}+\sqrt[3]{2-2x^3}+2^{2/3}\right)-2\log\left(\left(1-x^3\right)^{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]] + 2*2^(1/3)*Sqrt[3]*Arc
Tan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] + 4*Log[1 - (1 - x^3)^(1/3)] - 2
*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] + 2^(1/3)*Log[2^(2/3) + (2 - 2*x^3)
^(1/3) + (1 - x^3)^(2/3)] - 2*Log[1 + (1 - x^3)^(1/3) + (1 - x^3)^(2/3)])/1
2
```

fricas [A] time = 0.75, size = 182, normalized size = 1.33

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+2\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-1)^(2
/3)*(-x^3 + 1)^(1/3) + 4^(1/3)*sqrt(3))) - 1/24*4^(2/3)*(-1)^(1/3)*log(-4^(
2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3) + 2*4^(1/3)*(-1)^(2/3) + 2*(-x^3 + 1)^(2/3
)) + 1/12*4^(2/3)*(-1)^(1/3)*log(4^(2/3)*(-1)^(1/3) + 2*(-x^3 + 1)^(1/3)) -
 1/3*sqrt(3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3)) - 1/6*log((
-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log((-x^3 + 1)^(1/3) - 1)
```

giac [A] time = 0.24, size = 149, normalized size = 1.09

$$\frac{1}{6}\,\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\,\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\left(-x^{3}+1\right)^{\frac{1}{3}}+1\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3
))) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/12*2^(1/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) - 1/6*2^(1/3)
*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/6*log((-x^3 + 1)^(2/3) + (-x^3 +
 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [F] time = 2.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{2}{3}}\left(x^{3}+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x), x)
```

mupad [B] time = 4.92, size = 344, normalized size = 2.51

$$\frac{\ln\left(5-5\left(1-x^{3}\right)^{1/3}\right)}{3}-\frac{2^{1/3}\ln\left(6\left(1-x^{3}\right)^{1/3}-\frac{2^{1/3}\left(\frac{2^{2/3}\left(243\,2^{1/3}+243\left(1-x^{3}\right)^{1/3}\right)}{36}+9\right)}{6}\right)}{6}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right)\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1i}{6}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] log(5 - 5*(1 - x^3)^(1/3))/3 - (2^(1/3)*log(6*(1 - x^3)^(1/3) - (2^(1/3)*((
2^(2/3)*(243*2^(1/3) + 243*(1 - x^3)^(1/3)))/36 + 9))/6))/6 + log(((3^(1/2)
```

```
*1i)/6 - 1/6)*(((3^(1/2)*1i)/6 - 1/6)^2*(243*(1 - x^3)^(1/3) - 3^(1/2)*243i
 + 243) + 9) + 6*(1 - x^3)^(1/3))*((3^(1/2)*1i)/6 - 1/6) - log(6*(1 - x^3)^
(1/3) - ((3^(1/2)*1i)/6 + 1/6)*(((3^(1/2)*1i)/6 + 1/6)^2*(3^(1/2)*243i + 24
3*(1 - x^3)^(1/3) + 243) + 9))*((3^(1/2)*1i)/6 + 1/6) + ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(((-1)^(2/3)*2^(2/3)*(243*(-1)
^(1/3)*2^(1/3) - 243*(1 - x^3)^(1/3)))/36 - 9))/6))/6 - ((-1)^(1/3)*2^(1/3)
*log(6*(1 - x^3)^(1/3) - ((-1)^(1/3)*2^(1/3)*(3^(1/2)*1i + 1)*(((-1)^(2/3)*
2^(2/3)*(3^(1/2)*1i + 1)^2*(243*(1 - x^3)^(1/3) + (243*(-1)^(1/3)*2^(1/3)*(
3^(1/2)*1i + 1))/2))/144 + 9))/12)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.628 $\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=158

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -1/3*(-x^3+1)^(1/3)/x^3+1/6*ln(x)-1/12*ln(x^3+1)*2^(1/3)-1/6*ln(1-(-x^3+1)^
(1/3))+1/4*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1+2*(-x^3+1)^
(1/3))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 103, 156, 57, 618, 204, 31, 617}

$$-\frac{\sqrt[3]{1-x^3}}{3x^3}-\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(1/3)/(3*x^3) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/(3*Sqrt[
3]) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log
[x]/6 - Log[1 + x^3]/(6*2^(2/3)) - Log[1 - (1 - x^3)^(1/3)]/6 + Log[2^(1/3)
 - (1 - x^3)^(1/3)]/(2*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x^2(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{\frac{1}{3}-\frac{2x}{3}}{(1-x)^{2/3}x(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} - \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}x}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx,\right.$$

$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{1}{6}\operatorname{Subs}$$

$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} - \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{2\,2^{2/3}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{3x^3} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log(x)}{6} - \frac{\log\left(1+x^3\right)}{6\,2^{2/3}} -$$

Mathematica [A] time = 0.10, size = 196, normalized size = 1.24

$$\frac{1}{36}\left(-\frac{12\sqrt[3]{1-x^3}}{x^3} - 4\log\left(1-\sqrt[3]{1-x^3}\right) + 6\sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 3\sqrt[3]{2}\log\left(\left(1-x^3\right)^{2/3} + \sqrt[3]{2-2x^3} + 2^{2/3}\right) -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((-12*(1 - x^3)^(1/3))/x^3 + 4*Sqrt[3]*ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[
3]] - 6*2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]] - 4*L
og[1 - (1 - x^3)^(1/3)] + 6*2^(1/3)*Log[2^(1/3) - (1 - x^3)^(1/3)] - 3*2^(1
/3)*Log[2^(2/3) + (2 - 2*x^3)^(1/3) + (1 - x^3)^(2/3)] + 2*Log[1 + (1 - x^3
)^(1/3) + (1 - x^3)^(2/3)])/36
```

fricas [A] time = 0.60, size = 195, normalized size = 1.23

$$-\frac{12 \cdot 4^{\frac{1}{6}} \sqrt{3} x^3 \arctan\left(\frac{1}{6} \cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}} \sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}} + 4^{\frac{1}{3}} \sqrt{3}\right)\right) + 3 \cdot 4^{\frac{2}{3}} x^3 \log\left(4^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{2}{3}} + 2 \cdot 4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(12*4^(1/6)*sqrt(3)*x^3*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*(-x^3 + 1
)^(1/3) + 4^(1/3)*sqrt(3))) + 3*4^(2/3)*x^3*log(4^(2/3)*(-x^3 + 1)^(1/3) +
2*(-x^3 + 1)^(2/3) + 2*4^(1/3)) - 6*4^(2/3)*x^3*log(-4^(2/3) + 2*(-x^3 + 1)
^(1/3)) - 8*sqrt(3)*x^3*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/3*sqrt(3))
- 4*x^3*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 8*x^3*log((-x^3 + 1)
^(1/3) - 1) + 24*(-x^3 + 1)^(1/3))/x^3
```

giac [A] time = 0.22, size = 163, normalized size = 1.03

$$-\frac{1}{6} \sqrt{3}\, 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3}\, 2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) + \frac{1}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}\left(2\left(-x^3+1\right)^{\frac{1}{3}} + 1\right)\right) - \frac{1}{12} \cdot 2^{\frac{1}{3}} \log\left(2^{\frac{2}{3}} + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/12*2^(1
/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/6*2^(1/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) - 1/3*(-x^3 + 1)^(1/3)/x^3 + 1/18*l
og((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 1/9*log(abs((-x^3 + 1)^(1/3)
- 1))
```

maple [F] time = 3.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}} \left(x^3+1\right) x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^4), x)
```

mupad [B] time = 5.07, size = 368, normalized size = 2.33

$$\frac{2^{1/3}\,\ln\left(\frac{10\,{\left(1-x^3\right)}^{1/3}}{9}-\frac{2^{1/3}\,\left(\frac{2^{2/3}\,\left(243\,2^{1/3}+27\,{\left(1-x^3\right)}^{1/3}\right)}{36}-\frac{25}{3}\right)}{6}\right)}{6}-\frac{{\left(1-x^3\right)}^{1/3}}{3\,x^3}-\frac{\ln\left(\frac{31\,{\left(1-x^3\right)}^{1/3}}{243}-\frac{31}{243}\right)}{9}-\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1i}{18}\right)\left(\left(-\frac{1}{18}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*((2^(2/3)*(243*2^(1/3) + 27*
(1 - x^3)^(1/3)))/36 - 25/3))/6))/6 - (1 - x^3)^(1/3)/(3*x^3) - log((31*(1
- x^3)^(1/3))/243 - 31/243)/9 - log((((3^(1/2)*1i)/18 - 1/18)*(((3^(1/2)*1i)
/18 - 1/18)^2*(27*(1 - x^3)^(1/3) - 3^(1/2)*81i + 81) - 25/3) + (10*(1 - x^
3)^(1/3))/9)*((3^(1/2)*1i)/18 - 1/18) + log((10*(1 - x^3)^(1/3))/9 - ((3^(1
/2)*1i)/18 + 1/18)*(((3^(1/2)*1i)/18 + 1/18)^2*(3^(1/2)*81i + 27*(1 - x^3)^
(1/3) + 81) - 25/3))*((3^(1/2)*1i)/18 + 1/18) + (2^(1/3)*log((10*(1 - x^3)^
(1/3))/9 - (2^(1/3)*(3^(1/2)*1i - 1)*((2^(2/3)*(3^(1/2)*1i - 1)^2*((243*2^(
1/3)*(3^(1/2)*1i - 1))/2 + 27*(1 - x^3)^(1/3)))/144 - 25/3))/12)*(3^(1/2)*1
i - 1))/12 - (2^(1/3)*log((10*(1 - x^3)^(1/3))/9 - (2^(1/3)*(3^(1/2)*1i + 1
)*((2^(2/3)*(3^(1/2)*1i + 1)^2*((243*2^(1/3)*(3^(1/2)*1i + 1))/2 - 27*(1 -
x^3)^(1/3)))/144 + 25/3))/12)*(3^(1/2)*1i + 1))/12
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(-\left(x-1\right) \left(x^{2}+x+1\right)\right)^{\frac{2}{3}} \left(x+1\right) \left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.629 $\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=160

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}+\frac{1}{6}\log\left(-\sqrt[3]{1-x^3}-x\right)-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\ 2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{1}{3}\sqrt[3]{1-x^3}$$

```
[Out] -1/3*x^2*(-x^3+1)^(1/3)+1/12*ln(x^3+1)*2^(1/3)+1/6*ln(-x-(-x^3+1)^(1/3))-1/
4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)+1/9*arctan(1/3*(1-2*x/(-x^3+1)^(1/3
))*3^(1/2))*3^(1/2)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*
2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 228, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {494, 470, 584, 634, 618, 204, 628, 292, 31, 617}

$$-\frac{1}{3}\sqrt[3]{1-x^3}\,x^2-\frac{1}{18}\log\left(\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{1}{9}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\ldots\right.}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(x^2*(1 - x^3)^(1/3))/3 + ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(3*S
qrt[3]) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt
[3]) - Log[1 + x^2/(1 - x^3)^(2/3) - x/(1 - x^3)^(1/3)]/18 + Log[1 + x/(1 -
 x^3)^(1/3)]/9 + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x
^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^7}{\left(1+x^3\right)^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int \frac{x\left(2+x^3\right)}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{3}\text{Subst}\left(\int\left(\frac{1}{3(1+x)} + \frac{-1-x}{3\left(1-x+x^2\right)} + \frac{3x}{1+2x^3}\right)dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\text{Subst}\left(\int \frac{-1-x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) +$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log}{}$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{1}{9}\log\left(\right.$$

$$= -\frac{1}{3}x^2\sqrt[3]{1-x^3} + \frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{1}{18}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}} - \right.$$

Mathematica [C] time = 0.09, size = 78, normalized size = 0.49

$$\frac{1}{15}x^2\left(x^3\left(-F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)\right) + \frac{5\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{\left(x^3+1\right)^{2/3}} - 5\sqrt[3]{1-x^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(-5*(1 - x^3)^(1/3) - x^3*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3] + (5*H
ypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(1 + x^3)^(2/3)))/15
```

fricas [A] time = 0.64, size = 232, normalized size = 1.45

$$-\frac{1}{3}\left(-x^3+1\right)^{\frac{1}{3}}x^2 + \frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\arctan\left(\frac{4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(-x^3+1\right)^{\frac{1}{3}} - 4^{\frac{1}{3}}\sqrt{3}\,x\right)}{6\,x}\right) + \frac{1}{12}\cdot 4^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\log\left(-\frac{4^{\frac{2}{3}}\,(-1)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*(-x^3 + 1)^(1/3)*x^2 + 1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/
6)*(4^(2/3)*sqrt(3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 4^(1/3)*sqrt(3)*x)/x) + 1
/12*4^(2/3)*(-1)^(1/3)*log(-(4^(2/3)*(-1)^(1/3)*x - 2*(-x^3 + 1)^(1/3))/x)
```

```
- 1/24*4^(2/3)*(-1)^(1/3)*log((2*4^(1/3)*(-1)^(2/3)*x^2 + 4^(2/3)*(-1)^(1/3
)*(-x^3 + 1)^(1/3)*x + 2*(-x^3 + 1)^(2/3))/x^2) + 1/9*sqrt(3)*arctan(-1/3*(
sqrt(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 1/9*log((x + (-x^3 + 1)^(1/3))
/x) - 1/18*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [F] time = 2.04, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(-x^3+1)^{\frac{2}{3}}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(x^7/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(2/3)/(x**3+1),x)
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.630 $\int \frac{x^4}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal. Leaf size=139

$$-\frac{\log(x^3+1)}{6\ 2^{2/3}}-\frac{1}{2}\log\left(-\sqrt[3]{1-x^3}-x\right)+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*ln(x^3+1)*2^(1/3)-1/2*ln(-x-(-x^3+1)^(1/3))+1/4*ln(-2^(1/3)*x-(-x^3+1
)^(1/3))*2^(1/3)-1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/6
*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 207, normalized size of antiderivative = 1.49, number of steps used = 14, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {494, 481, 292, 31, 634, 618, 204, 628, 617}

$$\frac{1}{6}\log\left(\frac{x^2}{(1-x^3)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{1}{3}\log\left(\frac{x}{\sqrt[3]{1-x^3}}+1\right)-\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*x)/(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3]) + ArcTan[(1 - (2*2^(
1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]) + Log[1 + x^2/(1 - x^3)
^(2/3) - x/(1 - x^3)^(1/3)]/6 - Log[1 + x/(1 - x^3)^(1/3)]/3 - Log[1 + (2^(
2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(2/3)) + Log[
1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x^4}{\left(1+x^3\right)\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right) - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\,2^{2/3}} + \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right.$$

$$= \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log\left(1+\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}} - \right.}{6\,2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{-1+\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{1}{6}\log\left(1+\frac{x^2}{\left(1-x^3\right)^{2/3}}-\frac{x}{\sqrt[3]{1-x^3}}\right) - \frac{1}{3}\log$$

Mathematica [C] time = 0.05, size = 26, normalized size = 0.19

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(2/3)*(1 + x^3)),x]
[Out] (x^5*AppellF1[5/3, 2/3, 1, 8/3, x^3, -x^3])/5
```

fricas [A] time = 0.68, size = 197, normalized size = 1.42

$$\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(4^{\frac{1}{3}}\sqrt{3}x-4^{\frac{2}{3}}\sqrt{3}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}x+2\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(\frac{2\cdot 4^{\frac{1}{3}}x^2-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] 1/6*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(4^(1/3)*sqrt(3)*x - 4^(2/3)*sqrt(3
)*(-x^3 + 1)^(1/3))/x) + 1/12*4^(2/3)*log((4^(2/3)*x + 2*(-x^3 + 1)^(1/3))/
x) - 1/24*4^(2/3)*log((2*4^(1/3)*x^2 - 4^(2/3)*(-x^3 + 1)^(1/3)*x + 2*(-x^3
 + 1)^(2/3))/x^2) - 1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(-x^3 +
1)^(1/3))/x) - 1/3*log((x + (-x^3 + 1)^(1/3))/x) + 1/6*log((x^2 - (-x^3 + 1
)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{2}{3}} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^4/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\left(1-x^3\right)^{2/3} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^4/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}} (x+1)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.631 $\int \frac{x}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=88

$$\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] 1/12*ln(x^3+1)*2^(1/3)-1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 122, normalized size of antiderivative = 1.39, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{2^{2/3}x^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3])) +
Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx\right.}{2\sqrt[3]{2}}$$

$$= \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.43

$$\frac{x^2\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{2x^3}{x^3+1}\right)}{2\left(x^3+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^2*Hypergeometric2F1[2/3, 2/3, 5/3, (2*x^3)/(1 + x^3)])/(2*(1 + x^3)^(2/3
))
```

fricas [B] time = 3.02, size = 283, normalized size = 3.22

$$-\frac{1}{18} \cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}} \arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(19\,x^8-16\,x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-12\sqrt{3}\,(-1)^{\frac{1}{3}}\left(5\,x^7+4\,x^4-x\right)\left(-x\right.\right.}{6\left(109\,x^9-105\,x^6+3\,x^3+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/18*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(-1
)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(1/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) + 1/36*4^(2/3)*(-1)^(1/3)*log(-
(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^3 + 1) -
 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) - 1/72*4^(2/3)*(-1)^(1/3)*log((6*4^(1/3)*
(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6 - 16*x
^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [C] time = 4.24, size = 938, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/6*RootOf(_Z^3+2)*ln((54*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z
^2)^2*RootOf(_Z^3+2)^3*x^3+6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36
*_Z^2)*RootOf(_Z^3+2)^4*x^3-12*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-27*RootOf(RootOf(_Z^3+2)^2+6*_Z*
RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3-3*x^3*RootOf(_Z^3+2)^2+6*RootOf(
RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-4*RootOf(_
Z^3+2)*(-x^3+1)^(1/3)*x^2+5*(-x^3+1)^(2/3)*x+9*RootOf(RootOf(_Z^3+2)^2+6*_Z
*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)+RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))-
1/6*ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_
Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf
(_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2
)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x^3
+1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^
3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(_Z
^3+2)-ln(-(72*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf
(_Z^3+2)^3*x^3+18*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*Root
Of(_Z^3+2)^4*x^3-24*(-x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2
+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)*x^3+3*x^3*RootOf(_Z^3+2)^2-60*RootOf(RootOf(_Z^
3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*(-x^3+1)^(1/3)*x^2-8*RootOf(_Z^3+2)*(-x
^3+1)^(1/3)*x^2-2*(-x^3+1)^(2/3)*x-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_
Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)-3*RootOf(_Z^3+2)^2)/(x+1)/(x^2-x+1))*RootOf(
RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.632 $\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(x^3+1\right)}{6\,2^{2/3}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{2\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

```
[Out] -(-x^3+1)^(1/3)/x-1/12*ln(x^3+1)*2^(1/3)+1/4*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*
2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 137, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{1-x^3}}{x}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -((1 - x^3)^(1/3)/x) + ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]/
(2^(2/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/(1
- x^3)^(1/3)]/(6*2^(2/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx = \text{Subst}\left(\int \frac{1+x^3}{x^2\left(1+2x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \text{Subst}\left(\int \frac{x}{1+2x^3}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{3\sqrt[3]{2}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{\cdots}{1\cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, \cdots\right.}{2^{2/3}}$$

$$= -\frac{\sqrt[3]{1-x^3}}{x} + \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} - \frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}} + \frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 0.55, size = 81, normalized size = 0.79

$$-\frac{5\left(-3x^6+x^3+2\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{2x^3}{x^3-1}\right)-12x^3\left(x^3+1\right)\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{2x^3}{x^3-1}\right)}{10x\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/10*(5*(2 + x^3 - 3*x^6)*Hypergeometric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3
)] - 12*x^3*(1 + x^3)*Hypergeometric2F1[5/3, 2, 8/3, (2*x^3)/(-1 + x^3)])/(
x*(1 - x^3)^(5/3))
```

fricas [B] time = 2.65, size = 272, normalized size = 2.64

$$4 \cdot 4^{\frac{1}{6}} \sqrt{3} x \arctan\left(\frac{4^{\frac{1}{6}} \left(6 \cdot 4^{\frac{2}{3}} \sqrt{3} \left(19 x^{8}-16 x^{5}+x^{2}\right) \left(-x^{3}+1\right)^{\frac{1}{3}}+12 \sqrt{3} \left(5 x^{7}+4 x^{4}-x\right) \left(-x^{3}+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}} \sqrt{3} \left(71 x^{9}-111 x^{6}+33 x^{3}-1\right)\right)}{6 \left(109 x^{9}-105 x^{6}+3 x^{3}+1\right)}\right) + 2 \cdot 4^{\frac{2}{3}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(4*4^(1/6)*sqrt(3)*x*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(19*x^8 - 1
6*x^5 + x^2)*(-x^3 + 1)^(1/3) + 12*sqrt(3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(
2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6
+ 3*x^3 + 1)) + 2*4^(2/3)*x*log((3*4^(2/3)*(-x^3 + 1)^(1/3)*x^2 + 6*(-x^3 +
 1)^(2/3)*x + 4^(1/3)*(x^3 + 1))/(x^3 + 1)) - 4^(2/3)*x*log((6*4^(1/3)*(5*x
^4 - x)*(-x^3 + 1)^(2/3) + 4^(2/3)*(19*x^6 - 16*x^3 + 1) - 24*(2*x^5 - x^2)
*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 72*(-x^3 + 1)^(1/3))/x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}} x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

maple [C] time = 4.06, size = 1387, normalized size = 13.47

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] (x^3-1)/x/(-x^3+1)^(2/3)+(-1/6*ln((18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_
Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootO
f(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2-9*(x^6-2*x^3+1
)^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)
*x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-
2)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3
-2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf
(_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)
*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^
3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootO
f(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x
+1)/(x^2-x+1))*RootOf(_Z^3-2)-ln((18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z
^3-2)+36*_Z^2)^2*RootOf(_Z^3-2)^2*x^6+6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf
(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^3*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(
```

```
_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2-9*(x^6-2*x^3+1)
^(1/3)*RootOf(_Z^3-2)*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^4-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*RootOf(_Z^3-2
)^2*x^3-6*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-
2)^3*x^3-3*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^6-RootOf(
_Z^3-2)*x^6+3*(x^6-2*x^3+1)^(2/3)*x^2+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)*
RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x+6*RootOf(RootOf(_Z^3
-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+2*RootOf(_Z^3-2)*x^3-3*RootOf(RootOf
(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-RootOf(_Z^3-2))/(x-1)/(x^2+x+1)/(x+
1)/(x^2-x+1))*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)+1/6*Root
Of(_Z^3-2)*ln(-(-36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*
RootOf(_Z^3-2)^2*x^6-12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2
)*RootOf(_Z^3-2)^3*x^6+18*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+6*_Z*
RootOf(_Z^3-2)+36*_Z^2)*RootOf(_Z^3-2)^2*x^2+3*(x^6-2*x^3+1)^(1/3)*RootOf(_
Z^3-2)^2*x^4+36*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)^2*Root
Of(_Z^3-2)^2*x^3+12*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*Ro
otOf(_Z^3-2)^3*x^3-18*RootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*
x^6-6*RootOf(_Z^3-2)*x^6-3*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x+24*RootOf
(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)*x^3+8*RootOf(_Z^3-2)*x^3-6*R
ootOf(RootOf(_Z^3-2)^2+6*_Z*RootOf(_Z^3-2)+36*_Z^2)-2*RootOf(_Z^3-2))/(x-1)
/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^2*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.633 $\int \frac{1}{x^5(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=124

$$\frac{\sqrt[3]{1-x^3}}{4x}+\frac{\log\left(x^3+1\right)}{6\ 2^{2/3}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{2\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}-\frac{\sqrt[3]{1-x^3}}{4x^4}$$

```
[Out] -1/4*(-x^3+1)^(1/3)/x^4+1/4*(-x^3+1)^(1/3)/x+1/12*ln(x^3+1)*2^(1/3)-1/4*ln(
-2^(1/3)*x-(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(
1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$-\frac{\left(1-x^3\right)^{4/3}}{4x^4}+\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -(1 - x^3)^(4/3)/(4*x^4) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[
3]]/(2^(2/3)*Sqrt[3]) + Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)
/(1 - x^3)^(1/3)]/(6*2^(2/3)) - Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(3*2^(
2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5 \left(1-x^3\right)^{2/3} \left(1+x^3\right)} \, dx = \text{Subst}\left(\int \frac{\left(1+x^3\right)^2}{x^5 \left(1+2 x^3\right)} \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{x^5}+\frac{x}{1+2 x^3}\right) \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4 x^4}+\text{Subst}\left(\int \frac{x}{1+2 x^3} \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4 x^4}-\frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{2} x} \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)}{3 \sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1+\sqrt[3]{2} x}{1-\sqrt[3]{2} x+2^{2/3} x^2} \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)}{3 \sqrt[3]{2}}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4 x^4}-\frac{\log \left(1+\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{2}+2\ 2^{2/3} x}{1-\sqrt[3]{2} x+2^{2/3} x^2} \, dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4 x^4}+\frac{\log \left(1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log \left(1+\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2} \, dx\right.}{2^{2/3}}$$

$$= -\frac{\left(1-x^3\right)^{4/3}}{4 x^4}-\frac{\tan ^{-1}\left(\frac{1-\frac{2 \sqrt[3]{2} x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3}}+\frac{\log \left(1+\frac{2^{2/3} x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log \left(1+\frac{\sqrt[3]{2} x}{\sqrt[3]{1-x^3}}\right)}{3\ 2^{2/3}}$$

Mathematica [C] time = 13.90, size = 145, normalized size = 1.17

$$-\frac{81\left(x^3+1\right)^2 x^3\, {}_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{8}{3};\frac{2x^3}{x^3-1}\right)+216\left(x^9+x^6\right) {}_3F_2\left(\frac{2}{3},2,2;1,\frac{8}{3};\frac{2x^3}{x^3-1}\right)+5\left(\left(9x^9-20x^6-13x^3+4\right.\right.}{60x^4\left(1-x^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -1/60*(5*(-1 - 9*x^3 + x^6 + 9*x^9 + (4 - 13*x^3 - 20*x^6 + 9*x^9)*Hypergeo
metric2F1[2/3, 1, 5/3, (2*x^3)/(-1 + x^3)]) + 216*(x^6 + x^9)*Hypergeometri
cPFQ[{2/3, 2, 2}, {1, 8/3}, (2*x^3)/(-1 + x^3)] + 81*x^3*(1 + x^3)^2*Hyperg
eometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, (2*x^3)/(-1 + x^3)])/(x^4*(1 - x^3
)^(5/3))
```

fricas [B] time = 3.02, size = 312, normalized size = 2.52

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^4\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(19\,x^8-16\,x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-12\,\sqrt{3}\,(-1)^{\frac{1}{3}}\left(5\,x^7+4\,x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-4^{\frac{1}{3}}\sqrt{3}\left(71\,x^9-111\,x^6\right.\right.}{6\left(109\,x^9-105\,x^6+3\,x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^4*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - 12*sqrt(3)*(-1)^(
1/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(71*x^9 - 111*x
^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x
^4*log(-(3*4^(2/3)*(-1)^(1/3)*(-x^3 + 1)^(1/3)*x^2 - 4^(1/3)*(-1)^(2/3)*(x^
3 + 1) - 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 4^(2/3)*(-1)^(1/3)*x^4*log((6*4
^(1/3)*(-1)^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(19*x^6
 - 16*x^3 + 1) - 24*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) - 18
*(x^3 - 1)*(-x^3 + 1)^(1/3))/x^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}} x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

maple [C] time = 3.51, size = 1386, normalized size = 11.18

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] -1/4*(x^6-2*x^3+1)/x^4/(-x^3+1)^(2/3)+(-1/6*ln((18*RootOf(RootOf(_Z^3+2)^2+
6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+6*RootOf(RootOf(_Z^3+2)
^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-18*RootOf(RootOf(_Z^3+
2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z
^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^(
```

```
2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*
x^2-9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)*x^4+3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^
2)*x^6+RootOf(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootO
f(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*R
ootOf(_Z^3+2)+36*_Z^2)*x^3-2*RootOf(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^(2/3)*x^2+3
*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(_Z^3+2))/(x-1)
/(x^2+x+1)/(x+1)/(x^2-x+1))*RootOf(_Z^3+2)-ln((18*RootOf(RootOf(_Z^3+2)^2+6
*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+6*RootOf(RootOf(_Z^3+2)^
2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-18*RootOf(RootOf(_Z^3+2
)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-6*RootOf(RootOf(_Z^
3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3-9*(x^6-2*x^3+1)^(2
/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x
^2-9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf
(_Z^3+2)+36*_Z^2)*x^4+3*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2
)*x^6+RootOf(_Z^3+2)*x^6+9*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)*RootOf(RootOf
(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*Ro
otOf(_Z^3+2)+36*_Z^2)*x^3-2*RootOf(_Z^3+2)*x^3+3*(x^6-2*x^3+1)^(2/3)*x^2+3*
RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)+RootOf(_Z^3+2))/(x-1)/
(x^2+x+1)/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_
Z^2)+1/6*RootOf(_Z^3+2)*ln((36*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+
36*_Z^2)^2*RootOf(_Z^3+2)^2*x^6+12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3
+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^6-36*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z
^3+2)+36*_Z^2)^2*RootOf(_Z^3+2)^2*x^3-12*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootO
f(_Z^3+2)+36*_Z^2)*RootOf(_Z^3+2)^3*x^3+18*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+
2)^2*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^2-18*RootOf(Roo
tOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)*x^6-3*(x^6-2*x^3+1)^(1/3)*RootOf
(_Z^3+2)^2*x^4-6*RootOf(_Z^3+2)*x^6+24*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(
_Z^3+2)+36*_Z^2)*x^3+3*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3+2)^2*x+8*RootOf(_Z^3
+2)*x^3-6*RootOf(RootOf(_Z^3+2)^2+6*_Z*RootOf(_Z^3+2)+36*_Z^2)-2*RootOf(_Z^
3+2))/(x-1)/(x^2+x+1)/(x+1)/(x^2-x+1)))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{2}{3}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/(x^5*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.634 $\int \frac{x^6}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=291

$$-\frac{1}{2}\sqrt[3]{1-x^3}\,x+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\cdots\right)}{12\,2^{2/3}}$$

```
[Out] -1/2*x*(-x^3+1)^(1/3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)-1/12*l
n(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/
6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1/3)+(1-x)^2/(-x^
3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan(1/3*(1-2*2^(1/3
)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arctan(1/3*(1+2^(1/3)
*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 2/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.16, size = 115, normalized size = 0.40

$$\frac{1}{2}x\sqrt[3]{1-x^3}\left(-\frac{4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{\left(x^3+1\right)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x*(1 - x^3)^(1/3)*(-1 - (4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 + x
^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/3,
 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/2
```

fricas [A] time = 3.16, size = 356, normalized size = 1.22

$$\frac{1}{36}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(-\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{1}{3}}-48\sqrt{3}\left(x^{14}-2\,x^{11}\right.\right.}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/36*4^(1/6)*sqrt(3)*arctan(-1/6*4^(1/6)*(6*4^(2/3)*sqrt(3)*(x^16 - 33*x^13
 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1)^(1/3) - 48*sqrt(3)*(x^14 - 2
*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*
x^15 - 417*x^12 + 812*x^9 - 417*x^6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x
^12 - 628*x^9 + 447*x^6 - 102*x^3 + 1)) + 1/72*4^(2/3)*log(-(12*(-x^3 + 1)^
(2/3)*x^2 - 3*4^(2/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(x^6 + 2*x^3 + 1
))/(x^6 + 2*x^3 + 1)) - 1/144*4^(2/3)*log((24*4^(1/3)*(x^8 - 4*x^5 + x^2)*(
-x^3 + 1)^(2/3) + 4^(2/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 + 1) + 12*(x^10
- 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x^6 + 4*x^3 + 1)
) - 1/2*(-x^3 + 1)^(1/3)*x
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [C] time = 14.51, size = 696, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/2*x*(x^3-1)/(-x^3+1)^(2/3)+(1/12*RootOf(_Z^3-2)*ln((6*RootOf(RootOf(_Z^3-
2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^3+36*RootOf(RootOf(_Z^3
-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2*x^3-RootOf(_Z^3-2)*x^6
-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^6-9*(x^6-2*x^3+1)^
(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^2*
x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2+6*RootOf(_Z^3-2)*x^3+36*RootOf
(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x-R
ootOf(_Z^3-2)-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2))/(x+1)^
2/(x^2-x+1)^2)+1/4*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*ln(-
(12*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)*RootOf(_Z^3-2)^3*x^
3+18*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_Z^2)^2*RootOf(_Z^3-2)^2
*x^3+2*RootOf(_Z^3-2)*x^6+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+9*_
Z^2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2)+
9*_Z^2)*RootOf(_Z^3-2)^2*x-6*(x^6-2*x^3+1)^(1/3)*RootOf(_Z^3-2)^2*x^2-18*Ro
otOf(_Z^3-2)*(x^6-2*x^3+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z^3-2
)+9*_Z^2)*x^2-4*RootOf(_Z^3-2)*x^3-6*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z
^3-2)+9*_Z^2)*x^3+2*RootOf(_Z^3-2)+3*RootOf(RootOf(_Z^3-2)^2+3*_Z*RootOf(_Z
^3-2)+9*_Z^2))/(x+1)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(x^3+1)(-x^3+1)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{(1-x^3)^{2/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.635 $\int \frac{x^3}{(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=294

$$\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\,2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\,2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\,2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2}\ldots}{\ldots}\right)}{12\,2^{2\ldots}}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)+1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)-1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.04, size = 26, normalized size = 0.09

$$\frac{1}{4}x^4F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (x^4*AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])/4
```

fricas [F] time = 3.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}x^3}{x^6-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral(-(-x^3 + 1)^(1/3)*x^3/(x^6 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] int(x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(x^3/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.636 $\int \frac{1}{(1-x^3)^{2/3}(1+x^3)} dx$

Optimal. Leaf size=293

$$\frac{1}{2}x\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};x^3\right)+\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}-\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2}}{}\right.}{12\ 2^{2}}$$

```
[Out] 1/2*x*hypergeom([1/3, 2/3],[4/3],x^3)+1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3)
)*2^(1/3)-1/12*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(
1/3))*2^(1/3)+1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/24*ln(2*2^(1
/3)+(1-x)^2/(-x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/6*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)+1/12*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 0.07, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(1-x^3)^{2/3}(1+x^3)}\,dx = xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.21, size = 111, normalized size = 0.38

$$-\frac{4xF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)}{(1-x^3)^{2/3}(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};x^3,-x^3\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] (-4*x*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3])/((1 - x^3)^(2/3)*(1 + x^3)*(-4
*AppellF1[1/3, 2/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, 2/3, 2, 7/3,
x^3, -x^3] - 2*AppellF1[4/3, 5/3, 1, 7/3, x^3, -x^3])))
```

fricas [F] time = 3.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(-x^3+1\right)^{\frac{1}{3}}}{x^6-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
[Out] integral(-(-x^3 + 1)^(1/3)/(x^6 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

maple [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{2}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
[Out] int(1/(-x^3+1)^(2/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(1-x^3\right)^{2/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(2/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{2}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.637 $\int \frac{1}{x^3(1-x^3)^{2/3}(1+x^3)}\,dx$

Optimal. Leaf size=294

$$-\frac{\log\left(2^{2/3}-\frac{1-x}{\sqrt[3]{1-x^3}}\right)}{6\ 2^{2/3}}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\ 2^{2/3}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{3\ 2^{2/3}}+\frac{\log\left(\frac{(1-x)^2}{(1-x^3)^{2/3}}+\frac{2^{2/3}(1-x)}{\sqrt[3]{1-x^3}}+2\sqrt[3]{2}\right)}{12\ 2^{2/3}}-\frac{\tan^{-1}\left(\cdots\right.}{2\cdots}$$

```
[Out] -1/2*(-x^3+1)^(1/3)/x^2-1/12*ln(2^(2/3)+(-1+x)/(-x^3+1)^(1/3))*2^(1/3)+1/12
*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-
1/6*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)+1/24*ln(2*2^(1/3)+(1-x)^2/(-
x^3+1)^(2/3)+2^(2/3)*(1-x)/(-x^3+1)^(1/3))*2^(1/3)-1/6*arctan(1/3*(1-2*2^(1
/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)-1/12*arctan(1/3*(1+2^(1/
3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(1/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-2/3, 2/3, 1, 1/3, x^3, -x^3]/(2*x^2)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{2/3}\left(1+x^3\right)}\,dx=-\frac{F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};x^3,-x^3\right)}{2x^2}$$

Mathematica [C] time = 0.10, size = 120, normalized size = 0.41

$$\frac{\sqrt[3]{1-x^3}\left(\frac{4x^3F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)}{(x^3+1)\left(x^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};x^3,-x^3\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};x^3,-x^3\right)\right)-4F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};x^3,-x^3\right)\right)}-1\right)}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(2/3)*(1 + x^3)),x]
```

```
[Out] ((1 - x^3)^(1/3)*(-1 + (4*x^3*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3])/((1 +
 x^3)*(-4*AppellF1[1/3, -1/3, 1, 4/3, x^3, -x^3] + x^3*(3*AppellF1[4/3, -1/
3, 2, 7/3, x^3, -x^3] + AppellF1[4/3, 2/3, 1, 7/3, x^3, -x^3])))))/(2*x^2)
```

fricas [A] time = 3.63, size = 396, normalized size = 1.35

$$-\frac{4\cdot 4^{\frac{1}{6}}\sqrt{3}\,(-1)^{\frac{1}{3}}\,x^2\arctan\left(\frac{4^{\frac{1}{6}}\left(6\cdot 4^{\frac{2}{3}}\sqrt{3}\,(-1)^{\frac{2}{3}}\left(x^{16}-33\,x^{13}+110\,x^{10}-110\,x^{7}+33\,x^{4}-x\right)\left(-x^3+1\right)^{\frac{1}{3}}+48\,\sqrt{3}\,(-1)^{\frac{1}{3}}\left(x^{14}-2\,x^{11}-6\,x^{8}-2\,x^{5}+x^{2}\right)\right.}{6\left(x^{18}-102\,x^{15}+447\,x^{12}-628\,x^{9}+447\,x^{6}-102\,x\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/144*(4*4^(1/6)*sqrt(3)*(-1)^(1/3)*x^2*arctan(1/6*4^(1/6)*(6*4^(2/3)*sqrt
(3)*(-1)^(2/3)*(x^16 - 33*x^13 + 110*x^10 - 110*x^7 + 33*x^4 - x)*(-x^3 + 1
)^(1/3) + 48*sqrt(3)*(-1)^(1/3)*(x^14 - 2*x^11 - 6*x^8 - 2*x^5 + x^2)*(-x^3
 + 1)^(2/3) - 4^(1/3)*sqrt(3)*(x^18 + 42*x^15 - 417*x^12 + 812*x^9 - 417*x^
6 + 42*x^3 + 1))/(x^18 - 102*x^15 + 447*x^12 - 628*x^9 + 447*x^6 - 102*x^3
+ 1)) + 4^(2/3)*(-1)^(1/3)*x^2*log((24*4^(1/3)*(-1)^(2/3)*(x^8 - 4*x^5 + x^
2)*(-x^3 + 1)^(2/3) - 4^(2/3)*(-1)^(1/3)*(x^12 - 32*x^9 + 78*x^6 - 32*x^3 +
 1) + 12*(x^10 - 11*x^7 + 11*x^4 - x)*(-x^3 + 1)^(1/3))/(x^12 + 4*x^9 + 6*x
^6 + 4*x^3 + 1)) - 2*4^(2/3)*(-1)^(1/3)*x^2*log(-(12*(-x^3 + 1)^(2/3)*x^2 +
 3*4^(2/3)*(-1)^(1/3)*(x^4 - x)*(-x^3 + 1)^(1/3) + 4^(1/3)*(-1)^(2/3)*(x^6
+ 2*x^3 + 1))/(x^6 + 2*x^3 + 1)) + 72*(-x^3 + 1)^(1/3))/x^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

maple [C] time = 14.43, size = 991, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x)
```

```
[Out] 1/2*(x^3-1)/x^2/(-x^3+1)^(2/3)+(-1/12*ln((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*Ro
otOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^3-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*R
ootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2*x^3+RootOf(_Z^3+2)*x^6-3*RootOf(Ro
otOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf
(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x-18*RootOf(
RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)*(x^6-2*x^3+1)^(
1/3)*x^2-6*RootOf(_Z^3+2)*x^3+18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2
)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x+RootOf(_Z^3+2)-3*RootOf(RootOf(_Z^3+2
)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(x+1)^2/(x^2-x+1)^2)*RootOf(_Z^3+2)-1/4*ln
((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf(_Z^3+2)^3*x^
3-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*RootOf(_Z^3+2)^2
*x^3+RootOf(_Z^3+2)*x^6-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^
2)*x^6+9*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z^3+2)^2+3*_Z*
RootOf(_Z^3+2)+9*_Z^2)*x-18*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2)*RootOf(_Z^3+2)*(x^6-2*x^3+1)^(1/3)*x^2-6*RootOf(_Z^3+2)*x^3+18*RootOf(
RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x^3-6*(x^6-2*x^3+1)^(2/3)*x+Ro
otOf(_Z^3+2)-3*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2))/(x+1)^2
/(x^2-x+1)^2)*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)+1/12*Root
```

```
Of(_Z^3+2)*ln((6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*RootOf
(_Z^3+2)^3*x^3+36*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)^2*Roo
tOf(_Z^3+2)^2*x^3-RootOf(_Z^3+2)*x^6-6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(
_Z^3+2)+9*_Z^2)*x^6-9*(x^6-2*x^3+1)^(2/3)*RootOf(_Z^3+2)^2*RootOf(RootOf(_Z
^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_Z^2)*x-6*RootOf(_Z^3+2)^2*(x^6-2*x^3+1)^(1/3
)*x^2+2*RootOf(_Z^3+2)*x^3+12*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9
*_Z^2)*x^3-RootOf(_Z^3+2)-6*RootOf(RootOf(_Z^3+2)^2+3*_Z*RootOf(_Z^3+2)+9*_
Z^2))/(x+1)^2/(x^2-x+1)^2))/(-x^3+1)^(2/3)*((x^3-1)^2)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{2}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(2/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(2/3)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(1-x^3\right)}^{2/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^3*(1 - x^3)^(2/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{2}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(2/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(2/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.638 $\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=141

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+(-x^3+1)^(2/3)-2/5*(-x^3+1)^(5/3)+1/8*(-x^3+1)^(8/3)-1/2
4*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*
(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 627, 51, 55, 617, 204, 31}

$$\frac{1}{8}\left(1-x^3\right)^{8/3}-\frac{2}{5}\left(1-x^3\right)^{5/3}+\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)
^(8/3)/8 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]
) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\int \frac{x^{14}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}} - \frac{x^2}{\sqrt[3]{1-x}} + \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right) dx, x, x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{1-x}}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx, x, x^{\ldots}\right.$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - 2(1-x)^{2/3} + (1-x)^{5/3}\right) dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\ldots\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\ldots\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{4}\operatorname{Subst}\left(\int\ldots\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2} - \ldots\right.}{4\sqrt[3]{2}\ldots}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \left(1-x^3\right)^{2/3} - \frac{2}{5}\left(1-x^3\right)^{5/3} + \frac{1}{8}\left(1-x^3\right)^{8/3} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(\ldots\right.}{12\ldots}$$

Mathematica [C] time = 0.04, size = 53, normalized size = 0.38

$$\frac{20\,_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-5x^9-x^6-23x^3+29}{40\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (29 - 23*x^3 - x^6 - 5*x^9 + 20*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2
])/(40*(1 - x^3)^(1/3))
```

fricas [A] time = 0.71, size = 140, normalized size = 0.99

$$\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-5\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+\right.\right.}{120\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/120*(10*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 5*2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x
^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 10*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x
^3 + 1)^(1/3)) + 3*(5*x^9 + x^6 + 23*x^3 - 49)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.19, size = 136, normalized size = 0.96

$$\frac{1}{8}\left(x^3-1\right)^2\left(-x^3+1\right)^{\frac{2}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^3+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/8*(x^3 - 1)^2*(-x^3 + 1)^(2/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*
2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/
3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3
)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)
^(1/3)
```

maple [C] time = 3.78, size = 682, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/40*(5*x^9+x^6+23*x^3-49)/(-x^3+1)^(1/3)-1/12*ln((18*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(
_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*
RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-
1/2*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(
_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-1
2*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-4
```

```
2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(
x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+1/2*Ro
otOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln((18*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-6*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*RootOf(_Z^3-4)*x^3+21*RootOf
(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))
```

maxima [A] time = 1.07, size = 128, normalized size = 0.91

$$\frac{1}{8}\left(-x^{3}+1\right)^{\frac{8}{3}}+\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)-\frac{2}{5}\left(-x^{3}+1\right)^{\frac{5}{3}}-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/8*(-x^3 + 1)^(8/3) + 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(
1/3) + 2*(-x^3 + 1)^(1/3))) - 2/5*(-x^3 + 1)^(5/3) - 1/24*2^(2/3)*log(2^(2/
3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 1/12*2^(2/3)*log(-2^(1/
3) + (-x^3 + 1)^(1/3)) + (-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

mupad [B] time = 4.86, size = 148, normalized size = 1.05

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\left(1-x^3\right)^{2/3}-\frac{2\left(1-x^3\right)^{5/3}}{5}+\frac{\left(1-x^3\right)^{8/3}}{8}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3) - (2*(1 - x^3)^(5/3))/5 + (1 - x^3)^(8/3)/8 + (2^(2/3)*log((
1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 -
(2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*
1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**14/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.639 $\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=130

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/5*(-x^3+1)^(5/3)+1/24*ln(x^3+1)*2^(
2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^
3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 87, 43, 783, 78, 55, 617, 204, 31}

$$-\frac{1}{5}\left(1-x^3\right)^{5/3}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - ArcTan[(1 +
 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2
^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 783

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rubi steps

$$\int \frac{x^{11}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{x}{\sqrt[3]{1-x}} - \frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\right) dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}}\,dx, x, x^3\right)\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1-x}\left(-1+x^2\right)}\,dx, x, x^3\right)$$

$$= -\left(\frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt[3]{1-x}} - (1-x)^{2/3}\right) dx, x, x^3\right)\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(-1-x)(1-x)^{4/3}}\,dx\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(-1-x)\sqrt[3]{1-x}}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+\cdots}\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \cdots$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{2}\left(1-x^3\right)^{2/3} - \frac{1}{5}\left(1-x^3\right)^{5/3} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \cdots$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.37

$$\frac{-5\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-2x^6-x^3+13}{10\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (13 - x^3 - 2*x^6 - 5*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(10*(1
- x^3)^(1/3))
```

fricas [A] time = 0.58, size = 159, normalized size = 1.22

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+5\cdot 2^{\frac{2}{3}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}(-1)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/120*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(
6)*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 5*2^(2/3)*(-1)^(1/3)*(
x^3 - 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x
^3 + 1)^(2/3)) - 10*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) +
(-x^3 + 1)^(1/3)) - 12*(2*x^6 + x^3 - 8)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.18, size = 120, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1
/3))) + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

maple [C] time = 3.79, size = 497, normalized size = 3.82

$$-\frac{\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\_Z^3-4\right)^2\right)\ln\left(\frac{72x^3\,\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/10*(2*x^6+x^3-8)/(-x^3+1)^(1/3)-1/12*RootOf(_Z^3-4)*ln((-6*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-45*RootOf(Root
Of(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+2*RootOf(_
Z^3-4)*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+63*R
ootOf(_Z^3-4)*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)+21*(-x^3+1)^(2/3)-14*RootOf(_Z^3-4)-105*RootOf(RootOf(_Z^3-4)^2+6*_Z
```

```
*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z
*RootOf(_Z^3-4)+36*_Z^2)*ln((15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+72*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-
4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+15*RootOf(_Z^3-4)*x^3+72*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+126*RootOf(_Z^3-4)*(-x^3+1)^(1/3)
*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+42*(-x^3+1)^(2/3)-35*
RootOf(_Z^3-4)-168*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x
+1)/(x^2-x+1))
```

maxima [A] time = 1.27, size = 119, normalized size = 0.92

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{5}\left(-x^3+1\right)^{\frac{5}{3}}+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) - 1/5*(-x^3 + 1)^(5/3) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1
)^(1/3) + (-x^3 + 1)^(2/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3))
 + 1/2*(-x^3 + 1)^(2/3) + 1/2/(-x^3 + 1)^(1/3)
```

mupad [B] time = 5.15, size = 139, normalized size = 1.07

$$\frac{1}{2\left(1-x^3\right)^{1/3}}-\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{\left(1-x^3\right)^{2/3}}{2}-\frac{\left(1-x^3\right)^{5/3}}{5}-\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + (
1 - x^3)^(2/3)/2 - (1 - x^3)^(5/3)/5 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^
(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/24 + (2^(2/3)*log((1 - x^3)
^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**11/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.640 $\int \frac{x^8}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=115

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*(-x^3+1)^(2/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)
-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2)
)*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 87, 627, 51, 55, 617, 204, 31}

$$\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + (1 - x^3)^(2/3)/2 + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(
1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3
) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rubi steps

$$\int \frac{x^8}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1-x}}+\frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\right)dx,x,x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\left(1-x^2\right)}\,dx,x,x^3\right)$$

$$= \frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx,x,x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx,x,\sqrt[3]{1-\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{2}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\left(1-x^3\right)^{2/3}+\frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x\right.}{4\sqrt[3]{2}}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.36

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)-x^3+1}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (1 - x^3 + Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2])/(2*(1 - x^3)^(1/3)
)
```

fricas [A] time = 0.74, size = 130, normalized size = 1.13

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right.}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) + 12*(x^3 - 2)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.18, size = 109, normalized size = 0.95

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2*(-x^3 + 1)^(
2/3) + 1/2/(-x^3 + 1)^(1/3)
```

maple [C] time = 3.78, size = 672, normalized size = 5.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] -1/2*(x^3-2)/(-x^3+1)^(1/3)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*Roo
tOf(_Z^3-4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*R
ootOf(_Z^3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
x^3-5*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/
3)+42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4
))/(x+1)/(x^2-x+1))-1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3
)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((18*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21
*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)
```

```
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(
RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [A] time = 1.27, size = 108, normalized size = 0.94

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(-2^{\frac{1}{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2*(-x^3 + 1)^(2/3)
+ 1/2/(-x^3 + 1)^(1/3)
```

mupad [B] time = 4.89, size = 128, normalized size = 1.11

$$\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{\left(1-x^3\right)^{2/3}}{2}+\frac{2^{2/3}\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}-\frac{2^{2/3}\ln\left(\frac{(1-}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
1 - x^3)^(2/3)/2 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i -
1)^2)/16)*(3^(1/2)*1i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*
(3^(1/2)*1i + 1)^2)/16)*(3^(1/2)*1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**8/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.641 $\int \frac{x^5}{(1-x^3)^{4/3}(1+x^3)} dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log(x^3+1)}{12\sqrt[3]{2}}-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) + Log[1 + x^3]/(12*2^(1/3)) - Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{x}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right) \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) + \frac{\text{Subst}\left(\right.}{} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1}\right.}{2\sqrt[3]{2}} \\
&= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\,\sqrt{3}} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 0.95

$$\frac{1}{24}\left(\frac{12}{\sqrt[3]{1-x^3}} + 2^{2/3}\log\left(x^3+1\right) - 3\,2^{2/3}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right) - 2\,2^{2/3}\sqrt{3}\,\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (12/(1 - x^3)^(1/3) - 2*2^(2/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3)
)/Sqrt[3]] + 2^(2/3)*Log[1 + x^3] - 3*2^(2/3)*Log[2^(1/3) - (1 - x^3)^(1/3)
])/24
```

fricas [B] time = 0.73, size = 148, normalized size = 1.48

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\left(-x^3\right.\right.}{24\left(x^3-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.18, size = 98, normalized size = 0.98

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^
(1/3)
```

maple [C] time = 3.76, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-
4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*Roo
tOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/
(x^2-x+1))+1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^
3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootO
f(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)+1/2*ln((18*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_
Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [A] time = 1.53, size = 97, normalized size = 0.97

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

mupad [B] time = 4.85, size = 117, normalized size = 1.17

$$\frac{1}{2\left(1-x^{3}\right)^{1/3}}-\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}-\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{16}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{24}+\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^{3}\right)^{1/3}}{4}-\frac{2^{1/3}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) - (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 - (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
i - 1))/24 + (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**5/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.642 $\int \frac{x^2}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=100

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {444, 51, 55, 617, 204, 31}

$$\frac{1}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^
(1/3)*Sqrt[3]) - Log[1 + x^3]/(12*2^(1/3)) + Log[2^(1/3) - (1 - x^3)^(1/3)]
/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
```

```
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right)$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right) - \frac{\text{Subst}\left(\int\right.}{}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1}\right.}{2\sqrt[3]{2}}$$
$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.34

$$\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2]/(2*(1 - x^3)^(1/3))
```

fricas [A] time = 0.66, size = 125, normalized size = 1.25

$$\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}+2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-2^{\frac{2}{3}}\left(x^3-1\right)\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)-}{24\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2*s
qrt(6)*(-x^3 + 1)^(1/3))) - 2^(2/3)*(x^3 - 1)*log(2^(2/3) + 2^(1/3)*(-x^3 +
 1)^(1/3) + (-x^3 + 1)^(2/3)) + 2*2^(2/3)*(x^3 - 1)*log(-2^(1/3) + (-x^3 +
1)^(1/3)) - 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.19, size = 98, normalized size = 0.98

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-2^{\frac{1}{3}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/2/(-x^3 + 1)^(
1/3)
```

maple [C] time = 3.87, size = 667, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-
4)^2*x^3+15*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^
3-4)^3*x^3-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-5*Roo
tOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)+42*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+35*RootOf(_Z^3-4))/(x+1)/
(x^2-x+1))-1/12*ln((18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^
3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)+28*RootO
f(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln((18*RootOf(RootOf(_Z^3-4)
^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-12*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-12*RootOf(_Z^3-4)*x^3+21*RootOf(_
Z^3-4)^2*(-x^3+1)^(1/3)+42*(-x^3+1)^(2/3)-42*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)+28*RootOf(_Z^3-4))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [A] time = 1.34, size = 97, normalized size = 0.97

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{24}\cdot2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}+\left(-x^3+1\right)^{\frac{2}{3}}\right)+\frac{1}{12}\cdot2^{\frac{2}{3}}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(-2^(1/3) + (-x^3 + 1)^(1/3)) + 1/2/(-x^3 + 1)^(1/3)
```

mupad [B] time = 4.90, size = 117, normalized size = 1.17

$$\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}}{4}\right)}{12}+\frac{1}{2\left(1-x^3\right)^{1/3}}+\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/3}\left(-1+\sqrt{3}\,1i\right)^2}{16}\right)\left(-1+\sqrt{3}\,1i\right)}{24}-\frac{2^{2/3}\,\ln\left(\frac{\left(1-x^3\right)^{1/3}}{4}-\frac{2^{1/}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] (2^(2/3)*log((1 - x^3)^(1/3)/4 - 2^(1/3)/4))/12 + 1/(2*(1 - x^3)^(1/3)) + (
2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1
```

```
i - 1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/4 - (2^(1/3)*(3^(1/2)*1i + 1)^2)/
16)*(3^(1/2)*1i + 1))/24
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**2/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.643 $\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=154

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2/(-x^3+1)^(1/3)-1/2*ln(x)+1/24*ln(x^3+1)*2^(2/3)+1/2*ln(1-(-x^3+1)^(1/3)
)-1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/3*arctan(1/3*(1+2*(-x^3+1)^(1/3)
)*3^(1/2))*3^(1/2)-1/12*arctan(1/3*(1+2^(2/3)*(-x^3+1)^(1/3))*3^(1/2))*2^(2
/3)*3^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {446, 85, 156, 55, 618, 204, 31, 617}

$$\frac{1}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right)-\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)^(1/3))/Sqrt[3]]/Sqrt[3] - A
rcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*Sqrt[3]) - Log[x]/2
 + Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 - x^3)^(1/3)]/2 - Log[2^(1/3) - (
1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{1}{6}\operatorname{Subst}\left(\int \frac{2+x}{\sqrt[3]{1-x}\,x(1+x)}\,dx, x, x^3\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,(1+x)}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right.$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}\,x+x^2}\,dx, x, \sqrt[3]{1-x^3}\right)$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}\log\left(1-\sqrt[3]{1-x^3}\right) - \frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}$$

$$= \frac{1}{2\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{2} + \frac{\log\left(1+x^3\right)}{12\sqrt[3]{2}} + \frac{1}{2}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.35

$$\frac{2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right) - {}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}\left(1-x^3\right)\right)}{2\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*Hypergeometric2F1[-1/3,
1, 2/3, 1 - x^3])/(2*(1 - x^3)^(1/3))
```

fricas [A] time = 0.78, size = 226, normalized size = 1.47

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(2\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\right)\right)+2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^{3}-1\right)\log\left(2^{\frac{1}{3}}\,(-1)^{\frac{2}{3}}\,(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(2*sqrt(6)
*(-1)^(1/3)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3))) + 2^(2/3)*(-1)^(1/3)*(x^3
- 1)*log(2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3) - 2^(2/3)*(-1)^(1/3) + (-x^3 +
 1)^(2/3)) - 2*2^(2/3)*(-1)^(1/3)*(x^3 - 1)*log(-2^(1/3)*(-1)^(2/3) + (-x^3
 + 1)^(1/3)) - 8*sqrt(3)*(x^3 - 1)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/3) + 1/
3*sqrt(3)) + 4*(x^3 - 1)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) - 8*(
x^3 - 1)*log((-x^3 + 1)^(1/3) - 1) + 12*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [A] time = 0.19, size = 160, normalized size = 1.04

$$-\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x^{3}+1\right)^{\frac{1}{3}}\right)\right)+\frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x^{3}+1\right)^{\frac{1}{3}}+\left(-x^{3}+1\right)^{\frac{2}{3}}\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1
/3))) + 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2
/3)) - 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/3*sqrt(3)*arc
tan(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) + 1/2/(-x^3 + 1)^(1/3) - 1/6*log(
(-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) + 1/3*log(abs((-x^3 + 1)^(1/3) - 1
))
```

maple [F] time = 2.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^{3}+1\right)^{\frac{4}{3}}\left(x^{3}+1\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x), x)
```

mupad [B] time = 5.40, size = 253, normalized size = 1.64

$$\frac{\ln\left(\frac{17}{4}-\frac{17\left(1-x^3\right)^{1/3}}{4}\right)}{3}+\ln\left(\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\left(1458\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)^2-\frac{459\left(1-x^3\right)^{1/3}}{4}\right)-\frac{63}{4}\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)-\ln\left(\left(\frac{1}{6}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log(17/4 - (17*(1 - x^3)^(1/3))/4)/3 + log(((3^(1/2)*1i)/6 - 1/6)*(1458*((3
^(1/2)*1i)/6 - 1/6)^2 - (459*(1 - x^3)^(1/3))/4) - 63/4)*((3^(1/2)*1i)/6 -
1/6) - log(((3^(1/2)*1i)/6 + 1/6)*(1458*((3^(1/2)*1i)/6 + 1/6)^2 - (459*(1
- x^3)^(1/3))/4) + 63/4)*((3^(1/2)*1i)/6 + 1/6) - (2^(2/3)*log((2^(2/3)*((8
1*2^(1/3))/4 - (459*(1 - x^3)^(1/3))/4))/12 + 63/4))/12 + 1/(2*(1 - x^3)^(1
/3)) + ((-1)^(1/3)*2^(2/3)*log(((-1)^(1/3)*2^(2/3)*((81*(-1)^(2/3)*2^(1/3))
/4 - (459*(1 - x^3)^(1/3))/4))/12 - 63/4))/12 - ((-1)^(1/3)*2^(2/3)*log(((-
1)^(1/3)*2^(2/3)*(3^(1/2)*1i + 1)*((459*(1 - x^3)^(1/3))/4 - (81*(-1)^(2/3)
*2^(1/3)*(3^(1/2)*1i + 1)^2)/16))/24 - 63/4)*(3^(1/2)*1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.644 $\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=175

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan}{}$$

```
[Out] 5/6/(-x^3+1)^(1/3)-1/3/x^3/(-x^3+1)^(1/3)-1/6*ln(x)-1/24*ln(x^3+1)*2^(2/3)+
1/6*ln(1-(-x^3+1)^(1/3))+1/8*ln(2^(1/3)-(-x^3+1)^(1/3))*2^(2/3)+1/9*arctan(
1/3*(1+2*(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1+2^(2/3)*(-x^3+
1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {446, 103, 156, 51, 55, 618, 204, 31, 617}

$$-\frac{1}{3x^3\sqrt[3]{1-x^3}}+\frac{5}{6\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right)+\frac{\log\left(\sqrt[3]{2}-\sqrt[3]{1-x^3}\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{1-x^3}+1}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] 5/(6*(1 - x^3)^(1/3)) - 1/(3*x^3*(1 - x^3)^(1/3)) + ArcTan[(1 + 2*(1 - x^3)
^(1/3))/Sqrt[3]]/(3*Sqrt[3]) + ArcTan[(1 + 2^(2/3)*(1 - x^3)^(1/3))/Sqrt[3]
]/(2*2^(1/3)*Sqrt[3]) - Log[x]/6 - Log[1 + x^3]/(12*2^(1/3)) + Log[1 - (1 -
 x^3)^(1/3)]/6 + Log[2^(1/3) - (1 - x^3)^(1/3)]/(4*2^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4(1-x^3)^{4/3}(1+x^3)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{4/3}x^2(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{1}{3}\operatorname{Subst}\left(\int \frac{-\frac{1}{3}-\frac{4x}{3}}{(1-x)^{4/3}x(1+x)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{4/3}x}\,dx, x, x^3\right) + \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{(1-x)^{4/3}(1+\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{1}{9}\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{1-x}\,x}\,dx, x, x^3\right) + \frac{1}{6}\operatorname{Subst}\left(\int \frac{}{\sqrt[3]{1}}\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} - \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{}\right.$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} - \frac{\log(x)}{6} - \frac{\log(1+x^3)}{12\sqrt[3]{2}} + \frac{1}{6}\log\left(1-\sqrt[3]{1-x^3}\right) + \frac{\log}{}$$

$$= \frac{5}{6\sqrt[3]{1-x^3}} - \frac{1}{3x^3\sqrt[3]{1-x^3}} + \frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x^3}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}} - \frac{\log(x)}{6}$$

Mathematica [C] time = 0.03, size = 64, normalized size = 0.37

$$\frac{3x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{1}{2}(1-x^3)\right) + 2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-x^3\right) - 2}{6x^3\sqrt[3]{1-x^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (1 - x^3)/2] + 2*x^3*Hypergeome
tric2F1[-1/3, 1, 2/3, 1 - x^3])/(6*x^3*(1 - x^3)^(1/3))
```

fricas [A] time = 0.57, size = 238, normalized size = 1.36

$$6\sqrt{6}\,2^{\frac{1}{6}}(x^6-x^3)\arctan\left(\frac{1}{6}\cdot 2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}} + 2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - 3\cdot 2^{\frac{2}{3}}(x^6-x^3)\log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + \left(-x^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(x^6 - x^3)*arctan(1/6*2^(1/6)*(sqrt(6)*2^(1/3) + 2
*sqrt(6)*(-x^3 + 1)^(1/3))) - 3*2^(2/3)*(x^6 - x^3)*log(2^(2/3) + 2^(1/3)*(
-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/3)) + 6*2^(2/3)*(x^6 - x^3)*log(-2^(1/3) +
(-x^3 + 1)^(1/3)) + 8*sqrt(3)*(x^6 - x^3)*arctan(2/3*sqrt(3)*(-x^3 + 1)^(1/
3) + 1/3*sqrt(3)) - 4*(x^6 - x^3)*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) +
 1) + 8*(x^6 - x^3)*log((-x^3 + 1)^(1/3) - 1) - 12*(5*x^3 - 2)*(-x^3 + 1)^(
2/3))/(x^6 - x^3)
```

giac [A] time = 0.21, size = 181, normalized size = 1.03

$$\frac{1}{12}\sqrt{3}\,2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2\left(-x^3+1\right)^{\frac{1}{3}}\right)\right) - \frac{1}{24}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}} + \left(-x^3+1\right)^{\frac{2}{3}}\right) + \frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(\left|-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x^3 + 1)^(1/
3))) - 1/24*2^(2/3)*log(2^(2/3) + 2^(1/3)*(-x^3 + 1)^(1/3) + (-x^3 + 1)^(2/
3)) + 1/12*2^(2/3)*log(abs(-2^(1/3) + (-x^3 + 1)^(1/3))) + 1/9*sqrt(3)*arct
an(1/3*sqrt(3)*(2*(-x^3 + 1)^(1/3) + 1)) - 1/6*(5*x^3 - 2)/((-x^3 + 1)^(4/3
) - (-x^3 + 1)^(1/3)) - 1/18*log((-x^3 + 1)^(2/3) + (-x^3 + 1)^(1/3) + 1) +
 1/9*log(abs((-x^3 + 1)^(1/3) - 1))
```

maple [F] time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^4), x)
```

mupad [B] time = 5.02, size = 399, normalized size = 2.28

$$\frac{\ln\left(\frac{11\left(1-x^3\right)^{1/3}}{972}-\frac{11}{972}\right)}{9}+\frac{2^{2/3}\ln\left(\frac{2^{1/3}\left(\frac{2^{2/3}\left(\frac{81\,2^{1/3}}{4}-\frac{75\left(1-x^3\right)^{1/3}}{4}\right)}{12}-\frac{35}{12}\right)}{72}+\frac{\left(1-x^3\right)^{1/3}}{27}\right)}{12}+\ln\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)^2\left(\left(-\frac{1}{18}+\frac{\sqrt{3}\,1\mathrm{i}}{18}\right)\left(1\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] log((11*(1 - x^3)^(1/3))/972 - 11/972)/9 + (2^(2/3)*log((2^(1/3)*((2^(2/3)*
((81*2^(1/3))/4 - (75*(1 - x^3)^(1/3))/4))/12 - 35/12))/72 + (1 - x^3)^(1/3
)/27))/12 + log(((3^(1/2)*1i)/18 - 1/18)^2*(((3^(1/2)*1i)/18 - 1/18)*(1458*
((3^(1/2)*1i)/18 - 1/18)^2 - (75*(1 - x^3)^(1/3))/4) - 35/12) + (1 - x^3)^(
1/3)/27)*((3^(1/2)*1i)/18 - 1/18) - log((1 - x^3)^(1/3)/27 - ((3^(1/2)*1i)/
18 + 1/18)^2*(((3^(1/2)*1i)/18 + 1/18)*(1458*((3^(1/2)*1i)/18 + 1/18)^2 - (
75*(1 - x^3)^(1/3))/4) + 35/12))*((3^(1/2)*1i)/18 + 1/18) + ((5*x^3)/6 - 1/
3)/((1 - x^3)^(1/3) - (1 - x^3)^(4/3)) + (2^(2/3)*log((1 - x^3)^(1/3)/27 +
(2^(1/3)*(3^(1/2)*1i - 1)^2*((2^(2/3)*(3^(1/2)*1i - 1)*((81*2^(1/3)*(3^(1/2
)*1i - 1)^2)/16 - (75*(1 - x^3)^(1/3))/4))/24 - 35/12))/288)*(3^(1/2)*1i -
1))/24 - (2^(2/3)*log((1 - x^3)^(1/3)/27 - (2^(1/3)*(3^(1/2)*1i + 1)^2*((2^
```

```
(2/3)*(3^(1/2)*1i + 1)*((81*2^(1/3)*(3^(1/2)*1i + 1)^2)/16 - (75*(1 - x^3)^
(1/3))/4))/24 + 35/12))/288)*(3^(1/2)*1i + 1))/24
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**4*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.645 $\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=174

$$\frac{5}{6}\left(1-x^3\right)^{2/3}x+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{6}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2*x^4/(-x^3+1)^(1/3)+5/6*x*(-x^3+1)^(2/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-
2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/6*ln(x+(-x^3+1)^(1/3))+1/9*arctan(1/3*(
1-2*x/(-x^3+1)^(1/3))*3^(1/2))*3^(1/2)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3
+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.15, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^10*AppellF1[10/3, 4/3, 1, 13/3, x^3, -x^3])/10
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{10}x^{10}F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.25, size = 152, normalized size = 0.87

$$\frac{1}{72}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)-\frac{12\left(2x^3-5\right)x}{\sqrt[3]{1-x^3}}-5\,2^{2/3}\left(2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-12*x*(-5 + 2*x^3))/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^
3, -x^3] - 5*2^(2/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3)
)/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3
)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/72
```

fricas [B] time = 0.78, size = 271, normalized size = 1.56

$$6\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x+2\,\sqrt{6}\,(-1)^{\frac{1}{3}}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\,x}\right)+6\cdot 2^{\frac{2}{3}}\,(-1)^{\frac{1}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{1}{3}}(-1)^{\frac{2}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/72*(6*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(sqrt(6)*2^
(1/3)*x + 2*sqrt(6)*(-1)^(1/3)*(-x^3 + 1)^(1/3))/x) + 6*2^(2/3)*(-1)^(1/3)*
(x^3 - 1)*log((2^(1/3)*(-1)^(2/3)*x + (-x^3 + 1)^(1/3))/x) - 3*2^(2/3)*(-1)
^(1/3)*(x^3 - 1)*log(-(2^(2/3)*(-1)^(1/3)*x^2 + 2^(1/3)*(-1)^(2/3)*(-x^3 +
1)^(1/3)*x - (-x^3 + 1)^(2/3))/x^2) + 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt
(3)*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) - 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1
/3))/x) + 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2
) + 12*(2*x^4 - 5*x)*(-x^3 + 1)^(2/3))/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^9/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**9/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.646 $\int \frac{x^6}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=153

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{1}{2}\log\left(\sqrt[3]{1-x^3}+x\right)+\frac{\tan^{-1}\left(\frac{1-\frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/2*ln(x+(-x^3+1)^(1/3))+1/3*arctan(1/3*(1-2*x/(-x^3+1)^(1/3))*3
^(1/2))*3^(1/2)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(
2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.17, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^7*AppellF1[7/3, 4/3, 1, 10/3, x^3, -x^3])/7
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{7}x^7F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.16, size = 142, normalized size = 0.93

$$\frac{1}{24}\left(-6x^4F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};x^3,-x^3\right)+\frac{12x}{\sqrt[3]{1-x^3}}+2^{2/3}\left(-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((12*x)/(1 - x^3)^(1/3) - 6*x^4*AppellF1[4/3, 1/3, 1, 7/3, x^3, -x^3] + 2^(
2/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Lo
g[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Lo
g[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3)]))/24
```

fricas [B] time = 0.67, size = 239, normalized size = 1.56

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^3-1\right)\arctan\left(-\frac{2^{\frac{1}{6}}\left(\sqrt{6}\,2^{\frac{1}{3}}x-2\sqrt{6}\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6x}\right)-2\cdot 2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{1}{3}}x+\left(-x^3+1\right)^{\frac{1}{3}}}{x}\right)+2^{\frac{2}{3}}\left(x^3-1\right)\log\left(\frac{2^{\frac{2}{3}}x^2-2^{\frac{1}{3}}\cdots}{\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/24*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(-1/6*2^(1/6)*(sqrt(6)*2^(1/3)*x -
 2*sqrt(6)*(-x^3 + 1)^(1/3))/x) - 2*2^(2/3)*(x^3 - 1)*log((2^(1/3)*x + (-x^
3 + 1)^(1/3))/x) + 2^(2/3)*(x^3 - 1)*log((2^(2/3)*x^2 - 2^(1/3)*(-x^3 + 1)^
(1/3)*x + (-x^3 + 1)^(2/3))/x^2) - 8*sqrt(3)*(x^3 - 1)*arctan(-1/3*(sqrt(3)
*x - 2*sqrt(3)*(-x^3 + 1)^(1/3))/x) + 8*(x^3 - 1)*log((x + (-x^3 + 1)^(1/3)
)/x) - 4*(x^3 - 1)*log((x^2 - (-x^3 + 1)^(1/3)*x + (-x^3 + 1)^(2/3))/x^2) +
 12*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 1.68, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^6/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**6/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.647 $\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 0.36, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{1+x^3}\right)}{4\left(1+x^3\right)^{4/3}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.36

$$\frac{x^4\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{2x^3}{x^3+1}\right)}{4\left(x^3+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^4*Hypergeometric2F1[4/3, 4/3, 7/3, (2*x^3)/(1 + x^3)])/(4*(1 + x^3)^(4/3
))
```

fricas [B] time = 3.55, size = 318, normalized size = 3.00

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^3-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-11\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)
*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1)^
(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9 -
111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^(1
/3)*(x^3 - 1)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*(-1)
^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3)*(x
^3 - 1)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(1/3)*(
-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 +
 2*x^3 + 1)) + 36*(-x^3 + 1)^(2/3)*x)/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [C] time = 4.23, size = 627, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(
_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)
+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+30*(-x^3
+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-
4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+
36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2
+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*RootOf(_Z^3-4)*ln((-3*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-2
7*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x
^3+6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*Ro
otOf(_Z^3-4)^2*x+2*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-3*(-x^3+1)^(1/3)*Roo
tOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-3*Root
Of(_Z^3-4)*x^3-27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+
5*(-x^3+1)^(2/3)*x+RootOf(_Z^3-4)+9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(x^3/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**3/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.648 $\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=106

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

```
[Out] 1/2*x/(-x^3+1)^(1/3)-1/24*ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3
))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*
3^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x/(2*(1 - x^3)^(1/3)) - ArcTan[(1 - (2*2^(1/3)*x)/(1 - x^3)^(1/3))/Sqrt[3]]
/(2*2^(1/3)*Sqrt[3]) - Log[1 + (2^(2/3)*x^2)/(1 - x^3)^(2/3) - (2^(1/3)*x)/
(1 - x^3)^(1/3)]/(12*2^(1/3)) + Log[1 + (2^(1/3)*x)/(1 - x^3)^(1/3)]/(6*2^(
1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
```

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx &= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\int \frac{1}{\sqrt[3]{1-x^3}\left(1+x^3\right)}\,dx\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+2x^3}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1+\sqrt[3]{2}x}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{2-\sqrt[3]{2}x}{1-\sqrt[3]{2}x+2^{2/3}x^2}\right.\\
&= \frac{x}{2\sqrt[3]{1-x^3}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt[3]{2}x+2^{2/3}x^2}\,dx,x,\frac{x}{\sqrt[3]{1-x^3}}\right)-\frac{\text{Su}}{}\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}+\frac{\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1\right.}{2\sqrt[3]{2}}\\
&= \frac{x}{2\sqrt[3]{1-x^3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\log\left(1+\frac{2^{2/3}x^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{12\sqrt[3]{2}}+\frac{\log\left(1+\frac{\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}\right)}{6\sqrt[3]{2}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 100, normalized size = 0.94

$$
\frac{-7\left(3x^3+4\right)\left(x^3-1\right)^2\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{2x^3}{x^3-1}\right)-12\left(x^9+x^6\right)\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+7\left(3x^3+4\right)\left(x^3-1\right)^2}{14x^2\left(1-x^3\right)^{7/3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (7*(-1 + x^3)^2*(4 + 3*x^3) - 7*(-1 + x^3)^2*(4 + 3*x^3)*Hypergeometric2F1[
1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] - 12*(x^6 + x^9)*Hypergeometric2F1[2, 7/3,
 10/3, (2*x^3)/(-1 + x^3)])/(14*x^2*(1 - x^3)^(7/3))
```

fricas [B] time = 3.24, size = 288, normalized size = 2.72

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\left(x^{3}-1\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^{7}+4x^{4}-x\right)\left(-x^{3}+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^{9}-111x^{6}+33x^{3}-1\right)+12\sqrt{6}\left(19x^{8}-16x^{5}+x^{2}\right)\left(-x^{3}+1\right)^{\frac{1}{3}}\right)}{6\left(109x^{9}-105x^{6}+3x^{3}+1\right)}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(x^3 - 1)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3)*(5
*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6 + 33
*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x^9 -
 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(x^3 - 1)*log((6*2^(1/3)*(-x^3 + 1)^(1/3
)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(x^3
 - 1)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*x^6 - 16*x^
3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) + 36*(-x^3 +
 1)^(2/3)*x)/(x^3 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [C] time = 3.01, size = 944, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2/(-x^3+1)^(1/3)*x+1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*ln((9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4
)^3*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z
^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
6*_Z^2)*RootOf(_Z^3-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1
)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)
*x^2+9*RootOf(_Z^3-4)*x^3+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36
*_Z^2)*x^3+10*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/12*ln(-(6*RootOf(RootOf(_Z^
3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x
+RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4
)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x
+2*RootOf(_Z^3-4)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(
x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(-(6*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
```

```
Of(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*Ro
otOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootO
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+RootOf(_Z^3-4)^
2*(-x^3+1)^(1/3)*x^2-24*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(
_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-2*RootOf(_Z^3-4)*x^3+6*RootOf(RootOf(_Z
^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x+2*RootOf(_Z^3-4
)-6*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*
RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+1)(-x^3+1)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-x^3)^{4/3}\,(x^3+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.649 $\int \frac{1}{x^3(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=124

$$\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{\left(1-x^3\right)^{2/3}}{x^2}+\frac{1}{2x^2\sqrt[3]{1-x^3}}$$

```
[Out] 1/2/x^2/(-x^3+1)^(1/3)-(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^
(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/
3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 8.12, antiderivative size = 204, normalized size of antiderivative = 1.65, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{-18\left(x^3+1\right)^2x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-54x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-84x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-30x^6\,{}_2F_1}{14x^5\left(1-x\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(14 + 56*x^3 - 91*x^6 - 42*x^9 + 63*x^12 - 7*(1 - x^3)^2*(2 + 12*x^3 + 9*x
^6)*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 30*x^6*Hypergeomet
ric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 84*x^9*Hypergeometric2F1[2, 7/3,
 10/3, (-2*x^3)/(1 - x^3)] - 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^
3)/(1 - x^3)] - 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}
, (-2*x^3)/(1 - x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{14+56x^3-91x^6-42x^9+63x^{12}-7\left(1-x^3\right)^2\left(2+12x^3+9x^6\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\right.}{}$$

Mathematica [C] time = 3.11, size = 192, normalized size = 1.55

$$\frac{18\left(x^3+1\right)^2x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{2x^3}{x^3-1}\right)+54x^{12}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+84x^9\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{2x^3}{x^3-1}\right)+30x^6\,{}_2F_1\left(2,\frac{7}{3};\right.}{14x^5\left(1-x^3\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-14 - 56*x^3 + 91*x^6 + 42*x^9 - 63*x^12 + 7*(-1 + x^3)^2*(2 + 12*x^3 + 9*
x^6)*Hypergeometric2F1[1/3, 1, 4/3, (2*x^3)/(-1 + x^3)] + 30*x^6*Hypergeome
tric2F1[2, 7/3, 10/3, (2*x^3)/(-1 + x^3)] + 84*x^9*Hypergeometric2F1[2, 7/3
, 10/3, (2*x^3)/(-1 + x^3)] + 54*x^12*Hypergeometric2F1[2, 7/3, 10/3, (2*x^
3)/(-1 + x^3)] + 18*x^6*(1 + x^3)^2*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3
}, (2*x^3)/(-1 + x^3)])/(14*x^5*(1 - x^3)^(7/3))
```

fricas [B] time = 3.94, size = 340, normalized size = 2.74

$$-\frac{2\sqrt{6}\,2^{\frac{1}{6}}\,(-1)^{\frac{1}{3}}\left(x^5-x^2\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}\,(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/72*(2*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^5 - x^2)*arctan(1/6*2^(1/6)*(6*sqrt(
6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*(-1
)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x^9
- 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 2*2^(2/3)*(-1)^
(1/3)*(x^5 - x^2)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(2/3)*
(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 2^(2/3)*(-1)^(1/3
)*(x^5 - x^2)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) - 2^(
1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/
(x^6 + 2*x^3 + 1)) + 36*(2*x^3 - 1)*(-x^3 + 1)^(2/3))/(x^5 - x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

maple [C] time = 2.92, size = 570, normalized size = 4.60

$$-\frac{\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\_Z^3-4\right)^2\right)\ln\left(-\frac{-36x^3\,\mathrm{RootOf}\left(36\_Z^2+6\_Z\,\mathrm{RootOf}\left(\_Z^3-4\right)+\mathrm{RootOf}\left(\right.\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/2*(2*x^3-1)/x^2/(-x^3+1)^(1/3)-1/12*RootOf(_Z^3-4)*ln((-3*RootOf(RootOf(_
Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-9*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3-9*(-x^3+1)^(1
/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2
-3*RootOf(_Z^3-4)*x^3-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2
)*x^3+3*(-x^3+1)^(2/3)*x+RootOf(_Z^3-4)+3*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/2*RootOf(RootOf(_Z^3-4)^2+6*_Z*Root
Of(_Z^3-4)+36*_Z^2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*
_Z^2)*RootOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+3
```

```
6*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*
_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-x^3+1)^(
1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-4)^2+6*_
Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-12*RootOf
(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^3*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(1/(x**3*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.650 $\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=144

$$-\frac{\log(x^3+1)}{12\sqrt[3]{2}}+\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}x\right)}{4\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{7(1-x^3)^{2/3}}{10x^5}+\frac{1}{2x^5\sqrt[3]{1-x^3}}-\frac{4(1-x^3)^{2/3}}{5x^2}$$

```
[Out] 1/2/x^5/(-x^3+1)^(1/3)-7/10*(-x^3+1)^(2/3)/x^5-4/5*(-x^3+1)^(2/3)/x^2-1/24*
ln(x^3+1)*2^(2/3)+1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 8.32, antiderivative size = 397, normalized size of antiderivative = 2.76, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{54(x^3+1)^2(6x^3+1)x^6\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+54(x^3+1)^3x^6\,{}_4F_3\left(2,2,2,\frac{7}{3};1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)+567x^{15}\,{}_2F_1\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(28 - 182*x^3 - 476*x^6 + 819*x^9 + 378*x^12 - 567*x^15 - 28*Hypergeometri
c2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 182*x^3*Hypergeometric2F1[1/3, 1, 4
/3, (-2*x^3)/(1 - x^3)] + 476*x^6*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(
1 - x^3)] - 819*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 37
8*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 567*x^15*Hyperg
eometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 36*x^6*Hypergeometric2F1[2,
7/3, 10/3, (-2*x^3)/(1 - x^3)] + 342*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-
2*x^3)/(1 - x^3)] + 972*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 -
x^3)] + 594*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 54*x
^6*(1 + x^3)^2*(1 + 6*x^3)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^
3)/(1 - x^3)] + 54*x^6*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1,
 10/3}, (-2*x^3)/(1 - x^3)])/(70*x^8*(1 - x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^6(1-x^3)^{4/3}(1+x^3)}\,dx=-\frac{28-182x^3-476x^6+819x^9+378x^{12}-567x^{15}-28\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};-\frac{2x^3}{1-x^3}\right)+182x^3}{}$$

Mathematica [A] time = 5.16, size = 133, normalized size = 0.92

$$\frac{2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)-\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{(x^3-1)^{2/3}}+1\right)}{12\sqrt[3]{2}}-\frac{-8x^6+x^3+2}{10x^5\sqrt[3]{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/10*(2 + x^3 - 8*x^6)/(x^5*(1 - x^3)^(1/3)) + (2*Sqrt[3]*ArcTan[(-1 + (2*
2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] - Log[1 + (2^(2/3)*x^2)/(-1 + x^3)^(2
/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] + 2*Log[1 + (2^(1/3)*x)/(-1 + x^3)^(1/3
)])/(12*2^(1/3))
```

fricas [B] time = 4.99, size = 316, normalized size = 2.19

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}\left(x^8-x^5\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-111x^6+33x^3-1\right)+12\sqrt{6}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}\right)}{6\left(109x^9-105x^6+3x^3+1\right)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(x^8 - x^5)*arctan(1/6*2^(1/6)*(6*sqrt(6)*2^(2/3
)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - sqrt(6)*2^(1/3)*(71*x^9 - 111*x^6
+ 33*x^3 - 1) + 12*sqrt(6)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3))/(109*x
^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(x^8 - x^5)*log((6*2^(1/3)*(-x^3 +
1)^(1/3)*x^2 + 2^(2/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(
2/3)*(x^8 - x^5)*log((3*2^(2/3)*(5*x^4 - x)*(-x^3 + 1)^(2/3) + 2^(1/3)*(19*
x^6 - 16*x^3 + 1) - 12*(2*x^5 - x^2)*(-x^3 + 1)^(1/3))/(x^6 + 2*x^3 + 1)) +
 36*(8*x^6 - x^3 - 2)*(-x^3 + 1)^(2/3))/(x^8 - x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

maple [C] time = 4.02, size = 851, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/10*(8*x^6-x^3-2)/x^5/(-x^3+1)^(1/3)+1/12*RootOf(_Z^3-4)*ln(-(3*RootOf(Roo
tOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+54*RootOf(R
ootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^
3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3
-4)^2*x-5*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2-6*(-x^3+1)^(1/3)*RootOf(RootO
```

```
f(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2-RootOf(_Z^3-4)*
x^3-18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^
(2/3)*x+RootOf(_Z^3-4)+18*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z
^2))/(x+1)/(x^2-x+1))-1/12*ln((3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4
)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+1
8*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootO
f(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))*RootOf(_Z^3-4)-1/2*ln(
(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^
3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^
2*x^3+3*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)*x^2+18*(-x^3+1)^(1/3)*RootOf(RootOf
(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)
*x^3+36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3+6*(-x^3+1)
^(2/3)*x-RootOf(_Z^3-4)-12*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2))/(x+1)/(x^2-x+1))*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^6\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^6*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**6*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.651 $\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=162

$$\frac{\log\left(x^3+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(-\sqrt[3]{1-x^3}-\sqrt[3]{2}\,x\right)}{4\sqrt[3]{2}}+\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{2}\sqrt{3}}-\frac{5\left(1-x^3\right)^{2/3}}{8x^8}+\frac{1}{2x^8\sqrt[3]{1-x^3}}-\frac{13\left(1-x^3\right)^{2/3}}{20x^5}-\frac{49\left(1-x\right.}{40x^{2}}$$

[Out] 1/2/x^8/(-x^3+1)^(1/3)-5/8*(-x^3+1)^(2/3)/x^8-13/20*(-x^3+1)^(2/3)/x^5-49/40*(-x^3+1)^(2/3)/x^2+1/24*ln(x^3+1)*2^(2/3)-1/8*ln(-2^(1/3)*x-(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*x/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)

Rubi [C] time = 10.59, antiderivative size = 643, normalized size of antiderivative = 3.97, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{-81x^{18}\,{}_5F_4\left(2,2,2,2,\frac{7}{3};1,1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-324x^{15}\,{}_5F_4\left(2,2,2,2,\frac{7}{3};1,1,1,\frac{10}{3};-\frac{2x^3}{1-x^3}\right)-486x^{12}\,{}_5F_4\left(2,2,2,2,\right.}{}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]

[Out] -(70 - 308*x^3 + 1162*x^6 + 2856*x^9 - 4914*x^12 - 2268*x^15 + 3402*x^18 - 70*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 308*x^3*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 1162*x^6*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 2856*x^9*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 4914*x^12*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] + 2268*x^15*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 3402*x^18*Hypergeometric2F1[1/3, 1, 4/3, (-2*x^3)/(1 - x^3)] - 66*x^6*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 312*x^9*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 2268*x^12*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 6696*x^15*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] - 4050*x^18*Hypergeometric2F1[2, 7/3, 10/3, (-2*x^3)/(1 - x^3)] + 27*x^6*(1 + x^3)^2*(7 - 18*x^3 - 105*x^6)*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (-2*x^3)/(1 - x^3)] + 54*x^6*(1 - 15*x^3)*(1 + x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^9*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 486*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 324*x^15*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)] - 81*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, (-2*x^3)/(1 - x^3)])/(280*x^11*(1 - x^3)^(7/3))

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx = -\frac{70-308x^3+1162x^6+2856x^9-4914x^{12}-2268x^{15}+3402x^{18}-70\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\right.}{}$$

Mathematica [A] time = 5.21, size = 136, normalized size = 0.84

$$\frac{1}{120}\left(5\,2^{2/3}\left(-2\log\left(\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+1\right)-2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}-1}{\sqrt{3}}\right)+\log\left(-\frac{\sqrt[3]{2}x}{\sqrt[3]{x^3-1}}+\frac{2^{2/3}x^2}{\left(x^3-1\right)^{2/3}}+1\right)\right)-\frac{3\left(-49x^9+\right.}{x^8\sqrt[3]{}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] ((-3*(5 + x^3 + 23*x^6 - 49*x^9))/(x^8*(1 - x^3)^(1/3)) + 5*2^(2/3)*(-2*Sqr
t[3]*ArcTan[(-1 + (2*2^(1/3)*x)/(-1 + x^3)^(1/3))/Sqrt[3]] + Log[1 + (2^(2/
3)*x^2)/(-1 + x^3)^(2/3) - (2^(1/3)*x)/(-1 + x^3)^(1/3)] - 2*Log[1 + (2^(1/
3)*x)/(-1 + x^3)^(1/3)]))/120
```

fricas [B] time = 4.96, size = 351, normalized size = 2.17

$$-\frac{10\sqrt{6}\,2^{\frac{1}{6}}(-1)^{\frac{1}{3}}\left(x^{11}-x^8\right)\arctan\left(\frac{2^{\frac{1}{6}}\left(6\sqrt{6}\,2^{\frac{2}{3}}(-1)^{\frac{2}{3}}\left(5x^7+4x^4-x\right)\left(-x^3+1\right)^{\frac{2}{3}}-12\sqrt{6}(-1)^{\frac{1}{3}}\left(19x^8-16x^5+x^2\right)\left(-x^3+1\right)^{\frac{1}{3}}-\sqrt{6}\,2^{\frac{1}{3}}\left(71x^9-11\right.\right.}{6\left(109x^9-105x^6+3x^3+1\right)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/360*(10*sqrt(6)*2^(1/6)*(-1)^(1/3)*(x^11 - x^8)*arctan(1/6*2^(1/6)*(6*sq
rt(6)*2^(2/3)*(-1)^(2/3)*(5*x^7 + 4*x^4 - x)*(-x^3 + 1)^(2/3) - 12*sqrt(6)*
(-1)^(1/3)*(19*x^8 - 16*x^5 + x^2)*(-x^3 + 1)^(1/3) - sqrt(6)*2^(1/3)*(71*x
^9 - 111*x^6 + 33*x^3 - 1))/(109*x^9 - 105*x^6 + 3*x^3 + 1)) - 10*2^(2/3)*(
-1)^(1/3)*(x^11 - x^8)*log((6*2^(1/3)*(-1)^(2/3)*(-x^3 + 1)^(1/3)*x^2 - 2^(
2/3)*(-1)^(1/3)*(x^3 + 1) + 6*(-x^3 + 1)^(2/3)*x)/(x^3 + 1)) + 5*2^(2/3)*(-
1)^(1/3)*(x^11 - x^8)*log(-(3*2^(2/3)*(-1)^(1/3)*(5*x^4 - x)*(-x^3 + 1)^(2/
3) - 2^(1/3)*(-1)^(2/3)*(19*x^6 - 16*x^3 + 1) + 12*(2*x^5 - x^2)*(-x^3 + 1)
^(1/3))/(x^6 + 2*x^3 + 1)) + 9*(49*x^9 - 23*x^6 - x^3 - 5)*(-x^3 + 1)^(2/3)
)/(x^11 - x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

maple [C] time = 2.68, size = 649, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] 1/40*(49*x^9-23*x^6-x^3-5)/x^8/(-x^3+1)^(1/3)-1/2*RootOf(RootOf(_Z^3-4)^2+6
*_Z*RootOf(_Z^3-4)+36*_Z^2)*ln(-(-9*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^
3-4)+36*_Z^2)*RootOf(_Z^3-4)^3*x^3-36*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_
Z^3-4)+36*_Z^2)^2*RootOf(_Z^3-4)^2*x^3+12*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3
-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x+4*RootOf(_Z^3-4)^2*(-
x^3+1)^(1/3)*x^2+30*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3
-4)+36*_Z^2)*RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+12*RootOf(RootOf(_Z^3-
4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*x^3-2*(-x^3+1)^(2/3)*x-3*RootOf(_Z^3-4)-1
2*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))-1/
12*RootOf(_Z^3-4)*ln(-(3*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^
2)*RootOf(_Z^3-4)^3*x^3+27*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_
Z^2)^2*RootOf(_Z^3-4)^2*x^3-6*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*R
ootOf(_Z^3-4)+36*_Z^2)*RootOf(_Z^3-4)^2*x-2*RootOf(_Z^3-4)^2*(-x^3+1)^(1/3)
*x^2+3*(-x^3+1)^(1/3)*RootOf(RootOf(_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2)*
RootOf(_Z^3-4)*x^2+3*RootOf(_Z^3-4)*x^3+27*RootOf(RootOf(_Z^3-4)^2+6*_Z*Roo
tOf(_Z^3-4)+36*_Z^2)*x^3-5*(-x^3+1)^(2/3)*x-RootOf(_Z^3-4)-9*RootOf(RootOf(
_Z^3-4)^2+6*_Z*RootOf(_Z^3-4)+36*_Z^2))/(x+1)/(x^2-x+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^9\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^9*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**9*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.652 $\int \frac{x^{10}}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=292

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-2\cdots}{\cdots}\right)}{2\sqrt[3]{2}\cdots}$$

```
[Out] 1/2*x^5/(-x^3+1)^(1/3)+3/4*x^2*(-x^3+1)^(2/3)-1/2*x^2*hypergeom([1/3, 2/3],
[5/3],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1
)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3
+1)^(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan
(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arcta
n(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{11}x^{11}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^11*AppellF1[11/3, 4/3, 1, 14/3, x^3, -x^3])/11
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{11}x^{11}F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.14, size = 71, normalized size = 0.24

$$\frac{1}{20}x^2\left(-4x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-15F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)-\frac{5\left(x^3-3\right)}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^10/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*((-5*(-3 + x^3))/(1 - x^3)^(1/3) - 15*AppellF1[2/3, 1/3, 1, 5/3, x^3,
-x^3] - 4*x^3*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/20
```

fricas [F] time = 2.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^{10}}{x^9-x^6-x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^10/(x^9 - x^6 - x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^10/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^10/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{10}}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^10/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**10/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x
)
```

3.653 $\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=274

$$-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{\tan^{-1}\left(\cdots\right.}{2\sqrt[3]{\cdots}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-3/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^8*AppellF1[8/3, 4/3, 1, 11/3, x^3, -x^3])/8
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^7}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{8}x^8F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.12, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(-3x^3F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - 3*x^3*A
ppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

fricas [F] time = 2.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^{3}+1\right)^{\frac{2}{3}} x^{7}}{x^{9}-x^{6}-x^{3}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^7/(x^9 - x^6 - x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-x^{3}+1\right)^{\frac{4}{3}} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^7/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(x^{3}+1\right)\left(-x^{3}+1\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^7/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7}}{\left(1-x^{3}\right)^{4/3} \left(x^{3}+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^7/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}}{\left(-\left(x-1\right)\left(x^{2}+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^{2}-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**7/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.654 $\int \frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx$

Optimal. Leaf size=274

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}\,(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{1-2\ldots}{\ldots}\right)}{2\sqrt[3]{2}\ldots}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)-1/48*ln((1-x
)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {510}

$$\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^5*AppellF1[5/3, 4/3, 1, 8/3, x^3, -x^3])/5
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{5}x^5F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.09, size = 66, normalized size = 0.24

$$\frac{1}{10}x^2\left(x^3\left(-F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)\right)-5F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5}{\sqrt[3]{1-x^3}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*(5/(1 - x^3)^(1/3) - 5*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x^3] - x^3*App
ellF1[5/3, 1/3, 1, 8/3, x^3, -x^3]))/10
```

fricas [F] time = 3.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x^4}{x^9-x^6-x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x^4/(x^9 - x^6 - x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x^4/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x^4/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(1-x^3\right)}^{4/3}\,\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x^4/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}}\left(x+1\right)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(x**4/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.655 $\int \frac{x}{(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=274

$$-\frac{1}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{(1-x^3)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}+\frac{\tan^{-1}\left(\right.}{2\sqrt[3]{}}$$

```
[Out] 1/2*x^2/(-x^3+1)^(1/3)-1/4*x^2*hypergeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x
)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(
-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16
*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)
/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/
(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 0.09, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {510}

$$\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Int[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (x^2*AppellF1[2/3, 4/3, 1, 5/3, x^3, -x^3])/2
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=\frac{1}{2}x^2F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};x^3,-x^3\right)$$

Mathematica [C] time = 0.06, size = 45, normalized size = 0.16

$$\frac{x^2}{2\sqrt[3]{1-x^3}}-\frac{1}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] x^2/(2*(1 - x^3)^(1/3)) - (x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

fricas [F] time = 3.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}x}{x^9-x^6-x^3+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)*x/(x^9 - x^6 - x^3 + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(x/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(x/((x^3 + 1)*(-x^3 + 1)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(x/((1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+1)**(4/3)/(x**3+1),x)
[Out] Integral(x/((-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)), x)
```

3.656 $\int \frac{1}{x^2(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=292

$$-\frac{3}{4}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{3\left(1-x^3\right)^{2/3}}{2x}+\frac{1}{2\sqrt[3]{1-x^3}\,x}-\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}+\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(2^{2/3}\right.}{}$$

```
[Out] 1/2/x/(-x^3+1)^(1/3)-3/2*(-x^3+1)^(2/3)/x-3/4*x^2*hypergeom([1/3, 2/3],[5/3
],x^3)-1/48*ln((1-x)*(1+x)^2)*2^(2/3)-1/24*ln(1+2^(2/3)*(1-x)^2/(-x^3+1)^(2
/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)+1/12*ln(1+2^(1/3)*(1-x)/(-x^3+1)^
(1/3))*2^(2/3)+1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^(2/3)-1/12*arctan(1/3
*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)-1/24*arctan(1/
3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -(AppellF1[-1/3, 4/3, 1, 2/3, x^3, -x^3]/x)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};x^3,-x^3\right)}{x}$$

Mathematica [C] time = 0.06, size = 76, normalized size = 0.26

$$-\frac{3}{10}x^5F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)-x^2F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{3x^3-2}{2\sqrt[3]{1-x^3}\,x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] (-2 + 3*x^3)/(2*x*(1 - x^3)^(1/3)) - x^2*AppellF1[2/3, 1/3, 1, 5/3, x^3, -x
^3] - (3*x^5*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/10
```

fricas [F] time = 2.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{11}-x^8-x^5+x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
[Out] integral((-x^3 + 1)^(2/3)/(x^11 - x^8 - x^5 + x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{4}{3}}\left(x^3+1\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
[Out] int(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(1-x^3\right)^{4/3}\left(x^3+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)),x)
[Out] int(1/(x^2*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-(x-1)\left(x^2+x+1\right)\right)^{\frac{4}{3}}(x+1)\left(x^2-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**2*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.657 $\int \frac{1}{x^5(1-x^3)^{4/3}(1+x^3)}\,dx$

Optimal. Leaf size=308

$$-\frac{1}{2}x^2\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};x^3\right)-\frac{\left(1-x^3\right)^{2/3}}{x}+\frac{\log\left(\frac{2^{2/3}(1-x)^2}{\left(1-x^3\right)^{2/3}}-\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{12\sqrt[3]{2}}-\frac{\log\left(\frac{\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}+1\right)}{6\sqrt[3]{2}}-\frac{\log\left(2^{2/3}\sqrt[3]{1-x^3}+x-1\right)}{8\sqrt[3]{2}}$$

```
[Out] 1/2/x^4/(-x^3+1)^(1/3)-3/4*(-x^3+1)^(2/3)/x^4-(-x^3+1)^(2/3)/x-1/2*x^2*hype
rgeom([1/3, 2/3],[5/3],x^3)+1/48*ln((1-x)*(1+x)^2)*2^(2/3)+1/24*ln(1+2^(2/3
)*(1-x)^2/(-x^3+1)^(2/3)-2^(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/12*ln(1+2^
(1/3)*(1-x)/(-x^3+1)^(1/3))*2^(2/3)-1/16*ln(-1+x+2^(2/3)*(-x^3+1)^(1/3))*2^
(2/3)+1/12*arctan(1/3*(1-2*2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3
^(1/2)+1/24*arctan(1/3*(1+2^(1/3)*(1-x)/(-x^3+1)^(1/3))*3^(1/2))*2^(2/3)*3^
(1/2)
```

Rubi [C] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 0.08, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {510}

$$-\frac{F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -AppellF1[-4/3, 4/3, 1, -1/3, x^3, -x^3]/(4*x^4)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(1-x^3\right)^{4/3}\left(1+x^3\right)}\,dx=-\frac{F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};x^3,-x^3\right)}{4x^4}$$

Mathematica [C] time = 0.10, size = 79, normalized size = 0.26

$$-\frac{4x^9F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};x^3,-x^3\right)+5x^6F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};x^3,-x^3\right)+\frac{5\left(-4x^6+x^3+1\right)}{\sqrt[3]{1-x^3}}}{20x^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(1 - x^3)^(4/3)*(1 + x^3)),x]
```

```
[Out] -1/20*((5*(1 + x^3 - 4*x^6))/(1 - x^3)^(1/3) + 5*x^6*AppellF1[2/3, 1/3, 1,
5/3, x^3, -x^3] + 4*x^9*AppellF1[5/3, 1/3, 1, 8/3, x^3, -x^3])/x^4
```

fricas [F] time = 3.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-x^3+1\right)^{\frac{2}{3}}}{x^{14}-x^{11}-x^8+x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="fricas")
```

```
[Out] integral((-x^3 + 1)^(2/3)/(x^14 - x^11 - x^8 + x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-x^3+1\right)^{\frac{4}{3}} \left(x^3+1\right) x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

```
[Out] int(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(x^3+1\right)\left(-x^3+1\right)^{\frac{4}{3}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(-x^3+1)^(4/3)/(x^3+1),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 1)*(-x^3 + 1)^(4/3)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(1-x^3\right)^{4/3} \left(x^3+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)),x)
```

```
[Out] int(1/(x^5*(1 - x^3)^(4/3)*(x^3 + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(-\left(x-1\right)\left(x^2+x+1\right)\right)^{\frac{4}{3}} \left(x+1\right)\left(x^2-x+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(-x**3+1)**(4/3)/(x**3+1),x)
```

```
[Out] Integral(1/(x**5*(-(x - 1)*(x**2 + x + 1))**(4/3)*(x + 1)*(x**2 - x + 1)),
x)
```

3.658 $\int \frac{x^{11} \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=264

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^3\sqrt[3]{bc-}}{}$$

```
[Out] -c^3*(b*x^3+a)^(1/3)/d^4+1/4*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(4/3)/b^3/
d^3-1/7*(2*a*d+b*c)*(b*x^3+a)^(7/3)/b^3/d^2+1/10*(b*x^3+a)^(10/3)/b^3/d-1/6
*c^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^3*(-a*d+b*c)^(1/3)*ln((-a*
d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^3*(-a*d+b*c)^(1/3)*arc
tan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^
(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{4b^3d^3}-\frac{\left(a+bx^3\right)^{7/3}(2ad+bc)}{7b^3d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^3d}-\frac{c^3\sqrt[3]{a+bx^3}}{d^4}-\frac{c^3\sqrt[3]{bc-ad}\log\left(c+d\right.}{6d^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c^3*(a + b*x^3)^(1/3))/d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)
^(4/3))/(4*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(7/3))/(7*b^3*d^2) + (a +
b*x^3)^(10/3)/(10*b^3*d) - (c^3*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a
 + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^3*(b*
c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(13/3)) + (c^3*(b*c - a*d)^(1/3)*Log[(b
*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{4/3}}{b^2d^2} + \frac{(a+bx)^{7/3}}{b^2d} - \frac{c^3\sqrt[3]{a}}{d^3(c}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/3}}{10b^3d} - \frac{c^3\,\text{Subst}\left(\int\right.}{3}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/}}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/}}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/}}{10b^3d}$$

$$= -\frac{c^3\sqrt[3]{a+bx^3}}{d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{4/3}}{4b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{7/3}}{7b^3d^2} + \frac{\left(a+bx^3\right)^{10/}}{10b^3d}$$

Mathematica [A] time = 0.68, size = 270, normalized size = 1.02

$$\frac{\frac{105d\left(a+bx^3\right)^{4/3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3}-\frac{60d^2\left(a+bx^3\right)^{7/3}(2ad+bc)}{b^3}+\frac{42d^3\left(a+bx^3\right)^{10/3}}{b^3}-\frac{70c^3\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\right.\right.}{}}{420d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-420*c^3*(a + b*x^3)^(1/3) + (105*d*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x
^3)^(4/3))/b^3 - (60*d^2*(b*c + 2*a*d)*(a + b*x^3)^(7/3))/b^3 + (42*d^3*(a
+ b*x^3)^(10/3))/b^3 - (70*c^3*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^
(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
 a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(420*
d^4)
```

fricas [A] time = 0.53, size = 325, normalized size = 1.23

$$-\frac{140\sqrt{3}\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+70\,b^3c^3\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/420*(140*sqrt(3)*b^3*c^3*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b
*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 70*b^3*c^3*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/
3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 140*b^3*c^3*((b*c - a*d
)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(14*b^3*d^3*x
^9 - 2*(10*b^3*c*d^2 - a*b^2*d^3)*x^6 - 140*b^3*c^3 + 35*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 9*a^3*d^3 + (35*b^3*c^2*d - 5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^3)*
(b*x^3 + a)^(1/3))/(b^3*d^4)
```

giac [A] time = 0.29, size = 379, normalized size = 1.44

$$-\frac{\left(b^{34}c^4d^6-ab^{33}c^3d^7\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{34}cd^{10}-ab^{33}d^{11}\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\right.}{3\left(-\frac{bc-ad}{d}\right.}\right.}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^34*c^4*d^6 - a*b^33*c^3*d^7)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^34*c*d^10 - a*b^33*d^11) + 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^5 + 1/6*(-b*c*d^2 + a*d^
3)^(1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^30*c^3*d^
6 - 35*(b*x^3 + a)^(4/3)*b^29*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b^28*c*d^8 - 3
```

```
5*(b*x^3 + a)^(4/3)*a*b^28*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^27*d^9 + 40*(b*x
^3 + a)^(7/3)*a*b^27*d^9 - 35*(b*x^3 + a)^(4/3)*a^2*b^27*d^9)/(b^30*d^10)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{11}}{d\,x^{3}+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.98, size = 442, normalized size = 1.67

$$\left(\frac{3\,a^2}{4\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{4\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3}-\left(\frac{3\,a}{7\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{7\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}-{\left(b\,x^3+a\right)}^{1/3}\,\left(\frac{a^3}{b^3\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(4*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(4*b^3*d))*(a + b*x^3)^(4/3) - ((3*a)/(7*b^3*d) + (b^4*c - a*b^
3*d)/(7*b^6*d^2))*(a + b*x^3)^(7/3) - (a + b*x^3)^(1/3)*(a^3/(b^3*d) + (((3
*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a*b
^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(b^3*d)) + (a + b*x^3)^(10/3)/(10*b^3*d)
 - (c^3*log((a*d - b*c)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3
))/(3*d^(13/3)) - (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4 - a*c^3*d))/d^2 + (3
*c^3*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(4/3))/d^(7/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(1/3))/(3*d^(13/3)) + (c^3*log((3*(a + b*x^3)^(1/3)*(b*c^4
- a*c^3*d))/d^2 - (9*c^3*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(4/3))/d^(7/3))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(1/3))/d^(13/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \sqrt[3]{a + b x^{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.659 $\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=220

$$-\frac{(a+bx^3)^{4/3}(ad+bc)}{4b^2d^2}+\frac{(a+bx^3)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

```
[Out] c^2*(b*x^3+a)^(1/3)/d^3-1/4*(a*d+b*c)*(b*x^3+a)^(4/3)/b^2/d^2+1/7*(b*x^3+a)
^(7/3)/b^2/d+1/6*c^2*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(10/3)-1/2*c^2*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c^2*(-a*
d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1
/2))/d^(10/3)*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{(a+bx^3)^{4/3}(ad+bc)}{4b^2d^2}+\frac{(a+bx^3)^{7/3}}{7b^2d}+\frac{c^2\sqrt[3]{a+bx^3}}{d^3}+\frac{c^2\sqrt[3]{bc-ad}\log(c+dx^3)}{6d^{10/3}}-\frac{c^2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(1/3))/d^3 - ((b*c + a*d)*(a + b*x^3)^(4/3))/(4*b^2*d^2) +
 (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*(b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3
)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c^2
*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c^2*(b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2 \sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{(-bc-ad)\sqrt[3]{a+bx}}{bd^2} + \frac{(a+bx)^{4/3}}{bd} + \frac{c^2\sqrt[3]{a+bx}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} - \frac{\left(c^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,d\right.}{3d^3}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{\left(c^2\sqrt[3]{bc-\right.}{}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c^2\sqrt[3]{bc-}}{}$$

$$= \frac{c^2\sqrt[3]{a+bx^3}}{d^3} - \frac{(bc+ad)\left(a+bx^3\right)^{4/3}}{4b^2d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^2d} + \frac{c^2\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}} + \frac{c^{}}{}$$

Mathematica [A] time = 0.41, size = 230, normalized size = 1.05

$$\frac{-\frac{21d\left(a+bx^3\right)^{4/3}(ad+bc)}{b^2}+\frac{12d^2\left(a+bx^3\right)^{7/3}}{b^2}+\frac{14c^2\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+b\ldots}\right.\right.}{\sqrt[3]{d}}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (84*c^2*(a + b*x^3)^(1/3) - (21*d*(b*c + a*d)*(a + b*x^3)^(4/3))/b^2 + (12*
d^2*(a + b*x^3)^(7/3))/b^2 + (14*c^2*(b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1
 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*
(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))
/(84*d^3)
```

fricas [A] time = 0.51, size = 282, normalized size = 1.28

$$-\frac{28\sqrt{3}\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+14\,b^2c^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/84*(28*sqrt(3)*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d))
 + 14*b^2*c^2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1
/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 28*b^2*c^2*(-(b*c -
a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 3*(4*b^2*d^
2*x^6 + 28*b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2 - (7*b^2*c*d - a*b*d^2)*x^3)*(b*
x^3 + a)^(1/3))/(b^2*d^3)
```

giac [A] time = 0.25, size = 320, normalized size = 1.45

$$\frac{\left(b^{17}c^3d^4-ab^{16}c^2d^5\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{17}cd^7-ab^{16}d^8\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\ldots\right.\right.}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^17*c^3*d^4 - a*b^16*c^2*d^5)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^17*c*d^7 - a*b^16*d^8) - 1/3*sqrt(3
)*(-b*c*d^2 + a*d^3)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-
(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^4 - 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28*(28*(b*x^3 + a)^(1/3)*b^14*c^2*d^4 - 7
*(b*x^3 + a)^(4/3)*b^13*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^12*d^6 - 7*(b*x^3 + a
)^(4/3)*a*b^12*d^6)/(b^14*d^7)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{8}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.93, size = 336, normalized size = 1.53

$$\left(\frac{a^2}{b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\left(\frac{a}{2\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{4\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^2\,d}+\frac{c^2\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*b^
2*d))/(b^2*d))*(a + b*x^3)^(1/3) - (a/(2*b^2*d) + (b^3*c - a*b^2*d)/(4*b^4*
d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^2*d) + (c^2*log((a*d - b*c
)^(1/3) - d^(1/3)*(a + b*x^3)^(1/3))*(a*d - b*c)^(1/3))/(3*d^(10/3)) - (c^2
*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d - (3*c^2*((3^(1/2)*1i)/2 + 1
/2)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(
3*d^(10/3)) + (c^2*log((3*(a + b*x^3)^(1/3)*(b*c^3 - a*c^2*d))/d + (9*c^2*(
(3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(4/3))*((3^(1/2)*1i)/6 - 1/6)*(a
*d - b*c)^(1/3))/d^(10/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8} \sqrt[3]{a + b x^{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.660 $\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=186

$$-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{c\sqrt[3]{a}}{}$$

```
[Out] -c*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/b/d-1/6*c*(-a*d+b*c)^(1/3)*ln(d*
x^3+c)/d^(7/3)+1/2*c*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)
^(1/3))/d^(7/3)-1/3*c*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\sqrt[3]{a+bx^3}}{d^2}-\frac{c\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{c\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{c\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a}}{\sqrt[3]{}}}{\sqrt{}}\right)}{\sqrt{3}\,d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) - (c*(b*c - a*d)^(
1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(7/3)) - (c*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*d^(7/3)) + (c*
(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^
(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)\\
&= \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d}\\
&= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} + \frac{(c(bc-ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}\\
&= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{\left(c\sqrt[3]{bc-ad}\right)\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,d\right.}{2d^{7/3}}\\
&= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{7/3}} + \frac{c\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{\right.}}{2d^{7/3}}\\
&= -\frac{c\sqrt[3]{a+bx^3}}{d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4bd} - \frac{c\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}} - \frac{c\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{7/3}} +
\end{aligned}
$$

Mathematica [A] time = 0.47, size = 204, normalized size = 1.10

$$
\frac{c\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right.\right.}{6d^{7/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] -((c*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*b*d) + (c*(b*c - a*d)^(
1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))/(6*d^(7/3))
```

fricas [A] time = 0.61, size = 222, normalized size = 1.19

$$-\frac{4\sqrt{3}\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+2\,bc\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}\left(\frac{bc-ad}{d}\right.\right.}{12\,bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*b*c*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 +
a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*b*
c*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a
*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 4*b*c*((b*c - a*d)/d)^(1/3)*log((b*
x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)) - 3*(b*d*x^3 - 4*b*c + a*d)*(b*x^3
+ a)^(1/3))/(b*d^2)
```

giac [A] time = 0.25, size = 276, normalized size = 1.48

$$-\frac{\left(b^6c^2d^2-ab^5cd^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^6cd^4-ab^5d^5\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-}{d}\right.\right.}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right.}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^6*c^2*d^2 - a*b^5*c*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)
^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^6*c*d^4 - a*b^5*d^5) + 1/3*sqrt(3)*(-b
*c*d^2 + a*d^3)^(1/3)*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*c
*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c
- a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b^4*c*d^2 - (b*x^3 + a)^(4/
3)*b^3*d^3)/(b^4*d^4)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^5}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.62, size = 298, normalized size = 1.60

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b\,d}-\left(b\,x^3+a\right)^{1/3}\,\left(\frac{a}{b\,d}+\frac{b^2\,c-a\,b\,d}{b^2\,d^2}\right)-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,b\,c^2-3\,a\,c\,d\right)+\frac{c\,{\left(a\,d-b\,c\right)}^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,{\left(a\,d}{3\,d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b*d) - (a + b*x^3)^(1/3)*(a/(b*d) + (b^2*c - a*b*d)/(b
^2*d^2)) - (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*(a*d - b*c)^(1
/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b*c)^(1/3))/(3*d^(7/3)) - (c
*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d) + (c*((3^(1/2)*1i)/2 - 1/2)*(a*d
 - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 - 1/2)*(a
*d - b*c)^(1/3))/(3*d^(7/3)) + (c*log((a + b*x^3)^(1/3)*(3*b*c^2 - 3*a*c*d)
 - (c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^
(7/3)))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/(3*d^(7/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.661 $\int \frac{x^2 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=159

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{a+bx}}{d}$$

```
[Out] (b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^(4/3)-1/2*(-a*d+b*c)^(
1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)+1/3*(-a*d+b*c)^(1
/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4
/3)*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 50, 58, 617, 204, 31}

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6d^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}}+\frac{\sqrt[3]{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(1/3)/d + ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)) + ((b*c - a*d)^(1/3)
*Log[c + d*x^3])/(6*d^(4/3)) - ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d
^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{(bc \cdots}{\cdots}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{4/3}} - \frac{\sqrt[3]{bc-\cdots}}{\cdots}$$

$$= \frac{\sqrt[3]{a+bx^3}}{d} + \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}} + \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^{4/3}} - \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc \cdots\right.}{\cdots}$$

Mathematica [A] time = 0.29, size = 205, normalized size = 1.29

$$\frac{\sqrt[3]{bc-ad}\,\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a \cdots\right.}{6d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*
d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3
)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Lo
g[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)
*(a + b*x^3)^(2/3)])/(6*d^(4/3))
```

fricas [A] time = 0.53, size = 206, normalized size = 1.30

$$-\frac{2\,\sqrt{3}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(
1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + (-(b*c
- a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^
(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^
(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 + a)^(1/3))/d
```

giac [A] time = 0.27, size = 223, normalized size = 1.40

$$\frac{\left(bc-ad\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd-ad^2\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3\,d^2}+\frac{(bx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c -
 a*d)/d)^(1/3)))/(b*c*d - a*d^2) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d
)/d)^(1/3))/d^2 + (b*x^3 + a)^(1/3)/d - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/
d)^(2/3))/d^2
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}} x^2}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.62, size = 249, normalized size = 1.57

$$\frac{\left(b\,x^3+a\right)^{1/3}}{d}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-3\,b\,c\,d\right)-\frac{(a\,d-b\,c)^{1/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}}\right)\,(a\,d-b\,c)^{1/3}}{3\,d^{4/3}}-\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a\,d^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(1/3)/d + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) - ((a*d -
b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*(a*d - b*c)^(1/3))/(3*d^(4/3
)) - (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(
a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(4/3)))*((3^(1/2)*1i)/2 + 1/2)
*(a*d - b*c)^(1/3))/(3*d^(4/3)) + (log((a + b*x^3)^(1/3)*(3*a*d^2 - 3*b*c*d
) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3)*(9*a*d^3 - 9*b*c*d^2))/d^(4/3
))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(1/3))/d^(4/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.662 $\int \frac{\sqrt[3]{a+bx^3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log$$

```
[Out] -1/2*a^(1/3)*ln(x)/c-1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c/d^(1/3)+1/2*a^(1/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(1/3)-1/3*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(1/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 57, 617, 204, 31, 58}

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c\sqrt[3]{d}}+\frac{\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{d}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{d}}+\frac{\sqrt[3]{a}\log$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] -((a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt
[3]*c)) - ((b*c - a*d)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(1/3)) - (a^(1/3)*Log[x])/(2*c) - ((b
*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c*d^(1/3)) + (a^(1/3)*Log[a^(1/3) - (a +
 b*x^3)^(1/3)])/(2*c) + ((b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)])/(2*c*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{a^{2/3}\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6c\sqrt[3]{d}} + \frac{\sqrt[3]{a}\,\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} + \frac{\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-\right.}{2c\sqrt[3]{}}$$

$$= -\frac{\sqrt[3]{a}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{\sqrt[3]{bc-ad}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{d}} - \frac{\sqrt[3]{a}\,\log(x)}{2c} - \frac{\sqrt[3]{bc-ad}\,\log\left(c+\right.}{6c\sqrt[3]{d}}$$

Mathematica [A] time = 0.47, size = 268, normalized size = 1.09

$$\frac{\frac{\sqrt[3]{bc-ad}\left(-\log\left(-\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\,\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}-1}{\sqrt{3}}\right)\right)}{\sqrt[3]{d}} - \sqrt[3]{a}\left(\log\left(a^{2}\right.\right.}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x*(c + d*x^3)),x]
```

```
[Out] (-(a^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] -
 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/
3) + (a + b*x^3)^(2/3)])) + ((b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d
^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/d^(1/3))/(6*c)
```

fricas [A] time = 0.77, size = 276, normalized size = 1.12

$$-\frac{2\sqrt{3}\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+2\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+a^{\frac{1}{3}}\log\left(\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1
/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 2*sqrt(3)
*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) + sqrt(3)*a)/a) +
a^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + ((b*
c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)
^(1/3) + ((b*c - a*d)/d)^(2/3)) - 2*a^(1/3)*log((b*x^3 + a)^(1/3) - a^(1/3)
) - 2*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))
/c
```

giac [A] time = 0.80, size = 311, normalized size = 1.26

$$-\frac{\left(bc-ad\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3c}-\frac{a^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\right.}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b*c^2 - a*c*d) - 1/3*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*
(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/c - 1/6*a^(1/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c + 1/3*a^(1/3)*log(abs((b*x^3 +
 a)^(1/3) - a^(1/3)))/c + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*s
qrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3
))/(c*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x), x)
```

mupad [B] time = 4.74, size = 1607, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x*(c + d*x^3)),x)
```

```
[Out] log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) - (a/(27*c^3))^(1/3)*(((243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3) - (a + b*x^3)^(1/3)*(81*
a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4))*(a/(27*c^3))^(2/3) - 9*a*b^7*c^4*d^2 +
 27*a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4))*(a/(27*c^3))^(1/3) + log((a + b*
x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*
c^2*d^3) - (((243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5
)*(-(a*d - b*c)/(27*c^3*d))^(1/3) - (a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 8
1*a^2*b^5*c^4*d^4))*(-(a*d - b*c)/(27*c^3*d))^(2/3) - 9*a*b^7*c^4*d^2 + 27*
a^2*b^6*c^3*d^3 - 18*a^3*b^5*c^2*d^4)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(-(a
*d - b*c)/(27*c^3*d))^(1/3) + log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^
7*c^3*d^2 - 12*a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*
(-(a*d - b*c)/(27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 - 1/2)^2*((a + b*x^3)^(1/3
)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b
^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c
^3*d))^(1/3))*(-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^
6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d - b*c)/(27*c
^3*d))^(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*
a^3*b^5*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(
27*c^3*d))^(1/3)*(((3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5
*d^3 - 81*a^2*b^5*c^4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 72
9*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a*d - b*c)/(27*c^3*d))^(1/3))*(
-(a*d - b*c)/(27*c^3*d))^(2/3) + 9*a*b^7*c^4*d^2 - 27*a^2*b^6*c^3*d^3 + 18*
a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)/(27*c^3*d))^(1/3) +
log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5*c*d^4 +
 9*a^2*b^6*c^2*d^3) + ((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^(1/3)*(((3^(1/2)*
1i)/2 - 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^4*d^4) -
 ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*
b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d^2 - 27*
a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 - 1/2)*(a/(27*c^3))^
(1/3) - log((a + b*x^3)^(1/3)*(6*a^4*b^4*d^5 - 3*a*b^7*c^3*d^2 - 12*a^3*b^5
*c*d^4 + 9*a^2*b^6*c^2*d^3) - ((3^(1/2)*1i)/2 + 1/2)*(a/(27*c^3))^(1/3)*(((
3^(1/2)*1i)/2 + 1/2)^2*((a + b*x^3)^(1/3)*(81*a*b^6*c^5*d^3 - 81*a^2*b^5*c^
4*d^4) + ((3^(1/2)*1i)/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 +
486*a^3*b^4*c^4*d^5)*(a/(27*c^3))^(1/3))*(a/(27*c^3))^(2/3) + 9*a*b^7*c^4*d
^2 - 27*a^2*b^6*c^3*d^3 + 18*a^3*b^5*c^2*d^4))*((3^(1/2)*1i)/2 + 1/2)*(a/(2
7*c^3))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x*(c + d*x**3)), x)
```

3.663 $\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=340

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}$$

```
[Out] d*(b*x^3+a)^(1/3)/c^2+1/3*(-3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2-1/3*(b*x^3+a)^
(4/3)/a/c/x^3-1/6*(-3*a*d+b*c)*ln(x)/a^(2/3)/c^2+1/6*d^(2/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^2
-1/2*d^(2/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2-1/9*(-3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2)
)/a^(2/3)/c^2*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{(bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{2/3}c^2}-\frac{(bc-3ad)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}c^2}-\frac{\log(x)(bc-3ad)}{6a^{2/3}c^2}+\frac{d^{2/3}\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(1/3))/c^2 + ((b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*a*c^2) - (
a + b*x^3)^(4/3)/(3*a*c*x^3) - ((b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*x^
3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*c^2) + (d^(2/3)*(b*c - a*d
)^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3
]])/(Sqrt[3]*c^2) - ((b*c - 3*a*d)*Log[x])/(6*a^(2/3)*c^2) + (d^(2/3)*(b*c
- a*d)^(1/3)*Log[c + d*x^3])/(6*c^2) + ((b*c - 3*a*d)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(6*a^(2/3)*c^2) - (d^(2/3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(
1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
```

```
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^2(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{1}{3}(-bc+3ad)-\frac{bdx}{3}\right)}{x(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^2} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}\,dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{9c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\log(}{6c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\log(x)}{6a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc-ad}\,\log(}{6c^2}$$

$$= \frac{d\sqrt[3]{a+bx^3}}{c^2} + \frac{(bc-3ad)\sqrt[3]{a+bx^3}}{3ac^2} - \frac{\left(a+bx^3\right)^{4/3}}{3acx^3} - \frac{(bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}c^2} + \frac{d^{2/3}\sqrt[3]{bc}}{}$$

Mathematica [A] time = 1.24, size = 366, normalized size = 1.08

$$\frac{\frac{(bc-3ad)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{3c} + \frac{ad^{2/3}\left(\sqrt[3]{bc-ad}\,\log\left(-\sqrt[3]{d}\sqrt[3]{a+b}\right.\right.}{}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(4/3)/x^3) + ((b*c - 3*a*d)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1
/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b
*x^3)^(2/3)]))/2))/(3*c) + (a*d^(2/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt
[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d
)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c -
a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(2*c))/(3*a*c)
```

fricas [A] time = 0.73, size = 429, normalized size = 1.26

$$-\frac{6\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\arctan\left(-\frac{2\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+3\left(-bcd^2+ad^3\right)^{\frac{1}{3}}a^2x^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*arctan(-1/3*(2*sqrt(3)*(-
b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
- a*d^2)) + 3*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(2/3)*d^2 +
(-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
6*(-b*c*d^2 + a*d^3)^(1/3)*a^2*x^3*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*
d^3)^(1/3)) + 2*sqrt(3)*(a*b*c - 3*a^2*d)*x^3*sqrt(-(-a^2)^(1/3))*arctan(-1
/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt
(-(-a^2)^(1/3))/a^2) + (-a^2)^(2/3)*(b*c - 3*a*d)*x^3*log((b*x^3 + a)^(2/3)
*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*(b*c
 - 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a - (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)
*a^2*c)/(a^2*c^2*x^3)
```

giac [A] time = 0.78, size = 351, normalized size = 1.03

$$\frac{\left(bcd - ad^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3 - ac^2d\right)} - \frac{\sqrt{3}\,(bc - 3\,ad)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}c^2} - \frac{\sqrt{3}\left(-bcd^2 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d - a*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b
*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(b*c - 3*a*d)*arctan(1
/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(2/3)*c^2) - 1/3*sqr
t(3)*(-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^2 - 1/18*(b*c - 3*a*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^2) - 1/6
*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^2 + 1/9*(b*c - 3*a*d)*log(abs((
b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)/(c*x^3)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{1}{3}}}{\left(d\,x^3 + c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{\left(dx^3 + c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^4), x)
```

mupad [B] time = 9.99, size = 1917, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d
) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a
^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6))^(2/3))/81 - (b^5*d^4*(27*a^3*d
^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(
a^2*c^6))^(1/3))/9 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 +
 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*(-(27*a^3*
d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) + log(
- ((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) -
81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^
(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 +
17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3 -
 (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2
- 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((a*d^3 - b*c*d^2)/(27*c^6))^(
1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3
*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 81*a*b^4*c^4*d^3*((3
^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^
6)^(1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3
 + 17*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/
3 - (2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d
^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*((a*
d^3 - b*c*d^2)/(27*c^6))^(1/3) - log((2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d
^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*
c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d) + 81*a*b^4*c^4*d^3*((3^(1/
2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d^2*(a*d - b*c))/c^6)^(
1/3))*((d^2*(a*d - b*c))/c^6)^(2/3))/9 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 1
7*a*b^2*c^2*d - 45*a^2*b*c*d^2))/(3*c))*((d^2*(a*d - b*c))/c^6)^(1/3))/3)*(
(3^(1/2)*1i)/2 + 1/2)*((a*d^3 - b*c*d^2)/(27*c^6))^(1/3) + log((((3^(1/2)*1
i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a
^2*d^2 + b^2*c^2 - 5*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*
a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d
 - b*c)^3/(a^2*c^6))^(2/3))/81 + (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*
c^2*d - 45*a^2*b*c*d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9 - (2*
b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 - 32
*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d
^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1/3) - log((
2*b^4*d^5*(a + b*x^3)^(1/3)*(27*a^4*d^4 + 5*b^4*c^4 + 72*a^2*b^2*c^2*d^2 -
32*a*b^3*c^3*d - 72*a^3*b*c*d^3))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*(27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 + b^2*c^2 -
5*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 -
 3*a*b*c*d)*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))*(-(3*a*d - b*c)^3/(a^2*c^6)
)^(2/3))/81 - (b^5*d^4*(27*a^3*d^3 + b^3*c^3 + 17*a*b^2*c^2*d - 45*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - b*c)^3/(a^2*c^6))^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*
(-(27*a^3*d^3 - b^3*c^3 + 9*a*b^2*c^2*d - 27*a^2*b*c*d^2)/(729*a^2*c^6))^(1
/3) - (a + b*x^3)^(1/3)/(3*c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^4 \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**4/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**4*(c + d*x**3)), x)
```

3.664 $\int \frac{\sqrt[3]{a+bx^3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}+\frac{\log(x)\left(-9a\right.}{}$$

```
[Out] 1/9*(3*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^3-1/6*(b*x^3+a)^(4/3)/a/c/x^6+1/18*
(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(x)/a^(5/3)/c^3-1/6*d^(5/3)*(-a*d+b*c)^(1/
3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a)
^(1/3))/a^(5/3)/c^3+1/2*d^(5/3)*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3
)*(b*x^3+a)^(1/3))/c^3+1/27*(-9*a^2*d^2+3*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(1
/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(5/3)/c^3*3^(1/2)-1/3*d^(5/3)*(-a
*d+b*c)^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(
1/2))/c^3*3^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 149, 156, 57, 617, 204, 31, 58}

$$-\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{5/3}c^3}+\frac{\left(-9a^2d^2+3abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}a^{5/3}c^3}+\frac{\log(x)\left(-9a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 3*a*d)*(a + b*x^3)^(1/3))/(9*a*c^2*x^3) - (a + b*x^3)^(4/3)/(6*a*c*
x^6) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(
1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(5/3)*c^3) - (d^(5/3)*(b*c - a*d)^(1
/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/
(Sqrt[3]*c^3) + ((b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(5/3)*c^3)
 - (d^(5/3)*(b*c - a*d)^(1/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 3*a*b*c
*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(5/3)*c^3) + (d^(5/
3)*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2
*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
```

```
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(\frac{2}{3}(bc+3ad)+\frac{2bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} - \frac{\operatorname{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+3abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-6ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{6ac^2}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c^3} - \dots$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\,\log\dots}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\log(x)}{18a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{bc-ad}\,\log\dots}{6c^3}$$

$$= \frac{(bc+3ad)\sqrt[3]{a+bx^3}}{9ac^2x^3} - \frac{\left(a+bx^3\right)^{4/3}}{6acx^6} + \frac{\left(b^2c^2+3abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{5/3}c^3} - \frac{d^{5/3}\sqrt[3]{\dots}}{\dots}$$

Mathematica [A] time = 1.84, size = 411, normalized size = 1.11

$$-\frac{2\left(-9a^2d^2+3abcd+b^2c^2\right)\left(3\sqrt[3]{a+bx^3}-\frac{1}{2}\sqrt[3]{a}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)}{9ac^2}+\frac{ad^{5/3}\left(\sqrt[3]{b\dots}\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] -1/6*((a + b*x^3)^(4/3)/x^6 - (2*(b*c + 3*a*d)*(a + b*x^3)^(4/3))/(3*a*c*x^
3) + (2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(3*(a + b*x^3)^(1/3) - (a^(1/3)*(
2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/
3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*
x^3)^(2/3)]))/2))/(9*a*c^2) + (a*d^(5/3)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*S
qrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*
(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c
 - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/c^2)/(a*c)
```

fricas [A] time = 3.08, size = 472, normalized size = 1.28

$$-\frac{18\sqrt{3}\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\arctan\left(-\frac{2\sqrt{3}\left(bcd^2-ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right)+9\left(bcd^2-ad^3\right)^{\frac{1}{3}}a^3dx^6\log\left(\left(bx^3+a\dots\right.\right.}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*arctan(-1/3*(2*sqrt(3)*
(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d^2))/(b*c*d
 - a*d^2)) + 9*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(2/3)*d^2
- (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) -
18*(b*c*d^2 - a*d^3)^(1/3)*a^3*d*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a
*d^3)^(1/3)) - 2*sqrt(3)*(a*b^2*c^2 + 3*a^2*b*c*d - 9*a^3*d^2)*(a^2)^(1/6)*
x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(
1/3)*(a^2)^(2/3))/a^2) - (b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*
log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) +
2*(b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((b*x^3 + a)^(1/3)*a
 - (a^2)^(2/3)) + 3*(3*a^3*c^2 + (a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(
1/3))/(a^3*c^3*x^6)
```

giac [A] time = 0.81, size = 465, normalized size = 1.26

$$-\frac{\left(bcd^2-ad^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{3c^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2 - a*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (
-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^
(1/3)*d*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(
-(b*c - a*d)/d)^(1/3))/c^3 + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/c^3 + 1/27*sqrt(3)*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*arc
tan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^3) - 1/27*(
b^2*c^2 + 3*a*b*c*d - 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(
5/3)*c^3) + 1/54*(a^(1/3)*b^2*c^2 + 3*a^(4/3)*b*c*d - 9*a^(7/3)*d^2)*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*((b
*x^3 + a)^(4/3)*b^2*c + 2*(b*x^3 + a)^(1/3)*a*b^2*c - 6*(b*x^3 + a)^(4/3)*a
*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(a*b^2*c^2*x^6)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^7/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^7), x)
```

mupad [B] time = 12.55, size = 2767, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^7*(c + d*x^3)),x)
```

```
[Out] log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*
c))/c^9)^(1/3) + (9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a
*b^2*c^2*d - 14*a^2*b*c*d^2))/a)*(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 - (b^5*d
^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a
^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a
*d - b*c))/c^9)^(1/3))/3 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c
^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564
*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*(-(a*d
^6 - b*c*d^5)/(27*c^9))^(1/3) + log((((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(1
2*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^4*c^4*d^3*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729 -
(b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 +
 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(
b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 - (b^4*d^6*(a + b*x
^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^
3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*
b*c*d^6))/(243*a^3*c^8))*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9
*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - (((a + b*x^3)^(1/3
)*(b^2*c + 3*a*b*d))/(9*c^2) - (b*(a + b*x^3)^(4/3)*(6*a*d - b*c))/(18*a*c^
2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log(- (((3^(1/2)*1i)/2 - 1/2)
*((((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d -
14*a^2*b*c*d^2))/a + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d)*(-(d^5*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 + 1/2)*
(-(d^5*(a*d - b*c))/c^9)^(2/3))/9 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2
*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d -
1458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 - (b^4*d
^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3
*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d
- 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a*d^6 - b*c*d
^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((9*b^5*c^2*d^3*(a + b*
x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 81*a*
b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^5
*(a*d - b*c))/c^9)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(-(d^5*(a*d - b*c))/c^9)^(
2/3))/9 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3
*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3
*c^4))*(-(d^5*(a*d - b*c))/c^9)^(1/3))/3 + (b^4*d^6*(a + b*x^3)^(1/3)*(1458
*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^
3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*
a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a*d^6 - b*c*d^5)/(27*c^9))^(1/3) + log(
- (((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((9*b^5*c^2*d^3*(a + b*x
^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 + a*b^2*c^2*d - 14*a^2*b*c*d^2))/a + 9*a*b^
4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c
^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*
a*b*c*d)^3/(a^5*c^9))^(2/3))/729 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*
b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1
458*a^5*b*c*d^5))/(81*a^3*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*
c^9))^(1/3))/27 - (b^4*d^6*(a + b*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a
^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*
c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2
 - 1/2)*(-(b^6*c^6 - 729*a^6*d^6 - 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 72
```

```
9*a^5*b*c*d^5)/(19683*a^5*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(
1/2)*1i)/2 - 1/2)*((9*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(12*a^3*d^3 + b^3*c^3 +
 a*b^2*c^2*d - 14*a^2*b*c*d^2))/a - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*
(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^
5*c^9))^(1/3))*(-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(2/3))/729
- (b^5*d^4*(729*a^6*d^6 + b^6*c^6 - 9*a^2*b^4*c^4*d^2 - 135*a^3*b^3*c^3*d^3
 + 864*a^4*b^2*c^2*d^4 + 8*a*b^5*c^5*d - 1458*a^5*b*c*d^5))/(81*a^3*c^4))*(
-(b^2*c^2 - 9*a^2*d^2 + 3*a*b*c*d)^3/(a^5*c^9))^(1/3))/27 + (b^4*d^6*(a + b
*x^3)^(1/3)*(1458*a^7*d^7 + b^7*c^7 + 72*a^2*b^5*c^5*d^2 - 135*a^3*b^4*c^4*
d^3 - 1080*a^4*b^3*c^3*d^4 + 3564*a^5*b^2*c^2*d^5 + 8*a*b^6*c^6*d - 3888*a^
6*b*c*d^6))/(243*a^3*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(b^6*c^6 - 729*a^6*d^6
- 135*a^3*b^3*c^3*d^3 + 9*a*b^5*c^5*d + 729*a^5*b*c*d^5)/(19683*a^5*c^9))^(
1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{x^7 \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**7*(c + d*x**3)), x)
```

3.665 $\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=336

$$-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{18b^{5/3}d^3}-\frac{\left(-a^2d^2-3abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{5/3}d^3}-\frac{c^{5/3}\sqrt[3]{bc-ad}\ \mathrm{l}}{6a}$$

```
[Out] -1/18*(-a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/b/d^2+1/6*x^5*(b*x^3+a)^(1/3)/d-1/6*
c^(5/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^3-1/18*(-a^2*d^2-3*a*b*c*d+9*b^2*c^2
)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^3+1/2*c^(5/3)*(-a*d+b*c)^(1/3)*ln
((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(-a^2*d^2-3*a*b*c*d+9
*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^3*3
^(1/2)+1/3*c^(5/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [C] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(1/3)*AppellF1[8/3, -1/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^7 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x^8\sqrt[3]{a+bx^3}\,F_1\left(\frac{8}{3};-\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.37, size = 226, normalized size = 0.67

$$\frac{5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-ad)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)+\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}\left(ad-6bc+3bdx^3\right)\right)-2x^5\left(\frac{bx^3}{a}+1\right)^{2}\cdots}{90bcd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (-2*(-9*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^
3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^
2*((a + b*x^3)*(-6*b*c + a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b*c
- a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*
d)*x^3)/(a*(c + d*x^3))]))/(90*b*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2
/3))
```

fricas [A] time = 2.87, size = 494, normalized size = 1.47

$$\frac{18\sqrt{3}\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(bc^3-ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)+18\left(bc^3-ac^2d\right)^{\frac{1}{3}}b^3c\log\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}c-\left(bc^3-ac^2d\right)\cdots}{x}\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*arctan(-1/3*(sqrt(3)*(b*c^2
- a*c*d)*x + 2*sqrt(3)*(b*c^3 - a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 -
 a*c*d)*x)) + 18*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3 + a)^(1/3)*c - (
b*c^3 - a*c^2*d)^(1/3)*x)/x) - 9*(b*c^3 - a*c^2*d)^(1/3)*b^3*c*log(((b*x^3
+ a)^(2/3)*c^2 + (b*c^3 - a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)*c*x + (b*c^3 - a
*c^2*d)^(2/3)*x^2)/x^2) + 2*sqrt(3)*(9*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*(
b^2)^(1/6)*arctan(1/3*(sqrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3
)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*(9*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*(
b^2)^(2/3)*log(-((b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 3*a
*b*c*d - a^2*d^2)*(b^2)^(2/3)*log(((b^2)^(1/3)*b*x^2 + (b*x^3 + a)^(1/3)*(b
^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d - a
*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^3*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^7}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}} x^{7}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}} x^{7}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^7/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7}\left(b\,x^{3}+a\right)^{1/3}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x^7*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{7}\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.666 $\int \frac{x^4 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=276

$$\frac{(3bc-ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6b^{2/3}d^2}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^2}+\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\left(c+dx^3\right)}{6d^2}-\frac{c^{2/3}\sqrt[3]{bc-ad}\,\log\Big(}{}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/d+1/6*c^(2/3)*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/d^2+1/6*
(-a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^2-1/2*c^(2/3)*(-a*d+b*
c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1/9*(-a*d+3*b*c
)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)/d^2*3^(1/2)-1
/3*c^(2/3)*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [C] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx=\frac{\sqrt[3]{a+bx^3}\int \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$=\frac{x^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.19, size = 185, normalized size = 0.67

$$\frac{x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3}-a\left(\frac{bx^3}{a}+1\right)^{2/3}\right.\ 2}{15cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] ((-3*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1[5/
3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x^2*((a + b*x^3)*(1 + (d*
x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((
-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*c*d*(a + b*x^3)^(2/3)*(1 + (d*x^3
)/c)^(2/3))
```

fricas [B] time = 0.94, size = 452, normalized size = 1.64

$$\frac{6\left(bx^3+a\right)^{\frac{1}{3}}b^2dx^2+6\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{1}{3}}b^2\arctan\left(-\frac{\sqrt{3}\left(bc^2-acd\right)x+2\sqrt{3}\left(-bc^3+ac^2d\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}}{3\left(bc^2-acd\right)x}\right)+6\left(-bc^3+ac^2d\right)^{\frac{1}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*sqrt(3)*(-b*c^3 + a*c^2*d)^(1/3)*b^
2*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c^3 + a*c^2*d)^(2/
3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) + 6*(-b*c^3 + a*c^2*d)^(1/3)*b^2
*log(((b*x^3 + a)^(1/3)*c + (-b*c^3 + a*c^2*d)^(1/3)*x)/x) - 3*(-b*c^3 + a*
c^2*d)^(1/3)*b^2*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x
^3 + a)^(1/3)*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) - 2*sqrt(3)*(3*b^2*c
 - a*b*d)*sqrt(-(-b^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqr
t(3)*(b*x^3 + a)^(1/3)*(-b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) + 2*(-b^2
)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) - (-b^
2)^(2/3)*(3*b*c - a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^
(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^4}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)*x^4/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.667 $\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=234

$$-\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,d}+\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,d}+\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}\,d}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}x\right.}{2}$$

```
[Out] -1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(1/3)/d-1/2*b^(1/3)*ln(b^(1/3)*x-(b*x^3
+a)^(1/3))/d+1/2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(
1/3))/c^(1/3)/d-1/3*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1
/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/d*3^(1/2)
```

Rubi [C] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 0.27, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{x\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^2\sqrt[3]{a+bx^3}F_1\left(\frac{2}{3};-\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{a+bx^3}{a}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(1/3))
```

fricas [A] time = 0.62, size = 330, normalized size = 1.41

$$2\sqrt{3}\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3(bc-ad)x}\right)-2\sqrt{3}(-b)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}bx+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}}{3bx}\right)+2(-b)^{\frac{1}{3}}\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b*c - a*d)/c)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2
*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) - 2*sq
rt(3)*(-b)^(1/3)*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)
^(2/3))/(b*x)) + 2*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2
*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 + a)^(1/3))/x
) - (-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^
3 + a)^(2/3))/x^2) - ((b*c - a*d)/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) +
 (b*x^3 + a)^(1/3)*x*((b*c - a*d)/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+a\right)^{\frac{1}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(1/3))/(c + d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.668 $\int \frac{\sqrt[3]{a+bx^3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{4/3}}-\frac{\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}}-\frac{\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}}-\frac{\sqrt[3]{a+bx^3}}{cx}$$

```
[Out] -(b*x^3+a)^(1/3)/c/x+1/6*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(4/3)-1/2*(-a*d+b*c
)^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)-1/3*(-a*d+b*
c)^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/c^(4/3)*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 0.52, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3))
)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^2\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= -\frac{\sqrt[3]{a+bx^3}\sqrt[3]{1+\frac{dx^3}{c}}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 81, normalized size = 0.48

$$-\frac{\sqrt[3]{a+bx^3}\sqrt[3]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x]
[Out] -(((a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^2/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^2*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**2*(c + d*x**3)), x)
```

3.669 $\int \frac{\sqrt[3]{a+bx^3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=204

$$-\frac{d\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}+\frac{d\sqrt[3]{bc-ad}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}-\frac{\sqrt[3]{a+bx^3}}{4a}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x-1/6*d*(
-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(7/3)+1/2*d*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)+1/3*d*(-a*d+b*c)^(1/3)*arctan(1/3*(1
+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [C] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 0.71, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{x^3\left(c-3dx^3\right)(-(bc-ad))\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+2c\left(a+bx^3\right)}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(2*c*(a + b*x^3)*(c - 3*d*x^3) - (b*c - a*d)*x^3*(c - 3*d*x^3)*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)*x^3*
(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)])/(8*c^3*x^4*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{2c\left(a+bx^3\right)\left(c-3dx^3\right)-(bc-ad)x^3\left(c-3dx^3\right)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+3(bc-ad)x^3\left(c+\right.}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.78, size = 146, normalized size = 0.72

$$-\frac{x^3\left(3dx^3-c\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+2c\left(a+bx^3\right)\left(c-\right.}{8c^3x^4\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/8*(2*c*(a + b*x^3)*(c - 3*d*x^3) + (b*c - a*d)*x^3*(-c + 3*d*x^3)*Hyperg
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*(b*c - a*d)
*x^3*(c + d*x^3)*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(c^3*x^4*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^5*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**5/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**5*(c + d*x**3)), x)
```

3.670 $\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=258

$$\frac{\sqrt[3]{a+bx^3}\left(-28a^2d^2+7abcd+3b^2c^2\right)}{28a^2c^3x}+\frac{d^2\sqrt[3]{bc-ad}\log\left(c+dx^3\right)}{6c^{10/3}}-\frac{d^2\sqrt[3]{bc-ad}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}-\frac{d^2\sqrt[3]{b \cdots}}{\cdots}$$

```
[Out] -1/7*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^4+1/28
*(-28*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x+1/6*d^2*(-a*d+
b*c)^(1/3)*ln(d*x^3+c)/c^(10/3)-1/2*d^2*(-a*d+b*c)^(1/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)-1/3*d^2*(-a*d+b*c)^(1/3)*arctan(1/3*(
1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [C] time = 0.94, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-2x^3\left(2c^2-3cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2*x^6
+ 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*Hyperg
eometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3*x^3*
Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*a*c^
2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometr
icPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(56*c^4*x^
7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{8ac^3+8bc^3x^3-12ac^2dx^3-12bc^2dx^6+36acd^2x^6+36bcd^2x^9-2(bc-ad)x^3\left(2c^2-3cd\right.}{}$$

Mathematica [C] time = 2.75, size = 451, normalized size = 1.75

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{2}{3},2,2;1,\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+15bc^3x^3\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-2x^3\left(2c^2-3cdx^3+9d^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/56*(8*a*c^3 + 8*b*c^3*x^3 - 12*a*c^2*d*x^3 - 12*b*c^2*d*x^6 + 36*a*c*d^2
*x^6 + 36*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(2*c^2 - 3*c*d*x^3 + 9*d^2*x^6)*H
ypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*b*c^3
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15
*a*c^2*d*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] - 12*b*c^2*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 12*a*c*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hyperge
ometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*
x^7*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^8/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^8\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(x^8*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^8\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**8/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(x**8*(c + d*x**3)), x)
```

3.671 $\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=318

$$\frac{\sqrt[3]{a+bx^3}\left(-35a^2d^2+5abcd+3b^2c^2\right)}{140a^2c^3x^4}-\frac{\sqrt[3]{a+bx^3}\left(-140a^3d^3+35a^2bcd^2+15ab^2c^2d+9b^3c^3\right)}{140a^3c^4x}-\frac{d^3\sqrt[3]{bc-ad}\,\mathrm{lo}}{6c^{13/}}$$

```
[Out] -1/10*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+b*c)*(b*x^3+a)^(1/3)/a/c^2/x^7+1
/140*(-35*a^2*d^2+5*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(1/3)/a^2/c^3/x^4-1/140*(-
140*a^3*d^3+35*a^2*b*c*d^2+15*a*b^2*c^2*d+9*b^3*c^3)*(b*x^3+a)^(1/3)/a^3/c^
4/x-1/6*d^3*(-a*d+b*c)^(1/3)*ln(d*x^3+c)/c^(13/3)+1/2*d^3*(-a*d+b*c)^(1/3)*
ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)+1/3*d^3*(-a*d+b*c)^
(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(13/3)*3^(1/2)
```

Rubi [C] time = 2.64, antiderivative size = 905, normalized size of antiderivative = 2.85, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{-324bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{(bc}{c(l}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] -(56*a*c^4 + 56*b*c^4*x^3 - 72*a*c^3*d*x^3 - 72*b*c^3*d*x^6 + 108*a*c^2*d^2
*x^6 + 108*b*c^2*d^2*x^9 - 324*a*c*d^3*x^9 - 324*b*c*d^3*x^12 - 28*b*c^4*x^
3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 28*a*
c^3*d*x^3*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 36*b*c^3*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 36*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3
)/(c*(a + b*x^3))] - 54*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 54*a*c*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[2
/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 117*b*c^4*x^3*Hype
rgeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 117*a*c^3*d
*x^3*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99
*b*c^3*d*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 81*b*c^2*d^2*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] - 81*a*c*d^3*x^9*Hypergeometric2F1[2/3, 2, 5/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[2/3, 2
, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F
1[2/3, 2, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c - a*d)*x^3*(2*c
 - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{2/3, 2, 2}, {1, 5/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*Hypergeometri
cPFQ[{2/3, 2, 2, 2}, {1, 1, 5/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*
c^5*x^10*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^{11}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{56ac^4+56bc^4x^3-72ac^3dx^3-72bc^3dx^6+108ac^2d^2x^6+108bc^2d^2x^9-324acd^3x^9-324b}{}$$

Mathematica [C] time = 5.24, size = 214, normalized size = 0.67

$$\frac{\frac{\left(a+bx^3\right)\left(a^3\left(-14c^3+20c^2dx^3-35cd^2x^6+140d^3x^9\right)+a^2bcx^3\left(-2c^2+5cdx^3-35d^2x^6\right)+3ab^2c^2x^6\left(c-5dx^3\right)-9b^3c^3x^9\right)}{70a^3c^4x^{10}}+\frac{d^3x^2\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\right.}{c^5\left(\frac{dx^3}{c}+1\right)^{2/3}}}{2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] (((a + b*x^3)*(-9*b^3*c^3*x^9 + 3*a*b^2*c^2*x^6*(c - 5*d*x^3) + a^2*b*c*x^3
*(-2*c^2 + 5*c*d*x^3 - 35*d^2*x^6) + a^3*(-14*c^3 + 20*c^2*d*x^3 - 35*c*d^2
*x^6 + 140*d^3*x^9)))/(70*a^3*c^4*x^10) + (d^3*(-(b*c) + a*d)*x^2*(1 + (b*x
^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c +
d*x^3))])/(c^5*(1 + (d*x^3)/c)^(2/3)))/(2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{1}{3}}}{\left(d\,x^{3}+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
[Out] int((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{1}{3}}}{\left(dx^{3}+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^11/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{1/3}}{x^{11}\,\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^11*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^{3}}}{x^{11}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**11/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**11*(c + d*x**3)), x)
```

3.672 $\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} dx$

Optimal. Leaf size=64

$$\frac{x^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] 1/7*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-1/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -1/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{x^6 \sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x^7 \sqrt[3]{a+bx^3} F_1\left(\frac{7}{3};-\frac{1}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.64, size = 281, normalized size = 4.39

$$\frac{x\left(-\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(a^2d^2+5abcd-10b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(ad-5bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\right.}\right.}{40bd^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + a*d + 2*b*d*x^3) - ((-10*b^2*c^2 + 5*a*b*c*d +
a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + a*d)*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3,
 -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x
^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d
*x^3)/c)])))))/(40*b*d^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^6}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^6/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6 \left(b\,x^3+a\right)^{1/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**6*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.673 $\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=64

$$\frac{x^4 \sqrt[3]{a+bx^3} F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] 1/4*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-1/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4 \sqrt[3]{a+bx^3} F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \sqrt[3]{a+bx^3}}{c+dx^3} \, dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{x^3 \sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\
&= \frac{x^4 \sqrt[3]{a+bx^3} F_1\left(\frac{4}{3};-\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [B] time = 0.37, size = 240, normalized size = 3.75

$$\frac{x\left(4\left(\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+a+bx^3\right)+\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)}{}\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x]
```

```
[Out] (x*(((-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])/c + 4*(a + b*x^3 + (4*a^2*c^2*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}x^3}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*x^3/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{1/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(1/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt[3]{a+bx^{3}}}{c+dx^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.674 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] x*(b*x^3+a)^(1/3)*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^
(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} \, dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.04, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{1}{3}}}{d\,x^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{dx^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3 + a\right)^{1/3}}{d\,x^3 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(1/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

3.675 $\int \frac{\sqrt[3]{a+bx^3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -1/2*(b*x^3+a)^(1/3)*AppellF1(-2/3,-1/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1+b
*x^3/a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-2/3, -1/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/c)
])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^3\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{2}{3};-\frac{1}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.26, size = 327, normalized size = 5.11

$$\frac{\frac{c\left(16ac\left(a\left(c+3dx^3\right)+bdx^6\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}-bdx^6\left(\frac{bx^3}{a}+\right.}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (c*(16*a*c*(b*d*x^6 + a*(c + 3*d*x^3))*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)]))))/(8*c^2*x^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^3*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**3*(c + d*x**3)), x)
```

3.676 $\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -1/5*(b*x^3+a)^(1/3)*AppellF1(-5/3,-1/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(1+
b*x^3/a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)*AppellF1[-5/3, -1/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{x^6\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{1}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.46, size = 289, normalized size = 4.52

$$\frac{\frac{16x(-10a^2d^2+5abcd+b^2c^2)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}(5ad-bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{b}{}\right.}{ac^3}}{40\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + b*c*x^3 - 5*a*d*x^3))/(a*c^2*x^5) + (b*d*(-(b*c)
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/(a*c^3) + (16*(b^2*c^2 + 5*a*b*c*d - 10*a^2*d^2)*x*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/
3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7
/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{1}{3}}}{\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/((d*x^3 + c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{1/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(1/3)/(x^6*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx^3}}{x^6\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/x**6/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(1/3)/(x**6*(c + d*x**3)), x)
```

3.677 $\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=266

$$\frac{\left(a+bx^3\right)^{5/3}\left(a^2d^2+abcd+b^2c^2\right)}{5b^3d^3}-\frac{\left(a+bx^3\right)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3d}+\frac{c^3(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{14/3}}-\frac{c^3(bc\cdots}{\cdots}$$

```
[Out] -1/2*c^3*(b*x^3+a)^(2/3)/d^4+1/5*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/
b^3/d^3-1/8*(2*a*d+b*c)*(b*x^3+a)^(8/3)/b^3/d^2+1/11*(b*x^3+a)^(11/3)/b^3/d
+1/6*c^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(14/3)-1/2*c^3*(-a*d+b*c)^(2/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)-1/3*c^3*(-a*d+b*c)^(2/3)
*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(14/3
)*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{5/3}\left(a^2d^2+abcd+b^2c^2\right)}{5b^3d^3}-\frac{\left(a+bx^3\right)^{8/3}(2ad+bc)}{8b^3d^2}+\frac{\left(a+bx^3\right)^{11/3}}{11b^3d}-\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4}+\frac{c^3(bc-ad)^{2/3}\log\left(\cdots\right.}{6d^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -(c^3*(a + b*x^3)^(2/3))/(2*d^4) + ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^
3)^(5/3))/(5*b^3*d^3) - ((b*c + 2*a*d)*(a + b*x^3)^(8/3))/(8*b^3*d^2) + (a
+ b*x^3)^(11/3)/(11*b^3*d) - (c^3*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(14/3)) + (c^3*(
b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(14/3)) - (c^3*(b*c - a*d)^(2/3)*Log[
(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^3(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{\left(b^2c^2+abcd+a^2d^2\right)(a+bx)^{2/3}}{b^2d^3} + \frac{(-bc-2ad)(a+bx)^{5/3}}{b^2d^2} + \frac{(a+bx)^{8/3}}{b^2d} - \cdots\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{\left(a+bx^3\right)^{11/3}}{11b^3d} - \frac{c^3\,\text{Subst}(\cdots}{\cdots}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+bx\cdots}{11b\cdots}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+bx\cdots}{11b\cdots}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+bx\cdots}{11b\cdots}$$

$$= -\frac{c^3\left(a+bx^3\right)^{2/3}}{2d^4} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{8/3}}{8b^3d^2} + \frac{(a+bx\cdots}{11b\cdots}$$

Mathematica [C] time = 0.12, size = 148, normalized size = 0.56

$$\frac{\left(a+bx^3\right)^{2/3}\left(18a^3d^3+3a^2bd^2\left(11c-4dx^3\right)+220b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2ab^2d\left(44c^2-11cdx^3+5d^2x^6\right)+\right.}{440b^3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(18*a^3*d^3 + 3*a^2*b*d^2*(11*c - 4*d*x^3) + 2*a*b^2*d*(
44*c^2 - 11*c*d*x^3 + 5*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*d^2*
x^6 + 40*d^3*x^9) + 220*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^
3))/(-(b*c) + a*d)]))/(440*b^3*d^4)
```

fricas [B] time = 1.26, size = 455, normalized size = 1.71

$$-\frac{440\sqrt{3}\,b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+220\,b^3c^3\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/1320*(440*sqrt(3)*b^3*c^3*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*a
rctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
/d^2)^(1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 220*b^3*c^3*(-(b^2*c^2 -
2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) - 440*b^3*c^3*(-(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2
/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) - 3*(40*b^3*d^3*x^9 - 5*(11*b^3*c*d^2
- 2*a*b^2*d^3)*x^6 - 220*b^3*c^3 + 88*a*b^2*c^2*d + 33*a^2*b*c*d^2 + 18*a^3
*d^3 + 2*(44*b^3*c^2*d - 11*a*b^2*c*d^2 - 6*a^2*b*d^3)*x^3)*(b*x^3 + a)^(2/
3))/(b^3*d^4)
```

giac [A] time = 0.29, size = 409, normalized size = 1.54

$$-\frac{\left(b^{37}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{36}c^3d^8\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{37}cd^{11}-ab^{36}d^{12}\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^37*c^4*d^7*(-(b*c - a*d)/d)^(1/3) - a*b^36*c^3*d^8*(-(b*c - a*d)/d)
^(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^37*c*d^11 - a*b^36*d^12) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)
*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b
*c - a*d)/d)^(1/3))/d^6 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x^3 + a)^
(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/
d^6 - 1/440*(220*(b*x^3 + a)^(2/3)*b^33*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^32
*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^31*c*d^9 - 88*(b*x^3 + a)^(5/3)*a*b^31*c*
d^9 - 40*(b*x^3 + a)^(11/3)*b^30*d^10 + 110*(b*x^3 + a)^(8/3)*a*b^30*d^10 -
 88*(b*x^3 + a)^(5/3)*a^2*b^30*d^10)/(b^33*d^11)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}} x^{11}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.13, size = 490, normalized size = 1.84

$$\left(\frac{3\,a^2}{5\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{5\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}-\left(\frac{3\,a}{8\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{8\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{a^3}{2\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] ((3*a^2)/(5*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(5*b^3*d))*(a + b*x^3)^(5/3) - ((3*a)/(8*b^3*d) + (b^4*c - a*b^
3*d)/(8*b^6*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*(a^3/(2*b^3*d) + ((
(3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c - a
*b^3*d))/(b^3*d))*(b^4*c - a*b^3*d))/(2*b^3*d)) + (a + b*x^3)^(11/3)/(11*b^
3*d) - (c^3*log(((a + b*x^3)^(1/3)*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d))/d
^7 - (c^6*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(28/3)))*(a*d - b*c
)^(2/3))/(3*d^(14/3)) - (c^3*log((c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7
/3))/d^(22/3) + (c^6*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^7)*((3^(1/2)*1i)/2
- 1/2)*(a*d - b*c)^(2/3))/(3*d^(14/3)) + (c^3*log((c^6*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^7 - (c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(7/3))/(4*d^(22/3)))
*((3^(1/2)*1i)/6 + 1/6)*(a*d - b*c)^(2/3))/d^(14/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11} \left(a + b x^{3}\right)^{\frac{2}{3}}}{c + d x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.678 $\int \frac{x^8(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal. Leaf size=223

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+b\right.}}{2d^{11/3}}$$

[Out] 1/2*c^2*(b*x^3+a)^(2/3)/d^3-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^2/d^2+1/8*(b*x^3+a)^(8/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(11/3)+1/2*c^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)+1/3*c^2*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(11/3)*3^(1/2)

Rubi [A] time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 56, 617, 204, 31}

$$-\frac{(a+bx^3)^{5/3}(ad+bc)}{5b^2d^2}+\frac{(a+bx^3)^{8/3}}{8b^2d}+\frac{c^2(a+bx^3)^{2/3}}{2d^3}-\frac{c^2(bc-ad)^{2/3}\log(c+dx^3)}{6d^{11/3}}+\frac{c^2(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-\right.}}{2d^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (c^2*(a + b*x^3)^(2/3))/(2*d^3) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^2*d^2) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*(b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(11/3)) - (c^2*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(11/3)) + (c^2*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{2/3}}{bd^2}+\frac{(a+bx)^{5/3}}{bd}+\frac{c^2(a+bx)^{2/3}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+}\right.}{3d^3}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\frac{(c}{}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}-\frac{c^2(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{11/3}}+\frac{c^2}{}$$

$$= \frac{c^2\left(a+bx^3\right)^{2/3}}{2d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{5/3}}{5b^2d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^2d}+\frac{c^2(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right.}{\sqrt{3}\,d^{11/3}}$$

Mathematica [C] time = 0.10, size = 104, normalized size = 0.47

$$\frac{\left(a+bx^3\right)^{2/3}\left(-3a^2d^2-20b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2abd\left(dx^3-4c\right)+b^2\left(20c^2-8cdx^3+5d^2x^6\right)\right)}{40b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-3*a^2*d^2 + 2*a*b*d*(-4*c + d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6) - 20*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x
^3))/(-(b*c) + a*d)]))/(40*b^2*d^3)
```

fricas [B] time = 1.10, size = 398, normalized size = 1.78

$$\frac{40\sqrt{3}\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)-20\,b^2c^2\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/120*(40*sqrt(3)*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arcta
n(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)
^(1/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 20*b^2*c^2*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + 40*b^2*c^2*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/d^2)^(1/3)*log(-d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3
 + a)^(1/3)*(b*c - a*d)) + 3*(5*b^2*d^2*x^6 + 20*b^2*c^2 - 8*a*b*c*d - 3*a^
2*d^2 - 2*(4*b^2*c*d - a*b*d^2)*x^3)*(b*x^3 + a)^(2/3))/(b^2*d^3)
```

giac [A] time = 0.27, size = 350, normalized size = 1.57

$$\frac{\left(b^{19}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^{18}c^2d^6\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{19}cd^8-ab^{18}d^9\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^19*c^3*d^5*(-(b*c - a*d)/d)^(1/3) - a*b^18*c^2*d^6*(-(b*c - a*d)/d)^
(1/3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^
(1/3)))/(b^19*c*d^8 - a*b^18*d^9) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c^
2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
- a*d)/d)^(1/3))/d^5 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^3 + a)^(2/
3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^5
 + 1/40*(20*(b*x^3 + a)^(2/3)*b^16*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^15*c*d^6
 + 5*(b*x^3 + a)^(8/3)*b^14*d^7 - 8*(b*x^3 + a)^(5/3)*a*b^14*d^7)/(b^16*d^8
)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^8}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.11, size = 385, normalized size = 1.73

$$\left(\frac{a^2}{2\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{2\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{2\,a}{5\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{5\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^2\,d}+\frac{c^2\,\ln\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(2*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(2*b^2*d))*(a + b*x^3)^(2/3) - ((2*a)/(5*b^2*d) + (b^3*c - a*b^2*d)
/(5*b^4*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^2*d) + (c^2*log(((
a + b*x^3)^(1/3)*(b^2*c^6 + a^2*c^4*d^2 - 2*a*b*c^5*d))/d^5 - (c^4*(a*d - b
*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(22/3)))*(a*d - b*c)^(2/3))/(3*d^(11/
3)) - (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 - (c^4*((3^(1/2)*1
i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(16/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(2/3))/(3*d^(11/3)) + (c^2*log((c^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^5 -
 (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(16/3)))*((3^(1/2)*1i)/6 -
 1/6)*(a*d - b*c)^(2/3))/d^(11/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.679 $\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=188

$$\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}}-$$

```
[Out] -1/2*c*(b*x^3+a)^(2/3)/d^2+1/5*(b*x^3+a)^(5/3)/b/d+1/6*c*(-a*d+b*c)^(2/3)*l
n(d*x^3+c)/d^(8/3)-1/2*c*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^
3+a)^(1/3))/d^(8/3)-1/3*c*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 56, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{2/3}}{2d^2}+\frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}}-\frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}}-\frac{c(bc-ad)^{2/3}\tan^{-1}}{\sqrt{3}\,d^{8}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3))/(2*d^2) + (a + b*x^3)^(5/3)/(5*b*d) - (c*(b*c - a*d)
^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)) + (c*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*d^(8/3)) - (
c*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c\operatorname{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{(c(bc-ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{\left(c(bc-ad)^{2/3}\right)\operatorname{Subst}\left(\int \ldots\right.}{2d^{8/\ldots}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} + \frac{c(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{8/3}} - \frac{c(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}\ldots\right.}{2d^{8/3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}}{2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5bd} - \frac{c(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}} + \frac{c(bc-ad)^{2/3}\log\left(c\ldots\right.}{6d^{8/3}}$$

Mathematica [C] time = 0.05, size = 68, normalized size = 0.36

$$\frac{\left(a+bx^3\right)^{2/3}\left(5bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2ad-5bc+2bdx^3\right)}{10bd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(-5*b*c + 2*a*d + 2*b*d*x^3 + 5*b*c*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))/(10*b*d^2)
```

fricas [B] time = 0.83, size = 353, normalized size = 1.88

$$-\frac{10\sqrt{3}\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)+5\,bc\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/30*(10*sqrt(3)*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(
1/3) + sqrt(3)*(b*c - a*d))/(b*c - a*d)) + 5*b*c*(-(b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2
)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2 - 2*a
*b*c*d + a^2*d^2)/d^2)^(1/3)) - 10*b*c*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^
2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^
(1/3)*(b*c - a*d)) - 3*(2*b*d*x^3 - 5*b*c + 2*a*d)*(b*x^3 + a)^(2/3))/(b*d^
2)
```

giac [B] time = 0.55, size = 306, normalized size = 1.63

$$-\frac{\left(b^7c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ab^6cd^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^7cd^5-ab^6d^6\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c\,\mathrm{arcta}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^7*c^2*d^3*(-(b*c - a*d)/d)^(1/3) - a*b^6*c*d^4*(-(b*c - a*d)/d)^(1/
3))*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/
3)))/(b^7*c*d^5 - a*b^6*d^6) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*c*arcta
n(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/
d)^(1/3))/d^4 + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c*log((b*x^3 + a)^(2/3) + (b*x
^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 - 1/10*(
5*(b*x^3 + a)^(2/3)*b^5*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^4*d^4)/(b^5*d^5)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^5}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.06, size = 302, normalized size = 1.61

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b\,d}-\left(b\,x^3+a\right)^{2/3}\,\left(\frac{a}{2\,b\,d}+\frac{b^2\,c-a\,b\,d}{2\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c^2\,d^2-2\,a\,b\,c^3\,d+b^2\,c^4\right)}{d^3}-\frac{c^2\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{16/3}}\right)\,(a}{3\,d^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b*d) - (a + b*x^3)^(2/3)*(a/(2*b*d) + (b^2*c - a*b*d)/
(2*b^2*d^2)) - (c*log(((a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3
*d))/d^3 - (c^2*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(16/3)))*(a*d
 - b*c)^(2/3))/(3*d^(8/3)) - (c*log((c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)
^(7/3))/d^(10/3) + (c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^3)*((3^(1/2)*1i)
/2 - 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3)) + (c*log((c^2*(a + b*x^3)^(1/3)*(a
*d - b*c)^2)/d^3 - (c^2*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(10/3))
*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(2/3))/(3*d^(8/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.680 $\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=162

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}+$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/d-1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^(5/3)+1/2*(-a*d+b*
c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)+1/3*(-a*d+b*c
)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/
d^(5/3)*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {444, 50, 56, 617, 204, 31}

$$-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}}+\frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*
x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)) - ((b*c - a*d)^(
2/3)*Log[c + d*x^3])/(6*d^(5/3)) + ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx,x,x^3\right)}{3d}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx,x,\sqrt[3]{a+bx^3}\right.}{2d^{5/3}}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}} -$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2d} + \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}} - \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^{5/3}} + \frac{(bc-ad)^{2/\cdots}}{}$$

Mathematica [C] time = 0.03, size = 47, normalized size = 0.29

$$-\frac{\left(a+bx^3\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)-1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3)
)/(-(b*c) + a*d)]))/d
```

fricas [B] time = 0.99, size = 323, normalized size = 1.99

$$2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}-\sqrt{3}\,(bc-ad)}{3\,(bc-ad)}\right)-\left(\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{b^2\cdots}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) - sq
rt(3)*(b*c - a*d))/(b*c - a*d)) - ((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/
3)*log((b*x^3 + a)^(1/3)*d*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b
*x^3 + a)^(2/3)*(b*c - a*d) - (b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/
d^2)^(1/3)) + 2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*log(-d*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)) + 3*(b
*x^3 + a)^(2/3))/d
```

giac [B] time = 0.27, size = 259, normalized size = 1.60

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^2-ad^3\right)}+\frac{\left(bx^3+a\right)^{\frac{2}{3}}}{2d}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^2
 - a*d^3) + 1/2*(b*x^3 + a)^(2/3)/d + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/d^3 - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (
b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^2}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.05, size = 238, normalized size = 1.47

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,d}+\frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{d}-\frac{\left(a\,d-b\,c\right)^{4/3}\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{9\,d^{10/3}}\right)\left(a\,d-b\,c\right)^{2/3}}{3\,d^{5/3}}-\frac{\ln\left(\frac{\left(b\,x^3+a\right)^{1/3}\left(a\,d-b\,c\right)^2}{d}-\frac{\left(-\frac{1}{2}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*d) + (log(((a + b*x^3)^(1/3)*(a^2*d^2 + b^2*c^2 - 2*a*
b*c*d))/d - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(9*d^(10/3)))*(a*d -
b*c)^(2/3))/(3*d^(5/3)) - (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - (((3^(
1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d -
 b*c)^(2/3))/(3*d^(5/3)) + (log(((a + b*x^3)^(1/3)*(a*d - b*c)^2)/d - ((3^(
1/2)*1i - 1)^2*(a*d - b*c)^(7/3))/(4*d^(4/3)))*((3^(1/2)*1i)/6 - 1/6)*(a*d
- b*c)^(2/3))/d^(5/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + bx^3\right)^{\frac{2}{3}}}{c + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.681 $\int \frac{(a+bx^3)^{2/3}}{x(c+dx^3)}\,dx$

Optimal. Leaf size=245

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log}{}$$

```
[Out] -1/2*a^(2/3)*ln(x)/c+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c/d^(2/3)+1/2*a^(2/3)
*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(
1/3)*(b*x^3+a)^(1/3))/c/d^(2/3)+1/3*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)
^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(
1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(2/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 83, 55, 617, 204, 31, 56}

$$\frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}+\frac{a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{2/3}\log(x)}{2c}+\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}}-\frac{(bc-ad)^{2/3}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3
]*c) - ((b*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(2/3)) - (a^(2/3)*Log[x])/(2*c) + ((b*c
- a*d)^(2/3)*Log[c + d*x^3])/(6*c*d^(2/3)) + (a^(2/3)*Log[a^(1/3) - (a + b*
x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a
+ b*x^3)^(1/3)])/(2*c*d^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} - \frac{a^{2/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} + \frac{a\,\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= -\frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6cd^{2/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - \frac{(bc-ad)^{2/3}\log\left(\sqrt[3]{\cdots}\right.}{\cdots}$$

$$= \frac{a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} - \frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{2/3}} - \frac{a^{2/3}\log(x)}{2c} + \frac{(bc-ad)^{2/3}\log\left(c\cdots\right.}{6cd^{2/3}}$$

Mathematica [C] time = 0.12, size = 115, normalized size = 0.47

$$\frac{a^{2/3}\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)+3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x*(c + d*x^3)),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c)
 + a*d)] + a^(2/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sq
rt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*c)
```

fricas [B] time = 0.89, size = 425, normalized size = 1.73

$$-\frac{2\sqrt{3}\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{d^2}\right)^{\frac{1}{3}}+\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)-2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\sqrt{3}\left(bx\right.}{3\,a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)*arctan(-1/3*(2
*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(1/3) +
 sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt
(3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(1/3))/a) + (-(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/d^2)^(1/3)*log((b*x^3 + a)^(1/3)*d*(-(b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/d^2)^(2/3) - (b*x^3 + a)^(2/3)*(b*c - a*d) + (b*c - a*d)*(-(b^2*c^2
- 2*a*b*c*d + a^2*d^2)/d^2)^(1/3)) + (a^2)^(1/3)*log((b*x^3 + a)^(2/3)*a +
(a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a^2)^(2/3)) - 2*(-(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/d^2)^(1/3)*log(-d*(-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/d^2)^(2/3) -
(b*x^3 + a)^(1/3)*(b*c - a*d)) - 2*(a^2)^(1/3)*log((b*x^3 + a)^(1/3)*a - (a
^2)^(2/3)))/c
```

giac [A] time = 0.80, size = 341, normalized size = 1.39

$$-\frac{\left(bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*(-(b*c - a*d)/d)^(1/3) - a*d*(-(b*c - a*d)/d)^(1/3))*(-(b*c - a*d
)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^2 - a*
c*d) + 1/3*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3
))/a^(1/3))/c - 1/6*a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/
3) + a^(2/3))/c + 1/3*a^(2/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c - 1/3
*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) +
 (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) + 1/6*(-b*c*d^2 +
a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/
3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)
```

[Out] int((b*x^3+a)^(2/3)/x/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x), x)

mupad [B] time = 4.77, size = 1963, normalized size = 8.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x*(c + d*x^3)),x)

[Out] log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (a^2/(27*c^3))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - (-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - (243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 + 1/2)*(a^2/(27*c^3))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(a^2/(27*c^3))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(a^2/(27*c^3))^(2/3))*(a^2/(27*c^3))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(a^2/(27*c^3))^(1/3) - log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 + 1/2)^2*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) - 36*a^2*b^7*c^4*d^2 + 54*a^3*b^6*c^3*d^3 - 27*a^4*b^5*c^2*d^4 + 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(1/3) + log((a + b*x^3)^(1/3)*(2*a^5*b^5*d^4 - a^2*b^8*c^3*d - 5*a^4*b^6*c*d^3 + 4*a^3*b^7*c^2*d^2) - ((3^(1/2)*1i)/2 - 1/2)^2*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3)*(((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(54*a^2*b^6*c^4*d^3 - 108*a^3*b^5*c^3*d^4 + 54*a^4*b^4*c^2*d^5) - ((3^(1/2)*1i)/2 - 1/2)^2*(243*a

```
*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(-(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d)/(27*c^3*d^2))^(2/3))*(-(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(27
*c^3*d^2))^(1/3) + 36*a^2*b^7*c^4*d^2 - 54*a^3*b^6*c^3*d^3 + 27*a^4*b^5*c^2
*d^4 - 9*a*b^8*c^5*d))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d)/(27*c^3*d^2))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x*(c + d*x**3)), x)
```

3.682 $\int \frac{(a+bx^3)^{2/3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=347

$$\frac{d(a+bx^3)^{2/3}}{2c^2}+\frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}\,(bc-ad)^{2/3}\log(c+dx^3)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}\,c^2}+\frac{\sqrt[3]{d}\,(\ldots}{\ldots}$$

```
[Out] 1/2*d*(b*x^3+a)^(2/3)/c^2+1/6*(-3*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2-1/3*(b*x
^3+a)^(5/3)/a/c/x^3-1/6*(-3*a*d+2*b*c)*ln(x)/a^(1/3)/c^2-1/6*d^(1/3)*(-a*d+
b*c)^(2/3)*ln(d*x^3+c)/c^2+1/6*(-3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a
^(1/3)/c^2+1/2*d^(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/c^2+1/9*(-3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(1/3)/c^2*3^(1/2)+1/3*d^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(
1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2*3^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 55, 617, 204, 31, 56}

$$\frac{d(a+bx^3)^{2/3}}{2c^2}+\frac{(a+bx^3)^{2/3}(2bc-3ad)}{6ac^2}-\frac{\sqrt[3]{d}\,(bc-ad)^{2/3}\log(c+dx^3)}{6c^2}+\frac{(2bc-3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6\sqrt[3]{a}\,c^2}+\frac{\sqrt[3]{d}\,(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (d*(a + b*x^3)^(2/3))/(2*c^2) + ((2*b*c - 3*a*d)*(a + b*x^3)^(2/3))/(6*a*c^
2) - (a + b*x^3)^(5/3)/(3*a*c*x^3) + ((2*b*c - 3*a*d)*ArcTan[(a^(1/3) + 2*(
a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*c^2) + (d^(1/3)*(b
*c - a*d)^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]])/(Sqrt[3]*c^2) - ((2*b*c - 3*a*d)*Log[x])/(6*a^(1/3)*c^2) - (d^(
1/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^2) + ((2*b*c - 3*a*d)*Log[a^(1/
3) - (a + b*x^3)^(1/3)])/(6*a^(1/3)*c^2) + (d^(1/3)*(b*c - a*d)^(2/3)*Log[(
b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^4\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^2(c+dx)}\,dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(-2bc+3ad)-\frac{2bdx}{3}\right)}{x(c+dx)}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{c+dx}\,dx,x,x^3\right)}{3c^2} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x}\,dx,x,x^3\right)}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx,\right.}{9c^2}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2} - \frac{\sqrt[3]{d}\,(bc-ad}{}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} - \frac{(2bc-3ad)\log(x)}{6\sqrt[3]{a}\,c^2} - \frac{\sqrt[3]{d}\,(bc-ad}{}$$

$$= \frac{d\left(a+bx^3\right)^{2/3}}{2c^2} + \frac{(2bc-3ad)\left(a+bx^3\right)^{2/3}}{6ac^2} - \frac{\left(a+bx^3\right)^{5/3}}{3acx^3} + \frac{(2bc-3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,c^2} +$$

Mathematica [C] time = 0.16, size = 202, normalized size = 0.58

$$\frac{-9\sqrt[3]{a}\,dx^3\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) + 2\sqrt{3}\,x^3(2bc-3ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) - 6\sqrt[3]{a}\,c\left(a+bx^3\right)^{2/3} + 6bcx^3}{18\sqrt[3]{a}\,c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-6*a^(1/3)*c*(a + b*x^3)^(2/3) + 2*Sqrt[3]*(2*b*c - 3*a*d)*x^3*ArcTan[(1 +
 (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 9*a^(1/3)*d*x^3*(a + b*x^3)^(2/3
)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] - 6*b*c*x^
3*Log[x] + 9*a*d*x^3*Log[x] + 6*b*c*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)] -
9*a*d*x^3*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(1/3)*c^2*x^3)
```

fricas [A] time = 1.15, size = 1030, normalized size = 2.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(3*sqrt(1/3)*(2*a*b*c - 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3
 - 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) -
 6*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt
(3)*(b*c - a*d) - 2*sqrt(3)*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^
```

```
3 + a)^(1/3))/(b*c - a*d)) + (2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)
^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(2*b*c - 3*a*d)*(-a
)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1
/3)*a*x^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(2/3)) + 6*(b*x^3 + a)^(2/3)*a*c)/(a*c^2*x^3), 1/18*(6*sqrt(1/3
)*(2*a*b*c - 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 +
a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)^(1/3)*a*x^3*arctan(1/3*(sqrt(3)*(b*c - a*d) - 2*sqrt(3)*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) - (
2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3)) + 2*(2*b*c - 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1
/3) + (-a)^(1/3)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-
(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/
3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*a*x^3*log(-(b*x^3 + a)^(1/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) - 6*(b*x^3
+ a)^(2/3)*a*c)/(a*c^2*x^3)]
```

giac [A] time = 0.81, size = 400, normalized size = 1.15

$$\frac{\left(bcd\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{(2\,bc-3\,ad)\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(b\right.\right.}{18\,a^{\frac{1}{3}}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b*c*d*(-(b*c - a*d)/d)^(1/3) - a*d^2*(-(b*c - a*d)/d)^(1/3))*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 -
 a*c^2*d) - 1/18*(2*b*c - 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*
a^(1/3) + a^(2/3))/(a^(1/3)*c^2) + 1/9*sqrt(3)*(2*a^(2/3)*b*c - 3*a^(5/3)*d
)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a*c^2) + 1/9
*(2*a^(1/3)*b*c - 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^2) + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3
 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) - 1/6
*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
- a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c^2*d) - 1/3*(b*x^3 + a)^(2/3)/(
c*x^3)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{2}{3}}}{\left(d\,x^{3}+c\right)x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^4), x)
```

mupad [B] time = 10.57, size = 1908, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2
- 6*a*b*c*d) - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d
- b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3
*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c
)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2
+ 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/
(27*c^6))^(1/3) + log(- ((((6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^
2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a
*b*c*d)*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1
/3))/9 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*
d^2))/(3*c))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^
(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) - 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 + (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 3*a*b^4*c^4*d
^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d - 2*b*
c)^3/(a*c^6))^(2/3))*(-(3*a*d - 2*b*c)^3/(a*c^6))^(1/3))/9 - (a*b^5*d^4*(27
*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*(-(3*a*d -
 2*b*c)^3/(a*c^6))^(2/3))/81 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(
3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(2
7*a^3*d^3 - 8*b^3*c^3 + 36*a*b^2*c^2*d - 54*a^2*b*c*d^2)/(729*a*c^6))^(1/3)
 - (a + b*x^3)^(2/3)/(3*c*x^3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i
)/2 + 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^
2 - 6*a*b*c*d) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2/3))*((d*(a*d - b*c)^2)/c^6)^(1/3
))/3 + (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^
2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4
*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i
)/2 + 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2)/(27*c^6))^(1/3) + log((((3^
(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(6*b^4*d^3*(a + b*x^3)^(1/3)*(a
*d - b*c)^2*(9*a^2*d^2 + 2*b^2*c^2 - 6*a*b*c*d) + 27*a*b^4*c^4*d^3*((3^(1/2
)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((d*(a*d - b*c)^2)/c^6)^(2
/3))*((d*(a*d - b*c)^2)/c^6)^(1/3))/3 - (a*b^5*d^4*(27*a^3*d^3 - 19*b^3*c^3
 + 64*a*b^2*c^2*d - 72*a^2*b*c*d^2))/(3*c))*((d*(a*d - b*c)^2)/c^6)^(2/3))/
9 - (b^5*d^4*(a + b*x^3)^(1/3)*(4*a*d - 3*b*c)*(3*a^2*d^2 + 2*b^2*c^2 - 5*a
*b*c*d)^2)/(27*c^5))*((3^(1/2)*1i)/2 - 1/2)*((a^2*d^3 + b^2*c^2*d - 2*a*b*c
*d^2)/(27*c^6))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b x^3\right)^{\frac{2}{3}}}{x^4 \left(c+d x^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**4/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**4*(c + d*x**3)), x)
```

3.683 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=370

$$-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}+\frac{\log(x)\left(-9a^2d\right.}{18}$$

```
[Out] 1/18*(6*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^3-1/6*(b*x^3+a)^(5/3)/a/c/x^6+1/18
*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(x)/a^(4/3)/c^3+1/6*d^(4/3)*(-a*d+b*c)^(2
/3)*ln(d*x^3+c)/c^3-1/18*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(4/3)/c^3-1/2*d^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/
3)*(b*x^3+a)^(1/3))/c^3-1/27*(-9*a^2*d^2+6*a*b*c*d+b^2*c^2)*arctan(1/3*(a^(
1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c^3*3^(1/2)-1/3*d^(4/3)*(-
a*d+b*c)^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^
(1/2))/c^3*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {446, 103, 149, 156, 55, 617, 204, 31, 56}

$$-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{4/3}c^3}-\frac{\left(-9a^2d^2+6abcd+b^2c^2\right)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{9\sqrt{3}\,a^{4/3}c^3}+\frac{\log(x)\left(-9a^2d\right.}{18}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] ((b*c + 6*a*d)*(a + b*x^3)^(2/3))/(18*a*c^2*x^3) - (a + b*x^3)^(5/3)/(6*a*c
*x^6) - ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^
(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*a^(4/3)*c^3) - (d^(4/3)*(b*c - a*d)^(
2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*c^3) + ((b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*Log[x])/(18*a^(4/3)*c^3
) + (d^(4/3)*(b*c - a*d)^(2/3)*Log[c + d*x^3])/(6*c^3) - ((b^2*c^2 + 6*a*b*
c*d - 9*a^2*d^2)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(18*a^(4/3)*c^3) - (d^(4
/3)*(b*c - a*d)^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
```

```
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^7\left(c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{2/3}}{x^3(c+dx)}dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{2/3}\left(\frac{1}{3}(bc+6ad)+\frac{bdx}{3}\right)}{x^2(c+dx)}dx,x,x^3\right)}{6ac}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\frac{2}{9}\left(b^2c^2+6abcd-9a^2d^2\right)+\frac{2}{9}bd(bc-3ad)x}{x\sqrt[3]{a+bx}\,(c+dx)}dx,x,x^3\right)}{6ac^2}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}dx,x,x^3\right)}{3c^3} - \frac{(b^2}{}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\text{lo}}{6c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} + \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\log(x)}{18a^{4/3}c^3} + \frac{d^{4/3}(bc-ad)^{2/3}\text{lo}}{6c^3}$$

$$= \frac{(bc+6ad)\left(a+bx^3\right)^{2/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{5/3}}{6acx^6} - \frac{\left(b^2c^2+6abcd-9a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{9\sqrt{3}\,a^{4/3}c^3} - \frac{d^{4/3}(bc}{}$$

Mathematica [C] time = 0.38, size = 240, normalized size = 0.65

$$\frac{27a^{4/3}d^2x^6\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+3x^6\log(x)\left(-9a^2d^2+6abcd+b^2c^2\right)-3x^6\left(-9a^2d^2+6abcd+b^2c^2\right.}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*c*(a + b*x^3)^(2/3)*(-3*a*c - 2*b*c*x^3 + 6*a*d*x^3) - 2*Sqrt[3]
*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^
(1/3))/Sqrt[3]] + 27*a^(4/3)*d^2*x^6*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/
3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2
*d^2)*x^6*Log[x] - 3*(b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*x^6*Log[a^(1/3) - (a
 + b*x^3)^(1/3)])/(54*a^(4/3)*c^3*x^6)
```

fricas [A] time = 3.61, size = 1151, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arc
tan(-1/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d
^3)^(1/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a
^2*d^3)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*
```

```
d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 -
a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)
^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a
*b*c*d^2 - a^2*d^3)^(2/3)) + 3*sqrt(1/3)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d
^2)*x^6*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*a^
(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 + a)^(1/
3)*a^(2/3) + 3*a)/x^3) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log(
(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a
*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 3*(3*a^2
*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6), -1/54
*(18*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*a^2*d*x^6*arctan(-1
/3*(sqrt(3)*(b*c - a*d) + 2*sqrt(3)*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)*(b*x^3 + a)^(1/3))/(b*c - a*d)) + 9*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3
)^(1/3)*a^2*d*x^6*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*
a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^
3)^(2/3)*(b*x^3 + a)^(1/3)) - 18*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)
*a^2*d*x^6*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d
^2 - a^2*d^3)^(2/3)) - (b^2*c^2 + 6*a*b*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b
*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b^2*c^2 + 6*a*b
*c*d - 9*a^2*d^2)*a^(2/3)*x^6*log((b*x^3 + a)^(1/3) - a^(1/3)) + 6*sqrt(1/3
)*(a*b^2*c^2 + 6*a^2*b*c*d - 9*a^3*d^2)*x^6*arctan(sqrt(1/3)*(2*(b*x^3 + a)
^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) + 3*(3*a^2*c^2 + 2*(a*b*c^2 - 3*a^2*c*d)
*x^3)*(b*x^3 + a)^(2/3))/(a^2*c^3*x^6)]
```

giac [A] time = 0.78, size = 493, normalized size = 1.33

$$-\frac{\left(bcd^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}-ad^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4-ac^3d\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}}{}\right.}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b*c*d^2*(-(b*c - a*d)/d)^(1/3) - a*d^3*(-(b*c - a*d)/d)^(1/3))*(-(b*c
 - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^
4 - a*c^3*d) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(
b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 + 1/
6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c
 - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(b^2*c^2 + 6*
a*b*c*d - 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(4/3)*c^3) - 1/27*(a^(1/3)*b^2*c^2 + 6*a^(4/3)*b*c*d - 9*a^(7/3)*d
^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^3) + 1/54*(a^(2/3)*b^2
*c^2 + 6*a^(5/3)*b*c*d - 9*a^(8/3)*d^2)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^2*c^3) - 1/18*(2*(b*x^3 + a)^(5/3)*b^2*c + (b*
x^3 + a)^(2/3)*a*b^2*c - 6*(b*x^3 + a)^(5/3)*a*b*d + 6*(b*x^3 + a)^(2/3)*a^
2*b*d)/(a*b^2*c^2*x^6)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)
```

[Out] int((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(2/3)/x^7/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^7), x)

mupad [B] time = 15.19, size = 2788, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(2/3)/(x^7*(c + d*x^3)),x)

[Out] log(((((27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3) - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((((a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3 - (b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3) - (((a + b*x^3)^(2/3)*(b^2*c + 6*a*b*d))/(18*c^2) - (b*(a + b*x^3)^(5/3)*(3*a*d - b*c))/(9*a*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27*c^9))^(1/3) + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))*(-(d^4*(a*d - b*c)^2)/c^9)^(1/3))/3 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*a^2*c^4))*(-(d^4*(a*d - b*c)^2)/c^9)^(2/3))/9 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^

```
2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(a^2*d^6 + b^2*c^2*d^4 - 2*a*b*c*d^5)/(27
*c^9))^(1/3) - log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*((b^4*d
^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*
d^2 + 12*a*b^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) - (a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2
+ 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(
a^4*c^9))^(1/3))/27 - (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2
- 918*a^3*b^3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*
c*d^5))/(81*a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3
))/729 + (2*b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3
+ 5*a*b^2*c^2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 + 1/2)
*(-(b^6*c^6 - 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*
a^4*b^2*c^2*d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
 + log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*((b^4*d^3*(a + b*x^
3)^(1/3)*(a*d - b*c)^2*(162*a^4*d^4 + b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 12*a*b
^3*c^3*d - 108*a^3*b*c*d^3))/(3*a^2*c^2) + (a*b^4*c^4*d^3*((3^(1/2)*1i)/2 +
 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)
^3/(a^4*c^9))^(2/3))/3)*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(1
/3))/27 + (b^5*d^4*(729*a^6*d^6 + b^6*c^6 + 63*a^2*b^4*c^4*d^2 - 918*a^3*b^
3*c^3*d^3 + 2295*a^4*b^2*c^2*d^4 + 17*a*b^5*c^5*d - 2187*a^5*b*c*d^5))/(81*
a^2*c^4))*(-(b^2*c^2 - 9*a^2*d^2 + 6*a*b*c*d)^3/(a^4*c^9))^(2/3))/729 + (2*
b^5*d^7*(a + b*x^3)^(1/3)*(6*a*d - 5*b*c)*(9*a^3*d^3 + b^3*c^3 + 5*a*b^2*c^
2*d - 15*a^2*b*c*d^2)^2)/(729*a^2*c^10))*((3^(1/2)*1i)/2 - 1/2)*(-(b^6*c^6
- 729*a^6*d^6 + 81*a^2*b^4*c^4*d^2 - 108*a^3*b^3*c^3*d^3 - 729*a^4*b^2*c^2*
d^4 + 18*a*b^5*c^5*d + 1458*a^5*b*c*d^5)/(19683*a^4*c^9))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^7\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**7/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(x**7*(c + d*x**3)), x)
```

3.684 $\int \frac{x^6\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=334

$$-\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{18b^{4/3}d^3}+\frac{\left(-a^2d^2-6abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{4/3}d^3}-\frac{c^{4/3}(bc-ad)^{2/3}\,\mathrm{lo}}{6d^3}$$

```
[Out] -1/9*(-a*d+3*b*c)*x*(b*x^3+a)^(2/3)/b/d^2+1/6*x^4*(b*x^3+a)^(2/3)/d-1/6*c^(
4/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^3+1/2*c^(4/3)*(-a*d+b*c)^(2/3)*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/18*(-a^2*d^2-6*a*b*c*d+9*b^2*c
^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^3+1/27*(-a^2*d^2-6*a*b*c*d+9*b
^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^3*3^(
1/2)-1/3*c^(4/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3
)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{x^7\left(a+bx^3\right)^{2/3}F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^7*(a + b*x^3)^(2/3)*AppellF1[7/3, -2/3, 1, 10/3, -((b*x^3)/a), -((d*x^3)
/c)])/(7*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^6 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3} \int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^7 \left(a+bx^3\right)^{2/3} F_1\left(\frac{7}{3};-\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.76, size = 525, normalized size = 1.57

$$\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\sqrt[3]{bc-ad}\left(-a^2d^2-6abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-a^2\sqrt[3]{c}\,d\sqrt[3]{a+bx^3}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc}}{\sqrt[3]{ax^3}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (3*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*x^4*(1 + (b*x^3)/a)^
(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-18*a*b
*c*(b*c - a*d)^(1/3)*x + 6*a^2*d*(b*c - a*d)^(1/3)*x - 18*b^2*c*(b*c - a*d)
^(1/3)*x^4 + 15*a*b*d*(b*c - a*d)^(1/3)*x^4 + 9*b^2*d*(b*c - a*d)^(1/3)*x^7
 - 2*Sqrt[3]*a*c^(1/3)*(-3*b*c + a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c
 - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-3*b*
c + a*d)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^
(1/3)] + 3*a*b*c^(4/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x
^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] -
a^2*c^(1/3)*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b +
a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*b*c*
d^2*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

fricas [B] time = 4.14, size = 1164, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2*c*arctan(-
1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^
(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*c^2*d + a^2
*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*x - (b*x
^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(
1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2
+ (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x + (b*x^3 +
a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2
*b*d^2)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*
sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2
/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*d^2)*b^(2/3
)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d - a^2*d^
2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2) - 3*(3*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(b^2*d^3), -1/54*(18*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*b^2
```

```
*c*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d +
a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) - 18*(b^2*c^3 - 2*a*b*
c^2*d + a^2*c*d^2)^(1/3)*b^2*c*log(((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/
3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/x) + 9*(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2)^(1/3)*b^2*c*log(-((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(1/3)*(b*c
- a*d)*x^2 + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x
+ (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2) + 2*(9*b^2*c^2 - 6*a*b*c*d - a^2*
d^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c
*d - a^2*d^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x
^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d - a^2*b*d^2)*arc
tan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3
*b^2*d^2*x^4 - 2*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 + a)^(2/3))/(b^2*d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}} x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}} x^6}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}} x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^6/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.685 $\int \frac{x^3(a+bx^3)^{2/3}}{c+dx^3} dx$

Optimal. Leaf size=272

$$\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6d^2}-\frac{\sqrt[3]{c}\,(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d^2}+\frac{(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6\sqrt[3]{b}\,d^2}-\cdots$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/d+1/6*c^(1/3)*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/d^2-1/2*c^
(1/3)*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2+1
/6*(-2*a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(1/3)/d^2-1/9*(-2*a*d+3*
b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(1/3)/d^2*3^(1/2
)+1/3*c^(1/3)*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(
b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 0.24, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^4*(a + b*x^3)^(2/3)*AppellF1[4/3, -2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/
c)])/(4*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{x^4\left(a+bx^3\right)^{2/3}F_1\left(\frac{4}{3};-\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.59, size = 286, normalized size = 1.05

$$\frac{\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,(2ad-3bc)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{a+bx^3}}+\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6x\left(a+bx^3\right)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right.}{\sqrt[3]{bc-ad}}}{36d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] ((3*(-3*b*c + 2*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)
*x*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)
*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c
 - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^
(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)]))/(b*c - a*d)^(1/3))/(36*d)
```

fricas [B] time = 1.32, size = 1091, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - 2*a*b*d)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d
^2)^(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b
*c^2*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c -
2*a*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b
^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*
d))/x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*
a*b*c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^
2*c*d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2
))/(b*d^2), 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(1/3)*(3*b^2*c - 2*a*b*
d)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3
))*sqrt(-(-b)^(1/3)/b)/x) + 6*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^
(1/3)*b*arctan(1/3*(sqrt(3)*(b*c - a*d)*x - 2*sqrt(3)*(-b^2*c^3 + 2*a*b*c^2
*d - a^2*c*d^2)^(1/3)*(b*x^3 + a)^(1/3))/((b*c - a*d)*x)) + 2*(3*b*c - 2*a*
d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (3*b*c - 2*a*d)*(
-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a
```

```
)^(2/3))/x^2) + 6*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c
^3 + 2*a*b*c^2*d - a^2*c*d^2)^(2/3)*x - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d))/
x) - 3*(-b^2*c^3 + 2*a*b*c^2*d - a^2*c*d^2)^(1/3)*b*log(((-b^2*c^3 + 2*a*b*
c^2*d - a^2*c*d^2)^(1/3)*(b*c - a*d)*x^2 - (-b^2*c^3 + 2*a*b*c^2*d - a^2*c*
d^2)^(2/3)*(b*x^3 + a)^(1/3)*x - (b*x^3 + a)^(2/3)*(b*c^2 - a*c*d))/x^2))/(
b*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^3}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x^3/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.686 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{}\right.}{2c^{2/3}d}$$

```
[Out] -1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(2/3)/d+1/2*(-a*d+b*c)^(2/3)*ln((-a*d+b
*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/d-1/2*b^(2/3)*ln(-b^(1/3)*x+(b
*x^3+a)^(1/3))/d+1/3*b^(2/3)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(
1/2))/d*3^(1/2)-1/3*(-a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(
1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/d*3^(1/2)
```

Rubi [C] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 161, normalized size = 0.69

$$\frac{4acx\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [B] time = 1.01, size = 469, normalized size = 2.01

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\,\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,bx-2\,\sqrt{3}\left(bx^3\right.}{3\,bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(c + d*x^3),x)
[Out] int((a + b*x^3)^(2/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.687 $\int \frac{(a+bx^3)^{2/3}}{x^3(c+dx^3)}\,dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)^{2/3}\log(c+dx^3)}{6c^{5/3}}-\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}}+\frac{(bc-ad)^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}}-\frac{(a+bx^3)^{2/3}}{2cx^2}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/c/x^2+1/6*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(5/3)-1/2*(-a
*d+b*c)^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)+1/3*(-
a*d+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/c^(5/3)*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 89, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{(a+bx^3)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3, 1/3
, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(2*c*x^2*(1 + (b*x^3)/a)^(2/
3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^3\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}\left(1+\frac{dx^3}{c}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{2cx^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.05, size = 83, normalized size = 0.49

$$-\frac{\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{3};\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{2cx^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x]
[Out] -1/2*((a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Hypergeometric2F1[-2/3, -2/3,
 1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x^2*(1 + (b*x^3)/a)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^3/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^3*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**3/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**3*(c + d*x**3)), x)
```

3.688 $\int \frac{(a+bx^3)^{2/3}}{x^6(c+dx^3)}\,dx$

Optimal. Leaf size=206

$$-\frac{d(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{8/3}}+\frac{d(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}}-\frac{d(bc-ad)^{2/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}}-\frac{(a+\cdots}{}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/c/x^5-1/10*(-5*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^2-1/
6*d*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(8/3)+1/2*d*(-a*d+b*c)^(2/3)*ln((-a*d+b*
c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)-1/3*d*(-a*d+b*c)^(2/3)*arctan(1
/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)*3^(1/2
)
```

Rubi [C] time = 0.13, antiderivative size = 148, normalized size of antiderivative = 0.72, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{-2x^3\left(2c-3dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\cdots}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(c*(a + b*x^3)*(2*c - 3*d*x^3) - 2*(b*c - a*d)*x^3*(2*c - 3*d*x^3)*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c - a*d)*
x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(10*c^3*x^5*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx=\frac{\left(a+bx^3\right)^{2/3}\int\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^6\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$=-\frac{c\left(a+bx^3\right)\left(2c-3dx^3\right)-2(bc-ad)x^3\left(2c-3dx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)+6(bc-ad)x^3\left(c\right.}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 0.64, size = 148, normalized size = 0.72

$$-\frac{2x^3\left(3dx^3-2c\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+c\left(a+bx^3\right)\left(2c\right.}{10c^3x^5\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -1/10*(c*(a + b*x^3)*(2*c - 3*d*x^3) + 2*(b*c - a*d)*x^3*(-2*c + 3*d*x^3)*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6*(b*c -
 a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))])/(c^3*x^5*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^6/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^6*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^6\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**6/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**6*(c + d*x**3)), x)
```

3.689 $\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)} dx$

Optimal. Leaf size=257

$$\frac{(a+bx^3)^{2/3}(-20a^2d^2+8abcd+3b^2c^2)}{40a^2c^3x^2}+\frac{d^2(bc-ad)^{2/3}\log(c+dx^3)}{6c^{11/3}}-\frac{d^2(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}}+$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/c/x^8-1/20*(-4*a*d+b*c)*(b*x^3+a)^(2/3)/a/c^2/x^5+1/40
*(-20*a^2*d^2+8*a*b*c*d+3*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^2+1/6*d^2*(-a*
d+b*c)^(2/3)*ln(d*x^3+c)/c^(11/3)-1/2*d^2*(-a*d+b*c)^(2/3)*ln((-a*d+b*c)^(1
/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)+1/3*d^2*(-a*d+b*c)^(2/3)*arctan(1/3
*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)*3^(1/2)
```

Rubi [C] time = 0.97, antiderivative size = 451, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-9x^3(c+dx^3)^2(bc-ad)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-2x^3(5c^2-6cdx^3+9d^2x^6)(bc-ad)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^6 + 9
*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3*Hype
rgeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c^2*d*
x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6*b
*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[
{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*x^8*(a +
 b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{x^9(c+dx^3)}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^9(c+dx^3)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{5ac^3+5bc^3x^3-6ac^2dx^3-6bc^2dx^6+9acd^2x^6+9bcd^2x^9-2(bc-ad)x^3\left(5c^2-6cdx^3+\right.}{}$$

Mathematica [C] time = 2.20, size = 451, normalized size = 1.75

$$-\frac{-9x^3(c+dx^3)^2(bc-ad)\,{}_3F_2\left(\frac{1}{3},2,2;1,\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+21bc^3x^3\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-2x^3\left(5c^2-6cdx^3+9d^2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x]
```

```
[Out] -1/40*(5*a*c^3 + 5*b*c^3*x^3 - 6*a*c^2*d*x^3 - 6*b*c^2*d*x^6 + 9*a*c*d^2*x^
6 + 9*b*c*d^2*x^9 - 2*(b*c - a*d)*x^3*(5*c^2 - 6*c*d*x^3 + 9*d^2*x^6)*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*b*c^3*x^3
*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 21*a*c
^2*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 6*b*c^2*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 6*a*c*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] - 27*b*c*d^2*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] + 27*a*d^3*x^9*Hypergeometric2F1[1/3, 2, 4/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*Hypergeometri
cPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*x^8*(a
 + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^3+a)^{\frac{2}{3}}}{(d\,x^3+c)\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
[Out] int((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^9/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^9\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^9*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^9\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**9/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**9*(c + d*x**3)), x)
```

3.690 $\int \frac{(a+bx^3)^{2/3}}{x^{12}(c+dx^3)}\,dx$

Optimal. Leaf size=320

$$\frac{(a+bx^3)^{2/3}(-44a^2d^2+11abcd+6b^2c^2)}{220a^2c^3x^5}-\frac{(a+bx^3)^{2/3}(-220a^3d^3+88a^2bcd^2+33ab^2c^2d+18b^3c^3)}{440a^3c^4x^2}-\frac{d^3(bc-$$

```
[Out] -1/11*(b*x^3+a)^(2/3)/c/x^11-1/88*(-11*a*d+2*b*c)*(b*x^3+a)^(2/3)/a/c^2/x^8
+1/220*(-44*a^2*d^2+11*a*b*c*d+6*b^2*c^2)*(b*x^3+a)^(2/3)/a^2/c^3/x^5-1/440
*(-220*a^3*d^3+88*a^2*b*c*d^2+33*a*b^2*c^2*d+18*b^3*c^3)*(b*x^3+a)^(2/3)/a^
3/c^4/x^2-1/6*d^3*(-a*d+b*c)^(2/3)*ln(d*x^3+c)/c^(14/3)+1/2*d^3*(-a*d+b*c)^
(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(14/3)-1/3*d^3*(-a*d
+b*c)^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(
1/2))/c^(14/3)*3^(1/2)
```

Rubi [C] time = 2.63, antiderivative size = 819, normalized size of antiderivative = 2.56, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{-81bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-}{c(bx}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^2*d^2*
x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*x^3*Hy
pergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*a*c^3*
d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9
0*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1, 4/3,
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*Hyperg
eometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c^3*d*x
^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 99*b
*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*(b*c - a*d)*x^3*(5*c - 6*d*x^3)*(c + d*x
^3)^2*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2, 2, 2},
 {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(440*c^5*x^11*(a + b*x^3)
^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
- 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx = \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^{12}\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= -\frac{40ac^4+40bc^4x^3-45ac^3dx^3-45bc^3dx^6+54ac^2d^2x^6+54bc^2d^2x^9-81acd^3x^9-81bcd^3x^1}{}$$

Mathematica [C] time = 4.99, size = 819, normalized size = 2.56

$$-\frac{-81bcd^3x^{12}-162ad^4\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}+162bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{12}-297ad^4\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{(bc-ad)x}{c\left(bx^3+a\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x]
```

```
[Out] -1/440*(40*a*c^4 + 40*b*c^4*x^3 - 45*a*c^3*d*x^3 - 45*b*c^3*d*x^6 + 54*a*c^
2*d^2*x^6 + 54*b*c^2*d^2*x^9 - 81*a*c*d^3*x^9 - 81*b*c*d^3*x^12 - 80*b*c^4*
x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 80*
a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] + 90*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 90*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 108*b*c^2*d^2*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 108*a*c*d^3*x^9*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b*c*d^3*x^12*Hypergeometric2
F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a*d^4*x^12*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 198*b*c^4*x^3*
Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 198*a*c
^3*d*x^3*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 99*b*c^3*d*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 99*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 2, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeometric2F1[1/3, 2, 4/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hypergeometric2F1[1/3, 2, 4/3,
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^2*(-5*
c + 6*d*x^3)*HypergeometricPFQ[{1/3, 2, 2}, {1, 4/3}, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*HypergeometricPFQ[{1/3, 2,
 2, 2}, {1, 1, 4/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*x^11*(a + b*x
^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^12/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^12), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^{12}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(2/3)/(x^12*(c + d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**12/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.691 $\int \frac{x^7(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^8 \left(a+bx^3\right)^{2/3} F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] 1/8*x^8*(b*x^3+a)^(2/3)*AppellF1(8/3,-2/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(1+b*
x^3/a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^8 \left(a+bx^3\right)^{2/3} F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^8*(a + b*x^3)^(2/3)*AppellF1[8/3, -2/3, 1, 11/3, -((b*x^3)/a), -((d*x^3)
/c)])/(8*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7 \left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3} \int \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^8 \left(a+bx^3\right)^{2/3} F_1\left(\frac{8}{3};-\frac{2}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.24, size = 181, normalized size = 2.83

$$\frac{x^2\left(-2x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2+7abcd-14b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5ac\sqrt[3]{\frac{bx^3}{a}+1}(7bc-2ad)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};\right.\right.}{140bcd^2\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(5*c*(a + b*x^3)*(-7*b*c + 2*a*d + 4*b*d*x^3) + 5*a*c*(7*b*c - 2*a*d)*
(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)
] - 2*(-14*b^2*c^2 + 7*a*b*c*d + 2*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(140*b*c*d^2*(a + b*x^3
)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^7}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^7}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^7/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^7\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^7*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**7*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.692 $\int \frac{x^4(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] 1/5*x^5*(b*x^3+a)^(2/3)*AppellF1(5/3,-2/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^5*(a + b*x^3)^(2/3)*AppellF1[5/3, -2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/
c)])/(5*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^5\left(a+bx^3\right)^{2/3}F_1\left(\frac{5}{3};-\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.12, size = 141, normalized size = 2.20

$$\frac{2x^5\sqrt[3]{\frac{bx^3}{a}+1}\,(ad-2bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a+bx^3\right)}{20cd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*(-2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(
1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*c*d*(a + b
*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x^4}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*x^4/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3),x)
[Out] int((x^4*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.693 $\int \frac{x(a+bx^3)^{2/3}}{c+dx^3}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] 1/2*x^2*(b*x^3+a)^(2/3)*AppellF1(2/3,-2/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x
^3/a)^(2/3)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
```

```
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^3\right)^{2/3}}{c+dx^3}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= \frac{x^2\left(a+bx^3\right)^{2/3}F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{x^2 \left(a+bx^3\right)^{2/3} F_1\left(\frac{2}{3};-\frac{2}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\left(\frac{a+bx^3}{a}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^3)^(2/3))/(c + d*x^3),x]
[Out] (x^2*(a + b*x^3)^(2/3)*AppellF1[2/3, -2/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*((a + b*x^3)/a)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}x}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x*(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(2/3)*x/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\left(b\,x^3+a\right)^{2/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(2/3))/(c + d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(2/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(2/3)/(c + d*x**3), x)
```

3.694 $\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=62

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] -(b*x^3+a)^(2/3)*AppellF1(-1/3,-2/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(1+b*x^3/a
)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -(((a + b*x^3)^(2/3)*AppellF1[-1/3, -2/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c
)])/(c*x*(1 + (b*x^3)/a)^(2/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^2\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{1}{3};-\frac{2}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.14, size = 138, normalized size = 2.23

$$\frac{-5x^3\sqrt[3]{\frac{bx^3}{a}+1}(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10c\left(a+bx^3\right)}{10c^2x\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) - 5*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/
3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/
3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*c^2*x*(a + b
*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{2/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^2*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**2*(c + d*x**3)), x)
```

3.695 $\int \frac{(a+bx^3)^{2/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

```
[Out] -1/4*(b*x^3+a)^(2/3)*AppellF1(-4/3,-2/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(1+
b*x^3/a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(2/3)*AppellF1[-4/3, -2/3, 1, -1/3, -((b*x^3)/a), -((d*x^3)/c
)])/(4*c*x^4*(1 + (b*x^3)/a)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx &= \frac{\left(a+bx^3\right)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{x^5\left(c+dx^3\right)}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\\ &= -\frac{\left(a+bx^3\right)^{2/3}F_1\left(-\frac{4}{3};-\frac{2}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\left(1+\frac{bx^3}{a}\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.18, size = 181, normalized size = 2.83

$$\frac{5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2-4abcd+b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}(bc-2ad)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},\right.}{20ac^3x^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] (-5*c*(a + b*x^3)*(2*b*c*x^3 + a*(c - 4*d*x^3)) + 5*(b^2*c^2 - 4*a*b*c*d +
2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a
), -((d*x^3)/c)] + 2*b*d*(b*c - 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a*c^3*x^4*(a + b*x^3)^(1/
3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{2}{3}}}{\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/x^5/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/((d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{2/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(2/3)/(x^5*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/x**5/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(2/3)/(x**5*(c + d*x**3)), x)
```

3.696 $\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=251

$$-\frac{\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^2(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+\right.}{2d^{13/3}}$$

```
[Out] -c^2*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^4+1/4*c^2*(b*x^3+a)^(4/3)/d^3-1/7*(a*d+b*
c)*(b*x^3+a)^(7/3)/b^2/d^2+1/10*(b*x^3+a)^(10/3)/b^2/d-1/6*c^2*(-a*d+b*c)^(
4/3)*ln(d*x^3+c)/d^(13/3)+1/2*c^2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1
/3)*(b*x^3+a)^(1/3))/d^(13/3)-1/3*c^2*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(13/3)*3^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 88, 50, 58, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{7/3}(ad+bc)}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{c^2\sqrt[3]{a+bx^3}(bc-ad)}{d^4}-\frac{c^2(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{13/3}}+\frac{c^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -((c^2*(b*c - a*d)*(a + b*x^3)^(1/3))/d^4) + (c^2*(a + b*x^3)^(4/3))/(4*d^3
) - ((b*c + a*d)*(a + b*x^3)^(7/3))/(7*b^2*d^2) + (a + b*x^3)^(10/3)/(10*b^
2*d) - (c^2*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*
c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(13/3)) - (c^2*(b*c - a*d)^(4/3)*Log[c
 + d*x^3])/(6*d^(13/3)) + (c^2*(b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^
(1/3)*(a + b*x^3)^(1/3)])/(2*d^(13/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{(-bc-ad)(a+bx)^{4/3}}{bd^2}+\frac{(a+bx)^{7/3}}{bd}+\frac{c^2(a+bx)^{4/3}}{d^2(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{c^2\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{\left(c^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,d\right.}{3d^3}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}+\frac{\left(c^2(b\right.}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc}{}$$

$$= -\frac{c^2(bc-ad)\sqrt[3]{a+bx^3}}{d^4}+\frac{c^2\left(a+bx^3\right)^{4/3}}{4d^3}-\frac{(bc+ad)\left(a+bx^3\right)^{7/3}}{7b^2d^2}+\frac{\left(a+bx^3\right)^{10/3}}{10b^2d}-\frac{c^2(bc}{}$$

Mathematica [A] time = 0.54, size = 258, normalized size = 1.03

$$\frac{-\frac{60d(a+bx^3)^{7/3}(ad+bc)}{b^2}+\frac{42d^2(a+bx^3)^{10/3}}{b^2}-\frac{70c^2(bc-ad)\left(\sqrt[3]{bc-ad}\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}(a+bx^3)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\right.\right.\right.}{d^{4/3}}}{420d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (105*c^2*(a + b*x^3)^(4/3) - (60*d*(b*c + a*d)*(a + b*x^3)^(7/3))/b^2 + (42
*d^2*(a + b*x^3)^(10/3))/b^2 - (70*c^2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(
1/3) + (b*c - a*d)^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3
))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x
^3)^(1/3)] + Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^
(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])))/d^(4/3))/(420*d^3)
```

fricas [A] time = 1.07, size = 369, normalized size = 1.47

$$\frac{140\sqrt{3}\left(b^3c^3-ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+70\left(b^3c^3-ab^2c^2d\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/420*(140*sqrt(3)*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1
/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a
*d))/(b*c - a*d)) + 70*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3)) - 140*(b^3*c^3 - a*b^2*c^2*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(14*b^3*d^3*x^9 - 2*(10*b^3*c*d^2
- 11*a*b^2*d^3)*x^6 - 140*b^3*c^3 + 175*a*b^2*c^2*d - 20*a^2*b*c*d^2 - 6*a^
3*d^3 + (35*b^3*c^2*d - 40*a*b^2*c*d^2 + 2*a^2*b*d^3)*x^3)*(b*x^3 + a)^(1/3
))/(b^2*d^4)
```

giac [A] time = 0.30, size = 394, normalized size = 1.57

$$-\frac{\left(b^{24}c^4d^6-2\,ab^{23}c^3d^7+a^2b^{22}c^2d^8\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{23}cd^{10}-ab^{22}d^{11}\right)}+\frac{\sqrt{3}\left(bc^3-ac^2d\right)\left(-bcd^2+ad^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^24*c^4*d^6 - 2*a*b^23*c^3*d^7 + a^2*b^22*c^2*d^8)*(-(b*c - a*d)/d)^
(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^23*c*d^10 - a
*b^22*d^11) + 1/3*sqrt(3)*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*arctan
(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d
)^(1/3))/d^5 + 1/6*(b*c^3 - a*c^2*d)*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3
))/d^5 - 1/140*(140*(b*x^3 + a)^(1/3)*b^21*c^3*d^6 - 35*(b*x^3 + a)^(4/3)*b
^20*c^2*d^7 - 140*(b*x^3 + a)^(1/3)*a*b^20*c^2*d^7 + 20*(b*x^3 + a)^(7/3)*b
```

```
^19*c*d^8 - 14*(b*x^3 + a)^(10/3)*b^18*d^9 + 20*(b*x^3 + a)^(7/3)*a*b^18*d^
9)/(b^20*d^10)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}} x^8}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.12, size = 477, normalized size = 1.90

$$\left(\frac{a^2}{4\,b^2\,d}+\frac{\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b^3\,c-a\,b^2\,d\right)}{4\,b^2\,d}\right)\,{\left(b\,x^3+a\right)}^{4/3}-\left(\frac{2\,a}{7\,b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{7\,b^4\,d^2}\right)\,{\left(b\,x^3+a\right)}^{7/3}+\frac{{\left(b\,x^3+a\right)}^{10/3}}{10\,b^2\,d}+\frac{c^2}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a^2/(4*b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b^3*c - a*
b^2*d))/(4*b^2*d))*(a + b*x^3)^(4/3) - ((2*a)/(7*b^2*d) + (b^3*c - a*b^2*d)
/(7*b^4*d^2))*(a + b*x^3)^(7/3) + (a + b*x^3)^(10/3)/(10*b^2*d) + (c^2*log(
(3*(a + b*x^3)^(1/3)*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))/d^2 - (c^2*(a*d
 - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(13/3)))*(a*d - b*c)^(4/3))/(3*d^
(13/3)) - ((a^2/(b^2*d) + (((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))*(b
^3*c - a*b^2*d))/(b^2*d))*(a + b*x^3)^(1/3)*(b^3*c - a*b^2*d))/(b^2*d) - (c
^2*log((3*c^2*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(7/3) + (3*c^2*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2)*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3
))/(3*d^(13/3)) + (c^2*log((3*c^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d^2 - (9
*c^2*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(7/3))/d^(7/3))*((3^(1/2)*1i)/6 - 1
/6)*(a*d - b*c)^(4/3))/d^(13/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.697 $\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=211

$$\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}+\frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}}+$$

```
[Out] c*(-a*d+b*c)*(b*x^3+a)^(1/3)/d^3-1/4*c*(b*x^3+a)^(4/3)/d^2+1/7*(b*x^3+a)^(7
/3)/b/d+1/6*c*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^(10/3)-1/2*c*(-a*d+b*c)^(4/3)*
ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)+1/3*c*(-a*d+b*c)^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(10/
3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 80, 50, 58, 617, 204, 31}

$$-\frac{c\left(a+bx^3\right)^{4/3}}{4d^2}+\frac{c\sqrt[3]{a+bx^3}\,(bc-ad)}{d^3}+\frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}}-\frac{c(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (c*(b*c - a*d)*(a + b*x^3)^(1/3))/d^3 - (c*(a + b*x^3)^(4/3))/(4*d^2) + (a
+ b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b
*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(10/3)) + (c*(b*c - a*
d)^(4/3)*Log[c + d*x^3])/(6*d^(10/3)) - (c*(b*c - a*d)^(4/3)*Log[(b*c - a*d
)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{c\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx, x, x^3\right)}{3d}$$

$$= -\frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{(c(bc-ad))\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3d^2}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} - \frac{\left(c(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx}\right.}{3d^3}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{(c(bc}{}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{10/3}} - \frac{c(bc-}{}$$

$$= \frac{c(bc-ad)\sqrt[3]{a+bx^3}}{d^3} - \frac{c\left(a+bx^3\right)^{4/3}}{4d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7bd} + \frac{c(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{10/3}} -$$

Mathematica [A] time = 0.34, size = 255, normalized size = 1.21

$$\frac{c(bc-ad)\left(\sqrt[3]{bc-ad}\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\log\left(\sqrt[3]{bc-\right.\right.}{6d^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -1/4*(c*(a + b*x^3)^(4/3))/d^2 + (a + b*x^3)^(7/3)/(7*b*d) + (c*(b*c - a*d)
*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2
*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/
3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*L
og[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3
)*(a + b*x^3)^(2/3)]))/(6*d^(10/3))
```

fricas [A] time = 1.10, size = 298, normalized size = 1.41

$$28\sqrt{3}\left(b^2c^2-abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}\left(bc-ad\right)}{3\left(bc-ad\right)}\right)+14\left(b^2c^2-abcd\right)\left(\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left(bx^3+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/84*(28*sqrt(3)*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*s
qrt(3)*(b*x^3 + a)^(1/3)*d*((b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*
c - a*d)) + 14*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2
/3) - (b*x^3 + a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) - 28
*(b^2*c^2 - a*b*c*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c -
a*d)/d)^(1/3)) + 3*(4*b^2*d^2*x^6 + 28*b^2*c^2 - 35*a*b*c*d + 4*a^2*d^2 - (
7*b^2*c*d - 8*a*b*d^2)*x^3)*(b*x^3 + a)^(1/3))/(b*d^3)
```

giac [B] time = 0.29, size = 348, normalized size = 1.65

$$\frac{\left(b^{10}c^3d^4-2\,ab^9c^2d^5+a^2b^8cd^6\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^9cd^7-ab^8d^8\right)}-\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bc^2-acd\right)\mathrm{arc}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^10*c^3*d^4 - 2*a*b^9*c^2*d^5 + a^2*b^8*c*d^6)*(-(b*c - a*d)/d)^(1/3)
*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^9*c*d^7 - a*b^8*d^
8) - 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*arctan(1/3*sqrt(3
)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^
4 - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c^2 - a*c*d)*log((b*x^3 + a)^(2/3) + (b
*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^4 + 1/28
*(28*(b*x^3 + a)^(1/3)*b^8*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^7*c*d^5 - 28*(b*
x^3 + a)^(1/3)*a*b^7*c*d^5 + 4*(b*x^3 + a)^(7/3)*b^6*d^6)/(b^7*d^7)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}x^5}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.06, size = 348, normalized size = 1.65

$$\frac{\left(b\,x^3+a\right)^{7/3}}{7\,b\,d}-\left(b\,x^3+a\right)^{4/3}\,\left(\frac{a}{4\,b\,d}+\frac{b^2\,c-a\,b\,d}{4\,b^2\,d^2}\right)-\frac{c\,\ln\left(\frac{3\,\left(b\,x^3+a\right)^{1/3}\,\left(a^2\,c\,d^2-2\,a\,b\,c^2\,d+b^2\,c^3\right)}{d}-\frac{c\,{\left(a\,d-b\,c\right)}^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{10/3}}\right)\,(a}{3\,d^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(7/3)/(7*b*d) - (a + b*x^3)^(4/3)*(a/(4*b*d) + (b^2*c - a*b*d)/
(4*b^2*d^2)) - (c*log((3*(a + b*x^3)^(1/3)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2
*d))/d - (c*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(10/3)))*(a*d - b
*c)^(4/3))/(3*d^(10/3)) - (c*log((3*c*(a + b*x^3)^(1/3)*(a*d - b*c)^2)/d -
(3*c*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1
/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + (c*log((3*c*(a + b*x^3)^(1/3)*(a*d -
b*c)^2)/d + (3*c*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(7/3))/d^(4/3))*((3^(1/
2)*1i)/2 + 1/2)*(a*d - b*c)^(4/3))/(3*d^(10/3)) + ((a + b*x^3)^(1/3)*(b^2*c
 - a*b*d)*(a/(b*d) + (b^2*c - a*b*d)/(b^2*d^2)))/(b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.698 $\int \frac{x^2(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=187

$$-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-$$

```
[Out] -(-a*d+b*c)*(b*x^3+a)^(1/3)/d^2+1/4*(b*x^3+a)^(4/3)/d-1/6*(-a*d+b*c)^(4/3)*
ln(d*x^3+c)/d^(7/3)+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3
+a)^(1/3))/d^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(
1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 50, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{d^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}}-\frac{(bc-ad)^{4/3}\tan}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] -(((b*c - a*d)*(a + b*x^3)^(1/3))/d^2) + (a + b*x^3)^(4/3)/(4*d) - ((b*c -
a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqr
t[3]])/(Sqrt[3]*d^(7/3)) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*d^(7/3)) +
 ((b*c - a*d)^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*
d^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx=\frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}\,dx,x,x^3\right)$$

$$=\frac{\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx,x,x^3\right)}{3d}$$

$$=-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4d}+\frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3d^2}$$

$$=-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\,\text{Subst}\left(\int \ldots\right)}{2\ldots}$$

$$=-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6d^{7/3}}+\frac{(bc-ad)^{4/3}\log\left(\sqrt[3]{bc-\ldots}\right.}{2d^{\ldots}}$$

$$=-\frac{(bc-ad)\sqrt[3]{a+bx^3}}{d^2}+\frac{\left(a+bx^3\right)^{4/3}}{4d}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}}-\frac{(bc-ad)^{4/3}\log\ldots}{6d^{7/3}}$$

Mathematica [A] time = 0.32, size = 232, normalized size = 1.24

$$\frac{(ad-bc)\left(\sqrt[3]{bc-ad}\,\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\sqrt[3]{bc-ad}\,\log\left(\sqrt[3]{bc-ad}\ldots\right.\right.}{6d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + ((-(b*c) + a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) - 2*
Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
 a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*
c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)]))/(6*d^(7/3))
```

fricas [A] time = 1.05, size = 246, normalized size = 1.32

$$\frac{4\sqrt{3}(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}(bx^3+a)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2(bc-ad)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left((bx^3+a)^{\frac{2}{3}}+\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(
b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d
)) + 2*(b*c - a*d)*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 4*(b*c - a*d)*(
-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) + 3*(
b*d*x^3 - 4*b*c + 5*a*d)*(b*x^3 + a)^(1/3))/d^2
```

giac [A] time = 0.30, size = 297, normalized size = 1.59

$$-\frac{\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|(bx^3+a)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bcd^4-ad^5\right)}+\frac{\sqrt{3}\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\right.}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(-(b*c - a*d)/d)^(1/3)*log(abs((
b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c*d^4 - a*d^5) + 1/3*sqrt(3)
*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/
3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/d^3 + 1/6*(-b*c*d^2 +
a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c -
 a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/d^3 - 1/4*(4*(b*x^3 + a)^(1/3)*b*c
*d^2 - (b*x^3 + a)^(4/3)*d^3 - 4*(b*x^3 + a)^(1/3)*a*d^3)/d^4
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}x^2}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.72, size = 304, normalized size = 1.63

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,d}+\frac{\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(3\,a^2\,d^2-6\,a\,b\,c\,d+3\,b^2\,c^2\right)-\frac{(a\,d-b\,c)^{4/3}\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{7/3}}\right)\,(a\,d-b\,c)^{4/3}}{3\,d^{7/3}}+\frac{\left(b\,x^3+a\right)^{1/3}\,(}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*d) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6
*a*b*c*d) - ((a*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*(a*d - b
*c)^(4/3))/(3*d^(7/3)) + ((a + b*x^3)^(1/3)*(a*d - b*c))/d^2 - (log((a + b*
x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*c^2 - 6*a*b*c*d) + (((3^(1/2)*1i)/2 + 1/2)*(a
*d - b*c)^(4/3)*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)))*((3^(1/2)*1i)/2 + 1/2)*
(a*d - b*c)^(4/3))/(3*d^(7/3)) + (log((a + b*x^3)^(1/3)*(3*a^2*d^2 + 3*b^2*
c^2 - 6*a*b*c*d) - (((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3)*(9*a*d^3 - 9*b
*c*d^2))/d^(7/3))*((3^(1/2)*1i)/6 - 1/6)*(a*d - b*c)^(4/3))/d^(7/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.699 $\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=261

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log}{}$$

```
[Out] b*(b*x^3+a)^(1/3)/d-1/2*a^(4/3)*ln(x)/c+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c/
d^(4/3)+1/2*a^(4/3)*ln(a^(1/3)-(b*x^3+a)^(1/3))/c-1/2*(-a*d+b*c)^(4/3)*ln((
-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/d^(4/3)-1/3*a^(4/3)*arctan(1/3*(
a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/c*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*
arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/d^(4/3
)*3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 84, 156, 57, 617, 204, 31, 58}

$$\frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c}-\frac{a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}c}-\frac{a^{4/3}\log(x)}{2c}+\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}}-\frac{(bc-ad)^{4/3}\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (b*(a + b*x^3)^(1/3))/d - (a^(4/3)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*c) + ((b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*d^(4/3)) - (a^(4
/3)*Log[x])/(2*c) + ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c*d^(4/3)) + (a^(
4/3)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(2*c) - ((b*c - a*d)^(4/3)*Log[(b*c
- a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*d^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 84

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), I
nt[((b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*(e + f*x)^(p - 2))/
((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{\text{Subst}\left(\int \frac{a^2d+b(-bc+2ad)x}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3cd}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} - \frac{a^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c}$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\log(x)}{2c} + \frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6cd^{4/3}} + \frac{a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2c} - (bc-$$

$$= \frac{b\sqrt[3]{a+bx^3}}{d} - \frac{a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,c} + \frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,cd^{4/3}} - \frac{a^{4/3}\log(x)}{2c} + (bc$$

Mathematica [A] time = 0.67, size = 331, normalized size = 1.27

$$-\frac{\left(a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)\right)+\frac{(bc-a}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x*(c + d*x^3)),x]
```

```
[Out] (6*a*(a + b*x^3)^(1/3) - a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a
^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]) + ((b*c - a*d)*(6*d^(1/3)*(a
 + b*x^3)^(1/3) - 2*Sqrt[3]*(b*c - a*d)^(1/3)*ArcTan[(-1 + (2*d^(1/3)*(a +
b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*(b*c - a*d)^(1/3)*Log[(b*c -
a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + (b*c - a*d)^(1/3)*Log[(b*c - a*d)
^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^
(2/3)]))/d^(4/3))/(6*c)
```

fricas [A] time = 1.33, size = 320, normalized size = 1.23

$$-\frac{2\sqrt{3}\,a^{\frac{4}{3}}d\arctan\left(\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\,a^{\frac{4}{3}}d\log\left(\left(bx^3+a\right)^{\frac{1}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^(4/3)*d*arctan(1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + a^(4/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3)) - 2*a^(4/3)*d*log((b*x^3 + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(b*
c - a*d)*((b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*d*(
(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c - a*d)) - 6*(b*x^3 + a)^(1
/3)*b*c - (b*c - a*d)*((b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) - (b*x^3
+ a)^(1/3)*((b*c - a*d)/d)^(1/3) + ((b*c - a*d)/d)^(2/3)) + 2*(b*c - a*d)*(
(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) + ((b*c - a*d)/d)^(1/3)))/(c*d)
```

giac [A] time = 0.84, size = 357, normalized size = 1.37

$$-\frac{\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,c}-\frac{a^{\frac{4}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,c}+\frac{a^{\frac{4}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,c}+\frac{(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/c - 1/6*a^(4/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a
^(2/3))/c + 1/3*a^(4/3)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/c + 1/3*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3)
 - (-(b*c - a*d)/d)^(1/3)))/(b*c^2*d - a*c*d^2) + (b*x^3 + a)^(1/3)*b/d - 1
/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^
```

```
3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c*d^2) - 1/
6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^
(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(c*d^2)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}}}{\left(d\,x^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}}}{\left(dx^{3}+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x), x)
```

mupad [B] time = 6.08, size = 796, normalized size = 3.05

$$\ln\left(c\,d\left(-\frac{\left(a\,d-b\,c\right)^4}{c^3\,d^4}\right)^{1/3}+a\,d\,\left(b\,x^3+a\right)^{1/3}-b\,c\,\left(b\,x^3+a\right)^{1/3}\right)\,\left(-\frac{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4}{27\,c^3\,d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x*(c + d*x^3)),x)
```

```
[Out] log(c*d*(-(a*d - b*c)^4/(c^3*d^4))^(1/3) + a*d*(a + b*x^3)^(1/3) - b*c*(a +
 b*x^3)^(1/3))*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) + log(c*(a^4/c^3)^(1/3) - a*(a + b*x^3)^(
1/3))*(a^4/(27*c^3))^(1/3) + (b*(a + b*x^3)^(1/3))/d - log(c*(a^4/c^3)^(1/3
) + 2*a*(a + b*x^3)^(1/3) + 3^(1/2)*c*(a^4/c^3)^(1/3)*1i)*((3^(1/2)*1i)/2 +
 1/2)*(a^4/(27*c^3))^(1/3) + log(c*(a^4/c^3)^(1/3)*1i + a*(a + b*x^3)^(1/3)
*2i + 3^(1/2)*c*(a^4/c^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a^4/(27*c^3))^(1/3
) + log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6
*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d + 3*a^2*b^4*c*((3^(1/2
)*1i)/2 - 1/2)*(-(a*d - b*c)^4/(c^3*d^4))^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a
^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(
1/2)*1i)/2 - 1/2)*(-(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3) - log((3*a^2*b^4*(a + b*x^3)^(1/3)*(a*
d - b*c)^2*(2*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))/d - 3*a^2*b^4*c*((3^(1/2)*1i)/2 + 1/2)*(-(a*d - b*c)^4/(c^3*d^4)
)^(1/3)*(2*a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*
a*b^4*c^4*d - 6*a^4*b*c*d^4))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^4 + b^4*c^4 +
 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(27*c^3*d^4))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^{3}\right)^{\frac{4}{3}}}{x\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x*(c + d*x**3)), x)
```

3.700 $\int \frac{(a+bx^3)^{4/3}}{x^4(c+dx^3)}\,dx$

Optimal. Leaf size=399

$$\frac{d\left(a+bx^3\right)^{4/3}}{4c^2}+\frac{\left(a+bx^3\right)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}\,(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}}+\cdots$$

```
[Out] 1/3*(-3*a*d+4*b*c)*(b*x^3+a)^(1/3)/c^2-(-a*d+b*c)*(b*x^3+a)^(1/3)/c^2+1/4*d
*(b*x^3+a)^(4/3)/c^2+1/12*(-3*a*d+4*b*c)*(b*x^3+a)^(4/3)/a/c^2-1/3*(b*x^3+a
)^(7/3)/a/c/x^3-1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(x)/c^2-1/6*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^2/d^(1/3)+1/6*a^(1/3)*(-3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3
))/c^2+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^
2/d^(1/3)-1/9*a^(1/3)*(-3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))
/a^(1/3)*3^(1/2))/c^2*3^(1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*
(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/d^(1/3)*3^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 50, 57, 617, 204, 31, 58}

$$\frac{d\left(a+bx^3\right)^{4/3}}{4c^2}+\frac{\left(a+bx^3\right)^{4/3}(4bc-3ad)}{12ac^2}+\frac{\sqrt[3]{a+bx^3}\,(4bc-3ad)}{3c^2}-\frac{\sqrt[3]{a+bx^3}\,(bc-ad)}{c^2}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{d}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] ((4*b*c - 3*a*d)*(a + b*x^3)^(1/3))/(3*c^2) - ((b*c - a*d)*(a + b*x^3)^(1/3
))/c^2 + (d*(a + b*x^3)^(4/3))/(4*c^2) + ((4*b*c - 3*a*d)*(a + b*x^3)^(4/3)
)/(12*a*c^2) - (a + b*x^3)^(7/3)/(3*a*c*x^3) - (a^(1/3)*(4*b*c - 3*a*d)*Arc
Tan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*c^2) - (
(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]])/(Sqrt[3]*c^2*d^(1/3)) - (a^(1/3)*(4*b*c - 3*a*d)*Log[x])/(6*c
^2) - ((b*c - a*d)^(4/3)*Log[c + d*x^3])/(6*c^2*d^(1/3)) + (a^(1/3)*(4*b*c
- 3*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*c^2) + ((b*c - a*d)^(4/3)*Log
[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*d^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^4\left(c+dx^3\right)}dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^2(c+dx)}dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} - \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-4bc+3ad)-\frac{4bdx}{3}\right)}{x(c+dx)}dx, x, x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{d^2\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{c+dx}dx, x, x^3\right)}{3c^2} + \frac{(4bc-3ad)\operatorname{Subst}\left(\int \frac{(a+bx)^{4/3}}{x}dx, x, x^3\right)}{9ac^2}$$

$$= \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \frac{\left(a+bx^3\right)^{7/3}}{3acx^3} + \frac{(4bc-3ad)\operatorname{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{x}dx, x\right.}{9c^2}$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \cdots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \cdots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \cdots$$

$$= \frac{(4bc-3ad)\sqrt[3]{a+bx^3}}{3c^2} - \frac{(bc-ad)\sqrt[3]{a+bx^3}}{c^2} + \frac{d\left(a+bx^3\right)^{4/3}}{4c^2} + \frac{(4bc-3ad)\left(a+bx^3\right)^{4/3}}{12ac^2} - \cdots$$

Mathematica [A] time = 1.55, size = 389, normalized size = 0.97

$$\frac{(4bc-3ad)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)+3a\sqrt[3]{a+bx^3}+\frac{3}{4}\left(a+bx^3\right)^{4/3}\right)}{3c} + \frac{a\left(3d^{4/3}(a+b\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x]
```

```
[Out] (-((a + b*x^3)^(7/3)/x^3) + ((4*b*c - 3*a*d)*(3*a*(a + b*x^3)^(1/3) + (3*(a
 + b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/
a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1
/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2))/(3*c) + (a*(3*d^(4/3)*(a +
 b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(1
/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b
*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
+ b*x^3)^(2/3)]))))/(4*c*d^(1/3)))/(3*a*c)
```

fricas [A] time = 2.10, size = 383, normalized size = 0.96

$$\frac{6\sqrt{3}(bc-ad)x^3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}-\sqrt{3}(bc-ad)}{3(bc-ad)}\right)+2\sqrt{3}(4bc-3ad)(-a)^{\frac{1}{3}}x^3\arctan\left(\frac{2\sqrt{3}\left(bx\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*arctan(-1/3*(2*sqrt(
3)*(b*x^3 + a)^(1/3)*d*(-(b*c - a*d)/d)^(2/3) - sqrt(3)*(b*c - a*d))/(b*c -
 a*d)) + 2*sqrt(3)*(4*b*c - 3*a*d)*(-a)^(1/3)*x^3*arctan(1/3*(2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a)^(2/3) + sqrt(3)*a)/a) + (4*b*c - 3*a*d)*(-a)^(1/3)*x^3*
log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 3*(b*c
 - a*d)*x^3*(-(b*c - a*d)/d)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3)) - 2*(4*b*c - 3*a*d)*(-a)
^(1/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*c - a*d)*x^3*(-(b*c -
 a*d)/d)^(1/3)*log((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)) - 6*(b*x^3 +
 a)^(1/3)*a*c)/(c^2*x^3)
```

giac [A] time = 0.86, size = 394, normalized size = 0.99

$$-\frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3 - ac^2d\right)} - \frac{\sqrt{3}\left(4\,a^{\frac{1}{3}}bc - 3\,a^{\frac{4}{3}}d\right)\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}} + \cdots\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3
+ a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^3 - a*c^2*d) - 1/9*sqrt(3)*(4*a^
(1/3)*b*c - 3*a^(4/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))
/a^(1/3))/c^2 - 1/18*(4*a^(1/3)*b*c - 3*a^(4/3)*d)*log((b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/c^2 + 1/3*sqrt(3)*(-b*c*d^2 + a*d^3)^(
1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)
^(1/3))/(-(b*c - a*d)/d)^(1/3))/(c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*(b*c
 - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
(-(b*c - a*d)/d)^(2/3))/(c^2*d) + 1/9*(4*a*b*c - 3*a^2*d)*log(abs((b*x^3 +
a)^(1/3) - a^(1/3)))/(a^(2/3)*c^2) - 1/3*(b*x^3 + a)^(1/3)*a/(c*x^3)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^4/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^4), x)
```

mupad [B] time = 10.88, size = 2047, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^4*(c + d*x^3)),x)
```

```
[Out] log(c^2*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3) + 3*a*d*(a + b*x^3)^(1/3) - 4*b*
c*(a + b*x^3)^(1/3))*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^2*b^2*c^2*d - 108
*a^3*b*c*d^2)/(729*c^6))^(1/3) + log(((((81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*
c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))^(1/3) - 108*a*b^5*c^3*d^3*(a + b*x
^3)^(1/3)*(a*d - b*c)^2)*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*
a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^
4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^
4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^
3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^
4))*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3
)/(27*c^6*d))^(1/3) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*
(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 - 81*a*b^4*c^4*d^3*((3^(
1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d - b*c)^4/(c^6*d))
^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 + (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*
c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4
*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3 - (a*b^4*d^2*(a + b*x^3)
^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a
^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4))*((3^(1/2)*1i)
/2 - 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b
*c*d^3)/(27*c^6*d))^(1/3) - log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b*c)^2*
(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 171*
a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 + 1/2)*((((3^(1/
2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 + 81*a*b
^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((a*d -
 b*c)^4/(c^6*d))^(1/3))*((a*d - b*c)^4/(c^6*d))^(2/3))/9 - (a*b^5*d^2*(27*a
^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^2*d^3 + 162*a*b^4
*c^4*d - 153*a^4*b*c*d^4))/(3*c))*((a*d - b*c)^4/(c^6*d))^(1/3))/3)*((3^(1/
2)*1i)/2 + 1/2)*((a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3)/(27*c^6*d))^(1/3) + log((a*b^4*d^2*(a + b*x^3)^(1/3)*(a*d - b
*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3
+ 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*1i)/2 - 1/2)*(((
(3^(1/2)*1i)/2 + 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2 -
27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(
-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(2/3))/81 +
 (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2 + 332*a^3*b^2*c^
2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(1/3))/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^4*d^3 - 64*a*b^3*c^3 + 144*a^
2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - log((a*b^4*d^2*(a + b*x^3
)^(1/3)*(a*d - b*c)^2*(54*a^5*d^5 - 36*b^5*c^5 - 388*a^2*b^3*c^3*d^2 + 450*
a^3*b^2*c^2*d^3 + 171*a*b^4*c^4*d - 252*a^4*b*c*d^4))/(9*c^4) - (((3^(1/2)*
1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(108*a*b^5*c^3*d^3*(a + b*x^3)^(1/3)*
(a*d - b*c)^2 + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d)*(-(a*(3*a*d - 4*b*c)^3)/c^6)^(1/3))*(-(a*(3*a*d - 4*b*c)^3)/
c^6)^(2/3))/81 - (a*b^5*d^2*(27*a^5*d^5 - 27*b^5*c^5 - 341*a^2*b^3*c^3*d^2
+ 332*a^3*b^2*c^2*d^3 + 162*a*b^4*c^4*d - 153*a^4*b*c*d^4))/(3*c))*(-(a*(3*
a*d - 4*b*c)^3)/c^6)^(1/3))/9)*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^4*d^3 - 64*a*
b^3*c^3 + 144*a^2*b^2*c^2*d - 108*a^3*b*c*d^2)/(729*c^6))^(1/3) - (a*(a + b
*x^3)^(1/3))/(3*c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^4\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**4/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**4*(c + d*x**3)), x)
```

3.701 $\int \frac{(a+bx^3)^{4/3}}{x^7(c+dx^3)}\,dx$

Optimal. Leaf size=440

$$\frac{\sqrt[3]{a+bx^3}\,(9a^2d^2-12abcd+2b^2c^2)}{9ac^3}+\frac{(9a^2d^2-12abcd+2b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{(9a^2d^2-12abcd+2b^2c^2)}{9\sqrt{3}\,a^{2}}$$

```
[Out] d*(-a*d+b*c)*(b*x^3+a)^(1/3)/c^3+1/9*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*(b*x^
3+a)^(1/3)/a/c^3-1/18*(-6*a*d+b*c)*(b*x^3+a)^(4/3)/a/c^2/x^3-1/6*(b*x^3+a)^
(7/3)/a/c/x^6-1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^2)*ln(x)/a^(2/3)/c^3+1/6*d
^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^3+1/18*(9*a^2*d^2-12*a*b*c*d+2*b^2*c^
2)*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(2/3)/c^3-1/2*d^(2/3)*(-a*d+b*c)^(4/3)*ln(
(-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^3-1/27*(9*a^2*d^2-12*a*b*c*d+2*
b^2*c^2)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(2/3)/c^
3*3^(1/2)+1/3*d^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1
/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^3*3^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 440, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {446, 103, 149, 156, 50, 57, 617, 204, 31, 58}

$$\frac{\sqrt[3]{a+bx^3}\,(9a^2d^2-12abcd+2b^2c^2)}{9ac^3}+\frac{(9a^2d^2-12abcd+2b^2c^2)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{18a^{2/3}c^3}-\frac{(9a^2d^2-12abcd+2b^2c^2)}{9\sqrt{3}\,a^{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (d*(b*c - a*d)*(a + b*x^3)^(1/3))/c^3 + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^
2)*(a + b*x^3)^(1/3))/(9*a*c^3) - ((b*c - 6*a*d)*(a + b*x^3)^(4/3))/(18*a*c
^2*x^3) - (a + b*x^3)^(7/3)/(6*a*c*x^6) - ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*
d^2)*ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(9*Sqrt[3]*
a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3
)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^3) - ((2*b^2*c^2 - 12*a*b*
c*d + 9*a^2*d^2)*Log[x])/(18*a^(2/3)*c^3) + (d^(2/3)*(b*c - a*d)^(4/3)*Log[
c + d*x^3])/(6*c^3) + ((2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*Log[a^(1/3) - (
a + b*x^3)^(1/3)])/(18*a^(2/3)*c^3) - (d^(2/3)*(b*c - a*d)^(4/3)*Log[(b*c -
 a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^7\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+bx)^{4/3}}{x^3(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{4/3}\left(\frac{1}{3}(-bc+6ad)-\frac{bdx}{3}\right)}{x^2(c+dx)}\,dx, x, x^3\right)}{6ac}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}\left(-\frac{2}{9}\left(2b^2c^2-12abcd+9a^2d^2\right)-\frac{2}{9}bd(2bc-3ad)x\right)}{x(c+dx)}\right.}{6ac^2}$$

$$= -\frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+bx^3\right)^{7/3}}{6acx^6} + \frac{\left(d^2(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[3]{a+bx}}{c+dx}\,dx, x, x^3\right)}{3c^3} + \frac{\left(2b^2c^2\right.}{}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

$$= \frac{d(bc-ad)\sqrt[3]{a+bx^3}}{c^3} + \frac{\left(2b^2c^2-12abcd+9a^2d^2\right)\sqrt[3]{a+bx^3}}{9ac^3} - \frac{(bc-6ad)\left(a+bx^3\right)^{4/3}}{18ac^2x^3} - \frac{\left(a+\right.}{6}$$

Mathematica [A] time = 1.25, size = 429, normalized size = 0.98

$$\frac{x^6\left(4\left(9a^2d^2-12abcd+2b^2c^2\right)\left(-\frac{1}{2}a^{4/3}\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x]
```

```
[Out] (-18*a*c^2*(a + b*x^3)^(7/3) - 6*c*(b*c - 6*a*d)*x^3*(a + b*x^3)^(7/3) + x^
6*(4*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(3*a*(a + b*x^3)^(1/3) + (3*(a +
b*x^3)^(4/3))/4 - (a^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(
1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + Log[a^(2/3) + a^(1/3)
*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/2) - 9*a^2*d^(2/3)*(3*d^(4/3)*(a
+ b*x^3)^(4/3) - 2*(b*c - a*d)*(6*d^(1/3)*(a + b*x^3)^(1/3) + (b*c - a*d)^(
1/3)*(-2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/
3))/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(
b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a
 + b*x^3)^(2/3)])))))/(108*a^2*c^3*x^6)
```

fricas [A] time = 5.13, size = 503, normalized size = 1.14

$$18\sqrt{3}\left(a^2bc - a^3d\right)\left(bcd^2 - ad^3\right)^{\frac{1}{3}}x^6\arctan\left(-\frac{2\sqrt{3}\left(bcd^2-ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{1}{3}}-\sqrt{3}\left(bcd-ad^2\right)}{3\left(bcd-ad^2\right)}\right) - 2\sqrt{3}\left(2ab^2c^2 - 12a^2bcd + 9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(18*sqrt(3)*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(1/3)*x^6*arctan(-1/3*
(2*sqrt(3)*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3) - sqrt(3)*(b*c*d - a*d
^2))/(b*c*d - a*d^2)) - 2*sqrt(3)*(2*a*b^2*c^2 - 12*a^2*b*c*d + 9*a^3*d^2)*
(a^2)^(1/6)*x^6*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) - (2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(
a^2)^(2/3)*x^6*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(a^2)^(2/3)) + 2*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*(a^2)^(2/3)*x^6*log((
b*x^3 + a)^(1/3)*a - (a^2)^(2/3)) + 9*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1
/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 18*(a^2*b*c - a^3*d)*(b*c*d^2 - a*d^3)^(
1/3)*x^6*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(3*a^3*c^2
+ (7*a^2*b*c^2 - 6*a^3*c*d)*x^3)*(b*x^3 + a)^(1/3))/(a^2*c^3*x^6)
```

giac [A] time = 0.86, size = 481, normalized size = 1.09

$$\frac{\left(b^2c^2d - 2abcd^2 + a^2d^3\right)\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}} - \left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^4 - ac^3d\right)} - \frac{\sqrt{3}\left(-bcd^2 + ad^3\right)^{\frac{1}{3}}(bc-ad)\arctan\left(\frac{\sqrt{3}\left(2\right.}{}\right.}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x
^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c^4 - a*c^3*d) - 1/3*sqrt(3)*(-
b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3)
+ (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/c^3 - 1/6*(-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/c^3 - 1/27*sqrt(3)*(2*b^2*c^2 - 12*a*
b*c*d + 9*a^2*d^2)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/
3))/(a^(2/3)*c^3) - 1/54*(2*b^2*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log((b*x^3 +
a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(2/3)*c^3) + 1/27*(2*b^2
*c^2 - 12*a*b*c*d + 9*a^2*d^2)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c^3) - 1/18*(7*(b*x^3 + a)^(4/3)*b^2*c - 4*(b*x^3 + a)^(1/3)*a*b^2*c - 6
*(b*x^3 + a)^(4/3)*a*b*d + 6*(b*x^3 + a)^(1/3)*a^2*b*d)/(b^2*c^2*x^6)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a)^(4/3)/x^7/(d*x^3+c),x, algorithm="maxima")

[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^7), x)

mupad [B] time = 13.25, size = 2841, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^3)^(4/3)/(x^7*(c + d*x^3)),x)

[Out] log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/27 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((729*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3) + log(((((18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 - (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) + (((2*a*b^2*c - 3*a^2*b*d)*(a + b*x^3)^(1/3))/(9*c^2) + (b*(a + b*x^3)^(4/3)*(6*a*d - 7*b*c))/(18*c^2))/((a + b*x^3)^2 - 2*a*(a + b*x^3) + a^2) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*(-(a^4*d^6 + b^4*c^4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3) - log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 81*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))*(-(d^2*(a*d - b*c)^4)/c^9)^(2/3))/9 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*d^5))/(81*c^4))*(-(d^2*(a*d - b*c)^4)/c^9)^(1/3))/3 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 + 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*(-(a^4*d^6 + b^4*c^

```
4*d^2 - 4*a*b^3*c^3*d^3 + 6*a^2*b^2*c^2*d^4 - 4*a^3*b*c*d^5)/(27*c^9))^(1/3
) + log((((3^(1/2)*1i)/2 - 1/2)*((((3^(1/2)*1i)/2 + 1/2)*(18*b^5*c^2*d^3*(a
 + b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) + 9*a*b^4*c^4*d^3*((3^(1/2)*1i)
/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*
b*c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9
))^(2/3))/729 - (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 +
6939*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*
c*d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))
/27 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6
+ 6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 17
64*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 - 1/2)*((729
*a^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^
4*b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
- log((((3^(1/2)*1i)/2 + 1/2)*((((3^(1/2)*1i)/2 - 1/2)*(18*b^5*c^2*d^3*(a +
 b*x^3)^(1/3)*(a*d - b*c)^2*(6*a*d - b*c) - 9*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
 + 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*
c*d)^3/(a^2*c^9))^(1/3))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))
^(2/3))/729 + (b^5*d^4*(8*b^6*c^6 - 729*a^6*d^6 - 3258*a^2*b^4*c^4*d^2 + 69
39*a^3*b^3*c^3*d^3 - 7182*a^4*b^2*c^2*d^4 + 577*a*b^5*c^5*d + 3645*a^5*b*c*
d^5))/(81*c^4))*((9*a^2*d^2 + 2*b^2*c^2 - 12*a*b*c*d)^3/(a^2*c^9))^(1/3))/2
7 + (b^4*d^5*(a + b*x^3)^(1/3)*(a*d - b*c)^2*(1458*a^6*d^6 + 170*b^6*c^6 +
6561*a^2*b^4*c^4*d^2 - 12420*a^3*b^3*c^3*d^3 + 12798*a^4*b^2*c^2*d^4 - 1764
*a*b^5*c^5*d - 6804*a^5*b*c*d^5))/(243*c^8))*((3^(1/2)*1i)/2 + 1/2)*((729*a
^6*d^6 + 8*b^6*c^6 + 972*a^2*b^4*c^4*d^2 - 3024*a^3*b^3*c^3*d^3 + 4374*a^4*
b^2*c^2*d^4 - 144*a*b^5*c^5*d - 2916*a^5*b*c*d^5)/(19683*a^2*c^9))^(1/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**7/(d*x**3+c),x)
```

```
[Out] Timed out
```

3.702 $\int \frac{x^4 (a+bx^3)^{4/3}}{c+dx^3} \, dx$

Optimal. Leaf size=334

$$-\frac{(2a^2d^2-12abcd+9b^2c^2)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{18b^{2/3}d^3}-\frac{(2a^2d^2-12abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\,b^{2/3}d^3}-\frac{c^{2/3}(bc-ad)^{4/3}}{6d\cdots}$$

```
[Out] -1/18*(-7*a*d+6*b*c)*x^2*(b*x^3+a)^(1/3)/d^2+1/6*b*x^5*(b*x^3+a)^(1/3)/d-1/
6*c^(2/3)*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/d^3-1/18*(2*a^2*d^2-12*a*b*c*d+9*b^2
*c^2)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(2/3)/d^3+1/2*c^(2/3)*(-a*d+b*c)^(4/3
)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^3-1/27*(2*a^2*d^2-12*a*b
*c*d+9*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(2/3)
/d^3*3^(1/2)+1/3*c^(2/3)*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*
x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^3*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{ax^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^5*(a + b*x^3)^(1/3)*AppellF1[5/3, -4/3, 1, 8/3, -((b*x^3)/a), -((d*x^3
)/c)])/(5*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^4\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{ax^5\sqrt[3]{a+bx^3}\,F_1\left(\frac{5}{3};-\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.31, size = 225, normalized size = 0.67

$$\frac{2x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}\left(2a^2d^2-12abcd+9b^2c^2\right)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(6bc-7a\right.}{90cd^2\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*(9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x
^3)/c)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*c*x
^2*((a + b*x^3)*(-6*b*c + 7*a*d + 3*b*d*x^3)*(1 + (d*x^3)/c)^(2/3) + a*(6*b
*c - 7*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c)
 + a*d)*x^3)/(a*(c + d*x^3))]))/(90*c*d^2*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)
^(2/3))
```

fricas [A] time = 5.08, size = 550, normalized size = 1.65

$$2\sqrt{3}\left(9b^3c^2-12ab^2cd+2a^2bd^2\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-b^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-b^2\right)^{\frac{1}{3}}}}{3b^2x}\right)-18\sqrt{3}\left(b^3c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(9*b^3*c^2 - 12*a*b^2*c*d + 2*a^2*b*d^2)*sqrt(-(-b^2)^(1/3)
)*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2
)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 18*sqrt(3)*(b^3*c - a*b^2*d)*(-b*c^
3 + a*c^2*d)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c^2 - a*c*d)*x + 2*sqrt(3)*(-b*c
^3 + a*c^2*d)^(2/3)*(b*x^3 + a)^(1/3))/((b*c^2 - a*c*d)*x)) - 2*(9*b^2*c^2
- 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(
1/3)*b)/x) + (9*b^2*c^2 - 12*a*b*c*d + 2*a^2*d^2)*(-b^2)^(2/3)*log(-((-b^2)
^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2)
 - 18*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1/3)*log(((b*x^3 + a)^(1/3)*c +
 (-b*c^3 + a*c^2*d)^(1/3)*x)/x) + 9*(b^3*c - a*b^2*d)*(-b*c^3 + a*c^2*d)^(1
/3)*log(((b*x^3 + a)^(2/3)*c^2 - (-b*c^3 + a*c^2*d)^(1/3)*(b*x^3 + a)^(1/3)
*c*x + (-b*c^3 + a*c^2*d)^(2/3)*x^2)/x^2) + 3*(3*b^3*d^2*x^5 - (6*b^3*c*d -
 7*a*b^2*d^2)*x^2)*(b*x^3 + a)^(1/3))/(b^2*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^4}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x^{4}}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}} x^{4}}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^4/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] int((x^4*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + bx^{3}\right)^{\frac{4}{3}}}{c + dx^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.703 $\int \frac{x(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=277

$$\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,d^2}+\frac{\sqrt[3]{b}\,(3bc-4ad)\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{6d^2}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,d^2}+\cdots$$

```
[Out] 1/3*b*x^2*(b*x^3+a)^(1/3)/d+1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(1/3)/d^2+1/
6*b^(1/3)*(-4*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d^2-1/2*(-a*d+b*c)^(
4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(1/3)/d^2+1/9*b^(1/3)
*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^2*3^(
1/2)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3
+a)^(1/3))*3^(1/2))/c^(1/3)/d^2*3^(1/2)
```

Rubi [C] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 0.23, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{ax^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^2*(a + b*x^3)^(1/3)*AppellF1[2/3, -4/3, 1, 5/3, -((b*x^3)/a), -((d*x^3
)/c)])/(2*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{ax^2\sqrt[3]{a+bx^3}\,F_1\left(\frac{2}{3};-\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.32, size = 198, normalized size = 0.71

$$\frac{2bx^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(4ad-3bc)F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^2\left(a\left(\frac{bx^3}{a}+1\right)^{2/3}(3ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(a}{a(}\right.\right.}{30cd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (2*b*(-3*b*c + 4*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*Appel
lF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] + 5*x^2*(2*b*c*(a + b*x^3
)*(1 + (d*x^3)/c)^(2/3) + a*(-2*b*c + 3*a*d)*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(30*c*d*(a
 + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

fricas [A] time = 1.84, size = 396, normalized size = 1.43

$$6\left(bx^3+a\right)^{\frac{1}{3}}bdx^2-6\sqrt{3}\,(bc-ad)\left(\frac{bc-ad}{c}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}\,(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{bc-ad}{c}\right)^{\frac{2}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\,(3\,bc-4\,ad)\,(-b)^{\frac{1}{3}}\,\mathrm{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(1/3)*b*d*x^2 - 6*sqrt(3)*(b*c - a*d)*((b*c - a*d)/c)^(
1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b
*c - a*d)/c)^(2/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b*c - 4*a*d)*(-b)^(1/3)
*arctan(1/3*(sqrt(3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b)^(2/3))/(b*x)) -
 2*(3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 6
*(b*c - a*d)*((b*c - a*d)/c)^(1/3)*log(-(x*((b*c - a*d)/c)^(1/3) - (b*x^3 +
 a)^(1/3))/x) + (3*b*c - 4*a*d)*(-b)^(1/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a
)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(b*c - a*d)*((b*c - a*d)
/c)^(1/3)*log((x^2*((b*c - a*d)/c)^(2/3) + (b*x^3 + a)^(1/3)*x*((b*c - a*d)
/c)^(1/3) + (b*x^3 + a)^(2/3))/x^2))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{3}+a\right)^{\frac{4}{3}} x}{d\,x^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+a\right)^{\frac{4}{3}} x}{dx^{3}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^3)^(4/3))/(c + d*x^3),x)
```

```
[Out] int((x*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + bx^{3}\right)^{\frac{4}{3}}}{c + dx^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.704 $\int \frac{(a+bx^3)^{4/3}}{x^2(c+dx^3)}\,dx$

Optimal. Leaf size=254

$$-\frac{b^{4/3}\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{2d}-\frac{b^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{4/3}d}+\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\cdots}\right)}{2c^{4/3}d}$$

```
[Out] -a*(b*x^3+a)^(1/3)/c/x-1/6*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(4/3)/d-1/2*b^(4/
3)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/d+1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/d-1/3*b^(4/3)*arctan(1/3*(1+2*b^(1/3)*x
/(b*x^3+a)^(1/3))*3^(1/2))/d*3^(1/2)+1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3)/d*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] -((a*(a + b*x^3)^(1/3)*AppellF1[-1/3, -4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*x*(1 + (b*x^3)/a)^(1/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^2\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{1}{3};-\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [C] time = 0.55, size = 161, normalized size = 0.63

$$\frac{2b^2cx^6\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-\frac{5ax^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)\,{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{\left(\frac{dx^3}{c}+1\right)^{2/3}}-10ac\left(a+bx^3\right)}{10c^2x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x]
```

```
[Out] (-10*a*c*(a + b*x^3) + 2*b^2*c*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3,
 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)] - (5*a*(-2*b*c + a*d)*x^3*(1 + (b*x^3)
/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x
^3))])/(1 + (d*x^3)/c)^(2/3))/(10*c^2*x*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^2\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^2*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^2\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**2/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**2*(c + d*x**3)), x)
```

3.705 $\int \frac{(a+bx^3)^{4/3}}{x^5(c+dx^3)}\,dx$

Optimal. Leaf size=201

$$\frac{(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{7/3}}-\frac{(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}}-\frac{(bc-ad)^{4/3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}}-\frac{\sqrt[3]{a+bx^3}}{4}$$

```
[Out] -1/4*a*(b*x^3+a)^(1/3)/c/x^4-1/4*(-4*a*d+5*b*c)*(b*x^3+a)^(1/3)/c^2/x+1/6*(
-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(7/3)-1/2*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3
)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(7/3)-1/3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(
-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 0.45, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/3, -
1/3, -((c*((b*x^3)/a - (d*x^3)/c))/(c + d*x^3))])/(4*c*x^4*(1 + (b*x^3)/a)^
(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^5\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{a\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4cx^4\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.04, size = 84, normalized size = 0.42

$$-\frac{a\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{4}{3};-\frac{1}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{4cx^4\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x]
```

```
[Out] -1/4*(a*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3)*Hypergeometric2F1[-4/3, -4/
3, -1/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*x^4*(1 + (b*x^3)/a)^(1/3
))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^5/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^5\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^5*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^5\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**5/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**5*(c + d*x**3)), x)
```

3.706 $\int \frac{(a+bx^3)^{4/3}}{x^8(c+dx^3)} \, dx$

Optimal. Leaf size=250

$$-\frac{\sqrt[3]{a+bx^3}\left(28a^2d^2-35abcd+4b^2c^2\right)}{28ac^3x}-\frac{d(bc-ad)^{4/3}\log\left(c+dx^3\right)}{6c^{10/3}}+\frac{d(bc-ad)^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{10/3}}+\ldots$$

```
[Out] -1/7*a*(b*x^3+a)^(1/3)/c/x^7-1/28*(-7*a*d+8*b*c)*(b*x^3+a)^(1/3)/c^2/x^4-1/
28*(28*a^2*d^2-35*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x-1/6*d*(-a*d+b*
c)^(4/3)*ln(d*x^3+c)/c^(10/3)+1/2*d*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/
c^(1/3)-(b*x^3+a)^(1/3))/c^(10/3)+1/3*d*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-
a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(10/3)*3^(1/2)
```

Rubi [C] time = 0.51, antiderivative size = 169, normalized size of antiderivative = 0.68, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx^3\right)+5bcx^3\right)-2x^6\ldots\right.}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] (12*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)*Hypergeometric2F1[-1/3, 2, 2/
3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (4*c - 3*d*x^3)*(c*(a + b*x^3)*(5*b
*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(28*c^4*x^7*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^8\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{12c(bc-ad)x^3\left(a+bx^3\right)\left(c+dx^3\right){}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(a+bx^3)}\right)-\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(5b\right.\right.}{28c^4x^7\left(a+bx^3\right)^{2/3}}$$

Mathematica [C] time = 0.60, size = 179, normalized size = 0.72

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(12cx^3\left(a+bx^3\right)\left(c+dx^3\right)(ad-bc)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+\left(4c-3dx^3\right)\left(c\left(a+bx^3\right)\left(a\left(c-4dx\right.\right.\right.\right.}{28c^4x^7\left(a+bx^3\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x]
```

```
[Out] -1/28*(a*(1 + (b*x^3)/a)*(12*c*(-(b*c) + a*d)*x^3*(a + b*x^3)*(c + d*x^3)*H
ypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (4*c -
3*d*x^3)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])))/(c^4*
x^7*(a + b*x^3)^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{4}{3}}}{(dx^3+c)x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^8/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^8), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b\,x^3+a)^{4/3}}{x^8\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^8*(c + d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**8/(d*x**3+c),x)
[Out] Timed out
```

3.707 $\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=318

$$-\frac{\sqrt[3]{a+bx^3}\left(35a^2d^2-40abcd+2b^2c^2\right)}{140ac^3x^4}+\frac{\sqrt[3]{a+bx^3}\left(140a^3d^3-175a^2bcd^2+20ab^2c^2d+6b^3c^3\right)}{140a^2c^4x}+\frac{d^2(bc-ad)^{4/3}}{6c^1}$$

```
[Out] -1/10*a*(b*x^3+a)^(1/3)/c/x^10-1/70*(-10*a*d+11*b*c)*(b*x^3+a)^(1/3)/c^2/x^
7-1/140*(35*a^2*d^2-40*a*b*c*d+2*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^4+1/140*(
140*a^3*d^3-175*a^2*b*c*d^2+20*a*b^2*c^2*d+6*b^3*c^3)*(b*x^3+a)^(1/3)/a^2/c
^4/x+1/6*d^2*(-a*d+b*c)^(4/3)*ln(d*x^3+c)/c^(13/3)-1/2*d^2*(-a*d+b*c)^(4/3)
*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(13/3)-1/3*d^2*(-a*d+b*c)
^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))
/c^(13/3)*3^(1/2)
```

Rubi [C] time = 1.80, antiderivative size = 260, normalized size of antiderivative = 0.82, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{-18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(-\frac{1}{3},2,2;\frac{2}{3},1;\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6cx^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)(bc-}{1}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
```

```
[Out] (6*c*(b*c - a*d)*x^3*(a + b*x^3)*(11*c^2 + 2*c*d*x^3 - 9*d^2*x^6)*Hypergeom
etric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (14*c^2 - 12*c*
d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a*(c - 4*d*x^3)) - 2*(b*c -
a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
]) - 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{-1/3
, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(140*c^5*x^10*(a + b
*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{11}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{6c(bc-ad)x^3\left(a+bx^3\right)\left(11c^2+2cdx^3-9d^2x^6\right)\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{(bc-ad)x^3}{c\left(a+bx^3\right)}\right)-\left(14c^2-12cdx^3+\right.}{}$$

Mathematica [C] time = 2.70, size = 270, normalized size = 0.85

$$-\frac{a\left(\frac{bx^3}{a}+1\right)\left(18cx^3\left(a+bx^3\right)\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(-\frac{1}{3},2,2;\frac{2}{3},1;\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+6cx^3\left(a+bx^3\right)\left(-11c^2-2cdx^3+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x]
[Out] -1/140*(a*(1 + (b*x^3)/a)*(6*c*(b*c - a*d)*x^3*(a + b*x^3)*(-11*c^2 - 2*c*d
*x^3 + 9*d^2*x^6)*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + (14*c^2 - 12*c*d*x^3 + 9*d^2*x^6)*(c*(a + b*x^3)*(5*b*c*x^3 + a
*(c - 4*d*x^3)) - 2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))]) + 18*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3
)^2*HypergeometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))]))/(c^5*x^10*(a + b*x^3)^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="giac")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
[Out] int((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{\left(dx^3 + c\right)x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^11/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^11), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3 + a\right)^{4/3}}{x^{11}\,\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^11*(c + d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**11/(d*x**3+c),x)
[Out] Timed out
```

3.708 $\int \frac{(a+bx^3)^{4/3}}{x^{14}(c+dx^3)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[3]{a+bx^3}\,(130a^2d^2-143abcd+4b^2c^2)}{910ac^3x^7}+\frac{\sqrt[3]{a+bx^3}\,(455a^3d^3-520a^2bcd^2+26ab^2c^2d+12b^3c^3)}{1820a^2c^4x^4}-\frac{\sqrt[3]{a+bx^3}\,(18$$

```
[Out] -1/13*a*(b*x^3+a)^(1/3)/c/x^13-1/130*(-13*a*d+14*b*c)*(b*x^3+a)^(1/3)/c^2/x
^10-1/910*(130*a^2*d^2-143*a*b*c*d+4*b^2*c^2)*(b*x^3+a)^(1/3)/a/c^3/x^7+1/1
820*(455*a^3*d^3-520*a^2*b*c*d^2+26*a*b^2*c^2*d+12*b^3*c^3)*(b*x^3+a)^(1/3)
/a^2/c^4/x^4-1/1820*(1820*a^4*d^4-2275*a^3*b*c*d^3+260*a^2*b^2*c^2*d^2+78*a
*b^3*c^3*d+36*b^4*c^4)*(b*x^3+a)^(1/3)/a^3/c^5/x-1/6*d^3*(-a*d+b*c)^(4/3)*l
n(d*x^3+c)/c^(16/3)+1/2*d^3*(-a*d+b*c)^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(16/3)+1/3*d^3*(-a*d+b*c)^(4/3)*arctan(1/3*(1+2*(-a*d+b*
c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(16/3)*3^(1/2)
```

Rubi [C] time = 4.39, antiderivative size = 1446, normalized size of antiderivative = 3.69, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] -(140*a^2*c^5 + 840*a*b*c^5*x^3 - 686*a^2*c^4*d*x^3 + 700*b^2*c^5*x^6 - 131
6*a*b*c^4*d*x^6 + 612*a^2*c^3*d^2*x^6 - 630*b^2*c^4*d*x^9 + 1152*a*b*c^3*d^
2*x^9 - 513*a^2*c^2*d^3*x^9 + 540*b^2*c^3*d^2*x^12 - 918*a*b*c^2*d^3*x^12 +
 324*a^2*c*d^4*x^12 - 405*b^2*c^2*d^3*x^15 + 324*a*b*c*d^4*x^15 - 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 234*a*b*c^3*d^2*
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 32
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] - 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 504*a*b*c^3*d^2*x^9*Hyp
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
```

```
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(7*c - 6*d*x^3)*(c + d*x^3)^2*Hypergeo
metricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*c
*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2, 2
}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*x
^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^{14}\left(c+dx^3\right)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= -\frac{140a^2c^5+840abc^5x^3-686a^2c^4dx^3+700b^2c^5x^6-1316abc^4dx^6+612a^2c^3d^2x^6-630b}{}$$

Mathematica [C] time = 5.44, size = 1446, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x]
```

```
[Out] (-140*a^2*c^5 - 840*a*b*c^5*x^3 + 686*a^2*c^4*d*x^3 - 700*b^2*c^5*x^6 + 131
6*a*b*c^4*d*x^6 - 612*a^2*c^3*d^2*x^6 + 630*b^2*c^4*d*x^9 - 1152*a*b*c^3*d^
2*x^9 + 513*a^2*c^2*d^3*x^9 - 540*b^2*c^3*d^2*x^12 + 918*a*b*c^2*d^3*x^12 -
 324*a^2*c*d^4*x^12 + 405*b^2*c^2*d^3*x^15 - 324*a*b*c*d^4*x^15 + 828*a*b*c
^5*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
 828*a^2*c^4*d*x^3*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] + 828*b^2*c^5*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 918*a*b*c^4*d*x^6*Hypergeometric2F1[-1/3, 2, 2/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 90*a^2*c^3*d^2*x^6*Hypergeometric2F1[-1/
3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 90*b^2*c^4*d*x^9*Hypergeome
tric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 234*a*b*c^3*d^2*
x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 32
```

```
4*a^2*c^2*d^3*x^9*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 324*b^2*c^3*d^2*x^12*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 918*a*b*c^2*d^3*x^12*Hypergeometric2F1[-1/3, 2,
2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a^2*c*d^4*x^12*Hypergeometric
2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 594*b^2*c^2*d^3*x^15
*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 594*a
*b*c*d^4*x^15*Hypergeometric2F1[-1/3, 2, 2/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 280*b^2*c^5*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 560*a*b*c^4*d*x^6*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 280*a^2*c^3*d^2*x^6*Hypergeometric2F1[2/3, 1, 5
/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*b^2*c^4*d*x^9*Hypergeometric2F
1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 504*a*b*c^3*d^2*x^9*Hyp
ergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 252*a^2*c^
2*d^3*x^9*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 216*b^2*c^3*d^2*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 432*a*b*c^2*d^3*x^12*Hypergeometric2F1[2/3, 1, 5/3, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] + 216*a^2*c*d^4*x^12*Hypergeometric2F1[2/3, 1,
 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*b^2*c^2*d^3*x^15*Hypergeomet
ric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 324*a*b*c*d^4*x^15
*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 162*a^
2*d^5*x^15*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 54*c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^2*(-7*c + 6*d*x^3)*Hyperge
ometricPFQ[{-1/3, 2, 2}, {2/3, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 54*
c*(b*c - a*d)*x^3*(a + b*x^3)*(c + d*x^3)^3*HypergeometricPFQ[{-1/3, 2, 2,
2}, {2/3, 1, 1}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(1820*c^6*x^13*(a + b*
x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{\left(dx^3 + c\right)x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{4}{3}}}{\left(d\,x^3 + c\right)x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^14/(d*x^3+c),x, algorithm="maxima")
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^14), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^{14}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^14*(c + d*x^3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**14/(d*x**3+c),x)
[Out] Timed out
```

3.709 $\int \frac{x^6(a+bx^3)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=65

$$\frac{ax^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] 1/7*a*x^7*(b*x^3+a)^(1/3)*AppellF1(7/3,-4/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(1+
b*x^3/a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{ax^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^7*(a + b*x^3)^(1/3)*AppellF1[7/3, -4/3, 1, 10/3, -((b*x^3)/a), -((d*x^
3)/c)])/(7*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax^7\sqrt[3]{a+bx^3}\,F_1\left(\frac{7}{3};-\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.74, size = 343, normalized size = 5.28

$$\frac{x\left(\frac{16a^2c^2\left(a^2d^2-12abcd+10b^2c^2\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+2\left(a+bx^3\right)\left(2a^2d^2+3abd\left(3\right.\right.\right.}{80bd^3\left(a+bx^3\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(2*(a + b*x^3)*(2*a^2*d^2 + 3*a*b*d*(-8*c + 3*d*x^3) + b^2*(20*c^2 - 8*c
*d*x^3 + 5*d^2*x^6)) - ((20*b^3*c^3 - 30*a*b^2*c^2*d + 8*a^2*b*c*d^2 + a^3*
d^3)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/c + (16*a^2*c^2*(10*b^2*c^2 - 12*a*b*c*d + a^2*d^2)*AppellF1[1/
3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])))))/(80*b*d^3*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}x^6}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^6}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^6/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6 \left(b\,x^3+a\right)^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^6*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6 \left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.710 $\int \frac{x^3\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=65

$$\frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] 1/4*a*x^4*(b*x^3+a)^(1/3)*AppellF1(4/3,-4/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(1+b
*x^3/a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (a*x^4*(a + b*x^3)^(1/3)*AppellF1[4/3, -4/3, 1, 7/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax^4\sqrt[3]{a+bx^3}\,F_1\left(\frac{4}{3};-\frac{4}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.63, size = 280, normalized size = 4.31

$$\frac{x\left(\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}(4a^2d^2-15abcd+10b^2c^2)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}+\frac{16a^2c^2(6ad-5bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\cdots\right)\right)}\right)}{40d^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x]
```

```
[Out] (x*(4*(a + b*x^3)*(-5*b*c + 6*a*d + 2*b*d*x^3) + ((10*b^2*c^2 - 15*a*b*c*d
+ 4*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)
/a), -((d*x^3)/c)])/c + (16*a^2*c^2*(-5*b*c + 6*a*d)*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), -((d*x^3)/c)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1
, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a)
, -((d*x^3)/c)])))))/(40*d^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}x^3}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}x^3}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*x^3/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(b\,x^3+a\right)^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3),x)
[Out] int((x^3*(a + b*x^3)^(4/3))/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3*(a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.711 $\int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] a*x*(b*x^3+a)^(1/3)*AppellF1(1/3,-4/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a
)^(1/3)
```

Rubi [A] time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{ax\sqrt[3]{a+bx^3}\,F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [B] time = 0.41, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{4\left(bx^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(2a^2d+abdx^3+b^2x^3(c+dx^3)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+ \frac{bx \cdots}{\cdots}\right)}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{dx^3 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3 + a\right)^{\frac{4}{3}}}{d\,x^3 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{dx^3 + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,x^3+a\right)}^{4/3}}{d\,x^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

3.712 $\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)} dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -1/2*a*(b*x^3+a)^(1/3)*AppellF1(-2/3,-4/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(1
+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-2/3, -4/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)/
c)])/(2*c*x^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^3)^{4/3}}{x^3(c+dx^3)}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^3(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= -\frac{a\sqrt[3]{a+bx^3}F_1\left(-\frac{2}{3};-\frac{4}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.38, size = 341, normalized size = 5.25

$$-\frac{bx^6\left(\frac{bx^3}{a}+1\right)^{2/3}(ad-2bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4ac\left(x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\cdots\right.\right.\right.}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\cdots\right.\right.\right.}}{8c^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x]
```

```
[Out] -1/8*(b*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)] + (4*a*c*(-4*a*c*(a*c - 2*b*c*x^3 + 3*a*d*x^3
+ b*d*x^6)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a
+ b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x
^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/(
(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]
+ x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c
*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c^2*x^2*(a + b
*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^3/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^3\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)),x)
```

```
[Out] int((a + b*x^3)^(4/3)/(x^3*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^3\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**3/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(x**3*(c + d*x**3)), x)
```

3.713 $\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)} dx$

Optimal. Leaf size=65

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] -1/5*a*(b*x^3+a)^(1/3)*AppellF1(-5/3,-4/3,1,-2/3,-b*x^3/a,-d*x^3/c)/c/x^5/(
1+b*x^3/a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] -(a*(a + b*x^3)^(1/3)*AppellF1[-5/3, -4/3, 1, -2/3, -((b*x^3)/a), -((d*x^3)
/c)])/(5*c*x^5*(1 + (b*x^3)/a)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{x^6(c+dx^3)}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{x^6(c+dx^3)}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{a\sqrt[3]{a+bx^3}\,F_1\left(-\frac{5}{3};-\frac{4}{3},1;-\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5cx^5\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [B] time = 0.44, size = 286, normalized size = 4.40

$$\frac{-\frac{16ax\left(10a^2d^2-15abcd+4b^2c^2\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}(5ad-6bc)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};\right.}{c^3}}{40\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x]
```

```
[Out] ((-4*(a + b*x^3)*(2*a*c + 6*b*c*x^3 - 5*a*d*x^3))/(c^2*x^5) + (b*d*(-6*b*c
+ 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])/c^3 - (16*a*(4*b^2*c^2 - 15*a*b*c*d + 10*a^2*d^2)*x*AppellF
1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(c + d*x^3)*(-4*a*c*App
ellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4
/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(40*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^3+a\right)^{\frac{4}{3}}}{\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/x^6/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/((d*x^3 + c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\right)^{4/3}}{x^6\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)),x)
[Out] int((a + b*x^3)^(4/3)/(x^6*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+bx^3\right)^{\frac{4}{3}}}{x^6\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/x**6/(d*x**3+c),x)
[Out] Integral((a + b*x**3)**(4/3)/(x**6*(c + d*x**3)), x)
```

3.714 $\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=290

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}+\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right.}{11b^4c}$$

```
[Out] -1/2*(a*d+b*c)*(a^2*d^2+b^2*c^2)*(b*x^3+a)^(2/3)/b^4/d^4+1/5*(3*a^2*d^2+2*a
*b*c*d+b^2*c^2)*(b*x^3+a)^(5/3)/b^4/d^3-1/8*(3*a*d+b*c)*(b*x^3+a)^(8/3)/b^4
/d^2+1/11*(b*x^3+a)^(11/3)/b^4/d+1/6*c^4*ln(d*x^3+c)/d^(14/3)/(-a*d+b*c)^(1
/3)-1/2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(14/3)/(-a*d+b*c
)^(1/3)-1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3
^(1/2))/d^(14/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{5/3}\left(3a^2d^2+2abcd+b^2c^2\right)}{5b^4d^3}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)\left(a^2d^2+b^2c^2\right)}{2b^4d^4}-\frac{\left(a+bx^3\right)^{8/3}(3ad+bc)}{8b^4d^2}+\frac{\left(a+bx^3\right)^{1}}{11b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b*c + a*d)*(b^2*c^2 + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^4) + ((b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*(a + b*x^3)^(5/3))/(5*b^4*d^3) - ((b*c + 3*a*d)*
(a + b*x^3)^(8/3))/(8*b^4*d^2) + (a + b*x^3)^(11/3)/(11*b^4*d) - (c^4*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(14/3)*(b*c - a*d)^(1/3)) + (c^4*Log[c + d*x^3])/(6*d^(14/3)*(b*c - a*d)^
(1/3)) - (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(14/
3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{(bc+ad)\left(-b^2c^2-a^2d^2\right)}{b^3d^4\sqrt[3]{a+bx}} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)(a+bx)^{2/3}}{b^3d^3} + \frac{(-bc\right.$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+}{}$$

$$= -\frac{(bc+ad)\left(b^2c^2+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^4} + \frac{\left(b^2c^2+2abcd+3a^2d^2\right)\left(a+bx^3\right)^{5/3}}{5b^4d^3} - \frac{(bc+}{}$$

Mathematica [C] time = 0.26, size = 157, normalized size = 0.54

$$\frac{\left(a+bx^3\right)^{2/3}\left(\frac{-81a^3d^3+9a^2bd^2\left(6dx^3-11c\right)-3ab^2d\left(44c^2-22cdx^3+15d^2x^6\right)+b^3\left(-220c^3+88c^2dx^3-55cd^2x^6+40d^3x^9\right)}{b^4} + \frac{220c^4\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad}\right.}{440d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*((-81*a^3*d^3 + 9*a^2*b*d^2*(-11*c + 6*d*x^3) - 3*a*b^2*
d*(44*c^2 - 22*c*d*x^3 + 15*d^2*x^6) + b^3*(-220*c^3 + 88*c^2*d*x^3 - 55*c*
```

```
d^2*x^6 + 40*d^3*x^9))/b^4 + (220*c^4*Hypergeometric2F1[2/3, 1, 5/3, (d*(a
+ b*x^3))/(-(b*c) + a*d)])/(b*c - a*d)))/(440*d^4)
```

fricas [A] time = 0.82, size = 1004, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 + (
-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 4
40*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a
*d^3)^(1/3)) + 660*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt((-b*c*d^2 + a
*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(
2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a
*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)
/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c))
- 3*(220*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d
^5 - 81*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*
b^3*c*d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a
^2*b^2*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7
), 1/1320*(220*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(2/3)*d^2 +
 (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) -
 440*(-b*c*d^2 + a*d^3)^(2/3)*b^4*c^4*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 +
 a*d^3)^(1/3)) + 1320*sqrt(1/3)*(b^5*c^5*d - a*b^4*c^4*d^2)*sqrt(-(-b*c*d^2
 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-b*
c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(2
20*b^4*c^4*d^2 - 88*a*b^3*c^3*d^3 - 33*a^2*b^2*c^2*d^4 - 18*a^3*b*c*d^5 - 8
1*a^4*d^6 - 40*(b^4*c*d^5 - a*b^3*d^6)*x^9 + 5*(11*b^4*c^2*d^4 - 2*a*b^3*c*
d^5 - 9*a^2*b^2*d^6)*x^6 - 2*(44*b^4*c^3*d^3 - 11*a*b^3*c^2*d^4 - 6*a^2*b^2
*c*d^5 - 27*a^3*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(b^5*c*d^6 - a*b^4*d^7)]
```

giac [A] time = 0.33, size = 454, normalized size = 1.57

$$-\frac{b^{48}c^4d^7\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{49}cd^{11}-ab^{48}d^{12}\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^6-\sqrt{3}\,ad^7}+\frac{\left(-bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^48*c^4*d^7*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^49*c*d^11 - a*b^48*d^12) - (-b*c*d^2 + a*d^3)^(2/3)*c
^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c
 - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^6 - sqrt(3)*a*d^7) + 1/6*(-b*c*d^2 + a*d^3
)^(2/3)*c^4*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3
) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^6 - a*d^7) - 1/440*(220*(b*x^3 + a)^(2/3
)*b^43*c^3*d^7 - 88*(b*x^3 + a)^(5/3)*b^42*c^2*d^8 + 220*(b*x^3 + a)^(2/3)*
a*b^42*c^2*d^8 + 55*(b*x^3 + a)^(8/3)*b^41*c*d^9 - 176*(b*x^3 + a)^(5/3)*a*
b^41*c*d^9 + 220*(b*x^3 + a)^(2/3)*a^2*b^41*c*d^9 - 40*(b*x^3 + a)^(11/3)*b
^40*d^10 + 165*(b*x^3 + a)^(8/3)*a*b^40*d^10 - 264*(b*x^3 + a)^(5/3)*a^2*b^
40*d^10 + 220*(b*x^3 + a)^(2/3)*a^3*b^40*d^10)/(b^44*d^11)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.11, size = 438, normalized size = 1.51

$$\left(\frac{6\,a^2}{5\,b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\,\left(b^5\,c-a\,b^4\,d\right)}{5\,b^4\,d}\right)\,{\left(b\,x^3+a\right)}^{5/3}-\left(\frac{a}{2\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{8\,b^8\,d^2}\right)\,{\left(b\,x^3+a\right)}^{8/3}-{\left(b\,x^3+a\right)}^{2/3}\,\left(\frac{2\,a^3}{b^4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((6*a^2)/(5*b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c
- a*b^4*d))/(5*b^4*d))*(a + b*x^3)^(5/3) - (a/(2*b^4*d) + (b^5*c - a*b^4*d)
/(8*b^8*d^2))*(a + b*x^3)^(8/3) - (a + b*x^3)^(2/3)*((2*a^3)/(b^4*d) + (((6
*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c - a*b
^4*d))/(b^4*d))*(b^5*c - a*b^4*d))/(2*b^4*d)) + (a + b*x^3)^(11/3)/(11*b^4*
d) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^8*(a*d - b*c)^(1/3))/d^(22/3
)))/(3*d^(14/3)*(a*d - b*c)^(1/3)) - (log((c^8*(a + b*x^3)^(1/3))/d^7 - (c^
8*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*(3^(1/2)*c^4*1i + c^4
))/(6*d^(14/3)*(a*d - b*c)^(1/3)) + (c^4*log((c^8*(a + b*x^3)^(1/3))/d^7 -
(c^8*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(22/3)))*((3^(1/2)*1i)/6 -
1/6))/(d^(14/3)*(a*d - b*c)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\sqrt[3]{a + bx^3}\,\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.715 $\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=244

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}\right.}{2d^{11/3}\sqrt[3]{b}}$$

```
[Out] 1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3)/b^3/d^3-1/5*(2*a*d+b*c)*(b*x^
3+a)^(5/3)/b^3/d^2+1/8*(b*x^3+a)^(8/3)/b^3/d-1/6*c^3*ln(d*x^3+c)/d^(11/3)/(
-a*d+b*c)^(1/3)+1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/
3)/(-a*d+b*c)^(1/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b
*c)^(1/3))*3^(1/2))/d^(11/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^3d^3}-\frac{\left(a+bx^3\right)^{5/3}(2ad+bc)}{5b^3d^2}+\frac{\left(a+bx^3\right)^{8/3}}{8b^3d}-\frac{c^3\log\left(c+dx^3\right)}{6d^{11/3}\sqrt[3]{bc-ad}}+\frac{c^3\log\left(\sqrt[3]{bc-ad}\right.}{2d^{11/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^3*d^3) - ((b*c + 2*a
*d)*(a + b*x^3)^(5/3))/(5*b^3*d^2) + (a + b*x^3)^(8/3)/(8*b^3*d) + (c^3*Arc
Tan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3
]*d^(11/3)*(b*c - a*d)^(1/3)) - (c^3*Log[c + d*x^3])/(6*d^(11/3)*(b*c - a*d
)^(1/3)) + (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(1
1/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3\sqrt[3]{a+bx}} + \frac{(-bc-2ad)(a+bx)^{2/3}}{b^2d^2} + \frac{(a+bx)^{5/3}}{b^2d} - \frac{c}{d^3\sqrt[3]{a+b}}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\,\text{Subs}}{}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\log(}{6d^{11/3}\sqrt[3]{}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} - \frac{c^3\log(}{6d^{11/3}\sqrt[3]{}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{5/3}}{5b^3d^2} + \frac{\left(a+bx^3\right)^{8/3}}{8b^3d} + \frac{c^3\tan^{-}}{\sqrt{3}\,c}$$

Mathematica [C] time = 0.14, size = 145, normalized size = 0.59

$$-\frac{\left(a+bx^3\right)^{2/3}\left(9a^3d^3+3a^2bd^2\left(c-2dx^3\right)+20b^3c^3\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+ab^2d\left(8c^2-2cdx^3+5d^2x^6\right)+b^3c\left(-20\right.\right.}{40b^3d^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*((a + b*x^3)^(2/3)*(9*a^3*d^3 + 3*a^2*b*d^2*(c - 2*d*x^3) + b^3*c*(-2
0*c^2 + 8*c*d*x^3 - 5*d^2*x^6) + a*b^2*d*(8*c^2 - 2*c*d*x^3 + 5*d^2*x^6) +
20*b^3*c^3*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)]))
/(b^3*d^3*(b*c - a*d))
```

fricas [A] time = 0.86, size = 873, normalized size = 3.58

$$-\frac{20\left(bcd^2-ad^3\right)^{\frac{2}{3}}b^3c^3\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-40\left(bcd^2-ad^3\right)^{\frac{2}{3}}b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(2/3)*d^2 - (b*
c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 40*(b
*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(
1/3)) - 60*sqrt(1/3)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1
/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d
^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (
b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*
d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(20*b^3
*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d^5 + 5*(b^3*c*d^4 - a*b
^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2*b*d^5)*x^3)*(b*x^3 + a
)^(2/3))/(b^4*c*d^5 - a*b^3*d^6), -1/120*(20*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^
3*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d +
 (b*c*d^2 - a*d^3)^(2/3)) - 40*(b*c*d^2 - a*d^3)^(2/3)*b^3*c^3*log((b*x^3 +
 a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 120*sqrt(1/3)*(b^4*c^4*d - a*b^3*c
^3*d^2)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^
3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c
 - a*d))/d) - 3*(20*b^3*c^3*d^2 - 8*a*b^2*c^2*d^3 - 3*a^2*b*c*d^4 - 9*a^3*d
^5 + 5*(b^3*c*d^4 - a*b^2*d^5)*x^6 - 2*(4*b^3*c^2*d^3 - a*b^2*c*d^4 - 3*a^2
*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(b^4*c*d^5 - a*b^3*d^6)]
```

giac [A] time = 0.28, size = 371, normalized size = 1.52

$$\frac{b^{27}c^3d^5\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{28}cd^8-ab^{27}d^9\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^5-\sqrt{3}\,ad^6}-\frac{\left(-bcd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^27*c^3*d^5*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^28*c*d^8 - a*b^27*d^9) + (-b*c*d^2 + a*d^3)^(2/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^5 - sqrt(3)*a*d^6) - 1/6*(-b*c*d^2 + a*d^3)^(
2/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^5 - a*d^6) + 1/40*(20*(b*x^3 + a)^(2/3)*b^2
3*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^22*c*d^6 + 20*(b*x^3 + a)^(2/3)*a*b^22*c*
d^6 + 5*(b*x^3 + a)^(8/3)*b^21*d^7 - 16*(b*x^3 + a)^(5/3)*a*b^21*d^7 + 20*(
b*x^3 + a)^(2/3)*a^2*b^21*d^7)/(b^24*d^8)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{1}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.09, size = 339, normalized size = 1.39

$$\left(\frac{3\,a^2}{2\,b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{2\,b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\left(\frac{3\,a}{5\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{5\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{5/3}+\frac{{\left(b\,x^3+a\right)}^{8/3}}{8\,b^3\,d}-\frac{c^3\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(2*b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c
- a*b^3*d))/(2*b^3*d))*(a + b*x^3)^(2/3) - ((3*a)/(5*b^3*d) + (b^4*c - a*b^
3*d)/(5*b^6*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^3*d) - (c^3*lo
g((c^6*(a + b*x^3)^(1/3))/d^5 + (b*c^7 - a*c^6*d)/(d^(16/3)*(a*d - b*c)^(2/
3))))/(3*d^(11/3)*(a*d - b*c)^(1/3)) + (log((c^6*(a + b*x^3)^(1/3))/d^5 - (
c^6*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(16/3)))*(3^(1/2)*c^3*1i + c
^3))/(6*d^(11/3)*(a*d - b*c)^(1/3)) - (c^3*log((c^6*(a + b*x^3)^(1/3))/d^5
+ (c^6*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^(1/3))/d^(16/3))*((3^(1/2)*1i)/2
- 1/2))/(3*d^(11/3)*(a*d - b*c)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\sqrt[3]{a+bx^{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.716 $\int \frac{x^8}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=203

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt[3]{bc\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-\cdots}}$$

[Out] -1/2*(a*d+b*c)*(b*x^3+a)^(2/3)/b^2/d^2+1/5*(b*x^3+a)^(5/3)/b^2/d+1/6*c^2*ln(d*x^3+c)/d^(8/3)/(-a*d+b*c)^(1/3)-1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(8/3)/(-a*d+b*c)^(1/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(1/3)*3^(1/2)

Rubi [A] time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 56, 617, 204, 31}

$$-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^2d^2}+\frac{\left(a+bx^3\right)^{5/3}}{5b^2d}+\frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{8/3}\sqrt[3]{bc-ad}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{\cdots}}{\sqrt[3]{bc\cdots}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^2*d^2) + (a + b*x^3)^(5/3)/(5*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(1/3)) + (c^2*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(1/3)) - (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{bd} + \frac{c^2}{d^2\sqrt[3]{a+bx}\,(c+dx)}\right)dx, x, x^3\right) \\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}dx, x, x^3\right)}{3d^2} \\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc}}{}\cdots}\right.}{2\cdots} \\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\cdots\right.}{2d^{8/3}\sqrt[3]{bc-ad}\cdots} \\
&= -\frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^2d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{8/3}\sqrt[3]{bc-ad}} + \frac{c^2\log\left(c+dx^3\right)}{6d^{8/3}\sqrt[3]{bc-ad}} - \cdots
\end{aligned}$$

Mathematica [C] time = 0.09, size = 103, normalized size = 0.51

$$\frac{\left(a+bx^3\right)^{2/3}\left(3a^2d^2+5b^2c^2\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2abd\left(c-dx^3\right)+b^2c\left(2dx^3-5c\right)\right)}{10b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*(3*a^2*d^2 + 2*a*b*d*(c - d*x^3) + b^2*c*(-5*c + 2*d*x^3
) + 5*b^2*c^2*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)
]))/(10*b^2*d^2*(b*c - a*d))
```

fricas [B] time = 0.59, size = 768, normalized size = 3.78

$$\left[\frac{5\left(-bcd^2+ad^3\right)^{\frac{2}{3}}b^2c^2\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\right)-10\left(-bcd^2+ad^3\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c
*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-
b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)
^(1/3)) + 15*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((-b*c*d^2 + a*d^3)^
(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*
c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c
- a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(5
*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^2*c*d^3 - a*b*d^4)*x^3)*(b*x^
3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5), 1/30*(5*(-b*c*d^2 + a*d^3)^(2/3)*b^2
*c^2*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (-b*c*d^2 + a*d^3)^(2/3)) - 10*(-b*c*d^2 + a*d^3)^(2/3)*b^2*c^2*log((b
*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) + 30*sqrt(1/3)*(b^3*c^3*d - a
*b^2*c^2*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) - 3*(5*b^2*c^2*d^2 - 2*a*b*c*d^3 - 3*a^2*d^4 - 2*(b^
2*c*d^3 - a*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(b^3*c*d^4 - a*b^2*d^5)]
```

giac [A] time = 0.26, size = 313, normalized size = 1.54

$$-\frac{b^{12}c^2d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{13}cd^5-ab^{12}d^6\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^4-\sqrt{3}\,ad^5}+\frac{\left(-bc\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^12*c^2*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^13*c*d^5 - a*b^12*d^6) - (-b*c*d^2 + a*d^3)^(2/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) + 1/6*(-b*c*d^2 + a*d^3)^
(2/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) - 1/10*(5*(b*x^3 + a)^(2/3)*b^9
*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^8*d^4 + 5*(b*x^3 + a)^(2/3)*a*b^8*d^4)/(b^10
*d^5)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.11, size = 267, normalized size = 1.32

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^2\,d}-\left(\frac{a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{2\,b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{2/3}+\frac{c^2\,\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}+\frac{b\,c^5-a\,c^4\,d}{d^{10/3}\,\left(a\,d-b\,c\right)^{2/3}}\right)}{3\,d^{8/3}\,\left(a\,d-b\,c\right)^{1/3}}-\frac{\ln\left(\frac{c^4\,\left(b\,x^3+a\right)^{1/3}}{d^3}-\frac{c^4\,\left(1+\sqrt{3}\,1i\right.}{4\,c}\right.}{6\,d^{8/3}\,\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^2*d) - (a/(b^2*d) + (b^3*c - a*b^2*d)/(2*b^4*d^2))*(
a + b*x^3)^(2/3) + (c^2*log((c^4*(a + b*x^3)^(1/3))/d^3 + (b*c^5 - a*c^4*d)
/(d^(10/3)*(a*d - b*c)^(2/3))))/(3*d^(8/3)*(a*d - b*c)^(1/3)) - (log((c^4*(
a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i + 1)^2*(a*d - b*c)^(1/3))/(4*d^(10
/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(8/3)*(a*d - b*c)^(1/3)) + (c^2*log((c^4
*(a + b*x^3)^(1/3))/d^3 - (c^4*(3^(1/2)*1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(
10/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(8/3)*(a*d - b*c)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\sqrt[3]{a + bx^{3}}\left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.717 $\int \frac{x^5}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=168

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

```
[Out] 1/2*(b*x^3+a)^(2/3)/b/d-1/6*c*ln(d*x^3+c)/d^(5/3)/(-a*d+b*c)^(1/3)+1/2*c*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(5/3)/(-a*d+b*c)^(1/3)+1/3*c*a
rctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(
-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 56, 617, 204, 31}

$$-\frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}}+\frac{\left(a+bx^3\right)^{2/3}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b
*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(1/3)) - (c*Log[c +
 d*x^3])/(6*d^(5/3)*(b*c - a*d)^(1/3)) + (c*Log[(b*c - a*d)^(1/3) + d^(1/3)
*(a + b*x^3)^(1/3)])/(2*d^(5/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}} - \frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}} + x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^2} + \frac{c}{}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\,\mathrm{Subst}\left(\int \frac{1}{-3-}\right.}{d^5}$$

$$= \frac{\left(a+bx^3\right)^{2/3}}{2bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}\sqrt[3]{bc-ad}} - \frac{c\log\left(c+dx^3\right)}{6d^{5/3}\sqrt[3]{bc-ad}} + \frac{c\log\left(\sqrt[3]{bc-ad} + \sqrt[3]{d}\,\sqrt[3]{a}\right.}{2d^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 0.41

$$-\frac{\left(a+bx^3\right)^{2/3}\left(bc\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+ad-bc\right)}{2bd(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/2*((a + b*x^3)^(2/3)*(-(b*c) + a*d + b*c*Hypergeometric2F1[2/3, 1, 5/3,
(d*(a + b*x^3))/(-(b*c) + a*d)]))/(b*d*(b*c - a*d))
```

fricas [B] time = 0.65, size = 667, normalized size = 3.97

$$\left[-\frac{\left(bcd^2-ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}d^2-\left(bcd^2-ad^3\right)^{\frac{1}{3}}\left(bx^3+a\right)^{\frac{1}{3}}d+\left(bcd^2-ad^3\right)^{\frac{2}{3}}\right)-2\left(bcd^2-ad^3\right)^{\frac{2}{3}}bc\log\left(\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a
*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a
*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*lo
g((2*b*d^2*x^3 - b*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1
/3)*(b*c - a*d))*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 -
a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) - 3*(b*c*d^2 - a*d^3)*(b*x^3 +
 a)^(2/3))/(b^2*c*d^3 - a*b*d^4), -1/6*((b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 2*(b*c*d^2 - a*d^3)^(2/3)*b*c*log((b*x^3 + a)^(1/3)*d +
(b*c*d^2 - a*d^3)^(1/3)) + 6*sqrt(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt((b*c*d^
2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*
c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) - 3*(b*c
*d^2 - a*d^3)*(b*x^3 + a)^(2/3))/(b^2*c*d^3 - a*b*d^4)]
```

giac [A] time = 0.28, size = 257, normalized size = 1.53

$$\frac{\frac{2\,bcd\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{bcd^2-ad^3}+\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3\right.\right.}{bcd^3-ad}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*
d)/d)^(1/3)))/(b*c*d^2 - a*d^3) + 6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3
*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1
/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b*c*d^3 - a*d^4) + 3*(b*x^3 + a)^(2/3)/d)/b
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.10, size = 219, normalized size = 1.30

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b\,d}+\frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{4/3}}\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{5/3}\,{\left(a\,d-b\,c\right)}^{1/3}}+\frac{\ln\left(\frac{c^2\,\left(b\,x^3+a\right)^{1/3}}{d}-\frac{c^2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,{\left(a\,d-b\,c\right)}^{1/3}}{4\,d^{4/3}}\right)\,\left(c+\right.}{6\,d^{5/3}\,{\left(a\,d-b\,c\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b*d) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*
1i - 1)^2*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c - 3^(1/2)*c*1i))/(6*d^(5/3)*(a
*d - b*c)^(1/3)) + (log((c^2*(a + b*x^3)^(1/3))/d - (c^2*(3^(1/2)*1i + 1)^2
*(a*d - b*c)^(1/3))/(4*d^(4/3)))*(c + 3^(1/2)*c*1i))/(6*d^(5/3)*(a*d - b*c)
^(1/3)) - (c*log((c^2*(a + b*x^3)^(1/3))/d + (b*c^3 - a*c^2*d)/(d^(4/3)*(a*
d - b*c)^(2/3))))/(3*d^(5/3)*(a*d - b*c)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.718 $\int \frac{x^2}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=145

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b
*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(1/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+
a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 56, 617, 204, 31}

$$\frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(2/3)*(b*c - a*d)^(1/3))) + Log[c + d*x^3]/(6*d^(2/3)*(b*c - a*d)^(
1/3)) - Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(2/3)*(b*c
- a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{d}}+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x \cdots}\right.}{2d^{2/3}\sqrt[3]{b \cdots}}$$

$$= \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{d}\sqrt[3]{a \cdots}}{\sqrt[3]{bc- \cdots}}\right.}{d^{2/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c+dx^3\right)}{6d^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.34

$$\frac{\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) +
 a*d)])/(2*b*c - 2*a*d)
```

fricas [B] time = 0.75, size = 592, normalized size = 4.08

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bcd-ad^2\right)\sqrt{\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}}{bc-ad}}\,\log\left(\frac{2\,bd^2x^3-bcd+3\,ad^2+3\sqrt{\frac{1}{3}}\left(2\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(bcd-ad^2\right)+\left(-bcd^2+ad^3\right)^{\frac{1}{3}}(bc-ad) \cdots\right.}{dx^3+c}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)
)*log((2*b*d^2*x^3 - b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2
```

```
/3)*(b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d
^3)^(1/3)*(b*c - a*d))*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c
*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(1/3))/(d*x^3 + c)) + (-b*c*d^2 + a*d^3)^(2
/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*
d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^
(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)))/(b*c*d^2 - a*d^3), 1/6*(6*sqrt(1/3)*(b
*c*d - a*d^2)*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*
(2*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)
^(1/3)/(b*c - a*d))/d) + (-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
 - 2*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^
(1/3)))/(b*c*d^2 - a*d^3)]
```

giac [A] time = 0.29, size = 226, normalized size = 1.56

$$-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^2-\sqrt{3}\,ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\right.}{6\left(bcd^2-ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c
- a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt(3)*a*d^3)
+ 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-
(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3) - 1/3*(-(b
*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*
c - a*d)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.93, size = 208, normalized size = 1.43

$$\frac{\ln\left(d\left(b\,x^3+a\right)^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{9\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{2/3}\,{\left(a\,d-b\,c\right)}^{1/3}}+\frac{\ln\left(d\left(b\,x^3+a\right)^{1/3}-\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{36\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{2/3}\,{\left(a\,d-b\,c\right)}^{1/3}}-\frac{\ln\left(d\left(b\,x^3+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(d*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(9*d^(4/3)*(a*d - b*c)^(2/3
)))/(3*d^(2/3)*(a*d - b*c)^(1/3)) + (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*1i
 - 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i
- 1))/(6*d^(2/3)*(a*d - b*c)^(1/3)) - (log(d*(a + b*x^3)^(1/3) - ((3^(1/2)*
1i + 1)^2*(9*a*d^3 - 9*b*c*d^2))/(36*d^(4/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1
i + 1))/(6*d^(2/3)*(a*d - b*c)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.719 $\int \frac{1}{x\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=244

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c}+\cdots$$

```
[Out] -1/2*ln(x)/a^(1/3)/c-1/6*d^(1/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(1/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(1/3)/c+1/2*d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(1/3)/c*3^(1/2)+1/3*d^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 55, 617, 204, 31, 56}

$$-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}c\sqrt[3]{bc-ad}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}c}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(1/3)*
c) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(1/3)) - Log[x]/(2*a^(1/3)*c) - (d^(1/3)*L
og[c + d*x^3])/(6*c*(b*c - a*d)^(1/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(
2*a^(1/3)*c) + (d^(1/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*c*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3c} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2c} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\right.}{2c\sqrt[3]{bc-ad}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}\,c} + \frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c\sqrt[3]{bc-ad}} - \frac{\log(x)}{2\sqrt[3]{a}\,c} - \frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6c\sqrt[3]{bc-ad}} + \frac{\log\left(\right.}{}$$

Mathematica [C] time = 0.18, size = 140, normalized size = 0.57

$$\frac{3\sqrt[3]{a}\,d\left(a+bx^3\right)^{2/3}{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)-(bc-ad)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right.}{6\sqrt[3]{a}\,c(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^(1/3)*d*(a + b*x^3)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b*x^3
))/(-(b*c) + a*d)] - (b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3
))/a^(1/3))/Sqrt[3]] - 3*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/(6*a
^(1/3)*c*(-(b*c) + a*d))
```

fricas [A] time = 0.65, size = 628, normalized size = 2.57

$$\left[\frac{3\sqrt{\frac{1}{3}}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^3+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^3+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^3}\right)-2\sqrt{3}\,a\left(\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 +
 a)^(2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*
x^3 + a)^(1/3)*a^(2/3) + 3*a)/x^3) - 2*sqrt(3)*a*(d/(b*c - a*d))^(1/3)*arct
an(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3*sqrt(3)) - a*(
d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(2/
3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) + 2*a*(d/(b*c
 - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d)
 - a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2
*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c), -1/6*(2*sqrt(3)*a*(d/(b*c
 - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) -
 1/3*sqrt(3)) + a*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*
(d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(
1/3)) - 2*a*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3) + (
b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*a^(2/3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(
1/3) + a^(1/3))/a^(1/3)) + a^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)))/(a*c)]
```

giac [A] time = 0.74, size = 326, normalized size = 1.34

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2d-\sqrt{3}\,acd^2}-\frac{\left(-bcd^2+ad^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2*d - sqrt(3)*a*c*d^2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2
/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b
*c^2*d - a*c*d^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a
^(1/3))/a^(1/3))/(a^(1/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3
)*a^(1/3) + a^(2/3))/(a^(1/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3))
)/(a^(1/3)*c)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x), x)
```

mupad [B] time = 6.44, size = 702, normalized size = 2.88

$$\ln\left(b^{5}\,d^{4}\,\left(b\,x^{3}+a\right)^{1/3}-\frac{d\,\left(27\,b^{4}\,c^{2}\,d^{3}\,\left(b\,x^{3}+a\right)^{1/3}\,\left(2\,a^{2}\,d^{2}-2\,a\,b\,c\,d+b^{2}\,c^{2}\right)-243\,a\,b^{4}\,c^{4}\,d^{3}\,\left(\frac{d}{27\,b\,c^{4}-27\,a\,c^{3}\,d}\right)^{2/3}\,\left(2\right.\right.}{27\,b\,c^{4}-27\,a\,c^{3}\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2
*d^2 + b^2*c^2 - 2*a*b*c*d) - 243*a*b^4*c^4*d^3*(d/(27*b*c^4 - 27*a*c^3*d))
^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)))/(27*b*c^4 - 27*a*c^3*d))*(d/(27*
b*c^4 - 27*a*c^3*d))^(1/3) + log((a + b*x^3)^(1/3) - a*c^2*(1/(a*c^3))^(2/3
))*(1/(27*a*c^3))^(1/3) + (log(b^5*d^4*(a + b*x^3)^(1/3) - (d*(3^(1/2)*1i -
 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b^2*c^2 - 2*a*b*c*d) -
 (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(d/(27*b*c^4 - 27*a*c^3*d))^(2/3)*(2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27*a*c^3*d)))*(3^(1/2)*
1i - 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - (log(b^5*d^4*(a + b*x^3)^(1/
3) + (d*(3^(1/2)*1i + 1)^3*(27*b^4*c^2*d^3*(a + b*x^3)^(1/3)*(2*a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) - (243*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)^2*(d/(27*b*c^4 -
27*a*c^3*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/(8*(27*b*c^4 - 27
*a*c^3*d)))*(3^(1/2)*1i + 1)*(d/(27*b*c^4 - 27*a*c^3*d))^(1/3))/2 - log((a
+ b*x^3)^(1/3)*2i + a*c^2*(1/(a*c^3))^(2/3)*1i + 3^(1/2)*a*c^2*(1/(a*c^3))^
(2/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a*c^3))^(1/3) + log((a + b*x^3)^(1/3)*
2i + a*c^2*(1/(a*c^3))^(2/3)*1i - 3^(1/2)*a*c^2*(1/(a*c^3))^(2/3))*((3^(1/2
)*1i)/2 - 1/2)*(1/(27*a*c^3))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt[3]{a+bx^{3}} \left(c+dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.720 $\int \frac{1}{x^4 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=296

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\,\mathrm{l}}{}$$

```
[Out] -1/3*(b*x^3+a)^(2/3)/a/c/x^3+1/6*(3*a*d+b*c)*ln(x)/a^(4/3)/c^2+1/6*d^(4/3)*
ln(d*x^3+c)/c^2/(-a*d+b*c)^(1/3)-1/6*(3*a*d+b*c)*ln(a^(1/3)-(b*x^3+a)^(1/3)
)/a^(4/3)/c^2-1/2*d^(4/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c^2/
(-a*d+b*c)^(1/3)-1/9*(3*a*d+b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(
1/3)*3^(1/2))/a^(4/3)/c^2*3^(1/2)-1/3*d^(4/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^
3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 55, 617, 204, 31, 56}

$$-\frac{(3ad+bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{4/3}c^2}-\frac{(3ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}c^2}+\frac{\log(x)(3ad+bc)}{6a^{4/3}c^2}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}}-\frac{d^{4/3}\,\mathrm{l}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(3*a*c*x^3) - ((b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a + b*
x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)*c^2) - (d^(4/3)*ArcTan[(
1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c^2
*(b*c - a*d)^(1/3)) + ((b*c + 3*a*d)*Log[x])/(6*a^(4/3)*c^2) + (d^(4/3)*Log
[c + d*x^3])/(6*c^2*(b*c - a*d)^(1/3)) - ((b*c + 3*a*d)*Log[a^(1/3) - (a +
b*x^3)^(1/3)])/(6*a^(4/3)*c^2) - (d^(4/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^2\sqrt[3]{a+bx}\,(c+dx)}dx,x,x^3\right)$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(bc+3ad)+\frac{bdx}{3}}{x\sqrt[3]{a+bx}\,(c+dx)}dx,x,x^3\right)}{3ac}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}dx,x,x^3\right)}{3c^2} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\right.}{9ac^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\frac{\sqrt[3]{bc-}}{\sqrt[3]{a}}}\right.}{2c}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} + \frac{(bc+3ad)\log(x)}{6a^{4/3}c^2} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c^2\sqrt[3]{bc-ad}} - \frac{(bc+3ad)\log\left(\sqrt[3]{a}-\sqrt[3]{\right.}}{6a^{4/3}c^2}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{3acx^3} - \frac{(bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}c^2} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2\sqrt[3]{bc-ad}} + \frac{(bc+}{6}$$

Mathematica [C] time = 0.61, size = 156, normalized size = 0.53

$$-\frac{\frac{(3ad+bc)\left(3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-3\log(x)\right)}{a^{4/3}} - \frac{9d^2\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{bc-ad} + \frac{6c\left(a+bx^3\right)^{2/3}}{ax^3}}{18c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/18*((6*c*(a + b*x^3)^(2/3))/(a*x^3) - (9*d^2*(a + b*x^3)^(2/3)*Hypergeom
etric2F1[2/3, 1, 5/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(b*c - a*d) + ((b*c
+ 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 3
*Log[x] + 3*Log[a^(1/3) - (a + b*x^3)^(1/3)]))/a^(4/3))/c^2
```

fricas [A] time = 0.97, size = 837, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(3)*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3
 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c -
a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*
x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b
*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)
*d) + 3*sqrt(1/3)*(a*b*c + 3*a^2*d)*x^3*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3
*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/
3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) + (b*
c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1
```

```
/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) +
(-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*a*c)/(a^2*c^2*x^3), 1/18*(6*sqrt(3)*a^2*d
*x^3*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c -
 a*d))^(1/3) + 1/3*sqrt(3)) - 3*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log(-(b*x^
3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*
c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*a^2*d*x^3*(-d/(b*c - a*d))^(1/3)*log((
b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*sqrt(1/3)*(a*b
*c + 3*a^2*d)*x^3*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)
 - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (b*c + 3*a*d)*(-a)^(2/3)*x^3*log((b*x
^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(b*c + 3*a*d
)*(-a)^(2/3)*x^3*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^3 + a)^(2/3)*
a*c)/(a^2*c^2*x^3)]
```

giac [A] time = 0.88, size = 383, normalized size = 1.29

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}+\frac{\left(-bcd^2+ad\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) - (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/3*sqrt(3)*(
2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt
(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)
^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))
/(b*c^3 - a*c^2*d) + 1/18*(b*c + 3*a*d)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)
^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c^2) - 1/9*sqrt(3)*(a^(2/3)*b*c + 3*a^(5
/3)*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^2*c^2
) - 1/9*(a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(
a^(5/3)*c^2) - 1/3*(b*x^3 + a)^(2/3)/(a*c*x^3)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^4), x)
```

mupad [B] time = 11.36, size = 1929, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^
2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*(2*a^2*d^2 +
 b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*(-(3*a*d + b*c)^3
/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*
a^2*b*c*d^2))/(3*a^2*c))*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^
7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-(27*a^3*d^3 + b^3*c^3
+ 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log(- (-d^4/(27*b*
c^7 - 27*a*c^6*d))^(2/3)*((-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*
(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d
 - 12*a^3*b*c*d^3))/a^2 - 243*a*b^4*c^4*d^3*(-d^4/(27*b*c^7 - 27*a*c^6*d))^
(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)) + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 +
 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c)) - (4*b^5*d^7*(a + b*x^3)^(1/3)
*(3*a*d + b*c)^2)/(27*a^2*c^5))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3) - log(
(((((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2
+ 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 -
1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 - (b^5*d^4*(b^
3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*
1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3
)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3
 + b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + log((((
((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4
*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 + 3*a*b^4*c^4*d^3*((3^(1/2)*1i)/2 + 1/2
)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))*((3
^(1/2)*1i)/2 - 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(1/3))/9 + (b^5*d^4*(b^3*c
^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^2*c))*((3^(1/2)*1i)
/2 + 1/2)*(-(3*a*d + b*c)^3/(a^4*c^6))^(2/3))/81 - (4*b^5*d^7*(a + b*x^3)^(
1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 +
b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2)/(729*a^4*c^6))^(1/3) + (log(- ((3
^(1/2)*1i - 1)^2*(((3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3)*((
3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4 - 2*a^2*b^2*c^2*d^2 + 4*a
*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4*d^3*(3^(1/2)*1i - 1)^2*(
-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/4))/
2 + (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^
2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4*b^5*d^7*(a + b*x^3)^(1/3
)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i - 1)*(-d^4/(27*b*c^7 - 27*a*c^
6*d))^(1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(((3^(1/2)*1i + 1)*(-d^4/(27*b*c^
7 - 27*a*c^6*d))^(1/3)*((3*b^4*d^3*(a + b*x^3)^(1/3)*(18*a^4*d^4 + b^4*c^4
- 2*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 12*a^3*b*c*d^3))/a^2 - (243*a*b^4*c^4
*d^3*(3^(1/2)*1i + 1)^2*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3)*(2*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/4))/2 - (b^5*d^4*(b^3*c^3 - 27*a^3*d^3 + 8*a*b^2*c^2*d
 + 18*a^2*b*c*d^2))/(3*a^2*c))*(-d^4/(27*b*c^7 - 27*a*c^6*d))^(2/3))/4 - (4
*b^5*d^7*(a + b*x^3)^(1/3)*(3*a*d + b*c)^2)/(27*a^2*c^5))*(3^(1/2)*1i + 1)*
(-d^4/(27*b*c^7 - 27*a*c^6*d))^(1/3))/2 - (a + b*x^3)^(2/3)/(3*a*c*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt[3]{a + bx^3} \left(c + dx^3\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.721 $\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=273

$$\frac{(ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2}+\frac{c^{4/3}\log\left(c+dx^3\right)}{6d^2\sqrt[3]{bc-ad}}-\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\cdots}\right.}{2d^2\sqrt[3]{bc-ad}}$$

```
[Out] 1/3*x*(b*x^3+a)^(2/3)/b/d+1/6*c^(4/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(1/3)-1/2*
c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(1/3)
+1/6*(a*d+3*b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d^2-1/9*(a*d+3*b*c)
*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d^2*3^(1/2)+1/
3*c^(4/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/d^2/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 394, normalized size of antiderivative = 1.44, number of steps used = 15, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 522, 200, 31, 634, 617, 204, 628}

$$\frac{(ad+3bc)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2}-\frac{(ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{4/3}d^2}-\frac{(ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2}-\frac{c^{4/3}\log\left(\cdots\right.}{3d^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*b*d) - ((3*b*c + a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)*d^2) + (c^(4/3)*ArcTan[(c^(1/
3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3
]*d^2*(b*c - a*d)^(1/3)) + ((3*b*c + a*d)*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)])/(9*b^(4/3)*d^2) - ((3*b*c + a*d)*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(4/3)*d^2) - (c^(4/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^(1/3)) +
 (c^(4/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3
)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^6}{\left(1-bx^3\right)^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{c+(2bc+ad)x^3}{\left(1-bx^3\right)\left(c+(-bc+ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+ad)\,\text{Subst}\left(\int \frac{1}{1-b\cdots}\right.}{3bd^2}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2} + \frac{c^{4/3}\,\text{Subst}\left(\int \frac{2\cdots}{c^{2/3}+\sqrt[3]{c}\cdots}\right.}{}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{c^{4/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Su}\cdots}{}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} + \frac{(3bc+ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{4/3}d^2} - \frac{(3bc+ad)\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\cdots}{\sqrt[3]{a\cdots}}\right.}{18b^{4/3}d^2}$$

$$= \frac{x\left(a+bx^3\right)^{2/3}}{3bd} - \frac{(3bc+ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{4/3}d^2} + \frac{c^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2\sqrt[3]{bc-ad}} + \frac{(3bc+\cdots}{}$$

Mathematica [C] time = 0.82, size = 288, normalized size = 1.05

$$\frac{\frac{2\left(-a\sqrt[3]{c}\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)+6x\left(a+bx^3\right)^{2/3}\sqrt[3]{bc-ad}+2a\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)-2\sqrt{3}\,a\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}} - \frac{3x^4\sqrt[3]{\frac{bx^3}{a}+\cdots}}{\cdots}}{36bd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((-3*(3*b*c + a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((
b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^(1/3)) + (2*(6*(b*c - a*d)^(1/3)*x
*(a + b*x^3)^(2/3) - 2*Sqrt[3]*a*c^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*Log[c^(1/3) - ((b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*c^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(b*c - a*d)^(1/3))/(36*b*d)
```

fricas [A] time = 0.84, size = 826, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/18*(6*sqrt(3)*b^2*c*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*
d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)
```

```
/x) + 3*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))
^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 +
a)^(2/3)*c)/x^2) - 6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c + a*b*d
)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1
/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)
*sqrt(-1/b^(2/3)) + 2*a) - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3
 + a)^(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3
)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^2*d^2), -1/18*(6*sqrt(3)*b^2*c*(-
c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(
-c/(b*c - a*d))^(1/3))/x) - 6*b^2*c*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) + 3*b^2*c*(-c/(b*c - a
*d))^(1/3)*log(-((b*c - a*d)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)
*(b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2) - 6*(b*x^
3 + a)^(2/3)*b*d*x - 2*(3*b*c + a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^
(1/3))/x) + (3*b*c + a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1
/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*sqrt(1/3)*(3*b^2*c + a*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x^6/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**6/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.722 $\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=233

$$-\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d}$$

```
[Out] -1/6*c^(1/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(1/3)+1/2*c^(1/3)*ln((-a*d+b*c)^(1/3)
*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(1/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^
(1/3))/b^(1/3)/d+1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^
(1/3)/d*3^(1/2)-1/3*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x
^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{6\sqrt[3]{b}d}-\frac{\sqrt[3]{c}\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6d\sqrt[3]{bc-ad}}+\frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}}-\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}}\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)*d) -
 (c^(1/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(1/3)) - Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)]/(3*b^(1/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1
/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(1/3)*d) + (c^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(1/3)) - (c^(1/3)*Log[c^(2
/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3
)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = a\,\text{Subst}\left(\int \frac{x^3}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} + \frac{\text{Subst}\left(\int \frac{2+\sqrt[3]{b}x}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d} - \frac{\sqrt[3]{c}\,\text{Subst}\left(\ldots\right.}{}$$
$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{b}x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right.}{2d}$$
$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{b}d} + \frac{\sqrt[3]{c}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3d\sqrt[3]{bc-ad}} - \frac{\sqrt[3]{c}}{}$$
$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}d} - \frac{\sqrt[3]{c}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}d\sqrt[3]{bc-ad}} - \frac{\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}\right.}{6\sqrt[3]{b}d}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^4\sqrt[3]{\frac{a+bx^3}{a}}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*((a + b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(1/3))
```

fricas [A] time = 0.74, size = 761, normalized size = 3.27

$$\left[\frac{3\sqrt{\frac{1}{3}}b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\ldots}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-
b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*
(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*b*(c/
(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(
b*c - a*d))^(1/3))/x) + 2*b*(c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b
*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - b*(c/(b*c - a*d))^(1/3)*log(((
```

```
b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(
b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) - 2*(-b)^(2/3)*log(((-b)^(1/3
)*x + (b*x^3 + a)^(1/3))/x) + (-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^
(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b*d), -1/6*(6*sqrt(1/3)*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) - 2*sqrt(3)*b*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3
)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) - 2*b*(c/(b*c -
 a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*
c)/x) + b*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c - a*d))^(1/3)
+ (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)
*c)/x^2) + 2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (-b)^(2
/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3
))/x^2))/(b*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.723 $\int \frac{1}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=148

$$\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(1/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(1/3)+1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(2/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+}}\right.}{3c^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \frac{\text{Subst}}{}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x,\right.}{c^{2/3}\sqrt[3]{bc-}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.24, size = 168, normalized size = 1.14

$$\frac{\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.724 $\int \frac{1}{x^3 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=176

$$-\frac{d\log\left(c+dx^3\right)}{6c^{5/3}\sqrt[3]{bc-ad}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

```
[Out] -1/2*(b*x^3+a)^(2/3)/a/c/x^2-1/6*d*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(1/3)+1/2
*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(1/3)-
1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/
c^(5/3)/(-a*d+b*c)^(1/3)*3^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 235, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{5/3}\sqrt[3]{bc-ad}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}}-\frac{\left(a+bx^3\right)^{2/3}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(2/3)/(2*a*c*x^2) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*
x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(5/3)*(b*c - a*d)^(1/3
)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(5/3)*
(b*c - a*d)^(1/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c -
a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^3\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}} - \frac{d\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc}}\right.}{}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\right.}{2c^{4/3}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{5/3}\sqrt[3]{bc-ad}}$$

$$= -\frac{\left(a+bx^3\right)^{2/3}}{2acx^2} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}\sqrt[3]{bc-ad}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{5/3}\sqrt[3]{bc-ad}} - \frac{d\log\left(c^{2/3}+\right.}{6c}$$

Mathematica [C] time = 0.07, size = 124, normalized size = 0.70

$$\frac{-3x^3\left(c+dx^3\right)(bc-ad)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-4c\left(a+bx^3\right)\left(c+3dx^3\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)}{8c^3x^2\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-4*c*(a + b*x^3)*(c + 3*d*x^3)*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[4/
3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(8*c^3*x^2*(a + b*x^3)^(4/3)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^{1/3}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^3*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt[3]{a + bx^3} \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.725 $\int \frac{1}{x^6\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=214

$$\frac{(a+bx^3)^{2/3}(5ad+3bc)}{10a^2c^2x^2}+\frac{d^2\log\left(c+dx^3\right)}{6c^{8/3}\sqrt[3]{bc-ad}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}}-\frac{(a+bx^3)^{2/3}}{5acx^5}$$

```
[Out] -1/5*(b*x^3+a)^(2/3)/a/c/x^5+1/10*(5*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^2
+1/6*d^2*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(1/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x
/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(1/3)+1/3*d^2*arctan(1/3*(1+2*
(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/(-a*d+b*c)^(1/
3)*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$\frac{(a+bx^3)^{2/3}(ad+bc)}{2a^2c^2x^2}-\frac{(a+bx^3)^{5/3}}{5a^2cx^5}-\frac{d^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{8/3}\sqrt[3]{bc-ad}}+\frac{d^2\tan^{-1}\left(\cdots\right)}{\sqrt{3}\,c^{8/3}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*a^2*c^2*x^2) - (a + b*x^3)^(5/3)/(5*a^2*
c*x^5) + (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/
(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (d^2*Log[c^(1/3)
- ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(8/3)*(b*c - a*d)^(1/3)) +
 (d^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(8/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^6\left(c-(bc-ad)x^3\right)}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^6}+\frac{-bc-ad}{c^2x^3}+\frac{a^2d^2}{c^2\left(c-(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}} + \cdots$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cdots}{c^{2/3}+\cdots}\right.}{\cdots}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{8/3}\sqrt[3]{bc-ad}} + \frac{d^2\log\left(c^{2/3}+\frac{(bc\cdots}{(a\cdots}\right.}{6c^{8/3}\cdots}$$

$$= \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2a^2c^2x^2} - \frac{\left(a+bx^3\right)^{5/3}}{5a^2cx^5} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{8/3}\sqrt[3]{bc-ad}} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc\cdots}}{\sqrt[3]{a\cdots}}\right.}{3c^{8/3}\sqrt[3]{bc-a\cdots}}$$

Mathematica [C] time = 1.03, size = 207, normalized size = 0.97

$$-\frac{-9x^3\left(c+dx^3\right)^2(bc-ad)\,{}_3F_2\left(\frac{4}{3},2,2;1,\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right) - 3x^3\left(-c^2+8cdx^3+9d^2x^6\right)(bc-ad)\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\cdots}{40c^4x^5\left(a+bx^3\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/40*(8*c*(a + b*x^3)*(c^2 - 3*c*d*x^3 - 9*d^2*x^6)*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3*(b*c - a*d)*x^3*(-c^2 + 8*c
*d*x^3 + 9*d^2*x^6)*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 9*(b*c - a*d)*x^3*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2},
 {1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*x^5*(a + b*x^3)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^6}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{1}{3}} \left(d\,x^{3}+c\right) x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{1}{3}} \left(dx^{3}+c\right)x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{6}\,{\left(b\,x^{3}+a\right)}^{1/3}\,\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{6}\sqrt[3]{a+bx^{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(1/3)/(d*x**3+c),x)
[Out] Integral(1/(x**6*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.726 $\int \frac{1}{x^9 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=262

$$\frac{\left(a+bx^3\right)^{2/3}(4ad+3bc)}{20a^2c^2x^5}-\frac{\left(a+bx^3\right)^{2/3}\left(20a^2d^2+12abcd+9b^2c^2\right)}{40a^3c^3x^2}-\frac{d^3\log\left(c+dx^3\right)}{6c^{11/3}\sqrt[3]{bc-ad}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{11/3}\sqrt[3]{bc-ad}}$$

```
[Out] -1/8*(b*x^3+a)^(2/3)/a/c/x^8+1/20*(4*a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c^2/x^5
-1/40*(20*a^2*d^2+12*a*b*c*d+9*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^3/x^2-1/6*d^3
*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(1/3)+1/2*d^3*ln((-a*d+b*c)^(1/3)*x/c^(1/3
)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b*c)^(1/3)-1/3*d^3*arctan(1/3*(1+2*(-a*d+
b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(1/3)*3^
(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2a^3c^3x^2}+\frac{\left(a+bx^3\right)^{5/3}(ad+2bc)}{5a^3c^2x^5}-\frac{\left(a+bx^3\right)^{8/3}}{8a^3cx^8}+\frac{d^3\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{11/3}\sqrt[3]{bc-ad}}-\frac{d^3\log\left(\frac{x^2(bc-}{(a+bx}\right.}{6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(2/3))/(2*a^3*c^3*x^2) + ((2*b*
c + a*d)*(a + b*x^3)^(5/3))/(5*a^3*c^2*x^5) - (a + b*x^3)^(8/3)/(8*a^3*c*x^
8) - (d^3*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(11/3)*(b*c - a*d)^(1/3)) + (d^3*Log[c^(1/3) - (
(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(11/3)*(b*c - a*d)^(1/3)) - (
d^3*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c
 - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(11/3)*(b*c - a*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
```

```
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^9\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-bx^3)^3}{x^9(c-(bc-ad)x^3)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^9} + \frac{-2bc-ad}{c^2x^6} + \frac{b^2c^2+abcd+a^2d^2}{c^3x^3} + \frac{a^3d^3}{c^3(-c+(bc-ad)x^3)}\right) dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^3}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc+ad)(a+bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\ldots}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc+ad)(a+bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\ldots}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc+ad)(a+bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\ldots}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc+ad)(a+bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} + \frac{d^3}{\ldots}$$

$$= -\frac{(b^2c^2+abcd+a^2d^2)(a+bx^3)^{2/3}}{2a^3c^3x^2} + \frac{(2bc+ad)(a+bx^3)^{5/3}}{5a^3c^2x^5} - \frac{(a+bx^3)^{8/3}}{8a^3cx^8} - \frac{d^3}{\ldots}$$

Mathematica [C] time = 2.55, size = 821, normalized size = 3.13

$$-\frac{648bcd^3\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12} - 297ad^4\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12} + 297bcd^3\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{12} + 648\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -1/320*(40*a*c^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 40*b*c^4*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*
(a + b*x^3))] - 72*a*c^3*d*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*
x^3)/(c*(a + b*x^3))] - 72*b*c^3*d*x^6*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 216*a*c^2*d^2*x^6*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 216*b*c^2*d^2*x^9*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*a*c*d^3*x^9*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 648*b*c*d^3*
x^12*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*
b*c^4*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 9*a*c^3*d*x^3*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] - 45*b*c^3*d*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 45*a*c^2*d^2*x^6*Hypergeometric2F1[4/3, 2, 7/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 243*b*c^2*d^2*x^9*Hypergeometric2F1[4/3, 2, 7/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 243*a*c*d^3*x^9*Hypergeometric2F1[4/
3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*b*c*d^3*x^12*Hypergeome
tric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 297*a*d^4*x^12*Hy
pergeometric2F1[4/3, 2, 7/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 54*(b*c -
 a*d)*x^3*(c - 3*d*x^3)*(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 2}, {1, 7/
```

```
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*(b*c - a*d)*x^3*(c + d*x^3)^3*H
ypergeometricPFQ[{4/3, 2, 2, 2}, {1, 1, 7/3}, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))])/(c^5*x^8*(a + b*x^3)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\sqrt[3]{a+bx^{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.727 $\int \frac{x^7}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,1/3,1,11/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 1/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\sqrt[3]{1+\frac{bx^3}{a}}\,(c+dx^3)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{1}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8c\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 144, normalized size = 2.25

$$\frac{-2x^5\sqrt[3]{\frac{bx^3}{a}+1}\,(ad+2bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5cx^2\left(a+bx^3\right)}{20bcd\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*x^2*(a + b*x^3) - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1
, 5/3, -((b*x^3)/a), -((d*x^3)/c)] - 2*(2*b*c + a*d)*x^5*(1 + (b*x^3)/a)^(1
/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*b*c*d*(a +
b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^7}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x^7/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.728 $\int \frac{x^4}{\sqrt[3]{a+bx^3}\,(c+dx^3)}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,1/3,1,8/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt[3]{\frac{a+bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
[Out] (x^5*((a + b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3 + a\right)^{\frac{1}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^3 + a\right)^{\frac{1}{3}}\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
[Out] int(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3 + a\right)^{\frac{1}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.729 $\int \frac{x}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,1/3,1,5/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt[3]{\frac{a+bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2c\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*((a + b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*c*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)^{\frac{1}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3 + a\right)^{\frac{1}{3}}\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)^{\frac{1}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^3 + a\right)^{1/3}\,\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(x/((a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.730 $\int \frac{1}{x^2 \sqrt[3]{a+bx^3} \left(c+dx^3\right)} \, dx$

Optimal. Leaf size=62

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,1/3,1,2/3,-b*x^3/a,-d*x^3/c)/c/x/(b*x^3+a)^(1/3)

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]

[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 1/3, 1, 2/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*x*(a + b*x^3)^(1/3)))

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{1}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{cx\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 141, normalized size = 2.27

$$\frac{5x^3\sqrt[3]{\frac{bx^3}{a}+1}\,(bc-ad)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bdx^6\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10c\left(a+bx^3\right)}{10ac^2x\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (-10*c*(a + b*x^3) + 5*(b*c - a*d)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3,
1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^6*(1 + (b*x^3)/a)^(1/3)*
AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a*c^2*x*(a + b*
x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^3}\,\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.731 $\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,1/3,1,-1/3,-b*x^3/a,-d*x^3/c)/c/x^4/(b
*x^3+a)^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 1/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*c*x^4*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)}\,dx}{\sqrt[3]{a+bx^3}}\\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{4}{3};\frac{1}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4cx^4\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.22, size = 183, normalized size = 2.86

$$\frac{5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2-2abcd-b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}\,(2ad+bc)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\right.}{20a^2c^3x^4\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(a + b*x^3)*(-(a*c) + 2*b*c*x^3 + 4*a*d*x^3) + 5*(-(b^2*c^2) - 2*a*b*c
*d + 2*a^2*d^2)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x
^3)/a), -((d*x^3)/c)] - 2*b*d*(b*c + 2*a*d)*x^9*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(20*a^2*c^3*x^4*(a + b*x
^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{1}{3}}\left(d\,x^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5\,{\left(b\,x^3+a\right)}^{1/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(1/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

3.732 $\int \frac{x^{11}}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=241

$$\frac{\sqrt[3]{a+bx^3}\,(a^2d^2+abcd+b^2c^2)}{b^3d^3}-\frac{(a+bx^3)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{}\right.}{2d^{10/3}(bc-ad}$$

```
[Out] (a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(1/3)/b^3/d^3-1/4*(2*a*d+b*c)*(b*x^3+a)
^(4/3)/b^3/d^2+1/7*(b*x^3+a)^(7/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(10/3)/(-a*d
+b*c)^(2/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(10/3)/(
-a*d+b*c)^(2/3)+1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^
(1/3))*3^(1/2))/d^(10/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 58, 617, 204, 31}

$$\frac{\sqrt[3]{a+bx^3}\,(a^2d^2+abcd+b^2c^2)}{b^3d^3}-\frac{(a+bx^3)^{4/3}(2ad+bc)}{4b^3d^2}+\frac{(a+bx^3)^{7/3}}{7b^3d}+\frac{c^3\log(c+dx^3)}{6d^{10/3}(bc-ad)^{2/3}}-\frac{c^3\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{}\right.}{2d^{10/3}(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/(b^3*d^3) - ((b*c + 2*a*d
)*(a + b*x^3)^(4/3))/(4*b^3*d^2) + (a + b*x^3)^(7/3)/(7*b^3*d) + (c^3*ArcTa
n[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*
d^(10/3)*(b*c - a*d)^(2/3)) + (c^3*Log[c + d*x^3])/(6*d^(10/3)*(b*c - a*d)^
(2/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(10/
3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{b^2c^2+abcd+a^2d^2}{b^2d^3(a+bx)^{2/3}} + \frac{(-bc-2ad)\sqrt[3]{a+bx}}{b^2d^2} + \frac{(a+bx)^{4/3}}{b^2d} - \frac{}{d^3(a+\cdots}\right.\right.$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} - \frac{c^3\,\text{S}\cdots}{}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\,\cdots}{6d^{1\cdots}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\,\cdots}{6d^{1\cdots}}$$

$$= \frac{\left(b^2c^2+abcd+a^2d^2\right)\sqrt[3]{a+bx^3}}{b^3d^3} - \frac{(bc+2ad)\left(a+bx^3\right)^{4/3}}{4b^3d^2} + \frac{\left(a+bx^3\right)^{7/3}}{7b^3d} + \frac{c^3\,\text{ta}\cdots}{\sqrt{\cdots}}$$

Mathematica [A] time = 0.66, size = 251, normalized size = 1.04

$$\frac{\frac{84\sqrt[3]{a+bx^3}\left(a^2d^2+abcd+b^2c^2\right)}{b^3} - \frac{21d\left(a+bx^3\right)^{4/3}(2ad+bc)}{b^3} + \frac{12d^2\left(a+bx^3\right)^{7/3}}{b^3} + \frac{14c^3\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\cdots\right.}{\sqrt[3]{d}\,(bc-a\cdots}}{84d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((84*(b^2*c^2 + a*b*c*d + a^2*d^2)*(a + b*x^3)^(1/3))/b^3 - (21*d*(b*c + 2*
a*d)*(a + b*x^3)^(4/3))/b^3 + (12*d^2*(a + b*x^3)^(7/3))/b^3 + (14*c^3*(2*S
```

```
qrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*c - a*d)^
(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(
2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(84*d^3)
```

fricas [B] time = 0.89, size = 1322, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/84*(14*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 +
a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c
- a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28
*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*
(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 42*sqrt(1/3
)*(b^4*c^4*d - a*b^3*c^3*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/
3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*
sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2
- a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3))*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-
b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*
x^3 + c)) + 3*(28*b^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3
*b*c*d^4 + 18*a^4*d^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 -
 (7*b^4*c^3*d^2 - 8*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*
x^3 + a)^(1/3))/(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6), 1/84*(14*(-b^2
*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(2/3)*(b*c*d
 - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + (-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 28*(-b^2*c^2*d +
2*a*b*c*d^2 - a^2*d^3)^(2/3)*b^3*c^3*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2)
 - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 84*sqrt(1/3)*(b^4*c^4*d -
a*b^3*c^3*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*arctan(s
qrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) + 2*(-b^2*
c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt(-(-b^2*c^2*d +
 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + 3*(28*b
^4*c^4*d - 35*a*b^3*c^3*d^2 + 4*a^2*b^2*c^2*d^3 - 15*a^3*b*c*d^4 + 18*a^4*d
^5 + 4*(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 - (7*b^4*c^3*d^2 - 8
*a*b^3*c^2*d^3 - 5*a^2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(1/3))/(b^
5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)]
```

giac [A] time = 0.26, size = 372, normalized size = 1.54

$$\frac{b^{24}c^3d^4\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{25}cd^7-ab^{24}d^8\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^4-\sqrt{3}\,ad^5}-\frac{\left(-bcd^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*b^24*c^3*d^4*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
- a*d)/d)^(1/3)))/(b^25*c*d^7 - a*b^24*d^8) - (-b*c*d^2 + a*d^3)^(1/3)*c^3*
arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
a*d)/d)^(1/3))/(sqrt(3)*b*c*d^4 - sqrt(3)*a*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(
1/3)*c^3*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) +
 (-(b*c - a*d)/d)^(2/3))/(b*c*d^4 - a*d^5) + 1/28*(28*(b*x^3 + a)^(1/3)*b^2
```

```
0*c^2*d^4 - 7*(b*x^3 + a)^(4/3)*b^19*c*d^5 + 28*(b*x^3 + a)^(1/3)*a*b^19*c*
d^5 + 4*(b*x^3 + a)^(7/3)*b^18*d^6 - 14*(b*x^3 + a)^(4/3)*a*b^18*d^6 + 28*(
b*x^3 + a)^(1/3)*a^2*b^18*d^6)/(b^21*d^7)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.01, size = 331, normalized size = 1.37

$$\left(\frac{3\,a^2}{b^3\,d}+\frac{\left(\frac{3\,a}{b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{b^6\,d^2}\right)\,\left(b^4\,c-a\,b^3\,d\right)}{b^3\,d}\right)\,{\left(b\,x^3+a\right)}^{1/3}-\left(\frac{3\,a}{4\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{4\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{4/3}+\frac{{\left(b\,x^3+a\right)}^{7/3}}{7\,b^3\,d}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^3*d) + (((3*a)/(b^3*d) + (b^4*c - a*b^3*d)/(b^6*d^2))*(b^4*c -
a*b^3*d))/(b^3*d))*(a + b*x^3)^(1/3) - ((3*a)/(4*b^3*d) + (b^4*c - a*b^3*d)
/(4*b^6*d^2))*(a + b*x^3)^(4/3) + (a + b*x^3)^(7/3)/(7*b^3*d) + (log((3*c^3
*(a + b*x^3)^(1/3))/d + (3*c^3*(3^(1/2)*1i + 1)*(a*d - b*c)^(1/3))/(2*d^(4/
3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(10/3)*(a*d - b*c)^(2/3)) - (c^3*log((3*c
^3*(a + b*x^3)^(1/3))/d - (3*c^3*(a*d - b*c)^(1/3))/d^(4/3)))/(3*d^(10/3)*(
a*d - b*c)^(2/3)) - (c^3*log((3*c^3*(a + b*x^3)^(1/3))/d - (3*c^3*((3^(1/2)
*1i)/2 - 1/2)*(a*d - b*c)^(1/3))/d^(4/3))*((3^(1/2)*1i)/2 - 1/2))/(3*d^(10/
3)*(a*d - b*c)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.733 $\int \frac{x^8}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal. Leaf size=201

$$-\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2d^2}+\frac{(a+bx^3)^{4/3}}{4b^2d}-\frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}$$

```
[Out] -(a*d+b*c)*(b*x^3+a)^(1/3)/b^2/d^2+1/4*(b*x^3+a)^(4/3)/b^2/d-1/6*c^2*ln(d*x
^3+c)/d^(7/3)/(-a*d+b*c)^(2/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a
)^(1/3))/d^(7/3)/(-a*d+b*c)^(2/3)-1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)
^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(7/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 88, 58, 617, 204, 31}

$$-\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2d^2}+\frac{(a+bx^3)^{4/3}}{4b^2d}-\frac{c^2\log(c+dx^3)}{6d^{7/3}(bc-ad)^{2/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{7/3}(bc-ad)^{2/3}}-\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(((b*c + a*d)*(a + b*x^3)^(1/3))/(b^2*d^2)) + (a + b*x^3)^(4/3)/(4*b^2*d)
- (c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(7/3)*(b*c - a*d)^(2/3)) - (c^2*Log[c + d*x^3])/(6*d^(7/3)*(b
*c - a*d)^(2/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{-bc-ad}{bd^2(a+bx)^{2/3}} + \frac{\sqrt[3]{a+bx}}{bd} + \frac{c^2}{d^2(a+bx)^{2/3}(c+dx)}\right)dx, x, x^3\right)$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d^2}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\right.}{2d^{7/3}(bc-}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{7/3}(bc-ad)^{2/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\right.}{2d^{7/3}(bc-a}$$

$$= -\frac{(bc+ad)\sqrt[3]{a+bx^3}}{b^2d^2} + \frac{\left(a+bx^3\right)^{4/3}}{4b^2d} - \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{7/3}(bc-ad)^{2/3}} - \frac{c^2\log\left(c+dx^3\right.}{6d^{7/3}(bc-ad)^2}$$

Mathematica [A] time = 0.43, size = 211, normalized size = 1.05

$$\frac{-\frac{12\sqrt[3]{a+bx^3}\,(ad+bc)}{b^2} + \frac{3d\left(a+bx^3\right)^{4/3}}{b^2} - \frac{2c^2\left(\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-}\right.}{\sqrt[3]{d}\,(bc-ad)^{2/3}}}{12d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-12*(b*c + a*d)*(a + b*x^3)^(1/3))/b^2 + (3*d*(a + b*x^3)^(4/3))/b^2 - (2
*c^2*(2*Sqrt[3]*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3)
)/Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + Log[(b*
```

```
c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a +
 b*x^3)^(2/3)]))/(d^(1/3)*(b*c - a*d)^(2/3)))/(12*d^2)
```

fricas [B] time = 0.97, size = 1156, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a
)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c -
a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 4*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log(-(b*x^3 + a)^(1/3)*(b*c*
d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)) + 6*sqrt(1/3)*(b^3*
c^3*d - a*b^2*c^2*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)*l
og((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 + 3*sqrt(1/
3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a
)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c))
+ 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*a^3*d^4 - (b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))/(b^4*c^2*d^3 - 2*a*
b^3*c*d^4 + a^2*b^2*d^5), -1/12*(2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*b^2*c^2*log(-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2
 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(
b*x^3 + a)^(1/3)) - 4*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*b^2*c^2*log
(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(
2/3)) - 12*sqrt(1/3)*(b^3*c^3*d - a*b^2*c^2*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)^(1/3)*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3
+ a)^(1/3))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2)) + 3*(4*b^3*c^3*d - 5*a*b^2*c^2*d^2 - 2*a^2*b*c*d^3 + 3*
a^3*d^4 - (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(1/3))
/(b^4*c^2*d^3 - 2*a*b^3*c*d^4 + a^2*b^2*d^5)]
```

giac [A] time = 0.30, size = 312, normalized size = 1.55

$$-\frac{b^{10}c^2d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{11}cd^4-ab^{10}d^5\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd^3-\sqrt{3}\,ad^4}+\frac{\left(-bcd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b^10*c^2*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c
 - a*d)/d)^(1/3)))/(b^11*c*d^4 - a*b^10*d^5) + (-b*c*d^2 + a*d^3)^(1/3)*c^2
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b*c*d^3 - sqrt(3)*a*d^4) + 1/6*(-b*c*d^2 + a*d^3)^
(1/3)*c^2*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3)
+ (-(b*c - a*d)/d)^(2/3))/(b*c*d^3 - a*d^4) - 1/4*(4*(b*x^3 + a)^(1/3)*b^7*
c*d^2 - (b*x^3 + a)^(4/3)*b^6*d^3 + 4*(b*x^3 + a)^(1/3)*a*b^6*d^3)/(b^8*d^4
)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.69, size = 292, normalized size = 1.45

$$\frac{\left(b\,x^3+a\right)^{4/3}}{4\,b^2\,d}-\left(\frac{2\,a}{b^2\,d}+\frac{b^3\,c-a\,b^2\,d}{b^4\,d^2}\right)\,\left(b\,x^3+a\right)^{1/3}-\frac{\ln\left(3\,c^2\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(c^2+\sqrt{3}\,c^2\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c^2+\sqrt{3}\,c^2\right.}{6\,d^{7/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(4/3)/(4*b^2*d) - ((2*a)/(b^2*d) + (b^3*c - a*b^2*d)/(b^4*d^2))
*(a + b*x^3)^(1/3) - (log(3*c^2*(a + b*x^3)^(1/3) + ((3^(1/2)*c^2*1i + c^2)
*(9*a*d^3 - 9*b*c*d^2))/(6*d^(7/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*c^2*1i + c^
2))/(6*d^(7/3)*(a*d - b*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2
*(9*a*d^3 - 9*b*c*d^2))/(3*d^(7/3)*(a*d - b*c)^(2/3))))/(3*d^(7/3)*(a*d - b
*c)^(2/3)) + (c^2*log(3*c^2*(a + b*x^3)^(1/3) - (c^2*((3^(1/2)*1i)/6 - 1/6)
*(9*a*d^3 - 9*b*c*d^2))/(d^(7/3)*(a*d - b*c)^(2/3)))*((3^(1/2)*1i)/6 - 1/6)
)/(d^(7/3)*(a*d - b*c)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(a + b x^{3}\right)^{\frac{2}{3}} \left(c + d x^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.734 $\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=165

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

```
[Out] (b*x^3+a)^(1/3)/b/d+1/6*c*ln(d*x^3+c)/d^(4/3)/(-a*d+b*c)^(2/3)-1/2*c*ln((-a
*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(4/3)/(-a*d+b*c)^(2/3)+1/3*c*arcta
n(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(4/3)/(-a*d
+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 80, 58, 617, 204, 31}

$$\frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}}+\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}}+\frac{\sqrt[3]{a+bx^3}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) + (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c
 - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(4/3)*(b*c - a*d)^(2/3)) + (c*Log[c + d
*x^3])/(6*d^(4/3)*(b*c - a*d)^(2/3)) - (c*Log[(b*c - a*d)^(1/3) + d^(1/3)*(
a + b*x^3)^(1/3)])/(2*d^(4/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} - \frac{c\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3d}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subs}\ldots}{\ldots}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{4/3}(bc-ad)^{2/3}} - \frac{c\,\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= \frac{\sqrt[3]{a+bx^3}}{bd} + \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{4/3}(bc-ad)^{2/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{4/3}(bc-ad)^{2/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{a}\ldots\right.}{2d^{4/3}(bc-ad\ldots}$$

Mathematica [A] time = 0.18, size = 202, normalized size = 1.22

$$\frac{bc\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+6\sqrt[3]{d}\sqrt[3]{a+bx^3}(bc-ad)^{2/3}-2bc\log\left(\sqrt[3]{bc}\ldots\right.}{6bd^{4/3}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (6*d^(1/3)*(b*c - a*d)^(2/3)*(a + b*x^3)^(1/3) - 2*Sqrt[3]*b*c*ArcTan[(-1 +
 (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]] - 2*b*c*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] + b*c*Log[(b*c - a*d)^(2/3) - d^
(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(6*
b*d^(4/3)*(b*c - a*d)^(2/3))
```

fricas [B] time = 0.90, size = 1060, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(2/3)
*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c - a*d) +
 (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(-b^2*c^
2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^
2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 3*sqrt(1/3)*(b^2*c^2*d -
 a*b*c*d^2)*sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)*log((b^2*c^2
 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3 - 3*sqrt(1/3)*(2*(b*x^
3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(
b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*
sqrt((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d) - 3*(-b^2*c^2*d + 2*a*b*
c*d^2 - a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(d*x^3 + c)) + 6*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d^2 - 2*a*b^2*c
*d^3 + a^2*b*d^4), 1/6*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(
-(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(
1/3)*(b*c - a*d) + (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(
1/3)) - 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*b*c*log(-(b*x^3 + a)^(
1/3)*(b*c*d - a*d^2) - (-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)) - 6*sqrt
(1/3)*(b^2*c^2*d - a*b*c*d^2)*sqrt(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1
/3)/d)*arctan(sqrt(1/3)*((-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)*(b*c -
a*d) + 2*(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3))*sqrt
(-(-b^2*c^2*d + 2*a*b*c*d^2 - a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d + a^2*
d^2)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x^3 + a)^(1/3))/(b^3*c^2*d
^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)]
```

giac [A] time = 0.23, size = 253, normalized size = 1.53

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bcd^2-\sqrt{3}ad^3}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{bcd^2-ad^3}-\frac{2bc\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log}{}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(1/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d^2 - sqrt
(3)*a*d^3) + (-b*c*d^2 + a*d^3)^(1/3)*b*c*log((b*x^3 + a)^(2/3) + (b*x^3 +
a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d^2 - a*d^3)
 - 2*b*c*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c*d - a*d^2) - 6*(b*x^3 + a)^(1/3)/d)/b
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.68, size = 232, normalized size = 1.41

$$\frac{\left(b\,x^3+a\right)^{1/3}}{b\,d}-\frac{c\,\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}-\frac{c\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,c\,d\,\left(b\,x^3+a\right)^{1/3}+\frac{\left(9\,a\,d^3-9\,b\,c\,d^2\right)\,\left(c-\sqrt{3}\,c\,1{}\mathrm{i}\right)}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(c-\sqrt{\right.}}{6\,d^{4/3}\,{\left(a\,d-b\,c\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(1/3)/(b*d) - (c*log(3*c*d*(a + b*x^3)^(1/3) - (c*(9*a*d^3 - 9*
b*c*d^2))/(3*d^(4/3)*(a*d - b*c)^(2/3))))/(3*d^(4/3)*(a*d - b*c)^(2/3)) + (
log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c - 3^(1/2)*c*1i))/(6
*d^(4/3)*(a*d - b*c)^(2/3)))*(c - 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b*c)^(2/
3)) + (log(3*c*d*(a + b*x^3)^(1/3) + ((9*a*d^3 - 9*b*c*d^2)*(c + 3^(1/2)*c*
1i))/(6*d^(4/3)*(a*d - b*c)^(2/3)))*(c + 3^(1/2)*c*1i))/(6*d^(4/3)*(a*d - b
*c)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a + bx^{3}\right)^{\frac{2}{3}} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.735 $\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=145

$$-\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

```
[Out] -1/6*ln(d*x^3+c)/d^(1/3)/(-a*d+b*c)^(2/3)+1/2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(
b*x^3+a)^(1/3))/d^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1-2*d^(1/3)*(b*x^3
+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {444, 58, 617, 204, 31}

$$-\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}+\frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]]/(Sq
rt[3]*d^(1/3)*(b*c - a*d)^(2/3))) - Log[c + d*x^3]/(6*d^(1/3)*(b*c - a*d)^(
2/3)) + Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]/(2*d^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{d^{2/3}}-\cdots}\right.}{2d^{2/\cdots}}$$

$$= -\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\cdots\right.}{\sqrt[3]{d}\,(bc-ad\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{d}\,(bc-ad)^{2/3}} - \frac{\log\left(c+dx^3\right)}{6\sqrt[3]{d}\,(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.06, size = 164, normalized size = 1.13

$$\frac{-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+d^{2/3}\left(a+bx^3\right)^{2/3}\right)+2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\cdots}{6\sqrt[3]{d}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sq
rt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)] - Log[(b*c -
a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x
^3)^(2/3)])/(6*d^(1/3)*(b*c - a*d)^(2/3))
```

fricas [B] time = 0.63, size = 927, normalized size = 6.39

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(bcd-ad^2\right)\sqrt{-\frac{\left(b^2c^2d-2\,abcd^2+a^2d^3\right)^{\frac{1}{3}}}{d}}\,\log\left(\frac{b^2c^2-4\,abcd+3\,a^2d^2-2\left(b^2cd-abd^2\right)x^3+3\sqrt{\frac{1}{3}}\left(2\left(bx^3+a\right)^{\frac{2}{3}}\left(bcd-ad^2\right)-\left(b^2c^2d-2\,abc\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(b*c*d - a*d^2)*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3
)^(1/3)/d)*log((b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x^3
 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)^(1/3)*(b*c - a*d) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3
)*(b*x^3 + a)^(1/3))*sqrt(-(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d) + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*x^3 + a)^(1/3)*(b*c - a*d))/(
d*x^3 + c)) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) - 2*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3), 1/6*(6*sqrt(1/3)*(b*c*d - a*d^2)*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)^(1/3)/d)*arctan(-sqrt(1/3)*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)
*(b*c - a*d) - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3
))*sqrt((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)/d)/(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(2/
3)*(b*c*d - a*d^2) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(1/3)*(b*c - a*d)
+ (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*(b*x^3 + a)^(1/3)) + 2*(b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)*log(-(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) -
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)^(2/3)))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)]
```

giac [A] time = 0.29, size = 221, normalized size = 1.52

$$\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bcd-\sqrt{3}\,ad^{2}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{1}{3}}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc}{}\right.\right.}{6\left(bcd-ad^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c -
 a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b*c*d - sqrt(3)*a*d^2) + 1
/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*
c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c*d - a*d^2) - 1/3*(-(b*c -
a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b*c - a
*d)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.85, size = 213, normalized size = 1.47

$$\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{9\,a\,d^3-9\,b\,c\,d^2}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)}{3\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}+\frac{\ln\left(3\,d^2\,{\left(b\,x^3+a\right)}^{1/3}-\frac{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)\,\left(9\,a\,d^3-9\,b\,c\,d^2\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,d^{1/3}\,{\left(a\,d-b\,c\right)}^{2/3}}-\frac{\ln\left(3\,d^2\,\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(3*d^2*(a + b*x^3)^(1/3) - (9*a*d^3 - 9*b*c*d^2)/(3*d^(1/3)*(a*d - b*c)^
(2/3)))/(3*d^(1/3)*(a*d - b*c)^(2/3)) + (log(3*d^2*(a + b*x^3)^(1/3) - ((3^
(1/2)*1i - 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2
)*1i - 1))/(6*d^(1/3)*(a*d - b*c)^(2/3)) - (log(3*d^2*(a + b*x^3)^(1/3) + (
(3^(1/2)*1i + 1)*(9*a*d^3 - 9*b*c*d^2))/(6*d^(1/3)*(a*d - b*c)^(2/3)))*(3^(
1/2)*1i + 1))/(6*d^(1/3)*(a*d - b*c)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.736 $\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=245

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+$$

```
[Out] -1/2*ln(x)/a^(2/3)/c+1/6*d^(2/3)*ln(d*x^3+c)/c/(-a*d+b*c)^(2/3)+1/2*ln(a^(1
/3)-(b*x^3+a)^(1/3))/a^(2/3)/c-1/2*d^(2/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x
^3+a)^(1/3))/c/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3))/
a^(1/3)*3^(1/2))/a^(2/3)/c*3^(1/2)+1/3*d^(2/3)*arctan(1/3*(1-2*d^(1/3)*(b*x
^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 86, 57, 617, 204, 31, 58}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}c}-\frac{\log(x)}{2a^{2/3}c}+\frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}-\frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(2/3
)*c)) + (d^(2/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3
))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(2/3)) - Log[x]/(2*a^(2/3)*c) + (d^(2/3
)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(2/3)) + Log[a^(1/3) - (a + b*x^3)^(1/3)
]/(2*a^(2/3)*c) - (d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3
)])/(2*c*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{2/3}}\,dx, x, x^3\right)}{3c} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx, x, x^3\right)}{3c}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{2/3}c} - \frac{d^{2/3}\log\left(\sqrt[3]{bc-ad}\right.}{2c(bc-a}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}c} + \frac{d^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{2/3}} - \frac{\log(x)}{2a^{2/3}c} + \frac{d^{2/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.45, size = 308, normalized size = 1.26

$$-\frac{-\frac{2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{a^{2/3}} + \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)}{a^{2/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{(bc-ad)^{2/3}} - \frac{d^{2/3}\log\left(\right.}{}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/6*((2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/
3) + (2*Sqrt[3]*d^(2/3)*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a
*d)^(1/3))/Sqrt[3]])/(b*c - a*d)^(2/3) - (2*Log[a^(1/3) - (a + b*x^3)^(1/3)
])/a^(2/3) + (2*d^(2/3)*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(b*c - a*d)^(2/3) + Log[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^
(2/3)]/a^(2/3) - (d^(2/3)*Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)
*(a + b*x^3)^(1/3) + d^(2/3)*(a + b*x^3)^(2/3)])/(b*c - a*d)^(2/3))/c
```

fricas [B] time = 0.46, size = 472, normalized size = 1.93

$$-\frac{2\sqrt{3}\,a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)+a^2\left(-\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(\left(bx^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/
3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(2/3) - sqrt(3)*d)/d) + a^2*(-d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
1/3)*log((b*x^3 + a)^(2/3)*d^2 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2)*(-d^2/(b
^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-d^
2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 2*a^2*(-d^2/(b^2*c^2 - 2*a*b*c*
d + a^2*d^2))^(1/3)*log((b*x^3 + a)^(1/3)*d - (b*c - a*d)*(-d^2/(b^2*c^2 -
2*a*b*c*d + a^2*d^2))^(1/3)) + 2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*(a^2)^(1/
6)*(sqrt(3)*(a^2)^(1/3)*a + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(a^2)^(2/3))/a^2) +
 (a^2)^(2/3)*log((b*x^3 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*(a
^2)^(2/3)) - 2*(a^2)^(2/3)*log((b*x^3 + a)^(1/3)*a - (a^2)^(2/3)))/(a^2*c)
```

giac [A] time = 0.76, size = 321, normalized size = 1.31

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^2-\sqrt{3}\,acd}-\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b*c^2 - a*c*d) - (-b*c*d^2 + a*d^3)^(1/3)*arctan(1/3*sqrt(3)*(2*(b*
x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b
*c^2 - sqrt(3)*a*c*d) - 1/6*(-b*c*d^2 + a*d^3)^(1/3)*log((b*x^3 + a)^(2/3)
+ (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b*c^2
 - a*c*d) - 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/
a^(1/3))/(a^(2/3)*c) - 1/6*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3
) + a^(2/3))/(a^(2/3)*c) + 1/3*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(2/
3)*c)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x), x)
```

mupad [B] time = 4.94, size = 1413, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] log((((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (243*a*b^6*c
^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/3))
*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*d^5
*(a + b*x^3)^(1/3))*(1/(27*a^2*c^3))^(1/3) + log(- (((81*b^6*c^5*d^3 - 162*
a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - (-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*
a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^
4*d^5))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3) - 9*b^5*c
^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3) - 6*b^4*d
^5*(a + b*x^3)^(1/3))*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(
1/3) + log(((3^(1/2)*1i)/2 - 1/2)*(((3^(1/2)*1i)/2 - 1/2)^2*((81*b^6*c^5*d^
3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 - 1/2)*(243*a*b^
6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(1/(27*a^2*c^3))^(1/
3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c^3))^(1/3) + 6*b^4*
d^5*(a + b*x^3)^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^2*c^3))^(1/3) - log(
6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)*1i)/2 + 1/2)*(((3^(1/2)*1i)/2 + 1/2
)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) + ((3^(1/2)*1i)
/2 + 1/2)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^5)*(
1/(27*a^2*c^3))^(1/3))*(1/(27*a^2*c^3))^(2/3) - 9*b^5*c^2*d^4)*(1/(27*a^2*c
^3))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^2*c^3))^(1/3) + (log(6*b^4*d^5*
(a + b*x^3)^(1/3) + ((3^(1/2)*1i - 1)*(((3^(1/2)*1i - 1)^2*((81*b^6*c^5*d^3
 - 162*a*b^5*c^4*d^4)*(a + b*x^3)^(1/3) - ((3^(1/2)*1i - 1)*(-d^2/(27*b^2*c
^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5
*c^5*d^4 + 486*a^3*b^4*c^4*d^5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54
*a*b*c^4*d))^(2/3))/4 - 9*b^5*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 -
 54*a*b*c^4*d))^(1/3))/2)*(3^(1/2)*1i - 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d
^2 - 54*a*b*c^4*d))^(1/3))/2 - (log(6*b^4*d^5*(a + b*x^3)^(1/3) - ((3^(1/2)
*1i + 1)*(((3^(1/2)*1i + 1)^2*((81*b^6*c^5*d^3 - 162*a*b^5*c^4*d^4)*(a + b*
x^3)^(1/3) + ((3^(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*
c^4*d))^(1/3)*(243*a*b^6*c^6*d^3 - 729*a^2*b^5*c^5*d^4 + 486*a^3*b^4*c^4*d^
5))/2)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(2/3))/4 - 9*b^5
*c^2*d^4)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2)*(3^
(1/2)*1i + 1)*(-d^2/(27*b^2*c^5 + 27*a^2*c^3*d^2 - 54*a*b*c^4*d))^(1/3))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.737 $\int \frac{1}{x^4(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=299

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}$$

```
[Out] -1/3*(b*x^3+a)^(1/3)/a/c/x^3+1/6*(3*a*d+2*b*c)*ln(x)/a^(5/3)/c^2-1/6*d^(5/3
)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(2/3)-1/6*(3*a*d+2*b*c)*ln(a^(1/3)-(b*x^3+a)^(
1/3))/a^(5/3)/c^2+1/2*d^(5/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
c^2/(-a*d+b*c)^(2/3)+1/9*(3*a*d+2*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(1/3
))/a^(1/3)*3^(1/2))/a^(5/3)/c^2*3^(1/2)-1/3*d^(5/3)*arctan(1/3*(1-2*d^(1/3)
*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 103, 156, 57, 617, 204, 31, 58}

$$-\frac{(3ad+2bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{5/3}c^2}+\frac{(3ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}c^2}+\frac{\log(x)(3ad+2bc)}{6a^{5/3}c^2}-\frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(a + b*x^3)^(1/3)/(3*a*c*x^3) + ((2*b*c + 3*a*d)*ArcTan[(a^(1/3) + 2*(a +
b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*c^2) - (d^(5/3)*ArcTan
[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c
^2*(b*c - a*d)^(2/3)) + ((2*b*c + 3*a*d)*Log[x])/(6*a^(5/3)*c^2) - (d^(5/3)
*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(2/3)) - ((2*b*c + 3*a*d)*Log[a^(1/3) -
 (a + b*x^3)^(1/3)])/(6*a^(5/3)*c^2) + (d^(5/3)*Log[(b*c - a*d)^(1/3) + d^(
1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(2bc+3ad)+\frac{2bdx}{3}}{x(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3ac}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{2/3}(c+dx)}\,dx,x,x^3\right)}{3c^2} - \frac{(2bc+3ad)\,\text{Subst}\left(\int \frac{}{x(}\right.}{9ac^2}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} + \frac{d^{5/3}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}}\right.}{2c^2(bc}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\log(x)}{6a^{5/3}c^2} - \frac{d^{5/3}\log\left(c+dx^3\right)}{6c^2(bc-ad)^{2/3}} - \frac{(2bc+3ad)\log\left(\sqrt[3]{a}\right.}{6a^{5/3}c}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{3acx^3} + \frac{(2bc+3ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}c^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^2(bc-ad)^{2/3}} + \frac{(2}{}$$

Mathematica [A] time = 0.59, size = 303, normalized size = 1.01

$$\frac{\frac{(3ad+2bc)\left(\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^3}+\left(a+bx^3\right)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)\right)}{a^{2/3}c} + \frac{3ad^{5/3}\left(-\log\left(-\sqrt[3]{d}\sqrt[3]{a+bx^3}\sqrt[3]{bc-ad}+(bc-ac\right.\right.}{}}{18ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((-6*(a + b*x^3)^(1/3))/x^3 + ((2*b*c + 3*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(a
 + b*x^3)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a^(1/3) - (a + b*x^3)^(1/3)] + L
og[a^(2/3) + a^(1/3)*(a + b*x^3)^(1/3) + (a + b*x^3)^(2/3)]))/(a^(2/3)*c) +
 (3*a*d^(5/3)*(2*Sqrt[3]*ArcTan[(-1 + (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c -
a*d)^(1/3))/Sqrt[3]] + 2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)]
 - Log[(b*c - a*d)^(2/3) - d^(1/3)*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3) + d^
(2/3)*(a + b*x^3)^(2/3)]))/(c*(b*c - a*d)^(2/3)))/(18*a*c)
```

fricas [B] time = 1.13, size = 562, normalized size = 1.88

$$-\frac{6\sqrt{3}\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}-\sqrt{3}\,d}{3\,d}\right)+3\,a^3d\left(\frac{d^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}x^3\,\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(6*sqrt(3)*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*arct
an(-1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(d^2/(b^2*c^2 - 2*a*b*c*d
```

```
+ a^2*d^2))^(2/3) - sqrt(3)*d)/d) + 3*a^3*d*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2
*d^2))^(1/3)*x^3*log((b*x^3 + a)^(2/3)*d^2 - (b*x^3 + a)^(1/3)*(b*c*d - a*d
^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*(d^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)) - 6*a^3*d*(d^2/(b^2*c^2
 - 2*a*b*c*d + a^2*d^2))^(1/3)*x^3*log((b*x^3 + a)^(1/3)*d + (b*c - a*d)*(d
^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)) - 2*sqrt(3)*(2*a*b*c + 3*a^2*d)*
x^3*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*a - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) - (-a^2)^(2/3)*(2*b*c
 + 3*a*d)*x^3*log((b*x^3 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^3 + a)^(1/3)*
(-a^2)^(2/3)) + 2*(-a^2)^(2/3)*(2*b*c + 3*a*d)*x^3*log((b*x^3 + a)^(1/3)*a
- (-a^2)^(2/3)) + 6*(b*x^3 + a)^(1/3)*a^2*c)/(a^3*c^2*x^3)
```

giac [A] time = 0.76, size = 377, normalized size = 1.26

$$-\frac{d^2\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^3-ac^2d\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,bc^3-\sqrt{3}\,ac^2d}+\frac{\left(-bcd^2+ad^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(1/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b*c^3 - a*c^2*d) + (-b*c*d^2 + a*d^3)^(1/3)*d*arctan(1/3*sqrt(3)
*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sq
rt(3)*b*c^3 - sqrt(3)*a*c^2*d) + 1/6*(-b*c*d^2 + a*d^3)^(1/3)*d*log((b*x^3
+ a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2
/3))/(b*c^3 - a*c^2*d) + 1/9*sqrt(3)*(2*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*
(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(5/3)*c^2) + 1/18*(2*b*c + 3*a*d)*
log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(5/3)*c^2)
- 1/9*(2*b*c + 3*a*d)*log(abs((b*x^3 + a)^(1/3) - a^(1/3)))/(a^(5/3)*c^2) -
 1/3*(b*x^3 + a)^(1/3)/(a*c*x^3)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^4), x)
```

mupad [B] time = 11.14, size = 1959, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*
d))/a - 81*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/9 + (b^5*d^4*(8*b^3*c^3 -
 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d -
 b*c)^2))^(1/3))/3 - (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 +
 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(d^5/(27*b^2*c^8 + 27*a^2*c
^6*d^2 - 54*a*b*c^7*d))^(1/3) + log(- (((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)
*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(2*a^2*d^2 + b^2*c
^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))*(-(3*a*d + 2*b*c)^3/(
a^5*c^6))^(2/3))/81 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 1
8*a^2*b*c*d^2))/(3*a^3*c))*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4
*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*
c*d^2))/(9*a^3*c^4))*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*a^2*b*
c*d^2)/(729*a^5*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)*((((27*b^5*c^3*d^3*(a
+ b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - (81*a*b^4*c^4*d^3*(3^
(1/2)*1i - 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(
1/3))/2)*(3^(1/2)*1i - 1)^2*(d^5/(c^6*(a*d - b*c)^2))^(2/3))/36 + (b^5*d^4*
(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5
/(c^6*(a*d - b*c)^2))^(1/3))/6 + (2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 +
 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4))*(3^(1/2)*1i - 1
)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*a*b*c^7*d))^(1/3))/2 - (log(((3^(1
/2)*1i + 1)*((((27*b^5*c^3*d^3*(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a
*b*c*d))/a + (81*a*b^4*c^4*d^3*(3^(1/2)*1i + 1)*(2*a^2*d^2 + b^2*c^2 - 3*a*
b*c*d)*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/2)*(3^(1/2)*1i + 1)^2*(d^5/(c^6*(a*
d - b*c)^2))^(2/3))/36 + (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d
+ 18*a^2*b*c*d^2))/(3*a^3*c))*(d^5/(c^6*(a*d - b*c)^2))^(1/3))/6 - (2*b^4*d
^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*
d^2))/(9*a^3*c^4))*(3^(1/2)*1i + 1)*(d^5/(27*b^2*c^8 + 27*a^2*c^6*d^2 - 54*
a*b*c^7*d))^(1/3))/2 + log((2*b^4*d^6*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3
*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^2))/(9*a^3*c^4) - (((((27*b^5*c^3*d^3*
(a + b*x^3)^(1/3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a - 27*a*b^4*c^4*d^3*(
(3^(1/2)*1i)/2 - 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3
/(a^5*c^6))^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2
/3))/81 - (b^5*d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^
2))/(3*a^3*c))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))
/9)*((3^(1/2)*1i)/2 - 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^2*c^2*d + 54*
a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - log((((((27*b^5*c^3*d^3*(a + b*x^3)^(1/
3)*(4*a^2*d^2 - 2*b^2*c^2 + a*b*c*d))/a + 27*a*b^4*c^4*d^3*((3^(1/2)*1i)/2
+ 1/2)*(2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/
3))*((3^(1/2)*1i)/2 - 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(2/3))/81 + (b^5*
d^4*(8*b^3*c^3 - 27*a^3*d^3 + 28*a*b^2*c^2*d + 18*a^2*b*c*d^2))/(3*a^3*c))*
((3^(1/2)*1i)/2 + 1/2)*(-(3*a*d + 2*b*c)^3/(a^5*c^6))^(1/3))/9 - (2*b^4*d^6
*(a + b*x^3)^(1/3)*(27*a^3*d^3 + 4*b^3*c^3 + 18*a*b^2*c^2*d + 36*a^2*b*c*d^
2))/(9*a^3*c^4))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 8*b^3*c^3 + 36*a*b^
2*c^2*d + 54*a^2*b*c*d^2)/(729*a^5*c^6))^(1/3) - (a + b*x^3)^(1/3)/(3*a*c*x
^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.738 $\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=279

$$\frac{(2ad+3bc)\log\left(\sqrt[3]{b}x-\sqrt[3]{a+bx^3}\right)}{6b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2}+\frac{c^{5/3}\log\left(c+dx^3\right)}{6d^2(bc-ad)^{2/3}}-\frac{c^{5/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+\right.}{2d^2(bc-ad)^{2/3}}$$

```
[Out] 1/3*x^2*(b*x^3+a)^(1/3)/b/d+1/6*c^(5/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(2/3)+1/
6*(2*a*d+3*b*c)*ln(b^(1/3)*x-(b*x^3+a)^(1/3))/b^(5/3)/d^2-1/2*c^(5/3)*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(2/3)+1/9*(2*a*d+3
*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(5/3)/d^2*3^(1/
2)-1/3*c^(5/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*
3^(1/2))/d^2/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 400, normalized size of antiderivative = 1.43, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {494, 470, 584, 292, 31, 634, 617, 204, 628}

$$\frac{(2ad+3bc)\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2}-\frac{(2ad+3bc)\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)}{18b^{5/3}d^2}+\frac{(2ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2}-\frac{c^{5/3}\log\left(\right.}{3d^2(}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(a + b*x^3)^(1/3))/(3*b*d) + ((3*b*c + 2*a*d)*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(5/3)*d^2) - (c^(5/3)*ArcTan[(c
^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sq
rt[3]*d^2*(b*c - a*d)^(2/3)) + ((3*b*c + 2*a*d)*Log[1 - (b^(1/3)*x)/(a + b*
x^3)^(1/3)])/(9*b^(5/3)*d^2) - ((3*b*c + 2*a*d)*Log[1 + (b^(2/3)*x^2)/(a +
b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*b^(5/3)*d^2) - (c^(5/3)*
Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d^2*(b*c - a*d)^
(2/3)) + (c^(5/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
 (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*d^2*(b*c - a*d)^(2/3)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a^2\,\text{Subst}\left(\int \frac{x^7}{\left(1-bx^3\right)^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \frac{x\left(2c+(bc+2ad)x^3\right)}{\left(1-bx^3\right)\left(c+(-bc+ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{a\,\text{Subst}\left(\int \left(\frac{(3bc+2ad)x}{ad\left(1-bx^3\right)} + \frac{3bc^2x}{ad\left(-c+(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3bd}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^2\,\text{Subst}\left(\int \frac{x}{-c+(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d^2} - \frac{(3bc+2ad)\,\text{Subst}\left(\int \frac{x}{1-bx^3}\right.}{3bd^2}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} - \frac{c^{5/3}\,\text{Subst}\left(\int \frac{1}{-\sqrt[3]{c}+\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3d^2\sqrt[3]{bc-ad}} + \frac{c^{5/3}\,\text{Subst}\left(\int \frac{-\sqrt[3]{}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{}}\right.}{3d}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{c^{5/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d^2(bc-ad)^{2/3}} + \frac{c^{5/3}\,\text{Sub}}{}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{9b^{5/3}d^2} - \frac{(3bc+2ad)\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\right.}{18b^{5/3}d^2}$$

$$= \frac{x^2\sqrt[3]{a+bx^3}}{3bd} + \frac{(3bc+2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{5/3}d^2} - \frac{c^{5/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^2(bc-ad)^{2/3}} + \frac{(3bc+2}{}$$

Mathematica [C] time = 0.29, size = 190, normalized size = 0.68

$$\frac{5cx^2\left(\left(a+bx^3\right)\left(\frac{dx^3}{c}+1\right)^{2/3} - a\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)\right) - x^5\left(\frac{bx^3}{a}+1\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}(2ad+3bc)F_1\left(\frac{5}{3}\right.}{15bcd\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-((3*b*c + 2*a*d)*x^5*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(2/3)*AppellF1
[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]) + 5*c*x^2*((a + b*x^3)*(1 +
 (d*x^3)/c)^(2/3) - a*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))]))/(15*b*c*d*(a + b*x^3)^(2/3)*(1 +
(d*x^3)/c)^(2/3))
```

fricas [B] time = 1.14, size = 558, normalized size = 2.00

$$\frac{6\sqrt{3}\,b^3c\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(-\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right) + 6\left(bx^3+a\right)^{\frac{1}{3}}b^2dx^2 + 6\,b^3c\left(-\frac{}{b^2c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*b^3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-
1/3*(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a
^2*d^2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 6*(b*x^3 + a)^(1/3)*b^2*d*x^2 + 6*b^
3*c*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b*c - a*d)*(-c^2/(b^
2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x + (b*x^3 + a)^(1/3)*c)/x) - 3*b^3*c*(
-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*log(((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(2/3)*x^2 + (b*x^3 + a)^(2/3)*
c^2 - (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(-c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(1/3)*x)/x^2) - 2*sqrt(3)*(3*b^2*c + 2*a*b*d)*(b^2)^(1/6)*arctan(1/3*(s
qrt(3)*(b^2)^(1/3)*b*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(b^2)^(2/3))*(b^2)^(1/
6)/(b^2*x)) + 2*(b^2)^(2/3)*(3*b*c + 2*a*d)*log(-((b^2)^(2/3)*x - (b*x^3 +
a)^(1/3)*b)/x) - (b^2)^(2/3)*(3*b*c + 2*a*d)*log(((b^2)^(1/3)*b*x^2 + (b*x^
3 + a)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a)^(2/3)*b)/x^2))/(b^3*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b\,x^3 + a\right)^{\frac{2}{3}}\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^7/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^7/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.739 $\int \frac{x^4}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=234

$$-\frac{\log\left(\sqrt[3]{b}\,x-\sqrt[3]{a+bx^3}\right)}{2b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}-\frac{c^{2/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{2/3}}+\frac{c^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{2/3}}+\frac{c^{2/3}\tan^{-1}\left(\ldots\right)}{\sqrt{3}\,d(bc-\ldots}$$

```
[Out] -1/6*c^(2/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(2/3)-1/2*ln(b^(1/3)*x-(b*x^3+a)^(1/3
))/b^(2/3)/d+1/2*c^(2/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(
-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(
2/3)/d*3^(1/2)+1/3*c^(2/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^
3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 346, normalized size of antiderivative = 1.48, number of steps used = 14, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 481, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d}+\frac{\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1\right)}{6b^{2/3}d}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d}+\frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}}-\frac{c^{2/3}\log\left(\frac{x^2(bc-\ldots}{(a+b\ldots}\right)}{6\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(2/3)*d)
) + (c^(2/3)*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(
Sqrt[3]*c^(1/3))])/(Sqrt[3]*d*(b*c - a*d)^(2/3)) - Log[1 - (b^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(3*b^(2/3)*d) + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b
^(1/3)*x)/(a + b*x^3)^(1/3)]/(6*b^(2/3)*d) + (c^(2/3)*Log[c^(1/3) - ((b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*d*(b*c - a*d)^(2/3)) - (c^(2/3)*Log[c
^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(
1/3)*x)/(a + b*x^3)^(1/3)])/(6*d*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = a\,\text{Subst}\left(\int \frac{x^4}{\left(1-bx^3\right)\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x}{1-bx^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d} - \frac{c\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt[3]{b}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} - \frac{\text{Subst}\left(\int \frac{1-\sqrt[3]{b}\,x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}\,d} - \frac{c^{2/3}\,\text{Su}}{}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{b}+2b^{2/3}x}{1+\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x,\right.}{6b^{2/3}d}$$

$$= -\frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{6b^{2/3}d} + \frac{c^{2/3}\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3d(bc-ad)^{2/3}} -$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d} + \frac{c^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,d(bc-ad)^{2/3}} - \frac{\log\left(1-\frac{\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}\right)}{3b^{2/3}d} + \frac{\log\left(1+\right.}{}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.28

$$\frac{x^5\left(\frac{a+bx^3}{a}\right)^{2/3}F_1\left(\frac{5}{3};\frac{2}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5c\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*((a + b*x^3)/a)^(2/3)*AppellF1[5/3, 2/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*c*(a + b*x^3)^(2/3))
```

fricas [B] time = 0.46, size = 530, normalized size = 2.26

$$2\sqrt{3}\,b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(bc-ad)\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{2}{3}}+\sqrt{3}\,cx}{3\,cx}\right)+2\,b^2\left(\frac{c^2}{b^2c^2-2\,abcd+a^2d^2}\right)^{\frac{1}{3}}\log\left(-\frac{(bc-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*arctan(-1/3*
(2*sqrt(3)*(b*x^3 + a)^(1/3)*(b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^
2))^(2/3) + sqrt(3)*c*x)/(c*x)) + 2*b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2
))^(1/3)*log(-((b*c - a*d)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x -
(b*x^3 + a)^(1/3)*c)/x) - b^2*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*l
og(((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(
2/3)*x^2 + (b*x^3 + a)^(2/3)*c^2 + (b*x^3 + a)^(1/3)*(b*c^2 - a*c*d)*(c^2/(
b^2*c^2 - 2*a*b*c*d + a^2*d^2))^(1/3)*x)/x^2) + 2*sqrt(3)*b*sqrt(-(-b^2)^(1
/3))*arctan(-1/3*(sqrt(3)*(-b^2)^(1/3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-
```

```
b^2)^(2/3))*sqrt(-(-b^2)^(1/3))/(b^2*x)) - 2*(-b^2)^(2/3)*log(-((-b^2)^(2/3
)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(2/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*
x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2))/(b^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^4/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^4/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**4/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.740 $\int \frac{x}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=149

$$\frac{\log\left(c+dx^3\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

```
[Out] 1/6*ln(d*x^3+c)/c^(1/3)/(-a*d+b*c)^(2/3)-1/2*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-
(b*x^3+a)^(1/3))/c^(1/3)/(-a*d+b*c)^(2/3)-1/3*arctan(1/3*(1+2*(-a*d+b*c)^(1
/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(1/3)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 208, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {494, 292, 31, 634, 617, 204, 628}

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\,(bc-ad)^{2/3}}-\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[3]{c}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -(ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(
1/3))]/(Sqrt[3]*c^(1/3)*(b*c - a*d)^(2/3))) - Log[c^(1/3) - ((b*c - a*d)^(1
/3)*x)/(a + b*x^3)^(1/3)]/(3*c^(1/3)*(b*c - a*d)^(2/3)) + Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a +
 b*x^3)^(1/3)]/(6*c^(1/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
```

```
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{x}{(a+bx^3)^{2/3}(c+dx^3)}\,dx = \text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \sqrt[3]{\ldots}\right.}{3\sqrt[3]{c}\sqrt[3]{bc-ad}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} - \text{Subs}\ldots$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx\ldots\right.}{\sqrt[3]{c}(bc\ldots}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{c}(bc-ad)^{2/3}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{c}(bc-ad)^{2/3}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\ldots\right.}{6\sqrt[3]{c}(bc-ad)^{2/3}}$$

Mathematica [C] time = 0.04, size = 83, normalized size = 0.56

$$\frac{x^2\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{2c\left(a+bx^3\right)^{2/3}\left(\frac{dx^3}{c}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[2/3, 2/3, 5/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(2*c*(a + b*x^3)^(2/3)*(1 + (d*x^3)/c)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3 + a\right)^{\frac{2}{3}}\left(d\,x^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(x/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + bx^{3}\right)^{\frac{2}{3}}\left(c + dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.741 $\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=173

$$-\frac{d\log\left(c+dx^3\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

```
[Out] -(b*x^3+a)^(1/3)/a/c/x-1/6*d*ln(d*x^3+c)/c^(4/3)/(-a*d+b*c)^(2/3)+1/2*d*ln(
(-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(4/3)/(-a*d+b*c)^(2/3)+1/3*d*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(4/3
)/(-a*d+b*c)^(2/3)*3^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 232, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 453, 292, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{4/3}(bc-ad)^{2/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{acx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((a + b*x^3)^(1/3)/(a*c*x)) + (d*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)
/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(4/3)*(b*c - a*d)^(2/3))
 + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(4/3)*(b
*c - a*d)^(2/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/
3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(4/3)*(b*c - a*
d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx = \frac{\text{Subst}\left(\int \frac{1-bx^3}{x^2\left(c-(bc-ad)x^3\right)}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{x}{c+(-bc+ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}\sqrt[3]{bc-ad}} + \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{b}}\right.}{3c^{4}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}x^2}\right.}{6c^{4/3}(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{6c^{4/3}(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx^3}}{acx} + \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\,\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{4/3}(bc-ad)^{2/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}(bc-ad)^{2/3}} - \frac{d\log\left(c^{2/3}+\right.}{6c}$$

Mathematica [C] time = 0.06, size = 128, normalized size = 0.74

$$-\frac{\frac{6x^3(c+dx^3)(bc-ad)\,_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{a+bx^3}+5c\left(2c+3dx^3\right)\,_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{10c^3x\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/10*(5*c*(2*c + 3*d*x^3)*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + (6*(b*c - a*d)*x^3*(c + d*x^3)*Hypergeometric2F1[5/3, 2
, 8/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(a + b*x^3))/(c^3*x*(a + b*x^3)^
(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{\frac{2}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.742 $\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{a+bx^3}\,(4ad+3bc)}{4a^2c^2x}+\frac{d^2\log\left(c+dx^3\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}}-\frac{\sqrt[3]{a+bx^3}}{4acx^4}$$

```
[Out] -1/4*(b*x^3+a)^(1/3)/a/c/x^4+1/4*(4*a*d+3*b*c)*(b*x^3+a)^(1/3)/a^2/c^2/x+1/
6*d^2*ln(d*x^3+c)/c^(7/3)/(-a*d+b*c)^(2/3)-1/2*d^2*ln((-a*d+b*c)^(1/3)*x/c^
(1/3)-(b*x^3+a)^(1/3))/c^(7/3)/(-a*d+b*c)^(2/3)-1/3*d^2*arctan(1/3*(1+2*(-a
*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(7/3)/(-a*d+b*c)^(2/3)*
3^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 269, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {494, 461, 292, 31, 634, 617, 204, 628}

$$\frac{\sqrt[3]{a+bx^3}\,(ad+bc)}{a^2c^2x}-\frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4}-\frac{d^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}}+\frac{d^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{7/3}(bc-ad)^{2/3}}-\frac{d^2\tan^{-1}\left(\frac{2x\sqrt[3]{\ldots}}{\sqrt[3]{a\ldots}}\ldots\right)}{\sqrt{3}\,c^{7/3}(bc\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] ((b*c + a*d)*(a + b*x^3)^(1/3))/(a^2*c^2*x) - (a + b*x^3)^(4/3)/(4*a^2*c*x^
4) - (d^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqr
t[3]*c^(1/3))])/(Sqrt[3]*c^(7/3)*(b*c - a*d)^(2/3)) - (d^2*Log[c^(1/3) - ((
b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*c^(7/3)*(b*c - a*d)^(2/3)) + (d^
2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(7/3)*(b*c - a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

```
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 494

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)
, x_Symbol] :> With[{k = Denominator[p]}, Dist[(k*a^(p + (m + 1)/n))/n, Sub
st[Int[(x^((k*(m + 1))/n - 1)*(c - (b*c - a*d)*x^k)^q)/(1 - b*x^k)^(p + q +
 (m + 1)/n + 1), x], x, x^(n/k)/(a + b*x^n)^(1/k)], x]] /; FreeQ[{a, b, c,
d}, x] && IGtQ[n, 0] && RationalQ[m, p] && IntegersQ[p + (m + 1)/n, q] && L
tQ[-1, p, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}dx = \frac{\text{Subst}\left(\int \frac{\left(1-bx^3\right)^2}{x^5\left(c-(bc-ad)x^3\right)}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{cx^5}+\frac{-bc-ad}{c^2x^2}+\frac{a^2d^2x}{c^2\left(c-(bc-ad)x^3\right)}\right)dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{a^2}$$
$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{x}{c-(bc-ad)x^3}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{c^2}$$
$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}\sqrt[3]{bc-ad}} - \cdots$$
$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$
$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{7/3}(bc-ad)^{2/3}} + \frac{d^2\log\left(c^{2/3}+\cdots\right.}{6c^{7/3}\cdots}$$
$$= \frac{(bc+ad)\sqrt[3]{a+bx^3}}{a^2c^2x} - \frac{\left(a+bx^3\right)^{4/3}}{4a^2cx^4} - \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{7/3}(bc-ad)^{2/3}} - \frac{d^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc\cdots}}{\sqrt[3]{a\cdots}}\right.}{3c^{7/3}(bc-ad)\cdots}$$

Mathematica [C] time = 1.89, size = 267, normalized size = 1.24

$$-\frac{-81x^3\left(c+dx^3\right)^2(bc-ad)\,_4F_3\left(\frac{2}{3},2,2,2;1,1,\frac{8}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+216dx^6\left(c+dx^3\right)(ad-bc)\,_3F_2\left(\frac{2}{3},2,2;1,\frac{8}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -1/120*(-5*(2*c*(a + b*x^3)*(c^2 + 10*c*d*x^3 + 9*d^2*x^6) + (3*b*c*x^3*(-3
*c^2 + 2*c*d*x^3 + 9*d^2*x^6) + a*(-8*c^3 + 17*c^2*d*x^3 + 46*c*d^2*x^6 + 9
*d^3*x^9))*Hypergeometric2F1[2/3, 1, 5/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
]) + 216*d*(-(b*c) + a*d)*x^6*(c + d*x^3)*HypergeometricPFQ[{2/3, 2, 2}, {1
, 8/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*(b*c - a*d)*x^3*(c + d*x^3)
^2*HypergeometricPFQ[{2/3, 2, 2, 2}, {1, 1, 8/3}, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))])/(c^4*x^4*(a + b*x^3)^(5/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}} \left(d\,x^{3}+c\right) x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
[Out] int(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5}\,\left(b\,x^{3}+a\right)^{2/3}\,\left(d\,x^{3}+c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^5*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**5*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.743 $\int \frac{x^6}{(a+bx^3)^{2/3}(c+dx^3)} dx$

Optimal. Leaf size=64

$$\frac{x^7 \left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/7*x^7*(1+b*x^3/a)^(2/3)*AppellF1(7/3,2/3,1,10/3,-b*x^3/a,-d*x^3/c)/c/(b*x
^3+a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7 \left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(2/3)*AppellF1[7/3, 2/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*c*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)} dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3} \int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)} dx}{\left(a+bx^3\right)^{2/3}} \\
&= \frac{x^7\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{7}{3};\frac{2}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7c\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [B] time = 0.38, size = 249, normalized size = 3.89

$$\frac{x\left(4\left(\frac{4a^2c^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{b(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{a}{b}+x^3\right)-\frac{x^3\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+2}{}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-(((2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)])/(b*c)) + 4*(a/b + x^3 + (4*a^2*c^2*AppellF1[1/3
, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(b*(c + d*x^3)*(-4*a*c*AppellF1
[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2
/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]))))))/(8*d*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^6}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.744 $\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

```
[Out] 1/4*x^4*(1+b*x^3/a)^(2/3)*AppellF1(4/3,2/3,1,7/3,-b*x^3/a,-d*x^3/c)/c/(b*x^
3+a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = $\{511, 510\}$

$$\frac{x^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x^4\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{x^4\left(\frac{a+bx^3}{a}\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*((a + b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(4*c*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(x^3/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**3/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.745 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

```
[Out] x*(1+b*x^3/a)^(2/3)*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^(
2/3)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.04, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{2}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{2}{3}}\left(dx^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,x^3+a\right)}^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)),x)
```

```
[Out] int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.746 $\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=64

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] -1/2*(1+b*x^3/a)^(2/3)*AppellF1(-2/3,2/3,1,1/3,-b*x^3/a,-d*x^3/c)/c/x^2/(b*
x^3+a)^(2/3)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(2/3)*AppellF1[-2/3, 2/3, 1, 1/3, -((b*x^3)/a), -((d*x^3)
/c)])/(2*c*x^2*(a + b*x^3)^(2/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= -\frac{\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(-\frac{2}{3};\frac{2}{3},1;\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2cx^2\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.33, size = 338, normalized size = 5.28

$$\frac{\frac{4c\left(x^3\left(a+bx^3\right)\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(ac+3adx^3+2bcx^3+bdx^6\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}-bdx^6\left(\frac{b}{}\right.}{8ac^2x^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-(b*d*x^6*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)]) + (4*c*(-4*a*c*(a*c + 2*b*c*x^3 + 3*a*d*x^3 + b*d*x^6)*Appell
F1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(a + b*x^3)*(c + d*x
^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*A
ppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*
c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*Appel
lF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3
, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(8*a*c^2*x^2*(a + b*x^3)^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{2}{3}}\left(d\,x^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\left(b\,x^3+a\right)^{2/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(2/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)^{\frac{2}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(2/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

3.747 $\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=347

$$-\frac{a^4}{b^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right)^{5/3}}{5b^4 \ldots}$$

```
[Out] -a^4/b^4/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*a^2*(b*x^3+a)^(2/3)/b^4/d+1/2*a*(a*
d+b*c)*(b*x^3+a)^(2/3)/b^4/d^2+1/2*(a^2*d^2+a*b*c*d+b^2*c^2)*(b*x^3+a)^(2/3
)/b^4/d^3-2/5*a*(b*x^3+a)^(5/3)/b^4/d-1/5*(a*d+b*c)*(b*x^3+a)^(5/3)/b^4/d^2
+1/8*(b*x^3+a)^(8/3)/b^4/d-1/6*c^4*ln(d*x^3+c)/d^(11/3)/(-a*d+b*c)^(4/3)+1/
2*c^4*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(11/3)/(-a*d+b*c)^(4/3
)+1/3*c^4*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2)
)/d^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{\left(a+bx^3\right)^{2/3}\left(a^2d^2+abcd+b^2c^2\right)}{2b^4d^3}-\frac{a^4}{b^4\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d}+\frac{a\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^4d^2}-\frac{\left(a+bx^3\right)^{5/3}(\ldots}{5b^4d^2 \ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^4/(b^4*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a^2*(a + b*x^3)^(2/3))/(2*b^4
*d) + (a*(b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^4*d^2) + ((b^2*c^2 + a*b*c*d +
 a^2*d^2)*(a + b*x^3)^(2/3))/(2*b^4*d^3) - (2*a*(a + b*x^3)^(5/3))/(5*b^4*d
) - ((b*c + a*d)*(a + b*x^3)^(5/3))/(5*b^4*d^2) + (a + b*x^3)^(8/3)/(8*b^4*
d) + (c^4*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt
[3]])/(Sqrt[3]*d^(11/3)*(b*c - a*d)^(4/3)) - (c^4*Log[c + d*x^3])/(6*d^(11/
3)*(b*c - a*d)^(4/3)) + (c^4*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1
/3)])/(2*d^(11/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
```

```
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{a^4}{b^3(bc-ad)(a+bx)^{4/3}} + \frac{b^2c^2+abcd+a^2d^2}{b^3d^3\sqrt[3]{a+bx}} - \frac{(bc+ad)x}{b^2d^2\sqrt[3]{a+bx}} + \frac{}{bc}\right.\right.$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{\sqrt[3]{a+bx}}\right.}{3bd}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(b^2c^2+abcd+a^2d^2\right)\left(a+bx^3\right)^{2/3}}{2b^4d^3} - \frac{c^4\log\left(c+dx^3\right)}{6d^{11/3}(bc-ad)^{4/3}}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+ab\right.}{}$$

$$= -\frac{a^4}{b^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{a^2\left(a+bx^3\right)^{2/3}}{2b^4d} + \frac{a(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^4d^2} + \frac{\left(b^2c^2+ab\right.}{}$$

Mathematica [C] time = 0.27, size = 157, normalized size = 0.45

$$\frac{\frac{81a^3d^3+9a^2bd^2\left(8c+3dx^3\right)+3ab^2d\left(20c^2+8cdx^3-3d^2x^6\right)+b^3\left(40c^3+20c^2dx^3-8cd^2x^6+5d^3x^9\right)}{b^4}-\frac{40c^4\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d(bx^3+a)}{ad-bc}\right)}{bc-ad}}{40d^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((81*a^3*d^3 + 9*a^2*b*d^2*(8*c + 3*d*x^3) + 3*a*b^2*d*(20*c^2 + 8*c*d*x^3
- 3*d^2*x^6) + b^3*(40*c^3 + 20*c^2*d*x^3 - 8*c*d^2*x^6 + 5*d^3*x^9))/b^4 -
 (40*c^4*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(
b*c - a*d))/(40*d^4*(a + b*x^3)^(1/3))
```

fricas [B] time = 0.97, size = 1300, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/120*(60*sqrt(1/3)*(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c
^4*d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 -
b*c*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3)
 + (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d)
)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(
b*x^3 + a)^(1/3))/(d*x^3 + c)) + 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a
)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*
d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*
(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5
+ 81*a^5*d^6 + 5*(b^5*c^2*d^4 - 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c
^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*
c^4*d^2 - 16*a*b^4*c^3*d^3 - a^2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*
d^6)*x^3)*(b*x^3 + a)^(2/3))/(a*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7
 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^3), -1/120*(120*sqrt(1/3)*
(a*b^5*c^5*d - a^2*b^4*c^4*d^2 + (b^6*c^5*d - a*b^5*c^4*d^2)*x^3)*sqrt(-(-b
*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d
+ (-b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d)
+ 20*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/
3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^
(2/3)) - 40*(b^5*c^4*x^3 + a*b^4*c^4)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 +
 a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(20*a*b^4*c^4*d^2 - 8*a^2*b^3*c
^3*d^3 - 3*a^3*b^2*c^2*d^4 - 90*a^4*b*c*d^5 + 81*a^5*d^6 + 5*(b^5*c^2*d^4 -
 2*a*b^4*c*d^5 + a^2*b^3*d^6)*x^9 - (8*b^5*c^3*d^3 - 7*a*b^4*c^2*d^4 - 10*a
^2*b^3*c*d^5 + 9*a^3*b^2*d^6)*x^6 + (20*b^5*c^4*d^2 - 16*a*b^4*c^3*d^3 - a^
2*b^3*c^2*d^4 - 30*a^3*b^2*c*d^5 + 27*a^4*b*d^6)*x^3)*(b*x^3 + a)^(2/3))/(a
*b^6*c^2*d^5 - 2*a^2*b^5*c*d^6 + a^3*b^4*d^7 + (b^7*c^2*d^5 - 2*a*b^6*c*d^6
 + a^2*b^5*d^7)*x^3)]
```

giac [A] time = 0.26, size = 431, normalized size = 1.24

$$\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^5-2\sqrt{3}\,abcd^6+\sqrt{3}\,a^2d^7}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}c^4\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\right.}{6\left(b^2c^2d^5-2\,abcd^6+a^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^4*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^5 - 2*sqrt(3
)*a*b*c*d^6 + sqrt(3)*a^2*d^7) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^4*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7) + 1/3*c^4*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^3 - 2*a*
b*c*d^4 + a^2*d^5) - a^4/((b^5*c - a*b^4*d)*(b*x^3 + a)^(1/3)) + 1/40*(20*(
b*x^3 + a)^(2/3)*b^30*c^2*d^5 - 8*(b*x^3 + a)^(5/3)*b^29*c*d^6 + 40*(b*x^3
+ a)^(2/3)*a*b^29*c*d^6 + 5*(b*x^3 + a)^(8/3)*b^28*d^7 - 24*(b*x^3 + a)^(5/
3)*a*b^28*d^7 + 60*(b*x^3 + a)^(2/3)*a^2*b^28*d^7)/(b^32*d^8)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.19, size = 564, normalized size = 1.63

$$\left(\frac{3\,a^2}{b^4\,d}+\frac{\left(\frac{4\,a}{b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{b^8\,d^2}\right)\left(b^5\,c-a\,b^4\,d\right)}{2\,b^4\,d}\right)\left(b\,x^3+a\right)^{2/3}-\left(\frac{4\,a}{5\,b^4\,d}+\frac{b^5\,c-a\,b^4\,d}{5\,b^8\,d^2}\right)\left(b\,x^3+a\right)^{5/3}+\frac{\left(b\,x^3+a\right)^{8/3}}{8\,b^4\,d}+\frac{}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] ((3*a^2)/(b^4*d) + (((4*a)/(b^4*d) + (b^5*c - a*b^4*d)/(b^8*d^2))*(b^5*c -
a*b^4*d))/(2*b^4*d))*(a + b*x^3)^(2/3) - ((4*a)/(5*b^4*d) + (b^5*c - a*b^4*
d)/(5*b^8*d^2))*(a + b*x^3)^(5/3) + (a + b*x^3)^(8/3)/(8*b^4*d) + a^4/(b^4*
(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^4*log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*
c^9*d^4) - (c^8*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b
^2*c^2*d^13 - 36*a^3*b*c*d^14))/(9*d^(22/3)*(a*d - b*c)^(8/3))))/(3*d^(11/3
)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1/3)*(a*c^8*d^5 - b*c^9*d^4) - ((3
^(1/2)*c^4*1i + c^4)^2*(9*a^4*d^15 + 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 5
4*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14))/(36*d^(22/3)*(a*d - b*c)^(8/3)))*(3^
(1/2)*c^4*1i + c^4))/(6*d^(11/3)*(a*d - b*c)^(4/3)) + (c^4*log((a + b*x^3)^
```

```
(1/3)*(a*c^8*d^5 - b*c^9*d^4) - (c^8*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^15 +
 9*b^4*c^4*d^11 - 36*a*b^3*c^3*d^12 + 54*a^2*b^2*c^2*d^13 - 36*a^3*b*c*d^14
))/(d^(22/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/6))/(d^(11/3)*(a*d - b
*c)^(4/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**14/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.748 $\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=253

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-a\ldots}\right.}{2d^{8/3}(b\ldots}$$

```
[Out] a^3/b^3/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*a*(b*x^3+a)^(2/3)/b^3/d-1/2*(a*d+b*c
)*(b*x^3+a)^(2/3)/b^3/d^2+1/5*(b*x^3+a)^(5/3)/b^3/d+1/6*c^3*ln(d*x^3+c)/d^(
8/3)/(-a*d+b*c)^(4/3)-1/2*c^3*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/
d^(8/3)/(-a*d+b*c)^(4/3)-1/3*c^3*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-
a*d+b*c)^(1/3))*3^(1/2))/d^(8/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {446, 87, 43, 56, 617, 204, 31}

$$\frac{a^3}{b^3\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(ad+bc)}{2b^3d^2}-\frac{a\left(a+bx^3\right)^{2/3}}{2b^3d}+\frac{\left(a+bx^3\right)^{5/3}}{5b^3d}+\frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}}-\frac{c^3\log\left(\sqrt[3]{bc-a\ldots}\right.}{2d^{8/3}(b\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a^3/(b^3*(b*c - a*d)*(a + b*x^3)^(1/3)) - (a*(a + b*x^3)^(2/3))/(2*b^3*d) -
 ((b*c + a*d)*(a + b*x^3)^(2/3))/(2*b^3*d^2) + (a + b*x^3)^(5/3)/(5*b^3*d)
- (c^3*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(Sqrt[3]*d^(8/3)*(b*c - a*d)^(4/3)) + (c^3*Log[c + d*x^3])/(6*d^(8/3)*(b
*c - a*d)^(4/3)) - (c^3*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])
/(2*d^(8/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(-\frac{a^3}{b^2(bc-ad)(a+bx)^{4/3}} + \frac{-bc-ad}{b^2d^2\sqrt[3]{a+bx}} + \frac{x}{bd\sqrt[3]{a+bx}} - \frac{}{d^2(-bc+a}\right.\right.$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\text{Subst}\left(\int \frac{x}{\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3bd} + \frac{c^3\,\text{Su}}{}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{c^3\log\left(c+dx^3\right)}{6d^{8/3}(bc-ad)^{4/3}} + \frac{\text{Subst}\left(\int\left(-\frac{}{b}\right.\right.}{}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} + \frac{c}{6}$$

$$= \frac{a^3}{b^3(bc-ad)\sqrt[3]{a+bx^3}} - \frac{a\left(a+bx^3\right)^{2/3}}{2b^3d} - \frac{(bc+ad)\left(a+bx^3\right)^{2/3}}{2b^3d^2} + \frac{\left(a+bx^3\right)^{5/3}}{5b^3d} - \frac{c^3}{}$$

Mathematica [C] time = 0.11, size = 147, normalized size = 0.58

$$\frac{18a^3d^3 + 3a^2bd^2\left(2dx^3-c\right) + 10b^3c^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right) - ab^2d\left(5c^2+cdx^3+2d^2x^6\right) + b^3c\left(-10c^2-5cdx^3+\right.}{10b^3d^3\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (18*a^3*d^3 + 3*a^2*b*d^2*(-c + 2*d*x^3) + b^3*c*(-10*c^2 - 5*c*d*x^3 + 2*d
^2*x^6) - a*b^2*d*(5*c^2 + c*d*x^3 + 2*d^2*x^6) + 10*b^3*c^3*Hypergeometric
2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(10*b^3*d^3*(b*c - a*d)*
(a + b*x^3)^(1/3))
```

fricas [B] time = 1.14, size = 1141, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^
3*d^2)*x^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b
*c*d + 3*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
 (b*x^3 + a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*s
qrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^
3 + a)^(1/3))/(d*x^3 + c)) - 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)
*d + (b*c*d^2 - a*d^3)^(2/3)) + 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d
^3)^(2/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 3*(5*a*b^3*c
^3*d^2 - 2*a^2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 -
 2*a*b^3*c*d^4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^
2*b^2*c*d^4 + 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b
^4*c*d^5 + a^3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3),
1/30*(30*sqrt(1/3)*(a*b^4*c^4*d - a^2*b^3*c^3*d^2 + (b^5*c^4*d - a*b^4*c^3*
d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*
x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b
*c - a*d))/d) + 5*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log((b*
x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2
 - a*d^3)^(2/3)) - 10*(b^4*c^3*x^3 + a*b^3*c^3)*(b*c*d^2 - a*d^3)^(2/3)*log
((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 3*(5*a*b^3*c^3*d^2 - 2*a^
2*b^2*c^2*d^3 - 21*a^3*b*c*d^4 + 18*a^4*d^5 - 2*(b^4*c^2*d^3 - 2*a*b^3*c*d^
4 + a^2*b^2*d^5)*x^6 + (5*b^4*c^3*d^2 - 4*a*b^3*c^2*d^3 - 7*a^2*b^2*c*d^4 +
 6*a^3*b*d^5)*x^3)*(b*x^3 + a)^(2/3))/(a*b^5*c^2*d^4 - 2*a^2*b^4*c*d^5 + a^
3*b^3*d^6 + (b^6*c^2*d^4 - 2*a*b^5*c*d^5 + a^2*b^4*d^6)*x^3)]
```

giac [A] time = 0.26, size = 372, normalized size = 1.47

$$-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c^{2}d^{4}-2\sqrt{3}\,abcd^{5}+\sqrt{3}\,a^{2}d^{6}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{3}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)\right)}{6\left(b^{2}c^{2}d^{4}-2\,abcd^{5}+a^{2}d^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -(-b*c*d^2 + a*d^3)^(2/3)*c^3*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(
b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^4 - 2*sqrt(
3)*a*b*c*d^5 + sqrt(3)*a^2*d^6) + 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^3*log((b*x
^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)
^(2/3))/(b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6) - 1/3*c^3*(-(b*c - a*d)/d)^(2
/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d^2 - 2*a
```

```
*b*c*d^3 + a^2*d^4) + a^3/((b^4*c - a*b^3*d)*(b*x^3 + a)^(1/3)) - 1/10*(5*(
b*x^3 + a)^(2/3)*b^13*c*d^3 - 2*(b*x^3 + a)^(5/3)*b^12*d^4 + 10*(b*x^3 + a)
^(2/3)*a*b^12*d^4)/(b^15*d^5)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.16, size = 493, normalized size = 1.95

$$\frac{\left(b\,x^3+a\right)^{5/3}}{5\,b^3\,d}-\left(\frac{3\,a}{2\,b^3\,d}+\frac{b^4\,c-a\,b^3\,d}{2\,b^6\,d^2}\right)\,{\left(b\,x^3+a\right)}^{2/3}-\frac{a^3}{b^3\,{\left(b\,x^3+a\right)}^{1/3}\,\left(a\,d-b\,c\right)}-\frac{c^3\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^6\,d^4-b\,c^7\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(5/3)/(5*b^3*d) - ((3*a)/(2*b^3*d) + (b^4*c - a*b^3*d)/(2*b^6*d
^2))*(a + b*x^3)^(2/3) - a^3/(b^3*(a + b*x^3)^(1/3)*(a*d - b*c)) - (c^3*log
((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*(9*a^4*d^12 + 9*b^4*c^4*d
^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)
*(a*d - b*c)^(8/3))))/(3*d^(8/3)*(a*d - b*c)^(4/3)) + (log((a + b*x^3)^(1/3
)*(a*c^6*d^4 - b*c^7*d^3) - ((3^(1/2)*c^3*1i + c^3)^2*(9*a^4*d^12 + 9*b^4*c
^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*c^2*d^10 - 36*a^3*b*c*d^11))/(36*d^(
16/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^3*1i + c^3))/(6*d^(8/3)*(a*d - b*c)^(4
/3)) - (c^3*log((a + b*x^3)^(1/3)*(a*c^6*d^4 - b*c^7*d^3) - (c^6*((3^(1/2)*
1i)/2 - 1/2)^2*(9*a^4*d^12 + 9*b^4*c^4*d^8 - 36*a*b^3*c^3*d^9 + 54*a^2*b^2*
c^2*d^10 - 36*a^3*b*c*d^11))/(9*d^(16/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/
2 - 1/2))/(3*d^(8/3)*(a*d - b*c)^(4/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**11/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.749 $\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=203

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/2*(b*x^3+a)^(2/3)/b^2/d-1/6*c^2*ln(d*
x^3+c)/d^(5/3)/(-a*d+b*c)^(4/3)+1/2*c^2*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+
a)^(1/3))/d^(5/3)/(-a*d+b*c)^(4/3)+1/3*c^2*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a
)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^(5/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 87, 56, 617, 204, 31}

$$-\frac{a^2}{b^2\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\left(a+bx^3\right)^{2/3}}{2b^2d}-\frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{5/3}(bc-ad)^{4/3}}+\frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*(a + b*x^3)^(1/3))) + (a + b*x^3)^(2/3)/(2*b^2*d) +
(c^2*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])
/(Sqrt[3]*d^(5/3)*(b*c - a*d)^(4/3)) - (c^2*Log[c + d*x^3])/(6*d^(5/3)*(b*c
 - a*d)^(4/3)) + (c^2*Log[(b*c - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(
2*d^(5/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 87

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(
x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d
*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^{4/3}} + \frac{1}{bd\sqrt[3]{a+bx}} + \frac{c^2}{d(-bc+ad)\sqrt[3]{a+bx}\,(c+dx)}\right.\right.$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3d(bc-ad)}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,\right.}{2d^{5/3}(bc-}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/3}} + \frac{c^2\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{\right.}}{2d^{5/3}(bc-a}$$

$$= -\frac{a^2}{b^2(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\left(a+bx^3\right)^{2/3}}{2b^2d} + \frac{c^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{5/3}(bc-ad)^{4/3}} - \frac{c^2\log\left(c+dx^3\right)}{6d^{5/3}(bc-ad)^{4/}}$$

Mathematica [C] time = 0.07, size = 101, normalized size = 0.50

$$\frac{-3a^2d^2-2b^2c^2\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+abd\left(c-dx^3\right)+b^2c\left(2c+dx^3\right)}{2b^2d^2\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-3*a^2*d^2 + a*b*d*(c - d*x^3) + b^2*c*(2*c + d*x^3) - 2*b^2*c^2*Hypergeom
etric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)])/(2*b^2*d^2*(b*c - a
*d)*(a + b*x^3)^(1/3))
```

fricas [B] time = 0.99, size = 1004, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*
d^2)*x^3)*sqrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c
*d + 3*a*d^2 + 3*sqrt(1/3)*(2*(-b*c*d^2 + a*d^3)^(2/3)*(b*x^3 + a)^(2/3) +
(b*x^3 + a)^(1/3)*(b*c*d - a*d^2) + (-b*c*d^2 + a*d^3)^(1/3)*(b*c - a*d))*s
qrt((-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d)) - 3*(-b*c*d^2 + a*d^3)^(2/3)*(b*x
^3 + a)^(1/3))/(d*x^3 + c)) + (b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^
(2/3)*log((b*x^3 + a)^(2/3)*d^2 + (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (-b*c*d^2 + a*d^3)^(2/3)) - 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a
*d^3)^(2/3)*log((b*x^3 + a)^(1/3)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*
c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*
d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5
 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4 + a^2*b^3*d^5)*x^3), -1/6*(6*sqrt(1/3)*(a*b
^3*c^3*d - a^2*b^2*c^2*d^2 + (b^4*c^3*d - a*b^3*c^2*d^2)*x^3)*sqrt(-(-b*c*d
^2 + a*d^3)^(1/3)/(b*c - a*d))*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d + (-
b*c*d^2 + a*d^3)^(1/3))*sqrt(-(-b*c*d^2 + a*d^3)^(1/3)/(b*c - a*d))/d) + (b
^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3)*d^2
+ (-b*c*d^2 + a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (-b*c*d^2 + a*d^3)^(2/3))
- 2*(b^3*c^2*x^3 + a*b^2*c^2)*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(1/3
)*d - (-b*c*d^2 + a*d^3)^(1/3)) - 3*(a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 3*a^3*
d^4 + (b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^3)*(b*x^3 + a)^(2/3))/(a*
b^4*c^2*d^3 - 2*a^2*b^3*c*d^4 + a^3*b^2*d^5 + (b^5*c^2*d^3 - 2*a*b^4*c*d^4
+ a^2*b^3*d^5)*x^3)]
```

giac [A] time = 0.25, size = 325, normalized size = 1.60

$$\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{2}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c^{2}d^{3}-2\,\sqrt{3}\,abcd^{4}+\sqrt{3}\,a^{2}d^{5}}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}c^{2}\log\left(\left(bx^{3}+a\right)^{\frac{2}{3}}+\left(bx^{3}+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right.}{6\left(b^{2}c^{2}d^{3}-2\,abcd^{4}+a^{2}d^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] (-b*c*d^2 + a*d^3)^(2/3)*c^2*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b
*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^3 - 2*sqrt(3
)*a*b*c*d^4 + sqrt(3)*a^2*d^5) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*c^2*log((b*x^
3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^
(2/3))/(b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5) + 1/3*c^2*(-(b*c - a*d)/d)^(2/
3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3) - a^2/((b^3*c - a*b^2*d)*(b*x^3 + a)^(1/3)) + 1/2*(b*x^3 +
 a)^(2/3)/(b^2*d)
```

maple [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.99, size = 449, normalized size = 2.21

$$\frac{\left(b\,x^3+a\right)^{2/3}}{2\,b^2\,d}+\frac{a^2}{b^2\,\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}+\frac{c^2\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^4\,d^3-b\,c^5\,d^2\right)-\frac{c^4\,\left(9\,a^4\,d^9-36\,a^3\,b\,c\,d^8+54\,a^2\,b^2\,c^2\,d^7-36\ldots}{9\,d^{10/3}\,\left(a\,d-b\,c\right)^{8/3}}\right.}{3\,d^{5/3}\,\left(a\,d-b\,c\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (a + b*x^3)^(2/3)/(2*b^2*d) + a^2/(b^2*(a + b*x^3)^(1/3)*(a*d - b*c)) + (c^
2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*(9*a^4*d^9 + 9*b^4*c
^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(9*d^(10/
3)*(a*d - b*c)^(8/3))))/(3*d^(5/3)*(a*d - b*c)^(4/3)) - (log((a + b*x^3)^(1
/3)*(a*c^4*d^3 - b*c^5*d^2) - ((3^(1/2)*c^2*1i + c^2)^2*(9*a^4*d^9 + 9*b^4*
c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^2*d^7 - 36*a^3*b*c*d^8))/(36*d^(1
0/3)*(a*d - b*c)^(8/3)))*(3^(1/2)*c^2*1i + c^2))/(6*d^(5/3)*(a*d - b*c)^(4/
3)) + (c^2*log((a + b*x^3)^(1/3)*(a*c^4*d^3 - b*c^5*d^2) - (c^4*((3^(1/2)*1
i)/6 - 1/6)^2*(9*a^4*d^9 + 9*b^4*c^4*d^5 - 36*a*b^3*c^3*d^6 + 54*a^2*b^2*c^
2*d^7 - 36*a^3*b*c*d^8))/(d^(10/3)*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/6 - 1/
6))/(d^(5/3)*(a*d - b*c)^(4/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(a + bx^{3}\right)^{\frac{4}{3}} \left(c + dx^{3}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**8/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.750 $\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=174

$$\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

```
[Out] a/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c*ln(d*x^3+c)/d^(2/3)/(-a*d+b*c)^(4/3)-1
/2*c*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/d^(2/3)/(-a*d+b*c)^(4/3)-
1/3*c*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/d^
(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 56, 617, 204, 31}

$$\frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}}-\frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}}-\frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}}+\frac{a}{b\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] a/(b*(b*c - a*d)*(a + b*x^3)^(1/3)) - (c*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)
^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^(2/3)*(b*c - a*d)^(4/3)) +
(c*Log[c + d*x^3])/(6*d^(2/3)*(b*c - a*d)^(4/3)) - (c*Log[(b*c - a*d)^(1/3)
 + d^(1/3)*(a + b*x^3)^(1/3)])/(2*d^(2/3)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\operatorname{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \cdots$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \frac{c\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2d^{2/3}(bc-ad)^{4/3}} + \cdots$$

$$= \frac{a}{b(bc-ad)\sqrt[3]{a+bx^3}} - \frac{c\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,d^{2/3}(bc-ad)^{4/3}} + \frac{c\log\left(c+dx^3\right)}{6d^{2/3}(bc-ad)^{4/3}} - \cdots$$

Mathematica [C] time = 0.03, size = 77, normalized size = 0.44

$$\frac{bc\left(a+bx^3\right){}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+2a(bc-ad)}{2b\sqrt[3]{a+bx^3}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (2*a*(b*c - a*d) + b*c*(a + b*x^3)*Hypergeometric2F1[2/3, 1, 5/3, (d*(a + b
*x^3))/(-(b*c) + a*d)])/(2*b*(b*c - a*d)^2*(a + b*x^3)^(1/3))
```

fricas [B] time = 1.02, size = 872, normalized size = 5.01

$$-\frac{3\sqrt{\frac{1}{3}}\left(ab^2c^2d-a^2bcd^2+\left(b^3c^2d-ab^2cd^2\right)x^3\right)\sqrt{-\frac{\left(bcd^2-ad^3\right)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2\,bd^2x^3-bcd+3\,ad^2-3\sqrt{\frac{1}{3}}\left(2\left(bcd^2-ad^3\right)^{\frac{2}{3}}\left(bx^3+a\right)^{\frac{2}{3}}+(b\cdots\right.}{\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 + (b^3*c^2*d - a*b^2*c*d^2)*x
^3)*sqrt(-(b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*log((2*b*d^2*x^3 - b*c*d + 3
*a*d^2 - 3*sqrt(1/3)*(2*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(2/3) + (b*x^3
+ a)^(1/3)*(b*c*d - a*d^2) - (b*c*d^2 - a*d^3)^(1/3)*(b*c - a*d))*sqrt(-(b*
c*d^2 - a*d^3)^(1/3)/(b*c - a*d)) - 3*(b*c*d^2 - a*d^3)^(2/3)*(b*x^3 + a)^(
1/3))/(d*x^3 + c)) - (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3
 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3)*d + (b*c*d^2 -
a*d^3)^(2/3)) + 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2/3)*log((b*x^3 +
a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) - 6*(a*b*c*d^2 - a^2*d^3)*(b*x^3 + a)
^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^4*c^2*d^2 - 2*a*b
^3*c*d^3 + a^2*b^2*d^4)*x^3), 1/6*(6*sqrt(1/3)*(a*b^2*c^2*d - a^2*b*c*d^2 +
 (b^3*c^2*d - a*b^2*c*d^2)*x^3)*sqrt((b*c*d^2 - a*d^3)^(1/3)/(b*c - a*d))*a
rctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3)*d - (b*c*d^2 - a*d^3)^(1/3))*sqrt((b*c
*d^2 - a*d^3)^(1/3)/(b*c - a*d))/d) + (b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)
^(2/3)*log((b*x^3 + a)^(2/3)*d^2 - (b*c*d^2 - a*d^3)^(1/3)*(b*x^3 + a)^(1/3
)*d + (b*c*d^2 - a*d^3)^(2/3)) - 2*(b^2*c*x^3 + a*b*c)*(b*c*d^2 - a*d^3)^(2
/3)*log((b*x^3 + a)^(1/3)*d + (b*c*d^2 - a*d^3)^(1/3)) + 6*(a*b*c*d^2 - a^2
*d^3)*(b*x^3 + a)^(2/3))/(a*b^3*c^2*d^2 - 2*a^2*b^2*c*d^3 + a^3*b*d^4 + (b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^3)]
```

giac [B] time = 0.25, size = 301, normalized size = 1.73

$$-\frac{\frac{6\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d^2-2\sqrt{3}\,abcd^3+\sqrt{3}\,a^2d^4}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}bc\log\left(\left(bx^3+a\right)^{\frac{2}{3}}+\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\right)}{b^2c^2d^2-2\,abcd^3+a^2d^4}+\frac{2\,bc\left(-\frac{bc-ad}{d}\right)\cdots}{\cdots}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/6*(6*(-b*c*d^2 + a*d^3)^(2/3)*b*c*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3
) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(1/3))/(sqrt(3)*b^2*c^2*d^2 -
2*sqrt(3)*a*b*c*d^3 + sqrt(3)*a^2*d^4) - (-b*c*d^2 + a*d^3)^(2/3)*b*c*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)
/d)^(2/3))/(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4) + 2*b*c*(-(b*c - a*d)/d)^(
2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(1/3)))/(b^2*c^2 - 2*a*b*
c*d + a^2*d^2) - 6*a/((b*x^3 + a)^(1/3)*(b*c - a*d)))/b
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.98, size = 412, normalized size = 2.37

$$-\frac{a}{b\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}-\frac{c\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,c^2\,d^2-b\,c^3\,d\right)-\frac{c^2\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right)}{9\,d^{4/3}\,\left(a\,d-b\,c\right)^{8/3}}\right)}{3\,d^{2/3}\,\left(a\,d-b\,c\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - ((c - 3^(1/2)*c*1i)^2*(9*a^4
*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d
^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c - 3^(1/2)*c*1i))/(6*d^(2/3)*(a*d -
b*c)^(4/3)) - (c*log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d) - (c^2*(9*a^4*
d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^
5))/(9*d^(4/3)*(a*d - b*c)^(8/3))))/(3*d^(2/3)*(a*d - b*c)^(4/3)) - a/(b*(a
 + b*x^3)^(1/3)*(a*d - b*c)) + (log((a + b*x^3)^(1/3)*(a*c^2*d^2 - b*c^3*d)
 - ((c + 3^(1/2)*c*1i)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54
*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(36*d^(4/3)*(a*d - b*c)^(8/3)))*(c + 3^
(1/2)*c*1i))/(6*d^(2/3)*(a*d - b*c)^(4/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**5/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.751 $\int \frac{x^2}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=167

$$-\frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

```
[Out] -1/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)+1/2*
d^(1/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3*d
^(1/3)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {444, 51, 56, 617, 204, 31}

$$-\frac{1}{\sqrt[3]{a+bx^3}\,(bc-ad)}-\frac{\sqrt[3]{d}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x^3)^(1/3))) + (d^(1/3)*ArcTan[(1 - (2*d^(1/3)*(a +
 b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*(b*c - a*d)^(4/3)) - (
d^(1/3)*Log[c + d*x^3])/(6*(b*c - a*d)^(4/3)) + (d^(1/3)*Log[(b*c - a*d)^(1
/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3(bc-ad)}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{d}}+x}\,dx, x, \sqrt[3]{a+bx^3}\right.}{2(bc-ad)^{4/3}}$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\,\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}} -$$

$$= -\frac{1}{(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\sqrt[3]{d}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}} - \frac{\sqrt[3]{d}\,\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}} + \frac{\sqrt[3]{d}\,\log\left(\sqrt[3]{\right.}}{}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.30

$$-\frac{{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)}{\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)]/((b*c - a
*d)*(a + b*x^3)^(1/3)))
```

fricas [A] time = 0.71, size = 262, normalized size = 1.57

$$-\frac{2\sqrt{3}\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-\left(bx^3+a\right)\left(-\frac{d}{bc-ad}\right)^{\frac{1}{3}}\log\left(-\left(bx^3+a\right)^{\frac{1}{3}}(b\right.}{6\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(b*x^3 + a)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^
3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) - (b*x^3 + a)*(-d/(b*c -
 a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b
*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 2*(b*x^3 + a)*(-d
/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1
/3)*d) + 6*(b*x^3 + a)^(2/3))/((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)
```

giac [B] time = 0.26, size = 285, normalized size = 1.71

$$\frac{d\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^2-2\,abcd+a^2d^2\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^2d-2\sqrt{3}\,abcd^2+\sqrt{3}\,a^2d^3}-\frac{\left(-bcd^2+ad^3\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)^(
1/3)))/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*arctan(1/
3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*d)/d)^(
1/3))/(sqrt(3)*b^2*c^2*d - 2*sqrt(3)*a*b*c*d^2 + sqrt(3)*a^2*d^3) - 1/6*(-b
*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b*c - a*
d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3) -
 1/((b*x^3 + a)^(1/3)*(b*c - a*d))
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.92, size = 389, normalized size = 2.33

$$\frac{1}{\left(b\,x^3+a\right)^{1/3}\,\left(a\,d-b\,c\right)}+\frac{d^{1/3}\,\ln\left(\left(b\,x^3+a\right)^{1/3}\,\left(a\,d^4-b\,c\,d^3\right)-\frac{d^{2/3}\,\left(9\,a^4\,d^6-36\,a^3\,b\,c\,d^5+54\,a^2\,b^2\,c^2\,d^4-36\,a\,b^3\,c^3\,d^3+9\,b^4\,c^4\,d^2\right.}{9\,\left(a\,d-b\,c\right)^{8/3}}\right.}{3\,\left(a\,d-b\,c\right)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] 1/((a + b*x^3)^(1/3)*(a*d - b*c)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 -
 b*c*d^3) - (d^(2/3)*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2
*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3))))/(3*(a*d - b*c)^(4/3
)) - (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d^(2/3)*((3^(1/2)*
1i)/2 + 1/2)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d^3 + 54*a^2*b^2*c
^2*d^4 - 36*a^3*b*c*d^5))/(9*(a*d - b*c)^(8/3)))*((3^(1/2)*1i)/2 + 1/2))/(3
*(a*d - b*c)^(4/3)) + (d^(1/3)*log((a + b*x^3)^(1/3)*(a*d^4 - b*c*d^3) - (d
^(2/3)*((3^(1/2)*1i)/6 - 1/6)^2*(9*a^4*d^6 + 9*b^4*c^4*d^2 - 36*a*b^3*c^3*d
^3 + 54*a^2*b^2*c^2*d^4 - 36*a^3*b*c*d^5))/(a*d - b*c)^(8/3))*((3^(1/2)*1i)
/6 - 1/6))/(a*d - b*c)^(4/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + bx^3\right)^{\frac{4}{3}}\left(c + dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**2/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.752 $\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=271

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/2*ln(x)/a^(4/3)/c+1/6*d^(4/3)*ln(d*x^3+c)/
c/(-a*d+b*c)^(4/3)+1/2*ln(a^(1/3)-(b*x^3+a)^(1/3))/a^(4/3)/c-1/2*d^(4/3)*ln
((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3))/c/(-a*d+b*c)^(4/3)+1/3*arctan(1/
3*(a^(1/3)+2*(b*x^3+a)^(1/3))/a^(1/3)*3^(1/2))/a^(4/3)/c*3^(1/2)-1/3*d^(4/3
)*arctan(1/3*(1-2*d^(1/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c/(-a*
d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {446, 85, 156, 55, 617, 204, 31, 56}

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c}+\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}c}-\frac{\log(x)}{2a^{4/3}c}+\frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}}-\frac{d^{4/3}\log\left(\sqrt[3]{bc-ad}+\sqrt[3]{d}\sqrt[3]{a+bx^3}\right)}{2c(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] b/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) + ArcTan[(a^(1/3) + 2*(a + b*x^3)^(1/3)
)/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(4/3)*c) - (d^(4/3)*ArcTan[(1 - (2*d^(1/3)*
(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]])/(Sqrt[3]*c*(b*c - a*d)^(4/3
)) - Log[x]/(2*a^(4/3)*c) + (d^(4/3)*Log[c + d*x^3])/(6*c*(b*c - a*d)^(4/3)
) + Log[a^(1/3) - (a + b*x^3)^(1/3)]/(2*a^(4/3)*c) - (d^(4/3)*Log[(b*c - a*
d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{-bc+ad-bdx}{x\sqrt[3]{a+bx}\,(c+dx)}\,dx, x, x^3\right)}{3a(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt[3]{a+bx}}\,dx, x, x^3\right)}{3ac} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)}\right.}{3c(bc-ad)}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a}\right.}{2a^{4/3}c}$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{\log(x)}{2a^{4/3}c} + \frac{d^{4/3}\log\left(c+dx^3\right)}{6c(bc-ad)^{4/3}} + \frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{2a^{4/3}c} -$$

$$= \frac{b}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx^3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}c} - \frac{d^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}\sqrt[3]{a+bx^3}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt{3}\,c(bc-ad)^{4/3}} - \frac{\log(x)}{2a^{4/3}c}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 0.32

$$\frac{ad\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d\left(bx^3+a\right)}{ad-bc}\right)+(bc-ad)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)}{ac\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (a*d*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*d)] + (b*c
 - a*d)*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x^3)/a])/(a*c*(b*c - a*d)*(a
 + b*x^3)^(1/3))
```

fricas [B] time = 0.78, size = 975, normalized size = 3.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 3*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c -
 a^2*b*d)*x^3)*sqrt(-1/a^(2/3))*log((2*b*x^3 + 3*sqrt(1/3)*(2*(b*x^3 + a)^(
2/3)*a^(2/3) - (b*x^3 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^3 +
 a)^(1/3)*a^(2/3) + 3*a)/x^3) + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*c - a
*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3) - 1/3
*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(
2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a^3*d)
*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a^2*b*
d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(2/3)
```

```
+ (b*x^3 + a)^(1/3)*d))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^
3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*c + 2*sqrt(3)*(a^2*b*d*x^3 + a^3*d)*(d/(b*
c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(d/(b*c - a*d))^(1/3)
- 1/3*sqrt(3)) - ((b^2*c - a*b*d)*x^3 + a*b*c - a^2*d)*a^(2/3)*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*((b^2*c - a*b*d)*x^3 +
 a*b*c - a^2*d)*a^(2/3)*log((b*x^3 + a)^(1/3) - a^(1/3)) + (a^2*b*d*x^3 + a
^3*d)*(d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(d/(b*c - a*
d))^(2/3) + (b*x^3 + a)^(2/3)*d + (b*c - a*d)*(d/(b*c - a*d))^(1/3)) - 2*(a
^2*b*d*x^3 + a^3*d)*(d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(d/(b*c - a*d))^(
2/3) + (b*x^3 + a)^(1/3)*d) + 6*sqrt(1/3)*(a^2*b*c - a^3*d + (a*b^2*c - a^2
*b*d)*x^3)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3
))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^3)]
```

giac [A] time = 0.79, size = 389, normalized size = 1.44

$$-\frac{d^{2}\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^{3}+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^{2}c^{3}-2\,abc^{2}d+a^{2}cd^{2}\right)}-\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^{3}+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c^{3}-2\,\sqrt{3}\,abc^{2}d+\sqrt{3}\,a^{2}cd^{2}}+\frac{\left(-bcd^{2}+ad^{3}\right)^{\frac{2}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d
)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2) - (-b*c*d^2 + a*d^3)^(2/3)*ar
ctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c - a*
d)/d)^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) +
1/6*(-b*c*d^2 + a*d^3)^(2/3)*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*(-(b
*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^3 - 2*a*b*c^2*d + a^2*c
*d^2) + b/((b*x^3 + a)^(1/3)*(a*b*c - a^2*d)) + 1/3*sqrt(3)*arctan(1/3*sqrt
(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(1/3))/(a^(4/3)*c) - 1/6*log((b*x^3 +
 a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^(4/3)*c) + 1/3*log(abs(
(b*x^3 + a)^(1/3) - a^(1/3)))/(a^(4/3)*c)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}}\left(d\,x^{3}+c\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{4}{3}}\left(dx^{3}+c\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x), x)
```

mupad [B] time = 5.34, size = 3804, normalized size = 14.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log(9*a^7*b^14*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^
8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a
^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*
a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a
^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - (-d^4/(27*b^4*c^7 + 27*a^4*c^3*d
^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243
*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 -
51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^1
0*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7
*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4
*c^4*d^14))*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^
2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^
12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^
9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c
^3*d^12 - 27*a^16*b^5*c^2*d^13)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^
3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3) + log(9*a^7*b^1
4*c^11*d^4 - ((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d
^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*
d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*
d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d
^13 - 54*a^18*b^4*c^2*d^14) - (1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^
3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^
12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^1
6*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*
a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*(1/(27
*a^4*c^3))^(1/3) - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*
b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b
^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c
^2*d^13)*(1/(27*a^4*c^3))^(1/3) + (log(((3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7
+ 27*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*
d))^(1/3)*((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4
+ 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7
 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^1
0 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13
 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i - 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c
^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*
(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^
5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10
*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18
*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21
*b^4*c^4*d^14))/4))/2 - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^
9*b^12*c^9*d^6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^1
2*b^9*c^6*d^9 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b
^6*c^3*d^12 + 27*a^16*b^5*c^2*d^13)*(3^(1/2)*1i - 1)*(-d^4/(27*b^4*c^7 + 27
*a^4*c^3*d^4 - 108*a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^
(1/3))/2 - (log(((3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*
a^3*b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3)*((a + b*x^3)^
(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^
5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*
d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d
^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14
) - ((3^(1/2)*1i + 1)^2*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*b*c^4*
d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(2/3)*(243*a^10*b^15*c^15*d^3
 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^1
2*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16
*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a
```

```
^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))/4))/2 +
 9*a^7*b^14*c^11*d^4 - 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a
^10*b^11*c^8*d^7 + 1827*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^
13*b^8*c^5*d^10 - 765*a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b
^5*c^2*d^13)*(3^(1/2)*1i + 1)*(-d^4/(27*b^4*c^7 + 27*a^4*c^3*d^4 - 108*a^3*
b*c^4*d^3 + 162*a^2*b^2*c^5*d^2 - 108*a*b^3*c^6*d))^(1/3))/2 - b/((a + b*x^
3)^(1/3)*(a^2*d - a*b*c)) + log(((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 -
297*a^8*b^14*c^12*d^4 + 1485*a^9*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 +
8937*a^11*b^11*c^9*d^7 - 12663*a^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 -
 9855*a^14*b^8*c^6*d^10 + 5400*a^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 +
 486*a^17*b^5*c^3*d^13 - 54*a^18*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 - 1/2)^2*(
1/(27*a^4*c^3))^(2/3)*(243*a^10*b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 1
5795*a^12*b^13*c^13*d^5 - 51030*a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*
d^7 - 163296*a^15*b^10*c^10*d^8 + 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8
*c^8*d^10 + 69255*a^18*b^7*c^7*d^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b
^5*c^5*d^13 - 486*a^21*b^4*c^4*d^14))*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3
))^(1/3) - 9*a^7*b^14*c^11*d^4 + 90*a^8*b^13*c^10*d^5 - 405*a^9*b^12*c^9*d^
6 + 1071*a^10*b^11*c^8*d^7 - 1827*a^11*b^10*c^7*d^8 + 2079*a^12*b^9*c^6*d^9
 - 1575*a^13*b^8*c^5*d^10 + 765*a^14*b^7*c^4*d^11 - 216*a^15*b^6*c^3*d^12 +
 27*a^16*b^5*c^2*d^13)*((3^(1/2)*1i)/2 - 1/2)*(1/(27*a^4*c^3))^(1/3) - log(
((a + b*x^3)^(1/3)*(27*a^7*b^15*c^13*d^3 - 297*a^8*b^14*c^12*d^4 + 1485*a^9
*b^13*c^11*d^5 - 4455*a^10*b^12*c^10*d^6 + 8937*a^11*b^11*c^9*d^7 - 12663*a
^12*b^10*c^8*d^8 + 13041*a^13*b^9*c^7*d^9 - 9855*a^14*b^8*c^6*d^10 + 5400*a
^15*b^7*c^5*d^11 - 2052*a^16*b^6*c^4*d^12 + 486*a^17*b^5*c^3*d^13 - 54*a^18
*b^4*c^2*d^14) - ((3^(1/2)*1i)/2 + 1/2)^2*(1/(27*a^4*c^3))^(2/3)*(243*a^10*
b^15*c^15*d^3 - 2916*a^11*b^14*c^14*d^4 + 15795*a^12*b^13*c^13*d^5 - 51030*
a^13*b^12*c^12*d^6 + 109350*a^14*b^11*c^11*d^7 - 163296*a^15*b^10*c^10*d^8
+ 173502*a^16*b^9*c^9*d^9 - 131220*a^17*b^8*c^8*d^10 + 69255*a^18*b^7*c^7*d
^11 - 24300*a^19*b^6*c^6*d^12 + 5103*a^20*b^5*c^5*d^13 - 486*a^21*b^4*c^4*d
^14))*((3^(1/2)*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3) + 9*a^7*b^14*c^11*d^4 -
 90*a^8*b^13*c^10*d^5 + 405*a^9*b^12*c^9*d^6 - 1071*a^10*b^11*c^8*d^7 + 182
7*a^11*b^10*c^7*d^8 - 2079*a^12*b^9*c^6*d^9 + 1575*a^13*b^8*c^5*d^10 - 765*
a^14*b^7*c^4*d^11 + 216*a^15*b^6*c^3*d^12 - 27*a^16*b^5*c^2*d^13)*((3^(1/2)
*1i)/2 + 1/2)*(1/(27*a^4*c^3))^(1/3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.753 $\int \frac{1}{x^4(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=357

$$-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\cdots$$

```
[Out] -d^2/c^2/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-3*a*d-4*b*c)/a^2/c^2/(b*x^3+a)^(1
/3)-1/3/a/c/x^3/(b*x^3+a)^(1/3)+1/6*(3*a*d+4*b*c)*ln(x)/a^(7/3)/c^2-1/6*d^(
7/3)*ln(d*x^3+c)/c^2/(-a*d+b*c)^(4/3)-1/6*(3*a*d+4*b*c)*ln(a^(1/3)-(b*x^3+a
)^(1/3))/a^(7/3)/c^2+1/2*d^(7/3)*ln((-a*d+b*c)^(1/3)+d^(1/3)*(b*x^3+a)^(1/3
))/c^2/(-a*d+b*c)^(4/3)-1/9*(3*a*d+4*b*c)*arctan(1/3*(a^(1/3)+2*(b*x^3+a)^(
1/3))/a^(1/3)*3^(1/2))/a^(7/3)/c^2*3^(1/2)+1/3*d^(7/3)*arctan(1/3*(1-2*d^(1
/3)*(b*x^3+a)^(1/3)/(-a*d+b*c)^(1/3))*3^(1/2))/c^2/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {446, 103, 156, 51, 55, 617, 204, 31, 56}

$$-\frac{3ad+4bc}{3a^2c^2\sqrt[3]{a+bx^3}}-\frac{(3ad+4bc)\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^3}\right)}{6a^{7/3}c^2}-\frac{(3ad+4bc)\tan^{-1}\left(\frac{2\sqrt[3]{a+bx^3}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}c^2}+\frac{\log(x)(3ad+4bc)}{6a^{7/3}c^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(d^2/(c^2*(b*c - a*d)*(a + b*x^3)^(1/3))) - (4*b*c + 3*a*d)/(3*a^2*c^2*(a
+ b*x^3)^(1/3)) - 1/(3*a*c*x^3*(a + b*x^3)^(1/3)) - ((4*b*c + 3*a*d)*ArcTan
[(a^(1/3) + 2*(a + b*x^3)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*c^2
) + (d^(7/3)*ArcTan[(1 - (2*d^(1/3)*(a + b*x^3)^(1/3))/(b*c - a*d)^(1/3))/S
qrt[3]])/(Sqrt[3]*c^2*(b*c - a*d)^(4/3)) + ((4*b*c + 3*a*d)*Log[x])/(6*a^(7
/3)*c^2) - (d^(7/3)*Log[c + d*x^3])/(6*c^2*(b*c - a*d)^(4/3)) - ((4*b*c + 3
*a*d)*Log[a^(1/3) - (a + b*x^3)^(1/3)])/(6*a^(7/3)*c^2) + (d^(7/3)*Log[(b*c
 - a*d)^(1/3) + d^(1/3)*(a + b*x^3)^(1/3)])/(2*c^2*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{3}(4bc+3ad)+\frac{4bdx}{3}}{x(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3ac}$$

$$= -\frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{(a+bx)^{4/3}(c+dx)}\,dx, x, x^3\right)}{3c^2} - \frac{(4bc+3ad)\,\text{Subst}}{9}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{d^3\,\text{Subst}\left(\int \right.}{3c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\,\text{lo}}{6a^{7/3}c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} + \frac{(4bc+3ad)\,\text{lo}}{6a^{7/3}c^2}$$

$$= -\frac{d^2}{c^2(bc-ad)\sqrt[3]{a+bx^3}} - \frac{4bc+3ad}{3a^2c^2\sqrt[3]{a+bx^3}} - \frac{1}{3acx^3\sqrt[3]{a+bx^3}} - \frac{(4bc+3ad)\,\text{ta}}{3\sqrt{3}}$$

Mathematica [C] time = 0.05, size = 117, normalized size = 0.33

$$\frac{3a^2d^2x^3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{d(bx^3+a)}{ad-bc}\right)+(bc-ad)\left(x^3(3ad+4bc)\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx^3}{a}+1\right)+ac\right)}{3a^2c^2x^3\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*a^2*d^2*x^3*Hypergeometric2F1[-1/3, 1, 2/3, (d*(a + b*x^3))/(-(b*c) + a*
d)] + (b*c - a*d)*(a*c + (4*b*c + 3*a*d)*x^3*Hypergeometric2F1[-1/3, 1, 2/3
, 1 + (b*x^3)/a]))/(3*a^2*c^2*(-(b*c) + a*d)*x^3*(a + b*x^3)^(1/3))
```

fricas [B] time = 2.64, size = 1386, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*((4*a*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*
b^2*c^2 - a^3*b*c*d - 3*a^4*d^2)*x^3)*sqrt((-a)^(1/3)/a)*log((2*b*x^3 - 3*s
qrt(1/3)*(2*(b*x^3 + a)^(2/3)*(-a)^(2/3) - (b*x^3 + a)^(1/3)*a + (-a)^(1/3)
*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^3 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^3) - 6*sqr
t(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3
)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))^(1/3) + 1/3*sqrt(3)) + ((4*b^3*c^2 - a
*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-
```

```
a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
 - 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d
- 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(1/3) + (-a)^(1/3)) + 3*(a^3*b
*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c
- a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c
- a*d))^(1/3)) - 6*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log
((b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x^3 + a)^(1/3)*d) - 6*(a^2*b*c^2 -
 a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3
- a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^5*c^2*d)*x^3), -1/18*(6*sqrt(1/3)*((4*a
*b^3*c^2 - a^2*b^2*c*d - 3*a^3*b*d^2)*x^6 + (4*a^2*b^2*c^2 - a^3*b*c*d - 3*
a^4*d^2)*x^3)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x^3 + a)^(1/3) - (
-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 6*sqrt(3)*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(
-d/(b*c - a*d))^(1/3)*arctan(2/3*sqrt(3)*(b*x^3 + a)^(1/3)*(-d/(b*c - a*d))
^(1/3) + 1/3*sqrt(3)) - ((4*b^3*c^2 - a*b^2*c*d - 3*a^2*b*d^2)*x^6 + (4*a*b
^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*x^3 + a)^(2/3) - (b*
x^3 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*((4*b^3*c^2 - a*b^2*c*d - 3*a^2
*b*d^2)*x^6 + (4*a*b^2*c^2 - a^2*b*c*d - 3*a^3*d^2)*x^3)*(-a)^(2/3)*log((b*
x^3 + a)^(1/3) + (-a)^(1/3)) - 3*(a^3*b*d^2*x^6 + a^4*d^2*x^3)*(-d/(b*c - a
*d))^(1/3)*log(-(b*x^3 + a)^(1/3)*(b*c - a*d)*(-d/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*d - (b*c - a*d)*(-d/(b*c - a*d))^(1/3)) + 6*(a^3*b*d^2*x^6 +
a^4*d^2*x^3)*(-d/(b*c - a*d))^(1/3)*log((b*c - a*d)*(-d/(b*c - a*d))^(2/3)
+ (b*x^3 + a)^(1/3)*d) + 6*(a^2*b*c^2 - a^3*c*d + (4*a*b^2*c^2 - a^2*b*c*d)
*x^3)*(b*x^3 + a)^(2/3))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^6 + (a^4*b*c^3 - a^
5*c^2*d)*x^3)]
```

giac [A] time = 0.79, size = 486, normalized size = 1.36

$$\frac{d^3\left(-\frac{bc-ad}{d}\right)^{\frac{2}{3}}\log\left(\left|\left(bx^3+a\right)^{\frac{1}{3}}-\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2\right)}+\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\left(bx^3+a\right)^{\frac{1}{3}}+\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{bc-ad}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^2c^4-2\sqrt{3}\,abc^3d+\sqrt{3}\,a^2c^2d^2}-\frac{\left(-bcd^2+ad^3\right)^{\frac{2}{3}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*d^3*(-(b*c - a*d)/d)^(2/3)*log(abs((b*x^3 + a)^(1/3) - (-(b*c - a*d)/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) + (-b*c*d^2 + a*d^3)^(2/3)*d
*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + (-(b*c - a*d)/d)^(1/3))/(-(b*c -
 a*d)/d)^(1/3))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^
2) - 1/6*(-b*c*d^2 + a*d^3)^(2/3)*d*log((b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/
3)*(-(b*c - a*d)/d)^(1/3) + (-(b*c - a*d)/d)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d
+ a^2*c^2*d^2) - 1/3*(4*(b*x^3 + a)*b^2*c - 3*a*b^2*c - (b*x^3 + a)*a*b*d)/
((a^2*b*c^2 - a^3*c*d)*((b*x^3 + a)^(4/3) - (b*x^3 + a)^(1/3)*a)) - 1/9*sqr
t(3)*(4*b*c + 3*a*d)*arctan(1/3*sqrt(3)*(2*(b*x^3 + a)^(1/3) + a^(1/3))/a^(
1/3))/(a^(7/3)*c^2) - 1/9*(4*a^(1/3)*b*c + 3*a^(4/3)*d)*log(abs((b*x^3 + a)
^(1/3) - a^(1/3)))/(a^(8/3)*c^2) + 1/18*(4*a^(2/3)*b*c + 3*a^(5/3)*d)*log((
b*x^3 + a)^(2/3) + (b*x^3 + a)^(1/3)*a^(1/3) + a^(2/3))/(a^3*c^2)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^4), x)
```

mupad [B] time = 6.39, size = 5875, normalized size = 16.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] log((d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^
8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(
1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^1
6*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^
7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^
21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^
12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^
26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + (d^7/(27*b^4*c^10 + 27*a^4*
c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2
 - 108*a*b^3*c^9*d))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^1
3*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^
17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 10
7892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18
))*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8
*d^2 - 108*a*b^3*c^9*d))^(1/3) + (b^2/(a^2*d - a*b*c) + (b*(a + b*x^3)*(a*d
 - 4*b*c))/(3*a^2*c*(a*d - b*c)))/(a*(a + b*x^3)^(1/3) - (a + b*x^3)^(4/3))
 + log((-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729
*a^7*c^6))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((a + b*x^3)^(1/3)*(8975448*a
^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5
 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^
19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^
10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a
^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^1
5 + 1062882*a^27*b^4*c^9*d^16) + (-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2
*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*a^19*b^15*c^24*d^3 - 57
395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423490*a^22*
b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^10*c^19*d
^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10 + 1363146
165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*a^29*b^5*
```

```
c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a
*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) - 3254256*a^14*b^16*c^19
*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*
a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*
d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595
*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^1
5 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^
10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^
11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 22453
2*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 +
 26244*a^22*b^5*c^4*d^18))*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 1
08*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + (log(((3^(1/2)*1i - 1)^2*(d^7/(27*b^
4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b
^3*c^9*d))^(2/3)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i - 1)*(d^7/(27*b^
4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b
^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^21*d^4 - 944784*a^
14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531*a^17*b^14*c^19*d^
6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^17*d^8 + 42338133*a
^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 55092717*a^22*b^9*c^14*d
^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^12*d^13 + 25745364*
a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882*a^27*b^4*c^9*d^16
) + ((3^(1/2)*1i - 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*
d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782969*a^19*b^15*c^24
*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22*d^5 - 1004423
490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214155168*a^24*b^
10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*b^8*c^17*d^10
+ 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^12 + 100442349*
a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 - 3254256*a^14*b^16
*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 492
0750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*
c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 157
13595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^
9*d^15 + 177147*a^25*b^5*c^8*d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13
*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004
*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14
 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^
5*d^17 + 26244*a^22*b^5*c^4*d^18))*(3^(1/2)*1i - 1)*(d^7/(27*b^4*c^10 + 27*
a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(
1/3))/2 - (log(((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108
*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(((3^(1/2)*1
i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2
*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3)*((a + b*x^3)^(1/3)*(8975448*a^15*b^16*c^
21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 + 83790531
*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19*b^12*c^1
7*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10 + 550927
17*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^24*b^7*c^1
2*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15 + 1062882
*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i + 1)^2*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^
4 - 108*a^3*b*c^7*d^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(2/3)*(4782
969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*
c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 -
3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a
^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15
*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))/4))/2 +
 419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
```

d^16))/4 - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*
c^13*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356
*a^17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 +
107892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d
^18))*(3^(1/2)*1i + 1)*(d^7/(27*b^4*c^10 + 27*a^4*c^6*d^4 - 108*a^3*b*c^7*d
^3 + 162*a^2*b^2*c^8*d^2 - 108*a*b^3*c^9*d))^(1/3))/2 - log(((3^(1/2)*1i)/2
+ 1/2)^2*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(
729*a^7*c^6))^(2/3)*(((3^(1/2)*1i)/2 + 1/2)*((a + b*x^3)^(1/3)*(8975448*a^1
5*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*c^20*d^5 +
83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 56509893*a^19
*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10*c^15*d^10
+ 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38736144*a^2
4*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*c^10*d^15
+ 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 + 1/2)^2*(-(27*a^3*d^3 + 64*
b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)*(4782969*
a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21*b^13*c^22
*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*d^7 - 3214
155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 2582803260*a^26*
b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^6*c^15*d^1
2 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(-(27*a^3*d
^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) +
419904*a^13*b^17*c^20*d^4 - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^1
5*c^18*d^6 - 14781933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 155
29887*a^18*b^12*c^15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10
*c^13*d^11 + 13404123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 78
01029*a^23*b^7*c^10*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*
d^16) - (a + b*x^3)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^1
3*d^9 - 516132*a^15*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^
17*b^10*c^9*d^13 - 551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 10
7892*a^20*b^7*c^6*d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18
))*((3^(1/2)*1i)/2 + 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 10
8*a^2*b*c*d^2)/(729*a^7*c^6))^(1/3) + log(((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^
3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3
)*(419904*a^13*b^17*c^20*d^4 - ((3^(1/2)*1i)/2 - 1/2)*((a + b*x^3)^(1/3)*(8
975448*a^15*b^16*c^21*d^4 - 944784*a^14*b^17*c^22*d^3 - 36905625*a^16*b^15*
c^20*d^5 + 83790531*a^17*b^14*c^19*d^6 - 107173935*a^18*b^13*c^18*d^7 + 565
09893*a^19*b^12*c^17*d^8 + 42338133*a^20*b^11*c^16*d^9 - 93710763*a^21*b^10
*c^15*d^10 + 55092717*a^22*b^9*c^14*d^11 + 12105045*a^23*b^8*c^13*d^12 - 38
736144*a^24*b^7*c^12*d^13 + 25745364*a^25*b^6*c^11*d^14 - 8148762*a^26*b^5*
c^10*d^15 + 1062882*a^27*b^4*c^9*d^16) + ((3^(1/2)*1i)/2 - 1/2)^2*(-(27*a^3
*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6))^(2/3)
*(4782969*a^19*b^15*c^24*d^3 - 57395628*a^20*b^14*c^23*d^4 + 310892985*a^21
*b^13*c^22*d^5 - 1004423490*a^22*b^12*c^21*d^6 + 2152336050*a^23*b^11*c^20*
d^7 - 3214155168*a^24*b^10*c^19*d^8 + 3415039866*a^25*b^9*c^18*d^9 - 258280
3260*a^26*b^8*c^17*d^10 + 1363146165*a^27*b^7*c^16*d^11 - 478296900*a^28*b^
6*c^15*d^12 + 100442349*a^29*b^5*c^14*d^13 - 9565938*a^30*b^4*c^13*d^14))*(
-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(729*a^7*c^6
))^(1/3) - 3254256*a^14*b^16*c^19*d^5 + 10156428*a^15*b^15*c^18*d^6 - 14781
933*a^16*b^14*c^17*d^7 + 4920750*a^17*b^13*c^16*d^8 + 15529887*a^18*b^12*c^
15*d^9 - 22182741*a^19*b^11*c^14*d^10 + 5412825*a^20*b^10*c^13*d^11 + 13404
123*a^21*b^9*c^12*d^12 - 15713595*a^22*b^8*c^11*d^13 + 7801029*a^23*b^7*c^1
0*d^14 - 1889568*a^24*b^6*c^9*d^15 + 177147*a^25*b^5*c^8*d^16) - (a + b*x^3
)^(1/3)*(256608*a^14*b^13*c^12*d^10 - 46656*a^13*b^14*c^13*d^9 - 516132*a^1
5*b^12*c^11*d^11 + 347004*a^16*b^11*c^10*d^12 + 265356*a^17*b^10*c^9*d^13 -
551124*a^18*b^9*c^8*d^14 + 224532*a^19*b^8*c^7*d^15 + 107892*a^20*b^7*c^6*
d^16 - 113724*a^21*b^6*c^5*d^17 + 26244*a^22*b^5*c^4*d^18))*((3^(1/2)*1i)/2
- 1/2)*(-(27*a^3*d^3 + 64*b^3*c^3 + 144*a*b^2*c^2*d + 108*a^2*b*c*d^2)/(72
9*a^7*c^6))^(1/3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.754 $\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=322

$$\frac{(4ad+3bc)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{6b^{7/3}d^2}-\frac{(4ad+3bc)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{7/3}d^2}+\frac{x\left(a+bx^3\right)^{2/3}(bc-4ad)}{3b^2d(bc-ad)}+\frac{c^{7/3}\log\left(c+dx\right.}{6d^2(bc-ad)^{4/}}$$

```
[Out] a*x^4/b/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/3*(-4*a*d+b*c)*x*(b*x^3+a)^(2/3)/b^2/d
/(-a*d+b*c)+1/6*c^(7/3)*ln(d*x^3+c)/d^2/(-a*d+b*c)^(4/3)-1/2*c^(7/3)*ln((-a
*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d^2/(-a*d+b*c)^(4/3)+1/6*(4*a*d+3*
b*c)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)/d^2-1/9*(4*a*d+3*b*c)*arctan(1/
3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)/d^2*3^(1/2)+1/3*c^(7/3)*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d^2/(-
a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.21, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^{10}\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^10*(1 + (b*x^3)/a)^(1/3)*AppellF1[10/3, 4/3, 1, 13/3, -((b*x^3)/a), -((d
*x^3)/c)])/(10*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^9}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$
$$= \frac{x^{10}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{10}{3};\frac{4}{3},1;\frac{13}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{10ac\sqrt[3]{a+bx^3}}$$

Mathematica [C] time = 1.08, size = 504, normalized size = 1.57

$$-\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,\sqrt[3]{bc-ad}\,\left(-4a^2d^2+abcd+3b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2c\left(-4a^2\sqrt[3]{c}\,d\sqrt[3]{a+bx^3}\,\log\left(\frac{\sqrt[3]{c}\,x\sqrt[3]{bc}}{\sqrt[3]{ax^3+}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/36*(3*(b*c - a*d)^(1/3)*(3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c*(-
6*a*b*c*(b*c - a*d)^(1/3)*x + 24*a^2*d*(b*c - a*d)^(1/3)*x - 6*b^2*c*(b*c -
 a*d)^(1/3)*x^4 + 6*a*b*d*(b*c - a*d)^(1/3)*x^4 - 2*Sqrt[3]*a*c^(1/3)*(-(b*
c) + 4*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*c^(1/3)*(-(b*c) + 4*a*d)*(a + b*x^3)^(1/
3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + a*b*c^(4/3)*(a
+ b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c
^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - 4*a^2*c^(1/3)*d*(a + b*x^3
)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*
(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b^2*c*d*(b*c - a*d)^(4/3)*(a + b
*x^3)^(1/3))
```

fricas [B] time = 2.16, size = 1329, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^2 + (3*b^4*c^2 +
a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a
)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 -
2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) - 6*sqrt(3)*(b^4*c^2
*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(c/(b*c - a*d))^(1/3))/x) + 2*(3*a*b^2*c^2 + a^2*b*c*d - 4
*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)
*x - (b*x^3 + a)^(1/3))/x) - (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*
c^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^
(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/
(b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + 3*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c -
 a*d)*x^2*(c/(b*c - a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c -
 a*d))^(2/3) + (b*x^3 + a)^(2/3)*c)/x^2) + 6*((b^3*c*d - a*b^2*d^2)*x^4 + (
a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*d^3 +
 (b^5*c*d^2 - a*b^4*d^3)*x^3), -1/18*(6*sqrt(3)*(b^4*c^2*x^3 + a*b^3*c^2)*(
```

```
c/(b*c - a*d))^(1/3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*(c
/(b*c - a*d))^(1/3))/x) - 2*(3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c
^2 + a*b^2*c*d - 4*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1
/3))/x) + (3*a*b^2*c^2 + a^2*b*c*d - 4*a^3*d^2 + (3*b^3*c^2 + a*b^2*c*d - 4
*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (
b*x^3 + a)^(2/3))/x^2) + 6*(b^4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*
log(-((b*c - a*d)*x*(c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1/3)*c)/x) - 3*(b^
4*c^2*x^3 + a*b^3*c^2)*(c/(b*c - a*d))^(1/3)*log(((b*c - a*d)*x^2*(c/(b*c -
 a*d))^(1/3) + (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(c/(b*c - a*d))^(2/3) + (b*x
^3 + a)^(2/3)*c)/x^2) - 6*sqrt(1/3)*(3*a*b^3*c^2 + a^2*b^2*c*d - 4*a^3*b*d^
2 + (3*b^4*c^2 + a*b^3*c*d - 4*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*(b^(1/3)*
x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 6*((b^3*c*d - a*b^2*d^2)*x^
4 + (a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*c*d^2 - a^2*b^3*
d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x^9/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**9/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.755 $\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}\,x\right)}{2b^{4/3}d}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\,x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d}-\frac{c^{4/3}\log\left(c+dx^3\right)}{6d(bc-ad)^{4/3}}+\frac{c^{4/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2d(bc-ad)^{4/3}}-\frac{c^{4/3}\tan^{-1}\left(\ldots\right)}{\sqrt{3}\,d(bc-\ldots}$$

```
[Out] a*x/b/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*c^(4/3)*ln(d*x^3+c)/d/(-a*d+b*c)^(4/3)
+1/2*c^(4/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/d/(-a*d+b*c)^(4
/3)-1/2*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(4/3)/d+1/3*arctan(1/3*(1+2*b^(1/3
)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(4/3)/d*3^(1/2)-1/3*c^(4/3)*arctan(1/3*(1+2
*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/d/(-a*d+b*c)^(4/3)*3^
(1/2)
```

Rubi [C] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 0.26, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^7\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^7*(1 + (b*x^3)/a)^(1/3)*AppellF1[7/3, 4/3, 1, 10/3, -((b*x^3)/a), -((d*x
^3)/c)])/(7*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^6}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^7\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{7}{3};\frac{4}{3},1;\frac{10}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{7ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.45, size = 309, normalized size = 1.19

$$\frac{3x^4\sqrt[3]{\frac{bx^3}{a}+1}\,(bc-ad)^{4/3}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2ac\left(\sqrt[3]{c}\,\sqrt[3]{a+bx^3}\,\log\left(\frac{\sqrt[3]{c}\,x\,\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+c^{2/3}\right)-2\sqrt[3]{c}\right.}{12bc\sqrt[3]{a+bx^3}\,(bc-ad)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (3*(b*c - a*d)^(4/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*a*c*(-6*(b*c - a*d)^(1/3)*x + 2*Sqrt[3]*c^(1
/3)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)] + c^(1/3)*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((
b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b
+ a*x^3)^(1/3)]))/(12*b*c*(b*c - a*d)^(4/3)*(a + b*x^3)^(1/3))
```

fricas [B] time = 0.93, size = 1127, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x + 3*sqrt(1/3)*(a*b^2*c - a^2*b*d + (b^3*c
 - a*b^2*d)*x^3)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^
(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*
x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*sqrt(3)*(b^3*c*x
^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(b*
x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*c
- a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c - a
*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/
3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(b*
c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^(1
/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d)*
x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a*d
))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d -
 a*b^3*d^2)*x^3), 1/6*(6*(b*x^3 + a)^(2/3)*a*b*d*x - 6*sqrt(1/3)*(a*b^2*c -
 a^2*b*d + (b^3*c - a*b^2*d)*x^3)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-
b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*sqrt(3)*(b^3*c
*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*arctan(-1/3*(sqrt(3)*x - 2*sqrt(3)*(
b*x^3 + a)^(1/3)*(-c/(b*c - a*d))^(1/3))/x) - 2*((b^2*c - a*b*d)*x^3 + a*b*
c - a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + ((b^2*c -
 a*b*d)*x^3 + a*b*c - a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(
1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 2*(b^3*c*x^3 + a*b^2*c)*(-c/(
b*c - a*d))^(1/3)*log(-((b*c - a*d)*x*(-c/(b*c - a*d))^(2/3) - (b*x^3 + a)^
(1/3)*c)/x) + (b^3*c*x^3 + a*b^2*c)*(-c/(b*c - a*d))^(1/3)*log(-((b*c - a*d
)*x^2*(-c/(b*c - a*d))^(1/3) - (b*x^3 + a)^(1/3)*(b*c - a*d)*x*(-c/(b*c - a
*d))^(2/3) - (b*x^3 + a)^(2/3)*c)/x^2))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d
 - a*b^3*d^2)*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(bx^{3}+a\right)^{\frac{4}{3}} \left(dx^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^6/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(a + bx^{3}\right)^{\frac{4}{3}} \left(c + dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**6/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.756 $\int \frac{x^3}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=172

$$-\frac{x}{\sqrt[3]{a+bx^3}\,(bc-ad)}+\frac{\sqrt[3]{c}\log\left(c+dx^3\right)}{6(bc-ad)^{4/3}}-\frac{\sqrt[3]{c}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2(bc-ad)^{4/3}}+\frac{\sqrt[3]{c}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,(bc-ad)^{4/3}}$$

```
[Out] -x/(-a*d+b*c)/(b*x^3+a)^(1/3)+1/6*c^(1/3)*ln(d*x^3+c)/(-a*d+b*c)^(4/3)-1/2*
c^(1/3)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/(-a*d+b*c)^(4/3)+1/3
*c^(1/3)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2
))/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [C] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 0.53, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^4\sqrt[3]{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{dx^3+c}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((c*((b*x^3)/a
 - (d*x^3)/c))/(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3
))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{x^4\sqrt[3]{1+\frac{bx^3}{a}}\ {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{c\left(\frac{bx^3}{a}-\frac{dx^3}{c}\right)}{c+dx^3}\right)}{4ac\sqrt[3]{a+bx^3}\left(1+\frac{dx^3}{c}\right)^{4/3}}$$

Mathematica [C] time = 0.06, size = 86, normalized size = 0.50

$$\frac{x^4\sqrt[3]{\frac{bx^3}{a}+1}\ {}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{4ac\sqrt[3]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{4/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
[Out] (x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, ((-(b*c) + a*d)
*x^3)/(a*(c + d*x^3))])/(4*a*c*(a + b*x^3)^(1/3)*(1 + (d*x^3)/c)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(x^3/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**3/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.757 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^3+a)^(1/3)-1/6*d*ln(d*x^3+c)/c^(2/3)/(-a*d+b*c)^(4/3)
+1/2*d*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(2/3)/(-a*d+b*c)^(4
/3)-1/3*d*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/
2))/c^(2/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\,x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)}\,dx = \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}\,x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{2}{c^{2/3}+\sqrt[3]{c}\ldots}\right.}{\ldots}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}\,x+(bc-ad)^{2/3}\ldots}\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-a\ldots}}{\sqrt[3]{a+bx^3}}\right.}{6c^{2/3}(bc-ad)^{4/3}}$$

$$= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}\,x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\,c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}\ldots\right.}{\ldots}$$

Mathematica [C] time = 0.12, size = 256, normalized size = 1.43

$$-\frac{28c^3\left(a+bx^3\right)^2{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-28c^3\left(a+bx^3\right)^2+21c^2dx^3\left(a+bx^3\right)^2{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-21c^2dx^3}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21
*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2
, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(-(b*c) + a*d)*x^2*(a
 + b*x^3)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,x^3+a\right)^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.758 $\int \frac{1}{x^3\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=229

$$-\frac{\left(a+bx^3\right)^{2/3}(3bc-ad)}{2a^2cx^2(bc-ad)}+\frac{d^2\log\left(c+dx^3\right)}{6c^{5/3}(bc-ad)^{4/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{5/3}(bc-ad)^{4/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\,c^{5/3}(bc-ad)^{4/3}}+\frac{}{ax^2\sqrt[3]{a+b}}$$

```
[Out] b/a/(-a*d+b*c)/x^2/(b*x^3+a)^(1/3)-1/2*(-a*d+3*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^2+1/6*d^2*ln(d*x^3+c)/c^(5/3)/(-a*d+b*c)^(4/3)-1/2*d^2*ln((-a*d
+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(5/3)/(-a*d+b*c)^(4/3)+1/3*d^2*arc
tan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/(
-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [C] time = 1.29, antiderivative size = 542, normalized size of antiderivative = 2.37, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-9c^2x^6(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-9d^2x^{12}(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-18cdx^9(bc-ad)^2$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (28*c^4*(a + b*x^3)^2 + 168*c^3*d*x^3*(a + b*x^3)^2 + 126*c^2*d^2*x^6*(a +
b*x^3)^2 - 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 126*c^2*d^2*x^6*(a + b*x^3)^2
*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15*c^2
*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] - 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] - 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1
[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^2*x^6
*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] - 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^2*x^12*HypergeometricPF
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(b*c
- a*d)*x^5*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)} \, dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{x^3\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)} \, dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{28c^4\left(a+bx^3\right)^2+168c^3dx^3\left(a+bx^3\right)^2+126c^2d^2x^6\left(a+bx^3\right)^2-28c^4\left(a+bx^3\right)^2}{}$$

Mathematica [C] time = 0.83, size = 542, normalized size = 2.37

$$\frac{9c^2x^6(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+9d^2x^{12}(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)+18cdx^9(bc-ad)^2\,{}_3F_2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (-28*c^4*(a + b*x^3)^2 - 168*c^3*d*x^3*(a + b*x^3)^2 - 126*c^2*d^2*x^6*(a +
 b*x^3)^2 + 28*c^4*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d
)*x^3)/(c*(a + b*x^3))] + 168*c^3*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3
, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 126*c^2*d^2*x^6*(a + b*x^3)^
2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 15*c^
2*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 42*c*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 27*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F
1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^2*x^
6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 18*c*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^2*x^12*HypergeometricP
FQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(14*c^4*(-(b
*c) + a*d)*x^5*(a + b*x^3)^(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{3}+a\right)^{\frac{4}{3}} \left(d\,x^{3}+c\right) x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+a\right)^{\frac{4}{3}}\left(dx^{3}+c\right)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\left(b\,x^{3}+a\right)^{4/3}\left(d\,x^{3}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^3*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**3*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.759 $\int \frac{1}{x^6(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=287

$$-\frac{(a+bx^3)^{2/3}(6bc-ad)}{5a^2cx^5(bc-ad)}+\frac{(a+bx^3)^{2/3}(-5a^2d^2-3abcd+18b^2c^2)}{10a^3c^2x^2(bc-ad)}-\frac{d^3\log(c+dx^3)}{6c^{8/3}(bc-ad)^{4/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right.}{2c^{8/3}(bc-ad)^{4/3}}$$

```
[Out] b/a/(-a*d+b*c)/x^5/(b*x^3+a)^(1/3)-1/5*(-a*d+6*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^5+1/10*(-5*a^2*d^2-3*a*b*c*d+18*b^2*c^2)*(b*x^3+a)^(2/3)/a^3/c^
2/(-a*d+b*c)/x^2-1/6*d^3*ln(d*x^3+c)/c^(8/3)/(-a*d+b*c)^(4/3)+1/2*d^3*ln((-
a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(8/3)/(-a*d+b*c)^(4/3)-1/3*d^3*
arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3
)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [C] time = 3.86, antiderivative size = 950, normalized size of antiderivative = 3.31, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{297d^3(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+162d^3(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{15}+27d^3(bc-ad)^2\,{}_4F_3}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^6*(a +
 b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*x^3*(a
 + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeometric2
F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*d)^2*x
^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 171
*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 1
0/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*Hyperg
eometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c
- a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 7/3},
 {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d)^2*x^
12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81
*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hypergeometr
icPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27
*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))])/(70*c^5*(b*c - a*d)*x^8*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
```

```
b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^6\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^6\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{56c^5\left(a+bx^3\right)^2-252c^4dx^3\left(a+bx^3\right)^2-1512c^3d^2x^6\left(a+bx^3\right)^2-1134c^2d^3x^9}{}$$

Mathematica [C] time = 2.31, size = 950, normalized size = 3.31

$$-\frac{297d^3(bc-ad)^2\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{15}+162d^3(bc-ad)^2\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{10}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{15}+27d^3(bc-ad)^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -1/70*(56*c^5*(a + b*x^3)^2 - 252*c^4*d*x^3*(a + b*x^3)^2 - 1512*c^3*d^2*x^
6*(a + b*x^3)^2 - 1134*c^2*d^3*x^9*(a + b*x^3)^2 - 56*c^5*(a + b*x^3)^2*Hyp
ergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 252*c^4*d*
x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a +
b*x^3))] + 1512*c^3*d^2*x^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d^3*x^9*(a + b*x^3)^2*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c^3*(b*c - a*
d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 171*c^2*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] + 486*c*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2,
7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 297*d^3*(b*c - a*d)^2*x^15*
Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3
*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 216*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2,
 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 351*c*d^2*(b*c - a*d
)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a
+ b*x^3))] + 162*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1,
10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 27*c^3*(b*c - a*d)^2*x^6*Hyperg
eometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 81*c^2*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 81*c*d^2*(b*c - a*d)^2*x^12*Hyperg
eometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
] + 27*d^3*(b*c - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3
```

```
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^5*(-(b*c) + a*d)*x^8*(a + b*x^3)^
(7/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\left(b\,x^3+a\right)^{4/3}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^6*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**6*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.760 $\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=351

$$\frac{\left(a+bx^3\right)^{2/3}(9bc-4ad)(ad+3bc)}{20a^3c^2x^5(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}(9bc-ad)}{8a^2cx^8(bc-ad)}-\frac{\left(a+bx^3\right)^{2/3}\left(-20a^3d^3-12a^2bcd^2-9ab^2c^2d+81b^3c^3\right)}{40a^4c^3x^2(bc-ad)}$$

```
[Out] b/a/(-a*d+b*c)/x^8/(b*x^3+a)^(1/3)-1/8*(-a*d+9*b*c)*(b*x^3+a)^(2/3)/a^2/c/(
-a*d+b*c)/x^8+1/20*(-4*a*d+9*b*c)*(a*d+3*b*c)*(b*x^3+a)^(2/3)/a^3/c^2/(-a*d
+b*c)/x^5-1/40*(-20*a^3*d^3-12*a^2*b*c*d^2-9*a*b^2*c^2*d+81*b^3*c^3)*(b*x^3
+a)^(2/3)/a^4/c^3/(-a*d+b*c)/x^2+1/6*d^4*ln(d*x^3+c)/c^(11/3)/(-a*d+b*c)^(4
/3)-1/2*d^4*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/c^(11/3)/(-a*d+b
*c)^(4/3)+1/3*d^4*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3
))*3^(1/2))/c^(11/3)/(-a*d+b*c)^(4/3)*3^(1/2)
```

Rubi [C] time = 8.30, antiderivative size = 1486, normalized size of antiderivative = 4.23, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (280*c^6*(a + b*x^3)^2 - 672*c^5*d*x^3*(a + b*x^3)^2 + 3024*c^4*d^2*x^6*(a
+ b*x^3)^2 + 18144*c^3*d^3*x^9*(a + b*x^3)^2 + 13608*c^2*d^4*x^12*(a + b*x^
3)^2 - 280*c^6*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 672*c^5*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3024*c^4*d^2*x^6*(a + b*x^3)^2*H
ypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18144*c^
3*d^3*x^9*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c
*(a + b*x^3))] - 13608*c^2*d^4*x^12*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 66*c^4*(b*c - a*d)^2*x^6*Hyperge
ometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 312*c^3*d*(b*
c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 7/3, 10/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 6696*c*d^3*(b*c - a*d)^2*x^15*Hypergeome
tric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4050*d^4*(b*c -
a*d)^2*x^18*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 189*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] - 108*c^3*d*(b*c - a*d)^2*x^9*Hypergeomet
ricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3618*c^
2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c -
a*d)*x^3)/(c*(a + b*x^3))] - 6156*c*d^3*(b*c - a*d)^2*x^15*HypergeometricPF
Q[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2835*d^4*(b*
c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 7/3}, {1, 10/3}, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 54*c^4*(b*c - a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 7/
3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 648*c^3*d*(b*c - a*d
)^2*x^9*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 2268*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2,
2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2376*c*d^3*(b*c
 - a*d)^2*x^15*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)
*x^3)/(c*(a + b*x^3))] - 810*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2
, 2, 7/3}, {1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 81*c^4*(b*c -
 a*d)^2*x^6*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a
```

```
*d)*x^3)/(c*(a + b*x^3))] - 324*c^3*d*(b*c - a*d)^2*x^9*HypergeometricPFQ[{
2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 486
*c^2*d^2*(b*c - a*d)^2*x^12*HypergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1,
10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 324*c*d^3*(b*c - a*d)^2*x^15*Hy
pergeometricPFQ[{2, 2, 2, 2, 7/3}, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] - 81*d^4*(b*c - a*d)^2*x^18*HypergeometricPFQ[{2, 2, 2, 2, 7/3}
, {1, 1, 1, 10/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(560*c^6*(b*c - a*d)
*x^11*(a + b*x^3)^(7/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^9\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= \frac{280c^6\left(a+bx^3\right)^2-672c^5dx^3\left(a+bx^3\right)^2+3024c^4d^2x^6\left(a+bx^3\right)^2+18144c^3a}{}$$

Mathematica [A] time = 5.86, size = 278, normalized size = 0.79

$$\frac{\left(a+bx^3\right)^{2/3}\left(-\frac{x^6\left(20a^2d^2+32abcd+41b^2c^2\right)}{c^3}-\frac{5a^2}{c}+\frac{40b^4x^9}{\left(a+bx^3\right)(ad-bc)}+\frac{2ax^3(4ad+7bc)}{c^2}\right)}{40a^4x^8}+\frac{d^4\left(\log\left(\frac{\sqrt[3]{c}\,x\,\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] ((a + b*x^3)^(2/3)*((-5*a^2)/c + (2*a*(7*b*c + 4*a*d)*x^3)/c^2 - ((41*b^2*c
^2 + 32*a*b*c*d + 20*a^2*d^2)*x^6)/c^3 + (40*b^4*x^9)/((-(b*c) + a*d)*(a +
b*x^3))))/(40*a^4*x^8) + (d^4*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x
)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3
)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)
^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(6*c^(11/3)*(b*
c - a*d)^(4/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
[Out] int(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^9), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^9\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
[Out] int(1/(x^9*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{9}\left(a+bx^{3}\right)^{\frac{4}{3}}\left(c+dx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(1/(x**9*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.761 $\int \frac{x^{10}}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/11*x^11*(1+b*x^3/a)^(1/3)*AppellF1(11/3,4/3,1,14/3,-b*x^3/a,-d*x^3/c)/a/c
/(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^{11}\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^11*(1 + (b*x^3)/a)^(1/3)*AppellF1[11/3, 4/3, 1, 14/3, -((b*x^3)/a), -((d
*x^3)/c)])/(11*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{10}}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^{10}}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^{11}\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{11}{3};\frac{4}{3},1;\frac{14}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{11ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.29, size = 194, normalized size = 2.90

$$\frac{x^2\left(2x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(5a^2d^2-abcd-2b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5c\left(-5a^2d+ab\left(c-dx^3\right)+b^2cx^3\right)+5ac\sqrt[3]{\cdots}\right.}{20b^2cd\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(5*c*(-5*a^2*d + b^2*c*x^3 + a*b*(c - d*x^3)) + 5*a*c*(-(b*c) + 5*a*d)
*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*(-2*b^2*c^2 - a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF
1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(20*b^2*c*d*(b*c - a*d)*(
a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^10/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{10}}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x^10/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x**3+a)**(4/3)/(d*x**3+c),x)
[Out] Integral(x**10/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.762 $\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/8*x^8*(1+b*x^3/a)^(1/3)*AppellF1(8/3,4/3,1,11/3,-b*x^3/a,-d*x^3/c)/a/c/(b
*x^3+a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^8\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^8*(1 + (b*x^3)/a)^(1/3)*AppellF1[8/3, 4/3, 1, 11/3, -((b*x^3)/a), -((d*x
^3)/c)])/(8*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^7}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^8\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{8}{3};\frac{4}{3},1;\frac{11}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{8ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.14, size = 144, normalized size = 2.15

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,(bc-2ad)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5acx^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5acx^2}{5bc\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*a*c*x^2 - 5*a*c*x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((
b*x^3)/a), -((d*x^3)/c)] + (b*c - 2*a*d)*x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1
[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(5*b*c*(b*c - a*d)*(a + b*x
^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(b\,x^3+a)^{\frac{4}{3}}(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x^7/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**7/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.763 $\int \frac{x^4}{(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=67

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/5*x^5*(1+b*x^3/a)^(1/3)*AppellF1(5/3,4/3,1,8/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^5*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 4/3, 1, 8/3, -((b*x^3)/a), -((d*x^
3)/c)])/(5*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x^4}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^5\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{5}{3};\frac{4}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{5ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 129, normalized size = 1.93

$$\frac{x^2\left(dx^3\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5c\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-5c\right)}{5c\sqrt[3]{a+bx^3}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-5*c + 5*c*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)
/a), -((d*x^3)/c)] + d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3,
 -((b*x^3)/a), -((d*x^3)/c)]))/(5*c*(b*c - a*d)*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{{(b\,x^3+a)}^{4/3}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x^4/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x**4/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.764 $\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

```
[Out] 1/2*x^2*(1+b*x^3/a)^(1/3)*AppellF1(2/3,4/3,1,5/3,-b*x^3/a,-d*x^3/c)/a/c/(b*
x^3+a)^(1/3)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {511, 510}

$$\frac{x^2\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 4/3, 1, 5/3, -((b*x^3)/a), -((d*x^
3)/c)])/(2*a*c*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{x}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= \frac{x^2\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(\frac{2}{3};\frac{4}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{2ac\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 141, normalized size = 2.10

$$\frac{x^2\left(2bdx^3\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5\sqrt[3]{\frac{bx^3}{a}+1}\,(ad+bc)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-10bc\right)}{10ac\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (x^2*(-10*b*c + 5*(b*c + a*d)*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5
/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*d*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[
5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]))/(10*a*c*(-(b*c) + a*d)*(a +
 b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(x/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(x/((a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(x/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.765 $\int \frac{1}{x^2(a+bx^3)^{4/3}(c+dx^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

```
[Out] -(1+b*x^3/a)^(1/3)*AppellF1(-1/3,4/3,1,2/3,-b*x^3/a,-d*x^3/c)/a/c/x/(b*x^3+
a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(((1 + (b*x^3)/a)^(1/3)*AppellF1[-1/3, 4/3, 1, 2/3, -((b*x^3)/a), -((d*x^3
)/c)])/(a*c*x*(a + b*x^3)^(1/3)))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^2\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{1}{3};\frac{4}{3},1;\frac{2}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{acx\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.24, size = 193, normalized size = 2.97

$$\frac{-5x^3\sqrt[3]{\frac{bx^3}{a}+1}\left(a^2d^2-abcd+2b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{3},1;\frac{5}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+10c\left(-a^2d+ab\left(c-dx^3\right)+2b^2cx^3\right)+2bdx^6\cdots}{10a^2c^2x\sqrt[3]{a+bx^3}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (10*c*(-(a^2*d) + 2*b^2*c*x^3 + a*b*(c - d*x^3)) - 5*(2*b^2*c^2 - a*b*c*d +
 a^2*d^2)*x^3*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -((b*x^3)/a)
, -((d*x^3)/c)] + 2*b*d*(-2*b*c + a*d)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[5
/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)])/(10*a^2*c^2*(-(b*c) + a*d)*x*
(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

maple [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^3+a\right)^{\frac{4}{3}}\left(d\,x^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,{\left(b\,x^3+a\right)}^{4/3}\,\left(d\,x^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^2*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.766 $\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

```
[Out] -1/4*(1+b*x^3/a)^(1/3)*AppellF1(-4/3,4/3,1,-1/3,-b*x^3/a,-d*x^3/c)/a/c/x^4/
(b*x^3+a)^(1/3)
```

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = $\{511, 510\}$

$$-\frac{\sqrt[3]{\frac{bx^3}{a}+1}\,F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -((1 + (b*x^3)/a)^(1/3)*AppellF1[-4/3, 4/3, 1, -1/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*x^4*(a + b*x^3)^(1/3))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{x^5\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)}\,dx}{a\sqrt[3]{a+bx^3}}\\ &= -\frac{\sqrt[3]{1+\frac{bx^3}{a}}\,F_1\left(-\frac{4}{3};\frac{4}{3},1;-\frac{1}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acx^4\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [B] time = 0.37, size = 264, normalized size = 3.94

$$\frac{-2bdx^9\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^2d^2+abcd-5b^2c^2\right)F_1\left(\frac{5}{3};\frac{1}{3},1;\frac{8}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+5x^6\sqrt[3]{\frac{bx^3}{a}+1}\left(2a^3d^3-2a^2bcd^2-ab^2c^2d+5\right.}{20a^3c^3x^4\sqrt[3]{a+}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (5*c*(-10*b^3*c^2*x^6 + a*b^2*c*x^3*(-5*c + 2*d*x^3) + a^3*d*(-c + 4*d*x^3)
 + a^2*b*(c^2 + c*d*x^3 + 4*d^2*x^6)) + 5*(5*b^3*c^3 - a*b^2*c^2*d - 2*a^2*
b*c*d^2 + 2*a^3*d^3)*x^6*(1 + (b*x^3)/a)^(1/3)*AppellF1[2/3, 1/3, 1, 5/3, -
((b*x^3)/a), -((d*x^3)/c)] - 2*b*d*(-5*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x^9*(
1 + (b*x^3)/a)^(1/3)*AppellF1[5/3, 1/3, 1, 8/3, -((b*x^3)/a), -((d*x^3)/c)]
)/(20*a^3*c^3*(-(b*c) + a*d)*x^4*(a + b*x^3)^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^3+a)^{\frac{4}{3}}\,(d\,x^3+c)\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)^{\frac{4}{3}}(dx^3+c)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5\,(b\,x^3+a)^{4/3}\,(d\,x^3+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)),x)
```

```
[Out] int(1/(x^5*(a + b*x^3)^(4/3)*(c + d*x^3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a + bx^3\right)^{\frac{4}{3}} \left(c + dx^3\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

3.767 $\int \frac{x^{15}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=90

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

```
[Out] -1/4*(a*d+b*c)*x^4/b^2/d^2+1/8*x^8/b/d-1/4*a^3*ln(b*x^4+a)/b^3/(-a*d+b*c)+1
/4*c^3*ln(d*x^4+c)/d^3/(-a*d+b*c)
```

Rubi [A] time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}-\frac{x^4(ad+bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^3*
(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^3}{(a+bx)(c+dx)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d^2}+\frac{x}{bd}-\frac{a^3}{b^2(bc-ad)(a+bx)}-\frac{c^3}{d^2(-bc+ad)(c+dx)}\right)dx, x\\
&= -\frac{(bc+ad)x^4}{4b^2d^2}+\frac{x^8}{8bd}-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 1.02

$$-\frac{a^3\log\left(a+bx^4\right)}{4b^3(bc-ad)}+\frac{x^4(-ad-bc)}{4b^2d^2}+\frac{c^3\log\left(c+dx^4\right)}{4d^3(bc-ad)}+\frac{x^8}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(b*c) - a*d)*x^4)/(4*b^2*d^2) + x^8/(8*b*d) - (a^3*Log[a + b*x^4])/(4*b^
3*(b*c - a*d)) + (c^3*Log[c + d*x^4])/(4*d^3*(b*c - a*d))
```

fricas [A] time = 7.13, size = 100, normalized size = 1.11

$$\frac{\left(b^{3} c d^{2}-a b^{2} d^{3}\right) x^{8}-2 a^{3} d^{3} \log \left(b x^{4}+a\right)+2 b^{3} c^{3} \log \left(d x^{4}+c\right)-2\left(b^{3} c^{2} d-a^{2} b d^{3}\right) x^{4}}{8\left(b^{4} c d^{3}-a b^{3} d^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/8*((b^3*c*d^2 - a*b^2*d^3)*x^8 - 2*a^3*d^3*log(b*x^4 + a) + 2*b^3*c^3*log
(d*x^4 + c) - 2*(b^3*c^2*d - a^2*b*d^3)*x^4)/(b^4*c*d^3 - a*b^3*d^4)
```

giac [A] time = 0.17, size = 88, normalized size = 0.98

$$-\frac{a^{3} \log \left(\left|b x^{4}+a\right|\right)}{4\left(b^{4} c-a b^{3} d\right)}+\frac{c^{3} \log \left(\left|d x^{4}+c\right|\right)}{4\left(b c d^{3}-a d^{4}\right)}+\frac{b d x^{8}-2 b c x^{4}-2 a d x^{4}}{8 b^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a^3*log(abs(b*x^4 + a))/(b^4*c - a*b^3*d) + 1/4*c^3*log(abs(d*x^4 + c)
)/(b*c*d^3 - a*d^4) + 1/8*(b*d*x^8 - 2*b*c*x^4 - 2*a*d*x^4)/(b^2*d^2)
```

maple [A] time = 0.06, size = 89, normalized size = 0.99

$$\frac{x^{8}}{8 b d}-\frac{a x^{4}}{4 b^{2} d}-\frac{c x^{4}}{4 b d^{2}}+\frac{a^{3} \ln \left(b x^{4}+a\right)}{4(a d-b c) b^{3}}-\frac{c^{3} \ln \left(d x^{4}+c\right)}{4(a d-b c) d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*x^8/b/d-1/4/b^2/d*x^4*a-1/4/b/d^2*x^4*c-1/4*c^3/d^3/(a*d-b*c)*ln(d*x^4+
c)+1/4*a^3/b^3/(a*d-b*c)*ln(b*x^4+a)
```

maxima [A] time = 0.47, size = 84, normalized size = 0.93

$$-\frac{a^{3} \log \left(b x^{4}+a\right)}{4\left(b^{4} c-a b^{3} d\right)}+\frac{c^{3} \log \left(d x^{4}+c\right)}{4\left(b c d^{3}-a d^{4}\right)}+\frac{b d x^{8}-2(b c+a d) x^{4}}{8 b^{2} d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^3*log(b*x^4 + a)/(b^4*c - a*b^3*d) + 1/4*c^3*log(d*x^4 + c)/(b*c*d^3
 - a*d^4) + 1/8*(b*d*x^8 - 2*(b*c + a*d)*x^4)/(b^2*d^2)
```

mupad [B] time = 5.88, size = 88, normalized size = 0.98

$$\frac{x^8}{8\,b\,d}-\frac{c^3\,\ln\left(d\,x^4+c\right)}{4\,\left(a\,d^4-b\,c\,d^3\right)}-\frac{a^3\,\ln\left(b\,x^4+a\right)}{4\,\left(b^4\,c-a\,b^3\,d\right)}-\frac{x^4\,\left(a\,d+b\,c\right)}{4\,b^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] x^8/(8*b*d) - (c^3*log(c + d*x^4))/(4*(a*d^4 - b*c*d^3)) - (a^3*log(a + b*x
^4))/(4*(b^4*c - a*b^3*d)) - (x^4*(a*d + b*c))/(4*b^2*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.768 $\int \frac{x^{11}}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

```
[Out] 1/4*x^4/b/d+1/4*a^2*ln(b*x^4+a)/b^2/(-a*d+b*c)-1/4*c^2*ln(d*x^4+c)/d^2/(-a*
d+b*c)
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}+\frac{x^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^4/(4*b*d) + (a^2*Log[a + b*x^4])/(4*b^2*(b*c - a*d)) - (c^2*Log[c + d*x^4
])/(4*d^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^4\right)\\ &= \frac{x^4}{4bd}+\frac{a^2\log\left(a+bx^4\right)}{4b^2(bc-ad)}-\frac{c^2\log\left(c+dx^4\right)}{4d^2(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.94

$$\frac{a^2d^2\log\left(a+bx^4\right)-b\left(dx^4(ad-bc)+bc^2\log\left(c+dx^4\right)\right)}{4b^2d^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^2*d^2*Log[a + b*x^4] - b*(d*(-(b*c) + a*d)*x^4 + b*c^2*Log[c + d*x^4]))/
(4*b^2*d^2*(b*c - a*d))
```

fricas [A] time = 2.32, size = 72, normalized size = 1.03

$$\frac{\left(b^2cd - abd^2\right)x^4 + a^2d^2\log\left(bx^4 + a\right) - b^2c^2\log\left(dx^4 + c\right)}{4\left(b^3cd^2 - ab^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*((b^2*c*d - a*b*d^2)*x^4 + a^2*d^2*log(b*x^4 + a) - b^2*c^2*log(d*x^4 +
 c))/(b^3*c*d^2 - a*b^2*d^3)
```

giac [A] time = 0.20, size = 70, normalized size = 1.00

$$\frac{x^4}{4\,bd} + \frac{a^2\log\left(\left|bx^4 + a\right|\right)}{4\left(b^3c - ab^2d\right)} - \frac{c^2\log\left(\left|dx^4 + c\right|\right)}{4\left(bcd^2 - ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(abs(b*x^4 + a))/(b^3*c - a*b^2*d) - 1/4*c^2*log
(abs(d*x^4 + c))/(b*c*d^2 - a*d^3)
```

maple [A] time = 0.05, size = 65, normalized size = 0.93

$$\frac{x^4}{4bd} - \frac{a^2\ln\left(b\,x^4 + a\right)}{4\left(ad - bc\right)b^2} + \frac{c^2\ln\left(d\,x^4 + c\right)}{4\left(ad - bc\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*x^4/b/d+1/4*c^2/d^2/(a*d-b*c)*ln(d*x^4+c)-1/4*a^2/b^2/(a*d-b*c)*ln(b*x^
4+a)
```

maxima [A] time = 0.57, size = 68, normalized size = 0.97

$$\frac{x^4}{4\,bd} + \frac{a^2\log\left(bx^4 + a\right)}{4\left(b^3c - ab^2d\right)} - \frac{c^2\log\left(dx^4 + c\right)}{4\left(bcd^2 - ad^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/(b*d) + 1/4*a^2*log(b*x^4 + a)/(b^3*c - a*b^2*d) - 1/4*c^2*log(d*x^
4 + c)/(b*c*d^2 - a*d^3)
```

mupad [B] time = 5.63, size = 68, normalized size = 0.97

$$\frac{a^2\,\ln\left(b\,x^4 + a\right)}{4\,b^3\,c - 4\,a\,b^2\,d} + \frac{c^2\,\ln\left(d\,x^4 + c\right)}{4\,a\,d^3 - 4\,b\,c\,d^2} + \frac{x^4}{4\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (a^2*log(a + b*x^4))/(4*b^3*c - 4*a*b^2*d) + (c^2*log(c + d*x^4))/(4*a*d^3
- 4*b*c*d^2) + x^4/(4*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c),x)
[Out] Timed out
```

3.769 $\int \frac{x^7}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=53

$$\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}-\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}$$

```
[Out] -1/4*a*ln(b*x^4+a)/b/(-a*d+b*c)+1/4*c*ln(d*x^4+c)/d/(-a*d+b*c)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}-\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(a*Log[a + b*x^4])/(4*b*(b*c - a*d)) + (c*Log[c + d*x^4])/(4*d*(b*c - a*d)
)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)}+\frac{c}{(bc-ad)(c+dx)}\right)dx,x,x^4\right)\\ &= -\frac{a\log\left(a+bx^4\right)}{4b(bc-ad)}+\frac{c\log\left(c+dx^4\right)}{4d(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.81

$$-\frac{ad\log\left(a+bx^4\right)-bc\log\left(c+dx^4\right)}{4b^2cd-4abd^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((a*d*Log[a + b*x^4] - b*c*Log[c + d*x^4])/(4*b^2*c*d - 4*a*b*d^2))
```

fricas [A] time = 1.34, size = 42, normalized size = 0.79

$$-\frac{ad\log\left(bx^4+a\right)-bc\log\left(dx^4+c\right)}{4\left(b^2cd-abd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(a*d*log(b*x^4 + a) - b*c*log(d*x^4 + c))/(b^2*c*d - a*b*d^2)
```

giac [A] time = 0.19, size = 51, normalized size = 0.96

$$-\frac{a\log\left(\left|bx^4+a\right|\right)}{4\left(b^2c-abd\right)}+\frac{c\log\left(\left|dx^4+c\right|\right)}{4\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*a*log(abs(b*x^4 + a))/(b^2*c - a*b*d) + 1/4*c*log(abs(d*x^4 + c))/(b*c
*d - a*d^2)
```

maple [A] time = 0.05, size = 50, normalized size = 0.94

$$\frac{a\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,b}-\frac{c\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*c/(a*d-b*c)/d*ln(d*x^4+c)+1/4*a/(a*d-b*c)/b*ln(b*x^4+a)
```

maxima [A] time = 0.62, size = 49, normalized size = 0.92

$$-\frac{a\log\left(bx^4+a\right)}{4\left(b^2c-abd\right)}+\frac{c\log\left(dx^4+c\right)}{4\left(bcd-ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*a*log(b*x^4 + a)/(b^2*c - a*b*d) + 1/4*c*log(d*x^4 + c)/(b*c*d - a*d^2
)
```

mupad [B] time = 5.10, size = 51, normalized size = 0.96

$$-\frac{a\,\ln\left(b\,x^4+a\right)}{4\,b^2\,c-4\,a\,b\,d}-\frac{c\,\ln\left(d\,x^4+c\right)}{4\,a\,d^2-4\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (a*log(a + b*x^4))/(4*b^2*c - 4*a*b*d) - (c*log(c + d*x^4))/(4*a*d^2 - 4*
b*c*d)
```

sympy [B] time = 60.85, size = 144, normalized size = 2.72

$$\frac{a\log\left(x^4+\frac{\frac{a^3d^2}{b(ad-bc)}-\frac{2a^2cd}{ad-bc}+\frac{abc^2}{ad-bc}+2ac}{ad+bc}\right)}{4b\,(ad-bc)}-\frac{c\log\left(x^4+\frac{-\frac{a^2cd}{ad-bc}+\frac{2abc^2}{ad-bc}+2ac-\frac{b^2c^3}{d(ad-bc)}}{ad+bc}\right)}{4d\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] a*log(x**4 + (a**3*d**2/(b*(a*d - b*c)) - 2*a**2*c*d/(a*d - b*c) + a*b*c**2
/(a*d - b*c) + 2*a*c)/(a*d + b*c))/(4*b*(a*d - b*c)) - c*log(x**4 + (-a**2*
c*d/(a*d - b*c) + 2*a*b*c**2/(a*d - b*c) + 2*a*c - b**2*c**3/(d*(a*d - b*c)
))/(a*d + b*c))/(4*d*(a*d - b*c))
```

3.770 $\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=45

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

```
[Out] 1/4*ln(b*x^4+a)/(-a*d+b*c)-1/4*ln(d*x^4+c)/(-a*d+b*c)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 36, 31}

$$\frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[a + b*x^4]/(4*(b*c - a*d)) - Log[c + d*x^4]/(4*(b*c - a*d))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{(a+bx)(c+dx)}\,dx,x,x^4\right)\\ &= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^4\right)}{4(bc-ad)}-\frac{d\,\text{Subst}\left(\int \frac{1}{c+dx}\,dx,x,x^4\right)}{4(bc-ad)}\\ &= \frac{\log(a+bx^4)}{4(bc-ad)}-\frac{\log(c+dx^4)}{4(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.69

$$\frac{\log(a+bx^4)-\log(c+dx^4)}{4bc-4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Log[a + b*x^4] - Log[c + d*x^4])/(4*b*c - 4*a*d)
```

fricas [A] time = 0.61, size = 31, normalized size = 0.69

$$\frac{\log\left(bx^4 + a\right) - \log\left(dx^4 + c\right)}{4\,(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(log(b*x^4 + a) - log(d*x^4 + c))/(b*c - a*d)
```

giac [A] time = 0.34, size = 51, normalized size = 1.13

$$\frac{b\log\left(\left|bx^4 + a\right|\right)}{4\left(b^2c - abd\right)} - \frac{d\log\left(\left|dx^4 + c\right|\right)}{4\left(bcd - ad^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b*log(abs(b*x^4 + a))/(b^2*c - a*b*d) - 1/4*d*log(abs(d*x^4 + c))/(b*c*
d - a*d^2)
```

maple [A] time = 0.05, size = 42, normalized size = 0.93

$$-\frac{\ln\left(b\,x^4 + a\right)}{4\,(ad - bc)} + \frac{\ln\left(d\,x^4 + c\right)}{4ad - 4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4/(a*d-b*c)*ln(d*x^4+c)-1/4/(a*d-b*c)*ln(b*x^4+a)
```

maxima [A] time = 0.52, size = 41, normalized size = 0.91

$$\frac{\log\left(bx^4 + a\right)}{4\,(bc - ad)} - \frac{\log\left(dx^4 + c\right)}{4\,(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*log(b*x^4 + a)/(b*c - a*d) - 1/4*log(d*x^4 + c)/(b*c - a*d)
```

mupad [B] time = 4.99, size = 1012, normalized size = 22.49

$$-\frac{\operatorname{atan}\left(\frac{\left(\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\left(384\,a^2\,b^4\,d^6+\cdots\right.}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}\cdots\right)}{4\,a\,d-4\,b\,c}}{\frac{x^4\left(96\,c\,b^5\,d^4+96\,a\,b^4\,d^5\right)+\frac{\frac{x^4\left(512\,a^3\,b^4\,d^7+1536\,a^2\,b^5\,c\,d^6+1536\,a\,b^6\,c^2\,d^5+512\,b^7\,c^3\,d^4\right)+1024\,a\,b^6\,c^3\,d^4+1024\,a^3\,b^4\,c\,d^6+2048\,a^2\,b^5\,c^2\,d^5}{4\,a\,d-4\,b\,c}+x^4\left(384\,a^2\,b^4\,\cdots\right.}{4\,a\,d-4\,b\,c}}{4\,a\,d-4\,b\,c}\cdots}\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] -(atan(((((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512
*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^
4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a
^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a
^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4
*x^4)*1i)/(4*a*d - 4*b*c) - (((x^4*(96*a*b^4*d^5 + 96*b^5*c*d^4) - (x^4*(38
4*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b^5*c*d^5) - (x^4*(512*a^3*b^4*d^7
+ 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 + 1536*a^2*b^5*c*d^6) + 1024*a*b^6*c
^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a
*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d
- 4*b*c) - 8*b^4*d^4*x^4)*1i)/(4*a*d - 4*b*c))/(((x^4*(96*a*b^4*d^5 + 96*b^
5*c*d^4) + ((x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a
*b^5*c*d^5) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d - 4*b*c) + 64*a
*b^4*c*d^4)/(4*a*d - 4*b*c) + 8*b^4*d^4*x^4)/(4*a*d - 4*b*c) + ((x^4*(96*a*
b^4*d^5 + 96*b^5*c*d^4) - (x^4*(384*a^2*b^4*d^6 + 384*b^6*c^2*d^4 + 768*a*b
^5*c*d^5) - (x^4*(512*a^3*b^4*d^7 + 512*b^7*c^3*d^4 + 1536*a*b^6*c^2*d^5 +
1536*a^2*b^5*c*d^6) + 1024*a*b^6*c^3*d^4 + 1024*a^3*b^4*c*d^6 + 2048*a^2*b^
5*c^2*d^5)/(4*a*d - 4*b*c) + 512*a*b^5*c^2*d^4 + 512*a^2*b^4*c*d^5)/(4*a*d
- 4*b*c) + 64*a*b^4*c*d^4)/(4*a*d - 4*b*c) - 8*b^4*d^4*x^4)/(4*a*d - 4*b*c)
))*2i)/(4*a*d - 4*b*c)
```

sympy [B] time = 1.91, size = 138, normalized size = 3.07

$$\frac{\log\left(x^4+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}-\frac{\log\left(x^4+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{4(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] log(x**4 + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**
2/(a*d - b*c) + b*c)/(2*b*d))/(4*(a*d - b*c)) - log(x**4 + (a**2*d**2/(a*d
- b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d)
)/(4*(a*d - b*c))
```

3.771 $\int \frac{1}{x(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=62

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] ln(x)/a/c-1/4*b*ln(b*x^4+a)/a/(-a*d+b*c)+1/4*d*ln(d*x^4+c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^4])/(4*a*(b*c - a*d)) + (d*Log[c + d*x^4])/(4
*c*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx,x,x^4\right)\\
&= \frac{\log(x)}{ac}-\frac{b\log\left(a+bx^4\right)}{4a(bc-ad)}+\frac{d\log\left(c+dx^4\right)}{4c(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.87

$$\frac{-bc\log\left(a+bx^4\right)+ad\log\left(c+dx^4\right)-4ad\log(x)+4bc\log(x)}{4abc^2-4a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (4*b*c*Log[x] - 4*a*d*Log[x] - b*c*Log[a + b*x^4] + a*d*Log[c + d*x^4])/(4*
a*b*c^2 - 4*a^2*c*d)
```

fricas [A] time = 2.05, size = 54, normalized size = 0.87

$$-\frac{bc\log\left(bx^4+a\right)-ad\log\left(dx^4+c\right)-4\,(bc-ad)\log(x)}{4\left(abc^2-a^2cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*log(b*x^4 + a) - a*d*log(d*x^4 + c) - 4*(b*c - a*d)*log(x))/(a*b*
c^2 - a^2*c*d)
```

giac [A] time = 0.18, size = 73, normalized size = 1.18

$$-\frac{b^2\log\left(\left|bx^4+a\right|\right)}{4\left(ab^2c-a^2bd\right)}+\frac{d^2\log\left(\left|dx^4+c\right|\right)}{4\left(bc^2d-acd^2\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/4*b^2*log(abs(b*x^4 + a))/(a*b^2*c - a^2*b*d) + 1/4*d^2*log(abs(d*x^4 +
c))/(b*c^2*d - a*c*d^2) + 1/4*log(x^4)/(a*c)
```

maple [A] time = 0.06, size = 59, normalized size = 0.95

$$\frac{b\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,a}-\frac{d\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,c}+\frac{\ln(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/4*d/c/(a*d-b*c)*ln(d*x^4+c)+1/4*b/a/(a*d-b*c)*ln(b*x^4+a)+1/a/c*ln(x)
```

maxima [A] time = 0.71, size = 61, normalized size = 0.98

$$-\frac{b\log\left(bx^4+a\right)}{4\left(abc-a^2d\right)}+\frac{d\log\left(dx^4+c\right)}{4\left(bc^2-acd\right)}+\frac{\log\left(x^4\right)}{4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/4*b*log(b*x^4 + a)/(a*b*c - a^2*d) + 1/4*d*log(d*x^4 + c)/(b*c^2 - a*c*d
) + 1/4*log(x^4)/(a*c)
```

mupad [B] time = 5.49, size = 58, normalized size = 0.94

$$\frac{b\,\ln\left(b\,x^4+a\right)}{4\,a^2\,d-4\,a\,b\,c}+\frac{d\,\ln\left(d\,x^4+c\right)}{4\,b\,c^2-4\,a\,c\,d}+\frac{\ln(x)}{a\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (b*log(a + b*x^4))/(4*a^2*d - 4*a*b*c) + (d*log(c + d*x^4))/(4*b*c^2 - 4*a*
c*d) + log(x)/(a*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.772 $\int \frac{1}{x^5(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=87

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

```
[Out] -1/4/a/c/x^4-(a*d+b*c)*ln(x)/a^2/c^2+1/4*b^2*ln(b*x^4+a)/a^2/(-a*d+b*c)-1/4
*d^2*ln(d*x^4+c)/c^2/(-a*d+b*c)
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{\log(x)(ad+bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(4*a*c*x^4) - ((b*c + a*d)*Log[x])/(a^2*c^2) + (b^2*Log[a + b*x^4])/(4*a
^2*(b*c - a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2(a+bx)(c+dx)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{acx^2}+\frac{-bc-ad}{a^2c^2x}-\frac{b^3}{a^2(-bc+ad)(a+bx)}-\frac{d^3}{c^2(bc-ad)(c+dx)}\right)dx,\right.\\
&= -\frac{1}{4acx^4}-\frac{(bc+ad)\log(x)}{a^2c^2}+\frac{b^2\log\left(a+bx^4\right)}{4a^2(bc-ad)}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 88, normalized size = 1.01

$$-\frac{b^2\log\left(a+bx^4\right)}{4a^2(ad-bc)}+\frac{\log(x)(-ad-bc)}{a^2c^2}-\frac{d^2\log\left(c+dx^4\right)}{4c^2(bc-ad)}-\frac{1}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/4*1/(a*c*x^4) + ((-(b*c) - a*d)*Log[x])/(a^2*c^2) - (b^2*Log[a + b*x^4])
/(4*a^2*(-(b*c) + a*d)) - (d^2*Log[c + d*x^4])/(4*c^2*(b*c - a*d))
```

fricas [A] time = 8.11, size = 99, normalized size = 1.14

$$\frac{b^2c^2x^4\log\left(bx^4+a\right)-a^2d^2x^4\log\left(dx^4+c\right)-4\left(b^2c^2-a^2d^2\right)x^4\log(x)-abc^2+a^2cd}{4\left(a^2bc^3-a^3c^2d\right)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*c^2*x^4*log(b*x^4 + a) - a^2*d^2*x^4*log(d*x^4 + c) - 4*(b^2*c^2 -
 a^2*d^2)*x^4*log(x) - a*b*c^2 + a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^4)
```

giac [A] time = 0.19, size = 112, normalized size = 1.29

$$\frac{b^3\log\left(\left|bx^4+a\right|\right)}{4\left(a^2b^2c-a^3bd\right)}-\frac{d^3\log\left(\left|dx^4+c\right|\right)}{4\left(bc^3d-ac^2d^2\right)}-\frac{(bc+ad)\log\left(x^4\right)}{4\,a^2c^2}+\frac{bcx^4+adx^4-ac}{4\,a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*b^3*log(abs(b*x^4 + a))/(a^2*b^2*c - a^3*b*d) - 1/4*d^3*log(abs(d*x^4 +
 c))/(b*c^3*d - a*c^2*d^2) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) + 1/4*(b*c*
x^4 + a*d*x^4 - a*c)/(a^2*c^2*x^4)
```

maple [A] time = 0.06, size = 87, normalized size = 1.00

$$-\frac{b^2\ln\left(b\,x^4+a\right)}{4\,(ad-bc)\,a^2}+\frac{d^2\ln\left(d\,x^4+c\right)}{4\,(ad-bc)\,c^2}-\frac{d\ln(x)}{a\,c^2}-\frac{b\ln(x)}{a^2c}-\frac{1}{4ac\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/4*d^2/c^2/(a*d-b*c)*ln(d*x^4+c)-1/4*b^2/a^2/(a*d-b*c)*ln(b*x^4+a)-1/4/a/c
/x^4-1/a/c^2*ln(x)*d-1/a^2/c*ln(x)*b
```

maxima [A] time = 0.55, size = 87, normalized size = 1.00

$$\frac{b^2\log\left(bx^4+a\right)}{4\left(a^2bc-a^3d\right)}-\frac{d^2\log\left(dx^4+c\right)}{4\left(bc^3-ac^2d\right)}-\frac{(bc+ad)\log\left(x^4\right)}{4\,a^2c^2}-\frac{1}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*log(b*x^4 + a)/(a^2*b*c - a^3*d) - 1/4*d^2*log(d*x^4 + c)/(b*c^3 -
a*c^2*d) - 1/4*(b*c + a*d)*log(x^4)/(a^2*c^2) - 1/4/(a*c*x^4)
```

mupad [B] time = 6.21, size = 87, normalized size = 1.00

$$-\frac{b^2\,\ln\left(b\,x^4+a\right)}{4\,\left(a^3\,d-a^2\,b\,c\right)}-\frac{d^2\,\ln\left(d\,x^4+c\right)}{4\,\left(b\,c^3-a\,c^2\,d\right)}-\frac{1}{4\,a\,c\,x^4}-\frac{\ln(x)\,\left(a\,d+b\,c\right)}{a^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - (b^2*log(a + b*x^4))/(4*(a^3*d - a^2*b*c)) - (d^2*log(c + d*x^4))/(4*(b*c
^3 - a*c^2*d)) - 1/(4*a*c*x^4) - (log(x)*(a*d + b*c))/(a^2*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.773 $\int \frac{x^{13}}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=112

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

```
[Out] -1/2*(a*d+b*c)*x^2/b^2/d^2+1/6*x^6/b/d-1/2*a^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/b^(5/2)/(-a*d+b*c)+1/2*c^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(5/2)/(-a*
d+b*c)
```

Rubi [A] time = 0.27, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 479, 582, 522, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)}-\frac{x^2(ad+bc)}{2b^2d^2}+\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}+\frac{x^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -((b*c + a*d)*x^2)/(2*b^2*d^2) + x^6/(6*b*d) - (a^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*b^(5/2)*(b*c - a*d)) + (c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*d^(5/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13}}{(a+bx^4)(c+dx^4)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)\\
&= \frac{x^6}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2(3ac+3(bc+ad)x^2)}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6bd}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} + \frac{\text{Subst}\left(\int \frac{3ac(bc+ad)+3(b^2c^2+ad(bc+ad))x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{6b^2d^2}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b^2(bc-ad)} + \frac{c^3\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d^2(bc-ad)}\\
&= -\frac{(bc+ad)x^2}{2b^2d^2} + \frac{x^6}{6bd} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{5/2}(bc-ad)} + \frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.93

$$\frac{1}{6}\left(\frac{3a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{5/2}(ad-bc)} + \frac{x^2\left(-3ad-3bc+bdx^4\right)}{b^2d^2} + \frac{3c^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{d^{5/2}(bc-ad)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((x^2*(-3*b*c - 3*a*d + b*d*x^4))/(b^2*d^2) + (3*a^(5/2)*ArcTan[(Sqrt[b]*x^
2)/Sqrt[a]])/(b^(5/2)*(-(b*c) + a*d)) + (3*c^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqr
t[c]])/(d^(5/2)*(b*c - a*d)))/6
```

fricas [A] time = 2.61, size = 576, normalized size = 5.14

$$\left[\frac{2\left(b^2cd-abd^2\right)x^6 - 3\,a^2d^2\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) - 3\,b^2c^2\sqrt{-\frac{c}{d}}\,\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 6\left(b^2c^2-a^2d^2\right)x^2}{12\left(b^3cd^2-ab^2d^3\right)}, \frac{2\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 3*a^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^
2*sqrt(-a/b) - a)/(b*x^4 + a)) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*
sqrt(-c/d) - c)/(d*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^
2*d^3), 1/12*(2*(b^2*c*d - a*b*d^2)*x^6 - 6*a^2*d^2*sqrt(a/b)*arctan(b*x^2*
sqrt(a/b)/a) - 3*b^2*c^2*sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d
```

```
*x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/12*(2*(b
^2*c*d - a*b*d^2)*x^6 + 6*b^2*c^2*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) - 3*a
^2*d^2*sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) - 6*(b^
2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3), 1/6*((b^2*c*d - a*b*d^2)*x^6
 - 3*a^2*d^2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + 3*b^2*c^2*sqrt(c/d)*arct
an(d*x^2*sqrt(c/d)/c) - 3*(b^2*c^2 - a^2*d^2)*x^2)/(b^3*c*d^2 - a*b^2*d^3)]
```

giac [A] time = 0.19, size = 112, normalized size = 1.00

$$-\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^3c - ab^2d\right)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd^2 - ad^3\right)\sqrt{cd}} + \frac{b^2d^2x^6 - 3\,b^2cdx^2 - 3\,abd^2x^2}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b^2*d^2*x^6 - 3*
b^2*c*d*x^2 - 3*a*b*d^2*x^2)/(b^3*d^3)
```

maple [A] time = 0.06, size = 105, normalized size = 0.94

$$\frac{x^6}{6bd} + \frac{a^3 \arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\,(ad-bc)\,\sqrt{ab}\;b^2} - \frac{c^3 \arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2\,(ad-bc)\,\sqrt{cd}\;d^2} - \frac{a\,x^2}{2b^2d} - \frac{c\,x^2}{2b\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/6*x^6/b/d-1/2/b^2/d*x^2*a-1/2/b/d^2*x^2*c-1/2*c^3/d^2/(a*d-b*c)/(c*d)^(1/
2)*arctan(d*x^2/(c*d)^(1/2))+1/2*a^3/b^2/(a*d-b*c)/(a*b)^(1/2)*arctan(x^2*b
/(a*b)^(1/2))
```

maxima [A] time = 1.32, size = 100, normalized size = 0.89

$$-\frac{a^3 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^3c - ab^2d\right)\sqrt{ab}} + \frac{c^3 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd^2 - ad^3\right)\sqrt{cd}} + \frac{bdx^6 - 3\,(bc + ad)x^2}{6\,b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a^3*arctan(b*x^2/sqrt(a*b))/((b^3*c - a*b^2*d)*sqrt(a*b)) + 1/2*c^3*ar
ctan(d*x^2/sqrt(c*d))/((b*c*d^2 - a*d^3)*sqrt(c*d)) + 1/6*(b*d*x^6 - 3*(b*c
 + a*d)*x^2)/(b^2*d^2)
```

mupad [B] time = 5.70, size = 532, normalized size = 4.75

$$\frac{\ln\left(d^{10}\left(-a^5\,b^5\right)^{5/2} + b^{20}\,c^{10}\,\sqrt{-a^5\,b^5} - a^2\,b^{23}\,c^{10}\,x^2 - a^{12}\,b^{13}\,d^{10}\,x^2 + 2\,b^{10}\,c^5\,d^5\,\left(-a^5\,b^5\right)^{3/2} + 2\,a^7\,b^{18}\,c^5\,d^5\,x^2\right)}{4\,b^6\,c - 4\,a\,b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^10*(-a^5*b^5)^(5/2) + b^20*c^10*(-a^5*b^5)^(1/2) - a^2*b^23*c^10*x^2
 - a^12*b^13*d^10*x^2 + 2*b^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^7*b^18*c^5*d^
5*x^2)*(-a^5*b^5)^(1/2))/(4*b^6*c - 4*a*b^5*d) - (log(d^10*(-a^5*b^5)^(5/2)
```

```
 + b^20*c^10*(-a^5*b^5)^(1/2) + a^2*b^23*c^10*x^2 + a^12*b^13*d^10*x^2 + 2*
b^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^7*b^18*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(b^6*c - a*b^5*d)) - (log(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1
/2) + a^10*c^2*d^23*x^2 + b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3
/2) - 2*a^5*b^5*c^7*d^18*x^2)*(-c^5*d^5)^(1/2))/(4*(a*d^6 - b*c*d^5)) + (lo
g(b^10*(-c^5*d^5)^(5/2) + a^10*d^20*(-c^5*d^5)^(1/2) - a^10*c^2*d^23*x^2 -
b^10*c^12*d^13*x^2 + 2*a^5*b^5*d^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^7*d^18*x
^2)*(-c^5*d^5)^(1/2))/(4*a*d^6 - 4*b*c*d^5) + x^6/(6*b*d) - (x^2*(a*d + b*c
))/(2*b^2*d^2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.774 $\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

```
[Out] 1/2*x^2/b/d+1/2*a^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/b^(3/2)/(-a*d+b*c)-1/2*
c^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/d^(3/2)/(-a*d+b*c)
```

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 479, 522, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)}-\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}+\frac{x^2}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x^2/(2*b*d) + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*b^(3/2)*(b*c - a*d
)) - (c^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*d^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^9}{(a+bx^4)(c+dx^4)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^4}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)$$
$$= \frac{x^2}{2bd} - \frac{\text{Subst}\left(\int \frac{ac+(bc+ad)x^2}{(a+bx^2)(c+dx^2)}\,dx, x, x^2\right)}{2bd}$$
$$= \frac{x^2}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2b(bc-ad)} - \frac{c^2\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2d(bc-ad)}$$
$$= \frac{x^2}{2bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2b^{3/2}(bc-ad)} - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2d^{3/2}(bc-ad)}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.89

$$\frac{\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{b^{3/2}} + x^2\left(\frac{c}{d}-\frac{a}{b}\right) - \frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{d^{3/2}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-(a/b) + c/d)*x^2 + (a^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/b^(3/2) - (c^
(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/d^(3/2))/(2*b*c - 2*a*d)
```

fricas [A] time = 0.78, size = 416, normalized size = 4.52

$$\left[-\frac{ad\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4-2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right) + bc\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4+2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right) - 2\,(bc-ad)x^2}{4\left(b^2cd-abd^2\right)}, \frac{2\,ad\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right) - bc\sqrt{-\frac{c}{d}}\ldots}{4\left(b^2c\ldots\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + b
*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) - 2*(b*c -
a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/4*(2*a*d*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/
a) - b*c*sqrt(-c/d)*log((d*x^4 + 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)) + 2*(
b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), -1/4*(2*b*c*sqrt(c/d)*arctan(d*x^2*sqr
t(c/d)/c) + a*d*sqrt(-a/b)*log((b*x^4 - 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)
) - 2*(b*c - a*d)*x^2)/(b^2*c*d - a*b*d^2), 1/2*(a*d*sqrt(a/b)*arctan(b*x^2
*sqrt(a/b)/a) - b*c*sqrt(c/d)*arctan(d*x^2*sqrt(c/d)/c) + (b*c - a*d)*x^2)/
(b^2*c*d - a*b*d^2)]
```

giac [A] time = 0.18, size = 80, normalized size = 0.87

$$\frac{a^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(b^2c-abd\right)\sqrt{ab}} - \frac{c^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bcd-ad^2\right)\sqrt{cd}} + \frac{x^2}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

maple [A] time = 0.06, size = 81, normalized size = 0.88

$$-\frac{a^{2} \arctan\left(\frac{b\,x^{2}}{\sqrt{ab}}\right)}{2\,(ad-bc)\,\sqrt{ab}\;b}+\frac{c^{2} \arctan\left(\frac{d\,x^{2}}{\sqrt{cd}}\right)}{2\,(ad-bc)\,\sqrt{cd}\;d}+\frac{x^{2}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2/b/d*x^2+1/2*c^2/d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2
*a^2/b/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

maxima [A] time = 1.19, size = 80, normalized size = 0.87

$$\frac{a^{2} \arctan\left(\frac{bx^{2}}{\sqrt{ab}}\right)}{2\left(b^{2}c-abd\right)\sqrt{ab}}-\frac{c^{2} \arctan\left(\frac{dx^{2}}{\sqrt{cd}}\right)}{2\left(bcd-ad^{2}\right)\sqrt{cd}}+\frac{x^{2}}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*arctan(b*x^2/sqrt(a*b))/((b^2*c - a*b*d)*sqrt(a*b)) - 1/2*c^2*arcta
n(d*x^2/sqrt(c*d))/((b*c*d - a*d^2)*sqrt(c*d)) + 1/2*x^2/(b*d)
```

mupad [B] time = 5.72, size = 518, normalized size = 5.63

$$\frac{\ln\left(b^{9}\,c^{6}\,\sqrt{-a^{3}\,b^{3}}-a^{3}\,d^{6}\,\left(-a^{3}\,b^{3}\right)^{3/2}+a\,b^{11}\,c^{6}\,x^{2}+a^{7}\,b^{5}\,d^{6}\,x^{2}+2\,b^{3}\,c^{3}\,d^{3}\,\left(-a^{3}\,b^{3}\right)^{3/2}-2\,a^{4}\,b^{8}\,c^{3}\,d^{3}\,x^{2}\right)\,\sqrt{-a^{3}\,b}}{4\,b^{4}\,c-4\,a\,b^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(b^9*c^6*(-a^3*b^3)^(1/2) - a^3*d^6*(-a^3*b^3)^(3/2) + a*b^11*c^6*x^2 +
 a^7*b^5*d^6*x^2 + 2*b^3*c^3*d^3*(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*
(-a^3*b^3)^(1/2))/(4*b^4*c - 4*a*b^3*d) - (log(a^3*d^6*(-a^3*b^3)^(3/2) - b
^9*c^6*(-a^3*b^3)^(1/2) + a*b^11*c^6*x^2 + a^7*b^5*d^6*x^2 - 2*b^3*c^3*d^3*
(-a^3*b^3)^(3/2) - 2*a^4*b^8*c^3*d^3*x^2)*(-a^3*b^3)^(1/2))/(4*(b^4*c - a*b
^3*d)) - (log(b^6*c^3*(-c^3*d^3)^(3/2) - a^6*d^9*(-c^3*d^3)^(1/2) + a^6*c*d
^11*x^2 + b^6*c^7*d^5*x^2 - 2*a^3*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*
d^8*x^2)*(-c^3*d^3)^(1/2))/(4*(a*d^4 - b*c*d^3)) + (log(a^6*d^9*(-c^3*d^3)^
(1/2) - b^6*c^3*(-c^3*d^3)^(3/2) + a^6*c*d^11*x^2 + b^6*c^7*d^5*x^2 + 2*a^3
*b^3*d^3*(-c^3*d^3)^(3/2) - 2*a^3*b^3*c^4*d^8*x^2)*(-c^3*d^3)^(1/2))/(4*a*d
^4 - 4*b*c*d^3) + x^2/(2*b*d)
```

sympy [B] time = 29.70, size = 932, normalized size = 10.13

$$-\frac{\sqrt{-\frac{a^{3}}{b^{3}}}\log\left(x^{2}+\frac{-\frac{a^{4}d^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}-\frac{a^{3}b^{3}d^{6}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{a^{2}b^{4}cd^{5}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}+\frac{ab^{5}c^{2}d^{4}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{6}c^{3}d^{3}\left(-\frac{a^{3}}{b^{3}}\right)^{\frac{3}{2}}}{(ad-bc)^{3}}-\frac{b^{4}c^{4}\sqrt{-\frac{a^{3}}{b^{3}}}}{ad-bc}}{a^{3}cd^{2}+a^{2}bc^{2}d+ab^{2}c^{3}}\right)}{4\,(ad-bc)}+\frac{\sqrt{-\frac{a^{3}}{b^{3}}}\log\left(x^{2}+\frac{-\frac{a^{4}d^{4}}{a\cdots}}{}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-a**3/b**3)*log(x**2 + (-a**4*d**4*sqrt(-a**3/b**3)/(a*d - b*c) - a**
3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + a**2*b**4*c*d**5*(-a**3/b*
*3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/(a*d - b*c
)**3 - b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - b**4*c**4*sqrt(-
a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**3))/(4*(a*
d - b*c)) + sqrt(-a**3/b**3)*log(x**2 + (a**4*d**4*sqrt(-a**3/b**3)/(a*d -
b*c) + a**3*b**3*d**6*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a**2*b**4*c*d**5
*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4*(-a**3/b**3)**(3/2)/
(a*d - b*c)**3 + b**6*c**3*d**3*(-a**3/b**3)**(3/2)/(a*d - b*c)**3 + b**4*c
**4*sqrt(-a**3/b**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d + a*b**2*c**
3))/(4*(a*d - b*c)) - sqrt(-c**3/d**3)*log(x**2 + (-a**4*d**4*sqrt(-c**3/d*
*3)/(a*d - b*c) - a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a**2*
b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + a*b**5*c**2*d**4*(-c**3/d*
*3)**(3/2)/(a*d - b*c)**3 - b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*b*c**2*d +
 a*b**2*c**3))/(4*(a*d - b*c)) + sqrt(-c**3/d**3)*log(x**2 + (a**4*d**4*sqr
t(-c**3/d**3)/(a*d - b*c) + a**3*b**3*d**6*(-c**3/d**3)**(3/2)/(a*d - b*c)*
*3 - a**2*b**4*c*d**5*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 - a*b**5*c**2*d**4
*(-c**3/d**3)**(3/2)/(a*d - b*c)**3 + b**6*c**3*d**3*(-c**3/d**3)**(3/2)/(a
*d - b*c)**3 + b**4*c**4*sqrt(-c**3/d**3)/(a*d - b*c))/(a**3*c*d**2 + a**2*
b*c**2*d + a*b**2*c**3))/(4*(a*d - b*c)) + x**2/(2*b*d)
```

3.775 $\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx$

Optimal. Leaf size=79

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

```
[Out] -1/2*arctan(x^2*b^(1/2)/a^(1/2))*a^(1/2)/(-a*d+b*c)/b^(1/2)+1/2*arctan(x^2*
d^(1/2)/c^(1/2))*c^(1/2)/(-a*d+b*c)/d^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 481, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[b]*(b*c - a*d)) + (Sqrt[c]
*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[d]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx^4)(c+dx^4)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)(c+dx^2)} dx, x, x^2\right)\\ &= -\frac{a\,\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, x^2\right)}{2(bc-ad)}+\frac{c\,\text{Subst}\left(\int \frac{1}{c+dx^2} dx, x, x^2\right)}{2(bc-ad)}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2\sqrt{b}(bc-ad)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2\sqrt{d}(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{d}}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{b}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*(c + d*x^4)),x]
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[b]) + (Sqrt[c]*ArcTan[(Sqrt
[d]*x^2)/Sqrt[c]])/Sqrt[d])/(2*b*c - 2*a*d)
```

fricas [A] time = 0.53, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^4+2\,bx^2\sqrt{-\frac{a}{b}}-a}{bx^4+a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},-\frac{2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx^2\sqrt{\frac{a}{b}}}{a}\right)+\sqrt{-\frac{c}{d}}\log\left(\frac{dx^4-2\,dx^2\sqrt{-\frac{c}{d}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
[Out] [-1/4*(sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)) + sqrt(
-c/d)*log((d*x^4 - 2*d*x^2*sqrt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), -1/4*
(2*sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) + sqrt(-c/d)*log((d*x^4 - 2*d*x^2*sq
rt(-c/d) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(2*sqrt(c/d)*arctan(d*x^2*sqrt
(c/d)/c) - sqrt(-a/b)*log((b*x^4 + 2*b*x^2*sqrt(-a/b) - a)/(b*x^4 + a)))/(b
*c - a*d), -1/2*(sqrt(a/b)*arctan(b*x^2*sqrt(a/b)/a) - sqrt(c/d)*arctan(d*x
^2*sqrt(c/d)/c))/(b*c - a*d)]
```

giac [A] time = 0.17, size = 59, normalized size = 0.75

$$-\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

maple [A] time = 0.06, size = 60, normalized size = 0.76

$$\frac{a\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\,(ad-bc)\sqrt{ab}}-\frac{c\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2\,(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c),x)
[Out] -1/2*c/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)+1/2*a/(a*d-b*c)/(a
*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

maxima [A] time = 1.16, size = 59, normalized size = 0.75

$$-\frac{a\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}+\frac{c\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) + 1/2*c*arctan(d*x^2
/sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

mupad [B] time = 5.34, size = 379, normalized size = 4.80

$$\frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}+b^4\,c^2\,\sqrt{-a\,b}-b^5\,c^2\,x^2+2\,b^2\,c\,d\,{\left(-a\,b\right)}^{3/2}-a^2\,b^3\,d^2\,x^2+2\,a\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,b^2\,c-4\,a\,b\,d}-\frac{\ln\left(d^2\,{\left(-a\,b\right)}^{5/2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) - b^5*c^2*x^2 + 2*b^2*c*d*(-a*
b)^(3/2) - a^2*b^3*d^2*x^2 + 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*b^2*c - 4*a*
b*d) - (log(d^2*(-a*b)^(5/2) + b^4*c^2*(-a*b)^(1/2) + b^5*c^2*x^2 + 2*b^2*c
*d*(-a*b)^(3/2) + a^2*b^3*d^2*x^2 - 2*a*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*(b^2*
c - a*b*d)) - (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) + a^2*d^5*x^2 +
2*a*b*d^2*(-c*d)^(3/2) + b^2*c^2*d^3*x^2 - 2*a*b*c*d^4*x^2)*(-c*d)^(1/2))/(
4*(a*d^2 - b*c*d)) + (log(b^2*(-c*d)^(5/2) + a^2*d^4*(-c*d)^(1/2) - a^2*d^5
*x^2 + 2*a*b*d^2*(-c*d)^(3/2) - b^2*c^2*d^3*x^2 + 2*a*b*c*d^4*x^2)*(-c*d)^(
1/2))/(4*a*d^2 - 4*b*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.776 $\int \frac{x}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

```
[Out] 1/2*arctan(x^2*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-1/2*arctan(x^2*d
^(1/2)/c^(1/2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {465, 391, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*
ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*Sqrt[c]*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\
&= \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2(bc-ad)}-\frac{d\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2(bc-ad)}\\
&= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(bc-ad)}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2\sqrt{c}\,(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.84

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{\sqrt{c}}}{2bc-2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]
*x^2)/Sqrt[c]])/Sqrt[c])/(2*b*c - 2*a*d)
```

fricas [A] time = 0.53, size = 325, normalized size = 4.11

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)}{4\,(bc-ad)},\ \frac{2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right)-\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)}{4\,(bc-ad)},\ -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a)) + sqrt(
-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)))/(b*c - a*d), 1/4*(
2*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - sqrt(-b/a)*log((b*x^4 - 2*a*x^2*s
qrt(-b/a) - a)/(b*x^4 + a)))/(b*c - a*d), -1/4*(2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*x^2)) + sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c))
)/(b*c - a*d), -1/2*(sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - sqrt(d/c)*arct
an(c*sqrt(d/c)/(d*x^2)))/(b*c - a*d)]
```

giac [A] time = 0.18, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

maple [A] time = 0.06, size = 60, normalized size = 0.76

$$-\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\,(ad-bc)\sqrt{ab}}+\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(ad-bc)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2*b/(a*d-b*c)/(a*
b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

maxima [A] time = 1.28, size = 59, normalized size = 0.75

$$\frac{b\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\sqrt{ab}\,(bc-ad)}-\frac{d\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\,(bc-ad)\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b*arctan(b*x^2/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - 1/2*d*arctan(d*x^2/
sqrt(c*d))/((b*c - a*d)*sqrt(c*d))
```

mupad [B] time = 5.29, size = 399, normalized size = 5.05

$$\frac{\ln\left(a^2\,d^2\,{(-a\,b)}^{5/2}+b^2\,c^2\,{(-a\,b)}^{5/2}+2\,c\,d\,{(-a\,b)}^{7/2}-a^2\,b^5\,c^2\,x^2-a^4\,b^3\,d^2\,x^2+2\,a^3\,b^4\,c\,d\,x^2\right)\,\sqrt{-a\,b}}{4\,a^2\,d-4\,a\,b\,c}-\frac{\ln\left(a^2\,d^2\,(-a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b)^(7/2) - a^2
*b^5*c^2*x^2 - a^4*b^3*d^2*x^2 + 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/2))/(4*a^2*d
- 4*a*b*c) - (log(a^2*d^2*(-a*b)^(5/2) + b^2*c^2*(-a*b)^(5/2) + 2*c*d*(-a*b
)^(7/2) + a^2*b^5*c^2*x^2 + a^4*b^3*d^2*x^2 - 2*a^3*b^4*c*d*x^2)*(-a*b)^(1/
2))/(4*(a^2*d - a*b*c)) - (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*(-c*d)^(5/2)
+ 2*a*b*(-c*d)^(7/2) + a^2*c^2*d^5*x^2 + b^2*c^4*d^3*x^2 - 2*a*b*c^3*d^4*x^
2)*(-c*d)^(1/2))/(4*(b*c^2 - a*c*d)) + (log(a^2*d^2*(-c*d)^(5/2) + b^2*c^2*
(-c*d)^(5/2) + 2*a*b*(-c*d)^(7/2) - a^2*c^2*d^5*x^2 - b^2*c^4*d^3*x^2 + 2*a
*b*c^3*d^4*x^2)*(-c*d)^(1/2))/(4*b*c^2 - 4*a*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.777 $\int \frac{1}{x^3(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=92

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

```
[Out] -1/2/a/c/x^2-1/2*b^(3/2)*arctan(x^2*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)+1/2
*d^(3/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*c)
```

Rubi [A] time = 0.12, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {465, 480, 522, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}-\frac{1}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(2*a*c*x^2) - (b^(3/2)*ArcTan[(Sqrt[b]*x^2)/Sqrt[a]])/(2*a^(3/2)*(b*c -
a*d)) + (d^(3/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2acx^2} + \frac{\operatorname{Subst}\left(\int \frac{-bc-ad-bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx, x, x^2\right)}{2ac}$$
$$= -\frac{1}{2acx^2} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, x^2\right)}{2a(bc-ad)} + \frac{d^2\operatorname{Subst}\left(\int \frac{1}{c+dx^2}\,dx, x, x^2\right)}{2c(bc-ad)}$$
$$= -\frac{1}{2acx^2} - \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)}$$

Mathematica [A] time = 0.23, size = 169, normalized size = 1.84

$$\frac{-\frac{b^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{3/2}} - \frac{b^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{3/2}} + \frac{b}{a} + \frac{d^{3/2}x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{c^{3/2}} + \frac{d^{3/2}x^2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}+1\right)}{c^{3/2}} - \frac{d}{c}}{2x^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (b^(3/2)*x^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2)
- (b^(3/2)*x^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/2) + (d^(3/2)*
x^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2) + (d^(3/2)*x^2*ArcTan[
1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(3/2))/(2*(-(b*c) + a*d)*x^2)
```

fricas [A] time = 0.87, size = 432, normalized size = 4.70

$$\left[-\frac{bcx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx^4+2ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right) + adx^2\sqrt{-\frac{d}{c}}\log\left(\frac{dx^4-2cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right) + 2bc - 2ad}{4\left(abc^2-a^2cd\right)x^2}, -\frac{2adx^2\sqrt{\frac{d}{c}}\arctan\left(\frac{c\sqrt{\frac{d}{c}}}{dx^2}\right) + bc}{4(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/4*(b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 + a))
 + a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) + 2
*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), -1/4*(2*a*d*x^2*sqrt(d/c)*arctan(c
*sqrt(d/c)/(d*x^2)) + b*c*x^2*sqrt(-b/a)*log((b*x^4 + 2*a*x^2*sqrt(-b/a) -
a)/(b*x^4 + a)) + 2*b*c - 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2), 1/4*(2*b*c*x^2*
sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(-d/c)*log((d*x^4 - 2*c
*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) - 2*b*c + 2*a*d)/((a*b*c^2 - a^2*c*d)*x^2
), 1/2*(b*c*x^2*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - a*d*x^2*sqrt(d/c)*a
rctan(c*sqrt(d/c)/(d*x^2)) - b*c + a*d)/((a*b*c^2 - a^2*c*d)*x^2)]
```

giac [A] time = 0.21, size = 80, normalized size = 0.87

$$-\frac{b^2\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(abc-a^2d\right)\sqrt{ab}} + \frac{d^2\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^2-acd\right)\sqrt{cd}} - \frac{1}{2\,acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

maple [A] time = 0.06, size = 81, normalized size = 0.88

$$\frac{b^2 \arctan\left(\frac{b x^2}{\sqrt{ab}}\right)}{2 (ad - bc) \sqrt{ab}\, a} - \frac{d^2 \arctan\left(\frac{d x^2}{\sqrt{cd}}\right)}{2 (ad - bc) \sqrt{cd}\, c} - \frac{1}{2ac x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/2*d^2/c/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2/a/c/x^2+1/
2*b^2/a/(a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)
```

maxima [A] time = 1.20, size = 80, normalized size = 0.87

$$-\frac{b^2 \arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2 \left(abc - a^2 d\right)\sqrt{ab}} + \frac{d^2 \arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2 \left(bc^2 - acd\right)\sqrt{cd}} - \frac{1}{2\, acx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*arctan(b*x^2/sqrt(a*b))/((a*b*c - a^2*d)*sqrt(a*b)) + 1/2*d^2*arct
an(d*x^2/sqrt(c*d))/((b*c^2 - a*c*d)*sqrt(c*d)) - 1/2/(a*c*x^2)
```

mupad [B] time = 5.35, size = 354, normalized size = 3.85

$$\frac{\ln\left(c^3\, x^2\, \left(-a^3\, b^3\right)^{3/2} - a^8\, b\, d^3 + a^5\, b^4\, c^3 + a^6\, d^3\, x^2\, \sqrt{-a^3\, b^3}\right)\, \sqrt{-a^3\, b^3}}{4\, a^4\, d - 4\, a^3\, b\, c} - \frac{\ln\left(c^3\, x^2\, \left(-a^3\, b^3\right)^{3/2} + a^8\, b\, d^3 - a^5\, b^4\, c^3}{4\, \left(a^4\, d - a^3\, b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^3*x^2*(-a^3*b^3)^(3/2) - a^8*b*d^3 + a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3
*b^3)^(1/2))*(-a^3*b^3)^(1/2))/(4*a^4*d - 4*a^3*b*c) - (log(c^3*x^2*(-a^3*b
^3)^(3/2) + a^8*b*d^3 - a^5*b^4*c^3 + a^6*d^3*x^2*(-a^3*b^3)^(1/2))*(-a^3*b
^3)^(1/2))/(4*(a^4*d - a^3*b*c)) - 1/(2*a*c*x^2) - (log(a^3*x^2*(-c^3*d^3)^
(3/2) + b^3*c^8*d - a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^
(1/2))/(4*(b*c^4 - a*c^3*d)) + (log(a^3*x^2*(-c^3*d^3)^(3/2) - b^3*c^8*d +
a^3*c^5*d^4 + b^3*c^6*x^2*(-c^3*d^3)^(1/2))*(-c^3*d^3)^(1/2))/(4*b*c^4 - 4*
a*c^3*d)
```

sympy [B] time = 84.89, size = 1103, normalized size = 11.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] -sqrt(-b**3/a**3)*log(x**2 + (-a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d - b*
c)**3 + 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - 2*a**5*b**2
*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-b**3/a**3)/
(a*d - b*c) + 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - a**3*
b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-b**3/a**3)/(
```

```
a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d - b*
c)) + sqrt(-b**3/a**3)*log(x**2 + (a**7*c**3*d**4*(-b**3/a**3)**(3/2)/(a*d
- b*c)**3 - 2*a**6*b*c**4*d**3*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + 2*a**5*
b**2*c**5*d**2*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt(-b**3/a*
*3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + a
**3*b**4*c**7*(-b**3/a**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(-b**3/a**
3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*(a*d
- b*c)) - sqrt(-d**3/c**3)*log(x**2 + (-a**7*c**3*d**4*(-d**3/c**3)**(3/2)/
(a*d - b*c)**3 + 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - 2*
a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - a**5*d**5*sqrt(-d*
*3/c**3)/(a*d - b*c) + 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*c)**
3 - a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 - b**5*c**5*sqrt(-d**
3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))/(4*
(a*d - b*c)) + sqrt(-d**3/c**3)*log(x**2 + (a**7*c**3*d**4*(-d**3/c**3)**(3
/2)/(a*d - b*c)**3 - 2*a**6*b*c**4*d**3*(-d**3/c**3)**(3/2)/(a*d - b*c)**3
+ 2*a**5*b**2*c**5*d**2*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + a**5*d**5*sqrt
(-d**3/c**3)/(a*d - b*c) - 2*a**4*b**3*c**6*d*(-d**3/c**3)**(3/2)/(a*d - b*
c)**3 + a**3*b**4*c**7*(-d**3/c**3)**(3/2)/(a*d - b*c)**3 + b**5*c**5*sqrt(
-d**3/c**3)/(a*d - b*c))/(a**2*b**2*d**4 + a*b**3*c*d**3 + b**4*c**2*d**2))
/(4*(a*d - b*c)) - 1/(2*a*c*x**2)
```

3.778 $\int \frac{1}{x^7(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=112

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

```
[Out] -1/6/a/c/x^6+1/2*(a*d+b*c)/a^2/c^2/x^2+1/2*b^(5/2)*arctan(x^2*b^(1/2)/a^(1/
2))/a^(5/2)/(-a*d+b*c)-1/2*d^(5/2)*arctan(x^2*d^(1/2)/c^(1/2))/c^(5/2)/(-a*
d+b*c)
```

Rubi [A] time = 0.22, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 480, 583, 522, 205}

$$\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}+\frac{ad+bc}{2a^2c^2x^2}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}-\frac{1}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(6*a*c*x^6) + (b*c + a*d)/(2*a^2*c^2*x^2) + (b^(5/2)*ArcTan[(Sqrt[b]*x^2
)/Sqrt[a]])/(2*a^(5/2)*(b*c - a*d)) - (d^(5/2)*ArcTan[(Sqrt[d]*x^2)/Sqrt[c]
])/(2*c^(5/2)*(b*c - a*d))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
```

```
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\left(c+dx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{1}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3(bc+ad)-3bdx^2}{x^2\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}-\frac{\text{Subst}\left(\int \frac{-3\left(b^2c^2+abcd+a^2d^2\right)-3bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,x^2\right)}{2a^2(bc-ad)}-\frac{d^3\,\text{Subst}\left(\int \frac{1}{c+dx^2}\,dx,x,x^2\right)}{2c^2(bc-ad)}\\
&= -\frac{1}{6acx^6}+\frac{bc+ad}{2a^2c^2x^2}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(bc-ad)}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c}}\right)}{2c^{5/2}(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 193, normalized size = 1.72

$$\frac{\frac{3b^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}}+\frac{3b^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\right)}{a^{5/2}}-\frac{3b^2x^4}{a^2}+\frac{b}{a}-\frac{3d^{5/2}x^6\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{c^{5/2}}-\frac{3d^{5/2}x^6\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}+1\right)}{c^{5/2}}+\frac{3d^2x^4}{c^2}-\frac{d}{c}}{6x^6(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (b/a - d/c - (3*b^2*x^4)/a^2 + (3*d^2*x^4)/c^2 + (3*b^(5/2)*x^6*ArcTan[1 -
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(5/2) + (3*b^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/a^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 - (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/c^(5/2) - (3*d^(5/2)*x^6*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/c^(5/2))/(6*(-(b*c) + a*d)*x^6)
```

fricas [A] time = 3.46, size = 592, normalized size = 5.29

$$\left[-\frac{3\,b^2c^2x^6\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^4-2\,ax^2\sqrt{-\frac{b}{a}}-a}{bx^4+a}\right)+3\,a^2d^2x^6\sqrt{-\frac{d}{c}}\,\log\left(\frac{dx^4+2\,cx^2\sqrt{-\frac{d}{c}}-c}{dx^4+c}\right)-6\left(b^2c^2-a^2d^2\right)x^4+2\,abc^2-2\,a^2cd}{12\left(a^2bc^3-a^3c^2d\right)x^6},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^
4 + a)) + 3*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*
x^4 + c)) - 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3
- a^3*c^2*d)*x^6), 1/12*(6*a^2*d^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)
) - 3*b^2*c^2*x^6*sqrt(-b/a)*log((b*x^4 - 2*a*x^2*sqrt(-b/a) - a)/(b*x^4 +
```

```
a)) + 6*(b^2*c^2 - a^2*d^2)*x^4 - 2*a*b*c^2 + 2*a^2*c*d)/((a^2*b*c^3 - a^3*
c^2*d)*x^6), -1/12*(6*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) + 3
*a^2*d^2*x^6*sqrt(-d/c)*log((d*x^4 + 2*c*x^2*sqrt(-d/c) - c)/(d*x^4 + c)) -
 6*(b^2*c^2 - a^2*d^2)*x^4 + 2*a*b*c^2 - 2*a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d
)*x^6), -1/6*(3*b^2*c^2*x^6*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*x^2)) - 3*a^2*d
^2*x^6*sqrt(d/c)*arctan(c*sqrt(d/c)/(d*x^2)) - 3*(b^2*c^2 - a^2*d^2)*x^4 +
a*b*c^2 - a^2*c*d)/((a^2*b*c^3 - a^3*c^2*d)*x^6)]
```

giac [A] time = 0.18, size = 103, normalized size = 0.92

$$\frac{b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{ab}}-\frac{d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3-ac^2d\right)\sqrt{cd}}+\frac{3\,bcx^4+3\,adx^4-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*b*c*x^4 + 3*a*d
*x^4 - a*c)/(a^2*c^2*x^6)
```

maple [A] time = 0.06, size = 105, normalized size = 0.94

$$-\frac{b^3\arctan\left(\frac{b\,x^2}{\sqrt{ab}}\right)}{2\left(ad-bc\right)\sqrt{ab}\;a^2}+\frac{d^3\arctan\left(\frac{d\,x^2}{\sqrt{cd}}\right)}{2\left(ad-bc\right)\sqrt{cd}\;c^2}+\frac{d}{2a\,c^2x^2}+\frac{b}{2a^2c\,x^2}-\frac{1}{6ac\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/2*d^3/c^2/(a*d-b*c)/(c*d)^(1/2)*arctan(1/(c*d)^(1/2)*d*x^2)-1/2*b^3/a^2/(
a*d-b*c)/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^2)-1/6/a/c/x^6+1/2/a/c^2/x^2*
d+1/2/a^2/c/x^2*b
```

maxima [A] time = 1.51, size = 101, normalized size = 0.90

$$\frac{b^3\arctan\left(\frac{bx^2}{\sqrt{ab}}\right)}{2\left(a^2bc-a^3d\right)\sqrt{ab}}-\frac{d^3\arctan\left(\frac{dx^2}{\sqrt{cd}}\right)}{2\left(bc^3-ac^2d\right)\sqrt{cd}}+\frac{3\left(bc+ad\right)x^4-ac}{6\,a^2c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/2*b^3*arctan(b*x^2/sqrt(a*b))/((a^2*b*c - a^3*d)*sqrt(a*b)) - 1/2*d^3*arc
tan(d*x^2/sqrt(c*d))/((b*c^3 - a*c^2*d)*sqrt(c*d)) + 1/6*(3*(b*c + a*d)*x^4
 - a*c)/(a^2*c^2*x^6)
```

mupad [B] time = 5.53, size = 535, normalized size = 4.78

$$\frac{\ln\left(c^{10}\left(-a^5\,b^5\right)^{5/2}+a^{20}\,d^{10}\,\sqrt{-a^5\,b^5}-a^{12}\,b^{13}\,c^{10}\,x^2-a^{22}\,b^3\,d^{10}\,x^2+2\,a^{10}\,c^5\,d^5\left(-a^5\,b^5\right)^{3/2}+2\,a^{17}\,b^8\,c^5\,d^5\,x^2\right)}{4\,a^6\,d-4\,a^5\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] (log(c^10*(-a^5*b^5)^(5/2) + a^20*d^10*(-a^5*b^5)^(1/2) - a^12*b^13*c^10*x^
2 - a^22*b^3*d^10*x^2 + 2*a^10*c^5*d^5*(-a^5*b^5)^(3/2) + 2*a^17*b^8*c^5*d^
```

```
5*x^2)*(-a^5*b^5)^(1/2))/(4*a^6*d - 4*a^5*b*c) - (log(c^10*(-a^5*b^5)^(5/2)
 + a^20*d^10*(-a^5*b^5)^(1/2) + a^12*b^13*c^10*x^2 + a^22*b^3*d^10*x^2 + 2*
a^10*c^5*d^5*(-a^5*b^5)^(3/2) - 2*a^17*b^8*c^5*d^5*x^2)*(-a^5*b^5)^(1/2))/(
4*(a^6*d - a^5*b*c)) - (1/(6*a*c) - (x^4*(a*d + b*c))/(2*a^2*c^2))/x^6 - (l
og(a^10*(-c^5*d^5)^(5/2) + b^10*c^20*(-c^5*d^5)^(1/2) + a^10*c^12*d^13*x^2
+ b^10*c^22*d^3*x^2 + 2*a^5*b^5*c^10*(-c^5*d^5)^(3/2) - 2*a^5*b^5*c^17*d^8*
x^2)*(-c^5*d^5)^(1/2))/(4*(b*c^6 - a*c^5*d)) + (log(a^10*(-c^5*d^5)^(5/2) +
 b^10*c^20*(-c^5*d^5)^(1/2) - a^10*c^12*d^13*x^2 - b^10*c^22*d^3*x^2 + 2*a^
5*b^5*c^10*(-c^5*d^5)^(3/2) + 2*a^5*b^5*c^17*d^8*x^2)*(-c^5*d^5)^(1/2))/(4*
b*c^6 - 4*a*c^5*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.779 $\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=457

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}}{2}$$

```
[Out] x/b/d+1/4*a^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)+1/4*a^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(5/4)/(-a*d+b*c)*2
^(1/2)-1/4*c^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/4*c^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(5/4)/(-a*d+b*c)*
2^(1/2)-1/8*a^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(5
/4)/(-a*d+b*c)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*b^(1/2))/b^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1
/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(5/4)*ln(c^(1/4)*
d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {479, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{5/4}(bc-ad)}-\frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{5/4}(bc-ad)}+\frac{a^{5/4}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] x/(b*d) - (a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqr
t[2]*b^(5/4)*(b*c - a*d)) + (c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x
)/c^(1/4)])/(2*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (a^(5/4)*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5/4)*(b*c - a*d)) + (a^(5
/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(5
/4)*(b*c - a*d)) + (c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d)) - (c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c
^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
```

```
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^8}{(a+bx^4)(c+dx^4)}\,dx = \frac{x}{bd} - \frac{\int \frac{ac+(bc+ad)x^4}{(a+bx^4)(c+dx^4)}\,dx}{bd}$$

$$= \frac{x}{bd} + \frac{a^2\int \frac{1}{a+bx^4}\,dx}{b(bc-ad)} - \frac{c^2\int \frac{1}{c+dx^4}\,dx}{d(bc-ad)}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} + \frac{a^{3/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2b(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2d(bc-ad)} - \frac{c^{3/2}\int \frac{\sqrt{c}+\sqrt{d}}{c+dx^4}}{2d(bc-a}$$

$$= \frac{x}{bd} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} + \frac{a^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4b^{3/2}(bc-ad)} - \frac{a^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,b^{5/4}(bc-ad)}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}\,b^{5/4}(bc-ad)}$$

$$= \frac{x}{bd} - \frac{a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{a^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{5/4}(bc-ad)} + \frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{5/4}(bc-ad)}$$

Mathematica [A] time = 0.25, size = 377, normalized size = 0.82

$$\frac{-\frac{\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}} + \frac{\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{b^{5/4}} - \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{b^{5/4}} + \frac{2\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+\right.}{b^{5/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((-8*a*x)/b + (8*c*x)/d - (2*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/b^(5/4) + (2*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1
/4)])/b^(5/4) + (2*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/d^(5/4) - (2*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/d^(5
/4) - (Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
2])/b^(5/4) + (Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2])/b^(5/4) + (Sqrt[2]*c^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4
)*x + Sqrt[d]*x^2])/d^(5/4) - (Sqrt[2]*c^(5/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4
)*d^(1/4)*x + Sqrt[d]*x^2])/d^(5/4))/(8*b*c - 8*a*d)
```

fricas [B] time = 0.73, size = 1378, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^
3 + a^4*b^5*d^4))^(1/4)*b*d*arctan(((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*
d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a
^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*x - (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5
*c*d^2 - a^3*b^4*d^3)*(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 -
4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(3/4)*sqrt((a^2*x^2 + (b^4*c^2 - 2*a*b^3*c*
d + a^2*b^2*d^2)*sqrt(-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4
```

```
*a^3*b^6*c*d^3 + a^4*b^5*d^4)))/a^2))/a^4) - 4*(-c^5/(b^4*c^4*d^5 - 4*a*b^3
*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*arctan((
(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4*d^
5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4)*x
 - (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*(-c^5/(b^4*c^4
*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(3/4
)*sqrt((c^2*x^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*sqrt(-c^5/(b^4*c^4*
d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)))/c^2)
)/c^4) - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d
^3 + a^4*b^5*d^4))^(1/4)*b*d*log(a*x + (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a
^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (
-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b
^5*d^4))^(1/4)*b*d*log(a*x - (-a^5/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2
*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4))^(1/4)*(b^2*c - a*b*d)) + (-c^5/(b^4*
c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(
1/4)*b*d*log(c*x + (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7
 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - (-c^5/(b^4*c^4*d^5 -
4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9))^(1/4)*b*d*l
og(c*x - (-c^5/(b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9))^(1/4)*(b*c*d - a*d^2)) - 4*x)/(b*d)
```

giac [A] time = 0.22, size = 469, normalized size = 1.03

$$\frac{\left(ab^3\right)^{\frac14} a \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\left(\sqrt{2}\,b^3c-\sqrt{2}\,ab^2d\right)}+\frac{\left(ab^3\right)^{\frac14} a \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{2\left(\sqrt{2}\,b^3c-\sqrt{2}\,ab^2d\right)}-\frac{\left(cd^3\right)^{\frac14} c \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac14}\right)}{2\left(\frac{c}{d}\right)^{\frac14}}\right)}{2\left(\sqrt{2}\,bcd^2-\sqrt{2}\,ad^3\right)}-\frac{\left(cd^3\right)^{\frac14} c \cdots}{2\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) + 1/2*(a*b^3)^(1/4)*a*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^3*c - sqrt(2)*a*b^
2*d) - 1/2*(c*d^3)^(1/4)*c*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(
c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) - 1/2*(c*d^3)^(1/4)*c*arctan(
1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^2 - sqr
t(2)*a*d^3) + 1/4*(a*b^3)^(1/4)*a*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/
b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(a*b^3)^(1/4)*a*log(x^2 - sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^3*c - sqrt(2)*a*b^2*d) - 1/4*(c*d^
3)^(1/4)*c*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^2 -
sqrt(2)*a*d^3) + 1/4*(c*d^3)^(1/4)*c*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt
(c/d))/(sqrt(2)*b*c*d^2 - sqrt(2)*a*d^3) + x/(b*d)
```

maple [A] time = 0.06, size = 328, normalized size = 0.72

$$-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}-1\right)}{4\left(ad-bc\right)b}-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a \arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac14}}+1\right)}{4\left(ad-bc\right)b}-\frac{\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,a \ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac14}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)b}+\frac{\left(\frac{c}{d}\right)^{\frac14}\sqrt{2}\,c \arctan\cdots}{4\left(ad-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/b/d*x+1/8/d*c/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)
+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4/d*c/(a*d-b*c)*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4/d*c/(a*d-b*c)*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8/b*a/(a*d-b*c)*(a/b)^(1/4)*
```

```
2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)
*x+(a/b)^(1/2)))-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)
^(1/4)*x+1)-1/4/b*a/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4
)*x-1)
```

maxima [A] time = 1.22, size = 375, normalized size = 0.82

$$\frac{\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{b}x^{2}-\sqrt{2}\cdots\right)}{b^{\frac{1}{4}}}}{8\left(b^{2}c-abd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^
(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arc
tan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b
)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b^2*c - a*b*d) - 1/8*(2*sqrt(2)*c^(
3/2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2
*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*s
qrt(d)) + sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqr
t(c))/d^(1/4) - sqrt(2)*c^(5/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x
 + sqrt(c))/d^(1/4))/(b*c*d - a*d^2) + x/(b*d)
```

mupad [B] time = 5.63, size = 6361, normalized size = 13.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*
b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^
5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i - (-
a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*
d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c
^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c
^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b
^8*c^3*d))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i)/((-a^5/(256
*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 10
24*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a
^8*b*c^4*d^5))/(b*d) - (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3
*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c
^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 -
768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256
```

*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d
))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)) + (-a^5/(256*b^9*c^4 +
256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^
3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^
5))/(b*d) + (4*x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3
+ 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b^9*c^8*d^4 - 76
8*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5
*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^
4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4) +
(4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)
*2i - 2*atan(((-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1
536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3
*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*
a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d)
)^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 +
512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))
*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c
^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b
*d)) - (-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2
*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 -
a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5
*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)
*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6
*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/
(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2
- 1024*a*b^8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d)))/(
(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^
2*d^2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5
*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 -
1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a
^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^
5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^
9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*
a*b^8*c^3*d))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i + (-a^
5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^
2 - 1024*a*b^8*c^3*d))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8
*d - a^8*b*c^4*d^5))/(b*d) + (x*(-a^5/(256*b^9*c^4 + 256*a^4*b^5*d^4 - 1024
*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^8*c^3*d))^(3/4)*(256*a^3*b
^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^
7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4i)/(b*d))*(-a^5/(256*b^9*c^
4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 - 1024*a*b^
8*c^3*d))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))*1i))*(-a^5/(2
56*b^9*c^4 + 256*a^4*b^5*d^4 - 1024*a^3*b^6*c*d^3 + 1536*a^2*b^7*c^2*d^2 -
1024*a*b^8*c^3*d))^(1/4) + atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 102
4*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^
3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^
5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^
7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 5
12*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^
4*c^3*d^9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^
6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a
^8*c^4*d^4))/(b*d))*1i - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*
c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^
9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a
^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024
*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b
^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^
9))/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536

```
*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d
^4))/(b*d))*1i)/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6
+ 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*
c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (4*x*(-c^5/(256*a^4*d^9 +
 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c
*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d
^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d
))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2
*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) - (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*
d)) + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*
b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a
^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (4*x*(-c^5/(256*a^4*d^9 + 256*b^4*c^
4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4
)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c^6*d^6 + 512*a^
6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9))/(b*d))*(-c^5/(2
56*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 -
1024*a^3*b*c*d^8))^(1/4) + (4*x*(a^4*b^4*c^8 + a^8*c^4*d^4))/(b*d))))*(-c^5
/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7
 - 1024*a^3*b*c*d^8))^(1/4)*2i - 2*atan(((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d
^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(
((16*(a^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (
x*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*
c^2*d^7 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d
^5 + 512*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*
a^8*b^4*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b
^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4
*b^4*c^8 + a^8*c^4*d^4))/(b*d)) - (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 10
24*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a
^3*b^6*c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5
/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7
 - 1024*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 51
2*a^5*b^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4
*c^3*d^9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*
d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^
8 + a^8*c^4*d^4))/(b*d)))/((-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^
3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*
c^9 + a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) - (x*(-c^5/(256*a
^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024
*a^3*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b
^7*c^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^
9)*4i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1
536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i + (4*x*(a^4*b^4*c^8 + a^8
*c^4*d^4))/(b*d))*1i + (-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^
3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*(((16*(a^3*b^6*c^9
+ a^9*c^3*d^6 - a^4*b^5*c^8*d - a^8*b*c^4*d^5))/(b*d) + (x*(-c^5/(256*a^4*d
^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3
*b*c*d^8))^(3/4)*(256*a^3*b^9*c^8*d^4 - 768*a^4*b^8*c^7*d^5 + 512*a^5*b^7*c
^6*d^6 + 512*a^6*b^6*c^5*d^7 - 768*a^7*b^5*c^4*d^8 + 256*a^8*b^4*c^3*d^9)*4
i)/(b*d))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^6 + 1536*
a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4)*1i - (4*x*(a^4*b^4*c^8 + a^8*c^4
*d^4))/(b*d))*1i))*(-c^5/(256*a^4*d^9 + 256*b^4*c^4*d^5 - 1024*a*b^3*c^3*d^
6 + 1536*a^2*b^2*c^2*d^7 - 1024*a^3*b*c*d^8))^(1/4) + x/(b*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.780 $\int \frac{x^6}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}}{2}$$

```
[Out] -1/4*a^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(3/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(3/4)/(-a*d+b*c)*2^(1/
2)-1/8*a^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(3/4)/(
-a*d+b*c)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(3/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 297, 1162, 617, 204, 1165, 628}

$$-\frac{a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}b^{3/4}(bc-ad)}+\frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}b^{3/4}(bc-ad)}-\frac{a^{3/4}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) - (a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(3/
4)*(b*c - a*d)) - (c^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(3/4)*(b*c - a*d)) + (c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c - a*d)) + (a^(3/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(3/4)*(b*c -
 a*d)) + (c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(3/4)*(b*c - a*d)) - (c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{a\int \frac{x^2}{a+bx^4}\,dx}{bc-ad}+\frac{c\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{a\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)}-\frac{a\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{b}\,(bc-ad)}-\frac{c\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{d}\,(bc-ad)}+\frac{c\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{d}\,(bc-ad)}$$

$$= -\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4b(bc-ad)}-\frac{a^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\int \cdots}{4\sqrt{2}\,b^{3/4}\cdots}$$

$$= -\frac{a^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{a^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,b^{3/4}(bc-ad)}+\frac{c^{3/4}\cdots}{\cdots}$$

$$= \frac{a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{a^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,b^{3/4}(bc-ad)}-\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,d^{3/4}(bc-ad)}+\frac{c^{3/4}\tan\cdots}{2\sqrt{\cdots}}$$

Mathematica [A] time = 0.12, size = 340, normalized size = 0.76

$$-a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\cdots}{\sqrt[4]{\cdots}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(3/4)*d^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(3/4)*d^(3
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(3/4)*c^(3/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(3/4)*c^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] - a^(3/4)*d^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] - b^(3/4)*c^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(3/4)*d^(3/4)*(b*c - a*d))
```

fricas [B] time = 0.54, size = 1358, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4
*b^3*d^4))^(1/4)*arctan(((b^2*c - a*b*d)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6
*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*x - (b^2*c - a*b*d
)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^
4*b^3*d^4))^(1/4)*sqrt((a*x^2 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(-a
^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3
*d^4)))/a))/a) + (-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 -
 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*arctan(((b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3
 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*x
- (b*c*d - a*d^2)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5
- 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*sqrt((c*x^2 - (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*sqrt(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a
^3*b*c*d^6 + a^4*d^7)))/c))/c) - 1/4*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2
*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(1/4)*log(a^2*x + (b^5*c^3 -
 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^
3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4))^(3/4)) + 1/4*(-a^
3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*
d^4))^(1/4)*log(a^2*x - (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^
2*d^3)*(-a^3/(b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
 + a^4*b^3*d^4))^(3/4)) + 1/4*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*
b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(1/4)*log(c^2*x + (b^3*c^3*d^2 - 3*
a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d
^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(3/4)) - 1/4*(-c^3/(b^4*
c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7))^(
1/4)*log(c^2*x - (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*
(-c^3/(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 +
a^4*d^7))^(3/4))
```

giac [A] time = 0.24, size = 453, normalized size = 1.01

$$-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d\right)}-\frac{\left(ab^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^{4}c-\sqrt{2}\,ab^{3}d\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd^{3}-\sqrt{2}\,ad^{4}\right)}+\frac{\left(cd^{3}\right)^{\frac{3}{4}} a}{2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2
)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d
) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^
(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*
d^4) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt
(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^
(1/4) + sqrt(a/b))/(sqrt(2)*b^4*c - sqrt(2)*a*b^3*d) - 1/4*(c*d^3)^(3/4)*lo
g(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d^3 - sqrt(2)*a*d^4
) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c*d^3 - sqrt(2)*a*d^4)
```

maple [A] time = 0.05, size = 320, normalized size = 0.71

$$\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, a \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}+\frac{\sqrt{2}\, a \ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\, x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}} b}-\frac{\sqrt{2}\, c \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}} d}-\frac{\sqrt{2}\, c \arc}{4\left(ad\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*c/(a*d-b*c)/d/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^
(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4*c/(a*d-b*c)/d/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4*c/(a*d-b*c)/d/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)+1/8*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+1/4*a/(a*d-b*c)/b/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.42, size = 363, normalized size = 0.81

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4)
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))
/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4
)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) + 1/8*c*(2*sq
rt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(
c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)
*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(
c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x
+ sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

mupad [B] time = 5.72, size = 2553, normalized size = 5.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((4*b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c
*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4) + 4*a^3*b*d^3*x*(-a^
3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^
2 - 1024*a*b^6*c^3*d))^(1/4) + 2048*a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4) + 2048*b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^
3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a*b^7*
c^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*
a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4) - 8192*a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4) + 12288*a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4))/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256*a^4*b^
3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4
) - atan((b^4*c^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d
^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^3*b*d^3*x*(-a^3
/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2
 - 1024*a*b^6*c^3*d))^(1/4)*4i + a^4*b^4*d^7*x*(-a^3/(256*b^7*c^4 + 256*a^4
*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(
5/4)*2048i + b^8*c^4*d^3*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*
b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*2048i - a*b^7*c
^3*d^4*x*(-a^3/(256*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a
^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d))^(5/4)*8192i - a^3*b^5*c*d^6*x*(-a^3/(25
6*b^7*c^4 + 256*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1
024*a*b^6*c^3*d))^(5/4)*8192i + a^2*b^6*c^2*d^5*x*(-a^3/(256*b^7*c^4 + 256*
a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d)
)^(5/4)*12288i)/(a^3*d^2 + a*b^2*c^2 + a^2*b*c*d))*(-a^3/(256*b^7*c^4 + 256
*a^4*b^3*d^4 - 1024*a^3*b^4*c*d^3 + 1536*a^2*b^5*c^2*d^2 - 1024*a*b^6*c^3*d
))^(1/4)*2i - 2*atan((4*a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 10
24*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4) + 2048*b
^7*c^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 15
36*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 4*b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4) + 2048*a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d
^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) -
 8192*a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3
*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) - 8192*a^3*b^4*c*d^7
*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4) + 12288*a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4))/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a^4*d^7
+ 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*
c*d^6))^(1/4) - atan((a^3*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024
*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(1/4)*4i + b^7*c
^4*d^4*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a
^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*2048i + b^3*c^3*d*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(1/4)*4i + a^4*b^3*d^8*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3
 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*204
8i - a*b^6*c^3*d^5*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*
d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i - a^3*b^4*c*d^7
*x*(-c^3/(256*a^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2
```

```
*c^2*d^5 - 1024*a^3*b*c*d^6))^(5/4)*8192i + a^2*b^5*c^2*d^6*x*(-c^3/(256*a^
4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024*
a^3*b*c*d^6))^(5/4)*12288i)/(b^2*c^3 + a^2*c*d^2 + a*b*c^2*d))*(-c^3/(256*a
^4*d^7 + 256*b^4*c^4*d^3 - 1024*a*b^3*c^3*d^4 + 1536*a^2*b^2*c^2*d^5 - 1024
*a^3*b*c*d^6))^(1/4)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.781 $\int \frac{x^4}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\right.}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*a^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/b^(1/4)/(-a*d+b*c)*2^(1/2
)+1/4*c^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/4*c^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/d^(1/4)/(-a*d+b*c)*2^(1/
2)+1/8*a^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/b^(1/4)/(
-a*d+b*c)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1
/2))/b^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*c^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c
^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*c^(1/4)*ln(c^(1/4)*d^(1/
4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/d^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {481, 211, 1165, 628, 1162, 617, 204}

$$\frac{\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{b}(bc-ad)}-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\right.}{4\sqrt{2}\sqrt[4]{d}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*b^(1/
4)*(b*c - a*d)) - (c^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt
[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(2*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c - a*d)) - (a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*b^(1/4)*(b*c -
 a*d)) - (c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(
4*Sqrt[2]*d^(1/4)*(b*c - a*d)) + (c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*d^(1/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
```

```
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{a\int \frac{1}{a+bx^4}\,dx}{bc-ad} + \frac{c\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{a}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{a}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{c}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{c}\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)}$$

$$= -\frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}\,(bc-ad)} - \frac{\sqrt{a}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{b}\,(bc-ad)} + \frac{\sqrt[4]{a}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)} + \frac{\sqrt[4]{a}\ \ldots}{4\ldots}$$

$$= \frac{\sqrt[4]{a}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{c}\ \mathrm{l}\ldots}{}$$

$$= \frac{\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{a}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{b}\,(bc-ad)} - \frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{d}\,(bc-ad)} + \frac{\sqrt[4]{c}\ \mathrm{ta}\ldots}{2\sqrt{}\ldots}$$

Mathematica [A] time = 0.11, size = 340, normalized size = 0.76

$$\sqrt[4]{a}\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) - \sqrt[4]{a}\sqrt[4]{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) + 2\sqrt[4]{a}\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\ldots}{\sqrt[4]{a}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (2*a^(1/4)*d^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*a^(1/4)*d^(1
/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*b^(1/4)*c^(1/4)*ArcTan[1 -
(Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*b^(1/4)*c^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4
)*x)/c^(1/4)] + a^(1/4)*d^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqr
t[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2] + b^(1/4)*c^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(4*Sqrt[2]*b^(1/4)*d^(1/4)*(b*c - a*d))
```

fricas [B] time = 0.49, size = 1238, normalized size = 2.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4))^(1/4)*arctan(((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)
*x*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4
*b*d^4))^(3/4) - (b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sq
rt(x^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)))*(-a/(b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(3/4))/a) - (-c/(
b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))
^(1/4)*arctan(((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(3/4) - (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*sqrt(x^
2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)))*(-c/(b^4*c^4*d - 4*a*b^3*c^3
*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(3/4))/c) - 1/4*(-a/(b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^
(1/4)*log((b*c - a*d)*(-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4) + x) + 1/4*(-a/(b^5*c^4 - 4*a*b^4*c^3*d +
 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4))^(1/4)*log(-(b*c - a*d)*(
-a/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d
^4))^(1/4) + x) + 1/4*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
- 4*a^3*b*c*d^4 + a^4*d^5))^(1/4)*log((b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*
c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x) - 1/4*(-
c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^
5))^(1/4)*log(-(b*c - a*d)*(-c/(b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2
*d^3 - 4*a^3*b*c*d^4 + a^4*d^5))^(1/4) + x)
```

giac [A] time = 0.24, size = 437, normalized size = 0.97

$$-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}-\frac{\left(ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,b^2c-\sqrt{2}\,abd\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bcd-\sqrt{2}\,ad^2\right)}+\frac{\left(cd^3\right)^{\frac{1}{4}}\,\mathrm{a}}{2\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) - 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) +
```

```
1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) + 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1
/4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*b^2*
c - sqrt(2)*a*b*d) + 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*b^2*c - sqrt(2)*a*b*d) + 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt
(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(2)*a*d^2) - 1/4*(c*d^3
)^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c*d - sqrt(
2)*a*d^2)
```

maple [A] time = 0.06, size = 296, normalized size = 0.66

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4ad-4bc}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4ad-4bc}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\right.}{4\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2
))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(c/d)^(1/4)*x-1)+1/8/(a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(
1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/(a
*d-b*c)*(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/(a*d-b*c)*(
a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.39, size = 361, normalized size = 0.80

$$-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}\right.}{b^{\frac{1}{4}}}}{8\left(bc-ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b
^(1/4))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*ar
ctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(
b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(sqrt(b)*x^2 - sqrt(2
)*a^(1/4)*b^(1/4)*x + sqrt(a))/b^(1/4))/(b*c - a*d) + 1/8*(2*sqrt(2)*sqrt(c
)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*s
qrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*sqrt(c)*arctan(1/2*sqrt(2)*(2*sq
rt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt
(d)) + sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c
))/d^(1/4) - sqrt(2)*c^(1/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x +
sqrt(c))/d^(1/4))/(b*c - a*d)
```

mupad [B] time = 5.86, size = 5889, normalized size = 13.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)),x)
```

[Out] - atan((a^2*d^2*x*1i + b^2*c^2*x*1i - (a^6*b*d^6*x*256i)/(256*b^5*c^4 + 256 *a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a*b^6*c^5*d*x*256i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) + (a^5*b^2*c*d^5*x*768i)/(256*b^5* c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^ 4*c^3*d) + (a^2*b^5*c^4*d^2*x*768i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3 *b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - (a^3*b^4*c^3*d^3*x* 512i)/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2* d^2 - 1024*a*b^4*c^3*d) - (a^4*b^3*c^2*d^4*x*512i)/(256*b^5*c^4 + 256*a^4*b *d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))/((-a/ (256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((a*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6* c^5*d^2 - 6144*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^ 4 + 15360*a^4*b^3*c^2*d^5))/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d ^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a *b^2*c^2*d + 4*a^2*b*c*d^2)))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b ^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*2i - atan((a^2*d ^2*x*1i + b^2*c^2*x*1i - (b^6*c^6*d*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^5*b*c*d^ 6*x*256i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2* c^2*d^3 - 1024*a^3*b*c*d^4) + (a*b^5*c^5*d^2*x*768i)/(256*a^4*d^5 + 256*b^4 *c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a ^2*b^4*c^4*d^3*x*512i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - (a^3*b^3*c^3*d^4*x*512i)/(256*a^ 4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^ 3*b*c*d^4) + (a^4*b^2*c^2*d^5*x*768i)/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a *b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))/((-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c* d^4))^(1/4)*((c*(1024*a^6*b*d^7 + 1024*b^7*c^6*d - 6144*a*b^6*c^5*d^2 - 614 4*a^5*b^2*c*d^6 + 15360*a^2*b^5*c^4*d^3 - 20480*a^3*b^4*c^3*d^4 + 15360*a^4 *b^3*c^2*d^5))/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2 *b^2*c^2*d^3 - 1024*a^3*b*c*d^4) - 4*b^3*c^3 - 4*a^3*d^3 + 4*a*b^2*c^2*d + 4*a^2*b*c*d^2)))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 15 36*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*2i - 2*atan(((x*(4*a^2*b^5*c^ 4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^ 2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9 *c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d ^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256* a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^ (1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^ 6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2 *c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^ 5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^ 2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d ^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1 024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^ 3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 6 1440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d ^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4))/ ((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b* d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)* ((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4)*1i))*(-a/(256*b^5*c^4 + 256*a^4*b*d^4 - 1024*a^3*b^2*c*d^3 + 1536*a^2*b^3*c^2*d^2 - 1024*a*b^4*c^3*d))^(1/4) - 2*atan(((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4) + (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4))/((x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 - 3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 81920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i + 16*a^2*b^6*c^5*d^3 - 16*a^3*b^5*c^4*d^4 - 16*a^4*b^4*c^3*d^5 + 16*a^5*b^3*c^2*d^6)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*1i - (x*(4*a^2*b^5*c^4*d^3 + 4*a^4*b^3*c^2*d^5) - (-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d

```
^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)*((x*(1024*a^2*b^9*c^7*
d^4 - 3072*a^3*b^8*c^6*d^5 + 2048*a^4*b^7*c^5*d^6 + 2048*a^5*b^6*c^4*d^7 -
3072*a^6*b^5*c^3*d^8 + 1024*a^7*b^4*c^2*d^9) + (-c/(256*a^4*d^5 + 256*b^4*c
^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)
*(4096*a^2*b^10*c^8*d^4 - 24576*a^3*b^9*c^7*d^5 + 61440*a^4*b^8*c^6*d^6 - 8
1920*a^5*b^7*c^5*d^7 + 61440*a^6*b^6*c^4*d^8 - 24576*a^7*b^5*c^3*d^9 + 4096
*a^8*b^4*c^2*d^10)*1i)*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^
2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(3/4)*1i - 16*a^2*b^6*c^5*d^3
 + 16*a^3*b^5*c^4*d^4 + 16*a^4*b^4*c^3*d^5 - 16*a^5*b^3*c^2*d^6)*1i)*(-c/(2
56*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 10
24*a^3*b*c*d^4))^(1/4)*1i))*(-c/(256*a^4*d^5 + 256*b^4*c^4*d - 1024*a*b^3*c
^3*d^2 + 1536*a^2*b^2*c^2*d^3 - 1024*a^3*b*c*d^4))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.782 $\int \frac{x^2}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}\right.}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

```
[Out] 1/4*b^(1/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(1/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(1/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(1/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(1/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(1/4)/(-a*d+b*c)*2^(1/2
)+1/8*b^(1/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(1/4)/(-
a*d+b*c)*2^(1/2)-1/8*b^(1/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(1/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(1/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(1/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(1/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {482, 297, 1162, 617, 204, 1165, 628}

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}\sqrt[4]{a}(bc-ad)}-\frac{\sqrt[4]{d}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}\right.}{4\sqrt{2}\sqrt[4]{c}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*(b*c
- a*d)) + (b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1
/4)*(b*c - a*d)) + (d^(1/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(1/4)*(b*c - a*d)) - (d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (b^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c - a*d)) - (b^(1/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(1/4)*(b*c
- a*d)) - (d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(1/4)*(b*c - a*d)) + (d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(1/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
```

```
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{x^2}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{x^2}{c+dx^4}\,dx}{bc-ad}$$

$$= -\frac{\sqrt{b}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{b}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2(bc-ad)} + \frac{\sqrt{d}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)} - \frac{\sqrt{d}\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2(bc-ad)}$$

$$= \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} + \frac{\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)} - \frac{\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{d}}+x^2}\,dx}{4(bc-ad)}$$

$$= \frac{\sqrt[4]{b}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \frac{\sqrt[4]{b}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\,(bc-ad)} - \ldots$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{b}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\sqrt[4]{a}\,(bc-ad)} + \frac{\sqrt[4]{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\sqrt[4]{c}\,(bc-ad)} - \ldots$$

Mathematica [A] time = 0.14, size = 340, normalized size = 0.76

$$\sqrt[4]{b}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) - \sqrt[4]{b}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right) - 2\sqrt[4]{b}\sqrt[4]{c}\tan^{-1}\left(1-\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(1/4)*c^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(1/4)*c^(
1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(1/4)*d^(1/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(1/4)*d^(1/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] + b^(1/4)*c^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] - b^(1/4)*c^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] - a^(1/4)*d^(1/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] + a^(1/4)*d^(1/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(b*c - a*d))
```

fricas [B] time = 0.52, size = 1258, normalized size = 2.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] (-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*
d^4))^(1/4)*arctan((b*c - a*d)*x*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b
^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4) - (b*c - a*d)*sqrt((b*x^2 - (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)))/b)*(-b/(a*b^4*c^4 - 4*a^2*b^3*
c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)) - (-d/(b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*
arctan((b*c - a*d)*x*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a
^3*b*c^2*d^3 + a^4*c*d^4))^(1/4) - (b*c - a*d)*sqrt((d*x^2 - (b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2)*sqrt(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
- 4*a^3*b*c^2*d^3 + a^4*c*d^4)))/d)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^
2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)) + 1/4*(-b/(a*b^4*c^4 - 4*a
^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(1/4)*log(b*x
+ (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(-b/(a*b^4*c^4 -
4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4))^(3/4)) - 1/
4*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^
5*d^4))^(1/4)*log(b*x - (a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*
d^3)*(-b/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 +
 a^5*d^4))^(3/4)) - 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 -
4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x + (b^3*c^4 - 3*a*b^2*c^3*d + 3*
a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2
 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4)) + 1/4*(-d/(b^4*c^5 - 4*a*b^3*c^4*d
+ 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(1/4)*log(d*x - (b^3*c^
4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(-d/(b^4*c^5 - 4*a*b^3*c^4
*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4))^(3/4))
```

giac [A] time = 0.23, size = 477, normalized size = 1.06

$$\frac{\left(ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}+\frac{\left(ab^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,ab^3c-\sqrt{2}\,a^2b^2d\right)}-\frac{\left(cd^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^2d^2-\sqrt{2}\,acd^3\right)}-\frac{\left(cd^3\right)^{\frac{3}{4}} \operatorname{arcta}}{2\left(\sqrt{2}\,bc^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqr
t(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b^3*c - sqrt(2)*a^
```

```
2*b^2*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))
/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/2*(c*d^3)^(3/4)*arc
tan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2*d^2
 - sqrt(2)*a*c*d^3) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + s
qrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d) + 1/4*(a*b^3)^(3/4)*log(x^2
 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^3*c - sqrt(2)*a^2*b^2*d)
 + 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^2*d^2 - sqrt(2)*a*c*d^3) - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^
(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2*d^2 - sqrt(2)*a*c*d^3)
```

maple [A] time = 0.06, size = 296, normalized size = 0.66

$$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\,(ad-bc)\left(\frac{a}{b}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4\,(ad-bc)\left(\frac{c}{d}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\,\ln\left(\frac{x^2-\cdots}{x^2+\cdots}\right)}{8\,(ad-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)
)/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x-1)-1/8/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1
/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))-1/4/(a*
d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)-1/4/(a*d-b*c)/(a
/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.21, size = 363, normalized size = 0.81

$$\frac{b\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(2\,\sqrt{b}x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\,\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}-\frac{\sqrt{2}\,\log\left(\sqrt{b}x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\,\log\left(\sqrt{b}x^2-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x\cdots\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\cdots\right.}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)))/
(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/4)
*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(2)
*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(b*c - a*d) - 1/8*d*(2*sqr
t(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c
)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c
)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x +
 sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c - a*d)
```

mupad [B] time = 5.50, size = 6633, normalized size = 14.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)
*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^
2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10
 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 716
8*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4
*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 -
768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5
 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
 + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 40
96*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5
*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*
d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-b
/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 -
 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^
3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*
d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8
192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*
b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6
 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4
*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)
 - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^
4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(2
56*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d
+ 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 10
24*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*
b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*
d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768
*a^5*b^5*c^2*d^8))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d +
1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + 2*a*b^5*c*d^5))*(-b/(256*
a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*
a^4*b*c*d^3))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (
-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2
 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5*d^4 + 256*a
*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(
1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 71
68*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6
*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^
4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 2
56*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3
))^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*
a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^
(3/4)*(x*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b
^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c
*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7
+ 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256
*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^
3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^
2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4))/((x*(4*a*b^6
*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b
^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(x*(-b/(256*a^5*
```

d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*
b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c
^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a
^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^
2*d^8)*1i)*(-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^
5*c*d^6) + (-b/(256*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3
*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-b/(256*a^5
*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4
*b*c*d^3))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*
c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^
8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 51
2*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-b/(256
*a^5*d^4 + 256*a*b^4*c^4 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024
*a^4*b*c*d^3))^(1/4)*1i + 2*a*b^5*c*d^5))*(-b/(256*a^5*d^4 + 256*a*b^4*c^4
 - 1024*a^2*b^3*c^3*d + 1536*a^3*b^2*c^2*d^2 - 1024*a^4*b*c*d^3))^(1/4) + at
an(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d
^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(
x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*
d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 -
 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*
a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c
*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 76
8*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i + (x*(4*a*b^6*c^2*d^5 +
 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d
/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 -
 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096
*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b
^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^
5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(
256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1
024*a*b^3*c^4*d))^(1/4)*1i)/((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d/(
256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1
024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^
2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^
4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 819
2*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a*b^
9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 +
 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d
^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) -
 (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4
 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256
*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 +
1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024
*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^
7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9) + 256*a^6*b^4*c*d^
9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a
^5*b^5*c^2*d^8))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 15
36*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4) + 2*a*b^5*c*d^5))*(-d/(256*b^
4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*
b^3*c^4*d))^(1/4)*2i + 2*atan(((x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) + (-d
/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 -
 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c^5 + 256*a^4
*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/
4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6*d^5 + 7168
*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 - 4096*a^6*b

```
^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*
d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256
*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))
^(1/4) + (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^
4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3
/4)*(x*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2
*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d
^10 - 4096*a^2*b^9*c^6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 +
7168*a^5*b^6*c^3*d^8 - 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a
^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*
d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*
b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4))/((x*(4*a*b^6*c
^2*d^5 + 4*a^2*b^5*c*d^6) - (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c
^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(x*(-d/(256*b^4*c^
5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*
c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^6
*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8 -
 4096*a^6*b^5*c^2*d^9)*1i + 256*a*b^9*c^6*d^4 + 256*a^6*b^4*c*d^9 - 768*a^2
*b^8*c^5*d^5 + 512*a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*
d^8)*1i)*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)*1i - (x*(4*a*b^6*c^2*d^5 + 4*a^2*b^5*
c*d^6) + (-d/(256*b^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b
^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(3/4)*(256*a*b^9*c^6*d^4 - x*(-d/(256*b^4*c
^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3
*c^4*d))^(1/4)*(1024*a*b^10*c^7*d^4 + 1024*a^7*b^4*c*d^10 - 4096*a^2*b^9*c^
6*d^5 + 7168*a^3*b^8*c^5*d^6 - 8192*a^4*b^7*c^4*d^7 + 7168*a^5*b^6*c^3*d^8
- 4096*a^6*b^5*c^2*d^9)*1i + 256*a^6*b^4*c*d^9 - 768*a^2*b^8*c^5*d^5 + 512*
a^3*b^7*c^4*d^6 + 512*a^4*b^6*c^3*d^7 - 768*a^5*b^5*c^2*d^8)*1i)*(-d/(256*b
^4*c^5 + 256*a^4*c*d^4 - 1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a
*b^3*c^4*d))^(1/4)*1i + 2*a*b^5*c*d^5))*(-d/(256*b^4*c^5 + 256*a^4*c*d^4 -
1024*a^3*b*c^2*d^3 + 1536*a^2*b^2*c^3*d^2 - 1024*a*b^3*c^4*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.783 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}}{2}$$

```
[Out] 1/4*b^(3/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
+1/4*b^(3/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(3/4)/(-a*d+b*c)*2^(1/2)
-1/4*d^(3/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/4*d^(3/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(3/4)/(-a*d+b*c)*2^(1/2
)-1/8*b^(3/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(3/4)/(-
a*d+b*c)*2^(1/2)+1/8*b^(3/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/
2))/a^(3/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(3/4)*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^
(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(3/4)*ln(c^(1/4)*d^(1/4
)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(3/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)}+\frac{b^{3/4}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2\sqrt{a}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}\,dx}{2\sqrt{c}\,(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}\,(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,a^{3/4}(bc-ad)} - \frac{b^{3/4}\int}{4\sqrt{}}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}}{}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{3/4}(bc-ad)} - \frac{d^{3/4}}{2}$$

Mathematica [A] time = 0.20, size = 340, normalized size = 0.76

$$\frac{a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)-a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}\,x+\sqrt{c}+\sqrt{d}\,x^2\right)+2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}}{\sqrt[4]{c}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

fricas [B] time = 0.61, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

giac [A] time = 0.22, size = 437, normalized size = 0.97

$$\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}+\frac{\left(ab^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}abc-\sqrt{2}a^{2}d\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^{2}-\sqrt{2}acd\right)}-\frac{\left(cd^{3}\right)^{\frac{1}{4}}\mathrm{arc}}{2\left(\sqrt{}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) + 1/2*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1
```

```
/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) - 1/2*(c*d^3)^(1/4)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/
4*(a*b^3)^(1/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b*c
 - sqrt(2)*a^2*d) - 1/4*(a*b^3)^(1/4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqr
t(a/b))/(sqrt(2)*a*b*c - sqrt(2)*a^2*d) - 1/4*(c*d^3)^(1/4)*log(x^2 + sqrt(
2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2)*a*c*d) + 1/4*(c*d^3)
^(1/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^2 - sqrt(2
)*a*c*d)
```

maple [A] time = 0.06, size = 320, normalized size = 0.71

$$-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\,(ad-bc)\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\,(ad-bc)\,a}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b\ln\left(\frac{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\,(ad-bc)\,a}+\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d\,\mathrm{arctar}}{4\,(ad-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(
1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d/(a*d-b*c)*(c/d)^(1/4)/
c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*
ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)
^(1/2)))-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x
+1)-1/4*b/(a*d-b*c)*(a/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.38, size = 365, normalized size = 0.81

$$\frac{\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x\right.}{a^{\frac{3}{4}}}}{8\,(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4))
/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 + sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(sqrt(b)*x^2 -
 sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(b*c - a*d) - 1/8*(2*sqrt(2)
*d*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*
sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d*arctan(1/2*sqrt(2)*
(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqr
t(sqrt(c)*sqrt(d))) + sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(
1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(3/4)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/
4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c - a*d)
```

mupad [B] time = 5.85, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)),x)
```

[Out] - atan((((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a
^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3
*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)
*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*
c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^
11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7
 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10)
+ x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6
*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^
3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7
*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^
4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i - ((-d^3/(256*b^
4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*
a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*
d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 +
 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c
^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^
7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d
^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11
 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2
*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c
^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)
*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c
^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*1i)/(((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^
4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((
-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5
*d^2 - 1024*a*b^3*c^6*d))^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 +
256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^
6*d))^(1/4) + ((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +
1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*((-d^3/(256*b^4*c^7 + 256*a
^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))
^(3/4)*((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^
2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^
4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c
^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*
d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 40
96*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*
b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32
*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)))*(-d^3/(256*
b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 102
4*a*b^3*c^6*d))^(1/4)*2i - atan((((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 10
24*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(
256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*
b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^
8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^
6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 -
 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*
a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*
c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 -
16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^

3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*1i - ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 153
6*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*
b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3
/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b
^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c
*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*
d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^1
0) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*
a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7
*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*
b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b
^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*1i)/(((-b^3/(256
*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 10
24*a^6*b*c*d^3))^(1/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3
*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^
4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*
b*c*d^3))^(1/4)*(4096*a*b^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10
*c^7*d^5 + 36864*a^3*b^9*c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^
4*d^8 + 36864*a^6*b^6*c^3*d^9 - 20480*a^7*b^5*c^2*d^10) + x*(1024*a^7*b^4*d
^11 + 1024*b^11*c^7*d^4 - 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*
a^2*b^9*c^5*d^6 - 3072*a^3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^
6*c^2*d^9)) - 16*a^2*b^6*d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) + 8*b^7*d^7
x)(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^
2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) + ((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^
4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*((
-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2
*d^2 - 1024*a^6*b*c*d^3))^(3/4)*((-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 102
4*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*(4096*a*b
^11*c^8*d^4 + 4096*a^8*b^4*c*d^11 - 20480*a^2*b^10*c^7*d^5 + 36864*a^3*b^9*
c^6*d^6 - 20480*a^4*b^8*c^5*d^7 - 20480*a^5*b^7*c^4*d^8 + 36864*a^6*b^6*c^3
*d^9 - 20480*a^7*b^5*c^2*d^10) - x*(1024*a^7*b^4*d^11 + 1024*b^11*c^7*d^4 -
 4096*a*b^10*c^6*d^5 - 4096*a^6*b^5*c*d^10 + 6144*a^2*b^9*c^5*d^6 - 3072*a^
3*b^8*c^4*d^7 - 3072*a^4*b^7*c^3*d^8 + 6144*a^5*b^6*c^2*d^9)) - 16*a^2*b^6*
d^8 - 16*b^8*c^2*d^6 + 32*a*b^7*c*d^7) - 8*b^7*d^7*x)*(-b^3/(256*a^7*d^4 +
256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*
d^3))^(1/4)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1
536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4)*2i - 2*atan((b^3*d^3*x - (12
8*b^10*c^7*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^
5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (128*a^7*b^3*d^7*x)/(256*a^7*d^4 + 256*
a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3)
 - (768*a^2*b^8*c^5*d^2*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^
3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (384*a^3*b^7*c^4*d^3*x)/(2
56*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 -
1024*a^6*b*c*d^3) + (384*a^4*b^6*c^3*d^4*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4
 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) - (768*a^5*
b^5*c^2*d^5*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a
^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + (512*a*b^9*c^6*d*x)/(256*a^7*d^4 + 256
*a^3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3
) + (512*a^6*b^4*c*d^6*x)/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*c^3
*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))/((-b^3/(256*a^7*d^4 + 256*a^
3*b^4*c^4 - 1024*a^4*b^3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^
(1/4)*((b^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*
b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*
d^4 + 4608*a^6*b^2*c^3*d^5))/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^3*
c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3) + 2*a^2*b^2*d^4 + 2*b^4*c^
2*d^2 - 4*a*b^3*c*d^3)))*(-b^3/(256*a^7*d^4 + 256*a^3*b^4*c^4 - 1024*a^4*b^
3*c^3*d + 1536*a^5*b^2*c^2*d^2 - 1024*a^6*b*c*d^3))^(1/4) - 2*atan((b^3*d^3
*x - (128*a^7*d^10*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 +

```
 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^
7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^
3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^
3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d
^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^
5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*
c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (
768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3
+ 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c
^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b
^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3
*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7
+ 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*
c^6*d))^(1/4)*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2
560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*
b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*
a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 +
2*b^4*c^2*d^2 - 4*a*b^3*c*d^3)))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 102
4*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.784 $\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=460

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan}{2\sqrt{2}}$$

```
[Out] -1/a/c/x-1/4*b^(5/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)-1/4*b^(5/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(5/4)/(-a*d+b*c
)*2^(1/2)+1/4*d^(5/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
c)*2^(1/2)+1/4*d^(5/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(5/4)/(-a*d+b*
c)*2^(1/2)-1/8*b^(5/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a
^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(5/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+
x^2*b^(1/2))/a^(5/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(5/4)*ln(-c^(1/4)*d^(1/4)*x*2
^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(5/4)*ln(c^(1/
4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(5/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 584, 297, 1162, 617, 204, 1165, 628}

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{5/4}(bc-ad)}+\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{5/4}(bc-ad)}-\frac{b^{5/4}\tan}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(1/(a*c*x)) + (b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(5/4)*(b*c - a*d)) - (b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(5/4)*(b*c - a*d)) - (d^(5/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) + (d^(5/4)*ArcTan[1 + (Sqrt[2]*d^(1
/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (b^(5/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(5/4)*(b*c - a*d)) +
(b^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(5/4)*(b*c - a*d)) + (d^(5/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d)) - (d^(5/4)*Log[Sqrt[c] + Sqrt
[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(5/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{acx} + \frac{\int \frac{x^2(-bc-ad-bdx^4)}{(a+bx^4)(c+dx^4)}\,dx}{ac}$$

$$= -\frac{1}{acx} + \frac{\int \left(-\frac{b^2cx^2}{(bc-ad)(a+bx^4)} - \frac{ad^2x^2}{(-bc+ad)(c+dx^4)}\right)dx}{ac}$$

$$= -\frac{1}{acx} - \frac{b^2\int \frac{x^2}{a+bx^4}\,dx}{a(bc-ad)} + \frac{d^2\int \frac{x^2}{c+dx^4}\,dx}{c(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{3/2}\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)} - \frac{b^{3/2}\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a(bc-ad)} - \frac{d^{3/2}\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c(bc-ad)} + \frac{d^{3/2}\int \frac{\sqrt{c}+\cdots}{c+\cdots}}{2c(bc-\cdots}$$

$$= -\frac{1}{acx} - \frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)} - \frac{b\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a(bc-ad)} - \frac{b^{5/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}(bc-ad)} - \cdots$$

$$= -\frac{1}{acx} - \frac{b^{5/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)} + \frac{b^{5/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\cdots\right)}{4\sqrt{2}\,a^{5/4}(bc-ad)}$$

$$= -\frac{1}{acx} + \frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)} - \frac{b^{5/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}(bc-ad)} - \frac{d^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}\cdots}{\sqrt[4]{c}}\right)}{2\sqrt{2}\,c^{5/4}(bc-ad)}$$

Mathematica [A] time = 0.22, size = 385, normalized size = 0.84

$$\frac{\frac{\sqrt{2}\,b^{5/4}x\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{5/4}} - \frac{\sqrt{2}\,b^{5/4}x\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{a^{5/4}} - \frac{2\sqrt{2}\,b^{5/4}x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}} + \frac{2\sqrt{2}\,b^{5/4}x\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}+1\cdots\right)}{a^{5/4}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^
(1/4)])/a^(5/4) + (2*Sqrt[2]*b^(5/4)*x*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(5/4) + (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/c^(5/4) - (2*Sqrt[2]*d^(5/4)*x*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c
^(5/4) + (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[
b]*x^2])/a^(5/4) - (Sqrt[2]*b^(5/4)*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*x + Sqrt[b]*x^2])/a^(5/4) - (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] - Sqrt[2]*c^(1/
4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4) + (Sqrt[2]*d^(5/4)*x*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(5/4))/(-8*b*c*x + 8*a*d*x)
```

fricas [B] time = 0.87, size = 1395, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(1/4)*a*c*x*arctan(((-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d
+ 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*(a*b*c - a^2*d)*x - (
-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a
```

```
^9*d^4))^(1/4)*(a*b*c - a^2*d)*sqrt((b^3*x^2 - (a^3*b^2*c^2 - 2*a^4*b*c*d +
 a^5*d^2)*sqrt(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*
a^8*b*c*d^3 + a^9*d^4)))/b^3))/b) - 4*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^
2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*arctan(((-d^5/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
))^(1/4)*(b*c^2 - a*c*d)*x - (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7
*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*(b*c^2 - a*c*d)*sqrt((d^3*x^2
- (b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d
+ 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4)))/d^3))/d) + (-b^5/(a^
5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))
^(1/4)*a*c*x*log(b^4*x + (a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a
^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*
c*d^3 + a^9*d^4))^(3/4)) - (-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2
*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(1/4)*a*c*x*log(b^4*x - (a^4*b^3*c^3 -
 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3)*(-b^5/(a^5*b^4*c^4 - 4*a^6*b^3*
c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4))^(3/4)) - (-d^5/(b^4*c
^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1
/4)*a*c*x*log(d^4*x + (b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*
d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4))^(3/4)) + (-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2
- 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(1/4)*a*c*x*log(d^4*x - (b^3*c^7 - 3*a*b^
2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3)*(-d^5/(b^4*c^9 - 4*a*b^3*c^8*d + 6
*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4))^(3/4)) + 4)/(a*c*x)
```

giac [A] time = 0.21, size = 488, normalized size = 1.06

$$-\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)}-\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^2b^2c-\sqrt{2}a^3bd\right)}+\frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^3d-\sqrt{2}ac^2d^2\right)}+\frac{\left(cd^3\right)^{\frac{3}{4}}\,a\ldots}{2\left(\sqrt{\ldots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/
4))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/2*(a*b^3)^(3/4)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c - sqrt(2)
*a^3*b*d) + 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4)
)/(c/d)^(1/4))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/2*(c*d^3)^(3/4)*ar
ctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3*d
- sqrt(2)*a*c^2*d^2) + 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) +
sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d) - 1/4*(a*b^3)^(3/4)*log(x^
2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b^2*c - sqrt(2)*a^3*b*d
) - 1/4*(c*d^3)^(3/4)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)
*b*c^3*d - sqrt(2)*a*c^2*d^2) + 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3*d - sqrt(2)*a*c^2*d^2) - 1/(a*c*x)
```

maple [A] time = 0.06, size = 331, normalized size = 0.72

$$\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}+\frac{\sqrt{2}\,b\ln\left(\frac{x^2-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^2+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a}-\frac{\sqrt{2}\,d\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c}-\frac{\sqrt{2}\,d\ldots}{4\left(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8*d/c/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^
(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4*d/c/(a*d-b*c)/(c/d)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4*d/c/(a*d-b*c)/(c/d)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)+1/8*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)
*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b
)^(1/2)))+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*
x+1)+1/4*b/a/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)-
1/a/c/x
```

maxima [A] time = 1.27, size = 384, normalized size = 0.83

$$-\frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8\left(abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1
/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt
(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*b*c - a^2*d) + 1/8*d
^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(sq
rt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^
(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c^
(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^2 - a*c*d) - 1/(a*c*x)
```

mupad [B] time = 6.08, size = 5962, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] 2*atan(((-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^
2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b
^8*c^11*d^9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 +
1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256
*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d
))^(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^1
0*c^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^
7*c^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*
b^4*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a
^13*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^1
7*b^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-
d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*
d^2 - 1024*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^
9) - (-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b
^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d
^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(
```

1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6
 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9
 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d
^12)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c
^17*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13
*d^10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-d^5/(256*b
^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024
*a*b^3*c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5
/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2
 - 1024*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*
a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*
b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^
15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^
18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i -
256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 +
256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 76
8*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-d^5/(256*b^4*c^9
 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*
c^8*d))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-d^5/(256*b
^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024
*a*b^3*c^8*d))^(3/4)*(x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c
^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*(1024*a^12*b^12*c^
20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*
c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*
c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^1
1*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^1
4*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*
b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-d^5/(256*b^4*c^9 + 256*a
^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))
^(1/4) + atan((a^14*c*d^8*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6
*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^
8*c^9*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^
7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*1024i + a^6*b^4*d^5*x*(-b^5/(256*a
^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024
*a^8*b*c*d^3))^(1/4)*4i + a^5*b^5*c*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*
c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*
4i - a^7*b^7*c^8*d*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^
3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i - a^13*b*c^2*d^
7*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^
2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^8*b^6*c^7*d^2*x*(-b^5/(256*a
^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024
*a^8*b*c*d^3))^(5/4)*6144i - a^9*b^5*c^6*d^3*x*(-b^5/(256*a^9*d^4 + 256*a^5
*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(
5/4)*4096i + a^10*b^4*c^5*d^4*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024
*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*2048i - a^
11*b^3*c^4*d^5*x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d
 + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*4096i + a^12*b^2*c^3*d^6*
x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*
c^2*d^2 - 1024*a^8*b*c*d^3))^(5/4)*6144i)/(b^9*c^4 + a^4*b^5*d^4 + a^3*b^6*
c*d^3 + a^2*b^7*c^2*d^2 + a*b^8*c^3*d))*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^
4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*2i
 + atan((b^5*c^6*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^
6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i + a*b^8*c^14*x*(
-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7
*d^2 - 1024*a*b^3*c^8*d))^(5/4)*1024i + a^9*c^6*d^8*x*(-d^5/(256*b^4*c^9 +
256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^
8*d))^(5/4)*1024i + a*b^4*c^5*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 -
1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*4i - a
^2*b^7*c^13*d*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 +

1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i - a^8*b*c^7*d^7*x*(-
d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*
d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^3*b^6*c^12*d^2*x*(-d^5/(256*b^4*c^
9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^
3*c^8*d))^(5/4)*6144i - a^4*b^5*c^11*d^3*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5
*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)
*4096i + a^5*b^4*c^10*d^4*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3
*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*2048i - a^6*b^
3*c^9*d^5*x*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 153
6*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(5/4)*4096i + a^7*b^2*c^8*d^6*x*(-d^
5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 1024*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^
2 - 1024*a*b^3*c^8*d))^(5/4)*6144i)/(a^4*d^9 + b^4*c^4*d^5 + a*b^3*c^3*d^6
+ a^2*b^2*c^2*d^7 + a^3*b*c*d^8))*(-d^5/(256*b^4*c^9 + 256*a^4*c^5*d^4 - 10
24*a^3*b*c^6*d^3 + 1536*a^2*b^2*c^7*d^2 - 1024*a*b^3*c^8*d))^(1/4)*2i + 2*a
tan(((-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b
^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*
c^11*d^9) - (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 153
6*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^
5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^
(1/4)*(1024*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c
^18*d^6 - 4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c
^15*d^9 + 6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4
*c^12*d^12)*1i - 256*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13
*b^10*c^17*d^6 + 256*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b
^6*c^13*d^10 + 768*a^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i) + (-b^5
/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2
 - 1024*a^8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9)
- (-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*
c^2*d^2 - 1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4
- 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(102
4*a^12*b^12*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 -
4096*a^15*b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 +
6144*a^18*b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12
)*1i + 256*a^11*b^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17
*d^6 - 256*a^14*b^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^
10 - 768*a^18*b^5*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i))/((-b^5/(256*a^9*
d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^
8*b*c*d^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(2
56*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 -
1024*a^8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6
*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^1
2*c^20*d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*
b^9*c^17*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*
b^6*c^14*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i - 256
*a^11*b^12*c^19*d^4 + 768*a^12*b^11*c^18*d^5 - 768*a^13*b^10*c^17*d^6 + 256
*a^14*b^9*c^16*d^7 + 256*a^16*b^7*c^14*d^9 - 768*a^17*b^6*c^13*d^10 + 768*a
^18*b^5*c^12*d^11 - 256*a^19*b^4*c^11*d^12)*1i)*1i - (-b^5/(256*a^9*d^4 + 2
56*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d
^3))^(1/4)*(x*(4*a^11*b^9*c^12*d^8 + 4*a^12*b^8*c^11*d^9) - (-b^5/(256*a^9*
d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^
8*b*c*d^3))^(3/4)*(x*(-b^5/(256*a^9*d^4 + 256*a^5*b^4*c^4 - 1024*a^6*b^3*c^
3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1/4)*(1024*a^12*b^12*c^20*
d^4 - 4096*a^13*b^11*c^19*d^5 + 6144*a^14*b^10*c^18*d^6 - 4096*a^15*b^9*c^1
7*d^7 + 2048*a^16*b^8*c^16*d^8 - 4096*a^17*b^7*c^15*d^9 + 6144*a^18*b^6*c^1
4*d^10 - 4096*a^19*b^5*c^13*d^11 + 1024*a^20*b^4*c^12*d^12)*1i + 256*a^11*b
^12*c^19*d^4 - 768*a^12*b^11*c^18*d^5 + 768*a^13*b^10*c^17*d^6 - 256*a^14*b
^9*c^16*d^7 - 256*a^16*b^7*c^14*d^9 + 768*a^17*b^6*c^13*d^10 - 768*a^18*b^5
*c^12*d^11 + 256*a^19*b^4*c^11*d^12)*1i)*1i))*(-b^5/(256*a^9*d^4 + 256*a^5*
b^4*c^4 - 1024*a^6*b^3*c^3*d + 1536*a^7*b^2*c^2*d^2 - 1024*a^8*b*c*d^3))^(1

```
/4) - 1/(a*c*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.785 $\int \frac{1}{x^4(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=462

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}}{2\sqrt{2}\,a}$$

```
[Out] -1/3/a/c/x^3-1/4*b^(7/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)-1/4*b^(7/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)/(-a*d
+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
d+b*c)*2^(1/2)+1/4*d^(7/4)*arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(7/4)/(-a*
d+b*c)*2^(1/2)+1/8*b^(7/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2
))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(7/4)*ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1
/2)+x^2*b^(1/2))/a^(7/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(7/4)*ln(-c^(1/4)*d^(1/4)
*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)+1/8*d^(7/4)*ln(c
^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(7/4)/(-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {480, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(3*a*c*x^3) + (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (b^(7/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/
c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^
(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (b^(7/4)*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(7/4)*(b*c - a*d))
- (b^(7/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[
2]*a^(7/4)*(b*c - a*d)) - (d^(7/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d)) + (d^(7/4)*Log[Sqrt[c] + Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(7/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
```

```
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = -\frac{1}{3acx^3}+\frac{\int \frac{-3(bc+ad)-3bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx}{3ac}$$
$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{1}{a+bx^4}\,dx}{a(bc-ad)}+\frac{d^2\int \frac{1}{c+dx^4}\,dx}{c(bc-ad)}$$
$$= -\frac{1}{3acx^3}-\frac{b^2\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}-\frac{b^2\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{a+bx^4}\,dx}{2a^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}-\sqrt{d}\,x^2}{c+dx^4}\,dx}{2c^{3/2}(bc-ad)}+\frac{d^2\int \frac{\sqrt{c}+\sqrt{d}\,x^2}{c+dx^4}}{2c^{3/2}(bc-}$$
$$= -\frac{1}{3acx^3}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}-\frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx}{4a^{3/2}(bc-ad)}+\frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}-x^2}}{4\sqrt{2}\,a^{7/4}(bc-ad}$$
$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{}\right.}{4\sqrt{2}\,a^{7/4}(bc-ad)}$$
$$= -\frac{1}{3acx^3}+\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}(bc-ad)}-\frac{d^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{2\sqrt{2}\,c^{7/4}(bc-a}$$

Mathematica [A] time = 0.26, size = 406, normalized size = 0.88

$$\frac{-\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}+\frac{6\sqrt{2}b^{7/4}x^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{7/4}}-\frac{3\sqrt{2}b^{7/4}x^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{3\sqrt{2}b^{7/4}x^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{}\right.}{a^{7/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/
a^(1/4)])/a^(7/4) + (6*Sqrt[2]*b^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a
^(1/4)])/a^(7/4) + (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)])/c^(7/4) - (6*Sqrt[2]*d^(7/4)*x^3*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/c^(7/4) - (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/
4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*b^(7/4)*x^3*Log[Sqrt[a] + Sqrt[2]
*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (3*Sqrt[2]*d^(7/4)*x^3*Log[Sqr
t[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (3*Sqrt[2]*d^(7/
4)*x^3*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(24
*(-(b*c) + a*d)*x^3)
```

fricas [B] time = 5.31, size = 1415, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/12*(12*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*
b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*arctan(((a^5*b^3*c^3 - 3*a^6*b^2*c^2*d +
 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*
c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*x - (a^5*b^3*c^3 - 3*a^6*b^2*c^
2*d + 3*a^7*b*c*d^2 - a^8*d^3)*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9
*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(3/4)*sqrt((b^4*x^2 + (a^4*b^2*c
```

```
^2 - 2*a^5*b*c*d + a^6*d^2)*sqrt(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^
9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4)))/b^4))/b^5) - 12*(-d^7/(b^4*c^1
1 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1
/4)*a*c*x^3*arctan(((b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 +
 a^4*c^7*d^4))^(3/4)*x - (b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c
^5*d^3)*(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(3/4)*sqrt((d^4*x^2 + (b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*
d^2)*sqrt(-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8
*d^3 + a^4*c^7*d^4)))/d^4))/d^5) - 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d +
 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*x +
(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 +
 a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3
*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^11*d^4))^(1/4)*a*c*x^3*log(b^2*
x - (-b^7/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d
^3 + a^11*d^4))^(1/4)*(a^2*b*c - a^3*d)) + 3*(-d^7/(b^4*c^11 - 4*a*b^3*c^10
*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*log(
d^2*x + (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*
d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 3*(-d^7/(b^4*c^11 - 4*a*b^3*
c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*c^8*d^3 + a^4*c^7*d^4))^(1/4)*a*c*x^3*
log(d^2*x - (-d^7/(b^4*c^11 - 4*a*b^3*c^10*d + 6*a^2*b^2*c^9*d^2 - 4*a^3*b*
c^8*d^3 + a^4*c^7*d^4))^(1/4)*(b*c^3 - a*c^2*d)) - 4)/(a*c*x^3)
```

giac [A] time = 0.19, size = 472, normalized size = 1.02

$$-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^{2}bc-\sqrt{2}\,a^{3}d\right)}-\frac{\left(ab^{3}\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,a^{2}bc-\sqrt{2}\,a^{3}d\right)}+\frac{\left(cd^{3}\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}\,bc^{3}-\sqrt{2}\,ac^{2}d\right)}+\frac{(cd}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) - 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sq
rt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b*c - sqrt(2)*a
^3*d) + 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) + 1/2*(c*d^3)^(1/4)*d*arctan
(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^3 - sqrt
(2)*a*c^2*d) - 1/4*(a*b^3)^(1/4)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a
/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a^2*b*c - sqrt(2)*a^3*d) + 1/4*(c*d
^3)^(1/4)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^3 - s
qrt(2)*a*c^2*d) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqr
t(c/d))/(sqrt(2)*b*c^3 - sqrt(2)*a*c^2*d) - 1/3/(a*c*x^3)
```

maple [A] time = 0.06, size = 343, normalized size = 0.74

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)a^{2}}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,b^{2}\ln\left(\frac{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)a^{2}}-\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}\,d^{2}}{4\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] -1/8/c^2*d^2/(a*d-b*c)*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c
/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))-1/4/c^2*d^2/(a*d-b*c)*(
```

```
c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)-1/4/c^2*d^2/(a*d-b*c)*(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/3/a/c/x^3+1/8/a^2*b^2/(
a*d-b*c)*(a/b)^(1/4)*2^(1/2)*ln((x^2+(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^
2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2)))+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a/b)^(1/4)*x+1)+1/4/a^2*b^2/(a*d-b*c)*(a/b)^(1/4)*2^(1
/2)*arctan(2^(1/2)/(a/b)^(1/4)*x-1)
```

maxima [A] time = 1.36, size = 390, normalized size = 0.84

$$-\frac{\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\,b^2\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}\,x-\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}\,b^{\frac{7}{4}}\log\left(\sqrt{b}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}\cdots\right.}{a^{\frac{3}{4}}}}{8\left(abc-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/
4))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b^2*
arctan(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqr
t(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(7/4)*log(sqrt(b)*x^2 +
sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(7/4)*log(sqrt(b)*
x^2 - sqrt(2)*a^(1/4)*b^(1/4)*x + sqrt(a))/a^(3/4))/(a*b*c - a^2*d) + 1/8*(
2*sqrt(2)*d^2*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sq
rt(sqrt(c)*sqrt(d)))/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + 2*sqrt(2)*d^2*arctan
(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))
/(sqrt(c)*sqrt(sqrt(c)*sqrt(d))) + sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 + sqrt(2
)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4) - sqrt(2)*d^(7/4)*log(sqrt(d)*x^2 -
sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/c^(3/4))/(b*c^2 - a*c*d) - 1/3/(a*c*x^
3)
```

mupad [B] time = 6.19, size = 7459, normalized size = 16.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^2*b^16*c^11*x*256i)/(256*a
^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 102
4*a^10*b*c*d^3) - (a^4*b^14*c^9*d^2*x*1536i)/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^5*b
^13*c^8*d^3*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d +
 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^6*b^12*c^7*d^4*x*256i)/(256
*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1
024*a^10*b*c*d^3) - (a^7*b^11*c^6*d^5*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c
^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^8*
b^10*c^5*d^6*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^9*b^9*c^4*d^7*x*1536i)/(25
6*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 -
1024*a^10*b*c*d^3) + (a^10*b^8*c^3*d^8*x*1024i)/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) - (a^
11*b^7*c^2*d^9*x*256i)/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
 + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + (a^3*b^15*c^10*d*x*1024i)/(2
56*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 -
 1024*a^10*b*c*d^3))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*
c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((b^7*(1024*a^4*b^
```

8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d - 5120*a^11*b*c^5*d^7 + 10
240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10240*a^8*b^4*c^8*d^4 - 1126
4*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3) + 4*a^5*
b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4*b^4*c*d^7)))*(-b^7/(256*a
^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 102
4*a^10*b*c*d^3))^(1/4)*2i - atan((a^2*b^5*d^7*x*1i + b^7*c^2*d^5*x*1i - (a^
11*c^2*d^16*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^2*b^9*c^11*d^7*x*256i)/(256*
b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 10
24*a*b^3*c^10*d) + (a^3*b^8*c^10*d^8*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d
^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^4*
b^7*c^9*d^9*x*1536i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^5*b^6*c^8*d^10*x*1024i)/(25
6*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 -
1024*a*b^3*c^10*d) - (a^6*b^5*c^7*d^11*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*
d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) - (a^7
*b^4*c^6*d^12*x*256i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3
+ 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a^8*b^3*c^5*d^13*x*1024i)/(2
56*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 -
1024*a*b^3*c^10*d) - (a^9*b^2*c^4*d^14*x*1536i)/(256*b^4*c^11 + 256*a^4*c^
7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d) + (a
^10*b*c^3*d^15*x*1024i)/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^
3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))/((-d^7/(256*b^4*c^11 + 256*a
^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d)
)^(1/4)*((d^7*(1024*a^4*b^8*c^12 + 1024*a^12*c^4*d^8 - 5120*a^5*b^7*c^11*d
- 5120*a^11*b*c^5*d^7 + 10240*a^6*b^6*c^10*d^2 - 11264*a^7*b^5*c^9*d^3 + 10
240*a^8*b^4*c^8*d^4 - 11264*a^9*b^3*c^7*d^5 + 10240*a^10*b^2*c^6*d^6))/(256
*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1
024*a*b^3*c^10*d) + 4*a^5*b^3*d^8 + 4*b^8*c^5*d^3 - 4*a*b^7*c^4*d^4 - 4*a^4
*b^4*c*d^7)))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 +
1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*2i - 2*atan(((-d^7/(256*b^
4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024
*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^
7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d
^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 102
4*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024
*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4
096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12
+ 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*
a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13
*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 4505
6*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 4
0960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^1
2)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10
*d^11 - 16*a^14*b^7*c^9*d^12)*1i) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 -
1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(
4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*
c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(
1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2
*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096
*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1
024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 +
6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13
) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b
^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^1
4*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 4096
0*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 -

20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i))/((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i - 16*a^9*b^12*c^14*d^7 + 16*a^10*b^11*c^13*d^8 + 16*a^13*b^8*c^10*d^11 - 16*a^14*b^7*c^9*d^12)*1i)*1i - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*((-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(3/4)*(x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13) + (-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i)*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-d^7/(256*b^4*c^11 + 256*a^4*c^7*d^4 - 1024*a^3*b*c^8*d^3 + 1536*a^2*b^2*c^9*d^2 - 1024*a*b^3*c^10*d))^(1/4) - 2*atan(-((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i + x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 +

```
1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 +
 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^1
3))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^
11 + 16*a^14*b^7*c^9*d^12)*1i))/((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 10
24*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a
^9*b^11*c^11*d^9 + 4*a^11*b^9*c^9*d^11) - (-b^7/(256*a^11*d^4 + 256*a^7*b^4
*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4
)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^
2*c^2*d^2 - 1024*a^10*b*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^
4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(
4096*a^13*b^12*c^21*d^4 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d
^6 - 45056*a^16*b^9*c^18*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^1
6*d^9 + 40960*a^19*b^6*c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4
*c^13*d^12)*1i + x*(1024*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 614
4*a^13*b^11*c^18*d^6 - 4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1
024*a^16*b^8*c^15*d^9 - 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 -
 4096*a^19*b^5*c^12*d^12 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*
d^7 - 16*a^10*b^11*c^13*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)
*1i)*1i - (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536
*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(x*(4*a^9*b^11*c^11*d^9 + 4*a^
11*b^9*c^9*d^11) + (-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3
*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*((-b^7/(256*a^11*d^4
+ 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b
*c*d^3))^(3/4)*((-b^7/(256*a^11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d
+ 1536*a^9*b^2*c^2*d^2 - 1024*a^10*b*c*d^3))^(1/4)*(4096*a^13*b^12*c^21*d^4
 - 20480*a^14*b^11*c^20*d^5 + 40960*a^15*b^10*c^19*d^6 - 45056*a^16*b^9*c^1
8*d^7 + 40960*a^17*b^8*c^17*d^8 - 45056*a^18*b^7*c^16*d^9 + 40960*a^19*b^6*
c^15*d^10 - 20480*a^20*b^5*c^14*d^11 + 4096*a^21*b^4*c^13*d^12)*1i - x*(102
4*a^11*b^13*c^20*d^4 - 4096*a^12*b^12*c^19*d^5 + 6144*a^13*b^11*c^18*d^6 -
4096*a^14*b^10*c^17*d^7 + 1024*a^15*b^9*c^16*d^8 + 1024*a^16*b^8*c^15*d^9 -
 4096*a^17*b^7*c^14*d^10 + 6144*a^18*b^6*c^13*d^11 - 4096*a^19*b^5*c^12*d^1
2 + 1024*a^20*b^4*c^11*d^13))*1i + 16*a^9*b^12*c^14*d^7 - 16*a^10*b^11*c^13
*d^8 - 16*a^13*b^8*c^10*d^11 + 16*a^14*b^7*c^9*d^12)*1i)*1i))*(-b^7/(256*a^
11*d^4 + 256*a^7*b^4*c^4 - 1024*a^8*b^3*c^3*d + 1536*a^9*b^2*c^2*d^2 - 1024
*a^10*b*c*d^3))^(1/4) - 1/(3*a*c*x^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.786 $\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=479

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}}{2\sqrt{2}\,a}$$

```
[Out] -1/5/a/c/x^5+(a*d+b*c)/a^2/c^2/x+1/4*b^(9/4)*arctan(-1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)+1/4*b^(9/4)*arctan(1+b^(1/4)*x*2^(1/2)/a^
(1/4))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(-1+d^(1/4)*x*2^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)-1/4*d^(9/4)*arctan(1+d^(1/4)*x*2^(1/2)/c
^(1/4))/c^(9/4)/(-a*d+b*c)*2^(1/2)+1/8*b^(9/4)*ln(-a^(1/4)*b^(1/4)*x*2^(1/2
)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*b^(9/4)*ln(a^(1/4)*b^
(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(9/4)/(-a*d+b*c)*2^(1/2)-1/8*d^(9/4)
*ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(-a*d+b*c)*2^(1
/2)+1/8*d^(9/4)*ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(9/4)/(
-a*d+b*c)*2^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {480, 583, 584, 297, 1162, 617, 204, 1165, 628}

$$\frac{b^{9/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\,x+\sqrt{a}+\sqrt{b}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(bc-ad)}-\frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{9/4}(bc-ad)}+\frac{b^{9/4}\tan^{-1}}{2\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -1/(5*a*c*x^5) + (b*c + a*d)/(a^2*c^2*x) - (b^(9/4)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (b^(9/4)*ArcTan[1 + (Sq
rt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(b*c - a*d)) + (d^(9/4)*ArcTa
n[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)) - (d^(9
/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(9/4)*(b*c - a*d)
) + (b^(9/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqr
t[2]*a^(9/4)*(b*c - a*d)) - (b^(9/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(9/4)*(b*c - a*d)) - (d^(9/4)*Log[Sqrt[c] -
Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(9/4)*(b*c - a*d)) +
 (d^(9/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(9/4)*(b*c - a*d))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{x^6(a+bx^4)(c+dx^4)}\,dx = -\frac{1}{5acx^5} + \frac{\int \frac{-5(bc+ad)-5bdx^4}{x^2(a+bx^4)(c+dx^4)}\,dx}{5ac}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{\int \frac{x^2\left(-5(b^2c^2+abcd+a^2d^2)-5bd(bc+ad)x^4\right)}{(a+bx^4)(c+dx^4)}\,dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{\int \left(-\frac{5b^3c^2x^2}{(bc-ad)(a+bx^4)} - \frac{5a^2d^3x^2}{(-bc+ad)(c+dx^4)}\right)dx}{5a^2c^2}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^3\int \frac{x^2}{a+bx^4}\,dx}{a^2(bc-ad)} - \frac{d^3\int \frac{x^2}{c+dx^4}\,dx}{c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{b^{5/2}\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)} + \frac{b^{5/2}\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2a^2(bc-ad)} + \frac{d^{5/2}\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2c^2(bc-ad)}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)} + \frac{b^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4a^2(bc-ad)} + \frac{b^{9/4}\int \frac{\sqrt{2}}{-\frac{\sqrt{a}}{\sqrt{b}}-}}{4\sqrt{2}a^{9/4}}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} + \frac{b^{9/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{9/4}(bc-ad)} - \frac{b^{9/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{}\right.}{4\sqrt{2}a^{9/4}(bc}$$

$$= -\frac{1}{5acx^5} + \frac{bc+ad}{a^2c^2x} - \frac{b^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{b^{9/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{9/4}(bc-ad)} + \frac{d^{9/4}\tan}{2\sqrt{2}}$$

Mathematica [A] time = 0.38, size = 428, normalized size = 0.89

$$\frac{\frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{9/4}} - \frac{10\sqrt{2}b^{9/4}x^5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}+1\right)}{a^{9/4}} - \frac{5\sqrt{2}b^{9/4}x^5\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{9/4}} + \frac{5\sqrt{2}b^{9/4}x^5\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{}\right.}{a^{9/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] ((8*b)/a - (8*d)/c - (40*b^2*x^4)/a^2 + (40*d^2*x^4)/c^2 + (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*b^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(9/4) - (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) + (10*Sqrt[2]*d^(9
/4)*x^5*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/c^(9/4) - (5*Sqrt[2]*b^(9/
4)*x^5*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(9/4) + (5
*Sqrt[2]*b^(9/4)*x^5*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/a^(9/4) + (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/c^(9/4) - (5*Sqrt[2]*d^(9/4)*x^5*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(9/4))/(40*(-(b*c) + a*d)*x^5)
```

fricas [B] time = 8.92, size = 1456, normalized size = 3.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(20*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^1
2*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-b^9/(a^9*b^4*c^4 - 4*a^1
0*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b
*c - a^3*d)*x - (-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2
- 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*(a^2*b*c - a^3*d)*sqrt((b^5*x^2 - (a^5*
b^2*c^2 - 2*a^6*b*c*d + a^7*d^2)*sqrt(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d
+ 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4)))/b^5))/b^2) - 20*(-d^9/(
b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9
*d^4))^(1/4)*a^2*c^2*x^5*arctan(((-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b
^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*x -
(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 +
a^4*c^9*d^4))^(1/4)*(b*c^3 - a*c^2*d)*sqrt((d^5*x^2 - (b^2*c^7 - 2*a*b*c^6*
d + a^2*c^5*d^2)*sqrt(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2
- 4*a^3*b*c^10*d^3 + a^4*c^9*d^4)))/d^5))/d^2) + 5*(-b^9/(a^9*b^4*c^4 - 4*a
^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*
c^2*x^5*log(b^7*x + (a^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d
^3)*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c
*d^3 + a^13*d^4))^(3/4)) - 5*(-b^9/(a^9*b^4*c^4 - 4*a^10*b^3*c^3*d + 6*a^11
*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(1/4)*a^2*c^2*x^5*log(b^7*x - (a
^7*b^3*c^3 - 3*a^8*b^2*c^2*d + 3*a^9*b*c*d^2 - a^10*d^3)*(-b^9/(a^9*b^4*c^4
 - 4*a^10*b^3*c^3*d + 6*a^11*b^2*c^2*d^2 - 4*a^12*b*c*d^3 + a^13*d^4))^(3/4
)) - 5*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10
*d^3 + a^4*c^9*d^4))^(1/4)*a^2*c^2*x^5*log(d^7*x + (b^3*c^10 - 3*a*b^2*c^9*
d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2
*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 5*(-d^9/(b^4*c^13
 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*b*c^10*d^3 + a^4*c^9*d^4))^(
1/4)*a^2*c^2*x^5*log(d^7*x - (b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 -
a^3*c^7*d^3)*(-d^9/(b^4*c^13 - 4*a*b^3*c^12*d + 6*a^2*b^2*c^11*d^2 - 4*a^3*
b*c^10*d^3 + a^4*c^9*d^4))^(3/4)) + 20*(b*c + a*d)*x^4 - 4*a*c)/(a^2*c^2*x^
5)
```

giac [A] time = 0.24, size = 483, normalized size = 1.01

$$\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)}+\frac{\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^3bc-\sqrt{2}a^4d\right)}-\frac{\left(cd^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}bc^4-\sqrt{2}ac^3d\right)}-\frac{\left(cd^3\right)^{\frac{3}{4}}\text{arc}}{2(\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4
))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/2*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d)
 - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(
1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d) - 1/2*(c*d^3)^(3/4)*arctan(1/2*sqrt
(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3
*d) - 1/4*(a*b^3)^(3/4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(
2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(a*b^3)^(3/4)*log(x^2 - sqrt(2)*x*(a/b)^(
1/4) + sqrt(a/b))/(sqrt(2)*a^3*b*c - sqrt(2)*a^4*d) + 1/4*(c*d^3)^(3/4)*log
(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b*c^4 - sqrt(2)*a*c^3*d)
 - 1/4*(c*d^3)^(3/4)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*
b*c^4 - sqrt(2)*a*c^3*d) + 1/5*(5*b*c*x^4 + 5*a*d*x^4 - a*c)/(a^2*c^2*x^5)
```

maple [A] time = 0.07, size = 365, normalized size = 0.76

$$-\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{\sqrt{2}\,b^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)}{4\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}-\frac{\sqrt{2}\,b^{2}\ln\left(\frac{x^{2}-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}{x^{2}+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,x+\sqrt{\frac{a}{b}}}\right)}{8\left(ad-bc\right)\left(\frac{a}{b}\right)^{\frac{1}{4}}a^{2}}+\frac{\sqrt{2}\,d^{2}\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4\left(ad-bc\right)\left(\frac{c}{d}\right)^{\frac{1}{4}}c^{2}}+\frac{\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/c^2/(a*d-b*c)/(c/d)^(1/4)*2^(1/2)*ln((x^2-(c/d)^(1/4)*2^(1/2)*x+(c/
d)^(1/2))/(x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))+1/4*d^2/c^2/(a*d-b*c)/(c
/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d^2/c^2/(a*d-b*c)/(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b^2/a^2/(a*d-b*c)/(a/b
)^(1/4)*2^(1/2)*ln((x^2-(a/b)^(1/4)*2^(1/2)*x+(a/b)^(1/2))/(x^2+(a/b)^(1/4)
*2^(1/2)*x+(a/b)^(1/2)))-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a/b)^(1/4)*x+1)-1/4*b^2/a^2/(a*d-b*c)/(a/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a/b)^(1/4)*x-1)-1/5/a/c/x^5+1/a/c^2/x*d+1/a^2/c/x*b
```

maxima [A] time = 1.23, size = 405, normalized size = 0.85

$$\frac{b^{3}\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{b}x^{2}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{8\left(a^{2}bc-a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/8*b^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(b)*x + sqrt(2)*a^(1/4)*b^(1/4
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arcta
n(1/2*sqrt(2)*(2*sqrt(b)*x - sqrt(2)*a^(1/4)*b^(1/4))/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(b)*x^2 + sqrt(2)*a^(1/
4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(sqrt(b)*x^2 - sqrt(
2)*a^(1/4)*b^(1/4)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a^2*b*c - a^3*d) - 1/8*
d^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/s
qrt(sqrt(c)*sqrt(d)))/(sqrt(sqrt(c)*sqrt(d))*sqrt(d)) + 2*sqrt(2)*arctan(1/
2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/(s
qrt(sqrt(c)*sqrt(d))*sqrt(d)) - sqrt(2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d
^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)) + sqrt(2)*log(sqrt(d)*x^2 - sqrt(2)*c
^(1/4)*d^(1/4)*x + sqrt(c))/(c^(1/4)*d^(3/4)))/(b*c^3 - a*c^2*d) + 1/5*(5*(
b*c + a*d)*x^4 - a*c)/(a^2*c^2*x^5)
```

mupad [B] time = 6.01, size = 4547, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(a + b*x^4)*(c + d*x^4)),x)
```

```
[Out] - 2*atan((1024*a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 102
4*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^
11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d +
```

1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 1024*a^21*c^3*d^10*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^12*b^9*c^12*d*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 4*a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) + 6144*a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 1024*a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) - 4096*a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4) + 6144*a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4))/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4) - atan((a^11*b^10*c^13*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i + a^11*b^6*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^21*c^3*d^10*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^12*b^9*c^12*d*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i - a^20*b*c^4*d^9*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^8*b^9*c^3*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*4i + a^13*b^8*c^11*d^2*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i - a^14*b^7*c^10*d^3*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^15*b^6*c^9*d^4*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i + a^17*b^4*c^7*d^6*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*1024i - a^18*b^3*c^6*d^7*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*4096i + a^19*b^2*c^5*d^8*x*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(5/4)*6144i)/(b^16*c^8 + a^8*b^8*d^8 + a^7*b^9*c*d^7 + a^2*b^14*c^6*d^2 + a^3*b^13*c^5*d^3 + a^4*b^12*c^4*d^4 + a^5*b^11*c^3*d^5 + a^6*b^10*c^2*d^6 + a*b^15*c^7*d))*(-b^9/(256*a^13*d^4 + 256*a^9*b^4*c^4 - 1024*a^10*b^3*c^3*d + 1536*a^11*b^2*c^2*d^2 - 1024*a^12*b*c*d^3))^(1/4)*2i - 2*atan((4*b^9*c^11*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) + 1024*a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^13*c^11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^4*b^9*c^20*d*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 4*a^3*b^6*c^8*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1

```
/4) + 6144*a^5*b^8*c^19*d^2*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*
a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a
^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d
^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^7*b^6*c^17*
d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^
2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4) + 1024*a^9*b^4*c^15*d^6*x*(-d^9/
(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d
^2 - 1024*a*b^3*c^12*d))^(5/4) - 4096*a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^
13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a
*b^3*c^12*d))^(5/4) + 6144*a^11*b^2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^
4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d
))^(5/4))/(a^8*d^16 + b^8*c^8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*
b^5*c^5*d^11 + a^4*b^4*c^4*d^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7
*b*c*d^15))*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1
536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4) - atan((b^9*c^11*d^6*x*(-d
^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^1
1*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^3*b^10*c^21*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*1024i + a^13*c^11*d^10*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9
*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5
/4)*1024i - a^4*b^9*c^20*d*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a
^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i - a
^12*b*c^12*d^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^
3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^3*b^6*c^8*d
^9*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2
*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(1/4)*4i + a^5*b^8*c^19*d^2*x*(-d^9/(25
6*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2
- 1024*a*b^3*c^12*d))^(5/4)*6144i - a^6*b^7*c^18*d^3*x*(-d^9/(256*b^4*c^13
+ 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^
3*c^12*d))^(5/4)*4096i + a^7*b^6*c^17*d^4*x*(-d^9/(256*b^4*c^13 + 256*a^4*c
^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^
(5/4)*1024i + a^9*b^4*c^15*d^6*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 10
24*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*1024i
 - a^10*b^3*c^14*d^7*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c
^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*4096i + a^11*b^
2*c^13*d^8*x*(-d^9/(256*b^4*c^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 +
1536*a^2*b^2*c^11*d^2 - 1024*a*b^3*c^12*d))^(5/4)*6144i)/(a^8*d^16 + b^8*c^
8*d^8 + a*b^7*c^7*d^9 + a^2*b^6*c^6*d^10 + a^3*b^5*c^5*d^11 + a^4*b^4*c^4*d
^12 + a^5*b^3*c^3*d^13 + a^6*b^2*c^2*d^14 + a^7*b*c*d^15))*(-d^9/(256*b^4*c
^13 + 256*a^4*c^9*d^4 - 1024*a^3*b*c^10*d^3 + 1536*a^2*b^2*c^11*d^2 - 1024*
a*b^3*c^12*d))^(1/4)*2i - (1/(5*a*c) - (x^4*(a*d + b*c))/(a^2*c^2))/x^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b*x**4+a)/(d*x**4+c),x)
```

```
[Out] Timed out
```

3.787 $\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=93

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d+1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1
/2))*(-a*d+b*c)^(1/2)/b^(5/2)-1/2*a*(d*x^4+c)^(1/2)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 80, 50, 63, 208}

$$-\frac{a\sqrt{c+dx^4}}{2b^2}+\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}+\frac{\left(c+dx^4\right)^{3/2}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] -(a*Sqrt[c + d*x^4])/(2*b^2) + (c + d*x^4)^(3/2)/(6*b*d) + (a*Sqrt[b*c - a*
d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)$$
$$= \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{a\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)}{4b}$$
$$= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2}$$
$$= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d}$$
$$= -\frac{a\sqrt{c+dx^4}}{2b^2} + \frac{\left(c+dx^4\right)^{3/2}}{6bd} + \frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}}$$

Mathematica [A] time = 0.11, size = 88, normalized size = 0.95

$$\frac{a\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}} + \frac{\sqrt{c+dx^4}\left(b\left(c+dx^4\right)-3ad\right)}{6b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-3*a*d + b*(c + d*x^4)))/(6*b^2*d) + (a*Sqrt[b*c - a*d]*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2))
```

fricas [A] time = 0.44, size = 195, normalized size = 2.10

$$\left[\frac{3\,ad\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\left(bdx^4+bc-3\,ad\right)\sqrt{dx^4+c}}{12\,b^2d},\ \frac{3\,ad\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{\cdot}}{bc-ad}\right)}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*d*sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4
+ c)*b*sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*(b*d*x^4 + b*c - 3*a*d)*sqrt(d
*x^4 + c))/(b^2*d), 1/6*(3*a*d*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + (b*d*x^4 + b*c - 3*a*d)*sqrt(d*x^4 +
 c))/(b^2*d)]
```

giac [A] time = 0.16, size = 96, normalized size = 1.03

$$-\frac{\left(abc-a^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,b^2} + \frac{\left(dx^4+c\right)^{\frac{3}{2}}b^2d^2-3\sqrt{dx^4+c}\,abd^3}{6\,b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(a*b*c - a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-
b^2*c + a*b*d)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^2 - 3*sqrt(d*x^4 + c)*a*
b*d^3)/(b^3*d^3)
```

maple [B] time = 0.27, size = 1015, normalized size = 10.91

$$-\frac{a^2 d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^3}-\frac{a^2 d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2}{}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d-1/4*a/b^2*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/
b*d*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)+1/4*a/b^3*(-a*b)^(1/2)*d^(1/2)*
ln((-(-a*b)^(1/2)/b*d+d*(x^2+(-a*b)^(1/2)/b))/d^(1/2)+((x^2+(-a*b)^(1/2)/b)
^2*d-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^
3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1
/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/b*d*
(x^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d+1/4*a/b^2/
(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)/b-2*(-a*b)^(1/2)/b*d*(x^2+(-a*b)^(1/2
)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)/b*d*(x
^2+(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c-1/4*a/b^2*((
x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b
)^(1/2)-1/4*a/b^3*(-a*b)^(1/2)*d^(1/2)*ln(((-a*b)^(1/2)/b*d+d*(x^2-(-a*b)^(
1/2)/b))/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(
1/2)/b)-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*
c)/b+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(
-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/
2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(a*d-b*c)
/b+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a
*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)/b*d*(x^2-(-a*b)^(1/2)/b)-(a*d-b*c)/b)^(1/2)
)/(x^2-(-a*b)^(1/2)/b))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.69, size = 87, normalized size = 0.94

$$\frac{\left(d\,x^4+c\right)^{3/2}}{6\,b\,d}-\frac{a\,\sqrt{d\,x^4+c}}{2\,b^2}+\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\sqrt{a\,d-b\,c}}{a^2\,d-a\,b\,c}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d) - (a*(c + d*x^4)^(1/2))/(2*b^2) + (a*atan((a*b^(1
/2)*(c + d*x^4)^(1/2)*(a*d - b*c)^(1/2))/(a^2*d - a*b*c))*(a*d - b*c)^(1/2)
)/(2*b^(5/2))
```

sympy [A] time = 17.44, size = 90, normalized size = 0.97

$$\frac{2\left(-\frac{ad^2\sqrt{c+dx^4}}{4b^2}+\frac{ad^2(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^3\sqrt{\frac{ad-bc}{b}}}+\frac{d\left(c+dx^4\right)^{\frac{3}{2}}}{12b}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(-a*d**2*sqrt(c + d*x**4)/(4*b**2) + a*d**2*(a*d - b*c)*atan(sqrt(c + d*x
**4)/sqrt((a*d - b*c)/b))/(4*b**3*sqrt((a*d - b*c)/b)) + d*(c + d*x**4)**(3
/2)/(12*b))/d**2
```

3.788 $\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=120

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(1/2)-1/2*arcta
n(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)*(-a*d+b*c)^(1/2)/b^
2+1/4*x^2*(d*x^4+c)^(1/2)/b
```

Rubi [A] time = 0.15, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 478, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}+\frac{x^2\sqrt{c+dx^4}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b) - (Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a
*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b^2) + ((b*c - 2*a*d)*ArcTanh[(Sqrt
[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^5\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{c+dx^2}}{a+bx^2}\,dx,x,x^2\right)$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\text{Subst}\left(\int \frac{ac+(-bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,d\right.}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4b^2} - \frac{(a(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\right.}{2b^2}$$

$$= \frac{x^2\sqrt{c+dx^4}}{4b} - \frac{\sqrt{a}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{d}}$$

Mathematica [A] time = 0.19, size = 114, normalized size = 0.95

$$\frac{\frac{(bc-2ad)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}} - 2\sqrt{a}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right) + bx^2\sqrt{c+dx^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4] - 2*Sqrt[a]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*
x^2)/(Sqrt[a]*Sqrt[c + d*x^4])] + ((b*c - 2*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c
 + d*x^4]])/Sqrt[d])/(4*b^2)
```

fricas [A] time = 0.54, size = 714, normalized size = 5.95

$$\left[\frac{2\sqrt{dx^4+c}\,bdx^2 - (bc-2\,ad)\sqrt{d}\,\log\left(-2\,dx^4+2\sqrt{dx^4+c}\sqrt{d}\,x^2-c\right) + \sqrt{-abc+a^2d}\,d\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-\cdots}{\cdots}\right.}{8\,b^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sq
rt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-a*b*c + a^2*d)*d*log(((b^2*c^2 - 8*a
*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c
 - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2
*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 - 2*(b*c - 2*a*d)
*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + sqrt(-a*b*c + a^2*d)*d*log
(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a
^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*
d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(b^2*d), 1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 -
 2*sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))
 - (b*c - 2*a*d)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))
/(b^2*d), 1/4*(sqrt(d*x^4 + c)*b*d*x^2 - (b*c - 2*a*d)*sqrt(-d)*arctan(sqrt
(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(a*b*c - a^2*d)*d*arctan(1/2*((b*c - 2*a*d)
*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 +
(a*b*c^2 - a^2*c*d)*x^2)))/(b^2*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 0.28, size = 1066, normalized size = 8.88

$$-\frac{a^2 d \ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{a^2 d \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\cdots}{\cdots}\right)}{4\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b+1/4/b*c/d^(1/2)*ln(x^2*d^(1/2)+(d*x^4+c)^(1/2))+1
/4*a/b/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-
(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4*a/b/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c-1/4*a/b/(-a*b)^(
1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d
-b*c)/b)^(1/2)-1/4*a/b^2*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*
d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*
d-(a*d-b*c)/b)^(1/2))-1/4*a^2/b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*(
(x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/
```

```
b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4*a/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)
*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}\, x^5}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*x^5/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\, \sqrt{d\, x^4 + c}}{b\, x^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int((x^5*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \sqrt{c + dx^4}}{a + bx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(x**5*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.789 $\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/b^(
3/2)+1/2*(d*x^4+c)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 50, 63, 208}

$$\frac{\sqrt{c+dx^4}}{2b}-\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b) - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/
Sqrt[b*c - a*d]])/(2*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^3\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{a+bx}\,dx, x, x^4\right)$$
$$= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b}$$
$$= \frac{\sqrt{c+dx^4}}{2b} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2bd}$$
$$= \frac{\sqrt{c+dx^4}}{2b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}}$$

Mathematica [A] time = 0.05, size = 69, normalized size = 0.99

$$\frac{1}{2}\left(\frac{\sqrt{c+dx^4}}{b} - \frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[c + d*x^4]/b - (Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqr
t[b*c - a*d]])/b^(3/2))/2
```

fricas [A] time = 0.42, size = 156, normalized size = 2.23

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}}{4\,b}, -\frac{\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx^4+c}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c))/b, -1/2*(sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqr
t(d*x^4 + c))/b]
```

giac [A] time = 0.18, size = 66, normalized size = 0.94

$$\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b} + \frac{\sqrt{dx^4+c}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c
 + a*b*d)*b) + 1/2*sqrt(d*x^4 + c)/b
```

maple [B] time = 0.20, size = 988, normalized size = 14.11

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{4\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/4/b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2)-1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a
*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x^2+(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1
/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
)/(x^2+(-a*b)^(1/2)/b))*c+1/4/b*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b^2*(-a*b)^(1/2)*d^(1/2)*ln((
(x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/b^2/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*a*d-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.66, size = 54, normalized size = 0.77

$$\frac{\sqrt{d\,x^4+c}}{2\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b) - (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/
2))*(a*d - b*c)^(1/2))/(2*b^(3/2))
```

sympy [A] time = 8.18, size = 65, normalized size = 0.93

$$\frac{2\left(\frac{d\sqrt{c+dx^4}}{4b}-\frac{d(ad-bc)\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x**4)/(4*b) - d*(a*d - b*c)*atan(sqrt(c + d*x**4)/sqrt((a*d
 - b*c)/b))/(4*b**2*sqrt((a*d - b*c)/b)))/d
```

3.790 $\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=91

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))*d^(1/2)/b+1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/b/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {465, 402, 217, 206, 377, 205}

$$\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b}+\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(
2*Sqrt[a]*b) + (Sqrt[d]*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(2*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
```

```
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx^2}}{a+bx^2}\,dx, x, x^2\right)$$

$$= \frac{d\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{d\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} + \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

$$= \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2\sqrt{a}\,b} + \frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 0.98

$$\frac{\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{\sqrt{a}} + \sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] ((Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/
Sqrt[a] + Sqrt[d]*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/(2*b)
```

fricas [A] time = 0.49, size = 612, normalized size = 6.73

$$\left[\frac{2\sqrt{d}\,\log\left(-2\,dx^4-2\sqrt{dx^4+c}\,\sqrt{d}\,x^2-c\right)+\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(abc-2\,a^2d\right)x^6-\right.}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + sqrt(-(
b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, -1/8*(4*sqrt(-d
)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt(-(b*c - a*d)/a)*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4
*((a*b*c - 2*a^2*d)*x^6 - a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/
(b^2*x^8 + 2*a*b*x^4 + a^2)))/b, 1/4*(sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d
)*x^2 - c))/b, -1/4*(2*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - sqrt
((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((
b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 0.20, size = 1000, normalized size = 10.99

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{4\sqrt{-ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] -1/4/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/b*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)
^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*a*d+1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/(-a*b)^(1/2)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4
/b*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b
)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+
1/4/(-a*b)^(1/2)/b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/
b))*a*d-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2
*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^
(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}\,x}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.791 $\int \frac{\sqrt{c+dx^4}}{x(a+bx^4)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))*c^(1/2)/a+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*(-a*d+b*c)^(1/2)/a/b^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 83, 63, 208}

$$\frac{\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}-\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a) + (Sqrt[b*c - a*d]*ArcTan
h[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 83

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x)
, x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x
], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x\left(a+bx^4\right)}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(a+bx)}\,dx, x, x^4\right)$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}$$
$$= \frac{c\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2ad}$$
$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a} + \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{b}} - \sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x*(a + b*x^4)),x]
```

```
[Out] (-(Sqrt[c]*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]) + (Sqrt[b*c - a*d]*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b])/(2*a)
```

fricas [A] time = 0.46, size = 383, normalized size = 4.51

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx^4+a}\right) + \sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)}{4\,a}, \frac{2\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx^4+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right) + \right.}{4\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*
sqrt((b*c - a*d)/b))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*
sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x^4 + c)
*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 +
c)*sqrt(c) + 2*c)/x^4))/a, 1/4*(2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/
c) + sqrt((b*c - a*d)/b)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*b*s
qrt((b*c - a*d)/b))/(b*x^4 + a)))/a, 1/2*(sqrt(-(b*c - a*d)/b)*arctan(-sqrt
(d*x^4 + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) + sqrt(-c)*arctan(sqrt(d*x^
4 + c)*sqrt(-c)/c))/a]
```

giac [A] time = 0.18, size = 79, normalized size = 0.93

$$-\frac{(bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a} + \frac{c\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b*c - a*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*
c + a*b*d)*a) + 1/2*c*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [B] time = 0.26, size = 1037, normalized size = 12.20

$$\frac{c\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\; a}+\frac{c\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-b}{b}}}{x^2}\right)}{4\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x/(b*x^4+a),x)
```

```
[Out] -1/4/a*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2)+1/4/a/b*(-a*b)^(1/2)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-
a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)
^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+
(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1
/2))/(x^2+(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)
*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(
x^2+(-a*b)^(1/2)/b))*c-1/4/a*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a/b*(-a*b)^(1/2)*d^(1/2)*ln(((x^
2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/b/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4/a/(-(a*d-b*c)/b)^(1/2
)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b
)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c+1/2/a*(d*x^4+c)^(1/2)-1/2/a*c^(1
/2)*ln((2*c+2*c^(1/2)*(d*x^4+c)^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x), x)
```

mupad [B] time = 4.87, size = 199, normalized size = 2.34

$$\frac{\sqrt{c}\ \mathrm{atanh}\left(\frac{\sqrt{c}\left(\sqrt{d\,x^4+c}\,\left(\frac{a^2\,b\,d^4}{2}-a\,b^2\,c\,d^3+b^3\,c^2\,d^2\right)+\frac{c\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^4+c}}{16\,a^2}\right)}{2\,a\left(\frac{b^2\,c^2\,d^3}{4}-\frac{a\,b\,c\,d^4}{4}\right)}\right)}{2\,a}+\frac{\mathrm{atanh}\left(\frac{a\,b^2\,c\,d^3\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\left(\frac{a\,b^3\,c^2\,d^3}{4}-\frac{a^2\,b^2\,c\,d^4}{4}\right)}\right)\sqrt{b^2\,c}}{2\,a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x*(a + b*x^4)),x)
```

```
[Out] (c^(1/2)*atanh((c^(1/2)*((c + d*x^4)^(1/2)*((a^2*b*d^4)/2 + b^3*c^2*d^2 - a
*b^2*c*d^3) + (c*(8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2))/(16*
```

```
a^2)))/(2*a*((b^2*c^2*d^3)/4 - (a*b*c*d^4)/4))))/(2*a) + (atanh((a*b^2*c*d^
3*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*((a*b^3*c^2*d^3)/4 - (a^2*b^2
*c*d^4)/4)))*(b^2*c - a*b*d)^(1/2))/(2*a*b)
```

sympy [A] time = 12.05, size = 82, normalized size = 0.96

$$\frac{2\left(\frac{cd\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}+\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4ab\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x/(b*x**4+a),x)
```

```
[Out] 2*(c*d*atan(sqrt(c + d*x**4)/sqrt(-c))/(4*a*sqrt(-c)) + d*(a*d - b*c)*atan(
sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(4*a*b*sqrt((a*d - b*c)/b)))/d
```

3.792 $\int \frac{\sqrt{c+dx^4}}{x^3(a+bx^4)} dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

```
[Out] -1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)/
a^(3/2)-1/2*(d*x^4+c)^(1/2)/a/x^2
```

Rubi [A] time = 0.09, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 475, 12, 377, 205}

$$-\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{c+dx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*x^2) - (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/
(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^3\left(a+bx^4\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2\left(a+bx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}+\frac{\text{Subst}\left(\int \frac{-bc+ad}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}\\
&= -\frac{\sqrt{c+dx^4}}{2ax^2}-\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 53, normalized size = 0.70

$$-\frac{\sqrt{c+dx^4}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^3*(a + b*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c + d*x^4]*Hypergeometric2F1[-1/2, 1, 1/2, ((-(b*c) + a*d)*x^4)/
(a*(c + d*x^4))])/(a*x^2)
```

fricas [A] time = 0.46, size = 281, normalized size = 3.70

$$\left[\frac{x^2\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)-4\,\sqrt{dx^4+c}}{8\,ax^2},-\ x^2\sqrt{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(x^2*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((a*b*c - 2*a^2*d)*x^6 - a^2*c
*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a^2)) -
4*sqrt(d*x^4 + c))/(a*x^2), -1/4*(x^2*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c
- 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^
6 + (b*c^2 - a*c*d)*x^2)) + 2*sqrt(d*x^4 + c))/(a*x^2)]
```

giac [B] time = 1.29, size = 121, normalized size = 1.59

$$\frac{\left(bc\sqrt{d}-ad^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a}+\frac{c\sqrt{d}}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*c*sqrt(d) - a*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2
*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a) + c*
sqrt(d)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2 - c)*a)
```

maple [B] time = 0.26, size = 1075, normalized size = 14.14

$$\frac{bc\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{bc\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{a}{}}\cdots}{\cdots}\right)}{4\sqrt{-ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x)
```

```
[Out] -1/2/a/c/x^2*(d*x^4+c)^(3/2)+1/2/a/c*d*x^2*(d*x^4+c)^(1/2)+1/2/a*d^(1/2)*ln
(d^(1/2)*x^2+(d*x^4+c)^(1/2))+1/4/a*b/(-a*b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*
ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)^(1/2)/b)^
2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/(-a*b)^
(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4/a*
b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*
b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
*c-1/4/a*b/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+
(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*d+1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b
)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-
b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{x^3\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(x^3*(a + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^3 \left(a + bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**3/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**3*(a + b*x**4)), x)
```

3.793 $\int \frac{\sqrt{c+dx^4}}{x^5\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=115

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

```
[Out] 1/4*(-a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)-1/2*arctanh(b
^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)*(-a*d+b*c)^(1/2)/a^2-1/4*(
d*x^4+c)^(1/2)/a/x^4
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 99, 156, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}-\frac{\sqrt{b}\,\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}-\frac{\sqrt{c+dx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]
])/(4*a^2*Sqrt[c]) - (Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
^4])/Sqrt[b*c - a*d]])/(2*a^2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^5(a+bx^4)}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2(a+bx)}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-2bc+ad)-\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a}$$
$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{8a^2}$$
$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(b(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{4a^2d}$$
$$= -\frac{\sqrt{c+dx^4}}{4ax^4} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} - \frac{\sqrt{b}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 0.93

$$\frac{\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}} - 2\sqrt{b}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right) - \frac{a\sqrt{c+dx^4}}{x^4}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^5*(a + b*x^4)),x]
```

```
[Out] (-((a*Sqrt[c + d*x^4])/x^4) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c
]])/Sqrt[c] - 2*Sqrt[b]*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/S
qrt[b*c - a*d]])/(4*a^2)
```

fricas [A] time = 0.45, size = 513, normalized size = 4.46

$$\left[\frac{2\sqrt{b^2c-abd}\,cx^4\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right) - (2\,bc-ad)\sqrt{c}\,x^4\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right) - 2\,\sqrt{dx^4+c}\,ac}{8\,a^2cx^4},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(b^2*c - a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4
 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*
x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c
*x^4), 1/8*(4*sqrt(-b^2*c + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b*c - a*d)*sqrt(c)*x^4*log((d*x^4 - 2*sqrt(
d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), -1/4*(
```

```
(2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(b^2*c
- a*b*d)*c*x^4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c -
a*b*d))/(b*x^4 + a)) + sqrt(d*x^4 + c)*a*c)/(a^2*c*x^4), 1/4*(2*sqrt(-b^2*c
 + a*b*d)*c*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)
) - (2*b*c - a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) - sqrt(d*
x^4 + c)*a*c)/(a^2*c*x^4)]
```

giac [A] time = 0.17, size = 107, normalized size = 0.93

$$\frac{\left(b^2c-abd\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc-ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}-\frac{\sqrt{dx^4+c}}{4\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*c - a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b
^2*c + a*b*d)*a^2) - 1/4*(2*b*c - a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^
2*sqrt(-c)) - 1/4*sqrt(d*x^4 + c)/(a*x^4)
```

maple [B] time = 0.27, size = 1107, normalized size = 9.63

$$\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-}{b}}\cdots}{x\cdots}\right)}{4\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x)
```

```
[Out] -1/4/a/c/x^4*(d*x^4+c)^(3/2)-1/4/a/c^(1/2)*d*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1
/2))/x^2)+1/4/a/c*d*(d*x^4+c)^(1/2)+1/4/a^2*b*((x^2+(-a*b)^(1/2)/b)^2*d-2*(
-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/4/a^2*(-a*b)^(1/2
)*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2+(-a*b)
^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1
/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*d-1/4/a^2
*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^
2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/a^2*b
*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c
)/b)^(1/2)+1/4/a^2*(-a*b)^(1/2)*d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(
1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*
(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
^2-(-a*b)^(1/2)/b))*d-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))*c-1/2/a^2*b*(d*x^4+c)^(1/2)+1/2/a^2*b*c^(1/2)*ln((2*c+2*(d
*x^4+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^5/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^5), x)
```

mupad [B] time = 5.39, size = 269, normalized size = 2.34

$$\frac{\operatorname{atanh}\left(\frac{b^3 d^4 \sqrt{d x^4+c}\,\sqrt{b^2 c-a b d}}{16\left(\frac{a b^3 d^5}{16}-\frac{b^4 c d^4}{16}\right)}\right)\sqrt{b^2 c-a b d}}{2 a^2}-\frac{\sqrt{d x^4+c}}{4 a x^4}-\frac{\operatorname{atanh}\left(\frac{b^4\sqrt{c}\,d^4\sqrt{d x^4+c}}{16\left(\frac{b^4 c d^4}{16}-\frac{3 a b^3 d^5}{32}+\frac{a^2 b^2 d^6}{32 c}\right)}-\frac{3 b^3 d^5\sqrt{d x^4+c}}{32\sqrt{c}\left(\frac{a b^2 d^6}{32 c}-\frac{3 b^3 d^5}{32}+\frac{b^4 c d^4}{16 a}\right.} \cdots}{4 a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^5*(a + b*x^4)),x)
```

```
[Out] (atanh((b^3*d^4*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(16*((a*b^3*d^5)/1
6 - (b^4*c*d^4)/16)))*(b^2*c - a*b*d)^(1/2))/(2*a^2) - (c + d*x^4)^(1/2)/(4
*a*x^4) - (atanh((b^4*c^(1/2)*d^4*(c + d*x^4)^(1/2))/(16*((b^4*c*d^4)/16 -
(3*a*b^3*d^5)/32 + (a^2*b^2*d^6)/(32*c))) - (3*b^3*d^5*(c + d*x^4)^(1/2))/(
32*c^(1/2)*((a*b^2*d^6)/(32*c) - (3*b^3*d^5)/32 + (b^4*c*d^4)/(16*a))) + (b
^2*d^6*(c + d*x^4)^(1/2))/(32*c^(3/2)*((b^2*d^6)/(32*c) - (3*b^3*d^5)/(32*a
) + (b^4*c*d^4)/(16*a^2))))*(a*d - 2*b*c))/(4*a^2*c^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^5\left(a + bx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**5/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**5*(a + b*x**4)), x)
```

3.794 $\int \frac{\sqrt{c+dx^4}}{x^7(a+bx^4)}\,dx$

Optimal. Leaf size=110

$$\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}+\frac{\sqrt{c+dx^4}\,(3bc-ad)}{6a^2cx^2}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

```
[Out] 1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*(-a*d+b*c)^(1/2)
/a^(5/2)-1/6*(d*x^4+c)^(1/2)/a/x^6+1/6*(-a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/x
^2
```

Rubi [A] time = 0.16, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 475, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}\,(3bc-ad)}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}-\frac{\sqrt{c+dx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*x^6) + ((3*b*c - a*d)*Sqrt[c + d*x^4])/(6*a^2*c*x^2)
+ (b*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])
])/(2*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^7\left(a+bx^4\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^4\left(a+bx^2\right)}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{\operatorname{Subst}\left(\int \frac{-3bc+ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}-\frac{\operatorname{Subst}\left(\int -\frac{3bc(bc-ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a^2c}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{(b(bc-ad))\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6ax^6}+\frac{(3bc-ad)\sqrt{c+dx^4}}{6a^2cx^2}+\frac{b\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 5.33, size = 130, normalized size = 1.18

$$\frac{3bc^2x^4\sqrt{\frac{dx^4}{c}+1}\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)+\left(c+dx^4\right)\left(3bcx^4-a\left(c+dx^4\right)\right)}{6a^2cx^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^7*(a + b*x^4)),x]
```

```
[Out] ((c + d*x^4)*(3*b*c*x^4 - a*(c + d*x^4)) + 3*b*c^2*x^4*Sqrt[(b/a - d/c)*x^4
]*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(6
*a^2*c*x^6*Sqrt[c + d*x^4])
```

fricas [A] time = 0.52, size = 329, normalized size = 2.99

$$\left[\frac{3\,bcx^6\sqrt{-\frac{bc-ad}{a}}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(abc-2\,a^2d\right)x^6-a^2cx^2\right)\sqrt{dx^4+c}\,\sqrt{-\frac{bc-ad}{a}}}{b^2x^8+2\,abx^4+a^2}\right)+4\left((3\,bc-ad)x^4\right.}{24\,a^2cx^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*b*c*x^6*sqrt(-(b*c - a*d)/a)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2
)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((a*b*c - 2*a^2*d)*x^6
- a^2*c*x^2)*sqrt(d*x^4 + c)*sqrt(-(b*c - a*d)/a))/(b^2*x^8 + 2*a*b*x^4 + a
^2)) + 4*((3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6), 1/12*(3*b*
c*x^6*sqrt((b*c - a*d)/a)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
 c)*sqrt((b*c - a*d)/a)/((b*c*d - a*d^2)*x^6 + (b*c^2 - a*c*d)*x^2)) + 2*((
3*b*c - a*d)*x^4 - a*c)*sqrt(d*x^4 + c))/(a^2*c*x^6)]
```

giac [B] time = 1.75, size = 225, normalized size = 2.05

$$-\frac{\left(b^2c\sqrt{d}-abd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{2\,\sqrt{abcd-a^2d^2}\,a^2}-\frac{3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4bc\sqrt{d}-3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4ad^{\frac{3}{2}}\cdots}{3\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4}\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c*sqrt(d) - a*b*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 +
c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^
2) - 1/3*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c*sqrt(d) - 3*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d^(3/2) - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2*
sqrt(d) + 3*b*c^3*sqrt(d) - a*c^2*d^(3/2))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2 - c)^3*a^2)
```

maple [B] time = 0.26, size = 1116, normalized size = 10.15

$$\frac{bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a}-\frac{bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right.}{4\sqrt{-ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x)
```

```
[Out] 1/2/a^2*b/c/x^2*(d*x^4+c)^(3/2)-1/2/a^2*b/c*d*x^2*(d*x^4+c)^(1/2)-1/2/a^2*b
*d^(1/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))-1/4/a^2*b^2/(-a*b)^(1/2)*((x^2+(-a
*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)
+1/4/a^2*b*d^(1/2)*ln(((x^2+(-a*b)^(1/2)/b)*d-(-a*b)^(1/2)/b*d)/d^(1/2)+((x
^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)
^(1/2))-1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)
/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-
a*b)^(1/2)/b))*d+1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2
+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2))/(x^2+(-a*b)^(1/2)/b))*c+1/4/a^2*b^2/(-a*b)^(1/2)*((x^2-(-a*b)^(1/2)/b
)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/4/a^2*b*
d^(1/2)*ln(((x^2-(-a*b)^(1/2)/b)*d+(-a*b)^(1/2)/b*d)/d^(1/2)+((x^2-(-a*b)^(
1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))+1/4
/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-
```

```
a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b
))*d-1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-
a*b)^(1/2)/b))*c-1/6/a/x^6*(d*x^4+c)^(3/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{\left(bx^4 + a\right)x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^7/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4 + c}}{x^7\,\left(b\,x^4 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^7*(a + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{x^7\left(a + bx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**7/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**7*(a + b*x**4)), x)
```

3.795 $\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=857

$$\frac{\sqrt{dx^4+c}\,x^3}{5b}+\frac{(2bc-5ad)\sqrt{dx^4+c}\,x}{5b^2\sqrt{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b^2}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b^2}-\frac{\sqrt[4]{c}\,(2b}{}$$

```
[Out] 1/5*x^3*(d*x^4+c)^(1/2)/b+1/5*(-5*a*d+2*b*c)*x*(d*x^4+c)^(1/2)/b^2/d^(1/2)/
(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(
d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b^2-1/4*a*arctan(x*((-
a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/
b^(1/2))^(1/2)/b^2-1/5*c^(1/4)*(-5*a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/
4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/b^2/d^(3/4)/(d*x^4+c)^(1/2)+1/5*c^(1/4)*(-5*a^2*d^2+a*b*c*
d+b^2*c^2)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/d^(3/4)/(a*d+b
*c)/(d*x^4+c)^(1/2)+1/8*a*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(
1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((
-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(-a*d+b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^
(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/
2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2
)^(1/2)/b^(5/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x
^4+c)^(1/2)
```

Rubi [A] time = 1.86, antiderivative size = 1067, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {478, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{dx^4+c}\,x^3}{5b}+\frac{(2bc-5ad)\sqrt{dx^4+c}\,x}{5b^2\sqrt{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)}+\frac{(-a)^{3/4}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{4b^{9/4}}+\frac{(-a)^{3/4}\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{ad}}{\sqrt[4]{-a}\sqrt[4]{b}}\right)}{4b^{9/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x^3*Sqrt[c + d*x^4])/(5*b) + ((2*b*c - 5*a*d)*x*Sqrt[c + d*x^4])/(5*b^2*Sq
rt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b
*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(9/4)) + ((-a)^(3/
4)*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqr
t[c + d*x^4])])/(4*b^(9/4)) - (c^(1/4)*(2*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(5*b^2*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(2*b*c - 5*a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(10*b^2*d^(3/4)*Sqrt[c + d*x^4]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcT
an[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

```
+ (a*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) +
(Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*
Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(5/2)*c^(1/4)*d^(1/4)*(b*c +
 a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \frac{x^2\left(3ac+(-2bc+5ad)x^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} - \frac{\int \left(-\frac{(2bc-5ad)x^2}{b\sqrt{c+dx^4}} - \frac{5\left(-abc+a^2d\right)x^2}{b\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{5b}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{5b^2} - \frac{(a(bc-ad))\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{\left(\sqrt{c}\,(2bc-5ad)\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} - \frac{\left(\sqrt{c}\,(2bc-5ad)\right)\int \frac{1-\frac{\sqrt{d}\,x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{5b^2\sqrt{d}} + \frac{(a(bc-a}{}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\,(2bc-5ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\Big(2}{5b^2d^{3/4}\sqrt{c+dx^4}}$$

$$= \frac{x^3\sqrt{c+dx^4}}{5b} + \frac{(2bc-5ad)x\sqrt{c+dx^4}}{5b^2\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{(-a)^{3/4}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{9/4}} + \frac{(-a)^{3/}}{}$$

Mathematica [C] time = 0.12, size = 141, normalized size = 0.16

$$\frac{x^7\sqrt{\frac{dx^4}{c}+1}\,(2bc-5ad)F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) - 7acx^3\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + 7ax^3\left(c+dx^4\right)}{35ab\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (7*a*x^3*(c + d*x^4) - 7*a*c*x^3*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4, 1/2, 1,
7/4, -((d*x^4)/c), -((b*x^4)/a)] + (2*b*c - 5*a*d)*x^7*Sqrt[1 + (d*x^4)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(35*a*b*Sqrt[c + d
*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

maple [C] time = 0.77, size = 421, normalized size = 0.49

$$-\frac{\left(\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)\right)\sqrt{c}\,\sqrt{d}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{Elliptic}\ldots}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x\ldots}}\right.}{8\ldots}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/b*(1/5*x^3*(d*x^4+c)^(1/2)+2/5*I*c^(3/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c
^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)/d
^(1/2)*(EllipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(
1/2))^(1/2),I)))-a/b*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(1-I/c^
(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*(E
llipticF(x*(I/c^(1/2)*d^(1/2))^(1/2),I)-EllipticE(x*(I/c^(1/2)*d^(1/2))^(1/
2),I))-1/8/b^2*sum((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2
*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(
1/2))^(1/2)*_alpha^3*b/a*(1-I/c^(1/2)*d^(1/2)*x^2)^(1/2)*(1+I/c^(1/2)*d^(1/
2)*x^2)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(x*(I/c^(1/2)*d^(1/2))^(1/2),I*c^(1
/2)/d^(1/2)*_alpha^2/a*b,(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,x^6}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^6/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x^6*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**6*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.796 $\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=700

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)(b}{8b^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}\,(}$$

```
[Out] 1/3*x*(d*x^4+c)^(1/2)/b-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/
2))^(1/2)/(d*x^4+c)^(1/2))/b^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*(-a
*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b^2
/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/3*c^(3/4)*(-2*a*d+b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*
(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 904, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {478, 523, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^2\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt{-a}\sqrt[4]{d}\,(bc}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b) - ((-a)^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c -
a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]])/(4*b^(7/4)) + ((-a)^(1/4)*Sq
rt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c +
 d*x^4])]])/(4*b^(7/4)) + ((2*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(6*b^2*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt
[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*
x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*S
qrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[
```

```
-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sq
rt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^
2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{x\sqrt{c+dx^4}}{3b} - \frac{\int \frac{ac+(-2bc+3ad)x^4}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{3b}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2} - \frac{(a(bc-ad))\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b^2}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{(bc-ad)\int \ldots}{2\ldots}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} + \frac{(2bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}\ldots\right.\right.}{\ldots}$$

$$= \frac{x\sqrt{c+dx^4}}{3b} - \frac{\sqrt[4]{-a}\,\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}} + \frac{\sqrt[4]{-a}\,\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}} \ldots$$

Mathematica [C] time = 0.48, size = 241, normalized size = 0.34

$$\frac{x\left(5\left(\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}+c+dx^4\right)+\frac{x^4\sqrt{\frac{dx^4}{c}+1}\,(2bc-3ad)F\ldots}{\ldots}\right)}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (x*(((2*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/a + 5*(c + d*x^4 + (5*a^2*c^2*AppellF1[1/4, 1/2,
1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2
, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2,
9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)
/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,x^4}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

maple [C] time = 0.35, size = 368, normalized size = 0.53

$$-\frac{\left(\frac{\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,\frac{i\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{2}b}{a\sqrt{d}}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,a}\right.}{8b^{2}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] 1/b*(1/3*x*(d*x^4+c)^(1/2)+2/3*c/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1
/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF(
(I/c^(1/2)*d^(1/2))^(1/2)*x,I))-a/b*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1
/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*El
lipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/(
(-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)
/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1
/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi
((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(
1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}\, x^4}{bx^4 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^4/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{d\,x^4+c}}{b\,x^4+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x^4*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \sqrt{c + dx^{4}}}{a + bx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**4*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.797 $\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=786

$$-\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)(bc-ad)\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b^{3/2}\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}\left(\sqrt{-a}\,\sqrt{b}\,\sqrt{c}-a\sqrt{d}\right)}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)}{}$$

```
[Out] x*d^(1/2)*(d*x^4+c)^(1/2)/b/(c^(1/2)+x^2*d^(1/2))+1/4*arctan(x*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1
/2)/b+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*(
(-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b-c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/(d*x^4+c)^(1/2)+a*c^(1/4)*d^(5/4)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c
)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/
2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))
/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^
(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 1012, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {491, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{8\sqrt{-a}\,b^{3/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\,\tan}{4}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(b*(Sqrt[c] + Sqrt[d]*x^2)) + (Sqrt[b*c - a*d]*
ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(
1/4)*b^(5/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(1/4)*b^(5/4)) - (c^(1/4)*d^(1/4)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)
*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipt
icF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*Sqrt[c + d*x^4]) - ((Sqrt[c]
- (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*S
qrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c
^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqr
t[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
 + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)
], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sq
```

```
rt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(
4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]
*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
+ d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(8*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 491

```
Int[((x_)^2*Sqrt[(c_) + (d_.)*(x_)^4])/((a_) + (b_.)*(x_)^4), x_Symbol] :>
Dist[d/b, Int[x^2/Sqrt[c + d*x^4], x], x] + Dist[(b*c - a*d)/b, Int[x^2/((a
 + b*x^4)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a
*d, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} + \frac{(bc-ad)\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{(bc-ad}{}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{b\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\right.}{}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{b\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\,b^{5/4}} + \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\,b^{5/4}} - $$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.08

$$\frac{x^3\sqrt{c+dx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (x^3*Sqrt[c + d*x^4]*AppellF1[3/4, -1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)
])/(3*a*Sqrt[(c + d*x^4)/c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

maple [C] time = 0.29, size = 299, normalized size = 0.38

$$\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)\right)\sqrt{c}\,\sqrt{d}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1
/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF((I/c^(1/2)*d
^(1/2))^(1/2)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/8/b^2*sum((a
*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/
((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*
b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4
+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(
1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_
Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,x^2}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)*x^2/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4),x)
```

```
[Out] int((x^2*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.798 $\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=679

$$\frac{c^{3/4}d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{\sqrt{c+dx^4}\,(ad+bc)}+\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4ab\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{bc-\dots}{\sqrt{-a}\dots}}}{\sqrt{c+d\dots}}\right)}{4ab\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}$$

```
[Out] 1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2
))/a/b/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*(-a*d+b*c)*arctan(x*((-a*d+
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/b/((-a*d+b*c)/(-a)^(1/2)/
b^(1/2))^(1/2)+c^(3/4)*d^(3/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2
)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/
c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1
/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(-a*d+b*c)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1
/2)
```

Rubi [A] time = 0.98, antiderivative size = 881, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {406, 220, 409, 1217, 1707}

$$\frac{(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8ab\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}\,(bc-ad)\,(\dots}{\dots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] -(Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])])/(4*(-a)^(3/4)*b^(3/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
 a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*b^(3/4)) + (d
^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*Sqrt[c + d*x^4]
) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((
Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*
x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4
)*x)/c^(1/4)], 1/2])/(4*Sqrt[-a]*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] -
Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*
```

```
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + S
qrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(8*a*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{(-bc+ad)\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} - \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2ab} - \frac{(-b}{}$$

$$= \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\sqrt{c+dx^4}} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)(bc-ad)\right)\int}{2a\sqrt{b}\,(bc+ad)}$$

$$= -\frac{\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}b^{3/4}} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)}{2}$$

Mathematica [C] time = 0.21, size = 161, normalized size = 0.24

$$\frac{5acx\sqrt{c+dx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{\left(a+bx^4\right)\left(2x^4\left(adF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2bcF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[c + d*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4
)/a)])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(-2*b*c*AppellF1[5/4, -1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

maple [C] time = 0.27, size = 273, normalized size = 0.40

$$\frac{\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}b\,\mathrm{EllipticPi}\left(\cdots\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,a\cdots}\right.}{8b^{2}\,\mathrm{RootC}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d
^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I
)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_a
lpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2
))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x
^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha
^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)
)),_alpha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(a + b*x^4),x)
```

```
[Out] int((c + d*x^4)^(1/2)/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.799 $\int \frac{\sqrt{c+dx^4}}{x^2(a+bx^4)}\,dx$

Optimal. Leaf size=809

$$\frac{b\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)c^{5/4}}{a(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{dx^4+c}}$$

```
[Out] -(d*x^4+c)^(1/2)/a/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/(c^(1/2)+x^2*d^(1/2))-1/4*
arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(
-a)^(1/2)/b^(1/2))^(1/2)/a-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/
2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a-c^(1/4)*d^(1/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(d*x^4+c)^(1/2)+b*c^(5/4)*
d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c
^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x
^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(
1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(3/2)*b^(1/
2)*c^(1/2)+a^2*d^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^
2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/
4)/d^(1/4)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.51, antiderivative size = 1031, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {475, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8(-a)^{3/2}\sqrt{b}\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt{bc-ad}\tan}{4(}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*(Sqrt[c] + Sqrt[d
]*x^2)) + (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*S
qrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1/4)) + (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt
[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*b^(1
/4)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*Sqrt[c +
 d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*Sq
rt[c + d*x^4]) + ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d
)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c +
 d*x^4]) + ((Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - a*d)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
```

```
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[c] + Sqrt[d]
*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a*d)*(Sq
rt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx &= -\frac{\sqrt{c+dx^4}}{ax}+\frac{\int \frac{x^2\left(-bc+2ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}\\
&= -\frac{\sqrt{c+dx^4}}{ax}+\frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}}+\frac{(-bc+ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{a}\\
&= -\frac{\sqrt{c+dx^4}}{ax}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{a}+\frac{(-bc+ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}\\
&= -\frac{\sqrt{c+dx^4}}{ax}+\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{a}-\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{a}+\frac{(bc-ad)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx}}}{2a\sqrt{b}}\\
&= -\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{a\sqrt{c+dx^4}}+\\
&= -\frac{\sqrt{c+dx^4}}{ax}+\frac{\sqrt{d}x\sqrt{c+dx^4}}{a\left(\sqrt{c}+\sqrt{d}x^2\right)}+\frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt[4]{b}}+\frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-a}}\right.}{4(-a)^{5/4}\sqrt[4]{b}}
\end{aligned}$$

Mathematica [C] time = 0.13, size = 138, normalized size = 0.17

$$\frac{-7x^4\sqrt{\frac{dx^4}{c}+1}\,(bc-2ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-21a\left(c+dx^4\right)}{21a^2x\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^2*(a + b*x^4)),x]
```

```
[Out] (-21*a*(c + d*x^4) - 7*(b*c - 2*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*Ap
pellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*x*Sqrt[c + d
*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

maple [C] time = 0.29, size = 421, normalized size = 0.52

$$-\frac{\left(\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)\right)\sqrt{c}\,\sqrt{d}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{El}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right.}{}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x)
```

```
[Out] -1/a*b*(I*d^(1/2)/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x
^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF((I/c
^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/8/b^
2*sum((a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x
^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(
1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))+1/a*(-(d*x^4+c)^(1/2)/x+2*I*d^(1/2)*c^(1/2)/(I/c^(1/2)*d
^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1
/2)/(d*x^4+c)^(1/2)*(EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/
c^(1/2)*d^(1/2))^(1/2)*x,I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{(bx^4+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{x^2\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/(x^2*(a + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{x^2\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**2/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(x**2*(a + b*x**4)), x)
```

3.800 $\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=703

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{(bc-ad)\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\cdots\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}\cdots}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/x^3-1/4*(-a*d+b*c)*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^
(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*
(-a*d+b*c)*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/
a^2/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/3*d^(3/4)*(-a*d+2*b*c)*(cos(2*a
rctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1
/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d
^(1/2))/(d*x^4+c)^(1/2)-1/8*(-a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)
^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c
^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(
1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)
*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 893, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {475, 523, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8a^2\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{d}\,(bc-\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] -Sqrt[c + d*x^4]/(3*a*x^3) - (b^(1/4)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*
d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)) + (b^(1/4)*Sqrt
[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d
*x^4])])/(4*(-a)^(7/4)) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)
/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/
(6*a*c^(1/4)*Sqrt[c + d*x^4]) - (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d
]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(b*
c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*(-a)^(3/2)*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c
```

```
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - a
*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*
Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c
+ a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx = -\frac{\sqrt{c+dx^4}}{3ax^3}+\frac{\int \frac{-3bc+2ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3a}-\frac{(bc-ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}-\frac{(bc-ad)\int \frac{}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right.}}{2a^2}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6a\sqrt[4]{c}\sqrt{c+dx^4}}-\frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx^4}}{3ax^3}-\frac{\sqrt[4]{b}\sqrt{bc-ad}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}}+\frac{\sqrt[4]{b}\sqrt{-bc+ad}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right.}{4(-a)^{7/4}}$$

Mathematica [C] time = 0.30, size = 333, normalized size = 0.47

$$\frac{\frac{a\left(25ac\left(ac-adx^4+4bcx^4+bdx^8\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-10x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}{\left(a+bx^4\right)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}-bdx^8}{15a^2x^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(x^4*(a + b*x^4)),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (a*(25*a*c*(a*c + 4*b*c*x^4 - a*d*x^4 + b*d*x^8)*AppellF1[1/4
, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 10*x^4*(a + b*x^4)*(c + d*x^4)
*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*Appell
F1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(-5*a*c*Ap
pellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF
1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1,
 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*x^3*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="giac")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)

maple [C] time = 0.30, size = 370, normalized size = 0.53

$$-\frac{\left(\frac{\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,b}-\frac{(ad-bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,\frac{i\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{2}b\sqrt{c}}{a\sqrt{d}},\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,a}\right.}{8b^{2}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x)

[Out] 1/a*(-1/3*(d*x^4+c)^(1/2)/x^3+2/3*d/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/a*b*(d/b/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-1/8/b^2*sum((a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x^4+c)^(1/2)/x^4/(b*x^4+a),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*x^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d\,x^4+c}}{x^4\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)),x)

[Out] int((c + d*x^4)^(1/2)/(x^4*(a + b*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{x^4\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/x**4/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/(x**4*(a + b*x**4)), x)
```

3.801 $\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] 2/5*(e*x)^(5/2)*AppellF1(5/8,1,-1/2,13/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$\frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(5/2)*Sqrt[c + d*x^4]*AppellF1[5/8, 1, -1/2, 13/8, -((b*x^4)/a), -
((d*x^4)/c)])/(5*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(ex)^{3/2}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^4\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^4\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= \frac{2(ex)^{5/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};1,-\frac{1}{2};\frac{13}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{5ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.99

$$\frac{2x(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{5}{8};-\frac{1}{2},1;\frac{13}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e*x)^(3/2)*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (2*x*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[5/8, -1/2, 1, 13/8, -((d*x^4)/c),
 -((b*x^4)/a)])/(5*a*Sqrt[(c + d*x^4)/c])
```

fricas [F] time = 1.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,\sqrt{ex}\,ex}{bx^4+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)*e*x/(b*x^4 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,(ex)^{\frac{3}{2}}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(3/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*(e*x)^(3/2)/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^{3/2}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(3/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^{\frac{3}{2}}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(3/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral((e*x)**(3/2)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.802 $\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$

Optimal. Leaf size=71

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] 2/3*(e*x)^(3/2)*AppellF1(3/8,1,-1/2,11/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)
/a/e/(1+d*x^4/c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$\frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
```

```
[Out] (2*(e*x)^(3/2)*Sqrt[c + d*x^4]*AppellF1[3/8, 1, -1/2, 11/8, -((b*x^4)/a), -
((d*x^4)/c)])/(3*a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{ex}\sqrt{c+dx^4}}{a+bx^4}\,dx = \frac{2\,\text{Subst}\left(\int \frac{x^2\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$
$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{x^2\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$
$$= \frac{2(ex)^{3/2}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};1,-\frac{1}{2};\frac{11}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{3ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.99

$$\frac{2x\sqrt{ex}\sqrt{c+dx^4}\,F_1\left(\frac{3}{8};-\frac{1}{2},1;\frac{11}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[e*x]*Sqrt[c + d*x^4])/(a + b*x^4),x]
[Out] (2*x*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[3/8, -1/2, 1, 11/8, -((d*x^4)/c), -((b*x^4)/a)])/(3*a*Sqrt[(c + d*x^4)/c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\sqrt{ex}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
[Out] int((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}\,\sqrt{ex}}{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^(1/2)*(d*x^4+c)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)*sqrt(e*x)/(b*x^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{e\,x}\,\sqrt{d\,x^4+c}}{b\,x^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4),x)
[Out] int(((e*x)^(1/2)*(c + d*x^4)^(1/2))/(a + b*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex}\,\sqrt{c+dx^4}}{a+bx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**(1/2)*(d*x**4+c)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(e*x)*sqrt(c + d*x**4)/(a + b*x**4), x)
```

3.803 $\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] 2*AppellF1(1/8,1,-1/2,9/8,-b*x^4/a,-d*x^4/c)*(e*x)^(1/2)*(d*x^4+c)^(1/2)/a/
e/(1+d*x^4/c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 430, 429}

$$\frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
```

```
[Out] (2*Sqrt[e*x]*Sqrt[c + d*x^4]*AppellF1[1/8, 1, -1/2, 9/8, -((b*x^4)/a), -((d
*x^4)/c)])/(a*e*Sqrt[1 + (d*x^4)/c])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\left(a+bx^4\right)}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e}$$
$$= \frac{\left(2\sqrt{c+dx^4}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{a+\frac{bx^8}{e^4}}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$
$$= \frac{2\sqrt{ex}\,\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};1,-\frac{1}{2};\frac{9}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.99

$$\frac{2x\sqrt{c+dx^4}\,F_1\left(\frac{1}{8};-\frac{1}{2},1;\frac{9}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a\sqrt{ex}\,\sqrt{\frac{c+dx^4}{c}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/(Sqrt[e*x]*(a + b*x^4)),x]
[Out] (2*x*Sqrt[c + d*x^4]*AppellF1[1/8, -1/2, 1, 9/8, -((d*x^4)/c), -((b*x^4)/a)
])/(a*Sqrt[e*x]*Sqrt[(c + d*x^4)/c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^4+c}}{\sqrt{ex}\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(1/2)/(b*x^4+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*sqrt(e*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4+c}}{\sqrt{e\,x}\,\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)),x)
[Out] int((c + d*x^4)^(1/2)/((e*x)^(1/2)*(a + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^4}}{\sqrt{ex}\,\left(a+bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(1/2)/(b*x**4+a),x)
[Out] Integral(sqrt(c + d*x**4)/(sqrt(e*x)*(a + b*x**4)), x)
```

3.804 $\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\left(a+bx^4\right)}\,dx$

Optimal. Leaf size=69

$$-\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

```
[Out] -2*AppellF1(-1/8,1,-1/2,7/8,-b*x^4/a,-d*x^4/c)*(d*x^4+c)^(1/2)/a/e/(e*x)^(1
/2)/(1+d*x^4/c)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {466, 511, 510}

$$-\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{\frac{dx^4}{c}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (-2*Sqrt[c + d*x^4]*AppellF1[-1/8, 1, -1/2, 7/8, -((b*x^4)/a), -((d*x^4)/c)
])/(a*e*Sqrt[e*x]*Sqrt[1 + (d*x^4)/c])
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\sqrt{c+dx^4}}{(ex)^{3/2}\left(a+bx^4\right)}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{\sqrt{c+\frac{dx^8}{e^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e}$$

$$= \frac{\left(2\sqrt{c+dx^4}\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{dx^8}{ce^4}}}{x^2\left(a+\frac{bx^8}{e^4}\right)}\,dx, x, \sqrt{ex}\right)}{e\sqrt{1+\frac{dx^4}{c}}}$$

$$= -\frac{2\sqrt{c+dx^4}\,F_1\left(-\frac{1}{8};1,-\frac{1}{2};\frac{7}{8};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae\sqrt{ex}\sqrt{1+\frac{dx^4}{c}}}$$

Mathematica [B] time = 0.18, size = 143, normalized size = 2.07

$$\frac{x\left(-10x^4\sqrt{\frac{dx^4}{c}+1}\,(bc-4ad)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+14bdx^8\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-70a\left(c+d\right.\right.}{35a^2(ex)^{3/2}\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c + d*x^4]/((e*x)^(3/2)*(a + b*x^4)),x]
```

```
[Out] (x*(-70*a*(c + d*x^4) - 10*(b*c - 4*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^4)/c), -((b*x^4)/a)] + 14*b*d*x^8*Sqrt[1 + (d*x^4)
/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^4)/c), -((b*x^4)/a)]))/(35*a^2*(e*x
)^(3/2)*Sqrt[c + d*x^4])
```

fricas [F] time = 1.35, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{dx^4+c}\,\sqrt{ex}}{be^2x^6+ae^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*sqrt(e*x)/(b*e^2*x^6 + a*e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4+c}}{\left(bx^4+a\right)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d\,x^4+c}}{(ex)^{\frac{3}{2}}\left(b\,x^4+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx^4 + c}}{\left(bx^4 + a\right)(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^(1/2)/(e*x)^(3/2)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^4 + c)/((b*x^4 + a)*(e*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^4 + c}}{(e\,x)^{3/2}\,\left(b\,x^4 + a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)),x)
```

```
[Out] int((c + d*x^4)^(1/2)/((e*x)^(3/2)*(a + b*x^4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^4}}{(ex)^{\frac{3}{2}}\left(a + bx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**(1/2)/(e*x)**(3/2)/(b*x**4+a),x)
```

```
[Out] Integral(sqrt(c + d*x**4)/((e*x)**(3/2)*(a + b*x**4)), x)
```

3.805 $\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\,(ad+bc)}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}$$

```
[Out] 1/6*(d*x^4+c)^(3/2)/b/d^2-1/2*a^2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/2*(a*d+b*c)*(d*x^4+c)^(1/2)/b^2/d^2
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}\,(ad+bc)}{2b^2d^2}+\frac{\left(c+dx^4\right)^{3/2}}{6bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^4])/(2*b^2*d^2) + (c + d*x^4)^(3/2)/(6*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}} + \frac{a^2}{b^2(a+bx)\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{bd}\right)dx, x, x^4\right)$$
$$= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4b^2}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2b^2d}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^4}}{2b^2d^2} + \frac{(c+dx^4)^{3/2}}{6bd^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.26, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^4}\left(-3ad-2bc+bdx^4\right)}{6b^2d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-2*b*c - 3*a*d + b*d*x^4))/(6*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*b^(5/2)*Sqrt[b*c - a*d])
```

fricas [A] time = 1.24, size = 289, normalized size = 2.78

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right) - 2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2-\left(b^3cd-ab^2d^2\right)x^4\right)\sqrt{dx^4+\cdots}}{12\left(b^4cd^2-ab^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*
x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) - 2*(2*b^3*c^2 + a*b^2*c*d - 3*a
^2*b*d^2 - (b^3*c*d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/6*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^4 + b*c)) - (2*b^3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 - (b^3*c*
d - a*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

giac [A] time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}\,b^2} + \frac{(dx^4+c)^{\frac{3}{2}}b^2d^4 - 3\,\sqrt{dx^4+c}\,b^2cd^4 - 3\,\sqrt{dx^4+c}\,abd^5}{6\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/6*((d*x^4 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^4 + c)*b^2*c*d^4 - 3*s
qrt(d*x^4 + c)*a*b*d^5)/(b^3*d^6)
```

maple [B] time = 0.28, size = 378, normalized size = 3.63

$$\frac{\sqrt{d\,x^4+c}\;x^4}{6bd}-\frac{a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\;b^3}-\frac{a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b*(d*x^4+c)^(1/2)/d*x^4-1/3/b*(d*x^4+c)^(1/2)/d^2*c-1/2/b^2*a/d*(d*x^4+
c)^(1/2)-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))-1/4*a^2/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2
)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/
b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.82, size = 102, normalized size = 0.98

$$\frac{{\left(d\,x^4+c\right)}^{3/2}}{6\,b\,d^2}-\left(\frac{c}{b\,d^2}+\frac{2\,a\,d^3-2\,b\,c\,d^2}{4\,b^2\,d^4}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b*d^2) - (c/(b*d^2) + (2*a*d^3 - 2*b*c*d^2)/(4*b^2*d^4
))*(c + d*x^4)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1
/2)))/(2*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.806 $\int \frac{x^7}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

```
[Out] 1/2*a*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a
*d]])/(2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx=\frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)$$
$$=\frac{\sqrt{c+dx^4}}{2bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b}$$
$$=\frac{\sqrt{c+dx^4}}{2bd}-\frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2bd}$$
$$=\frac{\sqrt{c+dx^4}}{2bd}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{2}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/2
```

fricas [A] time = 1.55, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(b^2c-abd\right)}{4\left(b^3cd-ab^2d^2\right)},-\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b}}{bdx^4+b}\right.}{2\left(b^3cd-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c
)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/2*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^4 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - sqrt(d*x^4 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

giac [A] time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx^4+c}}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(a*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^4 + c)/b)/d
```

maple [B] time = 0.21, size = 335, normalized size = 4.53

$$\frac{a \ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,b^{2}}+\frac{a \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-b}{b}}\cdots}{x\cdots}\right)}{4\sqrt{-\frac{a}{\cdots}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b/d+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x
^2+(-a*b)^(1/2)/b))+1/4*a/b^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)
/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-
a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^4+c}}{2\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{2\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b*d) - (a*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)
^(1/2)))/(2*b^(3/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**7/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.807 $\int \frac{x^3}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,\sqrt{bc-ad}}$$

```
[Out] -1/2*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(2*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\,\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

fricas [A] time = 1.72, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)}{4\,\sqrt{b^2c-abd}},\ \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^4+c}\,\sqrt{-b^2c+abd}}{bdx^4+bc}\right)}{2\,\left(b^2c-abd\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b
*x^4 + a))/sqrt(b^2*c - a*b*d), 1/2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c))/(b^2*c - a*b*d)]
```

giac [A] time = 0.16, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [B] time = 0.21, size = 316, normalized size = 6.20

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\;b}-\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x^2+\cdots}\right)}{4\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*
(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)
*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/b/(
-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.80, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^4+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{2\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^4)^(1/2))/(a*b*d - b^2*c)^(1/2))/(2*(a*b*d - b^2*c)^(1/2))
```

sympy [A] time = 14.27, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*b*sqrt((a*d - b*c)/b))
```

3.808 $\int \frac{1}{x(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

```
[Out] -1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/2*arctanh(b^(1/2)*(d*x^4+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/(2*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^4\right)}{4a} - \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{2ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a\sqrt{c}} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a\sqrt{bc-ad}}$$

Mathematica [A] time = 0.09, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^4])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(2*a)
```

fricas [A] time = 1.36, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right)+\sqrt{c}\log\left(\frac{dx^4-2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right)}{4\,ac}, \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^4+c}\,(bc-ad}{bdx^4+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(c*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + sqrt(c)*log((d*x^4 - 2*sqrt
(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt
(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a*c), 1/4*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-
c)/c))/(a*c), 1/2*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^4 + c)*
sqrt(-c)/c))/(a*c)]
```

giac [A] time = 0.17, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a} + \frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [B] time = 0.29, size = 347, normalized size = 4.08

$$\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x^2+\cdots}\right)}{4\sqrt{-\frac{ad-\cdots}{b}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(
a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*
(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a/(-
(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2/a/c^(1/2)*ln(
(2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x), x)
```

mupad [B] time = 5.03, size = 652, normalized size = 7.67

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^4+c}}{\sqrt{c}}\right)}{2\,a\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{4\,\left(a^2\,d-a\,b\,c\right)}+\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\cdots}\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\,x^4+c}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(2\,a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^4+c}\,\sqrt{b^2\,c-a\,b\,d}}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}\right)}{4\,\left(a^2\,d-a\,b\,c\right)}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(b^3\,d^2\,\sqrt{d\cdots}\right.}{\cdots}}\right)\cdots}{2\,\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^4)^(1/2)/c^(1/2))/(2*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 -
((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2)
)/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a*b*c)) + ((b^
2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^2*c - a*b*d)^(1/2)*(2*a
^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(1/2)*(b^2*c -
 a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c)))*1i)/(4*(a^2*d - a
```

```
*b*c)))/(((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) - ((b^2*c - a*b*
d)^(1/2)*(2*a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^4)^(
1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b*c))))/(4*
(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*(b^3*d^2*(c + d*x^4)^(1/2) + ((b^
2*c - a*b*d)^(1/2)*(2*a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c
+ d*x^4)^(1/2)*(b^2*c - a*b*d)^(1/2))/(4*(a^2*d - a*b*c))))/(4*(a^2*d - a*b
*c))))/(4*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(2*(a^2*d - a*b*c))
```

sympy [**A**] time = 20.21, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{\frac{ad-bc}{b}}}\right)}{2a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^4}}{\sqrt{-c}}\right)}{2a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**4)/sqrt((a*d - b*c)/b))/(2*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**4)/sqrt(-c))/(2*a*sqrt(-c))
```

3.809 $\int \frac{1}{x^5(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

```
[Out] 1/4*(a*d+2*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/2*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/4*(d
*x^4+c)^(1/2)/a/c/x^4
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}}-\frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*a*c*x^4) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[
c]])/(4*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c
- a*d]])/(2*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)$$
$$= -\frac{\sqrt{c+dx^4}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$
$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, \ldots\right.}{8a^2c}$$
$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}} \ldots\right.}{4a^2cd}$$
$$= -\frac{\sqrt{c+dx^4}}{4acx^4} + \frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^2c^{3/2}} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.17, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{2a^2(ad-bc)} + \frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4ac^{3/2}} - \frac{\sqrt{c+dx^4}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/4*Sqrt[c + d*x^4]/(a*c*x^4) + (b*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(2*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/(4*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(2*a^2*
(-(b*c) + a*d))
```

fricas [A] time = 1.03, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^2x^4\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right) + (2\,bc+ad)\sqrt{c}\,x^4\log\left(\frac{dx^4+2\,\sqrt{dx^4+c}\,\sqrt{c}+2\,c}{x^4}\right) - 2\,\sqrt{dx^4+c}\,ac \ldots}{8\,a^2c^2x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d
*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + (2*b*c + a*d)*sqr
t(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*sqrt(d*x^4
+ c)*a*c)/(a^2*c^2*x^4), -1/8*(4*b*c^2*x^4*sqrt(-b/(b*c - a*d))*arctan(-sqr
t(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - (2*b*c + a
*d)*sqrt(c)*x^4*log((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*sqrt
```

```
(d*x^4 + c)*a*c)/(a^2*c^2*x^4), 1/4*(b*c^2*x^4*sqrt(b/(b*c - a*d))*log((b*d
*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*
x^4 + a)) - (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) -
 sqrt(d*x^4 + c)*a*c)/(a^2*c^2*x^4), -1/4*(2*b*c^2*x^4*sqrt(-b/(b*c - a*d))
*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
+ (2*b*c + a*d)*sqrt(-c)*x^4*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + sqrt(d*x^
4 + c)*a*c)/(a^2*c^2*x^4)]
```

giac [A] time = 0.17, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{2\sqrt{-b^2c+abd}\,a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}\,c} - \frac{\sqrt{dx^4+c}}{4\,acx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/4*(2*b*c + a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/4*sqrt(d*x^4 + c)/(a*c*x^4)
```

maple [B] time = 0.29, size = 402, normalized size = 3.44

$$-\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-\frac{ad-bc}{b}}\,a^2} - \frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad}{}}\right.}{}\right.}{4\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4*(d*x^4+c)^(1/2)/a/c/x^4+1/4/a*d/c^(3/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1
/2))/x^2)-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2
*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2
)/b))-1/4/a^2*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
+1/2/a^2*b/c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^5), x)
```

mupad [B] time = 5.35, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{4\,a^3\,d-4\,a^2\,b\,c} - \frac{\ln\left(\sqrt{d\,x^4+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-\right.}{4\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] (log((c + d*x^4)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(4*a^3*d - 4*a^2*b*c) - (log((c + d*x^4
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(4*(a^3*d - a^2*b*c)) - (c + d*x^4)^(1/2)/(4*a*c*x^4)
- (atan((b^4*d^4*(c + d*x^4)^(1/2)*3i)/(16*(c^3)^(1/2)*((3*b^4*d^4)/(16*c)
+ (5*a*b^3*d^5)/(32*c^2) + (a^2*b^2*d^6)/(32*c^3))) + (b^2*d^6*(c + d*x^4)^
(1/2)*1i)/(32*(c^3)^(1/2)*((5*b^3*d^5)/(32*a) + (b^2*d^6)/(32*c) + (3*b^4*c
*d^4)/(16*a^2))) + (b^3*d^5*(c + d*x^4)^(1/2)*5i)/(32*(c^3)^(1/2)*((3*b^4*d
^4)/(16*a) + (5*b^3*d^5)/(32*c) + (a*b^2*d^6)/(32*c^2))))*(a*d + 2*b*c)*1i)
/(4*a^2*(c^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a + bx^4\right) \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.810 $\int \frac{x^9}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

```
[Out] -1/4*(2*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(3/2)+1/2*a^(3/
2)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/4*x^2*(d*x^4+c)^(1/2)/b/d
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}+\frac{x^2\sqrt{c+dx^4}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4])/(4*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt
[a]*Sqrt[c + d*x^4])])/(2*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= \frac{x^2\sqrt{c+dx^4}}{4bd}-\frac{\operatorname{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4bd}\\ &= \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b^2}-\frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,\right.}{4b^2d}\\ &= \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2}-\frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,d\right.}{4b^2d}\\ &= \frac{x^2\sqrt{c+dx^4}}{4bd}+\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b^2\sqrt{bc-ad}}-\frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{4b^2d^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.21, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}}-\frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{d}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*x^2*Sqrt[c + d*x^4])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[
a]*Sqrt[c + d*x^4])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^2 + Sqrt[d]*
Sqrt[c + d*x^4]])/d^(3/2))/(4*b^2)
```

fricas [A] time = 2.82, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^4+c}\,bdx^2+ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^6-\left(abc^2-a^2cd\right)x^2\right)\sqrt{dx}}{b^2x^8+2\,abx^4+a^2}\right.}{8\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
- 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*
((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^
4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/8*(2
*sqrt(d*x^4 + c)*b*d*x^2 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 2*(b*c + 2*a*d)*sqrt(-
d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2), 1/8*(2*sqrt(d*x^4 + c)*
b*d*x^2 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)
*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b*c + 2*a*d)*s
qrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b^2*d^2), 1/4*(s
qrt(d*x^4 + c)*b*d*x^2 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*
d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + (b
*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)))/(b^2*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 0.32, size = 408, normalized size = 3.32

$$-\frac{a^2 \ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{a^2 \ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{4\sqrt{-a\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4*x^2*(d*x^4+c)^(1/2)/b/d-1/4/b*c/d^(3/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))
-1/2/b^2*a*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)+1/4*a^2/b^2/(-a*b)^(1/2)
/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4*a^2/b^2/(
-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*
d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(
1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x^9/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.811 $\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

```
[Out] 1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b/d^(1/2)-1/2*arctan(x^2*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2b}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2b}\\
&= -\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}} - \frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{\sqrt{bc-ad}}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/Sqrt[b
*c - a*d]) + Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]]/Sqrt[d])/(2*b)
```

fricas [A] time = 2.33, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\,\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^8-2\,(3\,abc^2-4\,a^2cd)x^4+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^6-(abc^2-a^2cd)x^2)\sqrt{dx^4+c}\,\sqrt{-\frac{a}{bc-ad}}}{b^2x^8+2\,abx^4+a^2}\right)+2\sqrt{d}\,\text{l}}{8\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^
2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^
2*x^8 + 2*a*b*x^4 + a^2)) + 2*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt
(d)*x^2 - c))/(b*d), 1/8*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d
+ 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 -
3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(
-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*sqrt(-d)*arctan(sqrt(-d)*
x^2/sqrt(d*x^4 + c)))/(b*d), 1/4*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c -
 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2))
 + sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(b*d), 1/4*(d
*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
```

```
sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) - 2*sqrt(-d)*arctan(sqrt(-d)*x^2/s
qrt(d*x^4 + c)))/(b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [B] time = 0.33, size = 356, normalized size = 3.91

$$\frac{a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\,\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-b}{b}} \cdots}{x^{2} \cdots}\right)}{4\sqrt{-ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)-1/4*a/b/(-a*b)^(1/2)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4*a/b/(-a*b)^(1/2)/(
-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)
/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a
*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(bx^{4}+a\right)\sqrt{dx^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5}}{\left(b\,x^{4}+a\right)\,\sqrt{d\,x^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.812 $\int \frac{x}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])]/(2*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 95, normalized size = 1.76

$$\frac{x^2\sqrt{\frac{dx^4}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{2a\sqrt{c+dx^4}\sqrt{\frac{dx^4}{c}-\frac{bx^4}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^4)/c]*ArcTanh[Sqrt[-((b*x^4)/a) + (d*x^4)/c]/Sqrt[1 + (d
*x^4)/c]])/(2*a*Sqrt[c + d*x^4]*Sqrt[-((b*x^4)/a) + (d*x^4)/c])
```

fricas [B] time = 0.99, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)}{8\left(abc-a^2d\right)}, \frac{\arctan\left(\frac{\left((bc-2\,ad)x^4\right.}{2\left((abcd-a^2\right.}\right.}{4\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(
3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt
(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2))/(a*b*c - a^2
*d), 1/4*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c -
a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2))/sqrt(a*b*c - a^
2*d)]
```

giac [A] time = 0.18, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{2\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

maple [B] time = 0.32, size = 322, normalized size = 5.96

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(\right.}}{x^2+\frac{\sqrt{-}}{b}}\right.}{4\sqrt{-ab}\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)
/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-
```

```
a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b
))-1/4/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)
/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(x/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(x/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.813 $\int \frac{1}{x^3(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

```
[Out] -1/2*b*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/2*(d*x^4+c)^(1/2)/a/c/x^2
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^4}}{2acx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(2*a*c*x^2) - (b*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqr
t[c + d*x^4])])/(2*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)$$
$$= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{\operatorname{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2ac}$$
$$= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{2a}$$
$$= -\frac{\sqrt{c+dx^4}}{2acx^2} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{3/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 4.83, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^4}{c}+1\right)\left(\frac{4x^4\left(c+dx^4\right)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^4}{c(bx^4+a)}\right)}{3c^2\left(a+bx^4\right)}+\frac{\left(c+2dx^4\right)\sin^{-1}\left(\sqrt{\frac{x^4(bc-ad)}{c(a+bx^4)}}\right)}{c\sqrt{\frac{ax^4(c+dx^4)(bc-ad)}{c^2(a+bx^4)^2}}}\right)}{2x^2\left(a+bx^4\right)\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/2*((1 + (d*x^4)/c)*(((c + 2*d*x^4)*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a +
 b*x^4))]])/(c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)^2)]) +
 (4*(b*c - a*d)*x^4*(c + d*x^4)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^4)/(c*(a + b*x^4))])/(3*c^2*(a + b*x^4))))/(x^2*(a + b*x^4)*Sqrt[c + d*x^4
])
```

fricas [B] time = 1.06, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^2\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\sqrt{dx^4+\ldots}}{8\left(a^2bc^2-a^3cd\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a*b*c + a^2*d)*b*c*x^2*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*
x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4
*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2), -1/4*(sqrt(a
*b*c - a^2*d)*b*c*x^2*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*
sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) +
2*sqrt(d*x^4 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^2)]
```

giac [A] time = 0.22, size = 116, normalized size = 1.45

$$\frac{1}{2} d^{\frac{3}{2}} \left(\frac{b \arctan\left(\frac{\left(\sqrt{d} x^2 - \sqrt{dx^4+c}\right)^2 b - bc + 2ad}{2\sqrt{abcd - a^2d^2}} \right)}{\sqrt{abcd - a^2d^2}\, ad} + \frac{2}{\left(\left(\sqrt{d}\,x^2 - \sqrt{dx^4+c}\right)^2 - c\right) ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2 - c)*a*d))
```

maple [B] time = 0.30, size = 350, normalized size = 4.38

$$\frac{b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d + \frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\; a} - \frac{b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b} - \frac{2(ad-bc)}{b} + 2\sqrt{-\frac{ad-bc}{b}}\ \cdots}{x^2+\cdots}\right)}{4\sqrt{-ab}\,\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2*(d*x^4+c)^(1/2)/a/c/x^2-1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((
-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/
2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b
*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)\sqrt{dx^4 + c}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(b\,x^4 + a\right)\,\sqrt{d\,x^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + b x^4\right) \sqrt{c + d x^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.814 $\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}\,(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

```
[Out] 1/2*b^2*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/6*(d*x^4+c)^(1/2)/a/c/x^6+1/6*(2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^
2/c^2/x^2
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^4}\,(2ad+3bc)}{6a^2c^2x^2}-\frac{\sqrt{c+dx^4}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(6*a*c*x^6) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^4])/(6*a^2*c^2
*x^2) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(2*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^7\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6ac}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}-\frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{6a^2c^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2a^2}\\
&= -\frac{\sqrt{c+dx^4}}{6acx^6}+\frac{(3bc+2ad)\sqrt{c+dx^4}}{6a^2c^2x^2}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{2a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 5.55, size = 137, normalized size = 1.19

$$\frac{\frac{b^2x^2\sqrt{\frac{dx^4}{c}+1}\,\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{a^3\sqrt{\frac{x^4(bc-ad)}{ac}}}+\frac{\left(c+dx^4\right)\left(-ac+2adx^4+3bcx^4\right)}{3a^2c^2x^6}}{2\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (((c + d*x^4)*(-(a*c) + 3*b*c*x^4 + 2*a*d*x^4))/(3*a^2*c^2*x^6) + (b^2*x^2*
Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(a^3
*Sqrt[((b*c - a*d)*x^4)/(a*c)]))/(2*Sqrt[c + d*x^4])
```

fricas [A] time = 1.13, size = 418, normalized size = 3.63

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^6\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-4\left((bc-2\,ad)x^6-acx^2\right)\sqrt{dx^4+c}\,\sqrt{-abc+a^2d}}{b^2x^8+2\,abx^4+a^2}\right)+4\left(a^2b\right.}{24\left(a^3bc^3-a^4c^2d\right)x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^6*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6
 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^
2)) + 4*(a^2*b*c^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*s
qrt(d*x^4 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^6), 1/12*(3*sqrt(a*b*c - a^2*d)*
b^2*c^2*x^6*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) - 2*(a^2*b*c
^2 - a^3*c*d - (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^4)*sqrt(d*x^4 + c))/
((a^3*b*c^3 - a^4*c^2*d)*x^6)]
```

giac [B] time = 1.68, size = 205, normalized size = 1.78

$$-\frac{1}{6}d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-6\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc-6\left(\sqrt{d}\,x^2\right.\right.}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2-c\right)^3a^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c +
 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*
c - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d)
*x^2 - sqrt(d*x^4 + c))^2 - c)^3*a^2*d^2))
```

maple [B] time = 0.27, size = 383, normalized size = 3.33

$$-\frac{b^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{4\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^2}+\frac{b^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-}{b}}\cdots}{x\cdots}\right.}{4\sqrt{-ab}\,\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/a^2*b/x^2*(d*x^4+c)^(1/2)/c+1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/
2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/a^2*b^2/(-a*b)^(1/2)/(-(a*d-
b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/6/a*(d*x^4+c)^(1/2)*
(-2*d*x^4+c)/x^6/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^7 \left(b x^4 + a\right) \sqrt{d x^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a + bx^4\right) \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.815 $\int \frac{x^8}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=872

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)a^2}{4b^2\sqrt[4]{c}\,(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{d}\,a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\ldots}{4b^2\sqrt[4]{c}\,(bc+ad\ldots}$$

```
[Out] -1/4*(-a)^(5/4)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/b^(7/4)/(-a*d+b*c)^(1/2)-1/4*(-a)^(5/4)*arctan(x*(a*d-b*c)^(1/2)/(-a)^(1
/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b*c)^(1/2)+1/3*x*(d*x^4+c)^(1/2)/
b/d-1/6*(3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/
4)/d^(5/4)/(d*x^4+c)^(1/2)+1/4*a^2*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4))
)^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*
x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d
*x^4+c)^(1/2)+1/4*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2
*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*
x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^
(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)
*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^
(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/
2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 872, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {479, 523, 220, 409, 1217, 1707}

$$\frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)a^2}{4b^2\sqrt[4]{c}\,(bc+ad)\sqrt{dx^4+c}}+\frac{\left(\sqrt{d}\,a+\sqrt{-a}\sqrt{b}\sqrt{c}\right)\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\ldots}{4b^2\sqrt[4]{c}\,(bc+ad\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(3*b*d) - ((-a)^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(7/4)*Sqrt[b*c - a*d]) - ((-a)^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(
7/4)*Sqrt[-(b*c) + a*d]) + (a^2*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1
/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ell
ipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^4]) + (a*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) -
 ((b*c + 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]
*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(6*b^2*c^(1/4)*d^(5
```

```
/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt
[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c
 + d*x^4]) + (a*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c]
 + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1
/4)*x)/c^(1/4)], 1/2])/(8*b^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{x\sqrt{c+dx^4}}{3bd} - \frac{\int \frac{ac+(bc+3ad)x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3bd}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} + \frac{a^2\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{b^2} - \frac{(bc+3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3b^2d}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\,d^{5/4}\sqrt{c+dx^4}} + \frac{a\int \frac{}{(1-}}{}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(bc+3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6b^2\sqrt[4]{c}\,d^{5/4}\sqrt{c+dx^4}} + \frac{\left(a\sqrt{c}\left(\right.\right.}{}$$

$$= \frac{x\sqrt{c+dx^4}}{3bd} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{bc-ad}} - \frac{(-a)^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{7/4}\sqrt{-bc+ad}} + \frac{a^2\left(\right.}{}$$

Mathematica [C] time = 0.35, size = 249, normalized size = 0.29

$$\frac{x\left(5\left(\frac{5a^2c^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{d\left(a+bx^4\right)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}+\frac{c}{d}+x^4\right)-\frac{x^4\sqrt{\frac{dx^4}{c}+1}\,(3ad+bc)F_1}{ad}\right.}{15b\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(-(((b*c + 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*d)) + 5*(c/d + x^4 + (5*a^2*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(d*(a + b*x^4)*(-5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1
/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -(
(d*x^4)/c), -((b*x^4)/a)]))))))/(15*b*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.32, size = 363, normalized size = 0.42

$$\frac{a^{2}\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3} b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,\frac{i\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{2} b\sqrt{c}}{a\sqrt{d}},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^{4}+c}\,a}-\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{2} d\,x^{2}+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^{4}+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right)}{8b^{3}\,\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b^2*(b*(1/3/d*x*(d*x^4+c)^(1/2)-1/3*c/d/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(
1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*E
llipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-a/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^
(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*
EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I))+1/8*a^2/b^3*sum(1/_alpha^3*(-1/((
-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/
(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/
2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi(
(I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1
/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(bx^{4}+a\right)\sqrt{dx^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{8}}{\left(b\,x^{4}+a\right)\,\sqrt{d\,x^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(a+bx^{4}\right)\sqrt{c+dx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.816 $\int \frac{x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$\frac{c^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{d}\sqrt{c+dx^4}\,(ad+bc)}-\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\cdots}{\cdots}$$

```
[Out] -1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/b/((a*d
-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2)
)^(1/2)/(d*x^4+c)^(1/2))/b/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*c^(3/4
)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/d^(1/4)/(a*d+b*c)/(d*x^4+c)
^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*
x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2
)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^
(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)
+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b
^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2
*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 837, normalized size of antiderivative = 1.31, number of steps used = 9, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {483, 220, 409, 1217, 1707}

$$-\frac{\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\cdots}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{d}\cdots}\right)}{4b^{3/4}\sqrt{bc-ad}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4]
)])/(4*b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)
/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(3/4)*Sqrt[-(b*c) + a*d]) + ((
Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elliptic
F[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*c^(1/4)*d^(1/4)*Sqrt[c + d*x^4]
) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x)/c^(1/4)], 1/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[-a]*S
qrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])
/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*S
qrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^
2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*
(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[
```

```
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d])
, 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqr
t[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2b} - \frac{\int \frac{}{\left(1+\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right.}}{}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2b\sqrt[4]{c}\,\sqrt[4]{d}\,\sqrt{c+dx^4}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\,\sqrt{d}}{\sqrt{b}}\right)\right)\int \frac{1+\frac{\sqrt{d}}{}}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)}}{2(bc+ad)}$$

$$= -\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{3/4}\sqrt{-bc+ad}} + \frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+}{\left(\sqrt{c}+\right.}}}{2b\sqrt[4]{c}}$$

Mathematica [C] time = 0.05, size = 65, normalized size = 0.10

$$\frac{x^5\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{5a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)
/a)])/(5*a*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.23, size = 265, normalized size = 0.42

$$-\frac{a\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x, \frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}}, \frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}\,a} - \frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right)}{8b^2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d
^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I
)-1/8*a/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2
*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/
2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^
(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b
*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.817 $\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=638

$$\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{c}\sqrt{c+dx^4}\,(ad+bc)}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}+\frac{\left(\sqrt{c}+\sqrt{d}\,x^2\right)\ldots}{\ldots}$$

```
[Out] 1/4*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a/((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))
^(1/2)/(d*x^4+c)^(1/2))/a/((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)+1/2*d^(3/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^4+c)^
(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)
+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+
x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2)
)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arct
an(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4
+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 742, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\tan\ldots\right.}{4\sqrt[4]{c}\sqrt{c+dx^4}\,(ad+bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])
/(4*(-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(3/4)*Sqrt[-(b*c) + a*d]) + ((
(Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqrt[b]*Sqr
t[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c]
 + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1
/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*
(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Ellipti
cPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqr
t[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d
)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^
4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx}{2a(bc+ad)} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right.}{2a(bc+a}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4(-a)^{3/4}\sqrt{-bc+ad}} + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\,(\sqrt{}}{}$$

Mathematica [C] time = 0.05, size = 161, normalized size = 0.25

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{(a+bx^4)\sqrt{c+dx^4}\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx}{c}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/((a + b*x
^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x
^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a
)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)\sqrt{dx^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.33, size = 191, normalized size = 0.30

$$\frac{\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x, \frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}}, \frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a} - \frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d\,x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{8b\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/
2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)\sqrt{dx^4 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.818 $\int \frac{1}{x^4(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=677

$$-\frac{b\tan^{-1}\left(\frac{x\sqrt{\frac{\sqrt{-a}\left(\frac{bc}{a}-d\right)}{\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{b\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4a^2\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}-\frac{b\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{-a}\sqrt{d}+\sqrt{b}\sqrt{c}\right)\Pi\left(-\frac{\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\right.}{4\sqrt{-a}\sqrt{b}\sqrt{}}\right.}{8a^2\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^4}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)}$$

```
[Out] -1/3*(d*x^4+c)^(1/2)/a/c/x^3-1/4*b*arctan(x*((b*c/a-d)*(-a)^(1/2)/b^(1/2))^
(1/2)/(d*x^4+c)^(1/2))/a^2/((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)-1/4*b*arcta
n(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))/a^2/((-a*d+b*c)/
(-a)^(1/2)/b^(1/2))^(1/2)-1/6*d^(3/4)*(a*d+4*b*c)*(cos(2*arctan(d^(1/4)*x/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d
^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a/c^(5/4)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/8*b*(cos(2*arcta
n(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(
sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(
1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))/(d*x^4+c)^(1/2)-1/
8*b*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/
4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+
x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))/(
d*x^4+c)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 864, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {480, 523, 220, 409, 1217, 1707}

$$-\frac{b\left(\sqrt{d}x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}x^2+\sqrt{c}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8a^2\sqrt[4]{c}\sqrt[4]{d}(bc+ad)\sqrt{dx^4+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{}}\right.}{4(-a)^{7/4}\sqrt{bc-a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(3*a*c*x^3) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1
/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*Ar
cTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^
(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(6*a*c^(5/4)*Sqrt[c + d*x^4]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])
*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2
]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*S
qrt[c + d*x^4]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a^2*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]
) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[
(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[
-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c
^(1/4)], 1/2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sq
```

```
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4
)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d]
)^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/
2])/(8*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{3acx^3}+\frac{\int \frac{-3bc-ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}-\frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{3ac}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}-\frac{b\int \frac{1}{\left(1-\frac{\sqrt{b}\,x^2}{\sqrt{-a}}\right)\cdots}}{2a\cdots}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{6ac^{5/4}\sqrt{c+dx^4}}-\frac{\left(b^{3/2}\sqrt{c}\left(\sqrt{\cdots}\right.\right.}{\cdots}$$

$$= -\frac{\sqrt{c+dx^4}}{3acx^3}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{bc-ad}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{7/4}\sqrt{-bc+ad}}-\frac{d^{3/4}\left(\sqrt{c}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.38, size = 337, normalized size = 0.50

$$\frac{\frac{5a\left(2x^4\left(a+bx^4\right)\left(c+dx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5ac\left(ac+2adx^4+4bcx^4+bdx^8\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}{\left(a+bx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)\right)}-bdx^8\sqrt{\cdots}}{15a^2cx^3\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-(b*d*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]) + (5*a*(-5*a*c*(a*c + 4*b*c*x^4 + 2*a*d*x^4 + b*d*x^8)*AppellF1
[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(a + b*x^4)*(c + d*x
^4)*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*App
ellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/((a + b*x^4)*(5*a*c*
AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)] - 2*x^4*(2*b*c*Appel
lF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2,
1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(15*a^2*c*x^3*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

maple [C] time = 0.31, size = 288, normalized size = 0.43

$$-\frac{\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,\frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a}-\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d\,x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{8a\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x)
[Out] 1/a*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3/c*d/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/
2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*Ell
ipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/8/a*sum(1/_alpha^3*(-1/((-a*d+b*c)
/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)
^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)
^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2
)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2
)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)\sqrt{dx^4+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.819 $\int \frac{x^6}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=804

$$\frac{\sqrt{dx^4+c}\,x}{b\sqrt{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)}-\frac{a\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b(bc-ad)}-\frac{a\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4b(bc-ad)}-\frac{\sqrt[4]{c}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4 \cdots}{\left(\sqrt{d}\,x^2 \cdots\right.}}}{bd^{3/4}\sqrt{\cdots}}$$

```
[Out] x*(d*x^4+c)^(1/2)/b/d^(1/2)/(c^(1/2)+x^2*d^(1/2))-1/4*a*arctan(x*((a*d-b*c)
/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^
(1/2)/b/(-a*d+b*c)-1/4*a*arctan(x*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*
x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/b/(-a*d+b*c)-c^(1/4)*(c
os(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*E
llipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^4+c)^(1/2)+1/2*c
^(1/4)*(2*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))
*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/d^(3/4)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*a*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))
),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(
1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2)
)*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/((-a)^(
1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1/8*a*(cos(2*arctan(d^(1/4)
*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2
/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(
1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/
c^(1/4)/d^(1/4)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 982, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {483, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{-a}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8b^{3/2}\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}+\frac{(-a)^{3/4}\tan^{-1}\left(\cdots\right.}{4b^{5/4}\sqrt{bc\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*Sqrt[c + d*x^4])/(b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^2)) + ((-a)^(3/4)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqr
t[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^4])])/(4*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sqrt[c] +
Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan
[(d^(1/4)*x)/c^(1/4)], 1/2])/(b*d^(3/4)*Sqrt[c + d*x^4]) + (c^(1/4)*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^4]) + (a*(Sqrt[c
] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(4*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (a*(Sqrt[c] + (Sqrt[-a]*Sq
rt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*c^(1/4
```

```
)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt
[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*
EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b^(3/2)*c^(1/4)*d^(1/
4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
```

```
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{(a+bx^4)\sqrt{c+dx^4}}\,dx = \frac{\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{b} - \frac{a\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx}{b}$$

$$= \frac{a\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} - \frac{a\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2b^{3/2}} + \frac{\sqrt{c}\int \frac{1}{\sqrt{c+dx^4}}\,dx}{b\sqrt{d}} - \frac{\sqrt{c}\int \frac{1-\sqrt{\cdot}}{\sqrt{c+\cdot}}}{b\sqrt{d}}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{bd^{3/4}\sqrt{c+dx^4}} + \frac{\sqrt[4]{c}\,(\cdots}{\cdots}$$

$$= \frac{x\sqrt{c+dx^4}}{b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4b^{5/4}\sqrt{-bc+ad}}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.08

$$\frac{x^7\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{7a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^4)*Sqrt[c + d*x^4]),x]
[Out] (x^7*Sqrt[(c + d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4
)/a)])/(7*a*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.34, size = 292, normalized size = 0.36

$$-\frac{a\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x, \frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}}, \frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}\,a} - \frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}\right)}{8b^2\,\mathrm{RootOf}\left(b\_Z^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] I/b*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c
^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)/d^(1/2)*(EllipticF((I/c^(1/2)*d
^(1/2))^(1/2)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/8*a/b^2*sum(
1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d
+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-
I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1
/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(
-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+
a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^6/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**6/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.820 $\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=656

$$\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{x\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}+\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\tan^{-1}\left(\frac{x\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\ldots\right.}{2\sqrt{c+dx^4}\,(ad+bc)}$$

```
[Out] 1/4*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-b*
c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*d+b*c)+1/4*arctan(x*((-a*d+b*c)/(-a)^(1/2)
/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(-a*
d+b*c)-1/2*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2
*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*
2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/b^(1/2)/((-a)^(1/2)
*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4)*x/c^(
1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^
(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^(1/2)*d^(1/2)/c^(1/2))^2/(-a)^
(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(
-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4
)/b^(1/2)/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 756, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {490, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)\ldots\right.}{4\sqrt[4]{c}\sqrt{c+dx^4}\,(ad+bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1
/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1/4)*b^
(1/4)*Sqrt[c + d*x^4])]/(4*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((Sqrt[
c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(4*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*c^(1/4)*(b*
c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c
] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(S
qrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[
c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]),
2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*Sqrt[-a]*Sqrt[b]*c^(1/4)*d^(1/4)*(b
*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}+\frac{\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2(bc+ad)}+\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)\right)\int}{2(bc+a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{4\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}}-\frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x\right.}{4\sqrt[4]{c}(bc}$$

Mathematica [C] time = 0.03, size = 65, normalized size = 0.10

$$\frac{x^3\sqrt{\frac{c+dx^4}{c}}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{3a\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

[Out] (x^3*Sqrt[(c + d*x^4)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)])/(3*a*Sqrt[c + d*x^4])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^4 + a)\sqrt{dx^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)

maple [C] time = 0.25, size = 191, normalized size = 0.29

$$\frac{\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x, \frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}}, \frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a} - \frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d\,x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{8b\,\mathrm{RootOf}\left(b\_Z^4+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)

[Out] 1/8/b*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^4 + a)\sqrt{dx^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((b*x^4 + a)*sqrt(d*x^4 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(b\,x^4+a)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + bx^4)\sqrt{c + dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.821 $\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=833

$$\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{ac\left(\sqrt{d}\,x^2+\sqrt{c}\right)}-\frac{b\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{-\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4a(bc-ad)}-\frac{b\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,\tan^{-1}\left(\frac{\sqrt{\frac{bc-ad}{\sqrt{-a}\sqrt{b}}}\,x}{\sqrt{dx^4+c}}\right)}{4a(bc-ad)}-\frac{\sqrt[4]{d}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{\ldots}\right.}}}{ac^{3/4}\sqrt{dx^4\ldots}}$$

```
[Out] -(d*x^4+c)^(1/2)/a/c/x+x*d^(1/2)*(d*x^4+c)^(1/2)/a/c/(c^(1/2)+x^2*d^(1/2))-
1/4*b*arctan(x*((a*d-b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((a*d-
b*c)/(-a)^(1/2)/b^(1/2))^(1/2)/a/(-a*d+b*c)-1/4*b*arctan(x*((-a*d+b*c)/(-a)
^(1/2)/b^(1/2))^(1/2)/(d*x^4+c)^(1/2))*((-a*d+b*c)/(-a)^(1/2)/b^(1/2))^(1/2
)/a/(-a*d+b*c)-d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arc
tan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(3
/4)/(d*x^4+c)^(1/2)+1/2*d^(1/4)*(a*d+2*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(
1/2))^2)^(1/2)/a/c^(3/4)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/8*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*ar
ctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1
/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(-d^(1/4)*(-a)^(1/2)/c^(1/
4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d
^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)-a*d^(1/2))/(d*x^4+c)^(1/2)-1
/8*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4
)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*c^(1/2)*(b^(1/2)-(-a)^
(1/2)*d^(1/2)/c^(1/2))^2/(-a)^(1/2)/b^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(d
^(1/4)*(-a)^(1/2)/c^(1/4)+c^(1/4)*b^(1/2)/d^(1/4))*(c^(1/2)+x^2*d^(1/2))*((
d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 1007, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {480, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{b}\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{8(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^4+c}}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc\ldots}}{\sqrt[4]{-a}\sqrt[4]{b}\ldots}\right.}{4(-a)^{5/4}\sqrt{bc-\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[c + d*x^4]/(a*c*x)) + (Sqrt[d]*x*Sqrt[c + d*x^4])/(a*c*(Sqrt[c] + Sq
rt[d]*x^2)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[
c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(4*(-a)^(5/4)*Sqrt[-(b*c)
 + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(a*c^(3/4)*Sqrt
[c + d*x^4]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
 Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(2*a*c^(3/4
)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqr
t[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4
]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x
```

```
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(4*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) + (Sqrt[b]*(S
qrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[
d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]
*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)]
, 1/2])/(8*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx = -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \frac{x^2\left(-bc+ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}}-\frac{bcx^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}-\frac{b\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{a}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{ac}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2a}-\frac{\sqrt{b}\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx}{2a}+\frac{\sqrt{d}\int}{a}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}}{\sqrt[4]{c}}\right.\right.}{ac^{3/4}\sqrt{c+dx^4}}$$

$$= -\frac{\sqrt{c+dx^4}}{acx}+\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{ac\left(\sqrt{c}+\sqrt{d}\,x^2\right)}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{4(-a)^{5/4}\sqrt{bc-ad}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}}\right.}{4(-a)^{5/4}\sqrt{-bc}}$$

Mathematica [C] time = 0.14, size = 141, normalized size = 0.17

$$\frac{7x^4\sqrt{\frac{dx^4}{c}+1}\,(ad-bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+3bdx^8\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-21a\left(c+dx^4\right)}{21a^2cx\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (-21*a*(c + d*x^4) + 7*(-(b*c) + a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*x^8*Sqrt[1 + (d*x^4)/c]*A
ppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(21*a^2*c*x*Sqrt[c
+ d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)\sqrt{dx^4 + c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

maple [C] time = 0.25, size = 310, normalized size = 0.37

$$-\frac{\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,\frac{i\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 b\sqrt{c}}{a\sqrt{d}},\frac{\sqrt{-\frac{i\sqrt{d}}{\sqrt{c}}}}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}}\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a}-\frac{\mathrm{arctanh}\left(\frac{2\,\mathrm{RootOf}\left(b\_Z^4+a\right)^2 d\,x^2+2c}{2\sqrt{\frac{-ad+bc}{b}}\,\sqrt{d\,x^4+c}}\right)}{\sqrt{\frac{-ad+bc}{b}}}}{8a\,\mathrm{RootOf}\left(b\_Z^4+a\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+
2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alp
ha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(
d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2
)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Roo
tOf(_Z^4*b+a))+1/a*(-1/c*(d*x^4+c)^(1/2)/x+I/c^(1/2)*d^(1/2)/(I/c^(1/2)*d^(
1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2
)/(d*x^4+c)^(1/2)*(EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/c^
(1/2)*d^(1/2))^(1/2)*x,I)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)\sqrt{dx^4 + c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)*sqrt(d*x^4 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^4\right) \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.822 $\int \frac{x^{15}}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=175

$$-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}+\frac{ax^8\sqrt{c+d}}{4b\left(a+bx^4\right)(b}$$

```
[Out] -1/4*a^2*(-5*a*d+6*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b
^(7/2)/(-a*d+b*c)^(3/2)+1/4*a*x^8*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)-1/
12*(4*b^2*c^2+8*a*b*c*d-15*a^2*d^2-b*d*(-5*a*d+2*b*c)*x^4)*(d*x^4+c)^(1/2)/
b^3/d^2/(-a*d+b*c)
```

Rubi [A] time = 0.17, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 98, 147, 63, 208}

$$-\frac{\sqrt{c+dx^4}\left(-15a^2d^2-bdx^4(2bc-5ad)+8abcd+4b^2c^2\right)}{12b^3d^2(bc-ad)}-\frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}}+\frac{ax^8\sqrt{c+d}}{4b\left(a+bx^4\right)(b}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^8*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[c + d*x^4]*(4
*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2 - b*d*(2*b*c - 5*a*d)*x^4))/(12*b^3*d^2*(
b*c - a*d)) - (a^2*(6*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b
*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{x^3}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x\left(2ac+\frac{1}{2}(-2bc+5ad)x\right)}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b(bc-ad)}$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} + \cdots$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} + \cdots$$

$$= \frac{ax^8\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}\left(4b^2c^2+8abcd-15a^2d^2-bd(2bc-5ad)x^4\right)}{12b^3d^2(bc-ad)} - \cdots$$

Mathematica [A] time = 0.31, size = 175, normalized size = 1.00

$$\frac{a^2(5ad-6bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{7/2}(bc-ad)^{3/2}} - \frac{\sqrt{c+dx^4}\left(-15a^3d^2+2a^2bd\left(4c-5dx^4\right)+2ab^2\left(2c^2+3cdx^4+d^2x^8\right)+2b^3cx \cdots\right.}{12b^3d^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -1/12*(Sqrt[c + d*x^4]*(-15*a^3*d^2 + 2*a^2*b*d*(4*c - 5*d*x^4) + 2*b^3*c*x
^4*(2*c - d*x^4) + 2*a*b^2*(2*c^2 + 3*c*d*x^4 + d^2*x^8)))/(b^3*d^2*(b*c -
a*d)*(a + b*x^4)) + (a^2*(-6*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])
/Sqrt[b*c - a*d]])/(4*b^(7/2)*(b*c - a*d)^(3/2))
```

fricas [A] time = 1.58, size = 622, normalized size = 3.55

$$\left[\frac{3\left(6a^3bcd^2-5a^4d^3+\left(6a^2b^2cd^2-5a^3bd^3\right)x^4\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2bc-ad-2\sqrt{dx^4+c}\sqrt{b^2c-abd}}{bx^4+a}\right)+2\left(2\left(b^5c^2d-2a \cdots\right.\right.}{24\left(ab^6c^2d^2-2a^2b^5cd^3+a^3 \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b^2*c*d^2 - 5*a^3*b*d^3)*x^4)*
sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2
*c - a*b*d))/(b*x^4 + a)) + 2*(2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*
x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d^3 - 2*(
2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqrt(d*x^4
+ c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2 - 2*a*b
^6*c*d^3 + a^2*b^5*d^4)*x^4), 1/12*(3*(6*a^3*b*c*d^2 - 5*a^4*d^3 + (6*a^2*b
^2*c*d^2 - 5*a^3*b*d^3)*x^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sq
rt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + (2*(b^5*c^2*d - 2*a*b^4*c*d^2 + a^2*b
^3*d^3)*x^8 - 4*a*b^4*c^3 - 4*a^2*b^3*c^2*d + 23*a^3*b^2*c*d^2 - 15*a^4*b*d
^3 - 2*(2*b^5*c^3 + a*b^4*c^2*d - 8*a^2*b^3*c*d^2 + 5*a^3*b^2*d^3)*x^4)*sqr
t(d*x^4 + c))/(a*b^6*c^2*d^2 - 2*a^2*b^5*c*d^3 + a^3*b^4*d^4 + (b^7*c^2*d^2
 - 2*a*b^6*c*d^3 + a^2*b^5*d^4)*x^4)]
```

giac [A] time = 0.17, size = 180, normalized size = 1.03

$$\frac{\sqrt{dx^4+c}\,a^3d}{4\left(b^4c-ab^3d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}+\frac{\left(6\,a^2bc-5\,a^3d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{-b^2c+abd}}+\frac{\left(dx^4+c\right)^{\frac{3}{2}}b^4d^4-3\,\sqrt{dx^4+c}\,b^4cd}{6\,b^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*a^3*d/((b^4*c - a*b^3*d)*((d*x^4 + c)*b - b*c + a*d)) +
 1/4*(6*a^2*b*c - 5*a^3*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(
(b^4*c - a*b^3*d)*sqrt(-b^2*c + a*b*d)) + 1/6*((d*x^4 + c)^(3/2)*b^4*d^4 -
3*sqrt(d*x^4 + c)*b^4*c*d^4 - 6*sqrt(d*x^4 + c)*a*b^3*d^5)/(b^6*d^6)
```

maple [B] time = 0.25, size = 923, normalized size = 5.27

$$\frac{a^3d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^4}+\frac{a^3d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{\cdots}}{\cdots}\right)}{8\left(ad-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/6/b^2*(d*x^4+c)^(1/2)/d*x^4-1/3/b^2*(d*x^4+c)^(1/2)/d^2*c-1/b^3*a/d*(d*x^
4+c)^(1/2)-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*
d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(
1/2)/b))-3/4*a^2/b^4/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1
/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2
*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2
)/b))-1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^
(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8
*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-
2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/
2)/b))+1/8*a^2/b^4*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)
^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/
```

```
8*a^3/b^4*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(
1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+
2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/
2)/b))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 5.19, size = 186, normalized size = 1.06

$$\frac{\left(d\,x^4+c\right)^{3/2}}{6\,b^2\,d^2}-\left(\frac{3\,c}{2\,b^2\,d^2}+\frac{a\,d-b\,c}{b^3\,d^2}\right)\,\sqrt{d\,x^4+c}+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(5\,a\,d-6\,b\,c\right)}{\sqrt{a\,d-b\,c}\,\left(5\,a^3\,d-6\,a^2\,b\,c\right)}\right)\,\left(5\,a\,d-6\,b\,c\right)}{4\,b^{7/2}\,\left(a\,d-b\,c\right)^{3/2}}-\frac{a^3}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(3/2)/(6*b^2*d^2) - ((3*c)/(2*b^2*d^2) + (a*d - b*c)/(b^3*d^2))
*(c + d*x^4)^(1/2) + (a^2*atan((a^2*b^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 6*b*
c))/((a*d - b*c)^(1/2)*(5*a^3*d - 6*a^2*b*c)))*(5*a*d - 6*b*c))/(4*b^(7/2)*
(a*d - b*c)^(3/2)) - (a^3*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^4*(c + d
*x^4) - 2*b^4*c + 2*a*b^3*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

3.823 $\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

```
[Out] 1/4*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/2*(d*x^4+c)^(1/2)/b^2/d-1/4*a^2*(d*x^4+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^4}}{4b^2\left(a+bx^4\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^4}}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] Sqrt[c + d*x^4]/(2*b^2*d) - (a^2*Sqrt[c + d*x^4])/(4*b^2*(b*c - a*d)*(a + b
*x^4)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*
d]])/(4*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)\\
&= -\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}+\frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{4b^2(bc-ad)}\\
&= \frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}-\frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8b^2(bc-ad)}\\
&= \frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}-\frac{(a(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4b^2d(bc-ad)}\\
&= \frac{\sqrt{c+dx^4}}{2b^2d}-\frac{a^2\sqrt{c+dx^4}}{4b^2(bc-ad)\left(a+bx^4\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{5/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}\left(\frac{a^2}{\left(a+bx^4\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((Sqrt[c + d*x^4]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^4))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/4
```

fricas [B] time = 1.00, size = 475, normalized size = 3.86

$$\left[\frac{\left(4\,a^2bcd-3\,a^3d^2+\left(4\,ab^2cd-3\,a^2bd^2\right)x^4\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\left(2\,ab^3c^2-5\,a^2b^2cd\right.}{8\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(b^2*
c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*
d))/(b*x^4 + a)) + 2*(2*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^
2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b
^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4), -1
/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^4)*sqrt(-b^2*c
 + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) - (2
*a*b^3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2 + 2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b
^2*d^2)*x^4)*sqrt(d*x^4 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
+ (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^4)]
```

giac [A] time = 0.17, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^4+c}\,a^2 d}{4\left(b^3c-ab^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^4+c}}{2\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d*x^4 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^4 + c)*b - b*c + a*d))
- 1/4*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/2*sqrt(d*x^4 + c)/(b^2*d)
```

maple [B] time = 0.19, size = 876, normalized size = 7.12

$$-\frac{a^2 d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^3}-\frac{a^2 d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\cdots}{\cdots}\right)}{8\left(ad\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2*(d*x^4+c)^(1/2)/b^2/d+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b
)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/
(x^2+(-a*b)^(1/2)/b))+1/2*a/b^3/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))+1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^
2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^
(1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(
-a*b)^(1/2)/b))-1/8*a/b^3*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2
-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(
1/2)-1/8*a^2/b^3*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(
-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a
*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.12, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^4+c}}{2\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^4+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{4\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b^3\,\left(d\,x^4+c\right)-2\,b^3\,c+2\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (c + d*x^4)^(1/2)/(2*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^4)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(4*b^(5/
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b^3*(c
+ d*x^4) - 2*b^3*c + 2*a*b^2*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

3.824 $\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/4*a*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.09, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx=\frac{1}{4}\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$

$$=\frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8b(bc-ad)}$$

$$=\frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}+\frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4bd(bc-ad)}$$

$$=\frac{a\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)}+\frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*b^(
3/2))
```

fricas [A] time = 0.94, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^4+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)}{8\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^4\right)},\ \frac{\left(\left(2\,b^2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((2*b^2*c - a*b*d)*x^4 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b*
d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) +
 2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^4), 1/4*(((2*b^2*c - a*b*d)
*x^4 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^4 + c)*sqrt(-b
^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d))/(a*b^
4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)
*x^4)]
```

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^4+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^4+c\right)b-bc+ad\right)}+\frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(d*x^4 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^4 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

maple [B] time = 0.20, size = 851, normalized size = 8.60

$$\frac{ad\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b^2}+\frac{ad\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{8\left(ad-b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/b^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-
2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/4/b
^2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-
b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/b^2*(-a*
b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^
(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/b^2*(-a*b)^(1
/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)
*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^2*d/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-
(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.93, size = 95, normalized size = 0.96

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^4+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(4*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^4)^(1/2))/(2*b*(a*d - b*c)*(2*b*(c +
 d*x^4) + 2*a*d - 2*b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Timed out
```

3.825 $\int \frac{x^3}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

```
[Out] 1/4*d*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/4*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -Sqrt[c + d*x^4]/(4*(b*c - a*d)*(a + b*x^4)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[b*c - a*d]])/(4*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^4\right)$$
$$= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^4\right)}{8(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^4}\right)}{4(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{4}\left(\frac{\sqrt{c+dx^4}}{\left(a+bx^4\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{ad-bc}}\right)}{\sqrt{b}(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]/((-(b*c) + a*d)*(a + b*x^4)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^4])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/4
```

fricas [B] time = 0.68, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^4+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^4+2\,bc-ad-2\,\sqrt{dx^4+c}\,\sqrt{b^2c-abd}}{bx^4+a}\right)+2\,\sqrt{dx^4+c}\left(b^2c-abd\right)}{8\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^4\right)}, -\frac{\left(bdx^4+ad\right)\sqrt{-b^2c+abd}}{4\left(ab^3c^2-2\,a^2b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((b*d*x^4 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^4 + 2*b*c - a*d - 2*s
qrt(d*x^4 + c)*sqrt(b^2*c - a*b*d))/(b*x^4 + a)) + 2*sqrt(d*x^4 + c)*(b^2*c
 - a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d
+ a^2*b^2*d^2)*x^4), -1/4*((b*d*x^4 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt
(d*x^4 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^4 + b*c)) + sqrt(d*x^4 + c)*(b^2*c
- a*b*d))/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*x^4)]
```

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,(bc-ad)} - \frac{\sqrt{dx^4+c}\,d}{4\left(\left(dx^4+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

[Out] -1/4*d*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/4*sqrt(d*x^4 + c)*d/(((d*x^4 + c)*b - b*c + a*d)*(b*c - a
*d))

maple [B] time = 0.21, size = 541, normalized size = 6.22

$$-\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,b}-\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ac}{}}}{}\right.}{8\left(ad-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)

[Out] 1/8*(-a*b)^(1/2)/a/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2
d-2(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8*(-a*b
)^(1/2)/a/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*
b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b*d/(a*d-b*c)/(-(a
*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+
2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)
^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?

mupad [B] time = 4.85, size = 84, normalized size = 0.97

$$\frac{d\,\sqrt{d\,x^4+c}}{2\,\left(a\,d-b\,c\right)\,\left(2\,b\,\left(d\,x^4+c\right)+2\,a\,d-2\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^4+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)

[Out] (d*(c + d*x^4)^(1/2))/(2*(a*d - b*c)*(2*b*(c + d*x^4) + 2*a*d - 2*b*c)) + (
d*atan((b^(1/2)*(c + d*x^4)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(1/2)*(a*d - b*
c)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)

[Out] Timed out

3.826 $\int \frac{1}{x(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

```
[Out] 1/4*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/2*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/4
*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}}+\frac{b\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ArcTanh[Sqrt[c + d*x^4]
/Sqrt[c]]/(2*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^4\right)}{4a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^4\right)}{4a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int\right.}{8a^2(bc-}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^4}\right)}{2a^2d} - \frac{(b(2bc-3ad))\,\text{Sul}}{}$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{2a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right.}{4a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.33, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) - (2*ArcTanh[Sqrt[c + d*x^
4]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^4])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(4*a^2)
```

fricas [A] time = 0.77, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^4+c}\,abc + \left(\left(2\,b^2c^2 - 3\,abcd\right)x^4 + 2\,abc^2 - 3\,a^2cd\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^4+2\,bc-ad+2\,\sqrt{dx^4+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^4+a}\right) + 2\left(\left(b\right.\right.}{8\left(a^3bc^2 - a^4cd + \left(a^2b^2c^2 - a^3bcd\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(d*x^4 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 -
3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d + 2*sqrt(d*x^4 +
c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*((b^2*c - a*b*d)*x^4 +
```

```
a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4))
/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4
 + b*c)) + ((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(c)*log((d*x^4 - 2*sqr
t(d*x^4 + c)*sqrt(c) + 2*c)/x^4))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3
*b*c*d)*x^4), 1/8*(2*sqrt(d*x^4 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^4 + a*b*c
 - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a*b
*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c
 - a*d + 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)))/(
a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4), 1/4*(sqrt(d*x^4 + c)*
a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^4 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*c
- a*d))*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 +
 b*c)) + 2*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^4
 + c)*sqrt(-c)/c))/(a^3*b*c^2 - a^4*c*d + (a^2*b^2*c^2 - a^3*b*c*d)*x^4)]
```

giac [A] time = 0.17, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^4+c}\,bd}{4\left(abc-a^2d\right)\left(\left(dx^4+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^4+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^4+c}}{\sqrt{-c}}\right)}{2\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(d*x^4 + c)*b*d/((a*b*c - a^2*d)*((d*x^4 + c)*b - b*c + a*d)) - 1/4
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/2*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

maple [B] time = 0.27, size = 880, normalized size = 6.67

$$\frac{d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\;a}+\frac{d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\ \cdots}{x^2+\cdots}\right)}{8\left(ad-bc\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/a^2/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2
*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/4/a^
2/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b
*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-
(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8/a^2*(-a*b
)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(
1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b
*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(
-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1
/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+1/8/a^2*(-a*b)^(1/2)/(
a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2
-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/a*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1
/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)
/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d
```

```
-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/2/a^2/c^(1/2)*ln((2*c+2*(d*x^4
+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x), x)
```

mupad [B] time = 5.87, size = 3017, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^
3*c*d^4 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*
x^4)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a
^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*
b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*
c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)
^(1/2)*1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) +
((((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*
(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4
 + a^4*b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1
/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*
c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*
c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2
)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3
*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*
1i)/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))/(((3*a*b
^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((((c +
 d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^
2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*
b^4*c^2*d^3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - ((c + d*x^4)^(1/2)*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 -
 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2
*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*
a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^
2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^
5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^4)^(1
/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(8*(a^4*d^2 + a^2*b^
2*c^2 - 2*a^3*b*c*d)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^
3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) + ((c + d*x^4)^(1/2)*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b
^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(64*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d
)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*
c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(8*(a^5*d^3 - a^
2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d -
b*c)^3)^(1/2)*1i)/(4*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2)) - (atan((((((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^
```

```
5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 -
256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^
(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4
)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a
^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((2*a^6*b^2*d^5 - 3*a^5*b
^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c
 + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 +
 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(4*a^2*c^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1
/2)))/(((3*a*b^3*d^4)/16 - (b^4*c*d^3)/8)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*
c*d) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^4*b^4*c^2*d^3)/(4*(a^5*d^2 +
a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^4)^(1/2)*(64*a^7*b^2*d^5 - 256*a^6*
b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*c^2*d^3))/(128*a^2*c^(1/2)*(a
^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c^(1/2)) - ((c + d*x^4)^(1/2)*
(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((2*a^6*b^2*d^5 - 3*a^5*b^3*c*d^4 + a^
4*b^4*c^2*d^3)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d*x^4)^(1/
2)*(64*a^7*b^2*d^5 - 256*a^6*b^3*c*d^4 - 128*a^4*b^5*c^3*d^2 + 320*a^5*b^4*
c^2*d^3))/(128*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(4*a^2*c
^(1/2)) + ((c + d*x^4)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d
^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2))))*1i)/(2*a^2
*c^(1/2)) - (b*d*(c + d*x^4)^(1/2))/(2*(a^2*d - a*b*c)*(2*b*(c + d*x^4) + 2
*a*d - 2*b*c))
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.827 $\int \frac{1}{x^5(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}\,(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

```
[Out] 1/4*(a*d+4*b*c)*arctanh((d*x^4+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/4*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^4+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/4*b*(-a*d+2*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^4+a)-1/4*
(d*x^4+c)^(1/2)/a/c/x^4/(b*x^4+a)
```

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{4a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}}-\frac{b\sqrt{c+dx^4}\,(2bc-ad)}{4a^2c\left(a+bx^4\right)(bc-ad)}-\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(a + b*x^4)) - Sqrt
[c + d*x^4]/(4*a*c*x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4
]/Sqrt[c]])/(4*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^4])/Sqrt[b*c - a*d]])/(4*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)$$

$$= -\frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^4\right)}{4ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{4a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c}}\right.}{8a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{4a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt{c+dx^4}}{4acx^4\left(a+bx^4\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{4a^3c^{3/2}} - \frac{b^{3/2}}{}$$

Mathematica [A] time = 0.70, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^4}\left(a^2d+ab\left(dx^4-c\right)-2b^2cx^4\right)}{x^4\left(a+bx^4\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^4}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^4}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*Sqrt[c + d*x^4]*(a^2*d - 2*b^2*c*x^4 + a*b*(-c + d*x^4)))/((b*c - a*d)*
x^4*(a + b*x^4)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^4]/Sqrt[c]])/Sqrt[c]
```

```
+ (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^4])/Sqrt[b*c -
a*d]])/(b*c - a*d)^(3/2))/(4*a^3*c)
```

fricas [A] time = 1.04, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)
*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d - 2*sqrt(d*x^4 + c)*(b*c -
a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*
d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*log((d*x^4 +
2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) - 2*(a^2*b*c^2 - a^3*c*d + (2*a*b^2*c
^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^8 + (a
^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a
*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^4 + c)
*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c)) - ((4*b^3*c^2 - 3*a*b^2*
c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(c)*l
og((d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(c) + 2*c)/x^4) + 2*(a^2*b*c^2 - a^3*c*d
+ (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a^4*b*c^2
*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/8*(2*((4*b^3*c^2 - 3*a*b^2*c*d -
 a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^4)*sqrt(-c)*arcta
n(sqrt(d*x^4 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^8 + (4*a*b^2
*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(b/(b*c - a*d))*log((b*d*x^4 + 2*b*c - a*d -
 2*sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^4 + a)) + 2*(a^2*b
*c^2 - a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*
c^3 - a^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4), -1/4*(((4*b^3*c^3 -
5*a*b^2*c^2*d)*x^8 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^4)*sqrt(-b/(b*c - a*d)
)*arctan(-sqrt(d*x^4 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^4 + b*c))
 + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^8 + (4*a*b^2*c^2 - 3*a^2*b*c*d
- a^3*d^2)*x^4)*sqrt(-c)*arctan(sqrt(d*x^4 + c)*sqrt(-c)/c) + (a^2*b*c^2 -
a^3*c*d + (2*a*b^2*c^2 - a^2*b*c*d)*x^4)*sqrt(d*x^4 + c))/((a^3*b^2*c^3 - a
^4*b*c^2*d)*x^8 + (a^4*b*c^3 - a^5*c^2*d)*x^4)]
```

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^{3}c-5\,ab^{2}d\right)\arctan\left(\frac{\sqrt{dx^{4}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{4\left(a^{3}bc-a^{4}d\right)\sqrt{-b^{2}c+abd}}-\frac{2\left(dx^{4}+c\right)^{\frac{3}{2}}b^{2}cd-2\sqrt{dx^{4}+c}\,b^{2}c^{2}d-\left(dx^{4}+c\right)^{\frac{3}{2}}abd^{2}+2\sqrt{dx^{4}+c}\,abcd}{4\left(a^{2}bc^{2}-a^{3}cd\right)\left(\left(dx^{4}+c\right)^{2}b-2\left(dx^{4}+c\right)bc+bc^{2}+\left(dx^{4}+c\right)a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^4 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/4*(2*(d*x^4 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^4 + c)*b^2*c^2*d - (d*x^4 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^4 + c
)*a*b*c*d^2 - sqrt(d*x^4 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^4 + c)^
2*b - 2*(d*x^4 + c)*b*c + b*c^2 + (d*x^4 + c)*a*d - a*c*d)) - 1/4*(4*b*c +
a*d)*arctan(sqrt(d*x^4 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

maple [B] time = 0.28, size = 938, normalized size = 5.07

$$-\frac{bd\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8\left(ad-bc\right)\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{bd\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{\cdots}}{\cdots}\right)}{8\left(ad-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/a^2/c/x^4*(d*x^4+c)^(1/2)+1/4/a^2*d/c^(3/2)*ln((2*c+2*(d*x^4+c)^(1/2)*
c^(1/2))/x^2)-1/2/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2
*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^
(1/2)/b))-1/2/a^3*b/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*
(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)
/b))+1/8/a^3*b*(-a*b)^(1/2)/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/
2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^
2*b*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/
b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a
*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b)
)-1/8/a^3*b*(-a*b)^(1/2)/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/
b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^2*b
*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b
*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^
(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+b/
a^3/c^(1/2)*ln((2*c+2*(d*x^4+c)^(1/2)*c^(1/2))/x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^5), x)
```

mupad [B] time = 7.05, size = 3822, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] (((c + d*x^4)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^4)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^4)*(2*a*d - 4*b*c) + 2*b*(c + d*x^4)^2 + 2*b*c^2 - 2*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c
)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*
c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((
-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^
5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(6
4*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^
4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5
*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2
)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a
^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(
a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4
*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^
3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a
^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2
```

d^5))/(64(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^4)^(1/2)*(5*a*d - 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)) + ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/32 + b^7*c^3*d^3 - (3*a*b^6*c^2*d^4)/2 + (3*a^2*b^5*c*d^5)/16)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)) + ((((c + d*x^4)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((a^9*b^2*c*d^6 + 2*a^6*b^5*c^4*d^3 - 4*a^7*b^4*c^3*d^4 + a^8*b^3*c^2*d^5)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^4)^(1/2)*(a*d + 4*b*c)*(128*a^6*b^5*c^5*d^2 - 320*a^7*b^4*c^4*d^3 + 256*a^8*b^3*c^3*d^4 - 64*a^9*b^2*c^2*d^5))/(64*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(8*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)/(4*a^3*(c^3)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.828 $\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=191

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}\,(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad}$$

```
[Out] 1/4*a^(3/2)*(-4*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1
/2))/b^3/(-a*d+b*c)^(3/2)-1/4*(4*a*d+b*c)*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/
2))/b^3/d^(3/2)+1/4*(-2*a*d+b*c)*x^2*(d*x^4+c)^(1/2)/b^2/d/(-a*d+b*c)+1/4*a
*x^6*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.37, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 470, 582, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^2}{\sqrt{c+dx^4}}\right)}{4b^3d^{3/2}}+\frac{x^2\sqrt{c+dx^4}\,(bc-2ad)}{4b^2d(bc-ad)}+\frac{ax^6\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((b*c - 2*a*d)*x^2*Sqrt[c + d*x^4])/(4*b^2*d*(b*c - a*d)) + (a*x^6*Sqrt[c +
 d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) + (a^(3/2)*(5*b*c - 4*a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^3*(b*c - a*d)^(3/2)) -
 ((b*c + 4*a*d)*ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]])/(4*b^3*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
```

```
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$

$$= \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(3ac-2(bc-2ad)x^2\right)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-2ac(bc-2ad)-2(bc-ad)(bc+4a}{\left(a+bx^2\right)\sqrt{c+dx^2}}\right.}{8b^2d(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{}}\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{a-(-bc+a}\right.}{4b^3(bc-ad)}$$

$$= \frac{(bc-2ad)x^2\sqrt{c+dx^4}}{4b^2d(bc-ad)} + \frac{ax^6\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^3(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.57, size = 150, normalized size = 0.79

$$\frac{\frac{a^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}}+bx^2\sqrt{c+dx^4}\left(\frac{a^2}{(a+bx^4)(ad-bc)}+\frac{1}{d}\right)-\frac{(4ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^4}+dx^2\right)}{d^{3/2}}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4]*(d^(-1) + a^2/((-(b*c) + a*d)*(a + b*x^4))) + (a^(3/
2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])
/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/d
^(3/2))/(4*b^3)
```

fricas [A] time = 2.75, size = 1386, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*
a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) +
 (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b
*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^4 + a^2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2
- a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4
+ a^2)) + 4*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqr
t(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), 1/
16*(4*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^2
*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(-a/(b*c - a*d))*log(
((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^
2*c^2 + 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)
*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*((b
^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*sqrt(d*x^4 + c))/(
a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4), -1/8*((5*a^2*b*c*
d^2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*ar
ctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*
d*x^6 + a*c*x^2)) - (a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b
^2*c*d - 4*a^2*b*d^2)*x^4)*sqrt(d)*log(-2*d*x^4 + 2*sqrt(d*x^4 + c)*sqrt(d)
*x^2 - c) - 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*x^2)*s
qrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*x^4),
1/8*(2*(a*b^2*c^2 + 3*a^2*b*c*d - 4*a^3*d^2 + (b^3*c^2 + 3*a*b^2*c*d - 4*a^
2*b*d^2)*x^4)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(d*x^4 + c)) - (5*a^2*b*c*d^
2 - 4*a^3*d^3 + (5*a*b^2*c*d^2 - 4*a^2*b*d^3)*x^4)*sqrt(a/(b*c - a*d))*arct
an(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a/(b*c - a*d))/(a*d*
x^6 + a*c*x^2)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 + (a*b^2*c*d - 2*a^2*b*d^2)*
x^2)*sqrt(d*x^4 + c))/(a*b^4*c*d^2 - a^2*b^3*d^3 + (b^5*c*d^2 - a*b^4*d^3)*
x^4)]
```

giac [B] time = 0.59, size = 337, normalized size = 1.76

$$-\frac{\left(5\,a^2bc\sqrt{d}\,-4\,a^3d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\left(b^4c-ab^3d\right)\sqrt{abcd-a^2d^2}}+\frac{\sqrt{dx^4+c}\,x^2}{4\,b^2d}+\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2a^2l}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(5*a^2*b*c*sqrt(d) - 4*a^3*d^(3/2))*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^4*c - a*b^3*d)*sq
rt(a*b*c*d - a^2*d^2)) + 1/4*sqrt(d*x^4 + c)*x^2/(b^2*d) + 1/2*((sqrt(d)*x^
2 - sqrt(d*x^4 + c))^2*a^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a^3*d^(3/2) - a^2*b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b -
2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))
^2*a*d + b*c^2)*(b^4*c - a*b^3*d)) + 1/8*(b*c*sqrt(d) + 4*a*d^(3/2))*log((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^3*d^2)
```

maple [B] time = 0.27, size = 953, normalized size = 4.99

$$-\frac{5a^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^3}+\frac{5a^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\cdots}}{\cdots}\right)}{8\sqrt{-ab}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/4/b^2*x^2/d*(d*x^4+c)^(1/2)-1/4/b^2*c/d^(3/2)*ln(d^(1/2)*x^2+(d*x^4+c)^(1
/2))-1/b^3*a*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)+5/8*a^2/b^3/(-a*b)^(1/
2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d
-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^
2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-5/8*a^2/b^3
/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8*a^2/b^3/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8*a^2/b^4*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8*a^2/b^3/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a^2/b^4*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b
)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13}}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^13/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a + bx^{4}\right)^{2} \sqrt{c + dx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**13/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.829 $\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

```
[Out] -1/4*(-2*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/2*arctanh(x^2*d^(1/2)/(d*x^4+c)^(1/2))/b^2/d^(
1/2)+1/4*a*x^2*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{ax^2\sqrt{c+dx^4}}{4b\left(a+bx^4\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x^2*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^2)/Sqrt[c + d*x^4]]/(2*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)\\ &= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4b(bc-ad)}\\ &= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,x^2\right)}{2b^2}-\frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{}{(a}\right.}{4b^2(bc-}\\ &= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}+\frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{2b^2}-\frac{(a(3bc-2ad))\,\text{Subst}\left(\int\right.}{4b^2(}\\ &= \frac{ax^2\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{4b^2(bc-ad)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^2}{\sqrt{c+dx^4}}\right)}{2b^2\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.22, size = 135, normalized size = 0.96

$$\frac{\frac{abx^2\sqrt{c+dx^4}}{\left(a+bx^4\right)(bc-ad)}+\frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^4}}\right)}{(bc-ad)^{3/2}}+\frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^4}+dx^2\right)}{\sqrt{d}}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((a*b*x^2*Sqrt[c + d*x^4])/((b*c - a*d)*(a + b*x^4)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^2 + Sqrt[d]*Sqrt[c + d*x^4]])/Sqrt[d])/(4*b^2)
```

fricas [A] time = 1.50, size = 1077, normalized size = 7.64

$$\left[\frac{4\sqrt{dx^4+c}\,abdx^2+4\left(\left(b^2c-abd\right)x^4+abc-a^2d\right)\sqrt{d}\,\log\left(-2\,dx^4-2\sqrt{dx^4+c}\,\sqrt{d}\,x^2-c\right)+\left(\left(3\,b^2cd-2\,abd^2\right.\right.}{16\left(ab^3cd-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 + 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4
*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c^2 - a^2*c*d)*x^2)*sqrt(d*x
^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a*b^3*c*d - a^
2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/16*(4*sqrt(d*x^4 + c)*a*b*d*x^2 -
 8*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^2/sqrt(
d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a
/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4
*a^2*c*d)*x^4 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^6 - (a*b*c
^2 - a^2*c*d)*x^2)*sqrt(d*x^4 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^8 + 2*a*b*x
^4 + a^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*s
qrt(d*x^4 + c)*a*b*d*x^2 + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2
*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4
+ c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)) + 2*((b^2*c - a*b*d)*x^4 + a*
b*c - a^2*d)*sqrt(d)*log(-2*d*x^4 - 2*sqrt(d*x^4 + c)*sqrt(d)*x^2 - c))/(a*
b^3*c*d - a^2*b^2*d^2 + (b^4*c*d - a*b^3*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*
a*b*d*x^2 - 4*((b^2*c - a*b*d)*x^4 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d
)*x^2/sqrt(d*x^4 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^4 + 3*a*b*c*d - 2*a^2*d
^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 +
c)*sqrt(a/(b*c - a*d))/(a*d*x^6 + a*c*x^2)))/(a*b^3*c*d - a^2*b^2*d^2 + (b^
4*c*d - a*b^3*d^2)*x^4)]
```

giac [B] time = 0.54, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{4\,\left(b^{3}c-ab^{2}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}\,x^{2}\cdots\right.}{2\left(\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}b-2\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}bc+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x
^4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c
 + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
4*log((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2)/(b^2*sqrt(d))
```

maple [B] time = 0.24, size = 893, normalized size = 6.33

$$\frac{3a\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,b^{2}}-\frac{3a\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\cdots}{x^{2}\cdots}\right.}{8\sqrt{-ab}\,\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/2/b^2*ln(d^(1/2)*x^2+(d*x^4+c)^(1/2))/d^(1/2)-3/8*a/b^2/(-a*b)^(1/2)/(-(a
*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b
+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b
)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+3/8*a/b^2/(-a*b)^(
1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x
^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8*a/b^2/
(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)
/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*
c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(
-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/8*a/b^2/(a*d
-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-
a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8*a/b^3*(-a*b)^(1/2)*d/(a*d-b*c)/(-(
a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/
b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*
b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^9}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^9/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**9/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.830 $\int \frac{x^5}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

```
[Out] 1/4*c*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/4*x^2*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^2\sqrt{c+dx^4}}{4\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(x^2*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\operatorname{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^2}{\sqrt{c+dx^4}}\right)}{4(bc-ad)}\\
&= -\frac{x^2\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^4}\left(-\frac{x^4(bc-ad)}{a+bx^4}-\frac{c\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^4\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^4}{c}+1}}\right)}{\sqrt{\frac{dx^4}{c}+1}}\right)}{4x^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[c + d*x^4]*(-(((b*c - a*d)*x^4)/(a + b*x^4)) - (c*Sqrt[(-(b/a) + d/c)
*x^4]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^4]/Sqrt[1 + (d*x^4)/c]])/Sqrt[1 + (d*x^
4)/c]))/(4*(b*c - a*d)^2*x^2)
```

fricas [B] time = 1.03, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^4+c}\left(abc-a^2d\right)x^2-\left(bcx^4+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2+4\left((bc-2\,ad)x\right.}{b^2x^8+2\,abx^4+a^2}\right.}{16\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(4*sqrt(d*x^4 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c^2 - 4*a^2
*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4 + c)*sqrt(
-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d +
 a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4), -1/8*(2*sqrt(d*x^4
 + c)*(a*b*c - a^2*d)*x^2 - (b*c*x^4 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1/2*
```

```
((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a
^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d
^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^4)]
```

giac [B] time = 1.63, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{4\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc\sqrt{d}-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2ad}{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\,x^2-\sqrt{dx^4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/2*((s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^2 - sqrt(d*x^4 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

maple [B] time = 0.24, size = 861, normalized size = 9.26

$$-\frac{\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}\,b}+\frac{\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(\right.}}{x^2+\frac{\sqrt{-}}{b}}\right.}{8\sqrt{-ab}\sqrt{-\frac{a}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/
2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*
(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)
/b))-1/8/b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)
^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*
d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(
1/2)/b))+1/8/b/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(
-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b^2*(-a*b)^(1/2
)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/
b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)
^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1
/8/b/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2
)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/b^2*(-a*b)^(1/2)*d/(a*d-b
*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*
d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x
^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.831 $\int \frac{x}{\left(a+bx^4\right)^2 \sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

```
[Out] 1/4*(-2*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/4*b*x^2*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}+\frac{bx^2\sqrt{c+dx^4}}{4a\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^2\right)$$

$$= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^2\right)}{4a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^2}{\sqrt{c+dx^4}}\right)}{4a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{3/2}(bc-ad)^{3/2}}$$

Mathematica [**C**] time = 6.01, size = 407, normalized size = 3.91

$$\frac{x^2\sqrt{c+dx^4}\left(-30dx^4\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}} - 45c\sqrt{\frac{ax^4\left(c+dx^4\right)(bc-ad)}{c^2\left(a+bx^4\right)^2}} + 16dx^4\left(\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^4\right)}{c\left(a+bx^4\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc\cdots}{c(b\cdots}\right.\right.}{60c^2\left(a+bx^4\right)^2\left(\frac{x^4\cdots}{c\cdots}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^4]*(-45*c*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a +
b*x^4)^2)] - 30*d*x^4*Sqrt[(a*(b*c - a*d)*x^4*(c + d*x^4))/(c^2*(a + b*x^4)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 30*d*x^4*ArcS
in[Sqrt[((b*c - a*d)*x^4)/(c*(a + b*x^4))]] + 16*c*(((b*c - a*d)*x^4)/(c*(a
 + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 16*d*x^4*(((b*c - a*d)*x^4)/
(c*(a + b*x^4)))^(5/2)*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(60*c^2*(((b*c - a*d)*x
^4)/(c*(a + b*x^4)))^(3/2)*(a + b*x^4)^2*Sqrt[(a*(c + d*x^4))/(c*(a + b*x^4
))])
```

fricas [**B**] time = 0.92, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^4+c}\left(ab^2c-a^2bd\right)x^2 - \left(\left(b^2c-2\,abd\right)x^4+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a\cdots\right.}{\cdots}\right.}{16\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 - ((b^2*c - 2*a*b*d)*x^4 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^8 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a
*c*x^2)*sqrt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)))
/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*
b*d^2)*x^4), 1/8*(2*sqrt(d*x^4 + c)*(a*b^2*c - a^2*b*d)*x^2 + ((b^2*c - 2*a
*b*d)*x^4 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*
x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (
```

```
a*b*c^2 - a^2*c*d)*x^2)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c
^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^4)]
```

giac [B] time = 0.38, size = 237, normalized size = 2.28

$$-\frac{1}{4}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc-2\left(\sqrt{d}\,x^2-\right.\right.}{\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2bc+4\left(\sqrt{d}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*
x^4 + c))^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

maple [B] time = 0.19, size = 867, normalized size = 8.34

$$\frac{\sqrt{-ab}\;d\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\,(ad-bc)\sqrt{-\frac{ad-bc}{b}}\;ab}-\frac{\sqrt{-ab}\;d\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-}{b}\right.}{8\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1
/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8/b/a*(-a*b)^(1/2)*d/(a*d
-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(
x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a/(-a
*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d
-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1
/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-1/8/
a/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b
)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-
1/8/a/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/
2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/b/a*(-a*b)^(1/2)*d/(a*d-
b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a
*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(
x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.832 $\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(3bc-2ad)}{4a^2cx^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

```
[Out] -1/4*b*(-4*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/4*(-2*a*d+3*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^4+a)
```

Rubi [A] time = 0.20, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^4}\,(3bc-2ad)}{4a^2cx^2(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^4}}{4ax^2\left(a+bx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x^2) + (b*Sqrt[c +
d*x^4])/(4*a*(b*c - a*d)*x^2*(a + b*x^4)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,d\right.}{4a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \right.}{4a^2(bc-}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{(b(3bc-4ad))\,\text{Subst}\left(\int \right.}{4a^2(bc}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x^2} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^2\left(a+bx^4\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-}}{\sqrt{a}\sqrt{}}\right.}{4a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 5.63, size = 155, normalized size = 1.04

$$\frac{a^2\left(c+dx^4\right)\left(\frac{b^2x^4}{\left(a+bx^4\right)(ad-bc)}-\frac{2}{c}\right)-\frac{bx^8\sqrt{\frac{dx^4}{c}+1}\,(3bc-4ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{4a^4x^2\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a^2*(c + d*x^4)*(-2/c + (b^2*x^4)/((-(b*c) + a*d)*(a + b*x^4))) - (b*(3*b*
c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*ArcSin[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b
*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))^(3/2)))/(4*a^4*x^2*Sqrt[c + d*x^4])
```

fricas [B] time = 1.04, size = 612, normalized size = 4.11

$$\left[-\frac{\left(\left(3\,b^{3}c^{2}-4\,ab^{2}cd\right)x^{6}+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x^{2}\right)\sqrt{-abc+a^{2}d}\,\log\left(\frac{\left(b^{2}c^{2}-8\,abcd+8\,a^{2}d^{2}\right)x^{8}-2\left(3\,abc^{2}-4\,a^{2}cd\right)x^{4}+a^{2}c^{2}+4\left((bc-2\right.}{b^{2}x^{8}+2\,abx^{4}+a^{2}}\right.}{16\left(\left(a^{3}b^{3}c^{3}-2\,a^{4}b^{2}c^{2}d+a^{5}bcd^{2}\right)x^{6}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2*(3*a*b*c
^2 - 4*a^2*c*d)*x^4 + a^2*c^2 + 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sqrt(d*x^4
+ c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) + 4*(2*a^2*b^2*c^2
- 4*a^3*b*c*d + 2*a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4
)*sqrt(d*x^4 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^
4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^2), -1/8*(((3*b^3*c^2 - 4*a*b^2*c*
d)*x^6 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*((
b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2
*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x^2)) + 2*(2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*
a^4*d^2 + (3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^4)*sqrt(d*x^4 + c))
/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^6 + (a^4*b^2*c^3 - 2*a^5*
b*c^2*d + a^6*c*d^2)*x^2)]
```

giac [B] time = 1.68, size = 418, normalized size = 2.81

$$\frac{1}{4}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^{2}c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\left(a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{abcd-a^{2}d^{2}}}+\frac{2\left(3\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}b^{2}c-4\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}ab\right.}{\left(\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{6}b-3\left(\sqrt{d}\,x^{2}-\sqrt{dx^{4}+c}\right)^{4}bc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b^2*c - 4*(sq
rt(d)*x^2 - sqrt(d*x^4 + c))^4*a*b*d - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b*c*d - 8*(sqrt(d)*x^2 - s
qrt(d*x^4 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^2 - sqrt(
d*x^4 + c))^6*b - 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b*c + 4*(sqrt(d)*x^2
- sqrt(d*x^4 + c))^4*a*d + 3*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c^2 - 4*(s
qrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

maple [B] time = 0.27, size = 885, normalized size = 5.94

$$\frac{3b\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)^{2}d+\frac{2\sqrt{-ab}\left(x^{2}-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^{2}-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^{2}}-\frac{3b\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^{2}+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\right.}{x^{2}}\right.}{8\sqrt{-ab}\,\sqrt{-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/2/a^2/x^2*(d*x^4+c)^(1/2)/c-3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*
ln((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)
^(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a
*d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))+3/8/a^2*b/(-a*b)^(1/2)/(-(a*d-b*c)/
b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d
-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b
)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2-(-
a*b)^(1/2)/b)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)
/b*d-(a*d-b*c)/b)^(1/2)-1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/
2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/
b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-
(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))+1/8/a^2*b/(a*d-b*c)/(x^2+(-a*b)^(
1/2)/b)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(
a*d-b*c)/b)^(1/2)+1/8/a^2*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln(
(-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1
/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-
b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.833 $\int \frac{1}{x^7 \left(a+bx^4\right)^2 \sqrt{c+dx^4}} \, dx$

Optimal. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{\sqrt{c+dx^4}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(b}$$

```
[Out] 1/4*b^2*(-6*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^4+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/12*(-2*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^6+1/12*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^4+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^2+1/4*b*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^4+a)
```

Rubi [A] time = 0.33, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^4}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{12a^3c^2x^2(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^4}}\right)}{4a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^4}\,(5bc-2ad)}{12a^2cx^6(bc-ad)}+\frac{b\sqrt{c+dx^4}}{4ax^6\left(a+bx^4\right)(b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^6) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^4])/(12*a^3*c^2*(b*c - a*d)*x^2) +
(b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x^6*(a + b*x^4)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^2)/(Sqrt[a]*Sqrt[c + d*x^4])])/(4*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^2\right)$$

$$= \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^2\right)}{4a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^6\left(a+bx^4\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a}{x^2\left(a+bx^2\right.}\right.}{12a^2c(b}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+}}{4a(bc-ad)x}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+}}{4a(bc-ad)x}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+}}{4a(bc-ad)x}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^6} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^4}}{12a^3c^2(bc-ad)x^2} + \frac{b\sqrt{c+}}{4a(bc-ad)x}$$

Mathematica [A] time = 5.87, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^4\right)\left(\frac{3b^3x^8}{\left(a+bx^4\right)(bc-ad)}+\frac{4x^4(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{12}\sqrt{\frac{dx^4}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^4\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^4}{a}+1}}\right)}{c\left(\frac{x^4(bc-ad)}{ac}\right)^{3/2}}}{12a^5x^6\sqrt{c+dx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a^2*(c + d*x^4)*((-2*a)/c + (4*(3*b*c + a*d)*x^4)/c^2 + (3*b^3*x^8)/((b*c
- a*d)*(a + b*x^4))) + (3*b^2*(5*b*c - 6*a*d)*x^12*Sqrt[1 + (d*x^4)/c]*ArcS
in[Sqrt[(b/a - d/c)*x^4]/Sqrt[1 + (b*x^4)/a]])/(c*(((b*c - a*d)*x^4)/(a*c))
^(3/2)))/(12*a^5*x^6*Sqrt[c + d*x^4])
```

fricas [A] time = 1.43, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5\,b^4c^3-6\,ab^3c^2d\right)x^{10}+\left(5\,ab^3c^3-6\,a^2b^2c^2d\right)x^6\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^8-2\left(3\,abc^2-4\,a^2cd\right)x^4+a^2c^2-}{b^2x^8+2\,abx^4+}\right.}{48\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^6)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^8 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^4 + a^2*c^2 - 4*((b*c - 2*a*d)*x^6 - a*c*x^2)*sq
rt(d*x^4 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^8 + 2*a*b*x^4 + a^2)) - 4*((15*a
*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^8 - 2*a^3*b^
2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d +
a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4 - 2*a^5*b^2*c^
3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*x^6
), 1/24*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^10 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2
*d)*x^6)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^4 - a*c)*sqrt(d*x^
4 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^6 + (a*b*c^2 - a^2*c*d)*x
^2)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)
*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a
^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^4)*sqrt(d*x^4 + c))/((a^4*b^3*c^4
 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^10 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a
^7*c^2*d^2)*x^6)]
```

giac [B] time = 2.04, size = 395, normalized size = 1.90

$$\frac{1}{12}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^2b^3c-2\,(\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}\,x^2-\sqrt{dx^4+c}\right)^4b-2\left(\sqrt{d}\,x^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^2 - sqrt(d*x^
4 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b^3*c - 2
*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^2 - sqrt(d*x^4 + c))^4*b - 2*(sqrt(d)*x^2 - sqrt(d*x^4 + c)
)^2*b*c + 4*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^2 - sqrt(d*x^4 + c))^4*b - 6*(sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*b*c - 3*(
sqrt(d)*x^2 - sqrt(d*x^4 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^2 - sq
rt(d*x^4 + c))^2 - c)^3*a^3*d^3))
```

maple [B] time = 0.25, size = 923, normalized size = 4.44

$$-\frac{5b^2\ln\left(\frac{\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{\left(x^2-\frac{\sqrt{-ab}}{b}\right)^2 d+\frac{2\sqrt{-ab}\left(x^2-\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{ad-bc}{b}}}{x^2-\frac{\sqrt{-ab}}{b}}\right)}{8\sqrt{-ab}\,\sqrt{-\frac{ad-bc}{b}}\,a^3}+\frac{5b^2\ln\left(\frac{-\frac{2\sqrt{-ab}\left(x^2+\frac{\sqrt{-ab}}{b}\right)d}{b}-\frac{2(ad-bc)}{b}+2 \cdots}{\cdots}\right)}{8\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] b/a^3/x^2*(d*x^4+c)^(1/2)/c+5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*c)/b)^(1/2)*l
n((-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^
(1/2)*((x^2+(-a*b)^(1/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*
d-b*c)/b)^(1/2))/(x^2+(-a*b)^(1/2)/b))-5/8/a^3*b^2/(-a*b)^(1/2)/(-(a*d-b*c)
/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*
d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/
b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-(-a*b)^(1/2)/b))-1/6/a^2*(d*x^4+c)^(1/2)*(-
2*d*x^4+c)/x^6/c^2-1/8/a^3*b^2/(a*d-b*c)/(x^2-(-a*b)^(1/2)/b)*((x^2-(-a*b)^
(1/2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)+1/8
/a^3*b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((2*(-a*b)^(1/2)*(x^
2-(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2-(-a*b)^(1/
2)/b)^2*d+2*(-a*b)^(1/2)*(x^2-(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2-
(-a*b)^(1/2)/b))-1/8/a^3*b^2/(a*d-b*c)/(x^2+(-a*b)^(1/2)/b)*((x^2+(-a*b)^(1
/2)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2)-1/8/a
^3*b*(-a*b)^(1/2)*d/(a*d-b*c)/(-(a*d-b*c)/b)^(1/2)*ln((-2*(-a*b)^(1/2)*(x^2
+(-a*b)^(1/2)/b)/b*d-2*(a*d-b*c)/b+2*(-(a*d-b*c)/b)^(1/2)*((x^2+(-a*b)^(1/2
)/b)^2*d-2*(-a*b)^(1/2)*(x^2+(-a*b)^(1/2)/b)/b*d-(a*d-b*c)/b)^(1/2))/(x^2+(
-a*b)^(1/2)/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.834 $\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=996

$$-\frac{(5bc-3ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32b^2\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt{-a}\sqrt[4]{}\cdots}{\cdots}$$

```
[Out] -1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4
)/(d*x^4+c)^(1/2))/b^(7/4)/(-a*d+b*c)^(3/2)+1/16*(-a)^(1/4)*(-3*a*d+5*b*c)*
arctan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/b^(7/4)/(a*d-b
*c)^(3/2)+1/4*a*x*(d*x^4+c)^(1/2)/b/(-a*d+b*c)/(b*x^4+a)+1/8*(-3*a*d+4*b*c)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1
/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(d*x^4+c)^(1/2)-1/16*a*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2
)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/(-a*d+
b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-3*a*d+5*b*c)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(-a)^(1/2)*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^2/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+5*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(
1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/32*(-3*a*
d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/
c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)
-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(
1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2
)+x^2*d^(1/2))^2)^(1/2)/b^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1
/2)
```

Rubi [A] time = 1.31, antiderivative size = 996, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {470, 523, 220, 409, 1217, 1707}

$$-\frac{(5bc-3ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32b^2\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}-\frac{\sqrt{-a}\sqrt[4]{}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (a*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(a + b*x^4)) - ((-a)^(1/4)*(5*b*c -
3*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*b^(7/4)*(b*c - a*d)^(3/2)) + ((-a)^(1/4)*(5*b*c - 3*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*b^(7/4)*(-(b*c) +
a*d)^(3/2)) + ((4*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sq
rt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*b
^2*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (a*((Sqrt[b]*Sqrt[c])/Sqr
t[-a] + Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
```

```
d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[-a]*
(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])*d^(1/4)*(5*b*c - 3*a*d)*(Sqrt[c] + Sqr
t[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x)/c^(1/4)], 1/2])/(16*b^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c +
d*x^4]) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 3*a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqr
t[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*
ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c
 + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c -
 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*b^2*c^(1/4)*d^(1/4)*
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
```

```
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{ac+(-4bc+3ad)x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b^2(bc-ad)} - \frac{(a(5bc-3ad))\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx}}}{4b^2(bc-ad)}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} + \frac{(4bc-3ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right.\right.}{8b^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{c+dx^4}}$$

$$= \frac{ax\sqrt{c+dx^4}}{4b(bc-ad)\left(a+bx^4\right)} - \frac{\sqrt[4]{-a}\,(5bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16b^{7/4}(bc-ad)^{3/2}} + \frac{\sqrt[4]{-a}\,(5bc-3a}{16b}$$

Mathematica [C] time = 0.35, size = 253, normalized size = 0.25

$$\frac{x\left(\frac{5a\left(\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}+c+dx^4\right)}{b\left(a+bx^4\right)} + \frac{x^4\sqrt{\frac{dx^4}{c}+1}\,(4bc-3ad)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right.}{ab}\right.}{20\sqrt{c+dx^4}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^8/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(((4*b*c - 3*a*d)*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)])/(a*b) + (5*a*(c + d*x^4 + (5*a*c^2*AppellF1[1/4,
1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4
, -((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)]))))/(b*(a + b*x^4))))/(20*(b*c - a*d)*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.31, size = 604, normalized size = 0.61

$$\frac{\left(-\frac{\sqrt{d\,x^4+c}\,bx}{4(ad-bc)\left(b\,x^4+a\right)a}-\frac{\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)}{4(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a}-\frac{(-5ad+3bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\,\mathrm{EllipticPi}\left(\sqrt{i\cdots}\right.}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a}\right.}{32ab(ad-bc)\,\mathrm{R}\cdots}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/b^2/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)
*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x
,I)-1/4*a/b^3*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha
^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(
1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1
)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a
*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*b+a))+a^2/b^2*(-1/4/a*b/(a*d-b*c)*x*(d*x^4+c)^(1/2)/(b*x^
4+a)-1/4*d/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)
^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)
*d^(1/2))^(1/2)*x,I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-
a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(
d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2
)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((
I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/
2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^8/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**8/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.835 $\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=908

$$\frac{\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32ab\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)\sqrt{dx^4+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc\cdots}}{\sqrt[4]{-a}\sqrt[4]{b}\cdots}\right.}{16(-a)^{3/4}b^{3/4}(bc-a\cdots}$$

```
[Out] -1/16*(a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2
))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/16*(a*d+b*c)*arctan(x*(a*d-b*c)^(1
/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3/2)-
1/4*x*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)-1/8*d^(3/4)*(cos(2*arctan(d^(1/4
)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1
/2)+x^2*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)
*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4))
)*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/
(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^
2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/16*d^(1/4)*(cos(2*arctan(d^
(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2
*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1
/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^
(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^
(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/
2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d
^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a
*d+b*c)/(d*x^4+c)^(1/2)+1/32*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-
1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/
2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/
(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.91, antiderivative size = 908, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {471, 523, 220, 409, 1217, 1707}

$$\frac{\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{32ab\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)\sqrt{dx^4+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc\cdots}}{\sqrt[4]{-a}\sqrt[4]{b}\cdots}\right.}{16(-a)^{3/4}b^{3/4}(bc-a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(x*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) - ((b*c + a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/
4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^(1
/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^(3/2))
 + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*
Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/
c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[-a]*Sqr
t[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/
(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(
16*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x
```

```
^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt
[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4
*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(3
2*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sq
rt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[
d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sq
rt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a*b*c^(1/4
)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} + \frac{\int \frac{c-dx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)} + \frac{(bc+ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \text{—}\ (b$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8b\sqrt[4]{c}(bc-ad)\sqrt{c+dx^4}} + \text{—}\ (\sqrt{}$$

$$= -\frac{x\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^4}}\right)}{16(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{-bc\cdots}}{\sqrt[4]{-a}\sqrt[4]{b}\cdots}\right)}{16(-a)^{3/4}b^{3/4}(-bc+a\cdots}$$

Mathematica [C] time = 0.19, size = 238, normalized size = 0.26

$$\frac{x\left(\frac{5\left(\frac{5ac^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}+c+dx^4\right)}{a+bx^4}+\frac{dx^4\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{a}\right)}{20\sqrt{c+dx^4}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*((d*x^4*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c), -((
b*x^4)/a)])/a + (5*(c + d*x^4 + (5*a*c^2*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^
4)/c), -((b*x^4)/a)])/(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b
*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)
/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(a +
b*x^4)))/(20*(-(b*c) + a*d)*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.26, size = 530, normalized size = 0.58

$$-\frac{\left(-\frac{\sqrt{d x^{4}+c}\, b x}{4(ad-bc)\left(b x^{4}+a\right)a}-\frac{\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\; d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x, i\right)}{4(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^{4}+c}\;a}-\frac{(-5ad+3bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^{2}}{\sqrt{c}}+1}\;\mathrm{RootOf}\left(b\_Z^{4}+a\right)^{3} b\,\mathrm{Elliptic}\cdots}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^{4}\cdots}}\cdots\right)}{32ab(ad\cdots}\cdots\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b^2*sum(1/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x
^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_
alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2
)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(
1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=
RootOf(_Z^4*b+a))-a/b*(-1/4/(a*d-b*c)*(d*x^4+c)^(1/2)/(b*x^4+a)/a*b*x-1/4*d
/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/
c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^
(1/2)*x,I)-1/32/b/a*sum((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b
)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(
1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(
1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*
d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/
(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(bx^{4}+a\right)^{2}\sqrt{dx^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{4}}{\left(b\,x^{4}+a\right)^{2}\,\sqrt{d\,x^{4}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.836 $\int \frac{1}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=983

$$\frac{(3bc-5ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt[4]{d}\,(3bc}{}$$

```
[Out] 1/16*b^(1/4)*(-5*a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/16*b^(1/4)*(-5*a*d+3*b*c)*arct
an(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(7/4)/(a*d-b*
c)^(3/2)+1/4*b*x*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)+1/8*d^(3/4)*(cos(2*
arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)
+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x
^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+
c)^(1/2)+1/16*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(
1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1
/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/(-a*d+b*c)/
(a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-5*a*d+3*b*c)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin
(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/
2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/
a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.03, antiderivative size = 983, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {414, 523, 220, 409, 1217, 1707}

$$\frac{(3bc-5ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{\sqrt[4]{d}\,(3bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) + (b^(1/4)*(3*b*c - 5*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(
-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(b*c)
 + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(7/4)*(-(b*c) +
a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] +
Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a*c^(1/4)
*(b*c - a*d)*Sqrt[c + d*x^4]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(
1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sq
rt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a*c^(1/4)*
```

```
(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqr
t[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*(
-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt
[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/
4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
 + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[
d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqr
t[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
```

```
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{-3bc+4ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(3bc-5ad)\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}} +$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c+dx^4}{\left(\sqrt{c}+\sqrt{d}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{8a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^4}} +$$

$$= \frac{bx\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^4}}\right)}{16(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}\,(3bc-5ad)}{16(-a)^{7}}$$

Mathematica [C] time = 0.39, size = 392, normalized size = 0.40

$$\frac{2bx^5\left(dx^4\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+5a\left(c+dx^4\right)\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\right.\right.}{20a^2\left(a+bx^4\right)\sqrt{c+dx^4}\,(bc-ad)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]*(5*a*(4*b*
c - 4*a*d + b*d*x^4) + b*d*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4
, 1/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]) + 2*b*x^5*(5*a*(c + d*x^4) + d*
x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^4)/c)
, -((b*x^4)/a)])*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/
a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))/(20*a^2*
(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4]*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4,
-((d*x^4)/c), -((b*x^4)/a)] + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*
x^4)/c), -((b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*
x^4)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.25, size = 333, normalized size = 0.34

$$-\frac{\sqrt{d\,x^4+c}\,bx}{4\,(ad-bc)\left(b\,x^4+a\right)a}-\frac{\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,d\,\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)}{4\,(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d\,x^4+c}\,a}-\frac{(-5ad+3bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/(a*d-b*c)*(d*x^4+c)^(1/2)/(b*x^4+a)/a*b*x-1/4*d/(a*d-b*c)/a/(I/c^(1/2)
*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^
(1/2)/(d*x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-1/32/b/a*sum
((-5*a*d+3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*
_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1
/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)
*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alp
ha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.837 $\int \frac{1}{x^4(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1046

$$-\frac{b(7bc-9ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32a^3\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{b\sqrt[4]{d}\,(7bc}{}$$

```
[Out] 1/16*b^(5/4)*(-9*a*d+7*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d
*x^4+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/16*b^(5/4)*(-9*a*d+7*b*c)*arc
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(11/4)/(a*d-
b*c)^(3/2)-1/12*(-4*a*d+7*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x^3+1/4*b*(
d*x^4+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^4+a)-1/24*d^(3/4)*(-4*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
(d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(d*x^4+c)^(
1/2)+1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/
2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a
*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/
4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1
/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/
2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)
^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)-1/16*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-1/32*b*(-9*a*d+7
*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(
1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-
a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2
)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x
^2*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.56, antiderivative size = 1046, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {472, 583, 523, 220, 409, 1217, 1707}

$$-\frac{b(7bc-9ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32a^3\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}}+\frac{b\sqrt[4]{d}\,(7bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^4])/(12*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
 d*x^4])/(4*a*(b*c - a*d)*x^3*(a + b*x^4)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4
)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(11/4)*(-(b*c) + a*d)^(3
/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(S
qrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(24
*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
```

```
Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2
)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
)/c^(1/4)], 1/2])/(16*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d
*x^4]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d)*(Sqrt[c]
 + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sq
rt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2
*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*
c + a*d)*Sqrt[c + d*x^4]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(7*b*
c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*a^3*c^(1/4)*d^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
```

```
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{\int \frac{-7bc+4ad-5bdx^4}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} + \frac{\int \frac{-21b^2c^2+20abcd+4a^2d^2-bd(7bc-}{\left(a+bx^4\right)\sqrt{c+dx^4}}}{12a^2c(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{(b(7bc-9ad))\int \frac{1}{\left(a+bx^4\right)\sqrt{c+d}}}{4a^2(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^{\cdots}\right.}{24a^2c^{5/4}(}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{d}\,x^{\cdots}\right.}{24a^2c^{5/4}(}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^4}}{12a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x^3\left(a+bx^4\right)} + \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-a}\,\sqrt[4]{}}\right.}{16(-a)^{11/4}(bc-ad)^3}$$

Mathematica [C] time = 0.70, size = 408, normalized size = 0.39

$$\frac{\frac{a\left(25ac\left(4a^2d\left(c+2dx^4\right)+4ab\left(-c^2+5cdx^4+d^2x^8\right)-7b^2cx^4\left(4c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+10x^4\left(c+dx^4\right)\left(-4a^2d+4ab\left(c-dx^4\right)+7b^2cx^4\right)\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\cdots\right.\right.\right.}{\left(a+bx^4\right)\left(2x^4\left(2bcF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+adF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}}{60a^3cx^3\sqrt{c+dx^4}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (b*d*(7*b*c - 4*a*d)*x^8*Sqrt[1 + (d*x^4)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((
d*x^4)/c), -((b*x^4)/a)] + (a*(25*a*c*(-7*b^2*c*x^4*(4*c + d*x^4) + 4*a^2*d
*(c + 2*d*x^4) + 4*a*b*(-c^2 + 5*c*d*x^4 + d^2*x^8))*AppellF1[1/4, 1/2, 1,
5/4, -((d*x^4)/c), -((b*x^4)/a)] + 10*x^4*(c + d*x^4)*(-4*a^2*d + 7*b^2*c*x
```

```
^4 + 4*a*b*(c - d*x^4))*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((
b*x^4)/a)] + a*d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))/
((a + b*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)]
 + 2*x^4*(2*b*c*AppellF1[5/4, 1/2, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)] + a*
d*AppellF1[5/4, 3/2, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))))/(60*a^3*c*(-(b
*c) + a*d)*x^3*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4 + a\right)^2 \sqrt{dx^4 + c}\, x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

maple [C] time = 0.25, size = 626, normalized size = 0.60

$$-\frac{\left(-\frac{\sqrt{d x^4+c}\, bx}{4(ad-bc)\left(b x^4+a\right)a} - \frac{\sqrt{-\frac{i\sqrt{d}\, x^2}{\sqrt{c}}+1}\, \sqrt{\frac{i\sqrt{d}\, x^2}{\sqrt{c}}+1}\, d\, \mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\, x, i\right)}{4(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\, \sqrt{d x^4+c}\, a} - \frac{(-5ad+3bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\, x^2}{\sqrt{c}}+1}\, \sqrt{\frac{i\sqrt{d}\, x^2}{\sqrt{c}}+1}\, \mathrm{RootOf}\left(b\_Z^4+a\right)^3 b\, \mathrm{Elliptic}\cdots}{\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\, \sqrt{d x^4}\cdots}\cdots\right.}{32ab(ad\cdots}\cdots\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/a^2*(-1/3/c*(d*x^4+c)^(1/2)/x^3-1/3/c*d/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(
1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*E
llipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/8/a^2*sum(1/_alpha^3*(-1/((-a*d+
b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^
4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^
2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^
(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^
(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a))-1/a*b*(-1/4/(a*d
-b*c)*(d*x^4+c)^(1/2)/(b*x^4+a)/a*b*x-1/4*d/(a*d-b*c)/a/(I/c^(1/2)*d^(1/2))
^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*
x^4+c)^(1/2)*EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-1/32/b/a*sum((-5*a*d+
```

```
3*b*c)/(a*d-b*c)/_alpha^3*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*
d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2
)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(
1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*
c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alp
ha=RootOf(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(1/(x**4*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.838 $\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1146

$$-\frac{\sqrt{dx^4+c}\,x^3}{4(bc-ad)\left(bx^4+a\right)}+\frac{\sqrt{d}\,\sqrt{dx^4+c}\,x}{4b(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad\ldots}}{\sqrt[4]{-a}\,\sqrt[4]{\ldots}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(ad-bc\ldots}$$

```
[Out] 1/16*(-a*d+3*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1
/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/16*(-a*d+3*b*c)*arctan(x*(a*d-b*
c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*c)^(
3/2)-1/4*x^3*(d*x^4+c)^(1/2)/(-a*d+b*c)/(b*x^4+a)+1/4*x*d^(1/2)*(d*x^4+c)^(
1/2)/b/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*c^(1/4)*d^(1/4)*(cos(2*arctan(d
^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE(sin(
2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/
(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/8*c^(1/4)*d^(
1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*
d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^4+c)^(
1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),
1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))*((d
*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(
d*x^4+c)^(1/2)-1/32*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)
/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*
d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+c)^(1/2)-1/16*d^(1/4)*(-a*d+3*b*c)*(cos(
2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*Elli
pticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(
b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^4+c)^(1/2)+1/32*(-a*d+3*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*
d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(
1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2
))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^4+
c)^(1/2)
```

Rubi [A] time = 1.56, antiderivative size = 1146, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {471, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{dx^4+c}\,x^3}{4(bc-ad)\left(bx^4+a\right)}+\frac{\sqrt{d}\,\sqrt{dx^4+c}\,x}{4b(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^4+c}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad\ldots}}{\sqrt[4]{-a}\,\sqrt[4]{\ldots}}\right)}{16\sqrt[4]{-a}\,b^{5/4}(ad-bc\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (Sqrt[d]*x*Sqrt[c + d*x^4])/(4*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) - (x^
3*Sqrt[c + d*x^4])/(4*(b*c - a*d)*(a + b*x^4)) + ((3*b*c - a*d)*ArcTan[(Sqr
t[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/
4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-a)^
```

```
(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d)^(3/2
)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + S
qrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*b*(b*c - a
*d)*Sqrt[c + d*x^4]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d
*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1
/2])/(8*b*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqr
t[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[
c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*b*c
^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a]*Sqr
t[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^
4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(16*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c
] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x
^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c
+ d*x^4]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c]
+ Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt
[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*A
rcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c
 - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\int \frac{x^2\left(3c+dx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\int \left(\frac{dx^2}{b\sqrt{c+dx^4}}+\frac{(3bc-ad)x^2}{b\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{4(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}+\frac{(3bc-ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}$$

$$= -\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}-\frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4b(bc-ad)}-\frac{(3bc-}{}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}-\frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c}{\left(\sqrt{c}\right.}}}{4b(bc-ad}$$

$$= \frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)}-\frac{x^3\sqrt{c+dx^4}}{4(bc-ad)\left(a+bx^4\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+}}\right.}{16\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.17, size = 162, normalized size = 0.14

$$\frac{dx^7\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+7cx^3\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-7ax^3\left(\right.}{28a\left(a+bx^4\right)\sqrt{c+dx^4}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (-7*a*x^3*(c + d*x^4) + 7*c*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + d*x^7*(a + b*x^4)*Sqrt[1 + (d
*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x^4)/a)])/(28*a*(b*
c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

fricas **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple **[C]** time = 0.36, size = 556, normalized size = 0.49

$$-\frac{\left(-\frac{\sqrt{d x^4+c}\, b x^3}{4(ad-bc)\left(b x^4+a\right)a}+\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)\right)\sqrt{c}\,\sqrt{d}}{4(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}\,a}-\frac{(-3ad+bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}\cdots}}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] 1/8/b^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2
+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_al
pha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/
(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/
2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*b+a))-a/b*(-1/4/a*b/(a*d-b*c)*x^3*(d*x^4+c)^(1/2)/(b*x^4+a)+1/4*I
*d^(1/2)/(a*d-b*c)/a*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*
x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF((I/
c^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/32/
b/a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*
```

```
(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d
^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1
/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_
alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^6/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^6/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**6/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.839 $\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1144

$$\frac{b\sqrt{dx^4+c}\,x^3}{4a(bc-ad)\left(bx^4+a\right)}-\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4a(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}\,(ad-bc)^{3/2}}$$

```
[Out] -1/16*(-3*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(
1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/16*(-3*a*d+b*c)*arctan(x*(a*d-b
*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b*c)^
(3/2)+1/4*b*x^3*(d*x^4+c)^(1/2)/a/(-a*d+b*c)/(b*x^4+a)-1/4*x*d^(1/2)*(d*x^4
+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))+1/4*c^(1/4)*d^(1/4)*(cos(2*arc
tan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticE
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^
4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4+c)^(1/2)-1/8*c^(1/4
)*d^(1/4)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^4
+c)^(1/2)+1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/
4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2)
,1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*
((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+
b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/32*(-3*a*d+b*c)*(cos(2*arctan(d^(1
/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*
arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)
^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*
c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a
)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^4+c)^(1/2)-1/16*d
^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2)
)*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^
(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/16*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*
arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2
^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+
c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^
(1/2)
```

Rubi [A] time = 1.44, antiderivative size = 1144, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {472, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{b\sqrt{dx^4+c}\,x^3}{4a(bc-ad)\left(bx^4+a\right)}-\frac{\sqrt{d}\sqrt{dx^4+c}\,x}{4a(bc-ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{dx^4+c}}\right)}{16(-a)^{5/4}\sqrt[4]{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -(Sqrt[d]*x*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2)) + (b
*x^3*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*(a + b*x^4)) - ((b*c - 3*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*
b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/(
```

```
(-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(5/4)*b^(1/4)*(-(b*c) + a*d)
^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c
] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(4*a*(b*
c - a*d)*Sqrt[c + d*x^4]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(
c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4
)], 1/2])/(8*a*(b*c - a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] - (Sqrt[-a]*Sqrt[d]
)/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(1
6*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[c] + (Sqrt[-a
]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(16*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - ((Sqrt[b]*S
qrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c
+ d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]
*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1
/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^4]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(S
qrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticP
i[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d
]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(
1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \frac{x^2\left(-bc+4ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{d\int \frac{x^2}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{(bc-3ad)\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx}{4a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\int \frac{1-\frac{\sqrt{d}x^2}{\sqrt{c}}}{\sqrt{c+dx^4}}\,dx}{4a(bc-ad)} - \frac{(bc-3ad)}{8\cdots}$$

$$= -\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} + \frac{\sqrt[4]{c}\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^2\right)\sqrt{\frac{c\cdots}{\left(\sqrt{c}\cdots\right.}}}{4a(bc-ad\cdots}$$

$$= -\frac{\sqrt{d}\,x\sqrt{c+dx^4}}{4a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^2\right)} + \frac{bx^3\sqrt{c+dx^4}}{4a(bc-ad)\left(a+bx^4\right)} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+\cdots}}\right.}{16(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}\cdots}$$

Mathematica [C] time = 0.15, size = 172, normalized size = 0.15

$$\frac{-3bdx^7\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right) + 7x^3\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\,(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\cdots\right.}{84a^2\left(a+bx^4\right)\sqrt{c+dx^4}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*b*x^3*(c + d*x^4) + 7*(b*c - 4*a*d)*x^3*(a + b*x^4)*Sqrt[1 + (d*x^4)/
c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] - 3*b*d*x^7*(a +
b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b*x
^4)/a)])/(84*a^2*(b*c - a*d)*(a + b*x^4)*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple [C] time = 0.25, size = 359, normalized size = 0.31

$$-\frac{\sqrt{d\,x^4+c}\;b\,x^3}{4\,(ad-bc)\left(b\,x^4+a\right)a}+\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\;\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\;\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\;x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\;x,i\right)\right)\sqrt{c}}{4\,(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\;\sqrt{d\,x^4+c}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/4/(a*d-b*c)*(d*x^4+c)^(1/2)/(b*x^4+a)/a*b*x^3+1/4*I*d^(1/2)/(a*d-b*c)/a*
c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/
2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*(EllipticF((I/c^(1/2)*d^(1/2))^(1/2
)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(1/2)*x,I))-1/32/b/a*sum((-3*a*d+b*c)/
(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c
)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^
3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x
^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d
^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf
(_Z^4*b+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int(x^2/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.840 $\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=1225

$$\frac{\sqrt{b}\,(5bc-7ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32(-a)^{5/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}} - b^{3/4}(5$$

```
[Out] -1/16*b^(3/4)*(-7*a*d+5*b*c)*arctan(x*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(
d*x^4+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/16*b^(3/4)*(-7*a*d+5*b*c)*arc
tan(x*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^4+c)^(1/2))/(-a)^(9/4)/(a*d-b
*c)^(3/2)-1/4*(-4*a*d+5*b*c)*(d*x^4+c)^(1/2)/a^2/c/(-a*d+b*c)/x+1/4*b*(d*x^
4+c)^(1/2)/a/(-a*d+b*c)/x/(b*x^4+a)+1/4*(-4*a*d+5*b*c)*x*d^(1/2)*(d*x^4+c)^
(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^2*d^(1/2))-1/4*d^(1/4)*(-4*a*d+5*b*c)*(co
s(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*El
lipticE(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)
^(1/2)+1/8*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2
)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4)
)),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(
1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^4+c)^(1/2)+1/32*(-7*a*d+5*b*c)*(cos(2*arct
an(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1
/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2)
)^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)-
1/32*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x/c^(1/4))),-1/4*(b^(1
/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*b^(1/2)*(c^(1/2)+x^2*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2
*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d
+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan
(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*EllipticF(si
n(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2))*(c^(1/2)-
(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)^(1/2)/a^2/c
^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)+1/16*b*d^(1/4)*(-7*a*d+5*b*c)*(
cos(2*arctan(d^(1/4)*x/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x/c^(1/4)))*
EllipticF(sin(2*arctan(d^(1/4)*x/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^2*d^(1/2
))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^4+c)/(c^(1/2)+x^2*d^(1/2))^2)
^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 1.90, antiderivative size = 1225, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {472, 583, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{b}\,(5bc-7ad)\left(\sqrt{d}\,x^2+\sqrt{c}\right)\sqrt{\frac{dx^4+c}{\left(\sqrt{d}\,x^2+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{32(-a)^{5/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^4+c}} - b^{3/4}(5$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*x) + (Sqrt[d]*(5*b*
c - 4*a*d)*x*Sqrt[c + d*x^4])/(4*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^2))
 + (b*Sqrt[c + d*x^4])/(4*a*(b*c - a*d)*x*(a + b*x^4)) - (b^(3/4)*(5*b*c -
```

```
7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(1
6*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(Sqrt[-(b
*c) + a*d]*x)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^4])])/(16*(-a)^(9/4)*(-(b*c)
 + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c +
 d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticE[2*ArcTan[(d^(1/4)*x)/c^(1/4)],
 1/2])/(4*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^4]) + (d^(1/4)*(5*b*c - 4*a*
d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[
c + d*x^4]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*
a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*El
lipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^4]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqr
t[d]*x^2)^2]*EllipticF[2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)
*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqr
t[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(
Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^
2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x)/c^(1/4)], 1/2]
)/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^4]) +
 (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] +
 Sqrt[d]*x^2)*Sqrt[(c + d*x^4)/(Sqrt[c] + Sqrt[d]*x^2)^2]*EllipticPi[(Sqrt[
b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*Ar
cTan[(d^(1/4)*x)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)
*(b*c + a*d)*Sqrt[c + d*x^4])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
```

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx = \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{\int \frac{-5bc+4ad-3bdx^4}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}dx}{4a(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \frac{x^2(-(bc-2ad)(5bc-2ad)+bd(5bc-4a}{\left(a+bx^4\right)\sqrt{c+dx^4}}}{4a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\int \left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}} + \frac{\left(-5b^2c^2+7abcd\right.}{\left(a+bx^4\right)\sqrt{c+dx}}\right.}{4a^2c(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} - \frac{(b(5bc-7ad))\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}}{4a^2(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)} + \frac{\left(\sqrt{b}(5bc-7ad)\right)\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x\right.}}{8a^2(bc-ad)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}$$

$$= -\frac{(5bc-4ad)\sqrt{c+dx^4}}{4a^2c(bc-ad)x} + \frac{\sqrt{d}(5bc-4ad)x\sqrt{c+dx^4}}{4a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}x^2\right)} + \frac{b\sqrt{c+dx^4}}{4a(bc-ad)x\left(a+bx^4\right)}$$

Mathematica [C] time = 0.33, size = 226, normalized size = 0.18

$$\frac{-7x^4\left(a+bx^4\right)\sqrt{\frac{dx^4}{c}+1}\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+21a\left(c+dx^4\right)\left(4a^2d-4ab\left(c-dx^4\right.\right.}{84a^3cx\left(a+bx^4\right)\sqrt{c+dx^4}(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (21*a*(c + d*x^4)*(4*a^2*d - 5*b^2*c*x^4 - 4*a*b*(c - d*x^4)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^4*(a + b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^4)/c), -((b*x^4)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^8*(a
+ b*x^4)*Sqrt[1 + (d*x^4)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^4)/c), -((b
*x^4)/a)])/(84*a^3*c*(b*c - a*d)*x*(a + b*x^4)*Sqrt[c + d*x^4])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

maple [C] time = 0.23, size = 674, normalized size = 0.55

$$-\frac{\left(-\frac{\sqrt{d x^4+c}\, b x^3}{4(ad-bc)\left(b x^4+a\right)a}+\frac{i\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\left(-\mathrm{EllipticE}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)+\mathrm{EllipticF}\left(\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,x,i\right)\right)\sqrt{c}\,\sqrt{d}}{4(ad-bc)\sqrt{\frac{i\sqrt{d}}{\sqrt{c}}}\,\sqrt{d x^4+c}\;a}-\frac{(-3ad+bc)\left(\frac{2\sqrt{-\frac{i\sqrt{d}\,x^2}{\sqrt{c}}+1}\,\sqrt{i\cdots}}{\cdots}\right.}{\cdots}\right.}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] -1/8/a^2*sum(1/_alpha*(-1/((-a*d+b*c)/b)^(1/2)*arctanh(1/2*(2*_alpha^2*d*x^
2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2))+2/(I/c^(1/2)*d^(1/2))^(1/2)*_a
lpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)
/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1/2))^(1/2)*x,I*_alpha^2/a*b*c^(1
/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2))),_alpha=R
ootOf(_Z^4*b+a))+1/a^2*(-(d*x^4+c)^(1/2)/c/x+I/c^(1/2)*d^(1/2)/(I/c^(1/2)*d
^(1/2))^(1/2)*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1
/2)/(d*x^4+c)^(1/2)*(EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/
c^(1/2)*d^(1/2))^(1/2)*x,I)))-1/a*b*(-1/4/(a*d-b*c)*(d*x^4+c)^(1/2)/(b*x^4+
a)/a*b*x^3+1/4*I*d^(1/2)/(a*d-b*c)/a*c^(1/2)/(I/c^(1/2)*d^(1/2))^(1/2)*(-I/
c^(1/2)*d^(1/2)*x^2+1)^(1/2)*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2
)*(EllipticF((I/c^(1/2)*d^(1/2))^(1/2)*x,I)-EllipticE((I/c^(1/2)*d^(1/2))^(
1/2)*x,I))-1/32/b/a*sum((-3*a*d+b*c)/(a*d-b*c)/_alpha*(-1/((-a*d+b*c)/b)^(1
/2)*arctanh(1/2*(2*_alpha^2*d*x^2+2*c)/((-a*d+b*c)/b)^(1/2)/(d*x^4+c)^(1/2)
)+2/(I/c^(1/2)*d^(1/2))^(1/2)*_alpha^3*b/a*(-I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)
*(I/c^(1/2)*d^(1/2)*x^2+1)^(1/2)/(d*x^4+c)^(1/2)*EllipticPi((I/c^(1/2)*d^(1
/2))^(1/2)*x,I*_alpha^2/a*b*c^(1/2)/d^(1/2),(-I/c^(1/2)*d^(1/2))^(1/2)/(I/c
^(1/2)*d^(1/2))^(1/2))),_alpha=RootOf(_Z^4*b+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^4+a\right)^2\sqrt{dx^4+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^2*sqrt(d*x^4 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(b\,x^4+a\right)^2\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^4\right)^2 \sqrt{c + dx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**4)**2*sqrt(c + d*x**4)), x)
```

3.841 $\int \frac{(ex)^m\left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(a^2d^2(m+3)(m+7)+bc(m+1)(bc(m+5)-2ad(m+7))\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{d^2e(m+1)(m+3)(m+7)\sqrt{c+dx^4}}-\frac{b\sqrt{c+}}{}$$

```
[Out] -b*(b*c*(5+m)-2*a*d*(7+m))*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d^2/e/(3+m)/(7+m)+b^
2*(e*x)^(5+m)*(d*x^4+c)^(1/2)/d/e^5/(7+m)+(a^2*d^2*(3+m)*(7+m)+b*c*(1+m)*(b
*c*(5+m)-2*a*d*(7+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-
d*x^4/c)*(1+d*x^4/c)^(1/2)/d^2/e/(1+m)/(3+m)/(7+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {464, 459, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(\frac{a^2d^2(m+7)}{m+1}+\frac{bc(bc(m+5)-2ad(m+7))}{m+3}\right)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{d^2e(m+7)\sqrt{c+dx^4}}-\frac{b\sqrt{c+dx^4}\,(ex)^{m+1}(bc(m+5)-2a}{d^2e(m+3)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] -((b*(b*c*(5 + m) - 2*a*d*(7 + m))*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d^2*e*(3
 + m)*(7 + m))) + (b^2*(e*x)^(5 + m)*Sqrt[c + d*x^4])/(d*e^5*(7 + m)) + (((
a^2*d^2*(7 + m))/(1 + m) + (b*c*(b*c*(5 + m) - 2*a*d*(7 + m)))/(3 + m))*(e*
x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)])/(d^2*e*(7 + m)*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> Simp[(d^2*(e*x)^(m + n + 1)*(a + b*x^n)^(p + 1))/(b*e^(n
 + 1)*(m + n*(p + 2) + 1)), x] + Dist[1/(b*(m + n*(p + 2) + 1)), Int[(e*x)^
m*(a + b*x^n)^p*Simp[b*c^2*(m + n*(p + 2) + 1) + d*((2*b*c - a*d)*(m + n +
```

```
1) + 2*b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
 && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && NeQ[m + n*(p + 2) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\sqrt{c+dx^4}}\,dx = \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\int \frac{(ex)^m\left(a^2d(7+m)-b(bc(5+m)-2ad(7+m))x^4\right)}{\sqrt{c+dx^4}}\,dx}{d(7+m)}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \left(-a^2 - \frac{bc(1+m}{}\right.$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} - \frac{\left(\left(-a^2 - \frac{bc(1+m)(l}{d^2}\right.\right.}{}$$

$$= -\frac{b(bc(5+m)-2ad(7+m))(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)(7+m)} + \frac{b^2(ex)^{5+m}\sqrt{c+dx^4}}{de^5(7+m)} + \frac{\left(a^2 + \frac{bc(1+m)(bc(5}{d^2(3+}\right.}{}$$

Mathematica [A] time = 0.18, size = 164, normalized size = 0.82

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m\left(a^2\left(m^2+14m+45\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\left(2a(m+9)\,{}_2F_1\left(\frac{1}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)+\right.\right.}{(m+1)(m+5)(m+9)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[1/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[1/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[1/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/((1 + m)*(5 + m)*(9
+ m)*Sqrt[c + d*x^4])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^8+2\,abx^4+a^2\right)(ex)^m}{\sqrt{dx^4+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(e*x)^m/sqrt(d*x^4 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^2(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)^2 (ex)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^2 (ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^2*(e*x)^m/sqrt(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x)^m \left(b\,x^4+a\right)^2}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2),x)
```

```
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(1/2), x)
```

sympy [C] time = 14.27, size = 185, normalized size = 0.92

$$\frac{a^2e^mxx^m\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}+\frac{abe^mx^5x^m\Gamma\left(\frac{m}{4}+\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{2\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}+\frac{b^2e^mx^9x^m\Gamma\left(\frac{m}{4}+\frac{9}{4}\right){}_2F_1\left(\right.}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
```

```
[Out] a**2*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x
**4*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + a*b*e**m*x**5*x**m*ga
mma(m/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)
/c)/(2*sqrt(c)*gamma(m/4 + 9/4)) + b**2*e**m*x**9*x**m*gamma(m/4 + 9/4)*hyp
er((1/2, m/4 + 9/4), (m/4 + 13/4,), d*x**4*exp_polar(I*pi)/c)/(4*sqrt(c)*ga
mma(m/4 + 13/4))
```

3.842 $\int \frac{(ex)^m(a+bx^4)}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=123

$$\frac{b\sqrt{c+dx^4}\,(ex)^{m+1}}{de(m+3)}-\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}(bc(m+1)-ad(m+3))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{de(m+1)(m+3)\sqrt{c+dx^4}}$$

```
[Out] b*(e*x)^(1+m)*(d*x^4+c)^(1/2)/d/e/(3+m)-(b*c*(1+m)-a*d*(3+m))*(e*x)^(1+m)*h
ypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1/2)/d/e/(1+m)
/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 115, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {459, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(\frac{a}{m+1}-\frac{bc}{d(m+3)}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e\sqrt{c+dx^4}}+\frac{b\sqrt{c+dx^4}\,(ex)^{m+1}}{de(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
```

```
[Out] (b*(e*x)^(1 + m)*Sqrt[c + d*x^4])/(d*e*(3 + m)) + ((a/(1 + m) - (b*c)/(d*(3
 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)])/(e*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ex)^m\left(a+bx^4\right)}{\sqrt{c+dx^4}}\,dx = \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \left(-a+\frac{bc(1+m)}{d(3+m)}\right)\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$$
$$= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} - \frac{\left(\left(-a+\frac{bc(1+m)}{d(3+m)}\right)\sqrt{1+\frac{dx^4}{c}}\right)\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}$$
$$= \frac{b(ex)^{1+m}\sqrt{c+dx^4}}{de(3+m)} + \frac{\left(a-\frac{bc(1+m)}{d(3+m)}\right)(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.10, size = 110, normalized size = 0.89

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m\left(a(m+5)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\,{}_2F_1\left(\frac{1}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)\right)}{(m+1)(m+5)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/Sqrt[c + d*x^4],x]
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[1/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/((1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 0.92, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)(ex)^m}{\sqrt{dx^4+c}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

maple [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)(ex)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)(ex)^m}{\sqrt{dx^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)*(e*x)^m/sqrt(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m\,(b\,x^4 + a)}{\sqrt{d\,x^4 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2),x)
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(1/2), x)
```

sympy [C] time = 4.92, size = 119, normalized size = 0.97

$$\frac{ae^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{m}{4}+\frac{5}{4}\\ \frac{m}{4}+\frac{9}{4}\end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(1/2),x)
[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(m
/4 + 5/4)*hyper((1/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/(
4*sqrt(c)*gamma(m/4 + 9/4))
```

3.843 $\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{e(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.97

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+1}{4}+1;-\frac{dx^4}{c}\right)}{(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/Sqrt[c + d*x^4],x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/((1 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex)^m}{\sqrt{dx^4+c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/sqrt(d*x^4 + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/sqrt(d*x^4 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(c + d*x^4)^(1/2),x)
```

```
[Out] int((e*x)^m/(c + d*x^4)^(1/2), x)
```

sympy [C] time = 1.07, size = 56, normalized size = 0.82

$$\frac{e^{m}xx^{m}\Gamma\left(\frac{m}{4}+\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{2},\frac{m}{4}+\frac{1}{4}\\ \frac{m}{4}+\frac{5}{4}\end{matrix}\middle|\frac{dx^{4}e^{i\pi}}{c}\right)}{4\sqrt{c}\,\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(d*x**4+c)**(1/2),x)
```

```
[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((1/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*e
xp_polar(I*pi)/c)/(4*sqrt(c)*gamma(m/4 + 5/4))
```

3.844 $\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{(a+bx^4)\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{(a+bx^4)\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ae(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A] time = 0.14, size = 125, normalized size = 1.54

$$\frac{x\sqrt{c+dx^4}\,(ex)^m\left(bcF_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-ad\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)\right)}{ac(m+1)\sqrt{\frac{dx^4}{c}+1}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b*c*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4, -((
d*x^4)/c), -((b*x^4)/a)] - a*d*Hypergeometric2F1[1/2, (1 + m)/4, (5 + m)/4,
 -((d*x^4)/c)]))/(a*c*(b*c - a*d)*(1 + m)*Sqrt[1 + (d*x^4)/c])
```

fricas [F] time = 1.12, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{bdx^8+(bc+ad)x^4+ac},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d*x^8 + (b*c + a*d)*x^4 + a*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(b\,x^4+a\right)\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^4+a)\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)\,\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(1/2),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**4)*sqrt(c + d*x**4)), x)
```

3.845 $\int \frac{(ex)^m}{(a+bx^4)^2\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};2,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^2\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};2,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2e(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [B] time = 0.23, size = 179, normalized size = 2.21

$$\frac{x\sqrt{c+dx^4}\,(ex)^m\left(a^2d^2\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)-abcdF_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+bc(bc-ad)F_1\left(\frac{m+1}{4};2,\right.\right.}{a^2c(m+1)\sqrt{\frac{dx^4}{c}+1}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(-(a*b*c*d*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/
4, -((d*x^4)/c), -((b*x^4)/a)]) + b*c*(b*c - a*d)*AppellF1[(1 + m)/4, 2, -1
/2, (5 + m)/4, -((b*x^4)/a), -((d*x^4)/c)] + a^2*d^2*Hypergeometric2F1[1/2,
 (1 + m)/4, (5 + m)/4, -((d*x^4)/c)]))/(a^2*c*(b*c - a*d)^2*(1 + m)*Sqrt[1
+ (d*x^4)/c])
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^2dx^{12}+\left(b^2c+2\,abd\right)x^8+\left(2\,abc+a^2d\right)x^4+a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d*x^12 + (b^2*c + 2*a*b*d)*x^8 + (2*a
*b*c + a^2*d)*x^4 + a^2*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)^2\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(1/2),x)
[Out] Timed out
```

3.846 $\int \frac{(ex)^m}{\left(a+bx^4\right)^3\sqrt{c+dx^4}}\,dx$

Optimal. Leaf size=81

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,1/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^3/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\left(a+bx^4\right)^3\sqrt{c+dx^4}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^3\sqrt{1+\frac{dx^4}{c}}}\,dx}{\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{1}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3e(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 0.95

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^mF_1\left(\frac{m+1}{4};3,\frac{1}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*Sqrt[c + d*x^4]),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 1/2, (5 + m)/4, -((b*
x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^3dx^{16}+\left(b^3c+3\,ab^2d\right)x^{12}+3\left(ab^2c+a^2bd\right)x^8+\left(3\,a^2bc+a^3d\right)x^4+a^3c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d*x^16 + (b^3*c + 3*a*b^2*d)*x^12 + 3
*(a*b^2*c + a^2*b*d)*x^8 + (3*a^2*b*c + a^3*d)*x^4 + a^3*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(bx^4+a\right)^3\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(b\,x^4+a\right)^3\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(bx^4+a\right)^3\sqrt{dx^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*sqrt(d*x^4 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(e\,x)^m}{\left(b\,x^4+a\right)^3\sqrt{d\,x^4+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(1/2),x)
[Out] Timed out
```

3.847 $\int \frac{(ex)^m(a+bx^4)^2}{(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=198

$$-\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc}{2cd^2e\sqrt{c+}}$$

```
[Out] 1/2*(-a*d+b*c)^2*(e*x)^(1+m)/c/d^2/e/(d*x^4+c)^(1/2)+b^2*(e*x)^(1+m)*(d*x^4
+c)^(1/2)/d^2/e/(3+m)-1/2*(2*b^2*c^2*(1+m)-(3+m)*(2*a^2*d^2-(-a*d+b*c)^2*(1
+m)))*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4
/c)^(1/2)/c/d^2/e/(1+m)/(3+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {463, 459, 365, 364}

$$-\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\left(2b^2c^2(m+1)-(m+3)\left(2a^2d^2-(m+1)(bc-ad)^2\right)\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cd^2e(m+1)(m+3)\sqrt{c+dx^4}}+\frac{(ex)^{m+1}(bc}{2cd^2e\sqrt{c+}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*(e*x)^(1 + m))/(2*c*d^2*e*Sqrt[c + d*x^4]) + (b^2*(e*x)^(1 +
 m)*Sqrt[c + d*x^4])/(d^2*e*(3 + m)) - ((2*b^2*c^2*(1 + m) - (3 + m)*(2*a^2
*d^2 - (b*c - a*d)^2*(1 + m)))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeome
tric2F1[1/2, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d^2*e*(1 + m)*(3 + m
)*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
```

```
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)^2}{\left(c+dx^4\right)^{3/2}}\,dx = \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} - \frac{\int \frac{(ex)^m\left(-2a^2d^2+(bc-ad)^2(1+m)-2b^2cdx^4\right)}{\sqrt{c+dx^4}}\,dx}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5+}{3+m}\right.}{2cd^2}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} - \frac{\left(\left(-a^2d^2(1-m)-2abcd(1+m)+\frac{b^2c^2(1+m)(5}{3+m}\right.\right.}{2cd^2\sqrt{c+dx^4}}$$

$$= \frac{(bc-ad)^2(ex)^{1+m}}{2cd^2e\sqrt{c+dx^4}} + \frac{b^2(ex)^{1+m}\sqrt{c+dx^4}}{d^2e(3+m)} + \frac{\left(a^2d^2(1-m)+2abcd(1+m)-\frac{b^2c^2(1+m)(5+m}{3+m}\right.}{2cd^2e(1+n}$$

Mathematica [A] time = 0.16, size = 167, normalized size = 0.84

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m\left(a^2\left(m^2+14m+45\right){}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)+b(m+1)x^4\left(2a(m+9)\,{}_2F_1\left(\frac{3}{2},\frac{m+5}{4};\frac{m+9}{4};-\frac{dx^4}{c}\right)+\right.\right.}{c(m+1)(m+5)(m+9)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a^2*(45 + 14*m + m^2)*Hypergeometric2F1[3/2
, (1 + m)/4, (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*(2*a*(9 + m)*Hypergeo
metric2F1[3/2, (5 + m)/4, (9 + m)/4, -((d*x^4)/c)] + b*(5 + m)*x^4*Hypergeo
metric2F1[3/2, (9 + m)/4, (13 + m)/4, -((d*x^4)/c)])))/(c*(1 + m)*(5 + m)*(
9 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^8+2\,abx^4+a^2\right)\sqrt{dx^4+c}\,(ex)^m}{d^2x^8+2\,cdx^4+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c
*d*x^4 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)^2(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^{4}+a\right)^{2} \left(ex\right)^{m}}{\left(d\,x^{4}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
[Out] int((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{4}+a\right)^{2} \left(ex\right)^{m}}{\left(dx^{4}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^4 + a)^2*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x\right)^m\,{\left(b\,x^4+a\right)}^2}{{\left(d\,x^4+c\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2),x)
[Out] int(((e*x)^m*(a + b*x^4)^2)/(c + d*x^4)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
[Out] Timed out
```

3.848 $\int \frac{(ex)^m(a+bx^4)}{(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}(ad(1-m)+bc(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

```
[Out] -1/2*(-a*d+b*c)*(e*x)^(1+m)/c/d/e/(d*x^4+c)^(1/2)+1/2*(a*d*(1-m)+b*c*(1+m))
*(e*x)^(1+m)*hypergeom([1/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(
1/2)/c/d/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {457, 365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}(ad(1-m)+bc(m+1))\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{2cde(m+1)\sqrt{c+dx^4}}-\frac{(ex)^{m+1}(bc-ad)}{2cde\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] -((b*c - a*d)*(e*x)^(1 + m))/(2*c*d*e*Sqrt[c + d*x^4]) + ((a*d*(1 - m) + b*
c*(1 + m))*(e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[1/2, (1 + m)
/4, (5 + m)/4, -((d*x^4)/c)])/(2*c*d*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rubi steps

$$\int \frac{(ex)^m \left(a+bx^4\right)}{\left(c+dx^4\right)^{3/2}} \, dx = -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}} + \frac{(-ad(-1+m)+bc(1+m)) \int \frac{(ex)^m}{\sqrt{c+dx^4}} \, dx}{2cd}$$

$$= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}} + \frac{\left((-ad(-1+m)+bc(1+m))\sqrt{1+\frac{dx^4}{c}}\right) \int \frac{(ex)^m}{\sqrt{1+\frac{dx^4}{c}}} \, dx}{2cd\sqrt{c+dx^4}}$$

$$= -\frac{(bc-ad)(ex)^{1+m}}{2cde\sqrt{c+dx^4}} + \frac{(ad(1-m)+bc(1+m))(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}} \, {}_2F_1\left(\frac{1}{2}, \frac{1+m}{4}; \frac{5+m}{4}; -\frac{dx^4}{c}\right)}{2cde(1+m)\sqrt{c+dx^4}}$$

Mathematica [A] time = 0.10, size = 113, normalized size = 0.86

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m \left(a(m+5)\,{}_2F_1\left(\frac{3}{2}, \frac{m+1}{4}; \frac{m+5}{4}; -\frac{dx^4}{c}\right) + b(m+1)x^4\,{}_2F_1\left(\frac{3}{2}, \frac{m+5}{4}; \frac{m+9}{4}; -\frac{dx^4}{c}\right)\right)}{c(m+1)(m+5)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*(a*(5 + m)*Hypergeometric2F1[3/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)] + b*(1 + m)*x^4*Hypergeometric2F1[3/2, (5 + m)/4,
 (9 + m)/4, -((d*x^4)/c)]))/(c*(1 + m)*(5 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4+a\right)\sqrt{dx^4+c}\,(ex)^m}{d^2x^8+2\,cdx^4+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,x^4+a\right)(ex)^m}{\left(d\,x^4+c\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^4+a\right)(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(e*x)^m/(d*x^4 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m\,\left(b\,x^4+a\right)}{\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2),x)
```

```
[Out] int(((e*x)^m*(a + b*x^4))/(c + d*x^4)^(3/2), x)
```

sympy [C] time = 61.44, size = 119, normalized size = 0.90

$$\frac{ae^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)} + \frac{be^m x^5 x^m \Gamma\left(\frac{m}{4}+\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{5}{4} \\ \frac{m}{4}+\frac{9}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

```
[Out] a*e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4
*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4)) + b*e**m*x**5*x**m*gamma(
m/4 + 5/4)*hyper((3/2, m/4 + 5/4), (m/4 + 9/4,), d*x**4*exp_polar(I*pi)/c)/
(4*c**(3/2)*gamma(m/4 + 9/4))
```

3.849 $\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*hypergeom([3/2, 1/4+1/4*m],[5/4+1/4*m],-d*x^4/c)*(1+d*x^4/c)^(1
/2)/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {365, 364}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)}{ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, (5 + m
)/4, -((d*x^4)/c)])/(c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{(c+dx^4)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m}{4};\frac{5+m}{4};-\frac{dx^4}{c}\right)}{ce(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.97

$$\frac{x\sqrt{\frac{dx^4}{c}+1}\,(ex)^m\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+1}{4}+1;-\frac{dx^4}{c}\right)}{c(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/(c + d*x^4)^(3/2),x]
```

```
[Out] (x*(e*x)^m*Sqrt[1 + (d*x^4)/c]*Hypergeometric2F1[3/2, (1 + m)/4, 1 + (1 + m
)/4, -((d*x^4)/c)])/(c*(1 + m)*Sqrt[c + d*x^4])
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{d^2x^8+2\,cdx^4+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(d^2*x^8 + 2*c*d*x^4 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

maple [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/(d*x^4 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(c + d*x^4)^(3/2),x)
```

[Out] int((e*x)^m/(c + d*x^4)^(3/2), x)

sympy [C] time = 1.49, size = 56, normalized size = 0.79

$$\frac{e^m x x^m \Gamma\left(\frac{m}{4}+\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{m}{4}+\frac{1}{4} \\ \frac{m}{4}+\frac{5}{4} \end{matrix}\middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4c^{\frac{3}{2}}\Gamma\left(\frac{m}{4}+\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x)**m/(d*x**4+c)**(3/2),x)

[Out] e**m*x*x**m*gamma(m/4 + 1/4)*hyper((3/2, m/4 + 1/4), (m/4 + 5/4,), d*x**4*exp_polar(I*pi)/c)/(4*c**(3/2)*gamma(m/4 + 5/4))

3.850 $\int \frac{(ex)^m}{(a+bx^4)(c+dx^4)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,1,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};1,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 1, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{\left(a+bx^4\right)\left(c+dx^4\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}\\ &= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};1,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ace(1+m)\sqrt{c+dx^4}}\end{aligned}$$

Mathematica [B] time = 0.21, size = 169, normalized size = 2.01

$$\frac{x\sqrt{c+dx^4}\,(ex)^m\left(b^2c^2F_1\left(\frac{m+1}{4};-\frac{1}{2},1;\frac{m+5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)+ad\left((ad-bc)\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{4};\frac{m+5}{4};-\frac{dx^4}{c}\right)-bc\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{4};\frac{m+}{4}\right.\right.\right.}{ac^2(m+1)\sqrt{\frac{dx^4}{c}+1}\,(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*Sqrt[c + d*x^4]*(b^2*c^2*AppellF1[(1 + m)/4, -1/2, 1, (5 + m)/4,
 -((d*x^4)/c), -((b*x^4)/a)] + a*d*(-(b*c*Hypergeometric2F1[1/2, (1 + m)/4,
 (5 + m)/4, -((d*x^4)/c)]) + (-(b*c) + a*d)*Hypergeometric2F1[3/2, (1 + m)/
4, (5 + m)/4, -((d*x^4)/c)])))/(a*c^2*(b*c - a*d)^2*(1 + m)*Sqrt[1 + (d*x^4
)/c])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{bd^2x^{12}+\left(2\,bcd+ad^2\right)x^8+\left(bc^2+2\,acd\right)x^4+ac^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b*d^2*x^12 + (2*b*c*d + a*d^2)*x^8 + (b*c
^2 + 2*a*c*d)*x^4 + a*c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(b\,x^4+a\right)\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)*(d*x^4 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)),x)
```

```
[Out] int((e*x)^m/((a + b*x^4)*(c + d*x^4)^(3/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a+bx^4)\,(c+dx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)/(d*x**4+c)**(3/2),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**4)*(c + d*x**4)**(3/2)), x)
```

3.851 $\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1} F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2 ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,2,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^2/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1} F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2 ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^2*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^m}{(a+bx^4)^2 (c+dx^4)^{3/2}} dx &= \frac{\sqrt{1+\frac{dx^4}{c}} \int \frac{(ex)^m}{(a+bx^4)^2\left(1+\frac{dx^4}{c}\right)^{3/2}} dx}{c\sqrt{c+dx^4}} \\
&= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\, F_1\left(\frac{1+m}{4};2,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2 ce(1+m)\sqrt{c+dx^4}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 77, normalized size = 0.92

$$\frac{x\left(\frac{dx^4}{c}+1\right)^{3/2}(ex)^m F_1\left(\frac{m+1}{4};2,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2(m+1)\left(c+dx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 2, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + m)*(c + d*x^4)^(3/2))
```

fricas [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^2d^2x^{16}+2\left(b^2cd+abd^2\right)x^{12}+\left(b^2c^2+4\,abcd+a^2d^2\right)x^8+2\left(abc^2+a^2cd\right)x^4+a^2c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^2*d^2*x^16 + 2*(b^2*c*d + a*b*d^2)*x^12
 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^8 + 2*(a*b*c^2 + a^2*c*d)*x^4 + a^2*c^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(b\,x^4+a\right)^2\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{\left(bx^4+a\right)^2\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^2/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^2*(d*x^4 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{\left(b\,x^4+a\right)^2\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^2*(c + d*x^4)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**2/(d*x**4+c)**(3/2),x)
[Out] Timed out
```

3.852 $\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(c+dx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

```
[Out] (e*x)^(1+m)*AppellF1(1/4+1/4*m,3,3/2,5/4+1/4*m,-b*x^4/a,-d*x^4/c)*(1+d*x^4/
c)^(1/2)/a^3/c/e/(1+m)/(d*x^4+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {511, 510}

$$\frac{\sqrt{\frac{dx^4}{c}+1}\,(ex)^{m+1}F_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(m+1)\sqrt{c+dx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] ((e*x)^(1 + m)*Sqrt[1 + (d*x^4)/c]*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -
((b*x^4)/a), -((d*x^4)/c)])/(a^3*c*e*(1 + m)*Sqrt[c + d*x^4])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(c+dx^4\right)^{3/2}}\,dx &= \frac{\sqrt{1+\frac{dx^4}{c}}\int \frac{(ex)^m}{\left(a+bx^4\right)^3\left(1+\frac{dx^4}{c}\right)^{3/2}}\,dx}{c\sqrt{c+dx^4}}\\
&= \frac{(ex)^{1+m}\sqrt{1+\frac{dx^4}{c}}\,F_1\left(\frac{1+m}{4};3,\frac{3}{2};\frac{5+m}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3ce(1+m)\sqrt{c+dx^4}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 0.92

$$\frac{x\left(\frac{dx^4}{c}+1\right)^{3/2}(ex)^mF_1\left(\frac{m+1}{4};3,\frac{3}{2};\frac{m+5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^3(m+1)\left(c+dx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x]
```

```
[Out] (x*(e*x)^m*(1 + (d*x^4)/c)^(3/2)*AppellF1[(1 + m)/4, 3, 3/2, (5 + m)/4, -((
b*x^4)/a), -((d*x^4)/c)])/(a^3*(1 + m)*(c + d*x^4)^(3/2))
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^4+c}\,(ex)^m}{b^3d^2x^{20}+\left(2\,b^3cd+3\,ab^2d^2\right)x^{16}+\left(b^3c^2+6\,ab^2cd+3\,a^2bd^2\right)x^{12}+\left(3\,ab^2c^2+6\,a^2bcd+a^3d^2\right)x^8+a^{3}\cdots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^4 + c)*(e*x)^m/(b^3*d^2*x^20 + (2*b^3*c*d + 3*a*b^2*d^2)*
x^16 + (b^3*c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^12 + (3*a*b^2*c^2 + 6*a^2*b*
c*d + a^3*d^2)*x^8 + a^3*c^2 + (3*a^2*b*c^2 + 2*a^3*c*d)*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(bx^4+a\right)^3\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(b\,x^4+a\right)^3\left(d\,x^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

```
[Out] int((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(ex)^m}{\left(bx^4+a\right)^3\left(dx^4+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(b*x^4+a)^3/(d*x^4+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^4 + a)^3*(d*x^4 + c)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(e\,x)^m}{\left(b\,x^4+a\right)^3\,\left(d\,x^4+c\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)),x)
[Out] int((e*x)^m/((a + b*x^4)^3*(c + d*x^4)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(b*x**4+a)**3/(d*x**4+c)**(3/2),x)
[Out] Timed out
```

3.853 $\int \frac{x^{17}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\,(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

```
[Out] 1/9*(d*x^6+c)^(3/2)/b/d^2-1/3*a^2*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c
)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/3*(a*d+b*c)*(d*x^6+c)^(1/2)/b^2/d^2
```

Rubi [A] time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}\,(ad+bc)}{3b^2d^2}+\frac{\left(c+dx^6\right)^{3/2}}{9bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^6])/(3*b^2*d^2) + (c + d*x^6)^(3/2)/(9*b*d^2) -
(a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*
c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= \frac{1}{6}\,\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}} + \frac{a^2}{b^2(a+bx)\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{bd}\right)dx, x, x^6\right)$$
$$= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2} + \frac{\left(c+dx^6\right)^{3/2}}{9bd^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2} + \frac{\left(c+dx^6\right)^{3/2}}{9bd^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3b^2d}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^6}}{3b^2d^2} + \frac{\left(c+dx^6\right)^{3/2}}{9bd^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.28, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^6}\left(-3ad-2bc+bdx^6\right)}{9b^2d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-2*b*c - 3*a*d + b*d*x^6))/(9*b^2*d^2) - (a^2*ArcTanh[(Sq
rt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*b^(5/2)*Sqrt[b*c - a*d])
```

fricas [A] time = 0.95, size = 288, normalized size = 2.77

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right) + 2\left(\left(b^3cd-ab^2d^2\right)x^6 - 2\,b^3c^2 - ab^2cd + 3\,a^2bd^2\right)\sqrt{dx^6+c}}{18\left(b^4cd^2-ab^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*
x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^6 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/9*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c
 + a*b*d)/(b*d*x^6 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^6 - 2*b^3*c^2 - a*b^2
*c*d + 3*a^2*b*d^2)*sqrt(d*x^6 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

giac [A] time = 0.17, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,b^2} + \frac{\left(dx^6+c\right)^{\frac{3}{2}}b^2d^4 - 3\,\sqrt{dx^6+c}\,b^2cd^4 - 3\,\sqrt{dx^6+c}\,abd^5}{9\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*a^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/9*((d*x^6 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^6 + c)*b^2*c*d^4 - 3*s
qrt(d*x^6 + c)*a*b*d^5)/(b^3*d^6)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.87, size = 103, normalized size = 0.99

$$\frac{\left(d\,x^6+c\right)^{3/2}}{9\,b\,d^2}-\left(\frac{2\,c}{3\,b\,d^2}+\frac{3\,a\,d^3-3\,b\,c\,d^2}{9\,b^2\,d^4}\right)\,\sqrt{d\,x^6+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(3/2)/(9*b*d^2) - ((2*c)/(3*b*d^2) + (3*a*d^3 - 3*b*c*d^2)/(9*b
^2*d^4))*(c + d*x^6)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b
*c)^(1/2)))/(3*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(a+bx^{6}\right)\sqrt{c+dx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**17/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.854 $\int \frac{x^{11}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

```
[Out] 1/3*a*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a
*d]])/(3*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b}$$
$$= \frac{\sqrt{c+dx^6}}{3bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3bd}$$
$$= \frac{\sqrt{c+dx^6}}{3bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 0.97

$$\frac{1}{3}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^6}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/3
```

fricas [A] time = 0.93, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(b^2c-abd\right)}{6\left(b^3cd-ab^2d^2\right)}, -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^6+c}\,\cdots}{bdx\cdots}\right.}{3\left(b^3cd\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c
)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/3*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^6 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - sqrt(d*x^6 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

giac [A] time = 0.19, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b} - \frac{\sqrt{dx^6+c}}{b}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^6 + c)/b)/d
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.75, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^6+c}}{3\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{3\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b*d) - (a*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)
^(1/2)))/(3*b^(3/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^{6}\right)\sqrt{c+dx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.855 $\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/3*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(3*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] -1/3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

fricas [A] time = 0.82, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)}{6\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^6+c}\,\sqrt{-b^2c+abd}}{bdx^6+bc}\right)}{3\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] [1/6*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b
*x^6 + a))/sqrt(b^2*c - a*b*d), 1/3*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c))/(b^2*c - a*b*d)]
```

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] 1/3*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.72, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^6+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{3\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] atan((b*(c + d*x^6)^(1/2))/(a*b*d - b^2*c)^(1/2))/(3*(a*b*d - b^2*c)^(1/2))
```

sympy [A] time = 23.76, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*b*sqrt((a*d - b*c)/b))
```

3.856 $\int \frac{1}{x(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

```
[Out] -1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/3*arctanh(b^(1/2)*(d*x^6+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/(3*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^6\right)}{6a} - \frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6a}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3ad}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a\sqrt{c}} + \frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a\sqrt{bc-ad}}$$

Mathematica [A] time = 0.13, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^6])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(3*a)
```

fricas [A] time = 0.96, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+\sqrt{c}\,\log\left(\frac{dx^6-2\,\sqrt{dx^6+c}\,\sqrt{c}+2\,c}{x^6}\right)}{6\,ac},\ \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{\sqrt{dx^6+c}\,(bc}{bd}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(c*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + sqrt(c)*log((d*x^6 - 2*sqrt
(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt
(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/(a*c), 1/6*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-
c)/c))/(a*c), 1/3*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^6 + c)*
sqrt(-c)/c))/(a*c)]
```

giac [A] time = 0.18, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\,\sqrt{-b^2c+abd}\,a} + \frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{6}+a\right)\sqrt{dx^{6}+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x), x)
```

mupad [B] time = 5.19, size = 652, normalized size = 7.67

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^6+c}}{\sqrt{c}}\right)}{3\,a\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)1\mathrm{i}}{a^2\,d-a\,b\,c}+\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d}\cdots}{27}\cdots\right.}{\cdots}}{\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d\,x^6+c}}{27}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,a^2\,b^2\,d^3}{9}-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\sqrt{d\,x^6+c}\,\sqrt{b^2\,c-a\,b\,d}}{36\,\left(a^2\,d-a\,b\,c\right)}\right)}{6\,\left(a^2\,d-a\,b\,c\right)}\right)}{a^2\,d-a\,b\,c}-\frac{\sqrt{b^2\,c-a\,b\,d}\left(\frac{2\,b^3\,d^2\,\sqrt{d}\cdots}{27}\cdots\right.}{\cdots}}\right)\cdots}{3\,\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^6)^(1/2)/c^(1/2))/(3*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^
2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a
*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)))*1i)/(a^2*d - a*b*c
) + ((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*
b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c)
))*1i)/(a^2*d - a*b*c))/(((b^2*c - a*b*d)^(1/2)*((2*b^3*d^2*(c + d*x^6)^(1/
2))/27 - ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 - ((8*a^3*b^2*d^3 - 16*a
^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(36*(a^2*d - a*b*c))
))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c) - ((b^2*c - a*b*d)^(1/2)*((2*b^3*d
^2*(c + d*x^6)^(1/2))/27 + ((b^2*c - a*b*d)^(1/2)*((2*a^2*b^2*d^3)/9 + ((8*
a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^6)^(1/2)*(b^2*c - a*b*d)^(1/2))/(3
6*(a^2*d - a*b*c))))/(6*(a^2*d - a*b*c))))/(a^2*d - a*b*c)))*(b^2*c - a*b*d
)^(1/2)*1i)/(3*(a^2*d - a*b*c))
```

sympy [A] time = 26.11, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{\frac{ad-bc}{b}}}\right)}{3a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^6}}{\sqrt{-c}}\right)}{3a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**6)/sqrt((a*d - b*c)/b))/(3*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**6)/sqrt(-c))/(3*a*sqrt(-c))
```

3.857 $\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

```
[Out] 1/6*(a*d+2*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/3*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/6*(d
*x^6+c)^(1/2)/a/c/x^6
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*a*c*x^6) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[
c]])/(6*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c
- a*d]])/(3*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx=\frac{1}{6}\operatorname{Subst}\left(\int\frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)$$

$$=-\frac{\sqrt{c+dx^6}}{6acx^6}-\frac{\operatorname{Subst}\left(\int\frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6ac}$$

$$=-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{b^2\operatorname{Subst}\left(\int\frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{6a^2}-\frac{(2bc+ad)\operatorname{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\right.}{12a^2c}$$

$$=-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{b^2\operatorname{Subst}\left(\int\frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{3a^2d}-\frac{(2bc+ad)\operatorname{Subst}\left(\int\frac{}{-\frac{c}{d}}\right.}{6a^2c}$$

$$=-\frac{\sqrt{c+dx^6}}{6acx^6}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.18, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{3a^2(ad-bc)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6ac^{3/2}}-\frac{\sqrt{c+dx^6}}{6acx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/6*Sqrt[c + d*x^6]/(a*c*x^6) + (b*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(3*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/(6*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(3*a^2*
(-(b*c) + a*d))
```

fricas [A] time = 1.15, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^2x^6\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right)+(2\,bc+ad)\sqrt{c}\,x^6\log\left(\frac{dx^6+2\,\sqrt{dx^6+c}\,\sqrt{c}+2\,c}{x^6}\right)-2\,\sqrt{dx^6+}}{12\,a^2c^2x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(
d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + (2*b*c + a*d)*sq
rt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*sqrt(d*x^6
 + c)*a*c)/(a^2*c^2*x^6), -1/12*(4*b*c^2*x^6*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^6*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*sq
```

```
rt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), 1/6*(b*c^2*x^6*sqrt(b/(b*c - a*d))*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^6 + a)) - (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c)
 - sqrt(d*x^6 + c)*a*c)/(a^2*c^2*x^6), -1/6*(2*b*c^2*x^6*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^6*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + sqrt(d*
x^6 + c)*a*c)/(a^2*c^2*x^6)]
```

giac [A] time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{3\sqrt{-b^2c+abd}\,a^2}-\frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{6\,a^2\sqrt{-c}\,c}-\frac{\sqrt{dx^6+c}}{6\,acx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*b^2*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/6*(2*b*c + a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/6*sqrt(d*x^6 + c)/(a*c*x^6)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^7), x)
```

mupad [B] time = 5.62, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{6\,a^3\,d-6\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^6+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-b^6\,c}{6\,\left(a^3\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] (log((c + d*x^6)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(6*a^3*d - 6*a^2*b*c) - (log((c + d*x^6
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(6*(a^3*d - a^2*b*c)) - (c + d*x^6)^(1/2)/(6*a*c*x^6)
- (atan((b^4*d^4*(c + d*x^6)^(1/2)*1i)/(18*(c^3)^(1/2)*((b^4*d^4)/(18*c) +
(5*a*b^3*d^5)/(108*c^2) + (a^2*b^2*d^6)/(108*c^3))) + (b^2*d^6*(c + d*x^6)^
```

```
(1/2)*1i)/(108*(c^3)^(1/2)*((5*b^3*d^5)/(108*a) + (b^2*d^6)/(108*c) + (b^4*
c*d^4)/(18*a^2))) + (b^3*d^5*(c + d*x^6)^(1/2)*5i)/(108*(c^3)^(1/2)*((b^4*d
^4)/(18*a) + (5*b^3*d^5)/(108*c) + (a*b^2*d^6)/(108*c^2))))*(a*d + 2*b*c)*1
i)/(6*a^2*(c^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^7 \left(a+bx^6\right) \sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**7*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.858 $\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

```
[Out] -1/6*(2*a*d+b*c)*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(3/2)+1/3*a^(3/
2)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/6*x^3*(d*x^6+c)^(1/2)/b/d
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}+\frac{x^3\sqrt{c+dx^6}}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6])/(6*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt
[a]*Sqrt[c + d*x^6])])/(3*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^3)/Sqrt[c + d*x^6]])/(6*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} - \frac{\operatorname{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6bd}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\right.}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\right.}{6b^2d}$$

$$= \frac{x^3\sqrt{c+dx^6}}{6bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{6b^2d^{3/2}}$$

Mathematica [A] time = 0.24, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{d^{3/2}} + \frac{bx^3\sqrt{c+dx^6}}{d}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ((b*x^3*Sqrt[c + d*x^6])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[
a]*Sqrt[c + d*x^6])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^3 + Sqrt[d]*
Sqrt[c + d*x^6]])/d^(3/2))/(6*b^2)
```

fricas [A] time = 1.00, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^6+c}\,bdx^3 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left(\left(b^2c^2-3\,abcd+2\,a^2d^2\right)x^9-\left(abc^2-a^2cd\right)x^3\right.}{b^2x^{12}+2\,abx^6+a^2}\right.}{12\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + (b*c + 2*a*d
)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2), 1/1
2*(2*sqrt(d*x^6 + c)*b*d*x^3 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8
*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((
b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6
+ c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*(b*c + 2*a*d)*
sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2), 1/12*(2*sqrt(d*x^
6 + c)*b*d*x^3 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6
 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + (b*c + 2
*a*d)*sqrt(d)*log(-2*d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b^2*d^2),
 1/6*(sqrt(d*x^6 + c)*b*d*x^3 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c
 - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3
)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqrt(d*x^6 + c)))/(b^2*d^2)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^14/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^14/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**14/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.859 $\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

```
[Out] 1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b/d^(1/2)-1/3*arctan(x^3*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b}\\
&= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{\sqrt{bc-ad}}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/Sqrt[b
*c - a*d]) + Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]]/Sqrt[d])/(3*b)
```

fricas [A] time = 1.08, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8abcd+8a^2d^2)x^{12}-2(3abc^2-4a^2cd)x^6+a^2c^2-4((b^2c^2-3abcd+2a^2d^2)x^9-(abc^2-a^2cd)x^3)\sqrt{dx^6+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{12}+2abx^6+a^2}\right)+2}{12bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(
b^2*x^12 + 2*a*b*x^6 + a^2)) + 2*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*s
qrt(d)*x^3 - c))/(b*d), 1/12*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*
c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c
^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*
sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*sqrt(-d)*arctan(sqr
t(-d)*x^3/sqrt(d*x^6 + c)))/(b*d), 1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*(
(b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c
*x^3)) + sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c))/(b*d),
1/6*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6
```

```
 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) - 2*sqrt(-d)*arctan(sqrt(-d)
*x^3/sqrt(d*x^6 + c)))/(b*d)]
```

giac [B] time = 0.23, size = 156, normalized size = 1.71

$$-\frac{\left(a\sqrt{-d}\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right)-\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)\right)\mathrm{sgn}(x)}{3\sqrt{abc-a^2d}\,b\sqrt{-d}}+\frac{a\arctan\left(\frac{a\sqrt{d+\frac{c}{x^6}}}{\sqrt{abc-a^2d}}\right)}{3\sqrt{abc-a^2d}\,b\mathrm{sgn}(x)}-\frac{\arctan\left(\frac{\sqrt{d+\frac{c}{x^6}}}{\sqrt{-d}}\right)}{3\,b\sqrt{-d}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(a*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - sqrt(a*b*c - a^2*d
)*arctan(sqrt(d)/sqrt(-d)))*sgn(x)/(sqrt(a*b*c - a^2*d)*b*sqrt(-d)) + 1/3*a
*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/(sqrt(a*b*c - a^2*d)*b*sgn(x
)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/(b*sqrt(-d)*sgn(x))
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^8}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^8/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**8/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.860 $\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/3*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])]/(3*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+bx^6)\sqrt{c+dx^6}} dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}} dx, x, x^3\right) \\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2} dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 95, normalized size = 1.76

$$\frac{x^3\sqrt{\frac{dx^6}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{3a\sqrt{c+dx^6}\sqrt{\frac{dx^6}{c}-\frac{bx^6}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^6)/c]*ArcTanh[Sqrt[-((b*x^6)/a) + (d*x^6)/c]/Sqrt[1 + (d
*x^6)/c]])/(3*a*Sqrt[c + d*x^6]*Sqrt[-((b*x^6)/a) + (d*x^6)/c])
```

fricas [B] time = 0.97, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)}{12\left(abc-a^2d\right)},\ \frac{\arctan\left(\frac{((bc-2\,ad)x\ldots}{2\left((abcd-\ldots\right.}\right.}{6\sqrt{a\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sq
rt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))/(a*b*c -
a^2*d), 1/6*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c
 - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))/sqrt(a*b*c -
 a^2*d)]
```

giac [A] time = 0.25, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x^3-\sqrt{dx^6+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{3\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x^2/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.861 $\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

```
[Out] -1/3*b*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/3*(d*x^6+c)^(1/2)/a/c/x^3
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^6}}{3acx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(3*a*c*x^3) - (b*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqr
t[c + d*x^6])])/(3*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^4(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{\operatorname{Subst}\left(\int \frac{bc}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3ac}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a}$$
$$= -\frac{\sqrt{c+dx^6}}{3acx^3} - \frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{3/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 1.17, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(\frac{4x^6(c+dx^6)(bc-ad)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^6}{c(bx^6+a)}\right)}{3c^2(a+bx^6)}+\frac{(c+2dx^6)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c(a+bx^6)}}\right)}{c\sqrt{\frac{ax^6(c+dx^6)(bc-ad)}{c^2(a+bx^6)^2}}}\right)}{3x^3(a+bx^6)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/3*((1 + (d*x^6)/c)*(((c + 2*d*x^6)*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a +
 b*x^6))]])/(c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)^2)]) +
 (4*(b*c - a*d)*x^6*(c + d*x^6)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^6)/(c*(a + b*x^6))])/(3*c^2*(a + b*x^6))))/(x^3*(a + b*x^6)*Sqrt[c + d*x^6
])
```

fricas [B] time = 1.08, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\,bcx^3\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)+4\sqrt{dx^6\cdots}}{12\left(a^2bc^2-a^3cd\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(sqrt(-a*b*c + a^2*d)*b*c*x^3*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*
c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2))
+ 4*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3), -1/6*(sqr
t(a*b*c - a^2*d)*b*c*x^3*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 +
c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3))
 + 2*sqrt(d*x^6 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{6}+a\right)\sqrt{dx^{6}+c}\,x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{4}\,\left(b\,x^{6}+a\right)\,\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\left(a+bx^{6}\right)\sqrt{c+dx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.862 $\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}\,(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

```
[Out] 1/3*b^2*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/9*(d*x^6+c)^(1/2)/a/c/x^9+1/9*(2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^
2/c^2/x^3
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^6}\,(2ad+3bc)}{9a^2c^2x^3}-\frac{\sqrt{c+dx^6}}{9acx^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(9*a*c*x^9) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^6])/(9*a^2*c^2
*x^3) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(3*a^
(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{10}\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{\text{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9ac}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}-\frac{\text{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{9a^2c^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3a^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3a^2}\\
&= -\frac{\sqrt{c+dx^6}}{9acx^9}+\frac{(3bc+2ad)\sqrt{c+dx^6}}{9a^2c^2x^3}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\sqrt{c+dx^6}}\right)}{3a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 4.65, size = 253, normalized size = 2.20

$$-\frac{\left(\frac{dx^6}{c}+1\right)\left(-\frac{8x^6\left(c+dx^6\right)^2(bc-ad)\,{}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)}{a+bx^6}+\frac{3c\left(c^2-4cdx^6-8d^2x^{12}\right)\sin^{-1}\left(\sqrt{\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}}\right)}{\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}}+\frac{24dx^{12}\left(c+dx^6\right)(ad-bc)\,{}_2F_1\left(2,2;\frac{5}{2};\frac{(bc\cdots}{c(b\cdots}\right)}{a+bx^6}\cdots}{27c^3x^9\left(a+bx^6\right)\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/27*((1 + (d*x^6)/c)*((3*c*(c^2 - 4*c*d*x^6 - 8*d^2*x^12)*ArcSin[Sqrt[((b
*c - a*d)*x^6)/(c*(a + b*x^6))]])/Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2
*(a + b*x^6)^2)] + (24*d*(-(b*c) + a*d)*x^12*(c + d*x^6)*Hypergeometric2F1[
2, 2, 5/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))])/(a + b*x^6) - (8*(b*c - a*d)
*x^6*(c + d*x^6)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^6)
/(c*(a + b*x^6))])/(a + b*x^6)))/(c^3*x^9*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [A] time = 1.06, size = 416, normalized size = 3.62

$$\left[-\frac{3\sqrt{-abc+a^2d}\,b^2c^2x^9\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2-4\left((bc-2\,ad)x^9-acx^3\right)\sqrt{dx^6+c}\,\sqrt{-abc+a^2d}}{b^2x^{12}+2\,abx^6+a^2}\right)-4\left((3\right.}{36\left(a^3bc^3-a^4c^2d\right)x^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/36*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^9*log(((b^2*c^2 - 8*a*b*c*d + 8*a^
2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^
9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 +
a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)
*sqrt(d*x^6 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^9), 1/18*(3*sqrt(a*b*c - a^2*d
)*b^2*c^2*x^9*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b
*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*
b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^6 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^6 + c)
)/((a^3*b*c^3 - a^4*c^2*d)*x^9)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{10}\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^10*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10} \left(a + b x^{6}\right) \sqrt{c + d x^{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**10*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.863 $\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,1,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x
^6+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 1, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a*Sqrt[c + d*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx &= \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{(a+bx^6)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}\\ &= \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};1,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{5a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^6)/c]*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6
)/a)])/(5*a*Sqrt[c + d*x^6])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^6+a\right)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx^6)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.864 $\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,1,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi **[A]** time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 1, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{x}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= \frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};1,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a\sqrt{c+dx^6}}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.02

$$\frac{x^4\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{4a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^4*Sqrt[(c + d*x^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)
/a)])/(4*a*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.865 $\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,1,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^
6+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 430, 429}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a*Sqrt[c + d*x^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{(a+bx^6)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{(a+bx^3)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};1,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a\sqrt{c+dx^6}}$$

Mathematica [A] time = 0.04, size = 65, normalized size = 1.02

$$\frac{x^2\sqrt{\frac{c+dx^6}{c}}\,F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{2a\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (x^2*Sqrt[(c + d*x^6)/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)
/a)])/(2*a*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^6+a)\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(x/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^6+a)\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
[Out] int(x/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^{6}\right)\sqrt{c+dx^{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(x/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.866 $\int \frac{1}{(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,1,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/(d*x^6+c)^(
1/2)
```

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 1, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a*Sqrt[c + d*x^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}\\ &= \frac{x\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{6};1,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] time = 0.20, size = 161, normalized size = 2.73

$$-\frac{7acxF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)}{\left(a+bx^6\right)\sqrt{c+dx^6}\left(3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)-7acF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};-\frac{dx^6}{c}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-7*a*c*x*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)])/((a + b*x
^6)*Sqrt[c + d*x^6]*(-7*a*c*AppellF1[1/6, 1/2, 1, 7/6, -((d*x^6)/c), -((b*x
^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/
a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6)/c), -((b*x^6)/a)])))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   Not i
ntegrable (provided residues have no relations)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6 + a\right)\sqrt{dx^6 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)\sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6 + a\right)\sqrt{dx^6 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.867 $\int \frac{1}{x^2(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,1,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x/(d*x^6+c)
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 1, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a*x*Sqrt[c + d*x^6]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx &= \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}\\ &= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{6};1,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{ax\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] time = 0.13, size = 141, normalized size = 2.27

$$\frac{-11x^6\sqrt{\frac{dx^6}{c}+1}\,(bc-2ad)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+10bdx^{12}\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-55a\left(c+\right.}{55a^2cx\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-55*a*(c + d*x^6) - 11*(b*c - 2*a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6,
 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*x^12*Sqrt[1 + (d*x^6)/c
]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*x^6)/a)])/(55*a^2*c*x*Sqr
t[c + d*x^6])
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^6+c}}{bdx^{14}+(bc+ad)x^8+acx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)/(b*d*x^14 + (b*c + a*d)*x^8 + a*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x^2\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.868 $\int \frac{1}{x^3(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,1,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^2/(d*
x^6+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 1, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a*x^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3 \left(a+bx^6\right) \sqrt{c+dx^6}}\, dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2 \left(a+bx^3\right) \sqrt{c+dx^3}}\, dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\, dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\, F_1\left(-\frac{1}{3};1,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2ax^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.12, size = 141, normalized size = 2.20

$$\frac{5x^6\sqrt{\frac{dx^6}{c}+1}\,(ad-2bc)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+2bdx^{12}\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-20a\left(c+dx^6\right)}{40a^2cx^2\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] (-20*a*(c + d*x^6) + 5*(-2*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3,
 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*x^12*Sqrt[1 + (d*x^6)/c]*
AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*x^6)/a)])/(40*a^2*c*x^2*Sqrt[
c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6 + a\right)\sqrt{dx^6 + c}\, x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \left(b\, x^6 + a\right) \sqrt{d\, x^6 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^6\right) \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.869 $\int \frac{1}{x^5(a+bx^6)\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,1,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a/x^4/(d*
x^6+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 1, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a*x^4*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};1,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4ax^4\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.16, size = 141, normalized size = 2.20

$$\frac{-4x^6\sqrt{\frac{dx^6}{c}+1}\,(ad+4bc)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-bdx^{12}\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-8a\left(c+dx^6\right)}{32a^2cx^4\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^6)*Sqrt[c + d*x^6]),x]
[Out] (-8*a*(c + d*x^6) - 4*(4*b*c + a*d)*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 1
/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] - b*d*x^12*Sqrt[1 + (d*x^6)/c]*Appe
llF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6)/a)])/(32*a^2*c*x^4*Sqrt[c +
d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)\sqrt{d\,x^6+c}\;x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
[Out] int(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)\sqrt{dx^6+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)*sqrt(d*x^6 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5\,\left(b\,x^6+a\right)\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^6)*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5\left(a+bx^6\right)\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**6+a)/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**6)*sqrt(c + d*x**6)), x)
```

3.870 $\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

```
[Out] 1/6*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/3*(d*x^6+c)^(1/2)/b^2/d-1/6*a^2*(d*x^6+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^6}}{6b^2\left(a+bx^6\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^6}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] Sqrt[c + d*x^6]/(3*b^2*d) - (a^2*Sqrt[c + d*x^6])/(6*b^2*(b*c - a*d)*(a + b
*x^6)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*
d]])/(6*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{17}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= -\frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6b^2(bc-ad)}$$
$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, \ldots\right.}{12b^2(bc-ad)}$$
$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} - \frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c\ldots}\right.}{6b^2d(bc-ad)}$$
$$= \frac{\sqrt{c+dx^6}}{3b^2d} - \frac{a^2\sqrt{c+dx^6}}{6b^2(bc-ad)\left(a+bx^6\right)} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{5/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.36, size = 107, normalized size = 0.87

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}\left(\frac{a^2}{\left(a+bx^6\right)(ad-bc)}+\frac{2}{d}\right)}{b^2} + \frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^17/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((Sqrt[c + d*x^6]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^6))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/6
```

fricas [B] time = 1.02, size = 475, normalized size = 3.86

$$\left[\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^6+4\,a^2bcd-3\,a^3d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\left(2\left(b^4c^2-2\,ab^3c\ldots\right.\right.}{12\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b
*d))/(b*x^6 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6), -
1/6*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^6 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^6 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^6)]
```

giac [A] time = 0.44, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^6+c}\,a^2d}{6\left(b^3c-ab^2d\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^6+c}}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(d*x^6 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^6 + c)*b - b*c + a*d))
- 1/6*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/3*sqrt(d*x^6 + c)/(b^2*d)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x^{17}}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^17/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.09, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^6+c}}{3\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^6+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{6\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b^3\,\left(d\,x^6+c\right)-3\,b^3\,c+3\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^17/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (c + d*x^6)^(1/2)/(3*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^6)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(6*b^(5/
```

```
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b^3*(c
+ d*x^6) - 3*b^3*c + 3*a*b^2*d))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**17/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

3.871 $\int \frac{x^{11}}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}} \, dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/6*a*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(6*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{11}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^6\right)$$

$$= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^6\right)}{12b(bc-ad)}$$

$$= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{(2bc-ad)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^6}\right)}{6bd(bc-ad)}$$

$$= \frac{a\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.10, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{6b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*b^(
3/2))
```

fricas [A] time = 1.02, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^6+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)}{12\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^6\right)},\ \frac{((2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((2*b^2*c - a*b*d)*x^6 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a))
+ 2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b
^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^6), 1/6*(((2*b^2*c - a*b*d
)*x^6 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^6 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d))/(a*b
^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2
)*x^6)]
```

giac [A] time = 0.29, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^6+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^6+c\right)b-bc+ad\right)} + \frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x^6 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^6 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{6}+a\right)^{2} \sqrt{d\,x^{6}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.99, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{6\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^6+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(6*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^6)^(1/2))/(2*b*(a*d - b*c)*(3*b*(c +
 d*x^6) + 3*a*d - 3*b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

3.872 $\int \frac{x^5}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=87

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

```
[Out] 1/6*d*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/6*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -Sqrt[c + d*x^6]/(6*(b*c - a*d)*(a + b*x^6)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[b*c - a*d]])/(6*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{12(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{6(bc-ad)}$$
$$= -\frac{\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6\sqrt{b}\,(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{6}\left(\frac{\sqrt{c+dx^6}}{\left(a+bx^6\right)(ad-bc)} + \frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]/((-(b*c) + a*d)*(a + b*x^6)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^6])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/6
```

fricas [B] time = 0.74, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^6+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^6+2\,bc-ad-2\,\sqrt{dx^6+c}\,\sqrt{b^2c-abd}}{bx^6+a}\right)+2\,\sqrt{dx^6+c}\left(b^2c-abd\right)}{12\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^6+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)}, -\frac{\left(bdx^6+ad\right)\sqrt{-b^2c+abd}}{6\left(\left(b^4c^2-2\,ab^3c\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*((b*d*x^6 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^6 + 2*b*c - a*d - 2*
sqrt(d*x^6 + c)*sqrt(b^2*c - a*b*d))/(b*x^6 + a)) + 2*sqrt(d*x^6 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/6*((b*d*x^6 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^6 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^6 + b*c)) + sqrt(d*x^6 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^6 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

giac [A] time = 0.30, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\sqrt{-b^2c+abd}\,(bc-ad)} - \frac{\sqrt{dx^6+c}\,d}{6\left(\left(dx^6+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/6*sqrt(d*x^6 + c)*d/(((d*x^6 + c)*b - b*c + a*d)*(b*c - a
*d))
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.93, size = 84, normalized size = 0.97

$$\frac{d\,\sqrt{d\,x^6+c}}{2\,\left(a\,d-b\,c\right)\,\left(3\,b\,\left(d\,x^6+c\right)+3\,a\,d-3\,b\,c\right)}+\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^6+c}}{\sqrt{a\,d-b\,c}}\right)}{6\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (d*(c + d*x^6)^(1/2))/(2*(a*d - b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c)) + (
d*atan((b^(1/2)*(c + d*x^6)^(1/2))/(a*d - b*c)^(1/2)))/(6*b^(1/2)*(a*d - b*
c)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

3.873 $\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

```
[Out] 1/6*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/3*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/6
*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.13, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}}+\frac{b\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) - ArcTanh[Sqrt[c + d*x^6]
/Sqrt[c]]/(3*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^6\right)}{6a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^6\right)}{6a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int \right.}{12a^2(bc\ }$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^6}\right)}{3a^2d} - \frac{(b(2bc-3ad))\,\text{Sul}}{}$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{3a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right.}{6a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.35, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) - (2*ArcTanh[Sqrt[c + d*x^
6]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^6])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(6*a^2)
```

fricas [A] time = 1.14, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^6+c}\,abc + \left(\left(2\,b^2c^2 - 3\,abcd\right)x^6 + 2\,abc^2 - 3\,a^2cd\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^6+2\,bc-ad+2\,\sqrt{dx^6+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^6+a}\right) + 2\left(\left(b\right.\right.}{12\left(\left(a^2b^2c^2 - a^3bcd\right)x^6 + a^3bc^2 - a^4cd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(2*sqrt(d*x^6 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d + 2*sqrt(d*x^6 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + 2*((b^2*c - a*b*d)*x^6
```

```
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
6 + b*c)) + ((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(c)*log((d*x^6 - 2*sq
rt(d*x^6 + c)*sqrt(c) + 2*c)/x^6))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c
^2 - a^4*c*d), 1/12*(2*sqrt(d*x^6 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^6 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b
*c - a*d + 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d), 1/6*(sqrt(d*x^6 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^6 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6
 + b*c)) + 2*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^6 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^6 + a^3*b*c^2 - a^4*c*d)]
```

giac [A] time = 0.41, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^6+c}\,bd}{6\left(abc-a^2d\right)\left(\left(dx^6+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^6+c}\,b}{\sqrt{-b^2c+abd}}\right)}{6\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^6+c}}{\sqrt{-c}}\right)}{3\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(d*x^6 + c)*b*d/((a*b*c - a^2*d)*((d*x^6 + c)*b - b*c + a*d)) - 1/6
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/3*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x), x)
```

mupad [B] time = 6.23, size = 3025, normalized size = 22.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b^2*d^5)/3 - 2*a
^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
 - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144*a^7*b^2
*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(216
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*
d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*
(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a
^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*
b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*a^6*b^2*d^5)
/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^
4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(144
*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^
3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a
^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(
12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b
*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d) - ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2
 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - ((((4*a^6*b
^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^2*c^2
 - 2*a^4*b*c*d) - ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/
2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4
*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2
*d^2 - 20*a*b^4*c*d^3))/(18*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + ((((4*
a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(a^5*d^2 + a^3*b^
2*c^2 - 2*a^4*b*c*d) + ((c + d*x^6)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3
)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^
5*b^4*c^2*d^3))/(216*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b
^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)
^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*
(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(12*(a^5*d^3 - a^2*b^3*c^3 + 3*a^
3*b^2*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i
)/(6*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((
(((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2
+ a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a
^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)
*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - ((c + d*x^6)^(1/
2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b
^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - (((((4*a^6*b^2*d^5)/3 - 2*a^5*b
^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))
 + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^
3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^
2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(
a^2*c^(1/2)))/(((a*b^3*d^4)/18 - (b^4*c*d^3)/27)/(a^5*d^2 + a^3*b^2*c^2 - 2
*a^4*b*c*d) + ((((4*a^6*b^2*d^5)/3 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/
3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^6)^(1/2)*(144*a^7*
b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(
648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) - (
(c + d*x^6)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + ((((4*a^6*b^2*d^5)/3
 - 2*a^5*b^3*c*d^4 + (2*a^4*b^4*c^2*d^3)/3)/(6*(a^5*d^2 + a^3*b^2*c^2 - 2*a
^4*b*c*d)) + ((c + d*x^6)^(1/2)*(144*a^7*b^2*d^5 - 576*a^6*b^3*c*d^4 - 288*
```

```
a^4*b^5*c^3*d^2 + 720*a^5*b^4*c^2*d^3))/(648*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2
*c^2 - 2*a^3*b*c*d)))/(6*a^2*c^(1/2)) + ((c + d*x^6)^(1/2)*(13*a^2*b^3*d^4
+ 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(108*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(a^2*c^(1/2))))*1i)/(3*a^2*c^(1/2)) - (b*d*(c + d*x^6)^(1/2))/(2*(a^2*
d - a*b*c)*(3*b*(c + d*x^6) + 3*a*d - 3*b*c))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.874 $\int \frac{1}{x^7(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}\,(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

```
[Out] 1/6*(a*d+4*b*c)*arctanh((d*x^6+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/6*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^6+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/6*b*(-a*d+2*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^6+a)-1/6*
(d*x^6+c)^(1/2)/a/c/x^6/(b*x^6+a)
```

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{6a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}}-\frac{b\sqrt{c+dx^6}\,(2bc-ad)}{6a^2c\left(a+bx^6\right)(bc-ad)}-\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*(a + b*x^6)) - Sqrt
[c + d*x^6]/(6*a*c*x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6
]/Sqrt[c]])/(6*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^6])/Sqrt[b*c - a*d]])/(6*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)$$

$$= -\frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^6\right)}{6ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{6a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c\cdot}}\right.}{12a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{6a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{c+dx^6}}{6acx^6\left(a+bx^6\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{6a^3c^{3/2}} - \frac{b^{3/2}}{}$$

Mathematica [A] time = 0.68, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^6}\left(a^2d+ab\left(dx^6-c\right)-2b^2cx^6\right)}{x^6\left(a+bx^6\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^6}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^6}}{\sqrt{c}}\right)}{\sqrt{c}}}{6a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*Sqrt[c + d*x^6]*(a^2*d - 2*b^2*c*x^6 + a*b*(-c + d*x^6)))/((b*c - a*d)*
x^6*(a + b*x^6)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^6]/Sqrt[c]])/Sqrt[c]
```

```
+ (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^6])/Sqrt[b*c -
a*d]])/(b*c - a*d)^(3/2))/(6*a^3*c)
```

fricas [A] time = 1.36, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(c)*log((d*x^6
 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12
+ (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^6 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sq
rt(c)*log((d*x^6 + 2*sqrt(d*x^6 + c)*sqrt(c) + 2*c)/x^6) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/12*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt
(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(b/(b*c - a*d))*log((b*d*x^6 + 2*
b*c - a*d - 2*sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^6 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6), -1/6*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^12 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^6)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^6 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^6 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^12 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^6)*sqrt(-c)*arctan(sqrt(d*x^6 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^6 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^6 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^12 + (a^4*b*c^3 - a^5*c^2*d)*x^6)]
```

giac [A] time = 0.41, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^{3}c-5\,ab^{2}d\right)\arctan\left(\frac{\sqrt{dx^{6}+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{6\,\left(a^{3}bc-a^{4}d\right)\sqrt{-b^{2}c+abd}}-\frac{2\,\left(dx^{6}+c\right)^{\frac{3}{2}}b^{2}cd-2\,\sqrt{dx^{6}+c}\,b^{2}c^{2}d-\left(dx^{6}+c\right)^{\frac{3}{2}}abd^{2}+2\,\sqrt{dx^{6}+c}\,abc}{6\,\left(a^{2}bc^{2}-a^{3}cd\right)\left(\left(dx^{6}+c\right)^{2}b-2\,\left(dx^{6}+c\right)bc+bc^{2}+\left(dx^{6}+c\right)a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^6 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/6*(2*(d*x^6 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^6 + c)*b^2*c^2*d - (d*x^6 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^6 + c
)*a*b*c*d^2 - sqrt(d*x^6 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^6 + c)^
2*b - 2*(d*x^6 + c)*b*c + b*c^2 + (d*x^6 + c)*a*d - a*c*d)) - 1/6*(4*b*c +
a*d)*arctan(sqrt(d*x^6 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

maple [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}\;x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^7), x)
```

mupad [B] time = 7.29, size = 3860, normalized size = 20.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] (((c + d*x^6)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^6)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^6)*(3*a*d - 6*b*c) + 3*b*(c + d*x^6)^2 + 3*b*c^2 - 3*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576
*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2
*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b
*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*
a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3
 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(12*(a^6*d^3 - a^3*b^3*c^3 +
3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*
b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 +
 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*
a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b^2*c
*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(21
6*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2
)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*
d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a^6*c
^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*
c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1
i)/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/((5*a^3*
b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(108*(a^6*b^
2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d
- 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d
^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2
- 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*((144*a^9*b
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^
(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*
c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3
*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(1
```

```
8*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2
)*(5*a*d - 4*b*c)*((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c
^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3
*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^6)^(1/2)*(5*a*d - 4*b*c)*(288*a
^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^
2*d^5))/(216*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(12*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2))))*(-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(6*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan((((((c + d
*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*
d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d))
 + (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a
^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d
*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*
a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 +
 a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d +
 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b
^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*
(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a
^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c
^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*
a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c
^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))
*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(12*a^3*(c^3)^(1/2)
))/((5*a^3*b^4*d^6 + 32*b^7*c^3*d^3 - 48*a*b^6*c^2*d^4 + 6*a^2*b^5*c*d^5)/(
108*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((((c + d*x^6)^(1/2)*(a^
4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^
5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + (((144*a^9*b
^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 576*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)
/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)) - ((c + d*x^6)^(1/2)*(a*
d + 4*b*c)*(288*a^6*b^5*c^5*d^2 - 720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4
 - 144*a^9*b^2*c^2*d^5))/(216*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 -
2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^
3*(c^3)^(1/2)) + ((((c + d*x^6)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*
b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(18*(a^4*b^2*c^4 + a^6
*c^2*d^2 - 2*a^5*b*c^3*d)) - (((144*a^9*b^2*c*d^6 + 288*a^6*b^5*c^4*d^3 - 5
76*a^7*b^4*c^3*d^4 + 144*a^8*b^3*c^2*d^5)/(216*(a^6*b^2*c^4 + a^8*c^2*d^2 -
 2*a^7*b*c^3*d)) + ((c + d*x^6)^(1/2)*(a*d + 4*b*c)*(288*a^6*b^5*c^5*d^2 -
720*a^7*b^4*c^4*d^3 + 576*a^8*b^3*c^3*d^4 - 144*a^9*b^2*c^2*d^5))/(216*a^3*
(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(1
2*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(12*a^3*(c^3)^(1/2))))*(a*d + 4*b*c)*1i)
/(6*a^3*(c^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Timed out
```

3.875 $\int \frac{x^{14}}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

```
[Out] -1/6*(-2*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/3*arctanh(x^3*d^(1/2)/(d*x^6+c)^(1/2))/b^2/d^(
1/2)+1/6*a*x^3*(d*x^6+c)^(1/2)/b/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}(3bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}}+\frac{ax^3\sqrt{c+dx^6}}{6b\left(a+bx^6\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a*x^3*Sqrt[c + d*x^6])/(6*b*(b*c - a*d)*(a + b*x^6)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^3)/Sqrt[c + d*x^6]]/(3*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{14}}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^3\right)$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^3\right)}{6b(bc-ad)}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^3\right)}{3b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int \frac{\ }{(a}\right.}{6b^2(bc-}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^3}{\sqrt{c+dx^6}}\right)}{3b^2} - \frac{(a(3bc-2ad))\operatorname{Subst}\left(\int\right.}{6b^2(}$$

$$= \frac{ax^3\sqrt{c+dx^6}}{6b(bc-ad)\left(a+bx^6\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^3}{\sqrt{c+dx^6}}\right)}{3b^2\sqrt{d}}$$

Mathematica [A] time = 0.28, size = 135, normalized size = 0.96

$$\frac{\frac{abx^3\sqrt{c+dx^6}}{\left(a+bx^6\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^6}+dx^3\right)}{\sqrt{d}}}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] ((a*b*x^3*Sqrt[c + d*x^6])/((b*c - a*d)*(a + b*x^6)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^3 + Sqrt[d]*Sqrt[c + d*x^6]])/Sqrt[d])/(6*b^2)
```

fricas [A] time = 2.01, size = 1077, normalized size = 7.64

$$\left[\frac{4\sqrt{dx^6+c}\,abdx^3 + 4\left(\left(b^2c-abd\right)x^6+abc-a^2d\right)\sqrt{d}\,\log\left(-2\,dx^6-2\sqrt{dx^6+c}\,\sqrt{d}\,x^3-c\right) + \left(\left(3\,b^2cd-2\,abd\right.\right.}{24\left(\left(b^4cd-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3 + 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*b*c^2 - a^2*c*d)*x^3)*sqrt(d*
x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((b^4*c*d - a
*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/24*(4*sqrt(d*x^6 + c)*a*b*d*x^3
 - 8*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^3/sqr
t(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(
-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2
- 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^9 - (a*
b*c^2 - a^2*c*d)*x^3)*sqrt(d*x^6 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^12 + 2*a
*b*x^6 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12
*(2*sqrt(d*x^6 + c)*a*b*d*x^3 + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d -
2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)) + 2*((b^2*c - a*b*d)*x^6
 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^6 - 2*sqrt(d*x^6 + c)*sqrt(d)*x^3 - c)
)/((b^4*c*d - a*b^3*d^2)*x^6 + a*b^3*c*d - a^2*b^2*d^2), 1/12*(2*sqrt(d*x^6
 + c)*a*b*d*x^3 - 4*((b^2*c - a*b*d)*x^6 + a*b*c - a^2*d)*sqrt(-d)*arctan(s
qrt(-d)*x^3/sqrt(d*x^6 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^6 + 3*a*b*c*d - 2
*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*
x^6 + c)*sqrt(a/(b*c - a*d))/(a*d*x^9 + a*c*x^3)))/((b^4*c*d - a*b^3*d^2)*x
^6 + a*b^3*c*d - a^2*b^2*d^2)]
```

giac [B] time = 0.51, size = 343, normalized size = 2.43

$$-\frac{\left(3\,abc\sqrt{-d}\,\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,a^2\sqrt{-d}\,d\arctan\left(\frac{a\sqrt{d}}{\sqrt{abc-a^2d}}\right) - 2\,\sqrt{abc-a^2d}\,bc\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 2\,\sqrt{abc-a^2d}\,ad}{6\left(\sqrt{abc-a^2d}\,b^3c\sqrt{-d} - \sqrt{abc-a^2d}\,ab^2\sqrt{-d}\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*a*b*c*sqrt(-d)*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*a^2*sqrt(-
d)*d*arctan(a*sqrt(d)/sqrt(a*b*c - a^2*d)) - 2*sqrt(a*b*c - a^2*d)*b*c*arct
an(sqrt(d)/sqrt(-d)) + 2*sqrt(a*b*c - a^2*d)*a*d*arctan(sqrt(d)/sqrt(-d)) +
 sqrt(a*b*c - a^2*d)*a*sqrt(-d)*sqrt(d))*sgn(x)/(sqrt(a*b*c - a^2*d)*b^3*c*
sqrt(-d) - sqrt(a*b*c - a^2*d)*a*b^2*sqrt(-d)*d) + 1/6*a*c*sqrt(d + c/x^6)/
((b^2*c*sgn(x) - a*b*d*sgn(x))*(b*c + a*(d + c/x^6) - a*d)) + 1/6*(3*a*b*c
- 2*a^2*d)*arctan(a*sqrt(d + c/x^6)/sqrt(a*b*c - a^2*d))/((b^3*c*sgn(x) - a
*b^2*d*sgn(x))*sqrt(a*b*c - a^2*d)) - 1/3*arctan(sqrt(d + c/x^6)/sqrt(-d))/
(b^2*sqrt(-d)*sgn(x))
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{14}}{\left(bx^6 + a\right)^2 \sqrt{dx^6 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^14/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{14}}{\left(b\,x^6 + a\right)^2\,\sqrt{d\,x^6 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^14/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

3.876 $\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

```
[Out] 1/6*c*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/6*x^3*(d*x^6+c)^(1/2)/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^3\sqrt{c+dx^6}}{6\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(x^3*Sqrt[c + d*x^6])/(6*(b*c - a*d)*(a + b*x^6)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{x^8}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$
$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6(bc-ad)}$$
$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6(bc-ad)}$$
$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{6(bc-ad)}$$
$$= -\frac{x^3\sqrt{c+dx^6}}{6(bc-ad)\left(a+bx^6\right)} + \frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6\sqrt{a}\,(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.58, size = 124, normalized size = 1.33

$$\frac{\sqrt{c+dx^6}\left(-\frac{x^6(bc-ad)}{a+bx^6}-\frac{c\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^6\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^6}{c}+1}}\right)}{\sqrt{\frac{dx^6}{c}+1}}\right)}{6x^3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (Sqrt[c + d*x^6]*(-(((b*c - a*d)*x^6)/(a + b*x^6)) - (c*Sqrt[(-(b/a) + d/c)
*x^6]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^6]/Sqrt[1 + (d*x^6)/c]])/Sqrt[1 + (d*x^
6)/c]))/(6*(b*c - a*d)^2*x^3)
```

fricas [B] time = 0.80, size = 426, normalized size = 4.58

$$\left[-\frac{4\sqrt{dx^6+c}\left(abc-a^2d\right)x^3-\left(bcx^6+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\left((bc-2\,ad)\right.}{b^2x^{12}+2\,abx^6+a^2}\right.}{24\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^6+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(4*sqrt(d*x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6 + c)*sqrt
(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c*
d + a^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/12*(2*sqrt(d*
x^6 + c)*(a*b*c - a^2*d)*x^3 - (b*c*x^6 + a*c)*sqrt(a*b*c - a^2*d)*arctan(1
```

```
/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
- a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)))/((a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*x^6 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
[Out] int(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{8}}{{\left(bx^{6}+a\right)}^{2}\sqrt{dx^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^8/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{8}}{\left(b\,x^{6}+a\right)^{2}\sqrt{d\,x^{6}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^8/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

3.877 $\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

```
[Out] 1/6*(-2*a*d+b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/6*b*x^3*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/(b*x^6+a)
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}+\frac{bx^3\sqrt{c+dx^6}}{6a\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (b*x^3*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*(a + b*x^6)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^3}{\sqrt{c+dx^6}}\right)}{6a(bc-ad)}$$

$$= \frac{bx^3\sqrt{c+dx^6}}{6a(bc-ad)\left(a+bx^6\right)} + \frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^3}{\sqrt{a}\,\sqrt{c+dx^6}}\right)}{6a^{3/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 1.00, size = 407, normalized size = 3.91

$$\frac{x^3\sqrt{c+dx^6}\left(-30dx^6\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}-45c\sqrt{\frac{ax^6\left(c+dx^6\right)(bc-ad)}{c^2\left(a+bx^6\right)^2}}+16dx^6\left(\frac{x^6(bc-ad)}{c\left(a+bx^6\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^6\right)}{c\left(a+bx^6\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)}{c(bx^6+}\right.\right.}{90c^2\left(a+bx^6\right)^2\left(\frac{x^6(bc-}{c(a+b}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^3*Sqrt[c + d*x^6]*(-45*c*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
b*x^6)^2)] - 30*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b*x^6)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 30*d*x^6*ArcS
in[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 16*c*(((b*c - a*d)*x^6)/(c*(a
 + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 16*d*x^6*(((b*c - a*d)*x^6)/
(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))]))/(90*c^2*(((b*c - a*d)*x
^6)/(c*(a + b*x^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6
))])
```

fricas [B] time = 0.90, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^6+c}\left(ab^2c-a^2bd\right)x^3-\left(\left(b^2c-2\,abd\right)x^6+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right.}{b^2x}\right.}{24\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^6\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 - ((b^2*c - 2*a*b*d)*x^6 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^12 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 -
a*c*x^3)*sqrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)
))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^
4*b*d^2)*x^6), 1/12*(2*sqrt(d*x^6 + c)*(a*b^2*c - a^2*b*d)*x^3 + ((b^2*c -
2*a*b*d)*x^6 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*
d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9
```

```
+ (a*b*c^2 - a^2*c*d)*x^3)))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^
3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^6)]
```

giac [B] time = 0.28, size = 237, normalized size = 2.28

$$-\frac{1}{6} d^{\frac{3}{2}} \left(\frac{(bc - 2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^3 - \sqrt{dx^6+c}\right)^2 b - bc + 2\,ad}{2\,\sqrt{abcd - a^2d^2}}\right)}{\left(abcd - a^2d^2\right)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}\,x^3 - \sqrt{dx^6+c}\right)^2 bc - 2\left(\sqrt{d}\,x^3\right.\right.}{\left(\left(\sqrt{d}\,x^3 - \sqrt{dx^6+c}\right)^4 b - 2\left(\sqrt{d}\,x^3 - \sqrt{dx^6+c}\right)^2 bc + 4\left(\right.\right.} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^3 - sqrt(d*x^6 + c))^2*b*c - 2*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^3 - sqrt(d*x^6 + c))^4*b - 2*(sqrt(d)*x^3 - sqrt(d*
x^6 + c))^2*b*c + 4*(sqrt(d)*x^3 - sqrt(d*x^6 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^6 + a\right)^2 \sqrt{d\,x^6 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^6 + a\right)^2 \sqrt{dx^6 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(b\,x^6 + a\right)^2 \sqrt{d\,x^6 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a + bx^6\right)^2 \sqrt{c + dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.878 $\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(3bc-2ad)}{6a^2cx^3(bc-ad)}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

```
[Out] -1/6*b*(-4*a*d+3*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/6*(-2*a*d+3*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^3/(b*x^6+a)
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^6}\,(3bc-2ad)}{6a^2cx^3(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^6}}{6ax^3\left(a+bx^6\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^6])/(6*a^2*c*(b*c - a*d)*x^3) + (b*Sqrt[c +
d*x^6])/(6*a*(b*c - a*d)*x^3*(a + b*x^6)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{\operatorname{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x\ldots\right.}{6a^2c(bc-ad)}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\operatorname{Subst}\left(\int \frac{\ldots}{(a+b\ldots}\right.}{6a^2(bc-a\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{(b(3bc-4ad))\operatorname{Subst}\left(\int \frac{\ldots}{a-(\ldots}\right.}{6a^2(bc-\ldots}$$

$$= -\frac{(3bc-2ad)\sqrt{c+dx^6}}{6a^2c(bc-ad)x^3} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^3\left(a+bx^6\right)} - \frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x\ldots}{\sqrt{a}\sqrt{c+d\ldots}}\right.}{6a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 5.40, size = 869, normalized size = 5.83

$$-\frac{\sqrt{dx^6+c}\left(120d^2\sin^{-1}\left(\sqrt{\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}}\right)x^{12}+96d^2\left(\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)^{5/2}\sqrt{\frac{a\left(dx^6+c\right)}{c\left(bx^6+a\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^6}{c\left(bx^6+a\right)}\right)x^{12}+32d^2\left(\frac{(bc-ad)\ldots}{c\left(bx^6+a\ldots\right.}\right.\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/90*(Sqrt[c + d*x^6]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(
a + b*x^6)^2)] - 180*c*d*x^6*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a +
 b*x^6)^2)] - 120*d^2*x^12*Sqrt[(a*(b*c - a*d)*x^6*(c + d*x^6))/(c^2*(a + b
*x^6)^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 180*c*
d*x^6*ArcSin[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 120*d^2*x^12*ArcSin
[Sqrt[((b*c - a*d)*x^6)/(c*(a + b*x^6))]] + 64*c^2*(((b*c - a*d)*x^6)/(c*(a
```

```
 + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 160*c*d*x^6*(((b*c - a*d)*x^
6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hypergeomet
ric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 96*d^2*x^12*(((b*c -
 a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*Hyp
ergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] + 32*c^2*(((b*
c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))]*
HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(c*(a + b*x^6))] +
 64*c*d*x^6*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqrt[(a*(c + d*x^6))/
(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^6)/(
c*(a + b*x^6))] + 32*d^2*x^12*(((b*c - a*d)*x^6)/(c*(a + b*x^6)))^(5/2)*Sqr
t[(a*(c + d*x^6))/(c*(a + b*x^6))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, (
(b*c - a*d)*x^6)/(c*(a + b*x^6))]))/(c^3*x^3*(((b*c - a*d)*x^6)/(c*(a + b*x
^6)))^(3/2)*(a + b*x^6)^2*Sqrt[(a*(c + d*x^6))/(c*(a + b*x^6))])
```

fricas [B] time = 1.04, size = 612, normalized size = 4.11

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^9+\left(3\,ab^2c^2-4\,a^2bcd\right)x^3\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a^2c^2+4\,(}{b^2x^{12}+2\,abx^6+a^2}\right.}{24\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*s
qrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 - 2*(3*a*b*
c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 + 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*sqrt(d*x^6
 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) + 4*((3*a*b^3*c^2
 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d
^2)*sqrt(d*x^6 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (
a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^3), -1/12*(((3*b^3*c^2 - 4*a*b^2
*c*d)*x^9 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^3)*sqrt(a*b*c - a^2*d)*arctan(1/2
*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d -
a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d)*x^3)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2*c*d
+ 2*a^3*b*d^2)*x^6 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x^6 +
c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^9 + (a^4*b^2*c^3 - 2*a
^5*b*c^2*d + a^6*c*d^2)*x^3)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6 + a\right)^2 \sqrt{dx^6 + c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4 \left(b\, x^6 + a\right)^2 \sqrt{d\, x^6 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^4*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

3.879 $\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{\sqrt{c+dx^6}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b\sqrt{c+}}{6ax^9\left(a+bx^6\right.}$$

```
[Out] 1/6*b^2*(-6*a*d+5*b*c)*arctan(x^3*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^6+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/18*(-2*a*d+5*b*c)*(d*x^6+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^9+1/18*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^6+c)^(1/2)/a^3/c^2/(-a*
d+b*c)/x^3+1/6*b*(d*x^6+c)^(1/2)/a/(-a*d+b*c)/x^9/(b*x^6+a)
```

Rubi [A] time = 0.31, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^6}\left(-4a^2d^2-8abcd+15b^2c^2\right)}{18a^3c^2x^3(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^3\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^6}}\right)}{6a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^6}\,(5bc-2ad)}{18a^2cx^9(bc-ad)}+\frac{b\sqrt{c+}}{6ax^9\left(a+bx^6\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^6])/(18*a^2*c*(b*c - a*d)*x^9) + ((15*b^2*c^
2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^6])/(18*a^3*c^2*(b*c - a*d)*x^3) +
(b*Sqrt[c + d*x^6])/(6*a*(b*c - a*d)*x^9*(a + b*x^6)) + (b^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x^3)/(Sqrt[a]*Sqrt[c + d*x^6])])/(6*a^(7/2)*(b*c
- a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^3\right)$$

$$= \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^3\right)}{6a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{b\sqrt{c+dx^6}}{6a(bc-ad)x^9\left(a+bx^6\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2d^2}{x^2\left(a+bx^2\right)\sqrt{}}\right.}{18a^2c(bc}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d}}{6a(bc-ad)x^9\left(\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d}}{6a(bc-ad)x^9\left(\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d}}{6a(bc-ad)x^9\left(\right.}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^6}}{18a^2c(bc-ad)x^9} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^6}}{18a^3c^2(bc-ad)x^3} + \frac{b\sqrt{c+d}}{6a(bc-ad)x^9\left(\right.}$$

Mathematica [A] time = 5.86, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^6\right)\left(\frac{3b^3x^{12}}{\left(a+bx^6\right)(bc-ad)}+\frac{4x^6(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{18}\sqrt{\frac{dx^6}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^6\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^6}{a}+1}}\right)}{c\left(\frac{x^6(bc-ad)}{ac}\right)^{3/2}}}{18a^5x^9\sqrt{c+dx^6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^10*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (a^2*(c + d*x^6)*((-2*a)/c + (4*(3*b*c + a*d)*x^6)/c^2 + (3*b^3*x^12)/((b*c
 - a*d)*(a + b*x^6))) + (3*b^2*(5*b*c - 6*a*d)*x^18*Sqrt[1 + (d*x^6)/c]*Arc
Sin[Sqrt[(b/a - d/c)*x^6]/Sqrt[1 + (b*x^6)/a]])/(c*(((b*c - a*d)*x^6)/(a*c)
)^(3/2)))/(18*a^5*x^9*Sqrt[c + d*x^6])
```

fricas [A] time = 1.37, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5\,b^4c^3-6\,ab^3c^2d\right)x^{15}+\left(5\,ab^3c^3-6\,a^2b^2c^2d\right)x^9\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{12}-2\left(3\,abc^2-4\,a^2cd\right)x^6+a}{b^2x^{12}+2\,a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/72*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^9)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^12 -
2*(3*a*b*c^2 - 4*a^2*c*d)*x^6 + a^2*c^2 - 4*((b*c - 2*a*d)*x^9 - a*c*x^3)*s
qrt(d*x^6 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^12 + 2*a*b*x^6 + a^2)) - 4*((15
*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^12 - 2*a^3
*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d
 + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3*c^4 - 2*a^5*b^2
*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2)*
x^9), 1/36*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^15 + (5*a*b^3*c^3 - 6*a^2*b^2*
c^2*d)*x^9)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^6 - a*c)*sqrt(d
*x^6 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^9 + (a*b*c^2 - a^2*c*d
)*x^3)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d
^3)*x^12 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2 + 2*(5*a^2*b^3*c^3 -
 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^6)*sqrt(d*x^6 + c))/((a^4*b^3
*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^15 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d
 + a^7*c^2*d^2)*x^9)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^10), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{10} \left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^10*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Timed out
```

3.880 $\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

```
[Out] 1/5*x^5*AppellF1(5/6,2,1/2,11/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d
*x^6+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{511, 510\}$

$$\frac{x^5\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^6)/c]*AppellF1[5/6, 2, 1/2, 11/6, -((b*x^6)/a), -((d*x^6
)/c)])/(5*a^2*Sqrt[c + d*x^6])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx &= \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}\\ &= \frac{x^5\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{5}{6};2,\frac{1}{2};\frac{11}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{5a^2\sqrt{c+dx^6}}\end{aligned}$$

Mathematica [B] time = 0.22, size = 169, normalized size = 2.64

$$\frac{x^5\left(-10bdx^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+11\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,(bc-6ad)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\right.\right.}{330a^2\left(a+bx^6\right)\sqrt{c+dx^6}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^5*(55*a*b*(c + d*x^6) + 11*(b*c - 6*a*d)*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]
*AppellF1[5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] - 10*b*d*x^6*(a +
b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c), -((b*
x^6)/a)]))/(330*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^6+c}\,x^4}{b^2dx^{18}+\left(b^2c+2\,abd\right)x^{12}+\left(2\,abc+a^2d\right)x^6+a^2c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)*x^4/(b^2*d*x^18 + (b^2*c + 2*a*b*d)*x^12 + (2*a*b*
c + a^2*d)*x^6 + a^2*c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^4/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**4/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.881 $\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

```
[Out] 1/4*x^4*AppellF1(2/3,2,1/2,5/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$\frac{x^4\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^6)/c]*AppellF1[2/3, 2, 1/2, 5/3, -((b*x^6)/a), -((d*x^6)
/c)])/(4*a^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx,x,x^2\right)$$
$$=\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{x}{\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx,x,x^2\right)}{2\sqrt{c+dx^6}}$$
$$=\frac{x^4\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{2}{3};2,\frac{1}{2};\frac{5}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.24, size = 168, normalized size = 2.62

$$-\frac{x^4\left(bdx^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{5}{3};\frac{1}{2},1;\frac{8}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-5\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,(bc-3ad)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{b}{}\right.\right.}{60a^2\left(a+bx^6\right)\sqrt{c+dx^6}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -1/60*(x^4*(-10*a*b*(c + d*x^6) - 5*(b*c - 3*a*d)*(a + b*x^6)*Sqrt[1 + (d*x
^6)/c]*AppellF1[2/3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^6*(a
+ b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b*
x^6)/a)]))/(a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(x^3/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(x**3/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.882 $\int \frac{x}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

```
[Out] 1/2*x^2*AppellF1(1/3,2,1/2,4/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*
x^6+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {465, 430, 429}

$$\frac{x^2\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x^2*Sqrt[1 + (d*x^6)/c]*AppellF1[1/3, 2, 1/2, 4/3, -((b*x^6)/a), -((d*x^6)
/c)])/(2*a^2*Sqrt[c + d*x^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2 \sqrt{c+dx^3}}\,dx, x, x^2\right)$$
$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2 \sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$
$$= \frac{x^2\sqrt{1+\frac{dx^6}{c}}\,F_1\left(\frac{1}{3};2,\frac{1}{2};\frac{4}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.19, size = 172, normalized size = 2.69

$$\frac{bdx^8\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,F_1\left(\frac{4}{3};\frac{1}{2},1;\frac{7}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+8x^2\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,(2bc-3ad)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+}{48a^2\left(a+bx^6\right)\sqrt{c+dx^6}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*b*x^2*(c + d*x^6) + 8*(2*b*c - 3*a*d)*x^2*(a + b*x^6)*Sqrt[1 + (d*x^6)
/c]*AppellF1[1/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*x^8*(a + b
*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b*x^6
)/a)])/(48*a^2*(b*c - a*d)*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^6+a\right)^2 \sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(b\,x^6+a\right)^2\,\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(x/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(x/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.883 $\int \frac{1}{(a+bx^6)^2 \sqrt{c+dx^6}} \, dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\, F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

```
[Out] x*AppellF1(1/6,2,1/2,7/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/(d*x^6+c)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^6}{c}+1}\, F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^6)/c]*AppellF1[1/6, 2, 1/2, 7/6, -((b*x^6)/a), -((d*x^6)/c
)])/(a^2*Sqrt[c + d*x^6])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^6)^2\sqrt{c+dx^6}}\,dx &= \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{(a+bx^6)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}\\
&= \frac{x\sqrt{1+\frac{dx^6}{c}}\, F_1\left(\frac{1}{6};2,\frac{1}{2};\frac{7}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2\sqrt{c+dx^6}}
\end{aligned}$$

Mathematica [B] time = 0.39, size = 329, normalized size = 5.58

$$\frac{x\left(-2bdx^6\sqrt{\frac{dx^6}{c}+1}\, F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)-\frac{7a\left(3bx^6(c+dx^6)\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)+7ac(6ad\cdots\right.}{(a+bx^6)\left(3x^6\left(2bcF_1\left(\frac{7}{6};\frac{1}{2},2;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+adF_1\left(\frac{7}{6};\frac{3}{2},1;\frac{13}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)\right)-7\cdots\right.}\right)}{42a^2\sqrt{c+dx^6}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (x*(-2*b*d*x^6*Sqrt[1 + (d*x^6)/c]*AppellF1[7/6, 1/2, 1, 13/6, -((d*x^6)/c)
, -((b*x^6)/a)] - (7*a*(7*a*c*(6*a*d - b*(6*c + d*x^6))*AppellF1[1/6, 1/2,
1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*b*x^6*(c + d*x^6)*(2*b*c*AppellF1[7
/6, 1/2, 2, 13/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)])))/((a + b*x^6)*(-7*a*c*AppellF1[1/6, 1/2,
 1, 7/6, -((d*x^6)/c), -((b*x^6)/a)] + 3*x^6*(2*b*c*AppellF1[7/6, 1/2, 2, 1
3/6, -((d*x^6)/c), -((b*x^6)/a)] + a*d*AppellF1[7/6, 3/2, 1, 13/6, -((d*x^6
)/c), -((b*x^6)/a)])))))/(42*a^2*(-(b*c) + a*d)*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^6\right)^2 \sqrt{c+dx^6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.884 $\int \frac{1}{x^2(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

```
[Out] -AppellF1(-1/6,2,1/2,5/6,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x/(d*x^6+
c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -((Sqrt[1 + (d*x^6)/c]*AppellF1[-1/6, 2, 1/2, 5/6, -((b*x^6)/a), -((d*x^6)/
c)])/(a^2*x*Sqrt[c + d*x^6]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{\sqrt{1+\frac{dx^6}{c}}\int \frac{1}{x^2\left(a+bx^6\right)^2\sqrt{1+\frac{dx^6}{c}}}\,dx}{\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{6};2,\frac{1}{2};\frac{5}{6};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{a^2x\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.29, size = 226, normalized size = 3.65

$$\frac{-11x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\left(12a^2d^2-24abcd+7b^2c^2\right)F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+55a\left(c+dx^6\right)\left(6a^2d-6ab\left(c\right.\right.}{330a^3cx\left(a+bx^6\right)\sqrt{c+dx^6}\left(b\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (55*a*(c + d*x^6)*(6*a^2*d - 7*b^2*c*x^6 - 6*a*b*(c - d*x^6)) - 11*(7*b^2*c
^2 - 24*a*b*c*d + 12*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[
5/6, 1/2, 1, 11/6, -((d*x^6)/c), -((b*x^6)/a)] + 10*b*d*(7*b*c - 6*a*d)*x^1
2*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[11/6, 1/2, 1, 17/6, -((d*x^6)/c)
, -((b*x^6)/a)])/(330*a^3*c*(b*c - a*d)*x*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^6+c}}{b^2dx^{20}+\left(b^2c+2\,abd\right)x^{14}+\left(2\,abc+a^2d\right)x^8+a^2cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^6 + c)/(b^2*d*x^20 + (b^2*c + 2*a*b*d)*x^14 + (2*a*b*c +
a^2*d)*x^8 + a^2*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^2*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^6\right)^2 \sqrt{c + dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.885 $\int \frac{1}{x^3(a+bx^6)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

```
[Out] -1/2*AppellF1(-1/3,2,1/2,2/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^2/(
d*x^6+c)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-1/3, 2, 1/2, 2/3, -((b*x^6)/a), -((d*x^6)/c
)])/(2*a^2*x^2*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx,x,x^2\right)$$

$$=\frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx,x,x^2\right)}{2\sqrt{c+dx^6}}$$

$$=-\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{1}{3};2,\frac{1}{2};\frac{2}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{2a^2x^2\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.37, size = 226, normalized size = 3.53

$$\frac{-5x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\,\left(3a^2d^2-15abcd+8b^2c^2\right)F_1\left(\frac{2}{3};\frac{1}{2},1;\frac{5}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+20a\left(c+dx^6\right)\left(3a^2d-3ab\left(c-\right.\right.}{120a^3cx^2\left(a+bx^6\right)\sqrt{c+dx^6}\,(bc-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (20*a*(c + d*x^6)*(3*a^2*d - 4*b^2*c*x^6 - 3*a*b*(c - d*x^6)) - 5*(8*b^2*c^
2 - 15*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[2/
3, 1/2, 1, 5/3, -((d*x^6)/c), -((b*x^6)/a)] + 2*b*d*(4*b*c - 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[5/3, 1/2, 1, 8/3, -((d*x^6)/c), -((b
*x^6)/a)])/(120*a^3*c*(b*c - a*d)*x^2*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^6 + a)^2 \sqrt{dx^6 + c}\, x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3 \left(b\, x^6 + a\right)^2 \sqrt{d\, x^6 + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^6\right)^2 \sqrt{c + dx^6}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)
[Out] Integral(1/(x**3*(a + b*x**6)**2*sqrt(c + d*x**6)), x)
```

3.886 $\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

```
[Out] -1/4*AppellF1(-2/3,2,1/2,1/3,-b*x^6/a,-d*x^6/c)*(1+d*x^6/c)^(1/2)/a^2/x^4/(
d*x^6+c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 511, 510}

$$-\frac{\sqrt{\frac{dx^6}{c}+1}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] -(Sqrt[1 + (d*x^6)/c]*AppellF1[-2/3, 2, 1/2, 1/3, -((b*x^6)/a), -((d*x^6)/c
)])/(4*a^2*x^4*Sqrt[c + d*x^6])
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^6\right)^2\sqrt{c+dx^6}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{c+dx^3}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{1+\frac{dx^6}{c}}\,\text{Subst}\left(\int \frac{1}{x^3\left(a+bx^3\right)^2\sqrt{1+\frac{dx^3}{c}}}\,dx, x, x^2\right)}{2\sqrt{c+dx^6}}$$

$$= -\frac{\sqrt{1+\frac{dx^6}{c}}\,F_1\left(-\frac{2}{3};2,\frac{1}{2};\frac{1}{3};-\frac{bx^6}{a},-\frac{dx^6}{c}\right)}{4a^2x^4\sqrt{c+dx^6}}$$

Mathematica [B] time = 0.28, size = 225, normalized size = 3.52

$$\frac{4x^6\left(a+bx^6\right)\sqrt{\frac{dx^6}{c}+1}\left(3a^2d^2+21abcd-20b^2c^2\right)F_1\left(\frac{1}{3};\frac{1}{2},1;\frac{4}{3};-\frac{dx^6}{c},-\frac{bx^6}{a}\right)+8a\left(c+dx^6\right)\left(3a^2d-3ab\left(c-dx^6\right)\right)}{96a^3cx^4\left(a+bx^6\right)\sqrt{c+dx^6}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^6)^2*Sqrt[c + d*x^6]),x]
```

```
[Out] (8*a*(c + d*x^6)*(3*a^2*d - 5*b^2*c*x^6 - 3*a*b*(c - d*x^6)) + 4*(-20*b^2*c
^2 + 21*a*b*c*d + 3*a^2*d^2)*x^6*(a + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[1
/3, 1/2, 1, 4/3, -((d*x^6)/c), -((b*x^6)/a)] + b*d*(-5*b*c + 3*a*d)*x^12*(a
 + b*x^6)*Sqrt[1 + (d*x^6)/c]*AppellF1[4/3, 1/2, 1, 7/3, -((d*x^6)/c), -((b
*x^6)/a)])/(96*a^3*c*(b*c - a*d)*x^4*(a + b*x^6)*Sqrt[c + d*x^6])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2\sqrt{dx^6+c}\,x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^6+a\right)^2\sqrt{d\,x^6+c}\;x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)
```

[Out] int(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^6+a\right)^2 \sqrt{dx^6+c}\, x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(b*x^6+a)^2/(d*x^6+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^6 + a)^2*sqrt(d*x^6 + c)*x^5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^5 \left(b\, x^6+a\right)^2 \sqrt{d\, x^6+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)),x)

[Out] int(1/(x^5*(a + b*x^6)^2*(c + d*x^6)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(a+bx^6\right)^2 \sqrt{c+dx^6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x**6+a)**2/(d*x**6+c)**(1/2),x)

[Out] Integral(1/(x**5*(a + b*x**6)**2*sqrt(c + d*x**6)), x)

3.887 $\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=104

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\,(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

```
[Out] 1/12*(d*x^8+c)^(3/2)/b/d^2-1/4*a^2*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*
c)^(1/2))/b^(5/2)/(-a*d+b*c)^(1/2)-1/4*(a*d+b*c)*(d*x^8+c)^(1/2)/b^2/d^2
```

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {446, 88, 63, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}\,(ad+bc)}{4b^2d^2}+\frac{(c+dx^8)^{3/2}}{12bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((b*c + a*d)*Sqrt[c + d*x^8])/(4*b^2*d^2) + (c + d*x^8)^(3/2)/(12*b*d^2) -
 (a^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b
*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{23}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)$$
$$= \frac{1}{8}\text{Subst}\left(\int \left(\frac{-bc-ad}{b^2d\sqrt{c+dx}} + \frac{a^2}{b^2(a+bx)\sqrt{c+dx}} + \frac{\sqrt{c+dx}}{bd}\right)dx,x,x^8\right)$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{(c+dx^8)^{3/2}}{12bd^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8b^2}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{(c+dx^8)^{3/2}}{12bd^2} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4b^2d}$$
$$= -\frac{(bc+ad)\sqrt{c+dx^8}}{4b^2d^2} + \frac{(c+dx^8)^{3/2}}{12bd^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.22, size = 91, normalized size = 0.88

$$\frac{\sqrt{c+dx^8}\left(-3ad-2bc+bdx^8\right)}{12b^2d^2} - \frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-2*b*c - 3*a*d + b*d*x^8))/(12*b^2*d^2) - (a^2*ArcTanh[(S
qrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[b*c - a*d])
```

fricas [A] time = 1.22, size = 288, normalized size = 2.77

$$\left[\frac{3\sqrt{b^2c-abd}\,a^2d^2\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right) + 2\left(\left(b^3cd-ab^2d^2\right)x^8 - 2\,b^3c^2 - ab^2cd + 3\,a^2bd^2\right)\sqrt{dx^8+\cdots}}{24\left(b^4cd^2-ab^3d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*sqrt(b^2*c - a*b*d)*a^2*d^2*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*
x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*((b^3*c*d - a*b^2*d^2)*x^8 -
 2*b^3*c^2 - a*b^2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d
^3), 1/12*(3*sqrt(-b^2*c + a*b*d)*a^2*d^2*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*
c + a*b*d)/(b*d*x^8 + b*c)) + ((b^3*c*d - a*b^2*d^2)*x^8 - 2*b^3*c^2 - a*b^
2*c*d + 3*a^2*b*d^2)*sqrt(d*x^8 + c))/(b^4*c*d^2 - a*b^3*d^3)]
```

giac [A] time = 0.16, size = 106, normalized size = 1.02

$$\frac{a^2\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,b^2} + \frac{(dx^8+c)^{\frac{3}{2}}b^2d^4 - 3\sqrt{dx^8+c}\,b^2cd^4 - 3\sqrt{dx^8+c}\,abd^5}{12\,b^3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*a^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*b^2) + 1/12*((d*x^8 + c)^(3/2)*b^2*d^4 - 3*sqrt(d*x^8 + c)*b^2*c*d^4 - 3*
sqrt(d*x^8 + c)*a*b*d^5)/(b^3*d^6)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.68, size = 103, normalized size = 0.99

$$\frac{\left(d\,x^8+c\right)^{3/2}}{12\,b\,d^2}-\left(\frac{c}{2\,b\,d^2}+\frac{4\,a\,d^3-4\,b\,c\,d^2}{16\,b^2\,d^4}\right)\sqrt{d\,x^8+c}+\frac{a^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(3/2)/(12*b*d^2) - (c/(2*b*d^2) + (4*a*d^3 - 4*b*c*d^2)/(16*b^2
*d^4))*(c + d*x^8)^(1/2) + (a^2*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c
)^(1/2)))/(4*b^(5/2)*(a*d - b*c)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.888 $\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=74

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

```
[Out] 1/4*a*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)/(-a*d+b*c)^
(1/2)+1/4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a
*d]])/(4*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8b}$$
$$= \frac{\sqrt{c+dx^8}}{4bd} - \frac{a\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4bd}$$
$$= \frac{\sqrt{c+dx^8}}{4bd} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.97

$$\frac{1}{4}\left(\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{\sqrt{c+dx^8}}{bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/(b*d) + (a*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(b^(3/2)*Sqrt[b*c - a*d]))/4
```

fricas [A] time = 0.81, size = 205, normalized size = 2.77

$$\left[\frac{\sqrt{b^2c-abd}\,ad\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\left(b^2c-abd\right)}{8\left(b^3cd-ab^2d^2\right)}, -\frac{\sqrt{-b^2c+abd}\,ad\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b}}{bdx^8+b}\right.}{4\left(b^3cd-a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b^2*c - a*b*d)*a*d*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c
)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2), -1/4*(sqrt(-b^2*c + a*b*d)*a*d*arctan(sqrt(d*x^8 + c)*
sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - sqrt(d*x^8 + c)*(b^2*c - a*b*d))/(b
^3*c*d - a*b^2*d^2)]
```

giac [A] time = 0.16, size = 64, normalized size = 0.86

$$-\frac{\frac{ad\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b} - \frac{\sqrt{dx^8+c}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(a*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b
*d)*b) - sqrt(d*x^8 + c)/b)/d
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.73, size = 58, normalized size = 0.78

$$\frac{\sqrt{d\,x^8+c}}{4\,b\,d}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,\sqrt{a\,d-b\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b*d) - (a*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)
^(1/2)))/(4*b^(3/2)*(a*d - b*c)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**15/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.889 $\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

```
[Out] -1/4*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(1/2)/(-a*d+b*c)^(
1/2)
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {444, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(4*Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4d}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]]/(Sqrt[b]*Sqrt[b*c -
 a*d])
```

fricas [A] time = 0.86, size = 130, normalized size = 2.55

$$\left[\frac{\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)}{8\,\sqrt{b^2c-abd}}, \frac{\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{dx^8+c}\,\sqrt{-b^2c+abd}}{bdx^8+bc}\right)}{4\,(b^2c-abd)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b
*x^8 + a))/sqrt(b^2*c - a*b*d), 1/4*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8
+ c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c))/(b^2*c - a*b*d)]
```

giac [A] time = 0.17, size = 40, normalized size = 0.78

$$\frac{\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.59, size = 40, normalized size = 0.78

$$\frac{\operatorname{atan}\left(\frac{b\,\sqrt{d\,x^8+c}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{4\,\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] atan((b*(c + d*x^8)^(1/2))/(a*b*d - b^2*c)^(1/2))/(4*(a*b*d - b^2*c)^(1/2))
```

sympy [A] time = 44.57, size = 37, normalized size = 0.73

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4b\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*b*sqrt((a*d - b*c)/b))
```

3.890 $\int \frac{1}{x(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

```
[Out] -1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a/c^(1/2)+1/4*arctanh(b^(1/2)*(d*x^8+
c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2)/a/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 86, 63, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/(4*a*Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 86

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{1}{8}\text{Subst}\left(\int \frac{1}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)$$
$$=\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^8\right)}{8a}-\frac{b\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a}$$
$$=\frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4ad}$$
$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a\sqrt{c}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a\sqrt{bc-ad}}$$

Mathematica [A] time = 0.10, size = 81, normalized size = 0.95

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-(ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]]/Sqrt[c]) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x^8])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d])/(4*a)
```

fricas [A] time = 0.75, size = 431, normalized size = 5.07

$$\left[\frac{c\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+\sqrt{c}\log\left(\frac{dx^8-2\,\sqrt{dx^8+c}\,\sqrt{c}+2\,c}{x^8}\right)}{8\,ac},\ \frac{2\,c\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{\sqrt{dx^8+c}\,(bc-ad\right.}{bdx^8+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(c*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*
(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + sqrt(c)*log((d*x^8 - 2*sqrt
(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(2*c*sqrt(-b/(b*c - a*d))*arcta
n(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt
(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/(a*c), 1/8*(c*sqrt(
b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*s
qrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-
c)/c))/(a*c), 1/4*(c*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*
d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) + sqrt(-c)*arctan(sqrt(d*x^8 + c)*
sqrt(-c)/c))/(a*c)]
```

giac [A] time = 0.16, size = 71, normalized size = 0.84

$$-\frac{b\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}\,a}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*a) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a*sqrt(-c))
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x), x)
```

mupad [B] time = 4.81, size = 652, normalized size = 7.67

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d\,x^8+c}}{\sqrt{c}}\right)}{4\,a\,\sqrt{c}}-\frac{\operatorname{atan}\left(\frac{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)\,1{}\mathrm{i}}{8\,\left(a^2\,d-a\,b\,c\right)}+\cdots}{\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(\frac{b^3\,d^2\,\sqrt{d\,x^8+c}}{4}-\frac{\sqrt{b^2\,c-a\,b\,d}\,\left(a^2\,b^2\,d^3-\frac{\left(8\,a^3\,b^2\,d^3-16\,a^2\,b^3\,c\,d^2\right)\,\sqrt{d\,x^8+c}\,\sqrt{b^2\,c-a\,b\,d}}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}\right)}{8\,\left(a^2\,d-a\,b\,c\right)}-\cdots}\right)\cdots}{4\,\left(a^2\,d-a\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] - atanh((c + d*x^8)^(1/2)/c^(1/2))/(4*a*c^(1/2)) - (atan((((b^2*c - a*b*d)^
(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3
- ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/
2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d - a*b*c)) + ((
b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 + ((b^2*c - a*b*d)^(1/2
)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*x^8)^(1/2)*(b^2
*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c)))*1i)/(8*(a^2*d
 - a*b*c)))/(((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2))/4 - ((b^2*
c - a*b*d)^(1/2)*(a^2*b^2*d^3 - ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*d^2)*(c + d*
x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2*d - a*b*c))
))/(8*(a^2*d - a*b*c)) - ((b^2*c - a*b*d)^(1/2)*((b^3*d^2*(c + d*x^8)^(1/2)
)/4 + ((b^2*c - a*b*d)^(1/2)*(a^2*b^2*d^3 + ((8*a^3*b^2*d^3 - 16*a^2*b^3*c*
d^2)*(c + d*x^8)^(1/2)*(b^2*c - a*b*d)^(1/2))/(8*(a^2*d - a*b*c))))/(8*(a^2
*d - a*b*c))))/(8*(a^2*d - a*b*c))))*(b^2*c - a*b*d)^(1/2)*1i)/(4*(a^2*d -
a*b*c))
```

sympy [A] time = 37.86, size = 66, normalized size = 0.78

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{\frac{ad-bc}{b}}}\right)}{4a\sqrt{\frac{ad-bc}{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{c+dx^8}}{\sqrt{-c}}\right)}{4a\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] -atan(sqrt(c + d*x**8)/sqrt((a*d - b*c)/b))/(4*a*sqrt((a*d - b*c)/b)) + ata
n(sqrt(c + d*x**8)/sqrt(-c))/(4*a*sqrt(-c))
```

3.891 $\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=117

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

```
[Out] 1/8*(a*d+2*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(3/2)-1/4*b^(3/2)*ar
ctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^2/(-a*d+b*c)^(1/2)-1/8*(d
*x^8+c)^(1/2)/a/c/x^8
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 103, 156, 63, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*a*c*x^8) + ((2*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[
c]])/(8*a^2*c^(3/2)) - (b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c
 - a*d]])/(4*a^2*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx=\frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)$$

$$=-\frac{\sqrt{c+dx^8}}{8acx^8}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(2bc+ad)+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}$$

$$=-\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8a^2}-\frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,\right.}{16a^2c}$$

$$=-\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{4a^2d}-\frac{(2bc+ad)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{8a^2cd}$$

$$=-\frac{\sqrt{c+dx^8}}{8acx^8}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^2c^{3/2}}-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2\sqrt{bc-ad}}$$

Mathematica [A] time = 0.14, size = 151, normalized size = 1.29

$$\frac{b^{3/2}\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{4a^2(ad-bc)}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8ac^{3/2}}-\frac{\sqrt{c+dx^8}}{8acx^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/8*Sqrt[c + d*x^8]/(a*c*x^8) + (b*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(4*a^
2*Sqrt[c]) + (d*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/(8*a*c^(3/2)) + (b^(3/2)*
Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(4*a^2*
(-(b*c) + a*d))
```

fricas [A] time = 0.73, size = 565, normalized size = 4.83

$$\left[\frac{2\,bc^2x^8\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right)+(2\,bc+ad)\sqrt{c}\,x^8\log\left(\frac{dx^8+2\,\sqrt{dx^8+c}\,\sqrt{c}+2\,c}{x^8}\right)-2\,\sqrt{dx^8+c}\,ac}{16\,a^2c^2x^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(
d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + (2*b*c + a*d)*sq
rt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*sqrt(d*x^8
 + c)*a*c)/(a^2*c^2*x^8), -1/16*(4*b*c^2*x^8*sqrt(-b/(b*c - a*d))*arctan(-s
qrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - (2*b*c +
 a*d)*sqrt(c)*x^8*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*sq
```

```
rt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), 1/8*(b*c^2*x^8*sqrt(b/(b*c - a*d))*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(
b*x^8 + a)) - (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c)
 - sqrt(d*x^8 + c)*a*c)/(a^2*c^2*x^8), -1/8*(2*b*c^2*x^8*sqrt(-b/(b*c - a*d
))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)
) + (2*b*c + a*d)*sqrt(-c)*x^8*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + sqrt(d*
x^8 + c)*a*c)/(a^2*c^2*x^8)]
```

giac [A] time = 0.18, size = 104, normalized size = 0.89

$$\frac{b^2 \arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,a^2} - \frac{(2\,bc+ad)\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{8\,a^2\sqrt{-c}\,c} - \frac{\sqrt{dx^8+c}}{8\,acx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*b^2*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d
)*a^2) - 1/8*(2*b*c + a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^2*sqrt(-c)*c
) - 1/8*sqrt(d*x^8 + c)/(a*c*x^8)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^9), x)
```

mupad [B] time = 5.51, size = 396, normalized size = 3.38

$$\frac{\ln\left(\sqrt{d\,x^8+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}+b^6\,c^2+a^2\,b^4\,d^2-2\,a\,b^5\,c\,d\right)\,\sqrt{b^4\,c-a\,b^3\,d}}{8\,a^3\,d-8\,a^2\,b\,c}-\frac{\ln\left(\sqrt{d\,x^8+c}\,\left(b^4\,c-a\,b^3\,d\right)^{3/2}-\right.}{8\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] (log((c + d*x^8)^(1/2)*(b^4*c - a*b^3*d)^(3/2) + b^6*c^2 + a^2*b^4*d^2 - 2*
a*b^5*c*d)*(b^4*c - a*b^3*d)^(1/2))/(8*a^3*d - 8*a^2*b*c) - (log((c + d*x^8
)^(1/2)*(b^4*c - a*b^3*d)^(3/2) - b^6*c^2 - a^2*b^4*d^2 + 2*a*b^5*c*d)*(b^4
*c - a*b^3*d)^(1/2))/(8*(a^3*d - a^2*b*c)) - (c + d*x^8)^(1/2)/(8*a*c*x^8)
- (atan((b^4*d^4*(c + d*x^8)^(1/2)*3i)/(128*(c^3)^(1/2)*((3*b^4*d^4)/(128*c
) + (5*a*b^3*d^5)/(256*c^2) + (a^2*b^2*d^6)/(256*c^3))) + (b^2*d^6*(c + d*x
```

```
^8)^(1/2)*1i)/(256*(c^3)^(1/2)*((5*b^3*d^5)/(256*a) + (b^2*d^6)/(256*c) + (
3*b^4*c*d^4)/(128*a^2))) + (b^3*d^5*(c + d*x^8)^(1/2)*5i)/(256*(c^3)^(1/2)*
((3*b^4*d^4)/(128*a) + (5*b^3*d^5)/(256*c) + (a*b^2*d^6)/(256*c^2))))*(a*d
+ 2*b*c)*1i)/(8*a^2*(c^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^9\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**9*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.892 $\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

```
[Out] -1/8*(2*a*d+b*c)*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(3/2)+1/4*a^(3/
2)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/b^2/(-a*d+b*c)^(1/2
)+1/8*x^4*(d*x^8+c)^(1/2)/b/d
```

Rubi [A] time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 479, 523, 217, 206, 377, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}+\frac{x^4\sqrt{c+dx^8}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8])/(8*b*d) + (a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt
[a]*Sqrt[c + d*x^8])])/(4*b^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[(Sq
rt[d]*x^4)/Sqrt[c + d*x^8]])/(8*b^2*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{19}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\operatorname{Subst}\left(\int \frac{x^4}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= \frac{x^4\sqrt{c+dx^8}}{8bd} - \frac{\operatorname{Subst}\left(\int \frac{ac+(bc+2ad)x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8bd}$$
$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,$$
$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx$$
$$= \frac{x^4\sqrt{c+dx^8}}{8bd} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{8b^2d^{3/2}}$$

Mathematica [A] time = 0.24, size = 118, normalized size = 0.96

$$\frac{\frac{2a^{3/2}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}} - \frac{(2ad+bc)\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{d^{3/2}} + \frac{bx^4\sqrt{c+dx^8}}{d}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ((b*x^4*Sqrt[c + d*x^8])/d + (2*a^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[
a]*Sqrt[c + d*x^8])])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*Log[d*x^4 + Sqrt[d]*
Sqrt[c + d*x^8]])/d^(3/2))/(8*b^2)
```

fricas [A] time = 0.92, size = 739, normalized size = 6.01

$$\left[\frac{2\sqrt{dx^8+c}\,bdx^4 + ad^2\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4\right)\sqrt{}}{b^2x^{16}+2\,abx^8+a^2}\right.}{16\,b^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 +
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + (b*c + 2*a*
d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*b*d*x^4 + a*d^2*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 -
8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*(
(b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^
8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*(b*c + 2*a*d
)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*b*d*x^4 - 2*a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x
^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + (b*c
+ 2*a*d)*sqrt(d)*log(-2*d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b^2*d^
2), 1/8*(sqrt(d*x^8 + c)*b*d*x^4 - a*d^2*sqrt(a/(b*c - a*d))*arctan(-1/2*((
b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c
*x^4)) + (b*c + 2*a*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sqrt(d*x^8 + c)))/(b^2*
d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(b\,x^{8}+a\right)\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{(a + bx^8)\sqrt{c + dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**19/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.893 $\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

```
[Out] 1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b/d^(1/2)-1/4*arctan(x^4*(-a*d+b*c
)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^(1/2)/b/(-a*d+b*c)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 483, 217, 206, 377, 205}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4*b*Sqr
t[b*c - a*d]) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
```

```
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rubi steps

$$\int \frac{x^{11}}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^2}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b}$$
$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4b\sqrt{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b\sqrt{d}}$$

Mathematica [A] time = 0.07, size = 90, normalized size = 0.99

$$\frac{\frac{\log\left(\sqrt{d}\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{\sqrt{bc-ad}}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-((Sqrt[a]*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/Sqrt[b
*c - a*d]) + Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]]/Sqrt[d])/(4*b)
```

fricas [A] time = 0.72, size = 632, normalized size = 6.95

$$\left[\frac{d\sqrt{-\frac{a}{bc-ad}}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\,(3\,abc^2-4\,a^2cd)x^8+a^2c^2-4\,((b^2c^2-3\,abcd+2\,a^2d^2)x^{12}-(abc^2-a^2cd)x^4)\sqrt{dx^8+c}\sqrt{-\frac{a}{bc-ad}}}{b^2x^{16}+2\,abx^8+a^2}\right)+2\sqrt{d}}{16\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*
d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/
(b^2*x^16 + 2*a*b*x^8 + a^2)) + 2*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*
sqrt(d)*x^4 - c))/(b*d), 1/16*(d*sqrt(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b
*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*
c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c
)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*sqrt(-d)*arctan(s
qrt(-d)*x^4/sqrt(d*x^8 + c)))/(b*d), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2
*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 +
a*c*x^4)) + sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c))/(b*d
), 1/8*(d*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*
```

```
x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) - 2*sqrt(-d)*arctan(sqrt
(-d)*x^4/sqrt(d*x^8 + c)))/(b*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad
 Argument Value
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^{8}+a\right)\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**11/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.894 $\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] 1/4*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(1/2)/(-a*d+b*c)
^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {465, 377, 205}

$$\frac{\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])]/(4*Sqrt[a]*Sqrt[b*c
 - a*d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4\sqrt{a}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 95, normalized size = 1.76

$$\frac{x^4\sqrt{\frac{dx^8}{c}+1}\tanh^{-1}\left(\frac{\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{4a\sqrt{c+dx^8}\sqrt{\frac{dx^8}{c}-\frac{bx^8}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[1 + (d*x^8)/c]*ArcTanh[Sqrt[-((b*x^8)/a) + (d*x^8)/c]/Sqrt[1 + (d
*x^8)/c]])/(4*a*Sqrt[c + d*x^8]*Sqrt[-((b*x^8)/a) + (d*x^8)/c])
```

fricas [B] time = 0.88, size = 245, normalized size = 4.54

$$\left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)}{16\left(abc-a^2d\right)}, \frac{\arctan\left(\frac{((bc-2}{2((ab}\right.}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2
*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*s
qrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))/(a*b*c -
 a^2*d), 1/8*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*
c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4))/sqrt(a*b*c
 - a^2*d)]
```

giac [A] time = 0.20, size = 72, normalized size = 1.33

$$-\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{4\,\sqrt{abcd-a^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*d)
/sqrt(a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

maple [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.895 $\int \frac{1}{x^5(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=80

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

```
[Out] -1/4*b*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/2)/(-a*d+b
*c)^(1/2)-1/4*(d*x^8+c)^(1/2)/a/c/x^4
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 480, 12, 377, 205}

$$-\frac{b\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx^8}}{4acx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(4*a*c*x^4) - (b*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqr
t[c + d*x^8])])/(4*a^(3/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rubi steps

$$\int \frac{1}{x^5 \left(a+bx^8\right) \sqrt{c+dx^8}} \, dx = \frac{1}{4} \text{Subst}\left(\int \frac{1}{x^2 \left(a+bx^2\right) \sqrt{c+dx^2}} \, dx, x, x^4\right)$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{\text{Subst}\left(\int \frac{bc}{\left(a+bx^2\right)\sqrt{c+dx^2}} \, dx, x, x^4\right)}{4ac}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b \, \text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}} \, dx, x, x^4\right)}{4a}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b \, \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2} \, dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a}$$
$$= -\frac{\sqrt{c+dx^8}}{4acx^4} - \frac{b \tan^{-1}\left(\frac{\sqrt{bc-ad}\, x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{3/2}\sqrt{bc-ad}}$$

Mathematica [C] time = 0.83, size = 179, normalized size = 2.24

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(\frac{4x^8\left(c+dx^8\right)(bc-ad)\, {}_2F_1\left(2,2;\frac{5}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)}{3c^2\left(a+bx^8\right)}+\frac{\left(c+2dx^8\right)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)}{c\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}}\right)}{4x^4\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/4*((1 + (d*x^8)/c)*(((c + 2*d*x^8)*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a +
 b*x^8))]])/(c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)^2)]) +
 (4*(b*c - a*d)*x^8*(c + d*x^8)*Hypergeometric2F1[2, 2, 5/2, ((b*c - a*d)*x
^8)/(c*(a + b*x^8))])/(3*c^2*(a + b*x^8))))/(x^4*(a + b*x^8)*Sqrt[c + d*x^8
])
```

fricas [B] time = 0.84, size = 332, normalized size = 4.15

$$\left[-\frac{\sqrt{-abc+a^2d}\, bcx^4 \log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)+4\sqrt{dx^8+c}}{16\left(a^2bc^2-a^3cd\right)x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(sqrt(-a*b*c + a^2*d)*b*c*x^4*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)
*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a
*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2))
 + 4*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4), -1/8*(sq
rt(a*b*c - a^2*d)*b*c*x^4*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 +
 c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4
)) + 2*sqrt(d*x^8 + c)*(a*b*c - a^2*d))/((a^2*b*c^2 - a^3*c*d)*x^4)]
```

giac [A] time = 0.23, size = 116, normalized size = 1.45

$$\frac{1}{4}\,d^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{\sqrt{abcd-a^{2}d^{2}}\,ad}+\frac{2}{\left(\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}-c\right)ad}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*d^(3/2)*(b*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a*d) + 2/(((sqrt(d)*x^
4 - sqrt(d*x^8 + c))^2 - c)*a*d))
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}\;x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}\,x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5}\,\left(b\,x^{8}+a\right)\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^5*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{5}\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.896 $\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=115

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}\,(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

```
[Out] 1/4*b^2*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(5/2)/(-a*d+
b*c)^(1/2)-1/12*(d*x^8+c)^(1/2)/a/c/x^12+1/12*(2*a*d+3*b*c)*(d*x^8+c)^(1/2)
/a^2/c^2/x^4
```

Rubi [A] time = 0.16, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 480, 583, 12, 377, 205}

$$\frac{b^2\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}+\frac{\sqrt{c+dx^8}\,(2ad+3bc)}{12a^2c^2x^4}-\frac{\sqrt{c+dx^8}}{12acx^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(12*a*c*x^12) + ((3*b*c + 2*a*d)*Sqrt[c + d*x^8])/(12*a^2*
c^2*x^4) + (b^2*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(4
*a^(5/2)*Sqrt[b*c - a*d])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
```

```
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{\operatorname{Subst}\left(\int \frac{-3bc-2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12ac}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}-\frac{\operatorname{Subst}\left(\int -\frac{3b^2c^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{12a^2c^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4a^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4a^2}\\
&= -\frac{\sqrt{c+dx^8}}{12acx^{12}}+\frac{(3bc+2ad)\sqrt{c+dx^8}}{12a^2c^2x^4}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{4a^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [C] time = 2.27, size = 253, normalized size = 2.20

$$-\frac{\left(\frac{dx^8}{c}+1\right)\left(-\frac{8x^8\left(c+dx^8\right)^2(bc-ad)\,{}_3F_2\left(2,2,2;1,\frac{5}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)}{a+bx^8}+\frac{3c\left(c^2-4cdx^8-8d^2x^{16}\right)\sin^{-1}\left(\sqrt{\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}}\right)}{\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}}+\frac{24dx^{16}\left(c+dx^8\right)(ad-bc)\,{}_2F_1\left(2,2;\frac{5}{2}\right.}{a+bx^8}\right.}{36c^3x^{12}\left(a+bx^8\right)\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/36*((1 + (d*x^8)/c)*((3*c*(c^2 - 4*c*d*x^8 - 8*d^2*x^16)*ArcSin[Sqrt[((b
*c - a*d)*x^8)/(c*(a + b*x^8))]])/Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2
*(a + b*x^8)^2)] + (24*d*(-(b*c) + a*d)*x^16*(c + d*x^8)*Hypergeometric2F1[
2, 2, 5/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))])/(a + b*x^8) - (8*(b*c - a*d)
*x^8*(c + d*x^8)^2*HypergeometricPFQ[{2, 2, 2}, {1, 5/2}, ((b*c - a*d)*x^8)
/(c*(a + b*x^8))])/(a + b*x^8)))/(c^3*x^12*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [A] time = 0.65, size = 416, normalized size = 3.62

$$\left[-\frac{3\,\sqrt{-abc+a^2d}\,b^2c^2x^{12}\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2-4\left((bc-2\,ad)x^{12}-acx^4\right)\sqrt{dx^8+c}\,\sqrt{-abc+a^2d}}{b^2x^{16}+2\,abx^8+a^2}\right)-4\left(\left(3\,a\right.\right.}{48\left(a^3bc^3-a^4c^2d\right)x^{12}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*sqrt(-a*b*c + a^2*d)*b^2*c^2*x^12*log(((b^2*c^2 - 8*a*b*c*d + 8*a
^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x
^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8
+ a^2)) - 4*((3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*
d)*sqrt(d*x^8 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12), 1/24*(3*sqrt(a*b*c - a^
2*d)*b^2*c^2*x^12*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt
(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*(
(3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x^8 - a^2*b*c^2 + a^3*c*d)*sqrt(d*x^8
 + c))/((a^3*b*c^3 - a^4*c^2*d)*x^12)]
```

giac [B] time = 1.69, size = 205, normalized size = 1.78

$$-\frac{1}{12}\,d^{\frac{5}{2}}\left(\frac{3\,b^2\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}\,a^2d^2}+\frac{2\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-6\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-6\left(\sqrt{d}\,x\right.\right.}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2-c\right)^3a^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*d^(5/2)*(3*b^2*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c
+ 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*a^2*d^2) + 2*(3*
(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
*c - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + 2*a*c*d)/(((sqrt(d
)*x^4 - sqrt(d*x^8 + c))^2 - c)^3*a^2*d^2))
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^13), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{13}\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^13*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{13}\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**13*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.897 $\int \frac{x^9}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=851

$$-\frac{\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16b\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}-\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{}}\right.}{8b^{3/4}\sqrt{bc-ad}}$$

```
[Out] -1/8*(-a)^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/b^(3/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a
)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(3/4)/(a*d-b*c)^(1/2)+1/4*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*
x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/d^(1/4)/(d*x^8+c)^(1/2)-1/8
*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(
b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/
2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)*(c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^
(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1
/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4
)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(
-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
b/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^
(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(
1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 851, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {465, 483, 220, 409, 1217, 1707}

$$-\frac{\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16b\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}-\frac{\sqrt[4]{-a}\,\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{}}\right.}{8b^{3/4}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((-a)^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(8*b^(3/4)*Sqrt[b*c - a*d]) - ((-a)^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(3/4)*Sqrt[-(b*c) + a*d])
+ ((Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elli
pticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*c^(1/4)*d^(1/4)*Sqrt[c +
d*x^8]) - (a*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt
[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^
(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sq
rt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] +
```

```
Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]
*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(
1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sq
rt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2
]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx,x,x\right.}{4b}$$

$$= \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{4b\sqrt[4]{c}\sqrt[4]{d}\sqrt{c+dx^8}} - \frac{\left(\sqrt{c}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\right)\text{Subst}\left(\int\right.}{4(bc+a}$$

$$= -\frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{bc-ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{-a}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8b^{3/4}\sqrt{-bc+ad}} + \frac{\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+}{\left(\sqrt{c}+\right.}}}{4b\sqrt[4]{c}}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.08

$$\frac{x^{10}\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{10a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^10*Sqrt[(c + d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8
)/a)])/(10*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^9/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^9/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**9/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.898 $\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=754

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\tan\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}\,(ad+bc)}$$

```
[Out] -1/8*b^(1/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(3/4)/(-a*d+b*c)^(1/2)-1/8*b^(1/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/(a*d-b*c)^(1/2)+1/8*d^(1/4)*(cos(2
*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*E
llipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(
-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*d^(1/4)*(cos(2*arctan(d^(1/4)
*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*
arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(
1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^
(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^
2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c
^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/
2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(
a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1
/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2
)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^
(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/d^(1/4)/(a*d+b
*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 754, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {465, 409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)F\left(2\tan\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}\,(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(b^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])
])/(8*(-a)^(3/4)*Sqrt[b*c - a*d]) - (b^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2
)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(3/4)*Sqrt[-(b*c) + a*d])
+ (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[-a]*Sqrt[
b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(S
qrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqr
t[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqr
t[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)
*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi
[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]
```

```
), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a}$$

$$= \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(1-\frac{\sqrt{b}x^2}{\sqrt{-a}}\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4a(bc+ad)} + \frac{\left(\sqrt{b}\sqrt{c}\left(\sqrt{b}\sqrt{c}\right.\right.}{}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8(-a)^{3/4}\sqrt{-bc+ad}} + \frac{\left(\frac{\sqrt{b}\sqrt{c}}{\sqrt{-a}}+\sqrt{d}\right)\sqrt[4]{d}\left(\sqrt{c}\right.}{8}$$

Mathematica [C] time = 0.05, size = 65, normalized size = 0.09

$$\frac{x^2\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{2a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[(c + d*x^8)/c]*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)
/a)])/(2*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.899 $\int \frac{1}{x^7\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=878

$$-\frac{b\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{\ldots}}\right)}{8(-a)^{7/4}\sqrt{bc-\ldots}}$$

```
[Out] -1/8*b^(5/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2)
)/(-a)^(7/4)/(-a*d+b*c)^(1/2)-1/8*b^(5/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(
1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*d-b*c)^(1/2)-1/6*(d*x^8+c)^(1/2
)/a/c/x^6-1/12*d^(3/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
a/c^(5/4)/(d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x
^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)-1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*((-a)^(1/2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2
))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(a*d+b*c)/(d*x^8+c
)^(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1
/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8
+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^
(1/2)-1/16*b*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1
/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/
2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+
c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(
1/2)
```

Rubi [A] time = 1.22, antiderivative size = 878, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 480, 523, 220, 409, 1217, 1707}

$$-\frac{b\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{\ldots}}\right)}{8(-a)^{7/4}\sqrt{bc-\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(6*a*c*x^6) - (b^(5/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^
(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(7/4)*Sqrt[b*c - a*d]) - (b^(5/4)*
ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(
-a)^(7/4)*Sqrt[-(b*c) + a*d]) - (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(12*a*c^(5/4)*Sqrt[c + d*x^8]) - (b*((Sqrt[b]*Sqrt[c])/Sqrt[-a] + S
qrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]
*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c
 + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a^2*c^(1/4)*(b*c + a*d)*Sqrt
```

```
[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*
x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sq
rt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a^2*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
```

```
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7 \left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} + \frac{\text{Subst}\left(\int \frac{-3bc-ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2a} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{6ac}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}} - \frac{b\,\text{Subst}\Big(\int}{}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{12ac^{5/4}\sqrt{c+dx^8}} - \frac{\left(b^{3/2}\sqrt{c}\,\left(\right.\right.}{}$$

$$= -\frac{\sqrt{c+dx^8}}{6acx^6} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{bc-ad}} - \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{7/4}\sqrt{-bc+ad}} - \frac{d^{3/4}\left(\sqrt{c}\right.}{}$$

Mathematica [C] time = 0.16, size = 141, normalized size = 0.16

$$\frac{-5x^8\sqrt{\frac{dx^8}{c}+1}\,(ad+3bc)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right) - bdx^{16}\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right) - 5a\left(c+dx^8\right)}{30a^2cx^6\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-5*a*(c + d*x^8) - 5*(3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[1/4, 1
/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] - b*d*x^16*Sqrt[1 + (d*x^8)/c]*Appe
llF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)])/(30*a^2*c*x^6*Sqrt[c +
d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)\sqrt{d\,x^{8}+c}\;x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
[Out] int(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{8}+a\right)\sqrt{dx^{8}+c}\,x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7}\,\left(b\,x^{8}+a\right)\,\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^7*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{7}\left(a+bx^{8}\right)\sqrt{c+dx^{8}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**7*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.900 $\int \frac{x^{13}}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1005

$$-\frac{\sqrt{-a}\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16b^{3/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}+\frac{(-a)^{3/4}\tan^{-1}\left(\right.}{8b^{5/4}\sqrt{b}}$$

```
[Out] 1/8*(-a)^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/
2))/b^(5/4)/(-a*d+b*c)^(1/2)-1/8*(-a)^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)
^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/b^(5/4)/(a*d-b*c)^(1/2)+1/2*x^2*(d*x^8+c)^(
1/2)/b/d^(1/2)/(c^(1/2)+x^4*d^(1/2))-1/2*c^(1/4)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticE(sin(2*arcta
n(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1
/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)+1/4*c^(1/4)*(cos(2*arct
an(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipt
icF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(
(d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/d^(3/4)/(d*x^8+c)^(1/2)-1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(
1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(-a)^(1/2)*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/
2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x
^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^
(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))
*(-a)^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(
d*x^8+c)^(1/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/c
os(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4
))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1
/2)+1/8*a*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)
/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.36, antiderivative size = 1005, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 483, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{-a}\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{16b^{3/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}+\frac{(-a)^{3/4}\tan^{-1}\left(\right.}{8b^{5/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*Sqrt[c + d*x^8])/(2*b*Sqrt[d]*(Sqrt[c] + Sqrt[d]*x^4)) + ((-a)^(3/4)*A
rcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/
4)*Sqrt[b*c - a*d]) - ((-a)^(3/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/
4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*b^(5/4)*Sqrt[-(b*c) + a*d]) - (c^(1/4)*(Sq
rt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[
2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*b*d^(3/4)*Sqrt[c + d*x^8]) + (c^(
1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
```

```
lipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(4*b*d^(3/4)*Sqrt[c + d*x^8]
) + (a*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^
4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*
x^2)/c^(1/4)], 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (a*(Sqrt[c
] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d
*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
 1/2])/(8*b*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + (Sqrt[-a]*(Sqrt[b]*Sqrt[
c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c]
+ Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqr
t[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[-a]*(Sqrt[b]*S
qrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*
Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(
16*b^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{2b}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} - \frac{a\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4b^{3/2}} + \cdots$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{\sqrt[4]{c}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{2bd^{3/4}\sqrt{c+dx^8}} + \frac{\sqrt[4]{c}\cdots}{\cdots}$$

$$= \frac{x^2\sqrt{c+dx^8}}{2b\sqrt{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{bc-ad}} - \frac{(-a)^{3/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8b^{5/4}\sqrt{-bc+ad}}\cdots$$

Mathematica [C] time = 0.07, size = 65, normalized size = 0.06

$$\frac{x^{14}\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{14a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^13/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^14*Sqrt[(c + d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^
8)/a)])/(14*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^13/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13}}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^13/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**13/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.901 $\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=768

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}$$

```
[Out] 1/8*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1
/4)/b^(1/4)/(-a*d+b*c)^(1/2)-1/8*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1
/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(1/4)/(a*d-b*c)^(1/2)-1/16*(cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticP
i(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1/2)+1/16*(co
s(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4))
)*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^
(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x
^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*
d^(1/2))^2)^(1/2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(-a)^(1/2)/b^(1/2)/(d*x^8+c)^(1
/2)-1/8*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/
2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(
c^(1/2)+x^4*d^(1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/8*d^(1/4)
*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^
4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(
1/2))^2)^(1/2)/c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 768, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 490, 1217, 220, 1707}

$$\frac{\tan^{-1}\left(\frac{x^2\sqrt{bc-ad}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}}-\frac{\tan^{-1}\left(\frac{x^2\sqrt{ad-bc}}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{ad-bc}}-\frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}x^4\right)^2}}\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}x^2}{\sqrt[4]{c}}\right.\right.}{8\sqrt[4]{c}\sqrt{c+dx^8}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^
(1/4)*b^(1/4)*Sqrt[b*c - a*d]) - ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4
)*b^(1/4)*Sqrt[c + d*x^8])]/(8*(-a)^(1/4)*b^(1/4)*Sqrt[-(b*c) + a*d]) - ((S
qrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(
c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1
/4)], 1/2])/(8*c^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] + (Sqrt[-a]
*Sqrt[d])/Sqrt[b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c
] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*c^(
1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellipt
icPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sq
rt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1/4)
*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d
])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*El
```

```
lipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]
*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*Sqrt[-a]*Sqrt[b]*c^(1
/4)*d^(1/4)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{(a+bx^8)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx^4)\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}} + \frac{\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4\sqrt{b}}$$

$$= -\frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{d}x^2}{\sqrt{c}}}{\left(\sqrt{-a}-\sqrt{b}x^2\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{4(bc+ad)} + \frac{\left(\sqrt{c}\left(\sqrt{b}\sqrt{c}+\right.\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{bc-ad}} - \frac{\tan^{-1}\left(\frac{\sqrt{-bc+ad}\,x^2}{\sqrt[4]{-a}\sqrt[4]{b}\sqrt{c+dx^8}}\right)}{8\sqrt[4]{-a}\sqrt[4]{b}\sqrt{-bc+ad}} - \frac{\left(\sqrt{c}-\frac{\sqrt{-a}\sqrt{d}}{\sqrt{b}}\right)\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}x^4\right)}{8\sqrt[4]{c}(bc+a}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.08

$$\frac{x^6\sqrt{\frac{c+dx^8}{c}}F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{6a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/((a + b*x^8)*Sqrt[c + d*x^8]),x]
[Out] (x^6*Sqrt[(c + d*x^8)/c]*AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)
/a)])/(6*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
[Out] int(x^5/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx^8)\,\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**5/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.902 $\int \frac{1}{x^3(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1032

$$-\frac{\sqrt{b}\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc}\cdots}{\sqrt[4]{-a}\sqrt[4]{\cdots}}\right)}{8(-a)^{5/4}\sqrt{bc\cdots}}$$

```
[Out] 1/8*b^(3/4)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))
/(-a)^(5/4)/(-a*d+b*c)^(1/2)-1/8*b^(3/4)*arctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1
/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/(a*d-b*c)^(1/2)-1/2*(d*x^8+c)^(1/2)
/a/c/x^2+1/2*x^2*d^(1/2)*(d*x^8+c)^(1/2)/a/c/(c^(1/2)+x^4*d^(1/2))-1/2*d^(1
/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(3/4)/(d*x^8+c)
^(1/2)+1/4*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^
(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^
(3/4)/(d*x^8+c)^(1/2)-1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4)
)),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^
(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d
^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/16*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2
)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*
x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2
)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^
(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/
2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/
c^(1/4)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/8*b*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c
^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/
2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(a*
d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.61, antiderivative size = 1032, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 480, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{b}\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{16(-a)^{3/2}\sqrt[4]{c}\sqrt[4]{d}\,(bc+ad)\sqrt{dx^8+c}}+\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc}\cdots}{\sqrt[4]{-a}\sqrt[4]{\cdots}}\right)}{8(-a)^{5/4}\sqrt{bc\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(2*a*c*x^2) + (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(2*a*c*(Sqrt[c
] + Sqrt[d]*x^4)) + (b^(3/4)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/
4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*Sqrt[b*c - a*d]) - (b^(3/4)*ArcTan[(Sqr
t[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(8*(-a)^(5/4)*S
qrt[-(b*c) + a*d]) - (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
t[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(2*
```

```
a*c^(3/4)*Sqrt[c + d*x^8]) + (d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x
^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1
/2])/(4*a*c^(3/4)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[
b])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4
)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a
*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(S
qrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF
[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*a*c^(1/4)*(b*c + a*d)*Sqrt[c + d
*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt
[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)*Sqrt
[c + d*x^8]) - (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + S
qrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]
*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcT
an[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*(-a)^(3/2)*c^(1/4)*d^(1/4)*(b*c + a*d)
*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} - \frac{bcx^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx,x,x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2a} + \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{2ac}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}-\sqrt{b}\,x^2\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{4a} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\left(\sqrt{-a}+\sqrt{b}\,x\right.}\right.}{4a}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{2ac^{3/4}\sqrt{c+dx^8}}$$

$$= -\frac{\sqrt{c+dx^8}}{2acx^2} + \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{2ac\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{8(-a)^{5/4}\sqrt{bc-ad}} - \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{-b}}{\sqrt[4]{-a}\,\sqrt[4]{}}\right.}{8(-a)^{5/4}\sqrt{-bc}}$$

Mathematica [C] time = 0.13, size = 141, normalized size = 0.14

$$\frac{7x^8\sqrt{\frac{dx^8}{c}+1}\,(ad-bc)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+3bdx^{16}\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-21a\left(c+dx^8\right)}{42a^2cx^2\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-21*a*(c + d*x^8) + 7*(-(b*c) + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)])/(42*a^2*c*x^2*Sqrt
[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)\sqrt{dx^8+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(a + bx^8\right) \sqrt{c + dx^8}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.903 $\int \frac{x^4}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,1,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^5\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 1, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};1,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 1.02

$$\frac{x^5\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{5a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[(c + d*x^8)/c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8
)/a)])/(5*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^8+a\right)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^8+a\right)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^8+a\right)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.904 $\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,1,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x
^8+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 1, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};1,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.02

$$\frac{x^3\sqrt{\frac{c+dx^8}{c}}\,F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{3a\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[(c + d*x^8)/c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8
)/a)])/(3*a*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^8+a)\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^8+a)\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+b x^8\right) \sqrt{c+d x^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(x**2/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.905 $\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,1,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/(d*x^8+c)^(
1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 1, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a*Sqrt[c + d*x^8])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx^8)\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{(a+bx^8)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};1,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.30, size = 161, normalized size = 2.73

$$-\frac{9acxF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)}{(a+bx^8)\sqrt{c+dx^8}\left(4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)-9acF_1\left(\frac{1}{8};\frac{1}{2},1;\frac{9}{8};-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-9*a*c*x*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)])/((a + b*x
^8)*Sqrt[c + d*x^8]*(-9*a*c*AppellF1[1/8, 1/2, 1, 9/8, -((d*x^8)/c), -((b*x
^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/
a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^8 + a)\,\sqrt{d\,x^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8 + a)\sqrt{dx^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b\,x^8 + a)\,\sqrt{d\,x^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.906 $\int \frac{1}{x^2(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,1,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x/(d*x^8+c)
^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 1, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a*x*Sqrt[c + d*x^8]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= -\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{1}{8};1,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{ax\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.15, size = 141, normalized size = 2.27

$$\frac{-5x^8\sqrt{\frac{dx^8}{c}+1}\,(bc-3ad)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7bdx^{16}\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{15}{8};\frac{1}{2},1;\frac{23}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-35a\left(c+dx^8\right)}{35a^2cx\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-35*a*(c + d*x^8) - 5*(b*c - 3*a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[7/8,
1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*x^16*Sqrt[1 + (d*x^8)/c]*
AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c), -((b*x^8)/a)])/(35*a^2*c*x*Sqrt[
c + d*x^8])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^8+c}}{bdx^{18}+(bc+ad)x^{10}+acx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b*d*x^18 + (b*c + a*d)*x^10 + a*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx^8+a)\sqrt{dx^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\,x^8+a)\,\sqrt{d\,x^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx^8+a)\sqrt{dx^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x^2\,(b\,x^8+a)\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^8\right) \sqrt{c + dx^8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a)/(d*x**8+c)**(1/2),x)
[Out] Integral(1/(x**2*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.907 $\int \frac{1}{x^4(a+bx^8)\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,1,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a/x^3/(d*
x^8+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 1, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a*x^3*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4\left(a+bx^8\right)\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= -\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};1,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3ax^3\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.14, size = 141, normalized size = 2.20

$$\frac{13x^8\sqrt{\frac{dx^8}{c}+1}\,(ad-3bc)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5bdx^{16}\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-65a\left(c+dx\right.}{195a^2cx^3\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^8)*Sqrt[c + d*x^8]),x]
```

```
[Out] (-65*a*(c + d*x^8) + 13*(-3*b*c + a*d)*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8
, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*x^16*Sqrt[1 + (d*x^8)/c
]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b*x^8)/a)])/(195*a^2*c*x^3*
Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)\sqrt{dx^8 + c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8 + a\right)\sqrt{d\,x^8 + c}\;x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)\sqrt{dx^8 + c}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)*sqrt(d*x^8 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4\,\left(b\,x^8 + a\right)\,\sqrt{d\,x^8 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^8)*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + bx^8\right) \sqrt{c + dx^8}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**4*(a + b*x**8)*sqrt(c + d*x**8)), x)
```

3.908 $\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

```
[Out] 1/8*a*(-3*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(5
/2)/(-a*d+b*c)^(3/2)+1/4*(d*x^8+c)^(1/2)/b^2/d-1/8*a^2*(d*x^8+c)^(1/2)/b^2/
(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.14, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {446, 89, 80, 63, 208}

$$-\frac{a^2\sqrt{c+dx^8}}{8b^2\left(a+bx^8\right)(bc-ad)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx^8}}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] Sqrt[c + d*x^8]/(4*b^2*d) - (a^2*Sqrt[c + d*x^8])/(8*b^2*(b*c - a*d)*(a + b
*x^8)) + (a*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*
d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{23}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{8}\text{Subst}\left(\int \frac{x^2}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)\\
&= -\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(2bc-ad)+b(bc-ad)x}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{8b^2(bc-ad)}\\
&= \frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,\right.}{16b^2(bc-ad)}\\
&= \frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}-\frac{(a(4bc-3ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c}\right.}{8b^2d(bc-ad)}\\
&= \frac{\sqrt{c+dx^8}}{4b^2d}-\frac{a^2\sqrt{c+dx^8}}{8b^2(bc-ad)\left(a+bx^8\right)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.26, size = 107, normalized size = 0.87

$$\frac{1}{8}\left(\frac{\sqrt{c+dx^8}\left(\frac{a^2}{\left(a+bx^8\right)(ad-bc)}+\frac{2}{d}\right)}{b^2}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{b^{5/2}(bc-ad)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^23/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((Sqrt[c + d*x^8]*(2/d + a^2/((-(b*c) + a*d)*(a + b*x^8))))/b^2 + (a*(4*b*c
 - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b^(5/2)*(b*c
 - a*d)^(3/2)))/8
```

fricas [B] time = 0.77, size = 475, normalized size = 3.86

$$\left[\frac{\left(\left(4\,ab^2cd-3\,a^2bd^2\right)x^8+4\,a^2bcd-3\,a^3d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\left(2\left(b^4c^2-2\,ab^3c\right.\right.}{16\left(ab^5c^2d-2\,a^2b^4cd^2+a^3b^3d^3+\left(b^6c^2d-2\,ab^5cd^2+a^2b^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(b^2
*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b
*d))/(b*x^8 + a)) + 2*(2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^
3*c^2 - 5*a^2*b^2*c*d + 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*
b^4*c*d^2 + a^3*b^3*d^3 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8), -
1/8*(((4*a*b^2*c*d - 3*a^2*b*d^2)*x^8 + 4*a^2*b*c*d - 3*a^3*d^2)*sqrt(-b^2*
c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) - (
2*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + 2*a*b^3*c^2 - 5*a^2*b^2*c*d +
 3*a^3*b*d^2)*sqrt(d*x^8 + c))/(a*b^5*c^2*d - 2*a^2*b^4*c*d^2 + a^3*b^3*d^3
 + (b^6*c^2*d - 2*a*b^5*c*d^2 + a^2*b^4*d^3)*x^8)]
```

giac [A] time = 0.18, size = 134, normalized size = 1.09

$$-\frac{\sqrt{dx^8+c}\,a^2d}{8\left(b^3c-ab^2d\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{\left(4\,abc-3\,a^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^3c-ab^2d\right)\sqrt{-b^2c+abd}}+\frac{\sqrt{dx^8+c}}{4\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(d*x^8 + c)*a^2*d/((b^3*c - a*b^2*d)*((d*x^8 + c)*b - b*c + a*d))
- 1/8*(4*a*b*c - 3*a^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
b^3*c - a*b^2*d)*sqrt(-b^2*c + a*b*d)) + 1/4*sqrt(d*x^8 + c)/(b^2*d)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^{23}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^23/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 5.02, size = 144, normalized size = 1.17

$$\frac{\sqrt{d\,x^8+c}}{4\,b^2\,d}-\frac{a\,\mathrm{atan}\left(\frac{a\,\sqrt{b}\,\sqrt{d\,x^8+c}\,\left(3\,a\,d-4\,b\,c\right)}{\left(3\,a^2\,d-4\,a\,b\,c\right)\,\sqrt{a\,d-b\,c}}\right)\,\left(3\,a\,d-4\,b\,c\right)}{8\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{a^2\,d\,\sqrt{d\,x^8+c}}{2\,\left(a\,d-b\,c\right)\,\left(4\,b^3\,\left(d\,x^8+c\right)-4\,b^3\,c+4\,a\,b^2\,d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^23/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (c + d*x^8)^(1/2)/(4*b^2*d) - (a*atan((a*b^(1/2)*(c + d*x^8)^(1/2)*(3*a*d -
 4*b*c))/((3*a^2*d - 4*a*b*c)*(a*d - b*c)^(1/2)))*(3*a*d - 4*b*c))/(8*b^(5/
```

```
2)*(a*d - b*c)^(3/2)) + (a^2*d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b^3*(c
+ d*x^8) - 4*b^3*c + 4*a*b^2*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**23/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.909 $\int \frac{x^{15}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=99

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*(-a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/b^(3/2)
/(-a*d+b*c)^(3/2)+1/8*a*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 208}

$$\frac{a\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}-\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - ((2*b*c - a*d)*ArcTanh[
(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{15}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$
$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{16b(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{(2bc-ad)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{8bd(bc-ad)}$$
$$= \frac{a\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.09, size = 98, normalized size = 0.99

$$\frac{\frac{a\sqrt{b}\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{(ad-2bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^15/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[b]*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + ((-2*b*c + a*d)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*b^(
3/2))
```

fricas [A] time = 1.22, size = 348, normalized size = 3.52

$$\left[\frac{\left(\left(2\,b^2c-abd\right)x^8+2\,abc-a^2d\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,\left(ab^2c-a^2bd\right)}{16\left(\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^8+ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)}, \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((2*b^2*c - a*b*d)*x^8 + 2*a*b*c - a^2*d)*sqrt(b^2*c - a*b*d)*log((b
*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a))
+ 2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*
d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2), 1/8*(((2*b^2*c - a*b*d
)*x^8 + 2*a*b*c - a^2*d)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(d*x^8 + c)*sqrt(-
b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d))/((b^
5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^8 + a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^
2*d^2)]
```

giac [A] time = 0.17, size = 116, normalized size = 1.17

$$\frac{\frac{\sqrt{dx^8+c}\,ad^2}{\left(b^2c-abd\right)\left(\left(dx^8+c\right)b-bc+ad\right)} + \frac{\left(2\,bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c-abd\right)\sqrt{-b^2c+abd}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(d*x^8 + c)*a*d^2/((b^2*c - a*b*d)*((d*x^8 + c)*b - b*c + a*d)) +
(2*b*c*d - a*d^2)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c -
a*b*d)*sqrt(-b^2*c + a*b*d)))/d
```

maple [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^{15}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^15/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.84, size = 95, normalized size = 0.96

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{d\,x^8+c}}{\sqrt{a\,d-b\,c}}\right)\,\left(a\,d-2\,b\,c\right)}{8\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{a\,d\,\sqrt{d\,x^8+c}}{2\,b\,\left(a\,d-b\,c\right)\,\left(4\,b\,\left(d\,x^8+c\right)+4\,a\,d-4\,b\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^15/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2))*(a*d - 2*b*c))/(8*b^(3
/2)*(a*d - b*c)^(3/2)) - (a*d*(c + d*x^8)^(1/2))/(2*b*(a*d - b*c)*(4*b*(c +
 d*x^8) + 4*a*d - 4*b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**15/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.910 $\int \frac{x^7}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=87

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b} \, (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

```
[Out] 1/8*d*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/(-a*d+b*c)^(3/2)/b^
(1/2)-1/8*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {444, 51, 63, 208}

$$\frac{d \tanh^{-1}\left(\frac{\sqrt{b} \sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b} \, (bc-ad)^{3/2}} - \frac{\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -Sqrt[c + d*x^8]/(8*(b*c - a*d)*(a + b*x^8)) + (d*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{x^7}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{8}\text{Subst}\left(\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)$$
$$=-\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{d\,\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx,x,x^8\right)}{16(bc-ad)}$$
$$=-\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx^8}\right)}{8(bc-ad)}$$
$$=-\frac{\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.98

$$\frac{1}{8}\left(\frac{\sqrt{c+dx^8}}{\left(a+bx^8\right)(ad-bc)}+\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]/((-(b*c) + a*d)*(a + b*x^8)) + (d*ArcTan[(Sqrt[b]*Sqrt[c +
 d*x^8])/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2)))/8
```

fricas [B] time = 1.06, size = 302, normalized size = 3.47

$$\left[-\frac{\left(bdx^8+ad\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx^8+2\,bc-ad-2\,\sqrt{dx^8+c}\,\sqrt{b^2c-abd}}{bx^8+a}\right)+2\,\sqrt{dx^8+c}\,\left(b^2c-abd\right)}{16\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^8+ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)},-\frac{\left(bdx^8+ad\right)\sqrt{-b^2c+abd}\cdots}{8\left(\left(b^4c^2-2\,ab^3c\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*d*x^8 + a*d)*sqrt(b^2*c - a*b*d)*log((b*d*x^8 + 2*b*c - a*d - 2*
sqrt(d*x^8 + c)*sqrt(b^2*c - a*b*d))/(b*x^8 + a)) + 2*sqrt(d*x^8 + c)*(b^2*
c - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*
b^2*c*d + a^3*b*d^2), -1/8*((b*d*x^8 + a*d)*sqrt(-b^2*c + a*b*d)*arctan(sqr
t(d*x^8 + c)*sqrt(-b^2*c + a*b*d)/(b*d*x^8 + b*c)) + sqrt(d*x^8 + c)*(b^2*c
 - a*b*d))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^8 + a*b^3*c^2 - 2*a^2*b
^2*c*d + a^3*b*d^2)]
```

giac [A] time = 0.16, size = 93, normalized size = 1.07

$$-\frac{d\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx^8+c}\,d}{8\left(\left(dx^8+c\right)b-bc+ad\right)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)
*(b*c - a*d)) - 1/8*sqrt(d*x^8 + c)*d/(((d*x^8 + c)*b - b*c + a*d)*(b*c - a
*d))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(b x^8 + a\right)^2 \sqrt{d x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.80, size = 84, normalized size = 0.97

$$\frac{d \sqrt{d x^8 + c}}{2\,(a d - b c)\,\left(4 b\,\left(d x^8 + c\right) + 4 a d - 4 b c\right)} + \frac{d \operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{d x^8+c}}{\sqrt{a d - b c}}\right)}{8 \sqrt{b}\,(a d - b c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (d*(c + d*x^8)^(1/2))/(2*(a*d - b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c)) + (
d*atan((b^(1/2)*(c + d*x^8)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.911 $\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=132

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

```
[Out] 1/8*(-3*a*d+2*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))*b^(1/2
)/a^2/(-a*d+b*c)^(3/2)-1/4*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^2/c^(1/2)+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.14, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {446, 103, 156, 63, 208}

$$\frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^2(bc-ad)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}}+\frac{b\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ArcTanh[Sqrt[c + d*x^8]
/Sqrt[c]]/(4*a^2*Sqrt[c]) + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^2*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{x(a+bx)^2\sqrt{c+dx}}\,dx, x, x^8\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{bc-ad+\frac{bdx}{2}}{x(a+bx)\sqrt{c+dx}}\,dx, x, x^8\right)}{8a(bc-ad)}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, x^8\right)}{8a^2} - \frac{(b(2bc-3ad))\,\text{Subst}\left(\int\right.}{16a^2(bc-}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx^8}\right)}{4a^2d} - \frac{(b(2bc-3ad))\,\text{Sul}}{}$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{4a^2\sqrt{c}} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right.}{8a^2(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.36, size = 123, normalized size = 0.93

$$\frac{\frac{ab\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{\sqrt{b}\,(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) - (2*ArcTanh[Sqrt[c + d*x^
8]/Sqrt[c]])/Sqrt[c] + (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d
*x^8])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2))/(8*a^2)
```

fricas [A] time = 1.22, size = 862, normalized size = 6.53

$$\left[\frac{2\sqrt{dx^8+c}\,abc + \left(\left(2\,b^2c^2 - 3\,abcd\right)x^8 + 2\,abc^2 - 3\,a^2cd\right)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx^8+2\,bc-ad+2\,\sqrt{dx^8+c}\,(bc-ad)\sqrt{\frac{b}{bc-ad}}}{bx^8+a}\right) + 2\left(\left(b\right.\right.}{16\left(\left(a^2b^2c^2 - a^3bcd\right)x^8 + a^3bc^2 - a^4cd\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(d*x^8 + c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 -
 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d + 2*sqrt(d*x^8 +
 c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + 2*((b^2*c - a*b*d)*x^8
```

```
+ a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8)
)/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 +
c)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b
*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^
8 + b*c)) + ((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(c)*log((d*x^8 - 2*sq
rt(d*x^8 + c)*sqrt(c) + 2*c)/x^8))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c
^2 - a^4*c*d), 1/16*(2*sqrt(d*x^8 + c)*a*b*c + 4*((b^2*c - a*b*d)*x^8 + a*b
*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) + ((2*b^2*c^2 - 3*a
*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b
*c - a*d + 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)))
/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d), 1/8*(sqrt(d*x^8 + c
)*a*b*c + ((2*b^2*c^2 - 3*a*b*c*d)*x^8 + 2*a*b*c^2 - 3*a^2*c*d)*sqrt(-b/(b*
c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8
 + b*c)) + 2*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-c)*arctan(sqrt(d*x
^8 + c)*sqrt(-c)/c))/((a^2*b^2*c^2 - a^3*b*c*d)*x^8 + a^3*b*c^2 - a^4*c*d)]
```

giac [A] time = 0.21, size = 139, normalized size = 1.05

$$\frac{\sqrt{dx^8+c}\,bd}{8\left(abc-a^2d\right)\left(\left(dx^8+c\right)b-bc+ad\right)}-\frac{\left(2\,b^2c-3\,abd\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(a^2bc-a^3d\right)\sqrt{-b^2c+abd}}+\frac{\arctan\left(\frac{\sqrt{dx^8+c}}{\sqrt{-c}}\right)}{4\,a^2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(d*x^8 + c)*b*d/((a*b*c - a^2*d)*((d*x^8 + c)*b - b*c + a*d)) - 1/8
*(2*b^2*c - 3*a*b*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((a^2*b
*c - a^3*d)*sqrt(-b^2*c + a*b*d)) + 1/4*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a
^2*sqrt(-c))
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x), x)
```

mupad [B] time = 5.82, size = 3017, normalized size = 22.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] (atan((((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^
3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d^5 - (3*a^5*b^
3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d) - (
(c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b^2*d^5
 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(512*(
a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2
*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^
3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a
*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4
*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^
4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^2*d^5 - (3*
a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*
d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(256*a^7*b
^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/
(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b
^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(
a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*
(-b*(a*d - b*c)^3)^(1/2)*1i)/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d -
 3*a^4*b*c*d^2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*
c^2 - 2*a^4*b*c*d) - ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((a^6*b^2*d
^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 - 2*
a^4*b*c*d) - ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*(2
56*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^
2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^3 +
 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2
))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d -
 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^
2*d - 3*a^4*b*c*d^2)) + ((((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^
2 - 20*a*b^4*c*d^3))/(32*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((a^6*b^
2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(a^5*d^2 + a^3*b^2*c^2 -
 2*a^4*b*c*d) + ((c + d*x^8)^(1/2)*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)
*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4
*c^2*d^3))/(512*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^3 - a^2*b^3*c^
3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(
1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)))*(3*a*
d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2))/(16*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2
*c^2*d - 3*a^4*b*c*d^2))))*(3*a*d - 2*b*c)*(-b*(a*d - b*c)^3)^(1/2)*1i)/(8*
(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (atan((((((a^6
*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2
*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c
*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*
d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c + d*x^8)^(1/2)*(13
*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2
 - 2*a^3*b*c*d)))*1i)/(a^2*c^(1/2)) - ((((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2
 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((c + d
*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1
280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*
d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 -
 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))*1i)/(a^2*c^(
1/2)))/(((3*a*b^3*d^4)/128 - (b^4*c*d^3)/64)/(a^5*d^2 + a^3*b^2*c^2 - 2*a^4
*b*c*d) + (((a^6*b^2*d^5 - (3*a^5*b^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a
^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5
- 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a
^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) - ((c +
d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d
^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(a^2*c^(1/2)) + (((a^6*b^2*d^5 - (3*a^5*b
^3*c*d^4)/2 + (a^4*b^4*c^2*d^3)/2)/(8*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)
) + ((c + d*x^8)^(1/2)*(256*a^7*b^2*d^5 - 1024*a^6*b^3*c*d^4 - 512*a^4*b^5*
```

```
c^3*d^2 + 1280*a^5*b^4*c^2*d^3))/(2048*a^2*c^(1/2)*(a^4*d^2 + a^2*b^2*c^2 -
 2*a^3*b*c*d)))/(8*a^2*c^(1/2)) + ((c + d*x^8)^(1/2)*(13*a^2*b^3*d^4 + 8*b^
5*c^2*d^2 - 20*a*b^4*c*d^3))/(256*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)))/(
a^2*c^(1/2))))*1i)/(4*a^2*c^(1/2)) - (b*d*(c + d*x^8)^(1/2))/(2*(a^2*d - a*
b*c)*(4*b*(c + d*x^8) + 4*a*d - 4*b*c))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.912 $\int \frac{1}{x^9(a+bx^8)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=185

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}(2bc-ad)}{8a^2c(a+bx^8)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)}$$

```
[Out] 1/8*(a*d+4*b*c)*arctanh((d*x^8+c)^(1/2)/c^(1/2))/a^3/c^(3/2)-1/8*b^(3/2)*(-
5*a*d+4*b*c)*arctanh(b^(1/2)*(d*x^8+c)^(1/2)/(-a*d+b*c)^(1/2))/a^3/(-a*d+b*
c)^(3/2)-1/8*b*(-a*d+2*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(b*x^8+a)-1/8*
(d*x^8+c)^(1/2)/a/c/x^8/(b*x^8+a)
```

Rubi [A] time = 0.23, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 103, 151, 156, 63, 208}

$$-\frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{8a^3(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}}-\frac{b\sqrt{c+dx^8}(2bc-ad)}{8a^2c(a+bx^8)(bc-ad)}-\frac{\sqrt{c+dx^8}}{8acx^8(a+bx^8)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(b*(2*b*c - a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(a + b*x^8)) - Sqrt
[c + d*x^8]/(8*a*c*x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8
]/Sqrt[c]])/(8*a^3*c^(3/2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x^8])/Sqrt[b*c - a*d]])/(8*a^3*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x^9\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{8}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)$$

$$= -\frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4bc+ad)+\frac{3bdx}{2}}{x(a+bx)^2\sqrt{c+dx}}\,dx,x,x^8\right)}{8ac}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-ad)(4bc+ad)+\frac{1}{2}bd(2bc-ad}{x(a+bx)\sqrt{c+dx}}\right.}{8a^2c(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{(a+bx)\sqrt{c\cdot}}\right.}{16a^3(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{\left(b^2(4bc-5ad)\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\right.}{8a^3d(bc-ad)}$$

$$= -\frac{b(2bc-ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{c+dx^8}}{8acx^8\left(a+bx^8\right)} + \frac{(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{8a^3c^{3/2}} - \frac{b^{3/2}}{}$$

Mathematica [A] time = 0.60, size = 163, normalized size = 0.88

$$\frac{\frac{a\sqrt{c+dx^8}\left(a^2d+ab\left(dx^8-c\right)-2b^2cx^8\right)}{x^8\left(a+bx^8\right)(bc-ad)}+\frac{b^{3/2}c(5ad-4bc)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx^8}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^8}}{\sqrt{c}}\right)}{\sqrt{c}}}{8a^3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^9*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*Sqrt[c + d*x^8]*(a^2*d - 2*b^2*c*x^8 + a*b*(-c + d*x^8)))/((b*c - a*d)*
x^8*(a + b*x^8)) + ((4*b*c + a*d)*ArcTanh[Sqrt[c + d*x^8]/Sqrt[c]])/Sqrt[c]
```

```
+ (b^(3/2)*c*(-4*b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^8])/Sqrt[b*c -
a*d]])/(b*c - a*d)^(3/2))/(8*a^3*c)
```

fricas [A] time = 1.27, size = 1236, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^
8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c
- a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*
b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(c)*log((d*x^8
 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) - 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8
 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16
+ (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
+ (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-b/(b*c - a*d))*arctan(-sqrt(d*x^
8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*d*x^8 + b*c)) - ((4*b^3*c^2 - 3*
a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sq
rt(c)*log((d*x^8 + 2*sqrt(d*x^8 + c)*sqrt(c) + 2*c)/x^8) + 2*((2*a*b^2*c^2
- a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a^3*b^2*c^3 - a^
4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/16*(2*((4*b^3*c^2 - 3*a*
b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2 - 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt
(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) - ((4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16
 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(b/(b*c - a*d))*log((b*d*x^8 + 2*
b*c - a*d - 2*sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(b/(b*c - a*d)))/(b*x^8 + a))
 + 2*((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))
/((a^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8), -1/8*(((
4*b^3*c^3 - 5*a*b^2*c^2*d)*x^16 + (4*a*b^2*c^3 - 5*a^2*b*c^2*d)*x^8)*sqrt(-
b/(b*c - a*d))*arctan(-sqrt(d*x^8 + c)*(b*c - a*d)*sqrt(-b/(b*c - a*d))/(b*
d*x^8 + b*c)) + ((4*b^3*c^2 - 3*a*b^2*c*d - a^2*b*d^2)*x^16 + (4*a*b^2*c^2
- 3*a^2*b*c*d - a^3*d^2)*x^8)*sqrt(-c)*arctan(sqrt(d*x^8 + c)*sqrt(-c)/c) +
 ((2*a*b^2*c^2 - a^2*b*c*d)*x^8 + a^2*b*c^2 - a^3*c*d)*sqrt(d*x^8 + c))/((a
^3*b^2*c^3 - a^4*b*c^2*d)*x^16 + (a^4*b*c^3 - a^5*c^2*d)*x^8)]
```

giac [A] time = 0.17, size = 257, normalized size = 1.39

$$\frac{\left(4\,b^3c - 5\,ab^2d\right)\arctan\left(\frac{\sqrt{dx^8+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(a^3bc - a^4d\right)\sqrt{-b^2c+abd}} - \frac{2\left(dx^8+c\right)^{\frac{3}{2}}b^2cd - 2\,\sqrt{dx^8+c}\,b^2c^2d - \left(dx^8+c\right)^{\frac{3}{2}}abd^2 + 2\,\sqrt{dx^8+c}\,abcd}{8\left(a^2bc^2 - a^3cd\right)\left(\left(dx^8+c\right)^2b - 2\left(dx^8+c\right)bc + bc^2 + \left(dx^8+c\right)a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*b^3*c - 5*a*b^2*d)*arctan(sqrt(d*x^8 + c)*b/sqrt(-b^2*c + a*b*d))/((
a^3*b*c - a^4*d)*sqrt(-b^2*c + a*b*d)) - 1/8*(2*(d*x^8 + c)^(3/2)*b^2*c*d -
 2*sqrt(d*x^8 + c)*b^2*c^2*d - (d*x^8 + c)^(3/2)*a*b*d^2 + 2*sqrt(d*x^8 + c
)*a*b*c*d^2 - sqrt(d*x^8 + c)*a^2*d^3)/((a^2*b*c^2 - a^3*c*d)*((d*x^8 + c)^
2*b - 2*(d*x^8 + c)*b*c + b*c^2 + (d*x^8 + c)*a*d - a*c*d)) - 1/8*(4*b*c +
a*d)*arctan(sqrt(d*x^8 + c)/sqrt(-c))/(a^3*sqrt(-c)*c)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^9), x)
```

mupad [B] time = 7.85, size = 3832, normalized size = 20.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^9*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] (((c + d*x^8)^(1/2)*(a^2*d^3 + 2*b^2*c^2*d - 2*a*b*c*d^2))/(2*a^2*(b*c^2 -
a*c*d)) + (b*(c + d*x^8)^(3/2)*(a*d^2 - 2*b*c*d))/(2*a^2*(b*c^2 - a*c*d)))/
((c + d*x^8)*(4*a*d - 8*b*c) + 4*b*(c + d*x^8)^2 + 4*b*c^2 - 4*a*c*d) + (at
an((((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^
3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^
2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((-b^3*(a*d - b*
c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b
^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*
d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3
*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a
^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*
c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((c
+ d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4
*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*
d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2 + a^6
*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*
c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/2)*(5
*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^
3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3
*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d
^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*1i)/(16*(a^6*d^3 - a^
3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^6)/256 + (b^7
*c^3*d^3)/8 - (3*a*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 +
a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*
(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^
3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b
*c^3*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(((a^9*b^2*c*d^6)/2
+ a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 +
 a^8*c^2*d^2 - 2*a^7*b*c^3*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x^8)^(1/
2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*
b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(
a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))/(16*(a^6*d^3 -
a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/
2)*(5*a*d - 4*b*c)*(((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a
*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^
```

```
6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*(
((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5
)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((-b^3*(a*d - b*c)^3)^(1
/2)*(c + d*x^8)^(1/2)*(5*a*d - 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c
^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*(a^4*b^2*c^4 + a
^6*c^2*d^2 - 2*a^5*b*c^3*d)*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^
5*b*c*d^2))))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
)))/(16*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(-b^3*
(a*d - b*c)^3)^(1/2)*(5*a*d - 4*b*c)*1i)/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)) + (atan(((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32
*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(3
2*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6
*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*
c^2*d^2 - 2*a^7*b*c^3*d) - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^
5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))
/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d +
4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c)*1i)/(16*a^3*(c^3)^(1/2)) + ((((
c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b
^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^
3*d)) - ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b
^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(
1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b
^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*
c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*
c)*1i)/(16*a^3*(c^3)^(1/2)))/(((5*a^3*b^4*d^6)/256 + (b^7*c^3*d^3)/8 - (3*a
*b^6*c^2*d^4)/16 + (3*a^2*b^5*c*d^5)/128)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^
7*b*c^3*d) - ((((c + d*x^8)^(1/2)*(a^4*b^3*d^6 + 32*b^7*c^4*d^2 - 64*a*b^6*
c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d^4))/(32*(a^4*b^2*c^4 + a^6*c^2
*d^2 - 2*a^5*b*c^3*d)) + ((((a^9*b^2*c*d^6)/2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4
*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4 + a^8*c^2*d^2 - 2*a^7*b*c^3*d)
 - ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6*b^5*c^5*d^2 - 1280*a^7*b^4*c^4
*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^2*d^5))/(512*a^3*(c^3)^(1/2)*(a
^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1
/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)) + ((((c + d*x^8)^(1/2)*(a^4*b^3*d
^6 + 32*b^7*c^4*d^2 - 64*a*b^6*c^3*d^3 + 6*a^3*b^4*c*d^5 + 26*a^2*b^5*c^2*d
^4))/(32*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)) - ((((a^9*b^2*c*d^6)/
2 + a^6*b^5*c^4*d^3 - 2*a^7*b^4*c^3*d^4 + (a^8*b^3*c^2*d^5)/2)/(a^6*b^2*c^4
 + a^8*c^2*d^2 - 2*a^7*b*c^3*d) + ((c + d*x^8)^(1/2)*(a*d + 4*b*c)*(512*a^6
*b^5*c^5*d^2 - 1280*a^7*b^4*c^4*d^3 + 1024*a^8*b^3*c^3*d^4 - 256*a^9*b^2*c^
2*d^5))/(512*a^3*(c^3)^(1/2)*(a^4*b^2*c^4 + a^6*c^2*d^2 - 2*a^5*b*c^3*d)))*
(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2)))*(a*d + 4*b*c))/(16*a^3*(c^3)^(1/2))))*
(a*d + 4*b*c)*1i)/(8*a^3*(c^3)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.913 $\int \frac{x^{19}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=141

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

```
[Out] -1/8*(-2*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))*a^
(1/2)/b^2/(-a*d+b*c)^(3/2)+1/4*arctanh(x^4*d^(1/2)/(d*x^8+c)^(1/2))/b^2/d^(
1/2)+1/8*a*x^4*(d*x^8+c)^(1/2)/b/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 470, 523, 217, 206, 377, 205}

$$-\frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}}+\frac{ax^4\sqrt{c+dx^8}}{8b\left(a+bx^8\right)(bc-ad)}+\frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a*x^4*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(a + b*x^8)) - (Sqrt[a]*(3*b*c - 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*b^2*(b*c
- a*d)^(3/2)) + ArcTanh[(Sqrt[d]*x^4)/Sqrt[c + d*x^8]]/(4*b^2*Sqrt[d])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\int \frac{x^{19}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{x^4}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$
$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{ac-2(bc-ad)x^2}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8b(bc-ad)}$$
$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, x^4\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{}{(a}\right.}{8b^2(bc-}$$
$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x^4}{\sqrt{c+dx^8}}\right)}{4b^2} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int\right.}{8b^2(}$$
$$= \frac{ax^4\sqrt{c+dx^8}}{8b(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt{a}\,(3bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8b^2(bc-ad)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{d}\,x^4}{\sqrt{c+dx^8}}\right)}{4b^2\sqrt{d}}$$

Mathematica [A] time = 0.26, size = 135, normalized size = 0.96

$$\frac{\frac{abx^4\sqrt{c+dx^8}}{\left(a+bx^8\right)(bc-ad)} + \frac{\sqrt{a}\,(2ad-3bc)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{(bc-ad)^{3/2}} + \frac{2\log\left(\sqrt{d}\,\sqrt{c+dx^8}+dx^4\right)}{\sqrt{d}}}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^19/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] ((a*b*x^4*Sqrt[c + d*x^8])/((b*c - a*d)*(a + b*x^8)) + (Sqrt[a]*(-3*b*c + 2
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(b*c - a*d)^
(3/2) + (2*Log[d*x^4 + Sqrt[d]*Sqrt[c + d*x^8]])/Sqrt[d])/(8*b^2)
```

fricas [A] time = 1.53, size = 1077, normalized size = 7.64

$$\left[\frac{4\sqrt{dx^8+c}\,abdx^4 + 4\left(\left(b^2c-abd\right)x^8+abc-a^2d\right)\sqrt{d}\,\log\left(-2\,dx^8-2\sqrt{dx^8+c}\,\sqrt{d}\,x^4-c\right) + \left(\left(3\,b^2cd-2\,abd\right.\right.}{32\left(\left(b^4cd-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^4 + 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d
)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 - c) + ((3*b^2*c*d -
 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt(-a/(b*c - a*d))*log(((b^2*c^2
 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 -
4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (a*b*c^2 - a^2*c*d)*x^4)*sqrt(d
*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((b^4*c*d -
a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/32*(4*sqrt(d*x^8 + c)*a*b*d*x^
4 - 8*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arctan(sqrt(-d)*x^4/sq
rt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d - 2*a^2*d^2)*sqrt
(-a/(b*c - a*d))*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2
 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b^2*c^2 - 3*a*b*c*d + 2*a^2*d^2)*x^12 - (
a*b*c^2 - a^2*c*d)*x^4)*sqrt(d*x^8 + c)*sqrt(-a/(b*c - a*d)))/(b^2*x^16 + 2
*a*b*x^8 + a^2)))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/
16*(2*sqrt(d*x^8 + c)*a*b*d*x^4 + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)) + 2*((b^2*c - a*b*d)*
x^8 + a*b*c - a^2*d)*sqrt(d)*log(-2*d*x^8 - 2*sqrt(d*x^8 + c)*sqrt(d)*x^4 -
 c))/((b^4*c*d - a*b^3*d^2)*x^8 + a*b^3*c*d - a^2*b^2*d^2), 1/16*(2*sqrt(d*
x^8 + c)*a*b*d*x^4 - 4*((b^2*c - a*b*d)*x^8 + a*b*c - a^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x^4/sqrt(d*x^8 + c)) + ((3*b^2*c*d - 2*a*b*d^2)*x^8 + 3*a*b*c*d
- 2*a^2*d^2)*sqrt(a/(b*c - a*d))*arctan(-1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt
(d*x^8 + c)*sqrt(a/(b*c - a*d))/(a*d*x^12 + a*c*x^4)))/((b^4*c*d - a*b^3*d^
2)*x^8 + a*b^3*c*d - a^2*b^2*d^2)]
```

giac [B] time = 0.65, size = 298, normalized size = 2.11

$$-\frac{\left(3\,abc\sqrt{d}-2\,a^{2}d^{\frac{3}{2}}\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}b-bc+2\,ad}{2\,\sqrt{abcd-a^{2}d^{2}}}\right)}{8\,\left(b^{3}c-ab^{2}d\right)\sqrt{abcd-a^{2}d^{2}}}-\frac{\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}abc\sqrt{d}-2\left(\sqrt{d}\,x^{4}\right.}{4\left(\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{4}b-2\left(\sqrt{d}\,x^{4}-\sqrt{dx^{8}+c}\right)^{2}bc+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*a*b*c*sqrt(d) - 2*a^2*d^(3/2))*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x
^8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((b^3*c - a*b^2*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/4*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*sqrt(d
) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a^2*d^(3/2) - a*b*c^2*sqrt(d))/(((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c
 + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(b^3*c - a*b^2*d)) - 1/
8*log((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2)/(b^2*sqrt(d))
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{19}}{\left(bx^{8}+a\right)^{2}\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^19/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^19/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19}}{\left(b\,x^8+a\right)^2\,\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^19/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**19/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.914 $\int \frac{x^{11}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=93

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

```
[Out] 1/8*c*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/(-a*d+b*c)^(3/2)
/a^(1/2)-1/8*x^4*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {465, 471, 12, 377, 205}

$$\frac{c\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}-\frac{x^4\sqrt{c+dx^8}}{8\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^4*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + (c*ArcTan[(Sqrt[b*c -
a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{11}}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int \frac{c}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8(bc-ad)}\\
&= -\frac{x^4\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)}+\frac{c\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\,\sqrt{c+dx^8}}\right)}{8\sqrt{a}\,(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.55, size = 124, normalized size = 1.33

$$
\frac{\sqrt{c+dx^8}\left(-\frac{x^8(bc-ad)}{a+bx^8}-\frac{c\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}\tanh^{-1}\left(\frac{\sqrt{x^8\left(\frac{d}{c}-\frac{b}{a}\right)}}{\sqrt{\frac{dx^8}{c}+1}}\right)}{\sqrt{\frac{dx^8}{c}+1}}\right)}{8x^4(bc-ad)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[c + d*x^8]*(-(((b*c - a*d)*x^8)/(a + b*x^8)) - (c*Sqrt[(-(b/a) + d/c)
*x^8]*ArcTanh[Sqrt[(-(b/a) + d/c)*x^8]/Sqrt[1 + (d*x^8)/c]])/Sqrt[1 + (d*x^
8)/c]))/(8*(b*c - a*d)^2*x^4)
```

fricas [B] time = 1.00, size = 426, normalized size = 4.58

$$
\left[-\frac{4\sqrt{dx^8+c}\left(abc-a^2d\right)x^4-\left(bcx^8+ac\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4\left((bc-2\,ad)\right.}{b^2x^{16}+2\,abx^8+a^2}\right.}{32\left(\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x^8+a^2b^2c^2-2\,a^3bcd+a^4d^2\right)}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(d*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(-a*b*c
 + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b*c^2 - 4*a^
2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x^8 + c)*sqr
t(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)))/((a*b^3*c^2 - 2*a^2*b^2*c
*d + a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2), -1/16*(2*sqrt(d
*x^8 + c)*(a*b*c - a^2*d)*x^4 - (b*c*x^8 + a*c)*sqrt(a*b*c - a^2*d)*arctan(
```

```
1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d
 - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)))/((a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^8 + a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)]
```

giac [B] time = 1.59, size = 244, normalized size = 2.62

$$\frac{c\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,\sqrt{abcd-a^2d^2}\,(bc-ad)}+\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc\sqrt{d}\,-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2ad}{4\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\sqrt{d}\,x^4-\sqrt{dx^8}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*c*sqrt(d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b - b*c + 2*a*
d)/sqrt(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) + 1/4*((s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c*sqrt(d) - 2*(sqrt(d)*x^4 - sqrt(d*x^8 +
 c))^2*a*d^(3/2) - b*c^2*sqrt(d))/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^
2*a*d + b*c^2)*(b^2*c - a*b*d))
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^11/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{11}}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^11/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.915 $\int \frac{x^3}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=104

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

```
[Out] 1/8*(-2*a*d+b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/a^(3/
2)/(-a*d+b*c)^(3/2)+1/8*b*x^4*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)
```

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {465, 382, 377, 205}

$$\frac{(bc-2ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}+\frac{bx^4\sqrt{c+dx^8}}{8a\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^4*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + ((b*c - 2*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)$$
$$=\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}$$
$$=\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x^4}{\sqrt{c+dx^8}}\right)}{8a(bc-ad)}$$
$$=\frac{bx^4\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^4}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{3/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.94, size = 407, normalized size = 3.91

$$\frac{x^4\sqrt{c+dx^8}\left(-30dx^8\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}-45c\sqrt{\frac{ax^8\left(c+dx^8\right)(bc-ad)}{c^2\left(a+bx^8\right)^2}}+16dx^8\left(\frac{x^8(bc-ad)}{c\left(a+bx^8\right)}\right)^{5/2}\sqrt{\frac{a\left(c+dx^8\right)}{c\left(a+bx^8\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)}{c\left(bx^8+\right.}\right.\right.}{120c^2\left(a+bx^8\right)^2\left(\frac{x^8(bc}{c(a+}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^4*Sqrt[c + d*x^8]*(-45*c*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
b*x^8)^2)] - 30*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a + b*x^8)
^2)] + 45*c*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 30*d*x^8*ArcS
in[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 16*c*(((b*c - a*d)*x^8)/(c*(a
 + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[2
, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 16*d*x^8*(((b*c - a*d)*x^8)/
(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric
2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(120*c^2*(((b*c - a*d)*
x^8)/(c*(a + b*x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^
8))])
```

fricas [B] time = 1.26, size = 467, normalized size = 4.49

$$\left[\frac{4\sqrt{dx^8+c}\left(ab^2c-a^2bd\right)x^4-\left(\left(b^2c-2\,abd\right)x^8+abc-2\,a^2d\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right.}{b^2}\right.}{32\left(\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x^8+a^3b^2c^2-2\,a^4bcd+a^5d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 - ((b^2*c - 2*a*b*d)*x^8 +
 a*b*c - 2*a^2*d)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^
2)*x^16 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 -
 a*c*x^4)*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2
)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x^8 + a^3*b^2*c^2 - 2*a^4*b*
c*d + a^5*d^2), 1/16*(2*sqrt(d*x^8 + c)*(a*b^2*c - a^2*b*d)*x^4 + ((b^2*c -
 2*a*b*d)*x^8 + a*b*c - 2*a^2*d)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a
*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^1
```

```
2 + (a*b*c^2 - a^2*c*d)*x^4)))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x
^8 + a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)]
```

giac [B] time = 0.39, size = 237, normalized size = 2.28

$$-\frac{1}{8}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(abcd-a^2d^2\right)^{\frac{3}{2}}}+\frac{2\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc-2\left(\sqrt{d}\,x^4\right.\right.}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2bc+4\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b
 - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sq
rt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*
d - b*c^2)/(((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*
x^8 + c))^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)*(a*b*c*d
 - a^2*d^2)))
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**3/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.916 $\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=149

$$-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(3bc-2ad)}{8a^2cx^4(bc-ad)}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

```
[Out] -1/8*b*(-4*a*d+3*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))/
a^(5/2)/(-a*d+b*c)^(3/2)-1/8*(-2*a*d+3*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c
)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^4/(b*x^8+a)
```

Rubi [A] time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$-\frac{\sqrt{c+dx^8}\,(3bc-2ad)}{8a^2cx^4(bc-ad)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{5/2}(bc-ad)^{3/2}}+\frac{b\sqrt{c+dx^8}}{8ax^4\left(a+bx^8\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((3*b*c - 2*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^4) + (b*Sqrt[c +
d*x^8])/(8*a*(b*c - a*d)*x^4*(a + b*x^8)) - (b*(3*b*c - 4*a*d)*ArcTan[(Sqrt
[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(5/2)*(b*c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^5\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{4}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx,x,x^4\right)$$

$$=\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{-3bc+2ad-2bdx^2}{x^2\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x,x^4\right)}{8a(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{bc(3bc-4ad)}{\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx,x\ldots\right)}{8a^2c(bc-ad)}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{\ldots}{(a+b\ldots}\ldots\right)}{8a^2(bc-a\ldots}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{(b(3bc-4ad))\,\text{Subst}\left(\int \frac{\ldots}{a-(\ldots}\ldots\right)}{8a^2(bc-\ldots}$$

$$=-\frac{(3bc-2ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^4}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^4\left(a+bx^8\right)}-\frac{b(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x\ldots}{\sqrt{a}\sqrt{c+d\ldots}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 2.03, size = 869, normalized size = 5.83

$$-\frac{\sqrt{dx^8+c}\left(120d^2\sin^{-1}\left(\sqrt{\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}}\right)x^{16}+96d^2\left(\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)^{5/2}\sqrt{\frac{a\left(dx^8+c\right)}{c\left(bx^8+a\right)}}\,{}_2F_1\left(2,3;\frac{7}{2};\frac{(bc-ad)x^8}{c\left(bx^8+a\right)}\right)x^{16}+32d^2\left(\frac{(bc-ad)\ldots}{c\left(bx^8+a\ldots\right.}\right.\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^5*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/120*(Sqrt[c + d*x^8]*(-45*c^2*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*
(a + b*x^8)^2)] - 180*c*d*x^8*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a
+ b*x^8)^2)] - 120*d^2*x^16*Sqrt[(a*(b*c - a*d)*x^8*(c + d*x^8))/(c^2*(a +
b*x^8)^2)] + 45*c^2*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 180*c
*d*x^8*ArcSin[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 120*d^2*x^16*ArcSi
n[Sqrt[((b*c - a*d)*x^8)/(c*(a + b*x^8))]] + 64*c^2*(((b*c - a*d)*x^8)/(c*(
```

```
a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeometric2F1[
2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 160*c*d*x^8*(((b*c - a*d)*x
^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hypergeome
tric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 96*d^2*x^16*(((b*c
- a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]*Hy
pergeometric2F1[2, 3, 7/2, ((b*c - a*d)*x^8)/(c*(a + b*x^8))] + 32*c^2*(((b
*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))]
*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/(c*(a + b*x^8))]
+ 64*c*d*x^8*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sqrt[(a*(c + d*x^8))
/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2}, ((b*c - a*d)*x^8)/
(c*(a + b*x^8))] + 32*d^2*x^16*(((b*c - a*d)*x^8)/(c*(a + b*x^8)))^(5/2)*Sq
rt[(a*(c + d*x^8))/(c*(a + b*x^8))]*HypergeometricPFQ[{2, 2, 3}, {1, 7/2},
((b*c - a*d)*x^8)/(c*(a + b*x^8))]))/(c^3*x^4*(((b*c - a*d)*x^8)/(c*(a + b*
x^8)))^(3/2)*(a + b*x^8)^2*Sqrt[(a*(c + d*x^8))/(c*(a + b*x^8))])
```

fricas [B] time = 1.27, size = 612, normalized size = 4.11

$$\left[-\frac{\left(\left(3\,b^3c^2-4\,ab^2cd\right)x^{12}+\left(3\,ab^2c^2-4\,a^2bcd\right)x^4\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+a^2c^2+4}{b^2x^{16}+2\,abx^8+}\right.}{32\left(\left(a^3b^3c^3-2\,a^4b^2c^2d+a^5bcd^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*b^3*c^2 - 4*a*b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*
sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 - 2*(3*a*b
*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 + 4*((b*c - 2*a*d)*x^12 - a*c*x^4)*sqrt(d*x
^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) + 4*((3*a*b^3*c
^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4
*d^2)*sqrt(d*x^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12
+ (a^4*b^2*c^3 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4), -1/16*(((3*b^3*c^2 - 4*a*
b^2*c*d)*x^12 + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x^4)*sqrt(a*b*c - a^2*d)*arctan
(1/2*((b*c - 2*a*d)*x^8 - a*c)*sqrt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*
d - a^2*d^2)*x^12 + (a*b*c^2 - a^2*c*d)*x^4)) + 2*((3*a*b^3*c^2 - 5*a^2*b^2
*c*d + 2*a^3*b*d^2)*x^8 + 2*a^2*b^2*c^2 - 4*a^3*b*c*d + 2*a^4*d^2)*sqrt(d*x
^8 + c))/((a^3*b^3*c^3 - 2*a^4*b^2*c^2*d + a^5*b*c*d^2)*x^12 + (a^4*b^2*c^3
 - 2*a^5*b*c^2*d + a^6*c*d^2)*x^4)]
```

giac [B] time = 1.76, size = 418, normalized size = 2.81

$$\frac{1}{8}\,d^{\frac{5}{2}}\left(\frac{\left(3\,b^2c-4\,abd\right)\arctan\left(\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^2bcd^2-a^3d^3\right)\sqrt{abcd-a^2d^2}}+\frac{2\left(3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b^2c-4\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)\right.}{\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^6b-3\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*d^(5/2)*((3*b^2*c - 4*a*b*d)*arctan(1/2*((sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c*d^2 - a^3*d^3)*sqrt
(a*b*c*d - a^2*d^2)) + 2*(3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b^2*c - 4*(sq
rt(d)*x^4 - sqrt(d*x^8 + c))^4*a*b*d - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*
b^2*c^2 + 14*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b*c*d - 8*(sqrt(d)*x^4 - s
qrt(d*x^8 + c))^2*a^2*d^2 + 3*b^2*c^3 - 2*a*b*c^2*d)/(((sqrt(d)*x^4 - sqrt(
d*x^8 + c))^6*b - 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b*c + 4*(sqrt(d)*x^4
```

```
- sqrt(d*x^8 + c))^4*a*d + 3*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c^2 - 4*(s
qrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*c*d - b*c^3)*(a^2*b*c*d^2 - a^3*d^3)))
```

maple [F] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2} \sqrt{d\,x^{8}+c}\; x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{8}+a\right)^{2} \sqrt{dx^{8}+c}\, x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^5), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5}\,\left(b\,x^{8}+a\right)^{2}\,\sqrt{d\,x^{8}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^5*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

3.917 $\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=208

$$\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{\sqrt{c+dx^8}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b\sqrt{c+}}{8ax^{12}\left(a+bx\right.}$$

```
[Out] 1/8*b^2*(-6*a*d+5*b*c)*arctan(x^4*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^8+c)^(1/2))
/a^(7/2)/(-a*d+b*c)^(3/2)-1/24*(-2*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b
*c)/x^12+1/24*(-4*a^2*d^2-8*a*b*c*d+15*b^2*c^2)*(d*x^8+c)^(1/2)/a^3/c^2/(-a
*d+b*c)/x^4+1/8*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^12/(b*x^8+a)
```

Rubi [A] time = 0.30, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {465, 472, 583, 12, 377, 205}

$$\frac{\sqrt{c+dx^8}\,\left(-4a^2d^2-8abcd+15b^2c^2\right)}{24a^3c^2x^4(bc-ad)}+\frac{b^2(5bc-6ad)\tan^{-1}\left(\frac{x^4\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^8}}\right)}{8a^{7/2}(bc-ad)^{3/2}}-\frac{\sqrt{c+dx^8}\,(5bc-2ad)}{24a^2cx^{12}(bc-ad)}+\frac{b\sqrt{c+}}{8ax^{12}\left(a+bx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((5*b*c - 2*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^12) + ((15*b^2*c
^2 - 8*a*b*c*d - 4*a^2*d^2)*Sqrt[c + d*x^8])/(24*a^3*c^2*(b*c - a*d)*x^4) +
 (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^12*(a + b*x^8)) + (b^2*(5*b*c - 6*a
*d)*ArcTan[(Sqrt[b*c - a*d]*x^4)/(Sqrt[a]*Sqrt[c + d*x^8])])/(8*a^(7/2)*(b*
c - a*d)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
```

```
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{13}\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{4}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx, x, x^4\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-5bc+2ad-4bdx^2}{x^4\left(a+bx^2\right)\sqrt{c+dx^2}}\,dx, x, x^4\right)}{8a(bc-ad)}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^{12}\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{-15b^2c^2+8abcd+4a^2c}{x^2\left(a+bx^2\right)}\right.}{24a^2c(bc}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+d}}{8a(bc-ad)x^{12}}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+d}}{8a(bc-ad)x^{12}}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+d}}{8a(bc-ad)x^{12}}$$

$$= -\frac{(5bc-2ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^{12}} + \frac{\left(15b^2c^2-8abcd-4a^2d^2\right)\sqrt{c+dx^8}}{24a^3c^2(bc-ad)x^4} + \frac{b\sqrt{c+d}}{8a(bc-ad)x^{12}}$$

Mathematica [A] time = 5.85, size = 175, normalized size = 0.84

$$\frac{a^2\left(c+dx^8\right)\left(\frac{3b^3x^{16}}{\left(a+bx^8\right)(bc-ad)}+\frac{4x^8(ad+3bc)}{c^2}-\frac{2a}{c}\right)+\frac{3b^2x^{24}\sqrt{\frac{dx^8}{c}+1}\,(5bc-6ad)\sin^{-1}\left(\frac{\sqrt{x^8\left(\frac{b}{a}-\frac{d}{c}\right)}}{\sqrt{\frac{bx^8}{a}+1}}\right)}{c\left(\frac{x^8(bc-ad)}{ac}\right)^{3/2}}}{24a^5x^{12}\sqrt{c+dx^8}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^13*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (a^2*(c + d*x^8)*((-2*a)/c + (4*(3*b*c + a*d)*x^8)/c^2 + (3*b^3*x^16)/((b*c
 - a*d)*(a + b*x^8))) + (3*b^2*(5*b*c - 6*a*d)*x^24*Sqrt[1 + (d*x^8)/c]*Arc
Sin[Sqrt[(b/a - d/c)*x^8]/Sqrt[1 + (b*x^8)/a]])/(c*(((b*c - a*d)*x^8)/(a*c)
)^(3/2)))/(24*a^5*x^12*Sqrt[c + d*x^8])
```

fricas [A] time = 1.66, size = 760, normalized size = 3.65

$$\left[-\frac{3\left(\left(5\,b^4c^3-6\,ab^3c^2d\right)x^{20}+\left(5\,ab^3c^3-6\,a^2b^2c^2d\right)x^{12}\right)\sqrt{-abc+a^2d}\,\log\left(\frac{\left(b^2c^2-8\,abcd+8\,a^2d^2\right)x^{16}-2\left(3\,abc^2-4\,a^2cd\right)x^8+}{b^2x^{16}+2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b^2*c^2*
d)*x^12)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^16 -
 2*(3*a*b*c^2 - 4*a^2*c*d)*x^8 + a^2*c^2 - 4*((b*c - 2*a*d)*x^12 - a*c*x^4)
*sqrt(d*x^8 + c)*sqrt(-a*b*c + a^2*d))/(b^2*x^16 + 2*a*b*x^8 + a^2)) - 4*((
15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^16 + 2*(
5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^8 - 2*a^3*b^2*
c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^4*b^3*c^4 - 2*a^5*b
^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*c^3*d + a^7*c^2*d^2
)*x^12), 1/48*(3*((5*b^4*c^3 - 6*a*b^3*c^2*d)*x^20 + (5*a*b^3*c^3 - 6*a^2*b
^2*c^2*d)*x^12)*sqrt(a*b*c - a^2*d)*arctan(1/2*((b*c - 2*a*d)*x^8 - a*c)*sq
rt(d*x^8 + c)*sqrt(a*b*c - a^2*d)/((a*b*c*d - a^2*d^2)*x^12 + (a*b*c^2 - a^
2*c*d)*x^4)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^16 + 2*(5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^
3)*x^8 - 2*a^3*b^2*c^3 + 4*a^4*b*c^2*d - 2*a^5*c*d^2)*sqrt(d*x^8 + c))/((a^
4*b^3*c^4 - 2*a^5*b^2*c^3*d + a^6*b*c^2*d^2)*x^20 + (a^5*b^2*c^4 - 2*a^6*b*
c^3*d + a^7*c^2*d^2)*x^12)]
```

giac [B] time = 2.28, size = 395, normalized size = 1.90

$$\frac{1}{24}\,d^{\frac{7}{2}}\left(\frac{3\left(5\,b^3c-6\,ab^2d\right)\arctan\left(-\frac{\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\left(a^3bcd^3-a^4d^4\right)\sqrt{abcd-a^2d^2}}-\frac{6\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^2b^3c-\right.}{\left(a^3bcd^3-a^4d^4\right)\left(\left(\sqrt{d}\,x^4-\sqrt{dx^8+c}\right)^4b-2\left(\sqrt{d}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*d^(7/2)*(3*(5*b^3*c - 6*a*b^2*d)*arctan(-1/2*((sqrt(d)*x^4 - sqrt(d*x^
8 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c*d^3 - a^4*d^4
)*sqrt(a*b*c*d - a^2*d^2)) - 6*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b^3*c - 2
*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*b^2*d - b^3*c^2)/((a^3*b*c*d^3 - a^4*d
^4)*((sqrt(d)*x^4 - sqrt(d*x^8 + c))^4*b - 2*(sqrt(d)*x^4 - sqrt(d*x^8 + c)
)^2*b*c + 4*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + b*c^2)) - 8*(3*(sqrt(d)
*x^4 - sqrt(d*x^8 + c))^4*b - 6*(sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*b*c - 3*(
sqrt(d)*x^4 - sqrt(d*x^8 + c))^2*a*d + 3*b*c^2 + a*c*d)/(((sqrt(d)*x^4 - sq
rt(d*x^8 + c))^2 - c)^3*a^3*d^3))
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\,x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}\, x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^13), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{13} \left(b\, x^8 + a\right)^2 \sqrt{d\, x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^13*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.918 $\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=924

$$\frac{\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{64ab\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{dx^8+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{}{\sqrt[4]{-}}\right.}{32(-a)^{3/4}b^{3/4}(bc}$$

```
[Out] -1/32*(a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1
/2))/(-a)^(3/4)/b^(3/4)/(-a*d+b*c)^(3/2)+1/32*(a*d+b*c)*arctan(x^2*(a*d-b*c
)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(3/4)/b^(3/4)/(a*d-b*c)^(3
/2)-1/8*x^2*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)-1/16*d^(3/4)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
F(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d
*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
+1/32*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2)
)*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/
4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*((-a)^(1/
2)*b^(1/2)*c^(1/2)+a*d^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4
*d^(1/2))^2)^(1/2)/a/b/c^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1
/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2)
)*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2
)^(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi
(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2
))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*
(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^
(1/2)/a/b/c^(1/4)/d^(1/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 924, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {465, 471, 523, 220, 409, 1217, 1707}

$$\frac{\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{64ab\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)\sqrt{dx^8+c}}-\frac{(bc+ad)\tan^{-1}\left(\frac{}{\sqrt[4]{-}}\right.}{32(-a)^{3/4}b^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(x^2*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) - ((b*c + a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b
^(3/4)*(b*c - a*d)^(3/2)) + ((b*c + a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/((
-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(3/4)*b^(3/4)*(-(b*c) + a*d)^
(3/2)) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]
*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt
[-a]*Sqrt[b]*Sqrt[c] + a*Sqrt[d])*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)
], 1/2])/(32*a*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) - (d^(3/4)*(Sqrt[c] +
```

```
Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTa
n[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*b*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
+ ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
 d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*
Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(
1/4)], 1/2])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + ((Sqrt[
b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2
/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
])/(64*a*b*c^(1/4)*d^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^9}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{c-dx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8b(bc-ad)} + \frac{(bc+ad)\,\text{Subst}\left(\int \frac{}{(a+bx}\right.}{8b(bc-a}$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}} +$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16b\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}} +$$

$$= -\frac{x^2\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{3/4}b^{3/4}(bc-ad)^{3/2}} + \frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt[4]{-a}}\right.}{32(-a)^{3/4}b^{3/4}(-bc}$$

Mathematica [C] time = 0.17, size = 159, normalized size = 0.17

$$-\frac{x^2\left(dx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-5c\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5a\left(\right.\right.}{40a\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^9/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/40*(x^2*(5*a*(c + d*x^8) - 5*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[
1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 +
(d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a)]))/(a*(b*c
 - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^9/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^9}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^9/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

3.919 $\int \frac{x}{(a+bx^8)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=999

$$\frac{(3bc-5ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\quad\sqrt[4]{d}\,(3bc}{64a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}+$$

```
[Out] 1/32*b^(1/4)*(-5*a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(7/4)/(-a*d+b*c)^(3/2)-1/32*b^(1/4)*(-5*a*d+3*b*c)*ar
ctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(7/4)/(a*
d-b*c)^(3/2)+1/8*b*x^2*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)+1/16*d^(3/4)*
(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/
4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c)/(d
*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)
))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/
4)*x^2/c^(1/4))),1/2*2^(1/2))*(b^(1/2)*c^(1/2)/(-a)^(1/2)+d^(1/2))*(c^(1/2)
+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*d^(1/4)*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(
2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*
c^(1/2)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^
(3/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-5*a*d+3*b*c)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1
/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4
*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^
(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)+1/64
*(-5*a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d
^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*
2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x
^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)
/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.41, antiderivative size = 999, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {465, 414, 523, 220, 409, 1217, 1707}

$$\frac{(3bc-5ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2\quad\sqrt[4]{d}\,(3bc}{64a^2\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}+$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (b*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) + (b^(1/4)*(3*b*c - 5
*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(
32*(-a)^(7/4)*(b*c - a*d)^(3/2)) - (b^(1/4)*(3*b*c - 5*a*d)*ArcTan[(Sqrt[-(
b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(7/4)*(-(b
*c) + a*d)^(3/2)) + (d^(3/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt
[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*
a*c^(1/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (((Sqrt[b]*Sqrt[c])/Sqrt[-a] + Sqr
t[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqr
```

```
t[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32
*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - S
qrt[-a]*Sqrt[d])*d^(1/4)*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c +
d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)]
, 1/2])/(32*(-a)^(3/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c
] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(
1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*S
qrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c - 5*a*d)*(
Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Elliptic
Pi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[
d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^2*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-3bc+4ad-bdx^4}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{(3bc-5ad)\,\text{Subst}\left(\int \frac{}{(a+}\right.}{8a(bc-}$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}} +$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{d^{3/4}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c+dx^8}{\left(\sqrt{c}+\sqrt{d}\,x^4\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)}{16a\sqrt[4]{c}\,(bc-ad)\sqrt{c+dx^8}} +$$

$$= \frac{bx^2\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\sqrt[4]{b}\,(3bc-5ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+dx^8}}\right)}{32(-a)^{7/4}(bc-ad)^{3/2}} - \frac{\sqrt[4]{b}\,(3bc-5ad)}{32(-a)^{7}}$$

Mathematica [C] time = 0.22, size = 169, normalized size = 0.17

$$\frac{x^2\left(bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,(3bc-4ad)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{}{}\right.\right.}{40a^2\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^2*(5*a*b*(c + d*x^8) + 5*(3*b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]
*AppellF1[1/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*x^8*(a + b*x^
8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b*x^8)/a
)]))/(40*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(x/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.920 $\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1060

$$-\frac{b(7bc-9ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64a^3\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}+\;b\sqrt[4]{d}$$

```
[Out] 1/32*b^(5/4)*(-9*a*d+7*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/
(d*x^8+c)^(1/2))/(-a)^(11/4)/(-a*d+b*c)^(3/2)-1/32*b^(5/4)*(-9*a*d+7*b*c)*a
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(11/4)/(
a*d-b*c)^(3/2)-1/24*(-4*a*d+7*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^6+1/8
*b*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^6/(b*x^8+a)-1/48*d^(3/4)*(-4*a*d+7*b*c)*(
cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4
)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*
d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(5/4)/(-a*d+b*c)/(
d*x^8+c)^(1/2)+1/32*b*d^(1/4)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1
/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^
(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a
)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/
4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elliptic
Pi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/
2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)
^(1/2)/a^3/c^(1/4)/d^(1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/32*b*d^(1/4
)*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(
d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1
/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((d*x^8+c)/
(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x
^8+c)^(1/2)-1/64*b*(-9*a*d+7*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^3/c^(1/4)/d^(1/4)/
(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.98, antiderivative size = 1060, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {465, 472, 583, 523, 220, 409, 1217, 1707}

$$-\frac{b(7bc-9ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\sqrt{c}+\sqrt{-a}\sqrt{d}\right)^2}{4\sqrt{-a}\sqrt{b}\sqrt{c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{c}-\sqrt{-a}\sqrt{d}\right)^2}{64a^3\sqrt[4]{c}\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}+\;b\sqrt[4]{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((7*b*c - 4*a*d)*Sqrt[c + d*x^8])/(24*a^2*c*(b*c - a*d)*x^6) + (b*Sqrt[c +
 d*x^8])/(8*a*(b*c - a*d)*x^6*(a + b*x^8)) + (b^(5/4)*(7*b*c - 9*a*d)*ArcTa
n[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11
/4)*(b*c - a*d)^(3/2)) - (b^(5/4)*(7*b*c - 9*a*d)*ArcTan[(Sqrt[-(b*c) + a*d
]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(11/4)*(-(b*c) + a*d
)^(3/2)) - (d^(3/4)*(7*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8
)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2
```

```
])/(48*a^2*c^(5/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[b]*Sqrt[c] - Sqr
t[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])*d^(1/4)*(7*b*c - 9*a*d)*(Sqrt[c] + Sqr
t[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d
^(1/4)*x^2)/c^(1/4)], 1/2])/(32*(-a)^(5/2)*c^(1/4)*(b*c - a*d)*(b*c + a*d)*
Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(7*b*c - 9*a*d
)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*Ellip
ticPi[-(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*S
qrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3*c^(1/4)*d^(1/4)*(b*
c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (b*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt
[d])^2*(7*b*c - 9*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] +
Sqrt[d]*x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-
a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*a^3
*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
```

```
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^7\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^4\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$= \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{-7bc+4ad-5bdx^4}{x^4\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{-21b^2c^2+20abcd+4}{\left(a+bx^4\right)}\right.}{24a^2c(b}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{(b(7bc-9ad))\,\text{Subst}\left(\int\right.}{8a^2(bc-}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{}\right.}{48a^2c}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} - \frac{d^{3/4}(7bc-4ad)\left(\sqrt{c}+\sqrt{}\right.}{48a^2c}$$

$$= -\frac{(7bc-4ad)\sqrt{c+dx^8}}{24a^2c(bc-ad)x^6} + \frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^6\left(a+bx^8\right)} + \frac{b^{5/4}(7bc-9ad)\tan^{-1}\left(\right.}{32(-a)^{11/4}(bc-a}$$

Mathematica [C] time = 0.39, size = 225, normalized size = 0.21

$$\frac{5x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(4a^2d^2+20abcd-21b^2c^2\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+5a\left(c+dx^8\right)\left(4a^2d-4ab\left(c-dx^8\right)\right)}{120a^3cx^6\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^7*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (5*a*(c + d*x^8)*(4*a^2*d - 7*b^2*c*x^8 - 4*a*b*(c - d*x^8)) + 5*(-21*b^2*c
^2 + 20*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[1
/4, 1/2, 1, 5/4, -((d*x^8)/c), -((b*x^8)/a)] + b*d*(-7*b*c + 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[5/4, 1/2, 1, 9/4, -((d*x^8)/c), -((b
*x^8)/a)])/(120*a^3*c*(b*c - a*d)*x^6*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^7), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^7\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^7*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.921 $\int \frac{x^{13}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=1164

$$-\frac{\sqrt{dx^8+c}\,x^6}{8(bc-ad)\left(bx^8+a\right)}+\frac{\sqrt{d}\,\sqrt{dx^8+c}\,x^2}{8b(bc-ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c}}\right.}{32\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/}}$$

```
[Out] 1/32*(-a*d+3*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^
(1/2))/(-a)^(1/4)/b^(5/4)/(-a*d+b*c)^(3/2)+1/32*(-a*d+3*b*c)*arctan(x^2*(a*
d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(1/4)/b^(5/4)/(a*d-b*
c)^(3/2)-1/8*x^6*(d*x^8+c)^(1/2)/(-a*d+b*c)/(b*x^8+a)+1/8*x^2*d^(1/2)*(d*x^
8+c)^(1/2)/b/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*c^(1/4)*d^(1/4)*(cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Elli
pticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))
*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/1
6*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan
(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(
1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b/(-a
*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2
/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2
)-(-a)^(1/2)*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(
1/4)/(-a^2*d^2+b^2*c^2)/(d*x^8+c)^(1/2)-1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(
1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(s
in(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^
2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/b^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)-
1/32*d^(1/4)*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(
2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4)))
,1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/(-a^2*d^2+b^2*c^2)/
(d*x^8+c)^(1/2)+1/64*(-a*d+3*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1
/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/
c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^
(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2
)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/b^(3/2)/c^(1/4)/d^(1
/4)/(-a*d+b*c)/(a*d+b*c)/(-a)^(1/2)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.94, antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 471, 584, 305, 220, 1196, 490, 1217, 1707}

$$-\frac{\sqrt{dx^8+c}\,x^6}{8(bc-ad)\left(bx^8+a\right)}+\frac{\sqrt{d}\,\sqrt{dx^8+c}\,x^2}{8b(bc-ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^8+c}}\right)}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{ad-bc}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c}}\right.}{32\sqrt[4]{-a}\,b^{5/4}(ad-bc)^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*b*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) - (
x^6*Sqrt[c + d*x^8])/(8*(b*c - a*d)*(a + b*x^8)) + ((3*b*c - a*d)*ArcTan[(S
qrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b
^(5/4)*(b*c - a*d)^(3/2)) + ((3*b*c - a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*x^2)/
```

```
((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(1/4)*b^(5/4)*(-(b*c) + a*d
)^(3/2)) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[
c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(8*b*
(b*c - a*d)*Sqrt[c + d*x^8]) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqr
t[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c
^(1/4)], 1/2])/(16*b*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[-a]*S
qrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*
x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)],
1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] +
(Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*S
qrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)
/c^(1/4)], 1/2])/(32*b*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (
(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(3*b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c]
- Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/
4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b
*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(3*b*c
 - a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]
*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt
[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*Sqrt[-a]*b^(3/2)*c
^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^{13}}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3c+dx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\text{Subst}\left(\int \left(\frac{dx^2}{b\sqrt{c+dx^4}} + \frac{(3bc-ad)x^2}{b\left(a+bx^4\right)\sqrt{c+dx^4}}\right) dx, x, x^2\right)}{8(bc-ad)}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} + \frac{(3bc-ad)\,\text{Subst}\left(\int \frac{}{(a+b}\right.}{8b(bc-}$$

$$= -\frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8b(bc-ad)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}}{8b(b}$$

$$= \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} - \frac{\sqrt[4]{c}\,\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c}{\left(\sqrt{c}\right.}}}{8b(bc-ad}$$

$$= \frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8b(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} - \frac{x^6\sqrt{c+dx^8}}{8(bc-ad)\left(a+bx^8\right)} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+}}\right.}{32\sqrt[4]{-a}\,b^{5/4}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.17, size = 159, normalized size = 0.14

$$\frac{x^6\left(dx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7c\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)-7a\left(c\right.\right.}{56a\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^13/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(-7*a*(c + d*x^8) + 7*c*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/4,
1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + d*x^8*(a + b*x^8)*Sqrt[1 + (d*x^
8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)/a)]))/(56*a*(b*c
- a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{13}}{\left(bx^{8}+a\right)^{2}\sqrt{dx^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(x^13/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13}}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(x^13/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

3.922 $\int \frac{x^5}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\, dx$

Optimal. Leaf size=1162

$$\frac{b\sqrt{dx^8+c}\,x^6}{8a(bc-ad)\left(bx^8+a\right)}-\frac{\sqrt{d}\,\sqrt{dx^8+c}\,x^2}{8a(bc-ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-}}{\sqrt[4]{-a}\,\sqrt[4]{b}}\right.}{32(-a)^{5/4}\sqrt[4]{b}\,(ad-bc}$$

```
[Out] -1/32*(-3*a*d+b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)
^(1/2))/(-a)^(5/4)/b^(1/4)/(-a*d+b*c)^(3/2)-1/32*(-3*a*d+b*c)*arctan(x^2*(a
*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(5/4)/b^(1/4)/(a*d-b
*c)^(3/2)+1/8*b*x^6*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(b*x^8+a)-1/8*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))+1/8*c^(1/4)*d^(1/4)*(cos
(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(
1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/(-a*d+b*c)/(d*x^8+c)^(1/2
)-1/16*c^(1/4)*d^(1/4)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*a
rctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/
2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/
a/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^
(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan
(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2
)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/
2)-(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/
2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/b^(1/2)/(d*x^8+c)^(1/2)-1/64*(-3*a*
d+b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(
1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*
(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(3/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)
/b^(1/2)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^
(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a/c^(1/4)/
(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/32*d^(1/4)*(-3*a*d+b*c)*(cos(2*arcta
n(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*Ellipti
cF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c
^(1/2)+(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2
)/a/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 1.91, antiderivative size = 1162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {465, 472, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{b\sqrt{dx^8+c}\,x^6}{8a(bc-ad)\left(bx^8+a\right)}-\frac{\sqrt{d}\,\sqrt{dx^8+c}\,x^2}{8a(bc-ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^2}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{dx^8+c}}\right)}{32(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}}-\frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{ad-}}{\sqrt[4]{-a}\,\sqrt[4]{b}}\right.}{32(-a)^{5/4}\sqrt[4]{b}\,(ad-bc}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[d]*x^2*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x^4)) +
(b*x^6*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*(a + b*x^8)) - ((b*c - 3*a*d)*ArcT
an[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5
/4)*b^(1/4)*(b*c - a*d)^(3/2)) - ((b*c - 3*a*d)*ArcTan[(Sqrt[-(b*c) + a*d]*
```

```
x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(5/4)*b^(1/4)*(-(b*c)
+ a*d)^(3/2)) + (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(
Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/
(8*a*(b*c - a*d)*Sqrt[c + d*x^8]) - (c^(1/4)*d^(1/4)*(Sqrt[c] + Sqrt[d]*x^4
)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x
^2)/c^(1/4)], 1/2])/(16*a*(b*c - a*d)*Sqrt[c + d*x^8]) - ((Sqrt[c] - (Sqrt[
-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c
 + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/
4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - ((Sqrt[
c] + (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1/4)*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x
^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)
*x^2)/c^(1/4)], 1/2])/(32*a*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]
) - ((Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d
]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqr
t[c] - Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(
d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*Sqrt[b]*c^(1/4)*d^(1/4)*(b*c -
a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + ((Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2
*(b*c - 3*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[
b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(3/2)*
Sqrt[b]*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx, x, x^2\right)$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(-bc+4ad+bdx^4\right)}{\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\text{Subst}\left(\int \left(\frac{dx^2}{\sqrt{c+dx^4}} + \frac{(-bc+3ad)x^2}{\left(a+bx^4\right)\sqrt{c+dx^4}}\right)dx, x, x^2\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{d\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx^4\right)\sqrt{\ldots}}\ldots\right)}{8a(bc-ad)}$$

$$= \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^4}}\,dx, x, x^2\right)}{8a(bc-ad)} + \frac{\left(\sqrt{c}\sqrt{d}\right)\text{Subst}\left(\int \frac{1}{\ldots}\ldots\right)}{8a(bc-\ldots}$$

$$= -\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} + \frac{\sqrt[4]{c}\,\sqrt[4]{d}\left(\sqrt{c}+\sqrt{d}\,x^4\right)\sqrt{\frac{c\ldots}{\left(\sqrt{c}\ldots\right.}}}{8a(bc-a\ldots}$$

$$= -\frac{\sqrt{d}\,x^2\sqrt{c+dx^8}}{8a(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)} + \frac{bx^6\sqrt{c+dx^8}}{8a(bc-ad)\left(a+bx^8\right)} - \frac{(bc-3ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x^{2}}{\sqrt[4]{-a}\,\sqrt[4]{b}\,\sqrt{c+\ldots}}\right.}{32(-a)^{5/4}\sqrt[4]{b}\,(bc-ad)^{3/2}\ldots}$$

Mathematica [C] time = 0.19, size = 169, normalized size = 0.15

$$\frac{x^6\left(-3bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},1;\frac{11}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+7\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,(bc-4ad)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{b\ldots}{\ldots}\right.\ldots}{168a^2\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^5/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^6*(21*a*b*(c + d*x^8) + 7*(b*c - 4*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*
AppellF1[3/4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] - 3*b*d*x^8*(a + b*x
^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((b*x^8)
/a)]))/(168*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**5/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.923 $\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=1243

$$\frac{\sqrt{b}\,(5bc-7ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{64(-a)^{5/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}-\frac{b^{3/4}(5b\ldots}{\ldots}$$

```
[Out] -1/32*b^(3/4)*(-7*a*d+5*b*c)*arctan(x^2*(-a*d+b*c)^(1/2)/(-a)^(1/4)/b^(1/4)
/(d*x^8+c)^(1/2))/(-a)^(9/4)/(-a*d+b*c)^(3/2)-1/32*b^(3/4)*(-7*a*d+5*b*c)*a
rctan(x^2*(a*d-b*c)^(1/2)/(-a)^(1/4)/b^(1/4)/(d*x^8+c)^(1/2))/(-a)^(9/4)/(a
*d-b*c)^(3/2)-1/8*(-4*a*d+5*b*c)*(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/x^2+1/8*b
*(d*x^8+c)^(1/2)/a/(-a*d+b*c)/x^2/(b*x^8+a)+1/8*(-4*a*d+5*b*c)*x^2*d^(1/2)*
(d*x^8+c)^(1/2)/a^2/c/(-a*d+b*c)/(c^(1/2)+x^4*d^(1/2))-1/8*d^(1/4)*(-4*a*d+
5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))*EllipticE(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1
/2)+x^4*d^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*
d+b*c)/(d*x^8+c)^(1/2)+1/16*d^(1/4)*(-4*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^
2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arc
tan(d^(1/4)*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*((d*x^8+c)/(c^
(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(3/4)/(-a*d+b*c)/(d*x^8+c)^(1/2)+1/64*(-7
*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/
4)*x^2/c^(1/4)))*EllipticPi(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/4*(b^(1/2)
*c^(1/2)+(-a)^(1/2)*d^(1/2))^2/(-a)^(1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/
2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2*((
d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*
c)/(a*d+b*c)/(d*x^8+c)^(1/2)-1/64*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/
c^(1/4)))^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticPi(sin(2*arct
an(d^(1/4)*x^2/c^(1/4))),-1/4*(b^(1/2)*c^(1/2)-(-a)^(1/2)*d^(1/2))^2/(-a)^(
1/2)/b^(1/2)/c^(1/2)/d^(1/2),1/2*2^(1/2))*b^(1/2)*(c^(1/2)+x^4*d^(1/2))*(b^
(1/2)*c^(1/2)+(-a)^(1/2)*d^(1/2))^2*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/
2)/(-a)^(5/2)/c^(1/4)/d^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8+c)^(1/2)+1/32*b*d
^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))^2)^(1/2)/cos(2*ar
ctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)*x^2/c^(1/4))),1/2
*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)-(-a)^(1/2)*d^(1/2)/b^(1/2))*((d*x^
8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c)/(a*d+b*c)/(d*x^8
+c)^(1/2)+1/32*b*d^(1/4)*(-7*a*d+5*b*c)*(cos(2*arctan(d^(1/4)*x^2/c^(1/4)))
^2)^(1/2)/cos(2*arctan(d^(1/4)*x^2/c^(1/4)))*EllipticF(sin(2*arctan(d^(1/4)
*x^2/c^(1/4))),1/2*2^(1/2))*(c^(1/2)+x^4*d^(1/2))*(c^(1/2)+(-a)^(1/2)*d^(1/
2)/b^(1/2))*((d*x^8+c)/(c^(1/2)+x^4*d^(1/2))^2)^(1/2)/a^2/c^(1/4)/(-a*d+b*c
)/(a*d+b*c)/(d*x^8+c)^(1/2)
```

Rubi [A] time = 2.38, antiderivative size = 1243, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {465, 472, 583, 584, 305, 220, 1196, 490, 1217, 1707}

$$\frac{\sqrt{b}\,(5bc-7ad)\left(\sqrt{d}\,x^4+\sqrt{c}\right)\sqrt{\frac{dx^8+c}{\left(\sqrt{d}\,x^4+\sqrt{c}\right)^2}}\,\Pi\left(\frac{\left(\sqrt{b}\,\sqrt{c}+\sqrt{-a}\,\sqrt{d}\right)^2}{4\sqrt{-a}\,\sqrt{b}\,\sqrt{c}\,\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{d}\,x^2}{\sqrt[4]{c}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\,\sqrt{c}-\sqrt{-a}\,\sqrt{d}\right)^2}{64(-a)^{5/2}\sqrt[4]{c}\,\sqrt[4]{d}\,(bc-ad)(bc+ad)\sqrt{dx^8+c}}-\frac{b^{3/4}(5b\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((5*b*c - 4*a*d)*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*x^2) + (Sqrt[d]*(5*
b*c - 4*a*d)*x^2*Sqrt[c + d*x^8])/(8*a^2*c*(b*c - a*d)*(Sqrt[c] + Sqrt[d]*x
```

```
^4)) + (b*Sqrt[c + d*x^8])/(8*a*(b*c - a*d)*x^2*(a + b*x^8)) - (b^(3/4)*(5*
b*c - 7*a*d)*ArcTan[(Sqrt[b*c - a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^
8])])/(32*(-a)^(9/4)*(b*c - a*d)^(3/2)) - (b^(3/4)*(5*b*c - 7*a*d)*ArcTan[(
Sqrt[-(b*c) + a*d]*x^2)/((-a)^(1/4)*b^(1/4)*Sqrt[c + d*x^8])])/(32*(-a)^(9/
4)*(-(b*c) + a*d)^(3/2)) - (d^(1/4)*(5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticE[2*ArcTan[(d^(1/4)*x^
2)/c^(1/4)], 1/2])/(8*a^2*c^(3/4)*(b*c - a*d)*Sqrt[c + d*x^8]) + (d^(1/4)*(
5*b*c - 4*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*
x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(16*a^2*c^(3/4)*(b
*c - a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] - (Sqrt[-a]*Sqrt[d])/Sqrt[b])*d^(1
/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqr
t[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(32*a^2*c^(1/
4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) + (b*(Sqrt[c] + (Sqrt[-a]*Sqrt[
d])/Sqrt[b])*d^(1/4)*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^
8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticF[2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/
2])/(32*a^2*c^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8]) - (Sqrt[b]*(Sq
rt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2*(5*b*c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)
*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)^2]*EllipticPi[-(Sqrt[b]*Sqrt[c] -
 Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*Sqrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4
)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1/4)*d^(1/4)*(b*c - a*d)*(b*c + a*
d)*Sqrt[c + d*x^8]) + (Sqrt[b]*(Sqrt[b]*Sqrt[c] - Sqrt[-a]*Sqrt[d])^2*(5*b*
c - 7*a*d)*(Sqrt[c] + Sqrt[d]*x^4)*Sqrt[(c + d*x^8)/(Sqrt[c] + Sqrt[d]*x^4)
^2]*EllipticPi[(Sqrt[b]*Sqrt[c] + Sqrt[-a]*Sqrt[d])^2/(4*Sqrt[-a]*Sqrt[b]*S
qrt[c]*Sqrt[d]), 2*ArcTan[(d^(1/4)*x^2)/c^(1/4)], 1/2])/(64*(-a)^(5/2)*c^(1
/4)*d^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[c + d*x^8])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 465

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k -
1)*(a + b*x^(n/k))^p*(c + d*x^(n/k))^q, x], x, x^k], x] /; k != 1] /; FreeQ
[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 490

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s
/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/(
```

```
(r - s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 584

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, p}, x] && IGtQ[n, 0]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx^4\right)^2\sqrt{c+dx^4}}\,dx,x,x^2\right)$$

$$=\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}-\frac{\text{Subst}\left(\int \frac{-5bc+4ad-3bdx^4}{x^2\left(a+bx^4\right)\sqrt{c+dx^4}}\,dx,x,x^2\right)}{8a(bc-ad)}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int \frac{x^2\left(-(bc-2ad)(5bc-\right.}{\left(a+bx^4\right.}\right.}{8a^2c(b}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}+\frac{\text{Subst}\left(\int\left(\frac{d(5bc-4ad)x^2}{\sqrt{c+dx^4}}+\right.\right.}{8a^2c(b}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}-\frac{(b(5bc-7ad))\,\text{Subst}\left(\int\right.}{8a^2(bc-}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+bx^8\right)}+\frac{\left(\sqrt{b}\,(5bc-7ad)\right)\text{Subst}\left(\right.}{16a^2}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+b\right.}$$

$$=-\frac{(5bc-4ad)\sqrt{c+dx^8}}{8a^2c(bc-ad)x^2}+\frac{\sqrt{d}\,(5bc-4ad)x^2\sqrt{c+dx^8}}{8a^2c(bc-ad)\left(\sqrt{c}+\sqrt{d}\,x^4\right)}+\frac{b\sqrt{c+dx^8}}{8a(bc-ad)x^2\left(a+b\right.}$$

Mathematica [C] time = 0.34, size = 226, normalized size = 0.18

$$\frac{-7x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(4a^2d^2-12abcd+5b^2c^2\right)F_1\left(\frac{3}{4};\frac{1}{2},1;\frac{7}{4};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+21a\left(c+dx^8\right)\left(4a^2d-4ab\left(c-\right.\right.}{168a^3cx^2\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (21*a*(c + d*x^8)*(4*a^2*d - 5*b^2*c*x^8 - 4*a*b*(c - d*x^8)) - 7*(5*b^2*c^
2 - 12*a*b*c*d + 4*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[3/
4, 1/2, 1, 7/4, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*(5*b*c - 4*a*d)*x^16*(a
 + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7/4, 1/2, 1, 11/4, -((d*x^8)/c), -((
b*x^8)/a)])/(168*a^3*c*(b*c - a*d)*x^2*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

maple [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
[Out] int(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/(x^3*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Timed out
```

3.924 $\int \frac{x^4}{(a+bx^8)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=64

$$\frac{x^5 \sqrt{\frac{dx^8}{c}+1}\, F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

```
[Out] 1/5*x^5*AppellF1(5/8,2,1/2,13/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {511, 510}

$$\frac{x^5 \sqrt{\frac{dx^8}{c}+1}\, F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*Sqrt[1 + (d*x^8)/c]*AppellF1[5/8, 2, 1/2, 13/8, -((b*x^8)/a), -((d*x^8
)/c)])/(5*a^2*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx^8)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^4}{(a+bx^8)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x^5\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{5}{8};2,\frac{1}{2};\frac{13}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{5a^2\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.20, size = 170, normalized size = 2.66

$$\frac{x^5\left(-5bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{13}{8};\frac{1}{2},1;\frac{21}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+13\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,(3bc-8ad)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\right.\right.}{520a^2\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^5*(65*a*b*(c + d*x^8) + 13*(3*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] - 5*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(520*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [**F**] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**4/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.925 $\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

```
[Out] 1/3*x^3*AppellF1(3/8,2,1/2,11/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d
*x^8+c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$\frac{x^3\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*Sqrt[1 + (d*x^8)/c]*AppellF1[3/8, 2, 1/2, 11/8, -((b*x^8)/a), -((d*x^8
)/c)])/(3*a^2*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{x^2}{\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x^3\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{3}{8};2,\frac{1}{2};\frac{11}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.22, size = 170, normalized size = 2.66

$$\frac{x^3\left(3bdx^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{11}{8};\frac{1}{2},1;\frac{19}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+11\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\,(5bc-8ad)F_1\left(\frac{3}{8};\frac{1}{2},1;\frac{11}{8};-\frac{dx^8}{c},\right.\right.}{264a^2\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-ad)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x^3*(33*a*b*(c + d*x^8) + 11*(5*b*c - 8*a*d)*(a + b*x^8)*Sqrt[1 + (d*x^8)/
c]*AppellF1[3/8, 1/2, 1, 11/8, -((d*x^8)/c), -((b*x^8)/a)] + 3*b*d*x^8*(a +
 b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[11/8, 1/2, 1, 19/8, -((d*x^8)/c), -((b
*x^8)/a)]))/(264*a^2*(b*c - a*d)*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^8+a\right)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+bx^8\right)^2 \sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(x**2/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.926 $\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

```
[Out] x*AppellF1(1/8,2,1/2,9/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/(d*x^8+c)
^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (x*Sqrt[1 + (d*x^8)/c]*AppellF1[1/8, 2, 1/2, 9/8, -((b*x^8)/a), -((d*x^8)/c
)])/(a^2*Sqrt[c + d*x^8])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= \frac{x\sqrt{1+\frac{dx^8}{c}}\,F_1\left(\frac{1}{8};2,\frac{1}{2};\frac{9}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.36, size = 328, normalized size = 5.56

$$-\frac{x\left(bdx^8\sqrt{\frac{dx^8}{c}+1}\,F_1\left(\frac{9}{8};\frac{1}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+\frac{3a\left(4bx^8\left(c+dx^8\right)\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)+9ac(8 \cdots}{\left(a+bx^8\right)\left(4x^8\left(2bcF_1\left(\frac{9}{8};\frac{1}{2},2;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+adF_1\left(\frac{9}{8};\frac{3}{2},1;\frac{17}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)\right)\cdots}\right)}{24a^2\sqrt{c+dx^8}\,(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -1/24*(x*(b*d*x^8*Sqrt[1 + (d*x^8)/c]*AppellF1[9/8, 1/2, 1, 17/8, -((d*x^8)
/c), -((b*x^8)/a)] + (3*a*(9*a*c*(8*a*d - b*(8*c + d*x^8))*AppellF1[1/8, 1/
2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*b*x^8*(c + d*x^8)*(2*b*c*AppellF
1[9/8, 1/2, 2, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1
, 17/8, -((d*x^8)/c), -((b*x^8)/a)])))/((a + b*x^8)*(-9*a*c*AppellF1[1/8, 1
/2, 1, 9/8, -((d*x^8)/c), -((b*x^8)/a)] + 4*x^8*(2*b*c*AppellF1[9/8, 1/2, 2
, 17/8, -((d*x^8)/c), -((b*x^8)/a)] + a*d*AppellF1[9/8, 3/2, 1, 17/8, -((d*
x^8)/c), -((b*x^8)/a)])))))/(a^2*(-(b*c) + a*d)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^8+a)^2\sqrt{dx^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(b\,x^{8}+a\right)^{2}\sqrt{d\,x^{8}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
[Out] int(1/((a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
[Out] Integral(1/((a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.927 $\int \frac{1}{x^2(a+bx^8)^2 \sqrt{c+dx^8}} \, dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\, F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2 x \sqrt{c+dx^8}}$$

```
[Out] -AppellF1(-1/8,2,1/2,7/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x/(d*x^8+
c)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\, F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2 x \sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -((Sqrt[1 + (d*x^8)/c]*AppellF1[-1/8, 2, 1/2, 7/8, -((b*x^8)/a), -((d*x^8)/
c)])/(a^2*x*Sqrt[c + d*x^8]))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^2\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= -\frac{\sqrt{1+\frac{dx^8}{c}}\, F_1\left(-\frac{1}{8};2,\frac{1}{2};\frac{7}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{a^2x\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.32, size = 226, normalized size = 3.65

$$\frac{-5x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(24a^2d^2-40abcd+9b^2c^2\right)F_1\left(\frac{7}{8};\frac{1}{2},1;\frac{15}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+35a\left(c+dx^8\right)\left(8a^2d-8ab\left(c-\right.\right.}{280a^3cx\left(a+bx^8\right)\sqrt{c+dx^8}\,(bc-}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (35*a*(c + d*x^8)*(8*a^2*d - 9*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 5*(9*b^2*c^
2 - 40*a*b*c*d + 24*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[7
/8, 1/2, 1, 15/8, -((d*x^8)/c), -((b*x^8)/a)] + 7*b*d*(9*b*c - 8*a*d)*x^16*
(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[15/8, 1/2, 1, 23/8, -((d*x^8)/c),
-((b*x^8)/a)])/(280*a^3*c*(b*c - a*d)*x*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{dx^8+c}}{b^2dx^{26}+\left(b^2c+2\,abd\right)x^{18}+\left(2\,abc+a^2d\right)x^{10}+a^2cx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^8 + c)/(b^2*d*x^26 + (b^2*c + 2*a*b*d)*x^18 + (2*a*b*c +
a^2*d)*x^10 + a^2*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}\;x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8+a\right)^2\sqrt{dx^8+c}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\left(b\,x^8+a\right)^2\sqrt{d\,x^8+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + bx^8)^2 \sqrt{c + dx^8}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**8)**2*sqrt(c + d*x**8)), x)
```

3.928 $\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

```
[Out] -1/3*AppellF1(-3/8,2,1/2,5/8,-b*x^8/a,-d*x^8/c)*(1+d*x^8/c)^(1/2)/a^2/x^3/(
d*x^8+c)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {511, 510}

$$-\frac{\sqrt{\frac{dx^8}{c}+1}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] -(Sqrt[1 + (d*x^8)/c]*AppellF1[-3/8, 2, 1/2, 5/8, -((b*x^8)/a), -((d*x^8)/c
)])/(3*a^2*x^3*Sqrt[c + d*x^8])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{c+dx^8}}\,dx &= \frac{\sqrt{1+\frac{dx^8}{c}}\int \frac{1}{x^4\left(a+bx^8\right)^2\sqrt{1+\frac{dx^8}{c}}}\,dx}{\sqrt{c+dx^8}}\\ &= -\frac{\sqrt{1+\frac{dx^8}{c}}\,F_1\left(-\frac{3}{8};2,\frac{1}{2};\frac{5}{8};-\frac{bx^8}{a},-\frac{dx^8}{c}\right)}{3a^2x^3\sqrt{c+dx^8}}\end{aligned}$$

Mathematica [B] time = 0.32, size = 226, normalized size = 3.53

$$\frac{-13x^8\left(a+bx^8\right)\sqrt{\frac{dx^8}{c}+1}\left(8a^2d^2-56abcd+33b^2c^2\right)F_1\left(\frac{5}{8};\frac{1}{2},1;\frac{13}{8};-\frac{dx^8}{c},-\frac{bx^8}{a}\right)+65a\left(c+dx^8\right)\left(8a^2d-8ab\left(\right.\right.}{1560a^3cx^3\left(a+bx^8\right)\sqrt{c+dx^8}\left(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a + b*x^8)^2*Sqrt[c + d*x^8]),x]
```

```
[Out] (65*a*(c + d*x^8)*(8*a^2*d - 11*b^2*c*x^8 - 8*a*b*(c - d*x^8)) - 13*(33*b^2
*c^2 - 56*a*b*c*d + 8*a^2*d^2)*x^8*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1
[5/8, 1/2, 1, 13/8, -((d*x^8)/c), -((b*x^8)/a)] + 5*b*d*(11*b*c - 8*a*d)*x^
16*(a + b*x^8)*Sqrt[1 + (d*x^8)/c]*AppellF1[13/8, 1/2, 1, 21/8, -((d*x^8)/c
), -((b*x^8)/a)])/(1560*a^3*c*(b*c - a*d)*x^3*(a + b*x^8)*Sqrt[c + d*x^8])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

maple [F] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}\; x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

```
[Out] int(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^8 + a\right)^2 \sqrt{dx^8 + c}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^8+a)^2/(d*x^8+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^8 + a)^2*sqrt(d*x^8 + c)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^4 \left(b\,x^8 + a\right)^2 \sqrt{d\,x^8 + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)),x)
```

```
[Out] int(1/(x^4*(a + b*x^8)^2*(c + d*x^8)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**8+a)**2/(d*x**8+c)**(1/2),x)
```

```
[Out] Timed out
```

3.929 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^5\,dx$

Optimal. Leaf size=123

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

```
[Out] 1/6*a*(c+d/x^2)^(3/2)*x^6/c-1/16*d^2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c
^(1/2))/c^(5/2)+1/16*d*(-a*d+2*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/8*(-a*d+2*b*c
)*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 51, 63, 208}

$$-\frac{d^2(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{5/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{16c^2}+\frac{x^4\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{8c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (d*(2*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c^2) + ((2*b*c - a*d)*Sqrt[c + d/
x^2]*x^4)/(8*c) + (a*(c + d/x^2)^(3/2)*x^6)/(6*c) - (d^2*(2*b*c - a*d)*ArcT
anh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^5\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x^4}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{\left(3bc-\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{6c}$$

$$= \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{(d(2bc-ad))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,\right.}{16c}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{\left(d^2(2bc-a\right.}{}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} + \frac{(d(2bc-ad}{}$$

$$= \frac{d(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c^2} + \frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^4}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^6}{6c} - \frac{d^2(2bc-ad}{}$$

Mathematica [A] time = 0.26, size = 121, normalized size = 0.98

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+2cdx^2-3d^2\right)+6bc\left(2cx^2+d\right)\right)+3d^{3/2}(ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{48c^{5/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(d + 2*c*x^2) + a*
(-3*d^2 + 2*c*d*x^2 + 8*c^2*x^4)) + 3*d^(3/2)*(-2*b*c + a*d)*ArcSinh[(Sqrt[
c]*x)/Sqrt[d]]))/(48*c^(5/2)*Sqrt[1 + (c*x^2)/d])
```

fricas [A] time = 0.93, size = 242, normalized size = 1.97

$$\left[-\frac{3\left(2\,bcd^{2}-ad^{3}\right)\sqrt{c}\,\log\left(-2\,cx^{2}-2\,\sqrt{c}\,x^{2}\sqrt{\frac{cx^{2}+d}{x^{2}}}-d\right)-2\left(8\,ac^{3}x^{6}+2\left(6\,bc^{3}+ac^{2}d\right)x^{4}+3\left(2\,bc^{2}d-acd^{2}\right)x^{2}\right)\cdots}{96\,c^{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(2*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^
2*d - a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^3, 1/48*(3*(2*b*c*d^2 - a*d^3)
*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c^3
*x^6 + 2*(6*b*c^3 + a*c^2*d)*x^4 + 3*(2*b*c^2*d - a*c*d^2)*x^2)*sqrt((c*x^2
 + d)/x^2))/c^3]
```

giac [A] time = 0.19, size = 143, normalized size = 1.16

$$\frac{1}{48}\left(2\left(4\,ax^{2}\mathrm{sgn}(x)+\frac{6\,bc^{4}\mathrm{sgn}(x)+ac^{3}d\mathrm{sgn}(x)}{c^{4}}\right)x^{2}+\frac{3\left(2\,bc^{3}d\mathrm{sgn}(x)-ac^{2}d^{2}\mathrm{sgn}(x)\right)}{c^{4}}\right)\sqrt{cx^{2}+d}\,x+\frac{\left(2\,bcd^{2}\mathrm{sgn}(x\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*x^2*sgn(x) + (6*b*c^4*sgn(x) + a*c^3*d*sgn(x))/c^4)*x^2 + 3*(2
*b*c^3*d*sgn(x) - a*c^2*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x + 1/16*(2*b*c*d^
2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(5/2) - 1
/32*(2*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(5/2)
```

maple [A] time = 0.06, size = 162, normalized size = 1.32

$$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(8\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\,c^{\frac{3}{2}}x^{3}+3a\,d^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)-6bc\,d^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)+3\sqrt{c\,x^{2}+d}\,a\sqrt{c}\,d\cdots\right.}{48\sqrt{c\,x^{2}+d}\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)*x*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^3*a-6*c^(1/2)*(c*
x^2+d)^(3/2)*x*a*d+12*c^(3/2)*(c*x^2+d)^(3/2)*x*b+3*c^(1/2)*(c*x^2+d)^(1/2)
*x*a*d^2-6*c^(3/2)*(c*x^2+d)^(1/2)*x*b*d+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*
d^3-6*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d^2)/(c*x^2+d)^(1/2)/c^(5/2)
```

maxima [B] time = 1.36, size = 243, normalized size = 1.98

$$-\frac{1}{96}\left(\frac{3\,d^{3}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}d^{3}-8\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}cd^{3}-3\sqrt{c+\frac{d}{x^{2}}}\,c^{2}d^{3}\right)}{\left(c+\frac{d}{x^{2}}\right)^{3}c^{2}-3\left(c+\frac{d}{x^{2}}\right)^{2}c^{3}+3\left(c+\frac{d}{x^{2}}\right)c^{4}-c^{5}}\right)a+\frac{1}{16}\left(\frac{d^{2}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^5*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c
^(5/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 - 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c
+ d/x^2)*c^2*d^3)/((c + d/x^2)^3*c^2 - 3*(c + d/x^2)^2*c^3 + 3*(c + d/x^2)*
c^4 - c^5))*a + 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2)
+ sqrt(c)))/c^(3/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c
 + d/x^2)^2*c - 2*(c + d/x^2)*c^2 + c^3))*b
```

mupad [B] time = 5.71, size = 134, normalized size = 1.09

$$\frac{a\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}+\frac{b\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{6\,c}-\frac{a\,x^6\,{\left(c+\frac{d}{x^2}\right)}^{5/2}}{16\,c^2}+\frac{b\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}-\frac{b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}-\frac{a\,d^3\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(1/2))/16 + (b*x^4*(c + d/x^2)^(1/2))/8 + (a*x^6*(c + d/
x^2)^(3/2))/(6*c) - (a*x^6*(c + d/x^2)^(5/2))/(16*c^2) + (b*x^4*(c + d/x^2)
^(3/2))/(8*c) - (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c^(5/2)
) - (b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

sympy [B] time = 72.29, size = 226, normalized size = 1.84

$$\frac{acx^7}{6\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+\frac{5a\sqrt{d}\,x^5}{24\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}x^3}{48c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{5}{2}}x}{16c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^3\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{16c^{\frac{5}{2}}}+\frac{bcx^5}{4\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+\frac{3b\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**5*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 5*a*sqrt(d)*x**5/(24*sqrt(c*x**2/
d + 1)) - a*d**(3/2)*x**3/(48*c*sqrt(c*x**2/d + 1)) - a*d**(5/2)*x/(16*c**2
*sqrt(c*x**2/d + 1)) + a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(5/2)) + b*c*
x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*sqrt(d)*x**3/(8*sqrt(c*x**2/d + 1
)) + b*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - b*d**2*asinh(sqrt(c)*x/sqrt(d)
)/(8*c**(3/2))
```

3.930 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^3\,dx$

Optimal. Leaf size=90

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

```
[Out] 1/4*a*(c+d/x^2)^(3/2)*x^4/c+1/8*d*(-a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1
/2))/c^(3/2)+1/8*(-a*d+4*b*c)*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-ad)}{8c}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] ((4*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(8*c) + (a*(c + d/x^2)^(3/2)*x^4)/(4*c)
 + (d*(4*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^3\,dx=-\left(\frac{1}{2}\,\text{Subst}\left(\int\frac{(a+bx)\sqrt{c+dx}}{x^3}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{\left(2bc-\frac{ad}{2}\right)\text{Subst}\left(\int\frac{\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)}{4c}$$

$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{(d(4bc-ad))\,\text{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{16c}$$

$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}-\frac{(4bc-ad)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{8c}$$

$$=\frac{(4bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{4c}+\frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{3/2}}$$

Mathematica [A] time = 0.14, size = 100, normalized size = 1.11

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\,\left(a\left(2cx^2+d\right)+4bc\right)+\sqrt{d}\,(4bc-ad)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{8c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(4*b*c + a*(d + 2*c*x^2))
 + Sqrt[d]*(4*b*c - a*d)*ArcSinh[(Sqrt[c]*x)/Sqrt[d]]))/(8*c^(3/2)*Sqrt[1 +
 (c*x^2)/d])
```

fricas [A] time = 0.62, size = 191, normalized size = 2.12

$$\left[-\frac{\left(4\,bcd-ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+\left(4\,bc^2+acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^2},-\frac{\left(4\,bcd-ad^2\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - a*d^2)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*x^2
+ d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/c^2, -1/8*((4*b*c*d - a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2
```

```
+ d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 + (4*b*c^2 + a*c*d)*x^2)*sqrt((c*x^2
+ d)/x^2))/c^2]
```

giac [A] time = 0.26, size = 105, normalized size = 1.17

$$\frac{1}{8}\left(2\,ax^2\mathrm{sgn}(x)+\frac{4\,bc^2\mathrm{sgn}(x)+acd\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2+d}\,x-\frac{\left(4\,bcd\mathrm{sgn}(x)-ad^2\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2+d}\right|\right)}{8\,c^{\frac{3}{2}}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*a*x^2*sgn(x) + (4*b*c^2*sgn(x) + a*c*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*
x - 1/8*(4*b*c*d*sgn(x) - a*d^2*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d
)))/c^(3/2) + 1/16*(4*b*c*d*log(abs(d)) - a*d^2*log(abs(d)))*sgn(x)/c^(3/2)
```

maple [A] time = 0.06, size = 122, normalized size = 1.36

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-a\,d^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)+4bcd\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)-\sqrt{c\,x^2+d}\;a\sqrt{c}\;dx+4\sqrt{c\,x^2+d}\;b\,c^{\frac{3}{2}}x+2\cdots\right.}{8\sqrt{c\,x^2+d}\;c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(1/2)*x*(2*(c*x^2+d)^(3/2)*c^(1/2)*x*a-(c*x^2+d)^(1/2)*
c^(1/2)*x*a*d+4*(c*x^2+d)^(1/2)*c^(3/2)*x*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a
*d^2+4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c*d)/(c*x^2+d)^(1/2)/c^(3/2)
```

maxima [B] time = 1.24, size = 159, normalized size = 1.77

$$\frac{1}{16}\left(\frac{d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2+\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c-2\left(c+\frac{d}{x^2}\right)c^2+c^3}\right)a+\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*(d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3
/2) + 2*((c + d/x^2)^(3/2)*d^2 + sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c -
2*(c + d/x^2)*c^2 + c^3))*a + 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c +
d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c))*b
```

mupad [B] time = 5.31, size = 93, normalized size = 1.03

$$\frac{a\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}+\frac{a\,x^4\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}{8\,c}+\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^4*(c + d/x^2)^(1/2))/8 + (b*x^2*(c + d/x^2)^(1/2))/2 + (a*x^4*(c + d/x
^2)^(3/2))/(8*c) + (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2)) - (a*
d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(3/2))
```

sympy [A] time = 59.05, size = 144, normalized size = 1.60

$$\frac{acx^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3a\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{3}{2}}x}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{3}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3*(c+d/x**2)**(1/2),x)
```

```
[Out] a*c*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*a*sqrt(d)*x**3/(8*sqrt(c*x**2/d
 + 1)) + a*d**(3/2)*x/(8*c*sqrt(c*x**2/d + 1)) - a*d**2*asinh(sqrt(c)*x/sqr
t(d))/(8*c**(3/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + b*d*asinh(sqrt(c)*x
/sqrt(d))/(2*sqrt(c))
```

3.931 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x\,dx$

Optimal. Leaf size=84

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

```
[Out] 1/2*a*(c+d/x^2)^(3/2)*x^2/c+1/2*(a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2)
)/c^(1/2)-1/2*(a*d+2*b*c)*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad+2bc)}{2c}+\frac{(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{3/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] -((2*b*c + a*d)*Sqrt[c + d/x^2])/(2*c) + (a*(c + d/x^2)^(3/2)*x^2)/(2*c) +
((2*b*c + a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*Sqrt[c])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x\,dx=-\left(\frac{1}{2}\,\text{Subst}\left(\int\frac{(a+bx)\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{(2bc+ad)\,\text{Subst}\left(\int\frac{\sqrt{c+dx}}{x}\,dx,x,\frac{1}{x^2}\right)}{4c}$$
$$=-\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{1}{4}(2bc+ad)\,\text{Subst}\left(\int\frac{1}{x\sqrt{c+dx}}\,dx,x,$$
$$=-\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}-\frac{(2bc+ad)\,\text{Subst}\left(\int\frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+$$
$$=-\frac{(2bc+ad)\sqrt{c+\frac{d}{x^2}}}{2c}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^2}{2c}+\frac{(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Mathematica [A] time = 0.19, size = 71, normalized size = 0.85

$$\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\left(\frac{x(ad+2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}\,\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}+ax^2-2b\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-2*b + a*x^2 + ((2*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[
d]])/(Sqrt[c]*Sqrt[d]*Sqrt[1 + (c*x^2)/d])))/2
```

fricas [A] time = 1.18, size = 155, normalized size = 1.85

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(acx^2-2\,bc\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\,c},-\frac{(2\,bc+ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{}}{cx^2+}\right.}{2\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x
^2) - d) + 2*(a*c*x^2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c, -1/2*((2*b*c + a*d
)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (a*c*x^
2 - 2*b*c)*sqrt((c*x^2 + d)/x^2))/c]
```

giac [A] time = 0.29, size = 92, normalized size = 1.10

$$\frac{1}{2}\sqrt{cx^2+d}\,ax\text{sgn}(x)+\frac{2\,b\sqrt{c}\,d\text{sgn}(x)}{\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2-d}-\frac{\left(2\,bc^{\frac{3}{2}}\text{sgn}(x)+a\sqrt{c}\,d\text{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*x*sgn(x) + 2*b*sqrt(c)*d*sgn(x)/((sqrt(c)*x - sqrt(c*
x^2 + d))^2 - d) - 1/4*(2*b*c^(3/2)*sgn(x) + a*sqrt(c)*d*sgn(x))*log((sqrt(
c)*x - sqrt(c*x^2 + d))^2)/c
```

maple [A] time = 0.06, size = 129, normalized size = 1.54

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(-a\,d^2x\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-2bcdx\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)-\sqrt{c\,x^2+d}\,a\sqrt{c}\,d\,x^2-2\sqrt{c\,x^2+d}\,b\,c^{\frac{3}{2}}\right.}{2\sqrt{c\,x^2+d}\,\sqrt{c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)*(-(c*x^2+d)^(1/2)*c^(1/2)*x^2*a*d-2*(c*x^2+d)^(1
/2)*c^(3/2)*x^2*b+2*(c*x^2+d)^(3/2)*c^(1/2)*b-ln(c^(1/2)*x+(c*x^2+d)^(1/2))
*x*a*d^2-2*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d)/(c*x^2+d)^(1/2)/d/c^(1/2)
```

maxima [A] time = 1.21, size = 108, normalized size = 1.29

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,x^2-\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\right)a-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^2}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*x^2 - d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/
x^2) + sqrt(c)))/sqrt(c))*a - 1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) + 2*sqrt(c + d/x^2))*b
```

mupad [B] time = 5.10, size = 68, normalized size = 0.81

$$\frac{a\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}-b\,\sqrt{c+\frac{d}{x^2}}+b\,\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^2*(c + d/x^2)^(1/2))/2 - b*(c + d/x^2)^(1/2) + b*c^(1/2)*atanh((c + d/
x^2)^(1/2)/c^(1/2)) + (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(1/2))
```

sympy [A] time = 60.43, size = 107, normalized size = 1.27

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}+\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2\sqrt{c}}+b\sqrt{c}\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bcx}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{b\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/2 + a*d*asinh(sqrt(c)*x/sqrt(d))/(2*sqrt(c))
 + b*sqrt(c)*asinh(sqrt(c)*x/sqrt(d)) - b*c*x/(sqrt(d)*sqrt(c*x**2/d + 1))
- b*sqrt(d)/(x*sqrt(c*x**2/d + 1))
```

3.932 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx$

Optimal. Leaf size=59

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d+a*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-a*(c+d/
x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$-a\sqrt{c+\frac{d}{x^2}}+a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] -(a*Sqrt[c + d/x^2]) - (b*(c + d/x^2)^(3/2))/(3*d) + a*Sqrt[c]*ArcTanh[Sqrt
[c + d/x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} - \frac{(ac)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -a\sqrt{c+\frac{d}{x^2}} - \frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d} + a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 82, normalized size = 1.39

$$
\frac{\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\sqrt{c}\sqrt{d}x^3\sinh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{\sqrt{\frac{cx^2}{d}+1}} - 3adx^2 - b\left(cx^2+d\right)\right)}{3dx^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-3*a*d*x^2 - b*(d + c*x^2) + (3*a*Sqrt[c]*Sqrt[d]*x^3*Arc
Sinh[(Sqrt[c]*x)/Sqrt[d]])/Sqrt[1 + (c*x^2)/d]))/(3*d*x^2)
```

fricas [A] time = 0.90, size = 166, normalized size = 2.81

$$
\left[\frac{3a\sqrt{c}dx^2\log\left(-2cx^2-2\sqrt{c}x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left((bc+3ad)x^2+bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{6dx^2}, -\frac{3a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}x^2\sqrt{\frac{cx^2}{x}}}{cx^2+d}\right.}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2)
- d) - 2*((b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2), -1/3*(3*
a*sqrt(-c)*d*x^2*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (
(b*c + 3*a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2))/(d*x^2)]
```

giac [B] time = 0.49, size = 163, normalized size = 2.76

$$-\frac{1}{2}\,a\sqrt{c}\,\log\left(\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^2\right)\mathrm{sgn}(x)+\frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4bc^{\frac{3}{2}}\mathrm{sgn}(x)+3\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^4a\sqrt{c}\,d\mathrm{sgn}\ldots\right.}{3\left(\left(\sqrt{c}x-\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*sqrt(c)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/3*(3*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(3/2)*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))
^4*a*sqrt(c)*d*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^2*sgn
(x) + b*c^(3/2)*d^2*sgn(x) + 3*a*sqrt(c)*d^3*sgn(x))/((sqrt(c)*x - sqrt(c*x
^2 + d))^2 - d)^3
```

maple [B] time = 0.06, size = 109, normalized size = 1.85

$$-\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(-3acd\,x^{3}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)-3\sqrt{c\,x^{2}+d}\,a\,c^{\frac{3}{2}}x^{4}+3\left(c\,x^{2}+d\right)^{\frac{3}{2}}a\sqrt{c}\,x^{2}+\left(c\,x^{2}+d\right)^{\frac{3}{2}}b\sqrt{c}\right)}{3\sqrt{c\,x^{2}+d}\,\sqrt{c}\,d\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x,x)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)/x^2*(-3*(c*x^2+d)^(1/2)*c^(3/2)*x^4*a+3*(c*x^2+d
)^(3/2)*c^(1/2)*x^2*a-3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d+(c*x^2+d)^(
3/2)*b*c^(1/2))/(c*x^2+d)^(1/2)/d/c^(1/2)
```

maxima [A] time = 1.23, size = 67, normalized size = 1.14

$$-\frac{1}{2}\left(\sqrt{c}\,\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)+2\sqrt{c+\frac{d}{x^{2}}}\right)a-\frac{b\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(c)*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))
+ 2*sqrt(c + d/x^2))*a - 1/3*b*(c + d/x^2)^(3/2)/d
```

mupad [B] time = 5.21, size = 57, normalized size = 0.97

$$a\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)-a\sqrt{c+\frac{d}{x^{2}}}-\frac{b\sqrt{c+\frac{d}{x^{2}}}\,\left(c\,x^{2}+d\right)}{3\,d\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x,x)
```

```
[Out] a*c^(1/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - a*(c + d/x^2)^(1/2) - (b*(c +
d/x^2)^(1/2)*(d + c*x^2))/(3*d*x^2)
```

sympy [A] time = 23.44, size = 75, normalized size = 1.27

$$\frac{a\left(-\frac{2c\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-2\sqrt{c+\frac{d}{x^2}}\right)}{2}+\frac{b\left(\begin{cases}-\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ -\frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x,x)
```

```
[Out] a*(-2*c*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - 2*sqrt(c + d/x**2))/2 +
b*Piecewise((-sqrt(c)/x**2, Eq(d, 0)), (-2*(c + d/x**2)**(3/2)/(3*d), True)
)/2
```

3.933 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

```
[Out] 1/3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^2-1/5*b*(c+d/x^2)^(5/2)/d^2
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^2) - (b*(c + d/x^2)^(5/2))/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^3}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int (a+bx)\sqrt{c+dx}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\,\text{Subst}\left(\int\left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\ &= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(5adx^2-2bcx^2+3bd\right)}{15d^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^3,x]
```

```
[Out] -1/15*(Sqrt[c + d/x^2]*(d + c*x^2)*(3*b*d - 2*b*c*x^2 + 5*a*d*x^2))/(d^2*x^
4)
```

fricas [A] time = 0.79, size = 60, normalized size = 1.30

$$\frac{\left(\left(2\,bc^2 - 5\,acd\right)x^4 - 3\,bd^2 - \left(bcd + 5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*c^2 - 5*a*c*d)*x^4 - 3*b*d^2 - (b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2)/(d^2*x^4)
```

giac [B] time = 0.92, size = 250, normalized size = 5.43

$$\frac{2\left(15\left(\sqrt{c}\,x - \sqrt{cx^2+d}\right)^8 ac^{\frac{3}{2}}\mathrm{sgn}(x) + 30\left(\sqrt{c}\,x - \sqrt{cx^2+d}\right)^6 bc^{\frac{5}{2}}\mathrm{sgn}(x) - 30\left(\sqrt{c}\,x - \sqrt{cx^2+d}\right)^6 ac^{\frac{3}{2}}d\mathrm{sgn}(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(3/2)*sgn(x) + 30*(sqrt(c)*x -
 sqrt(c*x^2 + d))^6*b*c^(5/2)*sgn(x) - 30*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a
*c^(3/2)*d*sgn(x) + 10*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d*sgn(x) +
 20*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^2*sgn(x) + 10*(sqrt(c)*x -
sqrt(c*x^2 + d))^2*b*c^(5/2)*d^2*sgn(x) - 10*(sqrt(c)*x - sqrt(c*x^2 + d))^
2*a*c^(3/2)*d^3*sgn(x) - 2*b*c^(5/2)*d^3*sgn(x) + 5*a*c^(3/2)*d^4*sgn(x))/(
(sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

maple [A] time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(5ad\,x^2 - 2bc\,x^2 + 3bd\right)\left(c\,x^2 + d\right)}{15d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x)
```

```
[Out] -1/15*((c*x^2+d)/x^2)^(1/2)*(5*a*d*x^2-2*b*c*x^2+3*b*d)*(c*x^2+d)/d^2/x^4
```

maxima [A] time = 0.72, size = 49, normalized size = 1.07

$$-\frac{1}{15}\,b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2} - \frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right) - \frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2)*c/d^2) - 1/3*a*(c +
d/x^2)^(3/2)/d
```

mupad [B] time = 4.82, size = 91, normalized size = 1.98

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(b\,c^2+a\,d\,c\right)}{5\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(5\,a\,d^2+b\,c\,d\right)}{15\,d^2\,x^2}-\frac{c\,\sqrt{c+\frac{d}{x^2}}\,\left(8\,a\,d+b\,c\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^3,x)
```

```
[Out] ((c + d/x^2)^(1/2)*(b*c^2 + a*c*d))/(5*d^2) - (b*(c + d/x^2)^(1/2))/(5*x^4)
 - ((c + d/x^2)^(1/2)*(5*a*d^2 + b*c*d))/(15*d^2*x^2) - (c*(c + d/x^2)^(1/2
)*(8*a*d + b*c))/(15*d^2)
```

sympy [A] time = 4.02, size = 58, normalized size = 1.26

$$-\frac{a\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for}\ d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**3,x)
```

```
[Out] -a*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True))
/2 - b*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2
```

3.934 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx$

Optimal. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

```
[Out] -1/3*c*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^3+1/5*(-a*d+2*b*c)*(c+d/x^2)^(5/2)/d^3-
1/7*b*(c+d/x^2)^(7/2)/d^3
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(2bc-ad)}{5d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(5/
2))/(5*d^3) - (b*(c + d/x^2)^(7/2))/(7*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^5}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)\sqrt{c+dx}}{d^2}+\frac{(-2bc+ad)(c+dx)^{3/2}}{d^2}+\frac{b(c+dx)^{5/2}}{d^2}\right) dx, x,\right.\right.\\
&= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 0.93

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(cx^2+d)\left(7adx^2\left(2cx^2-3d\right)+b\left(-8c^2x^4+12cdx^2-15d^2\right)\right)}{105d^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(7*a*d*x^2*(-3*d + 2*c*x^2) + b*(-15*d^2 + 12*
c*d*x^2 - 8*c^2*x^4)))/(105*d^3*x^6)
```

fricas [A] time = 0.65, size = 85, normalized size = 1.15

$$-\frac{\left(2\left(4\,bc^3-7\,ac^2d\right)x^6-\left(4\,bc^2d-7\,acd^2\right)x^4+15\,bd^3+3\left(bcd^2+7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/105*(2*(4*b*c^3 - 7*a*c^2*d)*x^6 - (4*b*c^2*d - 7*a*c*d^2)*x^4 + 15*b*d^
3 + 3*(b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^3*x^6)
```

giac [B] time = 1.38, size = 310, normalized size = 4.19

$$4\left(105\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{5}{2}}\mathrm{sgn}(x)+280\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}bc^{\frac{7}{2}}\mathrm{sgn}(x)-175\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{8}ac^{\frac{5}{2}}d\mathrm{sgn}(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/105*(105*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(5/2)*sgn(x) + 280*(sqrt(c)
*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*sgn(x) - 175*(sqrt(c)*x - sqrt(c*x^2 + d)
)^8*a*c^(5/2)*d*sgn(x) + 140*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d*sg
n(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(5/2)*d^2*sgn(x) + 84*(sqrt(c
)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*x^2
+ d))^4*a*c^(5/2)*d^3*sgn(x) - 28*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)
*d^3*sgn(x) + 49*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(5/2)*d^4*sgn(x) + 4*b
*c^(7/2)*d^4*sgn(x) - 7*a*c^(5/2)*d^5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d)
)^2 - d)^7
```

maple [A] time = 0.05, size = 70, normalized size = 0.95

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(14acd\,x^4-8b\,c^2x^4-21a\,d^2x^2+12bcd\,x^2-15b\,d^2\right)\left(c\,x^2+d\right)}{105d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*(14*a*c*d*x^4-8*b*c^2*x^4-21*a*d^2*x^2+12*b*c*d
*x^2-15*b*d^2)*(c*x^2+d)/d^3/x^6
```

maxima [A] time = 0.60, size = 84, normalized size = 1.14

$$-\frac{1}{105}\,b\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^3}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^3}\right)-\frac{1}{15}\,a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^2}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/105*b*(15*(c + d/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d
/x^2)^(3/2)*c^2/d^3) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^2 - 5*(c + d/x^2)^(3/2
)*c/d^2)
```

mupad [B] time = 4.92, size = 126, normalized size = 1.70

$$\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{15\,d^2}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{15\,d\,x^2}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}+\frac{4\,b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^5,x)
```

```
[Out] (2*a*c^2*(c + d/x^2)^(1/2))/(15*d^2) - (b*(c + d/x^2)^(1/2))/(7*x^6) - (a*(
c + d/x^2)^(1/2))/(5*x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*c*(c
 + d/x^2)^(1/2))/(15*d*x^2) - (b*c*(c + d/x^2)^(1/2))/(35*d*x^4) + (4*b*c^2
*(c + d/x^2)^(1/2))/(105*d^2*x^2)
```

sympy [A] time = 4.56, size = 78, normalized size = 1.05

$$-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**5,x)
```

```
[Out] -a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d
/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**3
```

3.935 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx$

Optimal. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

```
[Out] 1/3*c^2*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^4-1/5*c*(-2*a*d+3*b*c)*(c+d/x^2)^(5/2)
/d^4+1/7*(-a*d+3*b*c)*(c+d/x^2)^(7/2)/d^4-1/9*b*(c+d/x^2)^(9/2)/d^4
```

Rubi [A] time = 0.08, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{446, 77\}$

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-ad)}{7d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-2ad)}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(5/2))/(5*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^4) - (b*(c + d/x^
2)^(9/2))/(9*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^7}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int x^2(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\text{Subst}\left(\int\left(-\frac{c^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{3/2}}{d^3}+\frac{(-3bc+ad)(c+dx}{d^3}\right.\right.\right.\\ &= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right.}{9d^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.76

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-12cdx^2+15d^2\right)(6bc-9ad)-105bd^2\left(cx^2+d\right)\right)}{945d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-105*b*d^2*(d + c*x^2) + (6*b*c - 9*a*d)*x^2*(1 + (c*x^2)
/d)*(15*d^2 - 12*c*d*x^2 + 8*c^2*x^4)))/(945*d^3*x^8)
```

fricas [A] time = 0.65, size = 109, normalized size = 1.05

$$\frac{\left(8\left(2\,bc^4-3\,ac^3d\right)x^8-4\left(2\,bc^3d-3\,ac^2d^2\right)x^6-35\,bd^4+3\left(2\,bc^2d^2-3\,acd^3\right)x^4-5\left(bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(2*b*c^4 - 3*a*c^3*d)*x^8 - 4*(2*b*c^3*d - 3*a*c^2*d^2)*x^6 - 35*b
*d^4 + 3*(2*b*c^2*d^2 - 3*a*c*d^3)*x^4 - 5*(b*c*d^3 + 9*a*d^4)*x^2)*sqrt((c
*x^2 + d)/x^2)/(d^4*x^8)
```

giac [B] time = 2.06, size = 370, normalized size = 3.56

$$16\left(210\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}\mathrm{sgn}(x)+630\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{10}ac^{\frac{7}{2}}d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(7/2)*sgn(x) + 630*(sqrt(c
)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 +
d))^10*a*c^(7/2)*d*sgn(x) + 378*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d
*sgn(x) + 63*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(7/2)*d^2*sgn(x) + 168*(sq
rt(c)*x - sqrt(c*x^2 + d))^6*b*c^(9/2)*d^2*sgn(x) - 42*(sqrt(c)*x - sqrt(c*
x^2 + d))^6*a*c^(7/2)*d^3*sgn(x) - 72*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(
9/2)*d^3*sgn(x) + 108*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^4*sgn(x)
+ 18*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(9/2)*d^4*sgn(x) - 27*(sqrt(c)*x -
 sqrt(c*x^2 + d))^2*a*c^(7/2)*d^5*sgn(x) - 2*b*c^(9/2)*d^5*sgn(x) + 3*a*c^(
7/2)*d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^9
```

maple [A] time = 0.05, size = 94, normalized size = 0.90

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(24a\,c^2d\,x^6-16b\,c^3x^6-36ac\,d^2x^4+24b\,c^2d\,x^4+45a\,d^3x^2-30bc\,d^2x^2+35b\,d^3\right)\left(c\,x^2+d\right)}{315d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x)
```

```
[Out] -1/315*((c*x^2+d)/x^2)^(1/2)*(24*a*c^2*d*x^6-16*b*c^3*x^6-36*a*c*d^2*x^4+24
*b*c^2*d*x^4+45*a*d^3*x^2-30*b*c*d^2*x^2+35*b*d^3)*(c*x^2+d)/d^4/x^8
```

maxima [A] time = 0.57, size = 118, normalized size = 1.13

$$-\frac{1}{315}\,b\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^4}-\frac{135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^4}+\frac{189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^4}-\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3}{d^4}\right)-\frac{1}{105}\,a\left(\frac{15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^3}-\frac{42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*b*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7/2)*c/d^4 + 189*(c +
 d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4) - 1/105*a*(15*(c + d
/x^2)^(7/2)/d^3 - 42*(c + d/x^2)^(5/2)*c/d^3 + 35*(c + d/x^2)^(3/2)*c^2/d^3
)
```

mupad [B] time = 5.22, size = 168, normalized size = 1.62

$$\frac{16\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{8\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{105\,d^3}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^4}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}+\frac{4\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^2}+\frac{2\,b\,c^2\,\sqrt{c+}}{105\,d^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^7,x)
```

```
[Out] (16*b*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(9*x^8) - (8
*a*c^3*(c + d/x^2)^(1/2))/(105*d^3) - (a*(c + d/x^2)^(1/2))/(7*x^6) - (a*c*
(c + d/x^2)^(1/2))/(35*d*x^4) - (b*c*(c + d/x^2)^(1/2))/(63*d*x^6) + (4*a*c
^2*(c + d/x^2)^(1/2))/(105*d^2*x^2) + (2*b*c^2*(c + d/x^2)^(1/2))/(105*d^2*
x^4) - (8*b*c^3*(c + d/x^2)^(1/2))/(315*d^3*x^2)
```

sympy [A] time = 5.21, size = 112, normalized size = 1.08

$$-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**7,x)
```

```
[Out] -a*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**
(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)
/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4
```

3.936 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx$

Optimal. Leaf size=134

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

```
[Out] -1/3*c^3*(-a*d+b*c)*(c+d/x^2)^(3/2)/d^5+1/5*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(5
/2)/d^5-3/7*c*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^5+1/9*(-a*d+4*b*c)*(c+d/x^2)^(
9/2)/d^5-1/11*b*(c+d/x^2)^(11/2)/d^5
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-3ad)}{5d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{3/2}(bc-ad)}{3d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(4bc-ad)}{9d^5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (3*c*(2*b*c - a*d)*(c + d/x^2)^(7/2))/(7*d^5) + ((4*b
*c - a*d)*(c + d/x^2)^(9/2))/(9*d^5) - (b*(c + d/x^2)^(11/2))/(11*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^9}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int x^3(a+bx)\sqrt{c+dx}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)\sqrt{c+dx}}{d^4}-\frac{c^2(4bc-3ad)(c+dx)^{3/2}}{d^4}+\frac{3c(2bc-ad)(c+\right.\right.\right.}{d^4}\\
&= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}-\frac{3c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}+\frac{(4}{}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 90, normalized size = 0.67

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(x^2\left(\frac{cx^2}{d}+1\right)\left(-16c^3x^6+24c^2dx^4-30cd^2x^2+35d^3\right)(8bc-11ad)-315bd^3\left(cx^2+d\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-315*b*d^3*(d + c*x^2) + (8*b*c - 11*a*d)*x^2*(1 + (c*x^2
)/d)*(35*d^3 - 30*c*d^2*x^2 + 24*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x^10)
```

fricas [A] time = 0.82, size = 133, normalized size = 0.99

$$-\frac{\left(16\left(8\,bc^5-11\,ac^4d\right)x^{10}-8\left(8\,bc^4d-11\,ac^3d^2\right)x^8+6\left(8\,bc^3d^2-11\,ac^2d^3\right)x^6+315\,bd^5-5\left(8\,bc^2d^3-11\,acd^4\right)x^4\right.}{3465\,d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/3465*(16*(8*b*c^5 - 11*a*c^4*d)*x^10 - 8*(8*b*c^4*d - 11*a*c^3*d^2)*x^8
+ 6*(8*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 + 315*b*d^5 - 5*(8*b*c^2*d^3 - 11*a*c*
d^4)*x^4 + 35*(b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^10)
```

giac [B] time = 2.98, size = 430, normalized size = 3.21

$$32\left(3465\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\mathrm{sgn}(x)+11088\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{11}{2}}\mathrm{sgn}(x)-4851\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{12}ac\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/3465*(3465*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(9/2)*sgn(x) + 11088*(sq
rt(c)*x - sqrt(c*x^2 + d))^12*b*c^(11/2)*sgn(x) - 4851*(sqrt(c)*x - sqrt(c*
x^2 + d))^12*a*c^(9/2)*d*sgn(x) + 7392*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^2*sgn(x
) + 2640*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(11/2)*d^2*sgn(x) - 165*(sqrt(
c)*x - sqrt(c*x^2 + d))^8*a*c^(9/2)*d^3*sgn(x) - 1320*(sqrt(c)*x - sqrt(c*x
^2 + d))^6*b*c^(11/2)*d^3*sgn(x) + 1815*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c
^(9/2)*d^4*sgn(x) + 440*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^4*sgn(
x) - 605*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(9/2)*d^5*sgn(x) - 88*(sqrt(c)
*x - sqrt(c*x^2 + d))^2*b*c^(11/2)*d^5*sgn(x) + 121*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*a*c^(9/2)*d^6*sgn(x) + 8*b*c^(11/2)*d^6*sgn(x) - 11*a*c^(9/2)*d^7*
sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^11
```

maple [A] time = 0.05, size = 118, normalized size = 0.88

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(176a\,c^3d\,x^8-128b\,c^4x^8-264a\,c^2d^2x^6+192b\,c^3d\,x^6+330ac\,d^3x^4-240b\,c^2d^2x^4-385a\,d^4x^2+280bc\,d\right.}{3465d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*(176*a*c^3*d*x^8-128*b*c^4*x^8-264*a*c^2*d^2*x
^6+192*b*c^3*d*x^6+330*a*c*d^3*x^4-240*b*c^2*d^2*x^4-385*a*d^4*x^2+280*b*c*
d^3*x^2-315*b*d^4)*(c*x^2+d)/d^5/x^10
```

maxima [A] time = 0.47, size = 152, normalized size = 1.13

$$-\frac{1}{3465}\,b\left(\frac{315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^5}-\frac{1540\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^5}+\frac{2970\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^5}-\frac{2772\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^5}+\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4}{d^5}\right)-\frac{1}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/3465*b*(315*(c + d/x^2)^(11/2)/d^5 - 1540*(c + d/x^2)^(9/2)*c/d^5 + 2970
*(c + d/x^2)^(7/2)*c^2/d^5 - 2772*(c + d/x^2)^(5/2)*c^3/d^5 + 1155*(c + d/x
^2)^(3/2)*c^4/d^5) - 1/315*a*(35*(c + d/x^2)^(9/2)/d^4 - 135*(c + d/x^2)^(7
/2)*c/d^4 + 189*(c + d/x^2)^(5/2)*c^2/d^4 - 105*(c + d/x^2)^(3/2)*c^3/d^4)
```

mupad [B] time = 5.61, size = 210, normalized size = 1.57

$$\frac{16\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{128\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{3465\,d^5}-\frac{a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,d\,x^6}-\frac{b\,c\,\sqrt{c+\frac{d}{x^2}}}{99\,d\,x^8}+\frac{2\,a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d^2\,x^4}-\frac{8\,a\,c^3}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^9,x)
```

```
[Out] (16*a*c^4*(c + d/x^2)^(1/2))/(315*d^4) - (b*(c + d/x^2)^(1/2))/(11*x^10) -
(a*(c + d/x^2)^(1/2))/(9*x^8) - (128*b*c^5*(c + d/x^2)^(1/2))/(3465*d^5) -
(a*c*(c + d/x^2)^(1/2))/(63*d*x^6) - (b*c*(c + d/x^2)^(1/2))/(99*d*x^8) + (
2*a*c^2*(c + d/x^2)^(1/2))/(105*d^2*x^4) - (8*a*c^3*(c + d/x^2)^(1/2))/(315
*d^3*x^2) + (8*b*c^2*(c + d/x^2)^(1/2))/(693*d^2*x^6) - (16*b*c^3*(c + d/x^
2)^(1/2))/(1155*d^3*x^4) + (64*b*c^4*(c + d/x^2)^(1/2))/(3465*d^4*x^2)
```

sympy [A] time = 5.86, size = 146, normalized size = 1.09

$$-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{b\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)}{11}\right.}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**9,x)
```

```
[Out] -a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d
/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)
/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c +
 d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5
```

3.937 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx$

Optimal. Leaf size=150

$$-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{231c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(1\ldots}{99c^2}$$

```
[Out] -16/3465*d^3*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^3/c^5+8/1155*d^2*(-8*a*d+11*
b*c)*(c+d/x^2)^(3/2)*x^5/c^4-2/231*d*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^7/c^
3+1/99*(-8*a*d+11*b*c)*(c+d/x^2)^(3/2)*x^9/c^2+1/11*a*(c+d/x^2)^(3/2)*x^11/
c
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{1155c^4}-\frac{16d^3x^3\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{3465c^5}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{3/2}(11bc-8ad)}{99c^2}-\frac{2dx^7\left(c+\frac{d}{x^2}\right)^{3/2}(11b\ldots}{231c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (-16*d^3*(11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^3)/(3465*c^5) + (8*d^2*(11*b*
c - 8*a*d)*(c + d/x^2)^(3/2)*x^5)/(1155*c^4) - (2*d*(11*b*c - 8*a*d)*(c + d
/x^2)^(3/2)*x^7)/(231*c^3) + ((11*b*c - 8*a*d)*(c + d/x^2)^(3/2)*x^9)/(99*c
^2) + (a*(c + d/x^2)^(3/2)*x^11)/(11*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} + \frac{(11bc-8ad)\int \sqrt{c+\frac{d}{x^2}}\,x^8\,dx}{11c}$$

$$= \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c} - \frac{(2d(11bc-8ad))\int \sqrt{c+\frac{d}{x^2}}\,x^6}{33c^2}$$

$$= -\frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3} + \frac{(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^{11}}{11c}$$

$$= \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4} - \frac{2d(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{231c^3} + \frac{(11bc-8ad)\left(c\right.}{99c}$$

$$= -\frac{16d^3(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3465c^5} + \frac{8d^2(11bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{1155c^4} - \frac{2d(11bc-}{}$$

Mathematica [A] time = 0.08, size = 108, normalized size = 0.72

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(cx^2+d)\left(a\left(315c^4x^8-280c^3dx^6+240c^2d^2x^4-192cd^3x^2+128d^4\right)+11bc\left(35c^3x^6-30c^2dx^4+24c\right.\right.}{3465c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(11*b*c*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x
^4 + 35*c^3*x^6) + a*(128*d^4 - 192*c*d^3*x^2 + 240*c^2*d^2*x^4 - 280*c^3*d
*x^6 + 315*c^4*x^8)))/(3465*c^5)
```

fricas [A] time = 0.61, size = 131, normalized size = 0.87

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+ac^4d\right)x^9+5\left(11\,bc^4d-8\,ac^3d^2\right)x^7-6\left(11\,bc^3d^2-8\,ac^2d^3\right)x^5+8\left(11\,bc^2d^3-8\,acd^4\right)\right.}{3465\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + a*c^4*d)*x^9 + 5*(11*b*c^4*d - 8*a*
c^3*d^2)*x^7 - 6*(11*b*c^3*d^2 - 8*a*c^2*d^3)*x^5 + 8*(11*b*c^2*d^3 - 8*a*c
*d^4)*x^3 - 16*(11*b*c*d^4 - 8*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

giac [A] time = 0.22, size = 175, normalized size = 1.17

$$\frac{16\left(11\,bcd^{\frac{9}{2}}-8\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^5} + \frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\,\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\,\mathrm{sgn}(x)-1540\left(cx^2+d\right)^{\frac{9}{2}}ad\,\mathrm{sgn}(x)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 16/3465*(11*b*c*d^(9/2) - 8*a*d^(11/2))*sgn(x)/c^5 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1540*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 1485*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 2970*(c*x^2 + d)^(7/2
)*a*d^2*sgn(x) + 2079*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 2772*(c*x^2 + d)^(
5/2)*a*d^3*sgn(x) - 1155*(c*x^2 + d)^(3/2)*b*c*d^3*sgn(x) + 1155*(c*x^2 + d
)^(3/2)*a*d^4*sgn(x))/c^5
```

maple [A] time = 0.05, size = 113, normalized size = 0.75

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(315a\,x^8c^4-280a\,c^3d\,x^6+385b\,c^4x^6+240a\,c^2d^2x^4-330b\,c^3d\,x^4-192ac\,d^3x^2+264b\,c^2d^2x^2+128a\,d^4\right.}{3465c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(1/2)*x*(315*a*c^4*x^8-280*a*c^3*d*x^6+385*b*c^4*x^6
+240*a*c^2*d^2*x^4-330*b*c^3*d*x^4-192*a*c*d^3*x^2+264*b*c^2*d^2*x^2+128*a*
d^4-176*b*c*d^3)*(c*x^2+d)/c^5
```

maxima [A] time = 0.64, size = 158, normalized size = 1.05

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5-105\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^3x^3\right)b}{315\,c^4}+\frac{\left(315\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-1540\left(c+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^10*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)^(7/2)*d*x^7 + 189*(c + d/
x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^3)*b/c^4 + 1/3465*(315*(c
+ d/x^2)^(11/2)*x^11 - 1540*(c + d/x^2)^(9/2)*d*x^9 + 2970*(c + d/x^2)^(7/2
)*d^2*x^7 - 2772*(c + d/x^2)^(5/2)*d^3*x^5 + 1155*(c + d/x^2)^(3/2)*d^4*x^3
)*a/c^5
```

mupad [B] time = 4.59, size = 117, normalized size = 0.78

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^{11}}{11}+\frac{x\,\left(128\,a\,d^5-176\,b\,c\,d^4\right)}{3465\,c^5}+\frac{x^9\,\left(385\,b\,c^5+35\,a\,d\,c^4\right)}{3465\,c^5}-\frac{d\,x^7\,\left(8\,a\,d-11\,b\,c\right)}{693\,c^2}+\frac{2\,d^2\,x^5\,\left(8\,a\,d-11\right.}{1155\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^11)/11 + (x*(128*a*d^5 - 176*b*c*d^4))/(3465*c^5) +
 (x^9*(385*b*c^5 + 35*a*c^4*d))/(3465*c^5) - (d*x^7*(8*a*d - 11*b*c))/(693*
c^2) + (2*d^2*x^5*(8*a*d - 11*b*c))/(1155*c^3) - (8*d^3*x^3*(8*a*d - 11*b*c
))/(3465*c^4))
```

sympy [B] time = 6.44, size = 1386, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**10*(c+d/x**2)**(1/2),x)
```

```
[Out] 315*a*c**9*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860
*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**
5*d**20) + 1295*a*c**8*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*
x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**
2 + 3465*c**5*d**20) + 1990*a*c**7*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(3465
*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c*
*6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**6*d**(39/2)*x**12*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 343*a*c**5*d**(41/2)*x**10*s
qrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**
```

```
7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**4*d**(43/
2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 280*a*c*
*3*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**
17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20)
+ 560*a*c**2*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 448*a*c*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 128*a*d**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 35*b*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d
**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 11
0*b*c**6*d**(21/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*
d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 114*b*c**5*d**(23/2)*x
**10*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**
5*d**11*x**2 + 315*c**4*d**12) + 40*b*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) - 5*b*c**3*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 +
 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*b*c**2*d*
*(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
 945*c**5*d**11*x**2 + 315*c**4*d**12) - 40*b*c*d**(31/2)*x**2*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) - 16*b*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945
*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
```

3.938 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx$

Optimal. Leaf size=117

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

```
[Out] 8/315*d^2*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^3/c^4-4/105*d*(-2*a*d+3*b*c)*(c+
d/x^2)^(3/2)*x^5/c^3+1/21*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x^7/c^2+1/9*a*(c+d
/x^2)^(3/2)*x^9/c
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^3\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{315c^4}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{21c^2}-\frac{4dx^5\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{105c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{3/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (8*d^2*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(315*c^4) - (4*d*(3*b*c - 2*a
*d)*(c + d/x^2)^(3/2)*x^5)/(105*c^3) + ((3*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x
^7)/(21*c^2) + (a*(c + d/x^2)^(3/2)*x^9)/(9*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^8\,dx=\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\frac{(9bc-6ad)\int\sqrt{c+\frac{d}{x^2}}\,x^6\,dx}{9c}$$
$$=\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}-\frac{(4d(3bc-2ad))\int\sqrt{c+\frac{d}{x^2}}\,x^4\,dx}{21c^2}$$
$$=-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{21c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^9}{9c}+\ldots$$
$$=\frac{8d^2(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{315c^4}-\frac{4d(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{105c^3}+\frac{(3bc-2ad)\left(c+\ldots\right.}{21c^2}$$

Mathematica [A] time = 0.07, size = 86, normalized size = 0.74

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(35c^3x^6-30c^2dx^4+24cd^2x^2-16d^3\right)+3bc\left(15c^2x^4-12cdx^2+8d^2\right)\right)}{315c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(3*b*c*(8*d^2 - 12*c*d*x^2 + 15*c^2*x^4) + a
*(-16*d^3 + 24*c*d^2*x^2 - 30*c^2*d*x^4 + 35*c^3*x^6)))/(315*c^4)
```

fricas [A] time = 0.66, size = 107, normalized size = 0.91

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+ac^3d\right)x^7+3\left(3\,bc^3d-2\,ac^2d^2\right)x^5-4\left(3\,bc^2d^2-2\,acd^3\right)x^3+8\left(3\,bcd^3-2\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + a*c^3*d)*x^7 + 3*(3*b*c^3*d - 2*a*c^2*d^
2)*x^5 - 4*(3*b*c^2*d^2 - 2*a*c*d^3)*x^3 + 8*(3*b*c*d^3 - 2*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^4
```

giac [A] time = 0.17, size = 140, normalized size = 1.20

$$-\frac{8\left(3\,bcd^{\frac{7}{2}}-2\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^4}+\frac{35\left(cx^2+d\right)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\left(cx^2+d\right)^{\frac{7}{2}}bc\mathrm{sgn}(x)-135\left(cx^2+d\right)^{\frac{7}{2}}ad\mathrm{sgn}(x)-126\left(\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -8/315*(3*b*c*d^(7/2) - 2*a*d^(9/2))*sgn(x)/c^4 + 1/315*(35*(c*x^2 + d)^(9/
2)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 135*(c*x^2 + d)^(7/2)*a*d*s
gn(x) - 126*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 189*(c*x^2 + d)^(5/2)*a*d^2*sg
n(x) + 105*(c*x^2 + d)^(3/2)*b*c*d^2*sgn(x) - 105*(c*x^2 + d)^(3/2)*a*d^3*s
gn(x))/c^4
```

maple [A] time = 0.05, size = 89, normalized size = 0.76

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(35a\,x^6c^3-30a\,c^2d\,x^4+45b\,c^3x^4+24ac\,d^2x^2-36b\,c^2d\,x^2-16a\,d^3+24bc\,d^2\right)\left(c\,x^2+d\right)x}{315c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(1/2)*x*(35*a*c^3*x^6-30*a*c^2*d*x^4+45*b*c^3*x^4+24*
a*c*d^2*x^2-36*b*c^2*d*x^2-16*a*d^3+24*b*c*d^2)*(c*x^2+d)/c^4
```

maxima [A] time = 0.56, size = 124, normalized size = 1.06

$$\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)b}{105\,c^3}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-135\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+189\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2\right.}{315\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^8*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^
2)^(3/2)*d^2*x^3)*b/c^3 + 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 135*(c + d/x^2)
^(7/2)*d*x^7 + 189*(c + d/x^2)^(5/2)*d^2*x^5 - 105*(c + d/x^2)^(3/2)*d^3*x^
3)*a/c^4
```

mupad [B] time = 4.52, size = 97, normalized size = 0.83

$$\sqrt{c+\frac{d}{x^2}}\left(\frac{a\,x^9}{9}-\frac{x\,\left(16\,a\,d^4-24\,b\,c\,d^3\right)}{315\,c^4}+\frac{x^7\,\left(45\,b\,c^4+5\,a\,d\,c^3\right)}{315\,c^4}-\frac{d\,x^5\,\left(2\,a\,d-3\,b\,c\right)}{105\,c^2}+\frac{4\,d^2\,x^3\,\left(2\,a\,d-3\,b\,c\right)}{315\,c^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^9)/9 - (x*(16*a*d^4 - 24*b*c*d^3))/(315*c^4) + (x^7
*(45*b*c^4 + 5*a*c^3*d))/(315*c^4) - (d*x^5*(2*a*d - 3*b*c))/(105*c^2) + (4
*d^2*x^3*(2*a*d - 3*b*c))/(315*c^3))
```

sympy [B] time = 4.92, size = 910, normalized size = 7.78

$$\frac{35ac^7d^{\frac{19}{2}}x^{14}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}}+\frac{110ac^6d^{\frac{21}{2}}x^{12}\sqrt{\frac{cx^2}{d}+1}}{315c^7d^9x^6+945c^6d^{10}x^4+945c^5d^{11}x^2+315c^4d^{12}}+\frac{}{315c^7d^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**8*(c+d/x**2)**(1/2),x)
```

```
[Out] 35*a*c**7*d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**6*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**5*d**(23/2)*x**10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**4*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 5*a*c**3
*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 30*a*c**2*d**(29/2)*x**4*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 40*a*c*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d*
*9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*
a*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 +
945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(9/2)*x**10*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 33*b*c
**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x
**2 + 105*c**3*d**6) + 17*b*c**3*d**(13/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**
```

```
5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**2*d**(15/2)*x**4
*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**
6) + 12*b*c*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) + 8*b*d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

3.939 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^6\,dx$

Optimal. Leaf size=84

$$-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3}+\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

```
[Out] -2/105*d*(-4*a*d+7*b*c)*(c+d/x^2)^(3/2)*x^3/c^3+1/35*(-4*a*d+7*b*c)*(c+d/x^
2)^(3/2)*x^5/c^2+1/7*a*(c+d/x^2)^(3/2)*x^7/c
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{35c^2}-\frac{2dx^3\left(c+\frac{d}{x^2}\right)^{3/2}(7bc-4ad)}{105c^3}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (-2*d*(7*b*c - 4*a*d)*(c + d/x^2)^(3/2)*x^3)/(105*c^3) + ((7*b*c - 4*a*d)*(
c + d/x^2)^(3/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(3/2)*x^7)/(7*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^6\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}+\frac{(7bc-4ad)\int \sqrt{c+\frac{d}{x^2}}\,x^4\,dx}{7c}\\
&= \frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}-\frac{(2d(7bc-4ad))\int \sqrt{c+\frac{d}{x^2}}\,x^2\,dx}{35c^2}\\
&= -\frac{2d(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{105c^3}+\frac{(7bc-4ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^7}{7c}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 64, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(a\left(15c^2x^4-12cdx^2+8d^2\right)+7bc\left(3cx^2-2d\right)\right)}{105c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(7*b*c*(-2*d + 3*c*x^2) + a*(8*d^2 - 12*c*d*
x^2 + 15*c^2*x^4)))/(105*c^3)
```

fricas [A] time = 0.58, size = 82, normalized size = 0.98

$$\frac{\left(15\,ac^3x^7+3\left(7\,bc^3+ac^2d\right)x^5+\left(7\,bc^2d-4\,acd^2\right)x^3-2\left(7\,bcd^2-4\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*a*c^3*x^7 + 3*(7*b*c^3 + a*c^2*d)*x^5 + (7*b*c^2*d - 4*a*c*d^2)*x
^3 - 2*(7*b*c*d^2 - 4*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^3
```

giac [A] time = 0.17, size = 105, normalized size = 1.25

$$\frac{2\left(7\,bcd^{\frac{5}{2}}-4\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{105\,c^3}+\frac{15\left(cx^2+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+21\left(cx^2+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-42\left(cx^2+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)-35\left(cx^2\right.}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*b*c*d^(5/2) - 4*a*d^(7/2))*sgn(x)/c^3 + 1/105*(15*(c*x^2 + d)^(7/2
)*a*sgn(x) + 21*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 42*(c*x^2 + d)^(5/2)*a*d*sgn
(x) - 35*(c*x^2 + d)^(3/2)*b*c*d*sgn(x) + 35*(c*x^2 + d)^(3/2)*a*d^2*sgn(x)
)/c^3
```

maple [A] time = 0.06, size = 65, normalized size = 0.77

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(15a\,x^4c^2-12acd\,x^2+21b\,c^2x^2+8a\,d^2-14bcd\right)\left(c\,x^2+d\right)x}{105c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/105*((c*x^2+d)/x^2)^(1/2)*x*(15*a*c^2*x^4-12*a*c*d*x^2+21*b*c^2*x^2+8*a*d
^2-14*b*c*d)*(c*x^2+d)/c^3
```

maxima [A] time = 0.53, size = 90, normalized size = 1.07

$$\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3\right)b}{15\,c^2}+\frac{\left(15\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-42\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5+35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3\right)a}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^6*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3)*b/c^2 + 1/105*(1
5*(c + d/x^2)^(7/2)*x^7 - 42*(c + d/x^2)^(5/2)*d*x^5 + 35*(c + d/x^2)^(3/2)
*d^2*x^3)*a/c^3
```

mupad [B] time = 4.49, size = 77, normalized size = 0.92

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{a\,x^7}{7}+\frac{x\,\left(8\,a\,d^3-14\,b\,c\,d^2\right)}{105\,c^3}+\frac{x^5\,\left(21\,b\,c^3+3\,a\,d\,c^2\right)}{105\,c^3}-\frac{d\,x^3\,\left(4\,a\,d-7\,b\,c\right)}{105\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^7)/7 + (x*(8*a*d^3 - 14*b*c*d^2))/(105*c^3) + (x^5*
(21*b*c^3 + 3*a*c^2*d))/(105*c^3) - (d*x^3*(4*a*d - 7*b*c))/(105*c^2))
```

sympy [B] time = 3.99, size = 422, normalized size = 5.02

$$\frac{15ac^5d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^4d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^3d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{}{105c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**6*(c+d/x**2)**(1/2),x)
```

```
[Out] 15*a*c**5*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**4*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**3*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**2*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c*d**(17/2)*x**2*sqrt(c*x**2/d
 + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(1
9/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + b*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(3/2)*x**2*sqrt(c*x**2/
d + 1)/(15*c) - 2*b*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

3.940 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^4\,dx$

Optimal. Leaf size=53

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

```
[Out] 1/15*(-2*a*d+5*b*c)*(c+d/x^2)^(3/2)*x^3/c^2+1/5*a*(c+d/x^2)^(3/2)*x^5/c
```

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^3\left(c+\frac{d}{x^2}\right)^{3/2}(5bc-2ad)}{15c^2}+\frac{ax^5\left(c+\frac{d}{x^2}\right)^{3/2}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] ((5*b*c - 2*a*d)*(c + d/x^2)^(3/2)*x^3)/(15*c^2) + (a*(c + d/x^2)^(3/2)*x^5
)/(5*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^4\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}+\frac{(5bc-2ad)\int \sqrt{c+\frac{d}{x^2}}\,x^2\,dx}{5c}\\ &= \frac{(5bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{15c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^5}{5c}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(3acx^2-2ad+5bc\right)}{15c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)*(5*b*c - 2*a*d + 3*a*c*x^2))/(15*c^2)
```

fricas [A] time = 0.53, size = 57, normalized size = 1.08

$$\frac{\left(3\,ac^{2}x^{5}+\left(5\,bc^{2}+acd\right)x^{3}+\left(5\,bcd-2\,ad^{2}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 + a*c*d)*x^3 + (5*b*c*d - 2*a*d^2)*x)*sqrt((c*
x^2 + d)/x^2)/c^2
```

giac [A] time = 0.21, size = 72, normalized size = 1.36

$$-\frac{\left(5\,bcd^{\frac{3}{2}}-2\,ad^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c^{2}}+\frac{3\left(cx^{2}+d\right)^{\frac{5}{2}}a\mathrm{sgn}(x)+5\left(cx^{2}+d\right)^{\frac{3}{2}}bc\mathrm{sgn}(x)-5\left(cx^{2}+d\right)^{\frac{3}{2}}ad\mathrm{sgn}(x)}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(5*b*c*d^(3/2) - 2*a*d^(5/2))*sgn(x)/c^2 + 1/15*(3*(c*x^2 + d)^(5/2)*
a*sgn(x) + 5*(c*x^2 + d)^(3/2)*b*c*sgn(x) - 5*(c*x^2 + d)^(3/2)*a*d*sgn(x))
/c^2
```

maple [A] time = 0.05, size = 43, normalized size = 0.81

$$\frac{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\left(3a\,x^{2}c-2ad+5bc\right)\left(c\,x^{2}+d\right)x}{15c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(1/2)*x*(3*a*c*x^2-2*a*d+5*b*c)*(c*x^2+d)/c^2
```

maxima [A] time = 0.66, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}x^{3}}{3\,c}+\frac{\left(3\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}x^{5}-5\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}dx^{3}\right)a}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(c + d/x^2)^(3/2)*x^3/c + 1/15*(3*(c + d/x^2)^(5/2)*x^5 - 5*(c + d/x^
2)^(3/2)*d*x^3)*a/c^2
```

mupad [B] time = 4.44, size = 54, normalized size = 1.02

$$\sqrt{c+\frac{d}{x^{2}}}\,\left(\frac{a\,x^{5}}{5}-\frac{x\,\left(2\,a\,d^{2}-5\,b\,c\,d\right)}{15\,c^{2}}+\frac{x^{3}\,\left(5\,b\,c^{2}+a\,d\,c\right)}{15\,c^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((a*x^5)/5 - (x*(2*a*d^2 - 5*b*c*d))/(15*c^2) + (x^3*(5*b
*c^2 + a*c*d))/(15*c^2))
```

sympy [B] time = 3.04, size = 119, normalized size = 2.25

$$\frac{a\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{15c}-\frac{2ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{15c^2}+\frac{b\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/(1
5*c) - 2*a*d**(5/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*sqrt(d)*x**2*sqrt(c*x*
*2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c)
```

3.941 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^2\,dx$

Optimal. Leaf size=66

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

```
[Out] 1/3*a*(c+d/x^2)^(3/2)*x^3/c-b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)+b*
x*(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {451, 242, 277, 217, 206}

$$\frac{ax^3\left(c+\frac{d}{x^2}\right)^{3/2}}{3c}+bx\sqrt{c+\frac{d}{x^2}}-b\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
```

```
[Out] b*Sqrt[c + d/x^2]*x + (a*(c + d/x^2)^(3/2)*x^3)/(3*c) - b*Sqrt[d]*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,x^2\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}+b\int \sqrt{c+\frac{d}{x^2}}\,dx$$
$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx,x,\frac{1}{x}\right)$$
$$= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)$$
$$= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-(bd)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$
$$= b\sqrt{c+\frac{d}{x^2}}\,x+\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x^3}{3c}-b\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$

Mathematica [A] time = 0.06, size = 84, normalized size = 1.27

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\,\left(a\left(cx^2+d\right)+3bc\right)-3bc\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{3c\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2]*x^2,x]
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[d + c*x^2]*(3*b*c + a*(d + c*x^2)) - 3*b*c*Sqrt[d]
*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(3*c*Sqrt[d + c*x^2])
```

fricas [A] time = 0.72, size = 156, normalized size = 2.36

$$\left[\frac{3\,bc\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(acx^3+(3\,bc+ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{6\,c},\frac{3\,bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(acx^3\right.}{3\,c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] [1/6*(3*b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/
x^2) + 2*(a*c*x^3 + (3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c, 1/3*(3*b*c*s
qrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (a*c*x^3 + (
3*b*c + a*d)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

giac [B] time = 0.19, size = 116, normalized size = 1.76

$$\frac{bd\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\text{sgn}(x)}{\sqrt{-d}}-\frac{\left(3\,bcd\arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right)+3\,bc\sqrt{-d}\,\sqrt{d}\,+a\sqrt{-d}\,d^{\frac{3}{2}}\right)\text{sgn}(x)}{3\,c\sqrt{-d}}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}ac^2\text{sgn}(x)+3}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*d*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/3*(3*b*c*d*arctan(
sqrt(d)/sqrt(-d)) + 3*b*c*sqrt(-d)*sqrt(d) + a*sqrt(-d)*d^(3/2))*sgn(x)/(c*
sqrt(-d)) + 1/3*((c*x^2 + d)^(3/2)*a*c^2*sgn(x) + 3*sqrt(c*x^2 + d)*b*c^3*s
gn(x))/c^3
```

maple [A] time = 0.06, size = 83, normalized size = 1.26

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(3bc\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-3\sqrt{c\,x^2+d}\,bc-\left(c\,x^2+d\right)^{\frac{3}{2}}a\right)x}{3\sqrt{c\,x^2+d}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/3*((c*x^2+d)/x^2)^(1/2)*x*(3*d^(1/2)*ln(2*(d^(1/2)*(c*x^2+d)^(1/2)+d)/x)
*b*c-a*(c*x^2+d)^(3/2)-3*(c*x^2+d)^(1/2)*b*c)/(c*x^2+d)^(1/2)/c
```

maxima [A] time = 1.38, size = 75, normalized size = 1.14

$$\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3}{3\,c}+\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(c + d/x^2)^(3/2)*x^3/c + 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

mupad [B] time = 4.72, size = 80, normalized size = 1.21

$$b\,x\,\sqrt{c+\frac{d}{x^2}}+\frac{a\,x\,\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)}{3\,c}+\frac{b\,\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1{}\mathrm{i}}{\sqrt{c}\,x}\right)\,\sqrt{c+\frac{d}{x^2}}\,1{}\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] b*x*(c + d/x^2)^(1/2) + (a*x*(c + d/x^2)^(1/2)*(d + c*x^2))/(3*c) + (b*d^(1
/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2
) + 1)^(1/2))
```

sympy [A] time = 3.27, size = 107, normalized size = 1.62

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c}+\frac{b\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-b\sqrt{d}\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{bd}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + a*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c) +
 b*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - b*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) +
b*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2)))
```

3.942 $\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

```
[Out] a*(c+d/x^2)^(3/2)*x/c-1/2*(2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^
(1/2)-1/2*(2*a*d+b*c)*(c+d/x^2)^(1/2)/c/x
```

Rubi [A] time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,(2ad+bc)}{2cx}-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] -((b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + (a*(c + d/x^2)^(3/2)*x)/c - ((b*
c + 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{(-bc-2ad)\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{c}$$
$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{1}{2}(-bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} + \frac{1}{2}(-bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+}}\right.$$
$$= -\frac{(bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{a\left(c+\frac{d}{x^2}\right)^{3/2}x}{c} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2\sqrt{d}}$$

Mathematica [A] time = 0.09, size = 75, normalized size = 0.88

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(-\frac{x^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{cx^2+d}}+2ax^2-b\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*(-b + 2*a*x^2 - ((b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]
/Sqrt[d]])/(Sqrt[d]*Sqrt[d + c*x^2])))/(2*x)
```

fricas [A] time = 0.72, size = 164, normalized size = 1.93

$$\left[\frac{(bc+2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,adx^2-bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\,dx}, \frac{(bc+2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+\left(\right.}{2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^
2) + 2*d)/x^2) + 2*(2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x), 1/2*((b*
c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d))
+ (2*a*d*x^2 - b*d)*sqrt((c*x^2 + d)/x^2))/(d*x)]
```

giac [A] time = 0.25, size = 76, normalized size = 0.89

$$\frac{2\sqrt{cx^2+d}\,ac\mathrm{sgn}(x) + \frac{\left(bc^2\mathrm{sgn}(x)+2\,acd\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{\sqrt{cx^2+d}\,bc\mathrm{sgn}(x)}{x^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(c*x^2 + d)*a*c*sgn(x) + (b*c^2*sgn(x) + 2*a*c*d*sgn(x))*arctan(
sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - sqrt(c*x^2 + d)*b*c*sgn(x)/x^2)/c
```

maple [A] time = 0.05, size = 135, normalized size = 1.59

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\left(2a\,d^{\frac{3}{2}}x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+bc\sqrt{d}\,x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-2\sqrt{c\,x^2+d}\,ad\,x^2-\sqrt{c\,x^2+d}\,bc\,x^2+\right.}{2\sqrt{c\,x^2+d}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*((c*x^2+d)/x^2)^(1/2)/x*(2*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)
*x^2*a+d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c-2*(c*x^2+d)^(1/2
)*x^2*a*d-(c*x^2+d)^(1/2)*x^2*b*c+(c*x^2+d)^(3/2)*b)/(c*x^2+d)^(1/2)/d
```

maxima [A] time = 1.22, size = 133, normalized size = 1.56

$$\frac{1}{2}\left(2\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}\,\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(c + d/x^2)*x + sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(
c + d/x^2)*x + sqrt(d))))*a - 1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 -
 d) - c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sq
rt(d))*b
```

mupad [B] time = 5.11, size = 97, normalized size = 1.14

$$a\,x\sqrt{c+\frac{d}{x^2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2\,x}-\frac{b\,c\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{2\sqrt{d}}+\frac{a\sqrt{d}\,\mathrm{asin}\left(\frac{\sqrt{d}\,1\mathrm{i}}{\sqrt{c}\,x}\right)\sqrt{c+\frac{d}{x^2}}\,1\mathrm{i}}{\sqrt{c}\,\sqrt{\frac{d}{c\,x^2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(1/2),x)
```

```
[Out] a*x*(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2))/(2*x) - (b*c*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/(2*d^(1/2)) + (a*d^(1/2)*asin((d^(1/2)*1i)/(c^(1/2)*x))
*(c + d/x^2)^(1/2)*1i)/(c^(1/2)*(d/(c*x^2) + 1)^(1/2))
```

sympy [A] time = 4.48, size = 107, normalized size = 1.26

$$\frac{a\sqrt{c}\,x}{\sqrt{1+\frac{d}{cx^2}}}-a\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{ad}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(c)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x)) + a
*d/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x)
- b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*sqrt(d))
```

3.943 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$

Optimal. Leaf size=91

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

```
[Out] -1/4*b*(c+d/x^2)^(3/2)/d/x+1/8*c*(-4*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(
1/2))/d^(3/2)+1/8*(-4*a*d+b*c)*(c+d/x^2)^(1/2)/d/x
```

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{3/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-4ad)}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] ((b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d*x) - (b*(c + d/x^2)^(3/2))/(4*d*x) + (
c*(b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
```

```
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx &= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(-bc+4ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx}{4d}\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}-\frac{(-bc+4ad)\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{(c(bc-4ad))\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d}\\
&= \frac{(bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{4dx}+\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 100, normalized size = 1.10

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(4adx^2+bcx^2+2bd\right)+cx^4\sqrt{\frac{cx^2}{d}+1}\,(4ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8dx^3\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^2,x]
```

```
[Out] -1/8*(Sqrt[c + d/x^2]*((d + c*x^2)*(2*b*d + b*c*x^2 + 4*a*d*x^2) + c*(-(b*c
) + 4*a*d)*x^4*Sqrt[1 + (c*x^2)/d]*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d*x^3*(d
 + c*x^2))
```

fricas [A] time = 0.61, size = 194, normalized size = 2.13

$$\left[-\frac{\left(bc^2-4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2+\left(bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^2x^3},-\frac{\left(bc^2-4\,acd\right)\sqrt{-d}\,x^3\,\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2
 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(d^2*x^3), -1/8*((b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-d)*x*
sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*b*d^2 + (b*c*d + 4*a*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/(d^2*x^3)]
```

giac [A] time = 0.24, size = 130, normalized size = 1.43

$$-\frac{\frac{\left(bc^3\mathrm{sgn}(x)-4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\mathrm{sgn}(x)+\sqrt{cx^2+d}\,bc^3d\mathrm{sgn}(x)-4\,\sqrt{cx^2+d}\,ac^2d^2\mathrm{sgn}(x)}{c^2dx^4}}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/8*((b*c^3*sgn(x) - 4*a*c^2*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/(s
qrt(-d)*d) + ((c*x^2 + d)^(3/2)*b*c^3*sgn(x) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*
sgn(x) + sqrt(c*x^2 + d)*b*c^3*d*sgn(x) - 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x
))/(c^2*d*x^4))/c
```

maple [B] time = 0.06, size = 175, normalized size = 1.92

$$-\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(4ac\,d^{\frac{3}{2}}x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-b\,c^2\sqrt{d}\,x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-4\sqrt{c\,x^2+d}\,acd\,x^4+\sqrt{c\,x^2+d}\,b\,c^2x}{8\sqrt{c\,x^2+d}\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(1/2)/x^3*(4*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/
x)*x^4*a*c-d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2-4*(c*x^2+d
)^(1/2)*x^4*a*c*d+(c*x^2+d)^(1/2)*x^4*b*c^2+4*(c*x^2+d)^(3/2)*x^2*a*d-(c*x^
2+d)^(3/2)*x^2*b*c+2*(c*x^2+d)^(3/2)*b*d)/(c*x^2+d)^(1/2)/d^2
```

maxima [B] time = 1.35, size = 193, normalized size = 2.12

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)x^2-d}-\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}\right)a-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*x^2 - d) - c*log((sqrt(c + d/x^2)*
x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d))*a - 1/16*(c^2*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*((c +
d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*d*x^4 - 2*(c
 + d/x^2)*d^2*x^2 + d^3))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^2, x)
```

sympy [A] time = 6.97, size = 144, normalized size = 1.58

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2\sqrt{d}}-\frac{bc^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{3}{2}}}-\frac{bd}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**2,x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*x) - a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*s
qrt(d)) - b*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*b*sqrt(c)/(8*x**3*sqr
t(1 + d/(c*x**2))) + b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - b*d/(
4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

3.944 $\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$

Optimal. Leaf size=123

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

```
[Out] -1/6*b*(c+d/x^2)^(3/2)/d/x^3-1/16*c^2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x
^2)^(1/2))/d^(5/2)+1/8*(-2*a*d+b*c)*(c+d/x^2)^(1/2)/d/x^3+1/16*c*(-2*a*d+b*
c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A] time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{16d^2x}+\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-2ad)}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] ((b*c - 2*a*d)*Sqrt[c + d/x^2])/(8*d*x^3) - (b*(c + d/x^2)^(3/2))/(6*d*x^3)
 + (c*(b*c - 2*a*d)*Sqrt[c + d/x^2])/(16*d^2*x) - (c^2*(b*c - 2*a*d)*ArcTan
h[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{(-3bc+6ad)\int \frac{\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}-\frac{(-3bc+6ad)\,\text{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{(c(bc-2ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{\left(c^2(bc-2ad)\right)\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{\left(c^2(bc-2ad)\right)\text{Subst}\left(\int \frac{}{1}\right.}{16d^2}$$

$$= \frac{(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{8dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{6dx^3}+\frac{c(bc-2ad)\sqrt{c+\frac{d}{x^2}}}{16d^2x}-\frac{c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+}}\right.}{16d^{5/2}}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.55

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)\left(c^2x^6(bc-2ad)\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{cx^2}{d}+1\right)-bd^3\right)}{6d^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*Sqrt[c + d/x^2])/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)*(-(b*d^3) + c^2*(b*c - 2*a*d)*x^6*Hypergeometr
ic2F1[3/2, 3, 5/2, 1 + (c*x^2)/d]))/(6*d^4*x^5)
```

fricas [A] time = 0.64, size = 244, normalized size = 1.98

$$\left[-\frac{3\left(bc^3-2\,ac^2d\right)\sqrt{d}\,x^5\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(bc^2d-2\,acd^2\right)x^4-8\,bd^3-2\left(bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{96\,d^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(
b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^5), 1/48*(3*(b*c^3 -
2*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)
) + (3*(b*c^2*d - 2*a*c*d^2)*x^4 - 8*b*d^3 - 2*(b*c*d^2 + 6*a*d^3)*x^2)*sqr
t((c*x^2 + d)/x^2))/(d^3*x^5)]
```

giac [A] time = 0.30, size = 153, normalized size = 1.24

$$\frac{\frac{3\left(bc^4\mathrm{sgn}(x)-2\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)-6\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)-8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-3\sqrt{cx^2+d}\,bc^4d^2\mathrm{sgn}(x)+6}{c^3d^2x^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(b*c^4*sgn(x) - 2*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/
(sqrt(-d)*d^2) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) - 6*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) - 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4
*d^2*sgn(x) + 6*sqrt(c*x^2 + d)*a*c^3*d^3*sgn(x))/(c^3*d^2*x^6))/c
```

maple [B] time = 0.06, size = 220, normalized size = 1.79

$$\frac{\sqrt{\frac{c\,x^2+d}{x^2}}\,\left(6a\,c^2d^{\frac{3}{2}}x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-3b\,c^3\sqrt{d}\,x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-6\sqrt{c\,x^2+d}\,a\,c^2d\,x^6+3\sqrt{c\,x^2+d}\,b}{48\sqrt{c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(1/2)/x^5*(6*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/
x)*x^6*a*c^2-3*d^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^6*b*c^3-6*(c*x
^2+d)^(1/2)*x^6*a*c^2*d+3*(c*x^2+d)^(1/2)*x^6*b*c^3+6*(c*x^2+d)^(3/2)*x^4*a
*c*d-3*(c*x^2+d)^(3/2)*x^4*b*c^2-12*(c*x^2+d)^(3/2)*x^2*a*d^2+6*(c*x^2+d)^(
3/2)*x^2*b*c*d-8*(c*x^2+d)^(3/2)*b*d^2)/(c*x^2+d)^(1/2)/d^3
```

maxima [B] time = 1.27, size = 277, normalized size = 2.25

$$-\frac{1}{16}\left(\frac{c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3+\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2dx^4-2\left(c+\frac{d}{x^2}\right)d^2x^2+d^3}\right)a+\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right.\right.}{\left(c+\frac{d}{x^2}\right)^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*(c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))
/d^(3/2) + 2*((c + d/x^2)^(3/2)*c^2*x^3 + sqrt(c + d/x^2)*c^2*d*x)/((c + d/
x^2)^2*d*x^4 - 2*(c + d/x^2)*d^2*x^2 + d^3))*a + 1/96*(3*c^3*log((sqrt(c +
d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^
2)^(5/2)*c^3*x^5 - 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d^
```

```
2*x)/((c + d/x^2)^3*d^2*x^6 - 3*(c + d/x^2)^2*d^3*x^4 + 3*(c + d/x^2)*d^4*x
^2 - d^5))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\sqrt{c+\frac{d}{x^2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(1/2))/x^4, x)
```

sympy [B] time = 11.42, size = 226, normalized size = 1.84

$$-\frac{ac^{\frac{3}{2}}}{8dx\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}}{8x^3\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{3}{2}}}-\frac{ad}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{5}{2}}}{16d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}}{48dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{5b\sqrt{c}}{24x^5\sqrt{1+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(1/2)/x**4,x)
```

```
[Out] -a*c**(3/2)/(8*d*x*sqrt(1 + d/(c*x**2))) - 3*a*sqrt(c)/(8*x**3*sqrt(1 + d/(
c*x**2))) + a*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(3/2)) - a*d/(4*sqrt(c)
*x**5*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(16*d**2*x*sqrt(1 + d/(c*x**2))) +
 b*c**(3/2)/(48*d*x**3*sqrt(1 + d/(c*x**2))) - 5*b*sqrt(c)/(24*x**5*sqrt(1
+ d/(c*x**2))) - b*c**3*asinh(sqrt(d)/(sqrt(c)*x))/(16*d**(5/2)) - b*d/(6*s
qrt(c)*x**7*sqrt(1 + d/(c*x**2)))
```

3.945 $\int \left(a+\frac{b}{x^2}\right) \left(c+\frac{d}{x^2}\right)^{3/2} x^5 \, dx$

Optimal. Leaf size=123

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(6bc-ad)}{16c}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

```
[Out] 1/24*(-a*d+6*b*c)*(c+d/x^2)^(3/2)*x^4/c+1/6*a*(c+d/x^2)^(5/2)*x^6/c+1/16*d^
2*(-a*d+6*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/16*d*(-a*d+6*b*c)
*x^2*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 47, 63, 208}

$$\frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{16c^{3/2}}+\frac{x^4\left(c+\frac{d}{x^2}\right)^{3/2}(6bc-ad)}{24c}+\frac{dx^2\sqrt{c+\frac{d}{x^2}}\,(6bc-ad)}{16c}+\frac{ax^6\left(c+\frac{d}{x^2}\right)^{5/2}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (d*(6*b*c - a*d)*Sqrt[c + d/x^2]*x^2)/(16*c) + ((6*b*c - a*d)*(c + d/x^2)^(
3/2)*x^4)/(24*c) + (a*(c + d/x^2)^(5/2)*x^6)/(6*c) + (d^2*(6*b*c - a*d)*Arc
Tanh[Sqrt[c + d/x^2]/Sqrt[c]])/(16*c^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^5\,dx=-\left(\frac{1}{2}\text{Subst}\left(\int\frac{(a+bx)(c+dx)^{3/2}}{x^4}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}-\frac{\left(3bc-\frac{ad}{2}\right)\text{Subst}\left(\int\frac{(c+dx)^{3/2}}{x^3}\,dx,x,\frac{1}{x^2}\right)}{6c}$$

$$=\frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}-\frac{(d(6bc-ad))\text{Subst}\left(\int\frac{\sqrt{c+dx}}{x^2}\,dx,x,\right.}{16c}$$

$$=\frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c}+\frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}-\frac{\left(d^2(6bc-\right.}{}$$

$$=\frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c}+\frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}-\frac{(d(6bc-a}{}$$

$$=\frac{d(6bc-ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{16c}+\frac{(6bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}x^4}{24c}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^6}{6c}+\frac{d^2(6bc-a}{}$$

Mathematica [A] time = 0.14, size = 123, normalized size = 1.00

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(\sqrt{c}\,x\sqrt{\frac{cx^2}{d}+1}\left(a\left(8c^2x^4+14cdx^2+3d^2\right)+6bc\left(2cx^2+5d\right)\right)-3d^{3/2}(ad-6bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)\right)}{48c^{3/2}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(Sqrt[c]*x*Sqrt[1 + (c*x^2)/d]*(6*b*c*(5*d + 2*c*x^2) +
a*(3*d^2 + 14*c*d*x^2 + 8*c^2*x^4)) - 3*d^(3/2)*(-6*b*c + a*d)*ArcSinh[(Sqr
t[c]*x)/Sqrt[d]]))/(48*c^(3/2)*Sqrt[1 + (c*x^2)/d])
```

fricas [A] time = 0.67, size = 243, normalized size = 1.98

$$\left[-\frac{3\left(6\,bcd^2-ad^3\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(8\,ac^3x^6+2\left(6\,bc^3+7\,ac^2d\right)x^4+3\left(10\,bc^2d+acd^2\right)x\right.}{96\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*b*c*d^2 - a*d^3)*sqrt(c)*log(-2*c*x^2 + 2*sqrt(c)*x^2*sqrt((c*
x^2 + d)/x^2) - d) - 2*(8*a*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b
*c^2*d + a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/c^2, -1/48*(3*(6*b*c*d^2 - a*
d^3)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (8*a
*c^3*x^6 + 2*(6*b*c^3 + 7*a*c^2*d)*x^4 + 3*(10*b*c^2*d + a*c*d^2)*x^2)*sqrt
((c*x^2 + d)/x^2))/c^2]
```

giac [A] time = 0.23, size = 144, normalized size = 1.17

$$\frac{1}{48}\left(2\left(4\,acx^2\mathrm{sgn}(x)+\frac{6\,bc^5\mathrm{sgn}(x)+7\,ac^4d\mathrm{sgn}(x)}{c^4}\right)x^2+\frac{3\left(10\,bc^4d\mathrm{sgn}(x)+ac^3d^2\mathrm{sgn}(x)\right)}{c^4}\right)\sqrt{cx^2+d}\,x-\frac{\left(6\,bcd \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*a*c*x^2*sgn(x) + (6*b*c^5*sgn(x) + 7*a*c^4*d*sgn(x))/c^4)*x^2 +
3*(10*b*c^4*d*sgn(x) + a*c^3*d^2*sgn(x))/c^4)*sqrt(c*x^2 + d)*x - 1/16*(6*b
*c*d^2*sgn(x) - a*d^3*sgn(x))*log(abs(-sqrt(c)*x + sqrt(c*x^2 + d)))/c^(3/2
) + 1/32*(6*b*c*d^2*log(abs(d)) - a*d^3*log(abs(d)))*sgn(x)/c^(3/2)
```

maple [A] time = 0.06, size = 162, normalized size = 1.32

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-3a\,d^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)+18bc\,d^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)-3\sqrt{c\,x^2+d}\;a\sqrt{c}\;d^2x+18\sqrt{c\,x^2+\cdots}\right.}{48\left(c\,x^2+d\right)^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x)
```

```
[Out] 1/48*((c*x^2+d)/x^2)^(3/2)*x^3*(8*c^(1/2)*(c*x^2+d)^(5/2)*x*a-2*(c*x^2+d)^(
3/2)*a*c^(1/2)*d*x+12*(c*x^2+d)^(3/2)*b*c^(3/2)*x-3*(c*x^2+d)^(1/2)*a*c^(1/
2)*d^2*x+18*(c*x^2+d)^(1/2)*b*c^(3/2)*d*x-3*a*d^3*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))+18*b*c*d^2*ln(c^(1/2)*x+(c*x^2+d)^(1/2)))/(c*x^2+d)^(3/2)/c^(3/2)
```

maxima [B] time = 1.30, size = 240, normalized size = 1.95

$$\frac{1}{96}\left(\frac{3\,d^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}cd^3-3\sqrt{c+\frac{d}{x^2}}\,c^2d^3\right)}{\left(c+\frac{d}{x^2}\right)^3c-3\left(c+\frac{d}{x^2}\right)^2c^2+3\left(c+\frac{d}{x^2}\right)c^3-c^4}\right)a-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{\sqrt{c}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/96*(3*d^3*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(3/2) + 2*(3*(c + d/x^2)^(5/2)*d^3 + 8*(c + d/x^2)^(3/2)*c*d^3 - 3*sqrt(c +
 d/x^2)*c^2*d^3)/((c + d/x^2)^3*c - 3*(c + d/x^2)^2*c^2 + 3*(c + d/x^2)*c^3
 - c^4))*a - 1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) +
 sqrt(c)))/sqrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/
((c + d/x^2)^2 - 2*(c + d/x^2)*c + c^2))*b
```

mupad [B] time = 5.78, size = 130, normalized size = 1.06

$$\frac{a\,x^6\left(c+\frac{d}{x^2}\right)^{3/2}}{6}+\frac{5\,b\,x^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8}+\frac{a\,x^6\left(c+\frac{d}{x^2}\right)^{5/2}}{16\,c}+\frac{3\,b\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{a\,c\,x^6\,\sqrt{c+\frac{d}{x^2}}}{16}-\frac{3\,b\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^6*(c + d/x^2)^(3/2))/6 + (5*b*x^4*(c + d/x^2)^(3/2))/8 + (a*x^6*(c + d
/x^2)^(5/2))/(16*c) + (a*d^3*atan(((c + d/x^2)^(1/2)*1i)/c^(1/2))*1i)/(16*c
^(3/2)) + (3*b*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(1/2)) - (a*c*x^6
*(c + d/x^2)^(1/2))/16 - (3*b*c*x^4*(c + d/x^2)^(1/2))/8
```

sympy [B] time = 100.22, size = 253, normalized size = 2.06

$$\frac{ac^2x^7}{6\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{11ac\sqrt{d}\,x^5}{24\sqrt{\frac{cx^2}{d}+1}}+\frac{17ad^{\frac{3}{2}}x^3}{48\sqrt{\frac{cx^2}{d}+1}}+\frac{ad^{\frac{5}{2}}x}{16c\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^3\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{16c^{\frac{3}{2}}}+\frac{bc^2x^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3bc\sqrt{d}\,x^3}{8\sqrt{\frac{cx^2}{d}+1}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**5,x)
```

```
[Out] a*c**2*x**7/(6*sqrt(d)*sqrt(c*x**2/d + 1)) + 11*a*c*sqrt(d)*x**5/(24*sqrt(c
*x**2/d + 1)) + 17*a*d**(3/2)*x**3/(48*sqrt(c*x**2/d + 1)) + a*d**(5/2)*x/(
16*c*sqrt(c*x**2/d + 1)) - a*d**3*asinh(sqrt(c)*x/sqrt(d))/(16*c**(3/2)) +
b*c**2*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*b*c*sqrt(d)*x**3/(8*sqrt(c*x
**2/d + 1)) + b*d**(3/2)*x*sqrt(c*x**2/d + 1)/2 + b*d**(3/2)*x/(8*sqrt(c*x*
*2/d + 1)) + 3*b*d**2*asinh(sqrt(c)*x/sqrt(d))/(8*sqrt(c))
```

3.946 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^3\,dx$

Optimal. Leaf size=115

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}\,(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

```
[Out] 1/8*(a*d+4*b*c)*(c+d/x^2)^(3/2)*x^2/c+1/4*a*(c+d/x^2)^(5/2)*x^4/c+3/8*d*(a*
d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-3/8*d*(a*d+4*b*c)*(c+d/x^
2)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {446, 78, 47, 50, 63, 208}

$$\frac{x^2\left(c+\frac{d}{x^2}\right)^{3/2}(ad+4bc)}{8c}-\frac{3d\sqrt{c+\frac{d}{x^2}}\,(ad+4bc)}{8c}+\frac{3d(ad+4bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\sqrt{c}}+\frac{ax^4\left(c+\frac{d}{x^2}\right)^{5/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (-3*d*(4*b*c + a*d)*Sqrt[c + d/x^2])/(8*c) + ((4*b*c + a*d)*(c + d/x^2)^(3/
2)*x^2)/(8*c) + (a*(c + d/x^2)^(5/2)*x^4)/(4*c) + (3*d*(4*b*c + a*d)*ArcTan
h[Sqrt[c + d/x^2]/Sqrt[c]])/(8*Sqrt[c])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^3\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{(4bc+ad)\,\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)}{8c}$$

$$= \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{(3d(4bc+ad))\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x\right.}{16c}$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{1}{16}(3d(4b$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} - \frac{1}{8}(3(4bc$$

$$= -\frac{3d(4bc+ad)\sqrt{c+\frac{d}{x^2}}}{8c} + \frac{(4bc+ad)\left(c+\frac{d}{x^2}\right)^{3/2} x^2}{8c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^4}{4c} + \frac{3d(4bc+}{}$$

Mathematica [A] time = 0.25, size = 89, normalized size = 0.77

$$\frac{1}{8}\sqrt{c+\frac{d}{x^2}}\left(\frac{3\sqrt{d}\,x(ad+4bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{\frac{cx^2}{d}+1}} + 2acx^4 + 5adx^2 + 4bcx^2 - 8bd\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^3,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-8*b*d + 4*b*c*x^2 + 5*a*d*x^2 + 2*a*c*x^4 + (3*Sqrt[d]*(
4*b*c + a*d)*x*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(Sqrt[c]*Sqrt[1 + (c*x^2)/d]))
)/8
```

fricas [A] time = 0.69, size = 203, normalized size = 1.77

$$\left[\frac{3\left(4\,bcd + ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2 - 2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) + 2\left(2\,ac^2x^4 - 8\,bcd + \left(4\,bc^2 + 5\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c}, -\frac{3\left(4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(4*b*c*d + a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2
 + d)/x^2) - d) + 2*(2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c, -1/8*(3*(4*b*c*d + a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*a*c^2*x^4 - 8*b*c*d + (4*b*c^2 + 5
*a*c*d)*x^2)*sqrt((c*x^2 + d)/x^2))/c]
```

giac [A] time = 0.29, size = 126, normalized size = 1.10

$$\frac{2\,b\sqrt{c}\,d^2\mathrm{sgn}(x)}{\left(\sqrt{c}\,x - \sqrt{cx^2 + d}\right)^2 - d} + \frac{1}{8}\left(2\,acx^2\mathrm{sgn}(x) + \frac{4\,bc^3\mathrm{sgn}(x) + 5\,ac^2d\mathrm{sgn}(x)}{c^2}\right)\sqrt{cx^2 + d}\,x - \frac{3\left(4\,bc^{\frac{3}{2}}d\mathrm{sgn}(x) + a\sqrt{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 2*b*sqrt(c)*d^2*sgn(x)/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d) + 1/8*(2*a*c*x
^2*sgn(x) + (4*b*c^3*sgn(x) + 5*a*c^2*d*sgn(x))/c^2)*sqrt(c*x^2 + d)*x - 3/
16*(4*b*c^(3/2)*d*sgn(x) + a*sqrt(c)*d^2*sgn(x))*log((sqrt(c)*x - sqrt(c*x^
2 + d))^2)/c
```

maple [A] time = 0.06, size = 174, normalized size = 1.51

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(3a\,d^3x\ln\left(\sqrt{c}\;x + \sqrt{c\,x^2 + d}\right) + 12bc\,d^2x\ln\left(\sqrt{c}\;x + \sqrt{c\,x^2 + d}\right) + 3\sqrt{c\,x^2 + d}\;a\sqrt{c}\;d^2x^2 + 12\sqrt{c\,x^2 -}\right.}{8\left(c\,x^2 + d\right)^{\frac{3}{2}}\sqrt{c}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x)
```

```
[Out] 1/8*((c*x^2+d)/x^2)^(3/2)*x^2*(8*c^(3/2)*(c*x^2+d)^(3/2)*x^2*b+12*c^(3/2)*(
c*x^2+d)^(1/2)*x^2*b*d+2*c^(1/2)*(c*x^2+d)^(3/2)*x^2*a*d-8*c^(1/2)*(c*x^2+d
)^(5/2)*b+3*c^(1/2)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2
))*x*a*d^3+12*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x*b*c*d^2)/(c*x^2+d)^(3/2)/d/c^
(1/2)
```

maxima [A] time = 1.33, size = 171, normalized size = 1.49

$$-\frac{1}{16}\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}} - \sqrt{c}}{\sqrt{c+\frac{d}{x^2}} + \sqrt{c}}\right)}{\sqrt{c}} - \frac{2\left(5\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}d^2 - 3\sqrt{c + \frac{d}{x^2}}\,cd^2\right)}{\left(c + \frac{d}{x^2}\right)^2 - 2\left(c + \frac{d}{x^2}\right)c + c^2}\right)a + \frac{1}{4}\left(2\sqrt{c + \frac{d}{x^2}}\,cx^2 - 3\sqrt{c}\,d\log\left(\frac{\sqrt{c + \frac{d}{x^2}} -}{\sqrt{c + \frac{d}{x^2}} +}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*d^2*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/s
qrt(c) - 2*(5*(c + d/x^2)^(3/2)*d^2 - 3*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)
^2 - 2*(c + d/x^2)*c + c^2))*a + 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d
*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c +
d/x^2)*d)*b
```

mupad [B] time = 5.70, size = 105, normalized size = 0.91

$$\frac{5\,a\,x^4\,\left(c+\frac{d}{x^2}\right)^{3/2}}{8}-b\,d\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,b\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,\sqrt{c}}-\frac{3\,a\,c\,x^4\,\sqrt{c+\frac{d}{x^2}}}{8}+\frac{b\,c\,x^2\,\sqrt{c+}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(3/2))/8 - b*d*(c + d/x^2)^(1/2) + (3*b*c^(1/2)*d*atan
h((c + d/x^2)^(1/2)/c^(1/2)))/2 + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2))
)/(8*c^(1/2)) - (3*a*c*x^4*(c + d/x^2)^(1/2))/8 + (b*c*x^2*(c + d/x^2)^(1/2
))/2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**3,x)
```

```
[Out] Timed out
```

3.947 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx$

Optimal. Leaf size=110

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\,(3ad+2bc)+\frac{1}{2}\sqrt{c}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

```
[Out] -1/6*(3*a*d+2*b*c)*(c+d/x^2)^(3/2)/c+1/2*a*(c+d/x^2)^(5/2)*x^2/c+1/2*(3*a*d
+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))*c^(1/2)-1/2*(3*a*d+2*b*c)*(c+d/x^2
)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 50, 63, 208}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3ad+2bc)}{6c}-\frac{1}{2}\sqrt{c+\frac{d}{x^2}}\,(3ad+2bc)+\frac{1}{2}\sqrt{c}\,(3ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)+\frac{ax^2\left(c+\frac{d}{x^2}\right)^{5/2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] -((2*b*c + 3*a*d)*Sqrt[c + d/x^2])/2 - ((2*b*c + 3*a*d)*(c + d/x^2)^(3/2))/
(6*c) + (a*(c + d/x^2)^(5/2)*x^2)/(2*c) + (Sqrt[c]*(2*b*c + 3*a*d)*ArcTanh[
Sqrt[c + d/x^2]/Sqrt[c]])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x\,dx = -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{\left(bc+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(2bc+3ad)\,\text{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx\right.$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{1}{4}(c(2bc+$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} - \frac{(c(2bc+3a}{}$$

$$= -\frac{1}{2}(2bc+3ad)\sqrt{c+\frac{d}{x^2}} - \frac{(2bc+3ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{6c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^2}{2c} + \frac{1}{2}\sqrt{c}\,(2bc+$$

Mathematica [C] time = 0.10, size = 78, normalized size = 0.71

$$\frac{1}{3}\sqrt{c+\frac{d}{x^2}}\left(-\frac{(3ad+2bc)\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{cx^2}{d}\right)}{\sqrt{\frac{cx^2}{d}+1}} - \frac{b\left(cx^2+d\right)^2}{dx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x,x]
```

```
[Out] (Sqrt[c + d/x^2]*(-((b*(d + c*x^2)^2)/(d*x^2)) - ((2*b*c + 3*a*d)*Hypergeom
etric2F1[-3/2, -1/2, 1/2, -((c*x^2)/d)])/Sqrt[1 + (c*x^2)/d]))/3
```

fricas [A] time = 0.65, size = 195, normalized size = 1.77

$$\left[\frac{3\,(2\,bc+3\,ad)\sqrt{c}\,x^2\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)+2\left(3\,acx^4-2\,(4\,bc+3\,ad)x^2-2\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x^2}, -\frac{3\,(2\,bc+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*b*c + 3*a*d)*sqrt(c)*x^2*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x
^2 + d)/x^2) - d) + 2*(3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 - 2*b*d)*sqrt((c*x
```

```
^2 + d)/x^2))/x^2, -1/6*(3*(2*b*c + 3*a*d)*sqrt(-c)*x^2*arctan(sqrt(-c)*x^2
*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (3*a*c*x^4 - 2*(4*b*c + 3*a*d)*x^2 -
2*b*d)*sqrt((c*x^2 + d)/x^2))/x^2]
```

giac [B] time = 0.54, size = 225, normalized size = 2.05

$$\frac{1}{2}\sqrt{cx^2+d}\,acx\mathrm{sgn}(x)-\frac{1}{4}\left(2\,bc^{\frac{3}{2}}\mathrm{sgn}(x)+3\,a\sqrt{c}\,d\mathrm{sgn}(x)\right)\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)+\frac{2\left(6\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + d)*a*c*x*sgn(x) - 1/4*(2*b*c^(3/2)*sgn(x) + 3*a*sqrt(c)*d*
sgn(x))*log((sqrt(c)*x - sqrt(c*x^2 + d))^2) + 2/3*(6*(sqrt(c)*x - sqrt(c*x
^2 + d))^4*b*c^(3/2)*d*sgn(x) + 3*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*sqrt(c)
*d^2*sgn(x) - 6*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(3/2)*d^2*sgn(x) - 6*(s
qrt(c)*x - sqrt(c*x^2 + d))^2*a*sqrt(c)*d^3*sgn(x) + 4*b*c^(3/2)*d^3*sgn(x)
 + 3*a*sqrt(c)*d^4*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^3
```

maple [B] time = 0.06, size = 216, normalized size = 1.96

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(9ac\,d^3x^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)+6b\,c^2d^2x^3\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)+9\sqrt{c\,x^2+d}\;a\,c^{\frac{3}{2}}d^2x^4+6\sqrt{c\,x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x,x)
```

```
[Out] 1/6*((c*x^2+d)/x^2)^(3/2)*(4*c^(5/2)*(c*x^2+d)^(3/2)*x^4*b+6*c^(5/2)*(c*x^2
+d)^(1/2)*x^4*b*d+6*c^(3/2)*(c*x^2+d)^(3/2)*x^4*a*d-4*c^(3/2)*(c*x^2+d)^(5/
2)*x^2*b+9*c^(3/2)*(c*x^2+d)^(1/2)*x^4*a*d^2-6*c^(1/2)*(c*x^2+d)^(5/2)*x^2*
a*d+9*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*x^3*a*c*d^3+6*ln(c^(1/2)*x+(c*x^2+d)^(1
/2))*x^3*b*c^2*d^2-2*c^(1/2)*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2/c^(1/
2)
```

maxima [A] time = 1.23, size = 134, normalized size = 1.22

$$\frac{1}{4}\left(2\sqrt{c+\frac{d}{x^2}}\,cx^2-3\sqrt{c}\,d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)-4\sqrt{c+\frac{d}{x^2}}\,d\right)a-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*sqrt(c + d/x^2)*c*x^2 - 3*sqrt(c)*d*log((sqrt(c + d/x^2) - sqrt(c))/
(sqrt(c + d/x^2) + sqrt(c))) - 4*sqrt(c + d/x^2)*d)*a - 1/6*(3*c^(3/2)*log(
(sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3
/2) + 6*sqrt(c + d/x^2)*c)*b
```

mupad [B] time = 5.65, size = 95, normalized size = 0.86

$$b\,c^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3}-a\,d\,\sqrt{c+\frac{d}{x^2}}-b\,c\,\sqrt{c+\frac{d}{x^2}}+\frac{3\,a\,\sqrt{c}\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2}+\frac{a\,c\,x^2\,\sqrt{c+\frac{d}{x^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] b*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (b*(c + d/x^2)^(3/2))/3 - a*d*
(c + d/x^2)^(1/2) - b*c*(c + d/x^2)^(1/2) + (3*a*c^(1/2)*d*atanh((c + d/x^2
)^(1/2)/c^(1/2)))/2 + (a*c*x^2*(c + d/x^2)^(1/2))/2
```

sympy [A] time = 56.16, size = 187, normalized size = 1.70

$$\frac{3a\sqrt{c}\,d\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2}+\frac{ac\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2}-\frac{ac\sqrt{d}\,x}{\sqrt{\frac{cx^2}{d}+1}}-\frac{ad^{\frac{3}{2}}}{x\sqrt{\frac{cx^2}{d}+1}}+bc^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)-\frac{bc^2x}{\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{bc\sqrt{d}}{x\sqrt{\frac{cx^2}{d}+1}}+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x,x)
```

```
[Out] 3*a*sqrt(c)*d*asinh(sqrt(c)*x/sqrt(d))/2 + a*c*sqrt(d)*x*sqrt(c*x**2/d + 1)
/2 - a*c*sqrt(d)*x/sqrt(c*x**2/d + 1) - a*d**(3/2)/(x*sqrt(c*x**2/d + 1)) +
 b*c**(3/2)*asinh(sqrt(c)*x/sqrt(d)) - b*c**2*x/(sqrt(d)*sqrt(c*x**2/d + 1)
) - b*c*sqrt(d)/(x*sqrt(c*x**2/d + 1)) + b*d*Piecewise((-sqrt(c)/(2*x**2),
Eq(d, 0)), (-(c + d/x**2)**(3/2)/(3*d), True))
```

3.948 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=76

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

```
[Out] -1/3*a*(c+d/x^2)^(3/2)-1/5*b*(c+d/x^2)^(5/2)/d+a*c^(3/2)*arctanh((c+d/x^2)^
(1/2)/c^(1/2))-a*c*(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 208}

$$ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2}-ac\sqrt{c+\frac{d}{x^2}}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -(a*c*Sqrt[c + d/x^2]) - (a*(c + d/x^2)^(3/2))/3 - (b*(c + d/x^2)^(5/2))/(5
*d) + a*c^(3/2)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(a+bx)(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}a\operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -\frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}(ac)\operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x}\,dx, x, \frac{1}{x^2}\right)\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{1}{2}\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} - \frac{\left(ac^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}\\
&= -ac\sqrt{c+\frac{d}{x^2}} - \frac{1}{3}a\left(c+\frac{d}{x^2}\right)^{3/2} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d} + ac^{3/2}\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)
\end{aligned}$$

Mathematica [C] time = 0.07, size = 90, normalized size = 1.18

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(5ad^2x^2\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{cx^2}{d}\right)+3b\left(cx^2+d\right)^2\sqrt{\frac{cx^2}{d}+1}\right)}{15dx^4\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x,x]
```

```
[Out] -1/15*(Sqrt[c + d/x^2]*(3*b*(d + c*x^2)^2*Sqrt[1 + (c*x^2)/d] + 5*a*d^2*x^2
*Hypergeometric2F1[-3/2, -3/2, -1/2, -((c*x^2)/d)]))/(d*x^4*Sqrt[1 + (c*x^2
)/d])
```

fricas [A] time = 0.64, size = 213, normalized size = 2.80

$$\left[\frac{15\,ac^{\frac{3}{2}}dx^4\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(\left(3\,bc^2+20\,acd\right)x^4+3\,bd^2+\left(6\,bcd+5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,dx^4}, -\frac{15\,a\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/30*(15*a*c^(3/2)*d*x^4*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2
) - d) - 2*((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d^2 + (6*b*c*d + 5*a*d^2)*x^2)*s
qrt((c*x^2 + d)/x^2))/(d*x^4), -1/15*(15*a*sqrt(-c)*c*d*x^4*arctan(sqrt(-c)
*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + ((3*b*c^2 + 20*a*c*d)*x^4 + 3*b*d
^2 + (6*b*c*d + 5*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^4)]
```

giac [B] time = 1.07, size = 254, normalized size = 3.34

$$-\frac{1}{2}\,ac^{\frac{3}{2}}\log\left(\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^2\right)\operatorname{sgn}(x)+\frac{2\left(15\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8bc^{\frac{5}{2}}\operatorname{sgn}(x)+30\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^8ac^{\frac{3}{2}}ds\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/2*a*c^(3/2)*log((sqrt(c)*x - sqrt(c*x^2 + d))^2)*sgn(x) + 2/15*(15*(sqrt
(c)*x - sqrt(c*x^2 + d))^8*b*c^(5/2)*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 +
d))^8*a*c^(3/2)*d*sgn(x) - 90*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(3/2)*d^2
*sgn(x) + 30*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(5/2)*d^2*sgn(x) + 110*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*a*c^(3/2)*d^3*sgn(x) - 70*(sqrt(c)*x - sqrt(c*
x^2 + d))^2*a*c^(3/2)*d^4*sgn(x) + 3*b*c^(5/2)*d^4*sgn(x) + 20*a*c^(3/2)*d^
5*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^5
```

maple [B] time = 0.07, size = 153, normalized size = 2.01

$$\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(15a\,c^2d^2x^5\ln\left(\sqrt{c}\,x+\sqrt{c\,x^2+d}\right)+15\sqrt{c\,x^2+d}\;a\,c^{\frac{5}{2}}d\,x^6+10\left(c\,x^2+d\right)^{\frac{3}{2}}a\,c^{\frac{5}{2}}x^6-10\left(c\,x^2+d\right)^{\frac{5}{2}}a\,c\right.}{15\left(c\,x^2+d\right)^{\frac{3}{2}}\sqrt{c}\;d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x,x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*(10*c^(5/2)*(c*x^2+d)^(3/2)*x^6*a+15*c^(5/2)*(c*
x^2+d)^(1/2)*x^6*a*d-10*c^(3/2)*(c*x^2+d)^(5/2)*x^4*a+15*ln(c^(1/2)*x+(c*x^
2+d)^(1/2))*x^5*a*c^2*d^2-5*(c*x^2+d)^(5/2)*a*c^(1/2)*d*x^2-3*(c*x^2+d)^(5/
2)*b*c^(1/2)*d)/x^2/(c*x^2+d)^(3/2)/d^2/c^(1/2)
```

maxima [A] time = 1.31, size = 80, normalized size = 1.05

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{6}\left(3\,c^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)+2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}+6\sqrt{c+\frac{d}{x^2}}\,c\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/5*b*(c + d/x^2)^(5/2)/d - 1/6*(3*c^(3/2)*log((sqrt(c + d/x^2) - sqrt(c))
/(sqrt(c + d/x^2) + sqrt(c))) + 2*(c + d/x^2)^(3/2) + 6*sqrt(c + d/x^2)*c)*
a
```

mupad [B] time = 5.83, size = 72, normalized size = 0.95

$$a\,c^{3/2}\operatorname{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)-\frac{a\left(c+\frac{d}{x^2}\right)^{3/2}}{3}-a\,c\sqrt{c+\frac{d}{x^2}}-\frac{b\sqrt{c+\frac{d}{x^2}}\,\left(c\,x^2+d\right)^2}{5\,d\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x,x)
```

```
[Out] a*c^(3/2)*atanh((c + d/x^2)^(1/2)/c^(1/2)) - (a*(c + d/x^2)^(3/2))/3 - a*c*
(c + d/x^2)^(1/2) - (b*(c + d/x^2)^(1/2)*(d + c*x^2)^2)/(5*d*x^4)
```

sympy [A] time = 54.14, size = 73, normalized size = 0.96

$$-\frac{ac^2\operatorname{atan}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}}-ac\sqrt{c+\frac{d}{x^2}}-\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*c**2*atan(sqrt(c + d/x**2)/sqrt(-c))/sqrt(-c) - a*c*sqrt(c + d/x**2) - a
*(c + d/x**2)**(3/2)/3 - b*(c + d/x**2)**(5/2)/(5*d)
```

3.949 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=46

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

```
[Out] 1/5*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^2-1/7*b*(c+d/x^2)^(7/2)/d^2
```

Rubi [A] time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] ((b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^2) - (b*(c + d/x^2)^(7/2))/(7*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^3}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int (a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.07

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(7adx^2-2bcx^2+5bd\right)}{35d^2x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x]
```

```
[Out] -1/35*(Sqrt[c + d/x^2]*(d + c*x^2)^2*(5*b*d - 2*b*c*x^2 + 7*a*d*x^2))/(d^2*
x^6)
```

fricas [B] time = 0.72, size = 84, normalized size = 1.83

$$\frac{\left(\left(2\,bc^{3}-7\,ac^{2}d\right)x^{6}-\left(bc^{2}d+14\,acd^{2}\right)x^{4}-5\,bd^{3}-\left(8\,bcd^{2}+7\,ad^{3}\right)x^{2}\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{35\,d^{2}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/35*((2*b*c^3 - 7*a*c^2*d)*x^6 - (b*c^2*d + 14*a*c*d^2)*x^4 - 5*b*d^3 - (8
*b*c*d^2 + 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^2*x^6)
```

giac [B] time = 1.87, size = 370, normalized size = 8.04

$$\frac{2\left(35\left(\sqrt{c}\,x-\sqrt{cx^{2}+d}\right)^{12}ac^{\frac{5}{2}}\mathrm{sgn}(x)+70\left(\sqrt{c}\,x-\sqrt{cx^{2}+d}\right)^{10}bc^{\frac{7}{2}}\mathrm{sgn}(x)-70\left(\sqrt{c}\,x-\sqrt{cx^{2}+d}\right)^{10}ac^{\frac{5}{2}}d\,\mathrm{sgn}(x)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(5/2)*sgn(x) + 70*(sqrt(c)*x
- sqrt(c*x^2 + d))^10*b*c^(7/2)*sgn(x) - 70*(sqrt(c)*x - sqrt(c*x^2 + d))^1
0*a*c^(5/2)*d*sgn(x) + 70*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(7/2)*d*sgn(x
) + 105*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(5/2)*d^2*sgn(x) + 140*(sqrt(c)
*x - sqrt(c*x^2 + d))^6*b*c^(7/2)*d^2*sgn(x) - 140*(sqrt(c)*x - sqrt(c*x^2
+ d))^6*a*c^(5/2)*d^3*sgn(x) + 28*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(7/2)
*d^3*sgn(x) + 77*(sqrt(c)*x - sqrt(c*x^2 + d))^4*a*c^(5/2)*d^4*sgn(x) + 14*
(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(7/2)*d^4*sgn(x) - 14*(sqrt(c)*x - sqrt
(c*x^2 + d))^2*a*c^(5/2)*d^5*sgn(x) - 2*b*c^(7/2)*d^5*sgn(x) + 7*a*c^(5/2)*
d^6*sgn(x))/((sqrt(c)*x - sqrt(c*x^2 + d))^2 - d)^7
```

maple [A] time = 0.05, size = 48, normalized size = 1.04

$$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(7ad\,x^{2}-2bc\,x^{2}+5bd\right)\left(c\,x^{2}+d\right)}{35d^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x)
```

```
[Out] -1/35*((c*x^2+d)/x^2)^(3/2)*(7*a*d*x^2-2*b*c*x^2+5*b*d)*(c*x^2+d)/d^2/x^4
```

maxima [A] time = 0.59, size = 49, normalized size = 1.07

$$-\frac{a\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}}{5\,d}-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}}{d^{2}}-\frac{7\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}c}{d^{2}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/5*a*(c + d/x^2)^(5/2)/d - 1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^
(5/2)*c/d^2)*b
```

mupad [B] time = 5.33, size = 122, normalized size = 2.65

$$\frac{2\,b\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{5\,d}-\frac{2\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{5\,x^2}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{5\,x^4}-\frac{8\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{b\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^3,x)
```

```
[Out] (2*b*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (a*c^2*(c + d/x^2)^(1/2))/(5*d) - (2
*a*c*(c + d/x^2)^(1/2))/(5*x^2) - (a*d*(c + d/x^2)^(1/2))/(5*x^4) - (8*b*c*
(c + d/x^2)^(1/2))/(35*x^4) - (b*d*(c + d/x^2)^(1/2))/(7*x^6) - (b*c^2*(c +
 d/x^2)^(1/2))/(35*d*x^2)
```

sympy [A] time = 14.16, size = 138, normalized size = 3.00

$$-\frac{ac\left(\begin{cases}\frac{\sqrt{c}}{x^2} & \text{for } d=0\\ \frac{2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)}{2}-\frac{a\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d}-\frac{bc\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{b\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\right.}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] -a*c*Piecewise((sqrt(c)/x**2, Eq(d, 0)), (2*(c + d/x**2)**(3/2)/(3*d), True
))/2 - a*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d - b*c*(-c*(c
+ d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - b*(c**2*(c + d/x**2)**(3
/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
```

3.950 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=74

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

```
[Out] -1/5*c*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^3+1/7*(-a*d+2*b*c)*(c+d/x^2)^(7/2)/d^3-
1/9*b*(c+d/x^2)^(9/2)/d^3
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{7/2}(2bc-ad)}{7d^3}-\frac{c\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] -(c*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^3) + ((2*b*c - a*d)*(c + d/x^2)^(7/
2))/(7*d^3) - (b*(c + d/x^2)^(9/2))/(9*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^5}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x(a+bx)(c+dx)^{3/2}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{c(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{(-2bc+ad)(c+dx)^{5/2}}{d^2}+\frac{b(c+dx)^{7/2}}{d^2}\right)dx, x
\right.\right.\\
&= -\frac{c(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 0.96

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(9adx^2\left(2cx^2-5d\right)+b\left(-8c^2x^4+20cdx^2-35d^2\right)\right)}{315d^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(9*a*d*x^2*(-5*d + 2*c*x^2) + b*(-35*d^2 + 2
0*c*d*x^2 - 8*c^2*x^4)))/(315*d^3*x^8)
```

fricas [A] time = 0.81, size = 109, normalized size = 1.47

$$-\frac{\left(2\left(4\,bc^4-9\,ac^3d\right)x^8-\left(4\,bc^3d-9\,ac^2d^2\right)x^6+35\,bd^4+3\left(bc^2d^2+24\,acd^3\right)x^4+5\left(10\,bcd^3+9\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,d^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(4*b*c^4 - 9*a*c^3*d)*x^8 - (4*b*c^3*d - 9*a*c^2*d^2)*x^6 + 35*b*
d^4 + 3*(b*c^2*d^2 + 24*a*c*d^3)*x^4 + 5*(10*b*c*d^3 + 9*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2)/(d^3*x^8)
```

giac [B] time = 2.69, size = 430, normalized size = 5.81

$$\frac{4\left(315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{7}{2}}\mathrm{sgn}(x)+840\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}bc^{\frac{9}{2}}\mathrm{sgn}(x)-315\left(\sqrt{c}x-\sqrt{cx^2+d}\right)^{12}ac^{\frac{7}{2}}d\,\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(7/2)*sgn(x) + 840*(sqrt(c)
*x - sqrt(c*x^2 + d))^12*b*c^(9/2)*sgn(x) - 315*(sqrt(c)*x - sqrt(c*x^2 + d
))^12*a*c^(7/2)*d*sgn(x) + 1260*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(9/2)*
d*sgn(x) + 315*(sqrt(c)*x - sqrt(c*x^2 + d))^10*a*c^(7/2)*d^2*sgn(x) + 1764
*(sqrt(c)*x - sqrt(c*x^2 + d))^8*b*c^(9/2)*d^2*sgn(x) - 819*(sqrt(c)*x - sq
rt(c*x^2 + d))^8*a*c^(7/2)*d^3*sgn(x) + 504*(sqrt(c)*x - sqrt(c*x^2 + d))^6
*b*c^(9/2)*d^3*sgn(x) + 441*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(7/2)*d^4*s
gn(x) + 144*(sqrt(c)*x - sqrt(c*x^2 + d))^4*b*c^(9/2)*d^4*sgn(x) - 9*(sqrt(
c)*x - sqrt(c*x^2 + d))^4*a*c^(7/2)*d^5*sgn(x) - 36*(sqrt(c)*x - sqrt(c*x^2
 + d))^2*b*c^(9/2)*d^5*sgn(x) + 81*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(7/2
)*d^6*sgn(x) + 4*b*c^(9/2)*d^6*sgn(x) - 9*a*c^(7/2)*d^7*sgn(x))/((sqrt(c)*x
 - sqrt(c*x^2 + d))^2 - d)^9
```

maple [A] time = 0.05, size = 70, normalized size = 0.95

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18acd\,x^4-8b\,c^2x^4-45a\,d^2x^2+20bcd\,x^2-35b\,d^2\right)\left(c\,x^2+d\right)}{315d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*(18*a*c*d*x^4-8*b*c^2*x^4-45*a*d^2*x^2+20*b*c*d
*x^2-35*b*d^2)*(c*x^2+d)/d^3/x^6
```

maxima [A] time = 0.64, size = 84, normalized size = 1.14

$$-\frac{1}{35}\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^2}-\frac{7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^2}\right)a-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/35*(5*(c + d/x^2)^(7/2)/d^2 - 7*(c + d/x^2)^(5/2)*c/d^2)*a - 1/315*(35*(
c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x^2)^(5/2)*c^
2/d^3)*b
```

mupad [B] time = 5.76, size = 164, normalized size = 2.22

$$\frac{2\,a\,c^3\,\sqrt{c+\frac{d}{x^2}}}{35\,d^2}-\frac{8\,b\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{8\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{35\,x^4}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{7\,x^6}-\frac{10\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{35\,d\,x^2}-\frac{b\,c^2\,\sqrt{}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^5,x)
```

```
[Out] (2*a*c^3*(c + d/x^2)^(1/2))/(35*d^2) - (8*b*c^4*(c + d/x^2)^(1/2))/(315*d^3
) - (8*a*c*(c + d/x^2)^(1/2))/(35*x^4) - (a*d*(c + d/x^2)^(1/2))/(7*x^6) -
(10*b*c*(c + d/x^2)^(1/2))/(63*x^6) - (b*d*(c + d/x^2)^(1/2))/(9*x^8) - (a*
c^2*(c + d/x^2)^(1/2))/(35*d*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) +
 (4*b*c^3*(c + d/x^2)^(1/2))/(315*d^2*x^2)
```

sympy [B] time = 15.61, size = 194, normalized size = 2.62

$$-\frac{ac\left(-\frac{c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}-\frac{a\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^2}-\frac{bc\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{b\left(-\frac{c^3\left(c+\frac{d}{x^2}\right.}{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -a*c*(-c*(c + d/x**2)**(3/2)/3 + (c + d/x**2)**(5/2)/5)/d**2 - a*(c**2*(c +
 d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)**(7/2)/7)/d**2
 - b*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**
2)**(7/2)/7)/d**3 - b*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(
5/2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3
```

3.951 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=104

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

```
[Out] 1/5*c^2*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^4-1/7*c*(-2*a*d+3*b*c)*(c+d/x^2)^(7/2)
/d^4+1/9*(-a*d+3*b*c)*(c+d/x^2)^(9/2)/d^4-1/11*b*(c+d/x^2)^(11/2)/d^4
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{9/2}(3bc-ad)}{9d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{7/2}(3bc-2ad)}{7d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (c^2*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (c*(3*b*c - 2*a*d)*(c + d/x^2
)^(7/2))/(7*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(9/2))/(9*d^4) - (b*(c + d/x^
2)^(11/2))/(11*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^7} dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{c(3bc-2ad)(c+dx)^{5/2}}{d^3}+\frac{(-3bc+ad)}{d^3}\right.\right.\right.\\ &= \frac{c^2(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{9d^4}-\frac{b\left(c\right.}{1}\end{aligned}$$

Mathematica [A] time = 0.04, size = 94, normalized size = 0.90

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(-11adx^2\left(8c^2x^4-20cdx^2+35d^2\right)-3b\left(-16c^3x^6+40c^2dx^4-70cd^2x^2+105d^3\right)\right)}{3465d^4x^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-11*a*d*x^2*(35*d^2 - 20*c*d*x^2 + 8*c^2*x^
4) - 3*b*(105*d^3 - 70*c*d^2*x^2 + 40*c^2*d*x^4 - 16*c^3*x^6)))/(3465*d^4*x
^10)
```

fricas [A] time = 0.80, size = 134, normalized size = 1.29

$$\frac{\left(8\left(6\,bc^5-11\,ac^4d\right)x^{10}-4\left(6\,bc^4d-11\,ac^3d^2\right)x^8+3\left(6\,bc^3d^2-11\,ac^2d^3\right)x^6-315\,bd^5-5\left(3\,bc^2d^3+110\,acd^4\right)x^4-\right.}{3465\,d^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/3465*(8*(6*b*c^5 - 11*a*c^4*d)*x^10 - 4*(6*b*c^4*d - 11*a*c^3*d^2)*x^8 +
3*(6*b*c^3*d^2 - 11*a*c^2*d^3)*x^6 - 315*b*d^5 - 5*(3*b*c^2*d^3 + 110*a*c*d
^4)*x^4 - 35*(12*b*c*d^4 + 11*a*d^5)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^10)
```

giac [B] time = 3.28, size = 490, normalized size = 4.71

$$16\left(2310\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{16}ac^{\frac{9}{2}}\mathrm{sgn}(x)+6930\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{14}bc^{\frac{11}{2}}\mathrm{sgn}(x)-1155\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{14}ac^{\frac{9}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 16/3465*(2310*(sqrt(c)*x - sqrt(c*x^2 + d))^16*a*c^(9/2)*sgn(x) + 6930*(sqr
t(c)*x - sqrt(c*x^2 + d))^14*b*c^(11/2)*sgn(x) - 1155*(sqrt(c)*x - sqrt(c*x
^2 + d))^14*a*c^(9/2)*d*sgn(x) + 12474*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c
^(11/2)*d*sgn(x) + 231*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(9/2)*d^2*sgn(x
) + 15246*(sqrt(c)*x - sqrt(c*x^2 + d))^10*b*c^(11/2)*d^2*sgn(x) - 4851*(sq
rt(c)*x - sqrt(c*x^2 + d))^10*a*c^(9/2)*d^3*sgn(x) + 4950*(sqrt(c)*x - sqrt
(c*x^2 + d))^8*b*c^(11/2)*d^3*sgn(x) + 2475*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*a*c^(9/2)*d^4*sgn(x) + 990*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(11/2)*d^4*
sgn(x) + 495*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(9/2)*d^5*sgn(x) - 330*(sq
rt(c)*x - sqrt(c*x^2 + d))^4*b*c^(11/2)*d^5*sgn(x) + 605*(sqrt(c)*x - sqrt(
c*x^2 + d))^4*a*c^(9/2)*d^6*sgn(x) + 66*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c
^(11/2)*d^6*sgn(x) - 121*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(9/2)*d^7*sgn(
x) - 6*b*c^(11/2)*d^7*sgn(x) + 11*a*c^(9/2)*d^8*sgn(x))/((sqrt(c)*x - sqrt(
c*x^2 + d))^2 - d)^11
```

maple [A] time = 0.05, size = 94, normalized size = 0.90

$$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(88a\,c^2d\,x^6-48b\,c^3x^6-220ac\,d^2x^4+120b\,c^2d\,x^4+385a\,d^3x^2-210bc\,d^2x^2+315b\,d^3\right)\left(c\,x^2+d\right)}{3465d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] -1/3465*((c*x^2+d)/x^2)^(3/2)*(88*a*c^2*d*x^6-48*b*c^3*x^6-220*a*c*d^2*x^4+
120*b*c^2*d*x^4+385*a*d^3*x^2-210*b*c*d^2*x^2+315*b*d^3)*(c*x^2+d)/d^4/x^8
```

maxima [A] time = 0.58, size = 118, normalized size = 1.13

$$-\frac{1}{315}\left(\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{d^3}-\frac{90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c}{d^3}+\frac{63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^2}{d^3}\right)a-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\right.}{d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/315*(35*(c + d/x^2)^(9/2)/d^3 - 90*(c + d/x^2)^(7/2)*c/d^3 + 63*(c + d/x
^2)^(5/2)*c^2/d^3)*a - 1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)
^(9/2)*c/d^4 + 495*(c + d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^
4)*b
```

mupad [B] time = 6.31, size = 206, normalized size = 1.98

$$\frac{16\,b\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{8\,a\,c^4\,\sqrt{c+\frac{d}{x^2}}}{315\,d^3}-\frac{10\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{63\,x^6}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{9\,x^8}-\frac{4\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{105\,d\,x^4}+\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^7,x)
```

```
[Out] (16*b*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(315*
d^3) - (10*a*c*(c + d/x^2)^(1/2))/(63*x^6) - (a*d*(c + d/x^2)^(1/2))/(9*x^8
) - (4*b*c*(c + d/x^2)^(1/2))/(33*x^8) - (b*d*(c + d/x^2)^(1/2))/(11*x^10)
- (a*c^2*(c + d/x^2)^(1/2))/(105*d*x^4) + (4*a*c^3*(c + d/x^2)^(1/2))/(315*
d^2*x^2) - (b*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*b*c^3*(c + d/x^2)^(1/
2))/(385*d^2*x^4) - (8*b*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2)
```

sympy [B] time = 17.96, size = 262, normalized size = 2.52

$$-\frac{ac\left(\frac{c^2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{2c\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}\right)}{d^3}-\frac{a\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^3}-\frac{bc\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2(}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -a*c*(c**2*(c + d/x**2)**(3/2)/3 - 2*c*(c + d/x**2)**(5/2)/5 + (c + d/x**2)
**(7/2)/7)/d**3 - a*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/
2)/5 - 3*c*(c + d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**3 - b*c*(-c**3
*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c + d/x**2)**(
7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - b*(c**4*(c + d/x**2)**(3/2)/3 - 4*c*
*3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c + d/x**2)*
*(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4
```

3.952 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=134

$$-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

```
[Out] -1/5*c^3*(-a*d+b*c)*(c+d/x^2)^(5/2)/d^5+1/7*c^2*(-3*a*d+4*b*c)*(c+d/x^2)^(7
/2)/d^5-1/3*c*(-a*d+2*b*c)*(c+d/x^2)^(9/2)/d^5+1/11*(-a*d+4*b*c)*(c+d/x^2)^
(11/2)/d^5-1/13*b*(c+d/x^2)^(13/2)/d^5
```

Rubi [A] time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{446, 77\}$

$$\frac{c^2\left(c+\frac{d}{x^2}\right)^{7/2}(4bc-3ad)}{7d^5}-\frac{c^3\left(c+\frac{d}{x^2}\right)^{5/2}(bc-ad)}{5d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{11/2}(4bc-ad)}{11d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{9/2}(2bc-ad)}{3d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] -(c^3*(b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^5) + (c^2*(4*b*c - 3*a*d)*(c + d/
x^2)^(7/2))/(7*d^5) - (c*(2*b*c - a*d)*(c + d/x^2)^(9/2))/(3*d^5) + ((4*b*c
 - a*d)*(c + d/x^2)^(11/2))/(11*d^5) - (b*(c + d/x^2)^(13/2))/(13*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^9}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^3(a+bx)(c+dx)^{3/2}\,dx,x,\frac{1}{x^2}\right)\right)\\ &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{c^3(bc-ad)(c+dx)^{3/2}}{d^4}-\frac{c^2(4bc-3ad)(c+dx)^{5/2}}{d^4}+\frac{3c(2bc-ad)(c+\right.\right.\right.\\ &= -\frac{c^3(bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5}+\frac{c^2(4bc-3ad)\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}-\frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{9/2}}{3d^5}+\frac{(4bc\end{aligned}$$

Mathematica [A] time = 0.06, size = 115, normalized size = 0.86

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(13adx^2\left(16c^3x^6-40c^2dx^4+70cd^2x^2-105d^3\right)+b\left(-128c^4x^8+320c^3dx^6-560c^2d^2x^4+8\right.\right.}{15015d^5x^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(13*a*d*x^2*(-105*d^3 + 70*c*d^2*x^2 - 40*c^
2*d*x^4 + 16*c^3*x^6) + b*(-1155*d^4 + 840*c*d^3*x^2 - 560*c^2*d^2*x^4 + 32
0*c^3*d*x^6 - 128*c^4*x^8)))/(15015*d^5*x^12)
```

fricas [A] time = 0.95, size = 157, normalized size = 1.17

$$-\frac{\left(16\left(8\,bc^6-13\,ac^5d\right)x^{12}-8\left(8\,bc^5d-13\,ac^4d^2\right)x^{10}+6\left(8\,bc^4d^2-13\,ac^3d^3\right)x^8+1155\,bd^6-5\left(8\,bc^3d^3-13\,ac\right.\right.}{15015\,d^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/15015*(16*(8*b*c^6 - 13*a*c^5*d)*x^12 - 8*(8*b*c^5*d - 13*a*c^4*d^2)*x^1
0 + 6*(8*b*c^4*d^2 - 13*a*c^3*d^3)*x^8 + 1155*b*d^6 - 5*(8*b*c^3*d^3 - 13*a
*c^2*d^4)*x^6 + 35*(b*c^2*d^4 + 52*a*c*d^5)*x^4 + 105*(14*b*c*d^5 + 13*a*d^
6)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^5*x^12)
```

giac [B] time = 4.63, size = 550, normalized size = 4.10

$$\frac{32\left(15015\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{18}ac^{\frac{11}{2}}\operatorname{sgn}(x)+48048\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right)^{16}bc^{\frac{13}{2}}\operatorname{sgn}(x)-3003\left(\sqrt{c}\,x-\sqrt{cx^2+d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 32/15015*(15015*(sqrt(c)*x - sqrt(c*x^2 + d))^18*a*c^(11/2)*sgn(x) + 48048*
(sqrt(c)*x - sqrt(c*x^2 + d))^16*b*c^(13/2)*sgn(x) - 3003*(sqrt(c)*x - sqrt
(c*x^2 + d))^16*a*c^(11/2)*d*sgn(x) + 96096*(sqrt(c)*x - sqrt(c*x^2 + d))^1
4*b*c^(13/2)*d*sgn(x) - 6006*(sqrt(c)*x - sqrt(c*x^2 + d))^14*a*c^(11/2)*d^
2*sgn(x) + 109824*(sqrt(c)*x - sqrt(c*x^2 + d))^12*b*c^(13/2)*d^2*sgn(x) -
28314*(sqrt(c)*x - sqrt(c*x^2 + d))^12*a*c^(11/2)*d^3*sgn(x) + 37752*(sqrt(
c)*x - sqrt(c*x^2 + d))^10*b*c^(13/2)*d^3*sgn(x) + 13728*(sqrt(c)*x - sqrt(
c*x^2 + d))^10*a*c^(11/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8
*b*c^(13/2)*d^4*sgn(x) + 5720*(sqrt(c)*x - sqrt(c*x^2 + d))^8*a*c^(11/2)*d^
5*sgn(x) - 2288*(sqrt(c)*x - sqrt(c*x^2 + d))^6*b*c^(13/2)*d^5*sgn(x) + 371
8*(sqrt(c)*x - sqrt(c*x^2 + d))^6*a*c^(11/2)*d^6*sgn(x) + 624*(sqrt(c)*x -
sqrt(c*x^2 + d))^4*b*c^(13/2)*d^6*sgn(x) - 1014*(sqrt(c)*x - sqrt(c*x^2 + d
))^4*a*c^(11/2)*d^7*sgn(x) - 104*(sqrt(c)*x - sqrt(c*x^2 + d))^2*b*c^(13/2)
*d^7*sgn(x) + 169*(sqrt(c)*x - sqrt(c*x^2 + d))^2*a*c^(11/2)*d^8*sgn(x) + 8
*b*c^(13/2)*d^8*sgn(x) - 13*a*c^(11/2)*d^9*sgn(x))/((sqrt(c)*x - sqrt(c*x^2
 + d))^2 - d)^13
```

maple [A] time = 0.05, size = 118, normalized size = 0.88

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(208a\,c^3d\,x^8-128b\,c^4x^8-520a\,c^2d^2x^6+320b\,c^3d\,x^6+910ac\,d^3x^4-560b\,c^2d^2x^4-1365a\,d^4x^2+84\right.}{15015d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*(208*a*c^3*d*x^8-128*b*c^4*x^8-520*a*c^2*d^2*
x^6+320*b*c^3*d*x^6+910*a*c*d^3*x^4-560*b*c^2*d^2*x^4-1365*a*d^4*x^2+840*b*
c*d^3*x^2-1155*b*d^4)*(c*x^2+d)/d^5/x^10
```

maxima [A] time = 0.63, size = 152, normalized size = 1.13

$$-\frac{1}{1155}\left(\frac{105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{d^4}-\frac{385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}c}{d^4}+\frac{495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}c^2}{d^4}-\frac{231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3}{d^4}\right)a-\frac{1}{15015}\left(\frac{1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}}{d^5}-546\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/1155*(105*(c + d/x^2)^(11/2)/d^4 - 385*(c + d/x^2)^(9/2)*c/d^4 + 495*(c
+ d/x^2)^(7/2)*c^2/d^4 - 231*(c + d/x^2)^(5/2)*c^3/d^4)*a - 1/15015*(1155*(
c + d/x^2)^(13/2)/d^5 - 5460*(c + d/x^2)^(11/2)*c/d^5 + 10010*(c + d/x^2)^(
9/2)*c^2/d^5 - 8580*(c + d/x^2)^(7/2)*c^3/d^5 + 3003*(c + d/x^2)^(5/2)*c^4/
d^5)*b
```

mupad [B] time = 6.81, size = 248, normalized size = 1.85

$$\frac{16\,a\,c^5\,\sqrt{c+\frac{d}{x^2}}}{1155\,d^4}-\frac{128\,b\,c^6\,\sqrt{c+\frac{d}{x^2}}}{15015\,d^5}-\frac{4\,a\,c\,\sqrt{c+\frac{d}{x^2}}}{33\,x^8}-\frac{a\,d\,\sqrt{c+\frac{d}{x^2}}}{11\,x^{10}}-\frac{14\,b\,c\,\sqrt{c+\frac{d}{x^2}}}{143\,x^{10}}-\frac{b\,d\,\sqrt{c+\frac{d}{x^2}}}{13\,x^{12}}-\frac{a\,c^2\,\sqrt{c+\frac{d}{x^2}}}{231\,d\,x^6}+\frac{2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^9,x)
```

```
[Out] (16*a*c^5*(c + d/x^2)^(1/2))/(1155*d^4) - (128*b*c^6*(c + d/x^2)^(1/2))/(15
015*d^5) - (4*a*c*(c + d/x^2)^(1/2))/(33*x^8) - (a*d*(c + d/x^2)^(1/2))/(11
*x^10) - (14*b*c*(c + d/x^2)^(1/2))/(143*x^10) - (b*d*(c + d/x^2)^(1/2))/(1
3*x^12) - (a*c^2*(c + d/x^2)^(1/2))/(231*d*x^6) + (2*a*c^3*(c + d/x^2)^(1/2
))/(385*d^2*x^4) - (8*a*c^4*(c + d/x^2)^(1/2))/(1155*d^3*x^2) - (b*c^2*(c +
 d/x^2)^(1/2))/(429*d*x^8) + (8*b*c^3*(c + d/x^2)^(1/2))/(3003*d^2*x^6) - (
16*b*c^4*(c + d/x^2)^(1/2))/(5005*d^3*x^4) + (64*b*c^5*(c + d/x^2)^(1/2))/(
15015*d^4*x^2)
```

sympy [B] time = 19.45, size = 326, normalized size = 2.43

$$-\frac{ac\left(-\frac{c^3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}+\frac{3c^2\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}-\frac{3c\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}\right)}{d^4}-\frac{a\left(\frac{c^4\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}-\frac{4c^3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}+\frac{6c^2\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7}-\frac{4c\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}}{9}+\frac{\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}}{11}\right.}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -a*c*(-c**3*(c + d/x**2)**(3/2)/3 + 3*c**2*(c + d/x**2)**(5/2)/5 - 3*c*(c +
 d/x**2)**(7/2)/7 + (c + d/x**2)**(9/2)/9)/d**4 - a*(c**4*(c + d/x**2)**(3/
2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 - 4*c*(c
 + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**4 - b*c*(c**4*(c + d/x**2
)**(3/2)/3 - 4*c**3*(c + d/x**2)**(5/2)/5 + 6*c**2*(c + d/x**2)**(7/2)/7 -
4*c*(c + d/x**2)**(9/2)/9 + (c + d/x**2)**(11/2)/11)/d**5 - b*(-c**5*(c + d
/x**2)**(3/2)/3 + c**4*(c + d/x**2)**(5/2) - 10*c**3*(c + d/x**2)**(7/2)/7
+ 10*c**2*(c + d/x**2)**(9/2)/9 - 5*c*(c + d/x**2)**(11/2)/11 + (c + d/x**2
)**(13/2)/13)/d**5
```

3.953 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx$

Optimal. Leaf size=150

$$-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{429c^3}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{14}$$

```
[Out] -16/15015*d^3*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^5/c^5+8/3003*d^2*(-8*a*d+13
*b*c)*(c+d/x^2)^(5/2)*x^7/c^4-2/429*d*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^9/c
^3+1/143*(-8*a*d+13*b*c)*(c+d/x^2)^(5/2)*x^11/c^2+1/13*a*(c+d/x^2)^(5/2)*x^
13/c
```

Rubi [A] time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{8d^2x^7\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{3003c^4}-\frac{16d^3x^5\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{15015c^5}+\frac{x^{11}\left(c+\frac{d}{x^2}\right)^{5/2}(13bc-8ad)}{143c^2}-\frac{2dx^9\left(c+\frac{d}{x^2}\right)^{5/2}}{429c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (-16*d^3*(13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^5)/(15015*c^5) + (8*d^2*(13*b
*c - 8*a*d)*(c + d/x^2)^(5/2)*x^7)/(3003*c^4) - (2*d*(13*b*c - 8*a*d)*(c +
d/x^2)^(5/2)*x^9)/(429*c^3) + ((13*b*c - 8*a*d)*(c + d/x^2)^(5/2)*x^11)/(14
3*c^2) + (a*(c + d/x^2)^(5/2)*x^13)/(13*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{12}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} + \frac{(13bc-8ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx}{13c}$$

$$= \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c} - \frac{(6d(13bc-8ad))\int \left(c+\frac{d}{x^2}\right)^{3/2}}{143c^2}$$

$$= -\frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{9}}{429c^3} + \frac{(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{143c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{13}}{13c}$$

$$= \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{7}}{3003c^4} - \frac{2d(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{9}}{429c^3} + \frac{(13bc-8ad)\left(c\right.}{143c}$$

$$= -\frac{16d^3(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{5}}{15015c^5} + \frac{8d^2(13bc-8ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^{7}}{3003c^4} - \frac{2d(13bc-8}{}$$

Mathematica [A] time = 0.11, size = 110, normalized size = 0.73

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(a\left(1155c^4x^8-840c^3dx^6+560c^2d^2x^4-320cd^3x^2+128d^4\right)+13bc\left(105c^3x^6-70c^2dx^4+40\right.\right.}{15015c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^12,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(13*b*c*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d
*x^4 + 105*c^3*x^6) + a*(128*d^4 - 320*c*d^3*x^2 + 560*c^2*d^2*x^4 - 840*c^
3*d*x^6 + 1155*c^4*x^8)))/(15015*c^5)
```

fricas [A] time = 0.69, size = 155, normalized size = 1.03

$$\frac{\left(1155\,ac^6x^{13}+105\left(13\,bc^6+14\,ac^5d\right)x^{11}+35\left(52\,bc^5d+ac^4d^2\right)x^9+5\left(13\,bc^4d^2-8\,ac^3d^3\right)x^7-6\left(13\,bc^3d^3-8\,ac\right.\right.}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*a*c^6*x^13 + 105*(13*b*c^6 + 14*a*c^5*d)*x^11 + 35*(52*b*c^5*
d + a*c^4*d^2)*x^9 + 5*(13*b*c^4*d^2 - 8*a*c^3*d^3)*x^7 - 6*(13*b*c^3*d^3 -
 8*a*c^2*d^4)*x^5 + 8*(13*b*c^2*d^4 - 8*a*c*d^5)*x^3 - 16*(13*b*c*d^5 - 8*a
*d^6)*x)*sqrt((c*x^2 + d)/x^2)/c^5
```

giac [A] time = 0.18, size = 175, normalized size = 1.17

$$\frac{16\left(13\,bcd^{\frac{11}{2}}-8\,ad^{\frac{13}{2}}\right)\mathrm{sgn}(x)}{15015\,c^5}+\frac{1155\left(cx^2+d\right)^{\frac{13}{2}}a\mathrm{sgn}(x)+1365\left(cx^2+d\right)^{\frac{11}{2}}bc\mathrm{sgn}(x)-5460\left(cx^2+d\right)^{\frac{11}{2}}ad\mathrm{sgn}(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="giac")
```

```
[Out] 16/15015*(13*b*c*d^(11/2) - 8*a*d^(13/2))*sgn(x)/c^5 + 1/15015*(1155*(c*x^2
 + d)^(13/2)*a*sgn(x) + 1365*(c*x^2 + d)^(11/2)*b*c*sgn(x) - 5460*(c*x^2 +
d)^(11/2)*a*d*sgn(x) - 5005*(c*x^2 + d)^(9/2)*b*c*d*sgn(x) + 10010*(c*x^2 +
 d)^(9/2)*a*d^2*sgn(x) + 6435*(c*x^2 + d)^(7/2)*b*c*d^2*sgn(x) - 8580*(c*x^
```

```
2 + d)^(7/2)*a*d^3*sgn(x) - 3003*(c*x^2 + d)^(5/2)*b*c*d^3*sgn(x) + 3003*(c
*x^2 + d)^(5/2)*a*d^4*sgn(x))/c^5
```

maple [A] time = 0.05, size = 115, normalized size = 0.77

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(1155a\,x^8c^4-840a\,c^3d\,x^6+1365b\,c^4x^6+560a\,c^2d^2x^4-910b\,c^3d\,x^4-320ac\,d^3x^2+520b\,c^2d^2x^2+12\right.}{15015c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x)
```

```
[Out] 1/15015*((c*x^2+d)/x^2)^(3/2)*x^3*(1155*a*c^4*x^8-840*a*c^3*d*x^6+1365*b*c^
4*x^6+560*a*c^2*d^2*x^4-910*b*c^3*d*x^4-320*a*c*d^3*x^2+520*b*c^2*d^2*x^2+1
28*a*d^4-208*b*c*d^3)*(c*x^2+d)/c^5
```

maxima [A] time = 0.63, size = 158, normalized size = 1.05

$$\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}d^2x^7-231\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5\right)b}{1155\,c^4}+\frac{\left(1155\left(c+\frac{d}{x^2}\right)^{\frac{13}{2}}x^{13}-546\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/x^2)^(9/2)*d*x^9 + 495*(c
+ d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^3*x^5)*b/c^4 + 1/15015*(11
55*(c + d/x^2)^(13/2)*x^13 - 5460*(c + d/x^2)^(11/2)*d*x^11 + 10010*(c + d/
x^2)^(9/2)*d^2*x^9 - 8580*(c + d/x^2)^(7/2)*d^3*x^7 + 3003*(c + d/x^2)^(5/2
)*d^4*x^5)*a/c^5
```

mupad [B] time = 4.66, size = 137, normalized size = 0.91

$$\sqrt{c+\frac{d}{x^2}}\left(\frac{x\,\left(128\,a\,d^6-208\,b\,c\,d^5\right)}{15015\,c^5}+\frac{x^{11}\,\left(1365\,b\,c^6+1470\,a\,d\,c^5\right)}{15015\,c^5}+\frac{a\,c\,x^{13}}{13}+\frac{d\,x^9\,\left(a\,d+52\,b\,c\right)}{429\,c}-\frac{d^2\,x^7\,\left(8\,a\right.}{300}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(128*a*d^6 - 208*b*c*d^5))/(15015*c^5) + (x^11*(1365*
b*c^6 + 1470*a*c^5*d))/(15015*c^5) + (a*c*x^13)/13 + (d*x^9*(a*d + 52*b*c))
/(429*c) - (d^2*x^7*(8*a*d - 13*b*c))/(3003*c^2) + (2*d^3*x^5*(8*a*d - 13*b
*c))/(5005*c^3) - (8*d^4*x^3*(8*a*d - 13*b*c))/(15015*c^4))
```

sympy [B] time = 12.60, size = 3351, normalized size = 22.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**12,x)
```

```
[Out] 693*a*c**12*d**(51/2)*x**22*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45
045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 4504
5*c**7*d**29*x**2 + 9009*c**6*d**30) + 3528*a*c**11*d**(53/2)*x**20*sqrt(c*
x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d
**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30
) + 7175*a*c**10*d**(55/2)*x**18*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10
 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 +
```

45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 7290*a*c**9*d**(57/2)*x**16*sqr
t(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c*
*9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d
30) + 315*a*c9*d**(35/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) + 3699*a*c**8*d**(59/2)*x**14*sqrt(c*x**2/d + 1)/(9009*c**
11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**
8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 1295*a*c**8*d**(3
7/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 756*a
*c**7*d**(61/2)*x**12*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c*
*10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7
*d**29*x**2 + 9009*c**6*d**30) + 1990*a*c**7*d**(39/2)*x**14*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 63*a*c**6*d**(63/2)*x**10*sqrt
(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**
9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d*
*30) + 1358*a*c**6*d**(41/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8
 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 +
3465*c**5*d**20) - 630*a*c**5*d**(65/2)*x**8*sqrt(c*x**2/d + 1)/(9009*c**11
*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*
d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 343*a*c**5*d**(43/2
)*x**10*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 1680*a*c
4*d(67/2)*x**6*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10
*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d*
*29*x**2 + 9009*c**6*d**30) + 35*a*c**4*d**(45/2)*x**8*sqrt(c*x**2/d + 1)/(
3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 1386
0*c**6*d**19*x**2 + 3465*c**5*d**20) - 2016*a*c**3*d**(69/2)*x**4*sqrt(c*x*
*2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**
27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30)
+ 280*a*c**3*d**(47/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) - 1152*a*c**2*d**(71/2)*x**2*sqrt(c*x**2/d + 1)/(9009*c**11*d**25
*x**10 + 45045*c**10*d**26*x**8 + 90090*c**9*d**27*x**6 + 90090*c**8*d**28*
x**4 + 45045*c**7*d**29*x**2 + 9009*c**6*d**30) + 560*a*c**2*d**(49/2)*x**4
*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c
7*d18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 256*a*c*d**(73/
2)*sqrt(c*x**2/d + 1)/(9009*c**11*d**25*x**10 + 45045*c**10*d**26*x**8 + 90
090*c**9*d**27*x**6 + 90090*c**8*d**28*x**4 + 45045*c**7*d**29*x**2 + 9009*
c**6*d**30) + 448*a*c*d**(51/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 128*a*d**(53/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x*
*8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2
+ 3465*c**5*d**20) + 315*b*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c
9*d16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6
*d**19*x**2 + 3465*c**5*d**20) + 1295*b*c**9*d**(35/2)*x**16*sqrt(c*x**2/d
+ 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4
+ 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1990*b*c**8*d**(37/2)*x**14*sq
rt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7
*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 1358*b*c**7*d**(39
/2)*x**12*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6
+ 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*b*c
7*d(21/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10
*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 343*b*c**6*d**(41/2)*x**10*
sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c*
*7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*b*c**6*d**(2
3/2)*x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 9
45*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**5*d**(43/2)*x**8*sqrt(c*x**2

```
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 114*b*c**5*d**(25/2)*x**10*
sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**
11*x**2 + 315*c**4*d**12) + 280*b*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 40*b*c**4*d**(27/2)*x**8*sqrt(c*x**2/
d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 31
5*c**4*d**12) + 560*b*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**
16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*
x**2 + 3465*c**5*d**20) - 5*b*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 448*b*c**2*d**(49/2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 138
60*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c
**5*d**20) - 30*b*c**2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**
6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*b*c*d
**(51/2)*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*b*c*
d**(33/2)*x**2*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4
 + 945*c**5*d**11*x**2 + 315*c**4*d**12) - 16*b*d**(35/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12)
```

3.954 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx$

Optimal. Leaf size=117

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

```
[Out] 8/3465*d^2*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^5/c^4-4/693*d*(-6*a*d+11*b*c)*
(c+d/x^2)^(5/2)*x^7/c^3+1/99*(-6*a*d+11*b*c)*(c+d/x^2)^(5/2)*x^9/c^2+1/11*a
*(c+d/x^2)^(5/2)*x^11/c
```

Rubi [A] time = 0.06, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {453, 271, 264}

$$\frac{8d^2x^5\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{3465c^4}+\frac{x^9\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{99c^2}-\frac{4dx^7\left(c+\frac{d}{x^2}\right)^{5/2}(11bc-6ad)}{693c^3}+\frac{ax^{11}\left(c+\frac{d}{x^2}\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (8*d^2*(11*b*c - 6*a*d)*(c + d/x^2)^(5/2)*x^5)/(3465*c^4) - (4*d*(11*b*c -
6*a*d)*(c + d/x^2)^(5/2)*x^7)/(693*c^3) + ((11*b*c - 6*a*d)*(c + d/x^2)^(5/
2)*x^9)/(99*c^2) + (a*(c + d/x^2)^(5/2)*x^11)/(11*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^{10}\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} + \frac{(11bc-6ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx}{11c}$$

$$= \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^{11}}{11c} - \frac{(4d(11bc-6ad))\int \left(c+\frac{d}{x^2}\right)^{3/}}{99c^2}$$

$$= -\frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{99c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x}{11c}$$

$$= \frac{8d^2(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{3465c^4} - \frac{4d(11bc-6ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{693c^3} + \frac{(11bc-6ad)}{9}$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.76

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(cx^2+d)^2\left(3a\left(105c^3x^6-70c^2dx^4+40cd^2x^2-16d^3\right)+11bc\left(35c^2x^4-20cdx^2+8d^2\right)\right)}{3465c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^10,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(11*b*c*(8*d^2 - 20*c*d*x^2 + 35*c^2*x^4)
+ 3*a*(-16*d^3 + 40*c*d^2*x^2 - 70*c^2*d*x^4 + 105*c^3*x^6)))/(3465*c^4)
```

fricas [A] time = 0.69, size = 132, normalized size = 1.13

$$\frac{\left(315\,ac^5x^{11}+35\left(11\,bc^5+12\,ac^4d\right)x^9+5\left(110\,bc^4d+3\,ac^3d^2\right)x^7+3\left(11\,bc^3d^2-6\,ac^2d^3\right)x^5-4\left(11\,bc^2d^3-6\,a\right.\right.}{3465\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/3465*(315*a*c^5*x^11 + 35*(11*b*c^5 + 12*a*c^4*d)*x^9 + 5*(110*b*c^4*d +
3*a*c^3*d^2)*x^7 + 3*(11*b*c^3*d^2 - 6*a*c^2*d^3)*x^5 - 4*(11*b*c^2*d^3 - 6
*a*c*d^4)*x^3 + 8*(11*b*c*d^4 - 6*a*d^5)*x)*sqrt((c*x^2 + d)/x^2)/c^4
```

giac [A] time = 0.18, size = 140, normalized size = 1.20

$$-\frac{8\left(11\,bcd^{\frac{9}{2}}-6\,ad^{\frac{11}{2}}\right)\mathrm{sgn}(x)}{3465\,c^4}+\frac{315\left(cx^2+d\right)^{\frac{11}{2}}a\mathrm{sgn}(x)+385\left(cx^2+d\right)^{\frac{9}{2}}bc\mathrm{sgn}(x)-1155\left(cx^2+d\right)^{\frac{9}{2}}ad\mathrm{sgn}(x)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="giac")
```

```
[Out] -8/3465*(11*b*c*d^(9/2) - 6*a*d^(11/2))*sgn(x)/c^4 + 1/3465*(315*(c*x^2 + d
)^(11/2)*a*sgn(x) + 385*(c*x^2 + d)^(9/2)*b*c*sgn(x) - 1155*(c*x^2 + d)^(9/
2)*a*d*sgn(x) - 990*(c*x^2 + d)^(7/2)*b*c*d*sgn(x) + 1485*(c*x^2 + d)^(7/2)
*a*d^2*sgn(x) + 693*(c*x^2 + d)^(5/2)*b*c*d^2*sgn(x) - 693*(c*x^2 + d)^(5/2
)*a*d^3*sgn(x))/c^4
```

maple [A] time = 0.05, size = 91, normalized size = 0.78

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(315a\,x^6c^3-210a\,c^2d\,x^4+385b\,c^3x^4+120ac\,d^2x^2-220b\,c^2d\,x^2-48a\,d^3+88bc\,d^2\right)\left(c\,x^2+d\right)x^3}{3465c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x)
```

```
[Out] 1/3465*((c*x^2+d)/x^2)^(3/2)*x^3*(315*a*c^3*x^6-210*a*c^2*d*x^4+385*b*c^3*x
^4+120*a*c*d^2*x^2-220*b*c^2*d*x^2-48*a*d^3+88*b*c*d^2)*(c*x^2+d)/c^4
```

maxima [A] time = 0.49, size = 124, normalized size = 1.06

$$\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)b}{315\,c^3}+\frac{\left(105\left(c+\frac{d}{x^2}\right)^{\frac{11}{2}}x^{11}-385\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}dx^9+495\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}\right.}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^
2)^(5/2)*d^2*x^5)*b/c^3 + 1/1155*(105*(c + d/x^2)^(11/2)*x^11 - 385*(c + d/
x^2)^(9/2)*d*x^9 + 495*(c + d/x^2)^(7/2)*d^2*x^7 - 231*(c + d/x^2)^(5/2)*d^
3*x^5)*a/c^4
```

mupad [B] time = 4.57, size = 118, normalized size = 1.01

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x^9\,\left(385\,b\,c^5+420\,a\,d\,c^4\right)}{3465\,c^4}-\frac{x\,\left(48\,a\,d^5-88\,b\,c\,d^4\right)}{3465\,c^4}+\frac{a\,c\,x^{11}}{11}+\frac{d\,x^7\,\left(3\,a\,d+110\,b\,c\right)}{693\,c}-\frac{d^2\,x^5\,\left(6\,a\,d-11\right.}{1155\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^9*(385*b*c^5 + 420*a*c^4*d))/(3465*c^4) - (x*(48*a*d^
5 - 88*b*c*d^4))/(3465*c^4) + (a*c*x^11)/11 + (d*x^7*(3*a*d + 110*b*c))/(69
3*c) - (d^2*x^5*(6*a*d - 11*b*c))/(1155*c^2) + (4*d^3*x^3*(6*a*d - 11*b*c))
/(3465*c^3))
```

sympy [B] time = 9.65, size = 2304, normalized size = 19.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**10,x)
```

```
[Out] 315*a*c**10*d**(33/2)*x**18*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 1386
0*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c*
*5*d**20) + 1295*a*c**9*d**(35/2)*x**16*sqrt(c*x**2/d + 1)/(3465*c**9*d**16
*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x*
*2 + 3465*c**5*d**20) + 1990*a*c**8*d**(37/2)*x**14*sqrt(c*x**2/d + 1)/(346
5*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c
**6*d**19*x**2 + 3465*c**5*d**20) + 1358*a*c**7*d**(39/2)*x**12*sqrt(c*x**2
/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x*
*4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) + 35*a*c**7*d**(21/2)*x**14*s
qrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**1
1*x**2 + 315*c**4*d**12) + 343*a*c**6*d**(41/2)*x**10*sqrt(c*x**2/d + 1)/(3
465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860
*c**6*d**19*x**2 + 3465*c**5*d**20) + 110*a*c**6*d**(23/2)*x**12*sqrt(c*x**
2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 +
315*c**4*d**12) + 35*a*c**5*d**(43/2)*x**8*sqrt(c*x**2/d + 1)/(3465*c**9*d*
*16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19
*x**2 + 3465*c**5*d**20) + 114*a*c**5*d**(25/2)*x**10*sqrt(c*x**2/d + 1)/(3
15*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d*
```

```
*12) + 280*a*c**4*d**(45/2)*x**6*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 +
 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 34
65*c**5*d**20) + 40*a*c**4*d**(27/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9
*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 560*a
*c**3*d**(47/2)*x**4*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*
d**17*x**6 + 20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**2
0) - 5*a*c**3*d**(29/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 448*a*c**2*d**(49/
2)*x**2*sqrt(c*x**2/d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 +
20790*c**7*d**18*x**4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 30*a*c**
2*d**(31/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x*
*4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 128*a*c*d**(51/2)*sqrt(c*x**2/
d + 1)/(3465*c**9*d**16*x**8 + 13860*c**8*d**17*x**6 + 20790*c**7*d**18*x**
4 + 13860*c**6*d**19*x**2 + 3465*c**5*d**20) - 40*a*c*d**(33/2)*x**2*sqrt(c
*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**
2 + 315*c**4*d**12) - 16*a*d**(35/2)*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 35*b*c**8*
d**(19/2)*x**14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**
4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*b*c**7*d**(21/2)*x**12*sqrt
(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x
**2 + 315*c**4*d**12) + 114*b*c**6*d**(23/2)*x**10*sqrt(c*x**2/d + 1)/(315*
c**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12
) + 40*b*c**5*d**(25/2)*x**8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6 + 945*c
**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*b*c**5*d**(11/2
)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) - 5*b*c**4*d**(27/2)*x**6*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x**6
+ 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 33*b*c**4*d
**(13/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 +
 105*c**3*d**6) - 30*b*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(315*c**7*d**
9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 17*b
*c**3*d**(15/2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5
*x**2 + 105*c**3*d**6) - 40*b*c**2*d**(31/2)*x**2*sqrt(c*x**2/d + 1)/(315*c
**7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12)
 + 3*b*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**
4*d**5*x**2 + 105*c**3*d**6) - 16*b*c*d**(33/2)*sqrt(c*x**2/d + 1)/(315*c**
7*d**9*x**6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) +
 12*b*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d*
*5*x**2 + 105*c**3*d**6) + 8*b*d**(21/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*
x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6)
```

3.955 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}x^8\,dx$

Optimal. Leaf size=84

$$-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{315c^3}+\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{63c^2}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

```
[Out] -2/315*d*(-4*a*d+9*b*c)*(c+d/x^2)^(5/2)*x^5/c^3+1/63*(-4*a*d+9*b*c)*(c+d/x^
2)^(5/2)*x^7/c^2+1/9*a*(c+d/x^2)^(5/2)*x^9/c
```

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 264}

$$\frac{x^7\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{63c^2}-\frac{2dx^5\left(c+\frac{d}{x^2}\right)^{5/2}(9bc-4ad)}{315c^3}+\frac{ax^9\left(c+\frac{d}{x^2}\right)^{5/2}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (-2*d*(9*b*c - 4*a*d)*(c + d/x^2)^(5/2)*x^5)/(315*c^3) + ((9*b*c - 4*a*d)*(
c + d/x^2)^(5/2)*x^7)/(63*c^2) + (a*(c + d/x^2)^(5/2)*x^9)/(9*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^8\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c} + \frac{(9bc-4ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx}{9c}$$

$$= \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c} - \frac{(2d(9bc-4ad))\int \left(c+\frac{d}{x^2}\right)^{3/2} x^4}{63c^2}$$

$$= -\frac{2d(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{315c^3} + \frac{(9bc-4ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{63c^2} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^9}{9c}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.79

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(cx^2+d)^2\left(a\left(35c^2x^4-20cdx^2+8d^2\right)+9bc\left(5cx^2-2d\right)\right)}{315c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^8,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(9*b*c*(-2*d + 5*c*x^2) + a*(8*d^2 - 20*c*
d*x^2 + 35*c^2*x^4)))/(315*c^3)
```

fricas [A] time = 0.54, size = 106, normalized size = 1.26

$$\frac{\left(35\,ac^4x^9+5\left(9\,bc^4+10\,ac^3d\right)x^7+3\left(24\,bc^3d+ac^2d^2\right)x^5+\left(9\,bc^2d^2-4\,acd^3\right)x^3-2\left(9\,bcd^3-4\,ad^4\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/315*(35*a*c^4*x^9 + 5*(9*b*c^4 + 10*a*c^3*d)*x^7 + 3*(24*b*c^3*d + a*c^2*
d^2)*x^5 + (9*b*c^2*d^2 - 4*a*c*d^3)*x^3 - 2*(9*b*c*d^3 - 4*a*d^4)*x)*sqrt(
(c*x^2 + d)/x^2)/c^3
```

giac [A] time = 0.17, size = 105, normalized size = 1.25

$$\frac{2\left(9\,bcd^{\frac{7}{2}}-4\,ad^{\frac{9}{2}}\right)\mathrm{sgn}(x)}{315\,c^3}+\frac{35\left(cx^2+d\right)^{\frac{9}{2}}a\mathrm{sgn}(x)+45\left(cx^2+d\right)^{\frac{7}{2}}bc\mathrm{sgn}(x)-90\left(cx^2+d\right)^{\frac{7}{2}}ad\mathrm{sgn}(x)-63\left(cx^2\right.}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 2/315*(9*b*c*d^(7/2) - 4*a*d^(9/2))*sgn(x)/c^3 + 1/315*(35*(c*x^2 + d)^(9/2
)*a*sgn(x) + 45*(c*x^2 + d)^(7/2)*b*c*sgn(x) - 90*(c*x^2 + d)^(7/2)*a*d*sgn
(x) - 63*(c*x^2 + d)^(5/2)*b*c*d*sgn(x) + 63*(c*x^2 + d)^(5/2)*a*d^2*sgn(x)
)/c^3
```

maple [A] time = 0.06, size = 67, normalized size = 0.80

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(35a\,x^4c^2-20acd\,x^2+45b\,c^2x^2+8a\,d^2-18bcd\right)\left(c\,x^2+d\right)x^3}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x)
```

```
[Out] 1/315*((c*x^2+d)/x^2)^(3/2)*x^3*(35*a*c^2*x^4-20*a*c*d*x^2+45*b*c^2*x^2+8*a
*d^2-18*b*c*d)*(c*x^2+d)/c^3
```

maxima [A] time = 0.76, size = 90, normalized size = 1.07

$$\frac{\left(5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}x^7-7\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}dx^5\right)b}{35\,c^2}+\frac{\left(35\left(c+\frac{d}{x^2}\right)^{\frac{9}{2}}x^9-90\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}dx^7+63\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^2x^5\right)a}{315\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^2)^(5/2)*d*x^5)*b/c^2 + 1/315*(3
5*(c + d/x^2)^(9/2)*x^9 - 90*(c + d/x^2)^(7/2)*d*x^7 + 63*(c + d/x^2)^(5/2)
*d^2*x^5)*a/c^3
```

mupad [B] time = 4.55, size = 97, normalized size = 1.15

$$\sqrt{c+\frac{d}{x^2}}\,\left(\frac{x\,\left(8\,a\,d^4-18\,b\,c\,d^3\right)}{315\,c^3}+\frac{x^7\,\left(45\,b\,c^4+50\,a\,d\,c^3\right)}{315\,c^3}+\frac{a\,c\,x^9}{9}+\frac{d\,x^5\,\left(a\,d+24\,b\,c\right)}{105\,c}-\frac{d^2\,x^3\,\left(4\,a\,d-9\,b\,c\right)}{315\,c^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x*(8*a*d^4 - 18*b*c*d^3))/(315*c^3) + (x^7*(45*b*c^4 +
50*a*c^3*d))/(315*c^3) + (a*c*x^9)/9 + (d*x^5*(a*d + 24*b*c))/(105*c) - (d^
2*x^3*(4*a*d - 9*b*c))/(315*c^2))
```

sympy [B] time = 7.64, size = 1340, normalized size = 15.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x***8,x)
```

```
[Out] 35*a*c**8*d**(19/2)*x***14*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x***6 + 945*c**6
*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 110*a*c**7*d**(21/2)*
x**12*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x***6 + 945*c**6*d**10*x**4 + 945*c*
*5*d**11*x**2 + 315*c**4*d**12) + 114*a*c**6*d**(23/2)*x***10*sqrt(c*x**2/d
+ 1)/(315*c**7*d**9*x***6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*
c**4*d**12) + 40*a*c**5*d**(25/2)*x***8*sqrt(c*x**2/d + 1)/(315*c**7*d**9*x*
*6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 15*a*c**
5*d**(11/2)*x***10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x*
*2 + 105*c**3*d**6) - 5*a*c**4*d**(27/2)*x***6*sqrt(c*x**2/d + 1)/(315*c**7*
d**9*x***6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**12) + 3
3*a*c**4*d**(13/2)*x***8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d
**5*x**2 + 105*c**3*d**6) - 30*a*c**3*d**(29/2)*x**4*sqrt(c*x**2/d + 1)/(31
5*c**7*d**9*x***6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**4*d**
12) + 17*a*c**3*d**(15/2)*x***6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210
*c**4*d**5*x**2 + 105*c**3*d**6) - 40*a*c**2*d**(31/2)*x**2*sqrt(c*x**2/d +
 1)/(315*c**7*d**9*x***6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c
**4*d**12) + 3*a*c**2*d**(17/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
 + 210*c**4*d**5*x**2 + 105*c**3*d**6) - 16*a*c*d**(33/2)*sqrt(c*x**2/d + 1
)/(315*c**7*d**9*x***6 + 945*c**6*d**10*x**4 + 945*c**5*d**11*x**2 + 315*c**
4*d**12) + 12*a*c*d**(19/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 2
10*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*d**(21/2)*sqrt(c*x**2/d + 1)/(105*
c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 15*b*c**6*d**(9/2)*x
```

```
**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3
*d**6) + 33*b*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 +
210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*b*c**4*d**(13/2)*x**6*sqrt(c*x**2/
d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 3*b*c**3
*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2
 + 105*c**3*d**6) + 12*b*c**2*d**(17/2)*x**2*sqrt(c*x**2/d + 1)/(105*c**5*d
**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*b*c*d**(19/2)*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + b*d**(
3/2)*x**4*sqrt(c*x**2/d + 1)/5 + b*d**(5/2)*x**2*sqrt(c*x**2/d + 1)/(15*c)
- 2*b*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2)
```

3.956 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx$

Optimal. Leaf size=53

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2}(7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

```
[Out] 1/35*(-2*a*d+7*b*c)*(c+d/x^2)^(5/2)*x^5/c^2+1/7*a*(c+d/x^2)^(5/2)*x^7/c
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 264}

$$\frac{x^5\left(c+\frac{d}{x^2}\right)^{5/2}(7bc-2ad)}{35c^2}+\frac{ax^7\left(c+\frac{d}{x^2}\right)^{5/2}}{7c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] ((7*b*c - 2*a*d)*(c + d/x^2)^(5/2)*x^5)/(35*c^2) + (a*(c + d/x^2)^(5/2)*x^7
)/(7*c)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^6\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}+\frac{(7bc-2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx}{7c}\\ &= \frac{(7bc-2ad)\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{35c^2}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^7}{7c}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.83

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(5acx^2-2ad+7bc\right)}{35c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^6,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(d + c*x^2)^2*(7*b*c - 2*a*d + 5*a*c*x^2))/(35*c^2)
```

fricas [A] time = 0.51, size = 80, normalized size = 1.51

$$\frac{\left(5\,ac^{3}x^{7}+\left(7\,bc^{3}+8\,ac^{2}d\right)x^{5}+\left(14\,bc^{2}d+acd^{2}\right)x^{3}+\left(7\,bcd^{2}-2\,ad^{3}\right)x\right)\sqrt{\frac{cx^{2}+d}{x^{2}}}}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*a*c^3*x^7 + (7*b*c^3 + 8*a*c^2*d)*x^5 + (14*b*c^2*d + a*c*d^2)*x^3
+ (7*b*c*d^2 - 2*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

giac [A] time = 0.18, size = 72, normalized size = 1.36

$$-\frac{\left(7\,bcd^{\frac{5}{2}}-2\,ad^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{35\,c^{2}}+\frac{5\left(cx^{2}+d\right)^{\frac{7}{2}}a\mathrm{sgn}(x)+7\left(cx^{2}+d\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)-7\left(cx^{2}+d\right)^{\frac{5}{2}}ad\mathrm{sgn}(x)}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] -1/35*(7*b*c*d^(5/2) - 2*a*d^(7/2))*sgn(x)/c^2 + 1/35*(5*(c*x^2 + d)^(7/2)*
a*sgn(x) + 7*(c*x^2 + d)^(5/2)*b*c*sgn(x) - 7*(c*x^2 + d)^(5/2)*a*d*sgn(x))
/c^2
```

maple [A] time = 0.04, size = 45, normalized size = 0.85

$$\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(5a\,x^{2}c-2ad+7bc\right)\left(c\,x^{2}+d\right)x^{3}}{35c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x)
```

```
[Out] 1/35*((c*x^2+d)/x^2)^(3/2)*x^3*(5*a*c*x^2-2*a*d+7*b*c)*(c*x^2+d)/c^2
```

maxima [A] time = 0.65, size = 55, normalized size = 1.04

$$\frac{b\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}x^{5}}{5\,c}+\frac{\left(5\left(c+\frac{d}{x^{2}}\right)^{\frac{7}{2}}x^{7}-7\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}dx^{5}\right)a}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b*(c + d/x^2)^(5/2)*x^5/c + 1/35*(5*(c + d/x^2)^(7/2)*x^7 - 7*(c + d/x^
2)^(5/2)*d*x^5)*a/c^2
```

mupad [B] time = 4.63, size = 77, normalized size = 1.45

$$\sqrt{c+\frac{d}{x^{2}}}\,\left(\frac{x^{5}\,\left(7\,b\,c^{3}+8\,a\,d\,c^{2}\right)}{35\,c^{2}}-\frac{x\,\left(2\,a\,d^{3}-7\,b\,c\,d^{2}\right)}{35\,c^{2}}+\frac{a\,c\,x^{7}}{7}+\frac{d\,x^{3}\,\left(a\,d+14\,b\,c\right)}{35\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (c + d/x^2)^(1/2)*((x^5*(7*b*c^3 + 8*a*c^2*d))/(35*c^2) - (x*(2*a*d^3 - 7*b
*c*d^2))/(35*c^2) + (a*c*x^7)/7 + (d*x^3*(a*d + 14*b*c))/(35*c))
```

sympy [B] time = 5.95, size = 498, normalized size = 9.40

$$\frac{15ac^6d^{\frac{9}{2}}x^{10}\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{33ac^5d^{\frac{11}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{17ac^4d^{\frac{13}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{105c^5d^4x^4+210c^4d^5x^2+105c^3d^6}+\frac{}{105c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**6,x)
```

```
[Out] 15*a*c**6*d**(9/2)*x**10*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*
d**5*x**2 + 105*c**3*d**6) + 33*a*c**5*d**(11/2)*x**8*sqrt(c*x**2/d + 1)/(1
05*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 17*a*c**4*d**(13/
2)*x**6*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c
**3*d**6) + 3*a*c**3*d**(15/2)*x**4*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4
+ 210*c**4*d**5*x**2 + 105*c**3*d**6) + 12*a*c**2*d**(17/2)*x**2*sqrt(c*x**
2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105*c**3*d**6) + 8*a*c*
d**(19/2)*sqrt(c*x**2/d + 1)/(105*c**5*d**4*x**4 + 210*c**4*d**5*x**2 + 105
*c**3*d**6) + a*d**(3/2)*x**4*sqrt(c*x**2/d + 1)/5 + a*d**(5/2)*x**2*sqrt(c
*x**2/d + 1)/(15*c) - 2*a*d**(7/2)*sqrt(c*x**2/d + 1)/(15*c**2) + b*c*sqrt(
d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*b*d**(3/2)*x**2*sqrt(c*x**2/d + 1)/5 + b*d
**(5/2)*sqrt(c*x**2/d + 1)/(5*c)
```

3.957 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx$

Optimal. Leaf size=86

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+bdx\sqrt{c+\frac{d}{x^2}}+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}$$

```
[Out] 1/3*b*(c+d/x^2)^(3/2)*x^3+1/5*a*(c+d/x^2)^(5/2)*x^5/c-b*d^(3/2)*arctanh(d^(
1/2)/x/(c+d/x^2)^(1/2))+b*d*x*(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {451, 335, 277, 217, 206}

$$\frac{ax^5\left(c+\frac{d}{x^2}\right)^{5/2}}{5c}-bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{1}{3}bx^3\left(c+\frac{d}{x^2}\right)^{3/2}+bdx\sqrt{c+\frac{d}{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] b*d*Sqrt[c + d/x^2]*x + (b*(c + d/x^2)^(3/2)*x^3)/3 + (a*(c + d/x^2)^(5/2)*
x^5)/(5*c) - b*d^(3/2)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rubi steps

$$
\begin{aligned}
\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^4\,dx &= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} + b\int \left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx \\
&= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - b\,\text{Subst}\left(\int \frac{\left(c+dx^2\right)^{3/2}}{x^4}\,dx, x, \frac{1}{x}\right) \\
&= \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - (bd)\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\right. \\
&= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - \left(bd^2\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx\right. \\
&= bd\sqrt{c+\frac{d}{x^2}}\,x + \frac{1}{3}b\left(c+\frac{d}{x^2}\right)^{3/2} x^3 + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2} x^5}{5c} - bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 81, normalized size = 0.94

$$
\frac{1}{15}x\sqrt{c+\frac{d}{x^2}}\left(\frac{3a\left(cx^2+d\right)^2}{c} - \frac{15bd^{3/2}\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{\sqrt{cx^2+d}} + 5b\left(cx^2+4d\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*x*((3*a*(d + c*x^2)^2)/c + 5*b*(4*d + c*x^2) - (15*b*d^(3/
2)*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]])/Sqrt[d + c*x^2]))/15
```

fricas [A] time = 0.79, size = 203, normalized size = 2.36

$$
\left[\frac{15\,bcd^{\frac{3}{2}}\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right) + 2\left(3\,ac^2x^5 + \left(5\,bc^2+6\,acd\right)x^3 + \left(20\,bcd+3\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{30\,c}, \frac{15\,bc\sqrt{-d}\,d\,\text{arc}\ldots}{\ldots}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] [1/30*(15*b*c*d^(3/2)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d
)/x^2) + 2*(3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20*b*c*d + 3*a*d^2)*x)
*sqrt((c*x^2 + d)/x^2))/c, 1/15*(15*b*c*sqrt(-d)*d*arctan(sqrt(-d)*x*sqrt((
```

```
c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*a*c^2*x^5 + (5*b*c^2 + 6*a*c*d)*x^3 + (20
*b*c*d + 3*a*d^2)*x)*sqrt((c*x^2 + d)/x^2))/c]
```

giac [A] time = 0.20, size = 140, normalized size = 1.63

$$\frac{bd^2 \arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)\mathrm{sgn}(x)}{\sqrt{-d}} - \frac{\left(15\,bcd^2 \arctan\left(\frac{\sqrt{d}}{\sqrt{-d}}\right) + 20\,bc\sqrt{-d}\,d^{\frac{3}{2}} + 3\,a\sqrt{-d}\,d^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15\,c\sqrt{-d}} + \frac{3\left(cx^2+d\right)^{\frac{5}{2}}ac^4\mathrm{sgr}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] b*d^2*arctan(sqrt(c*x^2 + d)/sqrt(-d))*sgn(x)/sqrt(-d) - 1/15*(15*b*c*d^2*a
rctan(sqrt(d)/sqrt(-d)) + 20*b*c*sqrt(-d)*d^(3/2) + 3*a*sqrt(-d)*d^(5/2))*s
gn(x)/(c*sqrt(-d)) + 1/15*(3*(c*x^2 + d)^(5/2)*a*c^4*sgn(x) + 5*(c*x^2 + d)
^(3/2)*b*c^5*sgn(x) + 15*sqrt(c*x^2 + d)*b*c^5*d*sgn(x))/c^5
```

maple [A] time = 0.06, size = 99, normalized size = 1.15

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(-15bc\,d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+15\sqrt{c\,x^2+d}\;bcd+5\left(c\,x^2+d\right)^{\frac{3}{2}}bc+3\left(c\,x^2+d\right)^{\frac{5}{2}}a\right)x^3}{15\left(c\,x^2+d\right)^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x)
```

```
[Out] 1/15*((c*x^2+d)/x^2)^(3/2)*x^3*(3*a*(c*x^2+d)^(5/2)+5*(c*x^2+d)^(3/2)*b*c-1
5*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c+15*(c*x^2+d)^(1/2)*b*c*d)
/(c*x^2+d)^(3/2)/c
```

maxima [A] time = 1.35, size = 91, normalized size = 1.06

$$\frac{a\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5}{5\,c}+\frac{1}{6}\left(2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3+6\sqrt{c+\frac{d}{x^2}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/5*a*(c + d/x^2)^(5/2)*x^5/c + 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d
/x^2)*d*x + 3*d^(3/2)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
+ sqrt(d))))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

sympy [B] time = 5.25, size = 184, normalized size = 2.14

$$\frac{ac\sqrt{d}\,x^4\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{2ad^{\frac{3}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{5}+\frac{ad^{\frac{5}{2}}\sqrt{\frac{cx^2}{d}+1}}{5c}+\frac{b\sqrt{c}\,dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{bc\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{bd^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-bd^{\frac{3}{2}}\,\mathrm{asinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**4,x)
```

```
[Out] a*c*sqrt(d)*x**4*sqrt(c*x**2/d + 1)/5 + 2*a*d**(3/2)*x**2*sqrt(c*x**2/d + 1
)/5 + a*d**(5/2)*sqrt(c*x**2/d + 1)/(5*c) + b*sqrt(c)*d*x/sqrt(1 + d/(c*x**
2)) + b*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3 + b*d**(3/2)*sqrt(c*x**2/d + 1)
/3 - b*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) + b*d**2/(sqrt(c)*x*sqrt(1 + d/(
c*x**2)))
```

3.958 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx$

Optimal. Leaf size=121

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}\,(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

```
[Out] 1/3*(2*a*d+3*b*c)*(c+d/x^2)^(3/2)*x/c+1/3*a*(c+d/x^2)^(5/2)*x^3/c-1/2*(2*a*
d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))*d^(1/2)-1/2*d*(2*a*d+3*b*c)*(c+
d/x^2)^(1/2)/c/x
```

Rubi [A] time = 0.06, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {453, 242, 277, 195, 217, 206}

$$\frac{x\left(c+\frac{d}{x^2}\right)^{3/2}(2ad+3bc)}{3c}-\frac{d\sqrt{c+\frac{d}{x^2}}\,(2ad+3bc)}{2cx}-\frac{1}{2}\sqrt{d}\,(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)+\frac{ax^3\left(c+\frac{d}{x^2}\right)^{5/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] -(d*(3*b*c + 2*a*d)*Sqrt[c + d/x^2])/(2*c*x) + ((3*b*c + 2*a*d)*(c + d/x^2)
^(3/2)*x)/(3*c) + (a*(c + d/x^2)^(5/2)*x^3)/(3*c) - (Sqrt[d]*(3*b*c + 2*a*d
)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
```

```
nomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2} x^2\,dx = \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} + \frac{(3bc+2ad)\int \left(c+\frac{d}{x^2}\right)^{3/2}dx}{3c}$$

$$= \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} - \frac{(3bc+2ad)\,\mathrm{Subst}\left(\int \frac{\left(c+dx^2\right)^{3/2}}{x^2}dx, x, \frac{1}{x}\right)}{3c}$$

$$= \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} - \frac{(d(3bc+2ad))\,\mathrm{Subst}\left(\int \sqrt{c+dx^2}\right.}{c}$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} - \frac{1}{2}(d(3bc$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} - \frac{1}{2}(d(3bc$$

$$= -\frac{d(3bc+2ad)\sqrt{c+\frac{d}{x^2}}}{2cx} + \frac{(3bc+2ad)\left(c+\frac{d}{x^2}\right)^{3/2}x}{3c} + \frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x^3}{3c} - \frac{1}{2}\sqrt{d}\,(3bc$$

Mathematica [A] time = 0.09, size = 105, normalized size = 0.87

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(\sqrt{cx^2+d}\left(2acx^4+8adx^2+6bcx^2-3bd\right)-3\sqrt{d}\,x^2(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)\right)}{6x\sqrt{cx^2+d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2)*x^2,x]
```

```
[Out] (Sqrt[c + d/x^2]*(Sqrt[d + c*x^2]*(-3*b*d + 6*b*c*x^2 + 8*a*d*x^2 + 2*a*c*x
^4) - 3*Sqrt[d]*(3*b*c + 2*a*d)*x^2*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]))/(6*x
*Sqrt[d + c*x^2])
```

fricas [A] time = 0.66, size = 190, normalized size = 1.57

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,acx^4+2\,(3\,bc+4\,ad)x^2-3\,bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{12\,x}, \frac{3\,(3\,bc+2\,ad)\sqrt{-d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(3*b*c + 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 +
d)/x^2) + 2*d)/x^2) + 2*(2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d)*sqrt((c
*x^2 + d)/x^2))/x, 1/6*(3*(3*b*c + 2*a*d)*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt
((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c*x^4 + 2*(3*b*c + 4*a*d)*x^2 - 3*b*d
)*sqrt((c*x^2 + d)/x^2))/x]
```

giac [A] time = 0.27, size = 115, normalized size = 0.95

$$\frac{2\left(cx^{2}+d\right)^{\frac{3}{2}}ac\operatorname{sgn}(x)+6\sqrt{cx^{2}+d}\,bc^{2}\operatorname{sgn}(x)+6\sqrt{cx^{2}+d}\,acd\operatorname{sgn}(x)-\frac{3\sqrt{cx^{2}+d}\,bcd\operatorname{sgn}(x)}{x^{2}}+\frac{3\left(3\,bc^{2}d\operatorname{sgn}(x)+2\,acd^{2}\operatorname{sg}\right.}{\sqrt{-}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/6*(2*(c*x^2 + d)^(3/2)*a*c*sgn(x) + 6*sqrt(c*x^2 + d)*b*c^2*sgn(x) + 6*sq
rt(c*x^2 + d)*a*c*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c*d*sgn(x)/x^2 + 3*(3*b*c^
2*d*sgn(x) + 2*a*c*d^2*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d))/c
```

maple [A] time = 0.06, size = 170, normalized size = 1.40

$$-\frac{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}\left(6a\,d^{\frac{5}{2}}x^{2}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)+9bc\,d^{\frac{3}{2}}x^{2}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)-6\sqrt{c\,x^{2}+d}\,a\,d^{2}x^{2}-9\sqrt{c\,x^{2}+d}\,bcd\,x\right.}{6\left(c\,x^{2}+d\right)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x)
```

```
[Out] -1/6*((c*x^2+d)/x^2)^(3/2)*x*(-2*(c*x^2+d)^(3/2)*a*d*x^2-3*(c*x^2+d)^(3/2)*
b*c*x^2+6*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*a+9*d^(3/2)*ln(2*
(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c+3*(c*x^2+d)^(5/2)*b-6*(c*x^2+d)^(1/2
)*x^2*a*d^2-9*(c*x^2+d)^(1/2)*x^2*b*c*d)/(c*x^2+d)^(3/2)/d
```

maxima [A] time = 1.36, size = 163, normalized size = 1.35

$$\frac{1}{6}\left(2\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}x^{3}+6\sqrt{c+\frac{d}{x^{2}}}\,dx+3\,d^{\frac{3}{2}}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)\right)a+\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^{2}}}\,cx-\frac{2\sqrt{c+\frac{d}{x^{2}}}\,cdx}{\left(c+\frac{d}{x^{2}}\right)x^{2}-d}+3\,c\sqrt{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(c + d/x^2)^(3/2)*x^3 + 6*sqrt(c + d/x^2)*d*x + 3*d^(3/2)*log((sqrt(
c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*a + 1/4*(4*sqrt(c +
 d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d) + 3*c*sqrt(d)*l
og((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{2}\left(a+\frac{b}{x^{2}}\right)\left(c+\frac{d}{x^{2}}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b/x^2)*(c + d/x^2)^(3/2), x)
```

sympy [A] time = 7.65, size = 202, normalized size = 1.67

$$\frac{a\sqrt{c}\,dx}{\sqrt{1+\frac{d}{cx^2}}}+\frac{ac\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3}+\frac{ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3}-ad^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)+\frac{ad^2}{\sqrt{c}\,x\sqrt{1+\frac{d}{cx^2}}}+\frac{bc^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{b\sqrt{c}\,d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)*x**2,x)
```

```
[Out] a*sqrt(c)*d*x/sqrt(1 + d/(c*x**2)) + a*c*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/3
+ a*d**(3/2)*sqrt(c*x**2/d + 1)/3 - a*d**(3/2)*asinh(sqrt(d)/(sqrt(c)*x)) +
 a*d**2/(sqrt(c)*x*sqrt(1 + d/(c*x**2))) + b*c**(3/2)*x/sqrt(1 + d/(c*x**2)
) - b*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x) + b*sqrt(c)*d/(x*sqrt(1 + d/(c*x
**2))) - 3*b*c*sqrt(d)*asinh(sqrt(d)/(sqrt(c)*x))/2
```

3.959 $\int \left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}\,(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

```
[Out] -1/4*(4*a*d+b*c)*(c+d/x^2)^(3/2)/c/x+a*(c+d/x^2)^(5/2)*x/c-3/8*c*(4*a*d+b*c
)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)-3/8*(4*a*d+b*c)*(c+d/x^2)^(1/2
)/x
```

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {375, 453, 195, 217, 206}

$$-\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(4ad+bc)}{4cx}-\frac{3\sqrt{c+\frac{d}{x^2}}\,(4ad+bc)}{8x}-\frac{3c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8\sqrt{d}}+\frac{ax\left(c+\frac{d}{x^2}\right)^{5/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (-3*(b*c + 4*a*d)*Sqrt[c + d/x^2])/(8*x) - ((b*c + 4*a*d)*(c + d/x^2)^(3/2)
)/(4*c*x) + (a*(c + d/x^2)^(5/2)*x)/c - (3*c*(b*c + 4*a*d)*ArcTanh[Sqrt[d]/
(Sqrt[c + d/x^2]*x)])/(8*Sqrt[d])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
```

```
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}dx=-\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)\left(c+dx^2\right)^{3/2}}{x^2}dx,x,\frac{1}{x}\right)$$

$$=\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}+\frac{(-bc-4ad)\operatorname{Subst}\left(\int\left(c+dx^2\right)^{3/2}dx,x,\frac{1}{x}\right)}{c}$$

$$=-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{4}(3(bc+4ad))\operatorname{Subst}\left(\int\sqrt{c+dx^2}\,dx,x\right.$$

$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{8}(3c(bc+4ad))$$

$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{1}{8}(3c(bc+4ad))$$

$$=-\frac{3(bc+4ad)\sqrt{c+\frac{d}{x^2}}}{8x}-\frac{(bc+4ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{4cx}+\frac{a\left(c+\frac{d}{x^2}\right)^{5/2}x}{c}-\frac{3c(bc+4ad)\tan}{8\sqrt{}}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.61

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(cx^4(4ad+bc)\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^2\right)}{20d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)*(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^2 + c*(b*c + 4*a*d)*x^4*Hypergeometr
ic2F1[2, 5/2, 7/2, 1 + (c*x^2)/d]))/(20*d^3*x^3)
```

fricas [A] time = 0.81, size = 216, normalized size = 1.93

$$\left[\frac{3\left(bc^2+4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(8\,acdx^4-2\,bd^2-\left(5\,bcd+4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,dx^3},\ \frac{3\left(bc^2+4\,acd\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b*c^2 + 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^
2 + d)/x^2) + 2*d)/x^2) + 2*(8*a*c*d*x^4 - 2*b*d^2 - (5*b*c*d + 4*a*d^2)*x^
```

```
2)*sqrt((c*x^2 + d)/x^2))/(d*x^3), 1/8*(3*(b*c^2 + 4*a*c*d)*sqrt(-d)*x^3*ar
ctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (8*a*c*d*x^4 - 2*b*d^2
 - (5*b*c*d + 4*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(d*x^3)]
```

giac [A] time = 0.26, size = 145, normalized size = 1.29

$$\frac{8\sqrt{cx^2+d}\,ac^2\mathrm{sgn}(x) + \frac{3\left(bc^3\mathrm{sgn}(x)+4\,ac^2d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}} - \frac{5\left(cx^2+d\right)^{\frac{3}{2}}bc^3\mathrm{sgn}(x)+4\left(cx^2+d\right)^{\frac{3}{2}}ac^2d\mathrm{sgn}(x)-3\sqrt{cx^2+d}\,bc^3d\mathrm{sgn}}{c^2x^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(c*x^2 + d)*a*c^2*sgn(x) + 3*(b*c^3*sgn(x) + 4*a*c^2*d*sgn(x))*a
rctan(sqrt(c*x^2 + d)/sqrt(-d))/sqrt(-d) - (5*(c*x^2 + d)^(3/2)*b*c^3*sgn(x
) + 4*(c*x^2 + d)^(3/2)*a*c^2*d*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^3*d*sgn(x) -
 4*sqrt(c*x^2 + d)*a*c^2*d^2*sgn(x))/(c^2*x^4))/c
```

maple [B] time = 0.06, size = 213, normalized size = 1.90

$$-\frac{\left(\frac{cx^2+d}{x^2}\right)^{\frac{3}{2}}\left(12ac\,d^{\frac{5}{2}}x^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)+3bc^2d^{\frac{3}{2}}x^4\ln\left(\frac{2d+2\sqrt{cx^2+d}\,\sqrt{d}}{x}\right)-12\sqrt{cx^2+d}\,acd^2x^4-3\sqrt{cx^2+d}\right.}{8\left(cx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2),x)
```

```
[Out] -1/8*((c*x^2+d)/x^2)^(3/2)/x*(12*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x
)*x^4*a*c+3*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2-4*(c*x^2+
d)^(3/2)*a*c*d*x^4-(c*x^2+d)^(3/2)*b*c^2*x^4+4*(c*x^2+d)^(5/2)*x^2*a*d+(c*x
^2+d)^(5/2)*x^2*b*c-12*(c*x^2+d)^(1/2)*x^4*a*c*d^2-3*(c*x^2+d)^(1/2)*x^4*b*
c^2*d+2*(c*x^2+d)^(5/2)*b*d)/(c*x^2+d)^(3/2)/d^2
```

maxima [B] time = 1.42, size = 207, normalized size = 1.85

$$\frac{1}{4}\left(4\sqrt{c+\frac{d}{x^2}}\,cx - \frac{2\sqrt{c+\frac{d}{x^2}}\,cdx}{\left(c+\frac{d}{x^2}\right)x^2-d} + 3c\sqrt{d}\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)\right)a + \frac{1}{16}\left(\frac{3c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}} - \frac{2\left(5\left(c+\frac{d}{x}\right.\right.}{\left(c+\frac{d}{x^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*sqrt(c + d/x^2)*c*x - 2*sqrt(c + d/x^2)*c*d*x/((c + d/x^2)*x^2 - d)
+ 3*c*sqrt(d)*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d
))))*a + 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x +
 sqrt(d)))/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2
*d*x)/((c + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*b
```

mupad [B] time = 5.86, size = 78, normalized size = 0.70

$$\frac{a\,x\left(c\,x^2+d\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{d}{c\,x^2}\right)}{\left(\frac{d}{c}+x^2\right)^{3/2}} - \frac{b\left(c\,x^2+d\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{d}{c\,x^2}\right)}{x\left(\frac{d}{c}+x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)*(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x*(d + c*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -d/(c*x^2)))/(d/c + x^2
)^(3/2) - (b*(d + c*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -d/(c*x^2)))/(x*
(d/c + x^2)^(3/2))
```

sympy [B] time = 11.78, size = 216, normalized size = 1.93

$$\frac{ac^{\frac{3}{2}}x}{\sqrt{1+\frac{d}{cx^2}}}-\frac{a\sqrt{c}\,d\sqrt{1+\frac{d}{cx^2}}}{2x}+\frac{a\sqrt{c}\,d}{x\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac\sqrt{d}\,\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2}-\frac{bc^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{bc^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3b\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\,\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2),x)
```

```
[Out] a*c**(3/2)*x/sqrt(1 + d/(c*x**2)) - a*sqrt(c)*d*sqrt(1 + d/(c*x**2))/(2*x)
+ a*sqrt(c)*d/(x*sqrt(1 + d/(c*x**2))) - 3*a*c*sqrt(d)*asinh(sqrt(d)/(sqrt(
c)*x))/2 - b*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - b*c**(3/2)/(8*x*sqrt(1 +
 d/(c*x**2))) - 3*b*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asin
h(sqrt(d)/(sqrt(c)*x))/(8*sqrt(d)) - b*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x
**2)))
```

3.960 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=123

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

```
[Out] 1/24*(-6*a*d+b*c)*(c+d/x^2)^(3/2)/d/x-1/6*b*(c+d/x^2)^(5/2)/d/x+1/16*c^2*(-
6*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+1/16*c*(-6*a*d+b*c)*(
c+d/x^2)^(1/2)/d/x
```

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 195, 217, 206}

$$\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{16d^{3/2}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(bc-6ad)}{24dx}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-6ad)}{16dx}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] (c*(b*c - 6*a*d)*Sqrt[c + d/x^2])/(16*d*x) + ((b*c - 6*a*d)*(c + d/x^2)^(3/
2))/(24*d*x) - (b*(c + d/x^2)^(5/2))/(6*d*x) + (c^2*(b*c - 6*a*d)*ArcTanh[S
qrt[d]/(Sqrt[c + d/x^2]*x)])/(16*d^(3/2))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(-bc+6ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^2}\,dx}{6d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} - \frac{(-bc+6ad)\,\text{Subst}\left(\int \left(c+dx^2\right)^{3/2}\,dx, x, \frac{1}{x}\right)}{6d}$$

$$= \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{(c(bc-6ad))\,\text{Subst}\left(\int \sqrt{c+dx^2}\,dx, x, \frac{1}{x}\right)}{8d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{\left(c^2(bc-6ad)\right)\text{Subst}}{16d}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{\left(c^2(bc-6ad)\right)\text{Subst}}{1}$$

$$= \frac{c(bc-6ad)\sqrt{c+\frac{d}{x^2}}}{16dx} + \frac{(bc-6ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{24dx} - \frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{6dx} + \frac{c^2(bc-6ad)\tanh^{-1}}{16d^{3/2}}$$

Mathematica [A] time = 0.24, size = 126, normalized size = 1.02

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(\left(cx^2+d\right)\left(6adx^2\left(5cx^2+2d\right)+b\left(3c^2x^4+14cdx^2+8d^2\right)\right)+3c^2x^6\sqrt{\frac{cx^2}{d}+1}\,(6ad-bc)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}}\right.\right.}{48dx^5\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x]
```

```
[Out] -1/48*(Sqrt[c + d/x^2]*((d + c*x^2)*(6*a*d*x^2*(2*d + 5*c*x^2) + b*(8*d^2 +
 14*c*d*x^2 + 3*c^2*x^4)) + 3*c^2*(-(b*c) + 6*a*d)*x^6*Sqrt[1 + (c*x^2)/d]*
ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d*x^5*(d + c*x^2))
```

fricas [A] time = 0.88, size = 246, normalized size = 2.00

$$\left[-\frac{3\left(bc^3-6\,ac^2d\right)\sqrt{d}\,x^5\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(3\left(bc^2d+10\,acd^2\right)x^4+8\,bd^3+2\left(7\,bcd^2+6\,ad^3\right)x^2\right)\sqrt{\frac{cx^2}{x}}}{96\,d^2x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 6*a*c^2*d)*sqrt(d)*x^5*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c
*x^2 + d)/x^2) + 2*d)/x^2) + 2*(3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*
```

```
(7*b*c*d^2 + 6*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5), -1/48*(3*(b*c^
3 - 6*a*c^2*d)*sqrt(-d)*x^5*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2
+ d)) + (3*(b*c^2*d + 10*a*c*d^2)*x^4 + 8*b*d^3 + 2*(7*b*c*d^2 + 6*a*d^3)*x
^2)*sqrt((c*x^2 + d)/x^2))/(d^2*x^5)]
```

giac [A] time = 0.30, size = 173, normalized size = 1.41

$$-\frac{\frac{3\left(bc^4\mathrm{sgn}(x)-6\,ac^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d}+\frac{3\left(cx^2+d\right)^{\frac{5}{2}}bc^4\mathrm{sgn}(x)+30\left(cx^2+d\right)^{\frac{5}{2}}ac^3d\mathrm{sgn}(x)+8\left(cx^2+d\right)^{\frac{3}{2}}bc^4d\mathrm{sgn}(x)-48\left(cx^2+d\right)^{\frac{3}{2}}ac^3d^2\mathrm{sgn}}{c^3dx^6}}{48\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/48*(3*(b*c^4*sgn(x) - 6*a*c^3*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d))
/(sqrt(-d)*d) + (3*(c*x^2 + d)^(5/2)*b*c^4*sgn(x) + 30*(c*x^2 + d)^(5/2)*a*
c^3*d*sgn(x) + 8*(c*x^2 + d)^(3/2)*b*c^4*d*sgn(x) - 48*(c*x^2 + d)^(3/2)*a*
c^3*d^2*sgn(x) - 3*sqrt(c*x^2 + d)*b*c^4*d^2*sgn(x) + 18*sqrt(c*x^2 + d)*a*
c^3*d^3*sgn(x))/(c^3*d*x^6))/c
```

maple [B] time = 0.07, size = 259, normalized size = 2.11

$$-\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(18a\,c^2d^{\frac{5}{2}}x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-3b\,c^3d^{\frac{3}{2}}x^6\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-18\sqrt{c\,x^2+d}\,a\,c^2d^2x^6+3\sqrt{c\,x^2+}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -1/48*((c*x^2+d)/x^2)^(3/2)/x^3*(18*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2)
)/x)*x^6*a*c^2-3*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^6*b*c^3-6*(c
*x^2+d)^(3/2)*x^6*a*c^2*d+(c*x^2+d)^(3/2)*x^6*b*c^3+6*(c*x^2+d)^(5/2)*x^4*a
*c*d-(c*x^2+d)^(5/2)*x^4*b*c^2-18*(c*x^2+d)^(1/2)*x^6*a*c^2*d^2+3*(c*x^2+d)
^(1/2)*x^6*b*c^3*d+12*(c*x^2+d)^(5/2)*x^2*a*d^2-2*(c*x^2+d)^(5/2)*x^2*b*c*d
+8*(c*x^2+d)^(5/2)*b*d^2)/(c*x^2+d)^(3/2)/d^3
```

maxima [B] time = 1.38, size = 275, normalized size = 2.24

$$\frac{1}{16}\left(\frac{3\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{\sqrt{d}}-\frac{2\left(5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2x^3-3\sqrt{c+\frac{d}{x^2}}\,c^2dx\right)}{\left(c+\frac{d}{x^2}\right)^2x^4-2\left(c+\frac{d}{x^2}\right)dx^2+d^2}\right)a-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\right.\right.}{\left(c+\frac{d}{x^2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(3*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d))
)/sqrt(d) - 2*(5*(c + d/x^2)^(3/2)*c^2*x^3 - 3*sqrt(c + d/x^2)*c^2*d*x)/((c
 + d/x^2)^2*x^4 - 2*(c + d/x^2)*d*x^2 + d^2))*a - 1/96*(3*c^3*log((sqrt(c +
 d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2) + 2*(3*(c + d/x
^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3 - 3*sqrt(c + d/x^2)*c^3*d
^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*x^4 + 3*(c + d/x^2)*d^3*x^
2 - d^4))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right)\left(c + \frac{d}{x^2}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^2, x)
```

sympy [B] time = 18.95, size = 253, normalized size = 2.06

$$-\frac{ac^{\frac{3}{2}}\sqrt{1+\frac{d}{cx^2}}}{2x}-\frac{ac^{\frac{3}{2}}}{8x\sqrt{1+\frac{d}{cx^2}}}-\frac{3a\sqrt{c}\,d}{8x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3ac^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8\sqrt{d}}-\frac{ad^2}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}-\frac{bc^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17bc^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a*c**(3/2)*sqrt(1 + d/(c*x**2))/(2*x) - a*c**(3/2)/(8*x*sqrt(1 + d/(c*x**2
))) - 3*a*sqrt(c)*d/(8*x**3*sqrt(1 + d/(c*x**2))) - 3*a*c**2*asinh(sqrt(d)/
(sqrt(c)*x))/(8*sqrt(d)) - a*d**2/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2))) - b
*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*b*c**(3/2)/(48*x**3*sqrt(1 + d
/(c*x**2))) - 11*b*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + b*c**3*asinh(
sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - b*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c*x
**2)))
```

3.961 $\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=159

$$-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{128d^2x}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c\right.}{}$$

```
[Out] 1/48*(-8*a*d+3*b*c)*(c+d/x^2)^(3/2)/d/x^3-1/8*b*(c+d/x^2)^(5/2)/d/x^3-1/128
*c^3*(-8*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)+1/64*c*(-8*a
*d+3*b*c)*(c+d/x^2)^(1/2)/d/x^3+1/128*c^2*(-8*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^
2/x
```

Rubi [A] time = 0.09, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 279, 321, 217, 206}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{128d^2x}-\frac{c^3(3bc-8ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{128d^{5/2}}+\frac{c\sqrt{c+\frac{d}{x^2}}\,(3bc-8ad)}{64dx^3}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-8ad)}{48dx^3}-\frac{b\left(c\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (c*(3*b*c - 8*a*d)*Sqrt[c + d/x^2])/(64*d*x^3) + ((3*b*c - 8*a*d)*(c + d/x^
2)^(3/2))/(48*d*x^3) - (b*(c + d/x^2)^(5/2))/(8*d*x^3) + (c^2*(3*b*c - 8*a*
d)*Sqrt[c + d/x^2])/(128*d^2*x) - (c^3*(3*b*c - 8*a*d)*ArcTanh[Sqrt[d]/(Sqr
t[c + d/x^2]*x)])/(128*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx = -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(-3bc+8ad)\int \frac{\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx}{8d}$$

$$= -\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}-\frac{(-3bc+8ad)\,\text{Subst}\left(\int x^2\left(c+dx^2\right)^{3/2}\,dx,x,\frac{1}{x}\right)}{8d}$$

$$= \frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{(c(3bc-8ad))\,\text{Subst}\left(\int x^2\sqrt{c+dx^2}\,dx,x,\frac{1}{x}\right)}{16d}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{\left(c^2(3bc-8ad)\right)\text{Su}}{}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+}}{128d^2x}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+}}{128d^2x}$$

$$= \frac{c(3bc-8ad)\sqrt{c+\frac{d}{x^2}}}{64dx^3}+\frac{(3bc-8ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{48dx^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{8dx^3}+\frac{c^2(3bc-8ad)\sqrt{c+}}{128d^2x}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.45

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(cx^2+d\right)^2\left(c^3x^8(8ad-3bc)\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{cx^2}{d}+1\right)-5bd^4\right)}{40d^5x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x]
```

```
[Out] (Sqrt[c + d/x^2]*(d + c*x^2)^2*(-5*b*d^4 + c^3*(-3*b*c + 8*a*d)*x^8*Hyperge
ometric2F1[5/2, 4, 7/2, 1 + (c*x^2)/d]))/(40*d^5*x^7)
```

fricas [A] time = 0.60, size = 298, normalized size = 1.87

$$\left[-\frac{3\left(3\,bc^4-8\,ac^3d\right)\sqrt{d}\,x^7\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(3\,bc^3d-8\,ac^2d^2\right)x^6-48\,bd^4-2\left(3\,bc^2d^2+56\,acd\right.\right.}{768\,d^3x^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(d)*x^7*log(-(c*x^2 + 2*sqrt(d)*x*sqrt
((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(3*b*c^3*d - 8*a*c^2*d^2)*x^6 - 48*b*d
^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*a*d^4)*x^2)*sqrt((
c*x^2 + d)/x^2))/(d^3*x^7), 1/384*(3*(3*b*c^4 - 8*a*c^3*d)*sqrt(-d)*x^7*arc
tan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(3*b*c^3*d - 8*a*c^2
*d^2)*x^6 - 48*b*d^4 - 2*(3*b*c^2*d^2 + 56*a*c*d^3)*x^4 - 8*(9*b*c*d^3 + 8*
a*d^4)*x^2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^7)]
```

giac [A] time = 0.37, size = 214, normalized size = 1.35

$$\frac{\frac{3\left(3\,bc^5\mathrm{sgn}(x)-8\,ac^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{cx^2+d}}{\sqrt{-d}}\right)}{\sqrt{-d}\,d^2}+\frac{9\left(cx^2+d\right)^{\frac{7}{2}}bc^5\mathrm{sgn}(x)-24\left(cx^2+d\right)^{\frac{7}{2}}ac^4d\mathrm{sgn}(x)-33\left(cx^2+d\right)^{\frac{5}{2}}bc^5d\mathrm{sgn}(x)-40\left(cx^2+d\right)^{\frac{5}{2}}ac^4d^2\mathrm{sgn}}{}}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/384*(3*(3*b*c^5*sgn(x) - 8*a*c^4*d*sgn(x))*arctan(sqrt(c*x^2 + d)/sqrt(-d
))/(sqrt(-d)*d^2) + (9*(c*x^2 + d)^(7/2)*b*c^5*sgn(x) - 24*(c*x^2 + d)^(7/2
)*a*c^4*d*sgn(x) - 33*(c*x^2 + d)^(5/2)*b*c^5*d*sgn(x) - 40*(c*x^2 + d)^(5/
2)*a*c^4*d^2*sgn(x) - 33*(c*x^2 + d)^(3/2)*b*c^5*d^2*sgn(x) + 88*(c*x^2 + d
)^(3/2)*a*c^4*d^3*sgn(x) + 9*sqrt(c*x^2 + d)*b*c^5*d^3*sgn(x) - 24*sqrt(c*x
^2 + d)*a*c^4*d^4*sgn(x))/(c^4*d^2*x^8))/c
```

maple [B] time = 0.07, size = 302, normalized size = 1.90

$$\frac{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}\left(24a\,c^3d^{\frac{5}{2}}x^8\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-9b\,c^4d^{\frac{3}{2}}x^8\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-24\sqrt{c\,x^2+d}\,a\,c^3d^2x^8+9\sqrt{c\,x^2+d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/384*((c*x^2+d)/x^2)^(3/2)/x^5*(24*d^(5/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2)
)/x)*x^8*a*c^3-9*d^(3/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^8*b*c^4-8*(c
*x^2+d)^(3/2)*x^8*a*c^3*d+3*(c*x^2+d)^(3/2)*x^8*b*c^4+8*(c*x^2+d)^(5/2)*x^6
*a*c^2*d-3*(c*x^2+d)^(5/2)*x^6*b*c^3-24*(c*x^2+d)^(1/2)*x^8*a*c^3*d^2+9*(c*
x^2+d)^(1/2)*x^8*b*c^4*d+16*(c*x^2+d)^(5/2)*x^4*a*c*d^2-6*(c*x^2+d)^(5/2)*x
^4*b*c^2*d-64*(c*x^2+d)^(5/2)*x^2*a*d^3+24*(c*x^2+d)^(5/2)*x^2*b*c*d^2-48*(
c*x^2+d)^(5/2)*b*d^3)/(c*x^2+d)^(3/2)/d^4
```

maxima [B] time = 1.57, size = 354, normalized size = 2.23

$$-\frac{1}{96}\left(\frac{3\,c^3\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3x^5+8\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^3dx^3-3\sqrt{c+\frac{d}{x^2}}\,c^3d^2x\right)}{\left(c+\frac{d}{x^2}\right)^3dx^6-3\left(c+\frac{d}{x^2}\right)^2d^2x^4+3\left(c+\frac{d}{x^2}\right)d^3x^2-d^4}\right)a+\frac{1}{256}\left(\frac{3\,c^4\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\cdots}{\sqrt{c+\frac{d}{x^2}}\cdots}\right)}{d^{\frac{5}{2}}}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/96*(3*c^3*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)
))/d^(3/2) + 2*(3*(c + d/x^2)^(5/2)*c^3*x^5 + 8*(c + d/x^2)^(3/2)*c^3*d*x^3
 - 3*sqrt(c + d/x^2)*c^3*d^2*x)/((c + d/x^2)^3*d*x^6 - 3*(c + d/x^2)^2*d^2*
x^4 + 3*(c + d/x^2)*d^3*x^2 - d^4))*a + 1/256*(3*c^4*log((sqrt(c + d/x^2)*x
 - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) + 2*(3*(c + d/x^2)^(7/2)
*c^4*x^7 - 11*(c + d/x^2)^(5/2)*c^4*d*x^5 - 11*(c + d/x^2)^(3/2)*c^4*d^2*x^
3 + 3*sqrt(c + d/x^2)*c^4*d^3*x)/((c + d/x^2)^4*d^2*x^8 - 4*(c + d/x^2)^3*d
^3*x^6 + 6*(c + d/x^2)^2*d^4*x^4 - 4*(c + d/x^2)*d^5*x^2 + d^6))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)\left(c+\frac{d}{x^2}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b/x^2)*(c + d/x^2)^(3/2))/x^4, x)
```

sympy [B] time = 29.16, size = 287, normalized size = 1.81

$$-\frac{ac^{\frac{5}{2}}}{16dx\sqrt{1+\frac{d}{cx^2}}}-\frac{17ac^{\frac{3}{2}}}{48x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{11a\sqrt{c}\,d}{24x^5\sqrt{1+\frac{d}{cx^2}}}+\frac{ac^3\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{16d^{\frac{3}{2}}}-\frac{ad^2}{6\sqrt{c}\,x^7\sqrt{1+\frac{d}{cx^2}}}+\frac{3bc^{\frac{7}{2}}}{128d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\cdots}{128dx^3\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] -a*c**(5/2)/(16*d*x*sqrt(1 + d/(c*x**2))) - 17*a*c**(3/2)/(48*x**3*sqrt(1 +
 d/(c*x**2))) - 11*a*sqrt(c)*d/(24*x**5*sqrt(1 + d/(c*x**2))) + a*c**3*asin
h(sqrt(d)/(sqrt(c)*x))/(16*d**(3/2)) - a*d**2/(6*sqrt(c)*x**7*sqrt(1 + d/(c
*x**2))) + 3*b*c**(7/2)/(128*d**2*x*sqrt(1 + d/(c*x**2))) + b*c**(5/2)/(128
*d*x**3*sqrt(1 + d/(c*x**2))) - 13*b*c**(3/2)/(64*x**5*sqrt(1 + d/(c*x**2))
) - 5*b*sqrt(c)*d/(16*x**7*sqrt(1 + d/(c*x**2))) - 3*b*c**4*asinh(sqrt(d)/(
sqrt(c)*x))/(128*d**(5/2)) - b*d**2/(8*sqrt(c)*x**9*sqrt(1 + d/(c*x**2)))
```

3.962 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=90

$$-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{8c^2}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

```
[Out] -1/8*d*(-3*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/8*(-3*a*d+
4*b*c)*x^2*(c+d/x^2)^(1/2)/c^2+1/4*a*x^4*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{x^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{8c^2}-\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}+\frac{ax^4\sqrt{c+\frac{d}{x^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] ((4*b*c - 3*a*d)*Sqrt[c + d/x^2]*x^2)/(8*c^2) + (a*Sqrt[c + d/x^2]*x^4)/(4*
c) - (d*(4*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\sqrt{c+\frac{d}{x^2}}}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{a+bx}{x^3\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{\left(2bc-\frac{3ad}{2}\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(d(4bc-3ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{16c^2}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} + \frac{(4bc-3ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{8c^2}\\
&= \frac{(4bc-3ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^4}{4c} - \frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 95, normalized size = 1.06

$$\frac{\sqrt{c}\,x\left(cx^2+d\right)\left(2acx^2-3ad+4bc\right)+d\sqrt{cx^2+d}\,(3ad-4bc)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right)}{8c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c]*x*(d + c*x^2)*(4*b*c - 3*a*d + 2*a*c*x^2) + d*(-4*b*c + 3*a*d)*Sqr
t[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[d + c*x^2]])/(8*c^(5/2)*Sqrt[c + d/x^
2]*x)
```

fricas [A] time = 0.55, size = 192, normalized size = 2.13

$$\left[-\frac{\left(4\,bcd-3\,ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^2x^4+\left(4\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,c^3}, \frac{\left(4\,bcd-3\,ad^2\right)\cdot}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*b*c*d - 3*a*d^2)*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^
2 + d)/x^2) - d) - 2*(2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 +
d)/x^2))/c^3, 1/8*((4*b*c*d - 3*a*d^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c
*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a*c^2*x^4 + (4*b*c^2 - 3*a*c*d)*x^2)*sqrt(
(c*x^2 + d)/x^2))/c^3]
```

giac [A] time = 0.26, size = 113, normalized size = 1.26

$$\frac{1}{8}\sqrt{cx^4+dx^2}\left(\frac{2\,ax^2}{c}+\frac{4\,bc-3\,ad}{c^2}\right)+\frac{\left(4\,bcd-3\,ad^2\right)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)\sqrt{c}-d\right|\right)}{16\,c^{\frac{5}{2}}}-\frac{4\,bcd\log\left(|d|\right)-}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + d*x^2)*(2*a*x^2/c + (4*b*c - 3*a*d)/c^2) + 1/16*(4*b*c*d -
 3*a*d^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*sqrt(c) - d))/c^(5
/2) - 1/16*(4*b*c*d*log(abs(d)) - 3*a*d^2*log(abs(d)))/c^(5/2)
```

maple [A] time = 0.06, size = 129, normalized size = 1.43

$$\frac{\sqrt{c\,x^2+d}\,\left(2\sqrt{c\,x^2+d}\;a\,c^{\frac{5}{2}}x^3+3ac\,d^2\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)-4b\,c^2d\ln\left(\sqrt{c}\;x+\sqrt{c\,x^2+d}\right)-3\sqrt{c\,x^2+d}\;a\,\cdots\right.}{8\sqrt{\frac{c\,x^2+d}{x^2}}\;c^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/8*(c*x^2+d)^(1/2)*(2*(c*x^2+d)^(1/2)*c^(5/2)*x^3*a-3*(c*x^2+d)^(1/2)*c^(3
/2)*x*a*d+4*(c*x^2+d)^(1/2)*c^(5/2)*x*b+3*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c
*d^2-4*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d)/x^2)^(1/2)/x/c^(7/
2)
```

maxima [B] time = 1.17, size = 178, normalized size = 1.98

$$\frac{1}{4}\,b\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,d}{\left(c+\frac{d}{x^2}\right)c-c^2}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{1}{16}\,a\left(\frac{3\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2-5\sqrt{c+\frac{d}{x^2}}\,cd^2\right)}{\left(c+\frac{d}{x^2}\right)^2c^2-2\left(c+\frac{d}{x^2}\right)c^3+c^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/16*a*(3*d^2*log((sqrt(c
 + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/2) + 2*(3*(c + d/x^2
)^(3/2)*d^2 - 5*sqrt(c + d/x^2)*c*d^2)/((c + d/x^2)^2*c^2 - 2*(c + d/x^2)*c
^3 + c^4))
```

mupad [B] time = 5.35, size = 99, normalized size = 1.10

$$\frac{5\,a\,x^4\sqrt{c+\frac{d}{x^2}}}{8\,c}-\frac{3\,a\,x^4\left(c+\frac{d}{x^2}\right)^{3/2}}{8\,c^2}+\frac{b\,x^2\sqrt{c+\frac{d}{x^2}}}{2\,c}-\frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{3/2}}+\frac{3\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (5*a*x^4*(c + d/x^2)^(1/2))/(8*c) - (3*a*x^4*(c + d/x^2)^(3/2))/(8*c^2) + (
b*x^2*(c + d/x^2)^(1/2))/(2*c) - (b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*
c^(3/2)) + (3*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(5/2))
```

sympy [A] time = 71.06, size = 150, normalized size = 1.67

$$\frac{ax^5}{4\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{a\sqrt{d}\,x^3}{8c\sqrt{\frac{cx^2}{d}+1}}-\frac{3ad^{\frac{3}{2}}x}{8c^2\sqrt{\frac{cx^2}{d}+1}}+\frac{3ad^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{5}{2}}}+\frac{b\sqrt{d}\,x\sqrt{\frac{cx^2}{d}+1}}{2c}-\frac{bd\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] a*x**5/(4*sqrt(d)*sqrt(c*x**2/d + 1)) - a*sqrt(d)*x**3/(8*c*sqrt(c*x**2/d +
 1)) - 3*a*d**(3/2)*x/(8*c**2*sqrt(c*x**2/d + 1)) + 3*a*d**2*asinh(sqrt(c)*
x/sqrt(d))/(8*c**(5/2)) + b*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - b*d*asinh(
sqrt(c)*x/sqrt(d))/(2*c**(3/2))
```

3.963 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=59

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+1/2*a*x^2*(c+d/x^
2)^(1/2)/c
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 78, 63, 208}

$$\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}+\frac{ax^2\sqrt{c+\frac{d}{x^2}}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x^2)/(2*c) + ((2*b*c - a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt
[c]])/(2*c^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^2\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{2c}$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} - \frac{\left(bc-\frac{ad}{2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+\frac{d}{x^2}}\right)}{cd}$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x^2}{2c} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{3/2}}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 1.34

$$\frac{\sqrt{cx^2+d}\,(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right)+a\sqrt{c}\,x\left(cx^2+d\right)}{2c^{3/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c]*x*(d + c*x^2) + (2*b*c - a*d)*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x
)/Sqrt[d + c*x^2]])/(2*c^(3/2)*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.65, size = 146, normalized size = 2.47

$$\left[\frac{2\,acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{4\,c^2},\ \frac{acx^2\sqrt{\frac{cx^2+d}{x^2}}-(2\,bc-ad)\sqrt{-c}\,\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\ldots}}{cx^2\ldots}\right)}{2\,c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*c*x^2*sqrt((c*x^2 + d)/x^2) - (2*b*c - a*d)*sqrt(c)*log(-2*c*x^2
+ 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/c^2, 1/2*(a*c*x^2*sqrt((c*x^2 +
 d)/x^2) - (2*b*c - a*d)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)
/(c*x^2 + d)))/c^2]
```

giac [A] time = 0.26, size = 88, normalized size = 1.49

$$\frac{\sqrt{cx^4+dx^2}\,a}{2\,c}-\frac{(2\,bc-ad)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)\sqrt{c}-d\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{2\,bc\log\left(|d|\right)-ad\log\left(|d|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + d*x^2)*a/c - 1/4*(2*b*c - a*d)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + d*x^2))*sqrt(c) - d))/c^(3/2) + 1/4*(2*b*c*log(abs(d)) - a*d*lo
g(abs(d)))/c^(3/2)
```

maple [A] time = 0.05, size = 90, normalized size = 1.53

$$\frac{\sqrt{c\,x^{2}+d}\,\left(-acd\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)+2b\,c^{2}\ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)+\sqrt{c\,x^{2}+d}\,a\,c^{\frac{3}{2}}x\right)}{2\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,c^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/2*(c*x^2+d)^(1/2)*(c^(3/2)*(c*x^2+d)^(1/2)*x*a+2*b*ln(c^(1/2)*x+(c*x^2+d)
^(1/2))*c^2-ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d)/((c*x^2+d)/x^2)^(1/2)/x/c^
(5/2)
```

maxima [B] time = 1.20, size = 109, normalized size = 1.85

$$\frac{1}{4}\,a\left(\frac{2\sqrt{c+\frac{d}{x^{2}}}\,d}{\left(c+\frac{d}{x^{2}}\right)c-c^{2}}+\frac{d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)-\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*sqrt(c + d/x^2)*d/((c + d/x^2)*c - c^2) + d*log((sqrt(c + d/x^2) -
 sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2)) - 1/2*b*log((sqrt(c + d/x^2
) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
```

mupad [B] time = 5.08, size = 59, normalized size = 1.00

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{a\,x^{2}\,\sqrt{c+\frac{d}{x^{2}}}}{2\,c}-\frac{a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{2\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) + (a*x^2*(c + d/x^2)^(1/2))/(2
*c) - (a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2*c^(3/2))
```

sympy [A] time = 84.41, size = 66, normalized size = 1.12

$$\frac{a\sqrt{d}\,x\sqrt{\frac{cx^{2}}{d}+1}}{2c}-\frac{ad\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{3}{2}}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x*sqrt(c*x**2/d + 1)/(2*c) - a*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(
3/2)) + b*asinh(sqrt(c)*x/sqrt(d))/sqrt(c)
```

3.964 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x}\, dx$

Optimal. Leaf size=43

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(1/2)-b*(c+d/x^2)^(1/2)/d

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {446, 80, 63, 208}

$$\frac{a \tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]

[Out] -((b*Sqrt[c + d/x^2])/d) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/Sqrt[c]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{d} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{\sqrt{c}}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.70

$$\frac{adx\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{cx^2+d}}\right) - b\sqrt{c}\left(cx^2+d\right)}{\sqrt{c}\,dx^2\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x),x]
```

```
[Out] (-(b*Sqrt[c]*(d + c*x^2)) + a*d*x*Sqrt[d + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[
d + c*x^2]])/(Sqrt[c]*d*Sqrt[c + d/x^2]*x^2)
```

fricas [A] time = 0.63, size = 130, normalized size = 3.02

$$\left[\frac{a\sqrt{c}\,d\log\left(-2\,cx^2 - 2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}} - d\right) - 2\,bc\sqrt{\frac{cx^2+d}{x^2}}}{2\,cd}, -\frac{a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right) + bc\sqrt{\frac{cx^2+d}{x^2}}}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) -
 2*b*c*sqrt((c*x^2 + d)/x^2))/(c*d), -(a*sqrt(-c)*d*arctan(sqrt(-c)*x^2*sqr
t((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*c*sqrt((c*x^2 + d)/x^2))/(c*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, choosing root of [1,0,%%%{-2,[1,2,0]%%%}+%%%{-2,[1,0,0]%%%}
```

```
+%%%{-2,[0,1,1]%%%},0,%%%{1,[2,4,0]%%%}+%%%{-2,[2,2,0]%%%}+%%%{1,[2,0,0]%%%
}+%%%{2,[1,3,1]%%%}+%%%{-2,[1,1,1]%%%}+%%%{1,[0,2,2]%%%}] at parameters val
ues [86,-97,-82]Warning, choosing root of [1,0,%%%{-2,[1,0,1]%%%}+%%%{-4,[0
,1,0]%%%},0,%%%{1,[2,0,2]%%%}] at parameters values [-64,61.7937478349,70]S
ign error (%%%{d,0%%%}+%%%{-2*sqrt(c)*sqrt(d),1/2%%%}+%%%{2*c,1%%%}+%%%{-c*
sqrt(c)*sqrt(d)/d,3/2%%%}+%%%{c^2*sqrt(c)*sqrt(d)/(4*d^2),5/2%%%}+%%%{undef
,7/2%%%})Limit: Max order reached or unable to make series expansion Error:
 Bad Argument Value
```

maple [A] time = 0.05, size = 70, normalized size = 1.63

$$-\frac{\sqrt{c\,x^{2}+d}\,\left(-a d x \ln\left(\sqrt{c}\,x+\sqrt{c\,x^{2}+d}\right)+\sqrt{c\,x^{2}+d}\,b\sqrt{c}\right)}{\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,\sqrt{c}\,d\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x/(c+d/x^2)^(1/2),x)
```

```
[Out] -(c*x^2+d)^(1/2)*(-a*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*d*x+b*(c*x^2+d)^(1/2)*c^
(1/2))/((c*x^2+d)/x^2)^(1/2)/x^2/c^(1/2)/d
```

maxima [A] time = 1.10, size = 54, normalized size = 1.26

$$-\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{2\sqrt{c}}-\frac{b\sqrt{c+\frac{d}{x^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/sqrt(c)
 - b*sqrt(c + d/x^2)/d
```

mupad [B] time = 4.85, size = 35, normalized size = 0.81

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\,\sqrt{c+\frac{d}{x^{2}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(1/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(1/2) - (b*(c + d/x^2)^(1/2))/d
```

sympy [A] time = 22.34, size = 63, normalized size = 1.47

$$-\frac{a\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{c}}\sqrt{c+\frac{d}{x^{2}}}}\right)}{c\sqrt{-\frac{1}{c}}}+\frac{b\left(\begin{cases}-\frac{1}{\sqrt{c}x^{2}} & \text{for}\: d=0\\-\frac{2\sqrt{c+\frac{d}{x^{2}}}}{d} & \text{otherwise}\end{cases}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*atan(1/(sqrt(-1/c)*sqrt(c + d/x**2)))/(c*sqrt(-1/c)) + b*Piecewise((-1/(
sqrt(c)*x**2), Eq(d, 0)), (-2*sqrt(c + d/x**2)/d, True))/2
```

3.965 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^2+(-a*d+b*c)*(c+d/x^2)^(1/2)/d^2
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
```

```
[Out] ((b*c - a*d)*Sqrt[c + d/x^2])/d^2 - (b*(c + d/x^2)^(3/2))/(3*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^3}\,dx &= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{a+bx}{\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= \frac{(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^2}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\left(3adx^2+b\left(d-2cx^2\right)\right)}{3d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^3),x]
[Out] -1/3*(Sqrt[c + d/x^2]*(3*a*d*x^2 + b*(d - 2*c*x^2)))/(d^2*x^2)
```

fricas [A] time = 0.65, size = 39, normalized size = 0.91

$$\frac{\left((2\,bc - 3\,ad)x^2 - bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*((2*b*c - 3*a*d)*x^2 - b*d)*sqrt((c*x^2 + d)/x^2)/(d^2*x^2)
```

giac [B] time = 0.41, size = 88, normalized size = 2.05

$$\frac{3\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + dx^2}\right)^2 a + 3\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + dx^2}\right)b\sqrt{c} + bd}{3\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + dx^2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] 1/3*(3*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*a + 3*(sqrt(c)*x^2 - sqrt(c*x^
4 + d*x^2))*b*sqrt(c) + b*d)/(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3
```

maple [A] time = 0.05, size = 47, normalized size = 1.09

$$-\frac{\left(3ad\,x^2 - 2bc\,x^2 + bd\right)\left(c\,x^2 + d\right)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\;d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x)
[Out] -1/3*(3*a*d*x^2-2*b*c*x^2+b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^2/x^4
```

maxima [A] time = 0.53, size = 48, normalized size = 1.12

$$-\frac{1}{3}\,b\left(\frac{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2} - \frac{3\,\sqrt{c + \frac{d}{x^2}}\,c}{d^2}\right) - \frac{a\sqrt{c + \frac{d}{x^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^3/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2) - a*sqrt(c + d/x^2
)/d
```

mupad [B] time = 4.56, size = 35, normalized size = 0.81

$$-\frac{\sqrt{c + \frac{d}{x^2}}\;\left(b\,d + 3\,a\,d\,x^2 - 2\,b\,c\,x^2\right)}{3\,d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d + 3*a*d*x^2 - 2*b*c*x^2))/(3*d^2*x^2)
```

sympy [**A**] time = 7.05, size = 138, normalized size = 3.21

$$\frac{\begin{cases} \frac{-\frac{a}{x^2}-\frac{b}{2x^4}}{\sqrt{c}} & \text{for } d = 0 \\ \frac{\frac{2ac}{\sqrt{c+\frac{d}{x^2}}}+2a\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)+\frac{2bc\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2b\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{otherwise} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**3/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/x**2 - b/(2*x**4))/sqrt(c), Eq(d, 0)), ((2*a*c/sqrt(c + d/x*
*2) + 2*a*(-c/sqrt(c + d/x**2) - sqrt(c + d/x**2)) + 2*b*c*(-c/sqrt(c + d/x
**2) - sqrt(c + d/x**2))/d + 2*b*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x*
*2) - (c + d/x**2)**(3/2)/3)/d)/d, True))/2
```

3.966 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx$

Optimal. Leaf size=72

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

```
[Out] 1/3*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^3-1/5*b*(c+d/x^2)^(5/2)/d^3-c*(-a*d+b*c)
*(c+d/x^2)^(1/2)/d^3
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{3d^3}-\frac{c\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] -((c*(b*c - a*d)*Sqrt[c + d/x^2])/d^3) + ((2*b*c - a*d)*(c + d/x^2)^(3/2))/
(3*d^3) - (b*(c + d/x^2)^(5/2))/(5*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^5}\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)}{\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2\sqrt{c+dx}}+\frac{(-2bc+ad)\sqrt{c+dx}}{d^2}+\frac{b(c+dx)^{3/2}}{d^2}\right)dx,x,\frac{1}{x^2}\right)\right)\\
&= -\frac{c(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}+\frac{(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.83

$$\frac{\sqrt{c+\frac{d}{x^2}}\left(b\left(-8c^2x^4+4cdx^2-3d^2\right)-5adx^2\left(d-2cx^2\right)\right)}{15d^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^5),x]
```

```
[Out] (Sqrt[c + d/x^2]*(-5*a*d*x^2*(d - 2*c*x^2) + b*(-3*d^2 + 4*c*d*x^2 - 8*c^2*
x^4)))/(15*d^3*x^4)
```

fricas [A] time = 0.48, size = 62, normalized size = 0.86

$$-\frac{\left(2\left(4\,bc^2-5\,acd\right)x^4+3\,bd^2-\left(4\,bcd-5\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*b*c^2 - 5*a*c*d)*x^4 + 3*b*d^2 - (4*b*c*d - 5*a*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2)/(d^3*x^4)
```

giac [B] time = 0.50, size = 153, normalized size = 2.12

$$\frac{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^3a\sqrt{c}+20\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^2bc+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^2ad+15\left(\sqrt{c}\,x^2-\sqrt{cx^4\cdots}\right.}{15\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(15*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3*a*sqrt(c) + 20*(sqrt(c)*x^2
- sqrt(c*x^4 + d*x^2))^2*b*c + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*a*d
+ 15*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))*b*sqrt(c)*d + 3*b*d^2)/(sqrt(c)*x^
2 - sqrt(c*x^4 + d*x^2))^5
```

maple [A] time = 0.05, size = 70, normalized size = 0.97

$$\frac{\left(10acd\,x^4-8b\,c^2x^4-5a\,d^2x^2+4bcd\,x^2-3b\,d^2\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\;d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15*(10*a*c*d*x^4-8*b*c^2*x^4-5*a*d^2*x^2+4*b*c*d*x^2-3*b*d^2)*(c*x^2+d)/(
(c*x^2+d)/x^2)^(1/2)/d^3/x^6
```

maxima [A] time = 0.49, size = 83, normalized size = 1.15

$$-\frac{1}{15}b\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^3}\right)-\frac{1}{3}a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^2}-\frac{3\sqrt{c+\frac{d}{x^2}}\,c}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^5/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*b*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c +
 d/x^2)*c^2/d^3) - 1/3*a*((c + d/x^2)^(3/2)/d^2 - 3*sqrt(c + d/x^2)*c/d^2)
```

mupad [B] time = 4.68, size = 58, normalized size = 0.81

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(8\,b\,c^2\,x^4-10\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+5\,a\,d^2\,x^2+3\,b\,d^2\right)}{15\,d^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(1/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^2 + 5*a*d^2*x^2 + 8*b*c^2*x^4 - 10*a*c*d*x^4 - 4
*b*c*d*x^2))/(15*d^3*x^4)
```

sympy [A] time = 13.17, size = 204, normalized size = 2.83

$$\frac{\begin{cases} \frac{-\frac{a}{2x^4}-\frac{b}{3x^6}}{\sqrt{c}} \\ \frac{\frac{2ac\left(-\frac{c}{\sqrt{c+\frac{d}{x^2}}}-\sqrt{c+\frac{d}{x^2}}\right)}{d}+\frac{2a\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d}+\frac{2bc\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2b\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**5/(c+d/x**2)**(1/2),x)
```

```
[Out] Piecewise(((-a/(2*x**4) - b/(3*x**6))/sqrt(c), Eq(d, 0)), ((2*a*c*(-c/sqrt(
c + d/x**2) - sqrt(c + d/x**2))/d + 2*a*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c
 + d/x**2) - (c + d/x**2)**(3/2)/3)/d + 2*b*c*(c**2/sqrt(c + d/x**2) + 2*c*
sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*b*(-c**3/sqrt(c + d/x**2
) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5
)/d**2)/d, True))/2
```

3.967 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^7}\, dx$

Optimal. Leaf size=101

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

```
[Out] -1/3*c*(-2*a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4+1/5*(-a*d+3*b*c)*(c+d/x^2)^(5/2)/
d^4-1/7*b*(c+d/x^2)^(7/2)/d^4+c^2*(-a*d+b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(bc-ad)}{d^4}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(3bc-ad)}{5d^4}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-2ad)}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] (c^2*(b*c - a*d)*Sqrt[c + d/x^2])/d^4 - (c*(3*b*c - 2*a*d)*(c + d/x^2)^(3/2
))/(3*d^4) + ((3*b*c - a*d)*(c + d/x^2)^(5/2))/(5*d^4) - (b*(c + d/x^2)^(7/
2))/(7*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\, x^7}\, dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{\sqrt{c+dx}}\, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{c(3bc-2ad)\sqrt{c+dx}}{d^3}+\frac{(-3bc+ad)(c+dx)^{3/2}}{d^3}+\frac{b(c+dx)^{5/2}}{d^3}\right.\right.\right.\\
&= \frac{c^2(bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^4}-\frac{c(3bc-2ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^4}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 91, normalized size = 0.90

$$\frac{\left(\frac{cx^2}{d}+1\right)\left(8c^2x^4-4cdx^2+3d^2\right)(6bc-7ad)}{105d^3x^6\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(cx^2+d\right)}{7dx^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^7),x]
```

```
[Out] -1/7*(b*(d + c*x^2))/(d*Sqrt[c + d/x^2]*x^8) + ((6*b*c - 7*a*d)*(1 + (c*x^2
)/d)*(3*d^2 - 4*c*d*x^2 + 8*c^2*x^4))/(105*d^3*Sqrt[c + d/x^2]*x^6)
```

fricas [A] time = 0.62, size = 86, normalized size = 0.85

$$\frac{\left(8\left(6\,bc^3-7\,ac^2d\right)x^6-4\left(6\,bc^2d-7\,acd^2\right)x^4-15\,bd^3+3\left(6\,bcd^2-7\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{105\,d^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*b*c^3 - 7*a*c^2*d)*x^6 - 4*(6*b*c^2*d - 7*a*c*d^2)*x^4 - 15*b*d
^3 + 3*(6*b*c*d^2 - 7*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(d^4*x^6)
```

giac [B] time = 0.62, size = 219, normalized size = 2.17

$$\frac{140\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^4ac+210\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^3bc^{\frac{3}{2}}+105\left(\sqrt{c}\,x^2-\sqrt{cx^4+dx^2}\right)^3a\sqrt{c}\,d+252\left(\sqrt{c}\right.}{105\left(\sqrt{c}\,x^2-\sqrt{cx^4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(140*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^4*a*c + 210*(sqrt(c)*x^2 - s
qrt(c*x^4 + d*x^2))^3*b*c^(3/2) + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^3
*a*sqrt(c)*d + 252*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^2*b*c*d + 21*(sqrt(c
)*x^2 - sqrt(c*x^4 + d*x^2))^2*a*d^2 + 105*(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^
2))*b*sqrt(c)*d^2 + 15*b*d^3)/(sqrt(c)*x^2 - sqrt(c*x^4 + d*x^2))^7
```

maple [A] time = 0.05, size = 94, normalized size = 0.93

$$-\frac{\left(56a\,c^2d\,x^6-48b\,c^3x^6-28ac\,d^2x^4+24b\,c^2d\,x^4+21a\,d^3x^2-18bc\,d^2x^2+15b\,d^3\right)\left(c\,x^2+d\right)}{105\sqrt{\frac{c\,x^2+d}{x^2}}\;d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/105*(56*a*c^2*d*x^6-48*b*c^3*x^6-28*a*c*d^2*x^4+24*b*c^2*d*x^4+21*a*d^3*
x^2-18*b*c*d^2*x^2+15*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/d^4/x^8
```

maxima [A] time = 0.58, size = 118, normalized size = 1.17

$$-\frac{1}{35}b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^4}-\frac{21\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^4}+\frac{35\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^4}-\frac{35\sqrt{c+\frac{d}{x^2}}\,c^3}{d^4}\right)-\frac{1}{15}a\left(\frac{3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^3}-\frac{10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/x^7/(c+d/x^2)^(1/2),x, algorithm="maxima")

[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^4 - 21*(c + d/x^2)^(5/2)*c/d^4 + 35*(c + d/x^2)^(3/2)*c^2/d^4 - 35*sqrt(c + d/x^2)*c^3/d^4) - 1/15*a*(3*(c + d/x^2)^(5/2)/d^3 - 10*(c + d/x^2)^(3/2)*c/d^3 + 15*sqrt(c + d/x^2)*c^2/d^3)

mupad [B] time = 4.72, size = 102, normalized size = 1.01

$$\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3-56\,a\,c^2\,d\right)}{105\,d^4}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{7\,d\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(24\,b\,c^2-28\,a\,c\,d\right)}{105\,d^3\,x^2}-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(7\,a\,d-6\,b\,c\right)}{35\,d^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(1/2)),x)

[Out] ((c + d/x^2)^(1/2)*(48*b*c^3 - 56*a*c^2*d))/(105*d^4) - (b*(c + d/x^2)^(1/2))/(7*d*x^6) - ((c + d/x^2)^(1/2)*(24*b*c^2 - 28*a*c*d))/(105*d^3*x^2) - ((c + d/x^2)^(1/2)*(7*a*d - 6*b*c))/(35*d^2*x^4)

sympy [A] time = 18.00, size = 269, normalized size = 2.66

$$\frac{\begin{cases} \frac{-\frac{a}{3x^6}-\frac{b}{4x^8}}{\sqrt{c}} \\ \frac{\frac{2ac\left(\frac{c^2}{\sqrt{c+\frac{d}{x^2}}}+2c\sqrt{c+\frac{d}{x^2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3}\right)}{d^2}+\frac{2a\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{2bc\left(-\frac{c^3}{\sqrt{c+\frac{d}{x^2}}}-3c^2\sqrt{c+\frac{d}{x^2}}+c\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5}\right)}{d^3}+\frac{2b\left(\frac{c^4}{\sqrt{c+\frac{d}{x^2}}}+4c^3\right.}{}}{d} \end{cases}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x**2)/x**7/(c+d/x**2)**(1/2),x)

[Out] Piecewise(((-a/(3*x**6) - b/(4*x**8))/sqrt(c), Eq(d, 0)), ((2*a*c*(c**2/sqrt(c + d/x**2) + 2*c*sqrt(c + d/x**2) - (c + d/x**2)**(3/2)/3)/d**2 + 2*a*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**2 + 2*b*c*(-c**3/sqrt(c + d/x**2) - 3*c**2*sqrt(c + d/x**2) + c*(c + d/x**2)**(3/2) - (c + d/x**2)**(5/2)/5)/d**3 + 2*b*(c**4/sqrt(c + d/x**2) + 4*c**3*sqrt(c + d/x**2) - 2*c**2*(c + d/x**2)**(3/2) + 4*c*(c + d/x**2)**(5/2)/5 - (c + d/x**2)**(7/2)/7)/d**3)/d, True))/2

3.968 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=82

$$-\frac{2dx\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^3}+\frac{x^3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^2}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

```
[Out] -2/15*d*(-4*a*d+5*b*c)*x*(c+d/x^2)^(1/2)/c^3+1/15*(-4*a*d+5*b*c)*x^3*(c+d/x
^2)^(1/2)/c^2+1/5*a*x^5*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 271, 191}

$$\frac{x^3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^2}-\frac{2dx\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{15c^3}+\frac{ax^5\sqrt{c+\frac{d}{x^2}}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*(5*b*c - 4*a*d)*Sqrt[c + d/x^2]*x)/(15*c^3) + ((5*b*c - 4*a*d)*Sqrt[c
 + d/x^2]*x^3)/(15*c^2) + (a*Sqrt[c + d/x^2]*x^5)/(5*c)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\sqrt{c+\frac{d}{x^2}}}\,dx = \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} + \frac{(5bc-4ad)\int \frac{x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx}{5c}$$

$$= \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c} - \frac{(2d(5bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^2}$$

$$= -\frac{2d(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^3} + \frac{(5bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x^3}{15c^2} + \frac{a\sqrt{c+\frac{d}{x^2}}\,x^5}{5c}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.68

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(a\left(3c^2x^4-4cdx^2+8d^2\right)+5bc\left(cx^2-2d\right)\right)}{15c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[c + d/x^2]*x*(5*b*c*(-2*d + c*x^2) + a*(8*d^2 - 4*c*d*x^2 + 3*c^2*x^4
)))/(15*c^3)
```

fricas [A] time = 0.77, size = 59, normalized size = 0.72

$$\frac{\left(3\,ac^2x^5+\left(5\,bc^2-4\,acd\right)x^3-2\left(5\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^2*x^5 + (5*b*c^2 - 4*a*c*d)*x^3 - 2*(5*b*c*d - 4*a*d^2)*x)*sqrt
((c*x^2 + d)/x^2)/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 67, normalized size = 0.82

$$\frac{\left(3a\,x^4c^2-4acd\,x^2+5b\,c^2x^2+8a\,d^2-10bcd\right)\left(c\,x^2+d\right)}{15\sqrt{\frac{c\,x^2+d}{x^2}}\;c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] 1/15/x*(3*a*c^2*x^4-4*a*c*d*x^2+5*b*c^2*x^2+8*a*d^2-10*b*c*d)*(c*x^2+d)/((c
*x^2+d)/x^2)^(1/2)/c^3
```

maxima [A] time = 0.49, size = 85, normalized size = 1.04

$$\frac{\left(\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-3\sqrt{c+\frac{d}{x^2}}\,dx\right)b}{3\,c^2}+\frac{\left(3\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}x^5-10\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}dx^3+15\sqrt{c+\frac{d}{x^2}}\,d^2x\right)a}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)*b/c^2 + 1/15*(3*(c + d/
x^2)^(5/2)*x^5 - 10*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)*a/c
^3
```

mupad [B] time = 5.31, size = 53, normalized size = 0.65

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,\left(3\,a\,c^2\,x^4+5\,b\,c^2\,x^2-4\,a\,c\,d\,x^2-10\,b\,c\,d+8\,a\,d^2\right)}{15\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(8*a*d^2 + 3*a*c^2*x^4 + 5*b*c^2*x^2 - 10*b*c*d - 4*a*
c*d*x^2))/(15*c^3)
```

sympy [B] time = 3.55, size = 338, normalized size = 4.12

$$\frac{3ac^4d^{\frac{9}{2}}x^8\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{2ac^3d^{\frac{11}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{3ac^2d^{\frac{13}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{15c^5d^4x^4+30c^4d^5x^2+15c^3d^6}+\frac{12acd^{\frac{15}{2}}}{15c^5d^4x^4+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] 3*a*c**4*d**(9/2)*x**8*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5
*x**2 + 15*c**3*d**6) + 2*a*c**3*d**(11/2)*x**6*sqrt(c*x**2/d + 1)/(15*c**5
*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 3*a*c**2*d**(13/2)*x**4*sq
rt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x**2 + 15*c**3*d**6) + 1
2*a*c*d**(15/2)*x**2*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 + 30*c**4*d**5*x
**2 + 15*c**3*d**6) + 8*a*d**(17/2)*sqrt(c*x**2/d + 1)/(15*c**5*d**4*x**4 +
 30*c**4*d**5*x**2 + 15*c**3*d**6) + b*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c
) - 2*b*d**(3/2)*sqrt(c*x**2/d + 1)/(3*c**2)
```

3.969 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=51

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

```
[Out] 1/3*(-2*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^2+1/3*a*x^3*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {453, 191}

$$\frac{x\sqrt{c+\frac{d}{x^2}}\,(3bc-2ad)}{3c^2}+\frac{ax^3\sqrt{c+\frac{d}{x^2}}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] ((3*b*c - 2*a*d)*Sqrt[c + d/x^2]*x)/(3*c^2) + (a*Sqrt[c + d/x^2]*x^3)/(3*c)
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\sqrt{c+\frac{d}{x^2}}}\,dx&=\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}+\frac{(3bc-2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c}\\&=\frac{(3bc-2ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^2}+\frac{a\sqrt{c+\frac{d}{x^2}}\,x^3}{3c}\end{aligned}$$

Mathematica [A] time = 0.06, size = 34, normalized size = 0.67

$$\frac{x\sqrt{c+\frac{d}{x^2}}\left(acx^2-2ad+3bc\right)}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/Sqrt[c + d/x^2],x]
[Out] (Sqrt[c + d/x^2]*x*(3*b*c - 2*a*d + a*c*x^2))/(3*c^2)
```

fricas [A] time = 0.67, size = 36, normalized size = 0.71

$$\frac{\left(acx^3 + (3\,bc - 2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(a*c*x^3 + (3*b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 44, normalized size = 0.86

$$\frac{\left(a\,x^2c - 2ad + 3bc\right)\left(c\,x^2 + d\right)}{3\sqrt{\frac{c\,x^2+d}{x^2}}\;c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x)
[Out] 1/3/x*(a*c*x^2-2*a*d+3*b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(1/2)/c^2
```

maxima [A] time = 0.59, size = 49, normalized size = 0.96

$$\frac{b\sqrt{c + \frac{d}{x^2}}\,x}{c} + \frac{\left(\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}x^3 - 3\sqrt{c + \frac{d}{x^2}}\,dx\right)a}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
[Out] b*sqrt(c + d/x^2)*x/c + 1/3*((c + d/x^2)^(3/2)*x^3 - 3*sqrt(c + d/x^2)*d*x)
*a/c^2
```

mupad [B] time = 4.92, size = 67, normalized size = 1.31

$$\frac{a\,x^3\,\sqrt{c + \frac{d}{x^2}}\,\left(c - \frac{2\,d}{x^2}\right)}{3\,c^2} + \frac{b\,x\,\sqrt{\frac{c\,x^2}{d} + 1}}{\sqrt{c + \frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d} + 1} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x^3*(c + d/x^2)^(1/2)*(c - (2*d)/x^2))/(3*c^2) + (b*x*((c*x^2)/d + 1)^(1
/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
```

sympy [**A**] time = 2.76, size = 70, normalized size = 1.37

$$\frac{a\sqrt{d}\,x^2\sqrt{\frac{cx^2}{d}+1}}{3c}-\frac{2ad^{\frac{3}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^2}+\frac{b\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*x**2*sqrt(c*x**2/d + 1)/(3*c) - 2*a*d**(3/2)*sqrt(c*x**2/d + 1)/(
3*c**2) + b*sqrt(d)*sqrt(c*x**2/d + 1)/c
```

3.970 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=47

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

```
[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(1/2)+a*x*(c+d/x^2)^(1/2)/c
```

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {375, 451, 217, 206}

$$\frac{ax\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[c + d/x^2]*x)/c - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/Sqrt[d]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\operatorname{Subst}\left(\int \frac{a+bx^2}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\operatorname{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\operatorname{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)$$
$$= \frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{\sqrt{d}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.51

$$\frac{a\sqrt{d}\left(cx^2+d\right) - bc\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{c\sqrt{d}\,x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/Sqrt[c + d/x^2],x]
```

```
[Out] (a*Sqrt[d]*(d + c*x^2) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d
]])/(c*Sqrt[d]*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.74, size = 131, normalized size = 2.79

$$\left[\frac{2\,adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)}{2\,cd}, \frac{adx\sqrt{\frac{cx^2+d}{x^2}} + bc\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*d*x*sqrt((c*x^2 + d)/x^2) + b*c*sqrt(d)*log(-(c*x^2 - 2*sqrt(d)*x
*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c*d), (a*d*x*sqrt((c*x^2 + d)/x^2) + b
*c*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
```

```
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.05, size = 73, normalized size = 1.55

$$\frac{\sqrt{c\,x^2+d}\,\left(-bc\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+\sqrt{c\,x^2+d}\,a\sqrt{d}\right)}{\sqrt{\frac{c\,x^2+d}{x^2}}\,c\sqrt{d}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(1/2),x)
```

```
[Out] (c*x^2+d)^(1/2)*(a*(c*x^2+d)^(1/2)*d^(1/2)-b*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2
))/x)*c)/((c*x^2+d)/x^2)^(1/2)/x/c/d^(1/2)
```

maxima [A] time = 1.28, size = 58, normalized size = 1.23

$$\frac{a\sqrt{c+\frac{d}{x^2}}\,x}{c}+\frac{b\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a*sqrt(c + d/x^2)*x/c + 1/2*b*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d
/x^2)*x + sqrt(d)))/sqrt(d)
```

mupad [B] time = 5.00, size = 65, normalized size = 1.38

$$\frac{a\,x\,\sqrt{\frac{c\,x^2}{d}+1}}{\sqrt{c+\frac{d}{x^2}}\,\left(\sqrt{\frac{c\,x^2}{d}+1}+1\right)}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(1/2),x)
```

```
[Out] (a*x*((c*x^2)/d + 1)^(1/2))/((c + d/x^2)^(1/2)*(((c*x^2)/d + 1)^(1/2) + 1))
 - (b*log((c + d/x^2)^(1/2) + d^(1/2)/x))/d^(1/2)
```

sympy [A] time = 2.86, size = 39, normalized size = 0.83

$$\frac{a\sqrt{d}\,\sqrt{\frac{cx^2}{d}+1}}{c}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(1/2),x)
```

```
[Out] a*sqrt(d)*sqrt(c*x**2/d + 1)/c - b*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d)
```

3.971 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx$

Optimal. Leaf size=61

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)-1/2*b*(c+d/x^2)
^(1/2)/d/x
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {459, 335, 217, 206}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{3/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(2*d*x) + ((b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x
^2]*x)])/(2*d^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx = -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} + \frac{(-bc+2ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2}\,dx}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} - \frac{(-bc+2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} - \frac{(-bc+2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d}$$
$$= -\frac{b\sqrt{c+\frac{d}{x^2}}}{2dx} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{3/2}}$$

Mathematica [A] time = 0.06, size = 80, normalized size = 1.31

$$\frac{x^2\sqrt{cx^2+d}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right) - b\sqrt{d}\,\left(cx^2+d\right)}{2d^{3/2}x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^2),x]
```

```
[Out] (-(b*Sqrt[d]*(d + c*x^2)) + (b*c - 2*a*d)*x^2*Sqrt[d + c*x^2]*ArcTanh[Sqrt[
d + c*x^2]/Sqrt[d]])/(2*d^(3/2)*Sqrt[c + d/x^2]*x^3)
```

fricas [A] time = 0.71, size = 144, normalized size = 2.36

$$\left[-\frac{(bc-2\,ad)\sqrt{d}\,x\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\,bd\sqrt{\frac{cx^2+d}{x^2}}}{4\,d^2x}, -\frac{(bc-2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x\sqrt{\frac{cx^2+d}{x^2}}}{cx^2+d}\right)+bd\sqrt{\frac{cx^2+d}{x^2}}}{2\,d^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c - 2*a*d)*sqrt(d)*x*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x
^2) + 2*d)/x^2) + 2*b*d*sqrt((c*x^2 + d)/x^2))/(d^2*x), -1/2*((b*c - 2*a*d)
*sqrt(-d)*x*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + b*d*sqrt
((c*x^2 + d)/x^2))/(d^2*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
```

```
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.06, size = 105, normalized size = 1.72

$$-\frac{\sqrt{c\,x^2+d}\,\left(2a\,d^2x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)-bcd\,x^2\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\,\sqrt{d}}{x}\right)+\sqrt{c\,x^2+d}\,b\,d^{\frac{3}{2}}\right)}{2\sqrt{\frac{c\,x^2+d}{x^2}}\,d^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/2*(c*x^2+d)^(1/2)*(2*a*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*d^2-ln(2*
(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^2*b*c*d+(c*x^2+d)^(1/2)*d^(3/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^3/d^(5/2)
```

maxima [B] time = 1.18, size = 121, normalized size = 1.98

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^2}}\,cx}{\left(c+\frac{d}{x^2}\right)dx^2-d^2}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)b+\frac{a\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{2\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^2/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*b + 1/2*a*log((sqr
t(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/sqrt(d)
```

mupad [B] time = 5.53, size = 94, normalized size = 1.54

$$\begin{cases} -\frac{3\,a\,x^2+b}{3\,\sqrt{c}\,x^3} & \text{if } d=0 \\ \frac{b\,c\,\ln\left(2\,\sqrt{c+\frac{d}{x^2}}+\frac{2\,\sqrt{d}}{x}\right)}{2\,d^{3/2}}-\frac{b\,\sqrt{c+\frac{d}{x^2}}}{2\,d\,x}-\frac{a\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{\sqrt{d}} & \text{if } d\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(1/2)),x)
```

```
[Out] piecewise(d == 0, -(b + 3*a*x^2)/(3*c^(1/2)*x^3), d ~= 0, - (a*log((c + d/x
^2)^(1/2) + d^(1/2)/x))/d^(1/2) - (b*(c + d/x^2)^(1/2))/(2*d*x) + (b*c*log(
2*(c + d/x^2)^(1/2) + (2*d^(1/2))/x))/(2*d^(3/2)))
```

sympy [A] time = 4.92, size = 66, normalized size = 1.08

$$-\frac{a\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{\sqrt{d}}-\frac{b\sqrt{c}\,\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**2/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*asinh(sqrt(d)/(sqrt(c)*x))/sqrt(d) - b*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d
*x) + b*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(3/2))
```

3.972 $\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx$

Optimal. Leaf size=93

$$-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}+\frac{\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{8d^2x}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

```
[Out] -1/8*c*(-4*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/4*b*(c+d
/x^2)^(1/2)/d/x^3+1/8*(-4*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^2/x
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {459, 335, 321, 217, 206}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{5/2}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -(b*Sqrt[c + d/x^2])/(4*d*x^3) + ((3*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^2*x
) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
```

```
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx &= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(-3bc+4ad)\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^4}\,dx}{4d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}-\frac{(-3bc+4ad)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{4d}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{8d^2}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{(c(3bc-4ad))\,\mathrm{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^2}\\
&= -\frac{b\sqrt{c+\frac{d}{x^2}}}{4dx^3}+\frac{(3bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^2x}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{8d^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 107, normalized size = 1.15

$$-\frac{\left(cx^2+d\right)\left(d\sqrt{\frac{cx^2}{d}+1}\left(4adx^2-3bcx^2+2bd\right)+cx^4(3bc-4ad)\tanh^{-1}\left(\sqrt{\frac{cx^2}{d}+1}\right)\right)}{8d^3x^5\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{cx^2}{d}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(Sqrt[c + d/x^2]*x^4),x]
```

```
[Out] -1/8*((d + c*x^2)*(d*Sqrt[1 + (c*x^2)/d]*(2*b*d - 3*b*c*x^2 + 4*a*d*x^2) +
c*(3*b*c - 4*a*d)*x^4*ArcTanh[Sqrt[1 + (c*x^2)/d]]))/(d^3*Sqrt[c + d/x^2]*x
^5*Sqrt[1 + (c*x^2)/d])
```

fricas [A] time = 0.80, size = 201, normalized size = 2.16

$$\left[-\frac{\left(3\,bc^2-4\,acd\right)\sqrt{d}\,x^3\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(2\,bd^2-\left(3\,bcd-4\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{16\,d^3x^3},\ \frac{\left(3\,bc^2-4\,acd\right)\sqrt{-d}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b*c^2 - 4*a*c*d)*sqrt(d)*x^3*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x
^2 + d)/x^2) + 2*d)/x^2) + 2*(2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^2)*sqrt((c*x^
2 + d)/x^2))/(d^3*x^3), 1/8*((3*b*c^2 - 4*a*c*d)*sqrt(-d)*x^3*arctan(sqrt(-
```

```
d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) - (2*b*d^2 - (3*b*c*d - 4*a*d^2)*x^
2)*sqrt((c*x^2 + d)/x^2))/(d^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 146, normalized size = 1.57

$$-\frac{\sqrt{c\,x^{2}+d}\,\left(-4ac\,d^{2}x^{4}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)+3b\,c^{2}d\,x^{4}\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\,\sqrt{d}}{x}\right)+4\sqrt{c\,x^{2}+d}\,a\,d^{\frac{5}{2}}x^{2}-3\sqrt{c\,x^{2}+d}\,bc\,d\cdots\right)}{8\sqrt{\frac{c\,x^{2}+d}{x^{2}}}\,d^{\frac{7}{2}}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^2+d)^(1/2)*(-4*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*a*c*d^2+3*
ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2*d+4*(c*x^2+d)^(1/2)*d^(5/2)*x
^2*a-3*(c*x^2+d)^(1/2)*d^(3/2)*x^2*b*c+2*(c*x^2+d)^(1/2)*d^(5/2)*b)/((c*x^2
+d)/x^2)^(1/2)/x^5/d^(7/2)
```

maxima [B] time = 1.23, size = 200, normalized size = 2.15

$$-\frac{1}{4}\left(\frac{2\sqrt{c+\frac{d}{x^{2}}}\,cx}{\left(c+\frac{d}{x^{2}}\right)dx^{2}-d^{2}}+\frac{c\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}\right)a+\frac{1}{16}\,b\left(\frac{3\,c^{2}\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}+\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}c^{2}x^{3}-5\sqrt{c+\frac{d}{x^{2}}}\,c^{2}d\cdots\right)}{\left(c+\frac{d}{x^{2}}\right)^{2}d^{2}x^{4}-2\left(c+\frac{d}{x^{2}}\right)d^{3}x^{2}+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/x^4/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(c + d/x^2)*c*x/((c + d/x^2)*d*x^2 - d^2) + c*log((sqrt(c + d/x
^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(3/2))*a + 1/16*b*(3*c^2*
log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2) +
2*(3*(c + d/x^2)^(3/2)*c^2*x^3 - 5*sqrt(c + d/x^2)*c^2*d*x)/((c + d/x^2)^2*
d^2*x^4 - 2*(c + d/x^2)*d^3*x^2 + d^4))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^{2}}}{x^{4}\,\sqrt{c+\frac{d}{x^{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)),x)
```

```
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(1/2)), x)
```

sympy [A] time = 8.46, size = 150, normalized size = 1.61

$$-\frac{a\sqrt{c}\sqrt{1+\frac{d}{cx^2}}}{2dx}+\frac{ac\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{3}{2}}}+\frac{3bc^{\frac{3}{2}}}{8d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{b\sqrt{c}}{8dx^3\sqrt{1+\frac{d}{cx^2}}}-\frac{3bc^2\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{8d^{\frac{5}{2}}}-\frac{b}{4\sqrt{c}\,x^5\sqrt{1+\frac{d}{cx^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/x**4/(c+d/x**2)**(1/2),x)
```

```
[Out] -a*sqrt(c)*sqrt(1 + d/(c*x**2))/(2*d*x) + a*c*asinh(sqrt(d)/(sqrt(c)*x))/(2
*d**(3/2)) + 3*b*c**(3/2)/(8*d**2*x*sqrt(1 + d/(c*x**2))) + b*sqrt(c)/(8*d*
x**3*sqrt(1 + d/(c*x**2))) - 3*b*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(5/2
)) - b/(4*sqrt(c)*x**5*sqrt(1 + d/(c*x**2)))
```

3.973 $\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{3d(4bc-5ad)}{8c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^2(4bc-5ad)}{8c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -3/8*d*(-5*a*d+4*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(7/2)+3/8*d*(-5*a*
d+4*b*c)/c^3/(c+d/x^2)^(1/2)+1/8*(-5*a*d+4*b*c)*x^2/c^2/(c+d/x^2)^(1/2)+1/4
*a*x^4/c/(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 78, 51, 63, 208}

$$\frac{3x^2\sqrt{c+\frac{d}{x^2}}(4bc-5ad)}{8c^3}-\frac{x^2(4bc-5ad)}{4c^2\sqrt{c+\frac{d}{x^2}}}-\frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}+\frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((4*b*c - 5*a*d)*x^2)/(4*c^2*Sqrt[c + d/x^2]) + (3*(4*b*c - 5*a*d)*Sqrt[c
+ d/x^2]*x^2)/(8*c^3) + (a*x^4)/(4*c*Sqrt[c + d/x^2]) - (3*d*(4*b*c - 5*a*d
)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(8*c^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^3}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x^3(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)\right)$$

$$= \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(2bc-\frac{5ad}{2}\right)\text{Subst}\left(\int \frac{1}{x^2(c+dx)^{3/2}}\,dx,x,\frac{1}{x^2}\right)}{4c}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{(3(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{c+dx}}\,dx,x,\frac{1}{x^2}\right)}{8c^2}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3d(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+}}\right.}{16c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} + \frac{(3(4bc-5ad))\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\right.}{8c^3}$$

$$= -\frac{(4bc-5ad)x^2}{4c^2\sqrt{c+\frac{d}{x^2}}} + \frac{3(4bc-5ad)\sqrt{c+\frac{d}{x^2}}\,x^2}{8c^3} + \frac{ax^4}{4c\sqrt{c+\frac{d}{x^2}}} - \frac{3d(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8c^{7/2}}$$

Mathematica [A] time = 0.19, size = 111, normalized size = 0.94

$$\frac{\sqrt{c}\,x\left(a\left(2c^2x^4-5cdx^2-15d^2\right)+4bc\left(cx^2+3d\right)\right)+3d^{3/2}\sqrt{\frac{cx^2}{d}+1}\,(5ad-4bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{7/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^3)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(4*b*c*(3*d + c*x^2) + a*(-15*d^2 - 5*c*d*x^2 + 2*c^2*x^4)) + 3*
d^(3/2)*(-4*b*c + 5*a*d)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/
(8*c^(7/2)*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.83, size = 304, normalized size = 2.58

$$\left[-\frac{3\left(4\,bcd^2-5\,ad^3+\left(4\,bc^2d-5\,acd^2\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(2\,ac^3x^6+\left(4\,bc^3-5\,ac^2d\right)x^4\right.}{16\left(c^5x^2+c^4d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(c)*log(-
2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(2*a*c^3*x^6 + (4*b*
c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt((c*x^2 + d)/x^2)
)/(c^5*x^2 + c^4*d), 1/8*(3*(4*b*c*d^2 - 5*a*d^3 + (4*b*c^2*d - 5*a*c*d^2)*
x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (2*a
*c^3*x^6 + (4*b*c^3 - 5*a*c^2*d)*x^4 + 3*(4*b*c^2*d - 5*a*c*d^2)*x^2)*sqrt(
(c*x^2 + d)/x^2))/(c^5*x^2 + c^4*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.06, size = 140, normalized size = 1.19

$$\frac{\left(c\,x^2+d\right)\left(2a\,c^{\frac{7}{2}}x^5-5a\,c^{\frac{5}{2}}d\,x^3+4b\,c^{\frac{7}{2}}x^3-15a\,c^{\frac{3}{2}}d^2x+12b\,c^{\frac{5}{2}}dx+15\sqrt{c\,x^2+d}\,\,ac\,d^2\ln\left(\sqrt{c}\,\,x+\sqrt{c\,x^2+d}\,\right)-12\right.}{8\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^{\frac{9}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/8*(c*x^2+d)*(2*c^(7/2)*x^5*a-5*c^(5/2)*x^3*a*d+4*c^(7/2)*x^3*b-15*c^(3/2)
*x*a*d^2+12*c^(5/2)*x*b*d+15*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*
a*c*d^2-12*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*b*c^2*d)/((c*x^2+d
)/x^2)^(3/2)/x^3/c^(9/2)
```

maxima [B] time = 1.38, size = 215, normalized size = 1.82

$$-\frac{1}{16}\,a\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2d^2-25\left(c+\frac{d}{x^2}\right)cd^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c^3-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^4+\sqrt{c+\frac{d}{x^2}}\,c^5}+\frac{15\,d^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^2}}+\sqrt{c}}\right)}{c^{\frac{7}{2}}}\right)+\frac{1}{4}\,b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2-\sqrt{c+\frac{d}{x^2}}\,c^3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^3/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*a*(2*(15*(c + d/x^2)^2*d^2 - 25*(c + d/x^2)*c*d^2 + 8*c^2*d^2)/((c +
d/x^2)^(5/2)*c^3 - 2*(c + d/x^2)^(3/2)*c^4 + sqrt(c + d/x^2)*c^5) + 15*d^2*
log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(7/2)) + 1/4
*b*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)*c^
3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(5/
2))
```

mupad [B] time = 6.34, size = 134, normalized size = 1.14

$$\frac{a\,x^4}{4\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{15\,a\,d^2}{8\,c^3\,\sqrt{c+\frac{d}{x^2}}}+\frac{b\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,b\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{15\,a\,d^2\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{8\,c^{7/2}}+\frac{3\,b\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}-\frac{5\,a}{8\,c^2\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (a*x^4)/(4*c*(c + d/x^2)^(1/2)) - (15*a*d^2)/(8*c^3*(c + d/x^2)^(1/2)) + (b
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*b*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (15*a*d^2*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(8*c^(7/2)) + (3*b*
d)/(2*c^2*(c + d/x^2)^(1/2)) - (5*a*d*x^2)/(8*c^2*(c + d/x^2)^(1/2))
```

sympy [A] time = 103.90, size = 177, normalized size = 1.50

$$a\left(\frac{x^5}{4c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}-\frac{5\sqrt{d}\,x^3}{8c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{15d^{\frac{3}{2}}x}{8c^3\sqrt{\frac{cx^2}{d}+1}}+\frac{15d^2\operatorname{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{8c^{\frac{7}{2}}}\right)+b\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**3/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**5/(4*c*sqrt(d)*sqrt(c*x**2/d + 1)) - 5*sqrt(d)*x**3/(8*c**2*sqrt(c*x*
*2/d + 1)) - 15*d**(3/2)*x/(8*c**3*sqrt(c*x**2/d + 1)) + 15*d**2*asinh(sqrt
(c)*x/sqrt(d))/(8*c**(7/2))) + b*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3
*sqrt(d)*x/(2*c**2*sqrt(c*x**2/d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c*
*(5/2)))
```

3.974 $\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] 1/2*(-3*a*d+2*b*c)*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(5/2)+1/2*(3*a*d-2*b*
c)/c^2/(c+d/x^2)^(1/2)+1/2*a*x^2/c/(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {446, 78, 51, 63, 208}

$$-\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}+\frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] -(2*b*c - 3*a*d)/(2*c^2*Sqrt[c + d/x^2]) + (a*x^2)/(2*c*Sqrt[c + d/x^2]) +
((2*b*c - 3*a*d)*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/(2*c^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{a+bx}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{\left(bc-\frac{3ad}{2}\right)\mathrm{Subst}\left(\int \frac{1}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{4c^2}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} - \frac{(2bc-3ad)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{2c^2d}$$

$$= -\frac{2bc-3ad}{2c^2\sqrt{c+\frac{d}{x^2}}} + \frac{ax^2}{2c\sqrt{c+\frac{d}{x^2}}} + \frac{(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2c^{5/2}}$$

Mathematica [A] time = 0.22, size = 89, normalized size = 1.03

$$\frac{\sqrt{c}\,x\left(acx^2+3ad-2bc\right)-\sqrt{d}\sqrt{\frac{cx^2}{d}+1}\,(3ad-2bc)\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{5/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x)/(c + d/x^2)^(3/2),x]
```

```
[Out] (Sqrt[c]*x*(-2*b*c + 3*a*d + a*c*x^2) - Sqrt[d]*(-2*b*c + 3*a*d)*Sqrt[1 + (
c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/Sqrt[d]])/(2*c^(5/2)*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.73, size = 249, normalized size = 2.90

$$\left[-\frac{\left(2\,bcd-3\,ad^2+\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{c}\,\log\left(-2\,cx^2+2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)-2\left(ac^2x^4-\left(2\,bc^2-3\,acd\right)x^2\right)\sqrt{\frac{c}{}}}{4\left(c^4x^2+c^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*b*c*d - 3*a*d^2 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(c)*log(-2*c*x^2 +
 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d) - 2*(a*c^2*x^4 - (2*b*c^2 - 3*a*c
*d)*x^2)*sqrt((c*x^2 + d)/x^2))/(c^4*x^2 + c^3*d), -1/2*((2*b*c*d - 3*a*d^2
 + (2*b*c^2 - 3*a*c*d)*x^2)*sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x
^2)/(c*x^2 + d)) - (a*c^2*x^4 - (2*b*c^2 - 3*a*c*d)*x^2)*sqrt((c*x^2 + d)/x
^2))/(c^4*x^2 + c^3*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.06, size = 115, normalized size = 1.34

$$-\frac{\left(c\,x^{2}+d\right)\left(-a\,c^{\frac{5}{2}}x^{3}-3a\,c^{\frac{3}{2}}dx+2b\,c^{\frac{5}{2}}x+3\sqrt{c\,x^{2}+d}\;acd\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+d}\right)-2\sqrt{c\,x^{2}+d}\;b\,c^{2}\ln\left(\sqrt{c}\;x+\sqrt{}\right.\right.}{2\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{\frac{7}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x/(c+d/x^2)^(3/2),x)
```

```
[Out] -1/2*(c*x^2+d)*(-c^(5/2)*x^3*a-3*c^(3/2)*x*a*d+2*c^(5/2)*x*b+3*(c*x^2+d)^(1
/2)*ln(c^(1/2)*x+(c*x^2+d)^(1/2))*a*c*d-2*(c*x^2+d)^(1/2)*ln(c^(1/2)*x+(c*x
^2+d)^(1/2))*b*c^2)/((c*x^2+d)/x^2)^(3/2)/x^3/c^(7/2)
```

maxima [B] time = 1.20, size = 144, normalized size = 1.67

$$\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^{2}}\right)d-2\,cd\right)}{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}c^{2}-\sqrt{c+\frac{d}{x^{2}}}\,c^{3}}+\frac{3\,d\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*a*(2*(3*(c + d/x^2)*d - 2*c*d)/((c + d/x^2)^(3/2)*c^2 - sqrt(c + d/x^2)
*c^3) + 3*d*log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^
(5/2)) - 1/2*b*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c))
)/c^(3/2) + 2/(sqrt(c + d/x^2)*c))
```

mupad [B] time = 5.61, size = 90, normalized size = 1.05

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{b}{c\,\sqrt{c+\frac{d}{x^2}}}+\frac{a\,x^2}{2\,c\,\sqrt{c+\frac{d}{x^2}}}-\frac{3\,a\,d\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{2\,c^{5/2}}+\frac{3\,a\,d}{2\,c^2\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (b*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - b/(c*(c + d/x^2)^(1/2)) + (a
*x^2)/(2*c*(c + d/x^2)^(1/2)) - (3*a*d*atanh((c + d/x^2)^(1/2)/c^(1/2)))/(2
*c^(5/2)) + (3*a*d)/(2*c^2*(c + d/x^2)^(1/2))
```

sympy [B] time = 52.42, size = 264, normalized size = 3.07

$$a\left(\frac{x^3}{2c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{3\sqrt{d}\,x}{2c^2\sqrt{\frac{cx^2}{d}+1}}-\frac{3d\,\mathrm{asinh}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{2c^{\frac{5}{2}}}\right)+b\left(-\frac{2c^3x^2\sqrt{1+\frac{d}{cx^2}}}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}-\frac{c^3x^2\log\left(\frac{d}{cx^2}\right)}{2c^{\frac{9}{2}}x^2+2c^{\frac{7}{2}}d}+\frac{2c^3x^2\log\left(\sqrt{1\right.}}{2c^{\frac{9}{2}}x^2+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**3/(2*c*sqrt(d)*sqrt(c*x**2/d + 1)) + 3*sqrt(d)*x/(2*c**2*sqrt(c*x**2/
d + 1)) - 3*d*asinh(sqrt(c)*x/sqrt(d))/(2*c**(5/2))) + b*(-2*c**3*x**2*sqrt
(1 + d/(c*x**2))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**3*x**2*log(d/(c*x**2
))/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) + 2*c**3*x**2*log(sqrt(1 + d/(c*x**2))
+ 1)/(2*c**(9/2)*x**2 + 2*c**(7/2)*d) - c**2*d*log(d/(c*x**2))/(2*c**(9/2)*
x**2 + 2*c**(7/2)*d) + 2*c**2*d*log(sqrt(1 + d/(c*x**2)) + 1)/(2*c**(9/2)*x
**2 + 2*c**(7/2)*d))
```

3.975 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x} \, dx$

Optimal. Leaf size=52

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

```
[Out] a*arctanh((c+d/x^2)^(1/2)/c^(1/2))/c^(3/2)+(-a*d+b*c)/c/d/(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {446, 78, 63, 208}

$$\frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]) + (a*ArcTanh[Sqrt[c + d/x^2]/Sqrt[c]])/c^
(3/2)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{a+bx}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx, x, \frac{1}{x^2}\right)}{2c}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} - \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+\frac{d}{x^2}}\right)}{cd}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{c+\frac{d}{x^2}}}{\sqrt{c}}\right)}{c^{3/2}}$$

Mathematica [A] time = 0.09, size = 73, normalized size = 1.40

$$\frac{\sqrt{c}\,x(bc-ad)+ad^{3/2}\sqrt{\frac{cx^2}{d}+1}\,\sinh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)}{c^{3/2}dx\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x),x]
```

```
[Out] (Sqrt[c]*(b*c - a*d)*x + a*d^(3/2)*Sqrt[1 + (c*x^2)/d]*ArcSinh[(Sqrt[c]*x)/
Sqrt[d]])/(c^(3/2)*d*Sqrt[c + d/x^2]*x)
```

fricas [B] time = 0.64, size = 200, normalized size = 3.85

$$\left[\frac{2\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}+\left(acdx^2+ad^2\right)\sqrt{c}\,\log\left(-2\,cx^2-2\,\sqrt{c}\,x^2\sqrt{\frac{cx^2+d}{x^2}}-d\right)}{2\left(c^3dx^2+c^2d^2\right)}, \frac{\left(bc^2-acd\right)x^2\sqrt{\frac{cx^2+d}{x^2}}-\left(acdx^2\right.}{c^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) + (a*c*d*x^2 + a*d^2)*sqr
t(c)*log(-2*c*x^2 - 2*sqrt(c)*x^2*sqrt((c*x^2 + d)/x^2) - d))/(c^3*d*x^2 +
c^2*d^2), ((b*c^2 - a*c*d)*x^2*sqrt((c*x^2 + d)/x^2) - (a*c*d*x^2 + a*d^2)*
sqrt(-c)*arctan(sqrt(-c)*x^2*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^3*d*x^2
 + c^2*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{-2,[1]%%%},[2
,1,2]%%%}+%%%{%%{[4,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1,3]%%%}+%%%{-2,[0,1,4]%%
%} / %%%{%%%{1,[2]%%%},[2,0,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[1,0,1]%%%}+%%%{%%%{1,[1]%%%},[0,0,2]%%%} Error: Bad Argument Val
ue
```

maple [A] time = 0.06, size = 75, normalized size = 1.44

$$\frac{\left(c\,x^{2}+d\right)\left(-a\,c^{\frac{3}{2}}dx+b\,c^{\frac{5}{2}}x+\sqrt{c\,x^{2}+d}\;acd\ln\left(\sqrt{c}\;x+\sqrt{c\,x^{2}+d}\right)\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{\frac{5}{2}}d\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x,x)
```

```
[Out] (c*x^2+d)*(-a*c^(3/2)*d*x+b*c^(5/2)*x+(c*x^2+d)^(1/2)*a*c*d*ln(c^(1/2)*x+(c
*x^2+d)^(1/2)))/((c*x^2+d)/x^2)^(3/2)/x^3/c^(5/2)/d
```

maxima [A] time = 1.26, size = 69, normalized size = 1.33

$$-\frac{1}{2}\,a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}-\sqrt{c}}{\sqrt{c+\frac{d}{x^{2}}}+\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,c}\right)+\frac{b}{\sqrt{c+\frac{d}{x^{2}}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*(log((sqrt(c + d/x^2) - sqrt(c))/(sqrt(c + d/x^2) + sqrt(c)))/c^(3/2
) + 2/(sqrt(c + d/x^2)*c)) + b/(sqrt(c + d/x^2)*d)
```

mupad [B] time = 5.06, size = 54, normalized size = 1.04

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{c}}\right)}{c^{3/2}}-\frac{a}{c\,\sqrt{c+\frac{d}{x^{2}}}}+\frac{b\,\sqrt{x^{2}}}{d\,\sqrt{c\,x^{2}+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x*(c + d/x^2)^(3/2)),x)
```

```
[Out] (a*atanh((c + d/x^2)^(1/2)/c^(1/2)))/c^(3/2) - a/(c*(c + d/x^2)^(1/2)) + (b
*(x^2)^(1/2))/(d*(d + c*x^2)^(1/2))
```

sympy [A] time = 20.33, size = 49, normalized size = 0.94

$$-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{c+\frac{d}{x^{2}}}}{\sqrt{-c}}\right)}{c\sqrt{-c}}-\frac{ad-bc}{cd\sqrt{c+\frac{d}{x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x,x)
```

```
[Out] -a*atan(sqrt(c + d/x**2)/sqrt(-c))/(c*sqrt(-c)) - (a*d - b*c)/(c*d*sqrt(c +
 d/x**2))
```

3.976 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx$

Optimal. Leaf size=42

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

```
[Out] (a*d-b*c)/d^2/(c+d/x^2)^(1/2)-b*(c+d/x^2)^(1/2)/d^2
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {444, 43}

$$-\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
```

```
[Out] -((b*c - a*d)/(d^2*Sqrt[c + d/x^2])) - (b*Sqrt[c + d/x^2])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^3} \, dx &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{a+bx}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right) dx, x, \frac{1}{x^2}\right)\right)\\
&= -\frac{bc-ad}{d^2\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{c+\frac{d}{x^2}}}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.86

$$\frac{adx^2-b\left(2cx^2+d\right)}{d^2x^2\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^3),x]
[Out] (a*d*x^2 - b*(d + 2*c*x^2))/(d^2*Sqrt[c + d/x^2]*x^2)
```

fricas [A] time = 0.70, size = 46, normalized size = 1.10

$$-\frac{\left((2\,bc - ad)x^2 + bd\right)\sqrt{\frac{cx^2+d}{x^2}}}{cd^2x^2 + d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="fricas")
[Out] -((2*b*c - a*d)*x^2 + b*d)*sqrt((c*x^2 + d)/x^2)/(c*d^2*x^2 + d^3)
```

giac [A] time = 0.26, size = 37, normalized size = 0.88

$$-\frac{\frac{(2\,bc-ad)x^2}{d^2} + \frac{b}{d}}{\sqrt{cx^4 + dx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="giac")
[Out] -((2*b*c - a*d)*x^2/d^2 + b/d)/sqrt(c*x^4 + d*x^2)
```

maple [A] time = 0.05, size = 46, normalized size = 1.10

$$\frac{\left(ad\,x^2 - 2bc\,x^2 - bd\right)\left(c\,x^2 + d\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x)
[Out] (a*d*x^2-2*b*c*x^2-b*d)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^2/x^4
```

maxima [A] time = 0.46, size = 46, normalized size = 1.10

$$-b\left(\frac{\sqrt{c + \frac{d}{x^2}}}{d^2} + \frac{c}{\sqrt{c + \frac{d}{x^2}}\,d^2}\right) + \frac{a}{\sqrt{c + \frac{d}{x^2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^3,x, algorithm="maxima")
[Out] -b*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2)) + a/(sqrt(c + d/x^2)*d)
```

mupad [B] time = 4.52, size = 46, normalized size = 1.10

$$\frac{x\,\sqrt{c + \frac{d}{x^2}}\,\left(x^2\,\left(\frac{a}{d} - \frac{2\,b\,c}{d^2}\right) - \frac{b}{d}\right)}{c\,x^3 + d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^3*(c + d/x^2)^(3/2)),x)
```

```
[Out] (x*(c + d/x^2)^(1/2)*(x^2*(a/d - (2*b*c)/d^2) - b/d))/(d*x + c*x^3)
```

sympy [A] time = 3.42, size = 68, normalized size = 1.62

$$\begin{cases} \frac{a}{d\sqrt{c+\frac{d}{x^2}}} - \frac{2bc}{d^2\sqrt{c+\frac{d}{x^2}}} - \frac{b}{dx^2\sqrt{c+\frac{d}{x^2}}} & \text{for } d \neq 0 \\ \frac{-\frac{a}{2x^2} - \frac{b}{4x^4}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**3,x)
```

```
[Out] Piecewise((a/(d*sqrt(c + d/x**2)) - 2*b*c/(d**2*sqrt(c + d/x**2)) - b/(d*x*
*2*sqrt(c + d/x**2)), Ne(d, 0)), ((-a/(2*x**2) - b/(4*x**4))/c**(3/2), True
))
```

3.977 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx$

Optimal. Leaf size=68

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

```
[Out] -1/3*b*(c+d/x^2)^(3/2)/d^3+c*(-a*d+b*c)/d^3/(c+d/x^2)^(1/2)+(-a*d+2*b*c)*(c
+d/x^2)^(1/2)/d^3
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{\sqrt{c+\frac{d}{x^2}}\,(2bc-ad)}{d^3}+\frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (c*(b*c - a*d))/(d^3*Sqrt[c + d/x^2]) + ((2*b*c - a*d)*Sqrt[c + d/x^2])/d^3
 - (b*(c + d/x^2)^(3/2))/(3*d^3)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^5} \, dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{x(a+bx)}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)}{d^2(c+dx)^{3/2}}+\frac{-2bc+ad}{d^2\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d^2}\right) dx, x, \frac{1}{x^2}\right)\right)\\
&= \frac{c(bc-ad)}{d^3\sqrt{c+\frac{d}{x^2}}}+\frac{(2bc-ad)\sqrt{c+\frac{d}{x^2}}}{d^3}-\frac{b\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.88

$$\frac{b\left(8c^2x^4+4cdx^2-d^2\right)-3adx^2\left(2cx^2+d\right)}{3d^3x^4\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^5),x]
```

```
[Out] (-3*a*d*x^2*(d + 2*c*x^2) + b*(-d^2 + 4*c*d*x^2 + 8*c^2*x^4))/(3*d^3*Sqrt[c
 + d/x^2]*x^4)
```

fricas [A] time = 0.60, size = 73, normalized size = 1.07

$$\frac{\left(2\left(4\,bc^2-3\,acd\right)x^4-bd^2+\left(4\,bcd-3\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(cd^3x^4+d^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(4*b*c^2 - 3*a*c*d)*x^4 - b*d^2 + (4*b*c*d - 3*a*d^2)*x^2)*sqrt((c*x
^2 + d)/x^2)/(c*d^3*x^4 + d^4*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^5), x)
```

maple [A] time = 0.05, size = 69, normalized size = 1.01

$$-\frac{\left(6acd\,x^4-8b\,c^2x^4+3a\,d^2x^2-4bcd\,x^2+b\,d^2\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x)
```

```
[Out] -1/3*(6*a*c*d*x^4-8*b*c^2*x^4+3*a*d^2*x^2-4*b*c*d*x^2+b*d^2)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^3/x^6
```

maxima [A] time = 0.61, size = 81, normalized size = 1.19

$$-\frac{1}{3}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\,\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)-a\left(\frac{\sqrt{c+\frac{d}{x^2}}}{d^2}+\frac{c}{\sqrt{c+\frac{d}{x^2}}\,d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/3*b*((c + d/x^2)^(3/2)/d^3 - 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d
/x^2)*d^3)) - a*(sqrt(c + d/x^2)/d^2 + c/(sqrt(c + d/x^2)*d^2))
```

mupad [B] time = 4.64, size = 66, normalized size = 0.97

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(-8\,b\,c^2\,x^4+6\,a\,c\,d\,x^4-4\,b\,c\,d\,x^2+3\,a\,d^2\,x^2+b\,d^2\right)}{3\,d^3\,x^2\,\left(c\,x^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^5*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(b*d^2 + 3*a*d^2*x^2 - 8*b*c^2*x^4 + 6*a*c*d*x^4 - 4*b*
c*d*x^2))/(3*d^3*x^2*(d + c*x^2))
```

sympy [A] time = 11.07, size = 61, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{3d^3}-\frac{c\,(ad-bc)}{d^3\sqrt{c+\frac{d}{x^2}}}-\frac{\sqrt{c+\frac{d}{x^2}}\,(ad-2bc)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**5,x)
```

```
[Out] -b*(c + d/x**2)**(3/2)/(3*d**3) - c*(a*d - b*c)/(d**3*sqrt(c + d/x**2)) - s
qrt(c + d/x**2)*(a*d - 2*b*c)/d**3
```

3.978 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx$

Optimal. Leaf size=100

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

```
[Out] 1/3*(-a*d+3*b*c)*(c+d/x^2)^(3/2)/d^4-1/5*b*(c+d/x^2)^(5/2)/d^4-c^2*(-a*d+b*
c)/d^4/(c+d/x^2)^(1/2)-c*(-2*a*d+3*b*c)*(c+d/x^2)^(1/2)/d^4
```

Rubi [A] time = 0.07, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{3/2}(3bc-ad)}{3d^4}-\frac{c\sqrt{c+\frac{d}{x^2}}(3bc-2ad)}{d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] -((c^2*(b*c - a*d))/(d^4*Sqrt[c + d/x^2])) - (c*(3*b*c - 2*a*d)*Sqrt[c + d/
x^2])/d^4 + ((3*b*c - a*d)*(c + d/x^2)^(3/2))/(3*d^4) - (b*(c + d/x^2)^(5/2
))/(5*d^4)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^7} \, dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx)}{(c+dx)^{3/2}} \, dx, x, \frac{1}{x^2}\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{c^2(bc-ad)}{d^3(c+dx)^{3/2}}+\frac{c(3bc-2ad)}{d^3\sqrt{c+dx}}+\frac{(-3bc+ad)\sqrt{c+dx}}{d^3}+\frac{b(c+dx)^{3/2}}{d^3}\right) dx,\right.\right.\\
&= -\frac{c^2(bc-ad)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{c(3bc-2ad)\sqrt{c+\frac{d}{x^2}}}{d^4}+\frac{(3bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{3d^4}-\frac{b\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^4}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.81

$$\frac{-5adx^2\left(-8c^2x^4-4cdx^2+d^2\right)-3b\left(16c^3x^6+8c^2dx^4-2cd^2x^2+d^3\right)}{15d^4x^6\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^7),x]
```

```
[Out] (-5*a*d*x^2*(d^2 - 4*c*d*x^2 - 8*c^2*x^4) - 3*b*(d^3 - 2*c*d^2*x^2 + 8*c^2*
d*x^4 + 16*c^3*x^6))/(15*d^4*Sqrt[c + d/x^2]*x^6)
```

fricas [A] time = 0.72, size = 98, normalized size = 0.98

$$-\frac{\left(8\left(6\,bc^3-5\,ac^2d\right)x^6+4\left(6\,bc^2d-5\,acd^2\right)x^4+3\,bd^3-\left(6\,bcd^2-5\,ad^3\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(cd^4x^6+d^5x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/15*(8*(6*b*c^3 - 5*a*c^2*d)*x^6 + 4*(6*b*c^2*d - 5*a*c*d^2)*x^4 + 3*b*d^
3 - (6*b*c*d^2 - 5*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2)/(c*d^4*x^6 + d^5*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^7), x)
```

maple [A] time = 0.05, size = 94, normalized size = 0.94

$$\frac{\left(40a\,c^2d\,x^6-48b\,c^3x^6+20ac\,d^2x^4-24b\,c^2d\,x^4-5a\,d^3x^2+6bc\,d^2x^2-3b\,d^3\right)\left(c\,x^2+d\right)}{15\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x)
```

```
[Out] 1/15*(40*a*c^2*d*x^6-48*b*c^3*x^6+20*a*c*d^2*x^4-24*b*c^2*d*x^4-5*a*d^3*x^2
+6*b*c*d^2*x^2-3*b*d^3)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/d^4/x^8
```

maxima [A] time = 0.62, size = 116, normalized size = 1.16

$$-\frac{1}{5}\,b\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c}{d^4}+\frac{15\sqrt{c+\frac{d}{x^2}}\,c^2}{d^4}+\frac{5\,c^3}{\sqrt{c+\frac{d}{x^2}}\,d^4}\right)-\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}{d^3}-\frac{6\sqrt{c+\frac{d}{x^2}}\,c}{d^3}-\frac{3\,c^2}{\sqrt{c+\frac{d}{x^2}}\,d^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/5*b*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c + d/x
^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4)) - 1/3*a*((c + d/x^2)^(3/2)/d^3 -
 6*sqrt(c + d/x^2)*c/d^3 - 3*c^2/(sqrt(c + d/x^2)*d^3))
```

mupad [B] time = 4.84, size = 91, normalized size = 0.91

$$-\frac{\sqrt{c+\frac{d}{x^2}}\,\left(48\,b\,c^3\,x^6-40\,a\,c^2\,d\,x^6+24\,b\,c^2\,d\,x^4-20\,a\,c\,d^2\,x^4-6\,b\,c\,d^2\,x^2+5\,a\,d^3\,x^2+3\,b\,d^3\right)}{15\,d^4\,x^4\,\left(c\,x^2+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^7*(c + d/x^2)^(3/2)),x)
```

```
[Out] -((c + d/x^2)^(1/2)*(3*b*d^3 + 5*a*d^3*x^2 + 48*b*c^3*x^6 - 20*a*c*d^2*x^4
- 40*a*c^2*d*x^6 - 6*b*c*d^2*x^2 + 24*b*c^2*d*x^4))/(15*d^4*x^4*(d + c*x^2)
)
```

sympy [A] time = 13.28, size = 90, normalized size = 0.90

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{5d^4}+\frac{c^2\left(ad-bc\right)}{d^4\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(ad-3bc\right)}{3d^4}-\frac{\sqrt{c+\frac{d}{x^2}}\left(-2acd+3bc^2\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**7,x)
```

```
[Out] -b*(c + d/x**2)**(5/2)/(5*d**4) + c**2*(a*d - b*c)/(d**4*sqrt(c + d/x**2))
- (c + d/x**2)**(3/2)*(a*d - 3*b*c)/(3*d**4) - sqrt(c + d/x**2)*(-2*a*c*d +
 3*b*c**2)/d**4
```

3.979 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9} \, dx$

Optimal. Leaf size=126

$$\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{d^5}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

```
[Out] -c*(-a*d+2*b*c)*(c+d/x^2)^(3/2)/d^5+1/5*(-a*d+4*b*c)*(c+d/x^2)^(5/2)/d^5-1/
7*b*(c+d/x^2)^(7/2)/d^5+c^3*(-a*d+b*c)/d^5/(c+d/x^2)^(1/2)+c^2*(-3*a*d+4*b*
c)*(c+d/x^2)^(1/2)/d^5
```

Rubi [A] time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 77}

$$\frac{c^2\sqrt{c+\frac{d}{x^2}}\,(4bc-3ad)}{d^5}+\frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}}+\frac{\left(c+\frac{d}{x^2}\right)^{5/2}(4bc-ad)}{5d^5}-\frac{c\left(c+\frac{d}{x^2}\right)^{3/2}(2bc-ad)}{d^5}-\frac{b\left(c+\frac{d}{x^2}\right)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (c^3*(b*c - a*d))/(d^5*Sqrt[c + d/x^2]) + (c^2*(4*b*c - 3*a*d)*Sqrt[c + d/x
^2])/d^5 - (c*(2*b*c - a*d)*(c + d/x^2)^(3/2))/d^5 + ((4*b*c - a*d)*(c + d/
x^2)^(5/2))/(5*d^5) - (b*(c + d/x^2)^(7/2))/(7*d^5)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^9}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{x^3(a+bx)}{(c+dx)^{3/2}}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \left(\frac{c^3(bc-ad)}{d^4(c+dx)^{3/2}} - \frac{c^2(4bc-3ad)}{d^4\sqrt{c+dx}} + \frac{3c(2bc-ad)\sqrt{c+dx}}{d^4} + \frac{(-4bc+ad)(c+\right.\right.\right.$$

$$= \frac{c^3(bc-ad)}{d^5\sqrt{c+\frac{d}{x^2}}} + \frac{c^2(4bc-3ad)\sqrt{c+\frac{d}{x^2}}}{d^5} - \frac{c(2bc-ad)\left(c+\frac{d}{x^2}\right)^{3/2}}{d^5} + \frac{(4bc-ad)\left(c+\frac{d}{x^2}\right)^{5/2}}{5d^5} - \ldots$$

Mathematica [A] time = 0.04, size = 104, normalized size = 0.83

$$\frac{b\left(128c^4x^8+64c^3dx^6-16c^2d^2x^4+8cd^3x^2-5d^4\right)-7adx^2\left(16c^3x^6+8c^2dx^4-2cd^2x^2+d^3\right)}{35d^5x^8\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^9),x]
```

```
[Out] (-7*a*d*x^2*(d^3 - 2*c*d^2*x^2 + 8*c^2*d*x^4 + 16*c^3*x^6) + b*(-5*d^4 + 8*
c*d^3*x^2 - 16*c^2*d^2*x^4 + 64*c^3*d*x^6 + 128*c^4*x^8))/(35*d^5*Sqrt[c +
d/x^2]*x^8)
```

fricas [A] time = 0.66, size = 121, normalized size = 0.96

$$\frac{\left(16\left(8\,bc^4-7\,ac^3d\right)x^8+8\left(8\,bc^3d-7\,ac^2d^2\right)x^6-5\,bd^4-2\left(8\,bc^2d^2-7\,acd^3\right)x^4+\left(8\,bcd^3-7\,ad^4\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{35\left(cd^5x^8+d^6x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/35*(16*(8*b*c^4 - 7*a*c^3*d)*x^8 + 8*(8*b*c^3*d - 7*a*c^2*d^2)*x^6 - 5*b*
d^4 - 2*(8*b*c^2*d^2 - 7*a*c*d^3)*x^4 + (8*b*c*d^3 - 7*a*d^4)*x^2)*sqrt((c*
x^2 + d)/x^2)/(c*d^5*x^8 + d^6*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)/((c + d/x^2)^(3/2)*x^9), x)
```

maple [A] time = 0.05, size = 118, normalized size = 0.94

$$-\frac{\left(112a\,c^3d\,x^8-128b\,c^4x^8+56a\,c^2d^2x^6-64b\,c^3d\,x^6-14ac\,d^3x^4+16b\,c^2d^2x^4+7a\,d^4x^2-8bc\,d^3x^2+5b\,d^4\right)\left(c\,x^2\ldots\right.}{35\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}} d^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x)
```

```
[Out] -1/35*(112*a*c^3*d*x^8-128*b*c^4*x^8+56*a*c^2*d^2*x^6-64*b*c^3*d*x^6-14*a*c
*d^3*x^4+16*b*c^2*d^2*x^4+7*a*d^4*x^2-8*b*c*d^3*x^2+5*b*d^4)*(c*x^2+d)/((c*
x^2+d)/x^2)^(3/2)/d^5/x^10
```

maxima [A] time = 0.55, size = 151, normalized size = 1.20

$$-\frac{1}{35}\,b\left(\frac{5\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{d^5}-\frac{28\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}c}{d^5}+\frac{70\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}c^2}{d^5}-\frac{140\sqrt{c+\frac{d}{x^2}}\,c^3}{d^5}-\frac{35\,c^4}{\sqrt{c+\frac{d}{x^2}}\,d^5}\right)-\frac{1}{5}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}}{d^4}-\frac{5\left(c+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] -1/35*b*(5*(c + d/x^2)^(7/2)/d^5 - 28*(c + d/x^2)^(5/2)*c/d^5 + 70*(c + d/x
^2)^(3/2)*c^2/d^5 - 140*sqrt(c + d/x^2)*c^3/d^5 - 35*c^4/(sqrt(c + d/x^2)*d
^5)) - 1/5*a*((c + d/x^2)^(5/2)/d^4 - 5*(c + d/x^2)^(3/2)*c/d^4 + 15*sqrt(c
 + d/x^2)*c^2/d^4 + 5*c^3/(sqrt(c + d/x^2)*d^4))
```

mupad [B] time = 4.92, size = 154, normalized size = 1.22

$$\frac{c\sqrt{c+\frac{d}{x^2}}\;(21\,a\,d-29\,b\,c)}{35\,d^4\,x^2}-\frac{b\sqrt{c+\frac{d}{x^2}}}{7\,d^2\,x^6}-\frac{\sqrt{c+\frac{d}{x^2}}\;\left(7\,a\,d^2-13\,b\,c\,d\right)}{35\,d^4\,x^4}-\frac{\sqrt{c+\frac{d}{x^2}}\;\left(x^2\left(\frac{58\,b\,c^4-42\,a\,c^3\,d}{35\,d^5}+\frac{2\,c^3\,(77\,a\,d-9}{35\,d^5}\right.\right.}{c\,x^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^9*(c + d/x^2)^(3/2)),x)
```

```
[Out] (c*(c + d/x^2)^(1/2)*(21*a*d - 29*b*c))/(35*d^4*x^2) - (b*(c + d/x^2)^(1/2)
)/(7*d^2*x^6) - ((c + d/x^2)^(1/2)*(7*a*d^2 - 13*b*c*d))/(35*d^4*x^4) - ((c
 + d/x^2)^(1/2)*(x^2*((58*b*c^4 - 42*a*c^3*d)/(35*d^5) + (2*c^3*(77*a*d - 9
3*b*c))/(35*d^5)) + (c^2*(77*a*d - 93*b*c))/(35*d^4)))/(d + c*x^2)
```

sympy [A] time = 16.14, size = 122, normalized size = 0.97

$$-\frac{b\left(c+\frac{d}{x^2}\right)^{\frac{7}{2}}}{7d^5}-\frac{c^3\,(ad-bc)}{d^5\sqrt{c+\frac{d}{x^2}}}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}(ad-4bc)}{5d^5}-\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}\left(-3acd+6bc^2\right)}{3d^5}-\frac{\sqrt{c+\frac{d}{x^2}}\left(3ac^2d-4bc^3\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**9,x)
```

```
[Out] -b*(c + d/x**2)**(7/2)/(7*d**5) - c**3*(a*d - b*c)/(d**5*sqrt(c + d/x**2))
- (c + d/x**2)**(5/2)*(a*d - 4*b*c)/(5*d**5) - (c + d/x**2)**(3/2)*(-3*a*c*
d + 6*b*c**2)/(3*d**5) - sqrt(c + d/x**2)*(3*a*c**2*d - 4*b*c**3)/d**5
```

3.980 $\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$-\frac{8dx\sqrt{c+\frac{d}{x^2}}\,(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] 4/15*d*(-6*a*d+5*b*c)*x/c^3/(c+d/x^2)^(1/2)+1/15*(-6*a*d+5*b*c)*x^3/c^2/(c+
d/x^2)^(1/2)+1/5*a*x^5/c/(c+d/x^2)^(1/2)-8/15*d*(-6*a*d+5*b*c)*x*(c+d/x^2)^
(1/2)/c^4
```

Rubi [A] time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {453, 271, 192, 191}

$$\frac{x^3(5bc-6ad)}{15c^2\sqrt{c+\frac{d}{x^2}}}-\frac{8dx\sqrt{c+\frac{d}{x^2}}\,(5bc-6ad)}{15c^4}+\frac{4dx(5bc-6ad)}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (4*d*(5*b*c - 6*a*d)*x)/(15*c^3*Sqrt[c + d/x^2]) - (8*d*(5*b*c - 6*a*d)*Sqr
t[c + d/x^2]*x)/(15*c^4) + ((5*b*c - 6*a*d)*x^3)/(15*c^2*Sqrt[c + d/x^2]) +
 (a*x^5)/(5*c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^4}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = \frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)\int \frac{x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{5c}$$

$$=\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(4d(5bc-6ad))\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{15c^2}$$

$$=\frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}-\frac{(8d(5bc-6ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{15c^3}$$

$$=\frac{4d(5bc-6ad)x}{15c^3\sqrt{c+\frac{d}{x^2}}}-\frac{8d(5bc-6ad)\sqrt{c+\frac{d}{x^2}}\,x}{15c^4}+\frac{(5bc-6ad)x^3}{15c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^5}{5c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A] time = 0.08, size = 80, normalized size = 0.72

$$\frac{3a\left(c^3x^6-2c^2dx^4+8cd^2x^2+16d^3\right)+5bc\left(c^2x^4-4cdx^2-8d^2\right)}{15c^4x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^4)/(c + d/x^2)^(3/2),x]
```

```
[Out] (5*b*c*(-8*d^2 - 4*c*d*x^2 + c^2*x^4) + 3*a*(16*d^3 + 8*c*d^2*x^2 - 2*c^2*d
*x^4 + c^3*x^6))/(15*c^4*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.78, size = 95, normalized size = 0.86

$$\frac{\left(3\,ac^3x^7+\left(5\,bc^3-6\,ac^2d\right)x^5-4\left(5\,bc^2d-6\,acd^2\right)x^3-8\left(5\,bcd^2-6\,ad^3\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{15\left(c^5x^2+c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*a*c^3*x^7 + (5*b*c^3 - 6*a*c^2*d)*x^5 - 4*(5*b*c^2*d - 6*a*c*d^2)*x
^3 - 8*(5*b*c*d^2 - 6*a*d^3)*x)*sqrt((c*x^2 + d)/x^2)/(c^5*x^2 + c^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
```

```
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 91, normalized size = 0.82

$$\frac{\left(3a\,x^{6}c^{3}-6a\,c^{2}d\,x^{4}+5b\,c^{3}x^{4}+24ac\,d^{2}x^{2}-20b\,c^{2}d\,x^{2}+48a\,d^{3}-40bc\,d^{2}\right)\left(c\,x^{2}+d\right)}{15\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/15*(3*a*c^3*x^6-6*a*c^2*d*x^4+5*b*c^3*x^4+24*a*c*d^2*x^2-20*b*c^2*d*x^2+4
8*a*d^3-40*b*c*d^2)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^4
```

maxima [A] time = 0.56, size = 128, normalized size = 1.15

$$\frac{1}{3}b\left(\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}x^{3}-6\sqrt{c+\frac{d}{x^{2}}}dx}{c^{3}}-\frac{3d^{2}}{\sqrt{c+\frac{d}{x^{2}}}c^{3}x}\right)+\frac{1}{5}a\left(\frac{5d^{3}}{\sqrt{c+\frac{d}{x^{2}}}c^{4}x}+\frac{\left(c+\frac{d}{x^{2}}\right)^{\frac{5}{2}}x^{5}-5\left(c+\frac{d}{x^{2}}\right)^{\frac{3}{2}}dx^{3}+15\sqrt{c+\frac{d}{x^{2}}}}{c^{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^4/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*(((c + d/x^2)^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c
+ d/x^2)*c^3*x)) + 1/5*a*(5*d^3/(sqrt(c + d/x^2)*c^4*x) + ((c + d/x^2)^(5/2
)*x^5 - 5*(c + d/x^2)^(3/2)*d*x^3 + 15*sqrt(c + d/x^2)*d^2*x)/c^4)
```

mupad [B] time = 5.77, size = 79, normalized size = 0.71

$$\frac{3\,a\,c^{3}\,x^{6}+5\,b\,c^{3}\,x^{4}-6\,a\,c^{2}\,d\,x^{4}-20\,b\,c^{2}\,d\,x^{2}+24\,a\,c\,d^{2}\,x^{2}-40\,b\,c\,d^{2}+48\,a\,d^{3}}{15\,c^{4}\,x\,\sqrt{c+\frac{d}{x^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (48*a*d^3 + 3*a*c^3*x^6 + 5*b*c^3*x^4 - 40*b*c*d^2 + 24*a*c*d^2*x^2 - 6*a*c
^2*d*x^4 - 20*b*c^2*d*x^2)/(15*c^4*x*(c + d/x^2)^(1/2))
```

sympy [B] time = 7.83, size = 561, normalized size = 5.05

$$a\left(\frac{c^{5}d^{\frac{19}{2}}x^{10}\sqrt{\frac{cx^{2}}{d}+1}}{5c^{7}d^{9}x^{6}+15c^{6}d^{10}x^{4}+15c^{5}d^{11}x^{2}+5c^{4}d^{12}}+\frac{5c^{3}d^{\frac{23}{2}}x^{6}\sqrt{\frac{cx^{2}}{d}+1}}{5c^{7}d^{9}x^{6}+15c^{6}d^{10}x^{4}+15c^{5}d^{11}x^{2}+5c^{4}d^{12}}+\frac{30c^{2}d^{\frac{25}{2}}}{5c^{7}d^{9}x^{6}+15c^{6}d^{10}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**4/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**5*d**(19/2)*x**10*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**1
0*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 5*c**3*d**(23/2)*x**6*sqrt(c*
x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5
*c**4*d**12) + 30*c**2*d**(25/2)*x**4*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6
+ 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12) + 40*c*d**(27/2)*
x**2*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**6 + 15*c**6*d**10*x**4 + 15*c**5*d*
*11*x**2 + 5*c**4*d**12) + 16*d**(29/2)*sqrt(c*x**2/d + 1)/(5*c**7*d**9*x**
```

```
6 + 15*c**6*d**10*x**4 + 15*c**5*d**11*x**2 + 5*c**4*d**12)) + b*(c**3*d**(
9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*
d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4
*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d + 1)/(3*c**5*
d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(c*x**2/d + 1
)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6))
```

3.981 $\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -1/3*(-4*a*d+3*b*c)*x/c^2/(c+d/x^2)^(1/2)+1/3*a*x^3/c/(c+d/x^2)^(1/2)+2/3*(
-4*a*d+3*b*c)*x*(c+d/x^2)^(1/2)/c^3
```

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {453, 192, 191}

$$\frac{2x\sqrt{c+\frac{d}{x^2}}\,(3bc-4ad)}{3c^3}-\frac{x(3bc-4ad)}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((3*b*c - 4*a*d)*x)/(3*c^2*Sqrt[c + d/x^2]) + (2*(3*b*c - 4*a*d)*Sqrt[c +
d/x^2]*x)/(3*c^3) + (a*x^3)/(3*c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)x^2}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx=\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx}{3c}$$

$$=-\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}+\frac{(2(3bc-4ad))\int \frac{1}{\sqrt{c+\frac{d}{x^2}}}\,dx}{3c^2}$$

$$=-\frac{(3bc-4ad)x}{3c^2\sqrt{c+\frac{d}{x^2}}}+\frac{2(3bc-4ad)\sqrt{c+\frac{d}{x^2}}\,x}{3c^3}+\frac{ax^3}{3c\sqrt{c+\frac{d}{x^2}}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.72

$$\frac{a\left(c^2x^4-4cdx^2-8d^2\right)+3bc\left(cx^2+2d\right)}{3c^3x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b/x^2)*x^2)/(c + d/x^2)^(3/2),x]
[Out] (3*b*c*(2*d + c*x^2) + a*(-8*d^2 - 4*c*d*x^2 + c^2*x^4))/(3*c^3*Sqrt[c + d/
x^2]*x)
```

fricas [A] time = 0.69, size = 70, normalized size = 0.89

$$\frac{\left(ac^2x^5+\left(3\,bc^2-4\,acd\right)x^3+2\left(3\,bcd-4\,ad^2\right)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{3\left(c^4x^2+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(a*c^2*x^5 + (3*b*c^2 - 4*a*c*d)*x^3 + 2*(3*b*c*d - 4*a*d^2)*x)*sqrt((c
*x^2 + d)/x^2)/(c^4*x^2 + c^3*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 66, normalized size = 0.84

$$\frac{\left(a\,x^4c^2-4acd\,x^2+3b\,c^2x^2-8a\,d^2+6bcd\right)\left(c\,x^2+d\right)}{3\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x)
```

```
[Out] 1/3*(a*c^2*x^4-4*a*c*d*x^2+3*b*c^2*x^2-8*a*d^2+6*b*c*d)*(c*x^2+d)/((c*x^2+d
)/x^2)^(3/2)/c^3/x^3
```

maxima [A] time = 0.53, size = 90, normalized size = 1.14

$$b\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)+\frac{1}{3}\,a\left(\frac{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}x^3-6\,\sqrt{c+\frac{d}{x^2}}\,dx}{c^3}-\frac{3\,d^2}{\sqrt{c+\frac{d}{x^2}}\,c^3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)*x^2/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) + 1/3*a*(((c + d/x^2)
^(3/2)*x^3 - 6*sqrt(c + d/x^2)*d*x)/c^3 - 3*d^2/(sqrt(c + d/x^2)*c^3*x))
```

mupad [B] time = 5.20, size = 81, normalized size = 1.03

$$\frac{b\,c^2\,x^4+3\,b\,c\,d\,x^2+2\,b\,d^2}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}-\frac{-a\,c^2\,x^4+4\,a\,c\,d\,x^2+8\,a\,d^2}{3\,c^3\,x\,\sqrt{c+\frac{d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b/x^2))/(c + d/x^2)^(3/2),x)
```

```
[Out] (2*b*d^2 + b*c^2*x^4 + 3*b*c*d*x^2)/(c^2*x^3*(c + d/x^2)^(3/2)) - (8*a*d^2
- a*c^2*x^4 + 4*a*c*d*x^2)/(3*c^3*x*(c + d/x^2)^(1/2))
```

sympy [B] time = 7.31, size = 267, normalized size = 3.38

$$a\left(\frac{c^3d^{\frac{9}{2}}x^6\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{3c^2d^{\frac{11}{2}}x^4\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{12cd^{\frac{13}{2}}x^2\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+3c^3d^6}-\frac{8d^{\frac{15}{2}}\sqrt{\frac{cx^2}{d}+1}}{3c^5d^4x^4+6c^4d^5x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)*x**2/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(c**3*d**(9/2)*x**6*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x*
*2 + 3*c**3*d**6) - 3*c**2*d**(11/2)*x**4*sqrt(c*x**2/d + 1)/(3*c**5*d**4*x
**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 12*c*d**(13/2)*x**2*sqrt(c*x**2/d +
 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6) - 8*d**(15/2)*sqrt(
c*x**2/d + 1)/(3*c**5*d**4*x**4 + 6*c**4*d**5*x**2 + 3*c**3*d**6)) + b*(x**
2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)))
```

3.982 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=45

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] (2*a*d-b*c)/c^2/x/(c+d/x^2)^(1/2)+a*x/c/(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 453, 191}

$$\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}-\frac{bc-2ad}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] -((b*c - 2*a*d)/(c^2*Sqrt[c + d/x^2]*x)) + (a*x)/(c*Sqrt[c + d/x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{a+bx^2}{x^2\left(c+dx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)\\
&= \frac{ax}{c\sqrt{c+\frac{d}{x^2}}}+\frac{(-bc+2ad)\,\text{Subst}\left(\int \frac{1}{\left(c+dx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{c}\\
&= -\frac{bc-2ad}{c^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{ax}{c\sqrt{c+\frac{d}{x^2}}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.73

$$\frac{acx^2+2ad-bc}{c^2x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2)^(3/2),x]
```

```
[Out] (-(b*c) + 2*a*d + a*c*x^2)/(c^2*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.67, size = 47, normalized size = 1.04

$$\frac{\left(acx^3-(bc-2\,ad)x\right)\sqrt{\frac{cx^2+d}{x^2}}}{c^3x^2+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (a*c*x^3 - (b*c - 2*a*d)*x)*sqrt((c*x^2 + d)/x^2)/(c^3*x^2 + c^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.04, size = 43, normalized size = 0.96

$$\frac{\left(a\,x^2c+2ad-bc\right)\left(c\,x^2+d\right)}{\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2),x)
```

```
[Out] (a*c*x^2+2*a*d-b*c)*(c*x^2+d)/((c*x^2+d)/x^2)^(3/2)/x^3/c^2
```

maxima [A] time = 0.68, size = 53, normalized size = 1.18

$$a\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x}{c^2}+\frac{d}{\sqrt{c+\frac{d}{x^2}}\,c^2x}\right)-\frac{b}{\sqrt{c+\frac{d}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*(sqrt(c + d/x^2)*x/c^2 + d/(sqrt(c + d/x^2)*c^2*x)) - b/(sqrt(c + d/x^2)*
c*x)
```

mupad [B] time = 4.90, size = 38, normalized size = 0.84

$$\frac{\left(c\,x^2+d\right)\,\left(a\,c\,x^2+2\,a\,d-b\,c\right)}{c^2\,x^3\,{\left(c+\frac{d}{x^2}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(c + d/x^2)^(3/2),x)
```

```
[Out] ((d + c*x^2)*(2*a*d - b*c + a*c*x^2))/(c^2*x^3*(c + d/x^2)^(3/2))
```

sympy [A] time = 7.51, size = 65, normalized size = 1.44

$$a\left(\frac{x^2}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+\frac{2\sqrt{d}}{c^2\sqrt{\frac{cx^2}{d}+1}}\right)-\frac{b}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2),x)
```

```
[Out] a*(x**2/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + 2*sqrt(d)/(c**2*sqrt(c*x**2/d + 1)
)) - b/(c*sqrt(d)*sqrt(c*x**2/d + 1))
```

3.983 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx$

Optimal. Leaf size=59

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

[Out] -b*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(3/2)+(-a*d+b*c)/c/d/x/(c+d/x^2)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {452, 335, 217, 206}

$$\frac{bc-ad}{cdx\sqrt{c+\frac{d}{x^2}}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]

[Out] (b*c - a*d)/(c*d*Sqrt[c + d/x^2]*x) - (b*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/d^(3/2)

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^2} \, dx = \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} + \frac{b\int \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x^2} \, dx}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}} \, dx, x, \frac{1}{x}\right)}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-dx^2} \, dx, x, \frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d}$$

$$= \frac{bc-ad}{cd\sqrt{c+\frac{d}{x^2}}\,x} - \frac{b\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{d^{3/2}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.20

$$\frac{\sqrt{d}\,(bc-ad) - bc\sqrt{cx^2+d}\,\tanh^{-1}\left(\frac{\sqrt{cx^2+d}}{\sqrt{d}}\right)}{cd^{3/2}x\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^2),x]
```

```
[Out] (Sqrt[d]*(b*c - a*d) - b*c*Sqrt[d + c*x^2]*ArcTanh[Sqrt[d + c*x^2]/Sqrt[d]]
)/(c*d^(3/2)*Sqrt[c + d/x^2]*x)
```

fricas [A] time = 0.63, size = 195, normalized size = 3.31

$$\left[\frac{2\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}} + \left(bc^2x^2+bcd\right)\sqrt{d}\,\log\left(-\frac{cx^2-2\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2d}{x^2}\right)}{2\left(c^2d^2x^2+cd^3\right)}, \frac{\left(bcd-ad^2\right)x\sqrt{\frac{cx^2+d}{x^2}} + \left(bc^2x^2+bcd\right)\sqrt{}}{c^2d^2x^2+cd^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*sqrt(
d)*log(-(c*x^2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2))/(c^2*d^2*x^
2 + c*d^3), ((b*c*d - a*d^2)*x*sqrt((c*x^2 + d)/x^2) + (b*c^2*x^2 + b*c*d)*
sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)))/(c^2*d^2*x^2
 + c*d^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 79, normalized size = 1.34

$$-\frac{\left(c\,x^{2}+d\right)\left(\sqrt{c\,x^{2}+d}\;bcd\ln\left(\frac{2d+2\sqrt{c\,x^{2}+d}\;\sqrt{d}}{x}\right)+a\,d^{\frac{5}{2}}-bc\,d^{\frac{3}{2}}\right)}{\left(\frac{c\,x^{2}+d}{x^{2}}\right)^{\frac{3}{2}}c\,d^{\frac{5}{2}}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x)
```

```
[Out] -(c*x^2+d)*((c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*b*c*d+a*d^(
5/2)-d^(3/2)*b*c)/((c*x^2+d)/x^2)^(3/2)/x^3/c/d^(5/2)
```

maxima [A] time = 1.21, size = 80, normalized size = 1.36

$$\frac{1}{2}\,b\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^{2}}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^{2}}}\,x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^{2}}}\,dx}\right)-\frac{a}{\sqrt{c+\frac{d}{x^{2}}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqrt(d)))/d^(
3/2) + 2/(sqrt(c + d/x^2)*d*x)) - a/(sqrt(c + d/x^2)*c*x)
```

mupad [B] time = 5.11, size = 60, normalized size = 1.02

$$\frac{b}{d\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{a}{c\,x\,\sqrt{c+\frac{d}{x^2}}}-\frac{b\,\ln\left(\sqrt{c+\frac{d}{x^2}}+\frac{\sqrt{d}}{x}\right)}{d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^2*(c + d/x^2)^(3/2)),x)
```

```
[Out] b/(d*x*(c + d/x^2)^(1/2)) - a/(c*x*(c + d/x^2)^(1/2)) - (b*log((c + d/x^2)^
(1/2) + d^(1/2)/x))/d^(3/2)
```

sympy [B] time = 11.71, size = 206, normalized size = 3.49

$$-\frac{a}{c\sqrt{d}\sqrt{\frac{cx^2}{d}+1}}+b\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\right.}}{2cd^{\frac{7}{2}}x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**2,x)
```

```
[Out] -a/(c*sqrt(d)*sqrt(c*x**2/d + 1)) + b*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7
/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x**2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**
(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sqrt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2
*d**(9/2)) + d**3*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*l
og(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)))
```

3.984 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^4} \, dx$

Optimal. Leaf size=92

$$\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] 1/2*(-2*a*d+3*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(5/2)-1/2*b/d/x^3/(
c+d/x^2)^(1/2)+1/2*(2*a*d-3*b*c)/d^2/x/(c+d/x^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {459, 335, 288, 217, 206}

$$-\frac{3bc-2ad}{2d^2x\sqrt{c+\frac{d}{x^2}}}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{2d^{5/2}}-\frac{b}{2dx^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] -b/(2*d*Sqrt[c + d/x^2]*x^3) - (3*b*c - 2*a*d)/(2*d^2*Sqrt[c + d/x^2]*x) +
((3*b*c - 2*a*d)*ArcTanh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(2*d^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx &= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}+\frac{(-3bc+2ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2}x^4}\,dx}{2d}\\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{(-3bc+2ad)\,\text{Subst}\left(\int \frac{x^2}{\left(c+dx^2\right)^{3/2}}\,dx,x,\frac{1}{x}\right)}{2d}\\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c+dx^2}}\,dx,x,\frac{1}{x}\right)}{2d^2}\\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{1}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^2}\\
&= -\frac{b}{2d\sqrt{c+\frac{d}{x^2}}\,x^3}-\frac{3bc-2ad}{2d^2\sqrt{c+\frac{d}{x^2}}\,x}+\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x}\right)}{2d^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.62

$$\frac{x^2(2ad-3bc)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd}{2d^2x^3\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^4),x]
```

```
[Out] (-(b*d) + (-3*b*c + 2*a*d)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (c*x^2)/
d])/(2*d^2*Sqrt[c + d/x^2]*x^3)
```

fricas [A] time = 0.56, size = 248, normalized size = 2.70

$$\left[-\frac{\left(\left(3\,bc^2-2\,acd\right)x^3+\left(3\,bcd-2\,ad^2\right)x\right)\sqrt{d}\,\log\left(-\frac{cx^2-2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)+2\left(bd^2+\left(3\,bcd-2\,ad^2\right)x^2\right)\sqrt{\frac{cx^2+d}{x^2}}}{4\left(cd^3x^3+d^4x\right)},\cdot\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*b*c^2 - 2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(d)*log(-(c*x^
2 - 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) + 2*(b*d^2 + (3*b*c*d - 2
*a*d^2)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^3*x^3 + d^4*x), -1/2*(((3*b*c^2 -
2*a*c*d)*x^3 + (3*b*c*d - 2*a*d^2)*x)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^
2 + d)/x^2)/(c*x^2 + d)) + (b*d^2 + (3*b*c*d - 2*a*d^2)*x^2)*sqrt((c*x^2 +
d)/x^2))/(c*d^3*x^3 + d^4*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sq
rt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,cons
t vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.06, size = 132, normalized size = 1.43

$$\frac{\left(c x^2+d\right)\left(-2\sqrt{c x^2+d}\, a d^2 x^2 \ln\left(\frac{2d+2\sqrt{c x^2+d}\,\sqrt{d}}{x}\right)+3\sqrt{c x^2+d}\, bcd\, x^2 \ln\left(\frac{2d+2\sqrt{c x^2+d}\,\sqrt{d}}{x}\right)+2a d^{\frac{5}{2}} x^2-3bc\, d^{\frac{3}{2}} x^2 \cdots\right.}{2\left(\frac{c x^2+d}{x^2}\right)^{\frac{3}{2}} d^{\frac{7}{2}} x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x)
```

```
[Out] 1/2*(c*x^2+d)*(2*d^(5/2)*x^2*a-3*d^(3/2)*x^2*b*c-2*ln(2*(d+(c*x^2+d)^(1/2)*
d^(1/2))/x)*(c*x^2+d)^(1/2)*x^2*a*d^2+3*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)
*(c*x^2+d)^(1/2)*x^2*b*c*d-d^(5/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^5/d^(7/2)
```

maxima [B] time = 1.22, size = 162, normalized size = 1.76

$$-\frac{1}{4}\, b\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)c x^2-2\, c d\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}} d^2 x^3-\sqrt{c+\frac{d}{x^2}}\, d^3 x}+\frac{3\, c \log\left(\frac{\sqrt{c+\frac{d}{x^2}}\, x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\, x+\sqrt{d}}\right)}{d^{\frac{5}{2}}}\right)+\frac{1}{2}\, a\left(\frac{\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\, x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\, x+\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{\sqrt{c+\frac{d}{x^2}}\, d x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/4*b*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)*d^2*x^3 - sqrt(c
 + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x
 + sqrt(d)))/d^(5/2)) + 1/2*a*(log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c +
d/x^2)*x + sqrt(d)))/d^(3/2) + 2/(sqrt(c + d/x^2)*d*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^4\left(c+\frac{d}{x^2}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)),x)
```

```
[Out] int((a + b/x^2)/(x^4*(c + d/x^2)^(3/2)), x)
```

sympy [B] time = 18.44, size = 262, normalized size = 2.85

$$a\left(\frac{cd^2x^2\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2cd^2x^2\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{2d^3\sqrt{\frac{cx^2}{d}+1}}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}+\frac{d^3\log\left(\frac{cx^2}{d}\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}-\frac{2d^3\log\left(\sqrt{\frac{cx^2}{d}+1}+1\right)}{2cd^{\frac{7}{2}}x^2+2d^{\frac{9}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**4,x)
```

```
[Out] a*(c*d**2*x**2*log(c*x**2/d)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) - 2*c*d**2*x*
*2*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + 2*d**3*sq
rt(c*x**2/d + 1)/(2*c*d**(7/2)*x**2 + 2*d**(9/2)) + d**3*log(c*x**2/d)/(2*c
*d**(7/2)*x**2 + 2*d**(9/2)) - 2*d**3*log(sqrt(c*x**2/d + 1) + 1)/(2*c*d**(
7/2)*x**2 + 2*d**(9/2))) + b*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) +
3*c*asinh(sqrt(d)/(sqrt(c)*x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 +
d/(c*x**2))))
```

3.985 $\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} \, dx$

Optimal. Leaf size=123

$$-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}+\frac{3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

```
[Out] -3/8*c*(-4*a*d+5*b*c)*arctanh(d^(1/2)/x/(c+d/x^2)^(1/2))/d^(7/2)-1/4*b/d/x^
5/(c+d/x^2)^(1/2)+1/4*(4*a*d-5*b*c)/d^2/x^3/(c+d/x^2)^(1/2)+3/8*(-4*a*d+5*b
*c)*(c+d/x^2)^(1/2)/d^3/x
```

Rubi [A] time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {459, 335, 288, 321, 217, 206}

$$\frac{3\sqrt{c+\frac{d}{x^2}}\,(5bc-4ad)}{8d^3x}-\frac{5bc-4ad}{4d^2x^3\sqrt{c+\frac{d}{x^2}}}-\frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}}{x\sqrt{c+\frac{d}{x^2}}}\right)}{8d^{7/2}}-\frac{b}{4dx^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

```
[Out] -b/(4*d*Sqrt[c + d/x^2]*x^5) - (5*b*c - 4*a*d)/(4*d^2*Sqrt[c + d/x^2]*x^3)
+ (3*(5*b*c - 4*a*d)*Sqrt[c + d/x^2])/(8*d^3*x) - (3*c*(5*b*c - 4*a*d)*ArcT
anh[Sqrt[d]/(Sqrt[c + d/x^2]*x)])/(8*d^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{a+\frac{b}{x^2}}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} dx = -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} + \frac{(-5bc+4ad)\int \frac{1}{\left(c+\frac{d}{x^2}\right)^{3/2} x^6} dx}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{(-5bc+4ad)\,\text{Subst}\left(\int \frac{x^4}{(c+dx^2)^{3/2}} dx, x, \frac{1}{x}\right)}{4d}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{(3(5bc-4ad))\,\text{Subst}\left(\int \frac{x^2}{\sqrt{c+dx^2}} dx, x, \frac{1}{x}\right)}{4d^2}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\text{Subst}\left(\int \frac{}{\sqrt{}}\right.}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{(3c(5bc-4ad))\,\text{Subst}\left(\int \frac{}{1-}\right.}{8d^3}$$

$$= -\frac{b}{4d\sqrt{c+\frac{d}{x^2}}\,x^5} - \frac{5bc-4ad}{4d^2\sqrt{c+\frac{d}{x^2}}\,x^3} + \frac{3(5bc-4ad)\sqrt{c+\frac{d}{x^2}}}{8d^3x} - \frac{3c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{c+}}\right.}{8d^{7/2}}$$

Mathematica [C] time = 0.03, size = 60, normalized size = 0.49

$$\frac{cx^4(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{cx^2}{d}+1\right)-bd^2}{4d^3x^5\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/((c + d/x^2)^(3/2)*x^6),x]
```

[Out] (-(b*d^2) + c*(5*b*c - 4*a*d)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (c*x^2)/d])/(4*d^3*Sqrt[c + d/x^2]*x^5)

fricas [A] time = 0.74, size = 314, normalized size = 2.55

$$\left[-\frac{3\left(\left(5\,bc^3-4\,ac^2d\right)x^5+\left(5\,bc^2d-4\,acd^2\right)x^3\right)\sqrt{d}\,\log\left(-\frac{cx^2+2\,\sqrt{d}\,x\sqrt{\frac{cx^2+d}{x^2}}+2\,d}{x^2}\right)-2\left(3\left(5\,bc^2d-4\,acd^2\right)x^4-2\,bd^3+\right.}{16\left(cd^4x^5+d^5x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="fricas")

[Out] [-1/16*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(d)*log(-(c*x^2 + 2*sqrt(d)*x*sqrt((c*x^2 + d)/x^2) + 2*d)/x^2) - 2*(3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3), 1/8*(3*((5*b*c^3 - 4*a*c^2*d)*x^5 + (5*b*c^2*d - 4*a*c*d^2)*x^3)*sqrt(-d)*arctan(sqrt(-d)*x*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d)) + (3*(5*b*c^2*d - 4*a*c*d^2)*x^4 - 2*b*d^3 + (5*b*c*d^2 - 4*a*d^3)*x^2)*sqrt((c*x^2 + d)/x^2))/(c*d^4*x^5 + d^5*x^3)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [sign(t_nostep),sign(t_nostep+sqrt(d)/c*sign(t_nostep))]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [A] time = 0.06, size = 157, normalized size = 1.28

$$\frac{\left(c\,x^2+d\right)\left(12\sqrt{c\,x^2+d}\;ac\,d^2x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\;\sqrt{d}}{x}\right)-15\sqrt{c\,x^2+d}\;b\,c^2d\,x^4\ln\left(\frac{2d+2\sqrt{c\,x^2+d}\;\sqrt{d}}{x}\right)-12ac\,d^{\frac{5}{2}}x^4+15b\,c\right.}{8\left(\frac{c\,x^2+d}{x^2}\right)^{\frac{3}{2}}d^{\frac{9}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x)

[Out] 1/8*(c*x^2+d)*(12*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*a*c*d^2-15*(c*x^2+d)^(1/2)*ln(2*(d+(c*x^2+d)^(1/2)*d^(1/2))/x)*x^4*b*c^2*d-12*d^(5/2)*x^4*a*c+15*d^(3/2)*x^4*b*c^2-4*d^(7/2)*x^2*a+5*d^(5/2)*x^2*b*c-2*d^(7/2)*b)/((c*x^2+d)/x^2)^(3/2)/x^7/d^(9/2)

maxima [B] time = 1.28, size = 243, normalized size = 1.98

$$\frac{1}{16}\,b\left(\frac{2\left(15\left(c+\frac{d}{x^2}\right)^2c^2x^4-25\left(c+\frac{d}{x^2}\right)c^2dx^2+8\,c^2d^2\right)}{\left(c+\frac{d}{x^2}\right)^{\frac{5}{2}}d^3x^5-2\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^4x^3+\sqrt{c+\frac{d}{x^2}}\,d^5x}+\frac{15\,c^2\log\left(\frac{\sqrt{c+\frac{d}{x^2}}\,x-\sqrt{d}}{\sqrt{c+\frac{d}{x^2}}\,x+\sqrt{d}}\right)}{d^{\frac{7}{2}}}\right)-\frac{1}{4}\,a\left(\frac{2\left(3\left(c+\frac{d}{x^2}\right)cx^2-2\,cd\right.}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}d^2x^3-\sqrt{c+\frac{d}{x^2}}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/16*b*(2*(15*(c + d/x^2)^2*c^2*x^4 - 25*(c + d/x^2)*c^2*d*x^2 + 8*c^2*d^2)
/((c + d/x^2)^(5/2)*d^3*x^5 - 2*(c + d/x^2)^(3/2)*d^4*x^3 + sqrt(c + d/x^2)
*d^5*x) + 15*c^2*log((sqrt(c + d/x^2)*x - sqrt(d))/(sqrt(c + d/x^2)*x + sqr
t(d)))/d^(7/2)) - 1/4*a*(2*(3*(c + d/x^2)*c*x^2 - 2*c*d)/((c + d/x^2)^(3/2)
*d^2*x^3 - sqrt(c + d/x^2)*d^3*x) + 3*c*log((sqrt(c + d/x^2)*x - sqrt(d))/(
sqrt(c + d/x^2)*x + sqrt(d)))/d^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{b}{x^2}}{x^6\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)),x)
```

```
[Out] int((a + b/x^2)/(x^6*(c + d/x^2)^(3/2)), x)
```

sympy [A] time = 29.81, size = 180, normalized size = 1.46

$$a\left(-\frac{3\sqrt{c}}{2d^2x\sqrt{1+\frac{d}{cx^2}}}+\frac{3c\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{c}\,x}\right)}{2d^{\frac{5}{2}}}-\frac{1}{2\sqrt{c}\,dx^3\sqrt{1+\frac{d}{cx^2}}}\right)+b\left(\frac{15c^{\frac{3}{2}}}{8d^3x\sqrt{1+\frac{d}{cx^2}}}+\frac{5\sqrt{c}}{8d^2x^3\sqrt{1+\frac{d}{cx^2}}}-\frac{15c^2\operatorname{asinh}}{8d^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2)**(3/2)/x**6,x)
```

```
[Out] a*(-3*sqrt(c)/(2*d**2*x*sqrt(1 + d/(c*x**2))) + 3*c*asinh(sqrt(d)/(sqrt(c)*
x))/(2*d**(5/2)) - 1/(2*sqrt(c)*d*x**3*sqrt(1 + d/(c*x**2)))) + b*(15*c**(3
/2)/(8*d**3*x*sqrt(1 + d/(c*x**2))) + 5*sqrt(c)/(8*d**2*x**3*sqrt(1 + d/(c*
x**2))) - 15*c**2*asinh(sqrt(d)/(sqrt(c)*x))/(8*d**(7/2)) - 1/(4*sqrt(c)*d*
x**5*sqrt(1 + d/(c*x**2))))
```

3.986 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m \, dx$

Optimal. Leaf size=105

$$\frac{(ex)^{m+1}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2}(-m-1);-p,-q;\frac{1-m}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e(m+1)}$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1+m)*AppellF1(-1/2-1/2*m,-p,-q,1/2-1/2*m,-b/
a/x^2,-d/c/x^2)/e/(1+m)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{497, 511, 510\}$

$$\frac{x(ex)^m\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2}(-m-1);-p,-q;\frac{1-m}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(-1 - m)/2, -p, -q, (1 - m)
/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + m)*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))
^q)
```

Rule 497

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> -Dist[(e*x)^m*(x^(-1))^m, Subst[Int[((a + b/x^n)^p*(c
+ d/x^n)^q)/x^(m + 2), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e, m, p, q}, x
] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^m\,dx = -\left(\left(\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-2-m}\left(1+\frac{bx^2}{a}\right)^p\left(c+dx\right.\right.$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\left(\frac{1}{x}\right)^m (ex)^m\right)\text{Subst}\left(\int x^{-}\right.\right.$$
$$= \frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q} x(ex)^m F_1\left(\frac{1}{2}(-1-m);-p,-q;\frac{1-n}{2}\right.}{1+m}$$

Mathematica [A] time = 0.15, size = 115, normalized size = 1.10

$$\frac{x(ex)^m\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2}(m-2p-2q+1);-p,-q;\frac{1}{2}(m-2p-2q+3);-\frac{ax^2}{b},-\frac{c}{}\right.}{m-2p-2q+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^m,x]
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^m*AppellF1[(1 + m - 2*p - 2*q)/2, -p,
-q, (3 + m - 2*p - 2*q)/2, -((a*x^2)/b), -((c*x^2)/d)])/((1 + m - 2*p - 2*q
)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left((ex)^m\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="fricas")
[Out] integral((e*x)^m*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="giac")
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (ex)^m\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^m,x, algorithm="maxima")
[Out] integrate((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^m \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^m*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**m,x)
[Out] Timed out
```

3.987 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$

Optimal. Leaf size=84

$$\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/5*(a+b/x^2)^p*(c+d/x^2)^q*x^5*AppellF1(-5/2,-p,-q,-3/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$\frac{1}{5}x^5\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{2};-p,-q;-\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[-5/2, -p, -q, -3/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(5*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^6}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^6}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^6}\right.\right.$$
$$= \frac{1}{5}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^5 F_1\left(-\frac{5}{2}; -p, -q; -\frac{3}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right.$$

Mathematica [A] time = 0.20, size = 106, normalized size = 1.26

$$-\frac{x^5 \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{5}{2}; -p, -q; -p-q+\frac{7}{2}; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{2p+2q-5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^4,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^5*AppellF1[5/2 - p - q, -p, -q, 7/2 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c
*x^2)/d)^q))
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(x^4 \left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="fricas")
```

```
[Out] integral(x^4*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x^4 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^4,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^4*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**4,x)
[Out] Timed out
```

3.988 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx$

Optimal. Leaf size=100

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

```
[Out] 1/2*b^2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,3,-q,2+p,(a+b/x^2)/a,-d*(a
+b/x^2)/(-a*d+b*c))/a^3/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$\frac{b^2\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,3;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
```

```
[Out] (b^2*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 3, 2 + p, -((d*(
a + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^3*(1 + p)*((b*(c + d/x^2))/(
b*c - a*d))^q)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^3\,dx = -\left(\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\mathrm{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^3}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{b^2\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,3;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^3(1+p)}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 1.00

$$-\frac{x^4\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+2;-p,-q;-p-q+3;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q-2)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^3,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^4*AppellF1[2 - p - q, -p, -q, 3 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-2 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(x^3\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="fricas")
[Out] integral(x^3*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^3\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^3,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^3*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**3,x)
[Out] Timed out
```

3.989 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$

Optimal. Leaf size=84

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] 1/3*(a+b/x^2)^p*(c+d/x^2)^q*x^3*AppellF1(-3/2,-p,-q,-1/2,-b/a/x^2,-d/c/x^2)
/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$\frac{1}{3}x^3\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[-3/2, -p, -q, -1/2, -(b/(a*x^2)),
 -(d/(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^4}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^4}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^4}\right.\right.$$
$$= \frac{1}{3}\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x^3 F_1\left(-\frac{3}{2};-p,-q;-\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right.$$

Mathematica [A] time = 0.11, size = 106, normalized size = 1.26

$$-\frac{x^3 \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{3}{2};-p,-q;-p-q+\frac{5}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2p+2q-3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x^3*AppellF1[3/2 - p - q, -p, -q, 5/2 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(x^2 \left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="fricas")
```

```
[Out] integral(x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^2 \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x^2,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x^2*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x**2,x)
[Out] Timed out
```

3.990 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx$

Optimal. Leaf size=98

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

```
[Out] -1/2*b*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,2,-q,2+p,(a+b/x^2)/a,-d*(a+
b/x^2)/(-a*d+b*c))/a^2/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] time = 0.07, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {446, 137, 136}

$$-\frac{b\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
```

```
[Out] -(b*(a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a
 + b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a^2*(1 + p)*((b*(c + d/x^2))/(b
*c - a*d))^q)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q x\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x^2}\right)\right)$$

$$= -\frac{b\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a^2(1+p)}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 1.02

$$-\frac{x^2\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+1;-p,-q;-p-q+2;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*x,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*x^2*AppellF1[1 - p - q, -p, -q, 2 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)
/d)^q)
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(x\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="fricas")
[Out] integral(x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int x\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*x,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int(x*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*x,x)
[Out] Timed out
```

3.991 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] (a+b/x^2)^p*(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/
a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.08, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 511, 510}

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\text{Subst}\left(\int \frac{(a+bx^2)^p (c+dx^2)^q}{x^2} dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p (c+dx^2)^q}{x^2} dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^2} d\right.\right.$$
$$= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} xF_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Mathematica [A] time = 0.10, size = 104, normalized size = 1.32

$$-\frac{x\left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{1}{2};-p,-q;-p-q+\frac{3}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2p+2q-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
[Out] Timed out
```

3.992 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x} dx$

Optimal. Leaf size=97

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

```
[Out] 1/2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*AppellF1(1+p,1,-q,2+p,(a+b/x^2)/a,-d*(a+b/x
^2)/(-a*d+b*c))/a/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {446, 137, 136}

$$\frac{\left(a+\frac{b}{x^2}\right)^{p+1}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,1;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
```

```
[Out] ((a + b/x^2)^(1 + p)*(c + d/x^2)^q*AppellF1[1 + p, -q, 1, 2 + p, -((d*(a +
b/x^2))/(b*c - a*d)), (a + b/x^2)/a])/(2*a*(1 + p)*((b*(c + d/x^2))/(b*c -
a*d))^q)
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx = -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x}\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= \frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,1;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad},\frac{a+\frac{b}{x^2}}{a}\right)}{2a(1+p)}$$

Mathematica [A] time = 0.15, size = 95, normalized size = 0.98

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q;-p,-q;-p-q+1;-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{2(p+q)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x,x]
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-p - q, -p, -q, 1 - p - q, -((a*
x^2)/b), -((c*x^2)/d)])/((p + q)*(1 + (a*x^2)/b)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x,x)
```

```
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x,x)
```

```
[Out] Timed out
```

3.993 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$

Optimal. Leaf size=82

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

```
[Out] -(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/2,-p,-q,3/2,-b/a/x^2,-d/c/x^2)/((1+b/a/
x^2)^p)/((1+d/c/x^2)^q)/x
```

Rubi [A] time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 430, 429}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/2, -p, -q, 3/2, -(b/(a*x^2)), -(d
/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx = -\operatorname{Subst}\left(\int \left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\operatorname{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\operatorname{Subst}\left(\int \left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\right.\right.$$
$$= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{x}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 1.29

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{1}{2};-p,-q;-p-q+\frac{1}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{x(2p+2q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x]
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-1/2 - p - q, -p, -q, 1/2 - p - q,
-((a*x^2)/b), -((c*x^2)/d)])/((1 + 2*p + 2*q)*x*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^2,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**2,x)
[Out] Timed out
```

3.994 $\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} \, dx$

Optimal. Leaf size=85

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

```
[Out] -1/2*(a+b/x^2)^(1+p)*(c+d/x^2)^q*hypergeom([-q, 1+p],[2+p],-d*(a+b/x^2)/(-a
*d+b*c))/b/(1+p)/((b*(c+d/x^2)/(-a*d+b*c))^q)
```

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 70, 69}

$$-\frac{\left(a+\frac{b}{x^2}\right)^{p+1} \left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} {}_2F_1\left(p+1,-q;p+2;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -((a + b/x^2)^(1 + p)*(c + d/x^2)^q*Hypergeometric2F1[1 + p, -q, 2 + p, -((
d*(a + b/x^2))/(b*c - a*d))])/(2*b*(1 + p)*((b*(c + d/x^2))/(b*c - a*d))^q)
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] &&  !In
tegerQ[m] &&  !IntegerQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{x^3} dx = -\left(\frac{1}{2}\operatorname{Subst}\left(\int (a+bx)^p(c+dx)^q\,dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\left(\frac{1}{2}\left(\left(c+\frac{d}{x^2}\right)^q \left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q}\right)\operatorname{Subst}\left(\int (a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q dx, x, \frac{1}{x^2}\right)\right)$$
$$= -\frac{\left(a+\frac{b}{x^2}\right)^{1+p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{b\left(c+\frac{d}{x^2}\right)}{bc-ad}\right)^{-q} {}_2F_1\left(1+p,-q;2+p;-\frac{d\left(a+\frac{b}{x^2}\right)}{bc-ad}\right)}{2b(1+p)}$$

Mathematica [A] time = 0.08, size = 110, normalized size = 1.29

$$-\frac{\left(cx^2+d\right)\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(\frac{cx^2}{d}+1\right)^p\left(c+\frac{d}{x^2}\right)^q {}_2F_1\left(-p,-p-q-1;-p-q;\frac{(bc-ad)x^2}{b\left(cx^2+d\right)}\right)}{2dx^2(p+q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x]
```

```
[Out] -1/2*((a + b/x^2)^p*(c + d/x^2)^q*(d + c*x^2)*(1 + (c*x^2)/d)^p*Hypergeomet
ric2F1[-p, -1 - p - q, -p - q, ((b*c - a*d)*x^2)/(b*(d + c*x^2))])/(d*(1 +
p + q)*x^2*(1 + (a*x^2)/b)^p)
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^3,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**3,x)
[Out] Timed out
```

3.995 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$

Optimal. Leaf size=84

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

```
[Out] -1/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/2,-p,-q,5/2,-b/a/x^2,-d/c/x^2)/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/x^3
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {495, 511, 510}

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
```

```
[Out] -((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/2, -p, -q, 5/2, -(b/(a*x^2)), -(d/
(c*x^2))])/(3*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*x^3)
```

Rule 495

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_),
 x_Symbol] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^(m + 2), x], x, 1/
x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0] && In
tegerQ[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx = -\text{Subst}\left(\int x^2\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx,x,\frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int x^2\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right.\right.\right.$$
$$= -\frac{\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{2};-p,-q;\frac{5}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3x^3}$$

Mathematica [A] time = 0.37, size = 106, normalized size = 1.26

$$-\frac{\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{3}{2};-p,-q;-p-q-\frac{1}{2};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{x^3(2p+2q+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x]
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*AppellF1[-3/2 - p - q, -p, -q, -1/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((3 + 2*p + 2*q)*x^3*(1 + (a*x^2)/b)^p*(1 + (
c*x^2)/d)^q))
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="fricas")
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/x^4,x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4,x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/x**4,x)
[Out] Timed out
```

3.996 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

```
[Out] 2/7*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(7/2)*AppellF1(-7/4,-p,-q,-3/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{7/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{7e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(7/2)*AppellF1[-7/4, -p, -q, -3/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(7*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{5/2}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^8}\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{7/2}F_1\left(-\frac{7}{4};-p,-q;-\frac{3}{4};-\frac{b}{ax^2}\right.}{7e}$$

Mathematica [A] time = 0.20, size = 111, normalized size = 1.22

$$-\frac{2x(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{7}{4};-p,-q;-p-q+\frac{11}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(5/2)*AppellF1[7/4 - p - q, -p, -q,
 11/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-7 + 4*p + 4*q)*(1 + (a*x^2)/
b)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{ex}\,e^2x^2\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x)*e^2*x^2*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,6,1,1,0]%%%} / %%%{1,[0,0,0,0,3]%%%} Error: Bad Argument Value
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{5}{2}}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{5}{2}}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2),x, algorithm="maxima")
[Out] integrate((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^(5/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(5/2),x)
[Out] Timed out
```

3.997 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

```
[Out] 2/5*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(5/2)*AppellF1(-5/4,-p,-q,-1/4,-b/a/x^2,-
d/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{5/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{5e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(5/2)*AppellF1[-5/4, -p, -q, -1/4, -(b
/(a*x^2)), -(d/(c*x^2))])/(5*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q (ex)^{3/2}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^6}dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^6}dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^6}d\right.}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{5/2}F_1\left(-\frac{5}{4};-p,-q;-\frac{1}{4};-\frac{b}{ax^2},-\right.}{5e}$$

Mathematica [A] time = 0.18, size = 111, normalized size = 1.22

$$-\frac{2x(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{5}{4};-p,-q;-p-q+\frac{9}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*(e*x)^(3/2)*AppellF1[5/4 - p - q, -p, -q,
 9/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-5 + 4*p + 4*q)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex}\,ex\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*e*x*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1,[0,4,1,1,0]%%%} / %%%{1,[0,0,0,0,2]%%%} Error: Bad Argument Value
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{3}{2}}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex)^{\frac{3}{2}} \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,x)^{3/2}\,{\left(a+\frac{b}{x^2}\right)}^p\,{\left(c+\frac{d}{x^2}\right)}^q \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
```

```
[Out] int((e*x)^(3/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(3/2),x)
```

```
[Out] Timed out
```

3.998 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx$

Optimal. Leaf size=91

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

```
[Out] 2/3*(a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(3/2)*AppellF1(-3/4,-p,-q,1/4,-b/a/x^2,-d
/c/x^2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2(ex)^{3/2}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*(e*x)^(3/2)*AppellF1[-3/4, -p, -q, 1/4, -(b/
(a*x^2)), -(d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q \sqrt{ex}\,dx = -\frac{2\,\mathrm{Subst}\left(\int \frac{(a+be^2x^4)^p(c+de^2x^4)^q}{x^4}dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p(c+de^2x^4)^q}{x^4}dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\mathrm{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^4}\right.}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}(ex)^{3/2}F_1\left(-\frac{3}{4};-p,-q;\frac{1}{4};-\frac{b}{ax^2},-\right.}{3e}$$

Mathematica [A] time = 0.15, size = 111, normalized size = 1.22

$$-\frac{2x\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{3}{4};-p,-q;-p-q+\frac{7}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{4p+4q-3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*Sqrt[e*x]*AppellF1[3/4 - p - q, -p, -q, 7
/4 - p - q, -((a*x^2)/b), -((c*x^2)/d)])/((-3 + 4*p + 4*q)*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q*(e*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(e*x)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,x}\,\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q,x)
[Out] int((e*x)^(1/2)*(a + b/x^2)^p*(c + d/x^2)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q*(e*x)**(1/2),x)
[Out] Timed out
```

3.999 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$

Optimal. Leaf size=89

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

```
[Out] 2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(-1/4,-p,-q,3/4,-b/a/x^2,-d/c/x^2)*(e*x)^
(1/2)/e/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)
```

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$\frac{2\sqrt{ex}\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (2*(a + b/x^2)^p*(c + d/x^2)^q*Sqrt[e*x]*AppellF1[-1/4, -p, -q, 3/4, -(b/(a
*x^2)), -(d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx = -\frac{2\,\text{Subst}\left(\int \frac{\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q}{x^2}dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q}{x^2}dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \frac{\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q}{x^2}dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= \frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\sqrt{ex}\,F_1\left(-\frac{1}{4};-p,-q;\frac{3}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e}$$

Mathematica [A] time = 0.16, size = 111, normalized size = 1.25

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q+\frac{1}{4};-p,-q;-p-q+\frac{5}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{\sqrt{ex}\,(4p+4q-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/Sqrt[e*x],x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/4 - p - q, -p, -q, 5/4 - p - q
, -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 4*p + 4*q)*Sqrt[e*x]*(1 + (a*x^2)/b)^
p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{ex}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

```
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/sqrt(e*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{\sqrt{e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2),x)
```

```
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(1/2),x)
```

```
[Out] Timed out
```

3.1000 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

```
[Out] -2*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(1/4,-p,-q,5/4,-b/a/x^2,-d/c/x^2)/e/((1+
b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {496, 430, 429}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[1/4, -p, -q, 5/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*Sqrt[e*x])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{3/2}}\,dx = -\frac{2\,\text{Subst}\left(\int \left(a+be^2x^4\right)^p \left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{be^2x^4}{a}\right)^p \left(c+de^2x^4\right)^q\,dx, x, \frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right)\text{Subst}\left(\int \left(1+\frac{be^2x^4}{a}\right)^p \left(1+\frac{de^2x^4}{c}\right)^q\right.}{e}$$

$$= -\frac{2\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{e\sqrt{ex}}$$

Mathematica [A] time = 0.17, size = 111, normalized size = 1.25

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q-\frac{1}{4};-p,-q;-p-q+\frac{3}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(ex)^{3/2}(4p+4q+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x]
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/4 - p - q, -p, -q, 3/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((1 + 4*p + 4*q)*(e*x)^(3/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p \left(\frac{cx^2+d}{x^2}\right)^q}{e^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="giac")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(3/2),x)
[Out] Timed out
```

3.1001 $\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

```
[Out] -2/3*(a+b/x^2)^p*(c+d/x^2)^q*AppellF1(3/4,-p,-q,7/4,-b/a/x^2,-d/c/x^2)/e/((
1+b/a/x^2)^p)/((1+d/c/x^2)^q)/(e*x)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {496, 511, 510}

$$-\frac{2\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*AppellF1[3/4, -p, -q, 7/4, -(b/(a*x^2)), -(
d/(c*x^2))])/(3*e*(1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q*(e*x)^(3/2))
```

Rule 496

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{g = Denominator[m]}, -Dist[g/e, Subst[Int[((a +
b/(e^n*x^(g*n)))^p*(c + d/(e^n*x^(g*n)))^q)/x^(g*(m + 1) + 1), x], x, 1/(e*
x)^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && ILtQ[n, 0] && Fraction
Q[m]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q}{(ex)^{5/2}}\,dx = -\frac{2\,\mathrm{Subst}\left(\int x^2\left(a+be^2x^4\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\right)\mathrm{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(c+de^2x^4\right)^q\,dx,x,\frac{1}{\sqrt{ex}}\right)}{e}$$

$$= -\frac{\left(2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}\right)\mathrm{Subst}\left(\int x^2\left(1+\frac{be^2x^4}{a}\right)^p\left(1+\frac{de^2x^4}{c}\right)^q\right.}{e}$$

$$= -\frac{2\left(a+\frac{b}{x^2}\right)^p\left(1+\frac{b}{ax^2}\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(1+\frac{d}{cx^2}\right)^{-q}F_1\left(\frac{3}{4};-p,-q;\frac{7}{4};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)}{3e(ex)^{3/2}}$$

Mathematica [A] time = 0.18, size = 111, normalized size = 1.22

$$-\frac{2x\left(a+\frac{b}{x^2}\right)^p\left(\frac{ax^2}{b}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{cx^2}{d}+1\right)^{-q}F_1\left(-p-q-\frac{3}{4};-p,-q;-p-q+\frac{1}{4};-\frac{ax^2}{b},-\frac{cx^2}{d}\right)}{(ex)^{5/2}(4p+4q+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x]
```

```
[Out] (-2*(a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-3/4 - p - q, -p, -q, 1/4 - p -
q, -((a*x^2)/b), -((c*x^2)/d)])/((3 + 4*p + 4*q)*(e*x)^(5/2)*(1 + (a*x^2)/b
)^p*(1 + (c*x^2)/d)^q)
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex}\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q}{e^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x)*((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q/(e^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
[Out] int((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q}{(ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q/(e*x)^(5/2),x, algorithm="maxima")
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q/(e*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{x^2}\right)^p\,\left(c+\frac{d}{x^2}\right)^q}{{\left(e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2),x)
[Out] int(((a + b/x^2)^p*(c + d/x^2)^q)/(e*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q/(e*x)**(5/2),x)
[Out] Timed out
```

3.1002 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{5}{64}$$

```
[Out] -5/64*arccosh(x^(1/2))-5/96*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/
24*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/4*x^(7/2)*(-1+x^(1/2))^(1
/2)*(1+x^(1/2))^(1/2)-5/64*x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {280, 323, 330, 52}

$$\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{7/2}-\frac{1}{24}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{5}{96}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{5}{64}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{5}{64}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (-5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/64 - (5*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]]*x^(3/2))/96 - (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(
5/2))/24 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2))/4 - (5*ArcCosh[Sq
rt[x]])/64
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}\,dx = \frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2} - \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$$
$$= -\frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2} - \frac{5}{48}\int \frac{}{\sqrt{-1+}}$$
$$= -\frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{1}{4}\sqrt{-1+\sqrt{x}}$$
$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{}}$$
$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{}}$$
$$= -\frac{5}{64}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x} - \frac{5}{96}\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{24}\sqrt{-1+\sqrt{}}$$

Mathematica [A] time = 0.07, size = 111, normalized size = 0.82

$$\frac{\sqrt{\sqrt{x}+1}\,\sqrt{x}\left(48x^{7/2}-8x^{5/2}-10x^{3/2}-48x^3+8x^2+10x-15\sqrt{x}+15\right)+30\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{192\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 - 15*Sqrt[x] + 10*x - 10*x^(3/2) + 8*x^2 - 8
*x^(5/2) - 48*x^3 + 48*x^(7/2)) + 30*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt
[x]]/Sqrt[2]])/(192*Sqrt[-1 + Sqrt[x]])
```

fricas [A] time = 0.68, size = 62, normalized size = 0.46

$$\frac{1}{192}\left(48\,x^3-8\,x^2-10\,x-15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}+\frac{5}{128}\,\log\left(2\,\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}-2\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/192*(48*x^3 - 8*x^2 - 10*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) -
 1) + 5/128*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [A] time = 0.29, size = 162, normalized size = 1.20

$$\frac{1}{6720}\left(\left(2\left(\left(4\left(5\left(6\left(7\sqrt{x}-50\right)\left(\sqrt{x}+1\right)+1219\right)\left(\sqrt{x}+1\right)-12463\right)\left(\sqrt{x}+1\right)+64233\right)\left(\sqrt{x}+1\right)-53963\right)\left(\sqrt{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6720*((2*((4*(5*(6*(7*sqrt(x) - 50)*(sqrt(x) + 1) + 1219)*(sqrt(x) + 1) -
 12463)*(sqrt(x) + 1) + 64233)*(sqrt(x) + 1) - 53963)*(sqrt(x) + 1) + 59465
```

```
)*(sqrt(x) + 1) - 23205)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/840*((2*((
4*(5*(6*sqrt(x) - 37)*(sqrt(x) + 1) + 661)*(sqrt(x) + 1) - 4551)*(sqrt(x) +
 1) + 4781)*(sqrt(x) + 1) - 6335)*(sqrt(x) + 1) + 2835)*sqrt(sqrt(x) + 1)*s
qrt(sqrt(x) - 1) + 5/32*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [A] time = 0.05, size = 75, normalized size = 0.56

$$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-48\sqrt{x-1}\,x^{\frac{7}{2}}+8\sqrt{x-1}\,x^{\frac{5}{2}}+10\sqrt{x-1}\,x^{\frac{3}{2}}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)+15\sqrt{x-1}\,\sqrt{x}\right)}{192\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/192*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-48*x^(7/2)*(x-1)^(1/2)+8*x^(5/
2)*(x-1)^(1/2)+10*x^(3/2)*(x-1)^(1/2)+15*(x-1)^(1/2)*x^(1/2)+15*ln(x^(1/2)+
(x-1)^(1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.55, size = 57, normalized size = 0.42

$$\frac{1}{4}\,(x-1)^{\frac{3}{2}}x^{\frac{5}{2}}+\frac{5}{24}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{5}{32}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{5}{64}\,\sqrt{x-1}\,\sqrt{x}-\frac{5}{64}\,\log\left(2\,\sqrt{x-1}\,+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/4*(x - 1)^(3/2)*x^(5/2) + 5/24*(x - 1)^(3/2)*x^(3/2) + 5/32*(x - 1)^(3/2)
*sqrt(x) + 5/64*sqrt(x - 1)*sqrt(x) - 5/64*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 52.03, size = 831, normalized size = 6.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] ((1723*((x^(1/2) - 1)^(1/2) - 1i)^5)/(48*((x^(1/2) + 1)^(1/2) - 1)^5) - (23
5*((x^(1/2) - 1)^(1/2) - 1i)^3)/(48*((x^(1/2) + 1)^(1/2) - 1)^3) + (72283*(
(x^(1/2) - 1)^(1/2) - 1i)^7)/(16*((x^(1/2) + 1)^(1/2) - 1)^7) + (848801*((x
^(1/2) - 1)^(1/2) - 1i)^9)/(16*((x^(1/2) + 1)^(1/2) - 1)^9) + (4181067*((x^
(1/2) - 1)^(1/2) - 1i)^11)/(16*((x^(1/2) + 1)^(1/2) - 1)^11) + (10994181*((
x^(1/2) - 1)^(1/2) - 1i)^13)/(16*((x^(1/2) + 1)^(1/2) - 1)^13) + (17457599*
((x^(1/2) - 1)^(1/2) - 1i)^15)/(16*((x^(1/2) + 1)^(1/2) - 1)^15) + (1745759
9*((x^(1/2) - 1)^(1/2) - 1i)^17)/(16*((x^(1/2) + 1)^(1/2) - 1)^17) + (10994
181*((x^(1/2) - 1)^(1/2) - 1i)^19)/(16*((x^(1/2) + 1)^(1/2) - 1)^19) + (418
1067*((x^(1/2) - 1)^(1/2) - 1i)^21)/(16*((x^(1/2) + 1)^(1/2) - 1)^21) + (84
8801*((x^(1/2) - 1)^(1/2) - 1i)^23)/(16*((x^(1/2) + 1)^(1/2) - 1)^23) + (72
283*((x^(1/2) - 1)^(1/2) - 1i)^25)/(16*((x^(1/2) + 1)^(1/2) - 1)^25) + (172
3*((x^(1/2) - 1)^(1/2) - 1i)^27)/(48*((x^(1/2) + 1)^(1/2) - 1)^27) - (235*(
(x^(1/2) - 1)^(1/2) - 1i)^29)/(48*((x^(1/2) + 1)^(1/2) - 1)^29) + (5*((x^(1
/2) - 1)^(1/2) - 1i)^31)/(16*((x^(1/2) + 1)^(1/2) - 1)^31) + (5*((x^(1/2) -
 1)^(1/2) - 1i))/(16*((x^(1/2) + 1)^(1/2) - 1)))/((120*((x^(1/2) - 1)^(1/2)
 - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (16*((x^(1/2) - 1)^(1/2) - 1i)^2)/(
(x^(1/2) + 1)^(1/2) - 1)^2 - (560*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2) +
 1)^(1/2) - 1)^6 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2)
 - 1)^8 - (4368*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
 + (8008*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (114
40*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + (12870*((x
```

```
^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (11440*((x^(1/2)
 - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (8008*((x^(1/2) - 1)^(
1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (4368*((x^(1/2) - 1)^(1/2) -
1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + (1820*((x^(1/2) - 1)^(1/2) - 1i)^24)
/((x^(1/2) + 1)^(1/2) - 1)^24 - (560*((x^(1/2) - 1)^(1/2) - 1i)^26)/((x^(1/
2) + 1)^(1/2) - 1)^26 + (120*((x^(1/2) - 1)^(1/2) - 1i)^28)/((x^(1/2) + 1)^
(1/2) - 1)^28 - (16*((x^(1/2) - 1)^(1/2) - 1i)^30)/((x^(1/2) + 1)^(1/2) - 1
)^30 + ((x^(1/2) - 1)^(1/2) - 1i)^32/((x^(1/2) + 1)^(1/2) - 1)^32 + 1) - (5
*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/16
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1003 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/8*arccosh(x^(1/2))-1/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3
*x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/8*x^(1/2)*(-1+x^(1/2))^(1/2
)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {280, 323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{5/2}-\frac{1}{12}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{8}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 - (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2))
/3 - ArcCosh[Sqrt[x]]/8
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2} - \frac{1}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$

$$= -\frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2} - \frac{1}{8}\int \frac{}{\sqrt{-1+}}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}$$

$$= -\frac{1}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} - \frac{1}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}$$

Mathematica [A] time = 0.06, size = 99, normalized size = 0.95

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(8x^{5/2}-2x^{3/2}-8x^2+2x-3\sqrt{x}+3\right)+6\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{24\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2),x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 - 3*Sqrt[x] + 2*x - 2*x^(3/2) - 8*x^2 + 8*x^(
5/2)) + 6*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(24*Sqrt[-1
+ Sqrt[x]])
```

fricas [A] time = 0.68, size = 57, normalized size = 0.55

$$\frac{1}{24}\left(8x^2-2x-3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{16}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 - 2*x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/16*l
og(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [A] time = 0.22, size = 127, normalized size = 1.22

$$\frac{1}{120}\left(\left(2\left(\left(4\left(5\sqrt{x}-26\right)\left(\sqrt{x}+1\right)+321\right)\left(\sqrt{x}+1\right)-451\right)\left(\sqrt{x}+1\right)+745\right)\left(\sqrt{x}+1\right)-405\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*((4*(5*sqrt(x) - 26)*(sqrt(x) + 1) + 321)*(sqrt(x) + 1) - 451)*(s
qrt(x) + 1) + 745)*(sqrt(x) + 1) - 405)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
 + 1/60*((2*(3*(4*sqrt(x) - 17)*(sqrt(x) + 1) + 133)*(sqrt(x) + 1) - 295)*(
sqrt(x) + 1) + 195)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/4*log(sqrt(sqrt
(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [A] time = 0.05, size = 65, normalized size = 0.62

$$-\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(-8\sqrt{x-1}\,x^{\frac{5}{2}}+2\sqrt{x-1}\,x^{\frac{3}{2}}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)+3\sqrt{x-1}\,\sqrt{x}\right)}{24\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-8*(x-1)^(1/2)*x^(5/2)+2*(x-1)^(
1/2)*x^(3/2)+3*(x-1)^(1/2)*x^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.49, size = 47, normalized size = 0.45

$$\frac{1}{3}\,(x-1)^{\frac{3}{2}}x^{\frac{3}{2}}+\frac{1}{4}\,(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{8}\,\sqrt{x-1}\,\sqrt{x}-\frac{1}{8}\,\log\left(2\,\sqrt{x-1}+2\,\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*(x - 1)^(3/2)*x^(3/2) + 1/4*(x - 1)^(3/2)*sqrt(x) + 1/8*sqrt(x - 1)*sqr
t(x) - 1/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 31.39, size = 632, normalized size = 6.08

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{\frac{35\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{757\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{7339\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{41929\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25661\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\cdots}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] - atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1))/2 - ((35*((x^
(1/2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (757*((x^(1/2)
- 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) + (7339*((x^(1/2) - 1)^
(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (41929*((x^(1/2) - 1)^(1/2
) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25661*((x^(1/2) - 1)^(1/2) -
1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25661*((x^(1/2) - 1)^(1/2) - 1i)^13
)/((x^(1/2) + 1)^(1/2) - 1)^13 + (41929*((x^(1/2) - 1)^(1/2) - 1i)^15)/(3*(
(x^(1/2) + 1)^(1/2) - 1)^15) + (7339*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*((x^
(1/2) + 1)^(1/2) - 1)^17) + (757*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(1/2
) + 1)^(1/2) - 1)^19) + (35*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2) + 1
)^(1/2) - 1)^21) - ((x^(1/2) - 1)^(1/2) - 1i)^23/(2*((x^(1/2) + 1)^(1/2) -
1)^23) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1/2) - 1)))/((66*((x
^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((x^(1/2) - 1)
^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) - 1)^(1/2) - 1
i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^
(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1
)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2)
- 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14
+ (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)^16 - (220*(
(x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (66*((x^(1/2)
- 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/2) - 1)^(1/2
```

```
) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2) - 1i)^24/((
x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}} \sqrt{\sqrt{x} - 1} \sqrt{\sqrt{x} + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1004 $\int \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -1/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)-1/4*
x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {280, 323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,x^{3/2}-\frac{1}{4}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] -(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sqr
t[1 + Sqrt[x]]*x^(3/2))/2 - ArcCosh[Sqrt[x]]/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}dx$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{8}\int \frac{}{\sqrt{-1+\sqrt{}}}$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} - \frac{1}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.08, size = 87, normalized size = 1.19

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(2x^{3/2}-2x-\sqrt{x}+1\right)+2\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{4\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x],x]
```

```
[Out] (Sqrt[1 + Sqrt[x]]*Sqrt[x]*(1 - Sqrt[x] - 2*x + 2*x^(3/2)) + 2*Sqrt[1 - Sqr
t[x]]*ArcSin[Sqrt[1 - Sqrt[x]]/Sqrt[2]])/(4*Sqrt[-1 + Sqrt[x]])
```

fricas [A] time = 0.63, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} + \frac{1}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1} - 2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [B] time = 0.19, size = 92, normalized size = 1.26

$$\frac{1}{12}\left(\left(2\left(3\sqrt{x}-10\right)\left(\sqrt{x}+1\right)+43\right)\left(\sqrt{x}+1\right)-39\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{3}\left(\left(2\sqrt{x}-5\right)\left(\sqrt{x}+1\right)+9\right)\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*((2*(3*sqrt(x) - 10)*(sqrt(x) + 1) + 43)*(sqrt(x) + 1) - 39)*sqrt(sqrt
(x) + 1)*sqrt(sqrt(x) - 1) + 1/3*((2*sqrt(x) - 5)*(sqrt(x) + 1) + 9)*sqrt(s
qrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))
```

maple [A] time = 0.05, size = 52, normalized size = 0.71

$$-\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(-2\sqrt{x-1}\,x^{\frac{3}{2}}+\ln\left(\sqrt{x}+\sqrt{x-1}\right)+\sqrt{x-1}\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2),x)
```

```
[Out] -1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(-2*(x-1)^(1/2)*x^(3/2)+(x-1)^(1/2
)*x^(1/2)+ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.58, size = 37, normalized size = 0.51

$$\frac{1}{2}(x-1)^{\frac{3}{2}}\sqrt{x}+\frac{1}{4}\sqrt{x-1}\sqrt{x}-\frac{1}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*(x - 1)^(3/2)*sqrt(x) + 1/4*sqrt(x - 1)*sqrt(x) - 1/4*log(2*sqrt(x - 1)
 + 2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2),x)
```

```
[Out] int(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1), x)
```

3.1005 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=37

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] -arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {280, 330, 52}

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] - ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx &= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}-\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.95

$$\frac{\sqrt{\sqrt{x}+1}\sqrt{x}\left(\sqrt{x}-1\right)+2\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x],x]
```

```
[Out] ((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*Sqrt[1 - Sqrt[x]]*ArcSin[Sqrt
[1 - Sqrt[x]]/Sqrt[2]])/Sqrt[-1 + Sqrt[x]]
```

fricas [A] time = 0.64, size = 46, normalized size = 1.24

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [B] time = 0.18, size = 57, normalized size = 1.54

$$\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\left(\sqrt{x}-2\right)+2\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)*(sqrt(x) - 2) + 2*sqrt(sqrt(x) + 1)*sqr
t(sqrt(x) - 1) + 2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [B] time = 0.05, size = 72, normalized size = 1.95

$$-\frac{\sqrt{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}}+\sqrt{\sqrt{x}-1}\,\left(\sqrt{x}+1\right)^{\frac{3}{2}}-\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2),x)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(3/2)-(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)-((x
^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^(1/2)
+(x-1)^(1/2))
```

maxima [A] time = 0.49, size = 26, normalized size = 0.70

$$\sqrt{x-1}\sqrt{x}-\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) - log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 5.07, size = 41, normalized size = 1.11

$$\sqrt{x}\,\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}-\ln\left(\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2),x)
```

```
[Out] x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2) - log((x^(1/2) - 1)^(1/2)*(
x^(1/2) + 1)^(1/2) + x^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/sqrt(x), x)
```

3.1006 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\,\sqrt{x}\,\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))+2*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(1/2)-2*x^(1/2)
*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {327, 280, 330, 52}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{\sqrt{x}}-2\sqrt{\sqrt{x}-1}\,\sqrt{x}\,\sqrt{\sqrt{x}+1}+2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/Sqrt[x] - 2*Sqrt[-1 + Sqrt[x]]
*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 280

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^p*(a2 + b2*x^n)
^p)/(c*(m + 2*n*p + 1)), x] + Dist[(2*a1*a2*n*p)/(m + 2*n*p + 1), Int[(c*x)
^m*(a1 + b1*x^n)^(p - 1)*(a2 + b2*x^n)^(p - 1), x], x] /; FreeQ[{a1, b1, a2
, b2, c, m}, x] && EqQ[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[p, 0] && Ne
Q[m + 2*n*p + 1, 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2
*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(
a1*a2*c^(2*n)*(m + 1)), Int[(c*x)^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p
, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2, 0] && IG
tQ[2*n, 0] && LtQ[m, -1] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{3/2}}\,dx = \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}\,dx$$
$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1-}$$
$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\,\text{Subst}\left(\int \frac{}{\sqrt{-1+}}\right.$$
$$= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{\sqrt{x}} - 2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + 2\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.08, size = 74, normalized size = 1.10

$$\frac{2\left(-\frac{\sqrt{\sqrt{x}+1}\left(\sqrt{x}-1\right)}{\sqrt{x}} - 2\sqrt{1-\sqrt{x}}\,\sin^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{2}}\right)\right)}{\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(3/2),x]
```

```
[Out] (2*(-(((-1 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/Sqrt[x]) - 2*Sqrt[1 - Sqrt[x]]*Arc
Sin[Sqrt[1 - Sqrt[x]]/Sqrt[2]]))/Sqrt[-1 + Sqrt[x]]
```

fricas [A] time = 0.66, size = 55, normalized size = 0.82

$$-\frac{x\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)+2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="fricas
")
```

```
[Out] -(x*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1) + 2*sqrt(x
)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x)/x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.05, size = 47, normalized size = 0.70

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\sqrt{x}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)-\sqrt{x-1}\right)}{\sqrt{x-1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(3/2),x)
```

```
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(ln(x^(1/2)+(x-1)^(1/2))*x^(1/2)-(x-1
)^(1/2))/(x-1)^(1/2)/x^(1/2)
```

maxima [A] time = 1.28, size = 27, normalized size = 0.40

$$-\frac{2\sqrt{x-1}}{\sqrt{x}}+2\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2),x, algorithm="maxima
")
```

```
[Out] -2*sqrt(x - 1)/sqrt(x) + 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 6.25, size = 129, normalized size = 1.93

$$8\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)-\frac{\frac{5\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^2}{2\left(\sqrt{\sqrt{x}+1}-1\right)^2}+\frac{1}{2}}{\frac{\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}}-\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{2\left(\sqrt{\sqrt{x}+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(3/2),x)
```

```
[Out] 8*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) - ((5*((x^(1/
2) - 1)^(1/2) - 1i)^2)/(2*((x^(1/2) + 1)^(1/2) - 1)^2) + 1/2)/(((x^(1/2) -
1)^(1/2) - 1i)^3/((x^(1/2) + 1)^(1/2) - 1)^3 + ((x^(1/2) - 1)^(1/2) - 1i)/(
(x^(1/2) + 1)^(1/2) - 1)) - ((x^(1/2) - 1)^(1/2) - 1i)/(2*((x^(1/2) + 1)^(1
/2) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(3/2), x)
```

3.1007 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

```
[Out] 2/3*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx=\frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(5/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(3*x^(3/2))
```

fricas [A] time = 0.70, size = 30, normalized size = 0.97

$$\frac{2\left((x-1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x^2\right)}{3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="fricas
")
```

```
[Out] 2/3*((x - 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x^2)/x^2
```

giac [B] time = 0.23, size = 48, normalized size = 1.55

$$\frac{16\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+16\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] 16/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 16)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

maple [A] time = 0.05, size = 23, normalized size = 0.74

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(x-1\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(5/2),x)
```

```
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)/x^(3/2)
```

maxima [A] time = 1.32, size = 10, normalized size = 0.32

$$\frac{2\,(x-1)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2),x, algorithm="maxima
")
```

```
[Out] 2/3*(x - 1)^(3/2)/x^(3/2)
```

mupad [B] time = 5.26, size = 31, normalized size = 1.00

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{3}-\frac{2\,\sqrt{\sqrt{x}+1}}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(5/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/3 - (2*(x^(1/2) + 1)^(1/2))
/3))/x^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(5/2), x)
```

3.1008 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

```
[Out] 2/5*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)+4/15*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{15x^{3/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(5*x^(5/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(15*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{2}{5}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{5x^{5/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.57

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}(2x+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(7/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(3 + 2*x))/(15*x^(5/2))
```

fricas [A] time = 0.75, size = 37, normalized size = 0.59

$$\frac{2\left(2\,x^3+\left(2\,x^2+x-3\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="fricas
")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + x - 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1))/
x^3
```

giac [B] time = 0.28, size = 90, normalized size = 1.43

$$\frac{128\left(15\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}-20\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+80\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+64\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 128/15*(15*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^12 - 20*(sqrt(sqrt(x) +
1) - sqrt(sqrt(x) - 1))^8 + 80*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
64)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)^5
```

maple [A] time = 0.05, size = 28, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(x-1\right)\left(2x+3\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(7/2),x)
```

```
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(2*x+3)/x^(5/2)
```

maxima [A] time = 1.31, size = 21, normalized size = 0.33

$$\frac{4\left(x-1\right)^{\frac{3}{2}}}{15\,x^{\frac{3}{2}}}+\frac{2\left(x-1\right)^{\frac{3}{2}}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(7/2),x, algorithm="maxima
")
```

```
[Out] 4/15*(x - 1)^(3/2)/x^(3/2) + 2/5*(x - 1)^(3/2)/x^(5/2)
```

mupad [B] time = 5.07, size = 43, normalized size = 0.68

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{15}-\frac{2\,\sqrt{\sqrt{x}+1}}{5}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{15}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(7/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/15 - (2*(x^(1/2) + 1)^(1/2)
)/5 + (4*x^2*(x^(1/2) + 1)^(1/2))/15))/x^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)/x**(7/2), x)
```

3.1009 $\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx$

Optimal. Leaf size=94

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

```
[Out] 2/7*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(7/2)+8/35*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(5/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{3/2}}+\frac{8\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{35x^{5/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(7*x^(7/2)) + (8*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(35*x^(5/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{4}{7}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{7/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{8}{35}\int \frac{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{x^{5/2}}\,dx\\ &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{7x^{7/2}}+\frac{8\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{35x^{5/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{105x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.44

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(8x^2+12x+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(9/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(15 + 12*x + 8*x^2))/(105*x^(7/
2))
```

fricas [A] time = 0.56, size = 44, normalized size = 0.47

$$\frac{2\left(8\,x^4+\left(8\,x^3+4\,x^2+3\,x-15\right)\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}\right)}{105\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="fricas
")
```

```
[Out] 2/105*(8*x^4 + (8*x^3 + 4*x^2 + 3*x - 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sq
rt(x) - 1))/x^4
```

giac [A] time = 0.29, size = 111, normalized size = 1.18

$$\frac{4096\left(35\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}-70\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+168\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{8}+224\left(\right.\right.}{105\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 4096/105*(35*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^16 - 70*(sqrt(sqrt(x)
+ 1) - sqrt(sqrt(x) - 1))^12 + 168*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^
8 + 224*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 128)/((sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 4)^7
```

maple [A] time = 0.05, size = 33, normalized size = 0.35

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(x-1\right)\left(8x^2+12x+15\right)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(9/2),x)
```

```
[Out] 2/105*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(8*x^2+12*x+15)/x^(7/2)
```

maxima [A] time = 1.55, size = 31, normalized size = 0.33

$$\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{3}{2}}}+\frac{8\,(x-1)^{\frac{3}{2}}}{35\,x^{\frac{5}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{7\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(9/2),x, algorithm="maxima
")
```

```
[Out] 16/105*(x - 1)^(3/2)/x^(3/2) + 8/35*(x - 1)^(3/2)/x^(5/2) + 2/7*(x - 1)^(3/
2)/x^(7/2)
```

mupad [B] time = 5.04, size = 55, normalized size = 0.59

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{35}-\frac{2\,\sqrt{\sqrt{x}+1}}{7}+\frac{8\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{105}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(9/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/35 - (2*(x^(1/2) + 1)^(1/2)
)/7 + (8*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/105))/
x^(7/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

3.1010 $\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx$

Optimal. Leaf size=125

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

```
[Out] 2/9*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(9/2)+4/21*(-1+x^(1/2))^(3/2)*(1
+x^(1/2))^(3/2)/x^(7/2)+16/105*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(5/2)
+32/315*(-1+x^(1/2))^(3/2)*(1+x^(1/2))^(3/2)/x^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{32\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{315x^{3/2}}+\frac{16\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{105x^{5/2}}+\frac{4\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{21x^{7/2}}+\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}}{9x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2))/(9*x^(9/2)) + (4*(-1 + Sqrt[x]
)^(3/2)*(1 + Sqrt[x])^(3/2))/(21*x^(7/2)) + (16*(-1 + Sqrt[x])^(3/2)*(1 + S
qrt[x])^(3/2))/(105*x^(5/2)) + (32*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)
)/(315*x^(3/2))
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{11/2}}\,dx &= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{2}{3}\int \frac{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{x^{9/2}}\,dx\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{8}{21}\int \frac{\sqrt{-1+\sqrt{x}}}{x^{7/}}\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}(}{105x^{5/2}}\\
&= \frac{2\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{9x^{9/2}}+\frac{4\left(-1+\sqrt{x}\right)^{3/2}\left(1+\sqrt{x}\right)^{3/2}}{21x^{7/2}}+\frac{16\left(-1+\sqrt{x}\right)^{3/2}(}{105x^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.37

$$\frac{2\left(\sqrt{x}-1\right)^{3/2}\left(\sqrt{x}+1\right)^{3/2}\left(16x^3+24x^2+30x+35\right)}{315x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/x^(11/2),x]
```

```
[Out] (2*(-1 + Sqrt[x])^(3/2)*(1 + Sqrt[x])^(3/2)*(35 + 30*x + 24*x^2 + 16*x^3))/
(315*x^(9/2))
```

fricas [A] time = 0.68, size = 49, normalized size = 0.39

$$\frac{2\left(16\,x^5+\left(16\,x^4+8\,x^3+6\,x^2+5\,x-35\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{315\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="frica
s")
```

```
[Out] 2/315*(16*x^5 + (16*x^4 + 8*x^3 + 6*x^2 + 5*x - 35)*sqrt(x)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1))/x^5
```

giac [A] time = 0.43, size = 132, normalized size = 1.06

$$\frac{16384\left(315\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{20}-756\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{16}+1344\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{12}+230\cdots\right.}{315\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="giac"
)
```

```
[Out] 16384/315*(315*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^20 - 756*(sqrt(sqrt(
x) + 1) - sqrt(sqrt(x) - 1))^16 + 1344*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^12 + 2304*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 2304*(sqrt(sqrt(x
) + 1) - sqrt(sqrt(x) - 1))^4 + 1024)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) -
1))^4 + 4)^9
```

maple [A] time = 0.06, size = 38, normalized size = 0.30

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(x-1\right)\left(16x^3+24x^2+30x+35\right)}{315x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(11/2),x)
```

```
[Out] 2/315*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(x-1)*(16*x^3+24*x^2+30*x+35)/x^(
9/2)
```

maxima [A] time = 1.22, size = 41, normalized size = 0.33

$$\frac{32\,(x-1)^{\frac{3}{2}}}{315\,x^{\frac{3}{2}}}+\frac{16\,(x-1)^{\frac{3}{2}}}{105\,x^{\frac{5}{2}}}+\frac{4\,(x-1)^{\frac{3}{2}}}{21\,x^{\frac{7}{2}}}+\frac{2\,(x-1)^{\frac{3}{2}}}{9\,x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(11/2),x, algorithm="maxim
a")
```

```
[Out] 32/315*(x - 1)^(3/2)/x^(3/2) + 16/105*(x - 1)^(3/2)/x^(5/2) + 4/21*(x - 1)^
(3/2)/x^(7/2) + 2/9*(x - 1)^(3/2)/x^(9/2)
```

mupad [B] time = 5.02, size = 67, normalized size = 0.54

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{2\,x\,\sqrt{\sqrt{x}+1}}{63}-\frac{2\,\sqrt{\sqrt{x}+1}}{9}+\frac{4\,x^2\,\sqrt{\sqrt{x}+1}}{105}+\frac{16\,x^3\,\sqrt{\sqrt{x}+1}}{315}+\frac{32\,x^4\,\sqrt{\sqrt{x}+1}}{315}\right)}{x^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(11/2),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((2*x*(x^(1/2) + 1)^(1/2))/63 - (2*(x^(1/2) + 1)^(1/2)
)/9 + (4*x^2*(x^(1/2) + 1)^(1/2))/105 + (16*x^3*(x^(1/2) + 1)^(1/2))/315 +
(32*x^4*(x^(1/2) + 1)^(1/2))/315))/x^(9/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/2))**(1/2)*(1+x**(1/2))**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

3.1011 $\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 5/8*arccosh(x^(1/2))+5/12*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+1/3*
x^(5/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+5/8*x^(1/2)*(-1+x^(1/2))^(1/2)
*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{3}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{5/2}+\frac{5}{12}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{5}{8}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{5}{8}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (5*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/8 + (5*Sqrt[-1 + Sqrt[x]]*
Sqrt[1 + Sqrt[x]]*x^(3/2))/12 + (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/
2))/3 + (5*ArcCosh[Sqrt[x]])/8
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{5}{6}\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$
$$= \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{}}$$
$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+}$$
$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+}$$
$$= \frac{5}{8}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{5}{12}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{1}{3}\sqrt{-1+\sqrt{x}}\sqrt{1+}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.64

$$\frac{1}{24}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}\left(8x^2+10x+15\right)+30\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(5/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(15 + 10*x + 8*x^2) + 30*ArcT
anh[Sqrt[(-1 + Sqrt[x])/(1 + Sqrt[x])]])/24
```

fricas [A] time = 0.57, size = 57, normalized size = 0.55

$$\frac{1}{24}\left(8x^2+10x+15\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{5}{16}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/24*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/16
*log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [A] time = 0.23, size = 76, normalized size = 0.73

$$\frac{1}{24}\left(\left(2\left(\left(4\left(\sqrt{x}+1\right)\left(\sqrt{x}-4\right)+45\right)\left(\sqrt{x}+1\right)-55\right)\left(\sqrt{x}+1\right)+85\right)\left(\sqrt{x}+1\right)-33\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{5}{4}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*((4*(sqrt(x) + 1)*(sqrt(x) - 4) + 45)*(sqrt(x) + 1) - 55)*(sqrt(x)
 + 1) + 85)*(sqrt(x) + 1) - 33)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 5/4*l
og(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [A] time = 0.06, size = 65, normalized size = 0.62

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(8\sqrt{x-1}\,x^{\frac{5}{2}}+10\sqrt{x-1}\,x^{\frac{3}{2}}+15\ln\left(\sqrt{x}+\sqrt{x-1}\right)+15\sqrt{x-1}\,\sqrt{x}\right)}{24\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 1/24*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*(x-1)^(1/2)*x^(5/2)+10*(x-1)^(1
/2)*x^(3/2)+15*(x-1)^(1/2)*x^(1/2)+15*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.61, size = 47, normalized size = 0.45

$$\frac{1}{3}\sqrt{x-1}\,x^{\frac{5}{2}}+\frac{5}{12}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{5}{8}\sqrt{x-1}\,\sqrt{x}+\frac{5}{8}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/3*sqrt(x - 1)*x^(5/2) + 5/12*sqrt(x - 1)*x^(3/2) + 5/8*sqrt(x - 1)*sqrt(x
) + 5/8*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 27.09, size = 632, normalized size = 6.08

$$\frac{5\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)}{2}-\frac{-\frac{175\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{6\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{311\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{2\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{8361\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{2\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{42259\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{3\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{25295\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\cdots}{1+\frac{66\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{220\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{495\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{792\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{924\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (5*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)))/2 - ((311*(
(x^(1/2) - 1)^(1/2) - 1i)^5)/(2*((x^(1/2) + 1)^(1/2) - 1)^5) - (175*((x^(1/
2) - 1)^(1/2) - 1i)^3)/(6*((x^(1/2) + 1)^(1/2) - 1)^3) + (8361*((x^(1/2) -
1)^(1/2) - 1i)^7)/(2*((x^(1/2) + 1)^(1/2) - 1)^7) + (42259*((x^(1/2) - 1)^(
1/2) - 1i)^9)/(3*((x^(1/2) + 1)^(1/2) - 1)^9) + (25295*((x^(1/2) - 1)^(1/2)
 - 1i)^11)/((x^(1/2) + 1)^(1/2) - 1)^11 + (25295*((x^(1/2) - 1)^(1/2) - 1i)
^13)/((x^(1/2) + 1)^(1/2) - 1)^13 + (42259*((x^(1/2) - 1)^(1/2) - 1i)^15)/(
3*((x^(1/2) + 1)^(1/2) - 1)^15) + (8361*((x^(1/2) - 1)^(1/2) - 1i)^17)/(2*(
(x^(1/2) + 1)^(1/2) - 1)^17) + (311*((x^(1/2) - 1)^(1/2) - 1i)^19)/(2*((x^(
1/2) + 1)^(1/2) - 1)^19) - (175*((x^(1/2) - 1)^(1/2) - 1i)^21)/(6*((x^(1/2)
 + 1)^(1/2) - 1)^21) + (5*((x^(1/2) - 1)^(1/2) - 1i)^23)/(2*((x^(1/2) + 1)^
(1/2) - 1)^23) + (5*((x^(1/2) - 1)^(1/2) - 1i))/(2*((x^(1/2) + 1)^(1/2) - 1
)))/((66*((x^(1/2) - 1)^(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (12*((
x^(1/2) - 1)^(1/2) - 1i)^2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (220*((x^(1/2) -
1)^(1/2) - 1i)^6)/((x^(1/2) + 1)^(1/2) - 1)^6 + (495*((x^(1/2) - 1)^(1/2) -
 1i)^8)/((x^(1/2) + 1)^(1/2) - 1)^8 - (792*((x^(1/2) - 1)^(1/2) - 1i)^10)/(
(x^(1/2) + 1)^(1/2) - 1)^10 + (924*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2)
 + 1)^(1/2) - 1)^12 - (792*((x^(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1
/2) - 1)^14 + (495*((x^(1/2) - 1)^(1/2) - 1i)^16)/((x^(1/2) + 1)^(1/2) - 1)
^16 - (220*((x^(1/2) - 1)^(1/2) - 1i)^18)/((x^(1/2) + 1)^(1/2) - 1)^18 + (6
6*((x^(1/2) - 1)^(1/2) - 1i)^20)/((x^(1/2) + 1)^(1/2) - 1)^20 - (12*((x^(1/
2) - 1)^(1/2) - 1i)^22)/((x^(1/2) + 1)^(1/2) - 1)^22 + ((x^(1/2) - 1)^(1/2)
 - 1i)^24/((x^(1/2) + 1)^(1/2) - 1)^24 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(5/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1012 $\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=73

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 3/4*arccosh(x^(1/2))+1/2*x^(3/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)+3/4*x
^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\frac{1}{2}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,x^{3/2}+\frac{3}{4}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+\frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (3*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x])/4 + (Sqrt[-1 + Sqrt[x]]*Sq
rt[1 + Sqrt[x]]*x^(3/2))/2 + (3*ArcCosh[Sqrt[x]])/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx = \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}\,dx$$
$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{3}{8}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1}}$$
$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{3}{4}\text{Subst}\left(\int \frac{}{\sqrt{-1+}}\right.$$
$$= \frac{3}{4}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x} + \frac{1}{2}\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2} + \frac{3}{4}\cosh^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.85

$$\frac{1}{4}\left(\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}(2x+3)+6\tanh^{-1}\left(\sqrt{\frac{\sqrt{x}-1}{\sqrt{x}+1}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^(3/2)/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] (Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]*(3 + 2*x) + 6*ArcTanh[Sqrt[(-
1 + Sqrt[x])/(1 + Sqrt[x])]])/4
```

fricas [A] time = 0.62, size = 52, normalized size = 0.71

$$\frac{1}{4}(2x+3)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{3}{8}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 3/8*log(2*sqrt(
x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [A] time = 0.23, size = 59, normalized size = 0.81

$$\frac{1}{4}\left(\left(2\left(\sqrt{x}+1\right)\left(\sqrt{x}-2\right)+9\right)\left(\sqrt{x}+1\right)-5\right)\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{3}{2}\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((2*(sqrt(x) + 1)*(sqrt(x) - 2) + 9)*(sqrt(x) + 1) - 5)*sqrt(sqrt(x) +
1)*sqrt(sqrt(x) - 1) - 3/2*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [A] time = 0.06, size = 55, normalized size = 0.75

$$\frac{\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\left(2\sqrt{x-1}\,x^{\frac{3}{2}}+3\ln\left(\sqrt{x}+\sqrt{x-1}\right)+3\sqrt{x-1}\sqrt{x}\right)}{4\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 1/4*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*(x-1)^(1/2)*x^(3/2)+3*(x-1)^(1/2
)*x^(1/2)+3*ln(x^(1/2)+(x-1)^(1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.46, size = 37, normalized size = 0.51

$$\frac{1}{2}\sqrt{x-1}\,x^{\frac{3}{2}}+\frac{3}{4}\sqrt{x-1}\,\sqrt{x}+\frac{3}{4}\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] 1/2*sqrt(x - 1)*x^(3/2) + 3/4*sqrt(x - 1)*sqrt(x) + 3/4*log(2*sqrt(x - 1) +
 2*sqrt(x))
```

mupad [B] time = 18.76, size = 429, normalized size = 5.88

$$3\,\mathrm{atanh}\left(\frac{\sqrt{\sqrt{x}-1}-\mathrm{i}}{\sqrt{\sqrt{x}+1}-1}\right)+\frac{\frac{23\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^3}{\left(\sqrt{\sqrt{x}+1}-1\right)^3}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^5}{\left(\sqrt{\sqrt{x}+1}-1\right)^5}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^7}{\left(\sqrt{\sqrt{x}+1}-1\right)^7}+\frac{671\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^9}{\left(\sqrt{\sqrt{x}+1}-1\right)^9}+\frac{333\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{11}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{11}}+\cdots}{1+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^4}{\left(\sqrt{\sqrt{x}+1}-1\right)^4}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^6}{\left(\sqrt{\sqrt{x}+1}-1\right)^6}+\frac{70\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^8}{\left(\sqrt{\sqrt{x}+1}-1\right)^8}-\frac{56\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{10}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{10}}+\frac{28\left(\sqrt{\sqrt{x}-1}-\mathrm{i}\right)^{12}}{\left(\sqrt{\sqrt{x}+1}-1\right)^{12}}-\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 3*atanh(((x^(1/2) - 1)^(1/2) - 1i)/((x^(1/2) + 1)^(1/2) - 1)) + ((23*((x^(1
/2) - 1)^(1/2) - 1i)^3)/((x^(1/2) + 1)^(1/2) - 1)^3 + (333*((x^(1/2) - 1)^(
1/2) - 1i)^5)/((x^(1/2) + 1)^(1/2) - 1)^5 + (671*((x^(1/2) - 1)^(1/2) - 1i)
^7)/((x^(1/2) + 1)^(1/2) - 1)^7 + (671*((x^(1/2) - 1)^(1/2) - 1i)^9)/((x^(1
/2) + 1)^(1/2) - 1)^9 + (333*((x^(1/2) - 1)^(1/2) - 1i)^11)/((x^(1/2) + 1)^
(1/2) - 1)^11 + (23*((x^(1/2) - 1)^(1/2) - 1i)^13)/((x^(1/2) + 1)^(1/2) - 1
)^13 - (3*((x^(1/2) - 1)^(1/2) - 1i)^15)/((x^(1/2) + 1)^(1/2) - 1)^15 - (3*
((x^(1/2) - 1)^(1/2) - 1i))/((x^(1/2) + 1)^(1/2) - 1))/((28*((x^(1/2) - 1)^
(1/2) - 1i)^4)/((x^(1/2) + 1)^(1/2) - 1)^4 - (8*((x^(1/2) - 1)^(1/2) - 1i)^
2)/((x^(1/2) + 1)^(1/2) - 1)^2 - (56*((x^(1/2) - 1)^(1/2) - 1i)^6)/((x^(1/2
) + 1)^(1/2) - 1)^6 + (70*((x^(1/2) - 1)^(1/2) - 1i)^8)/((x^(1/2) + 1)^(1/2
) - 1)^8 - (56*((x^(1/2) - 1)^(1/2) - 1i)^10)/((x^(1/2) + 1)^(1/2) - 1)^10
+ (28*((x^(1/2) - 1)^(1/2) - 1i)^12)/((x^(1/2) + 1)^(1/2) - 1)^12 - (8*((x^
(1/2) - 1)^(1/2) - 1i)^14)/((x^(1/2) + 1)^(1/2) - 1)^14 + ((x^(1/2) - 1)^(1
/2) - 1i)^16/((x^(1/2) + 1)^(1/2) - 1)^16 + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(x**(3/2)/(sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1013 $\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx$

Optimal. Leaf size=35

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] arccosh(x^(1/2))+x^(1/2)*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {323, 330, 52}

$$\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 323

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> Simp[(c^(2*n - 1)*(c*x)^(m - 2*n + 1)*(a1 + b1*x^n
)^(p + 1)*(a2 + b2*x^n)^(p + 1))/(b1*b2*(m + 2*n*p + 1)), x] - Dist[(a1*a2*
c^(2*n)*(m - 2*n + 1))/(b1*b2*(m + 2*n*p + 1)), Int[(c*x)^(m - 2*n)*(a1 + b
1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ
[a2*b1 + a1*b2, 0] && IGtQ[2*n, 0] && GtQ[m, 2*n - 1] && NeQ[m + 2*n*p + 1,
 0] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m, p, x]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}\,dx &= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}}\,dx\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\,\sqrt{1+x}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,\sqrt{x}+\cosh^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 1.57

$$\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\sqrt{x}+2\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]),x]
```

```
[Out] Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x] + 2*ArcTanh[Sqrt[-1 + Sqrt[x]]
/Sqrt[1 + Sqrt[x]]]
```

fricas [A] time = 0.55, size = 46, normalized size = 1.31

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-\frac{1}{2}\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fricas
")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 1/2*log(2*sqrt(x)*sqrt(sqrt(x
) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [A] time = 0.22, size = 39, normalized size = 1.11

$$\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2\log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*log(sqrt(sqrt(x) + 1) - sqr
t(sqrt(x) - 1))
```

maple [A] time = 0.06, size = 41, normalized size = 1.17

$$\frac{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(\ln\left(\sqrt{x}+\sqrt{x-1}\right)+\sqrt{x-1}\,\sqrt{x}\right)}{\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] (x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*((x-1)^(1/2)*x^(1/2)+ln(x^(1/2)+(x-1)^(
1/2)))/(x-1)^(1/2)
```

maxima [A] time = 0.45, size = 24, normalized size = 0.69

$$\sqrt{x-1}\sqrt{x}+\log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxima
")
```

```
[Out] sqrt(x - 1)*sqrt(x) + log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{x}}{\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] int(x^(1/2)/((x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)), x)
```

sympy [C] time = 27.91, size = 83, normalized size = 2.37

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{x}\right)}{2\pi^{\frac{3}{2}}}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle|\frac{e^{2i\pi}}{x}\right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0),
()), 1/x)/(2*pi**(3/2)) - I*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()),
((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/x)/(2*pi**(3/2))
```

3.1014 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*arccosh(x^(1/2))
```

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {330, 52}

$$2\cosh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 2*ArcCosh[Sqrt[x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 330

```
Int[((c_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^
(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(
k*(m + 1) - 1)*(a1 + (b1*x^(k*n))/c^n)^p*(a2 + (b2*x^(k*n))/c^n)^p, x], x,
(c*x)^(1/k)], x]] /; FreeQ[{a1, b1, a2, b2, c, p}, x] && EqQ[a2*b1 + a1*b2,
 0] && IGtQ[2*n, 0] && FractionQ[m] && IntBinomialQ[a1*a2, b1*b2, c, 2*n, m
, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}\sqrt{1+x}}\,dx, x, \sqrt{x}\right)\\ &= 2\cosh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] time = 0.01, size = 26, normalized size = 3.25

$$4\tanh^{-1}\left(\frac{\sqrt{\sqrt{x}-1}}{\sqrt{\sqrt{x}+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*Sqrt[x]),x]
```

```
[Out] 4*ArcTanh[Sqrt[-1 + Sqrt[x]]/Sqrt[1 + Sqrt[x]]]
```

fricas [B] time = 0.67, size = 27, normalized size = 3.38

$$-\log\left(2\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}-2x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] -log(2*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) - 2*x + 1)
```

giac [B] time = 0.20, size = 20, normalized size = 2.50

$$-4 \log\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] -4*log(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))
```

maple [B] time = 0.05, size = 40, normalized size = 5.00

$$\frac{2\sqrt{\left(\sqrt{x}-1\right)\left(\sqrt{x}+1\right)}\,\ln\left(\sqrt{x}+\sqrt{x-1}\right)}{\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{x}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2*((x^(1/2)-1)*(x^(1/2)+1))^(1/2)/(x^(1/2)+1)^(1/2)/(x^(1/2)-1)^(1/2)*ln(x^
(1/2)+(x-1)^(1/2))
```

maxima [B] time = 0.65, size = 16, normalized size = 2.00

$$2 \log\left(2\sqrt{x-1}+2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*log(2*sqrt(x - 1) + 2*sqrt(x))
```

mupad [B] time = 5.29, size = 6, normalized size = 0.75

$$2\,\mathrm{acosh}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] 2*acosh(x^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1015 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

```
[Out] 2*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {265}

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx=\frac{2\sqrt{-1+\sqrt{x}}\sqrt{1+\sqrt{x}}}{\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(3/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/Sqrt[x]
```

fricas [A] time = 0.66, size = 25, normalized size = 0.86

$$\frac{2\left(\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2*(sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + x)/x
```

giac [A] time = 0.22, size = 25, normalized size = 0.86

$$\frac{16}{\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^{4}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 16/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)
```

maple [A] time = 0.06, size = 20, normalized size = 0.69

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)/x^(1/2)
```

maxima [A] time = 1.21, size = 10, normalized size = 0.34

$$\frac{2\sqrt{x-1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 2*sqrt(x - 1)/sqrt(x)
```

mupad [B] time = 5.56, size = 19, normalized size = 0.66

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] (2*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2))/x^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1016 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

```
[Out] 2/3*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(3/2)+4/3*(-1+x^(1/2))^(1/2)*(1+
x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3x^{3/2}}+\frac{4\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(3*x^(3/2)) + (4*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(3*Sqrt[x])
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{2}{3}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{3/2}}\,dx\\ &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3x^{3/2}}+\frac{4\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.57

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(5/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(1 + 2*x))/(3*x^(3/2))
```

fricas [A] time = 0.60, size = 34, normalized size = 0.54

$$\frac{2\left((2x+1)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}+2x^2\right)}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/3*((2*x + 1)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1) + 2*x^2)/x^2
```

giac [A] time = 0.22, size = 48, normalized size = 0.76

$$\frac{128\left(3\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)}{3\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 128/3*(3*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 + 4)/((sqrt(sqrt(x) + 1)
 - sqrt(sqrt(x) - 1))^4 + 4)^3
```

maple [A] time = 0.06, size = 25, normalized size = 0.40

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,(2x+1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2/3*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(2*x+1)/x^(3/2)
```

maxima [A] time = 1.20, size = 21, normalized size = 0.33

$$\frac{4\sqrt{x-1}}{3\sqrt{x}}+\frac{2\sqrt{x-1}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 4/3*sqrt(x - 1)/sqrt(x) + 2/3*sqrt(x - 1)/x^(3/2)
```

mupad [B] time = 5.50, size = 33, normalized size = 0.52

$$\frac{\sqrt{\sqrt{x}-1}\,\left(\frac{4x}{3}+\frac{2\sqrt{x}}{3}+\frac{4x^{3/2}}{3}+\frac{2}{3}\right)}{x^{3/2}\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((4*x)/3 + (2*x^(1/2))/3 + (4*x^(3/2))/3 + 2/3))/(x^(3
/2)*(x^(1/2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1017 $\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx$

Optimal. Leaf size=94

$$\frac{8\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

```
[Out] 2/5*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(5/2)+8/15*(-1+x^(1/2))^(1/2)*(1
+x^(1/2))^(1/2)/x^(3/2)+16/15*(-1+x^(1/2))^(1/2)*(1+x^(1/2))^(1/2)/x^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {272, 265}

$$\frac{8\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15x^{3/2}}+\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{5x^{5/2}}+\frac{16\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]])/(5*x^(5/2)) + (8*Sqrt[-1 + Sqrt[x]
]*Sqrt[1 + Sqrt[x]])/(15*x^(3/2)) + (16*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]
])/(15*Sqrt[x])
```

Rule 265

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b
2*x^n)^(p + 1))/(a1*a2*c*(m + 1)), x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}
, x] && EqQ[a2*b1 + a1*b2, 0] && EqQ[(m + 1)/(2*n) + p + 1, 0] && NeQ[m, -1
]
```

Rule 272

```
Int[(x_)^(m_)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[(x^(m + 1)*(a1 + b1*x^n)^(p + 1)*(a2 + b2*x^n)^(p + 1
))/(a1*a2*(m + 1)), x] - Dist[(b1*b2*(m + 2*n*(p + 1) + 1))/(a1*a2*(m + 1))
, Int[x^(m + 2*n)*(a1 + b1*x^n)^p*(a2 + b2*x^n)^p, x], x] /; FreeQ[{a1, b1,
 a2, b2, m, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && ILtQ[Simplify[(m + 1)/(2*
n) + p + 1], 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{7/2}}\,dx &= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{4}{5}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}\,x^{5/2}}\,dx\\
&= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{8}{15}\int \frac{1}{\sqrt{-1+\sqrt{x}}\,\sqrt{1+\\
&= \frac{2\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{5x^{5/2}}+\frac{8\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15x^{3/2}}+\frac{16\sqrt{-1+\sqrt{x}}\,\sqrt{1+\sqrt{x}}}{15\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.44

$$\frac{2\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}\,(8x^2+4x+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*x^(7/2)),x]
```

```
[Out] (2*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + Sqrt[x]]*(3 + 4*x + 8*x^2))/(15*x^(5/2))
```

fricas [A] time = 0.67, size = 39, normalized size = 0.41

$$\frac{2\left(8\,x^3+\left(8\,x^2+4\,x+3\right)\sqrt{x}\sqrt{\sqrt{x}+1}\sqrt{\sqrt{x}-1}\right)}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="fric
as")
```

```
[Out] 2/15*(8*x^3 + (8*x^2 + 4*x + 3)*sqrt(x)*sqrt(sqrt(x) + 1)*sqrt(sqrt(x) - 1)
)/x^3
```

giac [A] time = 0.26, size = 69, normalized size = 0.73

$$\frac{4096\left(5\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^8+10\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+8\right)}{15\left(\left(\sqrt{\sqrt{x}+1}-\sqrt{\sqrt{x}-1}\right)^4+4\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="giac
")
```

```
[Out] 4096/15*(5*(sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^8 + 10*(sqrt(sqrt(x) + 1
) - sqrt(sqrt(x) - 1))^4 + 8)/((sqrt(sqrt(x) + 1) - sqrt(sqrt(x) - 1))^4 +
4)^5
```

maple [A] time = 0.06, size = 30, normalized size = 0.32

$$\frac{2\sqrt{\sqrt{x}-1}\,\sqrt{\sqrt{x}+1}\,\left(8x^2+4x+3\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(x^(1/2)-1)^(1/2)/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 2/15*(x^(1/2)-1)^(1/2)*(x^(1/2)+1)^(1/2)*(8*x^2+4*x+3)/x^(5/2)
```

maxima [A] time = 1.30, size = 31, normalized size = 0.33

$$\frac{16\sqrt{x-1}}{15\sqrt{x}}+\frac{8\sqrt{x-1}}{15\,x^{\frac{3}{2}}}+\frac{2\sqrt{x-1}}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(-1+x^(1/2))^(1/2)/(1+x^(1/2))^(1/2),x, algorithm="maxi
ma")
```

```
[Out] 16/15*sqrt(x - 1)/sqrt(x) + 8/15*sqrt(x - 1)/x^(3/2) + 2/5*sqrt(x - 1)/x^(5
/2)
```

mupad [B] time = 5.66, size = 43, normalized size = 0.46

$$\frac{\sqrt{\sqrt{x}-1}\left(\frac{8x}{15}+\frac{16x^2}{15}+\frac{2\sqrt{x}}{5}+\frac{8x^{3/2}}{15}+\frac{16x^{5/2}}{15}+\frac{2}{5}\right)}{x^{5/2}\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(x^(1/2) - 1)^(1/2)*(x^(1/2) + 1)^(1/2)),x)
```

```
[Out] ((x^(1/2) - 1)^(1/2)*((8*x)/15 + (16*x^2)/15 + (2*x^(1/2))/5 + (8*x^(3/2))/
15 + (16*x^(5/2))/15 + 2/5))/(x^(5/2)*(x^(1/2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{\sqrt{x}-1}\sqrt{\sqrt{x}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(-1+x**(1/2))**(1/2)/(1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(sqrt(x) - 1)*sqrt(sqrt(x) + 1)), x)
```

3.1018 $\int x^2 (-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=78

$$\frac{1}{3}x^3 (bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/3*x^3*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 3/2/n],[1+3/2/n],b^2*x^(2*n
)/a^2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {366, 365, 364}

$$\frac{1}{3}x^3 (bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[3/(2*n), -p, 1 + 3/(2*n
), (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int x^2 (-a+bx^n)^p (a+bx^n)^p \, dx &= \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int x^2 \left(-a^2+b^2x^{2n}\right)^p \, dx\\
&= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int x^2 \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \, dx\\
&= \frac{1}{3}x^3 (-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 1.03

$$\frac{1}{3}x^3 (bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{3}{2n},-p;1+\frac{3}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^3*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{3/(2*n), -p}, {1 + 3/(
2*n)}, (b^2*x^(2*n))/a^2])/(3*(1 - (b^2*x^(2*n))/a^2)^p)
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p (bx^n - a)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (bx^n - a)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int x^2 \left(b\,x^n + a\right)^p \left(b\,x^n - a\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^n-a)^p*(b*x^n+a)^p,x)
```

```
[Out] int(x^2*(b*x^n-a)^p*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (bx^n - a)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,x^n\right)}^p\,{\left(b\,x^n-a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

```
[Out] int(x^2*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(-a + bx^{n}\right)^{p} \left(a + bx^{n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral(x**2*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1019 $\int x(-a+bx^n)^p(a+bx^n)^p\,dx$

Optimal. Leaf size=70

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] 1/2*x^2*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/n],[1+1/n],b^2*x^(2*n)/a^
2)/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {366, 365, 364}

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1),
 (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int x(-a+bx^n)^p(a+bx^n)^p\,dx &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int x\left(-a^2+b^2x^{2n}\right)^p\,dx\\ &= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int x\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\ &= \frac{1}{2}x^2(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 72, normalized size = 1.03

$$\frac{1}{2}x^2(bx^n-a)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x^2*(-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{n^(-1), -p}, {1 + n^(-
1)}, (b^2*x^(2*n))/a^2])/(2*(1 - (b^2*x^(2*n))/a^2)^p)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n + a)^p (bx^n - a)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (bx^n - a)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int x \left(b\,x^n + a\right)^p \left(b\,x^n - a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^n-a)^p*(b*x^n+a)^p,x)
```

```
[Out] int(x*(b*x^n-a)^p*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (bx^n - a)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,x^n\right)}^p\,{\left(b\,x^n-a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^n)^p*(b*x^n - a)^p,x)
```

```
[Out] int(x*(a + b*x^n)^p*(b*x^n - a)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(-a + bx^n\right)^p \left(a + bx^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral(x*(-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1020 $\int (-a+bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=73

$$x\left(bx^n-a\right)^p\left(a+bx^n\right)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] x*(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, 1/2/n],[1+1/2/n],b^2*x^(2*n)/a^2)
/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {253, 246, 245}

$$x\left(bx^n-a\right)^p\left(a+bx^n\right)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))
/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (-a+bx^n)^p(a+bx^n)^p\,dx &= \left((-a+bx^n)^p(a+bx^n)^p\left(-a^2+b^2x^{2n}\right)^{-p}\right)\int\left(-a^2+b^2x^{2n}\right)^p\,dx\\
&= \left((-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int\left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\
&= x(-a+bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.00

$$x\left(bx^n-a\right)^p\left(a+bx^n\right)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
 (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((bx^n+a)^p(bx^n-a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p(bx^n-a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int (b\,x^n+a)^p\,(b\,x^n-a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(b*x^n+a)^p,x)
```

```
[Out] int((b*x^n-a)^p*(b*x^n+a)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+a)^p(bx^n-a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x^n)^p\,(b\,x^n-a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p*(b*x^n - a)^p,x)
```

```
[Out] int((a + b*x^n)^p*(b*x^n - a)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-a+bx^n)^p\,(a+bx^n)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p, x)
```

3.1021 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x}\,dx$

Optimal. Leaf size=72

$$-\frac{\left(a^2-b^2x^{2n}\right)(bx^n-a)^p\,(a+bx^n)^p\;{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

```
[Out] -1/2*(-a+b*x^n)^p*(a+b*x^n)^p*(-b^2*x^(2*n)+a^2)*hypergeom([1, 1+p],[2+p],1
-b^2*x^(2*n)/a^2)/a^2/n/(1+p)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {267, 126, 266, 65}

$$-\frac{\left(a^2-b^2x^{2n}\right)(bx^n-a)^p\,(a+bx^n)^p\;{}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
```

```
[Out] -((-a + b*x^n)^p*(a + b*x^n)^p*(a^2 - b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 267

```
Int[(x_)^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)^(n_))^(
p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a1 + b1*
x)^p*(a2 + b2*x)^p, x], x, x^n], x] /; FreeQ[{a1, b1, a2, b2, m, n, p}, x]
&& EqQ[a2*b1 + a1*b2, 0] && IntegerQ[Simplify[(m + 1)/(2*n)]]
```

Rubi steps

$$\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x} dx = \frac{\text{Subst}\left(\int \frac{(-a+bx)^p(a+bx)^p}{x} dx, x, x^n\right)}{n}$$
$$= \frac{\left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(-a^2+b^2x^2\right)^p}{x} dx, x, x^n\right)}{n}$$
$$= \frac{\left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(-a^2+b^2x\right)^p}{x} dx, x, x^{2n}\right)}{2n}$$
$$= -\frac{(-a+bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right) {}_2F_1\left(1,1+p;2+p;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(1+p)}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 1.01

$$\frac{\left(b^2x^{2n}-a^2\right)(bx^n-a)^p (a+bx^n)^p {}_2F_1\left(1,p+1;p+2;1-\frac{b^2x^{2n}}{a^2}\right)}{2a^2n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x,x]
[Out] ((-a + b*x^n)^p*(a + b*x^n)^p*(-a^2 + b^2*x^(2*n))*Hypergeometric2F1[1, 1 +
 p, 2 + p, 1 - (b^2*x^(2*n))/a^2])/(2*a^2*n*(1 + p))
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p(bx^n-a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="fricas")
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^p(bx^n-a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="giac")
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

maple [F] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n+a)^p\,(b\,x^n-a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(b*x^n+a)^p/x,x)
[Out] int((b*x^n-a)^p*(b*x^n+a)^p/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p (bx^n - a)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\, x^n)^p\, (b\, x^n - a)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x,x)
```

```
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^n)^p\, (a + bx^n)^p}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x, x)
```

3.1022 $\int \frac{(-a+bx^n)^p(a+bx^n)^p}{x^2} dx$

Optimal. Leaf size=76

$$-\frac{(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

```
[Out] -(-a+b*x^n)^p*(a+b*x^n)^p*hypergeom([-p, -1/2/n],[1-1/2/n],b^2*x^(2*n)/a^2)
/x/((1-b^2*x^(2*n)/a^2)^p)
```

Rubi [A] time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {366, 365, 364}

$$-\frac{(bx^n-a)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[-1/(2*n), -p, 1 - 1/(2*n)
, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 366

```
Int[((c_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(n_))^(p_)*((a2_) + (b2_.)*(x_)
^(n_))^(p_), x_Symbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^Fra
cPart[p])/(a1*a2 + b1*b2*x^(2*n))^FracPart[p], Int[(c*x)^m*(a1*a2 + b1*b2*x
^(2*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, m, n, p}, x] && EqQ[a2*b1 +
a1*b2, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx^n)^p (a+bx^n)^p}{x^2} dx &= \left((-a+bx^n)^p (a+bx^n)^p \left(-a^2+b^2x^{2n}\right)^{-p}\right) \int \frac{\left(-a^2+b^2x^{2n}\right)^p}{x^2} dx \\
&= \left((-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \frac{\left(1-\frac{b^2x^{2n}}{a^2}\right)^p}{x^2} dx \\
&= -\frac{(-a+bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.03

$$-\frac{(bx^n-a)^p\,(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\,{}_2F_1\left(-\frac{1}{2n},-p;1-\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-a + b*x^n)^p*(a + b*x^n)^p)/x^2,x]
```

```
[Out] -(((-a + b*x^n)^p*(a + b*x^n)^p*HypergeometricPFQ[{-1/2*1/n, -p}, {1 - 1/(2
*n)}, (b^2*x^(2*n))/a^2])/(x*(1 - (b^2*x^(2*n))/a^2)^p))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n+a)^p(bx^n-a)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int\frac{(b\,x^n+a)^p\,(b\,x^n-a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n-a)^p*(b*x^n+a)^p/x^2,x)
```

```
[Out] int((b*x^n-a)^p*(b*x^n+a)^p/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx^n+a)^p(bx^n-a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x^n)^p*(a+b*x^n)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(b*x^n - a)^p/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,x^n)^p\,(b\,x^n-a)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2,x)
```

```
[Out] int(((a + b*x^n)^p*(b*x^n - a)^p)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^n)^p (a + bx^n)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*x**n)**p*(a+b*x**n)**p/x**2,x)
```

```
[Out] Integral((-a + b*x**n)**p*(a + b*x**n)**p/x**2, x)
```

3.1023 $\int \frac{1+x^6}{x(1-x^6)} dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

[Out] ln(x)-1/3*ln(-x^6+1)

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {446, 72}

$$\log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^6)/(x*(1 - x^6)),x]

[Out] Log[x] - Log[1 - x^6]/3

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^6}{x\left(1-x^6\right)}\,dx &= \frac{1}{6}\text{Subst}\left(\int \frac{1+x}{(1-x)x}\,dx,x,x^6\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right)dx,x,x^6\right)\\ &= \log(x)-\frac{1}{3}\log\left(1-x^6\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{3}\log\left(1-x^6\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^6)/(x*(1 - x^6)),x]

[Out] Log[x] - Log[1 - x^6]/3

fricas [A] time = 0.57, size = 11, normalized size = 0.73

$$-\frac{1}{3}\log\left(x^6-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^6 - 1) + log(x)
```

giac [A] time = 0.19, size = 16, normalized size = 1.07

$$\frac{1}{6}\log\left(x^6\right) - \frac{1}{3}\log\left(\left|x^6 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*log(x^6) - 1/3*log(abs(x^6 - 1))
```

maple [B] time = 0.05, size = 36, normalized size = 2.40

$$\ln(x) - \frac{\ln(x-1)}{3} - \frac{\ln(x+1)}{3} - \frac{\ln\left(x^2 - x + 1\right)}{3} - \frac{\ln\left(x^2 + x + 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6+1)/x/(-x^6+1),x)
```

```
[Out] -1/3*ln(x-1)-1/3*ln(x^2+x+1)-1/3*ln(x+1)+ln(x)-1/3*ln(x^2-x+1)
```

maxima [A] time = 0.46, size = 15, normalized size = 1.00

$$-\frac{1}{3}\log\left(x^6 - 1\right) + \frac{1}{6}\log\left(x^6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6+1)/x/(-x^6+1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^6 - 1) + 1/6*log(x^6)
```

mupad [B] time = 0.09, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln\left(x^6 - 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6 + 1)/(x*(x^6 - 1)),x)
```

```
[Out] log(x) - log(x^6 - 1)/3
```

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log\left(x^6 - 1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6+1)/x/(-x**6+1),x)
```

```
[Out] log(x) - log(x**6 - 1)/3
```

3.1024 $\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx$

Optimal. Leaf size=22

$$\frac{(ex)^{m+1}(a+bx^n)^{p+1}}{e}$$

```
[Out] (e*x)^(1+m)*(a+b*x^n)^(1+p)/e
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {449}

$$\frac{(ex)^{m+1}(a+bx^n)^{p+1}}{e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] ((e*x)^(1 + m)*(a + b*x^n)^(1 + p))/e
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int (ex)^m (a+bx^n)^p \left(a(1+m)+b(1+m+n+np)x^n\right) dx = \frac{(ex)^{1+m}(a+bx^n)^{1+p}}{e}$$

Mathematica [C] time = 0.17, size = 110, normalized size = 5.00

$$x(ex)^m (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left(\frac{bx^n(m+np+n+1)\,{}_2F_1\left(\frac{m+n+1}{n},-p;\frac{m+2n+1}{n};-\frac{bx^n}{a}\right)}{m+n+1}+a\,{}_2F_1\left(\frac{m+1}{n},-p;\frac{m+n+1}{n}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m*(a + b*x^n)^p*(a*(1 + m) + b*(1 + m + n + n*p)*x^n),x]
```

```
[Out] (x*(e*x)^m*(a + b*x^n)^p*(a*Hypergeometric2F1[(1 + m)/n, -p, (1 + m + n)/n,
 -((b*x^n)/a)] + (b*(1 + m + n + n*p)*x^n*Hypergeometric2F1[(1 + m + n)/n,
-p, (1 + m + 2*n)/n, -((b*x^n)/a)])/(1 + m + n)))/(1 + (b*x^n)/a)^p
```

fricas [A] time = 0.65, size = 40, normalized size = 1.82

$$\left(bxx^n e^{(m\log(e)+m\log(x))} + axe^{(m\log(e)+m\log(x))}\right)(bx^n+a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="fri
cas")
```

```
[Out] (b*x*x^n*e^(m*log(e) + m*log(x)) + a*x*e^(m*log(e) + m*log(x)))*(b*x^n + a)
^p
```

giac [A] time = 0.29, size = 38, normalized size = 1.73

$$(bx^n + a)^p bxx^m x^n e^m + (bx^n + a)^p axx^m e^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="gia
c")
```

```
[Out] (b*x^n + a)^p*b*x*x^m*x^n*e^m + (b*x^n + a)^p*a*x*x^m*e^m
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \left(\left(np + m + n + 1\right) b\, x^n + (m + 1)\, a\right) (ex)^m\, (b\, x^n + a)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n),x)
```

```
[Out] int((e*x)^m*(b*x^n+a)^p*(a*(m+1)+b*(n*p+m+n+1)*x^n),x)
```

maxima [A] time = 0.95, size = 36, normalized size = 1.64

$$\left(ae^m xx^m + be^m xe^{\left(m \log(x) + n \log(x)\right)}\right)(bx^n + a)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m*(a+b*x^n)^p*(a*(1+m)+b*(n*p+m+n+1)*x^n),x, algorithm="max
ima")
```

```
[Out] (a*e^m*x*x^m + b*e^m*x*e^(m*log(x) + n*log(x)))*(b*x^n + a)^p
```

mupad [B] time = 4.94, size = 31, normalized size = 1.41

$$\left(a\, x\, (e\, x)^m + b\, x^{n+1}\, (e\, x)^m\right)\, (a + b\, x^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m*(a*(m + 1) + b*x^n*(m + n + n*p + 1))*(a + b*x^n)^p,x)
```

```
[Out] (a*x*(e*x)^m + b*x^(n + 1)*(e*x)^m)*(a + b*x^n)^p
```

sympy [B] time = 8.43, size = 39, normalized size = 1.77

$$ae^m xx^m\, (a + bx^n)^p + be^m xx^m x^n\, (a + bx^n)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m*(a+b*x**n)**p*(a*(1+m)+b*(n*p+m+n+1)*x**n),x)
```

```
[Out] a*e**m*x*x**m*(a + b*x**n)**p + b*e**m*x*x**m*x**n*(a + b*x**n)**p
```

3.1025 $\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=114

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

```
[Out] b*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a/(-a*d+b*c)/e/(
1+m)-d*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b*c
)/e/(1+m)
```

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {508, 364}

$$\frac{b(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{ae(m+1)(bc-ad)}-\frac{d(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a
)])/(a*(b*c - a*d)*e*(1 + m)) - (d*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(ex)^m}{(a+bx^n)(c+dx^n)}\,dx &= \frac{b\int \frac{(ex)^m}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{(ex)^m}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{b(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)e(1+m)}-\frac{d(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)e(1+m)}\end{aligned}$$

Mathematica [A] time = 0.12, size = 88, normalized size = 0.77

$$\frac{x(ex)^m\left(ad\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)-bc\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)\right)}{ac(m+1)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*(-(b*c*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/
a)]) + a*d*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)]))/(
a*c*(-(b*c) + a*d)*(1 + m))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex)^m}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^n + a)\,(d\,x^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int((e*x)^m/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)*(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x)^m}{(a + b\,x^n)\,(c + d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int((e*x)^m/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(a + bx^n)\,(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral((e*x)**m/((a + b*x**n)*(c + d*x**n)), x)
```

3.1026 $\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=89

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

```
[Out] 1/3*b*x^3*hypergeom([1, 3/n],[(3+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/3*d*x^3*hyp
ergeom([1, 3/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a*(b*c - a*d)
) - (d*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c -
a*d))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx^n)(c+dx^n)}\,dx&=\frac{b\int \frac{x^2}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{x^2}{c+dx^n}\,dx}{bc-ad}\\&=\frac{bx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a(bc-ad)}-\frac{dx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 0.88

$$\frac{bcx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)-adx^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3abc^2-3a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)] - a*d*x^3*Hyper
geometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*a*b*c^2 - 3*a^2*c*d)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2}{bdx^{2n}+ac+(bc+ad)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

maple [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^n+a)\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int(x^2/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((b*x^n + a)*(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^2/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx^n)\,(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(x**2/((a + b*x**n)*(c + d*x**n)), x)
```

3.1027 $\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=89

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

```
[Out] 1/2*b*x^2*hypergeom([1, 2/n],[(2+n)/n],-b*x^n/a)/a/(-a*d+b*c)-1/2*d*x^2*hyp
ergeom([1, 2/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {508, 364}

$$\frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a*(b*c - a*d)
) - (d*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c -
a*d))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx^n)(c+dx^n)}\,dx &= \frac{b\int \frac{x}{a+bx^n}\,dx}{bc-ad}-\frac{d\int \frac{x}{c+dx^n}\,dx}{bc-ad}\\ &= \frac{bx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)}-\frac{dx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)}\end{aligned}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 0.88

$$\frac{bcx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)-adx^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2abc^2-2a^2cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)] - a*d*x^2*Hyper
geometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*a*b*c^2 - 2*a^2*c*d)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] integral(x/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n + a)\,(d\,x^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)/(d*x^n+c),x)
[Out] int(x/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] integrate(x/((b*x^n + a)*(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a + b\,x^n)\,(c + d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + bx^n)(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)/(c+d*x**n),x)
[Out] Integral(x/((a + b*x**n)*(c + d*x**n)), x)
```

3.1028 $\int \frac{1}{(a+bx^n)(c+dx^n)} dx$

Optimal. Leaf size=72

$$\frac{bx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] b*x*hypergeom([1, 1/n],[1+1/n],-b*x^n/a)/a/(-a*d+b*c)-d*x*hypergeom([1, 1/n
],[1+1/n],-d*x^n/c)/c/(-a*d+b*c)
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 245}

$$\frac{bx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^n)(c+dx^n)} dx &= \frac{b\int \frac{1}{a+bx^n} dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^n} dx}{bc-ad}\\ &= \frac{bx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 64, normalized size = 0.89

$$\frac{x\left(ad\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-bc\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,(d\,x^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int(1/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b\,x^n)\,(c + d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^n)\,(c + dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

3.1029 $\int \frac{1}{x(a+bx^n)(c+dx^n)} dx$

Optimal. Leaf size=63

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

```
[Out] ln(x)/a/c-b*ln(a+b*x^n)/a/(-a*d+b*c)/n+d*ln(c+d*x^n)/c/(-a*d+b*c)/n
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {446, 72}

$$-\frac{b\log(a+bx^n)}{an(bc-ad)}+\frac{d\log(c+dx^n)}{cn(bc-ad)}+\frac{\log(x)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] Log[x]/(a*c) - (b*Log[a + b*x^n])/(a*(b*c - a*d)*n) + (d*Log[c + d*x^n])/(c
*(b*c - a*d)*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^n)(c+dx^n)}dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)(c+dx)}dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{acx}+\frac{b^2}{a(-bc+ad)(a+bx)}+\frac{d^2}{c(bc-ad)(c+dx)}\right)dx, x, x^n\right)}{n}\\
&= \frac{\log(x)}{ac}-\frac{b\log(a+bx^n)}{a(bc-ad)n}+\frac{d\log(c+dx^n)}{c(bc-ad)n}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.89

$$\frac{-bc\log(a+bx^n)+ad\log(c+dx^n)-adn\log(x)+bcn\log(x)}{abc^2n-a^2cdn}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*n*Log[x] - a*d*n*Log[x] - b*c*Log[a + b*x^n] + a*d*Log[c + d*x^n])/(a*
b*c^2*n - a^2*c*d*n)
```

fricas [A] time = 0.71, size = 58, normalized size = 0.92

$$-\frac{bc\log\left(bx^{n}+a\right)-ad\log\left(dx^{n}+c\right)-(bc-ad)n\log(x)}{\left(abc^{2}-a^{2}cd\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(b*c*log(b*x^n + a) - a*d*log(d*x^n + c) - (b*c - a*d)*n*log(x))/((a*b*c^2
 - a^2*c*d)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx^{n}+a)(dx^{n}+c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x), x)
```

maple [A] time = 0.10, size = 69, normalized size = 1.10

$$\frac{b\ln\left(bx^{n}+a\right)}{(ad-bc)\,an}-\frac{d\ln\left(dx^{n}+c\right)}{(ad-bc)\,cn}+\frac{\ln\left(x^{n}\right)}{acn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] -1/n/c*d/(a*d-b*c)*ln(d*x^n+c)+1/n/a*b/(a*d-b*c)*ln(b*x^n+a)+1/n/c/a*ln(x^n
)
```

maxima [A] time = 0.57, size = 69, normalized size = 1.10

$$-\frac{b\log\left(\frac{bx^{n}+a}{b}\right)}{abcn-a^{2}dn}+\frac{d\log\left(\frac{dx^{n}+c}{d}\right)}{bc^{2}n-acdn}+\frac{\log(x)}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -b*log((b*x^n + a)/b)/(a*b*c*n - a^2*d*n) + d*log((d*x^n + c)/d)/(b*c^2*n -
 a*c*d*n) + log(x)/(a*c)
```

mupad [B] time = 5.72, size = 162, normalized size = 2.57

$$\frac{b\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^{n}+b\,c\,n\,x^{n}}{d\,x\left(a^{2}\,d\,n-a\,b\,c\,n\right)}\right)}{a^{2}\,d\,n-a\,b\,c\,n}+\frac{d\,\ln\left(-\frac{1}{b\,d\,x}-\frac{2\,a\,c\,n+a\,d\,n\,x^{n}+b\,c\,n\,x^{n}}{b\,x\left(b\,c^{2}\,n-a\,c\,d\,n\right)}\right)}{b\,c^{2}\,n-a\,c\,d\,n}+\frac{\ln(x)\,(n-1)}{a\,c\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] (b*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b*c*n*x^n)/(d*x*(a^2*d*n - a*b*
c*n))))/(a^2*d*n - a*b*c*n) + (d*log(- 1/(b*d*x) - (2*a*c*n + a*d*n*x^n + b
*c*n*x^n)/(b*x*(b*c^2*n - a*c*d*n))))/(b*c^2*n - a*c*d*n) + (log(x)*(n - 1)
)/(a*c*n)
```

sympy [A] time = 4.02, size = 332, normalized size = 5.27

$$\begin{cases} \frac{\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^n\right)}{an}}{c} & \text{for } d = 0 \\ \frac{-\frac{x^{-n}}{cn} + \frac{d\log\left(x^{-n}+\frac{d}{c}\right)}{c^2n}}{b} & \text{for } a = 0 \\ \frac{cn\log(x)}{ac^2n+acdnx^n} - \frac{c\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} + \frac{c}{ac^2n+acdnx^n} + \frac{dnx^n\log(x)}{ac^2n+acdnx^n} - \frac{dx^n\log\left(\frac{c}{d}+x^n\right)}{ac^2n+acdnx^n} & \text{for } b = \frac{ad}{c} \\ \frac{-\frac{x^{-n}}{an} + \frac{b\log\left(x^{-n}+\frac{b}{a}\right)}{a^2n}}{d} & \text{for } c = 0 \\ \frac{\log(x)}{(a+b)(c+d)} & \text{for } n = 0 \\ \frac{\frac{\log(x)}{c} - \frac{\log\left(\frac{c}{d}+x^n\right)}{cn}}{a} & \text{for } b = 0 \\ \frac{adn\log(x)}{a^2cdn-abc^2n} - \frac{ad\log\left(\frac{c}{d}+x^n\right)}{a^2cdn-abc^2n} - \frac{bcn\log(x)}{a^2cdn-abc^2n} + \frac{bc\log\left(\frac{a}{b}+x^n\right)}{a^2cdn-abc^2n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((log(x)/a - log(a/b + x**n)/(a*n))/c, Eq(d, 0)), ((-x**(-n)/(c*n
) + d*log(x**(-n) + d/c)/(c**2*n))/b, Eq(a, 0)), (c*n*log(x)/(a*c**2*n + a*
c*d*n*x**n) - c*log(c/d + x**n)/(a*c**2*n + a*c*d*n*x**n) + c/(a*c**2*n + a
*c*d*n*x**n) + d*n*x**n*log(x)/(a*c**2*n + a*c*d*n*x**n) - d*x**n*log(c/d +
 x**n)/(a*c**2*n + a*c*d*n*x**n), Eq(b, a*d/c)), ((-x**(-n)/(a*n) + b*log(x
**(-n) + b/a)/(a**2*n))/d, Eq(c, 0)), (log(x)/((a + b)*(c + d)), Eq(n, 0)),
 ((log(x)/c - log(c/d + x**n)/(c*n))/a, Eq(b, 0)), (a*d*n*log(x)/(a**2*c*d*
n - a*b*c**2*n) - a*d*log(c/d + x**n)/(a**2*c*d*n - a*b*c**2*n) - b*c*n*log
(x)/(a**2*c*d*n - a*b*c**2*n) + b*c*log(a/b + x**n)/(a**2*c*d*n - a*b*c**2*
n), True))
```

3.1030 $\int \frac{1}{x^2(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=90

$$\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

```
[Out] -b*hypergeom([1, -1/n],[(-1+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x+d*hypergeom([1,
-1/n],[(-1+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x
```

Rubi [A] time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{ax(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((b*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a*(b*c - a
*d)*x)) + (d*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*
(b*c - a*d)*x)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a+bx^n\right)\left(c+dx^n\right)}\,dx&=\frac{b\int \frac{1}{x^2(a+bx^n)}\,dx}{bc-ad}-\frac{d\int \frac{1}{x^2(c+dx^n)}\,dx}{bc-ad}\\&=-\frac{b\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)x}+\frac{d\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)x}\end{aligned}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.82

$$\frac{bc\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{bx^n}{a}\right)-ad\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{dx^n}{c}\right)}{acx(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((b*x^n)/a)] - a*d*Hypergeo
metric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)])/(a*c*(-(b*c) + a*d)*x)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bdx^2x^{2n} + (bc + ad)x^2x^n + acx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^2*x^(2*n) + (b*c + a*d)*x^2*x^n + a*c*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,(d\,x^n + c)\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int(1/x^2/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\left(a + b\,x^n\right)\,\left(c + d\,x^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)*(c + d*x^n)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + bx^n\right) \left(c + dx^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/(x**2*(a + b*x**n)*(c + d*x**n)), x)
```

3.1031 $\int \frac{1}{x^3(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

```
[Out] -1/2*b*hypergeom([1, -2/n],[(-2+n)/n],-b*x^n/a)/a/(-a*d+b*c)/x^2+1/2*d*hype
rgeom([1, -2/n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)/x^2
```

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {508, 364}

$$\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)}-\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2ax^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -(b*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a*(b*c - a*d
)*x^2) + (d*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 508

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, n, m}
, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\,(a+bx^n)\,(c+dx^n)}\,dx &= \frac{b\int \frac{1}{x^3(a+bx^n)}\,dx}{bc-ad}-\frac{d\int \frac{1}{x^3(c+dx^n)}\,dx}{bc-ad}\\ &= -\frac{b\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a(bc-ad)x^2}+\frac{d\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)x^2}\end{aligned}$$

Mathematica [A] time = 0.10, size = 77, normalized size = 0.81

$$\frac{bc\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{bx^n}{a}\right)-ad\,{}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{dx^n}{c}\right)}{2acx^2(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*c*Hypergeometric2F1[1, -2/n, (-2 + n)/n, -((b*x^n)/a)] - a*d*Hypergeomet
ric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)])/(2*a*c*(-(b*c) + a*d)*x^2)
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bdx^3x^{2n} + (bc + ad)x^3x^n + acx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^3*x^(2*n) + (b*c + a*d)*x^3*x^n + a*c*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)\,(d\,x^n + c)\,x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] int(1/x^3/(b*x^n+a)/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\left(a + b\,x^n\right)\,\left(c + d\,x^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1032 $\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=175

$$\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2}+\frac{d^2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2}+\frac{}{aen(b}$$

```
[Out] b*(e*x)^(1+m)/a/(-a*d+b*c)/e/n/(a+b*x^n)+b*(a*d*(1+m-2*n)-b*c*(1+m-n))*(e*x
)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/e/(1+
m)/n+d^2*(e*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-d*x^n/c)/c/(-a*d+b
*c)^2/e/(1+m)
```

Rubi [A] time = 0.29, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {504, 597, 364}

$$\frac{b(ex)^{m+1}(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2e(m+1)n(bc-ad)^2}+\frac{d^2(ex)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{ce(m+1)(bc-ad)^2}+\frac{}{aen(b}$$

Antiderivative was successfully verified.

```
[In] Int[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*(e*x)^(1 + m))/(a*(b*c - a*d)*e*n*(a + b*x^n)) + (b*(a*d*(1 + m - 2*n) -
 b*c*(1 + m - n))*(e*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*e*(1 + m)*n) + (d^2*(e*x)^(1 + m)*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((d*x^n)/c)])/(c*(b*c - a*d)^2
*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{(ex)^m}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \frac{(ex)^m(bc(1+m-n)+adn+bd(1+m-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} - \frac{\int \left(\frac{b(-ad(1+m-2n)+bc(1+m-n))(ex)^m}{(bc-ad)(a+bx^n)} + \frac{ad^2n(ex)^m}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{d^2\int \frac{(ex)^m}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1+m-2n)-bc(1+m-n)))\int}{a(bc-ad)^2n}$$

$$= \frac{b(ex)^{1+m}}{a(bc-ad)en(a+bx^n)} + \frac{b(ad(1+m-2n)-bc(1+m-n))(ex)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\right.}{a^2(bc-ad)^2e(1+m)n}$$

Mathematica [A] time = 0.29, size = 141, normalized size = 0.81

$$\frac{x(ex)^m\left(\frac{b^2c-abd}{a^2n+abnx^n} + \frac{b(ad(m-2n+1)-bc(m-n+1))\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{bx^n}{a}\right)}{a^2(m+1)n} + \frac{d^2\,{}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{dx^n}{c}\right)}{cm+c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*(e*x)^m*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 + m - 2*n) - b
*c*(1 + m - n))*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -((b*x^n)/a)
])/(a^2*(1 + m)*n) + (d^2*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, -(
(d*x^n)/c)])/(c + c*m)))/(b*c - a*d)^2
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(ex)^m}{b^2dx^{3n} + a^2c + (b^2c + 2\,abd)x^{2n} + (2\,abc + a^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((e*x)^m/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*
b*c + a^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((e*x)^m/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(ex)^m}{(b\,x^n+a)^2\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int((e*x)^m/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2e^m \int \frac{x^m}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx + \frac{be^m xx^m}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} - \left(b^2ce^m(m -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)^m/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*e^m*integrate(x^m/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*x^n), x) + b*e^m*x*x^m/(a^2*b*c*n - a^3*d*n + (a*b^2*c*
n - a^2*b*d*n)*x^n) - (b^2*c*e^m*(m - n + 1) - a*b*d*e^m*(m - 2*n + 1))*int
egrate(x^m/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^
2*b^2*c*d*n + a^3*b*d^2*n)*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(e\,x\right)}^m}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int((e*x)^m/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x)**m/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1033 $\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=142

$$\frac{bx^3(ad(3-2n)-bc(3-n))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}+\frac{bx^3}{an(bc-ad)\,(a+bx^n)}$$

```
[Out] b*x^3/a/(-a*d+b*c)/n/(a+b*x^n)+1/3*b*(a*d*(3-2*n)-b*c*(3-n))*x^3*hypergeom(
[1, 3/n],[(3+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/3*d^2*x^3*hypergeom([1, 3
/n],[(3+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$\frac{bx^3(ad(3-2n)-bc(3-n))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{bx^n}{a}\right)}{3a^2n(bc-ad)^2}+\frac{d^2x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)}{3c(bc-ad)^2}+\frac{bx^3}{an(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^3)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(3 - 2*n) - b*c*(3 - n))*x^
3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*x^n)/a)])/(3*a^2*(b*c - a*d)^2*
n) + (d^2*x^3*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])/(3*c*(b*c
 - a*d)^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{x^2}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^3}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{x^2(bc(3-n)+adn+bd(3-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$
$$= \frac{bx^3}{a(bc-ad)n(a+bx^n)} - \frac{\int \left(\frac{b(-ad(3-2n)+bc(3-n))x^2}{(bc-ad)(a+bx^n)} + \frac{ad^2nx^2}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}$$
$$= \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{x^2}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(3-2n)-bc(3-n)))\int \frac{x^2}{a+bx^n}\,dx}{a(bc-ad)^2n}$$
$$= \frac{bx^3}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(3-2n)-bc(3-n))x^3\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{bx^n}{a}\right)}{3a^2(bc-ad)^2n} + \frac{d^2x^3\,{}_2F_1}{3c}$$

Mathematica [A] time = 0.24, size = 135, normalized size = 0.95

$$\frac{x^3\left(a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{dx^n}{c}\right)+3bc(bc-ad)\right)+bc(a+bx^n)(ad(3-2n)+bc(n-3))\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\right.\right.}{3a^2cn(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^3*(b*c*(a*d*(3 - 2*n) + b*c*(-3 + n))*(a + b*x^n)*Hypergeometric2F1[1, 3
/n, (3 + n)/n, -((b*x^n)/a)] + a*(3*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*H
ypergeometric2F1[1, 3/n, (3 + n)/n, -((d*x^n)/c)])))/(3*a^2*c*(b*c - a*d)^2
*n*(a + b*x^n))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x^2}{b^2dx^{3n}+a^2c+\left(b^2c+2\,abd\right)x^{2n}+\left(2\,abc+a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c
+ a^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^2/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(b\,x^n+a)^2\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(x^2/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{bx^3}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} + d^2 \int \frac{x^2}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx - \left(abd(2\,n -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] b*x^3/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + d^2*integrate(x
^2/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*x^n), x) - (a*b*d*(2*n - 3) - b^2*c*(n - 3))*integrate(x^2/(a^2*b^2*c^2*n
- 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)
*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^2/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1034 $\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=143

$$\frac{bx^2(2ad(1-n)-bc(2-n))\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2}+\frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}+\frac{bx^2}{an(bc-ad)(a+bx^n)}$$

```
[Out] b*x^2/a/(-a*d+b*c)/n/(a+b*x^n)+1/2*b*(2*a*d*(1-n)-b*c*(2-n))*x^2*hypergeom(
[1, 2/n],[(2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+1/2*d^2*x^2*hypergeom([1, 2
/n],[(2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] time = 0.18, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {504, 597, 364}

$$\frac{bx^2(2ad(1-n)-bc(2-n))\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{bx^n}{a}\right)}{2a^2n(bc-ad)^2}+\frac{d^2x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)}{2c(bc-ad)^2}+\frac{bx^2}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x^2)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(2*a*d*(1 - n) - b*c*(2 - n))*x^
2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*
n) + (d^2*x^2*Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])/(2*c*(b*c
 - a*d)^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{x}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{x(bc(2-n)+adn+bd(2-n)x^n)}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$
$$= \frac{bx^2}{a(bc-ad)n(a+bx^n)} - \frac{\int \left(\frac{b(-2ad(1-n)+bc(2-n))x}{(bc-ad)(a+bx^n)} + \frac{ad^2nx}{(-bc+ad)(c+dx^n)}\right)dx}{a(bc-ad)n}$$
$$= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{x}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(2ad(1-n)-bc(2-n)))\int \frac{x}{a+bx^n}\,dx}{a(bc-ad)^2n}$$
$$= \frac{bx^2}{a(bc-ad)n(a+bx^n)} + \frac{b(2ad(1-n)-bc(2-n))x^2\,{}_2F_1\left(1,\frac{2}{n};\frac{2+n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2n} + \frac{d^2x^2\,{}_2}{}$$

Mathematica [A] time = 0.23, size = 134, normalized size = 0.94

$$\frac{x^2\left(a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n};-\frac{dx^n}{c}\right)+2bc(bc-ad)\right)+bc(a+bx^n)(bc(n-2)-2ad(n-1))\,{}_2F_1\left(1,\frac{2}{n};\frac{n+2}{n}\right.}{2a^2cn(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x^2*(b*c*(b*c*(-2 + n) - 2*a*d*(-1 + n))*(a + b*x^n)*Hypergeometric2F1[1,
2/n, (2 + n)/n, -((b*x^n)/a)] + a*(2*b*c*(b*c - a*d) + a*d^2*n*(a + b*x^n)*
Hypergeometric2F1[1, 2/n, (2 + n)/n, -((d*x^n)/c)])))/(2*a^2*c*(b*c - a*d)^
2*n*(a + b*x^n))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{x}{b^2dx^{3n}+a^2c+(b^2c+2abd)x^{2n}+(2abc+a^2d)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(x/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x}{(b\,x^n+a)^2\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(x/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{x}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx + \frac{bx^2}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} - \left(2\,abd(n-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(x/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) + b*x^2/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*
n)*x^n) - (2*a*b*d*(n - 1) - b^2*c*(n - 2))*integrate(x/(a^2*b^2*c^2*n - 2*
a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^n
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1035 $\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)(a+bx^n)}$$

```
[Out] b*x/a/(-a*d+b*c)/n/(a+b*x^n)+b*(a*d*(1-2*n)-b*c*(1-n))*x*hypergeom([1, 1/n]
,[1+1/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n+d^2*x*hypergeom([1, 1/n],[1+1/n],-d*x
^n/c)/c/(-a*d+b*c)^2
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n}$$

$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,{}_2F_1}{c}$$

Mathematica [A] time = 0.19, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2dx^{3n}+a^2c+\left(b^2c+2\,abd\right)x^{2n}+\left(2\,abc+a^2d\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^2(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n+a)^2\,(d\,x^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(1/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{1}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^n}\,dx - \left(abd(2\,n-1) - b^2c(n-1)\right)\int \frac{}{a^2b^2c^2n - 2\,a^3bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a+b\,x^n\right)^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1036 $\int \frac{1}{x(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=101

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

```
[Out] b/a/(-a*d+b*c)/n/(a+b*x^n)+ln(x)/a^2/c-b*(-2*a*d+b*c)*ln(a+b*x^n)/a^2/(-a*d
+b*c)^2/n-d^2*ln(c+d*x^n)/c/(-a*d+b*c)^2/n
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {446, 72}

$$-\frac{b(bc-2ad)\log(a+bx^n)}{a^2n(bc-ad)^2}+\frac{\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{cn(bc-ad)^2}+\frac{b}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*(a + b*x^n)) + Log[x]/(a^2*c) - (b*(b*c - 2*a*d)*Log[a +
 b*x^n])/(a^2*(b*c - a*d)^2*n) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+bx^n)^2(c+dx^n)}dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^2(c+dx)}dx, x, x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{a^2cx}+\frac{b^2}{a(-bc+ad)(a+bx)^2}+\frac{b^2(-bc+2ad)}{a^2(-bc+ad)^2(a+bx)}-\frac{d^3}{c(bc-ad)^2(c+dx)}\right)dx, x, x^n\right)}{n}\\
&= \frac{b}{a(bc-ad)n(a+bx^n)}+\frac{\log(x)}{a^2c}-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2n}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2n}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 97, normalized size = 0.96

$$\frac{-\frac{b(bc-2ad)\log(a+bx^n)}{a^2(bc-ad)^2}+\frac{n\log(x)}{a^2c}-\frac{d^2\log(c+dx^n)}{c(bc-ad)^2}+\frac{b}{a(bc-ad)(a+bx^n)}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b/(a*(b*c - a*d)*(a + b*x^n)) + (n*Log[x])/(a^2*c) - (b*(b*c - 2*a*d)*Log[
a + b*x^n])/(a^2*(b*c - a*d)^2) - (d^2*Log[c + d*x^n])/(c*(b*c - a*d)^2))/n
```

fricas [B] time = 0.67, size = 224, normalized size = 2.22

$$\frac{ab^2c^2 - a^2bcd + \left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)nx^n\log(x) + \left(ab^2c^2 - 2\,a^2bcd + a^3d^2\right)n\log(x) - \left(ab^2c^2 - 2\,a^2bcd + \left(\right.\right.}{\left(a^2b^3c^3 - 2\,a^3b^2c^2d + a^4bcd^2\right)nx^n + \left(a^3b^2c^3 - 2\,a^4bc^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2 - a^2*b*c*d + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*n*x^n*log(x) +
 (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*n*log(x) - (a*b^2*c^2 - 2*a^2*b*c*d +
(b^3*c^2 - 2*a*b^2*c*d)*x^n)*log(b*x^n + a) - (a^2*b*d^2*x^n + a^3*d^2)*log
(d*x^n + c))/((a^2*b^3*c^3 - 2*a^3*b^2*c^2*d + a^4*b*c*d^2)*n*x^n + (a^3*b^
2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x), x)
```

maple [A] time = 0.10, size = 131, normalized size = 1.30

$$\frac{2bd\ln(b\,x^n + a)}{(ad - bc)^2\,an} - \frac{b^2c\ln(b\,x^n + a)}{(ad - bc)^2\,a^2n} - \frac{d^2\ln(d\,x^n + c)}{(ad - bc)^2\,cn} - \frac{b}{(ad - bc)(b\,x^n + a)\,an} + \frac{\ln(x^n)}{a^2cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] -1/n/(a*d-b*c)^2/c*d^2*ln(d*x^n+c)-1/n/a*b/(a*d-b*c)/(b*x^n+a)+2/n*b/(a*d-b
*c)^2/a*ln(b*x^n+a)*d-1/n*b^2/(a*d-b*c)^2/a^2*ln(b*x^n+a)*c+1/n/c/a^2*ln(x^
n)
```

maxima [A] time = 0.67, size = 151, normalized size = 1.50

$$-\frac{d^2\log\left(\frac{dx^n+c}{d}\right)}{b^2c^3n - 2\,abc^2dn + a^2cd^2n} - \frac{\left(b^2c - 2\,abd\right)\log\left(\frac{bx^n+a}{b}\right)}{a^2b^2c^2n - 2\,a^3bcdn + a^4d^2n} + \frac{b}{a^2bcn - a^3dn + \left(ab^2cn - a^2bdn\right)x^n} + \frac{\log(x)}{a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -d^2*log((d*x^n + c)/d)/(b^2*c^3*n - 2*a*b*c^2*d*n + a^2*c*d^2*n) - (b^2*c
- 2*a*b*d)*log((b*x^n + a)/b)/(a^2*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n) +
 b/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x^n) + log(x)/(a^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,(a + b\,x^n)^2\,(c + d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/(x*(a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1037 $\int \frac{1}{x^2(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=142

$$-\frac{b(bc(n+1)-ad(2n+1))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)^2}+\frac{b}{anx(bc-ad)\,(a+bx^n)}$$

```
[Out] b/a/(-a*d+b*c)/n/x/(a+b*x^n)-b*(b*c*(1+n)-a*d*(1+2*n))*hypergeom([1, -1/n],
[(-1+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x-d^2*hypergeom([1, -1/n],[(-1+n)/n
],-d*x^n/c)/c/(-a*d+b*c)^2/x
```

Rubi [A] time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$-\frac{b(bc(n+1)-ad(2n+1))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2nx(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{dx^n}{c}\right)}{cx(bc-ad)^2}+\frac{b}{anx(bc-ad)\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x*(a + b*x^n)) - (b*(b*c*(1 + n) - a*d*(1 + 2*n))*Hyperg
eometric2F1[1, -n^(-1), -((1 - n)/n), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n*x
) - (d^2*Hypergeometric2F1[1, -n^(-1), -((1 - n)/n), -((d*x^n)/c)])/(c*(b*c
 - a*d)^2*x)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^2 (a+bx^n)^2 (c+dx^n)} dx = \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{\int \frac{adn-bc(1+n)-bd(1+n)x^n}{x^2(a+bx^n)(c+dx^n)} dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{\int \left(\frac{b(-bc(1+n)+ad(1+2n))}{(bc-ad)x^2(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^2(c+dx^n)}\right) dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} + \frac{d^2 \int \frac{1}{x^2(c+dx^n)} dx}{(bc-ad)^2} + \frac{(b(bc(1+n)-ad(1+2n))) \int \frac{1}{x^2(a+bx^n)}}{a(bc-ad)^2n}$$

$$= \frac{b}{a(bc-ad)nx(a+bx^n)} - \frac{b(bc(1+n)-ad(1+2n))\,{}_2F_1\left(1,-\frac{1}{n};-\frac{1-n}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2nx} - \frac{d^2\,{}_2}{}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.94

$$\frac{bc(a+bx^n)(ad(2n+1)-bc(n+1))\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{bx^n}{a}\right) - a\left(ad^2n(a+bx^n)\,{}_2F_1\left(1,-\frac{1}{n};\frac{n-1}{n};-\frac{dx^n}{c}\right) + bc(ad-bc\right.}{a^2cnx(bc-ad)^2(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(-(b*c*(1 + n)) + a*d*(1 + 2*n))*(a + b*x^n)*Hypergeometric2F1[1, -n^(
-1), (-1 + n)/n, -((b*x^n)/a)] - a*(b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n
)*Hypergeometric2F1[1, -n^(-1), (-1 + n)/n, -((d*x^n)/c)]))/(a^2*c*(b*c - a
*d)^2*n*x*(a + b*x^n))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2dx^2x^{3n} + a^2cx^2 + \left(b^2c + 2\,abd\right)x^2x^{2n} + \left(2\,abc + a^2d\right)x^2x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^2*x^(3*n) + a^2*c*x^2 + (b^2*c + 2*a*b*d)*x^2*x^(2*n) +
 (2*a*b*c + a^2*d)*x^2*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^2), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,x^n + a)^2 (d\,x^n + c)\,x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(1/x^2/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{1}{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^2x^n + \left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)x^2}\,dx - \left(abd(2\,n+1) - b^2c(n+1)\right) \int \frac{}{\left(ab^3c^2n - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^2*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^2), x) - (a*b*d*(2*n + 1) - b^2*c*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^2*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^2), x) + b/((a*b^2*c*n - a^2*b*d*n)*x*x^n
 + (a^2*b*c*n - a^3*d*n)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/(x^2*(a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1038 $\int \frac{1}{x^3(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=145

$$\frac{b(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2}+\frac{b}{anx^2(bc-ad)(a+bx^n)}$$

```
[Out] b/a/(-a*d+b*c)/n/x^2/(a+b*x^n)+1/2*b*(2*a*d*(1+n)-b*c*(2+n))*hypergeom([1,
-2/n],[(-2+n)/n],-b*x^n/a)/a^2/(-a*d+b*c)^2/n/x^2-1/2*d^2*hypergeom([1, -2/
n],[(-2+n)/n],-d*x^n/c)/c/(-a*d+b*c)^2/x^2
```

Rubi [A] time = 0.21, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {504, 597, 364}

$$\frac{b(2ad(n+1)-bc(n+2))\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2nx^2(bc-ad)^2}-\frac{d^2\,{}_2F_1\left(1,-\frac{2}{n};-\frac{2-n}{n};-\frac{dx^n}{c}\right)}{2cx^2(bc-ad)^2}+\frac{b}{anx^2(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] b/(a*(b*c - a*d)*n*x^2*(a + b*x^n)) + (b*(2*a*d*(1 + n) - b*c*(2 + n))*Hype
rgeometric2F1[1, -2/n, -((2 - n)/n), -((b*x^n)/a)])/(2*a^2*(b*c - a*d)^2*n*
x^2) - (d^2*Hypergeometric2F1[1, -2/n, -((2 - n)/n), -((d*x^n)/c)])/(2*c*(b
*c - a*d)^2*x^2)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 504

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ
[a, b, c, d, e, m, n, p, q, x]
```

Rule 597

```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n
_)))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((g*x)^m*(a
 + b*x^n)^p*(e + f*x^n))/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x]
```

Rubi steps

$$\int \frac{1}{x^3 (a+bx^n)^2 (c+dx^n)} dx = \frac{b}{a(bc-ad)nx^2 (a+bx^n)} - \frac{\int \frac{adn-bc(2+n)-bd(2+n)x^n}{x^3(a+bx^n)(c+dx^n)} dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} - \frac{\int \left(\frac{b(2ad(1+n)-bc(2+n))}{(bc-ad)x^3(a+bx^n)} + \frac{ad^2n}{(-bc+ad)x^3(c+dx^n)}\right) dx}{a(bc-ad)n}$$

$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} + \frac{d^2 \int \frac{1}{x^3(c+dx^n)} dx}{(bc-ad)^2} - \frac{(b(2ad(1+n)-bc(2+n))) \int \frac{}{x^3(}}{a(bc-ad)^2 n}$$

$$= \frac{b}{a(bc-ad)nx^2 (a+bx^n)} + \frac{b(2ad(1+n)-bc(2+n))\, {}_2F_1\left(1,-\frac{2}{n};\frac{2-n}{n};-\frac{bx^n}{a}\right)}{2a^2(bc-ad)^2nx^2} -$$

Mathematica [A] time = 0.23, size = 136, normalized size = 0.94

$$\frac{bc (a+bx^n) (2ad(n+1)-bc(n+2))\, {}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{bx^n}{a}\right) - a \left(ad^2n (a+bx^n)\, {}_2F_1\left(1,-\frac{2}{n};\frac{n-2}{n};-\frac{dx^n}{c}\right) + 2bc(ad\right.}{2a^2cnx^2(bc-ad)^2 (a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*c*(2*a*d*(1 + n) - b*c*(2 + n))*(a + b*x^n)*Hypergeometric2F1[1, -2/n, (
-2 + n)/n, -((b*x^n)/a)] - a*(2*b*c*(-(b*c) + a*d) + a*d^2*n*(a + b*x^n)*Hy
pergeometric2F1[1, -2/n, (-2 + n)/n, -((d*x^n)/c)]))/(2*a^2*c*(b*c - a*d)^2
*n*x^2*(a + b*x^n))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2dx^3x^{3n} + a^2cx^3 + \left(b^2c + 2abd\right)x^3x^{2n} + \left(2abc + a^2d\right)x^3x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^3*x^(3*n) + a^2*c*x^3 + (b^2*c + 2*a*b*d)*x^3*x^(2*n) +
 (2*a*b*c + a^2*d)*x^3*x^n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)*x^3), x)
```

maple [F] time = 1.29, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\, x^n + a)^2 (d\, x^n + c)\, x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] int(1/x^3/(b*x^n+a)^2/(d*x^n+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{1}{\left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x^3x^n + \left(b^2c^3 - 2\,abc^2d + a^2cd^2\right)x^3}\,dx + \left(b^2c(n+2) - 2\,abd(n+1)\right)\int \frac{}{\left(ab^3c^2n - 2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x^3*x^n + (b^2*c^3 - 2
*a*b*c^2*d + a^2*c*d^2)*x^3), x) + (b^2*c*(n + 2) - 2*a*b*d*(n + 1))*integr
ate(1/((a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a^3*b*d^2*n)*x^3*x^n + (a^2*b^2*c^2
*n - 2*a^3*b*c*d*n + a^4*d^2*n)*x^3), x) + b/((a*b^2*c*n - a^2*b*d*n)*x^2*x
^n + (a^2*b*c*n - a^3*d*n)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a+b\,x^n\right)}^2\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(1/(x^3*(a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1039 $\int \frac{x^{-1+2n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal. Leaf size=130

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{b^3x^{4n}}{4dn}$$

```
[Out] -(-a*d+b*c)^3*x^n/d^4/n+1/2*b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x^(2*n)/d^3/n-1
/3*b^2*(-3*a*d+b*c)*x^(3*n)/d^2/n+1/4*b^3*x^(4*n)/d/n+c*(-a*d+b*c)^3*ln(c+d
*x^n)/d^5/n
```

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{bx^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)}{2d^3n}-\frac{b^2x^{3n}(bc-3ad)}{3d^2n}-\frac{x^n(bc-ad)^3}{d^4n}+\frac{c(bc-ad)^3\log\left(c+dx^n\right)}{d^5n}+\frac{b^3x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)^3*x^n)/(d^4*n)) + (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2*
n))/(2*d^3*n) - (b^2*(b*c - 3*a*d)*x^(3*n))/(3*d^2*n) + (b^3*x^(4*n))/(4*d*
n) + (c*(b*c - a*d)^3*Log[c + d*x^n])/(d^5*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}\left(a+bx^n\right)^3}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{c+dx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^3}{d^4}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x}{d^3}-\frac{b^2(bc-3ad)x^2}{d^2}+\frac{b^3x^3}{d}+\frac{c(bc-ad)^3}{d^4(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{(bc-ad)^3x^n}{d^4n}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{2n}}{2d^3n}-\frac{b^2(bc-3ad)x^{3n}}{3d^2n}+\frac{b^3x^{4n}}{4dn}+\frac{c(bc-\end{aligned}$$

Mathematica [A] time = 0.35, size = 115, normalized size = 0.88

$$\frac{6bd^2x^{2n}\left(3a^2d^2-3abcd+b^2c^2\right)-4b^2d^3x^{3n}(bc-3ad)+12dx^n(ad-bc)^3+12c(bc-ad)^3\log\left(c+dx^n\right)+3b^3d^4}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (12*d*(-(b*c) + a*d)^3*x^n + 6*b*d^2*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(2
*n) - 4*b^2*d^3*(b*c - 3*a*d)*x^(3*n) + 3*b^3*d^4*x^(4*n) + 12*c*(b*c - a*d
)^3*Log[c + d*x^n])/(12*d^5*n)
```

fricas [A] time = 0.75, size = 177, normalized size = 1.36

$$\frac{3\,b^3d^4x^{4\,n} - 4\left(b^3cd^3 - 3\,ab^2d^4\right)x^{3\,n} + 6\left(b^3c^2d^2 - 3\,ab^2cd^3 + 3\,a^2bd^4\right)x^{2\,n} - 12\left(b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3c\right.}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^3*d^4*x^(4*n) - 4*(b^3*c*d^3 - 3*a*b^2*d^4)*x^(3*n) + 6*(b^3*c^2*
d^2 - 3*a*b^2*c*d^3 + 3*a^2*b*d^4)*x^(2*n) - 12*(b^3*c^3*d - 3*a*b^2*c^2*d^
2 + 3*a^2*b*c*d^3 - a^3*d^4)*x^n + 12*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^
2*d^2 - a^3*c*d^3)*log(d*x^n + c))/(d^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^3 x^{2\,n-1}}{dx^n + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(2*n - 1)/(d*x^n + c), x)
```

maple [B] time = 0.08, size = 284, normalized size = 2.18

$$-\frac{a^3c\ln\left(d\,e^{n\ln(x)} + c\right)}{d^2n} + \frac{a^3e^{n\ln(x)}}{dn} + \frac{3a^2b\,c^2\ln\left(d\,e^{n\ln(x)} + c\right)}{d^3n} - \frac{3a^2bc\,e^{n\ln(x)}}{d^2n} + \frac{3a^2b\,e^{2n\ln(x)}}{2dn} - \frac{3a\,b^2c^3\ln\left(d\,e^{n\ln(x)} + c\right)}{d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^3-3/d^2/n*exp(n*ln(x))*a^2*b*c+3/d^3/n*exp(n*ln(x))*a*
b^2*c^2-1/d^4/n*exp(n*ln(x))*b^3*c^3+1/4*b^3/d/n*exp(n*ln(x))^4+3/2*b/d/n*e
xp(n*ln(x))^2*a^2-3/2*b^2/d^2/n*exp(n*ln(x))^2*a*c+1/2*b^3/d^3/n*exp(n*ln(x
))^2*c^2+b^2/d/n*exp(n*ln(x))^3*a-1/3*b^3/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*ln
(d*exp(n*ln(x))+c)*a^3+3*c^2/d^3/n*ln(d*exp(n*ln(x))+c)*a^2*b-3*c^3/d^4/n*l
n(d*exp(n*ln(x))+c)*a*b^2+c^4/d^5/n*ln(d*exp(n*ln(x))+c)*b^3
```

maxima [A] time = 0.53, size = 231, normalized size = 1.78

$$a^3\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) + \frac{1}{12}\,b^3\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n} + \frac{3\,d^3x^{4\,n} - 4\,cd^2x^{3\,n} + 6\,c^2dx^{2\,n} - 12\,c^3x^n}{d^4n}\right) - \frac{1}{2}\,ab^2\left(\frac{6\,c^3\log\left(\frac{dx^n}{d}\right.}{d^4n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^3*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/12*b^3*(12*c^4*log((d*x^
n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n) + 6*c^2*d*x^(2*n) - 12
*c^3*x^n)/(d^4*n)) - 1/2*a*b^2*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x
```

```
^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 3/2*a^2*b*(2*c^2*log((d*x^n
+ c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1} (a + b x^n)^3}{c + d x^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

sympy [A] time = 135.87, size = 320, normalized size = 2.46

$$\begin{cases} \frac{(a+b)^3 \log(x)}{c} \\ \frac{\frac{a^3 x^{2n}}{2n} + \frac{a^2 b x^{3n}}{n} + \frac{3ab^2 x^{4n}}{4n} + \frac{b^3 x^{5n}}{5n}}{c} \\ \frac{(a+b)^3 \log(x)}{c+d} \\ -\frac{a^3 c \log\left(\frac{c}{d} + x^n\right)}{d^2 n} + \frac{a^3 x^n}{dn} + \frac{3a^2 bc^2 \log\left(\frac{c}{d} + x^n\right)}{d^3 n} - \frac{3a^2 bcx^n}{d^2 n} + \frac{3a^2 bx^{2n}}{2dn} - \frac{3ab^2 c^3 \log\left(\frac{c}{d} + x^n\right)}{d^4 n} + \frac{3ab^2 c^2 x^n}{d^3 n} - \frac{3ab^2 cx^{2n}}{2d^2 n} + \frac{ab^2 x^{3n}}{dn} + \frac{b^3 c^4 \log}{} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**3*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**3*x**(2*n)/(2*n)
 + a**2*b*x**(3*n)/n + 3*a*b**2*x**(4*n)/(4*n) + b**3*x**(5*n)/(5*n))/c, Eq
(d, 0)), ((a + b)**3*log(x)/(c + d), Eq(n, 0)), (-a**3*c*log(c/d + x**n)/(d
**2*n) + a**3*x**n/(d*n) + 3*a**2*b*c**2*log(c/d + x**n)/(d**3*n) - 3*a**2*
b*c*x**n/(d**2*n) + 3*a**2*b*x**(2*n)/(2*d*n) - 3*a*b**2*c**3*log(c/d + x**
n)/(d**4*n) + 3*a*b**2*c**2*x**n/(d**3*n) - 3*a*b**2*c*x**(2*n)/(2*d**2*n)
+ a*b**2*x**(3*n)/(d*n) + b**3*c**4*log(c/d + x**n)/(d**5*n) - b**3*c**3*x*
*n/(d**4*n) + b**3*c**2*x**(2*n)/(2*d**3*n) - b**3*c*x**(3*n)/(3*d**2*n) +
b**3*x**(4*n)/(4*d*n), True))
```

3.1040 $\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=90

$$-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}+\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}+\frac{b^2x^{3n}}{3dn}$$

```
[Out] (-a*d+b*c)^2*x^n/d^3/n-1/2*b*(-2*a*d+b*c)*x^(2*n)/d^2/n+1/3*b^2*x^(3*n)/d/n
-c*(-a*d+b*c)^2*ln(c+d*x^n)/d^4/n
```

Rubi [A] time = 0.09, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{x^n(bc-ad)^2}{d^3n}-\frac{bx^{2n}(bc-2ad)}{2d^2n}-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}+\frac{b^2x^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] ((b*c - a*d)^2*x^n)/(d^3*n) - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2*n) + (b^2*x^
(3*n))/(3*d*n) - (c*(b*c - a*d)^2*Log[c + d*x^n])/(d^4*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}(a+bx^n)^2}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{c+dx}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{(-bc+ad)^2}{d^3}-\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^2}{d}-\frac{c(bc-ad)^2}{d^3(c+dx)}\right)dx,x,x^n\right)}{n}\\
&= \frac{(bc-ad)^2x^n}{d^3n}-\frac{b(bc-2ad)x^{2n}}{2d^2n}+\frac{b^2x^{3n}}{3dn}-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4n}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.91

$$\frac{-\frac{c(bc-ad)^2\log(c+dx^n)}{d^4}+\frac{x^n(bc-ad)^2}{d^3}-\frac{bx^{2n}(bc-2ad)}{2d^2}+\frac{b^2x^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (((b*c - a*d)^2*x^n)/d^3 - (b*(b*c - 2*a*d)*x^(2*n))/(2*d^2) + (b^2*x^(3*n)
)/(3*d) - (c*(b*c - a*d)^2*Log[c + d*x^n])/d^4)/n
```

fricas [A] time = 0.58, size = 108, normalized size = 1.20

$$\frac{2\,b^2d^3x^{3\,n}-3\left(b^2cd^2-2\,abd^3\right)x^{2\,n}+6\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^n-6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^2*d^3*x^(3*n) - 3*(b^2*c*d^2 - 2*a*b*d^3)*x^(2*n) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*x^n - 6*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*log(d*x
^n + c))/(d^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^n+a\right)^2x^{2\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(2*n - 1)/(d*x^n + c), x)
```

maple [A] time = 0.08, size = 173, normalized size = 1.92

$$-\frac{a^2c\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^2n}+\frac{a^2\mathrm{e}^{n\ln(x)}}{dn}+\frac{2ab\,c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^3n}-\frac{2abc\,\mathrm{e}^{n\ln(x)}}{d^2n}+\frac{ab\,\mathrm{e}^{2n\ln(x)}}{dn}-\frac{b^2c^3\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{d^4n}+\frac{b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/d/n*exp(n*ln(x))*a^2-2/d^2/n*exp(n*ln(x))*a*b*c+1/d^3/n*exp(n*ln(x))*b^2*
c^2+1/3*b^2/d/n*exp(n*ln(x))^3+b/d/n*exp(n*ln(x))^2*a-1/2*b^2/d^2/n*exp(n*l
n(x))^2*c-c/d^2/n*ln(d*exp(n*ln(x))+c)*a^2+2*c^2/d^3/n*ln(d*exp(n*ln(x))+c)
*a*b-c^3/d^4/n*ln(d*exp(n*ln(x))+c)*b^2
```

maxima [A] time = 0.70, size = 150, normalized size = 1.67

$$a^2\left(\frac{x^n}{dn}-\frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right)-\frac{1}{6}\,b^2\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+ab\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2}{d^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) - 1/6*b^2*(6*c^3*log((d*x^n
+ c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + a*
b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

sympy [A] time = 63.51, size = 202, normalized size = 2.24

$$\begin{cases} \frac{(a+b)^2 \log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{a^2x^{2n}}{2n} + \frac{2abx^{3n}}{3n} + \frac{b^2x^{4n}}{4n}}{c} & \text{for } d = 0 \\ \frac{(a+b)^2 \log(x)}{c+d} & \text{for } n = 0 \\ -\frac{a^2c \log\left(\frac{c}{d}+x^n\right)}{d^2n} + \frac{a^2x^n}{dn} + \frac{2abc^2 \log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{2abcx^n}{d^2n} + \frac{abx^{2n}}{dn} - \frac{b^2c^3 \log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{b^2c^2x^n}{d^3n} - \frac{b^2cx^{2n}}{2d^2n} + \frac{b^2x^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)**2*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a**2*x**(2*n)/(2*n)
 + 2*a*b*x**(3*n)/(3*n) + b**2*x**(4*n)/(4*n))/c, Eq(d, 0)), ((a + b)**2*lo
g(x)/(c + d), Eq(n, 0)), (-a**2*c*log(c/d + x**n)/(d**2*n) + a**2*x**n/(d*n
) + 2*a*b*c**2*log(c/d + x**n)/(d**3*n) - 2*a*b*c*x**n/(d**2*n) + a*b*x**(2
*n)/(d*n) - b**2*c**3*log(c/d + x**n)/(d**4*n) + b**2*c**2*x**n/(d**3*n) -
b**2*c*x**(2*n)/(2*d**2*n) + b**2*x**(3*n)/(3*d*n), True))
```

3.1041 $\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx$

Optimal. Leaf size=60

$$\frac{c(bc-ad)\log(c+dx^n)}{d^3n}-\frac{x^n(bc-ad)}{d^2n}+\frac{bx^{2n}}{2dn}$$

```
[Out] -(-a*d+b*c)*x^n/d^2/n+1/2*b*x^(2*n)/d/n+c*(-a*d+b*c)*ln(c+d*x^n)/d^3/n
```

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{x^n(bc-ad)}{d^2n}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}+\frac{bx^{2n}}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] -(((b*c - a*d)*x^n)/(d^2*n)) + (b*x^(2*n))/(2*d*n) + (c*(b*c - a*d)*Log[c +
 d*x^n])/(d^3*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}(a+bx^n)}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{(bc-ad)x^n}{d^2n}+\frac{bx^{2n}}{2dn}+\frac{c(bc-ad)\log(c+dx^n)}{d^3n}\end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.83

$$\frac{dx^n(2ad-2bc+bdx^n)+2c(bc-ad)\log(c+dx^n)}{2d^3n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n))/(c + d*x^n),x]
```

[Out] (d*x^n*(-2*b*c + 2*a*d + b*d*x^n) + 2*c*(b*c - a*d)*Log[c + d*x^n])/(2*d^3*n)

fricas [A] time = 0.60, size = 56, normalized size = 0.93

$$\frac{bd^2x^{2n} - 2\left(bcd - ad^2\right)x^n + 2\left(bc^2 - acd\right)\log\left(dx^n + c\right)}{2\,d^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")

[Out] 1/2*(b*d^2*x^(2*n) - 2*(b*c*d - a*d^2)*x^n + 2*(b*c^2 - a*c*d)*log(d*x^n + c))/(d^3*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)x^{2n-1}}{dx^n + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")

[Out] integrate((b*x^n + a)*x^(2*n - 1)/(d*x^n + c), x)

maple [A] time = 0.07, size = 87, normalized size = 1.45

$$-\frac{ac\ln\left(d\,\mathrm{e}^{n\ln(x)} + c\right)}{d^2n} + \frac{a\,\mathrm{e}^{n\ln(x)}}{dn} + \frac{b\,c^2\ln\left(d\,\mathrm{e}^{n\ln(x)} + c\right)}{d^3n} - \frac{bc\,\mathrm{e}^{n\ln(x)}}{d^2n} + \frac{b\,\mathrm{e}^{2n\ln(x)}}{2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2*n-1)*(b*x^n+a)/(d*x^n+c),x)

[Out] 1/d/n*exp(n*ln(x))*a-1/d^2/n*exp(n*ln(x))*b*c+1/2*b/d/n*exp(n*ln(x))^2-c/d^2/n*ln(d*exp(n*ln(x))+c)*a+c^2/d^3/n*ln(d*exp(n*ln(x))+c)*b

maxima [A] time = 0.70, size = 83, normalized size = 1.38

$$a\left(\frac{x^n}{dn} - \frac{c\log\left(\frac{dx^n+c}{d}\right)}{d^2n}\right) + \frac{1}{2}\,b\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n} + \frac{dx^{2n} - 2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+2*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")

[Out] a*(x^n/(d*n) - c*log((d*x^n + c)/d)/(d^2*n)) + 1/2*b*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{2n-1}\,\left(a + b\,x^n\right)}{c + d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n),x)

[Out] int((x^(2*n - 1)*(a + b*x^n))/(c + d*x^n), x)

sympy [A] time = 26.97, size = 105, normalized size = 1.75

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d = 0 \wedge n = 0 \\ \frac{\frac{ax^{2n}}{2n} + \frac{bx^{3n}}{3n}}{c} & \text{for } d = 0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n = 0 \\ -\frac{ac\log\left(\frac{c}{d} + x^n\right)}{d^2 n} + \frac{ax^n}{dn} + \frac{bc^2\log\left(\frac{c}{d} + x^n\right)}{d^3 n} - \frac{bcx^n}{d^2 n} + \frac{bx^{2n}}{2dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(2*n)/(2*n) + b*x
**(3*n)/(3*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (-a*c*log(
c/d + x**n)/(d**2*n) + a*x**n/(d*n) + b*c**2*log(c/d + x**n)/(d**3*n) - b*c
*x**n/(d**2*n) + b*x**(2*n)/(2*d*n), True))
```

3.1042 $\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx$

Optimal. Leaf size=54

$$\frac{c\log(c+dx^n)}{dn(bc-ad)} - \frac{a\log(a+bx^n)}{bn(bc-ad)}$$

```
[Out] -a*ln(a+b*x^n)/b/(-a*d+b*c)/n+c*ln(c+d*x^n)/d/(-a*d+b*c)/n
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{c\log(c+dx^n)}{dn(bc-ad)} - \frac{a\log(a+bx^n)}{bn(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*Log[a + b*x^n])/(b*(b*c - a*d)*n)) + (c*Log[c + d*x^n])/(d*(b*c - a*d)
*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a+bx^n)(c+dx^n)} dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)(c+dx)} dx, x, x^n\right)}{n} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)} + \frac{c}{(bc-ad)(c+dx)}\right) dx, x, x^n\right)}{n} \\
&= -\frac{a\log(a+bx^n)}{b(bc-ad)n} + \frac{c\log(c+dx^n)}{d(bc-ad)n}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 44, normalized size = 0.81

$$-\frac{ad\log(a+bx^n) - bc\log(c+dx^n)}{b^2cdn - abd^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] -((a*d*Log[a + b*x^n] - b*c*Log[c + d*x^n])/(b^2*c*d*n - a*b*d^2*n))
```

fricas [A] time = 0.68, size = 45, normalized size = 0.83

$$-\frac{ad\log\left(bx^n+a\right)-bc\log\left(dx^n+c\right)}{\left(b^2cd-abd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
[Out] -(a*d*log(b*x^n + a) - b*c*log(d*x^n + c))/((b^2*c*d - a*b*d^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2\,n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

maple [A] time = 0.07, size = 59, normalized size = 1.09

$$\frac{a\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{(ad-bc)\,bn}-\frac{c\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{(ad-bc)\,dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)/(d*x^n+c),x)
[Out] a/(a*d-b*c)/b/n*ln(b*exp(n*ln(x))+a)-c/d/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)
```

maxima [A] time = 0.59, size = 60, normalized size = 1.11

$$-\frac{a\log\left(\frac{bx^n+a}{b}\right)}{b^2cn-abdn}+\frac{c\log\left(\frac{dx^n+c}{d}\right)}{bcdn-ad^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
[Out] -a*log((b*x^n + a)/b)/(b^2*c*n - a*b*d*n) + c*log((d*x^n + c)/d)/(b*c*d*n -
 a*d^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{2\,n-1}}{\left(a+b\,x^n\right)\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.1043 $\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=75

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

```
[Out] a/b/(-a*d+b*c)/n/(a+b*x^n)+c*ln(a+b*x^n)/(-a*d+b*c)^2/n-c*ln(c+d*x^n)/(-a*d
+b*c)^2/n
```

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{bn(bc-ad)(a+bx^n)}+\frac{c\log(a+bx^n)}{n(bc-ad)^2}-\frac{c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] a/(b*(b*c - a*d)*n*(a + b*x^n)) + (c*Log[a + b*x^n])/((b*c - a*d)^2*n) - (c
*Log[c + d*x^n])/((b*c - a*d)^2*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+2n}}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^2}+\frac{bc}{(bc-ad)^2(a+bx)}-\frac{cd}{(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{a}{b(bc-ad)n(a+bx^n)}+\frac{c\log(a+bx^n)}{(bc-ad)^2n}-\frac{c\log(c+dx^n)}{(bc-ad)^2n}\end{aligned}$$

Mathematica [A] time = 0.14, size = 58, normalized size = 0.77

$$\frac{\frac{a(bc-ad)}{b(a+bx^n)}+c\log(a+bx^n)-c\log(c+dx^n)}{n(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] ((a*(b*c - a*d))/(b*(a + b*x^n)) + c*Log[a + b*x^n] - c*Log[c + d*x^n])/((b
*c - a*d)^2*n)
```

fricas [A] time = 0.78, size = 120, normalized size = 1.60

$$\frac{abc - a^2d + \left(b^2cx^n + abc\right)\log\left(bx^n + a\right) - \left(b^2cx^n + abc\right)\log\left(dx^n + c\right)}{\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)nx^n + \left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a*b*c - a^2*d + (b^2*c*x^n + a*b*c)*log(b*x^n + a) - (b^2*c*x^n + a*b*c)*l
og(d*x^n + c))/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^n + (a*b^3*c^2 -
2*a^2*b^2*c*d + a^3*b*d^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\left(bx^n + a\right)^2\left(dx^n + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [A] time = 0.09, size = 109, normalized size = 1.45

$$\frac{c\ln\left(b\,e^{n\ln(x)} + a\right)}{\left(a^2d^2 - 2abcd + b^2c^2\right)n} - \frac{c\ln\left(d\,e^{n\ln(x)} + c\right)}{\left(a^2d^2 - 2abcd + b^2c^2\right)n} + \frac{e^{n\ln(x)}}{\left(ad - bc\right)\left(b\,e^{n\ln(x)} + a\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/(a*d-b*c)/n*exp(n*ln(x))/(b*exp(n*ln(x))+a)+c/n/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(b*exp(n*ln(x))+a)-c/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(d*exp(n*ln(x))+c
)
```

maxima [A] time = 0.52, size = 121, normalized size = 1.61

$$\frac{c\log\left(\frac{bx^n + a}{b}\right)}{b^2c^2n - 2\,abcdn + a^2d^2n} - \frac{c\log\left(\frac{dx^n + c}{d}\right)}{b^2c^2n - 2\,abcdn + a^2d^2n} + \frac{a}{ab^2cn - a^2bdn + \left(b^3cn - ab^2dn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c*log((b*x^n + a)/b)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) - c*log((d*x^n +
 c)/d)/(b^2*c^2*n - 2*a*b*c*d*n + a^2*d^2*n) + a/(a*b^2*c*n - a^2*b*d*n + (
b^3*c*n - a*b^2*d*n)*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{{\left(a + b\,x^n\right)}^2\,\left(c + d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1044 $\int \frac{x^{-1+2n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal. Leaf size=105

$$\frac{a}{2bn(bc-ad)\,(a+bx^n)^2}-\frac{c}{n(bc-ad)^2\,(a+bx^n)}-\frac{cd\log\,(a+bx^n)}{n(bc-ad)^3}+\frac{cd\log\,(c+dx^n)}{n(bc-ad)^3}$$

```
[Out] 1/2*a/b/(-a*d+b*c)/n/(a+b*x^n)^2-c/(-a*d+b*c)^2/n/(a+b*x^n)-c*d*ln(a+b*x^n)
/(-a*d+b*c)^3/n+c*d*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A] time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 77}

$$\frac{a}{2bn(bc-ad)\,(a+bx^n)^2}-\frac{c}{n(bc-ad)^2\,(a+bx^n)}-\frac{cd\log\,(a+bx^n)}{n(bc-ad)^3}+\frac{cd\log\,(c+dx^n)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] a/(2*b*(b*c - a*d)*n*(a + b*x^n)^2) - c/((b*c - a*d)^2*n*(a + b*x^n)) - (c*
d*Log[a + b*x^n])/((b*c - a*d)^3*n) + (c*d*Log[c + d*x^n])/((b*c - a*d)^3*n
)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a+bx^n)^3\,(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^3(c+dx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(bc-ad)(a+bx)^3}+\frac{bc}{(bc-ad)^2(a+bx)^2}-\frac{bcd}{(bc-ad)^3(a+bx)}+\frac{cd^2}{(bc-ad)^3(c+dx)}\right)dx,x,x^n\right)}{n}\\
&= \frac{a}{2b(bc-ad)n\,(a+bx^n)^2}-\frac{c}{(bc-ad)^2n\,(a+bx^n)}-\frac{cd\log\,(a+bx^n)}{(bc-ad)^3n}+\frac{cd\log\,(c+
\end{aligned}$$

Mathematica [A] time = 0.17, size = 97, normalized size = 0.92

$$\frac{\frac{a}{2b(bc-ad)(a+bx^n)^2}-\frac{c}{(bc-ad)^2(a+bx^n)}-\frac{cd\log(a+bx^n)}{(bc-ad)^3}+\frac{cd\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (a/(2*b*(b*c - a*d)*(a + b*x^n)^2) - c/((b*c - a*d)^2*(a + b*x^n)) - (c*d*L
og[a + b*x^n])/(b*c - a*d)^3 + (c*d*Log[c + d*x^n])/(b*c - a*d)^3)/n
```

fricas [B] time = 0.54, size = 267, normalized size = 2.54

$$-\frac{ab^2c^2 - a^3d^2 + 2\left(b^3c^2 - ab^2cd\right)x^n + 2\left(b^3cdx^{2n} + 2ab^2cdx^n + a^2bcd\right)\log\left(bx^n + a\right) - 2\left(b^3cdx^{2n} + 2ab^2cd\right.}{2\left(\left(b^6c^3 - 3ab^5c^2d + 3a^2b^4cd^2 - a^3b^3d^3\right)nx^{2n} + 2\left(ab^5c^3 - 3a^2b^4c^2d + 3a^3b^3cd^2 - a^4b^2d^3\right)nx^n + \left(a^2b^4c^3 - 3a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*c^2 - a^3*d^2 + 2*(b^3*c^2 - a*b^2*c*d)*x^n + 2*(b^3*c*d*x^(2*n
) + 2*a*b^2*c*d*x^n + a^2*b*c*d)*log(b*x^n + a) - 2*(b^3*c*d*x^(2*n) + 2*a*
b^2*c*d*x^n + a^2*b*c*d)*log(d*x^n + c))/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*
b^4*c*d^2 - a^3*b^3*d^3)*n*x^(2*n) + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*n*x^n + (a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^
2*c*d^2 - a^5*b*d^3)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\left(bx^n + a\right)^3\left(dx^n + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

maple [A] time = 0.11, size = 203, normalized size = 1.93

$$\frac{cd\ln\left(b\,e^{n\ln(x)} + a\right)}{\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)n} - \frac{cd\ln\left(d\,e^{n\ln(x)} + c\right)}{\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)n} + \frac{-\frac{bc\,e^{n\ln(x)}}{\left(a^2d^2 - 2abcd + b^2c^2\right)n} + \frac{\left(-abd - b^2c\right)a}{2\left(a^2d^2 - 2abcd + b^2\right.}}{\left(b\,e^{n\ln(x)} + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] (-b*c/(a^2*d^2-2*a*b*c*d+b^2*c^2)/n*exp(n*ln(x))+1/2*a*(-a*b*d-b^2*c)/(a^2*
d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*exp(n*ln(x))+a)^2+c*d/n/(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*exp(n*ln(x))+a)-c*d/n/(a^3*d^3-3*a^2*b*c*d
^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*exp(n*ln(x))+c)
```

maxima [B] time = 0.66, size = 243, normalized size = 2.31

$$-\frac{cd\log\left(\frac{bx^n + a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{cd\log\left(\frac{dx^n + c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} - \frac{}{2\left(a^2b^3c^2n - 2\,a^3b^2cdn + a^4b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -c*d*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^
3*d^3*n) + c*d*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*
d^2*n - a^3*d^3*n) - 1/2*(2*b^2*c*x^n + a*b*c + a^2*d)/(a^2*b^3*c^2*n - 2*a
```

```
^3*b^2*c*d*n + a^4*b*d^2*n + (b^5*c^2*n - 2*a*b^4*c*d*n + a^2*b^3*d^2*n)*x^
(2*n) + 2*(a*b^4*c^2*n - 2*a^2*b^3*c*d*n + a^3*b^2*d^2*n)*x^n)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a+b\,x^n)^3\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1045 $\int \frac{x^{-1+3n}(a+bx^n)^3}{c+dx^n}\,dx$

Optimal. Leaf size=158

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

```
[Out] c*(-a*d+b*c)^3*x^n/d^5/n-1/2*(-a*d+b*c)^3*x^(2*n)/d^4/n+1/3*b*(3*a^2*d^2-3*
a*b*c*d+b^2*c^2)*x^(3*n)/d^3/n-1/4*b^2*(-3*a*d+b*c)*x^(4*n)/d^2/n+1/5*b^3*x
^(5*n)/d/n-c^2*(-a*d+b*c)^3*ln(c+d*x^n)/d^6/n
```

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$\frac{bx^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)}{3d^3n}-\frac{b^2x^{4n}(bc-3ad)}{4d^2n}-\frac{c^2(bc-ad)^3\log\left(c+dx^n\right)}{d^6n}+\frac{cx^n(bc-ad)^3}{d^5n}-\frac{x^{2n}(bc-ad)^3}{2d^4n}+\frac{b^3x^{5n}}{5dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)^3*x^n)/(d^5*n) - ((b*c - a*d)^3*x^(2*n))/(2*d^4*n) + (b*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n))/(3*d^3*n) - (b^2*(b*c - 3*a*d)*x^(4*
n))/(4*d^2*n) + (b^3*x^(5*n))/(5*d*n) - (c^2*(b*c - a*d)^3*Log[c + d*x^n])/
(d^6*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}\left(a+bx^n\right)^3}{c+dx^n}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+bx)^3}{c+dx}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)^3}{d^5}+\frac{(-bc+ad)^3x}{d^4}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^2}{d^3}-\frac{b^2(bc-3ad)x^3}{d^2}+\frac{b^3x^4}{d}-\frac{c^2(bc-ad)^3}{d^5(c+dx)}\right)d
\end{aligned}$$

$$= \frac{c(bc-ad)^3x^n}{d^5n}-\frac{(bc-ad)^3x^{2n}}{2d^4n}+\frac{b\left(b^2c^2-3abcd+3a^2d^2\right)x^{3n}}{3d^3n}-\frac{b^2(bc-3ad)x^{4n}}{4d^2n}+\frac{b^3x}{5d}$$

Mathematica [A] time = 0.31, size = 138, normalized size = 0.87

$$\frac{20bd^3x^{3n}\left(3a^2d^2-3abcd+b^2c^2\right)-15b^2d^4x^{4n}(bc-3ad)-60c^2(bc-ad)^3\log\left(c+dx^n\right)+30d^2x^{2n}(ad-bc)^3+60c}{60d^6n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^3)/(c + d*x^n),x]
```

```
[Out] (60*c*d*(b*c - a*d)^3*x^n + 30*d^2*(-(b*c) + a*d)^3*x^(2*n) + 20*b*d^3*(b^2
*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x^(3*n) - 15*b^2*d^4*(b*c - 3*a*d)*x^(4*n) +
12*b^3*d^5*x^(5*n) - 60*c^2*(b*c - a*d)^3*Log[c + d*x^n])/(60*d^6*n)
```

fricas [A] time = 0.80, size = 230, normalized size = 1.46

$$\frac{12\,b^3d^5x^{5\,n}-15\left(b^3cd^4-3\,ab^2d^5\right)x^{4\,n}+20\left(b^3c^2d^3-3\,ab^2cd^4+3\,a^2bd^5\right)x^{3\,n}-30\left(b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^3*d^5*x^(5*n) - 15*(b^3*c*d^4 - 3*a*b^2*d^5)*x^(4*n) + 20*(b^3*c
^2*d^3 - 3*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^(3*n) - 30*(b^3*c^3*d^2 - 3*a*b^2*c
^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^(2*n) + 60*(b^3*c^4*d - 3*a*b^2*c^3*d^2
 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x^n - 60*(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b
*c^3*d^2 - a^3*c^2*d^3)*log(d*x^n + c))/(d^6*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^n+a\right)^3x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^3*x^(3*n - 1)/(d*x^n + c), x)
```

maple [B] time = 0.07, size = 342, normalized size = 2.16

$$\frac{a^3c^2\ln\left(x^n+\frac{c}{d}\right)}{d^3n}-\frac{a^3c\,x^n}{d^2n}+\frac{a^3x^{2n}}{2dn}-\frac{3a^2b\,c^3\ln\left(x^n+\frac{c}{d}\right)}{d^4n}+\frac{3a^2b\,c^2x^n}{d^3n}-\frac{3a^2bc\,x^{2n}}{2d^2n}+\frac{a^2b\,x^{3n}}{dn}+\frac{3a\,b^2c^4\ln\left(x^n+\frac{c}{d}\right)}{d^5n}-\frac{3a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] 1/5*b^3/d/n*(x^n)^5+3/4*b^2/d/n*(x^n)^4*a-1/4*b^3/d^2/n*(x^n)^4*c+b/d/n*(x^
n)^3*a^2-b^2/d^2/n*(x^n)^3*a*c+1/3*b^3/d^3/n*(x^n)^3*c^2+1/2/d/n*(x^n)^2*a^
3-3/2/d^2/n*(x^n)^2*a^2*b*c+3/2/d^3/n*(x^n)^2*a*b^2*c^2-1/2/d^4/n*(x^n)^2*b
^3*c^3-c/d^2/n*x^n*a^3+3*c^2/d^3/n*x^n*a^2*b-3*c^3/d^4/n*x^n*a*b^2+c^4/d^5/
n*x^n*b^3+c^2/d^3/n*ln(x^n+c/d)*a^3-3*c^3/d^4/n*ln(x^n+c/d)*a^2*b+3*c^4/d^5
/n*ln(x^n+c/d)*a*b^2-c^5/d^6/n*ln(x^n+c/d)*b^3
```

maxima [A] time = 0.66, size = 286, normalized size = 1.81

$$-\frac{1}{60}\,b^3\left(\frac{60\,c^5\log\left(\frac{dx^n+c}{d}\right)}{d^6n}-\frac{12\,d^4x^{5\,n}-15\,cd^3x^{4\,n}+20\,c^2d^2x^{3\,n}-30\,c^3dx^{2\,n}+60\,c^4x^n}{d^5n}\right)+\frac{1}{4}\,ab^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/60*b^3*(60*c^5*log((d*x^n + c)/d)/(d^6*n) - (12*d^4*x^(5*n) - 15*c*d^3*x
^(4*n) + 20*c^2*d^2*x^(3*n) - 30*c^3*d*x^(2*n) + 60*c^4*x^n)/(d^5*n)) + 1/4
*a*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3*n
```

```
) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/2*a^2*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^3*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}\,(a+b\,x^n)^3}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^3)/(c + d*x^n), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**3/(c+d*x**n),x)
[Out] Timed out
```

3.1046 $\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=118

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

```
[Out] -c*(-a*d+b*c)^2*x^n/d^4/n+1/2*(-a*d+b*c)^2*x^(2*n)/d^3/n-1/3*b*(-2*a*d+b*c)
*x^(3*n)/d^2/n+1/4*b^2*x^(4*n)/d/n+c^2*(-a*d+b*c)^2*ln(c+d*x^n)/d^5/n
```

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {446, 88}

$$\frac{c^2(bc-ad)^2\log(c+dx^n)}{d^5n}-\frac{cx^n(bc-ad)^2}{d^4n}+\frac{x^{2n}(bc-ad)^2}{2d^3n}-\frac{bx^{3n}(bc-2ad)}{3d^2n}+\frac{b^2x^{4n}}{4dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] -((c*(b*c - a*d)^2*x^n)/(d^4*n)) + ((b*c - a*d)^2*x^(2*n))/(2*d^3*n) - (b*(
b*c - 2*a*d)*x^(3*n))/(3*d^2*n) + (b^2*x^(4*n))/(4*d*n) + (c^2*(b*c - a*d)^
2*Log[c + d*x^n])/(d^5*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}(a+bx^n)^2}{c+dx^n}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^2}{c+dx}\,dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{c(bc-ad)^2}{d^4}+\frac{(-bc+ad)^2x}{d^3}-\frac{b(bc-2ad)x^2}{d^2}+\frac{b^2x^3}{d}+\frac{c^2(bc-ad)^2}{d^4(c+dx)}\right)dx, x, x^n\right)}{n}\\ &= -\frac{c(bc-ad)^2x^n}{d^4n}+\frac{(bc-ad)^2x^{2n}}{2d^3n}-\frac{b(bc-2ad)x^{3n}}{3d^2n}+\frac{b^2x^{4n}}{4dn}+\frac{c^2(bc-ad)^2\log(c+d}{d^5n}\end{aligned}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 0.87

$$\frac{12c^2(bc-ad)^2\log(c+dx^n)-4bd^3x^{3n}(bc-2ad)+6d^2x^{2n}(bc-ad)^2-12cdx^n(bc-ad)^2+3b^2d^4x^{4n}}{12d^5n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^2)/(c + d*x^n),x]
```

```
[Out] (-12*c*d*(b*c - a*d)^2*x^n + 6*d^2*(b*c - a*d)^2*x^(2*n) - 4*b*d^3*(b*c - 2
*a*d)*x^(3*n) + 3*b^2*d^4*x^(4*n) + 12*c^2*(b*c - a*d)^2*Log[c + d*x^n])/(1
2*d^5*n)
```

fricas [A] time = 0.63, size = 146, normalized size = 1.24

$$\frac{3\,b^2d^4x^{4\,n}-4\left(b^2cd^3-2\,abd^4\right)x^{3\,n}+6\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^{2\,n}-12\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^n+12\left(b^2c^4\right.}{12\,d^5n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*d^4*x^(4*n) - 4*(b^2*c*d^3 - 2*a*b*d^4)*x^(3*n) + 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*x^(2*n) - 12*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*x^n + 12*(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*log(d*x^n + c))/(d^5*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^n+a\right)^2x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2*x^(3*n - 1)/(d*x^n + c), x)
```

maple [B] time = 0.07, size = 236, normalized size = 2.00

$$\frac{a^2c^2\ln\left(d\,e^{n\ln(x)}+c\right)}{d^3n}-\frac{a^2c\,e^{n\ln(x)}}{d^2n}+\frac{a^2e^{2n\ln(x)}}{2dn}-\frac{2ab\,c^3\ln\left(d\,e^{n\ln(x)}+c\right)}{d^4n}+\frac{2ab\,c^2e^{n\ln(x)}}{d^3n}-\frac{abc\,e^{2n\ln(x)}}{d^2n}+\frac{2ab\,e^{3n\ln(x)}}{3dn}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] 1/4*b^2/d/n*exp(n*ln(x))^4+1/2/d/n*exp(n*ln(x))^2*a^2-1/d^2/n*exp(n*ln(x))^
2*a*b*c+1/2/d^3/n*exp(n*ln(x))^2*b^2*c^2+2/3*b/d/n*exp(n*ln(x))^3*a-1/3*b^2
/d^2/n*exp(n*ln(x))^3*c-c/d^2/n*exp(n*ln(x))*a^2+2*c^2/d^3/n*exp(n*ln(x))*a
*b-c^3/d^4/n*exp(n*ln(x))*b^2+c^2/d^3/n*ln(d*exp(n*ln(x))+c)*a^2-2*c^3/d^4/
n*ln(d*exp(n*ln(x))+c)*a*b+c^4/d^5/n*ln(d*exp(n*ln(x))+c)*b^2
```

maxima [A] time = 0.54, size = 192, normalized size = 1.63

$$\frac{1}{12}\,b^2\left(\frac{12\,c^4\log\left(\frac{dx^n+c}{d}\right)}{d^5n}+\frac{3\,d^3x^{4\,n}-4\,cd^2x^{3\,n}+6\,c^2dx^{2\,n}-12\,c^3x^n}{d^4n}\right)-\frac{1}{3}\,ab\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+}{d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] 1/12*b^2*(12*c^4*log((d*x^n + c)/d)/(d^5*n) + (3*d^3*x^(4*n) - 4*c*d^2*x^(3
*n) + 6*c^2*d*x^(2*n) - 12*c^3*x^n)/(d^4*n)) - 1/3*a*b*(6*c^3*log((d*x^n +
c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) + 6*c^2*x^n)/(d^3*n)) + 1/2*
a^2*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n) - 2*c*x^n)/(d^2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3\,n-1}\,{\left(a+b\,x^n\right)}^2}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^2)/(c + d*x^n), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**2/(c+d*x**n),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.1047 $\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n} dx$

Optimal. Leaf size=86

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

```
[Out] c*(-a*d+b*c)*x^n/d^3/n-1/2*(-a*d+b*c)*x^(2*n)/d^2/n+1/3*b*x^(3*n)/d/n-c^2*(
-a*d+b*c)*ln(c+d*x^n)/d^4/n
```

Rubi [A] time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {446, 77}

$$-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}+\frac{cx^n(bc-ad)}{d^3n}-\frac{x^{2n}(bc-ad)}{2d^2n}+\frac{bx^{3n}}{3dn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] (c*(b*c - a*d)*x^n)/(d^3*n) - ((b*c - a*d)*x^(2*n))/(2*d^2*n) + (b*x^(3*n))
/(3*d*n) - (c^2*(b*c - a*d)*Log[c + d*x^n])/(d^4*n)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}(a+bx^n)}{c+dx^n}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2(a+bx)}{c+dx}\,dx,x,x^n\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{c(bc-ad)}{d^3}+\frac{(-bc+ad)x}{d^2}+\frac{bx^2}{d}-\frac{c^2(bc-ad)}{d^3(c+dx)}\right)dx,x,x^n\right)}{n}\\
&= \frac{c(bc-ad)x^n}{d^3n}-\frac{(bc-ad)x^{2n}}{2d^2n}+\frac{bx^{3n}}{3dn}-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4n}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 78, normalized size = 0.91

$$\frac{-\frac{c^2(bc-ad)\log(c+dx^n)}{d^4}+\frac{cx^n(bc-ad)}{d^3}-\frac{x^{2n}(bc-ad)}{2d^2}+\frac{bx^{3n}}{3d}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n))/(c + d*x^n),x]
```

```
[Out] ((c*(b*c - a*d)*x^n)/d^3 - ((b*c - a*d)*x^(2*n))/(2*d^2) + (b*x^(3*n))/(3*d
) - (c^2*(b*c - a*d)*Log[c + d*x^n])/d^4)/n
```

fricas [A] time = 0.74, size = 82, normalized size = 0.95

$$\frac{2\,bd^3x^{3\,n}-3\left(bcd^2-ad^3\right)x^{2\,n}+6\left(bc^2d-acd^2\right)x^n-6\left(bc^3-ac^2d\right)\log\left(dx^n+c\right)}{6\,d^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d^3*x^(3*n) - 3*(b*c*d^2 - a*d^3)*x^(2*n) + 6*(b*c^2*d - a*c*d^2)*
x^n - 6*(b*c^3 - a*c^2*d)*log(d*x^n + c))/(d^4*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx^n+a)x^{3\,n-1}}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)*x^(3*n - 1)/(d*x^n + c), x)
```

maple [A] time = 0.07, size = 125, normalized size = 1.45

$$\frac{a\,c^2\ln\left(d\,e^{n\ln(x)}+c\right)}{d^3n}-\frac{ac\,e^{n\ln(x)}}{d^2n}+\frac{a\,e^{2n\ln(x)}}{2dn}-\frac{b\,c^3\ln\left(d\,e^{n\ln(x)}+c\right)}{d^4n}+\frac{b\,c^2e^{n\ln(x)}}{d^3n}-\frac{bc\,e^{2n\ln(x)}}{2d^2n}+\frac{b\,e^{3n\ln(x)}}{3dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] 1/3*b/d/n*exp(n*ln(x))^3+1/2/d/n*exp(n*ln(x))^2*a-1/2/d^2/n*exp(n*ln(x))^2*
b*c-c/d^2/n*exp(n*ln(x))*a+c^2/d^3/n*exp(n*ln(x))*b+c^2/d^3/n*ln(d*exp(n*ln
(x))+c)*a-c^3/d^4/n*ln(d*exp(n*ln(x))+c)*b
```

maxima [A] time = 0.53, size = 112, normalized size = 1.30

$$-\frac{1}{6}\,b\left(\frac{6\,c^3\log\left(\frac{dx^n+c}{d}\right)}{d^4n}-\frac{2\,d^2x^{3\,n}-3\,cdx^{2\,n}+6\,c^2x^n}{d^3n}\right)+\frac{1}{2}\,a\left(\frac{2\,c^2\log\left(\frac{dx^n+c}{d}\right)}{d^3n}+\frac{dx^{2\,n}-2\,cx^n}{d^2n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -1/6*b*(6*c^3*log((d*x^n + c)/d)/(d^4*n) - (2*d^2*x^(3*n) - 3*c*d*x^(2*n) +
 6*c^2*x^n)/(d^3*n)) + 1/2*a*(2*c^2*log((d*x^n + c)/d)/(d^3*n) + (d*x^(2*n)
 - 2*c*x^n)/(d^2*n))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3\,n-1}\,\left(a+b\,x^n\right)}{c+d\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n),x)
```

```
[Out] int((x^(3*n - 1)*(a + b*x^n))/(c + d*x^n), x)
```

sympy [A] time = 57.94, size = 139, normalized size = 1.62

$$\begin{cases} \frac{(a+b)\log(x)}{c} & \text{for } d=0 \wedge n=0 \\ \frac{\frac{ax^{3n}}{3n}+\frac{bx^{4n}}{4n}}{c} & \text{for } d=0 \\ \frac{(a+b)\log(x)}{c+d} & \text{for } n=0 \\ \frac{ac^2\log\left(\frac{c}{d}+x^n\right)}{d^3n} - \frac{acx^n}{d^2n} + \frac{ax^{2n}}{2dn} - \frac{bc^3\log\left(\frac{c}{d}+x^n\right)}{d^4n} + \frac{bc^2x^n}{d^3n} - \frac{bcx^{2n}}{2d^2n} + \frac{bx^{3n}}{3dn} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Piecewise(((a + b)*log(x)/c, Eq(d, 0) & Eq(n, 0)), ((a*x**(3*n)/(3*n) + b*x
**(4*n)/(4*n))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 0)), (a*c**2*lo
g(c/d + x**n)/(d**3*n) - a*c*x**n/(d**2*n) + a*x**(2*n)/(2*d*n) - b*c**3*lo
g(c/d + x**n)/(d**4*n) + b*c**2*x**n/(d**3*n) - b*c*x**(2*n)/(2*d**2*n) + b
*x**(3*n)/(3*d*n), True))
```

3.1048 $\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx$

Optimal. Leaf size=71

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

```
[Out] x^n/b/d/n+a^2*ln(a+b*x^n)/b^2/(-a*d+b*c)/n-c^2*ln(c+d*x^n)/d^2/(-a*d+b*c)/n
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 72}

$$\frac{a^2\log(a+bx^n)}{b^2n(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2n(bc-ad)}+\frac{x^n}{bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] x^n/(b*d*n) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)*n) - (c^2*Log[c + d*x^n
])/(d^2*(b*c - a*d)*n)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{bd}+\frac{a^2}{b(bc-ad)(a+bx)}+\frac{c^2}{d(-bc+ad)(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= \frac{x^n}{bdn}+\frac{a^2\log(a+bx^n)}{b^2(bc-ad)n}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)n}\end{aligned}$$

Mathematica [A] time = 0.11, size = 66, normalized size = 0.93

$$\frac{\frac{a^2\log(a+bx^n)}{b^2(bc-ad)}-\frac{c^2\log(c+dx^n)}{d^2(bc-ad)}+\frac{x^n}{bd}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x^n/(b*d) + (a^2*Log[a + b*x^n])/(b^2*(b*c - a*d)) - (c^2*Log[c + d*x^n])/
(d^2*(b*c - a*d)))/n
```

fricas [A] time = 0.79, size = 74, normalized size = 1.04

$$\frac{a^2d^2\log\left(bx^n+a\right)-b^2c^2\log\left(dx^n+c\right)+\left(b^2cd-abd^2\right)x^n}{\left(b^3cd^2-ab^2d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] (a^2*d^2*log(b*x^n + a) - b^2*c^2*log(d*x^n + c) + (b^2*c*d - a*b*d^2)*x^n)
/((b^3*c*d^2 - a*b^2*d^3)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3n-1}}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)*(d*x^n + c)), x)
```

maple [A] time = 0.10, size = 78, normalized size = 1.10

$$-\frac{a^2\ln\left(b\,\mathrm{e}^{n\ln(x)}+a\right)}{(ad-bc)\,b^2n}+\frac{c^2\ln\left(d\,\mathrm{e}^{n\ln(x)}+c\right)}{(ad-bc)\,d^2n}+\frac{\mathrm{e}^{n\ln(x)}}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)/(d*x^n+c),x)
```

```
[Out] 1/b/d/n*exp(n*ln(x))+c^2/d^2/n/(a*d-b*c)*ln(d*exp(n*ln(x))+c)-a^2/(a*d-b*c)
/b^2/n*ln(b*exp(n*ln(x))+a)
```

maxima [A] time = 0.63, size = 81, normalized size = 1.14

$$\frac{a^2\log\left(\frac{bx^n+a}{b}\right)}{b^3cn-ab^2dn}-\frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{bcd^2n-ad^3n}+\frac{x^n}{bdn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] a^2*log((b*x^n + a)/b)/(b^3*c*n - a*b^2*d*n) - c^2*log((d*x^n + c)/d)/(b*c*
d^2*n - a*d^3*n) + x^n/(b*d*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3n-1}}{(a+b\,x^n)\,(c+d\,x^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1049 $\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

```
[Out] -a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)-a*(-a*d+2*b*c)*ln(a+b*x^n)/b^2/(-a*d+b*c)^2
/n+c^2*ln(c+d*x^n)/d/(-a*d+b*c)^2/n
```

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{b^2n(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2n(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{dn(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] -(a^2/(b^2*(b*c - a*d)*n*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(
b^2*(b*c - a*d)^2*n) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+3n}}{(a+bx^n)^2(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^2(c+dx)}\,dx,x,x^n\right)}{n}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^2}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)}+\frac{c^2}{(bc-ad)^2(c+dx)}\right)dx,x,x^n\right)}{n}\\
&= -\frac{a^2}{b^2(bc-ad)n(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2n}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2n}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 90, normalized size = 0.95

$$\frac{-\frac{a^2}{b^2(bc-ad)(a+bx^n)}-\frac{a(2bc-ad)\log(a+bx^n)}{b^2(bc-ad)^2}+\frac{c^2\log(c+dx^n)}{d(bc-ad)^2}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (-(a^2/(b^2*(b*c - a*d)*(a + b*x^n))) - (a*(2*b*c - a*d)*Log[a + b*x^n])/(b
^2*(b*c - a*d)^2) + (c^2*Log[c + d*x^n])/(d*(b*c - a*d)^2))/n
```

fricas [A] time = 0.82, size = 166, normalized size = 1.75

$$-\frac{a^{2}bcd - a^{3}d^{2} + \left(2\,a^{2}bcd - a^{3}d^{2} + \left(2\,ab^{2}cd - a^{2}bd^{2}\right)x^{n}\right)\log\left(bx^{n} + a\right) - \left(b^{3}c^{2}x^{n} + ab^{2}c^{2}\right)\log\left(dx^{n} + c\right)}{\left(b^{5}c^{2}d - 2\,ab^{4}cd^{2} + a^{2}b^{3}d^{3}\right)nx^{n} + \left(ab^{4}c^{2}d - 2\,a^{2}b^{3}cd^{2} + a^{3}b^{2}d^{3}\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] -(a^2*b*c*d - a^3*d^2 + (2*a^2*b*c*d - a^3*d^2 + (2*a*b^2*c*d - a^2*b*d^2)*
x^n)*log(b*x^n + a) - (b^3*c^2*x^n + a*b^2*c^2)*log(d*x^n + c))/((b^5*c^2*d
 - 2*a*b^4*c*d^2 + a^2*b^3*d^3)*n*x^n + (a*b^4*c^2*d - 2*a^2*b^3*c*d^2 + a^
3*b^2*d^3)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\left(bx^{n} + a\right)^{2}\left(dx^{n} + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^2*(d*x^n + c)), x)
```

maple [A] time = 0.08, size = 163, normalized size = 1.72

$$\frac{a^{2}d\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{\left(a^{2}d^{2} - 2abcd + b^{2}c^{2}\right)b^{2}n} - \frac{2ac\ln\left(b\,\mathrm{e}^{n\ln(x)} + a\right)}{\left(a^{2}d^{2} - 2abcd + b^{2}c^{2}\right)bn} + \frac{c^{2}\ln\left(d\,\mathrm{e}^{n\ln(x)} + c\right)}{\left(a^{2}d^{2} - 2abcd + b^{2}c^{2}\right)dn} + \frac{a^{2}}{(ad - bc)\left(b\,\mathrm{e}^{n\ln(x)} + a\right)b^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^2/(d*x^n+c),x)
```

```
[Out] a^2/(a*d-b*c)/b^2/n/(b*exp(n*ln(x))+a)+c^2/d/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)*
ln(d*exp(n*ln(x))+c)+a^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n*ln(b*exp(n*ln(x)
)+a)*d-2*a/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/n*ln(b*exp(n*ln(x))+a)*c
```

maxima [A] time = 0.58, size = 147, normalized size = 1.55

$$\frac{c^{2}\log\left(\frac{dx^{n}+c}{d}\right)}{b^{2}c^{2}dn - 2\,abcd^{2}n + a^{2}d^{3}n} - \frac{a^{2}}{ab^{3}cn - a^{2}b^{2}dn + \left(b^{4}cn - ab^{3}dn\right)x^{n}} - \frac{\left(2\,abc - a^{2}d\right)\log\left(\frac{bx^{n}+a}{b}\right)}{b^{4}c^{2}n - 2\,ab^{3}cdn + a^{2}b^{2}d^{2}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((d*x^n + c)/d)/(b^2*c^2*d*n - 2*a*b*c*d^2*n + a^2*d^3*n) - a^2/(a*b
^3*c*n - a^2*b^2*d*n + (b^4*c*n - a*b^3*d*n)*x^n) - (2*a*b*c - a^2*d)*log((
b*x^n + a)/b)/(b^4*c^2*n - 2*a*b^3*c*d*n + a^2*b^2*d^2*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^{n}\right)}^{2}\,\left(c+d\,x^{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^2*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1050 $\int \frac{x^{-1+3n}}{(a+bx^n)^3(c+dx^n)}\,dx$

Optimal. Leaf size=120

$$-\frac{a^2}{2b^2n(bc-ad)\,(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2\,(a+bx^n)}+\frac{c^2\log\left(a+bx^n\right)}{n(bc-ad)^3}-\frac{c^2\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

```
[Out] -1/2*a^2/b^2/(-a*d+b*c)/n/(a+b*x^n)^2+a*(-a*d+2*b*c)/b^2/(-a*d+b*c)^2/n/(a+
b*x^n)+c^2*ln(a+b*x^n)/(-a*d+b*c)^3/n-c^2*ln(c+d*x^n)/(-a*d+b*c)^3/n
```

Rubi [A] time = 0.11, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {446, 88}

$$-\frac{a^2}{2b^2n(bc-ad)\,(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2n(bc-ad)^2\,(a+bx^n)}+\frac{c^2\log\left(a+bx^n\right)}{n(bc-ad)^3}-\frac{c^2\log\left(c+dx^n\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] -a^2/(2*b^2*(b*c - a*d)*n*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a*
d)^2*n*(a + b*x^n)) + (c^2*Log[a + b*x^n])/((b*c - a*d)^3*n) - (c^2*Log[c +
 d*x^n])/((b*c - a*d)^3*n)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+3n}}{(a+bx^n)^3\,(c+dx^n)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^3(c+dx)}\,dx,x,x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{b(bc-ad)(a+bx)^3}+\frac{a(-2bc+ad)}{b(bc-ad)^2(a+bx)^2}+\frac{bc^2}{(bc-ad)^3(a+bx)}-\frac{c^2d}{(bc-ad)^3(c+dx)}\right)dx,x,x^n\right)}{n}\\ &= -\frac{a^2}{2b^2(bc-ad)n\,(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2n\,(a+bx^n)}+\frac{c^2\log\left(a+bx^n\right)}{(bc-ad)^3n}-\frac{c^2\log\left(c+dx^n\right)}{(bc-ad)^3n}\end{aligned}$$

Mathematica [A] time = 0.27, size = 112, normalized size = 0.93

$$\frac{-\frac{a^2}{2b^2(bc-ad)(a+bx^n)^2}+\frac{a(2bc-ad)}{b^2(bc-ad)^2(a+bx^n)}+\frac{c^2\log(a+bx^n)}{(bc-ad)^3}-\frac{c^2\log(c+dx^n)}{(bc-ad)^3}}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^3*(c + d*x^n)),x]
```

```
[Out] (-1/2*a^2/(b^2*(b*c - a*d)*(a + b*x^n)^2) + (a*(2*b*c - a*d))/(b^2*(b*c - a
*d)^2*(a + b*x^n)) + (c^2*Log[a + b*x^n])/(b*c - a*d)^3 - (c^2*Log[c + d*x^
n])/(b*c - a*d)^3)/n
```

fricas [B] time = 0.62, size = 301, normalized size = 2.51

$$\frac{3\,a^2b^2c^2 - 4\,a^3bcd + a^4d^2 + 2\left(2\,ab^3c^2 - 3\,a^2b^2cd + a^3bd^2\right)x^n + 2\left(b^4c^2x^{2\,n} + 2\,ab^3c^2x^n + a^2b^2c^2\right)\log\left(bx^n + a\right) - 2}{2\left(\left(b^7c^3 - 3\,ab^6c^2d + 3\,a^2b^5cd^2 - a^3b^4d^3\right)nx^{2\,n} + 2\left(ab^6c^3 - 3\,a^2b^5c^2d + 3\,a^3b^4cd^2 - a^4b^3d^3\right)nx^n + \left(a^2b^5c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] 1/2*(3*a^2*b^2*c^2 - 4*a^3*b*c*d + a^4*d^2 + 2*(2*a*b^3*c^2 - 3*a^2*b^2*c*d
 + a^3*b*d^2)*x^n + 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log
(b*x^n + a) - 2*(b^4*c^2*x^(2*n) + 2*a*b^3*c^2*x^n + a^2*b^2*c^2)*log(d*x^n
 + c))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*n*x^(2*n)
 + 2*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*n*x^n +
(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\left(bx^n + a\right)^3\left(dx^n + c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^3*(d*x^n + c)), x)
```

maple [A] time = 0.12, size = 214, normalized size = 1.78

$$-\frac{c^2\ln\left(b\,e^{n\ln(x)} + a\right)}{\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)n} + \frac{c^2\ln\left(d\,e^{n\ln(x)} + c\right)}{\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)n} + \frac{\frac{(-ad+2bc)a\,e^{n\ln(x)}}{\left(a^2d^2-2abcd+b^2c^2\right)bn} + \frac{(-ad+3bc)a}{2\left(a^2d^2-2abcd+b\right.}}{\left(b\,e^{n\ln(x)} + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^3/(d*x^n+c),x)
```

```
[Out] ((-a*d+2*b*c)*a/n/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*exp(n*ln(x))+1/2*a^2*(-a*d+
3*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/n)/(b*exp(n*ln(x))+a)^2+c^2/n/(a^3*d
^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(d*exp(n*ln(x))+c)-c^2/n/(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*ln(b*exp(n*ln(x))+a)
```

maxima [B] time = 0.68, size = 262, normalized size = 2.18

$$\frac{c^2\log\left(\frac{bx^n+a}{b}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} - \frac{c^2\log\left(\frac{dx^n+c}{d}\right)}{b^3c^3n - 3\,ab^2c^2dn + 3\,a^2bcd^2n - a^3d^3n} + \frac{}{2\left(a^2b^4c^2n - 2\,a^3b^3cdn + a^4b^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^3/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] c^2*log((b*x^n + a)/b)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n - a^3
*d^3*n) - c^2*log((d*x^n + c)/d)/(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d
^2*n - a^3*d^3*n) + 1/2*(3*a^2*b*c - a^3*d + 2*(2*a*b^2*c - a^2*b*d)*x^n)/(
a^2*b^4*c^2*n - 2*a^3*b^3*c*d*n + a^4*b^2*d^2*n + (b^6*c^2*n - 2*a*b^5*c*d*
```

```
n + a^2*b^4*d^2*n)*x^(2*n) + 2*(a*b^5*c^2*n - 2*a^2*b^4*c*d*n + a^3*b^3*d^2
*n)*x^n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{{\left(a+b\,x^n\right)}^3\,\left(c+d\,x^n\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^3*(c + d*x^n)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**3/(c+d*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1051 $\int x^{13}(b+cx)^{13}(b+2cx)\,dx$

Optimal. Leaf size=14

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

```
[Out] 1/14*x^14*(c*x+b)^14
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{1}{14}x^{14}(b+cx)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (x^14*(b + c*x)^14)/14
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{13}(b+cx)^{13}(b+2cx)\,dx=\frac{1}{14}x^{14}(b+cx)^{14}$$

Mathematica [B] time = 0.01, size = 172, normalized size = 12.29

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13*(b + c*x)^13*(b + 2*c*x),x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

fricas [B] time = 0.43, size = 154, normalized size = 11.00

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^2+26x^{25}c^{11}b^3+\frac{143}{2}x^{24}c^{10}b^4+143x^{23}c^9b^5+\frac{429}{2}x^{22}c^8b^6+\frac{1716}{7}x^{21}c^7b^7+\frac{429}{2}x^{20}c^6b^8+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

giac [A] time = 0.15, size = 13, normalized size = 0.93

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

maple [B] time = 0.04, size = 155, normalized size = 11.07

$$\frac{1}{14}c^{14}x^{28}+b\,c^{13}x^{27}+\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(c*x+b)^13*(2*c*x+b),x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

maxima [B] time = 0.57, size = 154, normalized size = 11.00

$$\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}+\frac{429}{2}\,b^8c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13*(c*x+b)^13*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

mupad [B] time = 0.16, size = 154, normalized size = 11.00

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13*(b + c*x)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

sympy [B] time = 0.12, size = 175, normalized size = 12.50

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(c*x+b)**13*(2*c*x+b),x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

3.1052 $\int x^{27} \left(b+cx^2\right)^{13} \left(b+2cx^2\right) \, dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28} \left(b+cx^2\right)^{14}$$

[Out] 1/28*x^28*(c*x^2+b)^14

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{28}x^{28} \left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

[In] Int[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{27} \left(b+cx^2\right)^{13} \left(b+2cx^2\right) \, dx &= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx) \, dx, x, x^2\right)\\ &= \frac{1}{28}x^{28} \left(b+cx^2\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.01, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{1}{2}$$

Antiderivative was successfully verified.

[In] Integrate[x^27*(b + c*x^2)^13*(b + 2*c*x^2),x]

[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 + (c^14*x^56)/28

fricas [B] time = 0.44, size = 156, normalized size = 9.75

$$\frac{1}{28}x^{56}c^{14}+\frac{1}{2}x^{54}c^{13}b+\frac{13}{4}x^{52}c^{12}b^2+13x^{50}c^{11}b^3+\frac{143}{4}x^{48}c^{10}b^4+\frac{143}{2}x^{46}c^9b^5+\frac{429}{4}x^{44}c^8b^6+\frac{858}{7}x^{42}c^7b^7+\frac{429}{4}x^{40}c^6b^8+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

giac [B] time = 0.19, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

maple [B] time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(c*x^2+b)^13*(2*c*x^2+b),x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

maxima [B] time = 0.73, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}\,b^4c^{10}x^{48}+\frac{143}{2}\,b^5c^9x^{46}+\frac{429}{4}\,b^6c^8x^{44}+\frac{858}{7}\,b^7c^7x^{42}+\frac{429}{4}\,b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^27*(c*x^2+b)^13*(2*c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

mupad [B] time = 4.93, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^27*(b + c*x^2)^13*(b + 2*c*x^2),x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
```

```
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

sympy [B] time = 0.12, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^2x^{32}}{4}+13b^{11}c^3x^{34}+\frac{143b^{10}c^4x^{36}}{4}+\frac{143b^9c^5x^{38}}{2}+\frac{429b^8c^6x^{40}}{4}+\frac{858b^7c^7x^{42}}{7}+\frac{429b^6c^8x^{44}}{4}+\frac{14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**27*(c*x**2+b)**13*(2*c*x**2+b),x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

3.1053 $\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A] time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx, x, x^3\right)\\ &= \frac{1}{42}x^{42}\left(b+cx^3\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.02, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^41*(b + c*x^3)^13*(b + 2*c*x^3),x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

fricas [B] time = 0.45, size = 156, normalized size = 9.75

$$\frac{1}{42}x^{84}c^{14}+\frac{1}{3}x^{81}c^{13}b+\frac{13}{6}x^{78}c^{12}b^2+\frac{26}{3}x^{75}c^{11}b^3+\frac{143}{6}x^{72}c^{10}b^4+\frac{143}{3}x^{69}c^9b^5+\frac{143}{2}x^{66}c^8b^6+\frac{572}{7}x^{63}c^7b^7+\frac{143}{2}x^{60}c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

giac [B] time = 0.16, size = 156, normalized size = 9.75

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

maple [B] time = 0.04, size = 157, normalized size = 9.81

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}b\,c^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(c*x^3+b)^13*(2*c*x^3+b),x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

maxima [B] time = 0.62, size = 156, normalized size = 9.75

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^2c^{12}x^{78}+\frac{26}{3}\,b^3c^{11}x^{75}+\frac{143}{6}\,b^4c^{10}x^{72}+\frac{143}{3}\,b^5c^9x^{69}+\frac{143}{2}\,b^6c^8x^{66}+\frac{572}{7}\,b^7c^7x^{63}+\frac{143}{2}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^41*(c*x^3+b)^13*(2*c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

mupad [B] time = 4.77, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^41*(b + c*x^3)^13*(b + 2*c*x^3),x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
```

```
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

sympy [B] time = 0.12, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^2x^{48}}{6}+\frac{26b^{11}c^3x^{51}}{3}+\frac{143b^{10}c^4x^{54}}{6}+\frac{143b^9c^5x^{57}}{3}+\frac{143b^8c^6x^{60}}{2}+\frac{572b^7c^7x^{63}}{7}+\frac{143b^6c^8x^{66}}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**41*(c*x**3+b)**13*(2*c*x**3+b),x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

3.1054 $\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {446, 74}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx &= \frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}\\ &= \frac{x^{14n}(b+cx^n)^{14}}{14n}\end{aligned}$$

Mathematica [A] time = 0.19, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + 14*n)*(b + c*x^n)^13*(b + 2*c*x^n),x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

fricas [B] time = 0.55, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28n}+14\,bc^{13}x^{27n}+91\,b^2c^{12}x^{26n}+364\,b^3c^{11}x^{25n}+1001\,b^4c^{10}x^{24n}+2002\,b^5c^9x^{23n}+3003\,b^6c^8x^{22n}+3432}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

giac [B] time = 0.57, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^2c^{12}x^{26\,n} + 364\,b^3c^{11}x^{25\,n} + 1001\,b^4c^{10}x^{24\,n} + 2002\,b^5c^9x^{23\,n} + 3003\,b^6c^8x^{22\,n} + 34}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

maple [B] time = 0.08, size = 230, normalized size = 10.95

$$\frac{b^{14}x^{14n}}{14n}+\frac{b^{13}c\,x^{15n}}{n}+\frac{13b^{12}c^2x^{16n}}{2n}+\frac{26b^{11}c^3x^{17n}}{n}+\frac{143b^{10}c^4x^{18n}}{2n}+\frac{143b^9c^5x^{19n}}{n}+\frac{429b^8c^6x^{20n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{42}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

maxima [B] time = 0.64, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^2c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^3c^{11}x^{25\,n}}{n}+\frac{143\,b^4c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^5c^9x^{23\,n}}{n}+\frac{429\,b^6c^8x^{22\,n}}{2\,n}+\frac{1716\,b^7c^7x^{21\,n}}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+14*n)*(b+c*x^n)^13*(b+2*c*x^n),x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

mupad [B] time = 5.21, size = 229, normalized size = 10.90

$$\frac{b^{14}\,x^{14\,n}}{14\,n}+\frac{c^{14}\,x^{28\,n}}{14\,n}+\frac{13\,b^{12}\,c^2\,x^{16\,n}}{2\,n}+\frac{26\,b^{11}\,c^3\,x^{17\,n}}{n}+\frac{143\,b^{10}\,c^4\,x^{18\,n}}{2\,n}+\frac{143\,b^9\,c^5\,x^{19\,n}}{n}+\frac{429\,b^8\,c^6\,x^{20\,n}}{2\,n}+\frac{1716\,b^7\,c^7}{7\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(14*n - 1)*(b + c*x^n)^13*(b + 2*c*x^n),x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
```

```
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+14*n)*(b+c*x**n)**13*(b+2*c*x**n),x)
[Out] Timed out
```

3.1055 $\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)dx$

Optimal. Leaf size=13

$$x^m(a+bx^n)^p$$

```
[Out] x^m*(a+b*x^n)^p
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {449}

$$x^m(a+bx^n)^p$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] x^m*(a + b*x^n)^p
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+m}(a+bx^n)^{-1+p}\left(am+b(m+np)x^n\right)dx=x^m(a+bx^n)^p$$

Mathematica [C] time = 0.18, size = 107, normalized size = 8.23

$$\frac{x^m(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(bx^n(m+np)\,{}_2F_1\left(\frac{m+n}{n},1-p;\frac{m}{n}+2;-\frac{bx^n}{a}\right)+a(m+n)\,{}_2F_1\left(\frac{m}{n},1-p;\frac{m+n}{n};-\frac{bx^n}{a}\right)\right)}{a(m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*(a + b*x^n)^(-1 + p)*(a*m + b*(m + n*p)*x^n),x]
```

```
[Out] (x^m*(a + b*x^n)^p*(a*(m + n)*Hypergeometric2F1[m/n, 1 - p, (m + n)/n, -((b
*x^n)/a)] + b*(m + n*p)*x^n*Hypergeometric2F1[(m + n)/n, 1 - p, 2 + m/n, -(
(b*x^n)/a)]))/(a*(m + n)*(1 + (b*x^n)/a)^p)
```

fricas [B] time = 0.65, size = 32, normalized size = 2.46

$$\left(bxx^{m-1}x^n+axx^{m-1}\right)(bx^n+a)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="frica
s")
```

```
[Out] (b*x*x^(m - 1)*x^n + a*x*x^(m - 1))*(b*x^n + a)^(p - 1)
```

giac [B] time = 0.28, size = 70, normalized size = 5.38

$$bxx^ne^{\left(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x)\right)}+axe^{\left(p\log(bx^n+a)+m\log(x)-\log(bx^n+a)-\log(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="giac"
)
```

```
[Out] b*x*x^n*e^(p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x)) + a*x*e^(
p*log(b*x^n + a) + m*log(x) - log(b*x^n + a) - log(x))
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \left(am+\left(np+m\right)b\,x^{n}\right)x^{m-1}\left(b\,x^{n}+a\right)^{p-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m-1)*(b*x^n+a)^(p-1)*(a*m+b*(n*p+m)*x^n),x)
```

```
[Out] int(x^(m-1)*(b*x^n+a)^(p-1)*(a*m+b*(n*p+m)*x^n),x)
```

maxima [A] time = 0.84, size = 16, normalized size = 1.23

$$e^{\left(p\log\left(bx^{n}+a\right)+m\log\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*(a+b*x^n)^(-1+p)*(a*m+b*(n*p+m)*x^n),x, algorithm="maxim
a")
```

```
[Out] e^(p*log(b*x^n + a) + m*log(x))
```

mupad [B] time = 4.81, size = 25, normalized size = 1.92

$$\left(a\,x^{m}+b\,x^{m+n}\right)\,\left(a+b\,x^{n}\right)^{p-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*(a*m + b*x^n*(m + n*p))*(a + b*x^n)^(p - 1),x)
```

```
[Out] (a*x^m + b*x^(m + n))*(a + b*x^n)^(p - 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*(a+b*x**n)**(-1+p)*(a*m+b*(n*p+m)*x**n),x)
```

```
[Out] Timed out
```

3.1056 $\int \frac{b+2cx}{x(b+cx)} dx$

Optimal. Leaf size=8

$$\log(x(b+cx))$$

```
[Out] ln(x*(c*x+b))
```

Rubi [A] time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {72}

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x*(b + c*x)),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \frac{b+2cx}{x(b+cx)} dx &= \int \left(\frac{1}{x}+\frac{c}{b+cx}\right) dx \\ &= \log(x)+\log(b+cx) \end{aligned}$$

Mathematica [A] time = 0.02, size = 9, normalized size = 1.12

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x*(b + c*x)),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

fricas [A] time = 0.45, size = 10, normalized size = 1.25

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

giac [A] time = 0.15, size = 11, normalized size = 1.38

$$\log\left(|cx+b|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="giac")
```

```
[Out] log(abs(c*x + b)) + log(abs(x))
```

maple [**A**] time = 0.04, size = 9, normalized size = 1.12

$$\ln\left(\left(cx+b\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x/(c*x+b),x)
[Out] ln(x*(c*x+b))
```

maxima [**A**] time = 0.48, size = 9, normalized size = 1.12

$$\log\left(cx+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x, algorithm="maxima")
[Out] log(c*x + b) + log(x)
```

mupad [**B**] time = 4.66, size = 8, normalized size = 1.00

$$\ln\left(x\ \left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x*(b + c*x)),x)
[Out] log(x*(b + c*x))
```

sympy [**A**] time = 0.12, size = 8, normalized size = 1.00

$$\log\left(bx+cx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x/(c*x+b),x)
[Out] log(b*x + c*x**2)
```

3.1057 $\int \frac{b+2cx^2}{x(b+cx^2)} dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

[Out] ln(x)+1/2*ln(c*x^2+b)

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x^2)/(x*(b + c*x^2)),x]

[Out] Log[x] + Log[b + c*x^2]/2

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx,x,x^2\right)\\ &= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x^2)/(x*(b + c*x^2)),x]

[Out] Log[x] + Log[b + c*x^2]/2

fricas [A] time = 0.67, size = 13, normalized size = 0.87

$$\frac{1}{2}\log\left(cx^2+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

giac [A] time = 0.15, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|cx^2+b\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*log(x^2) + 1/2*log(abs(c*x^2 + b))
```

maple [A] time = 0.05, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x/(c*x^2+b),x)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

maxima [A] time = 0.49, size = 17, normalized size = 1.13

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x/(c*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

mupad [B] time = 0.06, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x*(b + c*x^2)),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

sympy [A] time = 0.18, size = 12, normalized size = 0.80

$$\log{\left(x \right)}+\frac{\log{\left(\frac{b}{c}+x^{2} \right)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x/(c*x**2+b),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

3.1058 $\int \frac{b+2cx^3}{x(b+cx^3)}\,dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x*(b + c*x^3)),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

fricas [A] time = 0.48, size = 13, normalized size = 0.87

$$\frac{1}{3}\log\left(cx^3+b\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

giac [A] time = 0.16, size = 15, normalized size = 1.00

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

maple [A] time = 0.05, size = 14, normalized size = 0.93

$$\ln(x)+\frac{\ln\left(c\,x^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x/(c*x^3+b),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

maxima [A] time = 0.55, size = 17, normalized size = 1.13

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x/(c*x^3+b),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

mupad [B] time = 4.63, size = 13, normalized size = 0.87

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x*(b + c*x^3)),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

sympy [A] time = 0.19, size = 12, normalized size = 0.80

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x/(c*x**3+b),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

3.1059 $\int \frac{b+2cx^n}{x(b+cx^n)} dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^n}{x(b+cx^n)} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)} dx, x, x^n\right)}{n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right) dx, x, x^n\right)}{n}\\ &= \log(x)+\frac{\log(b+cx^n)}{n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^n)/(x*(b + c*x^n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

fricas [A] time = 0.65, size = 17, normalized size = 1.13

$$\frac{n\log(x)+\log(cx^n+b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="fricas")
[Out] (n*log(x) + log(c*x^n + b))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\,cx^n + b}{(cx^n + b)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="giac")
[Out] integrate((2*c*x^n + b)/((c*x^n + b)*x), x)
```

maple [A] time = 0.05, size = 17, normalized size = 1.13

$$\frac{\ln\left((c\,x^n + b)\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b+2*c*x^n)/x/(b+c*x^n),x)
[Out] 1/n*ln(x^n*(b+c*x^n))
```

maxima [B] time = 0.49, size = 47, normalized size = 3.13

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x^n)/x/(b+c*x^n),x, algorithm="maxima")
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

mupad [B] time = 4.72, size = 15, normalized size = 1.00

$$\ln(x) + \frac{\ln\left(b + c\,x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x*(b + c*x^n)),x)
[Out] log(x) + log(b + c*x^n)/n
```

sympy [A] time = 0.62, size = 29, normalized size = 1.93

$$\begin{cases} \log{\left(x \right)} & \text{for}\: c = 0 \wedge n = 0 \\ \frac{\left(b + 2 c\right) \log{\left(x \right)}}{b + c} & \text{for}\: n = 0 \\ \log{\left(x \right)} & \text{for}\: c = 0 \\ \log{\left(x \right)} + \frac{\log{\left(\frac{b}{c} + x^{n} \right)}}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b+2*c*x**n)/x/(b+c*x**n),x)
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (log(x), Eq(c, 0)), (log(x) + log(b/c + x**n)/n, True))
```

3.1060 $\int \frac{b+2cx}{x^8(b+cx)^8} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{7x^7(b+cx)^7}$$

```
[Out] -1/7/x^7/(c*x+b)^7
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{b+2cx}{x^8(b+cx)^8} \, dx = -\frac{1}{7x^7(b+cx)^7}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(x^8*(b + c*x)^8),x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

fricas [B] time = 0.59, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^7x^{14} + 7\,bc^6x^{13} + 21\,b^2c^5x^{12} + 35\,b^3c^4x^{11} + 35\,b^4c^3x^{10} + 21\,b^5c^2x^9 + 7\,b^6cx^8 + b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

giac [A] time = 0.17, size = 13, normalized size = 0.93

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

maple [B] time = 0.06, size = 177, normalized size = 12.64

$$\frac{c^7}{7\,(cx+b)^7\,b^7}+\frac{c^7}{(cx+b)^6\,b^8}+\frac{4c^7}{(cx+b)^5\,b^9}+\frac{12c^7}{(cx+b)^4\,b^{10}}+\frac{30c^7}{(cx+b)^3\,b^{11}}+\frac{66c^7}{(cx+b)^2\,b^{12}}+\frac{132c^7}{(cx+b)\,b^{13}}-\frac{132c^6}{b^{13}x}+\frac{66c^5}{b^{12}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/x^8/(c*x+b)^8,x)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7
/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b
^9*c^2/x^5+1/b^8*c/x^6
```

maxima [B] time = 0.61, size = 81, normalized size = 5.79

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x^8/(c*x+b)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

mupad [B] time = 7.09, size = 12, normalized size = 0.86

$$-\frac{1}{7\,x^7\,(b+c\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(x^8*(b + c*x)^8),x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

sympy [B] time = 0.89, size = 87, normalized size = 6.21

$$-\frac{1}{7b^7x^7+49b^6cx^8+147b^5c^2x^9+245b^4c^3x^{10}+245b^3c^4x^{11}+147b^2c^5x^{12}+49bc^6x^{13}+7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/x**8/(c*x+b)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

3.1061 $\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/14/x^14/(c*x^2+b)^7
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

fricas [B] time = 0.44, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

giac [A] time = 0.15, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

maple [B] time = 0.06, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{2}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{2}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{2}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{2}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{2}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{2}+b\right)^{2}c}-\frac{132}{\left(c\,x^{2}+b\right)c}\right)c^{8}}{2b^{13}}-\frac{66c^{6}}{b^{13}x^{2}}+\frac{33c^{5}}{b^{12}x^{4}}-\frac{15c^{4}}{b^{11}x^{6}}+\frac{6c^{3}}{b^{10}x^{8}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^2+b)/x^15/(c*x^2+b)^8,x)
```

```
[Out] -1/2*c^8/b^13*(-4*b^4/c/(c*x^2+b)^5-66*b/c/(c*x^2+b)^2-1/7*b^6/c/(c*x^2+b)^
7-b^5/c/(c*x^2+b)^6-12*b^3/c/(c*x^2+b)^4-132/c/(c*x^2+b)-30*b^2/c/(c*x^2+b)
^3)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

maxima [B] time = 0.64, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^2+b)/x^15/(c*x^2+b)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

mupad [B] time = 2.32, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,\left(c\,x^{2}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^2)/(x^15*(b + c*x^2)^8),x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

sympy [B] time = 1.36, size = 87, normalized size = 5.44

$$-\frac{1}{14b^7x^{14} + 98b^6cx^{16} + 294b^5c^2x^{18} + 490b^4c^3x^{20} + 490b^3c^4x^{22} + 294b^2c^5x^{24} + 98bc^6x^{26} + 14c^7x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**2+b)/x**15/(c*x**2+b)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

3.1062 $\int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right) \\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7} \end{aligned}$$

Mathematica [A] time = 0.05, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

fricas [B] time = 0.54, size = 81, normalized size = 5.06

$$-\frac{1}{21\,\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

giac [A] time = 0.17, size = 15, normalized size = 0.94

$$-\frac{1}{21\,\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

maple [B] time = 0.06, size = 197, normalized size = 12.31

$$-\frac{\left(-\frac{b^{6}}{7\left(c\,x^{3}+b\right)^{7}c}-\frac{b^{5}}{\left(c\,x^{3}+b\right)^{6}c}-\frac{4b^{4}}{\left(c\,x^{3}+b\right)^{5}c}-\frac{12b^{3}}{\left(c\,x^{3}+b\right)^{4}c}-\frac{30b^{2}}{\left(c\,x^{3}+b\right)^{3}c}-\frac{66b}{\left(c\,x^{3}+b\right)^{2}c}-\frac{132}{\left(c\,x^{3}+b\right)c}\right)c^{8}}{3b^{13}}-\frac{44c^{6}}{b^{13}x^{3}}+\frac{22c^{5}}{b^{12}x^{6}}-\frac{10c^{4}}{b^{11}x^{9}}+\frac{4c}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x^3+b)/x^22/(c*x^3+b)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-4*b^4/c/(c*x^3+b)^5-66*b/c/(c*x^3+b)^2-1/7*b^6/c/(c*x^3+b)^
7-b^5/c/(c*x^3+b)^6-12*b^3/c/(c*x^3+b)^4-132/c/(c*x^3+b)-30*b^2/c/(c*x^3+b)
^3)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

maxima [B] time = 0.61, size = 81, normalized size = 5.06

$$-\frac{1}{21\,\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x^3+b)/x^22/(c*x^3+b)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

mupad [B] time = 9.98, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,\left(c\,x^{3}+b\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^3)/(x^22*(b + c*x^3)^8),x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

<u>**sympy**</u> [**B**] time = 1.91, size = 87, normalized size = 5.44

$$-\frac{1}{21b^7x^{21} + 147b^6cx^{24} + 441b^5c^2x^{27} + 735b^4c^3x^{30} + 735b^3c^4x^{33} + 441b^2c^5x^{36} + 147bc^6x^{39} + 21c^7x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x**3+b)/x**22/(c*x**3+b)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

3.1063 $\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {446, 74}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]

[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8} dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8} dx, x, x^n\right)}{n}\\ &= -\frac{x^{-7n}}{7n(b+cx^n)^7}\end{aligned}$$

Mathematica [A] time = 0.25, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 - 7*n)*(b + 2*c*x^n))/(b + c*x^n)^8,x]

[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)

fricas [B] time = 0.56, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^7nx^{14n}+7bc^6nx^{13n}+21b^2c^5nx^{12n}+35b^3c^4nx^{11n}+35b^4c^3nx^{10n}+21b^5c^2nx^{9n}+7b^6cnx^{8n}+b^7nx^{7n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\,cx^n + b)x^{-7\,n-1}}{(cx^n + b)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^n + b)*x^(-7*n - 1)/(c*x^n + b)^8, x)
```

maple [B] time = 0.10, size = 203, normalized size = 9.67

$$-\frac{x^{-7n}}{7b^7n}+\frac{c\,x^{-6n}}{b^8n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{66c^5x^{-2n}}{b^{12}n}+\frac{(9009b^5c\,x^n+20020b^4c^2x^{2n}+24024b^3c^3x^{3n}+16380}{7\,(c\,x^n+b)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-7*n)*(b+2*c*x^n)/(c*x^n+b)^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(c*x^n+b)^7
```

maxima [B] time = 0.79, size = 612, normalized size = 29.14

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^2c^{11}x^{11\,n}+9579570\,b^3c^{10}x^{10\,n}+8270262\,b^4c^9x^{9\,n}+401}{b^{14}c^7nx^{14\,n}+7\,b^{15}c^6nx^{13\,n}+21\,b^{16}c^5nx^{12\,n}+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-7*n)*(b+2*c*x^n)/(b+c*x^n)^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

mupad [B] time = 4.99, size = 105, normalized size = 5.00

$$-\frac{1}{7\,x^{7\,n}\,\left(b^7\,n+c^7\,n\,x^{7\,n}+7\,b^6\,c\,n\,x^n+7\,b\,c^6\,n\,x^{6\,n}+21\,b^5\,c^2\,n\,x^{2\,n}+35\,b^4\,c^3\,n\,x^{3\,n}+35\,b^3\,c^4\,n\,x^{4\,n}+21\,b^2\,c^5\,n\,x^5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x^n)/(x^(7*n + 1)*(b + c*x^n)^8),x)
```

```
[Out] -1/(7*x^(7*n)*(b^7*n + c^7*n*x^(7*n) + 7*b^6*c*n*x^n + 7*b*c^6*n*x^(6*n) +
21*b^5*c^2*n*x^(2*n) + 35*b^4*c^3*n*x^(3*n) + 35*b^3*c^4*n*x^(4*n) + 21*b^2
*c^5*n*x^(5*n)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-7*n)*(b+2*c*x**n)/(b+c*x**n)**8,x)
```

```
[Out] Timed out
```

3.1064 $\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/24*(x^16+1)^(3/2)+1/8*arctanh(1/2*(x^16+1)^(1/2)*2^(1/2))*2^(1/2)-1/8*(x
^16+1)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {446, 80, 50, 63, 206}

$$-\frac{1}{24}\left(x^{16}+1\right)^{3/2}-\frac{\sqrt{x^{16}+1}}{8}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] -Sqrt[1 + x^16]/8 - (1 + x^16)^(3/2)/24 + ArcTanh[Sqrt[1 + x^16]/Sqrt[2]]/(
4*Sqrt[2])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{31}\sqrt{1+x^{16}}}{1-x^{16}}\,dx &= \frac{1}{16}\text{Subst}\left(\int \frac{x\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right)\\ &= -\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{16}\text{Subst}\left(\int \frac{\sqrt{1+x}}{1-x}\,dx, x, x^{16}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{8}\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx, x, x^{16}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{1}{4}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x^{16}}\right)\\ &= -\frac{1}{8}\sqrt{1+x^{16}}-\frac{1}{24}\left(1+x^{16}\right)^{3/2}+\frac{\tanh^{-1}\left(\frac{\sqrt{1+x^{16}}}{\sqrt{2}}\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.85

$$\frac{1}{24}\left(3\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{x^{16}+1}}{\sqrt{2}}\right)-\sqrt{x^{16}+1}\left(x^{16}+4\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^31*Sqrt[1 + x^16])/(1 - x^16),x]
```

```
[Out] (-(Sqrt[1 + x^16]*(4 + x^16)) + 3*Sqrt[2]*ArcTanh[Sqrt[1 + x^16]/Sqrt[2]])/
24
```

fricas [A] time = 0.74, size = 46, normalized size = 0.88

$$-\frac{1}{24}\left(x^{16}+4\right)\sqrt{x^{16}+1}+\frac{1}{16}\sqrt{2}\log\left(\frac{x^{16}+2\sqrt{2}\sqrt{x^{16}+1}+3}{x^{16}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="fricas")
```

```
[Out] -1/24*(x^16 + 4)*sqrt(x^16 + 1) + 1/16*sqrt(2)*log((x^16 + 2*sqrt(2)*sqrt(x
^16 + 1) + 3)/(x^16 - 1))
```

giac [A] time = 0.15, size = 56, normalized size = 1.08

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{x^{16}+1}\right|}{2\left(\sqrt{2}+\sqrt{x^{16}+1}\right)}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="giac")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(x^16
+ 1))/(sqrt(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

maple [C] time = 1.15, size = 86, normalized size = 1.65

$$\frac{\text{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{x^{16}\,\text{RootOf}\left(\_Z^2-2\right)+3\,\text{RootOf}\left(\_Z^2-2\right)+4\sqrt{x^{16}+1}}{(x-1)(x+1)\left(x^2+1\right)\left(x^4+1\right)\left(x^8+1\right)}\right)}{16}-\frac{\left(x^{16}+4\right)\sqrt{x^{16}+1}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^31*(x^16+1)^(1/2)/(-x^16+1),x)
```

```
[Out] -1/24*(x^16+4)*(x^16+1)^(1/2)+1/16*RootOf(_Z^2-2)*ln(-(RootOf(_Z^2-2)*x^16+
4*(x^16+1)^(1/2)+3*RootOf(_Z^2-2))/(x-1)/(x+1)/(x^2+1)/(x^4+1)/(x^8+1))
```

maxima [A] time = 1.25, size = 53, normalized size = 1.02

$$-\frac{1}{24}\left(x^{16}+1\right)^{\frac{3}{2}}-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x^{16}+1}}{\sqrt{2}+\sqrt{x^{16}+1}}\right)-\frac{1}{8}\sqrt{x^{16}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^31*(x^16+1)^(1/2)/(-x^16+1),x, algorithm="maxima")
```

```
[Out] -1/24*(x^16 + 1)^(3/2) - 1/16*sqrt(2)*log(-(sqrt(2) - sqrt(x^16 + 1))/(sqrt
(2) + sqrt(x^16 + 1))) - 1/8*sqrt(x^16 + 1)
```

mupad [B] time = 4.82, size = 37, normalized size = 0.71

$$\frac{\sqrt{2}\,\text{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^{16}+1}}{2}\right)}{8}-\frac{\sqrt{x^{16}+1}}{8}-\frac{\left(x^{16}+1\right)^{3/2}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^31*(x^16 + 1)^(1/2))/(x^16 - 1),x)
```

```
[Out] (2^(1/2)*atanh((2^(1/2)*(x^16 + 1)^(1/2))/2))/8 - (x^16 + 1)^(1/2)/8 - (x^1
6 + 1)^(3/2)/24
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**31*(x**16+1)**(1/2)/(-x**16+1),x)
```

```
[Out] Timed out
```

3.1065 $\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\, x}\, dx$

Optimal. Leaf size=93

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(c^(1/2)*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))*c^(1/2)/a^(1/2)-2*ar
ctanh(d^(1/2)*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))*d^(1/2)/b^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {446, 105, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}}-\frac{2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a
] - (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])])/Sq
rt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx &= -\mathrm{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left(c\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\right) - d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)\\
&= -\left((2c)\,\mathrm{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\right) - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{b}\\
&= \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{b}\\
&= \frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{b}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 142, normalized size = 1.53

$$\frac{2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+\frac{b}{x}}}{\sqrt{a}\,\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}} - \frac{2\sqrt{d}\,x\sqrt{c+\frac{d}{x}}\,\sqrt{bc-ad}\,\sqrt{\frac{b(cx+d)}{x(bc-ad)}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{bcx+bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d/x]/(Sqrt[a + b/x]*x),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d/x]*x*Sqrt[(b*(d + c*x))/((b*c - a*d)
*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*d + b*c*x) + (2*S
qrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/Sqrt[a]
```

fricas [B] time = 1.01, size = 757, normalized size = 8.14

$$\left[\frac{1}{2}\sqrt{\frac{c}{a}}\,\log\left(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 - 4\left(2\,a^2cx^2 + \left(abc + a^2d\right)x\right)\sqrt{\frac{c}{a}}\,\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}} - 8\left(abc^2 + a^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^
2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)
- 8*(a*b*c^2 + a^2*c*d)*x) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt(
(a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), -sqrt(-c/a)
*arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
 + a*d)) + 1/2*sqrt(d/b)*log(-(8*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^2 - 4*(2*b^2*d*x + (b^2*c + a*b*d)*x^2)*sqrt(d/b)*sqrt((a*x + b)/x)*sqrt(
(c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2), sqrt(-d/b)*arctan(1/2*(2*b*d*
x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*
x^2 + b*d^2 + (b*c*d + a*d^2)*x)) + 1/2*sqrt(c/a)*log(-8*a^2*c^2*x^2 - b^2*
c^2 - 6*a*b*c*d - a^2*d^2 - 4*(2*a^2*c*x^2 + (a*b*c + a^2*d)*x)*sqrt(c/a)*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x), -sqrt(-c/a)*
arctan(2*a*x*sqrt(-c/a)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c
+ a*d)) + sqrt(-d/b)*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-d/b)*sqrt
((a*x + b)/x)*sqrt((c*x + d)/x)/(a*c*d*x^2 + b*d^2 + (b*c*d + a*d^2)*x))]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

maple [B] time = 0.10, size = 143, normalized size = 1.54

$$-\frac{\sqrt{\frac{ax+b}{x}}\,\sqrt{\frac{cx+d}{x}}\,\left(-\sqrt{bd}\,c\ln\left(\frac{2acx+ad+bc+2\sqrt{(cx+d)(ax+b)}\,\sqrt{ac}}{2\sqrt{ac}}\right)+\sqrt{ac}\,d\ln\left(\frac{adx+bcx+2bd+2\sqrt{bd}\,\sqrt{(cx+d)(ax+b)}}{x}\right)\right)x}{\sqrt{(cx+d)(ax+b)}\,\sqrt{bd}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x)
```

```
[Out] -((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln((a*d*x+b*c*x+2*(b*d)^(1/2)*((c*x
+d)*(a*x+b))^(1/2)+2*b*d)/x)*d*(a*c)^(1/2)-ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+
b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*c*(b*d)^(1/2))/((c*x+d)*(a*x+b)
)^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\frac{d}{x}}}{\sqrt{a+\frac{b}{x}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)/x/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c + d/x)/(sqrt(a + b/x)*x), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)),x)
[Out] int((c + d/x)^(1/2)/(x*(a + b/x)^(1/2)), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+\frac{d}{x}}}{x\sqrt{a+\frac{b}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)/x/(a+b/x)**(1/2),x)
[Out] Integral(sqrt(c + d/x)/(x*sqrt(a + b/x)), x)
```

3.1066 $\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=252

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{5(bc-ad)^2(ad+7bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n}+\frac{5(bc-ad)(ad+7bc)(a}{96bd^3}$$

```
[Out] 5/64*(-a*d+b*c)^3*(a*d+7*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*
x^n)^(1/2))/b^(3/2)/d^(9/2)/n+5/96*(-a*d+b*c)*(a*d+7*b*c)*(a+b*x^n)^(3/2)*(
c+d*x^n)^(1/2)/b/d^3/n-1/24*(a*d+7*b*c)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d
^2/n+1/4*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n-5/64*(-a*d+b*c)^2*(a*d+7*b*c
)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^4/n
```

Rubi [A] time = 0.23, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{5(bc-ad)^3(ad+7bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{64b^{3/2}d^{9/2}n}-\frac{(ad+7bc)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n}+\frac{5(bc-ad)(ad+7bc)(a+bx^n)^{3/2}}{96bd^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-5*(b*c - a*d)^2*(7*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(64*b*d^4*
n) + (5*(b*c - a*d)*(7*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(96*b*
d^3*n) - ((7*b*c + a*d)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(24*b*d^2*n) + (
(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(4*b*d*n) + (5*(b*c - a*d)^3*(7*b*c + a*
d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(3/2
)*d^(9/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} - \frac{(7bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{8bdn}$$

$$= -\frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a+bx^n)^{7/2}\sqrt{c+dx^n}}{4bdn} + \frac{(5(bc-ad)(7bc+ad))}{4\ldots}$$

$$= \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96bd^3n} - \frac{(7bc+ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24bd^2n} + \frac{(a\ldots}{\ldots}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

$$= -\frac{5(bc-ad)^2(7bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64bd^4n} + \frac{5(bc-ad)(7bc+ad)(a+bx^n)^{3/2}\sqrt{c+\ldots}}{96bd^3n}$$

Mathematica [A] time = 1.09, size = 223, normalized size = 0.88

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\left(15a^3d^3+a^2bd^2(118dx^n-191c)+ab^2d\left(265c^2-172cdx^n+136d^2x^{2n}\right)+b^3\left(-105c^3+7\ldots\right.\right.}{192b^2d^{9/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(15*a^3*d^3 + a^2*b*d^2*(-191*c + 11
8*d*x^n) + a*b^2*d*(265*c^2 - 172*c*d*x^n + 136*d^2*x^(2*n)) + b^3*(-105*c^
```

```
3 + 70*c^2*d*x^n - 56*c*d^2*x^(2*n) + 48*d^3*x^(3*n))) + 15*(b*c - a*d)^(7/
2)*(7*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a
+ b*x^n])/Sqrt[b*c - a*d]])/(192*b^2*d^(9/2)*n*Sqrt[c + d*x^n])
```

fricas [A] time = 0.86, size = 607, normalized size = 2.41

$$\left[-\frac{15\left(7\,b^4c^4 - 20\,ab^3c^3d + 18\,a^2b^2c^2d^2 - 4\,a^3bcd^3 - a^4d^4\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n} + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\,\sqrt{\right.\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/768*(15*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 - a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^
2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*
x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3
*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3
- 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d
^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^5*n), -1/384*(15*(7*b^4*c^
4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4)*sqrt(-b*
d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
 2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 265*a*b^3*c^2*d^2 - 191*a^2*b^2*c*
d^3 + 15*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 17*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2
*d^2 - 86*a*b^3*c*d^3 + 59*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c
))/(b^2*d^5*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{5}{2}}\,x^{2n-1}}{\sqrt{d\,x^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2\,n-1}\,{\left(a+b\,x^n\right)}^{5/2}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(2*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1067 $\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=199

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n}-\frac{(ad+5bc)(a+bx^n)^{3/2}\sqrt{c}}{12bd^2n}$$

```
[Out] -1/8*(-a*d+b*c)^2*(a*d+5*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*
x^n)^(1/2))/b^(3/2)/d^(7/2)/n-1/12*(a*d+5*b*c)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1
/2)/b/d^2/n+1/3*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b/d/n+1/8*(-a*d+b*c)*(a*d+5
*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^3/n
```

Rubi [A] time = 0.16, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 80, 50, 63, 217, 206}

$$-\frac{(bc-ad)^2(ad+5bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{8b^{3/2}d^{7/2}n}-\frac{(ad+5bc)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n}+\frac{(bc-ad)(ad+5bc)\sqrt{a+bx^n}\sqrt{c}}{8bd^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)*(5*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b*d^3*n) - (
(5*b*c + a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b*d^2*n) + ((a + b*x^n
)^(5/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)^2*(5*b*c + a*d)*ArcTanh[(
Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(8*b^(3/2)*d^(7/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} - \frac{(5bc+ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{6bdn}$$

$$= -\frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx^n)^{5/2}\sqrt{c+dx^n}}{3bdn} + \frac{((bc-ad)(5bc+ad))\,\text{S}}{8b}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

$$= \frac{(bc-ad)(5bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{8bd^3n} - \frac{(5bc+ad)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{12bd^2n} + \frac{(a+bx}{}$$

Mathematica [A] time = 0.67, size = 178, normalized size = 0.89

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\left(3a^2d^2+2abd\,(7dx^n-11c)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{5/2}(ad+5bc)\sqrt{\frac{b(}{}}}{24b^2d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(3*a^2*d^2 + 2*a*b*d*(-11*c + 7*d*x^
n) + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(5/2)*(5*b*
c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n]
)/Sqrt[b*c - a*d]])/(24*b^2*d^(7/2)*n*Sqrt[c + d*x^n])
```

fricas [A] time = 0.75, size = 469, normalized size = 2.36

$$\left[\frac{3\left(5\,b^3c^3-9\,ab^2c^2d+3\,a^2bcd^2+a^3d^3\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^
n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a
*b*d^2)*x^n) + 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2
*b*d^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)
)/(b^2*d^4*n), 1/48*(3*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3
)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqr
t(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^
2)*x^n)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 22*a*b^2*c*d^2 + 3*a^2*b*d
^3 - 2*(5*b^3*c*d^2 - 7*a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^2*d^4*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n + a)^{\frac{3}{2}}\, x^{2n-1}}{\sqrt{d\,x^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{2\,n-1}}{\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}\,(a + b\,x^n)^{3/2}}{\sqrt{c + d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

3.1068 $\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=146

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

```
[Out] 1/4*(-a*d+b*c)*(a*d+3*b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n
)^(1/2))/b^(3/2)/d^(5/2)/n+1/2*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d/n-1/4*(a
*d+3*b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d^2/n
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 80, 50, 63, 217, 206}

$$\frac{(bc-ad)(ad+3bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{3/2}d^{5/2}n}-\frac{(ad+3bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n}+\frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] -((3*b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b*d^2*n) + ((a + b*x^n)
^(3/2)*Sqrt[c + d*x^n])/(2*b*d*n) + ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqr
t[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(3/2)*d^(5/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{x\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{n}$$

$$= \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} - \frac{(3bc+ad)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{4bdn}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\operatorname{Subst}}{8bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\operatorname{Subst}}{4}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{((bc-ad)(3bc+ad))\operatorname{Subst}}{4b^2d^2}$$

$$= -\frac{(3bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4bd^2n} + \frac{(a+bx^n)^{3/2}\sqrt{c+dx^n}}{2bdn} + \frac{(bc-ad)(3bc+ad)\tanh^{-1}}{4b^{3/2}d^{5/2}n}$$

Mathematica [A] time = 0.51, size = 141, normalized size = 0.97

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(ad-3bc+2bdx^n)+(ad+3bc)(bc-ad)^{3/2}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{4b^2d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 2*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c + a*d + 2*b*d*x^n) + (b*c -
a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]
*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^2*d^(5/2)*n*Sqrt[c + d*x^n])
```

fricas [A] time = 0.78, size = 359, normalized size = 2.46

$$\left[-\frac{\left(3\,b^2c^2-2\,abcd-a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2-4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n+a}\right.}{16\,b^2d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) +
 b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(
b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(2*b
^2*d^2*x^n - 3*b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3
*n), -1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*sqrt(
-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^
2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n - 3*
b^2*c*d + a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^2*d^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}\, x^{2\, n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\, x^{n}+a}\; x^{2 n -1}}{\sqrt{d\, x^{n}+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + a}\, x^{2\, n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(2*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\, n-1}\, \sqrt{a+b\, x^n}}{\sqrt{c+d\, x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
```

```
[Out] int((x^(2*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}\sqrt{a + bx^{n}}}{\sqrt{c + dx^{n}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(2*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

3.1069 $\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=89

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

```
[Out] -(a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)
/d^(3/2)/n+(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 80, 63, 217, 206}

$$\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn}-\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(b*d*n) - ((b*c + a*d)*ArcTanh[(Sqrt[d]*S
qrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*d^(3/2)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^n\right)}{n}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,x^n\right)}{2bdn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx^n}\right)}{b^2dn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2dn}$$
$$= \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{bdn} - \frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}d^{3/2}n}$$

Mathematica [A] time = 0.32, size = 123, normalized size = 1.38

$$\frac{b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n) - \sqrt{bc-ad}\,(ad+bc)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{b^2d^{3/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n) - Sqrt[b*c - a*d]*(b*c + a*d)*Sqrt[(
b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*
d]])/(b^2*d^(3/2)*n*Sqrt[c + d*x^n])
```

fricas [A] time = 0.56, size = 281, normalized size = 3.16

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}\,bd + (bc+ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2n} + b^2c^2 + 6\,abcd + a^2d^2 - 4\left(2\sqrt{bd}\,bdx^n + (bc+ad)\sqrt{\right.\right.}{4\,b^2d^2n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a*d)*sqrt(b*d)*log(8*b
^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (
b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^
2)*x^n))/(b^2*d^2*n), 1/2*(2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*b*d + (b*c + a
*d)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*s
qrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*
d^2)*x^n)))/(b^2*d^2*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{b\,x^{n}+a}\,\sqrt{d\,x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(2*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{\sqrt{a+bx^{n}}\,\sqrt{c+dx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(2*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

3.1070 $\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=91

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(3/2)/n/d^(1/2
)+2*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {446, 78, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}+\frac{2a\sqrt{c+dx^n}}{bn(bc-ad)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(b*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (2*ArcTanh[(Sqrt[
d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(3/2)*Sqrt[d]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+2n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$
$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{bn}$$
$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx^n}\right)}{b^2n}$$
$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\right)}{b^2n}$$
$$= \frac{2a\sqrt{c+dx^n}}{b(bc-ad)n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{3/2}\sqrt{d}\,n}$$

Mathematica [A] time = 0.90, size = 122, normalized size = 1.34

$$\frac{2\left(\frac{ab(c+dx^n)}{(bc-ad)\sqrt{a+bx^n}} + \frac{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{d}}\right)}{b^2n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*((a*b*(c + d*x^n))/((b*c - a*d)*Sqrt[a + b*x^n]) + (Sqrt[b*c - a*d]*Sqrt
[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c -
a*d]])/Sqrt[d]))/(b^2*n*Sqrt[c + d*x^n])
```

fricas [B] time = 0.95, size = 408, normalized size = 4.48

$$\left[\frac{4\sqrt{bx^n+a}\sqrt{dx^n+c}\,abd + \left(\left(b^2c-abd\right)\sqrt{bd}\,x^n + \left(abc-a^2d\right)\sqrt{bd}\right)\log\left(8\,b^2d^2x^{2\,n} + b^2c^2 + 6\,abcd + a^2d^2 + 4\,(2\right.}{2\left(\left(b^4cd-ab^3d^2\right)nx^n + \left(ab^3cd-a^2b^2d^2\right)n\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(4*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d + ((b^2*c - a*b*d)*sqrt(b*d)*
x^n + (a*b*c - a^2*d)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*
d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^4*c*d - a*b^3*d^2)*n*x
^n + (a*b^3*c*d - a^2*b^2*d^2)*n), (2*sqrt(b*x^n + a)*sqrt(d*x^n + c)*a*b*d
 - ((b^2*c - a*b*d)*sqrt(-b*d)*x^n + (a*b*c - a^2*d)*sqrt(-b*d))*arctan(1/2
*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n
```

```
 + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^4*c*d - a
*b^3*d^2)*n*x^n + (a*b^3*c*d - a^2*b^2*d^2)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b\,x^n+a)^{\frac{3}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(2*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(bx^n+a)^{\frac{3}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2n-1}}{(a+b\,x^n)^{3/2}\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(2*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1071 $\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=95

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

```
[Out] 2/3*a*(c+d*x^n)^(1/2)/b/(-a*d+b*c)/n/(a+b*x^n)^(3/2)-2/3*(-a*d+3*b*c)*(c+d*
x^n)^(1/2)/b/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {446, 78, 37}

$$\frac{2a\sqrt{c+dx^n}}{3bn(bc-ad)\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3bn(bc-ad)^2\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*a*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)*n*(a + b*x^n)^(3/2)) - (2*(3*b*c - a
*d)*Sqrt[c + d*x^n])/(3*b*(b*c - a*d)^2*n*Sqrt[a + b*x^n])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+2n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}+\frac{(3bc-ad)\,\text{Subst}\left(\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx,x,x^n\right)}{3b(bc-ad)n}\\
&= \frac{2a\sqrt{c+dx^n}}{3b(bc-ad)n\,(a+bx^n)^{3/2}}-\frac{2(3bc-ad)\sqrt{c+dx^n}}{3b(bc-ad)^2n\sqrt{a+bx^n}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.60

$$\frac{2\sqrt{c+dx^n}\,(-2ac+adx^n-3bcx^n)}{3n(bc-ad)^2\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(-2*a*c - 3*b*c*x^n + a*d*x^n))/(3*(b*c - a*d)^2*n*(a + b*x^n)^(3/2))
```

fricas [A] time = 1.07, size = 135, normalized size = 1.42

$$-\frac{2\,(2\,ac+(3\,bc-ad)x^n)\sqrt{bx^n+a}\,\sqrt{dx^n+c}}{3\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)nx^{2\,n}+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)nx^n+\left(a^2b^2c^2-2\,a^3bcd+a^4d^2\right)n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*a*c + (3*b*c - a*d)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*n*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*n*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{x^{2n-1}}{(b\,x^n+a)^{\frac{5}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(2*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+2*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(2*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2\,n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(2*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+2*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Timed out
```

3.1072 $\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=358

$$\frac{(bc-ad)^2\left(3a^2d^2+14abcd+63b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{128b^2d^5n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\,\sqrt{}}{192b^2d^4n}$$

```
[Out] -1/128*(-a*d+b*c)^3*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*arctanh(d^(1/2)*(a+b*
x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(11/2)/n-1/192*(-a*d+b*c)*(3*
a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^4/n+1/
240*(3*a^2*d^2+14*a*b*c*d+63*b^2*c^2)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d
^3/n-3/40*(a*d+3*b*c)*(a+b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/5*x^n*(a+
b*x^n)^(7/2)*(c+d*x^n)^(1/2)/b/d/n+1/128*(-a*d+b*c)^2*(3*a^2*d^2+14*a*b*c*d
+63*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^5/n
```

Rubi [A] time = 0.40, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+14abcd+63b^2c^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] ((b*c - a*d)^2*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
 + d*x^n])/(128*b^2*d^5*n) - ((b*c - a*d)*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*
d^2)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(192*b^2*d^4*n) + ((63*b^2*c^2 + 14
*a*b*c*d + 3*a^2*d^2)*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n])/(240*b^2*d^3*n) -
(3*(3*b*c + a*d)*(a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(40*b^2*d^2*n) + (x^n*(
a + b*x^n)^(7/2)*Sqrt[c + d*x^n])/(5*b*d*n) - ((b*c - a*d)^3*(63*b^2*c^2 +
14*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c +
 d*x^n])])/(128*b^(5/2)*d^(11/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
```

```
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{5/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx,x,x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(-ac-\frac{3}{2}(3bc+ad)x\right)}{\sqrt{c+dx}}\,dx,x,x^n\right)}{5bdn}$$

$$= -\frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+dx^n}}{40b^2d^2n} + \frac{x^n(a+bx^n)^{7/2}\sqrt{c+dx^n}}{5bdn} + \frac{\left(63b^2c^2+14ab\right.}{}$$

$$= \frac{\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{240b^2d^3n} - \frac{3(3bc+ad)(a+bx^n)^{7/2}\sqrt{c+}}{40b^2d^2n}$$

$$= -\frac{(bc-ad)\left(63b^2c^2+14abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{192b^2d^4n} + \frac{\left(63b^2c^2+14abcd\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

$$= \frac{(bc-ad)^2\left(63b^2c^2+14abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{128b^2d^5n} - \frac{(bc-ad)\left(63b^2c^2+14\right.}{}$$

Mathematica [A] time = 2.02, size = 274, normalized size = 0.77

$$\frac{\sqrt{c+dx^n}\left(\frac{5(bc-ad)^3\left(3a^2d^2+14abcd+63b^2c^2\right)\left(-\frac{16d^3(a+bx^n)^3}{15(ad-bc)^3}-\frac{4d^2(a+bx^n)^2}{3(bc-ad)^2}-\frac{2d(a+bx^n)}{ad-bc}-\frac{2\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{4bd^5} - \frac{24(ad+3bc)(a+bx^n}{bd}\right.}{320bdn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(5/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (Sqrt[c + d*x^n]*((-24*(3*b*c + a*d)*(a + b*x^n)^4)/(b*d) + 64*x^n*(a + b*x
^n)^4 + (5*(b*c - a*d)^3*(63*b^2*c^2 + 14*a*b*c*d + 3*a^2*d^2)*((-2*d*(a +
b*x^n))/(-(b*c) + a*d) - (4*d^2*(a + b*x^n)^2)/(3*(b*c - a*d)^2) - (16*d^3*
(a + b*x^n)^3)/(15*(-(b*c) + a*d)^3) - (2*Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(
Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]))))/(4*b*d^5)))/(320*b*d*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.80, size = 771, normalized size = 2.15

$$\left[-\frac{15\left(63b^5c^5-175ab^4c^4d+150a^2b^3c^3d^2-30a^3b^2c^2d^3-5a^4bcd^4-3a^5d^5\right)\sqrt{bd}\log\left(8b^2d^2x^{2n}+b^2c^2+6a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(63*b^5*c^5 - 175*a*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b
^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) - 4*(384*b
^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^2 + 1564*a^2*b^3*c^2*d^3
- 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d^4 - 7*a*b^4*d^5)*x^(3*n)
 + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^3*d^5)*x^(2*n) - 2*(315*b
^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4 - 15*a^3*b^2*d^5)*x^n)*s
qrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n), 1/3840*(15*(63*b^5*c^5 - 175*a
*b^4*c^4*d + 150*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 5*a^4*b*c*d^4 - 3*a
^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d
))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d +
a*b*d^2)*x^n)) + 2*(384*b^5*d^5*x^(4*n) + 945*b^5*c^4*d - 2310*a*b^4*c^3*d^
2 + 1564*a^2*b^3*c^2*d^3 - 90*a^3*b^2*c*d^4 - 45*a^4*b*d^5 - 144*(3*b^5*c*d
^4 - 7*a*b^4*d^5)*x^(3*n) + 8*(63*b^5*c^2*d^3 - 148*a*b^4*c*d^4 + 93*a^2*b^
3*d^5)*x^(2*n) - 2*(315*b^5*c^3*d^2 - 749*a*b^4*c^2*d^3 + 481*a^2*b^3*c*d^4
 - 15*a^3*b^2*d^5)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^6*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{3n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{(b\, x^n + a)^{\frac{5}{2}}\, x^{3n-1}}{\sqrt{d\, x^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)*(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{5}{2}} x^{3n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(5/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3n-1}\, (a + b\, x^n)^{5/2}}{\sqrt{c + d\, x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(5/2))/(c + d*x^n)^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1073 $\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=291

$$-\frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96b^2d^3n}+\cdots$$

```
[Out] 1/64*(-a*d+b*c)^2*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^
n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(9/2)/n+1/96*(3*a^2*d^2+10*a*b*
c*d+35*b^2*c^2)*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b^2/d^3/n-1/24*(3*a*d+7*b*c
)*(a+b*x^n)^(5/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/4*x^n*(a+b*x^n)^(5/2)*(c+d*x^
n)^(1/2)/b/d/n-1/64*(-a*d+b*c)*(3*a^2*d^2+10*a*b*c*d+35*b^2*c^2)*(a+b*x^n)^
(1/2)*(c+d*x^n)^(1/2)/b^2/d^4/n
```

Rubi [A] time = 0.32, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(3a^2d^2+10abcd+35b^2c^2\right)(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{96b^2d^3n}-\frac{(bc-ad)\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{64b^2d^4n}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] -((b*c - a*d)*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c
+ d*x^n])/(64*b^2*d^4*n) + ((35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*(a + b*x^
n)^(3/2)*Sqrt[c + d*x^n])/(96*b^2*d^3*n) - ((7*b*c + 3*a*d)*(a + b*x^n)^(5/
2)*Sqrt[c + d*x^n])/(24*b^2*d^2*n) + (x^n*(a + b*x^n)^(5/2)*Sqrt[c + d*x^n]
)/(4*b*d*n) + ((b*c - a*d)^2*(35*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*ArcTanh[
(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(64*b^(5/2)*d^(9/2)*n
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}(a+bx^n)^{3/2}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(-ac-\frac{1}{2}(7bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{4bdn}$$

$$= -\frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+dx^n}}{24b^2d^2n} + \frac{x^n(a+bx^n)^{5/2}\sqrt{c+dx^n}}{4bdn} + \Big(35b^2c^2+10ab$$

$$= \frac{\left(35b^2c^2+10abcd+3a^2d^2\right)(a+bx^n)^{3/2}\sqrt{c+dx^n}}{96b^2d^3n} - \frac{(7bc+3ad)(a+bx^n)^{5/2}\sqrt{c+}}{24b^2d^2n}$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \Big(35b^2c^2+10abcd+3$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \Big(35b^2c^2+10abcd+3$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \Big(35b^2c^2+10abcd+3$$

$$= -\frac{(bc-ad)\left(35b^2c^2+10abcd+3a^2d^2\right)\sqrt{a+bx^n}\sqrt{c+dx^n}}{64b^2d^4n} + \Big(35b^2c^2+10abcd+3$$

Mathematica [A] time = 1.01, size = 241, normalized size = 0.83

$$\frac{3(bc-ad)^{5/2}\left(3a^2d^2+10abcd+35b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)-b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\left(9a^3d^3+3a^2bd^2\right.}{192b^3d^{9/2}n\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*(a + b*x^n)^(3/2))/Sqrt[c + d*x^n],x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(9*a^3*d^3 + 3*a^2*b*d^2*(5*c - 2*
d*x^n) - a*b^2*d*(145*c^2 - 92*c*d*x^n + 72*d^2*x^(2*n)) + b^3*(105*c^3 - 7
0*c^2*d*x^n + 56*c*d^2*x^(2*n) - 48*d^3*x^(3*n)))) + 3*(b*c - a*d)^(5/2)*(3
5*b^2*c^2 + 10*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSi
nh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(192*b^3*d^(9/2)*n*Sqrt[c +
d*x^n])
```

fricas [A] time = 0.65, size = 607, normalized size = 2.09

$$\left[\frac{3\left(35\,b^4c^4-60\,ab^3c^3d+18\,a^2b^2c^2d^2+4\,a^3bcd^3+3\,a^4d^4\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(35*b^4*c^4 - 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
 + 3*a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d
^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d
*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(48*b^4*d^4*x^(3*n) - 105*b^4*c^
3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 -
 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)
*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^5*n), -1/384*(3*(35*b^4*c^4 -
 60*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 + 3*a^4*d^4)*sqrt(-b*d
)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a
)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) -
2*(48*b^4*d^4*x^(3*n) - 105*b^4*c^3*d + 145*a*b^3*c^2*d^2 - 15*a^2*b^2*c*d^
3 - 9*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 9*a*b^3*d^4)*x^(2*n) + 2*(35*b^4*c^2*d^2
 - 46*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^5*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^{\frac{3}{2}}x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(b\,x^n+a)^{\frac{3}{2}}\,x^{3n-1}}{\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
[Out] int(x^(3*n-1)*(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^{\frac{3}{2}} x^{3n-1}}{\sqrt{dx^n + c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^n + a)^(3/2)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3n-1} (a + b x^n)^{3/2}}{\sqrt{c + d x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(3/2))/(c + d*x^n)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1074 $\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=221

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)(}{}$$

```
[Out] -1/8*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/
2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/d^(7/2)/n-1/12*(3*a*d+5*b*c)*(a+b*x^n)^
(3/2)*(c+d*x^n)^(1/2)/b^2/d^2/n+1/3*x^n*(a+b*x^n)^(3/2)*(c+d*x^n)^(1/2)/b/d
/n+1/8*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d^
3/n
```

Rubi [A] time = 0.25, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {446, 90, 80, 50, 63, 217, 206}

$$\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n}-\frac{(bc-ad)\left(a^2d^2+2abcd+5b^2c^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{8b^{5/2}d^{7/2}n}-\frac{(3ad+5bc)(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(8*b^2*
d^3*n) - ((5*b*c + 3*a*d)*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(12*b^2*d^2*n)
 + (x^n*(a + b*x^n)^(3/2)*Sqrt[c + d*x^n])/(3*b*d*n) - ((b*c - a*d)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[
c + d*x^n])])/(8*b^(5/2)*d^(7/2)*n)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{3bdn} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-ac-\frac{1}{2}(5bc+3ad)x\right)}{\sqrt{c+dx}}\,dx, x, x^n\right)}{3bdn}$$

$$= -\frac{(5bc+3ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12b^2d^2n} + \frac{x^n\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{3bdn} + \frac{\left(5b^2c^2+2abcd+\right.}{}$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12b^2d^2n} + \cdots$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12b^2d^2n} + \cdots$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12b^2d^2n} + \cdots$$

$$= \frac{\left(5b^2c^2+2abcd+a^2d^2\right)\sqrt{a+bx^n}\,\sqrt{c+dx^n}}{8b^2d^3n} - \frac{(5bc+3ad)\,(a+bx^n)^{3/2}\,\sqrt{c+dx^n}}{12b^2d^2n} + \cdots$$

Mathematica [A] time = 0.62, size = 191, normalized size = 0.86

$$\frac{b\sqrt{d}\,\sqrt{a+bx^n}\,(c+dx^n)\left(-3a^2d^2+2abd\,(dx^n-2c)+b^2\left(15c^2-10cdx^n+8d^2x^{2n}\right)\right)-3(bc-ad)^{3/2}\left(a^2d^2+2ab\cdots\right.}{24b^3d^{7/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + 3*n)*Sqrt[a + b*x^n])/Sqrt[c + d*x^n],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*a^2*d^2 + 2*a*b*d*(-2*c + d*x^n)
 + b^2*(15*c^2 - 10*c*d*x^n + 8*d^2*x^(2*n))) - 3*(b*c - a*d)^(3/2)*(5*b^2*
c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(24*b^3*d^(7/2)*n*Sqrt[c + d*x^n])
```

fricas [A] time = 1.06, size = 471, normalized size = 2.13

$$\left[-\frac{3\left(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^(2*n) + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n
 + (b*c + a*d)*sqrt(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*
b*d^2)*x^n) - 4*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b
*d^3 - 2*(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b
^3*d^4*n), 1/48*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt
(-b*d)*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^
n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n
)) + 2*(8*b^3*d^3*x^(2*n) + 15*b^3*c^2*d - 4*a*b^2*c*d^2 - 3*a^2*b*d^3 - 2*
(5*b^3*c*d^2 - a*b^2*d^3)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^4*n)
]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}\,x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n+a}\;x^{3n-1}}{\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)*(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}\,x^{3\,n-1}}{\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)*(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*x^(3*n - 1)/sqrt(d*x^n + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3\,n-1}\,\sqrt{a+b\,x^n}}{\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2),x)
[Out] int((x^(3*n - 1)*(a + b*x^n)^(1/2))/(c + d*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}\sqrt{a+bx^n}}{\sqrt{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)*(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
[Out] Integral(x**(3*n - 1)*sqrt(a + b*x**n)/sqrt(c + d*x**n), x)
```

3.1075 $\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=150

$$-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

```
[Out] -1/4*(4*a*b*c*d-3*(a*d+b*c)^2)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d
*x^n)^(1/2))/b^(5/2)/d^(5/2)/n-3/4*(a*d+b*c)*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2
)/b^2/d^2/n+1/2*x^n*(a+b*x^n)^(1/2)*(c+d*x^n)^(1/2)/b/d/n
```

Rubi [A] time = 0.15, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 90, 80, 63, 217, 206}

$$-\frac{3(ad+bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n}-\frac{\left(4abcd-3(ad+bc)^2\right)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{4b^{5/2}d^{5/2}n}+\frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (-3*(b*c + a*d)*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])/(4*b^2*d^2*n) + (x^n*Sqrt[
a + b*x^n]*Sqrt[c + d*x^n])/(2*b*d*n) - ((4*a*b*c*d - 3*(b*c + a*d)^2)*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(4*b^(5/2)*d^(5/2
)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{\sqrt{a+bx^n}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} + \frac{\text{Subst}\left(\int \frac{-ac-\frac{3}{2}(bc+ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{2bdn}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{}$$

$$= -\frac{3(bc+ad)\sqrt{a+bx^n}\sqrt{c+dx^n}}{4b^2d^2n} + \frac{x^n\sqrt{a+bx^n}\sqrt{c+dx^n}}{2bdn} - \frac{\left(4abcd-3(bc+ad)^2\right.}{4b^{5/2}}$$

Mathematica [A] time = 0.46, size = 157, normalized size = 1.05

$$\frac{\sqrt{bc-ad}\left(3a^2d^2+2abcd+3b^2c^2\right)\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)+b\sqrt{d}\sqrt{a+bx^n}\,(c+dx^n)\,(-3ad-3bc+2bd}{4b^3d^{5/2}n\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x^n]*(c + d*x^n)*(-3*b*c - 3*a*d + 2*b*d*x^n) + Sqrt[
b*c - a*d]*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*Sqrt[(b*(c + d*x^n))/(b*c -
a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(4*b^3*d^(5/2)*n*
Sqrt[c + d*x^n])
```

fricas [A] time = 0.80, size = 361, normalized size = 2.41

$$\left[\frac{\left(3\,b^2c^2+2\,abcd+3\,a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^{2\,n}+b^2c^2+6\,abcd+a^2d^2+4\left(2\,\sqrt{bd}\,bdx^n+(bc+ad)\sqrt{bd}\right)\sqrt{bx^n}\right.}{16\,b^3d^3n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n) + 4*(2*
b^2*d^2*x^n - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*
d^3*n), -1/8*((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*
sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c
)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)) - 2*(2*b^2*d^2*x^n
 - 3*b^2*c*d - 3*a*b*d^2)*sqrt(b*x^n + a)*sqrt(d*x^n + c))/(b^3*d^3*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{b\,x^{n}+a}\,\sqrt{d\,x^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(1/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{\sqrt{bx^{n}+a}\,\sqrt{dx^{n}+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(1/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/(sqrt(b*x^n + a)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3\,n-1}}{\sqrt{a+b\,x^{n}}\,\sqrt{c+d\,x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)),x)
```

```
[Out] int(x^(3*n - 1)/((a + b*x^n)^(1/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{\sqrt{a+bx^{n}}\,\sqrt{c+dx^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(1/2)/(c+d*x**n)**(1/2),x)
```

```
[Out] Integral(x**(3*n - 1)/(sqrt(a + b*x**n)*sqrt(c + d*x**n)), x)
```

3.1076 $\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=133

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

```
[Out] -(3*a*d+b*c)*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/
2)/d^(3/2)/n-2*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(1/2)+(a+b*x^
n)^(1/2)*(c+d*x^n)^(1/2)/b^2/d/n
```

Rubi [A] time = 0.16, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 89, 80, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{b^2n(bc-ad)\sqrt{a+bx^n}}-\frac{(3ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}+\frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(b^2*(b*c - a*d)*n*Sqrt[a + b*x^n]) + (Sqrt[a + b*
x^n]*Sqrt[c + d*x^n])/(b^2*d*n) - ((b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[a +
b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*d^(3/2)*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{3/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(bc-ad)+\frac{1}{2}b(bc-ad)x}{\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, x^n\right)}{b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\right.}{2b^2dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{b^3dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{b^3dn}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{b^2(bc-ad)n\sqrt{a+bx^n}} + \frac{\sqrt{a+bx^n}\sqrt{c+dx^n}}{b^2dn} - \frac{(bc+3ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}d^{3/2}n}$$

Mathematica [A] time = 0.52, size = 185, normalized size = 1.39

$$\frac{\sqrt{bc-ad}\left(-3a^2d^2+2abcd+b^2c^2\right)\sqrt{a+bx^n}\sqrt{\frac{b(c+dx^n)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right) - b\sqrt{d}\,(c+dx^n)\left(-3a^2d+ab\,(c-dx^n\right.}{b^3d^{3/2}n(ad-bc)\sqrt{a+bx^n}\sqrt{c+dx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(3/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-(b*Sqrt[d]*(c + d*x^n)*(-3*a^2*d + b^2*c*x^n + a*b*(c - d*x^n))) + Sqrt[b
*c - a*d]*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^n]*Sqrt[(b*(c + d*
x^n))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sqrt[b*c - a*d]])/(b^3
*d^(3/2)*(-(b*c) + a*d)*n*Sqrt[a + b*x^n]*Sqrt[c + d*x^n])
```

fricas [B] time = 0.99, size = 540, normalized size = 4.06

$$\left[\frac{4\left(ab^2cd - 3a^2bd^2 + \left(b^3cd - ab^2d^2\right)x^n\right)\sqrt{bx^n + a}\sqrt{dx^n + c} + \left(\left(b^3c^2 + 2ab^2cd - 3a^2bd^2\right)\sqrt{bd}\,x^n + \left(ab^2c^2 + 2\right.\right.}{4\left(\left(b^5cd^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b^2*c*d - 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n +
a)*sqrt(d*x^n + c) + ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(b*d)*x^n +
 (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n) + b
^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt(b*
d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^5*c*d
^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2 - a^2*b^3*d^3)*n), 1/2*(2*(a*b^2*c*d -
 3*a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) +
 ((b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*sqrt(-b*d)*x^n + (a*b^2*c^2 + 2*a^2
*b*c*d - 3*a^3*d^2)*sqrt(-b*d))*arctan(1/2*(2*sqrt(-b*d)*b*d*x^n + (b*c + a
*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c)/(b^2*d^2*x^(2*n) + a*b*c*d
+ (b^2*c*d + a*b*d^2)*x^n)))/((b^5*c*d^2 - a*b^4*d^3)*n*x^n + (a*b^4*c*d^2
- a^2*b^3*d^3)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{3}{2}}\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

maple [F] time = 1.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(b\,x^n + a)^{\frac{3}{2}}\sqrt{d\,x^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(3/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(bx^n + a)^{\frac{3}{2}}\sqrt{dx^n + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(3/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(3/2)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a + b\,x^n)^{3/2}\sqrt{c + d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(3/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(3/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1077 $\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx$

Optimal. Leaf size=147

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}$$

```
[Out] 2*arctanh(d^(1/2)*(a+b*x^n)^(1/2)/b^(1/2)/(c+d*x^n)^(1/2))/b^(5/2)/n/d^(1/2
)-2/3*a^2*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)/n/(a+b*x^n)^(3/2)+4/3*a*(-2*a*d+3*
b*c)*(c+d*x^n)^(1/2)/b^2/(-a*d+b*c)^2/n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {446, 89, 78, 63, 217, 206}

$$-\frac{2a^2\sqrt{c+dx^n}}{3b^2n(bc-ad)\,(a+bx^n)^{3/2}}+\frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2n(bc-ad)^2\sqrt{a+bx^n}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx^n}}{\sqrt{b}\,\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (-2*a^2*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)*n*(a + b*x^n)^(3/2)) + (4*a*(3*
b*c - 2*a*d)*Sqrt[c + d*x^n])/(3*b^2*(b*c - a*d)^2*n*Sqrt[a + b*x^n]) + (2*
ArcTanh[(Sqrt[d]*Sqrt[a + b*x^n])/(Sqrt[b]*Sqrt[c + d*x^n])])/(b^(5/2)*Sqrt
[d]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^{-1+3n}}{(a+bx^n)^{5/2}\sqrt{c+dx^n}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-ad)+\frac{3}{2}b(bc-ad)x}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx, x, x^n\right)}{3b^2(bc-ad)n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,d\right.}{b^2n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,d\right.}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{b^3n}$$

$$= -\frac{2a^2\sqrt{c+dx^n}}{3b^2(bc-ad)n\,(a+bx^n)^{3/2}} + \frac{4a(3bc-2ad)\sqrt{c+dx^n}}{3b^2(bc-ad)^2n\sqrt{a+bx^n}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{b}\sqrt{c+dx^n}}\right)}{b^{5/2}\sqrt{d}\,n}$$

Mathematica [A] time = 0.81, size = 217, normalized size = 1.48

$$\frac{2\sqrt{c+dx^n}\left(\frac{(3b^2c^2-a^2d^2)(a+bx^n)}{d(bc-ad)^2}+\frac{a^2}{ad-bc}-\frac{3(a+bx^n)\left(\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}-\sqrt{d}\sqrt{a+bx^n}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^n}}{\sqrt{bc-ad}}\right)\right)}{d\sqrt{bc-ad}\sqrt{\frac{b(c+dx^n)}{bc-ad}}}\right)}{3b^2n\,(a+bx^n)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*n)/((a + b*x^n)^(5/2)*Sqrt[c + d*x^n]),x]
```

```
[Out] (2*Sqrt[c + d*x^n]*(a^2/(-(b*c) + a*d) + ((3*b^2*c^2 - a^2*d^2)*(a + b*x^n)
)/(d*(b*c - a*d)^2) - (3*(a + b*x^n)*(Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/
(b*c - a*d)] - Sqrt[d]*Sqrt[a + b*x^n]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^n])/Sq
rt[b*c - a*d]]))/(d*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^n))/(b*c - a*d)])))/(3
*b^2*n*(a + b*x^n)^(3/2))
```

fricas [B] time = 0.92, size = 769, normalized size = 5.23

$$\left[\frac{4\left(5\,a^2b^2cd-3\,a^3bd^2+2\left(3\,ab^3cd-2\,a^2b^2d^2\right)x^n\right)\sqrt{bx^n+a}\,\sqrt{dx^n+c}\,+3\left(\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\sqrt{bd}\,x^{2\,n}\right.}{6\left(\left(b^7c^2d-2\,ab^6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(4*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^n)
*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)
*sqrt(b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(b*d)*x^
n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(b*d))*log(8*b^2*d^2*x^(2*n)
+ b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*sqrt(b*d)*b*d*x^n + (b*c + a*d)*sqrt
(b*d))*sqrt(b*x^n + a)*sqrt(d*x^n + c) + 8*(b^2*c*d + a*b*d^2)*x^n))/((b^7*
c^2*d - 2*a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5
*c*d^2 + a^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^
3)*n), 1/3*(2*(5*a^2*b^2*c*d - 3*a^3*b*d^2 + 2*(3*a*b^3*c*d - 2*a^2*b^2*d^2
)*x^n)*sqrt(b*x^n + a)*sqrt(d*x^n + c) - 3*((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*sqrt(-b*d)*x^(2*n) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*sqrt(
-b*d)*x^n + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*sqrt(-b*d))*arctan(1/2*(2
*sqrt(-b*d)*b*d*x^n + (b*c + a*d)*sqrt(-b*d))*sqrt(b*x^n + a)*sqrt(d*x^n +
c)/(b^2*d^2*x^(2*n) + a*b*c*d + (b^2*c*d + a*b*d^2)*x^n)))/((b^7*c^2*d - 2*
a*b^6*c*d^2 + a^2*b^5*d^3)*n*x^(2*n) + 2*(a*b^6*c^2*d - 2*a^2*b^5*c*d^2 + a
^3*b^4*d^3)*n*x^n + (a^2*b^5*c^2*d - 2*a^3*b^4*c*d^2 + a^4*b^3*d^3)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{x^{3n-1}}{(b\,x^n+a)^{\frac{5}{2}}\sqrt{d\,x^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

```
[Out] int(x^(3*n-1)/(b*x^n+a)^(5/2)/(d*x^n+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3\,n-1}}{(bx^n+a)^{\frac{5}{2}}\sqrt{dx^n+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+3*n)/(a+b*x^n)^(5/2)/(c+d*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3*n - 1)/((b*x^n + a)^(5/2)*sqrt(d*x^n + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3n-1}}{(a+b\,x^n)^{5/2}\,\sqrt{c+d\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)),x)
[Out] int(x^(3*n - 1)/((a + b*x^n)^(5/2)*(c + d*x^n)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+3*n)/(a+b*x**n)**(5/2)/(c+d*x**n)**(1/2),x)
[Out] Timed out
```

3.1078 $\int x^p(b+cx)^p(b+2cx)\,dx$

Optimal. Leaf size=20

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

```
[Out] x^(1+p)*(c*x+b)^(1+p)/(1+p)
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {74}

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(b + c*x)^p*(b + 2*c*x),x]
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^p(b+cx)^p(b+2cx)\,dx=\frac{x^{1+p}(b+cx)^{1+p}}{1+p}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{x^{p+1}(b+cx)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(b + c*x)^p*(b + 2*c*x),x]
[Out] (x^(1 + p)*(b + c*x)^(1 + p))/(1 + p)
```

fricas [A] time = 0.66, size = 25, normalized size = 1.25

$$\frac{\left(cx^2+bx\right)(cx+b)^p x^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="fricas")
[Out] (c*x^2 + b*x)*(c*x + b)^p*x^p/(p + 1)
```

giac [A] time = 0.16, size = 35, normalized size = 1.75

$$\frac{(cx+b)^p cx^2x^p+(cx+b)^p bxx^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="giac")
```

```
[Out] ((c*x + b)^p*c*x^2*x^p + (c*x + b)^p*b*x*x^p)/(p + 1)
```

maple [A] time = 0.05, size = 21, normalized size = 1.05

$$\frac{x^{p+1}\left(cx+b\right)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(c*x+b)^p*(2*c*x+b),x)
```

```
[Out] x^(p+1)*(c*x+b)^(p+1)/(p+1)
```

maxima [A] time = 0.82, size = 29, normalized size = 1.45

$$\frac{\left(cx^{2}+bx\right)e^{\left(p\log(cx+b)+p\log(x)\right)}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(c*x+b)^p*(2*c*x+b),x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)*e^(p*log(c*x + b) + p*log(x))/(p + 1)
```

mupad [B] time = 4.80, size = 22, normalized size = 1.10

$$\frac{x\,x^{p}\,{\left(b+c\,x\right)}^{p}\,\left(b+c\,x\right)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(b + c*x)^p*(b + 2*c*x),x)
```

```
[Out] (x*x^p*(b + c*x)^p*(b + c*x))/(p + 1)
```

sympy [A] time = 3.10, size = 46, normalized size = 2.30

$$\begin{cases} \frac{bxx^{p}(b+cx)^{p}}{p+1} + \frac{cx^{2}x^{p}(b+cx)^{p}}{p+1} & \text{for}\: p \neq -1 \\ \log\left(x\right) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(c*x+b)**p*(2*c*x+b),x)
```

```
[Out] Piecewise((b*x*x**p*(b + c*x)**p/(p + 1) + c*x**2*x**p*(b + c*x)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

3.1079 $\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

```
[Out] 1/2*x^(2+2*p)*(c*x^2+b)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{2(p+1)}\left(b+cx^2\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2*(1 + p))*(b + c*x^2)^(1 + p))/(2*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+2(1+p)}\left(b+cx^2\right)^p\left(b+2cx^2\right)\,dx=\frac{x^{2(1+p)}\left(b+cx^2\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.13, size = 97, normalized size = 3.59

$$\frac{x^{2p+2}\left(b+cx^2\right)^p\left(\frac{cx^2}{b}+1\right)^{-p}\left(2c(p+1)x^2\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 2*(1 + p))*(b + c*x^2)^p*(b + 2*c*x^2),x]
```

```
[Out] (x^(2 + 2*p)*(b + c*x^2)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2
)/b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

fricas [A] time = 0.69, size = 32, normalized size = 1.19

$$\frac{\left(cx^3+bx\right)\left(cx^2+b\right)^p x^{2p+1}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^3 + b*x)*(c*x^2 + b)^p*x^(2*p + 1)/(p + 1)
```

giac [B] time = 0.19, size = 52, normalized size = 1.93

$$\frac{\left(cx^2+b\right)^p cx^3 e^{\left(2\,p\log(x)+\log(x)\right)}+\left(cx^2+b\right)^p bxe^{\left(2\,p\log(x)+\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="giac")
[Out] 1/2*((c*x^2 + b)^p*c*x^3*e^(2*p*log(x) + log(x)) + (c*x^2 + b)^p*b*x*e^(2*p
*log(x) + log(x)))/(p + 1)
```

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{2p+2}\left(c\,x^2+b\right)^{p+1}}{2p+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p+1)*(c*x^2+b)^p*(2*c*x^2+b),x)
[Out] 1/2*x^(2*p+2)*(c*x^2+b)^(p+1)/(p+1)
```

maxima [A] time = 0.89, size = 35, normalized size = 1.30

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log(x)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*p)*(c*x^2+b)^p*(2*c*x^2+b),x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

mupad [B] time = 4.88, size = 47, normalized size = 1.74

$$\left(c\,x^2+b\right)^p\,\left(\frac{c\,x^{2\,p+1}\,x^3}{2\,p+2}+\frac{b\,x\,x^{2\,p+1}}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2*p + 1)*(b + c*x^2)^p*(b + 2*c*x^2),x)
[Out] (b + c*x^2)^p*((c*x^(2*p + 1)*x^3)/(2*p + 2) + (b*x*x^(2*p + 1))/(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*p)*(c*x**2+b)**p*(2*c*x**2+b),x)
[Out] Timed out
```

3.1080 $\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)dx$

Optimal. Leaf size=27

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

```
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(1+p)/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{3(p+1)}\left(b+cx^3\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3*(1 + p))*(b + c*x^3)^(1 + p))/(3*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+3(1+p)}\left(b+cx^3\right)^p\left(b+2cx^3\right)dx=\frac{x^{3(1+p)}\left(b+cx^3\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.10, size = 97, normalized size = 3.59

$$\frac{x^{3p+3}\left(b+cx^3\right)^p\left(\frac{cx^3}{b}+1\right)^{-p}\left(2c(p+1)x^3\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2)\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + 3*(1 + p))*(b + c*x^3)^p*(b + 2*c*x^3),x]
```

```
[Out] (x^(3 + 3*p)*(b + c*x^3)^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -
((c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3
)/b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

fricas [A] time = 0.57, size = 32, normalized size = 1.19

$$\frac{\left(cx^4+bx\right)\left(cx^3+b\right)^p x^{3p+2}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^4 + b*x)*(c*x^3 + b)^p*x^(3*p + 2)/(p + 1)
```

giac [B] time = 0.26, size = 56, normalized size = 2.07

$$\frac{\left(cx^3+b\right)^p cx^4 e^{\left(3\,p\log(x)+2\,\log(x)\right)} + \left(cx^3+b\right)^p bxe^{\left(3\,p\log(x)+2\,\log(x)\right)}}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="giac")
[Out] 1/3*((c*x^3 + b)^p*c*x^4*e^(3*p*log(x) + 2*log(x)) + (c*x^3 + b)^p*b*x*e^(3
*p*log(x) + 2*log(x)))/(p + 1)
```

maple [A] time = 0.04, size = 26, normalized size = 0.96

$$\frac{x^{3p+3}\left(c\,x^3+b\right)^{p+1}}{3p+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x)
[Out] 1/3*x^(3+3*p)*(c*x^3+b)^(p+1)/(p+1)
```

maxima [A] time = 0.69, size = 35, normalized size = 1.30

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3\,p\log(x)\right)}}{3\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+3*p)*(c*x^3+b)^p*(2*c*x^3+b),x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

mupad [B] time = 4.90, size = 47, normalized size = 1.74

$$\left(c\,x^3+b\right)^p\,\left(\frac{c\,x^{3\,p+2}\,x^4}{3\,p+3}+\frac{b\,x\,x^{3\,p+2}}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*p + 2)*(b + c*x^3)^p*(b + 2*c*x^3),x)
[Out] (b + c*x^3)^p*((c*x^(3*p + 2)*x^4)/(3*p + 3) + (b*x*x^(3*p + 2))/(3*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+3*p)*(c*x**3+b)**p*(2*c*x**3+b),x)
[Out] Timed out
```

3.1081 $\int x^{-1+n(1+p)}\left(b+cx^n\right)^p\left(b+2cx^n\right)\,dx$

Optimal. Leaf size=27

$$\frac{x^{n(p+1)}\left(b+cx^n\right)^{p+1}}{n(p+1)}$$

```
[Out] x^(n*(1+p))*(b+c*x^n)^(1+p)/n/(1+p)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {449}

$$\frac{x^{n(p+1)}\left(b+cx^n\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
[Out] (x^(n*(1 + p))*(b + c*x^n)^(1 + p))/(n*(1 + p))
```

Rule 449

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(
m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\int x^{-1+n(1+p)}\left(b+cx^n\right)^p\left(b+2cx^n\right)\,dx=\frac{x^{n(1+p)}\left(b+cx^n\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.16, size = 101, normalized size = 3.74

$$\frac{\left(b+cx^n\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right.\right.}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n*(1 + p))*(b + c*x^n)^p*(b + 2*c*x^n),x]
[Out] ((b + c*x^n)^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p, 2 + p,
 -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 + p, 3 +
 p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*(1 + (c*x^n)/b)^p)
```

fricas [A] time = 0.74, size = 35, normalized size = 1.30

$$\frac{\left(cxx^n+bx\right)\left(cx^n+b\right)^p x^{np+n-1}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="fricas")
[Out] (c*x*x^n + b*x)*(c*x^n + b)^p*x^(n*p + n - 1)/(n*p + n)
```

giac [B] time = 0.25, size = 66, normalized size = 2.44

$$\frac{(cx^n + b)^p cxx^n e^{(np\log(x)+n\log(x)-\log(x))} + (cx^n + b)^p bxe^{(np\log(x)+n\log(x)-\log(x))}}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="giac")
[Out] ((c*x^n + b)^p*c*x*x^n*e^(n*p*log(x) + n*log(x) - log(x)) + (c*x^n + b)^p*b
*x*e^(n*p*log(x) + n*log(x) - log(x)))/(n*p + n)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int (2c\,x^n + b)\,x^{(p+1)n-1}\,(c\,x^n + b)^p\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n*(p+1))*(c*x^n+b)^p*(b+2*c*x^n),x)
[Out] int(x^(-1+n*(p+1))*(c*x^n+b)^p*(b+2*c*x^n),x)
```

maxima [A] time = 0.96, size = 39, normalized size = 1.44

$$\frac{\left(cx^{2\,n} + bx^n\right)e^{\left(np\log(x)+p\log(cx^n+b)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n*(1+p))*(b+c*x^n)^p*(b+2*c*x^n),x, algorithm="maxima")
[Out] (c*x^(2*n) + b*x^n)*e^(n*p*log(x) + p*log(c*x^n + b))/(n*(p + 1))
```

mupad [B] time = 4.86, size = 54, normalized size = 2.00

$$\left(\frac{b\,x\,x^{n\,\left(p+1\right)-1}}{n\,\left(p+1\right)}+\frac{c\,x\,x^n\,x^{n\,\left(p+1\right)-1}}{n\,\left(p+1\right)}\right)\,{\left(b+c\,x^n\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n*(p + 1) - 1)*(b + c*x^n)^p*(b + 2*c*x^n),x)
[Out] ((b*x*x^(n*(p + 1) - 1))/(n*(p + 1)) + (c*x*x^n*x^(n*(p + 1) - 1))/(n*(p +
1)))*(b + c*x^n)^p
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n*(1+p))*(b+c*x**n)**p*(b+2*c*x**n),x)
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```